Image Filtering
Salvador Dali, “Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln”, 1976
Salvador Dali, “Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln”, 1976
Filtering noise

How can we “smooth” away noise in an image?

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Mean filtering

\[ F[x, y] \]

\[ G[x, y] \]
Mean filtering

\[ F[x, y] \]

\[ G[x, y] \]
Cross-correlation filtering

Let’s write this down as an equation. Assume the averaging window is \((2k+1) \times (2k+1)\):

\[
G[i, j] = \frac{1}{(2k + 1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i + u, j + v]
\]

We can generalize this idea by allowing different weights for different neighboring pixels:

\[
G[i, j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v] F[i + u, j + v]
\]

This is called a **cross-correlation** operation and written:

\[
G = H \otimes F
\]

\(H\) is called the “filter,” “kernel,” or “mask.”

The above allows negative filter indices. When you implement need to use: \(H[u+k,v+k]\) instead of \(H[u,v]\)
# Mean kernel

What’s the kernel for a 3x3 mean filter?

$$F'[x, y]$$

When can taking an un weighted mean be bad idea?
Gaussian filtering

A Gaussian kernel gives less weight to pixels further from the center of the window.

\[
F[x, y] = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2 + v^2}{\sigma^2}}
\]

This kernel is an approximation of a Gaussian function:

What happens if you increase \( \sigma \)?
Mean vs. Gaussian filtering