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A survey of formal methods for determining the centre of rotation of ball joints

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Abstract

The determination of an accurate centre of rotation (CoR) from segment marker positions is of interest across a wide range of applications, but particularly for clinical gait analysis and for estimating the hip joint centre during surgical intervention of the knee, for limb alignment purposes. For the first time in this survey of formal methods, we classify, analyse and compare different methods (geometric, algebraic, bias compensated algebraic, and Pratt sphere fit methods, as well as the centre transformation technique, the Holzreiter approach, the helical pivot technique, the Schwartz transformation techniques, the minimal amplitude point method and the Stoddart approach) for the determination of spherical joint centres from marker position data. In addition, we propose a new method, the symmetrical CoR estimation or SCoRE, in which the coordinates of the joint centre must only remain constant relative to each segment, thus not requiring the assumption that one segment should remain at rest.

For each method, 1000 CoR estimations were analysed with the application of isotropic, independent and identically distributed Gaussian noise (standard deviation 0.1 cm) to each of the marker positions, to all markers on the segment simultaneously and the two in combination. For the test conditions used here, most techniques were capable of determining the CoR to within 0.3 cm, as long as the spherical range of motion (RoM) of the joint was 45° or more. Under the most stringent conditions tested, however, the SCoRE was capable of best determining the CoR, to within approximately 1.2 mm with a RoM of 20°. The correct selection and application of these methodologies should help improve the accuracy of surgical navigation and clinical kinematic measurement.

Keywords: Centre of rotation; Ball joints; Joint centre

1. Introduction

The determination of joint kinematics during clinical motion analysis often includes assumptions regarding the point about which two segments move relative to one another. The determination of this so-called centre of rotation (CoR) can often be difficult to measure in vivo (Cappozzo et al., 2005; Croce et al., 2005), but knowledge of its exact location is important in clinical gait analysis settings, where the calculation of hip joint moments may form the basis of therapy. In addition, the ability to establish the hip joint centre for determining lower limb alignment axes during surgical intervention (Kinzl et al., 2004) of the knee is becoming increasingly important with the increasing popularity of navigation systems, with accuracy a premium.

Although specific bone landmarks and joint positions can be measured using techniques such as digital roentgen stereophotogrammetric (Vrooman et al., 1998) and video fluoroscopy analysis (Dennis et al., 1998), reflective marker positions determined using infra-red optical systems allow non-invasive measurement of kinematics in real time. During gait analysis or surgery, such markers may be fastened to the skin or more directly attached to the bone segments. The CoR at the hip may then be calculated from the three-

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dimensional (3D) coordinates of the markers, measured during manipulation of the femur relative to the pelvis. During measurement of these marker positions, the relative motion between the markers and bone (from, e.g. local shifting and deformation of the skin) and the accuracy of the measurement system have been shown to be main causes of error, or artefact (Leardini et al., 2005; Stagni et al., 2005; Taylor et al., 2005). Any computational method for determining the CoR must therefore be able to provide accurate results from noisy marker positions.

The most widely used methods for estimating the positions of joint centres are based on simplified models for the specific joint in question. Most of these approaches then apply empirical relations between externally palpable bone landmarks and the joint centres themselves (Bell et al., 1990; Kadaba et al., 1990; Davis et al., 1991; Frigo and Rabuffetti, 1998; Vaughan et al., 1999). The results of such approaches, termed predictive or regression methods, for human hip joints have been shown to be accurate to approximately 2 cm when compared with X-ray measurements (Bell et al., 1990; Neptune and Hull, 1995).

Recently, so-called formal methods have been proposed that do not refer to empirical correlations. The underlying mathematical approaches can be divided into two categories. The first includes variants of sphere fitting methods, where the centre and the radii of spheres are optimised to fit the trajectories of marker positions (Cappozzo, 1984; Bell et al., 1990; Shea et al., 1997; Silaghi et al., 1998; Leardini et al., 1999; Piazza et al., 2001; Gamage and Lasenby, 2002; Halvorsen, 2003). The second type of approach considers the distance between markers on each joint segment fixed (apart from skin artefacts and measurement errors), to enable the definition of a local coordinate system (Stoddart et al., 1999; Marin et al., 2003; Piazza et al., 2004; Schwartz and Rozumalski, 2005; Siston and Delp, in press). Appropriate transformation of these local systems for all time frames into a common reference system enables approximation of the joint centre at a fixed position. These techniques are considered "coordinate transformation methods".

A number of studies have compared these different predictive and formal methods (Bell et al., 1990; Seidel et al., 1995; Leardini et al., 1999; Camomilla et al., in press), but reached unclear conclusions, since testing has been performed under different conditions. Many approaches may only be used under the assumption that one body segment is at rest relative to the other. Whilst the appropriate transformation of one set of markers into the local coordinate system of the second is nearly always possible, the measurements of the segment at rest can lead to systematic errors, and the use of techniques that consider this disadvantage is therefore necessary. The goal of this study was to perform a systematic survey of formal techniques for estimating the CoR. In addition, we propose a new method of joint centre determination that is capable of the dynamic assessment of two body segments moving simultaneously.

2. Methods

2.1. The virtual hip joint

For appropriate, direct comparisons between different approaches to determine the CoR, including their statistical analyses, numerical simulations in which the exact joint position is known are most suitable. This allows the fast simulation of various geometric situations, marker numbers and placement, and error conditions. We have consequently used a virtual joint with positions that could easily represent markers used during gait analysis or lower limb surgery: using marker sets approximately 10 and 15 cm distant from the CoR. The configuration used in this study contains four markers attached to each of the simulated thigh and pelvis segments (Fig. 1).

In order to study the influence of marker artefacts and measurement errors, specific noise has been applied to the generated marker positions. Based on previous kinematic analyses (Taylor et al., 2005), which suggested that marker artefacts are composed of measurement error together with both elastic distortion, where markers move independently of the marker set as a whole, and synchronous shifting of the entire marker set, two distinct artificial error types have been included in this study. Firstly, isotropic, independent and identically distributed Gaussian noise (standard deviation 0.1 cm) was applied to each of the marker positions. Secondly, a similar Gaussian noise was applied, but with the deviations applied to all markers on the segment simultaneously (also standard deviation 0.1 cm). These two error conditions and their combination were applied to the generated data and assessed to determine the ability of each of the methods described below to calculate the CoR.

Two movement scenarios were additionally superimposed on the marker positions. In the first, one of the two segments could rotate randomly around the CoR, but within a specified angular cone of movement, or range of motion (RoM), and was affected by the aforementioned noise conditions. In this case, the other segment was held stationary (Fig. 1). This movement was assumed to be similar to a situation where the subject performs specific circular motions of the leg, i.e. flexion-extension together with abduction-abduction. In the second scenario, one of the two segments could again rotate randomly around the CoR, but now noise conditions were applied to both marker sets. Additionally, the complete joint configuration (CoR and marker

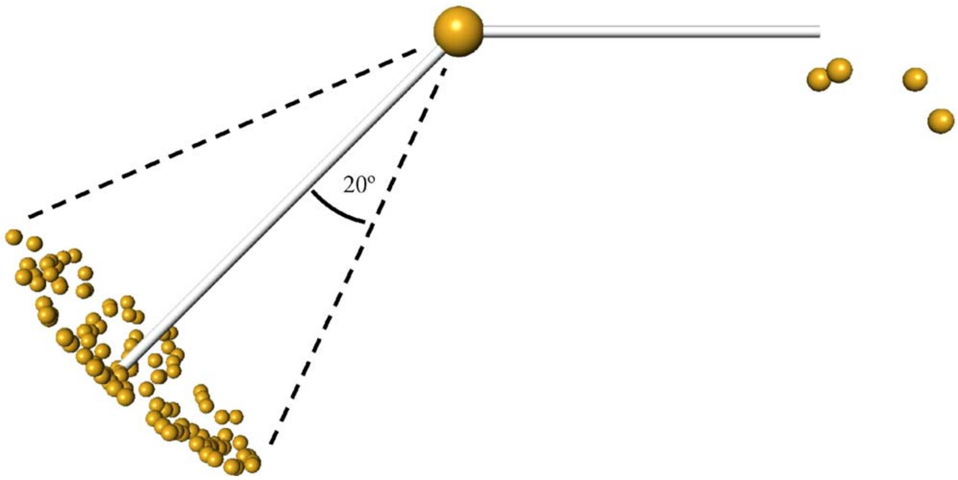

Fig. 1. Visualisation of an exemplary test configuration of the virtual hip, shown for a 20° range of motion. Four markers were considered for each segment (although only a single marker is displayed for each time point for the thigh marker set, left) and are shown attached to a common centre of rotation. Ranges of motion of 5°, 10°, 20°, 45° and 90° were examined in the study.

sets) was able to randomly translate in space, enabling the simulation of a moving CoR.

2.1.1. Sphere fit methods

Within this class of joint determination methods, it is assumed that the CoR is stationary, an assumption that is only reasonable if one segment is also at rest, which is highly unlikely during gait analysis. Sphere fit methods together with some of the coordinate transformation techniques discussed later are therefore usually only applicable after a transformation of the marker coordinates of one segment into a fixed coordinate system of the second segment. Under this condition the markers on, e.g. the femoral segment move on the surface of a sphere with specific radii around a common, pelvic, centre. Thus, this approach attempts to fit best both the radii and the position of the joint centre to the marker data. Presented as the basis of a number of biomechanical methods (Cappozzo, 1984; Bell et al., 1990; Shea et al., 1997; Silaghi et al., 1998; Leardini et al., 1999; Piazza et al., 2001; Gamage and Lasenby, 2002; Halvorsen, 2003), sphere fitting methodologies also arise in other scientific and technical disciplines (for an overview see Gander et al. (1994), Chernov and Lesort (2003) and Nievergelt (2003)).

The method supposes that the kinematics of two segments, $k = 1, 2$, are represented by markers $j = 1, \ldots, m$. The global marker positions p_{ij}^k are given in the time frames $i = 1, \ldots, n$. If one segment is assumed to be at rest, the index k is omitted. The most apparent approach is then to minimise the sum of the squared Euclidean (geometric) distances between the sphere and the marker positions,

$$f_{\text{geom}}(c, r_1, \ldots, r_m) = \sum_{j=1}^{m} \sum_{i=1}^{n} (\|p_{ij} - c\| - r_j)^2, \quad (1)$$

where c is the CoR and r_j the radius of the sphere on which marker j moves.

This technique is termed a geometric sphere fit method and has been evaluated by Piazza and co-workers (Piazza et al., 2001). The minimisation of (1) is a non-linear problem without closed solution, which is solved iteratively (Marquardt, 1963; Gander et al., 1994; Chernov and Lesort, 2003; Deuflhard and Hohmann, 2003). Note that the radii r_j in (1) can be computed directly as $r_j = (1/n)\sum_{i=1}^{n} \|p_{ij} - c\|$. Since at least an initial guess for c is required, other modified least-squares criterion methods have been proposed that do not require a starting estimate, originally by Delonge (1972) and Kåsa (1976):

$$f_{\text{alg}}(c, r_1, \ldots, r_m) = \sum_{j=1}^{m} \sum_{i=1}^{n} (\|p_{ij} - c\|^2 - r_j^2)^2. \quad (2)$$

Whilst the approach, called an algebraic sphere fit method or Kåsa–Delonge estimator, has the advantage that this minimisation task has a simple closed solution (Pratt, 1987; Chernov and Lesort, 2003), it is strongly biased, i.e. a systematic error exists such that even for a large number of trials, the mean of the computed CoRs do not converge to the true value (Zelniker and Clarkson, 2003; Chernov and Lesort, 2004). This technique has been employed in various forms (Cappozzo, 1984; Shea et al., 1997; Gamage and Lasenby, 2002). Furthermore, the Reuleaux method proposed by Halvorsen et al. (1999) was later identified by Cereatti and co-workers (Cereatti et al., 2004) as an incomplete algebraic sphere fit. To overcome the bias problem, Halvorsen proposed a modified approach, named the bias compensated algebraic sphere fit method, where the bias is iteratively reduced (Halvorsen, 2003).

Another form of sphere fit, hereafter named the Pratt sphere fit method, has also been proposed (Pratt, 1987), in which none of the bias associated with the algebraic sphere fit method is present:

$$f_{\text{Pratt}}(c, r_1, \ldots, r_m) = \sum_{j=1}^{m} \frac{1}{r_j^2} \sum_{i=1}^{n} (\|p_{ij} - c\|^2 - r_j^2)^2. \quad (3)$$

Here, we investigated different numerical techniques for the minimisation tasks in both the geometric sphere fit and the Pratt sphere fit methods (Marquardt, 1963; Gander et al., 1994; Chernov and Lesort, 2003; Deuflhard and Hohmann, 2003). With any reasonable initial estimate for the position of the CoR (i.e. one that is not more than a few centimeters from the true centre and therefore leads to a convergent solution), these techniques were comparably accurate and effective.

2.1.2. Transformation techniques—one sided approaches

Similar to the sphere fit methods, one sided transformation techniques assume that one segment and therefore the CoR is stationary. Assuming that at least three markers on the moving segment are present, it is possible to define rigid-body transformations, i.e. rotations R_i and translations t_i, $i = 1, \ldots, n$, which transform a given reference marker set onto the time varying marker configurations. If the CoR is stationary, all these transformations should map a single common fixed point, $\tilde{c}$, onto the actual joint positions. Thus, the joint centre is the particular point c, for which a $\tilde{c}$ exists, such that

$$c = R_i\tilde{c} + t_i, \quad i = 1, \ldots, n. \quad (4)$$

This approach can then be extended by defining a local coordinate system on the moving segment, in which the marker coordinates should remain stationary with time. The coordinate transformations from a global coordinate system into these local systems should always yield the same value for the joint centre, providing the advantage that no reference marker configuration is required. Thus, the t_i may be interpreted as the vectors from the global origin to the point of origin in the segment fixed system and R_i as rotation matrices that map the (translated) global coordinate system to these segment fixed systems (Fig. 2).

The most obvious approach, here named the centre transformation technique (CTT), is therefore to compute transformations (R_i, t_i) from global coordinates into local coordinates and then to determine c and $\tilde{c}$ for which the residual of (4) is minimal. Thus, the objective function

$$f_{\text{CTT}}(c, \tilde{c}) = \sum_{i=1}^{n} \|R_i\tilde{c} + t_i - c\|^2 \quad (5)$$

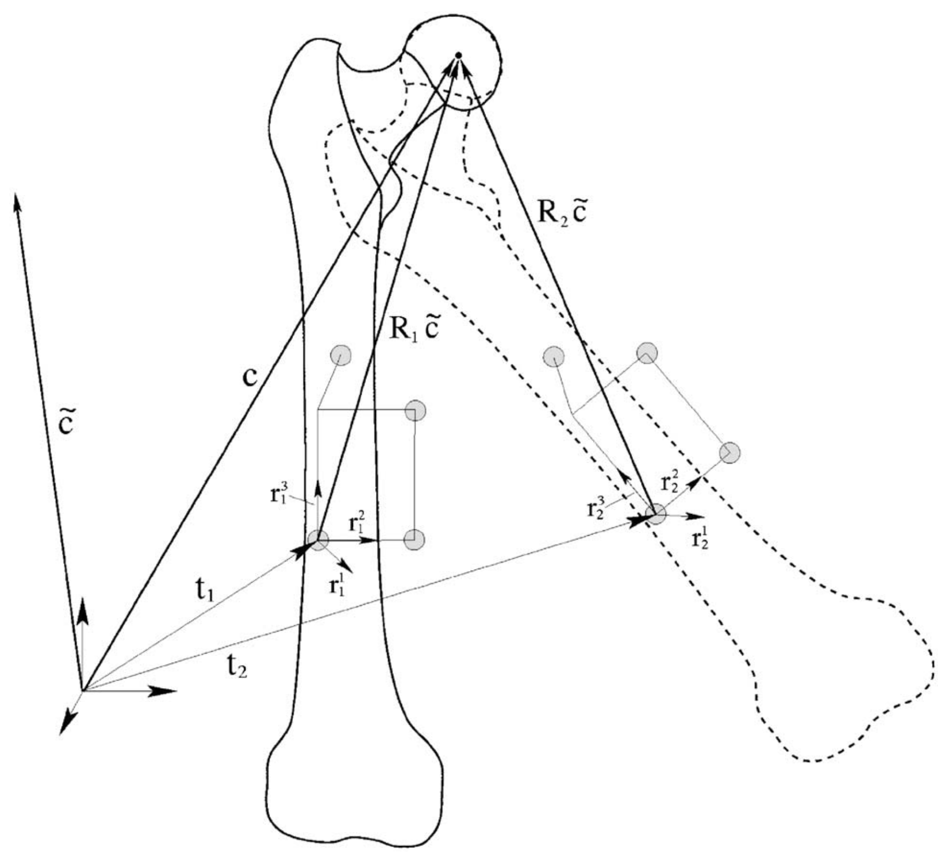

Fig. 2. Illustration of the construction of local coordinates for the femoral segment. The translations t_i, together with the rotations r_i^1, r_i^2, r_i^3, transform the CoR vector, $\tilde{c}$, from the global into the local coordinate systems.

must be minimised. This is equivalent to the linear least-squares problem:

$$\begin{pmatrix} R_1 & -I \\ \vdots & \vdots \\ R_n & -I \end{pmatrix} \begin{pmatrix} \tilde{c} \\ c \end{pmatrix} = - \begin{pmatrix} t_1 \\ \vdots \\ t_n \end{pmatrix}, \tag{6}$$

where I is the 3D identity matrix. This form of the problem can be easily solved using appropriate numerical techniques, i.e. normal equations:

$$\begin{pmatrix} nI & -\sum_{i=1}^{n} R_i^{\mathrm{T}} \\ -\sum_{i=1}^{n} R_i & nI \end{pmatrix} \begin{pmatrix} \tilde{c} \\ c \end{pmatrix} = \begin{pmatrix} -\sum_{i=1}^{n} R_i^{\mathrm{T}} t_i \\ \sum_{i=1}^{n} t_i \end{pmatrix}, \tag{7}$$

which, for c, yields the closed solution:

$$\left(n(n-1)I - \sum_{i,j=1,i\neq j}^{n} R_i R_j^{\mathrm{T}} \right) c = (n-1) \sum_{i=1}^{n} t_i - \sum_{i,j=1,i\neq j}^{n} R_i R_j^{\mathrm{T}} t_j. \tag{8}$$

Alternatively, a numerically more robust QR decomposition method (Deuflhard and Hohmann, 2003) may be used to solve Eq. (6). Similar methodologies have been evaluated with a mechanical linkage (Siston and Delp, 2005) and used in the determination of human hip joint positions during various common daily activities (Piazza et al., 2004).

A CTT related approach has been derived by Holzreiter (Holzreiter, 1991), here named the Holzreiter approach. Eliminating $\tilde{c}$ from two type (4) equations, one with index i and the other with j, yields

$$(R_i R_j^{\mathrm{T}} - I)c = R_i R_j^{\mathrm{T}} t_j - t_i. \tag{9}$$

Since $(R_i R_j^{\mathrm{T}} - I)$ is a matrix of rank 2 for $R_i \neq R_j, R_i, R_j \neq I$, Eq. (9) does not determine a unique centre c, but rather has a set of solutions that define the rotation axis of the marker set transformation from frame i to j. The CoR, c, may then be determined by minimising

$$f_{\mathrm{HR}}(c) = \sum_{i=1,j=1,i\neq j}^{n} \| (R_i R_j^{\mathrm{T}} - I)c - R_i R_j^{\mathrm{T}} t_j + t_i \|^2, \tag{10}$$

or, equivalently, by solving the linear least-squares problem

$$\begin{pmatrix} R_1 R_2^{\mathrm{T}} - I \\ \vdots \\ R_{n-1} R_n^{\mathrm{T}} - I \end{pmatrix} c = \begin{pmatrix} R_1 R_2^{\mathrm{T}} t_2 - t_1 \\ \vdots \\ R_{n-1} R_n^{\mathrm{T}} t_n - t_{n-1} \end{pmatrix}. \tag{11}$$

In a similar process used to calculate the CTT solution (from (6) to (8)), it is possible to derive a closed solution for c as

$$\left(2n(n-1)I - \sum_{i,j=1,i\neq j}^{n} (R_i R_j^{\mathrm{T}} + R_j R_i^{\mathrm{T}}) \right) c = 2(n-1) \sum_{i=1}^{n} t_i - \sum_{i,j=1,i\neq j}^{n} (R_i R_j^{\mathrm{T}} t_j + R_j R_i^{\mathrm{T}} t_i), \tag{12}$$

which produces the same outcome as Eq. (8), i.e. results in an identical CoR as the CTT. The drawback of the Holzreiter approach, however, is that it requires a much larger matrix of dimension $(3n(n-1)/2) \times 3$, rather than of $3n \times 6$ for the CTT.

Based on the original methodology from Woltring et al. (1985), a further approach, here named the helical pivot technique, determines the CoR as the point closest to all instantaneous helical axes (Woltring, 1990). The original helical axis approach describes the movement of a rigid-body as a rotation around and a translation parallel to a unique helical axis. For n time frames, there are $k = n(n-1)/2$ possible helical axes, which may be defined by the points nearest to the origin, $s_i, i = 1, \ldots, k$, and direction vectors $n_i, i = 1, \ldots, k$. By defining $Q_i = I - n_i n_i^{\mathrm{T}}$ as the projections onto the orthogonal complements of n_i, the CoR is approximated as the point nearest to all helical axes, using

$$\sum_{i=1}^{k} Q_i c = \sum_{i=1}^{k} s_i. \tag{13}$$

Since it is mostly assumed that the helical axis determination is very sensitive to small rotation angles, ϑ_i, several weighting terms w_i have been proposed (Woltring et al., 1985; Halvorsen, 2002; Camomilla et al., in press). Rather than in (13), the CoR was determined using

$$\sum_{i=1}^{k} w_i Q_i c = \sum_{i=1}^{k} w_i s_i, \tag{14}$$

with $w_i = \sin(\vartheta_i/2)$, as suggested in the original work (Woltring et al., 1985) or $w_i = \vartheta_i$ as in other studies (Camomilla et al., in press). We have proven, however, that when this variable, w_i, is set to $\sin^2(\vartheta_i/2)$, an equivalent formulation to the CTT is produced, i.e. Eq. (14) becomes identical to (12).

A further approach, named the Schwartz transformation technique (Schwartz and Rozumalski, 2005), is only presented here in a form that assumes one segment is at rest. This method is based on Eq. (9), taking on a similar construction to the Holzreiter approach. Here, however, the CoR is defined by the intersection or nearest point between the axis described by Eq. (9) and one further axis of rotation represented by

$$(R_k R_l^{\mathrm{T}} - I)c = R_k R_l^{\mathrm{T}} t_l - t_k, \tag{15}$$

where k and l are further time points. The CoR is therefore calculated by the minimisation of

$$f_{\mathrm{STT}}(c) = \left\| \left(R_i R_j^{\mathrm{T}} - I \right) c - R_i R_j^{\mathrm{T}} t_j + t_i \right\|^2 + \left\| \left(R_k R_l^{\mathrm{T}} - I \right) c - R_k R_l^{\mathrm{T}} t_l + t_k \right\|^2, \tag{16}$$

which possesses a closed solution similar to (12). In practice, this approach computes an approximation for c, for each group of four frames, where at least three must be different. Since the resulting $O(n^4)$ estimates for c have a distribution that can be non-normal and asymmetric, the authors proposed the use of the mode of this distribution as the final approximation for the joint centre position.

Another approach, the minimal amplitude point method, has been proposed by Marin and co-workers (Marin et al., 2003). Here, the transformations from the moving to the fixed segment (or global coordinates) are calculated:

$$\tilde{c} = R_i c + t_i, \quad i = 1, \ldots, n. \tag{17}$$

The CoR, c, is then defined as the point that "moves" the least under the transformations (R_i, t_i). The minimisation of the discontinuous objective function

$$f_{\mathrm{MAM}}(c) = \sum_{k=x,y,z} \left[\max_{i=1,\ldots,n} (R_i c + t_i)_k - \min_{i=1,\ldots,n} (R_i c + t_i)_k \right] \tag{18}$$

was then performed by the authors using a genetic algorithm (Houck et al., 1995). We found that it is alternatively possible to use simulated annealing (van Laarhoven and Aarts, 1987) to achieve faster optimisation.

The final approach within our classification of transformation methods, together with a review of similar methods in the field of computer vision, was published by Stoddart and co-workers (Stoddart et al., 1999). This method, here called the Stoddart approach, is not restricted by the assumption that one segment must remain at rest. Here, local reference coordinate systems are chosen on both segments as mean segment fixed systems, computed by averaging the coordinate systems over all time frames. This approach leads to a closed solution similar to (8).

In some of the previous approaches, e.g. Eqs. (8), (11) or (16), it follows that these methods may be implemented without using the transformations R_i directly, but rather only the transformations $R_i R_j^{\mathrm{T}}$, which may be understood as the direct rotations from frame j to frame i. Such rotations, together with the corresponding translations, can be easily computed using a variety of methods, e.g. (Söderkvist and Wedin, 1993; Eggert et al., 1997).

2.1.3. Transformation techniques—two sided approaches

In practice, contrary to the general assumptions of the last section, neither of the two segments will be fixed and a coordinate transformation from the global to the segment fixed coordinates usually precedes determination of the joint centre. This transformation, of course, is subject to the same sources of error and problems with defining local coordinate systems as those for the other segment. Here, approaches that can determine the CoR without requiring this initial transformation have been classified as two sided approaches. Although presented as one sided approaches for ease of description, only the Schwartz transformation technique and the Stoddart approach from the methods considered in the previous sections do not require the assumption that the CoR must remain stationary. Additionally, we present a novel approach here, named the symmetrical CoR estimation (SCoRE), that is also capable of considering a moving CoR.

All sphere fit methods inherently require a CoR fixed in the global coordinate system. For coordinate transformation methods, however, no stationary CoR is required, but it is necessary to define local coordinate systems on each segment. The philosophy of the approach presented here is that the coordinates of the CoR must remain constant relative to both segments. Mathematically, this leads to the objective function

$$f_{\mathrm{SCoRE}}(c_1, c_2) = \sum_{i=1}^{n} \| R_i c_1 + t_i - (S_i c_2 + d_i) \|^2, \tag{19}$$

which must be minimised, in which c_1, c_2 are the centres of rotation in the local coordinate systems and $(R_i, t_i), (S_i, d_i)$ the transformations from the local segment coordinates into an appropriate global system. Similar to (5), this can be written as a linear least-squares problem

$$\begin{pmatrix} R_1 & -S_1 \\ \vdots & \vdots \\ R_n & -S_n \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} d_1 - t_1 \\ \vdots \\ d_n - t_n \end{pmatrix}, \tag{20}$$

which has again a closed solution, but is better solved by numerically more robust methods (Deuflhard and Hohmann, 2003). In the global coordinate system, this approach yields a joint position for each segment and each time instant, $R_i c_1 + t_i$ and $S_i c_2 + d_i$, which are not necessarily exactly coincidental. The most obvious estimation for the actual global CoR is therefore to take the mean between these two positions.

2.2. Numerical simulation

In order to perform a fair comparison between the various aforementioned approaches, we repeated all simulations $n_t = 1000$ times, each with 200 time frames, with different random conditions, i.e. distribution of the marker positions within a specified RoM and Gaussian noise attributed to each marker. As a measure of the performance of each method, the root mean square (RMS) error

$$\sqrt{\frac{1}{n_t}\sum_{i=1}^{n_t}\|c_i - c_{\mathrm{ex}}\|^2} \tag{21}$$

was calculated, where c_i is the CoR estimation of the ith simulation and c_{ex} the exact centre position. The CoR was then determined for simulated movements of either one or both segments within five different specified angular cones of movement of 5°, 10°, 20°, 45° and 90°.

3. Results

For all approaches, the RMS errors increased approximately exponentially with decreasing RoM (Fig. 3, top) when one segment was held stationary and noise was applied independently to each marker on the moving segment to simulate skin elasticity conditions and measurement errors. Since movement was only applied to a single segment, the SCoRE approach reduces to the same formulation as the CTT and the two methods therefore delivered identical results. Under these conditions, the algebraic approach clearly led to unacceptably large errors when the RoM was small, but improved rapidly when the segments rotated more than approximately 20° relative to one another. On average, with only a 5° RoM, an error in the position of the CoR of more than 5 cm could be expected for all methods (Fig. 3, top) except for the geometric, the bias compensated algebraic and the Pratt sphere fit approaches, which would produce an error of about 1 cm. With a wider RoM, the accuracy of all methods increased rapidly, and one could expect to determine the position of the CoR within 1 cm using all approaches when the RoM increased beyond 20°. The geometric, the bias compensated algebraic and the Pratt sphere fit methods, which produced the best CoR estimation under these conditions, gave indistinguishable results from one another in every case tested.

Various weighting parameters, w_i, from Eq. (14) were checked for the helical pivot technique. It was found that the best results were always obtained with $w_i = \sin^2(\vartheta_i/2)$, even if the differences were small. Although approached from a different perspective, the use of this weighting factor ensures that the formulation of the helical pivot technique becomes completely equivalent to that of the CTT.

The Schwartz transformation technique had the disadvantage of prohibitively long computation times due to the large number of centre estimates. In this study, some 200 million CoR estimations were required for each single 200 time frame simulation (over 1 week was required on a 2 GHz computer for the required repetitions for each of the 20 RoM simulations) and it was therefore deemed too computationally expensive to be included for all measurement frames. A selection of individual comparisons was thus performed, each producing indistinguishable results from the CTT, Holzreiter, and helical pivot techniques (Fig. 3), even if the centre distributions were significantly asymmetric.

When Gaussian noise was applied only to the marker set as a whole, simulating synchronous shifting of the marker set, all methods were capable of determining the CoR much more accurately (Fig. 3, bottom). Again, under these conditions, the SCoRE, CTT, Holzreiter, Schwartz and helical pivot techniques produced indistinguishable results. Here, these approaches could determine the CoR to within 0.25 cm, even for very small ranges of motion, producing a clear improvement on the other techniques. The errors, in general, were much smaller than under the application of noise to individual markers, however, and, with the exception of the algebraic sphere fit method, all approaches were capable of reproducing the CoR accurately with a RoM of approximately 20°.

When a combination of individual and group marker noise was applied, the effects were approximately additive and the accuracy of the determination of the CoR was therefore dominated by the errors associated by the independent marker motion.

When both segments were subjected to motion and noise artefacts (Fig. 4), the positions of all markers on one segment had to be transformed into the local coordinate system of the second segment for all one sided approaches. As a result, the one sided and the two sided approaches now differed. Under these conditions, the SCoRE, CTT, Holzreiter, Schwartz and helical pivot techniques produced the most accurate results for small RoMs (approximately 0.5 cm error at 20° RoM). The geometric, the bias compensated algebraic and Pratt sphere fit approaches, which previously displayed good results when one segment was fixed, now yielded much larger errors (approximately 2 cm error at 20° RoM), particularly when noise was applied to individual markers. Once again, the algebraic fit determined the least accurate CoR at low RoMs. Similar to the conditions when one segment was fixed, the combination of individual and group marker noise produced almost additive results, the errors again dominated by individual marker noise.

4. Discussion

The ability to accurately determine the CoR is of importance across a number of disciplines, but particularly in the field of orthopaedics, where knowledge of

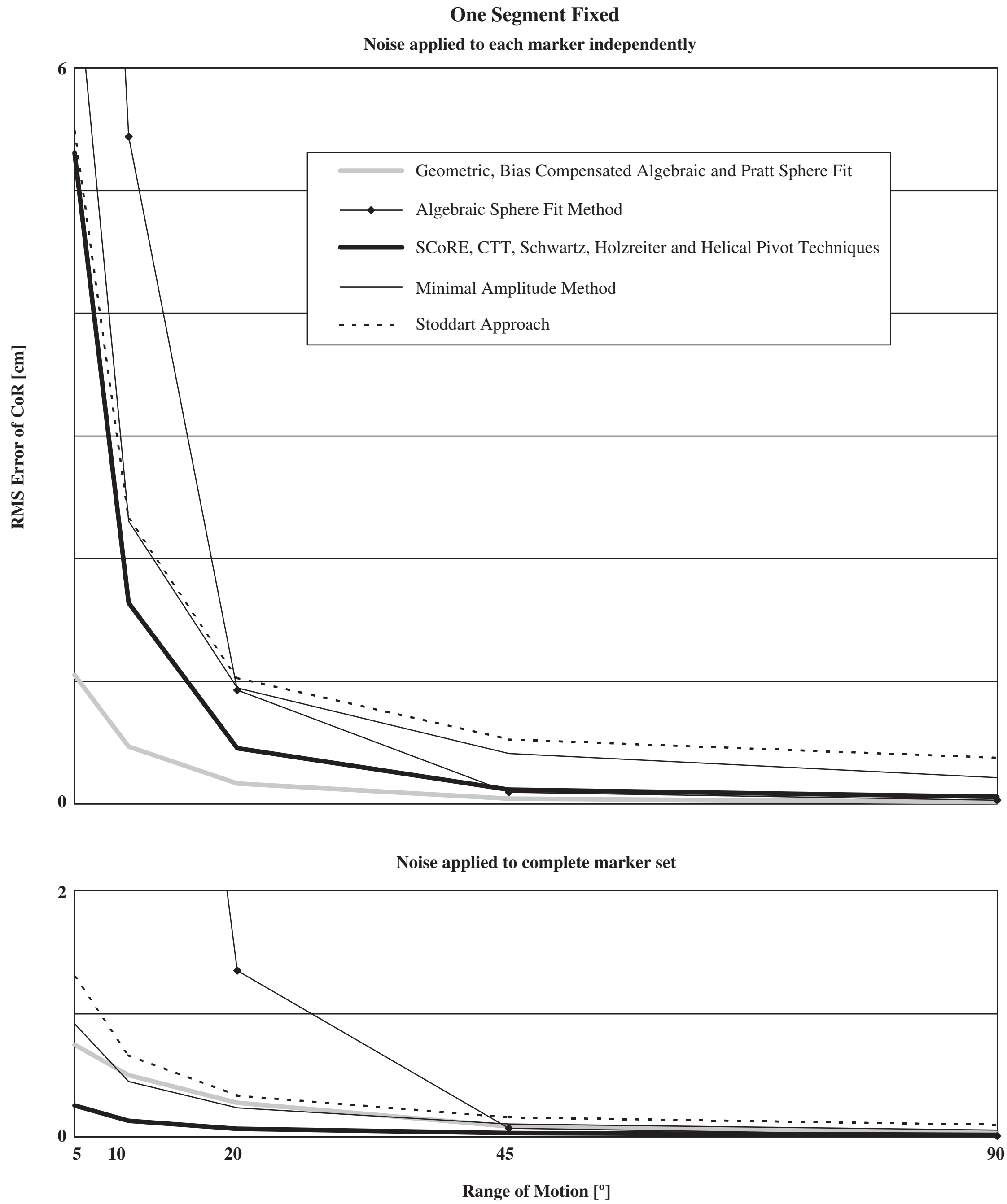

Fig. 3. RMS error of estimated CoR shown for different approaches over the 1000 simulations, assuming one static segment (no movement or Gaussian noise). Top: isotropic, independent and identically distributed Gaussian noise (standard deviation 0.1 cm) was applied to each marker position. Bottom: Gaussian noise (standard deviation 0.1 cm) was applied to all markers on the segment simultaneously. A combination of these two error sources led to an almost identical RMS error as the application of independent noise alone. Since one segment was held stationary, the SCoRE reduces to the coordinate transformation technique, producing identical results.

the hip joint centres can lead to improved clinical assessment, kinematic measurement, surgical navigation or positioning and orientation of replacement components. Whilst biomechanical literature has paid considerable attention to the issue of determining the CoR, this is, to the authors' knowledge, the first time that a complete, systematic classification and survey of formal techniques for estimating the centre about which two segments rotate has been performed. In addition, a new method, the SCoRE, has been presented, which is capable of considering two segments moving simultaneously.

From the marker set configurations, segment movements and artefact conditions applied to the marker

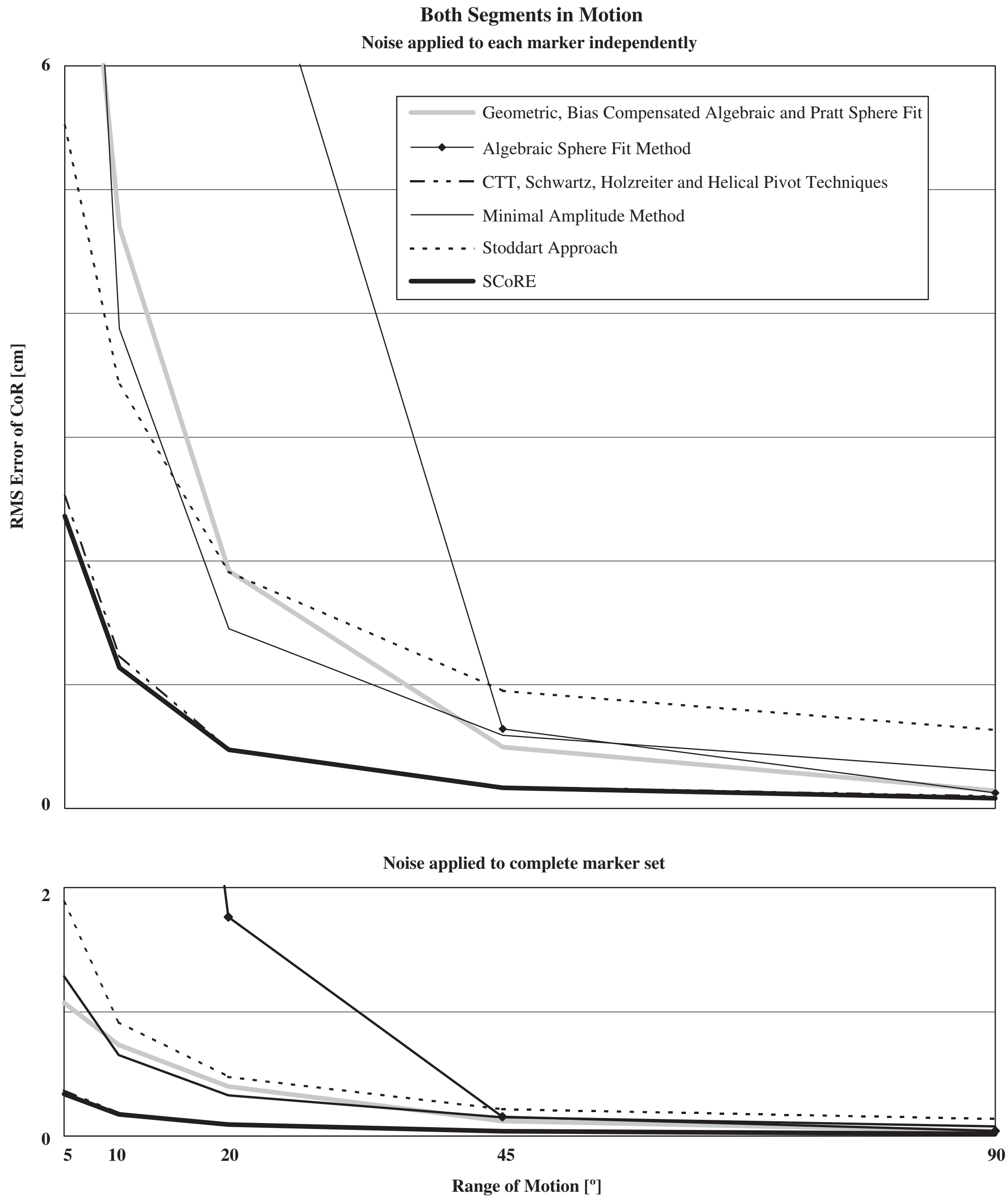

Fig. 4. RMS error of estimated CoR shown for different approaches, when adding noise to both segments. Top: isotropic, independent and identically distributed Gaussian noise (standard deviation 0.1 cm) was applied to each marker position. Bottom: Gaussian noise (standard deviation 0.1 cm) was applied to all markers on the segment simultaneously. A combination of these two error sources led to an almost identical RMS error as the application of independent noise (top).

sets, which were roughly based on the human hip joint and the errors associated with marker measurement, this study demonstrated that most commonly employed techniques were capable of determining the CoR to within 0.3 cm, as long as the RoM of the joint was 45° or more. Under more limited RoMs, however, the differences in accuracy between approaches became much more apparent, with only the SCoRE, CTT, Holzreiter, Schwartz and helical pivot approaches maintaining small errors: at 20°, these techniques were about three times better than the minimal amplitude point method, the next best approach (Fig. 4). It should be noted that the Schwartz and helical pivot approaches, which give almost identical results to the CTT, require more complex implementations, whereas the CTT is composed only of a few lines of code.

It was observed that the error conditions played a large role in the accuracy of each approach, particularly for the sphere fit methods, which performed well when one segment was held stationary, but became relatively much less accurate when movement and noise were applied to both segments. The SCoRE and other transformation-based approaches, however, fared relatively better with the additional movement of the second segment, producing the most accurate estimations of the CoR. These approaches, which consider the complete configurations of the two segment marker sets, have inherent advantages over sphere fits, where all markers are treated completely independently. It should be noted that for all sphere fit methods, the best results were obtained only if Eqs. (1), (2) or (3) were evaluated for every marker attached to the moving segment, rather than for the centroid of the set, i.e. when a concentric sphere was determined for each marker in the set, each with the identical CoR. A number of previous studies, e.g. (Leardini et al., 1999; Piazza et al., 2001) have failed to consider these conditions and have instead only used the averaged marker set coordinates. When these simplifications were examined in this study, and not all markers or only the centroid positions were considered, a substantial loss of accuracy was determined in the position of the CoR.

Few previous comparative assessments have been performed on multiple CoR approaches. The relative performance of different techniques is therefore difficult to ascertain due to varying test conditions and kinematics. Camomilla and co-workers studied a number of approaches also considered in this investigation with segment motion and error conditions similar to those examined in Fig. 3 (top), where one segment was assumed to be fixed and not subject to skin marker artefacts (Camomilla et al., in press). Although the marker configuration, joint geometry and joint motion were different, it was observed that their bias compensated algebraic fit slightly outperformed the "minimal linear displacement point" (here called Holzreiter approach) and the helical pivot approach—results that complement our observations for this general set of conditions. It must be noted that their results for the geometric fit (S2 in the Camomilla study) suffered from the inaccuracy described previously, in that only the centroid position, rather than all individual marker coordinates were used. This resulted in the apparent discrepancy for these specific results. Complete motion of the joint, including both axes, the CoR itself and noise applied to both sets of markers (Fig. 4), was not considered in the Camomilla study, and therefore these conditions could not be compared.

The same study (Camomilla et al., in press) examined a number of different regimes for finding the optimal motion of the marker set. Their findings demonstrated that the more equally the markers were distributed over the surface of the "sphere", the more accurate the calculation of the CoR. Since the markers in our study were equally distributed over the entire RoM, rather than limited to only a number of arcs as described by their "star arc", an extrapolation of their results would seem to support a more accurate CoR determination under a more even distribution of markers, as used in this study. Despite limiting their movements to these distinct paths, however, the errors of the estimations at the comparable 200 measurement frames were of the same order observed in this study.

Some of the more elaborate methods such as the Schwartz transformation technique, the minimal amplitude point method, and the Stoddard approach did not perform as well or as rapidly as the conceptually simpler algorithms. Due to its additional extreme computational cost, the Schwartz transformation technique was tested at only a limited number of points. Despite producing results that were almost identical to the CTT, the Holzreiter approach or the helical pivot technique, and more accurate than many of the other techniques, this method is prohibitively time-consuming with the computing power available and as such becomes currently impractical.

In this study, the moving marker set was allowed to rotate within a specific cone, or RoM, which varied between 5° and 90°. For any reasonable estimations of the CoR based on the conditions used in this study, relative motion of the two segments of at about 20° or more was required. In cases where, e.g. the surgical intervention of a joint is required, it may be possible that the patient is unable to rotate the joint and such ranges of motion are not possible. In these cases, the geometric, bias compensated algebraic and Pratt sphere fit approaches will have reduced accuracy, and coordinate transformation techniques such as the SCoRE should be used.

For the simulation of marker artefacts, normally associated with measurement inaccuracies, elastic distortion of the skin and synchronous shifting of the entire marker set, isotropic, independent and identically distributed Gaussian noise with a standard deviation of 0.1 cm was applied to each marker position and the whole segment marker set. Whilst the Gaussian noise error conditions induced to the marker positions in this study were not taken from actual clinical gait data, the amount of marker artefact was considered appropriate for a first comparative estimate of the accuracy of the approaches examined. It is possible, however, that the accuracy with which it is possible to determine the CoR is reduced in more obese patients, where the soft tissues covering the underlying bones cause additional shift and distortion errors in the marker positions and configurations. Although examining these effects was beyond the scope of this study, it may be possible to reduce the effect of skin stretching the marker set configuration by applying orthogonal distance regression techniques to remove relative marker to marker movements (Taylor et al., 2005).

The conditions used in this study (marker configuration, error conditions, segment movement etc.) should be seen as only a typical example for demonstrating the accuracy of each of the various approaches. With, e.g. a different number of markers, different marker placement or configuration, it may be entirely possible to achieve a more accurate joint position. Specific techniques may therefore offer further enhancement of the accuracy of the approaches investigated in this study by optimising the configuration and position of markers within the sets. The conditions chosen in this study, however, allowed a complete and fair comparison between the different methods. Furthermore, the relative performance of the different approaches investigated in this study is almost independent of the test conditions. If the specific conditions were known prior to a practical application, it could be that the optimal method of CoR determination can be chosen for those conditions. Although highly unlikely, it may be that certain applications require the determination of a CoR when one segment is rigidly fixed. Under such circumstances, the use of the geometric, bias compensated algebraic or Pratt sphere fit techniques may be justified. Under more normal conditions for kinematic assessment, but also in the operating theatre where it is almost impossible to hold one segment stationary relative to the global measurement system, the use of the SCoRE has here been shown to yield the best results. This method has the benefit that it is fast and just as, or more simple, to implement as any of the other techniques. It also has the distinct advantage that the position of the joint centre is provided in both of the local segment coordinate systems.

For the first time, a complete survey and classification of formal methods has been performed in this study. A new method, the SCoRE, has additionally been presented, for which no assumption of the segment movements relative to the CoR is required. In nearly all test scenarios investigated in this study, the SCoRE produced the smallest errors in the estimation of joint centre. Follow-up work to this study involves similar direct comparisons to be performed on clinical data.

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