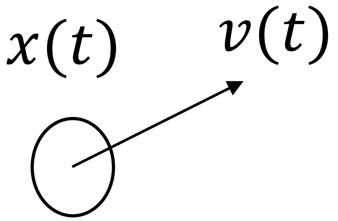
Rigid Bodies and Contacts

Yanzhe Yang
Feb 26, 2020

Rigid bodies and contacts

- Particle: State $Y = \binom{x}{v}$, State Derivative $\frac{d}{dt}Y = \frac{d}{dt}\binom{x}{v} = \binom{v}{F/m}$



Rigid bodies and contacts

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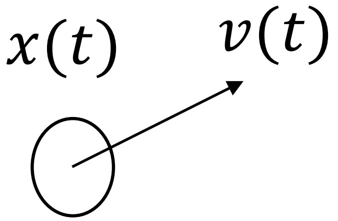
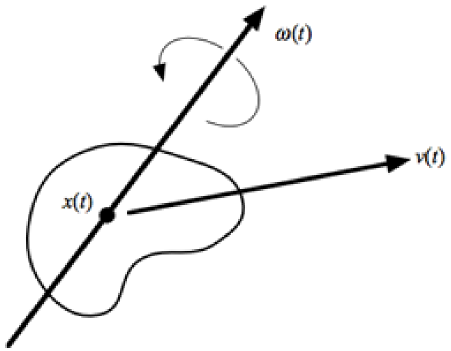


- Rigid body: State $Y = \begin{pmatrix} x \\ R \\ mv \\ Iw \end{pmatrix}$, State Derivative $\frac{d}{dt}Y = \frac{d}{dt}\begin{pmatrix} x \\ R \\ mv \\ Iw \end{pmatrix} = \begin{pmatrix} v \\ w \times R \\ F \\ \tau \end{pmatrix}$



R:	rotation matrix
mv:	linear momentum
Iw:	angular momentum

Rigid bodies and contacts

Image source:
https://noobtuts.com/unity/2d-angry-birds-game
https://giphy.com/explore/walking-in-the-snow

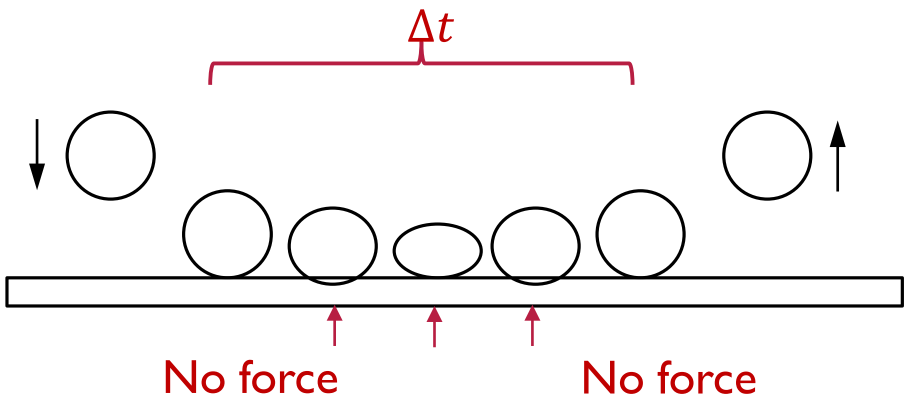
Today's topic:

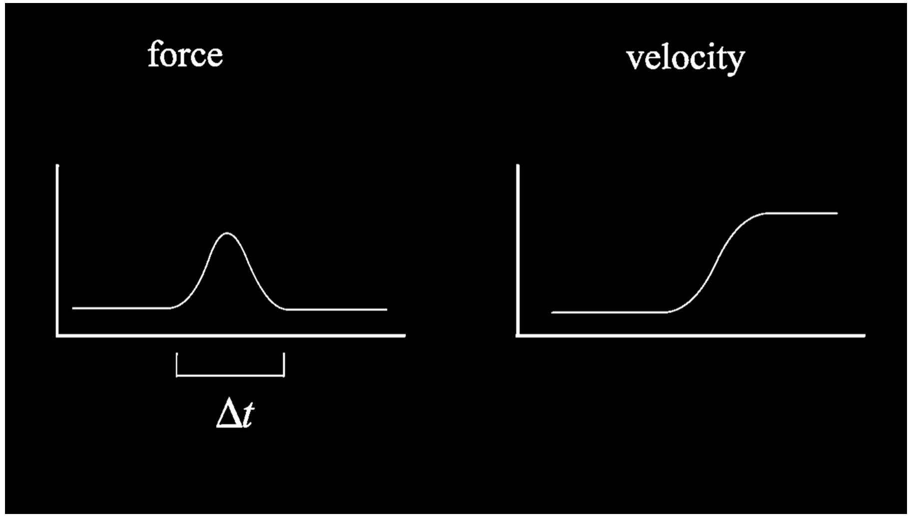
How to simulate contacts for rigid bodies?

Example: ball falling

Reference: Andrew Witkin and David Baraff, Physically Based Modeling: Principles and Practice, Online SIGGRAPH 1997 Course Notes, 1997
https://www.cs.cmu.edu/~baraff/sigcourse/

Soft collision





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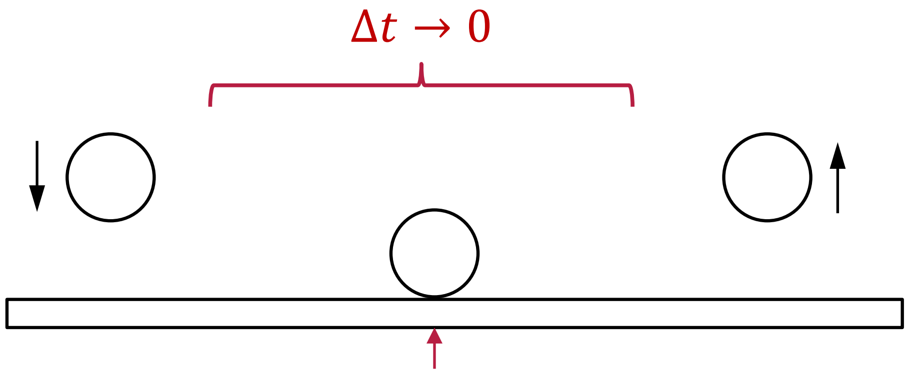
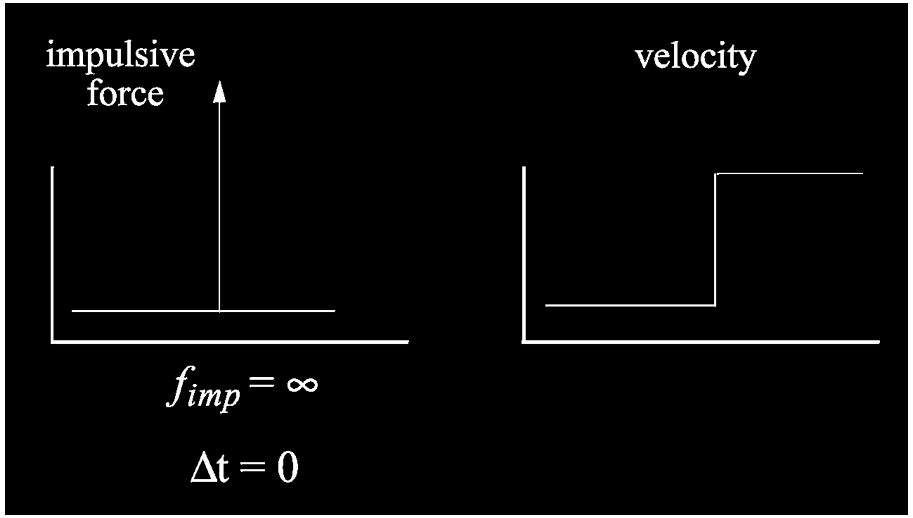
Rigid body collision





Discontinuity

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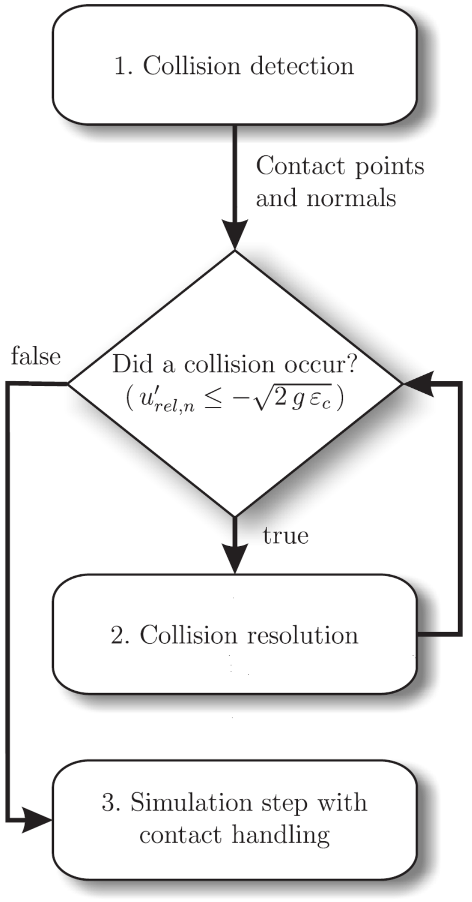
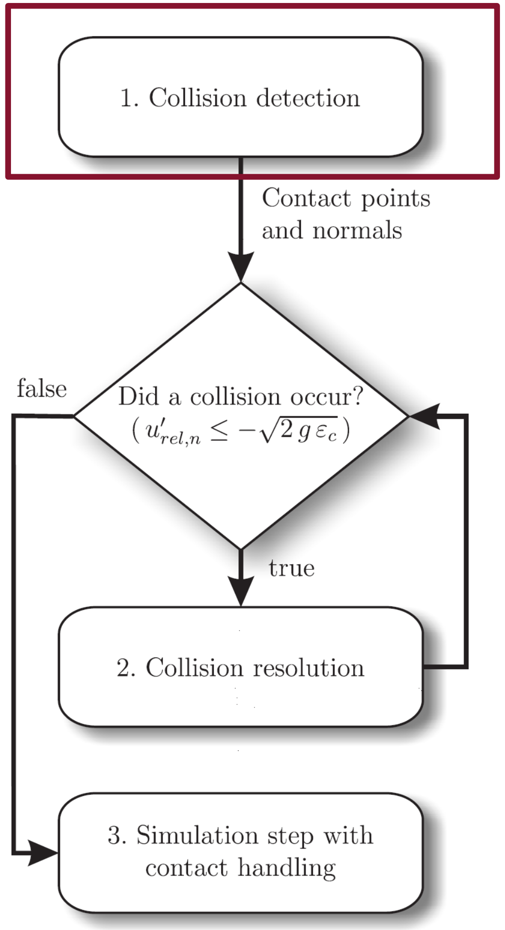
https://www.cs.cmu.edu/~baraff/sigcourse/

A typical simulation loop



Reference: J. Bender and A. Schmitt. Constraint-based collision and contact handling using impulses. In Proceddings of the 19th international conference on computer animation and social agents. July 2006

Questions



When and where collision happens?

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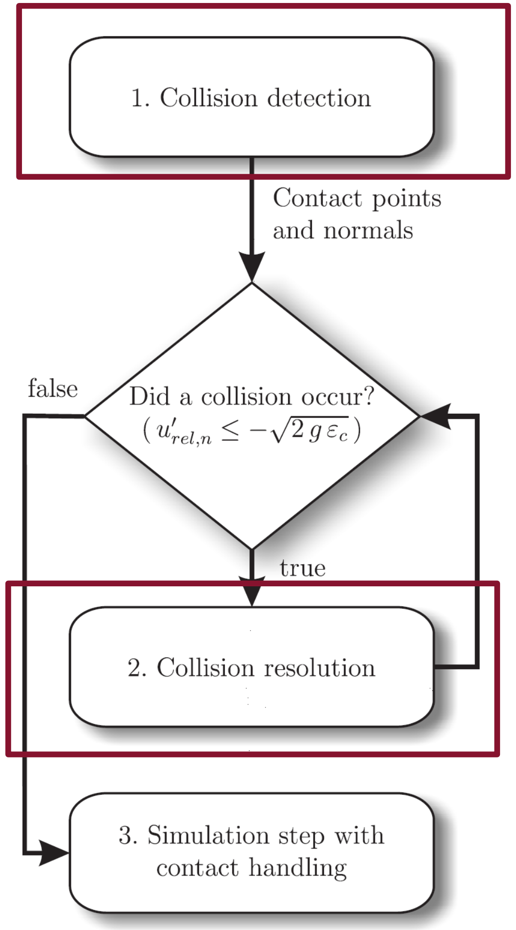

When and where collision happens?

How to compute impulses?

Reference: J. Bender and A. Schmitt. Constraint-based collision and contact handling using impulses. In Proceddings of the 19th international conference on computer animation and social agents. July 2006

Outline

- Collision detection basics
- Computing impulses using a coefficient of restitution
- Penalty based method
- Constraint based method
- Recent work

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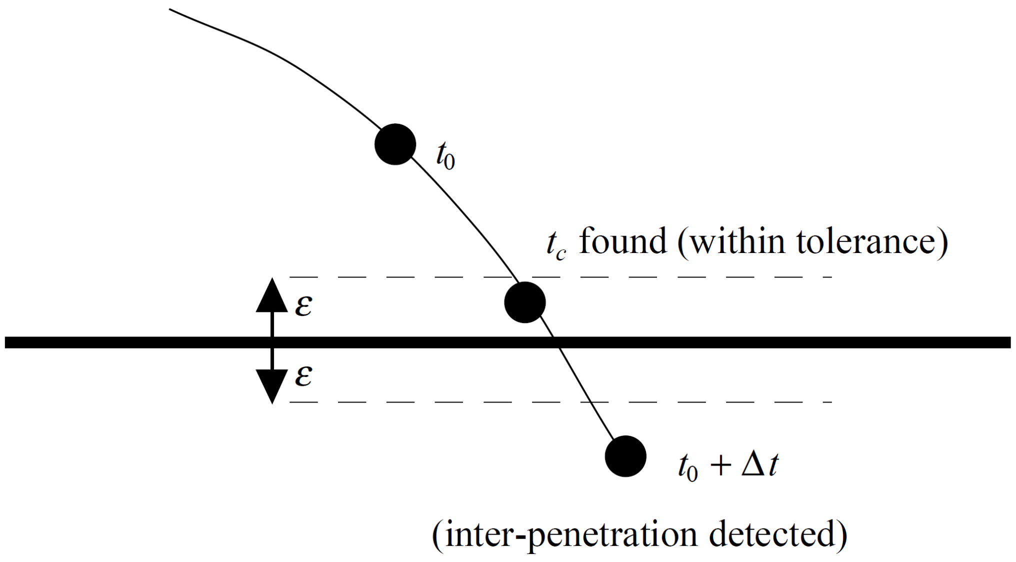
Finding the collision time by backtracking

- Bisection method.

  If we know collision time t_c happens within $[t_0, t_0 + \Delta t)$, then check $t_0 + \Delta t/2$



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Finding the collision time by backtracking

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- Easy to implement and quite robust, but a little slow. Faster convergence could be achieved using the *regula falsi* (false position) method

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- t_c is not always needed (discuss in penalty-based method)

Reference: David Baraff. Analytical Methods for Dynamic Simulation of Non-penetrating Rigid Bodies. SIGGRAPH 1989
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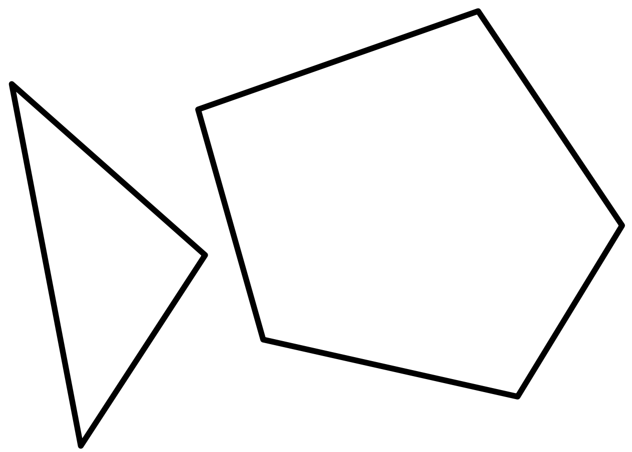
Detecting collision points

- If a separating plane is found, no inter-penetrating.



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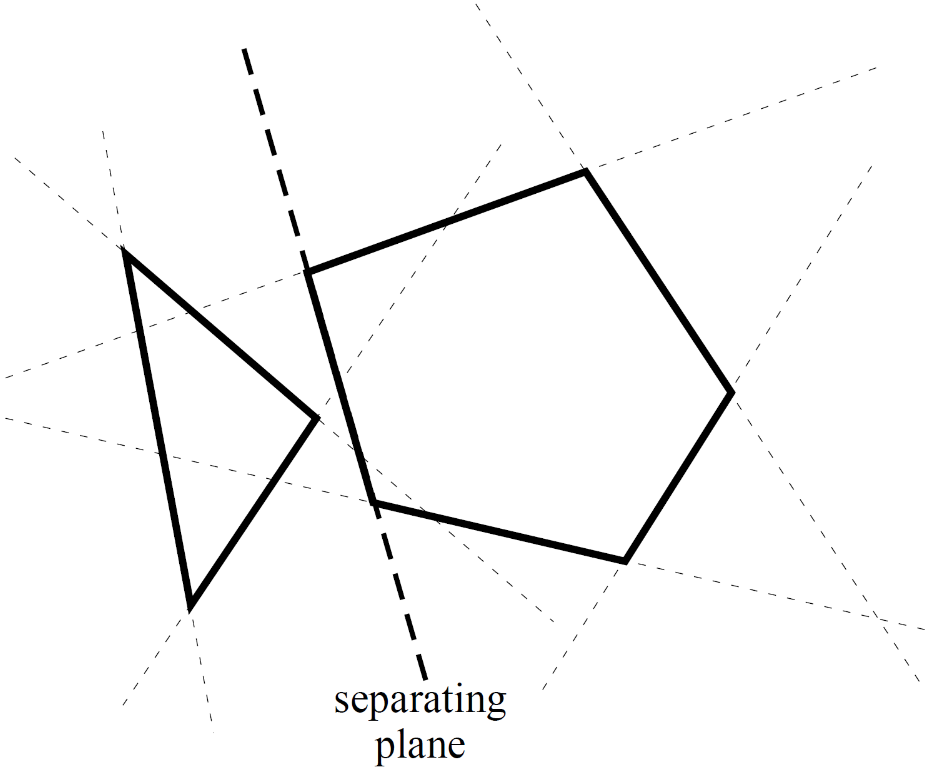
Detecting collision points: separating plane

- Exhaustive search for a separating plane considering two rigid bodies



A separating plane either contains a face of one of the convex polyhedral or contains an edge from one convex polyhedral and is parallel to an edge of the other convex polyhedral

Reference: David Baraff. Analytical Methods for Dynamic Simulation of Non-penetrating Rigid Bodies. SIGGRAPH 1989
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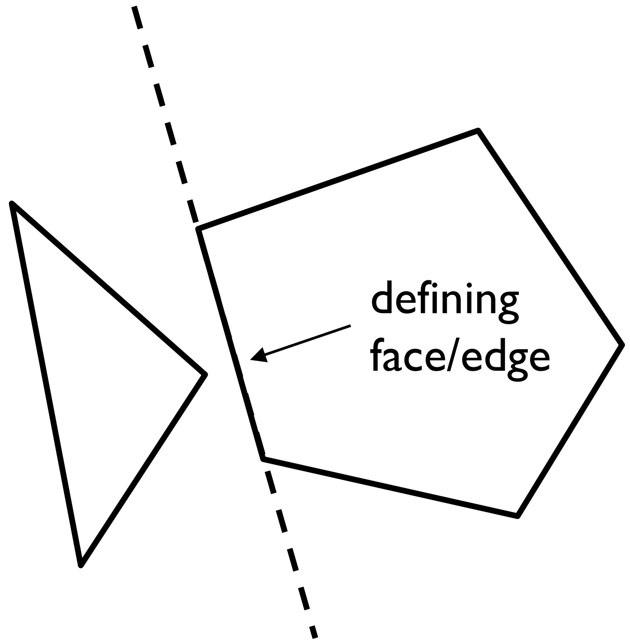
Detecting collision points: defining face/edge

- The face/edge that is contained in the separating plane is called *defining face/edge*



Reference: David Baraff. Analytical Methods for Dynamic Simulation of Non-penetrating Rigid Bodies. SIGGRAPH 1989
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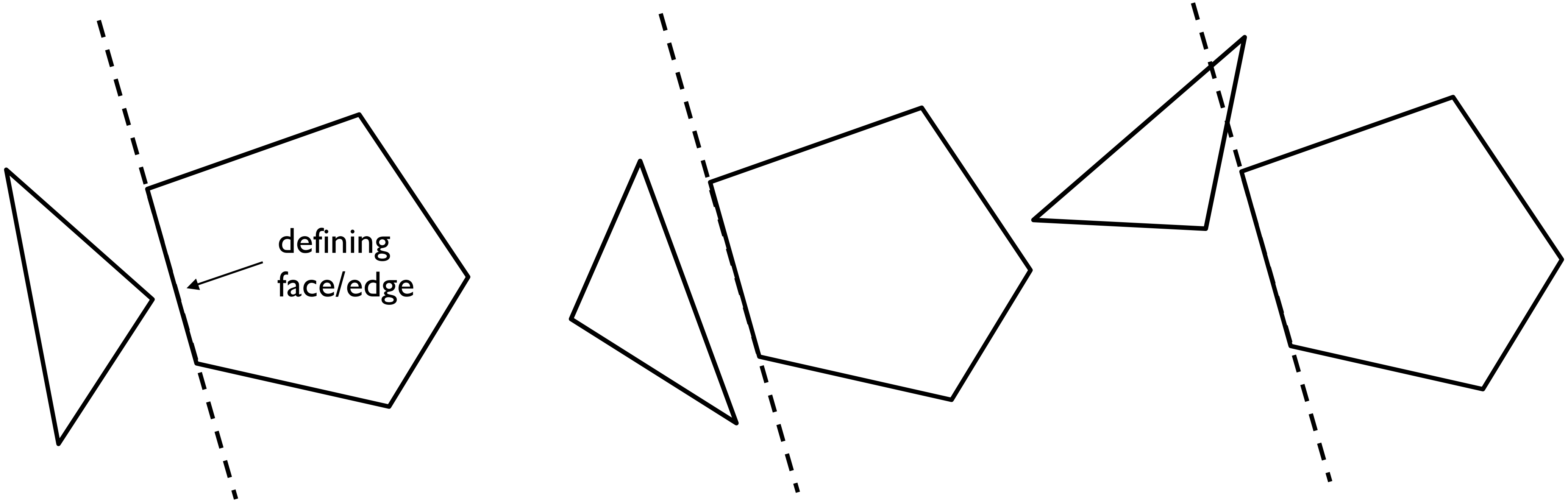
Detecting collision points: subsequent time steps

- For subsequent time steps, we will still use the defining face/edge to define a separating plane until it no longer does so.



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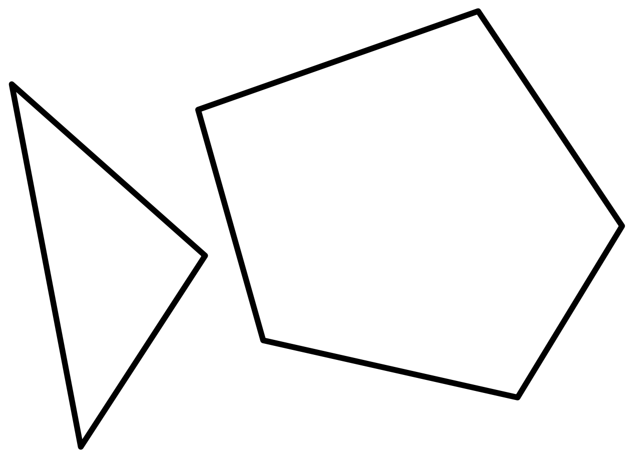
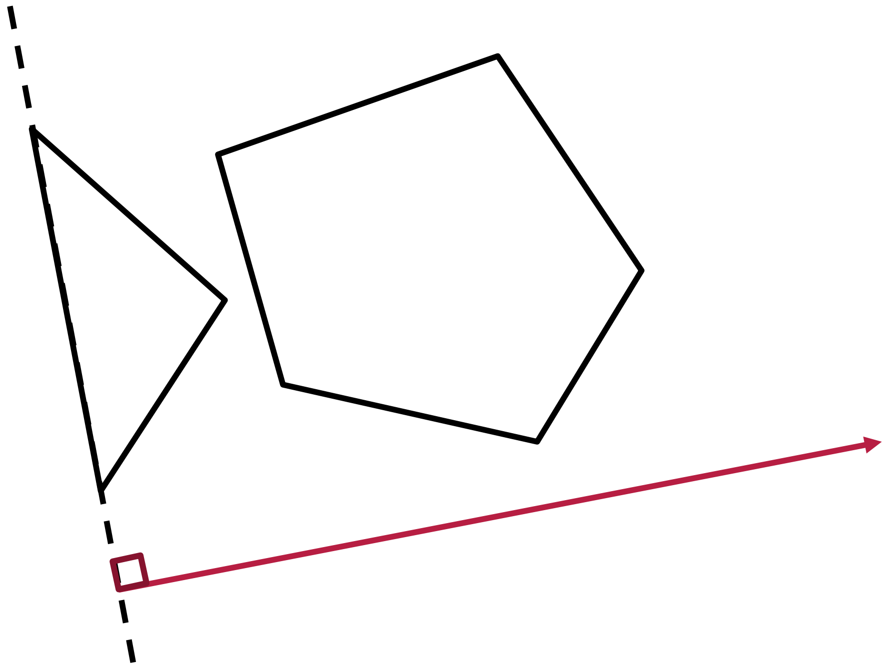
Separating axis theorem (SAT)

- Exhaustive search for projection gap in normal axes



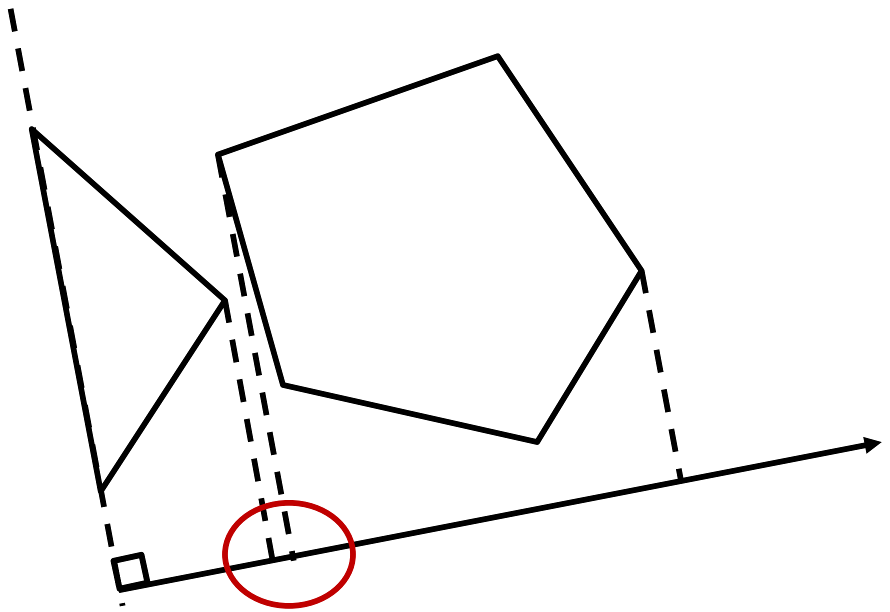
Separating axis theorem (SAT): axes

- In 2d cases, axes could be obtained using the unique normal of edges



Separating axis theorem (SAT): projection gap

- If there is a gap in projection region, there is no inter-penetration/collision
- Vertex projection can be computed using dot production



Separating axis theorem (SAT): 3d case

- If there is a gap in projection region, there is no inter-penetration/collision
- Vertex projection can be computed using dot production
- In 3d case, axes are normal of faces or the cross product of two edges (one from each object)

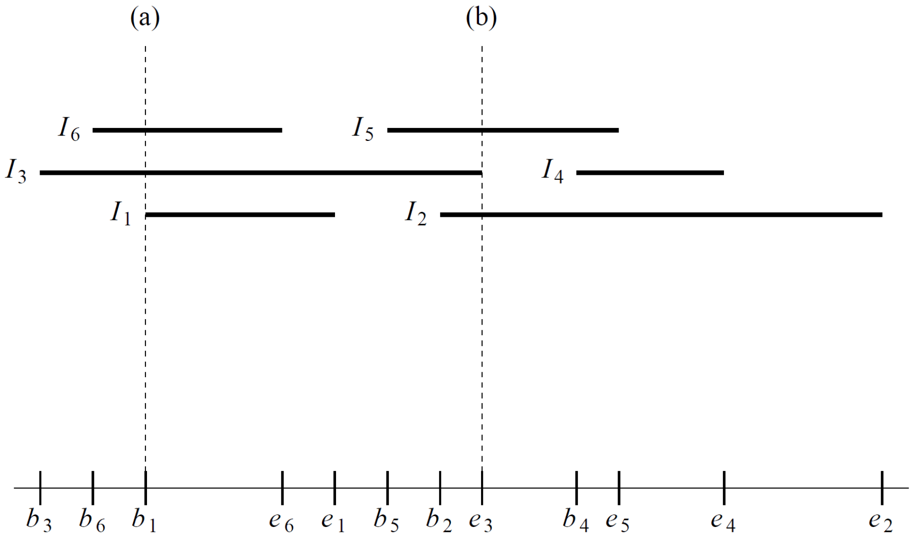
Collision detection is only done when bounding boxes overlap

- Naïve algorithm $O(n^2)$

Reference: David Baraff. Analytical Methods for Dynamic Simulation of Non-penetrating Rigid Bodies. SIGGRAPH 1989
https://www.cs.cmu.edu/~baraff/papers/sig89.pdf

Collision detection is only done when bounding boxes overlap

- Naïve algorithm $O(n^2)$
- Sweep/sort algorithm $O(n \log n + k)$



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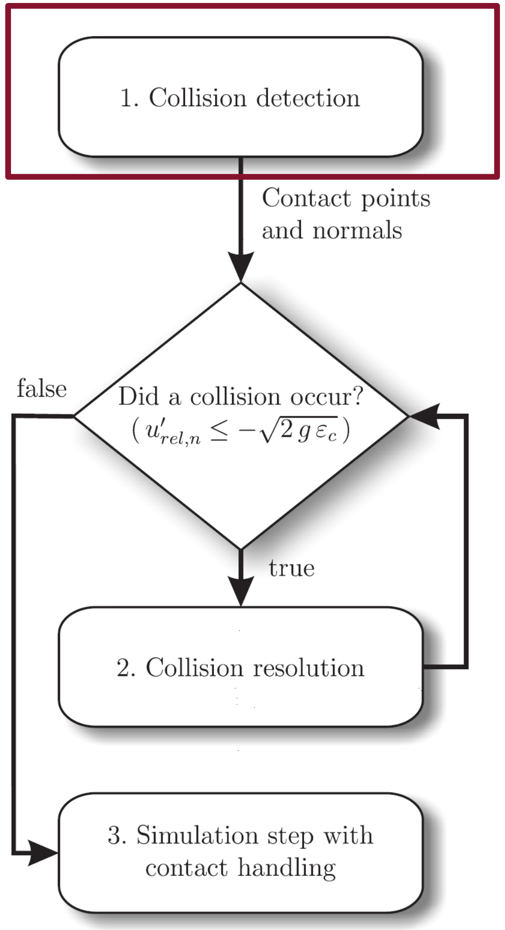
Questions



When and where collision happens?

Reference: J. Bender and A. Schmitt. Constraint-based collision and contact handling using impulses. In Proceddings of the 19th international conference on computer animation and social agents. July 2006

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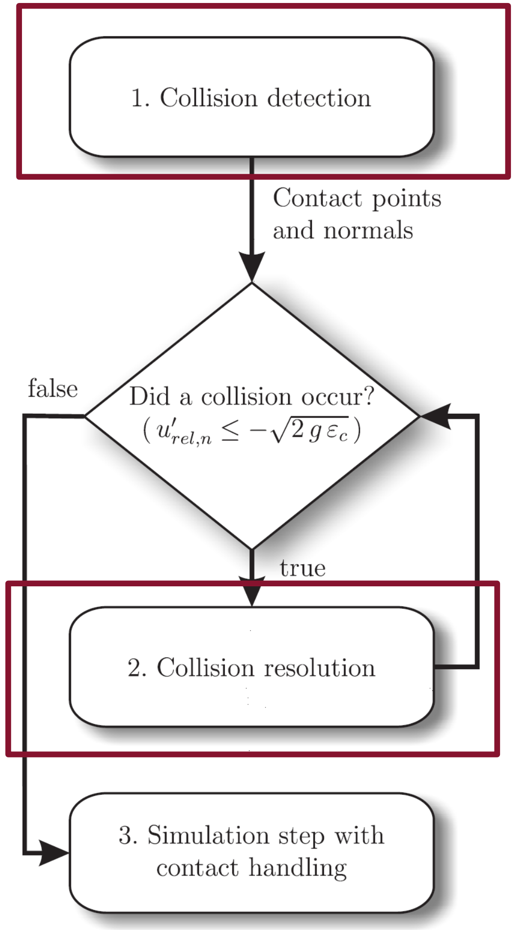

When and where collision happens?
How to resolve the collision?
(How to compute impulses or contact forces?)

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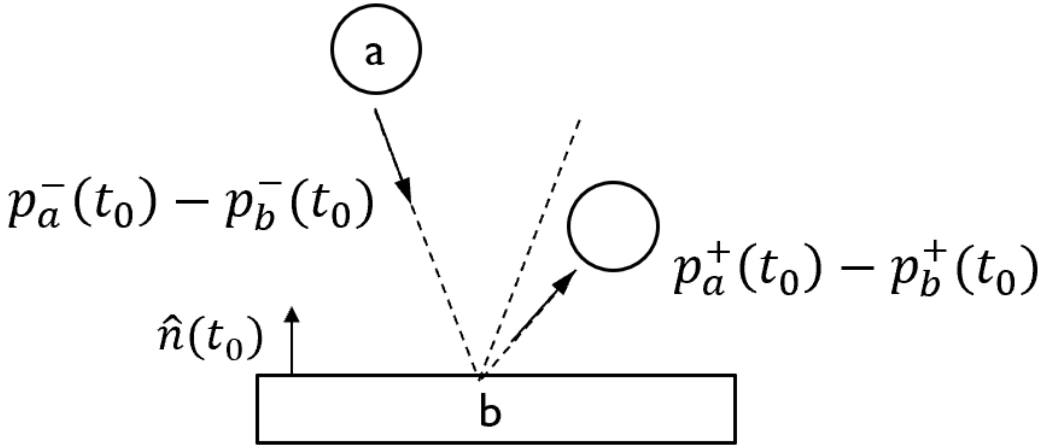
Outline

- Collision detection basics
- Computing impulses using a coefficient of restitution
- Penalty based method
- Constraint based method
- Recent work

Force based

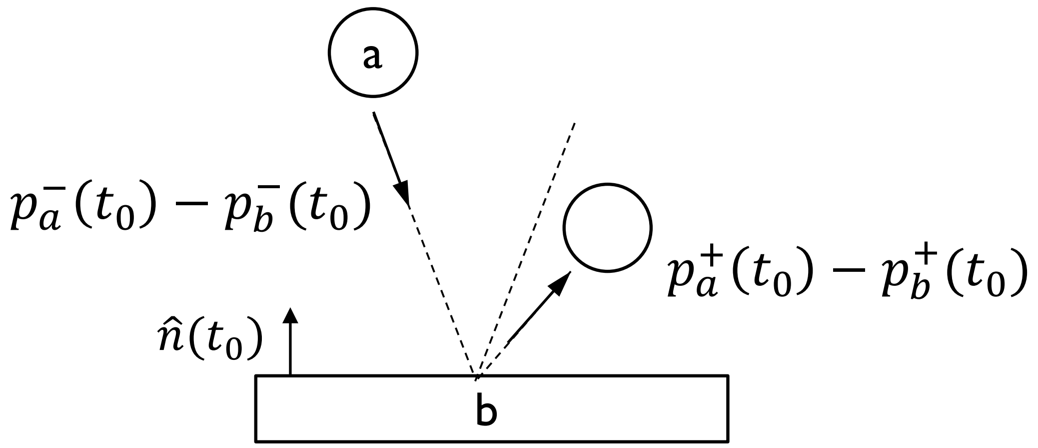
A simple example of collision



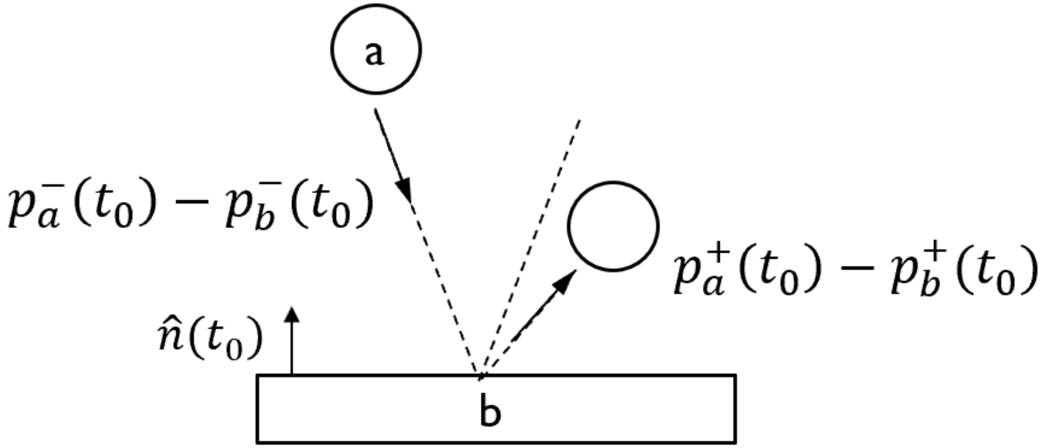
For rigid bodies, $p_a(t_0) = x_a(t_0) + R_a(t_0)r_a$
r_a is a vector from center of mass to collision point

Newton's law for restitution

- $v_{rel}^{+} = -\epsilon v_{rel}^{-}\ , \quad 0 \leq \epsilon \leq 1$
- $v_{rel}^{+} = \hat{n}(t_0)(\dot{p_a}^{+}(t_0) - \dot{p_b}^{+}(t_0))$



What is the impulse in this process?



Solve $J = j\hat{n}(t_0)$, given newton's law on restitution $v_a^+(t_0) = v_a^-(t_0) + j\hat{n}(t_0)/m_a$ and rigid body dynamics

Solving $J = j\hat{n}(t_0)$

$v_{rel}^{+} = \hat{n}(t_0)\left(\dot{p_a}^{+}(t_0) - \dot{p_b}^{+}(t_0)\right) = -\epsilon v_{rel}^{-}$ Newton's law for restitution

Solving $J = j\hat{n}(t_0)$

$$v_{rel}^{+} = \hat{n}(t_0)\left(\dot{p_a}^{+}(t_0) - \dot{p_b}^{+}(t_0)\right) = -\epsilon v_{rel}^{-}$$

Newton's law for restitution

$$\dot{p_a}^{+}(t_0) = v_a^{+}(t_0) + w_a^{+}(t_0) \times r_a$$
$$v_a^{+}(t_0) = v_a^{-}(t_0) + j\hat{n}(t_0)/m_a$$
$$w_a^{+}(t_0) = w_a^{-}(t_0) + I_a^{-1}(t_0)(r_a \times j\,\hat{n}(t_0))$$
$$\Rightarrow \dot{p}_a^{+}(t_0) = \dot{p}_a^{-}(t_0) + j(\frac{\hat{n}(t_0)}{m_a} + I_a^{-1}(t_0)\left(r_a \times \hat{n}(t_0)\right) \times r_a$$

Rigid body dynamics for a

Solving $J = j\hat{n}(t_0)$

$$v_{rel}^{+} = \hat{n}(t_0)\left(\dot{p_a}^{+}(t_0) - \dot{p_b}^{+}(t_0)\right) = -\epsilon v_{rel}^{-}$$

Newton's law for restitution

$$\dot{p_a}^{+}(t_0) = v_a^{+}(t_0) + w_a^{+}(t_0) \times r_a$$

$$v_a^{+}(t_0) = v_a^{-}(t_0) + j\hat{n}(t_0)/m_a$$

$$w_a^{+}(t_0) = w_a^{-}(t_0) + I_a^{-1}(t_0)(r_a \times j\,\hat{n}(t_0))$$

$$\Rightarrow \dot{p}_a^{+}(t_0) = \dot{p}_a^{-}(t_0) + j(\frac{\hat{n}(t_0)}{m_a} + I_a^{-1}(t_0)\big(r_a \times \hat{n}(t_0)\big) \times r_a$$

Rigid body dynamics for a

Similarly,

$$\dot{p}_b^{+}(t_0) = \dot{p}_b^{-}(t_0) - j(\frac{\hat{n}(t_0)}{m_b} + I_b^{-1}(t_0)\big(r_b \times \hat{n}(t_0)\big) \times r_b$$

Rigid body dynamics for b

Solving $J = j\hat{n}(t_0)$

$$j = \frac{-(1+\epsilon)v_{rel}^{-}}{\frac{1}{m_a} + \frac{1}{m_b} + \hat{n}(t_0) \cdot z_a + \hat{n}(t_0) \cdot z_b}$$

$$z_a = (I_a^{-1}(t_0)(r_a \times \hat{n}(t_0))) \times r_a$$

$$z_b = (I_b^{-1}(t_0)(r_b \times \hat{n}(t_0))) \times r_b$$

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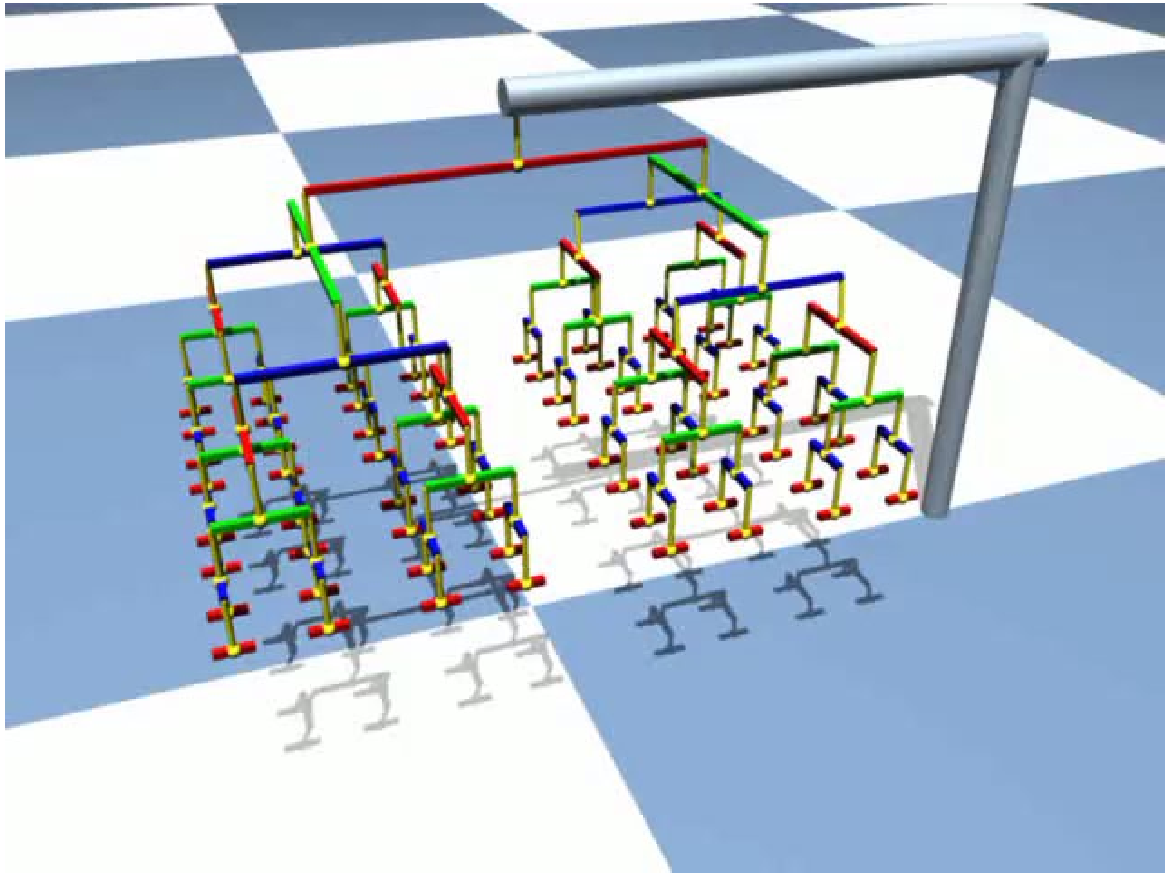
Impulse-based dynamic simulation in linear time



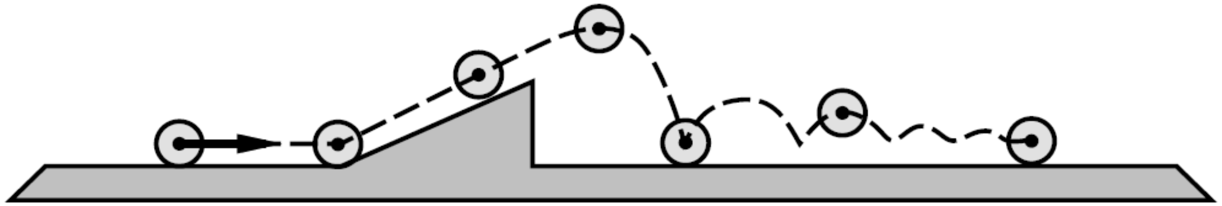
Reference: Jan Bender. Impulse-based dynamic simulation in linear time. Computer Animation and Virtual Worlds. 2007

Impulse-based methods: summary

- Newton's law of restitution: $v_{rel}^{+} = -\epsilon v_{rel}^{-}$, $0 \leq \epsilon \leq 1$
- $j = \dfrac{-(1+\epsilon)v_{rel}^{-}}{\frac{1}{m_a}+\frac{1}{m_b}+\hat{n}(t_0)\cdot(I_a^{-1}(t_0)(r_a\times\hat{n}(t_0)))\times r_a+\hat{n}(t_0)\cdot(I_b^{-1}(t_0)(r_b\times\hat{n}(t_0)))\times r_b}$

$$J = j\hat{n}(t_0)$$

- Impulses often applied in **local** contact resolution scheme



Reference:
James K. Hahn. Realistic animation of rigid bodies. Computer Graphics, Volume 22, Number 4, August 1988
Rick Parent. Compute Animation: Algorithm and Technique. Chapter 7.4
Brian Vincent Mirtich. Impulse-based Dynamic Simulation of Rigid Body Systems. PhD thesis, Fall 1996

Outline

- Collision detection basics
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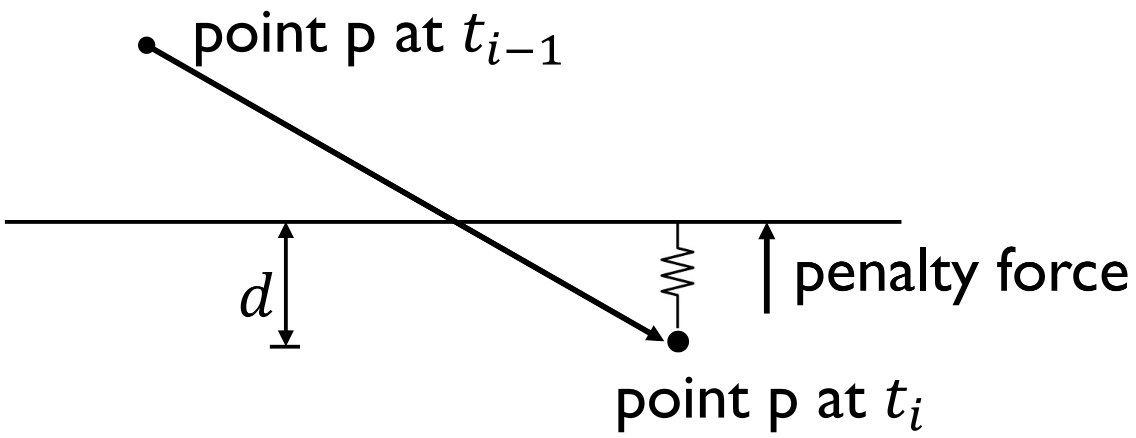
- Constraint based method
- Recent work

} Force based

Prevent penetration using a spring

- A spring with zero resting length is attached to prevent interpenetration.
- t_c is not needed



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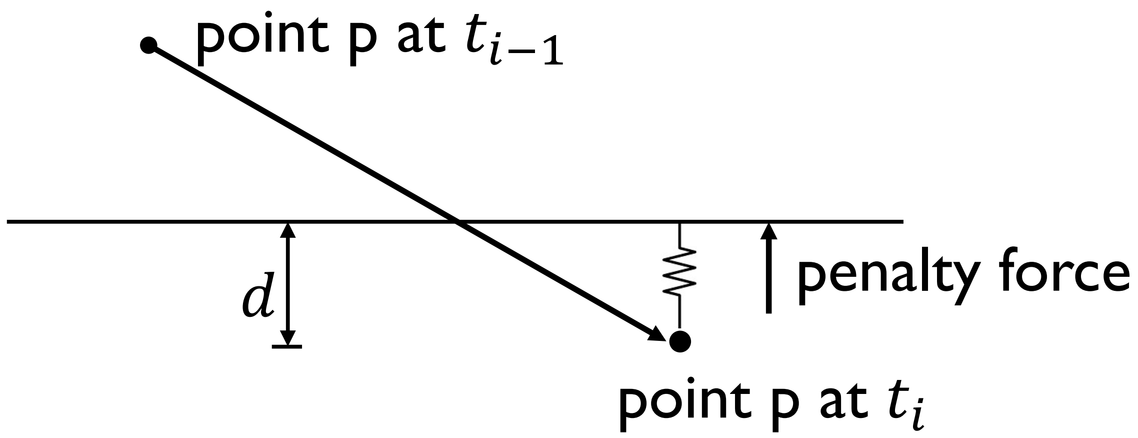
Control parameter m, k_p

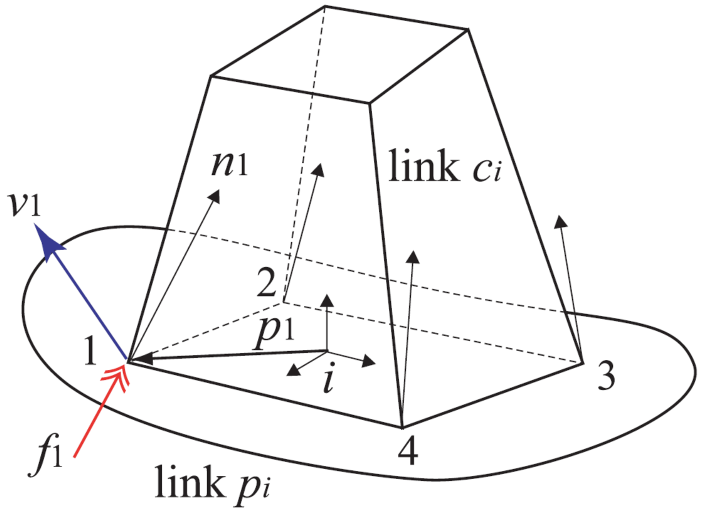
- $F = -k_p d,\ \ a = F/m.$ Here m, k_p are control parameter for the collision simulation



Reference: Rick Parent. Compute Animation: Algorithm and Technique. Chapter 7.4
Katsu Yamane and Yoshihiko Nakamura. Stable Penalty-based Model of Frictional Contacts. ICRA 2006

Control parameter m, k_p, k_d

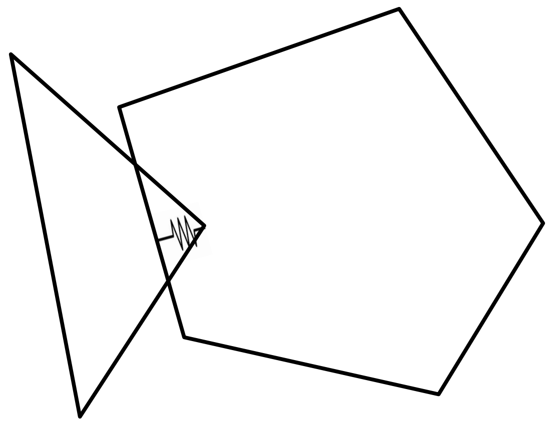
- $F = -\frac{d(k_p - k_d v_n)}{n_C}$. Here, v_n is the normal velocity and n_c is the number of contact points.



Reference: Katsu Yamane and Yoshihiko Nakamura. Stable Penalty-based Model of Frictional Contacts. ICRA 2006
https://ieeexplore.ieee.org/abstract/document/1641984

Penalty based methods: convex polyhedral

- The penalty force will give rise to torque when no acting in line with the center of mass of an object
- The spring is attached to both objects and imparts an equal but opposite force on the two to restore nonpenetration.



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Penalty-based methods: summary

Penalty-based methods compute the contact forces based on the penetration depth and normal velocity of a pair of objects using linear or nonlinear spring-damper model.

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Penalty-based methods compute the contact forces based on the penetration depth and normal velocity of a pair of objects using linear or nonlinear spring-damper model.

Pros:

- Easy to incorporate with other forces like gravity
- Scales well with the complexity of the scene

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- Unclear in determining where contact between objects should break

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There are advanced work to do it right

Cons:

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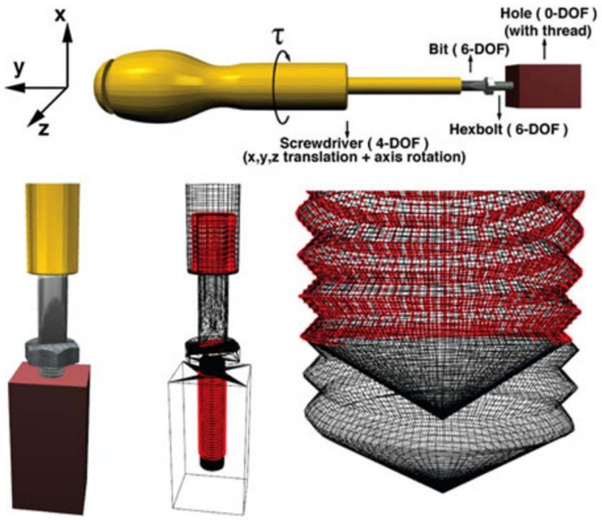
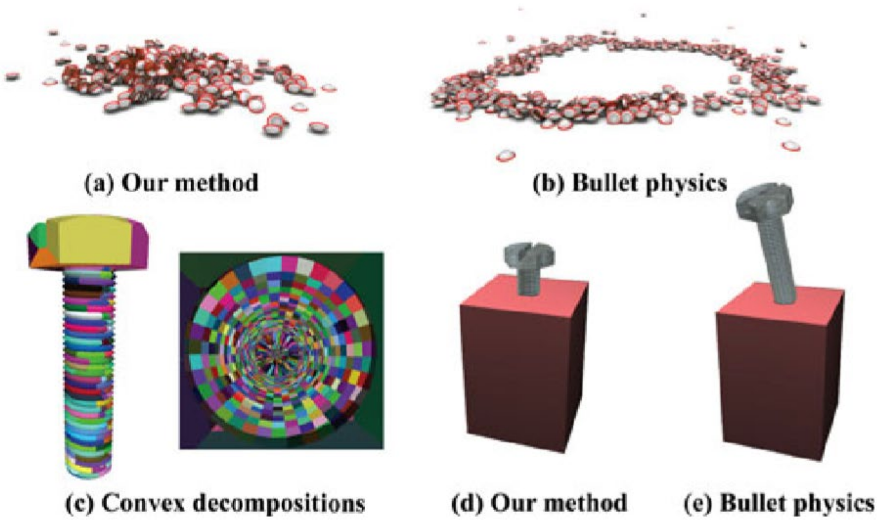
Katsu Yamane and Yoshihiko Nakamura. Stable Penalty-based Model of Frictional Contacts. ICRA 2006

IEEE TRANSACTIONS ON VISUALIZATION AND COMPUTER GRAPHICS, VOL. 20, NO. 9, SEPTEMBER 2014

Implicit Multibody Penalty-Based Distributed Contact

Hongyi Xu, *Member, IEEE*, Yili Zhao, *Member, IEEE*, and Jernej Barbič, *Member, IEEE*





IEEE TRANSACTIONS ON VISUALIZATION AND COMPUTER GRAPHICS, VOL. 20, NO. 9, SEPTEMBER 2014

Implicit Multibody Penalty-Based Distributed Contact

Hongyi Xu, *Member, IEEE*, Yili Zhao, *Member, IEEE*, and Jernej Barbič, *Member, IEEE*

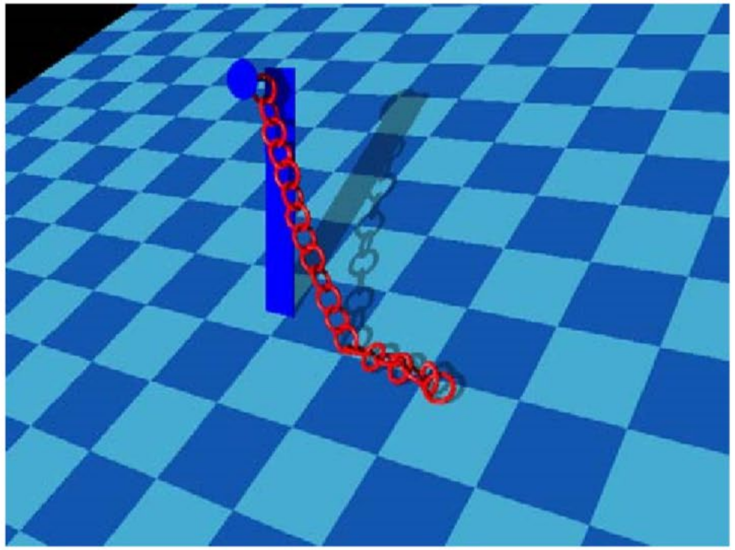
It addressed the stability problems due to the **highly variable and unpredictable net stiffness** by employing exact analytical contact gradients, symbolic Gaussian elimination, SVD solver, and semi-implicit integration.

Comparison to Bullet Physics

Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Stable Penalty-Based Model of Frictional Contacts

Katsu Yamane and Yoshihiko Nakamura
Department of Mechano-Informatics, University of Tokyo 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656 Japan
yamane@ynl.t.u-tokyo.ac.jp



Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Stable Penalty-Based Model of Frictional Contacts

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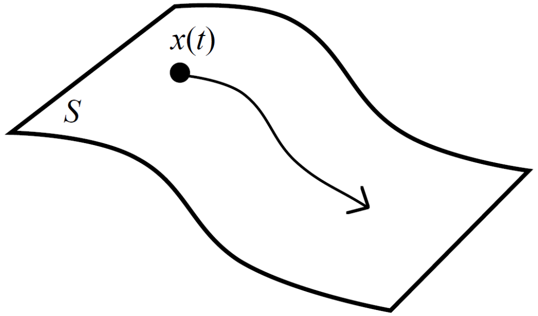
- It solves the problems in implementing Coulomb's friction model, i.e. how to handle static/dynamic friction forces.

Outline

- Collision detection basics
- Computing impulses using a coefficient of restitution
- Penalty based method
- Constraint based method
- Recent work

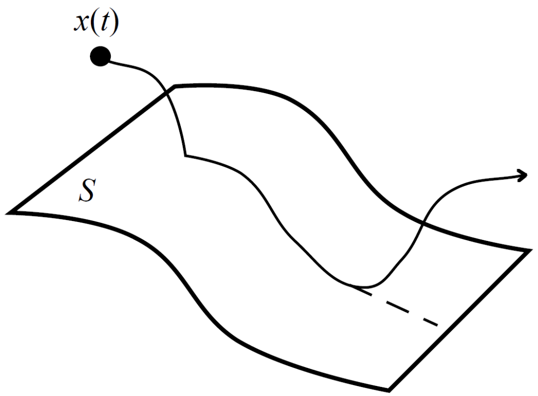
Constraint based method

- Computes constraint forces that are designed to exactly cancel any external accelerations that would result in interpenetration



$C(x(t)) = 0$

Equality constraints

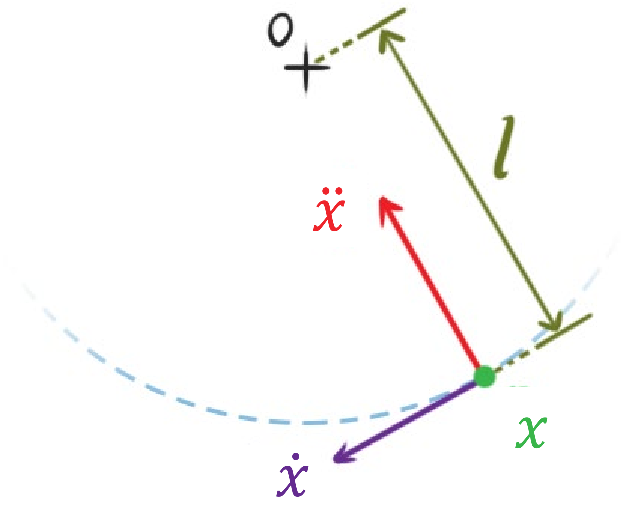


$C(x(t)) \geq 0$

Inequality constraints

Reference: David Baraff. Non-penetrating Rigid Body Simulation. Eurographics 1993 State of the Art Reports.
https://www.cs.cmu.edu/~baraff/papers/eg93.pdf

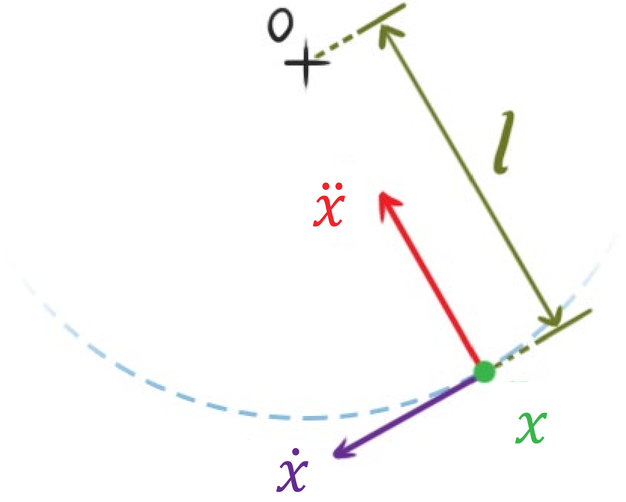
A simple example of equality constraint



- $C(x) = \frac{1}{2}(x \cdot x - l^2)$
- $C(x) = 0$
- In addition, $C(\dot{x}) = 0, C(\ddot{x}) = 0$

Reference: https://www.toptal.com/game/video-game-physics-part-iii-constrained-rigid-body-simulation

Solving equality constraints



- From equality constraints, we have

$$C(x) = \frac{1}{2}(x \cdot x - l^2) = 0 \quad [1]$$

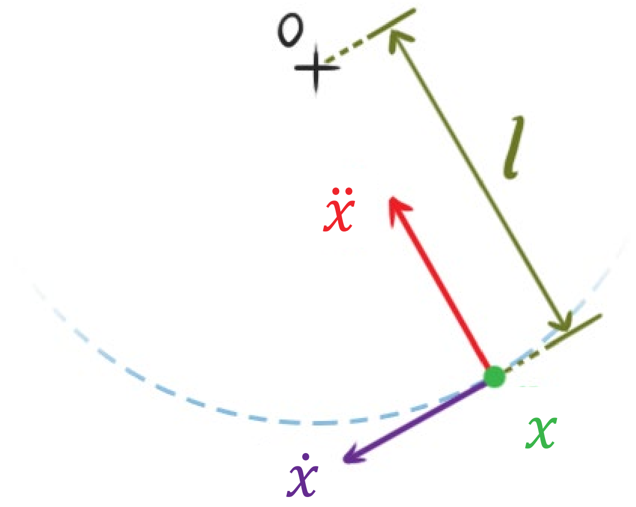
$$\dot{C(x)} = x \cdot \dot{x} = 0 \quad [2]$$

$$\ddot{C(x)} = \ddot{x} \cdot x + \dot{x} \cdot \dot{x} = 0 \quad [3]$$

- From newton's 2nd law: $\ddot{p} = (f_{ext} + f_C)/m \quad [4]$

Reference: https://www.toptal.com/game/video-game-physics-part-iii-constrained-rigid-body-simulation

Solving equality constraints



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- From [3][4], we can derive $f_C \cdot x = -f_{ext} \cdot x - m\,\dot{x} \cdot \dot{x}$ [5]

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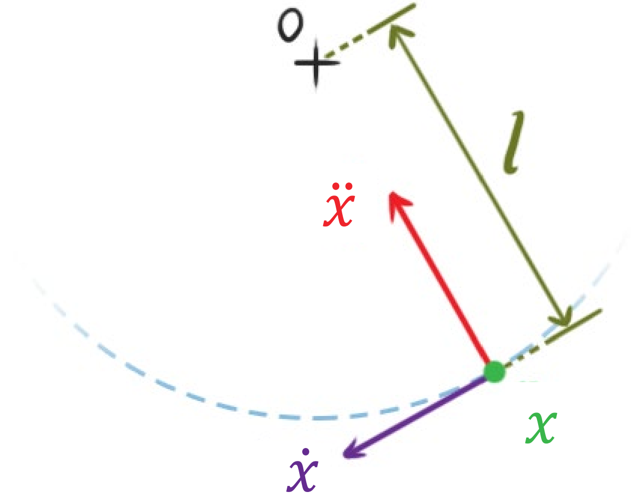

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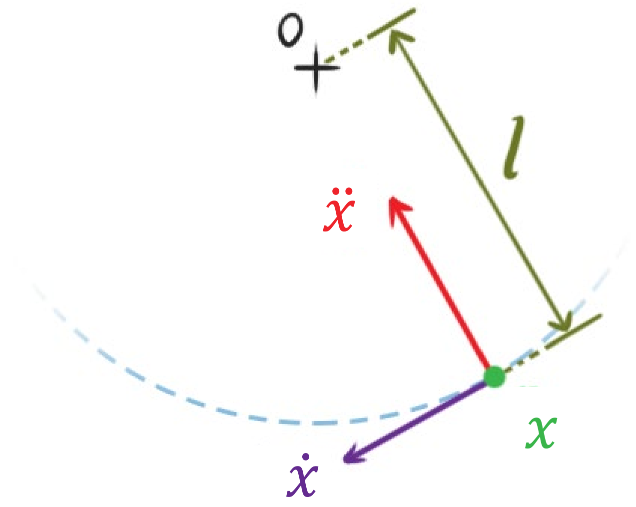

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- From [5][7], we can obtain $\lambda = \frac{-f_{ext} \cdot x - m\dot{x} \cdot \dot{x}}{x \cdot x}$

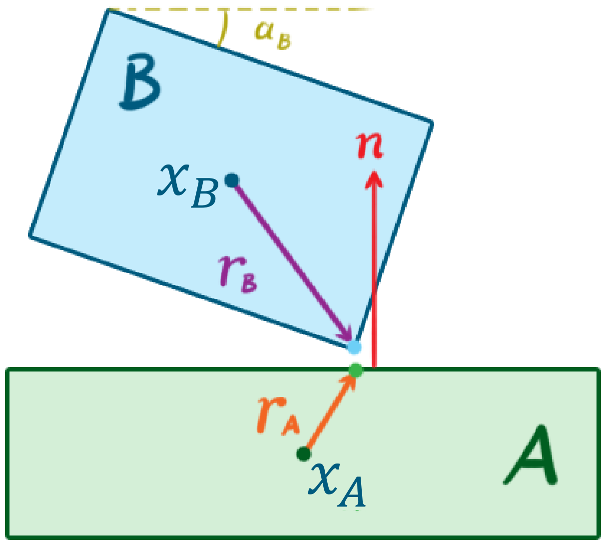
Reference: https://www.toptal.com/game/video-game-physics-part-iii-constrained-rigid-body-simulation

Solving equality constraints

- λ is also known as Lagrange multiplier
- For any constraints, the calculation involves determining the direction of force and its magnitude λ
- For rigid bodies, $JM^{-1}J^T\lambda = -\dot{J}\dot{q} - JM^{-1}F_{ext}$
  Here, q is a state vector for position and rotation, M is a matrix of mass and inertia, J is the Jacobian matrix of constraints

Reference: https://www.toptal.com/game/video-game-physics-part-iii-constrained-rigid-body-simulation

A simple example of inequality constraint



- A and B are colliding
- $R(\alpha_B) = \begin{bmatrix} \cos\alpha_B & -\sin\alpha_B \\ \sin\alpha_B & \cos\alpha_B \end{bmatrix}$

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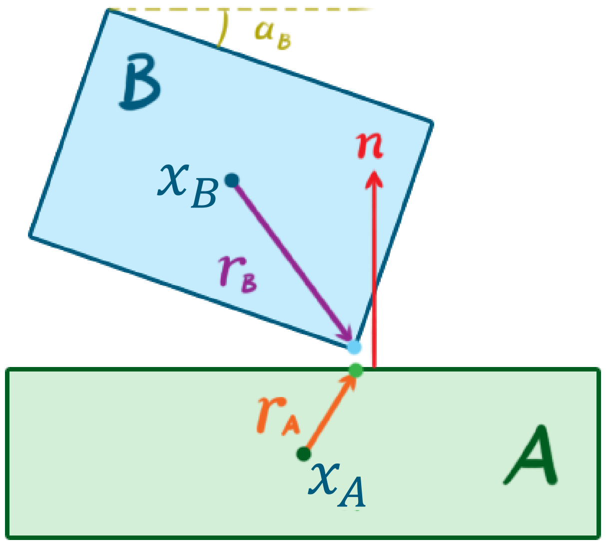
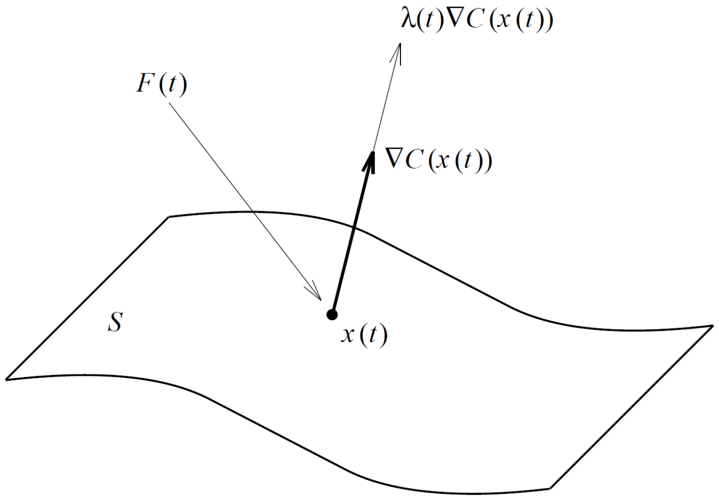


- A and B are colliding
- $R(\alpha_B) = \begin{bmatrix} \cos\alpha_B & -\sin\alpha_B \\ \sin\alpha_B & \cos\alpha_B \end{bmatrix}$
- $C(x_B, \alpha_B, x_A, \alpha_A) = (x_B + R(\alpha_B) r_B) - (x_A + R(\alpha_A) r_A)$
- $C(x_B, \alpha_B, x_A, \alpha_A) \geq 0$ (i.e. penetration depth is positive)

Reference: https://www.toptal.com/game/video-game-physics-part-iii-constrained-rigid-body-simulation

Constraint forces to avoid penetration

- Normal of surface $\nabla C(x(t))$
- $F_c(t) = \lambda(t)\ \nabla C\big(x(t)\big)$, where $\lambda(t)$ is a scalar. $\lambda(t) \geq 0$ so that the force will only push not pull the two objects



Reference: David Baraff. Non-penetrating Rigid Body Simulation. Eurographics 1993 State of the Art Reports.
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Formulate as Linear Complementarity Problem (LCP)

- The constraint problem could be formulated as an LCP problem
- Given a real matrix M and vector q, the linear complementarity problem LCP(M, q) seeks vectors z and w which satisfy the following constraints:

$$w, z \geq 0$$

$$z^T w = 0$$

$$w = Mz + q$$

Refernce: http://www.cs.unc.edu/~lin/COMP768-F07/LEC/rbd2.pdf

Resting contact

- Body separating (no response required)

$$v_{rel} > \epsilon$$

- Colliding contact

$$v_{rel} < -\epsilon$$

- Resting contact

$$-\epsilon < v_{rel} < \epsilon$$

Reference: http://www.cs.unc.edu/~lin/COMP768-F07/LEC/rbd2.pdf

Resting constraint forces properties

- Prevent interpenetration $\ddot{d}_i(t_0) \geq 0$
- Repulsive: $f_N(t) \geq 0$
- Is zero when bodies start to separate: $f_i \ddot{d}_i(t_0) = 0$

Refernce: http://www.cs.unc.edu/~lin/COMP768-F07/LEC/rbd2.pdf

Linear Complementarity Problem (LCP)

- $\ddot{d}_i(t_0) = a_{i1}f_1 + a_{i2}f_2 + \cdots + a_{in}f_n + b_i$

$$\begin{pmatrix} \ddot{d}_1(t_0) \\ \vdots \\ \ddot{d}_n(t_0) \end{pmatrix} = A \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} + \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$$\begin{cases} \ddot{d}_i(t_0) \geq 0 \\ f_i \geq 0 \\ f_i \ddot{d}_i(t_0) = 0 \end{cases}$$

Prevent interpenetration

Repulsive

Is zero when bodies are starting to take apart

Refernce: http://www.cs.unc.edu/~lin/COMP768-F07/LEC/rbd2.pdf

Solving Linear Complementarity Problem (LCP)

- Pivoting algorithms (like Gaussian elimination)
  - Need read and write access to matrices
  - Do not provide useful intermediate results
  - May exploit sparsity well
- Iterative algorithms (like Conjugate gradients)
  - Only need read access to matrices
  - Can stop early for approximation
  - Faster for large matrices
  - Can be warm started (i.e. from previous results)

Constraint-based methods: summary

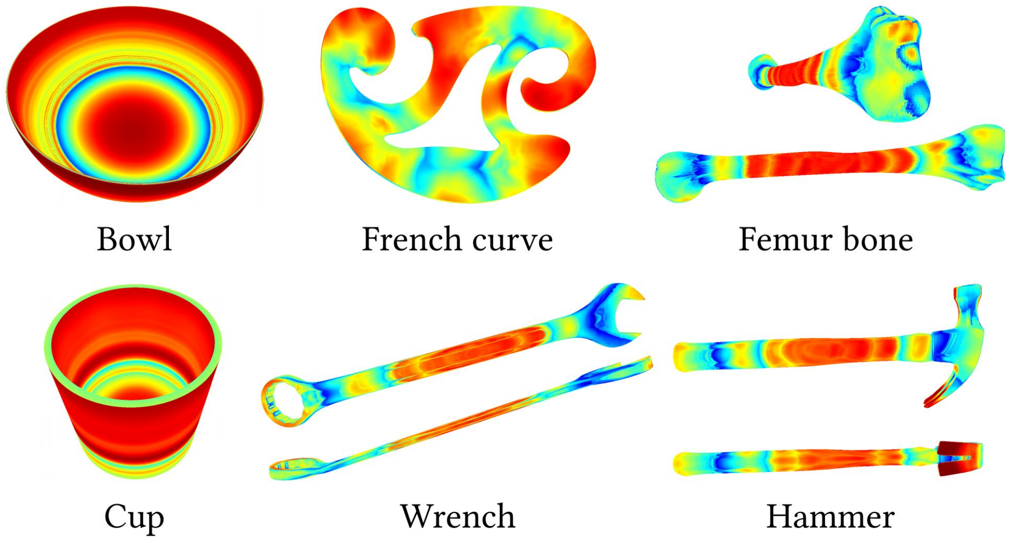
- Computes constraint forces that are designed to exactly cancel any external accelerations that would result in interpenetration
- Useful when the scene is composed of relatively small number of objects with simple shape
- Does not scale well with the complexity of the scene
(the penalty based method scales well with the complexity of the scene)

Outline

- Collision detection basics
- Computing impulses using a coefficient of restitution
- Penalty based method
- Constraint based method
- Recent work

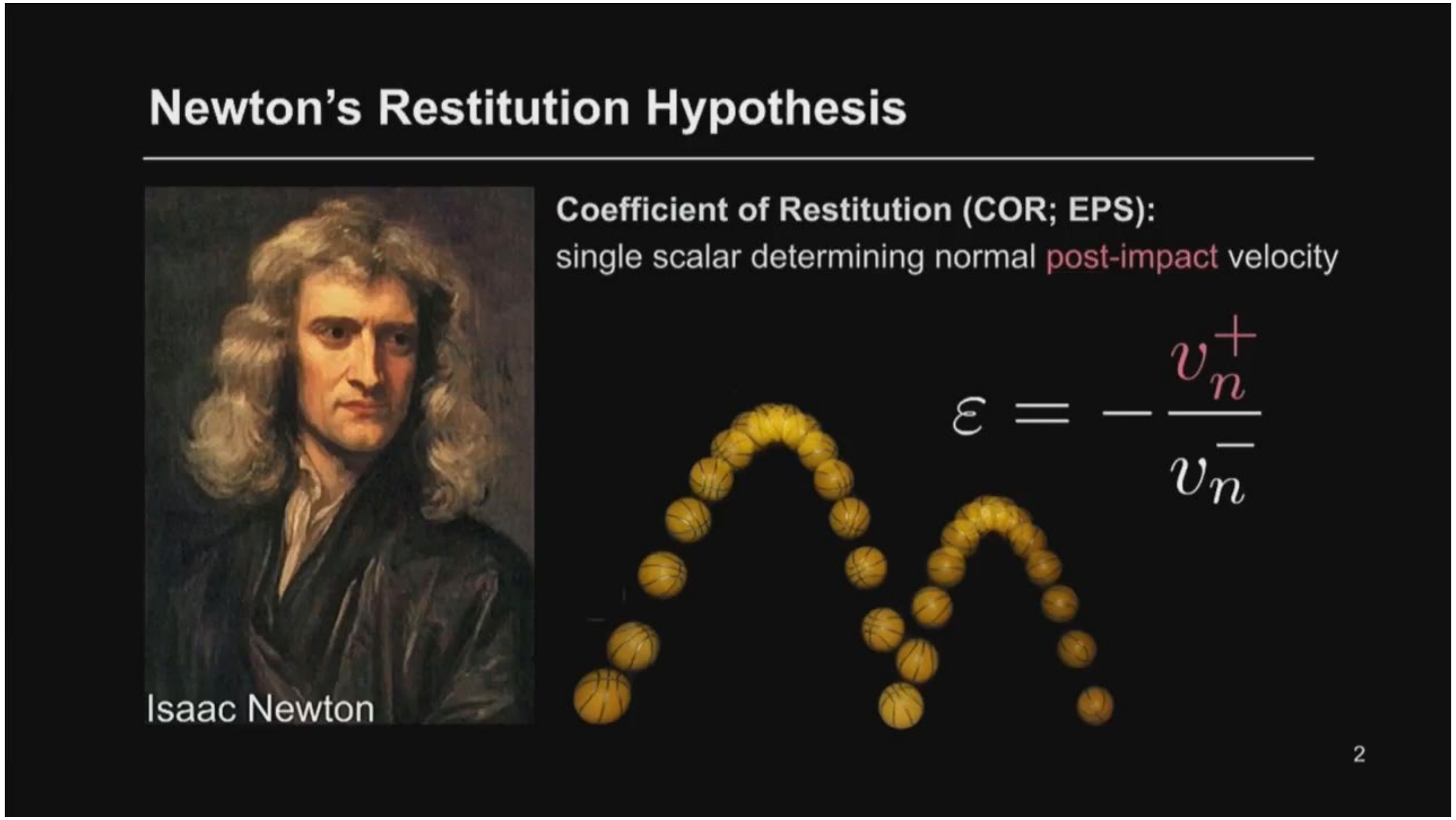
Bounce Maps

- Wang et al's work proposed a novel method to enrich standard rigid-body impact models with a spatially varying coefficient of restitution map, or Bounce Map



Jui-Hsien Wang , Rajsekhar Setaluri, Dinesh K. Pai, Doug L. James. Bounce Maps: An Improved Restitution Model for Real-Time Rigid-Body Impact. SIGGRAPH 2017

Bounce Maps



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Bounce Maps

Bounce Maps

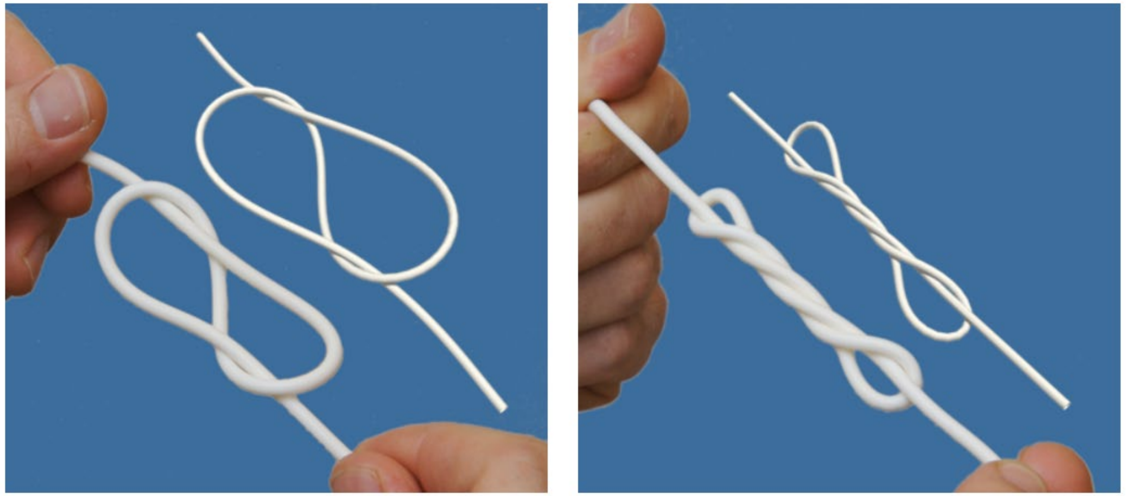
An Improved Restitution Model for Real-Time Rigid-Body Impact

JUI-HSIEN WANG
RAJSEKHAR SETALURI
DOUG L. JAMES
STANFORD UNIVERSITY

DINESH K. PAI
UNIVERSITY OF BRITISH COLUMBIA

Discrete Elastic Rods

- Representation configuration
  - Centerline (dynamic)
  - Adapted material frames (quasistatic)
- Bending forces, twist forces
- Fast projection method



M. Bergou, M. Wardetzky, S. Robinson, B. Audoly and Eitan Grinspun. SIGGRAPH 2008

Discrete Elastic Rods

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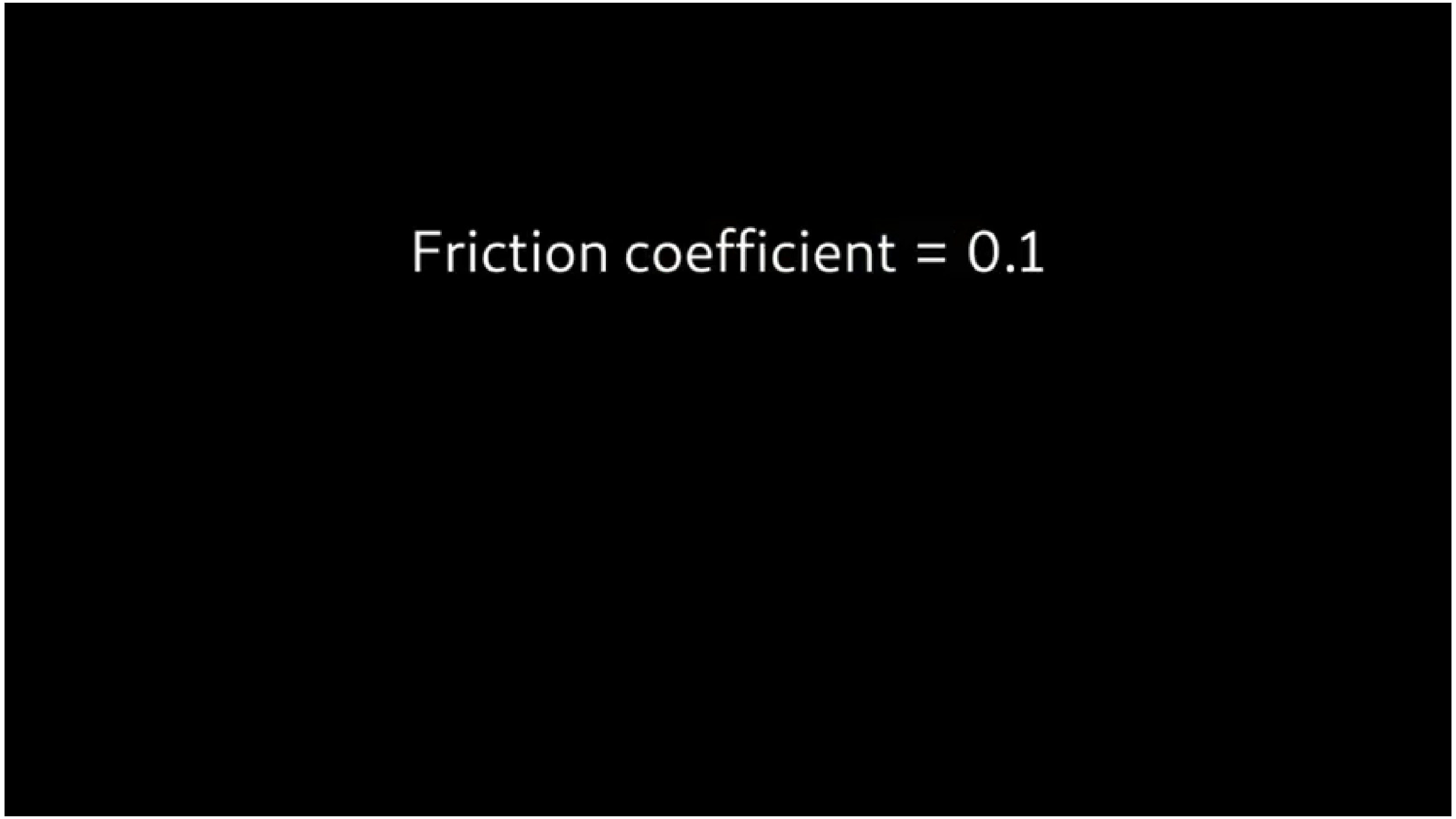
Efficient and accurate collision response

- Verschoor and et al present an efficient and accurate collision response by solving Mixed LCP directly (not constructing LCP matrices).
- Based on Conjugate Residual (CR) solver
- A good reference for constraint-based method

Mickeal Verschoor and Andrei C. Jalba. Efficient and Accurate Collision Response for Elastically Deformable Models. ACM Transactions on Graphics, Volume 38, Issue 2. March 2019

Efficient and accurate collision response



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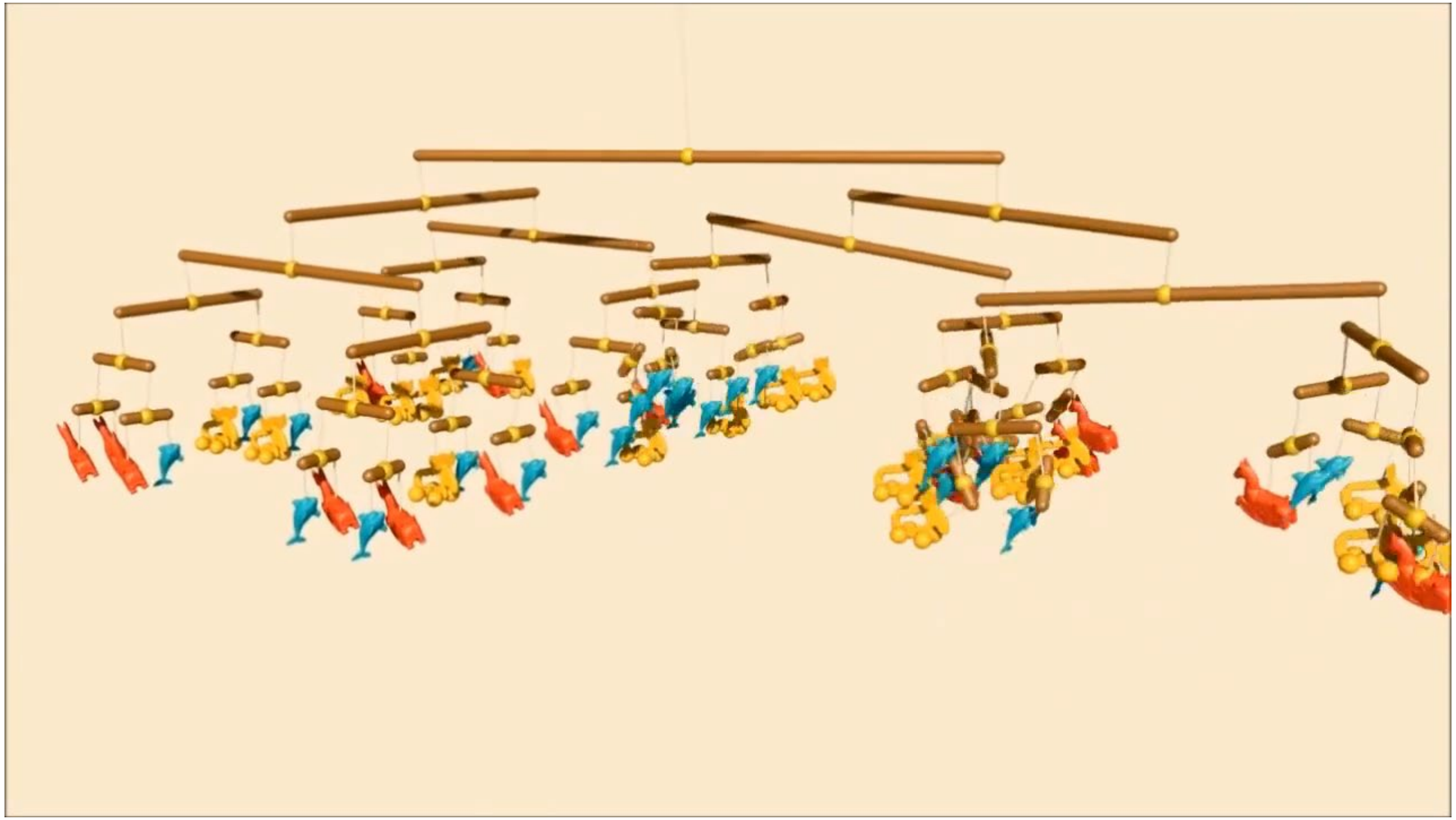
Position-Based Rigid Body Dynamics



A large-scale mobile simulation involving **127** constraints. A simulation step requires **0.83 ms** on average.

Reference: Crispin Deul, Patrick Charrier, Jan Bender. Position-Based Rigid Body Dynamics. Computer Animation and Virtual Worlds 2014

Summary

- Collision detection
  - backtracking t_c,
  - determining interpenetration using separation plane
  - determining interpeneration using separating axis theorem (SAT)
- Impulse based method. Computing impulse based on restitution coefficient ϵ
- Penalty based method
- Constraint based method

} Force based

References

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