More Single View Geometry

...with a lot of slides stolen from Steve Seitz

15-463: Computational Photography
Alexei Efros, CMU, Fall 2011
Quiz: which is 1,2,3-point perspective

Image A

Image B

Image C
Automatic Photo Pop-up

Original Image

Geometric Labels

Fit Segments

Cut and Fold

Novel View
Results

Input Image

Cut and Fold

Automatic Photo Pop-up
How can we model more complex scene?

1. Find world coordinates (X,Y,Z) for a few points
2. Connect the points with planes to model geometry
   • Texture map the planes
Finding world coordinates (X,Y,Z)

1. Define the ground plane (Z=0)
2. Compute points (X,Y,0) on that plane
3. Compute the *heights* Z of all other points
Measurements on planes

Approach: unwarp, then measure

What kind of warp is this?
Unwarp ground plane

Our old friend – the homography

Need 4 reference points with world coordinates

\[ p = (x, y) \]
\[ p' = (X, Y, 0) \]
Finding world coordinates \((X,Y,Z)\)

1. Define the ground plane \((Z=0)\)
2. Compute points \((X,Y,0)\) on that plane
3. Compute the \textit{heights} \(Z\) of all other points
Comparing heights
Perspective cues
Perspective cues
Comparing heights

Vanishing Point
Measuring height
Computing vanishing points (from lines)

Intersect $p_1 q_1$ with $p_2 q_2$

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Least squares version

- Better to use more than two lines and compute the “closest” point of intersection
- See notes by Bob Collins for one good way of doing this:
Vanishing point

Vanishing line

Vertical vanishing point (at infinity)

Vanishing point
Measuring height without a ruler

Compute $Z$ from image measurements

- Need more than vanishing points to do this
Measuring height

vanishing line (horizon)

\[ v \approx (b \times b_0) \times (v_x \times v_y) \]

\[ t \equiv (v \times t_0) \times (r \times b) \]
Measuring height

What if the point on the ground plane $b_0$ is not known?

- Here the guy is standing on the box
- Use one side of the box to help find $b_0$ as shown above
What if $v_z$ is not infinity?
The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points

\[
P_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}
\]

Can permute the point ordering

- \(4! = 24\) different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry
Measuring height

Scene points represented as
\[ \mathbf{p} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \]

Image points as
\[ \mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \]

Scene cross ratio
\[ \frac{\| \mathbf{T} - \mathbf{B} \|}{\| \mathbf{R} - \mathbf{B} \|} = \frac{H}{R} \]

Image cross ratio
\[ \frac{\| \mathbf{t} - \mathbf{b} \|}{\| \mathbf{r} - \mathbf{b} \|} = \frac{H}{R} \]
Measuring height

vanishing line (horizon)

\[ \frac{v_z - b}{v_z - t} = \frac{H}{R} \]

image cross ratio
Measuring heights of people

Here we go!

185.3 cm

reference
Forensic Science: measuring heights of suspects

Reference height

190.3 ± 3.34 cm
Assessing geometric accuracy

Are the heights of the 2 groups of people consistent with each other?

*Flagellatio*, Piero della Francesca

Estimated relative heights
Assessing geometric accuracy

The Marriage of the Virgin, Raphael

Estimated relative heights

-9.0%  -4.5%
+0.3%  +2.8%
Complete approach

- Load in an image
- Click on lines parallel to X axis
  - repeat for Y, Z axes
- Compute vanishing points
- Specify 3D and 2D positions of 4 points on reference plane
- Compute homography H
- Specify a reference height
- Compute 3D positions of several points
- Create a 3D model from these points
- Extract texture maps
  - Cut out objects
  - Fill in holes
- Output a VRML model
Interactive silhouette cut-out
Occlusion filling

Geometric filling by exploiting:
- symmetries
- repeated regular patterns

Texture synthesis
- repeated stochastic patterns
Complete 3D reconstruction

- Planar measurements
- Height measurements
- Automatic vanishing point/line computation
- Interactive segmentation
- Occlusion filling
- Object placement in 3D model
A virtual museum @ Microsoft

The Virtual Museum

A. Criminisi @ Microsoft, 2002

Thanks

A. Criminisi  http://research.microsoft.com/~antcrim/