Model Reduction
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Example

Vertices: 3321
Triangles: 6638
Overview

General Model Reduction
Model Reduction of Fluids
Model Reduction of Deformable Solids
Coupling
Questions
Overview

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Coupling

Questions
Full Space
\( u \in \mathbb{R}^n \)
\( n \approx 3,000,000 \)

Reduced Space
\( r \in \mathbb{R}^m \)
\( m \approx 64 \)

\[ B^T u = r \]
\[ u = Br \]
Full Space
\[ \mathbf{u} \in \mathbb{R}^n \quad n \approx 3,000,000 \]
\[ \dot{\mathbf{u}} = F(\mathbf{u}) \]

Reduced Space
\[ \mathbf{r} \in \mathbb{R}^m \quad m \approx 64 \]
\[ \dot{\mathbf{r}} = \hat{F}(\mathbf{r}) \]

\[ \mathbf{B}^T \mathbf{u} = \mathbf{r} \]
\[ \mathbf{u} = \mathbf{B} \mathbf{r} \]
Full Space
\[ u \in \mathbb{R}^n \]
\[ n \approx 3,000,000 \]

\[ \dot{u} = F(u) \]

Reduced Space
\[ r \in \mathbb{R}^m \]
\[ m \approx 64 \]

\[ \dot{r} = \hat{F}(r) \]

How?
(Blackboard)
Full Space
\[ u \in \mathbb{R}^n \]
\[ n \approx 3,000,000 \]

Reduced Space
\[ r \in \mathbb{R}^m \]
\[ m \approx 64 \]

\[ \dot{u} = F(u) \]
\[ \dot{r} = \hat{F}(r) \]
\[ \hat{F} = B^T \circ F \circ B \]
\[ \hat{F} = B^T \circ F \circ B \]

**Example:**

If: \[ F(u) = Mu \]

Then: \[ \hat{F}(r) = \underbrace{B^TMBr}_{\text{precompute}} = \hat{M}r \]
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Questions
\[ \dot{u} = -(u \cdot \nabla)u - \nu \nabla^2 u + \nabla p + f \]

- Advection
- Diffusion
- Projection
- Forces
\[ \mathbf{\dot{u}} = - (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \nabla^2 \mathbf{u} + \nabla p + \mathbf{f} \]

**Advection:**

\[ \mathbf{\hat{A}} \in \mathbb{R}^{6400000 \times 6400000} \times 3000000 \]

\[ \mathbf{r}' = E e^{\Delta t \Lambda} E^{-1} \mathbf{r} \]

(where)

\[ E \Lambda E^{-1} = \mathbf{\hat{A}} \otimes_1 \mathbf{r} \]
\[
\dot{u} = - (u \cdot \nabla) u - \nu \nabla^2 u + \nabla p + f
\]

**Diffusion:**

\[
\hat{D} \in \mathbb{R}^{3000000 \times 3000000}
\]

\[
r' = E e^{\Delta t \Lambda} E^{-1} r
\]

(\text{where})

\[
E \Lambda E^{-1} = \hat{D}
\]
\[ \dot{u} = -(u \cdot \nabla)u - \nu \nabla^2 u + \nabla p + f \]

**Projection:**

- Divergence free
- Boundary conditions

```c
void project(state *s) { return; }
```
\[ \dot{u} = -(u \cdot \nabla)u - \nu \nabla^2 u + \nabla p + f \]

**Forces:**

\[ r \quad + = \quad B^T f \]
\[ \dot{u} = -(u \cdot \nabla)u - \nu \nabla^2 u + \nabla p + f \]

**Summary:**

<table>
<thead>
<tr>
<th>Process</th>
<th>Full</th>
<th>Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advection</td>
<td>$O(n)$</td>
<td>$O(m^3)$</td>
</tr>
<tr>
<td>Diffusion</td>
<td>$O(n)$</td>
<td>$O(m^2)$</td>
</tr>
<tr>
<td>Projection</td>
<td>$O(n)$</td>
<td>$O(0)$</td>
</tr>
<tr>
<td>Forces</td>
<td>$O(n)$</td>
<td>$O(m)$</td>
</tr>
</tbody>
</table>

- **Time:**
  - Full: 8.8 ms/ frame
  - Reduced: 0.8 ms/ frame
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Questions
Reduced Deformation

**Full System:**

\[ M\ddot{u} + D(u, \dot{u}) + R(u) = f. \]

**Reduced System:**

\[ \ddot{q} + \tilde{D}(q, \dot{q}) + \tilde{R}(q) = \tilde{f} \]

\[ \tilde{D} = U^T D(Uq, U\dot{q}), \]
\[ \tilde{R}(q) = U^T R(Uq), \]
\[ \tilde{f} = U^T f. \]
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Vertices: 3321
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Vertices: 41361
Triangles: 59630
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Questions
Background $\rightarrow$ Object

\[ 0 = S(\hat{B}\hat{r} + Br) \]

\[ \hat{r} = Mr \]
Background → Object

Full

\[ 0 = S (\hat{B}\hat{r} + Br) \]

Reduced

\[ \hat{r} = Mr \]

Object → Background

Full

\[ r += B^T \hat{B}\hat{r} \]

Reduced

\[ r += N\hat{r} \]
QuickTime® and a decompressor are needed to see this picture.
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Questions
• Good Reviews :)
• What’s the next big thing?
• Machine Learning + Graphics:
  • Learning physics?
• Networking + Graphics:
  • Distributed physics?
• Biology + Graphics:
  • Animal Morphology?
• Other disciplines:
  • Urban planning?
  • What else?
What’s the next big thing?

- video tape an object under known lighting conditions to create a “data driven” model of its surface reflectance
- huge database of mocap data
- if we get the physics right, machine learning should be able to estimate parameters
  - how much force is being applied from a motion capture animation?
- distribute the calculation of massive fluid simulations across a network
- distributing computation between CPU and GPU
- once we solve biomechanics in graphics + then we can build robotic prothetic arms
- network-based graphics
  - separate the objects from the scene, render each separately
  - take advantage of the # of cores to do faster rendering
- computer graphics must wait for developments in machine learning for certain applications
- physics is parallel, computer cores are becoming parallel... how can we take advantage of this?
- sound synthesis
  - model reduction for sound synthesis
- haptics: glove with little actuators
- surface feeling “rendering” - “roughness rendering”
- wear and tear on an environment
Future Directions

- More Phenomena
- Coupling
- Control