15-464/15-664   Reference List for March 15, 2023

Eulerian finite difference based approach to solving Navier-Stokes equations

We did a quick introduction to the Navier-Stokes equations today.   I began by mentioning that a finite-difference based Eulerian fluid simulation was used to create the water simulation in the movie Antz:
https://www.youtube.com/watch?v=KfxQi9_BfHI
https://www.youtube.com/watch?v=fIoPcc1ngEg

The paper behind this technique is this one:

Foster, Nick, and Dimitri Metaxas. "Realistic animation of liquids." Graphical models and image processing 58, no. 5 (1996): 471-483.
http://www.cbim.rutgers.edu/dmdocuments/gmip96%20Foster.pdf

I have also include some notes from a previous class that may or may not be helpful for describing the governing equations in the following pages.

Navier Stokes Equations

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = \underbrace{-\nabla p + g + \nu \nabla^2 u}_{\text{"forces"}}$$

- $\frac{\partial u}{\partial t}$ ↑ acceleration
- $u \cdot \nabla u$ ↑ advection
- $-\nabla p$ ↑ pressure gradient
- g ↑ gravitational acceleration
- $\nu \nabla^2 u$ ↑ drag

∇ = gradient

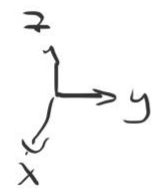
∇^2 = Laplacian

u = velocity p = pressure

g = gravity ν = viscosity

$f = ma$

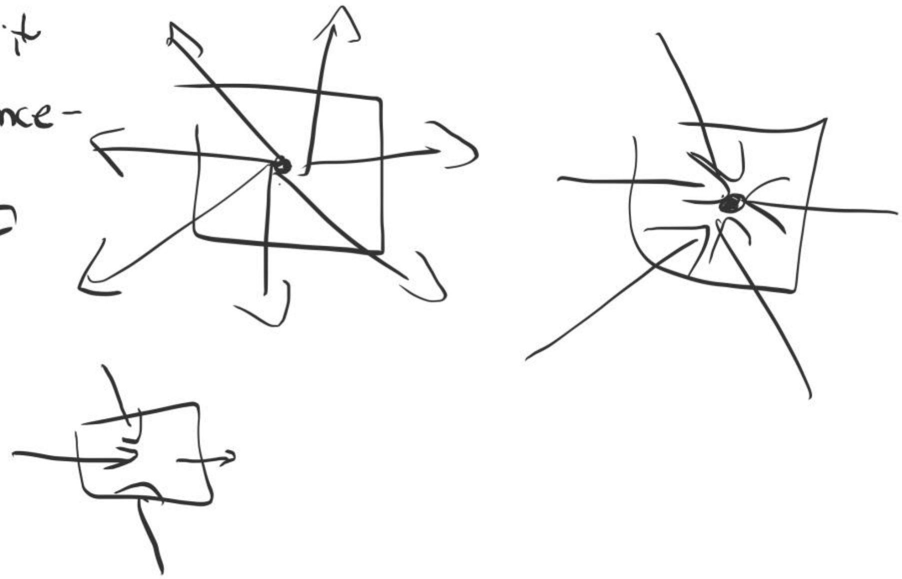
∇p = gradient $\begin{bmatrix} \partial p/\partial x \\ \partial p/\partial y \\ \partial p/\partial z \end{bmatrix}$



$\nabla^2 u$ = Laplacian $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$

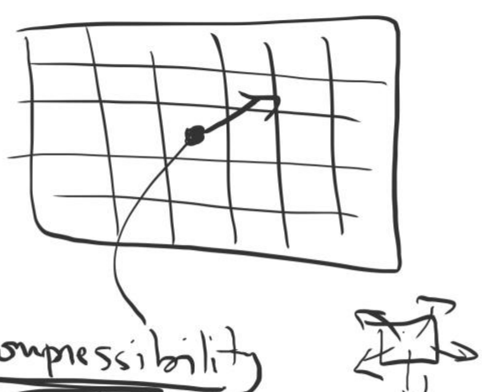
Step 1 ⇒ Apply forces (Navier-Stokes)

Step 2 ⇒ make it divergence-free



vortex confinement

Eulerian



incompressibility

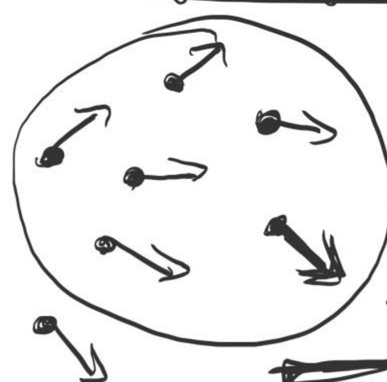
mass conservation ← goes away

diffusion

advection

PIC FLIP APIC

Lagrangian



f=ma

density

pressure

SPH