Lecture 5:

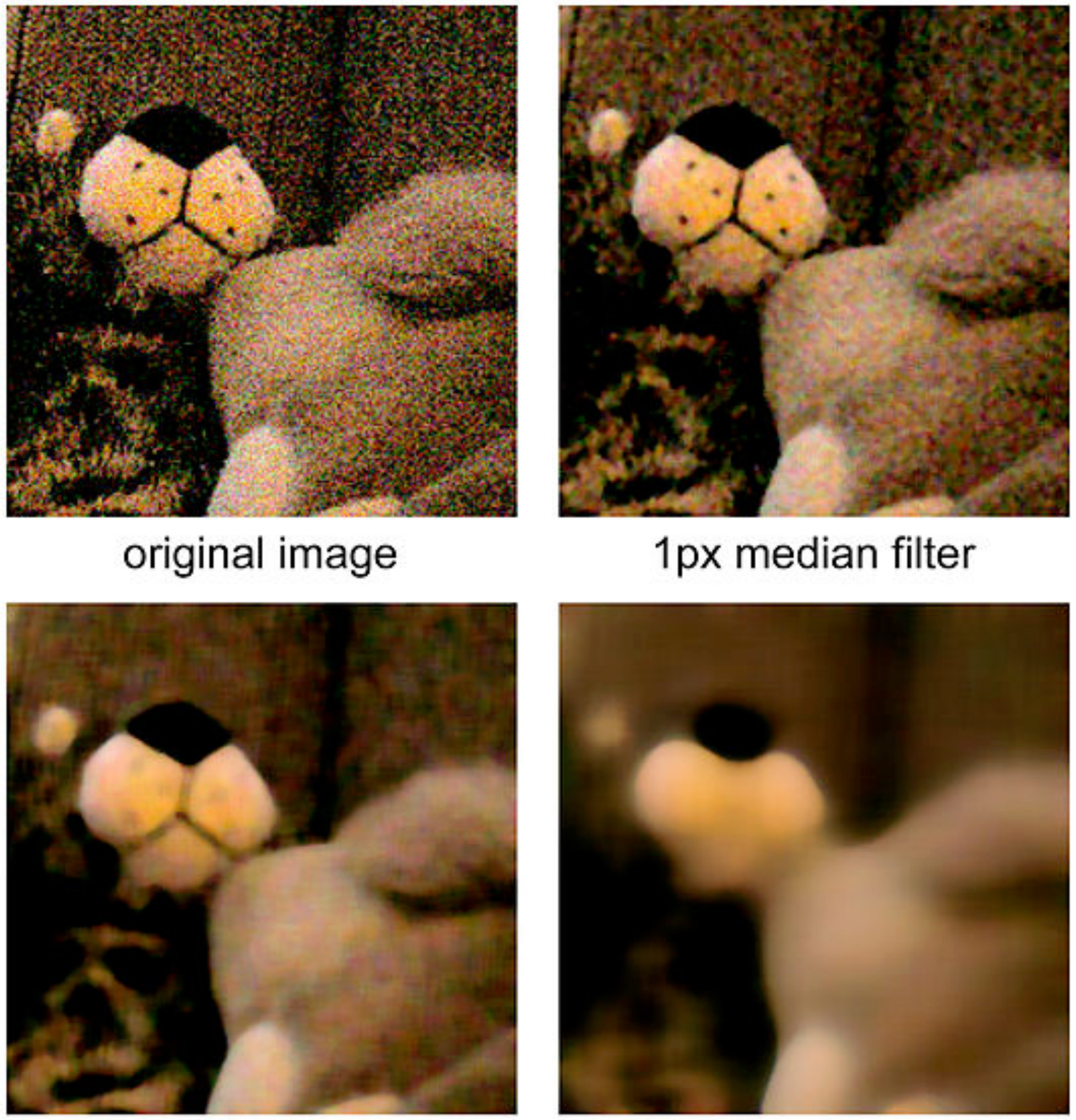
Image Processing Algorithm Grab Bag

Visual Computing Systems

CMU 15-769, Fall 2016

Median filter

- **Noise reduction filter**
  - **Unlike gaussian blur, one bright pixel doesn't drag up the average**
- **Not linear, not separable**
  - **Filter weights are 1 or 0 depending on image content**
- **Naive algorithm for width-N square kernel support region:**
  - **Sort N^2 elements in support region, pick median: $O(N^2 \log(N^2))$ work per pixel**



original image

1px median filter

3px median filter

10px median filter

5x5 median filter

- $O(N^2)$ work-per-pixel solution for 8-bit pixel data (radix sort 8 bit-integer data)
  - Bin all pixels in support region, then scan histogram bins to find median

```
int WIDTH = 1024;
int HEIGHT = 1024;
uint8 input[(WIDTH+2) * (HEIGHT+2)];
uint8 output[WIDTH * HEIGHT];
int histogram[256];

for (int j=0; j<HEIGHT; j++) {
  for (int i=0; i<WIDTH; i++) {
    // construct histogram of support region
    for (int ii=0; ii<256; ii++)
      histogram[ii] = 0;
    for (int jj=0; jj<5; jj++)
      for (int ii=0; ii<5; ii++)
         histogram[input[(j+jj)*(WIDTH+2) + (i+ii)]]++;

    // scan the 256 bins to find median
    // median value of 5x5=25 elements is bin containing 13th value
    int count = 0;
    for (int ii=0; ii<256; i++) {
       if (count + histogram[ii] >= 13)
         output[j*WIDTH + i] = uint8(ii);
       count += histogram[ii];
    }
  }
}
```

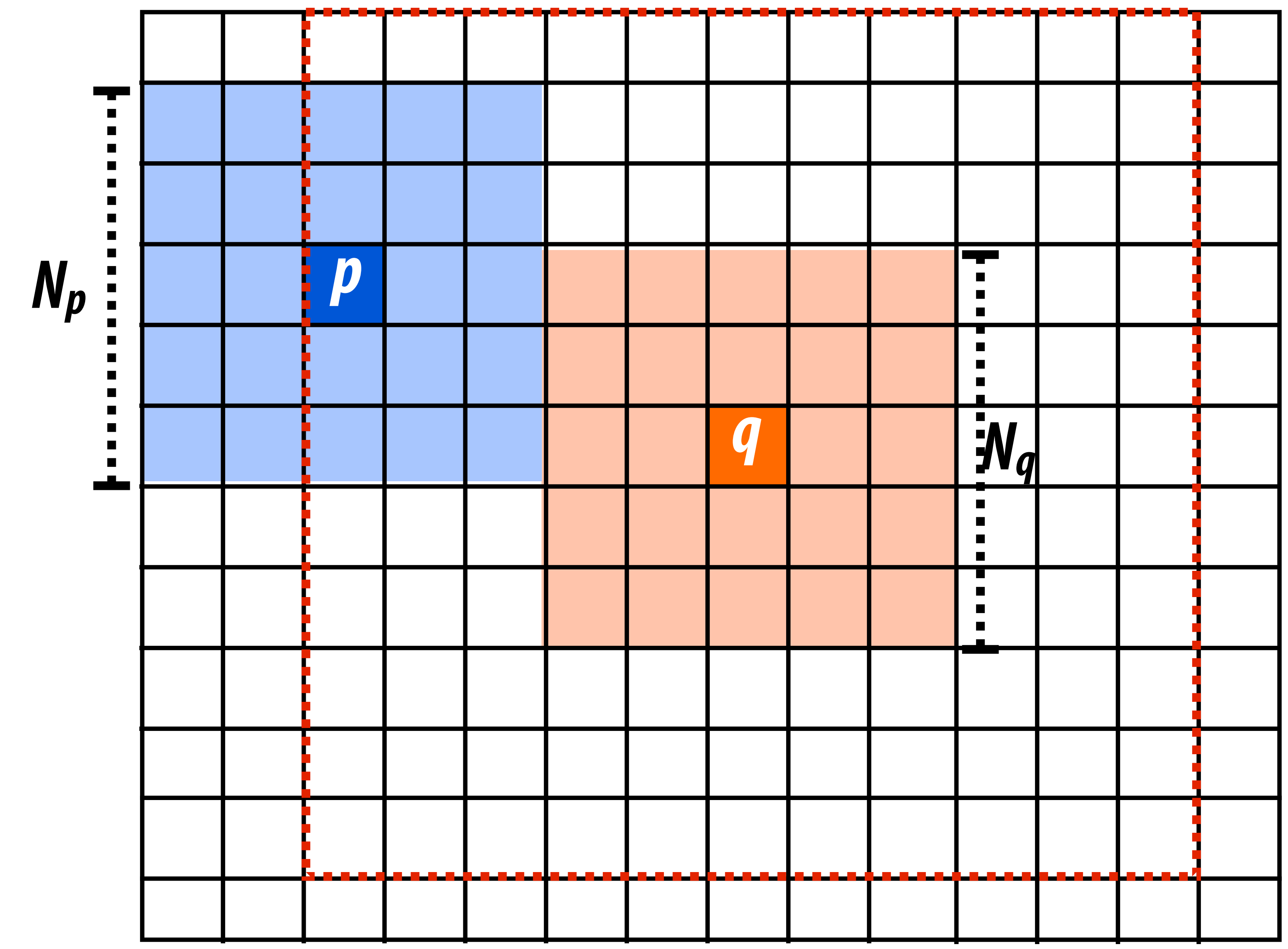
See Weiss [SIGGRAPH 2006] for O(lg N) work-per-pixel median filter (incrementally updates histogram)

Denoising using non-local means

- Main assumption: images have repeating texture
- Main idea: replace pixel with average value of nearby pixels that have a similar surrounding region

$$\mathrm{NL}[I](p) = \sum_{q \in S} w(p,q) I(q)$$

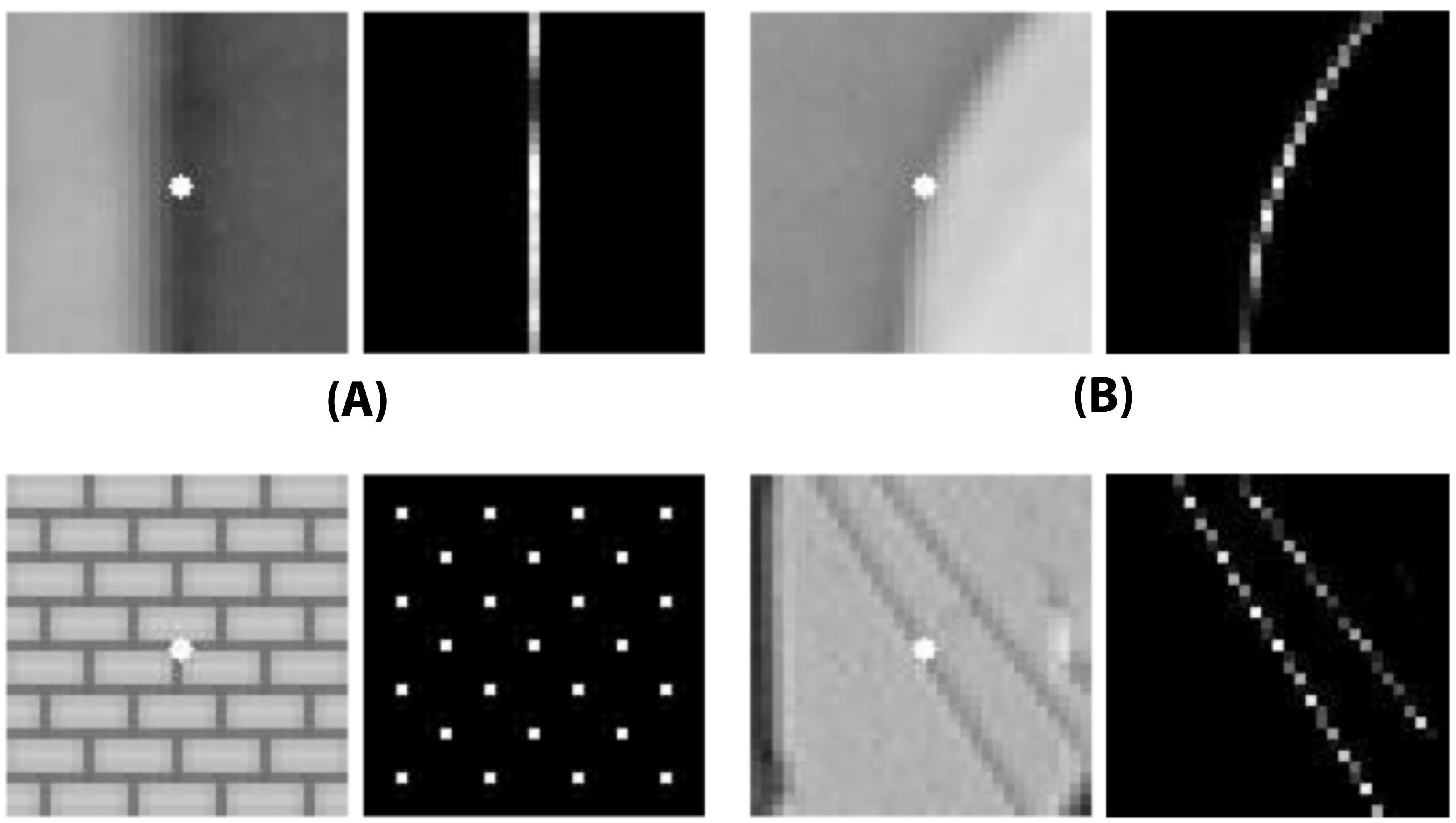
$$w(p,q) = \frac{1}{C_p} e^{\frac{-\| N_p - N_q \|^2}{h^2}}$$



- N_p and N_q are vectors of pixel values in square window around pixels p and q (highlighted regions in figure)
- Difference between N_p and P_q = "similarity" of surrounding regions (here: L2 distance)
- Cp is a normalization constant to ensure weights sum to one for pixel p.
- S is the search region (given by dotted red line in figure)

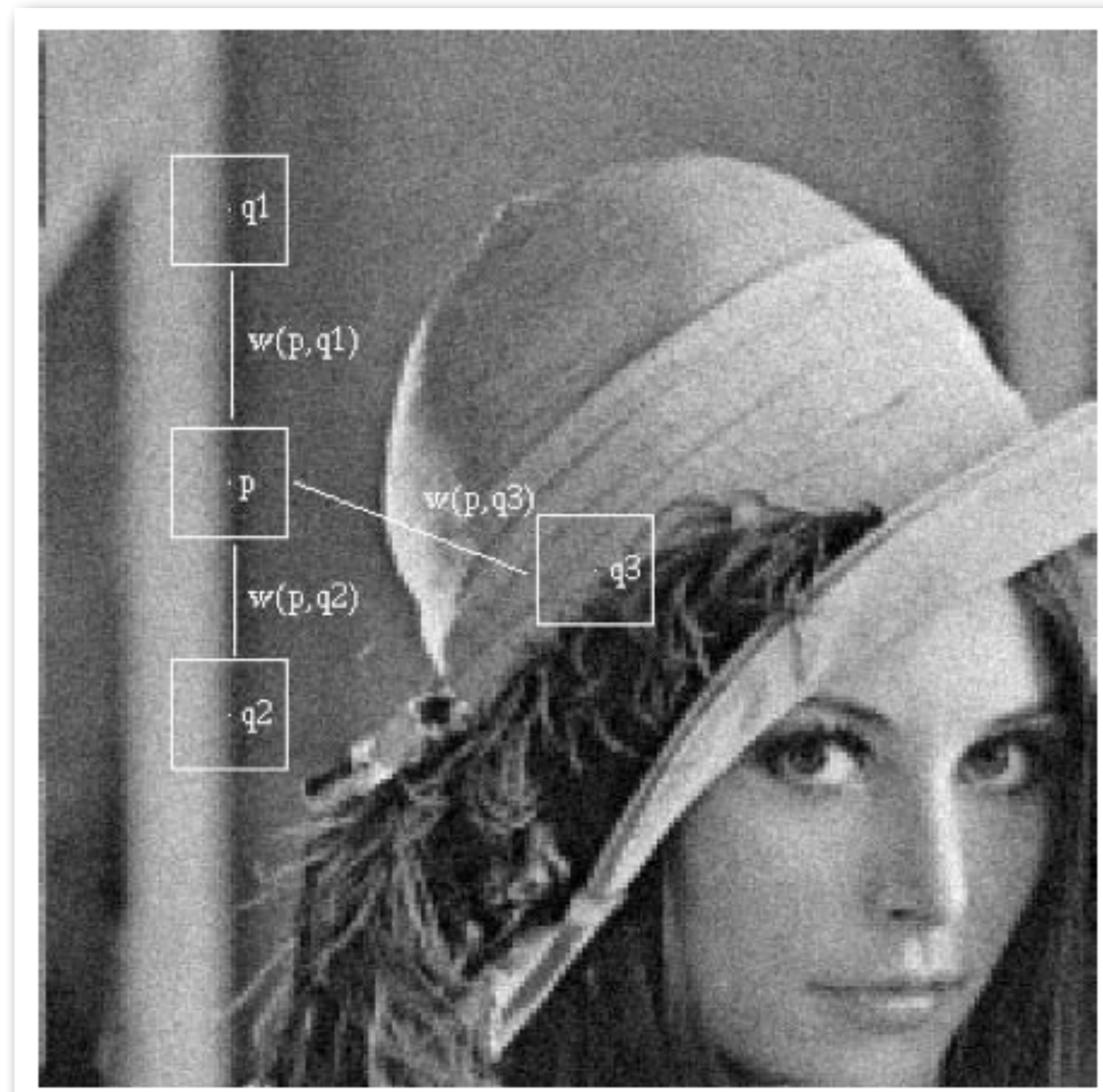
Denoising using non-local means

- **Large weight for input pixels that have similar neighborhood as *p***
  - Intuition: "filtered result is the average of pixels like this one"
  - In example below-right: *q1* and *q2* have high weight, *q3* has low weight



(A) (B) (C) (D)

In each image pair above:
- Image at left shows the pixel to denoise.
- Image at right shows weights of pixels in 21x21-pixel kernel support window.



Buades et al. CVPR 2005

Recall from last week: bilateral filter

$$\mathrm{BF}[I](p) = \frac{1}{W_p} \sum_{q \in S} f(|I_p - I_q|) G_\sigma(\|p - q\|) I(q)$$

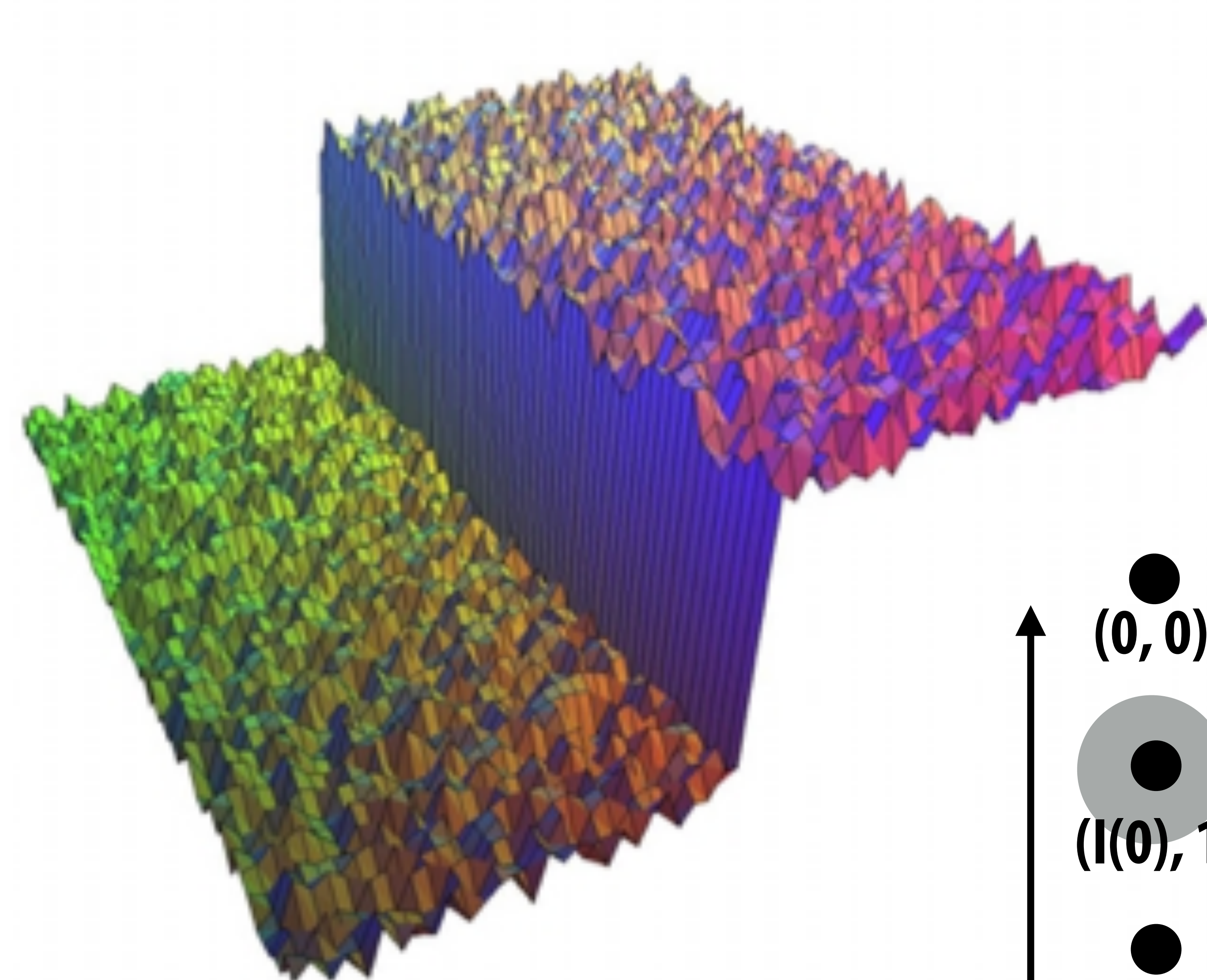
Normalization: $$W_p = \sum_{q \in S} f(|I_p - I_q|) G_\sigma(\|p - q\|)$$

Recall key property of a separable filter: can we decomposed into product of two (cheap-to-compute) 1D filters

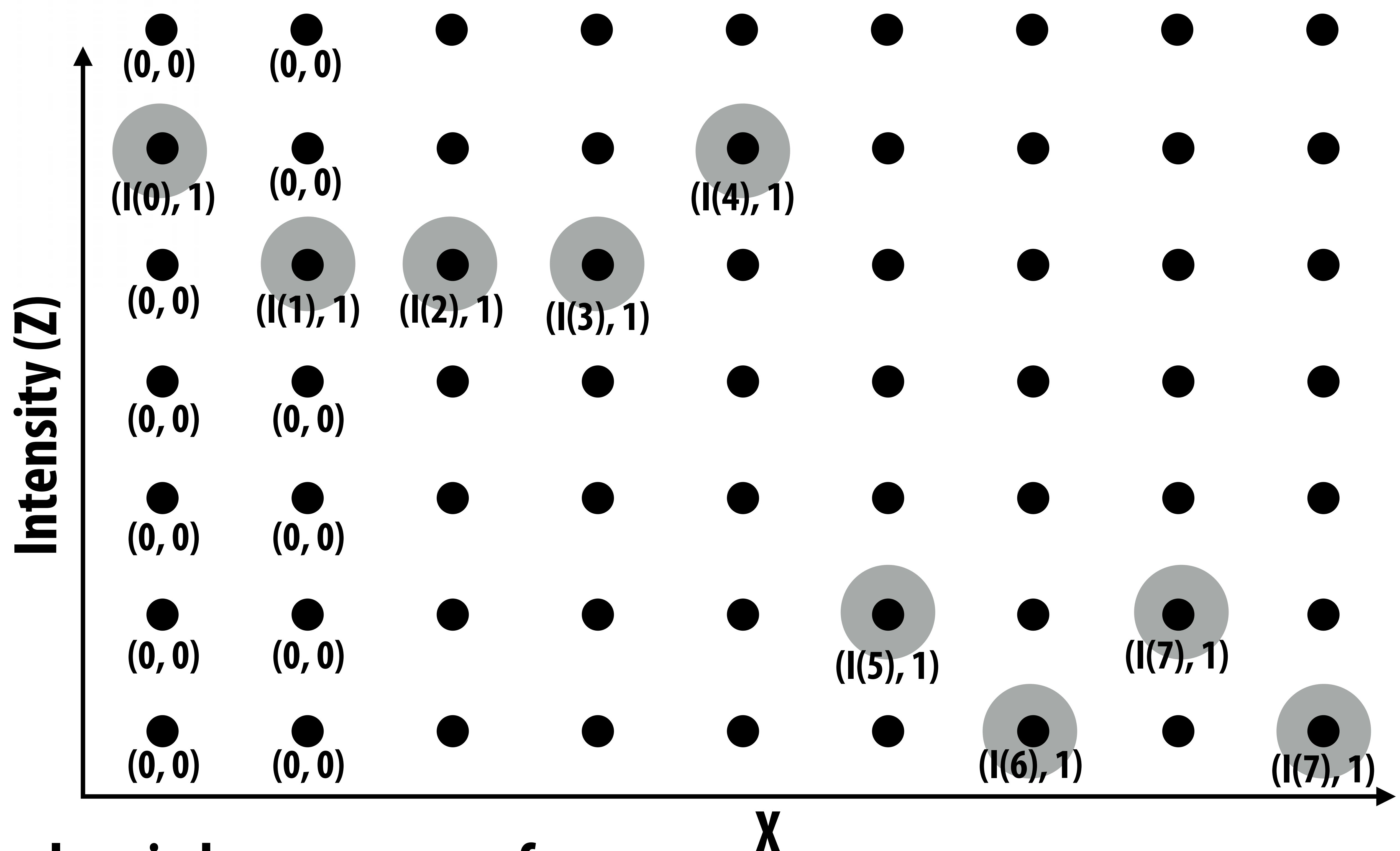
- **2D gaussian is a separable filter**

Bilateral filter is non-separable: execution has high cost (S^2) when used with large support region S (large σ)

Implementation via the bilateral grid



Consider image as surface embedded in 3D space
X,Y corresponds to position in image place
Z corresponds to image pixel value at (x,y)

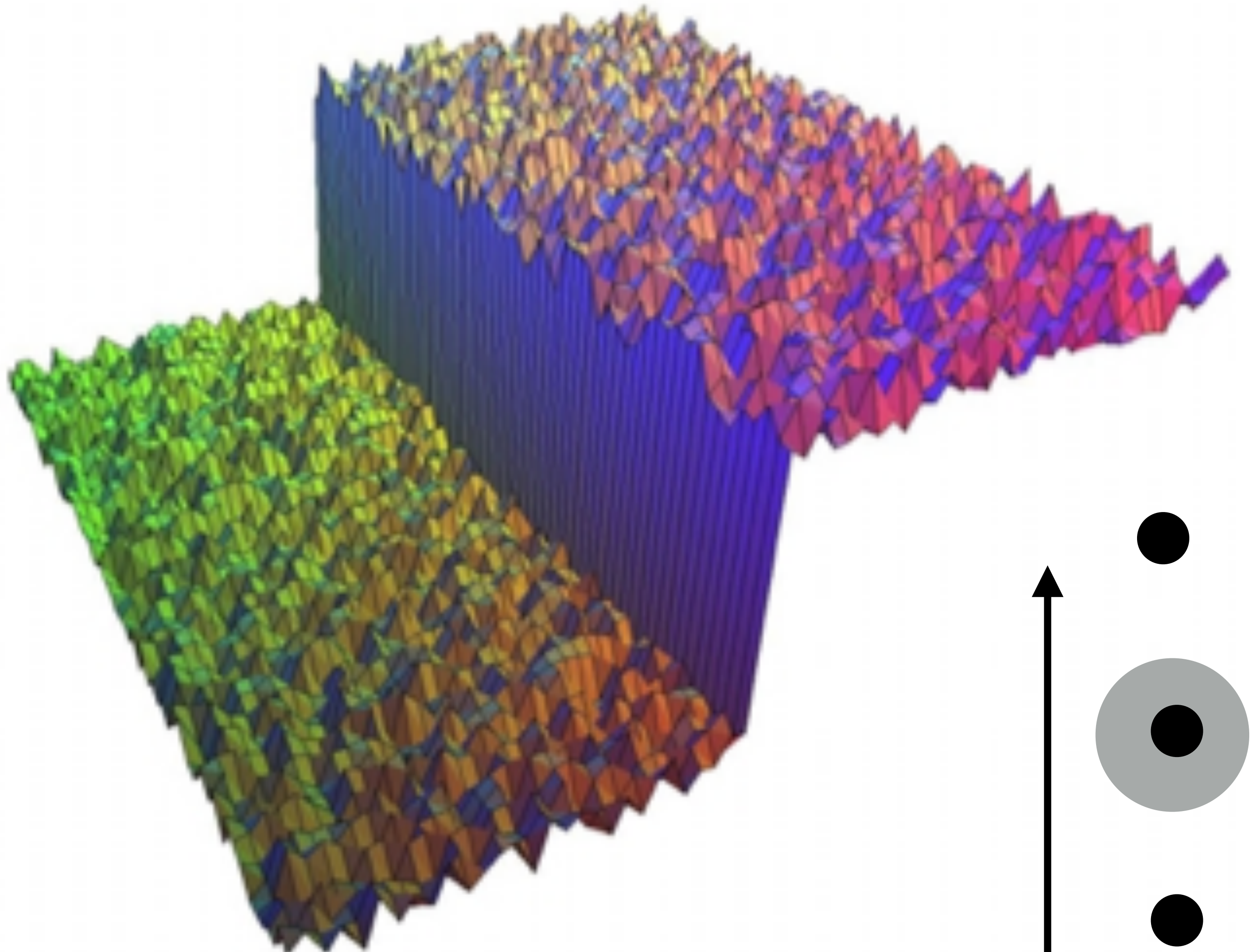


Bilateral grid stores image values in homogeneous form:

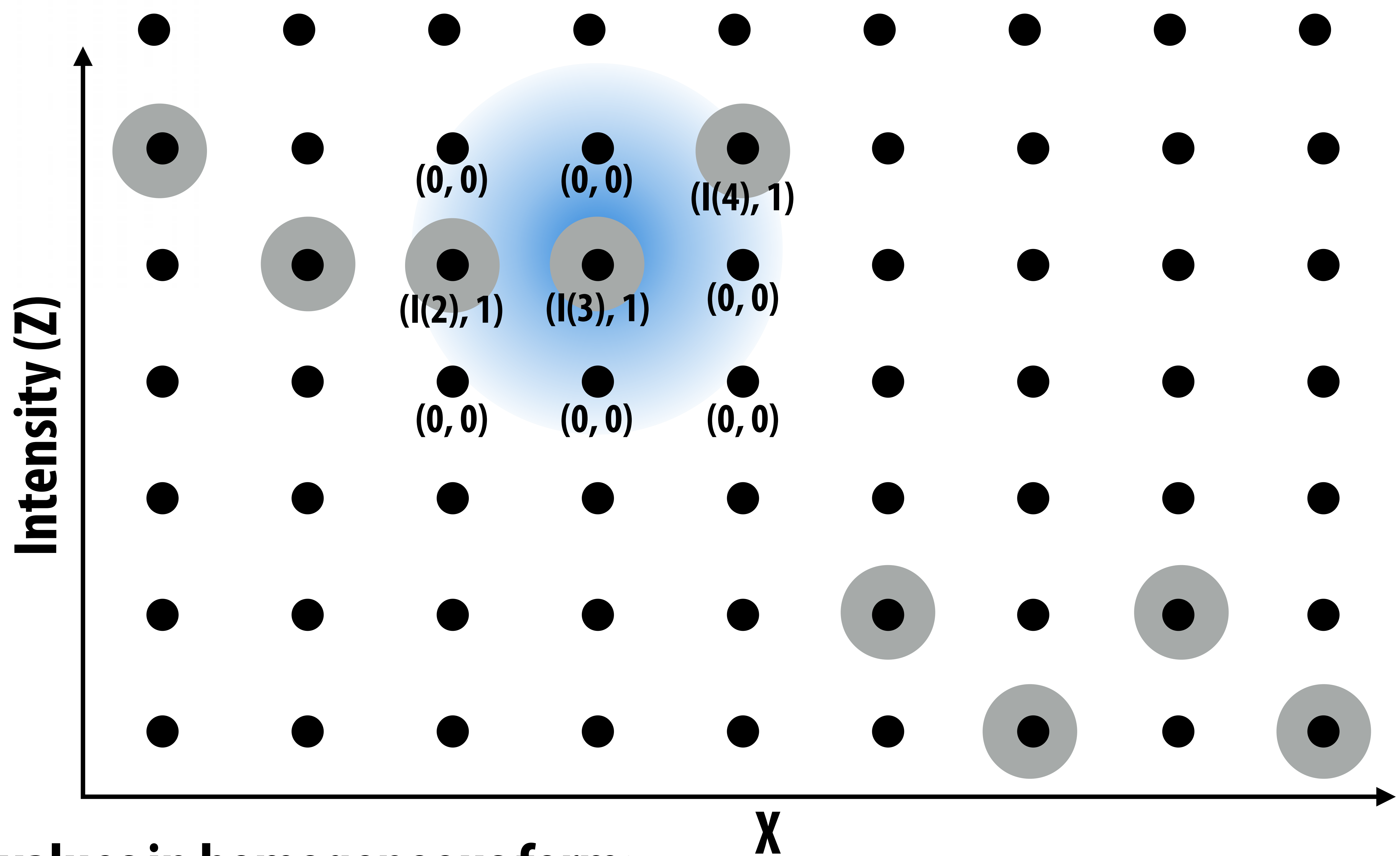
for all pixels (x,y) in image I(x,y): BG(x,y, I(x,y)) += (I(x,y), 1)

So… BG(x,y, I(x,y)) = (wI(x,y), w)

Implementation via the bilateral grid



Consider effect of applying 3D Gaussian filter to contents of 3D bilateral grid.
(Note: only 2D grid shown)



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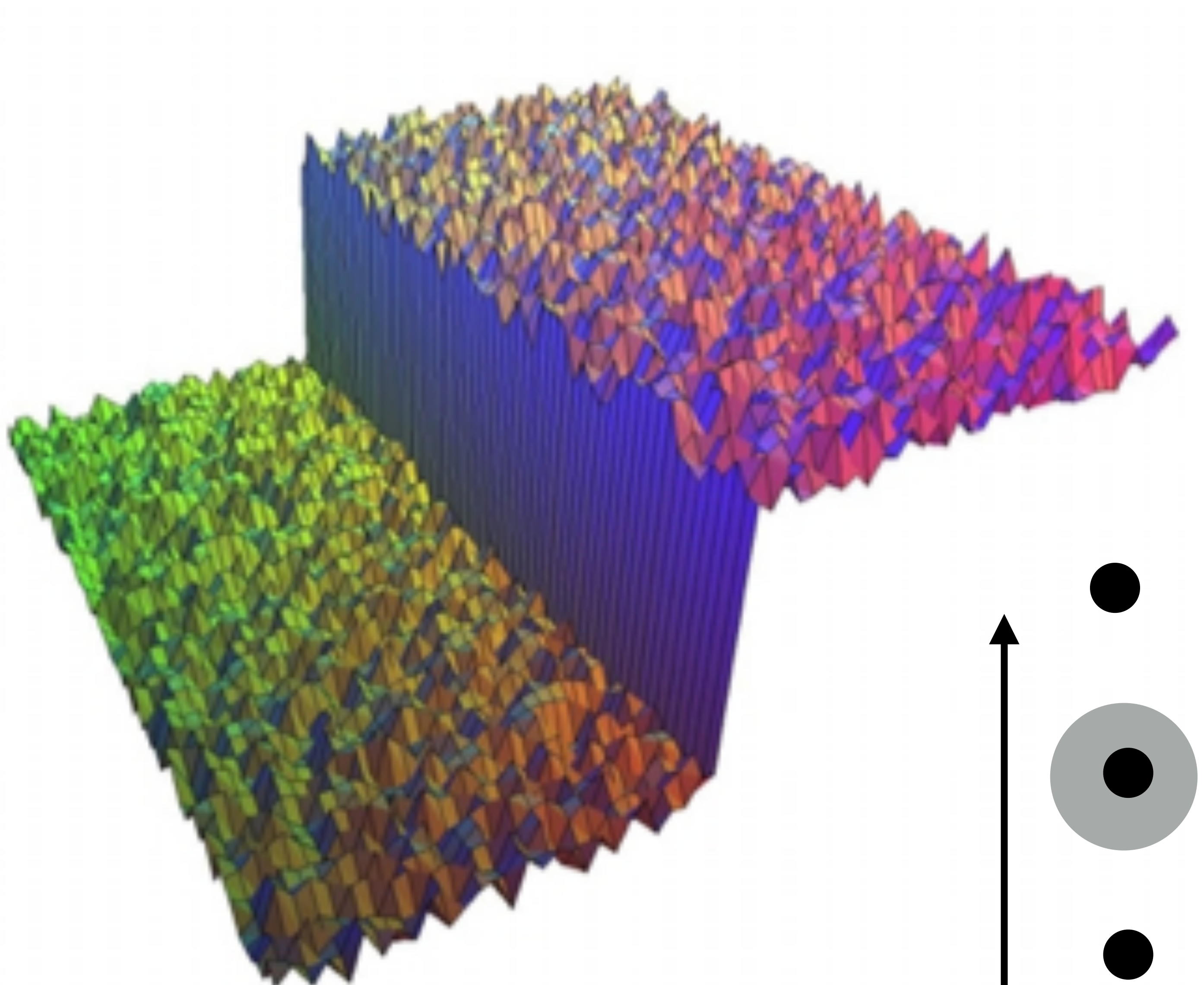
Implementation via the bilateral grid



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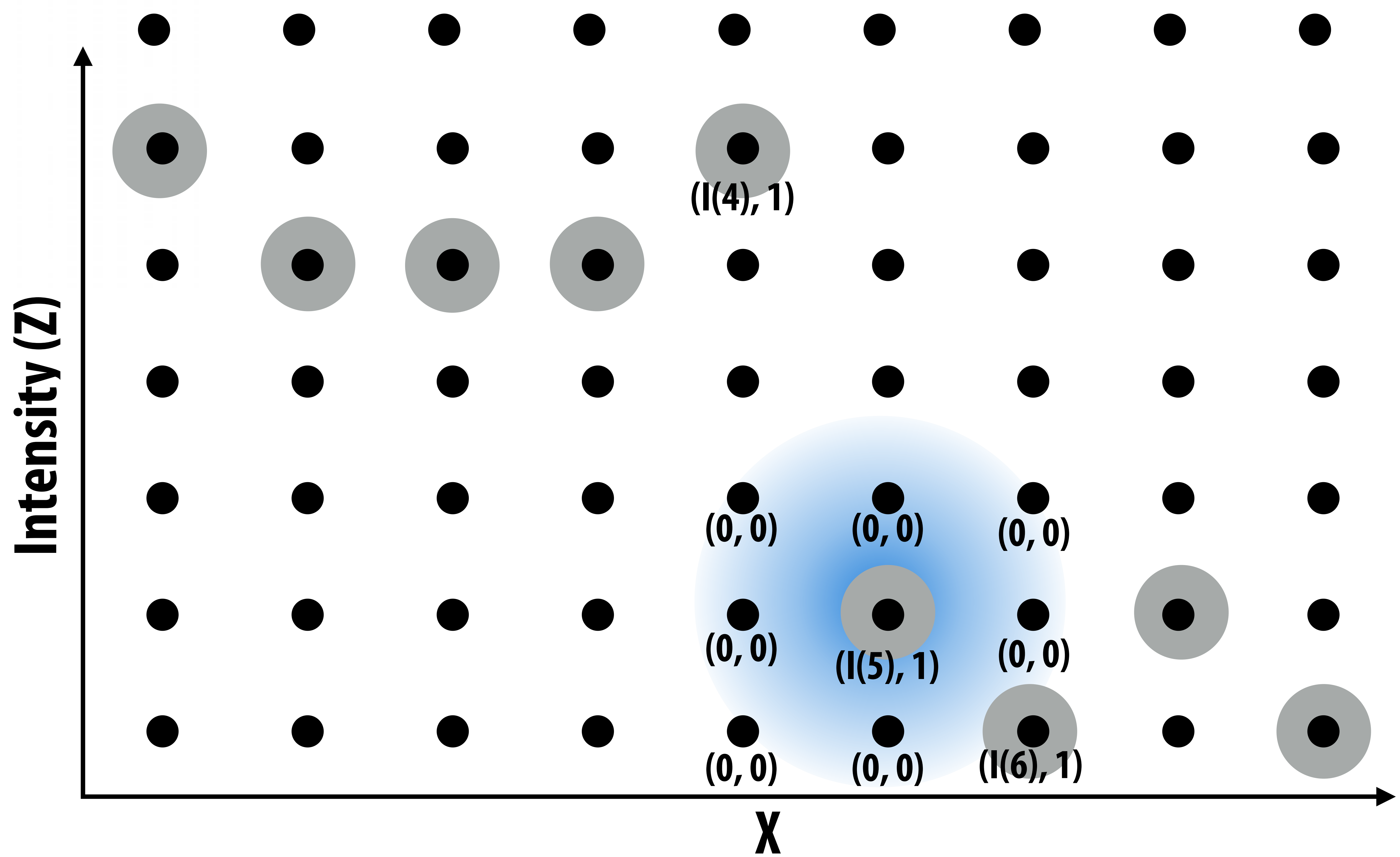
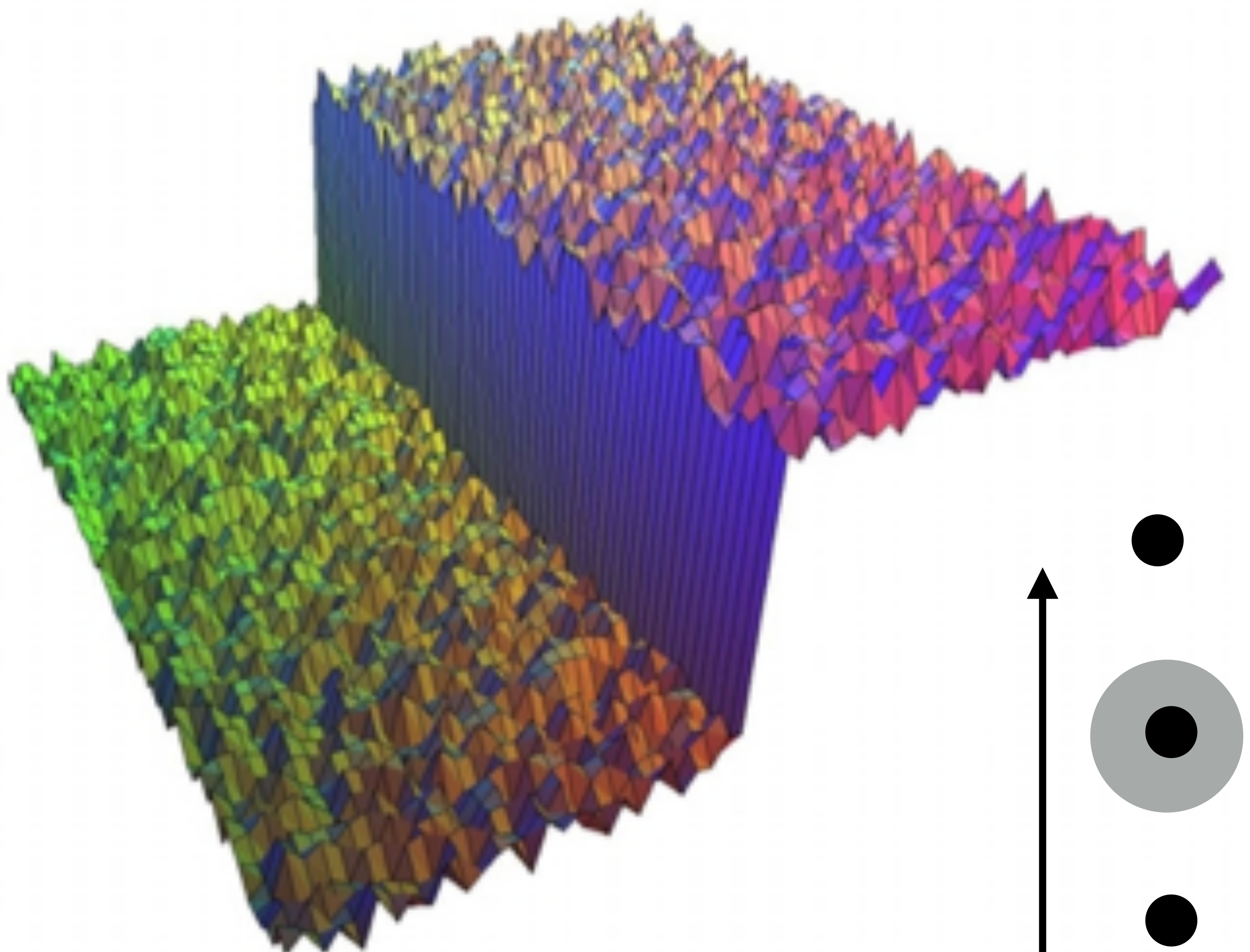
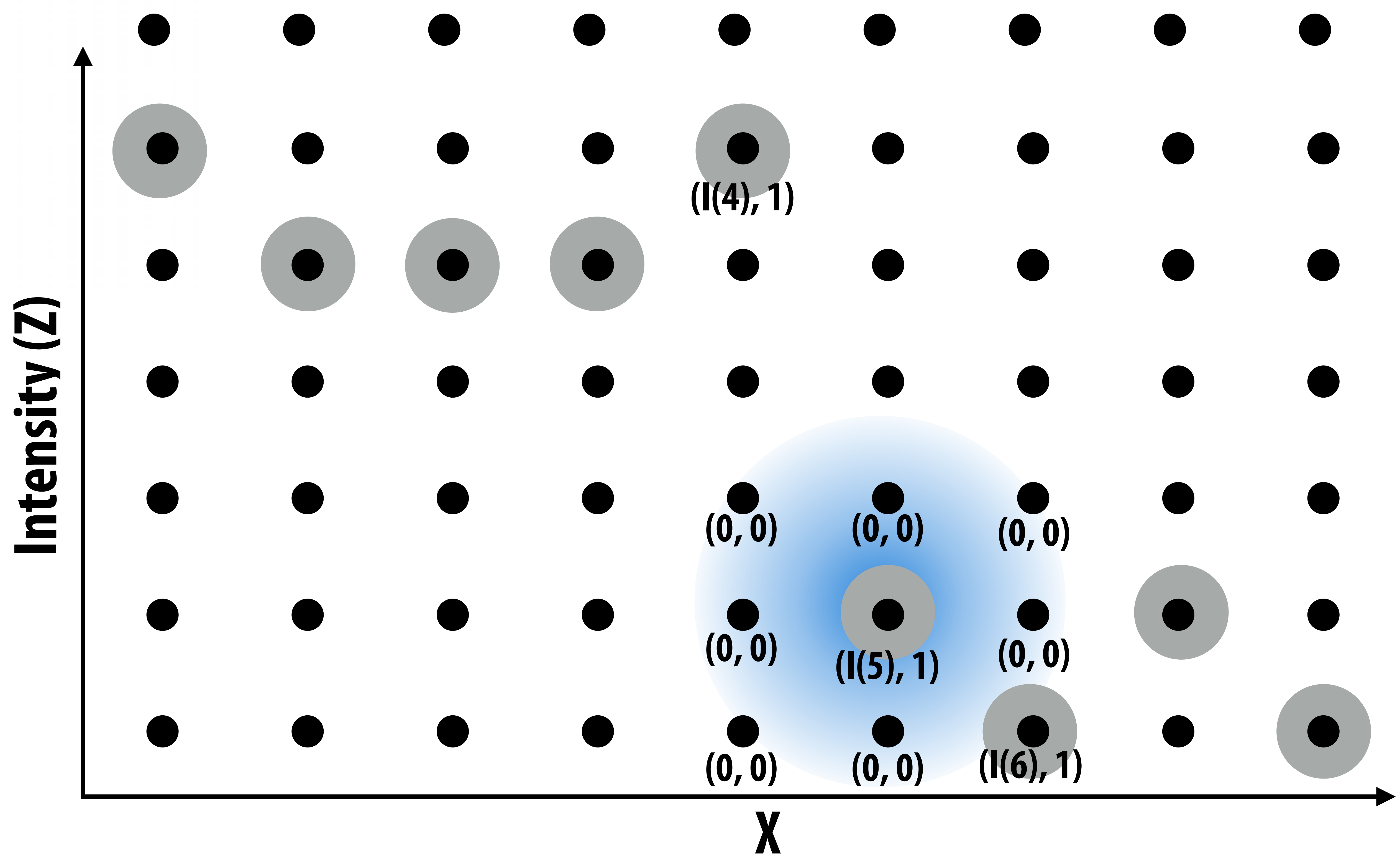


Notice how spatially adjacent (in x,y) values on the other side of the edge <u>do not</u> effect result of filtering via gaussian centered at position 5.

Implementation via the bilateral grid





"Slicing" step to recover final image (homogeneous divide)

BF[I](x,y) = BG(x,y,I(x,y).value) / BG(x,y,I(x,y).w)

Bilateral grid notes

- High performance comes from using lower resolution grid (spatial dimensions) than dimensionality of source image
  - Applicable when filter support region is large (okay to pre filter signal prior to convolving with wide Gaussian)
  - For small support region (e.g., for denoising) direct evaluation of bilateral filter is likely more efficient

- Bilateral filter using RGB distances represented as a 5D grid
  - Higher memory footprint and higher cost

Estimating Motion Using Optical flow

Optical flow

Goal: determine 2D screen-space velocity of visible objects in image

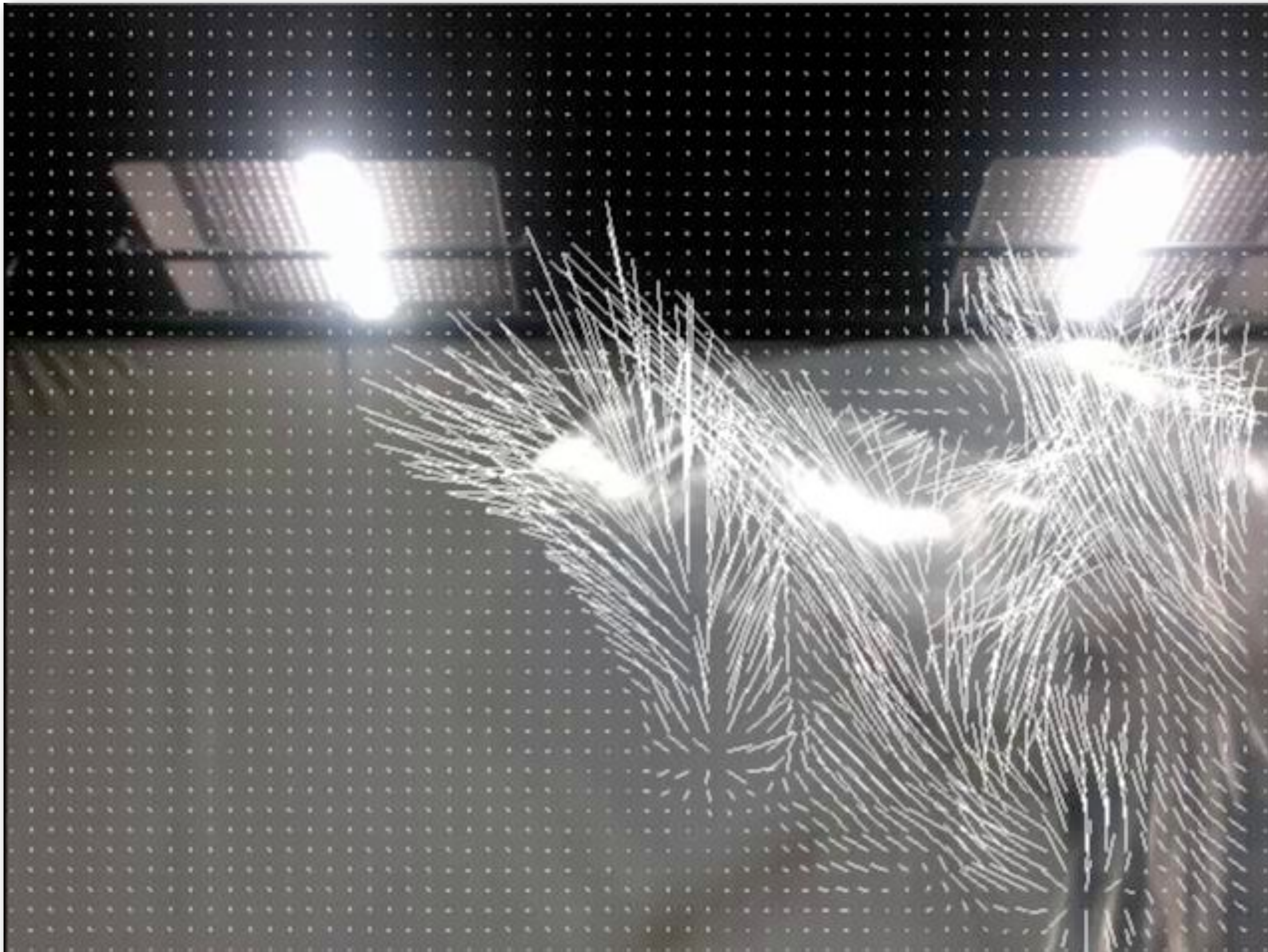


Image source: https://vimeo.com/28395792

Optical flow

- Given image A (at time t) and image B (at time $t + \Delta t$) compute the per-pixel motion needed to correspond the two images
- Major assumption 1: "brightness constancy"
  - The appearance of a scene surface point that is visible in both images A and B is the same in both images

$$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t)$$

The point observed at (x,y) at time t moves to (x+Δ, y+Δ) at t+Δt
(and has a constant appearance in both situations)

Taylor expansion

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t) + I_x(x, y, t)\Delta x + I_y(x, y, t)\Delta y + I_t(x, y, t)\Delta t + \text{higher order terms}$$

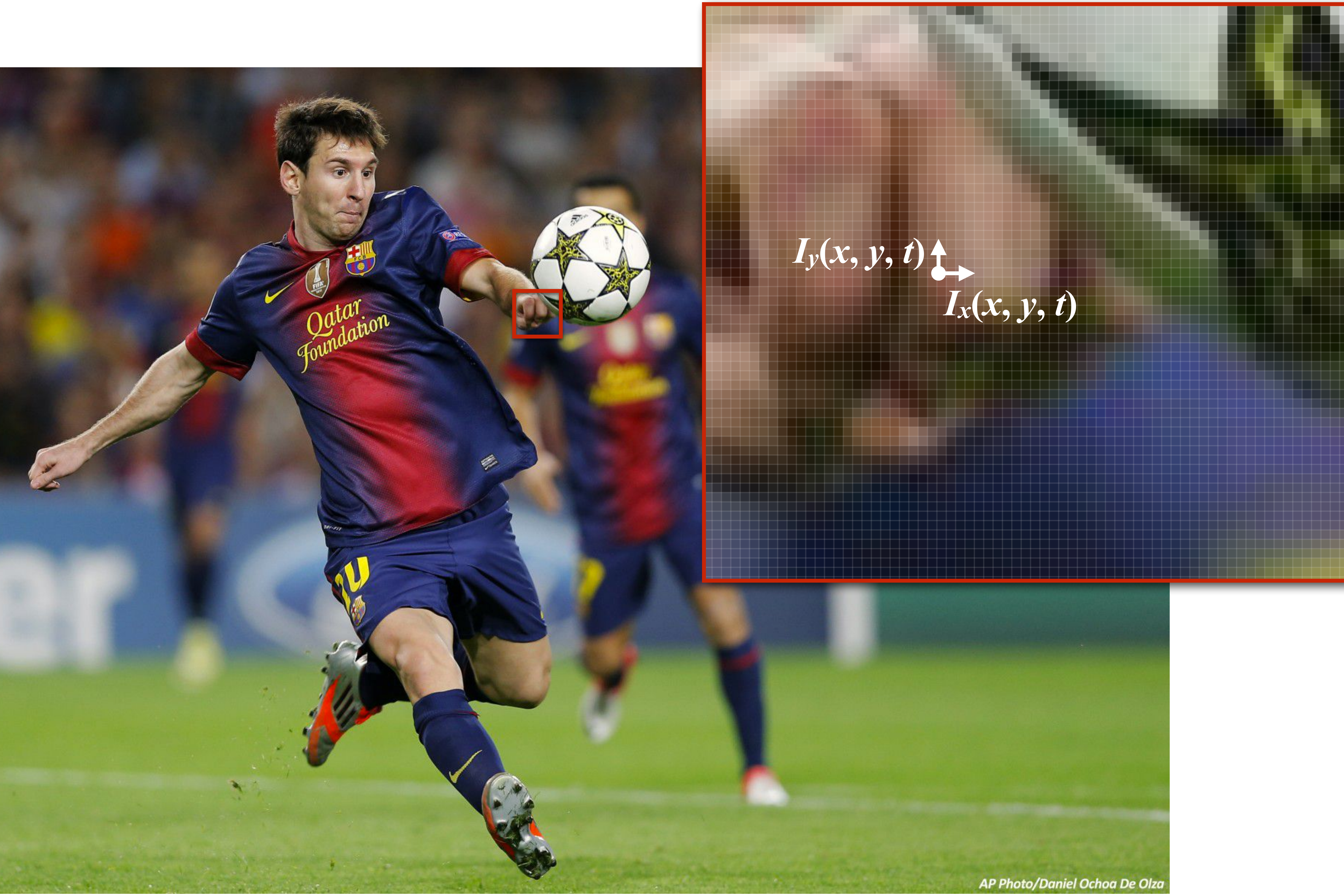
So...

$$I(x, y, t) \approx I(x, y, t) + I_x(x, y, t)\Delta x + I_y(x, y, t)\Delta y + I_t(x, y, t)\Delta t$$

$$\boxed{I_x(x, y, t)\Delta x + I_y(x, y, t)\Delta y} + \boxed{I_t(x, y, t)\Delta t} = 0$$

The observed change in pixel (x,y)

Is due to object motion at point by (Δx, Δy)


$I_y(x, y, t)$
$I_x(x, y, t)$
Qatar Foundation
AP Photo/Daniel Ochoa De Olza

Gradient-constraint equation for a pixel is underconstrained

Gradient-constraint equation is insufficient to solve for motion

One equation, two unknowns: (Δx, Δy)

$$\boxed{I_x(x, y, t)}\Delta x + \boxed{I_y(x, y, t)}\Delta y + \boxed{I_t(x, y, t)\Delta t} = 0$$

Known: observed change in pixel (x,y) over consecutive frames

Known: spatial image gradients in image A

Major assumption 2: nearby pixels have similar motion (Lucas-Kanade)

$$I_x(x_0, y_0, t)\Delta x + I_y(x_0, y_0, t)\Delta y + I_t(x_0, y_0, t)\Delta t = 0$$

$$I_x(x_1, y_1, t)\Delta x + I_y(x_1, y_1, t)\Delta y + I_t(x_1, y_1, t)\Delta t = 0$$

$$I_x(x_2, y_2, t)\Delta x + I_y(x_2, y_2, t)\Delta y + I_t(x_2, y_2, t)\Delta t = 0$$

$$\vdots$$

Now we have a overconstrained system, compute least squares solution

Weighted least-squares solution

Gradient-constraint equation is insufficient to solve for motion

One equation, two unknowns: (Δx, Δy)

$$I_x(x_0, y_0, t)\Delta x + I_y(x_0, y_0, t)\Delta y + I_t(x_0, y_0, t)\Delta t = 0$$

$$I_x(x_1, y_1, t)\Delta x + I_y(x_1, y_1, t)\Delta y + I_t(x_1, y_1, t)\Delta t = 0$$

$$I_x(x_2, y_2, t)\Delta x + I_y(x_2, y_2, t)\Delta y + I_t(x_2, y_2, t)\Delta t = 0$$

$$\vdots$$

Compute weighted least squares solution by minimizing:

$$E(\Delta x, \Delta y) = \sum_{x_i, y_i} w(x_i, y_i, x, y)\left[I_x(x_i, y_i, t)\Delta x + I_y(x_i, y_i, t)\Delta y + I_t(x_i, y_i, t)\Delta t\right]^2$$

(x_i, y_i) are pixels in region around (x,y).

Weighting function w() weights error contribution based on distance between (x_i, y_i) and (x, y) e.g., Gaussian fall-off.

Solving for motion

E (Δx, Δy) minimized when derivatives are zero:

$$\frac{dE(\Delta x,\Delta y)}{d(\Delta x)} = \sum_{x_i,y_i} w(x_i,y_i,x,y)\left[I_x^2 \Delta x + I_x I_y \Delta y + I_x I_t\right] = 0$$

$$\frac{dE(\Delta x,\Delta y)}{d(\Delta y)} = \sum_{x_i,y_i} w(x_i,y_i,x,y)\left[I_y^2 \Delta y + I_x I_y \Delta x + I_y I_t\right] = 0$$

Rewrite, now solve the following linear system for Δx, Δy:

$$\Delta x \overset{\text{A0}}{\boxed{\sum_{x_i,y_i} w(x_i,y_i,x,y) I_x^2}} + \Delta y \overset{\text{B0}}{\boxed{\sum_{x_i,y_i} w(x_i,y_i,x,y) I_x I_y}} + \overset{\text{C0}}{\boxed{\sum_{x_i,y_i} w(x_i,y_i,x,y) I_x I_t}} = 0$$

$$\Delta x \overset{\text{A1}}{\boxed{\sum_{x_i,y_i} w(x_i,y_i,x,y) I_x I_y}} + \Delta y \overset{\text{B1}}{\boxed{\sum_{x_i,y_i} w(x_i,y_i,x,y) I_y^2}} + \overset{\text{C1}}{\boxed{\sum_{x_i,y_i} w(x_i,y_i,x,y) I_y I_t}} = 0$$

Precompute partial derivatives I_x, I_y, I_t from original images A and B

For each pixel (*x*,*y*): evaluate A0, B0, C0, A1, B1, C1, then solve for (Δ*x*, Δ*y*) at (*x*,*y*)

Optical flow, implemented in practice

Gradient-constraint equation makes a linear motion assumption

$$I(x, y, t) \approx I(x, y, t) + I_x(x, y, t)\Delta x + I_y(x, y, t)\Delta y + I_t(x, y, t)\Delta t$$

$$\boxed{I_x(x, y, t)\Delta x + I_y(x, y, t)\Delta y} + \boxed{I_t(x, y, t)\Delta t} = 0$$

The observed change in pixel (x,y)

Is due to object motion at point by (Δx, Δy)

- **Improvement: iterative techniques use this original flow field to compute higher order residuals (to estimate non-linear motion)**

- **Question: why is it important for optical flow implementation to be very efficient?**
  - **Hint: consider linear-motion assumption**