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# Analyzing the Physical Correctness of Interpolated Human Motion

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**Abstract**

*Two human motions can be linearly interpolated to produce a new motion, giving the animator control over the length of a jump, the speed of walking, or the height of a kick. Over the past ten years, this simple technique has been shown to produce surprisingly natural looking results. In this paper, we analyze the motions produced by this technique for physical correctness and suggest small modifications to the standard interpolation technique that in some circumstances will produce significantly more natural looking motion.*

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Three-Dimensional Graphics and Realism]: Animation

## 1. Introduction

Over the past ten years, interpolation of motion capture data has been shown to be a very powerful technique for generating high quality and natural looking motion. This technique is successful in part because the naturalness of the original motions is not destroyed by the relatively small changes made in the process of interpolation. However, larger changes may also produce natural looking motion if interpolation is performed within a well-defined class of behaviors such as kicking [KG04] or walking [RCB98] where significant events such as foot contact can be aligned.

For these larger changes, in particular, it is not immediately obvious why interpolation should produce such good results. For example, straightforward linear interpolation could well introduce visually apparent errors in the physics of the motion. In this paper, we analyze the physical correctness of motions created by interpolating a few, presumably physically correct, human motions.

We analyze the interpolated motion in terms of a number of basic physical properties: (1) linear and angular momentum during flight; (2) foot contact, static balance and friction with the ground during stance; (3) continuity of position and velocity between phases. We assume that the motions used for interpolation are physically correct themselves, have the same skeleton, can be aligned in time by picking corresponding key events and that linear interpolation is used to interpolate parameters of motions between these key events.

Our analysis shows that with a few simple modifications to the straightforward interpolation technique proposed by others, we can prove that these physical properties are satisfied for a wide range of different kinds of motions. The interpolated motion will satisfy these physical properties if the motions used for interpolation do not include significant rotation during the flight phase (runs, forward and vertical jumps, for example), rotate around approximately the same principal axis by approximately the same amount (jumps with turns, for example) or have no flight phase (walks or kicks, for example).

The analysis presented in this paper should at least partially resolve a concern that has been raised about interpolation–that it is not a suitable technique for highly dynamic motions because the physics of the resulting motion is incorrect. While the main contribution of the paper lies in its analysis, the few simple modifications to the interpolation scheme that we describe can also significantly improve the visual quality of certain classes of interpolated motions while guaranteeing their physical correctness.

## 2. Background

Interpolation is a component of many different approaches to modeling, editing, and synthesizing human motions. In this section, however, we focus on research that uses interpolation as a standalone technique as this is when our analysis should provide insight. The key properties of such interpolation techniques are the amount of data to be interpolated, the selection method for that data, how the key events in the motions are aligned in time, and the representation of the motions used for interpolation.

Perlin [Per95] proposed one of the first systems that included interpolation. He used blending operations on a set of base motions to create new motions and transitions between them. Wiley and Hahn [WH97] and Guo and Roberge [GR96] used linear interpolation on a set of hand-selected example motions to produce modified motions within that set. For example, Wiley and Hahn were able to interpolate among a set of reaching and pointing motions to have the character point to other places in the space. The set of example motions was quite small in this work as motion capture data was not yet easy to obtain.

Rose and his colleagues [RCB98] implemented a very impressive system that used radial basis functions to represent motions for interpolation. The motions included a set of walks and runs of varying speed and emotion. The key events in the motions were selected by hand so that the motions could be appropriately aligned in time for interpolation.

Kovar and Gleicher [KG04] added a search technique for identifying a set of motions with similar time events that could then be interpolated. They also presented techniques for automatically registering the motions for interpolation [KG03].

Linear interpolation has been used extensively for creating transitions between motions [WB04, RCB98, Per95]. Transitions are created by blending portions of two motions with a weight that changes over time. The analysis in this paper, however, assumes a constant weight function and is therefore not applicable to transitions.

Abe et al. [ALP04] used optimization to synthesize a family of highly dynamic motions based on a given motion capture clip and then interpolated to create intermediate motions. They observed that the space of motions does not need to be sampled very densely for the optimization to produce good results. The analysis in this paper justifies their empirical observation.

Interpolation has also been used to solve other problems in animation. For example, Park and his colleagues [PSS02] used interpolation for on-the-fly generation of locomotion based on user parameters. Rose et al. [RSC01] used interpolation to perform inverse kinematics efficiently.

Postprocessing is often used to remove unwanted results of interpolation such as foot sliding [LS99, RCB98]. This additional editing would be hard to analyze. We instead propose an alternative, easy to analyze, technique that removes foot sliding by interpolating only non-redundant degrees of freedom.

## 3. Problem Description

The interpolation problem is defined as follows: Given $k$ human motions $M_1$, $M_2$,...,$M_k$ compute motion $M$ by interpolating the parameters of these example motions. Each motion is defined as a sequence of frames $M(t) = \{P_{root}(t), Q(t)\}$, where $P_{root}(t)$ is the position of the root segment of the character, $Q(t) = \{q_1(t)...q_n(t)\}$ is the orientation of the root and the relative angles of the character's joints and $t = 0..T$ is the time of a particular frame. In this work we use Euler angles to represent rotations although most of the analysis is independent of the rotation representation.

Using a technique proposed by a number of other researchers including [RCB98], we compute motion $M$ by interpolating the root positions and all the joint angles of the example motions. The example motions must be scaled in time, or time-warped, to align key events such as foot contacts. We use a time-warping scheme similar to the one proposed by Rose et al. [RCB98]. We assume that a set of matching key frames for the input motions is provided (either by the user or computed automatically) and that the motion segments between these key frames can be scaled uniformly.

In our work, as in most other approaches to interpolation, we automatically locate these key frames at changes in the contact with the environment because the physical laws governing the motion change with contact. Motions $M_1, M_2,...,M_k$ are split into phases based on these key frames and the corresponding phases are interpolated. For example, a jumping motion would consist of three phases: lift-off, flight and landing. Additional key frames can be added during long contact phases to better align the motions without violating the assumptions behind our analysis.

We compute each phase of motion $M$ by interpolating corresponding phases of motions $M_1$, $M_2$,...,$M_k$ with a constant set of weights, $w_1$, $w_2$,...,$w_k$:

$$M = w_1 M_1 + w_2 M_2 + ... + w_k M_k \quad (1)$$

where $\sum_{i=1}^{k} w_i = 1$. The analysis in this paper assumes that the weights sum to one so our results are limited to interpolation and do not generalize to extrapolation. The analysis is presented for interpolation of only two motions, $M_1$ and $M_2$ but generalizes to the interpolation of $k$ motions because equation 1 can be recursively computed by interpolating two motions at a time. The weights for each interpolation sum to 1 and the final interpolation produces a motion with the weighting given in equation 1.

Consider a particular phase $F$. At each time $t$ of that phase we compute motion $M(t, w)$ as follows:

$$M(t,w) = \begin{cases} P_{root}(t) = wP_{1root}(t_1) + (1-w)P_{2root}(t_2) \\ Q_i = wQ_{1i}(t_1) + (1-w)Q_{2i}(t_2), \quad \text{for } i = 1..n \end{cases} \quad (2)$$

where $w = 0..1$ is the interpolation weight, $T_1$, $T_2$ and $T$ are the time of phase $F$ in motions $M_1$, $M_2$ and $M$ respectively, and $t_1 = tT_1/T$ and $t_2 = tT_2/T$ are time indices into motions $M_1$ and $M_2$.

We use a right-handed coordinate system for all motions: the positive $X$ axis points right, the $Y$ axis points up, and the negative $Z$ axis points forward. Positive rotation is counter-clockwise about the axis of rotation.

We analyze the physical correctness of motions computed by linear interpolation of two motions with a constant weight $w$. This analysis includes: (1) the flight phases of the motion, (2) the contact phases and (3) the transitions between the flight and contact phases. During flight the only force acting on the character is gravity. During contact the feet of the character should not slide, contact forces should not require an unreasonably high coefficient of friction, and when the character is in static balance, the center of mass of the character should fall within the support polygon of the feet. The transition between contact and flight phases must maintain continuity (for example, the velocity and position at the end of the flight phase should match that at the beginning of the contact phase). In the next three sections, we present our analysis and suggest some improvements over existing interpolation schemes.

## 4. Analysis of the flight phase

In the next two sections, we analyze the linear and angular momentum of the interpolated motion during flight. We verify that during flight the net external force acting on the character is gravity and that for a restricted model of the character angular momentum is conserved.

### 4.1. Linear momentum during flight

Because the net external force acting on the character during flight is gravity, the trajectory of the center of mass should be a parabola:

$$R_{com}(t) = R_{0com} + V_{0com}t + 0.5Gt^2 \tag{3}$$

where $R_{0com}$ and $V_{0com}$ are position and velocity of the center of mass of the character at the start of the flight phase and $G = (0, -9.8, 0)$ is the acceleration due to gravity.

Figure 1 shows the $Z$ component of the trajectory of the center of mass when a forward jump with no turn and a forward jump with a 360 degree turn are interpolated using equation 2. Because gravity only acts in the vertical, $Y$, direction, the $Z$ component should be a straight line during flight but it is not. The trajectory appears to contain additional forces that act on the character during flight.

As the example in figure 1 shows, linear interpolation of the root position and the joint angles of the character can result in a *non-linear trajectory* for the center of mass. A simple fix is to interpolate the center of mass trajectories instead of the root positions. The root position can then be computed

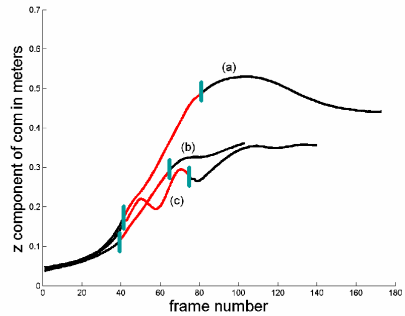


**Figure 1:** *The Z component of the trajectory of the center of mass for: (a) forward jump with no turn (motion $M_1$); (b) forward jump with 360 degree turn (motion $M_2$); (c) the motion that results from interpolating motions $M_1$ and $M_2$. Vertical bars are used to indicate the beginning and ending of the flight phase for each motion. The trajectory of the center of mass of the interpolated motion during flight is not a straight line as it should be.*

from the new center of mass position and joint angles (see Appendix A). The interpolation equation is now:

$$M(t,w) = \begin{cases} P_{com}(t) = wP_{1com}(t_1) + (1-w)P_{2com}(t_2) \\ Q_i(t) = wQ_{1i}(t_1) + (1-w)Q_{2i}(t_2), \text{ for } i = 1..n \\ P_{root}(t) = F(P_{com}(t), Q(t)) \end{cases} \tag{4}$$

where $F$ is the function that computes the root position from the center of mass and the joint angles. With this small change, we can now prove that the net external force acting on the character during flight is gravity. According to Newton's second law:

$$\frac{dP}{dt} = F_{net} = \bar{m}G \tag{5}$$

where $P$ is the total linear momentum of the character, $F_{net}$ is the net external force acting on the character, $\bar{m}$ is the total mass of the character and $G = (0, -9.8, 0)$ is the acceleration due to gravity.

**Proof:** Linear momentum of the articulated character $P = \bar{m}V_{com}$. Taking the derivative of $P$ with respect to time:

$$\begin{aligned} \frac{dP(t)}{dt} &= \bar{m}A_{com}(t) \\ &= \bar{m}(wA_{1com}(t_1)(\tfrac{T_1}{T})^2 + (1-w)A_{2com}(t_2)(\tfrac{T_2}{T})^2) \\ &= w(\tfrac{T_1}{T})^2\bar{m}A_{1com}(t_1) + (1-w)(\tfrac{T_2}{T})^2\bar{m}A_{2com}(t_2) \\ &= w(\tfrac{T_1}{T})^2\bar{m}G + (1-w)(\tfrac{T_2}{T})^2\bar{m}G \\ &= \bar{m}G(w(\tfrac{T_1}{T})^2 + (1-w)(\tfrac{T_2}{T})^2) \\ &= \bar{m}G \end{aligned} \tag{6}$$

The transition from the first to the second line is obtained by taking second derivative of the position of the center of mass in equation 4 with respect to time (see Appendix B). The transition from the second to the third line is obtained by rearranging terms in the equation. The transition

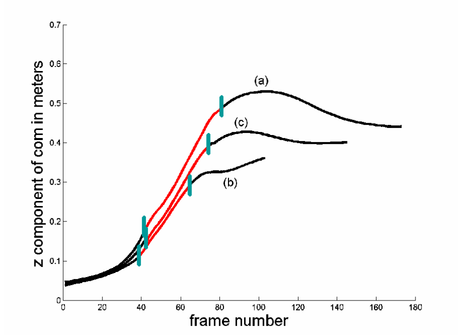


**Figure 2:** *Example from figure 1 but with the flight phase of the interpolated motion computed by interpolating the center of mass positions of the input motions instead of the root positions and with the time of the flight phase computed as* $T = \sqrt{T_1^2 w + T_2^2(1-w)}$.

from the third to the fourth line is obtained by substituting $\bar{m}A_{1com}(t_1) = \bar{m}G$ and $\bar{m}A_{2com}(t_2) = \bar{m}G$. This substitution is valid because we assume that motions $M_1$ and $M_2$ are physically correct. The transition from the fourth to the fifth line is obtained by rearranging terms in the equation. The transition from the fifth to the sixth line is obtained by substituting:

$$T = \sqrt{T_1^2 w + T_2^2(1-w)} \tag{7}$$

This equation defines the choice of the time, $T$, which will ensure that gravity is correct during flight.

In the literature, the time of an interpolated motion has generally been computed as: $T = wT_1 + (1-w)T_2$. But setting time in this way results in scaling gravity by:

$$\frac{wT_1^2 + (1-w)T_2^2}{(wT_1 + (1-w)T_2)^2} \tag{8}$$

In many cases this error will be small and will not be noticeable. Reitsma and Pollard [RP03] determined that if gravity is between $-9.0$ and $-12.7$ the error is not visible to the human observer.

Figure 2 shows the example from figure 1 with the interpolated motion during the flight phase computed according to equation 4 and with time $T = \sqrt{T_1^2 w + T_2^2(1-w)}$. The $Z$ component of the trajectory of the center of mass during flight is now a straight line.

The difference between the interpolation schemes in equation 4 and equation 2 becomes most apparent when interpolating dissimilar motions (as in example 1) or motions that involve significant movement of the root of the character with respect to the center of mass during flight. In our experiments, we found that for many motions linear interpolation of the root resulted in an almost linear interpolation of the center of mass. See Figure 3 for an example.

The pelvis is often chosen as the root of the character. Because it is generally very close to the center of mass of the entire body, it often moves similarly. Figure 4 compares the position of the center of mass to the position of the root for the three motions used in figures 1 and 3. For the forward jumps, the root moves similarly to the center of mass but for the jump with a 360 degree turn, the root moves along a different trajectory. As a result, interpolating the root positions of two forward jumps produces a natural looking motion and interpolating the root positions of a forward jump and a forward jump with a turn produces an unnatural looking result.

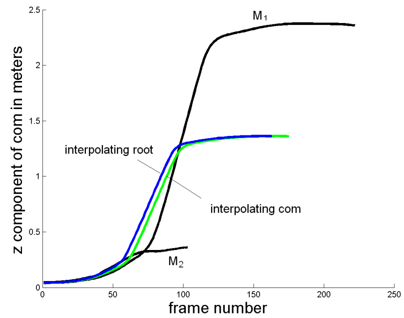


**Figure 3:** *Interpolating a very small forward jump, 0.4 meters, (motion* $M_1$*) and a very large forward jump, 2.5 meters, (motion* $M_2$*). The Z component of the center of mass is shown for motion* $M_1$*, motion* $M_2$ *and two interpolated motions, one computed by interpolating root positions and one computed by interpolating the center of mass positions. The two trajectories for the Z component of the center of mass are very similar.*

### 4.2. Angular momentum during flight

Because the only force acting on the system during flight is gravity, and gravity acts at the center of mass, the angular momentum of the system about the center of mass should be constant during flight. In general, angular momentum will not be constant for a motion computed by interpolating two arbitrary motions. For example, the upper row in figure 5 shows an interpolation between a forward jump and a vertical jump with a 360 degree turn. The angular momentum of the interpolated motion is not constant during flight.

However, even relatively large fluctuations in angular momentum are often unnoticed by the viewer if they do not create large changes in angular velocities. Because angular momentum, $H$, is equal to the product between inertia of the body and angular velocity ($H = I\Omega$), large changes in angular momentum result in small changes in angular velocity if the corresponding inertia is also large. For example, in figure 5 (upper row, rightmost image) angular momentum changes significantly around the $X$ axis but the motion still appears natural. The change in angular momentum is hard to detect because the inertia around the $X$ axis is large (because the longitudinal axis of the body is perpendicular to the $X$ axis) and the resulting change in angular velocity is small.

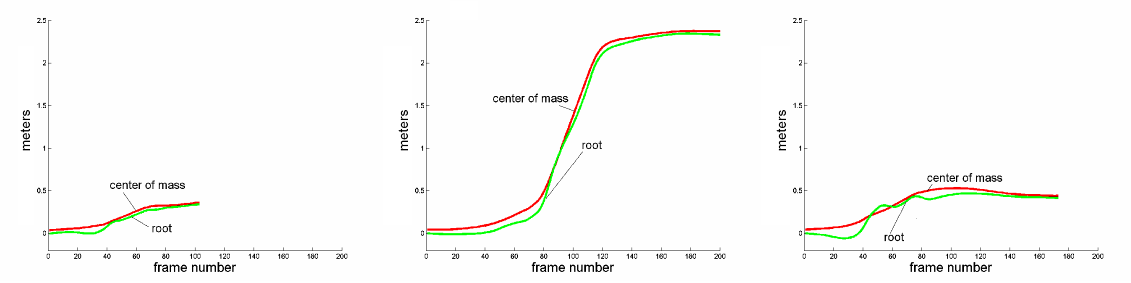


**Figure 4:** *Comparing the center of mass trajectory to the root trajectory for 3 different jumps. Z components are shown. Left: small forward jump, middle: large forward jump, right: forward jump with 360 degree turn. These jumps were used to compute the interpolated motions in figures 1 and 3.*

Although the angular momentum is not necessarily preserved when interpolating two arbitrary motions it is possible to show that for a single rigid body, angular momentum is conserved during flight if both motions rotate around the same principal axis or one or both contain no rotation. In many common motions, visible rotation (with large angular velocity) during flight is either absent (for example, a short forward jump or run) or happens around only one principal axis (for example, a longer forward jump, a flip or a vertical jump with a turn). Approximating the character with a rigid body is not an accurate model for most motions but this proof still provides some insight into when angular momentum will be preserved.

**Proof:** If a rigid body rotates around a principal axis then the angular momentum, $H$, is equal to the product of inertia of the body $I$, and the angular velocity of the body, $\Omega$ around the axis of rotation. Let $H_1 = I_1\Omega_1$ and $H_2 = I_2\Omega_2$ be the angular momentum for the first and second motions respectively. If we interpolate the center of mass positions and the rotation angles, the angular momentum of the interpolated motion is

$$H = I_3\Omega = I_3(w\Omega_1\frac{T_1}{T} + (1-w)\Omega_2\frac{T_2}{T}) = constant \quad (9)$$

The angular momentum $H$ is constant because $I_3$, $\Omega_1$ and $\Omega_2$ are constant during flight.

The bottom row of figure 5 shows the angular momentum for a motion computed by interpolating two forward jumps. Because both jumps involve a rotation around the same axis the angular momentum in the resulting motion remains relatively constant during flight.

## 5. Analysis of the contact phase

In this section, we analyze the physical correctness of the motion while one or both feet are in contact with the environment. The following conditions should hold for the motion to be physically valid: (1) the feet of the character should not slide; (2) when the character is in static balance its center of mass should fall within the support polygon of the feet; (3) the contact forces that correspond to the motion should not require an unreasonably high coefficient of friction. We now analyze each of these requirements.

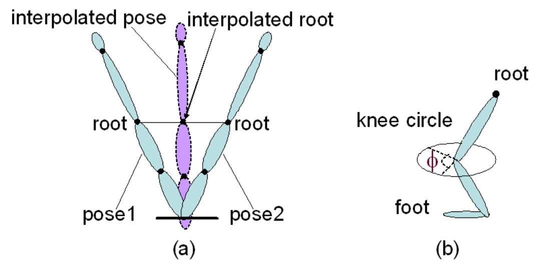


**Figure 6:** *(a)Two poses of a simplified character that have the same contact are interpolated with weight $w = 0.5$. The resulting pose penetrates the ground. (b) The redundant degrees of freedom of each leg can be intuitively parameterized by one parameter, $\Phi$, that represents the "knee circle" of the leg.*

### 5.1. Non-sliding Foot Contact

We assume that when one or both feet of the character are in contact with the ground, the position of the feet should not move (the character does not slip). This condition, however, does not hold for either the center of mass or root interpolation schemes presented above. Consider the example in figure 6(a): two poses of a simplified character that have the same contact point are interpolated with weight $w = 0.5$ resulting in a foot position below the ground.

Other researchers have addressed this problem by rooting the character at a foot that is in contact. This solution works well when there is only one foot in contact but may result in sliding of the other foot if that other foot is also in contact. In general, preserving the contact positions of both feet and computing joint angles via interpolation is not possible because the system is over-constrained. A common solution is to to eliminate foot sliding in the interpolated motion with a post processing step (see, for example [LS99] and [RCB98]). This additional editing would be hard to analyze for physical validity.

An alternative solution that preserves physics is to inter-

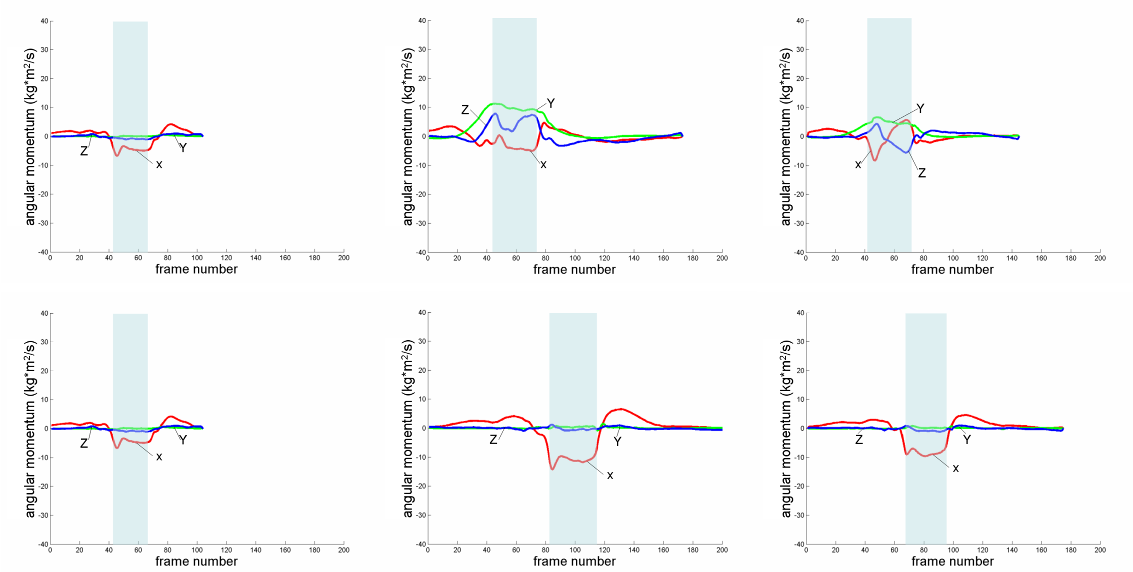


**Figure 5:** *Upper row, from left to right: Angular momentum curves for a forward jump, a vertical jump with a 360 degree turn about the vertical axis and a motion computed by interpolating those motions. Lower row, from left to right: Angular momentum curves for a small forward jump, a very large forward jump and a motion computed by interpolating those motions. X, Y and Z components of angular momentum are shown for each graph. The shaded area represents the flight phase.*

polate only the non-redundant degrees of freedom (dofs) of the character and the constraints. Each constraint reduces the number of available dofs. Therefore, if the character originally had $n+3$ dofs ($n$ rotational and 3 translational), then when both feet are in contact, the number of degrees of freedom is reduced by 12. Korein and Badler [KB82] and later Lee and Shin [LS99] showed that the degrees of freedom of a leg in contact with the ground can be controlled by just one parameter, $\Phi$, assuming that the hip position of the leg is also known. Intuitively, that parameter represents the "knee circle" of the leg (figure 6(b)). Thus, when both legs are in contact the non-redundant degrees of freedom of the character are (1) root position; (2) all the joint angles of the character except the legs; and (3) two "knee circle" parameters, one for each leg. We can now interpolate these non-redundant degrees of freedom and the constraints that include the positions and orientations of both feet and the feet will not slide.

Because there is no real advantage in interpolating the root as opposed to interpolating the center of mass on the ground, we can interpolate the center of mass as we did for the flight phase:

$$M(t,w) = \begin{cases} P_{com}(t) = wP_{1com}(t_1) + (1-w)P_{2com}(t_2) \\ Qnr_i(t) = wQnr_{1i}(t_1) + (1-w)Qnr_{2i}(t_2) \\ C_j(t) = wC_{1j}(t_1) + (1-w)C_{2j}(t_2) \\ P_{root}(t) = F2(P_{com}(t), Q_{nr}(t), C(t)) \end{cases} \quad (10)$$

where $Qnr_i$ are the non-redundant dofs of the character not including the root, $C_j$ are constraints such as feet positions and orientations, and $F2$ is the function that computes the root position of the character from the center of mass position, non-redundant dofs and the constraints (see Appendix C for details). To preserve continuity of the motion, we use equation 10 independent of whether one foot or both feet are in contact. With this interpolation scheme, the feet will not slide during contact and we can prove that the static balance condition holds and that the ground reaction forces are within the friction cone.

### 5.2. Static Balance

Static balance exists when the projection of the center of mass of the character onto the ground is within the support polygon of the feet. We do not assume that input motions are statically balanced but we show that if they are, the interpolated motion is as well. We assume that $M_1$ and $M_2$ have the same support polygon.

The position of the center of mass of the interpolated motion at time $t$ is equal to the interpolation of the center of mass of motion $M_1$ at time $t_1$ and of center of mass of motion $M_2$ at time $t_2$ (equation 10). Therefore, the center of mass of the interpolated motion, $P_{com}$, will lie on a segment connecting points $P_{1com}$ and $P_{2com}$. The projection of $P_{com}$ onto a plane of contact will also lie on a segment connecting the projections of $P_{1com}$ and $P_{2com}$. Let us call these projections $P^g_{com}$, $P^g_{1com}$ and $P^g_{2com}$, then:

$$P^g_{com}(t) = wP^g_{1com}(t_1) + (1-w)P^g_{2com}(t_2) \quad (11)$$

Because $P^g_{1com}$ and $P^g_{2com}$ lie within the support polygon of

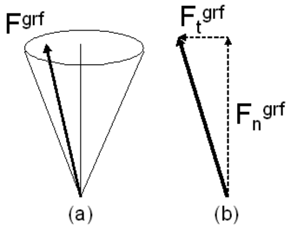


**Figure 7:** *(a) The ground reaction force must fall within a friction cone oriented along the contact normal. (b) The tangential, $F_t^{grf}$, and the normal, $F_n^{grf}$, components of the ground reaction force.*

the feet, $P^g_{com}$ will also lie within the support polygon assuming the support polygon is convex and $0 \leq w \leq 1$.

### 5.3. Friction cone

For the motion to be physically valid, ground contact should not require an unreasonably high coefficient of friction. We use a Coulomb friction model to analyze the ground contact. If contacting surfaces do not move with respect to each other (static friction) the ratio of the absolute values of the tangential component of the ground reaction force, $F_t^{grf}$, and the normal component of the ground reaction force, $F_n^{grf}$, should be smaller than the coefficient of static friction:

$$\frac{F_t^{grf}(t)}{F_n^{grf}(t)} < \mu_s \tag{12}$$

Geometrically this constraint means that the ground reaction force must fall within a friction cone oriented along the contact normal (figure 7).

Assuming a single support polygon, Newton's second law says that the ground reaction force, $F^{grf}(t)$ is

$$F^{grf}(t) = \bar{m}A_{com}(t) - \bar{m}G \tag{13}$$

where $\bar{m}$ is the total mass of the system, $A_{com}(t)$ is the acceleration of the center of mass of the system and $G = (0, -9.8, 0)$ is the acceleration due to gravity. The ground reaction force of the interpolated motion computed according to equation 10 is an interpolation of the ground reaction forces of motions $M_1$ and $M_2$ (see Appendix D for the proof):

$$F^{grf}(t) = wF_1^{grf}(t_1)\left(\frac{T_1}{T}\right)^2 + (1-w)F_2^{grf}(t_2)\left(\frac{T_2}{T}\right)^2 \tag{14}$$

The proof requires that we set the time $T$ of the contact phase as $T = \sqrt{T_1^2 w + T_2^2(1-w)}$, which is the same formula we used for the flight phase. Now we need to show that equation 12 holds for the interpolated motion. From equation 14, we know that the tangential and normal components of the ground reaction force can be computed by interpolating the corresponding components of motions $M_1$ and $M_2$:

$$\frac{F_t^{grf}(t)}{F_n^{grf}(t)} = \frac{wF_{t1}^{grf}(t_1)(\frac{T_1}{T})^2 + (1-w)F_{t2}^{grf}(t_2)(\frac{T_2}{T})^2}{wF_{n1}^{grf}(t_1)(\frac{T_1}{T})^2 + (1-w)F_{n2}^{grf}(t_2)(\frac{T_2}{T})^2} \tag{15}$$

To show that equation 12 holds for the interpolated motion we first show that for any positive numbers $a$,$b$,$c$ and $d$, if $\frac{a}{b} < \mu$ and $\frac{c}{d} < \mu$ then $\frac{a+c}{b+d} < \mu$ (see Appendix E for the proof). From this result, and because we know that equation 12 holds for motions $M_1$ and $M_2$ we can conclude that equation 12 holds for the interpolated motion.

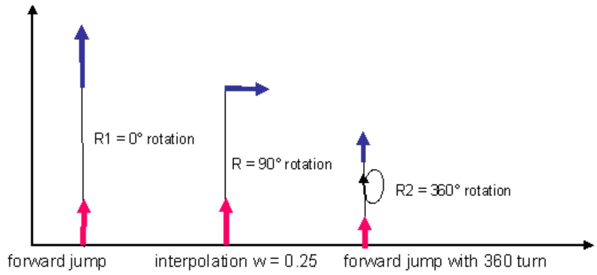


**Figure 8:** *Interpolation between a long forward jump and a forward jump with a 360 degree turn (weight $w = 0.75$). Motions are shown schematically: the center of mass is projected onto the ground; the arrows represent the facing direction of the character.*

### 6. Transition between phases

We have analyzed the motion of the character during flight and contact phases independently so we also need to analyze the continuity of the motion across the transition between phases. The continuity of the position of the center of mass follows trivially from the fact that it is computed by interpolating the center of mass of motions $M_1$ and $M_2$ which are themselves assumed to be continuous. The velocity of the center of mass may be discontinuous during the transition because different time scalings are applied to adjacent phases. We have found, however, that this discontinuity is not noticeable in practice.

Motions with rotation during the flight phase, however, may have significant discontinuities at the transition between flight and stance phases because the orientation of the interpolated motion may not match that of the original motions after the flight phase. For example, consider the interpolation of a long forward jump with a forward jump with a 360 degree turn (schematically shown in figure 8). In the original motions the character lands facing the positive $Z$ axis but in the interpolated motion, the character lands facing the positive $X$ axis (rotated 90 degrees clockwise about vertical). Both the original interpolation scheme (equation 2) and the modified scheme (equation 10) will have problems with this transition. The resulting motion will have either significant foot sliding because the motion of the root does not match the motion of the joint angles or a discontinuity in the joint angles.

To reduce these problems, the subsequent motion of the root (or center of mass) in the original motions can be rotated to align them with the interpolated motion at the end of the flight phase. This operation brings the root motion into alignment with the character's facing direction and joint movements but it may introduce a discontinuity in the velocity of the center of mass. For example, in figure 8, the landing velocity of the center of mass for the interpolated motion will be rotated instantaneously by 90 degrees in the transition to the stance phase. The discontinuity will be small when the required rotations are small or the ground plane components of the velocity at landing, $V_{landing}$, are small. The problem will be worse for motions with more complicated rotations such as forward or twisting flips.

## 7. Summary of Analysis

We have made three changes to the interpolation scheme of equation 2: (1) during flight we interpolate the center of mass positions instead of the root positions; (2) during ground contact we interpolate the positions of the feet, the center of mass positions and all non-redundant degrees of freedom to prevent the feet from sliding on the ground; (3) the timing of each phase is computed as $T = \sqrt{T_1^2 w + T_2^2 (1-w)}$. With these changes, we can prove the following properties about the physical correctness of the interpolated motion:

- The net force acting on the character during flight will be equal to gravity.
- During contact, the feet of the character will not slide.
- If the character is balanced in the original motions, it will also be balanced in the interpolated motion.
- If the ground reaction force in both original motions is within the friction cone, it will also be within the friction cone for the interpolated motion.
- If we interpolate two motions that do not have visible rotation during the flight phases (for example, runs, short forward jumps and vertical jumps) or motions that rotate about approximately the same principal axis (for example, flips and longer forward jumps), the angular momentum in the interpolated motion will be close to constant during flight. This analysis of angular momentum holds when the character can be reasonably approximated by a rigid body during flight.
- If we interpolate two motions that either (1) do not have visible rotation during their flight phases or (2) rotate by approximately the same angle about the vertical axis in both original motions or (3) occur mostly in the vertical direction (for example, a vertical jump), then the continuity of the velocity of the center of mass will be preserved during the transition from flight to contact (ignoring the discontinuity due to differences in time scaling of two phases).

## 8. Experimental results

Our experimental results consist of two parts. We first demonstrate that a variety of dynamical and non-dynamical motions can be successfully interpolated to generate realistic looking motions. The motions are generated by interpolating the trajectories of the center of mass and joint angles during the flight phases and during the stance phases placing the root at one of the feet in contact and interpolating the root and joint angles. The motions are also aligned as described in section 6. The motions are all included in the accompanying video.

We performed the following experiments: (1) the interpolation of two forward jumps of very different lengths with no rotation; (2) the interpolation of two forward jumps of different lengths, each with a 90 degree turn; (3) the interpolation of two vertical jumps of different heights and different amounts of rotation; (4) the interpolation of two motions in which the actor stepped over obstacles of different heights; (5) the interpolation of running and a running jump. In each of these experiments, the original motions had the properties required to guarantee the physical correctness of the interpolated motions according to our analysis. The interpolated motions did indeed look visually realistic.

We also compare linear interpolation using root positions during a flight with interpolation using the position of the center of mass. We interpolated root and center of mass for a forward jump with no turn and forward jump with 360 degree turn (the example in figure 1). This interpolation results in unnatural motion during the flight phase if the root is interpolated and natural looking motion if the center of mass is interpolated.

Interpolation of either the center of mass position or of the root position may cause the feet to slide or penetrate the ground. We demonstrated this by interpolating motions of a person sitting down on two seats of different heights. Interpolating root position results in significant sliding of the feet. Even simply placing the root at one foot of the character significantly reduces the problem although the second foot still moves slightly with respect to the ground.

Our last experiment demonstrated that if two motions with different amounts of rotation are interpolated there may be a visible discontinuity in the velocity of center of mass at landing (section 6). This phenomena is demonstrated on the interpolation of a forward jump and a vertical jump with a 360 degrees turn. The resulting motion is quite unnatural.

## 9. Discussion

We, like others who have experimented with interpolation, have observed that the matching of key events is crucial for good results. Some key events such as foot contact are relatively easy to detect automatically. Others such as oscillations in arm swing, are more difficult to detect and match accurately. However, if two jumps are interpolated, one with a double arm swing during the landing phase and one with a single arm swing, the resulting motion will not be natural. Problems such as this will have to be addressed in an automatic fashion to make interpolation useful in situations where the details of the original motions are not a good match.

The analysis presented here only looked at physical correctness. It will not catch unnatural motions like the ones described in the previous paragraph or intersections of the segments of the body that will sometimes arise when two natural motions are interpolated. Those errors will have to be detected and fixed using editing techniques which may themselves introduce errors in the physical correctness of the motion.

In our analysis we assumed a time warping based on a set of matching keys. A method for performing dynamic time warping is presented in [KG03]. Our analysis cannot be directly applied to this approach but it might be possible to extend the analysis.

This paper analyzes a number of physical properties of the interpolated motion. Another physical property that should be satisfied during contact is that the center of pressure should fall within the support polygon of the feet.

The results presented here lead to a number of interesting further questions. First, in situations in which the interpolated motion is not going to be physically correct, we need better guidelines on how much error is acceptable. Reitsma and Pollard took a step in that direction [RP03] and observed that errors in horizontal velocity were easier to detect than errors in vertical velocity but we need a much more complete understanding of what errors will be perceptible and which will not be noticed.

For example, the angular momentum of the interpolated motion is not conserved during flight in the general case. From our experiments, however, even relatively large fluctuations in angular momentum are not noticed by the viewer if they do not result in large changes in angular velocities. A deeper understanding of this observation might be useful in developing better guidelines for interpolation. Similarly, it would be useful to have a guideline for when the discontinuity in the transitions from flight to stance will be visible.

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**Appendix A:** Given the position of the center of mass of the character, $P_{com}$, and the values of all joint angles, $Q_i$, $i = 1..n$, we compute the position of the root of the character, $P_{root}$ as follows: (1) for the given $Q_i$ compute the center of mass of the character, $P^0_{com}$ assuming that the root is at the origin; this calculation gives us the relative position of the root of the character with respect to the center of mass of the character; (2) compute root position: $P_{root} = P_{com} - P^0_{com}$.

**Appendix B:** The velocity of center of mass is computed by taking the derivative of the position of the center of mass in equation 4 with respect to time. Similarly, acceleration of center of mass is computed by taking the derivative of the velocity:

$$\begin{aligned} V_{com}(t) &= \frac{dP_{com}(t)}{dt} \\ &= \frac{d(wP_{1com}(t_1)+(1-w)P_{2com}(t_2))}{dt} \\ &= wV_{1com}(t_1)\frac{T_1}{T} + (1-w)V_{2com}(t_2)\frac{T_2}{T} \end{aligned} \quad (16)$$

$$\begin{aligned} A_{com}(t) &= \frac{dV_{com}(t)}{dt} \\ &= \frac{d(wV_{1com}(t_1)\frac{T_1}{T}+(1-w)V_{2com}(t_2)\frac{T_2}{T})}{dt} \\ &= wA_{1com}(t_1)(\frac{T_1}{T})^2 + (1-w)V_{2com}(t_2)(\frac{T_2}{T})^2 \end{aligned} \quad (17)$$

where $w = 0..1$ is the interpolation weight, $T_1$, $T_2$ and $T$ are the overall time of phase $F$ in motions $M_1$, $M_2$ and $M$. $t_1 = tT_1/T$ and $t_2 = tT_2/T$ are scaled time indices into motions $M_1$ and $M_2$.

**Appendix C:** To obtain the root position given the center of mass position, the values of all non-redundant dofs and the values of constraints, such as feet position and orientation, we first note that the center of mass of the character can be decomposed into the summation of three quantities: the center of mass for the upper body, $P_{upcom}$, for the left leg, $P_{llcom}$ and for the right leg, $P_{rlcom}$. The center of mass of the entire body, $P_{com}$, is then $P_{com} = P_{upcom}M_{upcom} + P_{llcom}M_{llcom} + P_{rlcom}M_{rlcom}$. In this derivation we assume both legs are in contact but when only one leg is in contact the derivation is very similar. The position of the center of mass of the upper body can be re-expressed in terms of the root position (an unknown) and the center of mass of the upper body assuming the root is at the origin (see Appendix A):

$$P_{com} = (P_{root} + P^0_{upcom})M_{upcom} + P_{llcom}M_{llcom} + P_{rlcom}M_{rlcom} \quad (18)$$

The position of the center of mass for a leg can be expressed in terms of the position of the root, the position of a knee joint and the position of the foot joint. For example, for the left leg

$$P_{llcom} = 2P_{root} + 2/3(P_{lknee} - P_{root}) + 1/2(P_{lfoot} - P_{lknee}) + P_{lfcom} \quad (19)$$

where $P_{lknee}$ is the position of the left knee joint, $P_{lfoot}$ is the position of the left foot joint and $P_{lfcom}$ is the position of the center of mass of the left foot. $P_{lknee}$ in its turn can re-expressed as a function $P_{root}$ as was shown in [KB82], leaving us with $P_{root}$ as the only unknown in equation 18. Solving the equation for $P_{root}$ (either analytically if possible or numerically if not) will give us the desired result.

**Appendix D:** It can be shown that ground reaction force of an interpolated motion computed according to equation 10 is a time-scaled interpolation of the ground reaction forces of motions $M_1$ and $M_2$ if we compute time $T$ for that contact phase as $T = \sqrt{T_1^2 w + T_2^2(1-w)}$:

$$F^{grf}(t) = F_1^{grf}(t_1)(\frac{T_1}{T})^2 w + F_2^{grf}(t_2)(\frac{T_2}{T})^2(1-w) \quad (20)$$

**Proof:** Assuming a single support polygon, by Newton's second law the ground reaction force, $F^{grf}(t)$ is

$$F^{grf}(t) = \bar{m}A_{com}(t) - \bar{m}G \quad (21)$$

where $\bar{m}$ is the total mass of the system, $A_{com}(t)$ is the acceleration of the center of mass of the system and $G = (0, -9.8, 0)$ is an acceleration due to gravity. Substituting equation 17 into 21 yields:

$$F^{grf}(t) = \bar{m}A_{1com}(t_1)(\frac{T_1}{T})^2 w + \bar{m}A_{2com}(t_2)(\frac{T_2}{T})^2(1-w) - \bar{m}G \quad (22)$$

From Newton's second law $\bar{m}A_{1com}(t_1) = F_1^{grf}(t_1) + \bar{m}G$ and $\bar{m}A_{2com}(t_2) = F_2^{grf}(t_2) + \bar{m}G$. Substituting this into the equation above:

$$\begin{aligned} F^{grf}(t) &= (F_1^{grf}(t_1) + \bar{m}G)(\tfrac{T_1}{T})^2 w + \\ &\quad (F_2^{grf}(t_2) + \bar{m}G)(\tfrac{T_2}{T})^2(1-w) - \bar{m}G \end{aligned} \quad (23)$$

Rearranging the terms we have:

$$\begin{aligned} F^{grf}(t) &= F_1^{grf}(t_1)(\tfrac{T_1}{T})^2 w + F_2^{grf}(t_2)(\tfrac{T_2}{T})^2(1-w) + \\ &\quad \bar{m}G(\tfrac{T_1^2 w + T_2^2(1-w)}{T^2}) - \bar{m}G \end{aligned} \quad (24)$$

Because $T = \sqrt{T_1^2 w + T_2^2(1-w)}$, the ground reaction force for the interpolated motion is the interpolation of ground reaction forces from first and second motions:

$$F^{grf}(t) = F_1^{grf}(t_1)(\frac{T_1}{T})^2 w + F_2^{grf}(t_2)(\frac{T_2}{T})^2(1-w) \quad (25)$$

**Appendix E** It is easy to show that for any positive numbers $a$,$b$,$c$ and $d$, if $\frac{a}{b} < \mu$ and $\frac{c}{d} < \mu$ then $\frac{a+c}{b+d} < \mu$.

Adding equations $a < \mu b$ and $c < \mu d$ together we have: $(a+c) < \mu(b+d)$. Now rearranging terms yields: $\frac{a+c}{b+d} < \mu$

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