15-464/15-664 Reference List for March 18, 2020

Eulerian finite difference based approach to solving Navier-Stokes equations

We began by talking about the Navier-Stokes equations, and a very straightforward approach to solving them:

Foster, Nick, and Dimitri Metaxas. "Realistic animation of liquids." Graphical models and image processing 58, no. 5 (1996): 471-483.
http://www.cbim.rutgers.edu/dmdocuments/gmip96%20Foster.pdf

This algorithm was developed into the system used in the movie Antz:
https://www.youtube.com/watch?v=KfxQi9_BfHI

Making it stable

The next big development was to make this basic algorithm fast and stable, through straightforward innovations introduced by Jos Stam. The original paper is “Stable Fluids” from SIGGRAPH 1999. However, the paper below is quite clear in walking through the algorithm, providing detailed diagrams and pseudocode. The resulting system is still very usable today and produces nice looking smoke.

Stam, Jos. "Real-time fluid dynamics for games." In Proceedings of the game developer conference, vol. 18. 2003. http://www.autodeskresearch.com/pdf/GDC03.pdf
https://www.youtube.com/watch?v=t-erFRTMIWA
https://www.youtube.com/watch?v=fFI0FOiR3Nc

In class, I mentioned the blog page associated with the second video. Here is the direct link that walks through how these effects were created:
https://softologyblog.wordpress.com/2019/02/28/jos-stams-fluid-simulations-in-3d/

SPH Introductory paper

We didn't get to this paper, but I'll leave it in the references. The next major development was in an opposing particle based representation for fluids. This is considered the introductory paper and does a great job of walking through the details of how it works.

Müller, Matthias, David Charypar, and Markus Gross. "Particle-based fluid simulation for interactive applications." In *Proceedings of the 2003 ACM SIGGRAPH/Eurographics symposium on Computer animation*, pp. 154-159. Eurographics Association, 2003.
https://matthias-research.github.io/pages/publications/sca03.pdf
https://www.youtube.com/watch?v=aaWqybIa4r4

If you are interested in more introductory reading to understand the basics, this is a good course:

SIGGRAPH 2007 Course (Introductory)

Robert Bridson and Matthias Müller-Fischer, "Fluid Simulation for Computer Animation"
http://www.cs.ubc.ca/~rbridson/fluidsimulation/

Navier Stokes Equations

"forces"

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + g + \nu \nabla^2 u$$

- $\frac{\partial u}{\partial t}$: acceleration
- $u \cdot \nabla u$: advection
- $-\nabla p$: pressure gradient
- g: gravitational acceleration
- $\nu \nabla^2 u$: drag

∇ = gradient

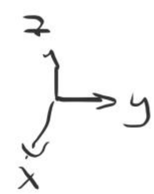
∇^2 = Laplacian

u = velocity p = pressure

g = gravity ν = viscosity

$f = ma$

∇p = gradient
$$\begin{bmatrix} \partial p / \partial x \\ \partial p / \partial y \\ \partial p / \partial z \end{bmatrix}$$



$\nabla^2 u$ = Laplacian
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Step 1 ⇒ Apply forces (Navier-Stoker)

Step 2 ⇒

make it divergence-free