Written Assignment #2
Transforms
15-462 Computer Graphics, Fall 2004

DUE: Tuesday, September 28, just before class

NOTE: Ok, I'm told we need a change of policy as late days **can** be used on the homeworks (although I wouldn't recommend it!) and yet I want them right at the beginning of class. So .. if you want to use late days on the homework, please let me know 24 hours in advance and I will hold off on going through solutions. **Otherwise late homeworks will not be accepted.**

The work must be your own. Please use any resources available to you (the book, the web, etc.), but write up the answers in your own words, explaining all of the steps. Also please cite any external references you use (other than the textbook) to come up with your answers.

Total: 24 points

1. **(3 points) Shirley 5.7.** Describe in words what this 2D transform matrix does:

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2. **(3 points) Shirley 5.8.** Write down the 3 by 3 matrix that rotates a 2D point by angle θ about a point $\mathbf{p} = (x_p, y_p)$.

3. **(3 points) Shirley 5.9.** Write down the 4 by 4 rotation matrix that takes the orthonormal 3D vectors $\mathbf{u} = (x_u, y_u, z_u)$, $\mathbf{v} = (x_v\ y_v, z_v)$ and $\mathbf{w} = (x_w, y_w, z_w)$, to orthonormal 3D vectors $\mathbf{a} = (x_a, y_a, z_a)$, $\mathbf{b} = (x_b, y_b, z_b)$ and $\mathbf{c} = (x_c, y_c, z_c)$. So $M\mathbf{u} = \mathbf{a}$, $M\mathbf{v} = \mathbf{b}$, and $M\mathbf{w} = \mathbf{c}$.

4. **(3 points) Shirley 6.7.** For the eye position $\mathbf{e} = (0, 1, 0)$, a gaze vector $\mathbf{g} = (0, -1, 0)$, and a view-up-vector $\mathbf{t} = (1, 1, 0)$, what is the resulting orthonormal **uvw** basis used for coordinate rotations?

5. **(9 points) Shadows**. OpenGL does not provide shadows. One way to create the shadow of an object onto a surface (e.g., the floor) is to place an imaginary camera at the light source and then calculate the projection of the object onto that surface. In other words, the surface is treated as the image plane. If the object is a single triangle, for example, its projection onto the floor will also (usually) be a triangle. This triangle can then be added to the scene, placed just above the floor. It can be given a dark color (or texture) so that it looks like a shadow.

a. You are given a triangle with vertices **a**, **b**, and **c**. You want to find the shadow of this triangle on the ground plane, located at y=3. Assume a distant (directional) light source, having light rays in the –**y** direction. Give a 4 by 4 matrix to project vertices **a**, **b**, and **c** onto the ground plane in order to create the shadow.

b. Give a 4 by 4 matrix to project vertices **a**, **b**, and **c** onto the same ground plane (y=3) in order to create the shadow if we have a directional light source point in any given direction **d**. (Hint: you can use the parametric equations of lines through the vertices in the **d** direction to figure out how to build the desired matrix.)

c. Give a 4 by 4 matrix (or a series of 4 by 4 matrices) to project vertices **a**, **b**, and **c** onto the ground plane (y=3) if we have a point light source at position **L**. (Hint: this is a standard projection problem with the camera at point **L** and pointing along the –**y** axis.)

6. **(3 points) Shirley 7.1**. Building a BSP tree sometimes requires splitting triangles. Each time a triangle is split, two new triangles are formed. Given N triangles, what is the minimum number of triangles that could end up in the resulting BSP tree? What is the maximum number (given a worst case arrangement of triangles)?