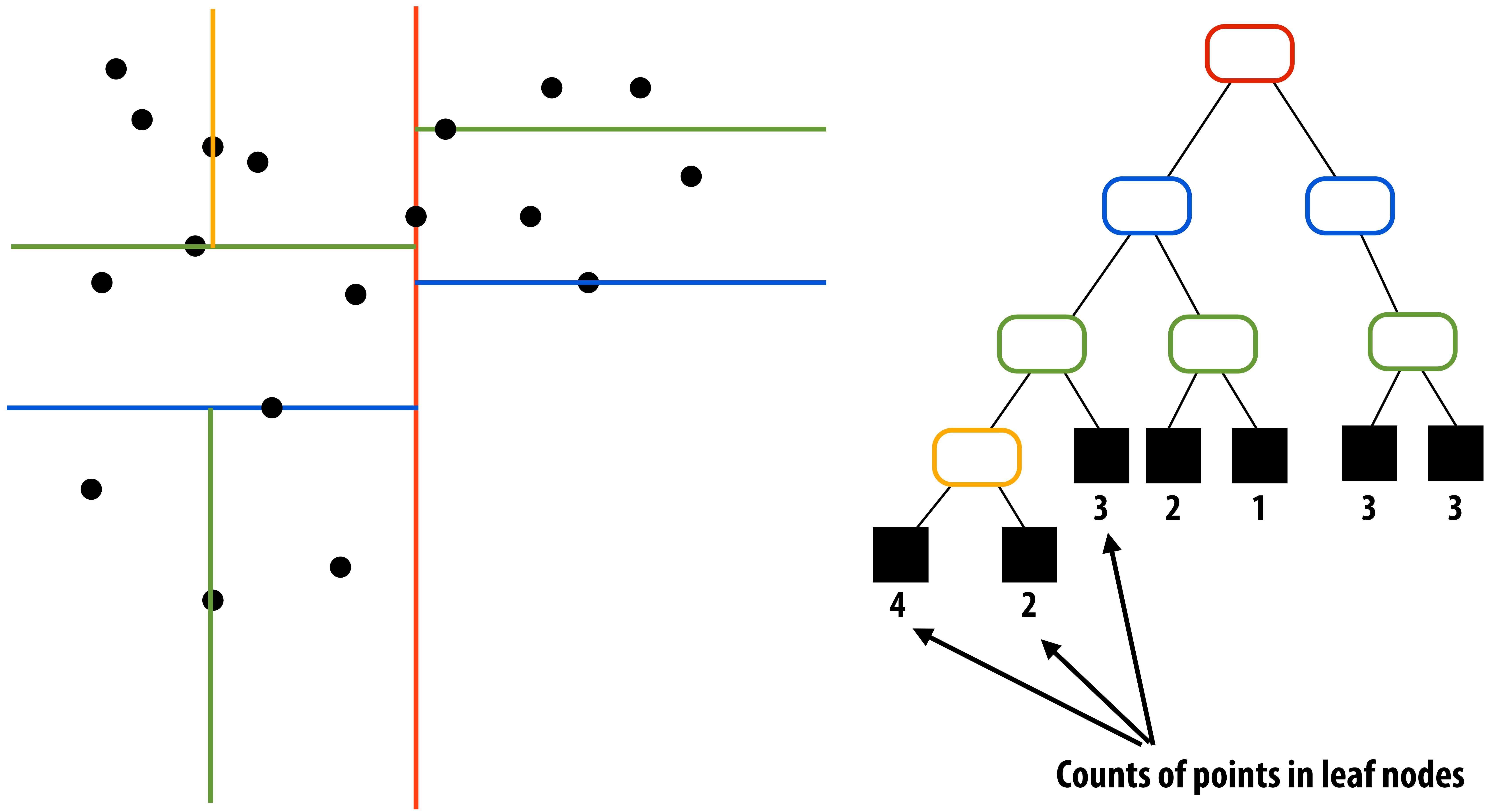
Lecture 24:

Image Retrieval: Part II

Visual Computing Systems
CMU 15-869, Fall 2013

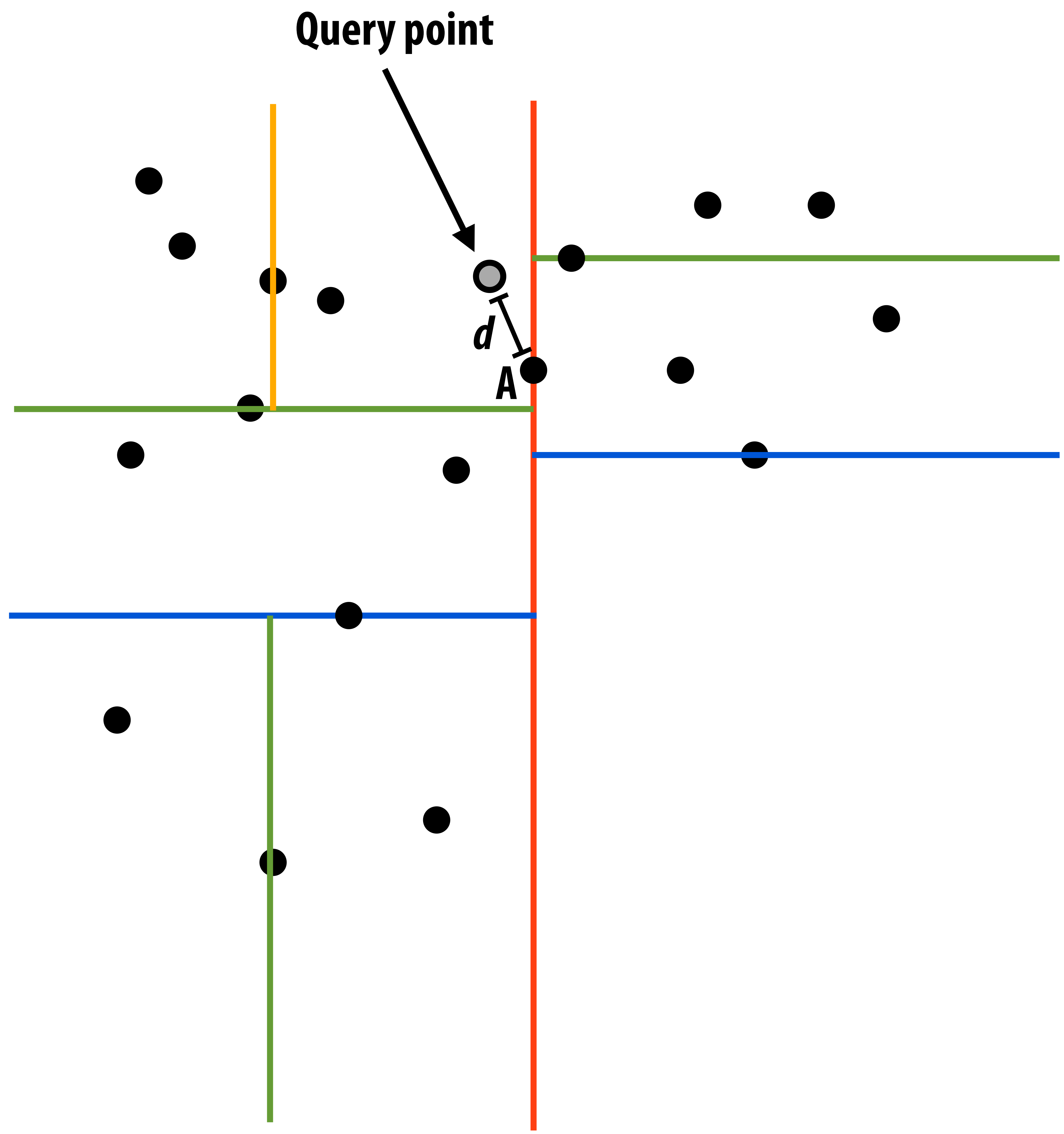
Review: K-D tree

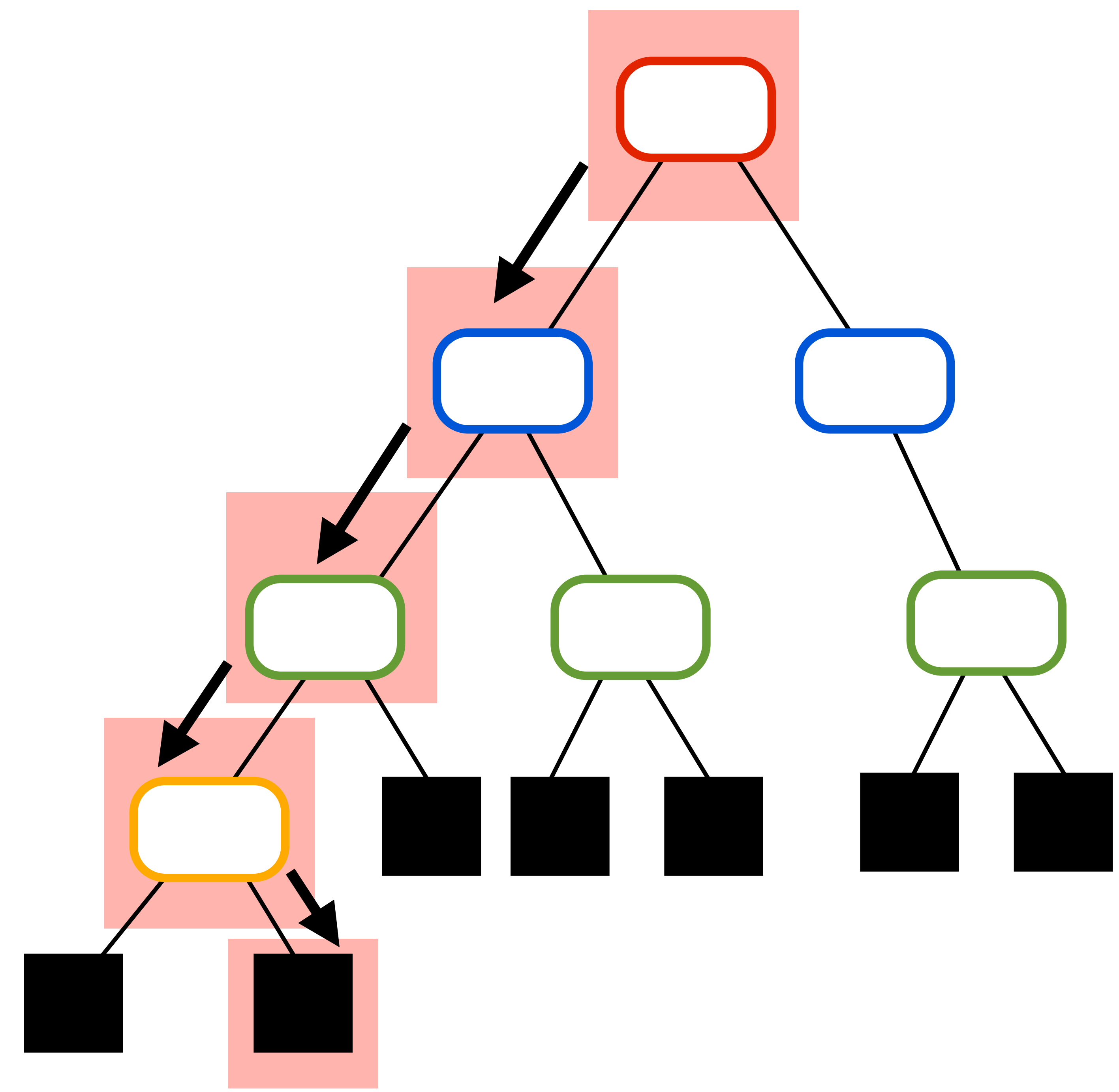
- Spatial partitioning hierarchy
- K = dimensionality of space (below: K = 2)



Nearest neighbor search with K-D tree

Step 1: traverse to leaf cell containing query: compute closest point in this cell to the query.

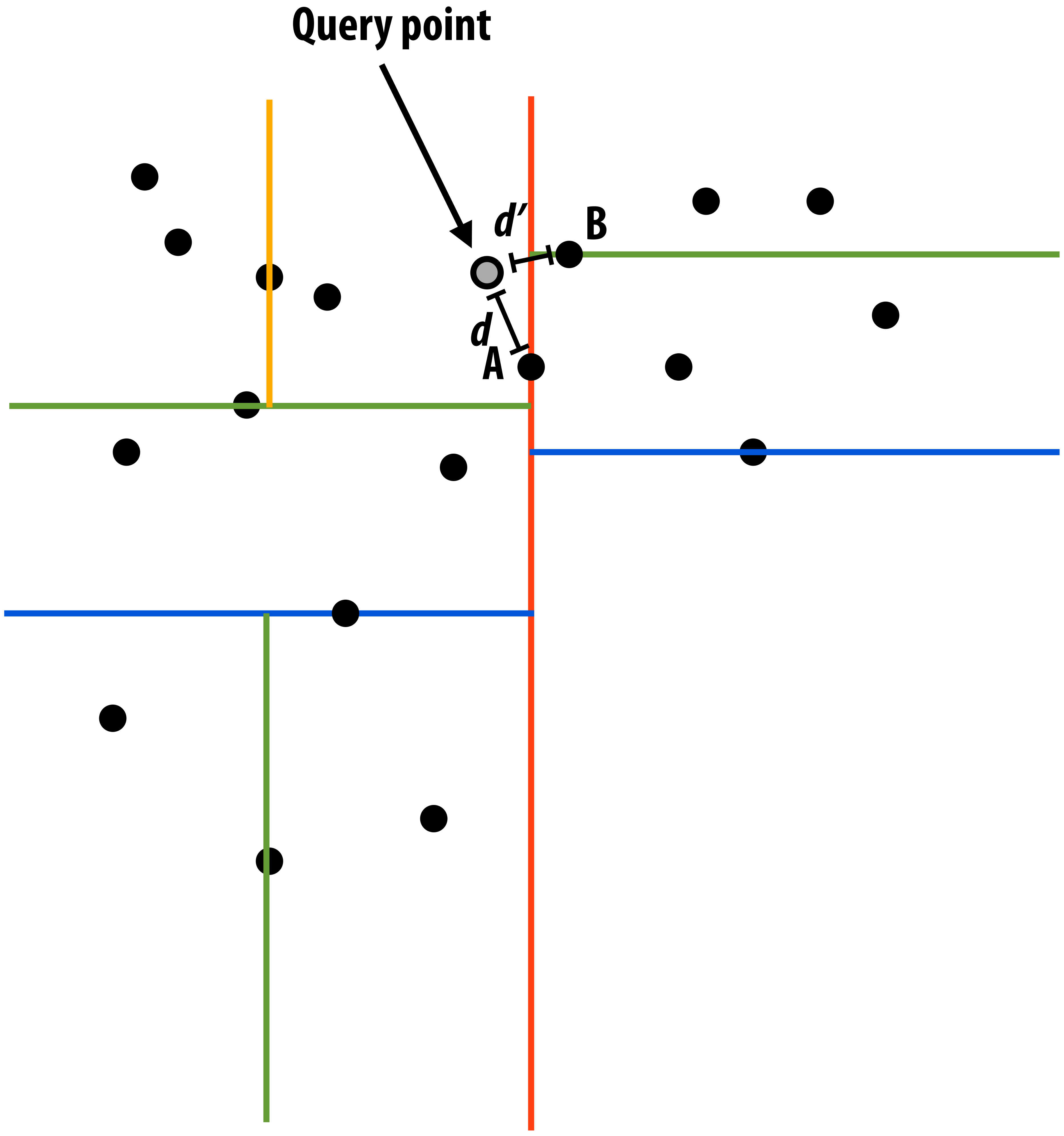


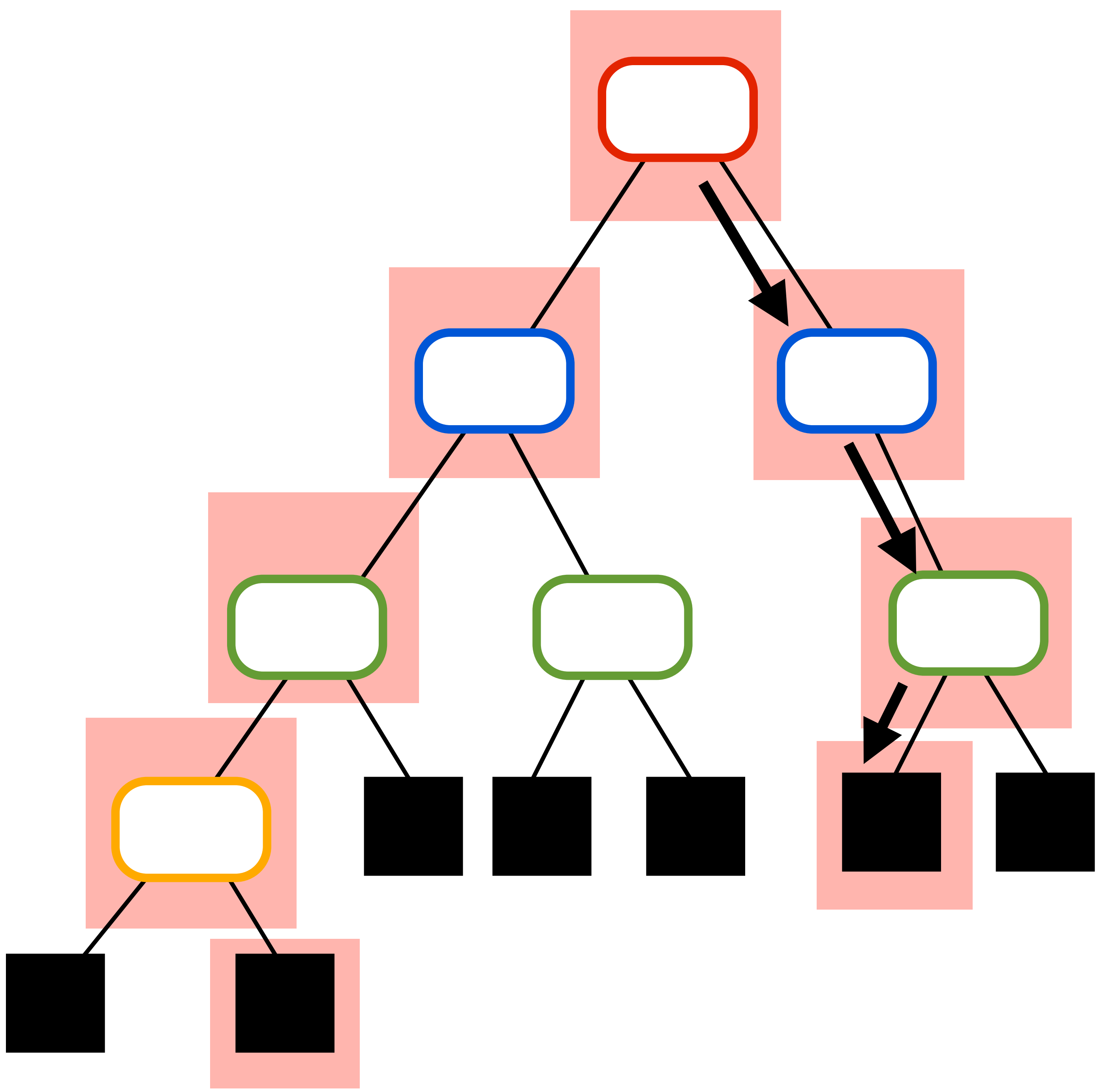


Closest so far: A (at distance *d*)

Nearest neighbor search with K-D tree

Step 2: backtrack: if distance to other cells is closer than distance to closest point found so far, must check points in this cell

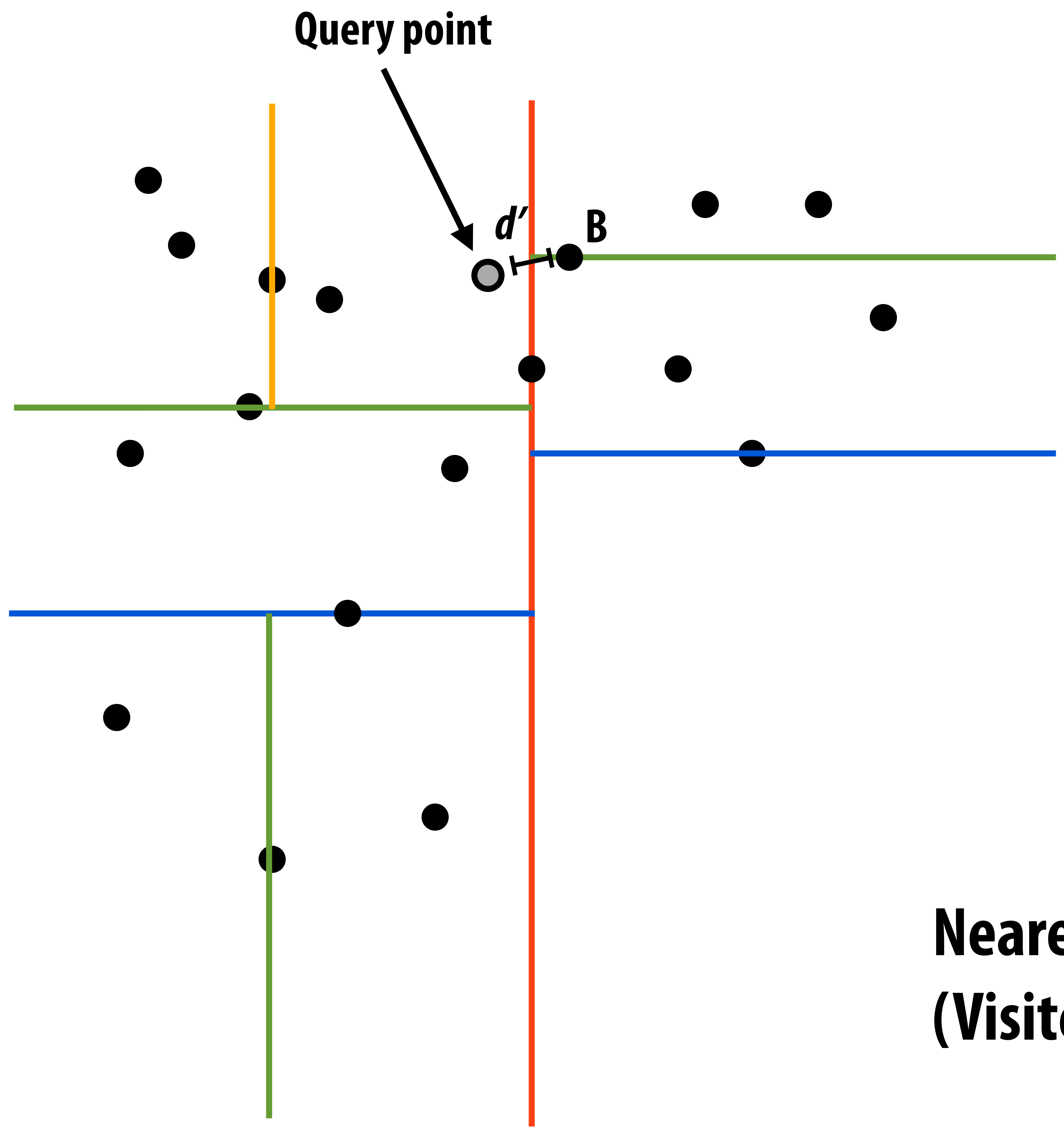


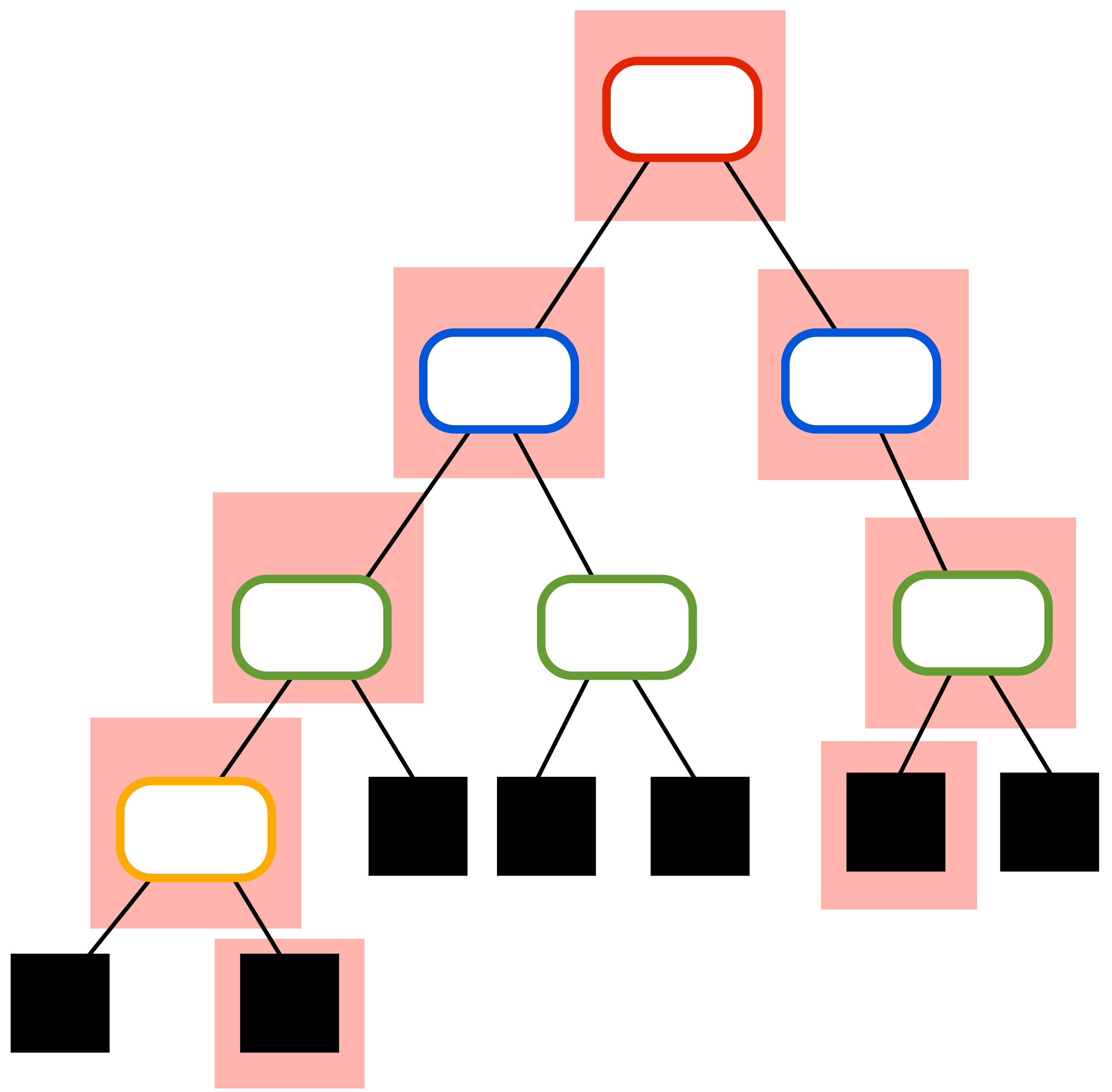


Closest so far: B (at distance *d′*)

Nearest neighbor search with K-D tree

Step 2: backtrack: if distance to other cells is closer than distance to closest point found so far, must check points in this cell

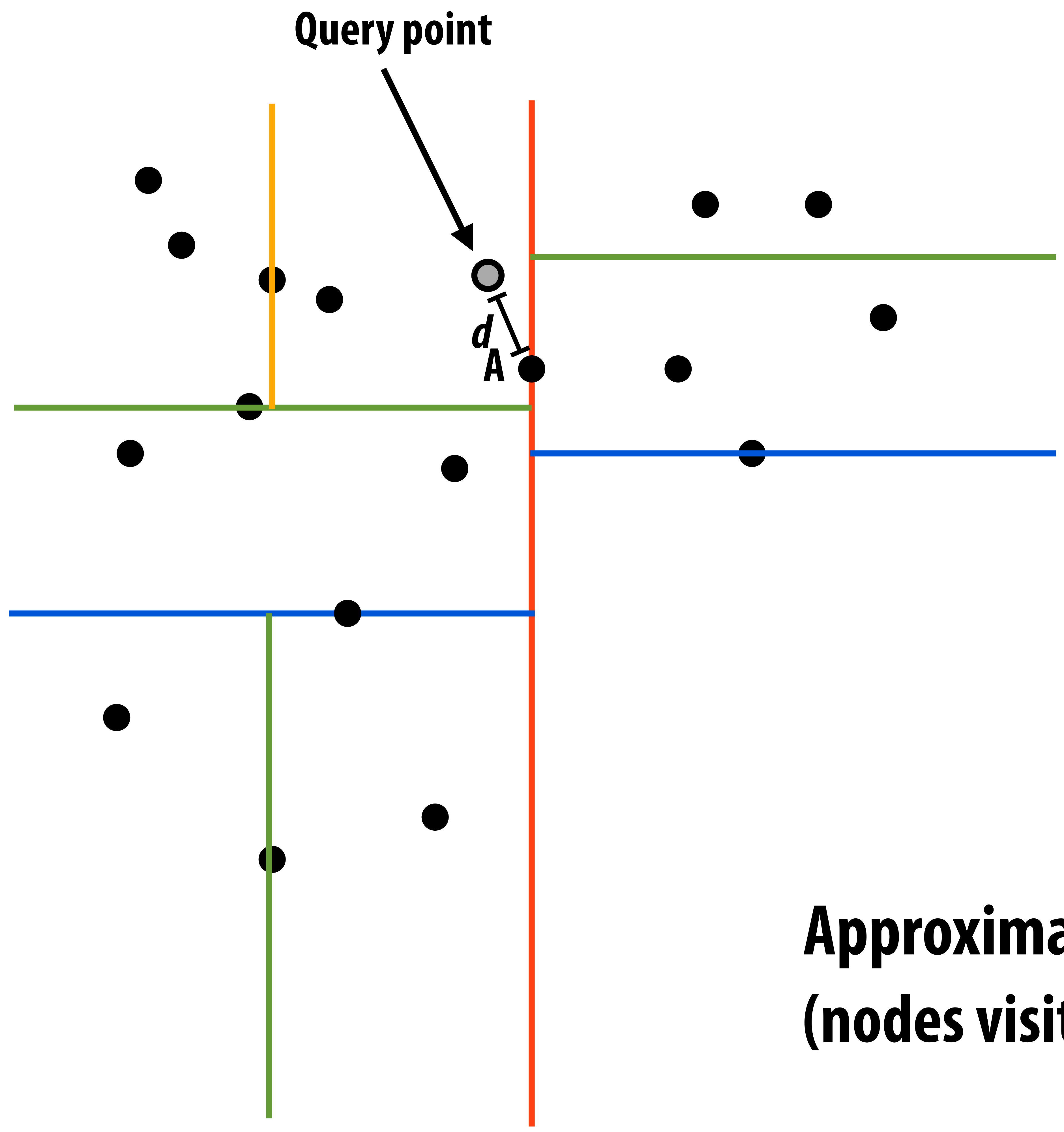


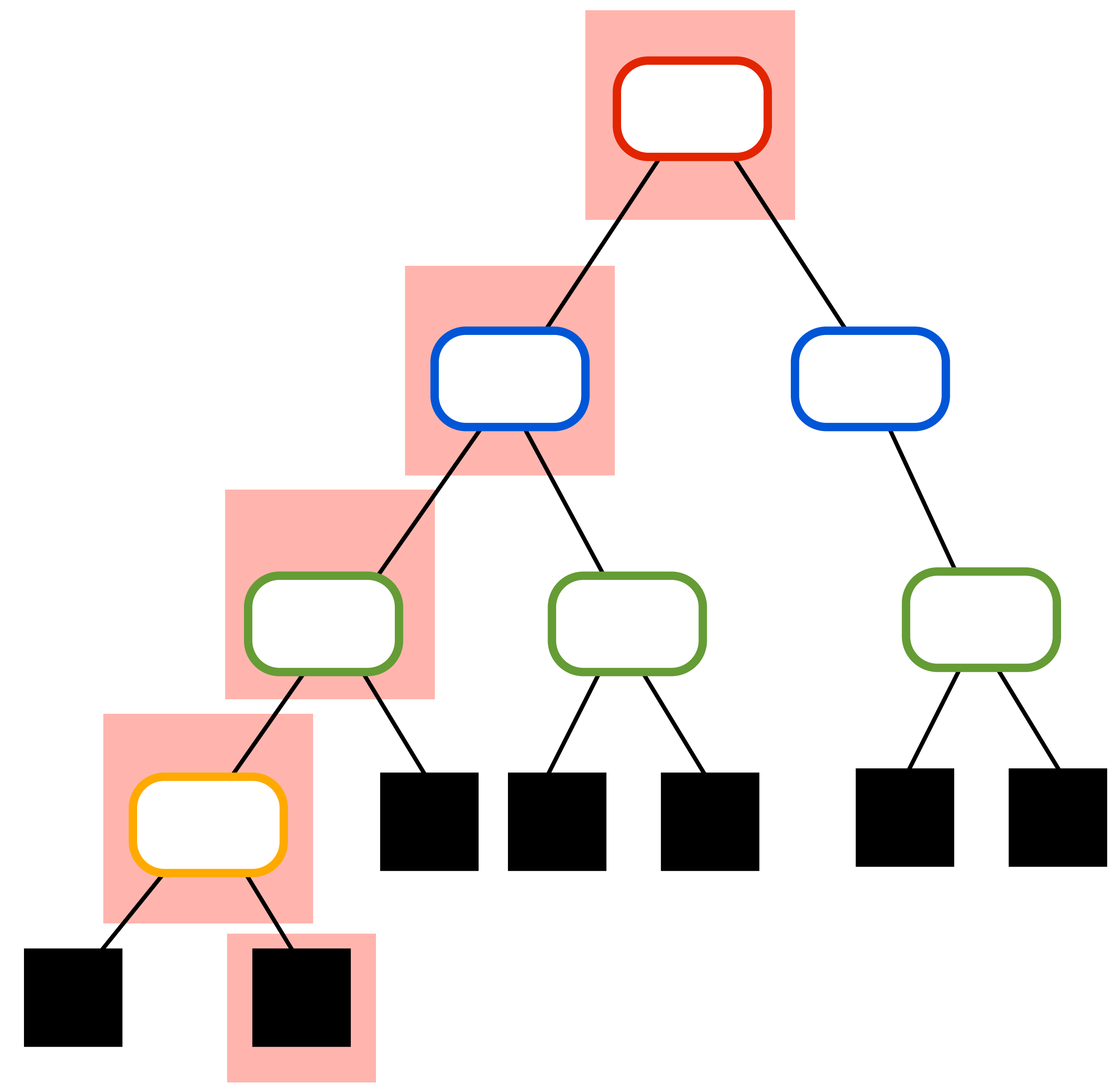


Nearest neighbor result: B (at distance *d′*)
(Visited nodes during query shown in pink)

Approximate nearest neighbor (ANN) search

One simple answer: just take closest point in leaf node containing query

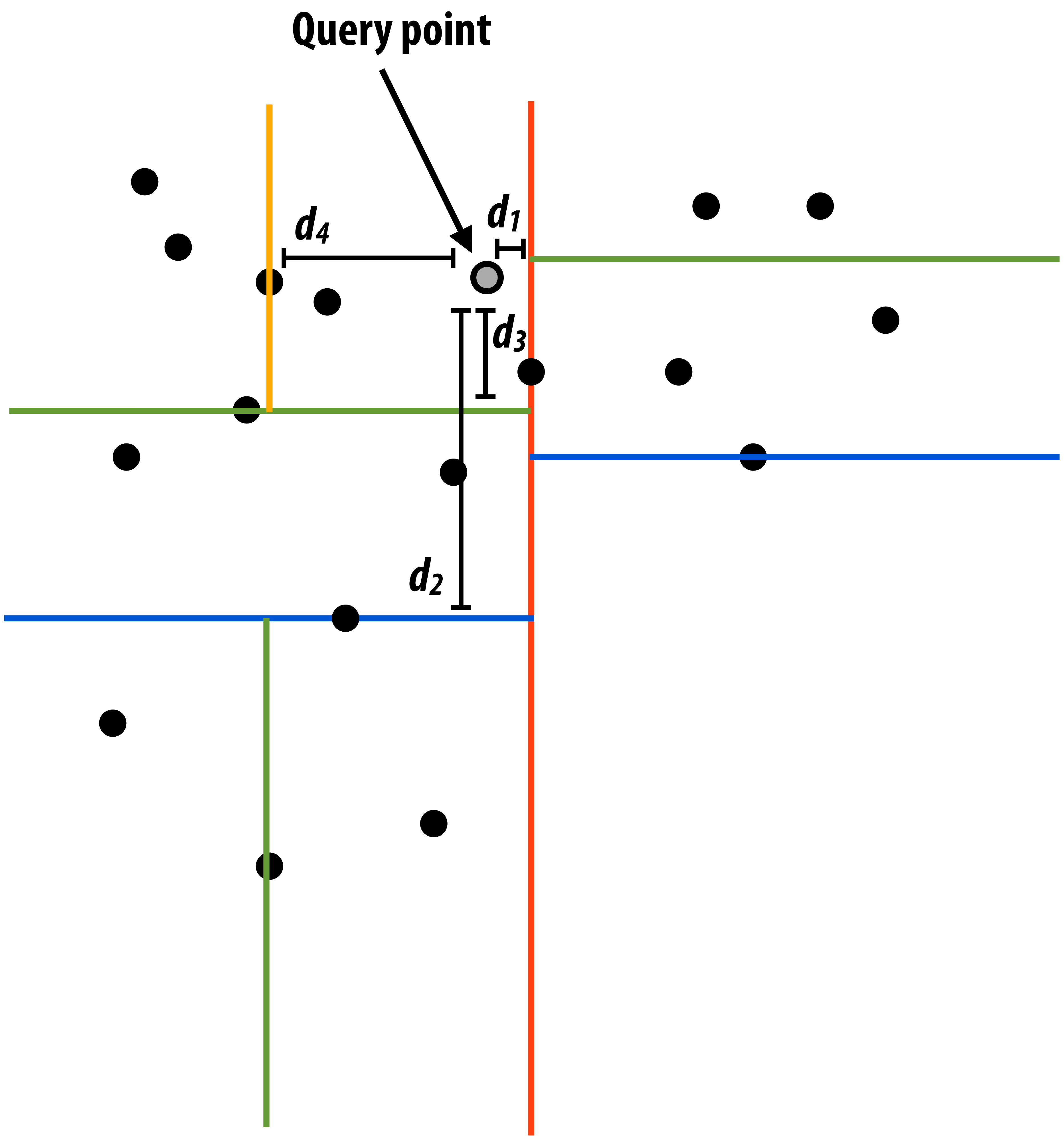


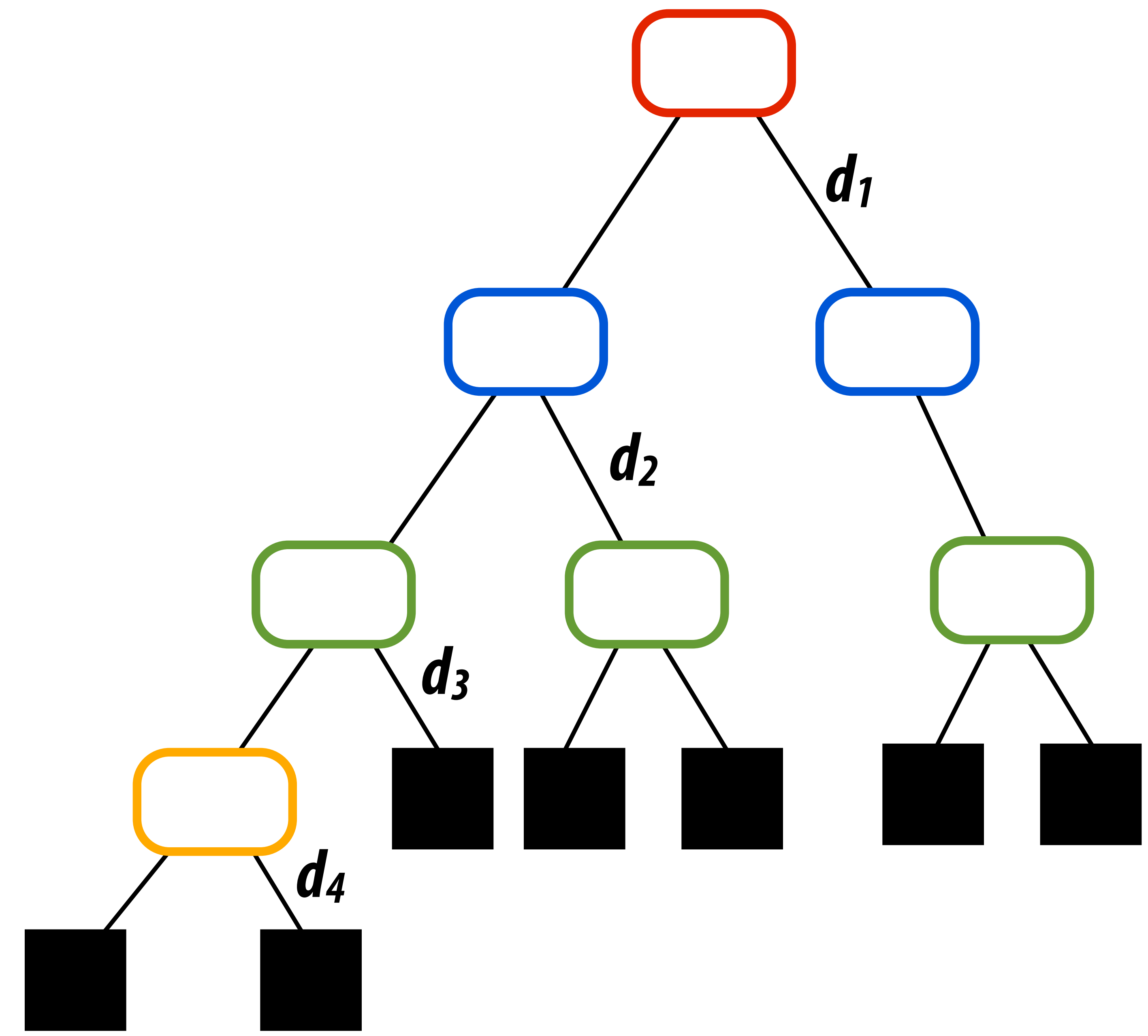


Approximate nearest neighbor: A (at distance *d*)
(nodes visited during query shown in pink)

Approximate nearest neighbor search

Improvement: place nodes in priority queue during downward traversal
Resume downward traversal from closest N nodes to query





Basic K-D tree build

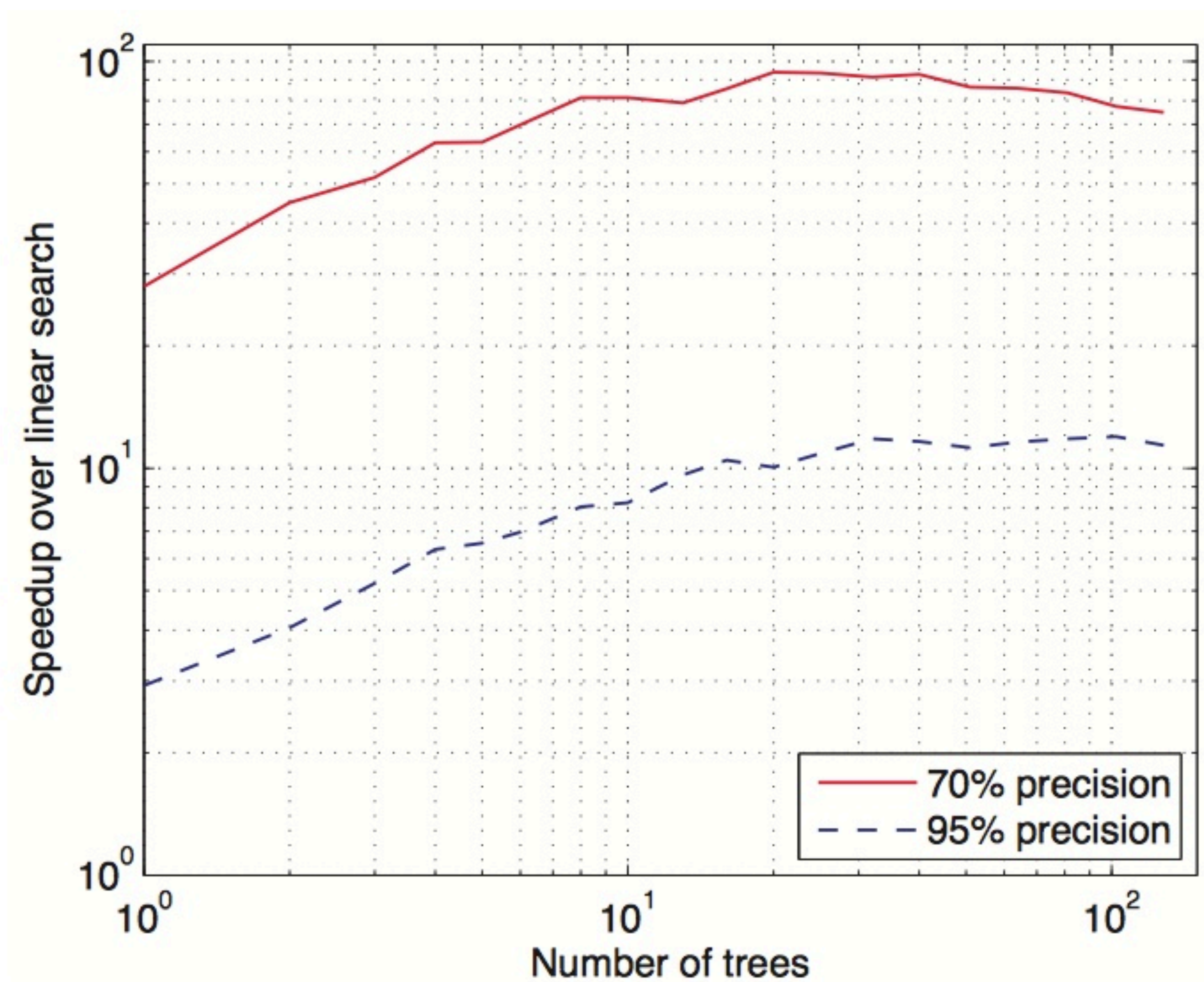
- **To find a partition for a node:**
  - Partition axis for which the variance of current data points is the highest
  - Split at the median of the current data points

Randomized K-D tree

- **To find a partition for a node:**
  - Randomly choose axis to partition
    - Draw from distribution weighted proportionally with variance of current data points is the highest
    - Simple solution: pick partition axis by uniformly sampling from top N axes with highest variance
  - Randomly choose partition point
    - Draw from distribution heavily weighted at the median of the current data points (make it likely to split near the median of the data points)

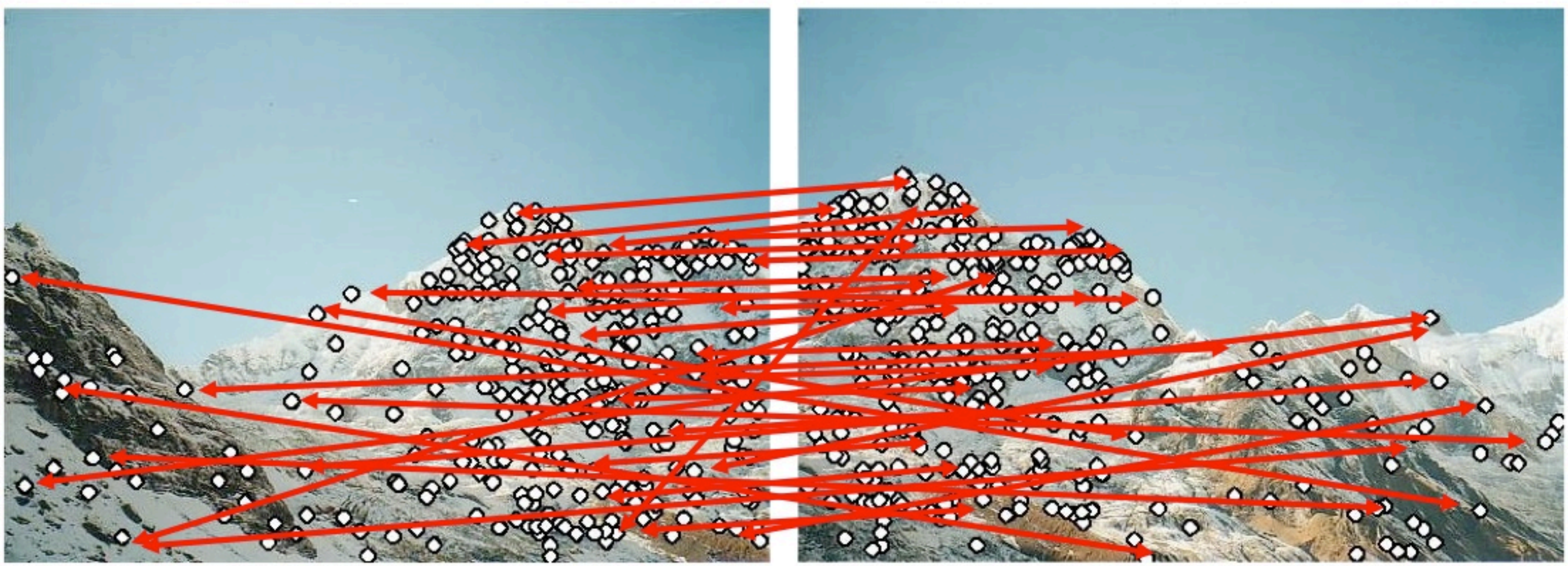
ANN search using a forest of randomized K-D trees

- Construct a set ("forest") of random K-D trees
- For each tree, find NN in leaf cell containing query
  - Add all nodes (across all trees) traversed along the way to a priority queue (node priority = distance from query to node)
- Take closest of all answers across all trees as an initial ANN
- For top D nodes in queue, resume downward search from that node (D = 5 in figure [Muja et al. 2009])
- Solution for approximate k-NN as well



K-D search application: feature correspondence

- Example: SIFT descriptor, K=128
- For all descriptors in image 1, find nearest neighbor in image 2



1 2

Application: approximate K-means clustering *

- **Assign N points to one of K clusters, subject to minimizing distance of points to their cluster centers**

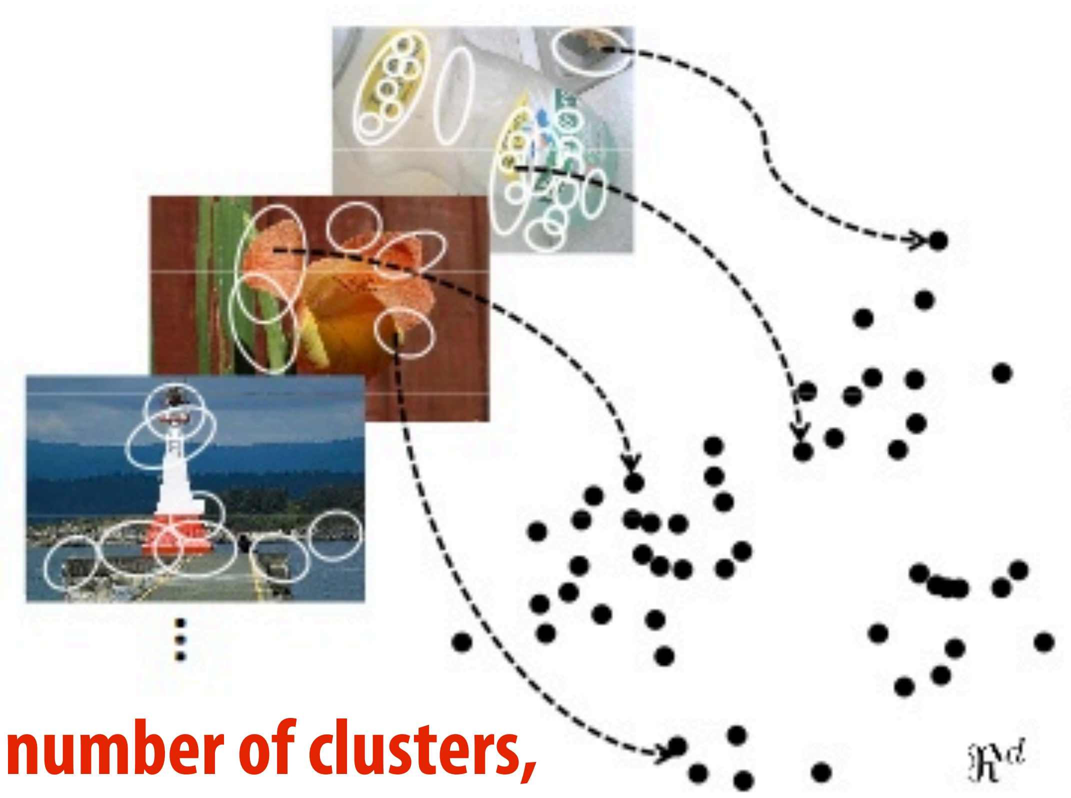
$$\underset{S}{\operatorname{argmin}} \sum_{i=1}^{k} \sum_{j \in S_i} \left\| p_j - \mu_i \right\|^2$$

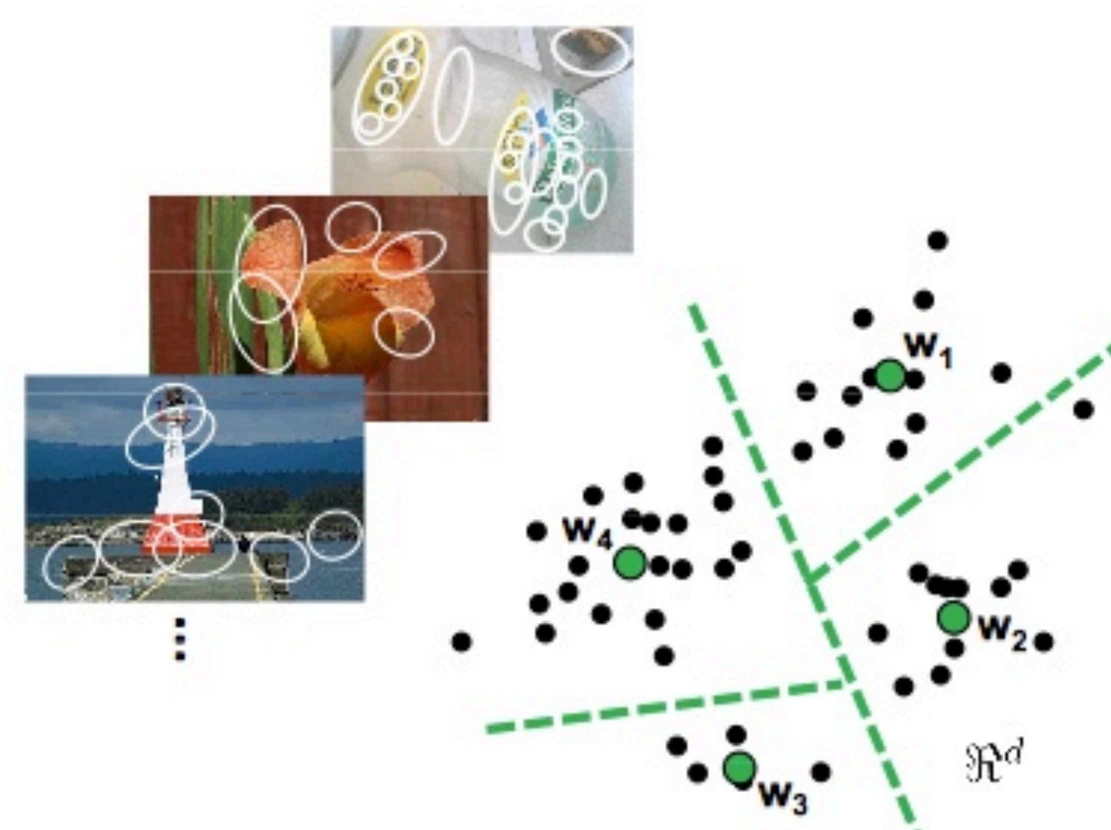
(for p_j in set of points in cluster i (S_i) and cluster center positions μ_i)

- **Basic algorithm: O(kN) per iteration**

```
randomly initialize cluster means
while (assignment of points to clusters continues to change)
  for each point p:
     for each cluster c:
        compute distance between p and mean(c)
     assign P to closest cluster
  recompute cluster means
```

- **Recall: clustering used to compute vocabulary for bag-of-words representation (given all features in database, assign each feature to one of K-clusters)**



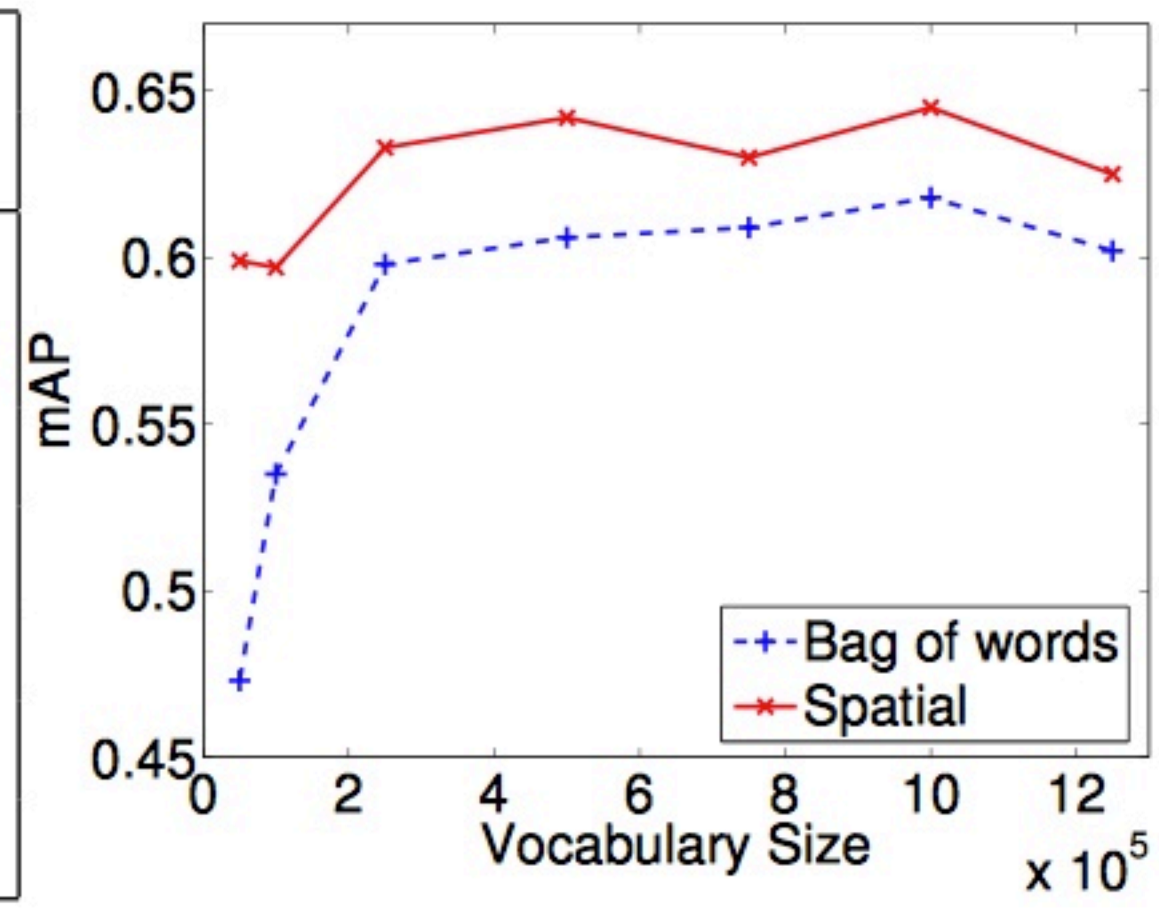


* On this slide: K is the number of clusters, not the dimensionality of the points!!!

Size matters: large vocabularies yield better retrieval performance

- **Consider bag of words implementation:**
  - K = 100,000 to 1,000,000 words
  - N ~ 10's of millions when generating datasets for large vocabularies (train on sampling of descriptors from millions of images)

Vocab Size	Bag of words	Spatial
50K	0.473	0.599
100K	0.535	0.597
250K	0.598	0.633
500K	0.606	0.642
750K	0.609	0.630
1M	**0.618**	**0.645**
1.25M	0.602	0.625



Results from object retrieval task (N = 16.7M for 1M word vocabulary)
mAP = mean average precision (average precision is precision averaged over all recall values)

[Philbin et al. CVPR 2007]

Basic K-means algorithm does not scale to large K

- **Consider bag of words implementation:**
  - K = 100,000 to 1,000,000 words
  - N ~ 1M (sampling of descriptors from millions of images)

- **Approximate K-means:**
  - Replace inner loop on previous slide with ANN search using K-D tree

```
randomly initialize cluster means
while (assignment of points to clusters continues to change)
  construct K-D tree from cluster means
  for each point p:
     use approximate NN search to find closest cluster center
     assign P to closest cluster
  recompute cluster means
```

  - Per-iteration run time: O(N lg k)
  - Enables construction of much larger vocabularies (~1M)

Approx. k-NN application to image retrieval

- **Full representation of database**
  - Search based on actual descriptor values, not quantized values
- **Database:**
  - K-D tree of features appearing in database images
  - e.g., SIFT descriptor: K = 128
- **Search procedure:**
  - Compute SIFT features for query image
  - For each descriptor
    - Find ANN descriptor in database (or k-NN)
    - Add "vote" for image containing feature (e.g., vote weighted by distance)
  - Rank database images by final score

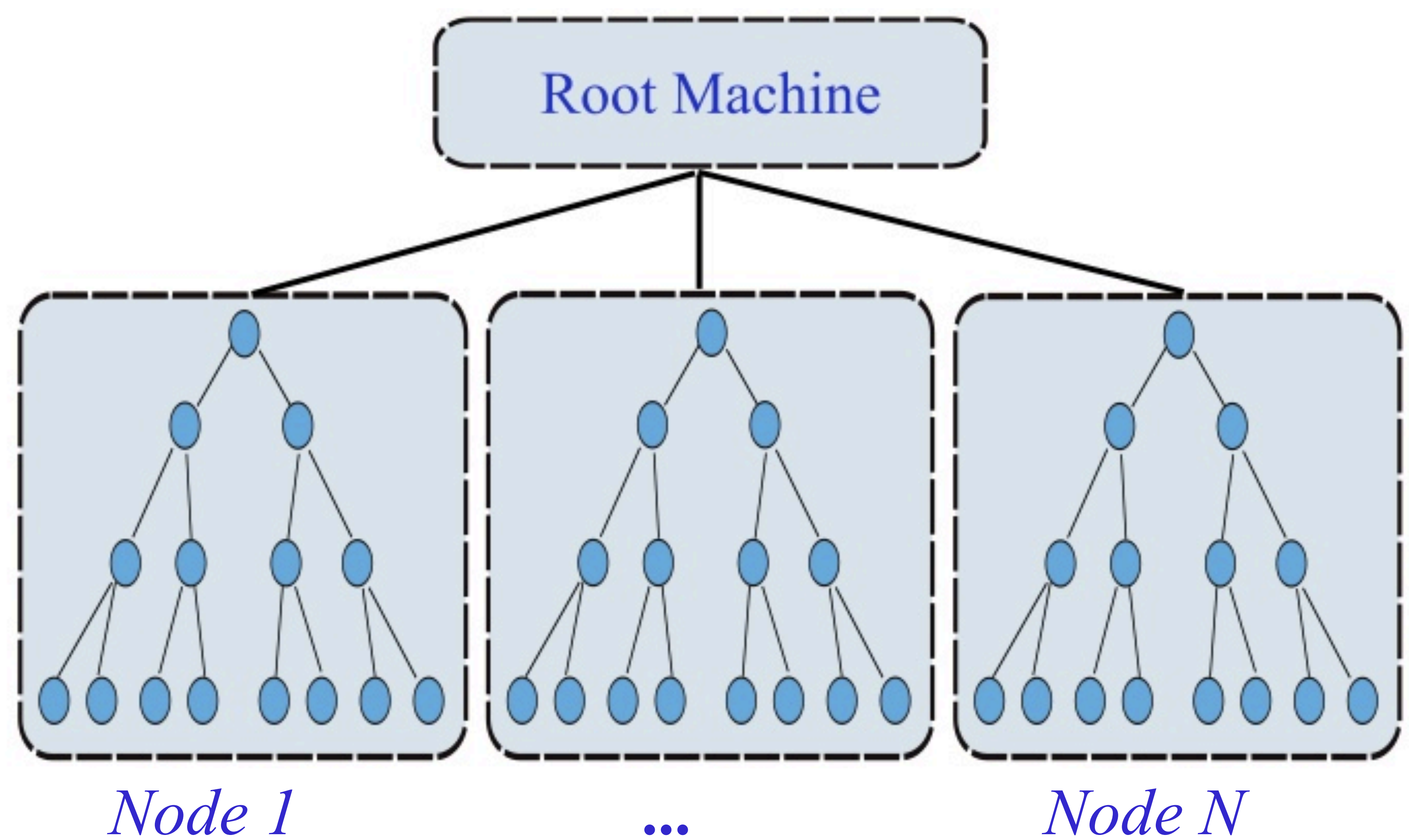
Nearest neighbor image retrieval

- **Good: no quantization of features like in bag of words**
  - Common problem: how many visual words to create?
  - Active research area is design of good vocabulary
- **Cost:**
  - Storage of K-D tree is much larger than inverted index
    - Must store descriptor values, not just a weight (tf-idf) for each descriptor
    - Also store tree structure itself, but this is much less (unless forest gets large)
  - 1 million images, ~1,000 descriptors per image, 128 bytes = 128 bytes per descriptor → 128 GB database!!!

Distributing a search tree

- Simple solution:
  - Partition dataset into chunks of data points that fit in memory on a node
  - Build K-D trees independently and in parallel on all nodes
  - For each query:
    - Broadcast query to all N nodes
    - Run N independent k-NN searches in parallel
    - Broadcast results to a master node
    - Master sorts results to produce overall k-NN

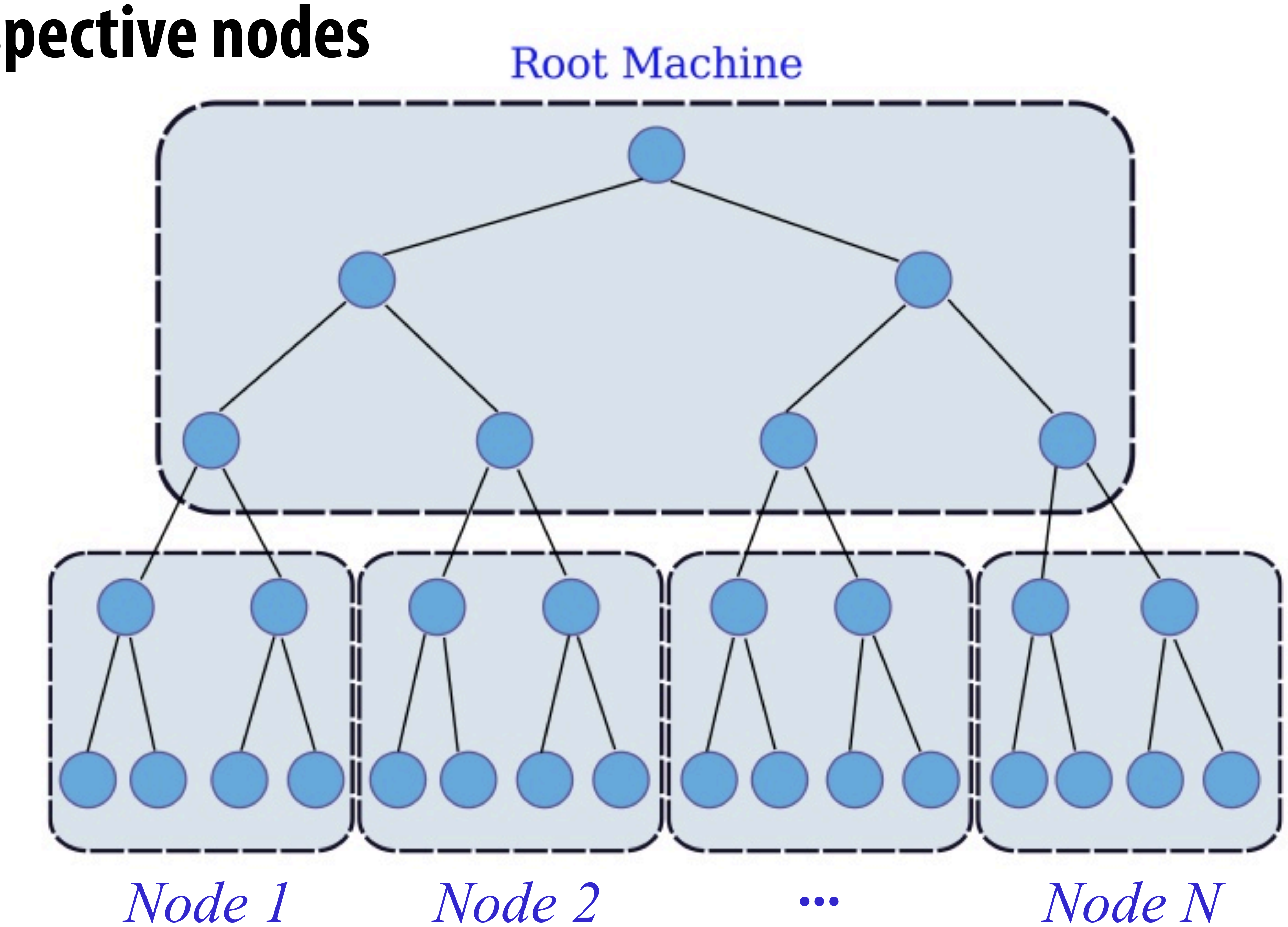
- Problems:
  - Lack of parallelism in the combine results stage
  - Less efficient structure
    - N independent K-D tree lookups
    - Search through single, large K-D tree would visit fewer nodes



[Figure credit: Aly et al. VISAPP 2011]

Distributing a search tree

- Idea: Store top part of tree in master, bottom parts of tree are distributed across nodes
- Tree construction:
  - Build top subtree using sampling of entire dataset that fits in memory
  - Top subtree height must be at least lg(N) (to generate N leaf trees for N machines)
  - For each remaining datapoint:
    - Use search to determine which subtree data belongs to
    - Build leaf trees in parallel on respective nodes



[Figure credit: Aly et al. VISAPP 2011]

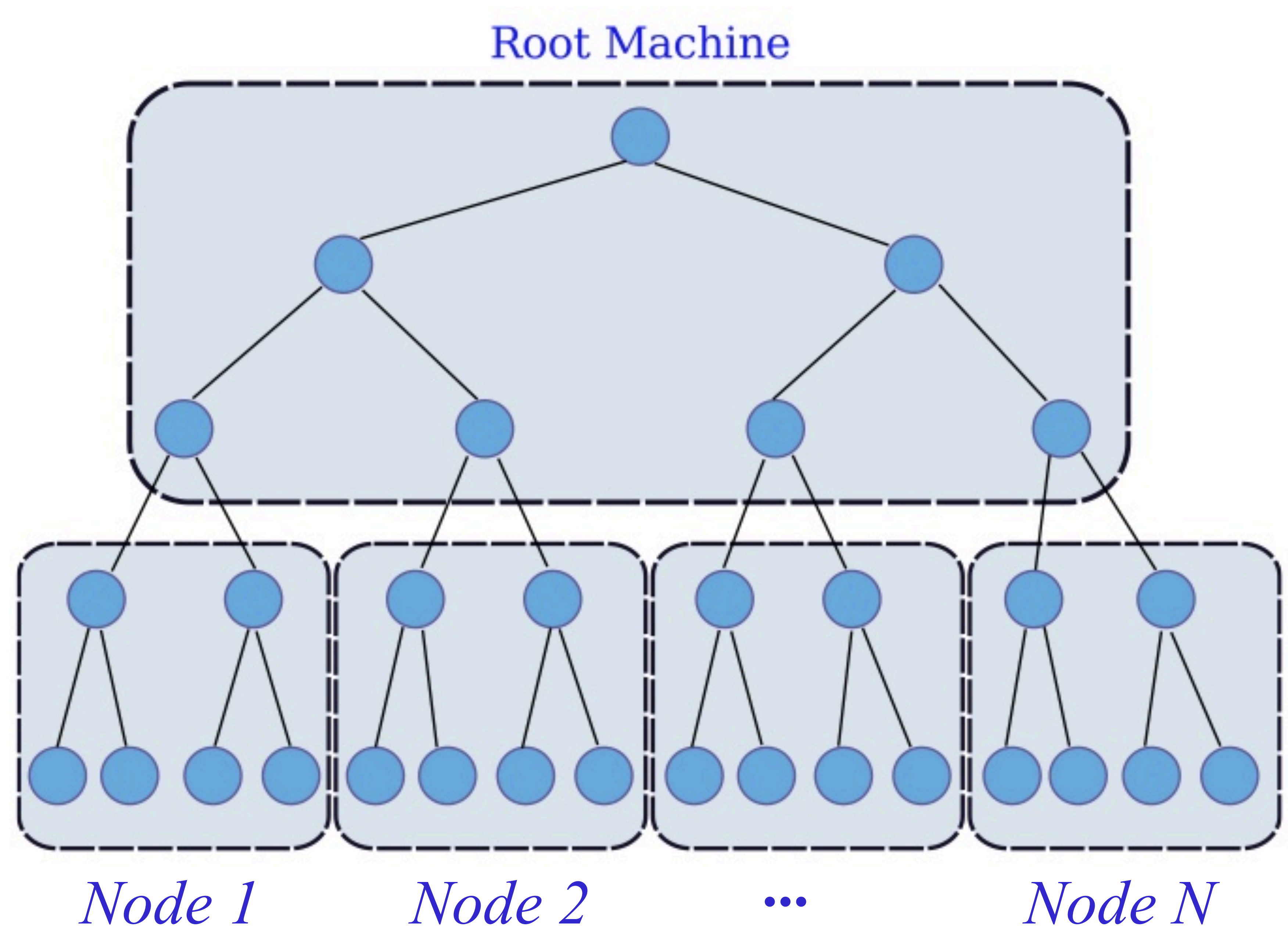
Distributing a search tree

- For each query:
  - Compute features, for each feature:
    - Search top of tree, find all leaf nodes within distance *d* to query
    - Send query to these leaf nodes
    - All leaf nodes carry out search in parallel
  - Send k-NN results back to master for combination

- Good:
  - Efficacy similar to single big tree (each node contains an actual subtree, not a random sampling of data points)

- Bad: serialization of work at root

- Optimizations:
  - Replicate root tree to increase over system throughput (but not individual query latency)



Computational characteristics

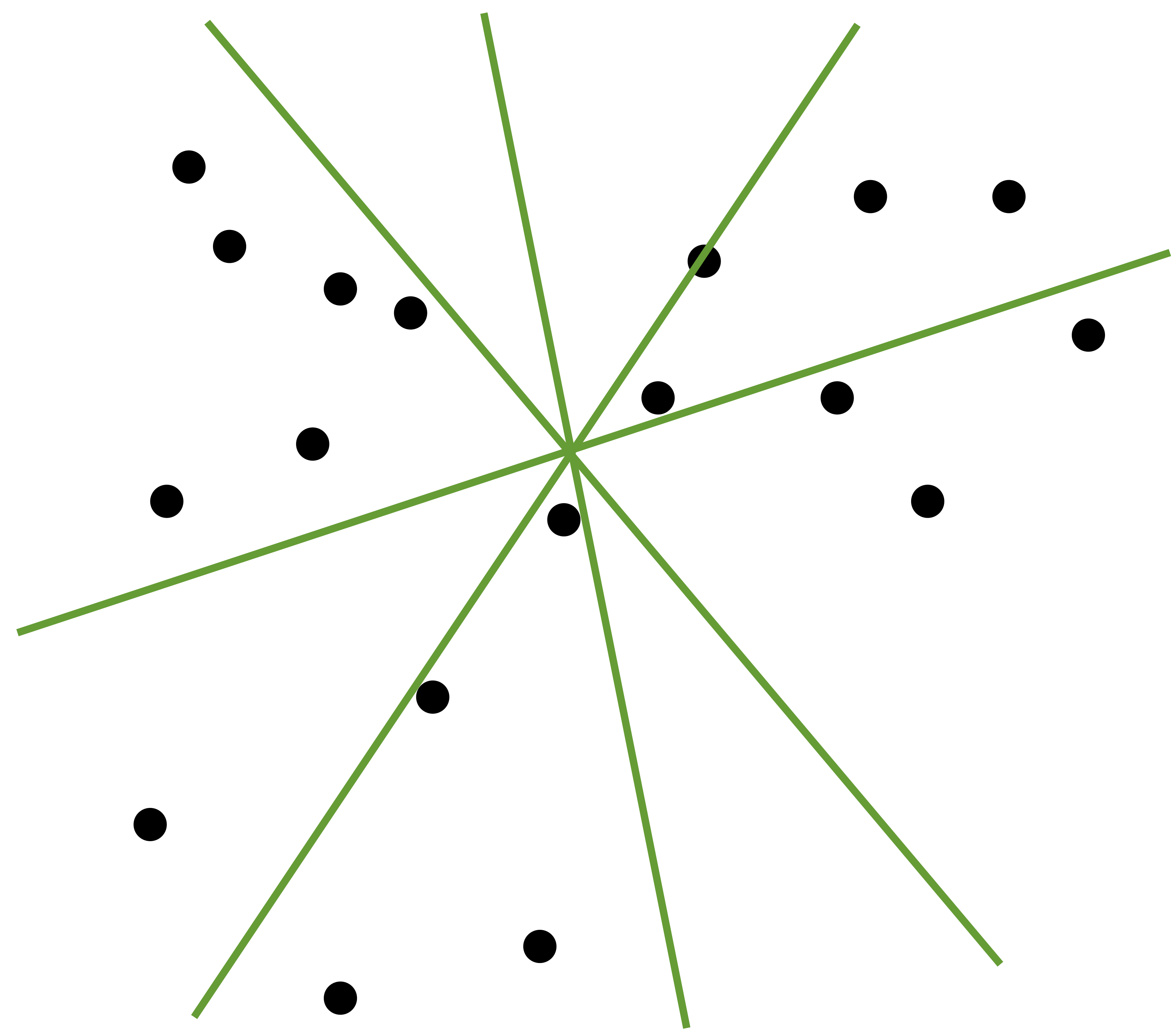
- **Inverted index**
  - Computation:
    - K-d tree lookup to quantize features into words (tree holds cluster centers)
    - Sparse dot products to compute image distances
  - Storage:
    - For each word, maintain list of documents and word TFIDF weight for word in each document: 4 to 8 bytes per descriptor
- **Full representation, approx k-NN search**
  - Computation:
    - K-d tree lookup to find k-NN
    - Dense dot products (e.g., 128-element vector) at the leaves
  - Storage:
    - Must store full descriptor representation (128 bytes for SIFT) for each occurrence
    - Also store tree structure (increasingly significant with a forest of trees)

Locality sensitive hashing

- **Basic intuition:**
  - Hash points into buckets, such that points nearby in space are likely to fall into the same (or nearby) buckets
- **Given x1 and x2 and distance *r***
  - If d(x1,x2) < *r*, then P(h(x1) = h(x2)) is high
  - If d(x1,x2) > α*r*, then P(h(x1) = h(x2) is low

Locality sensitive hashing

- Example: pick *m* random projections
  - For each input query, hash into *m* different hash keys (associated with *m* different hash tables)
  - Union of data points from matching bins is candidate nearest neighbor set
    - Compute full distance function on these points



Locality sensitive hashing (as an embedding)

- Example: pick *m* random projections
  - For each input query, compute 1 bit per projection
  - Query now reduced to m-bit string
  - 1 hash table containing (m-bit keys)
  - Check all hash bins with hamming distance similar to query!

Note: much better ways to determine set of hash functions than random projections (Learn them from the data)

Image retrieval summary

- **Key issues at scale:**
  - Quality of results
  - Speed of query
  - Space footprint of index