1. Let $\hat{n}$ be a normal vector of the plane which $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ lie in.

Then, $\vec{a} \times \vec{b} \parallel \hat{n}$ (parallel)

and $\vec{c} \times \vec{d} \parallel \hat{n}$.

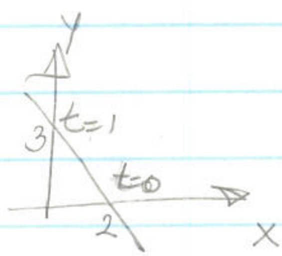
For any vectors $\vec{x}, \vec{y}, \vec{z}$, if $\vec{x} \parallel \vec{y}$ and $\vec{y} \parallel \vec{z}$ then $\vec{x} \parallel \vec{z}$.

Thus, $\vec{a} \times \vec{b} \parallel \vec{c} \times \vec{d}$.

We know that for any vectors $\vec{x} \parallel \vec{y}$, $\vec{x} \times \vec{y} = \vec{0}$.

Thus, $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$ #

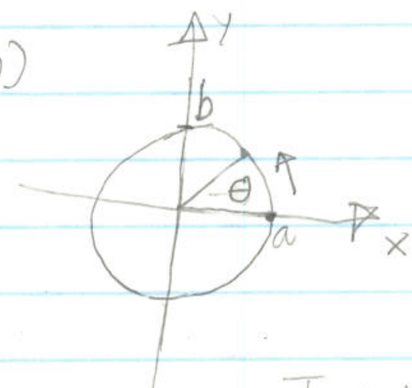
2. a)



We only need to pick 2 sample points to form a parametric equation for a line.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} t$$ #

b)



We know that for an ellipse rotating counterclockwise, its parametric equation is:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a\cos\theta \\ b\sin\theta \end{bmatrix}$$

To make it clockwise, simply set a negative parameter: $t = -\theta$.

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3\cos(-t) \\ 2\sin(-t) \end{bmatrix} = \begin{bmatrix} 3\cos t \\ -2\sin t \end{bmatrix}$$ #

3. Distance between 2 points $p_1(x_1, y_1, z_1)$ and $p_2(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

Thus, equation for the plane is

$$(x-2)^2 + (y-1)^2 + (z-1)^2 = (x-3)^2 + (y-1)^2 + (z-5)^2$$

$$-4x + 2y - 2z + 6 = -6x - 2y - 10z + 35$$

$$2x + 4y + 8z - 29 = 0$$ #

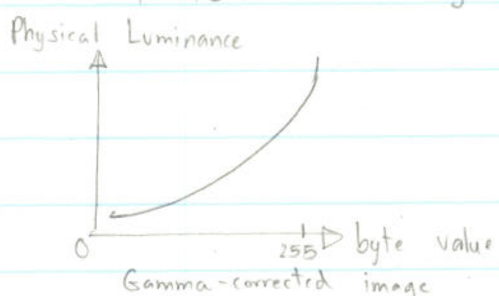
Parametric equation for plane consists of 2 parameters. Here, we pick s and t.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} s \\ t \\ \frac{29}{8} - \frac{s}{4} - \frac{t}{2} \end{bmatrix}$$ #

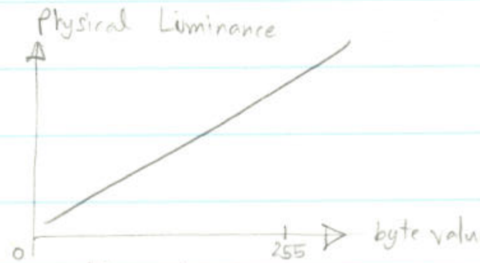
Normal vector of a plane $Ax + By + Cz + D = 0$ is $\begin{bmatrix} A \\ B \\ C \end{bmatrix}$.

Thus, normal vector of this plane is $\begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$ #

4. Consider 24-bit RGB images:

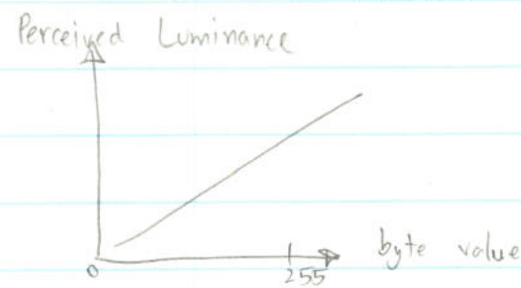


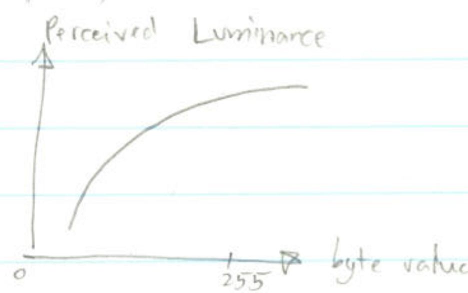
Gamma-corrected image



Linear image

Due to the nature of human's eye perception, we have:



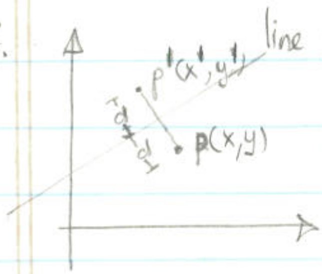


The quality of the images depends on how quickly pixel's byte value changes in relative to the change in perceived luminance. On the graph of byte value versus perceived luminance, the higher slope reflects a lower quality. As we can see from the graph, Gamma-corrected images has better image quality toward the low byte values as its slope is smaller than that of linear image.

5. We first draw the opaque object with the depth buffer on. Then, we use glDepthMark (GL_FALSE) to preserve the depth values. When translucent objects are drawn, their depth values are still compare but will not overwrite the preserved depth value and, as a result, will not eliminate the opaque objects previously drawn.

6. $$\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The matrix rotates 180° around the origin, shears in $-y$ direction by 1. and translates along $+x$ direction by 1. #

7. 

For point $p(x,y)$, its distance from line $y = mx + c$ is

$$d = \frac{mx - y + c}{\sqrt{m^2+1}}$$

and the line drawn from $p(x,y)$ to $p'(x',y')$ has a slope of

$$\text{slope} = -1/m$$

Thus,
$$y' = y - \frac{2d}{\sqrt{m^2+1}} = \frac{(m^2+1)y + 2(mx - y + c)}{m^2+1} = \frac{2mx + (m^2-1)y + 2c}{m^2+1}$$

$$x' = x - \frac{2dm}{\sqrt{m^2+1}} = \frac{(m^2+1)x - 2(mx - y + c)m}{m^2+1} = \frac{(1-m^2)x + 2m(y) - 2mb}{m^2+1}$$

Then,
$$\begin{bmatrix} x' \\ y' \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & 0 & \frac{-2mc}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & 0 & \frac{2c}{1+m^2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ #

8. The tranformation is equivalent to translating the origin to $(x_p, 0, z_p)$, rotating about the origin, and translating back. The matrix is

$$\begin{bmatrix} 1 & 0 & 0 & x_p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_p \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -z_p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & \sin\theta & (1-\cos\theta)x_p - \sin\theta\, z_p \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & (\sin\theta)x_p + (1-\cos\theta)z_p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ #

9. a). Rotation matrix always have the form $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Notice that the $\sin\theta = -\sin(-\theta)$ and $\cos\theta = \cos(-\theta)$.

Thus, the transpose of rotation matrix is the same as its inverse.

For rotation matrix R, $R^T = R^{-1} \Rightarrow R^T R = I_n$.

Using the given fact, we found that any rotation matrix is an orthogonal matrix. #

b). Point rotation: $p' = Mp$.

Normal rotation: $n' = Mn'$. because $M^T = M^{-1}$. for any orthogonal matrix. #