Radiosity

Measures of Illumination
The Radiosity Equation
Form Factors
Radiosity Algorithms
[Angel, Ch 13.4-13.5]

Alternative Notes
• SIGGRAPH 1993 Education Slide Set – Radiosity Overview, by Stephen Spencer

www.siggraph.org/education/materials/HyperGraph/radiosity/overview_1.htm
Limitations of Ray Tracing

Local vs. Global Illumination

- Local illumination: Phong model (OpenGL)
  - Light to surface to viewer
  - No shadows, interreflections
  - Fast enough for interactive graphics
- Global illumination: Ray tracing
  - Multiple specular reflections and transmissions
  - Only one step of diffuse reflection
- Global illumination: Radiosity
  - All diffuse interreflections; shadows
  - Advanced: combine with specular reflection
Image vs. Object Space

- **Image space**: Ray tracing
  - Trace backwards from viewer
  - View-dependent calculation
  - Result: rasterized image (pixel by pixel)
- **Object space**: Radiosity
  - Assume only diffuse-diffuse interactions
  - View-independent calculation
  - Result: 3D model, color for each surface patch
  - Can render with OpenGL

Classical Radiosity Method

- Divide surfaces into patches (elements)
- Model light transfer between patches as system of linear equations
- Important assumptions:
  - Reflection and emission are diffuse
  - Recall: diffuse reflection is equal in all directions
  - So radiance is independent of direction
  - No participating media (no fog)
  - No transmission (only opaque surfaces)
  - Radiosity is constant across each element
  - Solve for R, G, B separately
Balance of Energy

- Lambertian surfaces (ideal diffuse reflector)
- Divided into n elements
- Variables
  - $A_i$: Area of element $i$ (computable)
  - $B_i$: Radiosity of element $i$ (unknown)
  - $E_i$: Radiant emitted flux density of element $i$ (given)
  - $\rho_i$: Reflectance of element $i$ (given)
  - $F_{ji}$: Form factor from $j$ to $i$ (computable)

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{n} F_{ji} A_j B_j$$

Form Factors

- Form factor $F_{ij}$: Fraction of light leaving element $i$ arriving at element $j$
- Depends on
  - Shape of patches $i$ and $j$
  - Relative orientation of both patches
  - Distance between patches
  - Occlusion by other patches
Form Factor Equation

- Polar angles $\theta$ and $\theta'$ between normals and ray between $x$ and $y$
- Visibility function $v(x,y) = 0$ if ray from $x$ to $y$ is occluded, $v(x,y) = 1$ otherwise
- Distance $r$ between $x$ and $y$

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x,y) \, dy \, dx$$

Reciprocity

- Symmetry of form factor

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x,y) \, dy \, dx = A_j F_{ji}$$

- Divide earlier radiosity equation

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{n} F_{ji} A_j B_j$$

$$B_i = E_i + \rho_i \sum_{j} (F_{ji} A_j / A_i) B_j$$

$$= E_i + \rho_i \sum_{j} F_{ij} B_j$$
Radiosity as a Linear System

- Restate radiosity equation
- In matrix form

\[ B_i - \rho_i \sum_j F_{ij} B_j = E_i \]

- Known: reflectances \( \rho_i \), form factors \( F_i \), emissions \( E_i \)
- Unknown: Radiosities \( B_i \)
- \( n \) linear equations in \( n \) unknowns

Radiosity “Pipeline”

- Scene Geometry
- Reflectance Properties
- Form factor calculation
- Solution of Radiosity Eq
- Radiosity Image
- Visualization
- Viewing Conditions
Visualization

- Radiosity solution is viewer independent
- Can exploit graphics hardware to obtain image
- Convert color on patch to vertex color
- Easy part of radiosity method

Computing Form Factors

- Visibility critical
- Two principal methods
  - Hemicube: exploit z-buffer hardware
  - Ray casting (can be slow)
    - Both exhibit aliasing effects
- For inter-visible elements
  - Many special cases can be solved analytically
  - Avoid full numeric approximation of double integral
Hemicube Algorithm

- Render model onto a hemicube as seen from the center of a patch
- Store patch identifiers $j$ instead of color
- Use z-buffer to resolve visibility
- Efficiently implementable in hardware
- Examples of antialiasing [Chandran et al.]
No Intensity Interpolation

Wireframe
Radiosity Equation Revisited

- Direct form
  \[ B_i = E_i + \rho_i \sum_j F_{ij} B_j \]

- As matrix equation
  \[
  \begin{bmatrix}
  1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\
  -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  -\rho_n F_{n1} & \rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn}
  \end{bmatrix}
  \begin{bmatrix}
  B_1 \\
  B_2 \\
  \vdots \\
  B_n
  \end{bmatrix}
  =
  \begin{bmatrix}
  E_1 \\
  E_2 \\
  \vdots \\
  E_n
  \end{bmatrix}
  \]

- Unknown: radiosity \( B_i \)
- Known: emission \( E_i \), form factor \( F_{ij} \), reflect. \( \rho_i \)

Classical Radiosity Algorithms

- Matrix Radiosity
  - Diagonally dominant matrix
  - Use Gauss-Seidel iterative solution
  - Time and space complexity is \( O(n^2) \) for \( n \) elements
  - Memory cost excessive
- Progressive Refinement Radiosity
  - Solve equations incrementally with form factors
  - Time complexity is \( O(n \cdot s) \) for \( s \) iterations
  - Used more commonly (space complexity \( O(n) \))
Matrix Radiosity

- Compute all form factors $F_{ij}$
- Make initial approximation to radiosity
  - Emitting elements $B_i = E_i$
  - Other elements $B_i = 0$
- Apply equation to get next approximation
  \[ B_i^l = E_i + \rho_i \sum_j F_{ij} B_j \]
- Iterate with new approximation
- Intuitively
  - Gather incoming light for each element $i$
  - Base new estimate on previous estimate

Radiosity Summary

- Assumptions
  - Opaque Lambertian surfaces (ideal diffuse)
  - Radiosity constant across each element
- Radiosity computation structure
  - Break scene into patches
  - Compute form factors between patches
    - Lighting independent
  - Solve linear radiosity equation
    - Viewer independent
  - Render using standard hardware
Lecture Summary

- The Radiosity Equation
- Form Factors
- Radiosity Algorithms
Solid Angle

- **2D angle subtended by object O from point x:**
  - Length of projection onto unit circle at x
  - Measured in radians (0 to 2\pi)
- **3D solid angle subtended by O from point x:**
  - Area of of projection onto unit sphere at x
  - Measured in steradians (0 to 4\pi)

Radiant Power and Radiosity

- **Radiant power P**
  - Rate at which light energy is transmitted
  - Dimension: power = energy / time
- **Flux density Φ**
  - Radiant power per unit area of the surface
  - Dimension: power / area
- **Irradiance E:** incident flux density of surface
- **Radiosity B:** exitant flux density of surface
  - Dimension: power / area
- **Flux density at a point Φ(x) = dP/dA** (or dP/dx)
Power at Point in a Direction

- Radiant intensity $I$
  - Power radiated per unit solid angle by point source
  - Dimension: power / solid angle
- Radiant intensity in direction $\omega$
  - $I(\omega) = \frac{dP}{d\omega}$
- Radiance $L(x, \omega)$
  - Flux density at point $x$ in direction $\omega$
  - Dimension: power / (area × solid angle)

Radiance

- Measured across surface in direction $\omega$
- For angle $\theta$ between $\omega$ and normal $n$
  \[
  L(x, \omega) = \frac{d^2P}{d\omega dx'} = \frac{d^2P}{d\omega \cos \theta dx}
  \]
Radiosity and Radiance

- Radiosity $B(x) = \frac{dP}{dx}$
- Radiance $L(x, \omega) = \frac{d^2P}{d\omega \cos \theta} \, dx$
- Let $\Omega$ be set of all directions above $x$

\[ B(x) = \int_{\Omega} L(x, \omega) \cos \theta \, d\omega \]