The theme for today was simulation of deformable bodies. We looked at Shape Matching, As-Rigid-As-Possible, and Finite Element techniques.

**Shape Matching**

We started today by looking at the use of Shape Matching for fast, stable, deformable body simulation.

[https://www.youtube.com/watch?v=LAoQ1dhk1w](https://www.youtube.com/watch?v=LAoQ1dhk1w)

**As-Rigid-As-Possible**

We followed up with an exploration of the As-Rigid-As-Possible technique for animation. The three papers below cover the 2D, 3D, and 3D with simulation versions of this approach:

[https://www-ui.is.s.u-tokyo.ac.jp/~takeo/research/rigid/index.html](https://www-ui.is.s.u-tokyo.ac.jp/~takeo/research/rigid/index.html)

[https://igl.ethz.ch/projects/ARAP/](https://igl.ethz.ch/projects/ARAP/)

[https://www.youtube.com/watch?v=Ja1MSoMRKgs](https://www.youtube.com/watch?v=Ja1MSoMRKgs)
**Finite Element**

We then took a look at the finite element method, with reference to this paper. Whiteboard notes are attached below. You can find the video from the link that follows the paper reference.


The following paper discusses how to make the approach robust to large deformations, including element inversion:


The basic finite element technique discussed in these papers was extended to simulate goop. We couldn't play this video in class, but you can find it at the link below.


A question came up as to whether this approach is embarrassingly parallel. The answer would appear to be yes, but it is not as simple as it first appears. This paper contains an in-depth discussion of the issues we encountered.


We had time to have a quick look at one recent research paper on identifying simulation parameters for such simulations from observed motion. The paper is here:

FINITE ELEMENT SIMULATION

(1) Compute **Strain** (deformation)

- Green's strain tensor $\varepsilon$
- 3x3 matrix
- Strain rate tensor $\dot{\varepsilon}$

(2) Compute **Stress** (internal force)

- 3x3 matrix (tensor) $\sigma$
- $\sigma^{(0)}$ - elastic stress due to strain
- $\sigma^{(v)}$ - viscous stress due to strain rate

Simplifying assumption — isotropic material

4 parameters that determine behavior:

- $M$ - rigidity
- $N$ - resist volume
- $\phi, \psi$ - how quickly dissipates kinetic energy
(3) Compute node forces from stress

(4) Update node state from forces