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Technical Details

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1. Procrustes Formula for Computing Pelvis Rotation

In our 3D character model, the pelvis is a rigid T-structure, uniquely identified by the 3D positions of 3 virtual markers *pelvis root*, *left femur*, and *lowerback*, denoted $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3$ (Figure 1). Then, the vectors describing the orientation of the pelvis at a given time t are $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$.

$$\mathbf{p}_1 = \frac{\mathbf{X}_1 - \mathbf{X}_2}{||\mathbf{X}_1 - \mathbf{X}_2||} \quad (1)$$

$$\mathbf{p}_2^* = (\mathbf{X}_1 - \mathbf{X}_3) - \left(\frac{\mathbf{X}_1 - \mathbf{X}_3}{||\mathbf{X}_1 - \mathbf{X}_3||} \cdot \mathbf{p}_1\right) \quad (2)$$

$$\mathbf{p}_2 = \frac{\mathbf{p}_2^*}{||\mathbf{p}_2^*||} \quad (3)$$

$$\mathbf{p}_3 = \mathbf{p}_1 \times \mathbf{p}_2 \quad (4)$$

Let the corresponding vectors at the pelvis 'home' position (all rotations zero) be denoted by $\mathbf{h}_1$, $\mathbf{h}_2$ and $\mathbf{h}_3$. Then,

$$\mathbf{RH} = \mathbf{P} \quad (5)$$

$$\begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3] = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3] \quad (6)$$

Equation 5 can be rearranged as follows:

$$\begin{bmatrix} \mathbf{h}_1^T & 0,0,0 & 0,0,0 \\ 0,0,0 & \mathbf{h}_1^T & 0,0,0 \\ 0,0,0 & 0,0,0 & \mathbf{h}_1^T \\ \mathbf{h}_2^T & 0,0,0 & 0,0,0 \\ 0,0,0 & \mathbf{h}_2^T & 0,0,0 \\ 0,0,0 & 0,0,0 & \mathbf{h}_2^T \\ \mathbf{h}_3^T & 0,0,0 & 0,0,0 \\ 0,0,0 & \mathbf{h}_3^T & 0,0,0 \\ 0,0,0 & 0,0,0 & \mathbf{h}_3^T \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1^T \\ \mathbf{p}_2^T \\ \mathbf{p}_3^T \end{bmatrix} \quad (7)$$

(8)

We can solve for the elements of $\mathbf{R}$ by solving Equation 7 for the least squares solution. However, the computed matrix $\mathbf{R}$ may not be orthogonal. The procrustes formula [JCG04] can be used to find the closest orthogonal matrix.

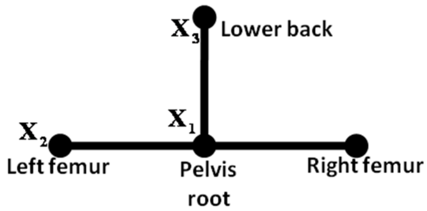

Figure 1: *The pelvis is a T-structure.*

$$\mathbf{R} = \mathbf{U\Sigma V}^T \quad (9)$$

$$\mathbf{R}^* = \mathbf{UV}^T \quad (10)$$

where $\mathbf{R}^*$ is the desired rotation matrix. We then use the standard rotation formulae [Cra89] to compute the pelvis orientation from the rotation matrix.

2. Converting Marker Positions to Joint Angles

Every limb is described by the three joint angles (roll, pitch and yaw) relative to its parent limb. Because the root joint for our character model is the pelvis, we start by recovering the rotation of the pelvis with respect to the world coordinate frame via procrustes analysis (as described in Section 1) and work our way down each hierarchical chain to generate the full pose $\mathbf{X}_{ja}$.

The marker positions $\mathbf{X}_{3D}$ give us two out of the three joint angles for a limb segment. We infer yaw rotation from motion capture data by observing that joint angles describe the rotation of the limb segment in the xyz ordering—it is possible to convert this description to the zyx ordering, in which case, θ_x and θ_y are functions of 3D marker positions, and θ_z is the 'yaw' angle, which can not be computed from marker positions. We look up the rotation for the corresponding limb segment in the best match motion capture pose, and simply use that θ_z to complete the generated 3D pose.

We will work out the method for a two limb kinematic

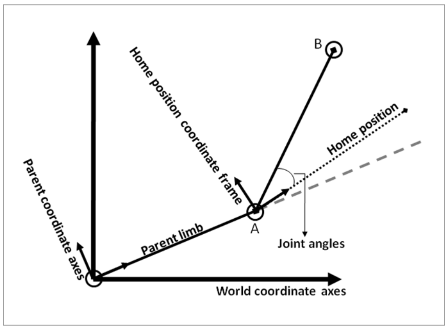

Figure 2: *Diagram of various coordinate systems.*

chain in detail. Doing the same for the full hierarchical model is an extension of these equations.

The rotation convention followed in all subsequent equations is

$$^{B}\mathbf{x} = _{B}\mathbf{R}_{A} \cdot ^{A}\mathbf{x} \tag{11}$$

rotates a vector expressed in the coordinate frame A to a vector expressed in the coordinate frame B. In addition,

$$_{B}\mathbf{R}_{A} = (_{A}\mathbf{R}_{B})^{T} \tag{12}$$

Let the 3D world positions of the virtual markers A and B be denoted by $^{w}\mathbf{x}_{A}$ and $^{w}\mathbf{x}_{B}$.The vector that lies along the limb segment is given as

$$^{w}\mathbf{x}_{AB} = \frac{^{w}\mathbf{x}_{B} - ^{w}\mathbf{x}_{A}}{||^{w}\mathbf{x}_{B} - ^{w}\mathbf{x}_{A}||} \tag{13}$$

The rotations that describe the coordinate frame attached to the parent limb (with respect to the world coordinate frame) are known, and therefore,

$$^{p}\mathbf{x}_{AB} = _{p}\mathbf{R}_{w} \cdot ^{w}\mathbf{x}_{AB} \tag{14}$$
$$= (_{w}\mathbf{R}_{p})^{T} \cdot ^{w}\mathbf{x}_{AB} \tag{15}$$

There are three rotation matrices associated with each limb—the first describes the parent limb with respect to the world ($_{w}\mathbf{R}_{p}$), the second defines the zero offset of the current limb, that is, the orientation of the limb segment when the corresponding joint angles are zero ($\mathbf{R}_{zo}$), and the third represents the joint angles($\mathbf{R}_{ja}$).

It follows that:

$$^{p}\mathbf{x}_{AB} = \mathbf{R}_{ja} \cdot \mathbf{R}_{zo} \cdot [0,0,1]^{T} \tag{16}$$
$$^{p}\mathbf{x}_{AB} = \mathbf{R}_{1} \cdot [0,0,1]^{T} \tag{17}$$

When $\mathbf{R}_1$ is represented in the zyx ordering, the rotation about the z-axis γ is the yaw rotation for the limb, while β and α are the roll and pitch respectively. From Equation 17 it is possible to determine β and α, but not γ. The equations give us two sets, (β_1, α_1) and (β_2, α_2), both of which satisfy the constraints. We choose (β_1, α_1) so that $-90^\circ < \beta_1 < 90^\circ$.

For ease of notation,

$$\mathbf{c} = ^{p}\mathbf{x}_{AB} = [c_1, c_2, c_3]^{T} \tag{18}$$

$$\beta_1 = Atan2(c_1, \sqrt{c_2^2 + c_3^2}), \tag{19}$$
$$\alpha_1 = Atan2(\frac{-c_2}{\cos\beta_1}, \frac{c_3}{\cos\beta_1}) \tag{20}$$

At this point, only the yaw rotation γ remains to be computed.

The matrix $\mathbf{R}_1$ can be determined completely for a motion capture pose because the joint angles (and therefore, $\mathbf{R}_{ja}$) are known. Let us call this matrix $\mathbf{R}_{1known}$. Using the rotation matrix formulae for the zyx ordering, we obtain two possible angles γ_{1known} and γ_{2known}, both of which satisfy the transcendental equations. We pick the yaw rotation that does not lead to wrapping in the mesh and call it γ.

Combining the two angles that were obtained from the 3D virtual marker positions with the angle inferred from the motion capture pose,

$$\mathbf{R}_{1new} = \mathbf{R}(\gamma, \beta_1, \alpha_1) \tag{21}$$
$$\mathbf{R}_{ja_{new}} = \mathbf{R}_{1new} \cdot \mathbf{R}_{zo}^{T} \tag{22}$$
$$\tag{23}$$

Finally, the joint angles are computed from $\mathbf{R}_{ja_{new}}$ using the standard rotation matrix equations [Cra89].

References

[Cra89] CRAIG J. G.: *Introduction to Robotics, Mechanics and Control*. Addison-Wesley Publishing Company, 1989.

[JCG04] JOHN C. GOWER G. B. D.: *Procrustes Problems*. Oxford University Press, USA, 2004.