15-464 / 16-464 HW1: On Bezier Quaternion Interpolation
due Tuesday, September 13, before class

Shoemake's 1985 SIGGRAPH paper gives an algorithm for finding the two additional control points required to define a spline that interpolates from one quaternion to another. (Also see Parent, pg 99-101.)

This algorithm requires the following operations (variable names from Shoemake, second figure on page 249):

$$\bar{q}_{n-1} = slerp(q_{n-1}, q_n, 2)$$
$$a_n = slerp(\bar{q}_{n-1}, q_{n+1}, 0.5)$$
$$b_n = slerp(a_n, q_n, 2)$$

Then to interpolate from quaternion q_{n-1} to q_n, we would use as the four control parameters: $q_{n-1}, a_{n-1}, b_n, q_{n.}$.

1.) Analyze the case where the quaternion keyframes are evenly spaced rotations about a single axis. If we use these control points, do we get constant velocity rotation as expected?

The Bezier curve definition for curves in Euclidean space places the control points at 1/3 the magnitude of the velocities at the data points (Parent, pg 463). A closer match to this placement would be:

$$\bar{q}_{n-1} = slerp(q_{n-1}, q_n, 2)$$
$$a_n = slerp(\bar{q}_{n-1}, q_{n+1}, 0.5)$$
$$a'_n = slerp(q_n, a_n, 1/3)$$
$$b'_n = slerp(a'_n, q_n, 2)$$

Now, to interpolate from quaternion q_{n-1} to q_n, we would use as the four control parameters: $q_{n-1}, a'_{n-1}, b'_n, q_{n.}$.

2.) Analyze the case where the quaternion keyframes are evenly spaced rotations about a single axis. If we use this second set of control points, how does the result change? Compare your answer to the answer for problem 1. What is the right thing to do here; i.e., how should a_n and b_n be chosen?