# Curves and Surfaces

Parametric Representations
Cubic Polynomial Forms
Hermite Curves
Bezier Curves and Surfaces
[Angel 10.1-10.6]

## Goals

- How do we draw surfaces?
  - Approximate with polygons
  - Draw polygons
- How do we specify a surface?
  - Explicit, implicit, parametric
- How do we approximate a surface?
  - Interpolation (use only points)
  - Hermite (use points and tangents)
  - Bezier (use points, and more points for tangents)

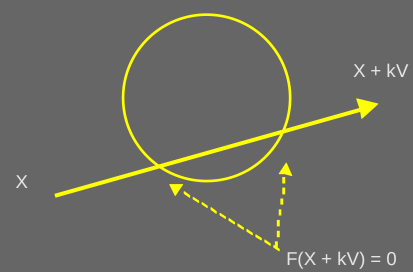
## Explicit Representation

- Curve in 2D: $y = f(x)$
- Curve in 3D: $y = f(x)$, $z = g(x)$
- Surface in 3D: $z = f(x,y)$
- Problems:
  - How about a vertical line $x = c$ as $y = f(x)$?
  - Circle $y = \pm (r^2 - x^2)^{1/2}$ two or zero values for x
- Too dependent on coordinate system
- Rarely used in computer graphics

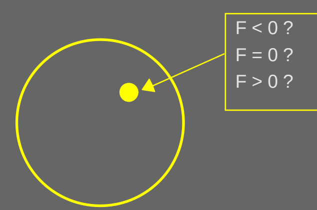
## Implicit Representation

- Curve in 2D: $f(x,y) = 0$
  - Line: $ax + by + c = 0$
  - Circle: $x^2 + y^2 - r^2 = 0$
- Surface in 3d: $f(x,y,z) = 0$
  - Plane: $ax + by + cz + d = 0$
  - Sphere: $x^2 + y^2 + z^2 - r^2 = 0$
- $f(x,y,z)$ can describe 3D object:
  - Inside: $f(x,y,z) < 0$
  - Surface: $f(x,y,z) = 0$
  - Outside: $f(x,y,z) > 0$

# What Implicit Functions are Good For

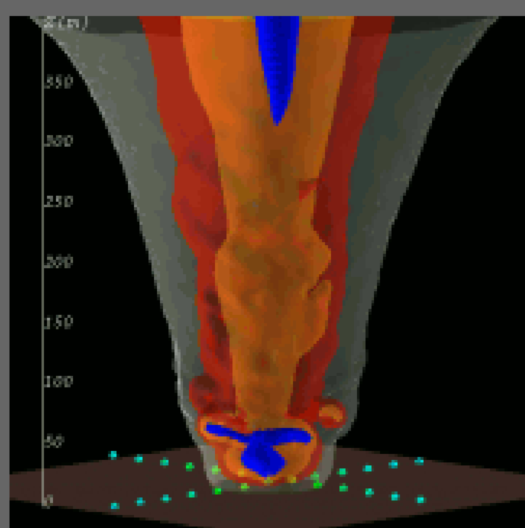



Ray - Surface Intersection Test




Inside/Outside Test

# Isosurfaces of Simulated Tornado

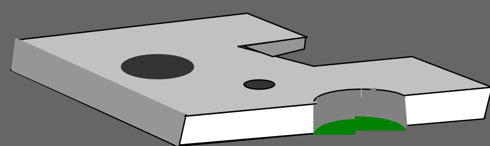


# Constructive Solid Geometry (CSG)

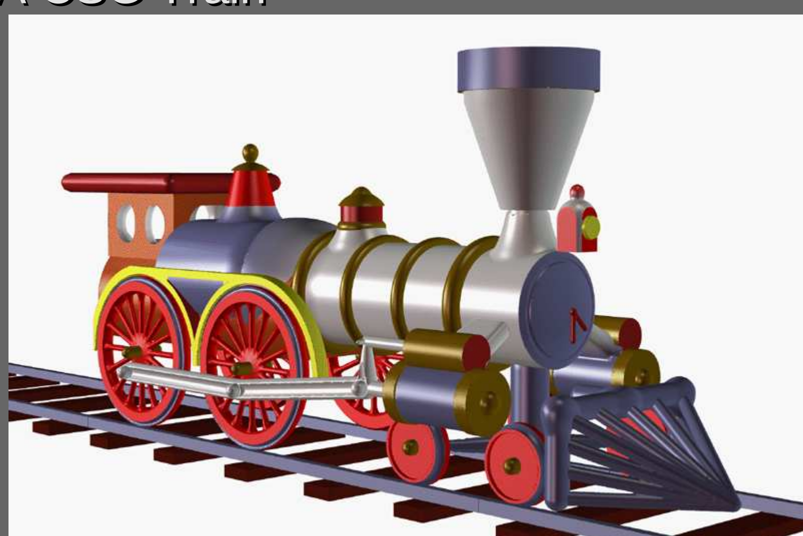
Generate complex shapes with basic building blocks

- machine an object - saw parts off, drill holes
- glue pieces together

This is sensible for objects that are actually made that way (human-made, particularly machined objects)



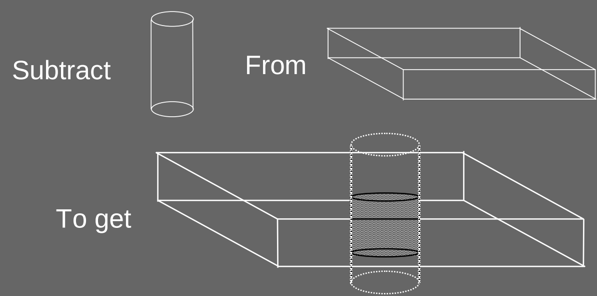
# A CSG Train



Brian Wyvill & students, Univ. of Calgary

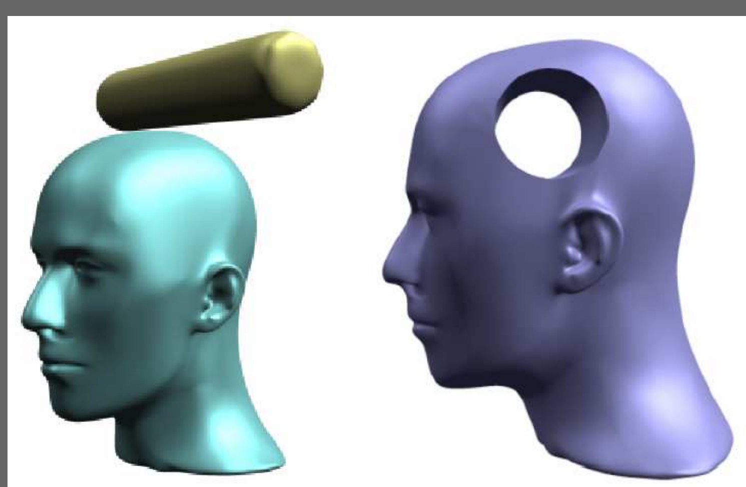
## CSG: Negative Objects

- Use point-by-point boolean functions
  - remove a volume by using a negative object
  - e.g. drill a hole by subtracting a cylinder




Inside(BLOCK-CYL) = Inside(BLOCK) And Not(Inside(CYL))

## CSG



## CSG

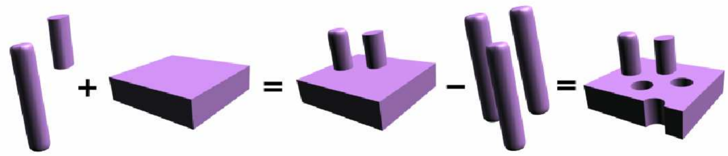


Figure 20: Unions and differences of piecewise-smooth surfaces. The resulting surfaces have creases and corners.

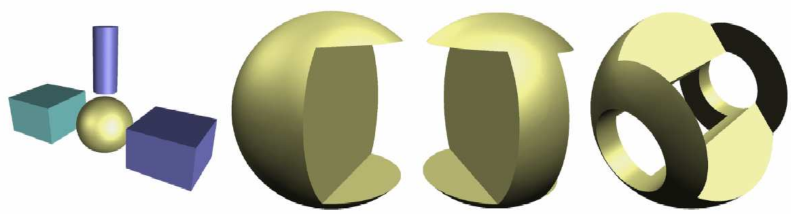
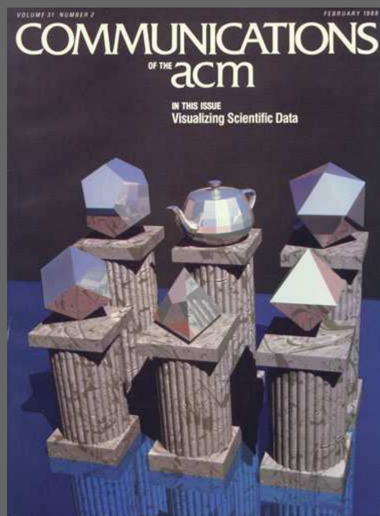


Figure 21: Sequence of difference operations: Subtracting two boxes and a cylinder from a sphere. The result has convex and concave corners. The subdivision scheme that we use [4] represents these features explicitely.

## The Utah Teapot

## The Utah Teapot




**The Six Platonic Solids**

(by Jim Arvo and Dave Kirk, from their '87 SIGGRAPH paper *Fast Ray Tracing by Ray Classification*.)

See
http://www.sjbaker.org/teapot/
for more history.

## The Utah Teapot

The teapot is just cubic Bezier patches!

```
Rim:
  { 102, 103, 104, 105,   4,   5,   6,   7,   8,   9,  10,  11,  12,  13,  14,  15 }
Body:
  {  12,  13,  14,  15,  16,  17,  18,  19,  20,  21,  22,  23,  24,  25,  26,  27 }
  {  24,  25,  26,  27,  29,  30,  31,  32,  33,  34,  35,  36,  37,  38,  39,  40 }
Lid:
  {  96,  96,  96,  96,  97,  98,  99, 100, 101, 101, 101, 101,   0,   1,   2,   3 }
  {   0,   1,   2,   3, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117 }
Handle:
  {  41,  42,  43,  44,  45,  46,  47,  48,  49,  50,  51,  52,  53,  54,  55,  56 }
  {  53,  54,  55,  56,  57,  58,  59,  60,  61,  62,  63,  64,  28,  65,  66,  67 }
Spout:
  {  68,  69,  70,  71,  72,  73,  74,  75,  76,  77,  78,  79,  80,  81,  82,  83 }
  {  80,  81,  82,  83,  84,  85,  86,  87,  88,  89,  90,  91,  92,  93,  94,  95 }

Vertices:

  {  0.2000,  0.0000, 2.70000 }, {  0.2000, -0.1120, 2.70000 }, {  0.1120, -0.2000, 2.70000 },
  {  0.0000, -0.2000, 2.70000 }, {  1.3375,  0.0000, 2.53125 }, {  1.3375, -0.7490, 2.53125 },
  {  0.7490, -1.3375, 2.53125 }, {  0.0000, -1.3375, 2.53125 }, {  1.4375,  0.0000, 2.53125 },
  {  1.4375, -0.8050, 2.53125 }, {  0.8050, -1.4375, 2.53125 }, {  0.0000, -1.4375, 2.53125 },
  {  1.5000,  0.0000, 2.40000 }, {  1.5000, -0.8400, 2.40000 }, {  0.8400, -1.5000, 2.40000 },
  {  0.0000, -1.5000, 2.40000 }, {  1.7500,  0.0000, 1.87500 }, {  1.7500, -0.9800, 1.87500 },
  {  0.9800, -1.7500, 1.87500 }, {  0.0000, -1.7500, 1.87500 }, {  2.0000,  0.0000, 1.35000 },
  {  2.0000, -1.1200, 1.35000 }, {  1.1200, -2.0000, 1.35000 }, {  0.0000, -2.0000, 1.35000 },
  {  2.0000,  0.0000, 0.90000 }, {  2.0000, -1.1200, 0.90000 }, {  1.1200, -2.0000, 0.90000 },
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  {  2.0000, -1.1200, 0.45000 }, {  1.1200, -2.0000, 0.45000 }, {  0.0000, -2.0000, 0.45000 },
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  {  0.0000, -1.5000, 0.22500 }, {  1.5000,  0.0000, 0.15000 }, {  1.5000, -0.8400, 0.15000 },
  {  0.8400, -1.5000, 0.15000 }, {  0.0000, -1.5000, 0.15000 }, { -1.6000,  0.0000, 2.02500 },
  { -1.6000, -0.3000, 2.02500 }, { -1.5000, -0.3000, 2.25000 }, { -1.5000,  0.0000, 2.25000 },
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  {  0.0000, -1.3000, 2.40000 }, {  0.0000,  0.0000, 0.00000 },
```

## Algebraic Surfaces

- Special case of implicit representation
- f(x,y,z) is polynomial in x, y, z
- Quadrics: degree of polynomial $\leq 2$
- Render more efficiently than arbitrary surfaces
- Implicit form often used in computer graphics
- How do we represent curves implicitly?

## Parametric Form for Curves

- Curves: single parameter u (e.g. time)
- x = x(u), y = y(u), z = z(u)
- Circle: x = cos(u), y = sin(u), z = 0
- Tangent described by derivative

$$\mathbf{p}(u) = \begin{bmatrix} x(u) \\ y(u) \\ z(u) \end{bmatrix} \qquad \frac{d\mathbf{p}(u)}{du} = \begin{bmatrix} \frac{dx(u)}{du} \\ \frac{dy(u)}{du} \\ \frac{dz(u)}{du} \end{bmatrix}$$

- Magnitude is “velocity”

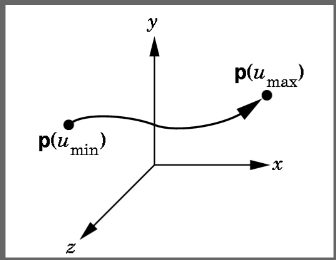
# Parametric Form for Surfaces

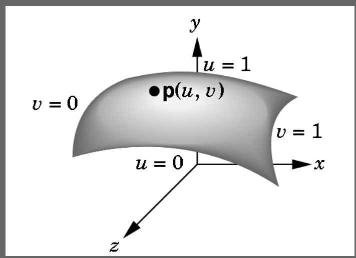
- Use parameters u and v
- x = x(u,v), y = y(u,v), z = z(u,v)
- Describes surface as both u and v vary
- Partial derivatives describe tangent plane at each point p(u,v) = [x(u,v)  y(u,v)  z(u,v)]$^T$

$$\frac{\partial \mathbf{p}(u,v)}{\partial u} = \begin{bmatrix} \frac{\partial x(u,v)}{\partial u} \\ \frac{\partial y(u,v)}{\partial u} \\ \frac{\partial z(u,v)}{\partial u} \end{bmatrix} \qquad \frac{\partial \mathbf{p}(u,v)}{\partial v} = \begin{bmatrix} \frac{\partial x(u,v)}{\partial v} \\ \frac{\partial y(u,v)}{\partial v} \\ \frac{\partial z(u,v)}{\partial v} \end{bmatrix}$$

# Assessment of Parametric Forms

- Parameters often have natural meaning
- Easy to define and calculate
  - Tangent and normal
  - Curves segments (for example, $0 \leq u \leq 1$)
  - Surface patches (for example, $0 \leq u,v \leq 1$)



## Parametric Polynomial Curves

- Restrict x(u), y(u), z(u) to be polynomial in u
- Fix degree n

$$\mathbf{p}(u) = \sum_{k=0}^{n} \mathbf{c}_k u^k$$

- Each $c_k$ is a column vector

$$\mathbf{c}_k = \begin{bmatrix} c_{xk} \\ c_{yk} \\ c_{zk} \end{bmatrix}$$

## Parametric Polynomial Surfaces

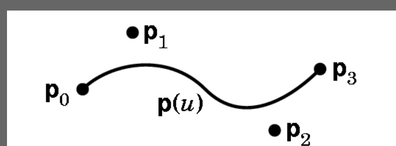
- Restrict x(u,v), y(u,v), z(u,v) to be polynomial of fixed degree n

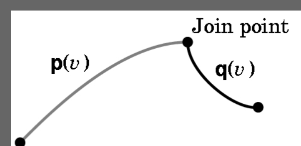
$$\mathbf{p}(u,v) = \begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{bmatrix} = \sum_{i=0}^{n} \sum_{k=0}^{n} \mathbf{c}_{ik} u^i v^k$$

- Each $c_{ik}$ is a 3-element column vector
- Restrict to simple case where $0 \leq u,v \leq 1$

## Approximating Surfaces

- Use parametric polynomial surfaces
- Important concepts:
  - Join points for segments and patches
  - Control points to interpolate
  - Tangents and smoothness
  - Blending functions to describe interpolation
- First curves, then surfaces




## Outline

- Parametric Representations
- Cubic Polynomial Forms
- Hermite Curves
- Bezier Curves and Surfaces

## Cubic Polynomial Form

- Degree 3 appears to be a useful compromise
- Curves:

$$\mathbf{p}(u) = \mathbf{c}_0 + \mathbf{c}_1 u + \mathbf{c}_2 u^2 + \mathbf{c}_3 u^3 = \sum_{k=0}^{3} \mathbf{c}_k u^k$$

- Each $c_k$ is a column vector $[c_{kx} \ \ c_{ky} \ \ c_{kz}]^T$
- From control information (points, tangents) derive 12 values $c_{kx}$, $c_{ky}$, $c_{kz}$ for $0 \le k \le 3$
- These determine cubic polynomial form

## Interpolation by Cubic Polynomials

- Simplest case, although rarely used
- Curves:
  - Given 4 control points $p_0$, $p_1$, $p_2$, $p_3$
  - All should lie on curve: 12 conditions, 12 unknowns
- Space $0 \le u \le 1$ evenly

$p_0 = p(0)$, $p_1 = p(1/3)$, $p_2 = p(2/3)$, $p_3 = p(1)$

## Equations to Determine $c_k$

- Plug in values for u = 0, 1/3, 2/3, 1

$$\mathbf{p}_0 = \mathbf{p}(0) = \mathbf{c}_0$$

$$\mathbf{p}_1 = \mathbf{p}(\tfrac{1}{3}) = \mathbf{c}_0 + \tfrac{1}{3}\mathbf{c}_1 + (\tfrac{1}{3})^2\mathbf{c}_2 + (\tfrac{1}{3})^3\mathbf{c}_3$$

$$\mathbf{p}_2 = \mathbf{p}(\tfrac{2}{3}) = \mathbf{c}_0 + \tfrac{2}{3}\mathbf{c}_1 + (\tfrac{2}{3})^2\mathbf{c}_2 + (\tfrac{2}{3})^3\mathbf{c}_3$$

$$\mathbf{p}_3 = \mathbf{p}(1) = \mathbf{c}_0 + \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3$$

$$\begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{1}{3} & (\frac{1}{3})^2 & (\frac{1}{3})^3 \\ 1 & \frac{2}{3} & (\frac{2}{3})^2 & (\frac{2}{3})^3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix}$$

Note:
$p_k$ and $c_k$ are vectors!

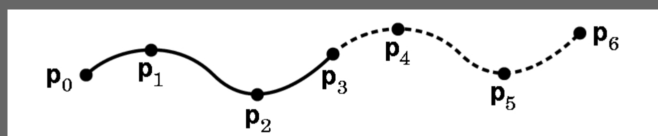
## Interpolating Geometry Matrix

- Invert A to obtain interpolating geometry matrix

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -5.5 & 9 & -4.5 & 1 \\ 9 & -22.5 & 18 & 4.5 \\ -4.5 & 13.5 & -13.5 & 4.5 \end{bmatrix} \qquad \mathbf{c} = A^{-1}\mathbf{p}$$

## Joining Interpolating Segments

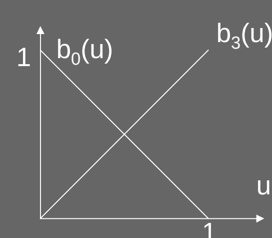
- Do not solve degree n for n points
- Divide into overlap sequences of 4 points
- $p_0$, $p_1$, $p_2$, $p_3$ then $p_3$, $p_4$, $p_5$, $p_6$, etc.




- At join points
  - Will be continuous ($C^0$ continuity)
  - Derivatives will usually not match (no $C^1$ continuity)

## Blending Functions

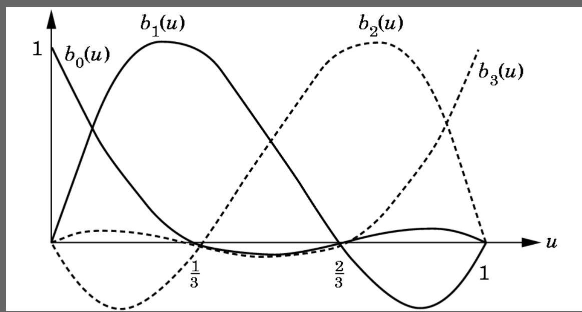
- Make explicit, how control points contribute
- Simplest example: straight line with control points $p_0$ and $p_3$
- $p(u) = (1 – u)\ p_0 + u\ p_3$
- $b_0(u) = 1 – u$, $b_3(u) = u$



## Blending Polynomials for Interpolation

- Each blending polynomial is a cubic
- Solve (see [Angel, p. 427]):

$$\mathrm{p}(u) = b_0(u)\mathrm{p}_0 + b_1(u)\mathrm{p}_1 + b_2(u)\mathrm{p}_2 + b_3(u)\mathrm{p}_3$$

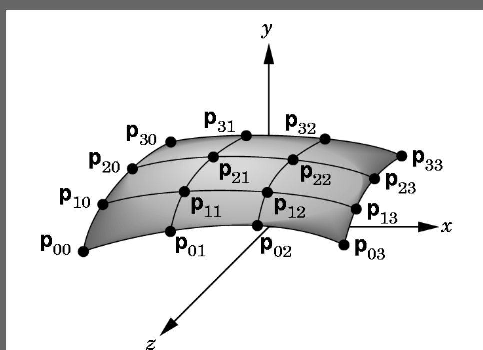



## Cubic Interpolation Patch

- Bicubic surface patch with 4 $\times$ 4 control points

$$\mathrm{p}(u,v) = \sum_{i=0}^{3}\sum_{k=0}^{3} u^i v^k \mathbf{c}_{ik}$$

Note: each $c_{ik}$ is 3 column vector (48 unknowns)



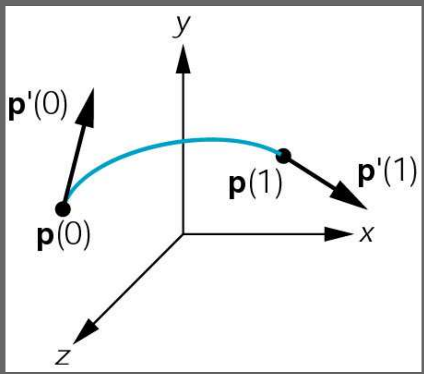

[Angel, Ch. 10.4.2]

## Outline

- Parametric Representations
- Cubic Polynomial Forms
- Hermite Curves
- Bezier Curves and Surfaces

## Hermite Curves

- Another cubic polynomial curve
- Specify two endpoints and their tangents



## Deriving the Hermite Form

- As before

$$\mathrm{p}(0) = \mathrm{p}_0 = \mathrm{c}_0$$
$$\mathrm{p}(1) = \mathrm{p}_3 = \mathrm{c}_0 + \mathrm{c}_1 + \mathrm{c}_2 + \mathrm{c}_3$$

- Calculate derivative

$$\mathrm{p}'(u) = \begin{bmatrix} \frac{dx}{du} \\ \frac{dy}{du} \\ \frac{dz}{du} \end{bmatrix} = \mathrm{c}_1 + 2u\mathrm{c}_2 + 3u^2\mathrm{c}_3$$

- Yields

$$\mathrm{p}'_0 = \mathrm{p}'(0) = \mathrm{c}_1$$
$$\mathrm{p}'_3 = \mathrm{p}'(1) = \mathrm{c}_1 + 2\mathrm{c}_2 + 3\mathrm{c}_3$$

## Summary of Hermite Equations

- Write in matrix form
- Remember $p_k$ and $p'_k$ and $c_k$ are vectors!

$$\begin{bmatrix} \mathrm{p}_0 \\ \mathrm{p}_3 \\ \mathrm{p}'_0 \\ \mathrm{p}'_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} \mathrm{c}_0 \\ \mathrm{c}_1 \\ \mathrm{c}_2 \\ \mathrm{c}_3 \end{bmatrix}$$

- Let $q = [p_0 \;\; p_3 \;\; p'_0 \;\; p'_3]^T$ and invert to find Hermite geometry matrix $M_H$ satisfying

$$\mathrm{c} = \mathrm{M}_H\mathrm{q}$$

## Blending Functions

- Explicit Hermite geometry matrix

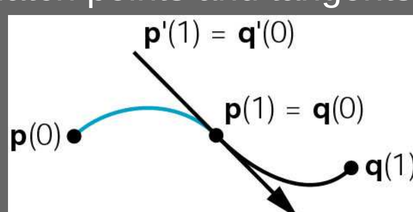
$$\mathrm{M}_H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$

- Blending functions for $\mathbf{u} = [1 \;\; u \;\; u^2 \;\; u^3]^T$

$$\mathrm{b}(u) = \mathrm{M}_H^T \mathbf{u} = \begin{bmatrix} 2u^3 - 3u^2 + 1 \\ -2u^3 + 3u^2 \\ u^3 - 2u^2 + u \\ u^3 - u^2 \end{bmatrix}$$

## Join Points for Hermite Curves

- Match points and tangents (derivatives)




- Much smoother than point interpolation
- How to obtain the tangents?
- Skip Hermite surface patch
- More widely used: Bezier curves and surfaces

## Parametric Continuity

- Matching endpoints ($C^0$ parametric continuity)

$$\mathbf{p}(1) = \begin{bmatrix} p_x(1) \\ p_y(1) \\ p_z(1) \end{bmatrix} = \begin{bmatrix} q_x(0) \\ q_y(0) \\ q_z(0) \end{bmatrix} = \mathbf{q}(0)$$

- Matching derivatives ($C^1$ parametric continuity)

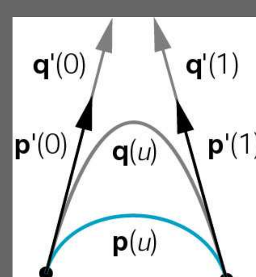
$$\mathbf{p}'(1) = \begin{bmatrix} p'_x(1) \\ p'_y(1) \\ p'_z(1) \end{bmatrix} = \begin{bmatrix} q'_x(0) \\ q'_y(0) \\ q'_z(0) \end{bmatrix} = \mathbf{q}'(0)$$

## Geometric Continuity

- For matching tangents, less is required

$$\mathbf{p}'(1) = \begin{bmatrix} p'_x(1) \\ p'_y(1) \\ p'_z(1) \end{bmatrix} = k \begin{bmatrix} q'_x(0) \\ q'_y(0) \\ q'_z(0) \end{bmatrix} = k\mathbf{q}'(0)$$

- $G^1$ geometric continuity
- Extends to higher derivatives



## Outline

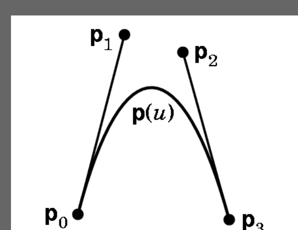
- Parametric Representations
- Cubic Polynomial Forms
- Hermite Curves
- Bezier Curves and Surfaces

## Bezier Curves

- Widely used in computer graphics
- Approximate tangents by using control points

$$p'(0) = 3(p_1 - p_0)$$

$$p'(1) = 3(p_3 - p_2)$$



## Equations for Bezier Curves

- Set up equations for cubic parametric curve
- Recall:

$$\mathbf{p}(u) = \mathbf{c}_0 + \mathbf{c}_1 u + \mathbf{c}_2 u^2 + \mathbf{c}_3 u^3$$
$$\mathbf{p}'(u) = \mathbf{c}_1 + 2\mathbf{c}_2 u + 3\mathbf{c}_3 u^2$$

- Solve for $c_k$

$$\mathbf{p}_0 = \mathbf{p}(0) = \mathbf{c}_0$$
$$\mathbf{p}_3 = \mathbf{p}(1) = \mathbf{c}_0 + \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3$$
$$\mathbf{p}'(0) = 3\mathbf{p}_1 - 3\mathbf{p}_0 = \mathbf{c}_1$$
$$\mathbf{p}'(1) = 3\mathbf{p}_3 - 3\mathbf{p}_2 = \mathbf{c}_1 + 2\mathbf{c}_2 + 3\mathbf{c}_3$$

## Bezier Geometry Matrix

- Calculate Bezier geometry matrix $M_B$

$$\begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix} = \mathrm{M}_B \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} \text{ so } \mathrm{M}_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

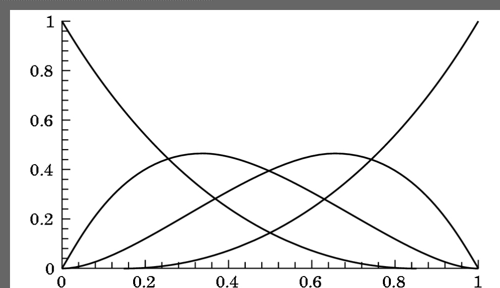
- Have $C^0$ continuity, not $C^1$ continuity
- Have $C^1$ continuity with additional condition

## Blending Polynomials

- Determine contribution of each control point

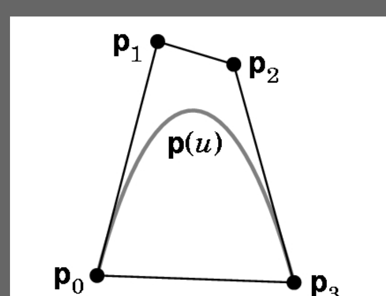
$$\mathrm{b}(u) = \mathrm{M}_B^T \mathbf{u} = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 3u^2(1-u) \\ u^3 \end{bmatrix}$$

Smooth blending polynomials



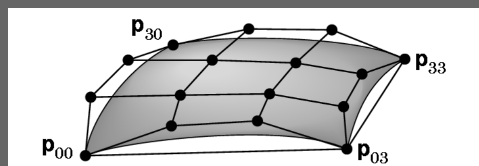

## Convex Hull Property

- Bezier curve contained entirely in convex hull of control points
- Determined choice of tangent coefficient (?)



## Bezier Surface Patches

- Specify Bezier patch with 4 $\times$ 4 control points




- Bezier curves along the boundary

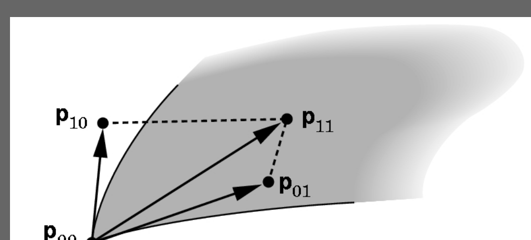
$$\mathbf{p}(0,0) = \mathbf{p}_{00}$$
$$\frac{\partial \mathbf{p}}{\partial u}(0,0) = 3(\mathbf{p}_{10} - \mathbf{p}_{00})$$
$$\frac{\partial \mathbf{p}}{\partial v}(0,0) = 3(\mathbf{p}_{01} - \mathbf{p}_{00})$$

## Twist

- Inner points determine twist at corner

$$\frac{\partial^2 \mathbf{p}}{\partial u\, \partial v}(0,0) = 9(\mathbf{p}_{00} - \mathbf{p}_{01} + p_{10} - p_{11})$$

- Flat means $p_{00}$, $p_{10}$, $p_{01}$, $p_{11}$ in one plane
- $(\partial^2 p/\partial u \partial v)(0,0) = 0$



## Summary

- Parametric Representations
- Cubic Polynomial Forms
- Hermite Curves
- Bezier Curves and Surfaces