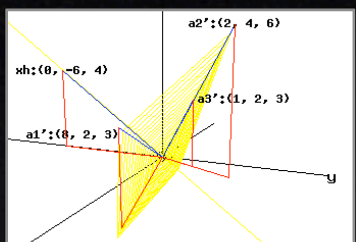
Model Reduction

Adrien Treuille

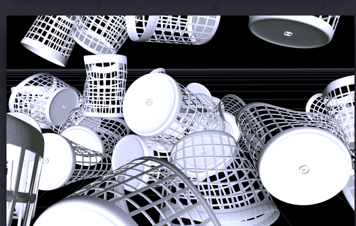
Example

Overview



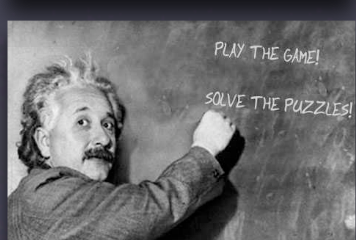
General Model Reduction

Model Reduction of Fluids



Model Reduction of Deformable Solids

Coupling



Questions

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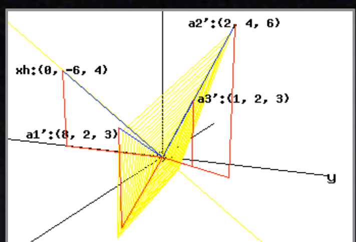

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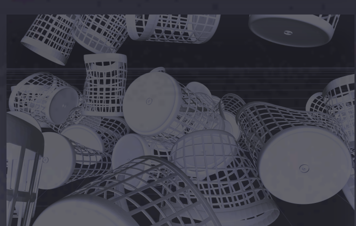

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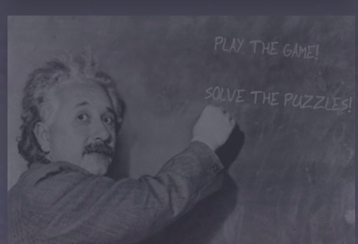

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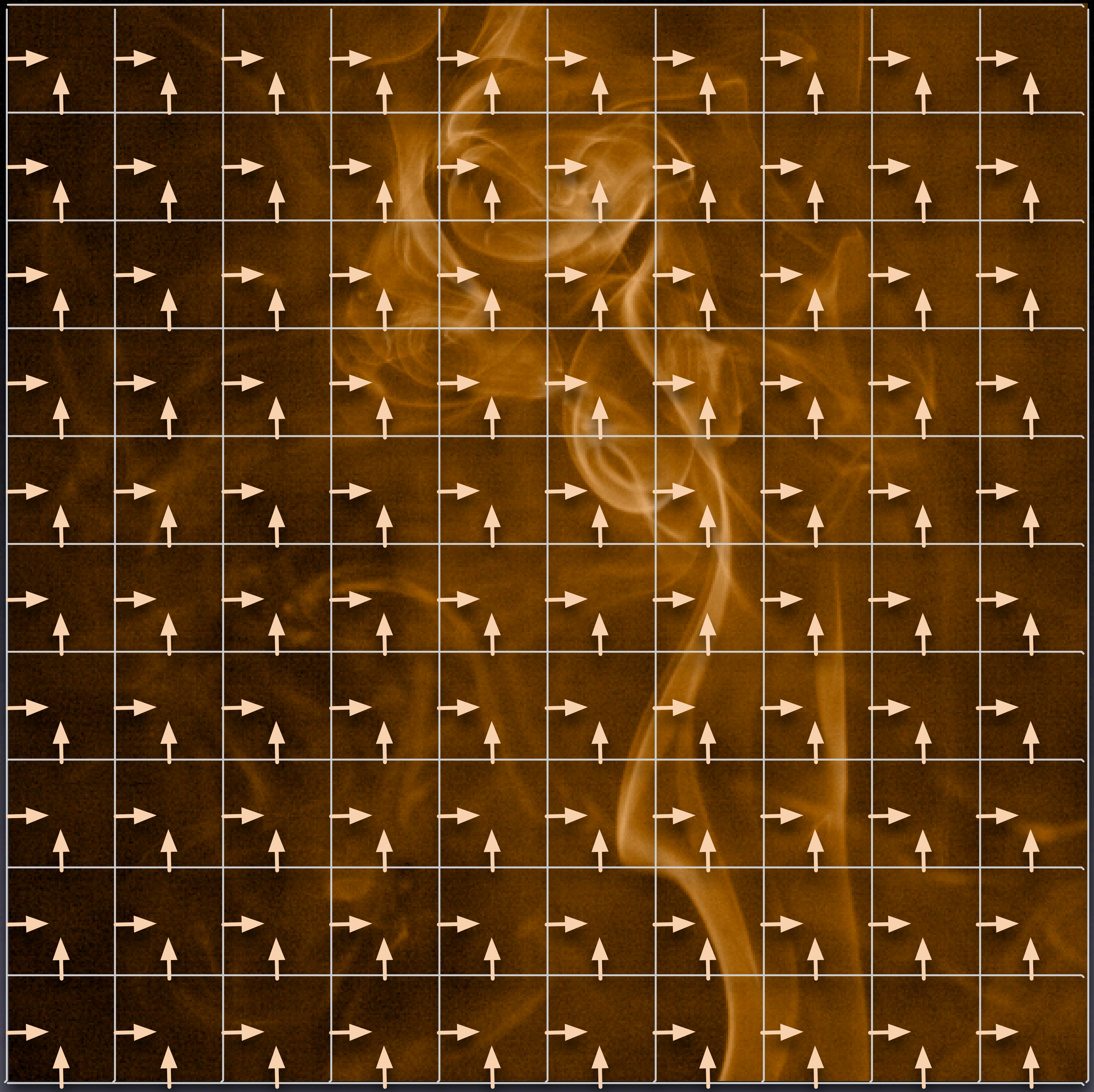

Full Space

$$\mathbf{u} \in \mathbf{R}^n$$

$n \approx 3{,}000{,}000$

$$\underline{B}^T \mathbf{u} = \mathbf{r}$$

$$\mathbf{u} = \underline{B}\mathbf{r}$$

Reduced Space

$$\mathbf{r} \in \mathbf{R}^m$$

$m \approx 64$

$\Bigg\{$  $\cdots \Bigg\}$

··· 64 ···

Full Space

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n ≈ 3,000,000

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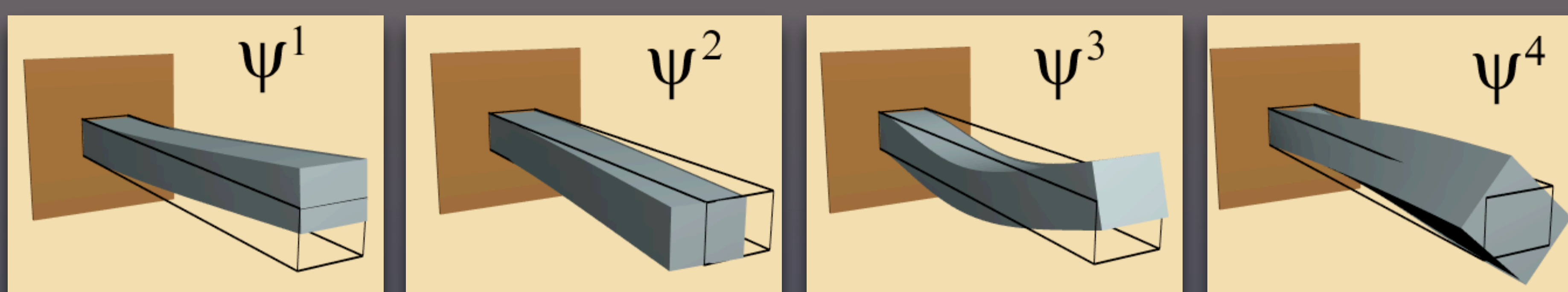
Reduced Space

$$\mathbf{r} \in \mathbf{R}^m$$

m ≈ 64

$$\dot{\mathbf{u}} = F(\mathbf{u})$$

$$\dot{\mathbf{r}} = \hat{F}(r)$$

$\{$


$\ldots \}$

··· 64 ···

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$$\mathbf{r} \in \mathbf{R}^m$$

$m \approx 64$

$$\dot{\mathbf{u}} = F(\mathbf{u})$$

$$\dot{\mathbf{r}} = \hat{F}(r)$$

How?

(Blackboard)

Full Space

$$\mathbf{u} \in \mathbf{R}^n$$

n ≈ 3,000,000

$$\dot{\mathbf{u}} = F(\mathbf{u})$$

Reduced Space

$$\mathbf{r} \in \mathbf{R}^m$$

m ≈ 64

$$\dot{\mathbf{r}} = \hat{F}(r)$$

$$\begin{tikzcd} \text{Full} & \bullet \arrow[rr, "F"] & & \bullet \arrow[d, "B^T"] \\ \text{Reduced} & \mathbf{r} \arrow[u, "B"] & & \dot{\mathbf{r}} \end{tikzcd}$$

$$\hat{F} = B^T \circ F \circ B$$

$$\begin{tikzcd}
\text{Full} & \bullet \arrow[rr, "F"] & & \bullet \arrow[d, "B^T"] \\
\text{Reduced} & \bullet \arrow[u, "B"] & & \bullet \\
& \mathbf{r} & & \dot{\mathbf{r}}
\end{tikzcd}$$

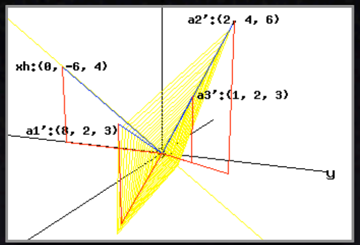
$$\hat{F} = B^T \circ F \circ B$$

Example:

If: $$F(\mathbf{u}) = M\mathbf{u}$$

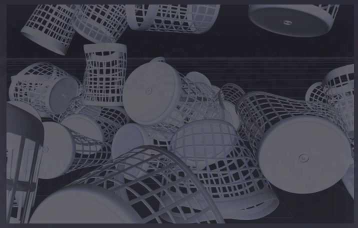
Then: $$\hat{F}(\mathbf{r}) = \underbrace{B^T M B}_{\text{precompute}}\mathbf{r} = \hat{M}\mathbf{r}$$

Overview



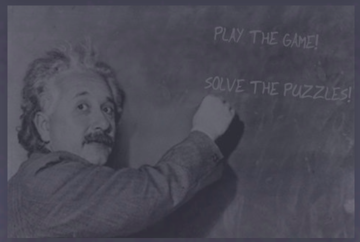
General Model Reduction

Model Reduction of Fluids



Model Reduction of Deformable Solids

Coupling



Questions

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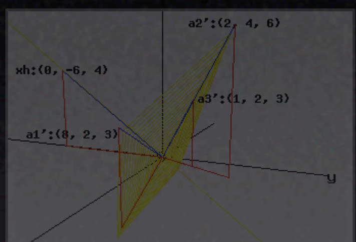

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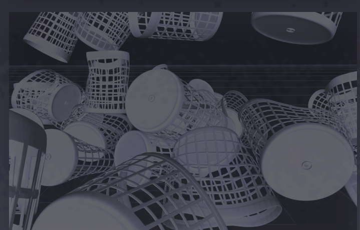

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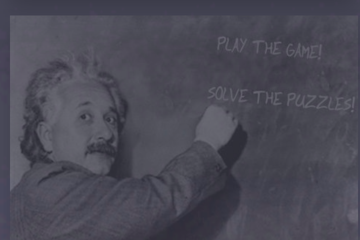


Questions

$$\dot{\mathbf{u}} = \underline{-(\mathbf{u} \cdot \nabla)\mathbf{u}} - \underline{\nu\nabla^2\mathbf{u}} + \underline{\nabla p} + \underline{\mathbf{f}}$$

- Advection
- Diffusion
- Projection
- Forces

$$\dot{\mathbf{u}} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nu\nabla^2\mathbf{u} + \nabla p + \mathbf{f}$$

Advection:

$$\hat{A} \in \mathbb{R}^{64\times 64\times 64 \quad 3000000\times 3000000}$$

$$\mathbf{r}' = E e^{\Delta t \Lambda} E^{-1}\mathbf{r}$$

(where)

$$E \Lambda E^{-1} = \hat{A} \otimes_1 \mathbf{r}$$

$$\dot{\mathbf{u}} = -\underline{(\mathbf{u} \cdot \nabla)\mathbf{u}} - \underline{\nu \nabla^2 \mathbf{u}} + \nabla p + \mathbf{f}$$

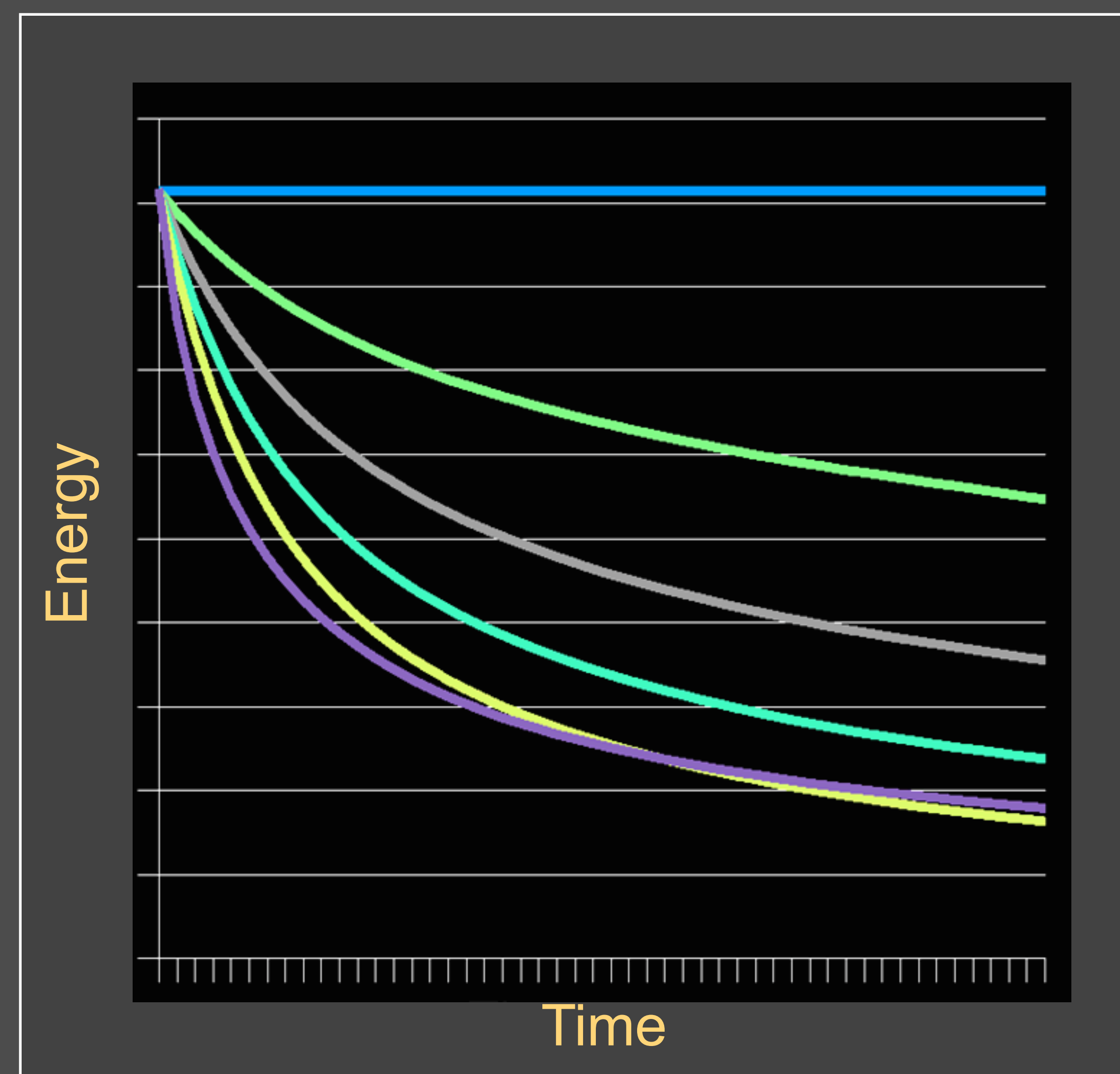
Diffusion:

$$\hat{D} \in \mathbf{R}^{3000000 \times 3000000}$$

$$\mathbf{r}' = E e^{\Delta t \Lambda} E^{-1} \mathbf{r}$$

(where)

$$E \Lambda E^{-1} = \hat{D}$$



$$\dot{\mathbf{u}} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \underline{\nu\nabla^2\mathbf{u}} + \underline{\nabla p} + \mathbf{f}$$

Projection:


```
void project(state *s) {
    return;
}
```

$$\dot{\mathbf{u}} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \nu\nabla^2\mathbf{u} + \nabla p + \mathbf{f}$$

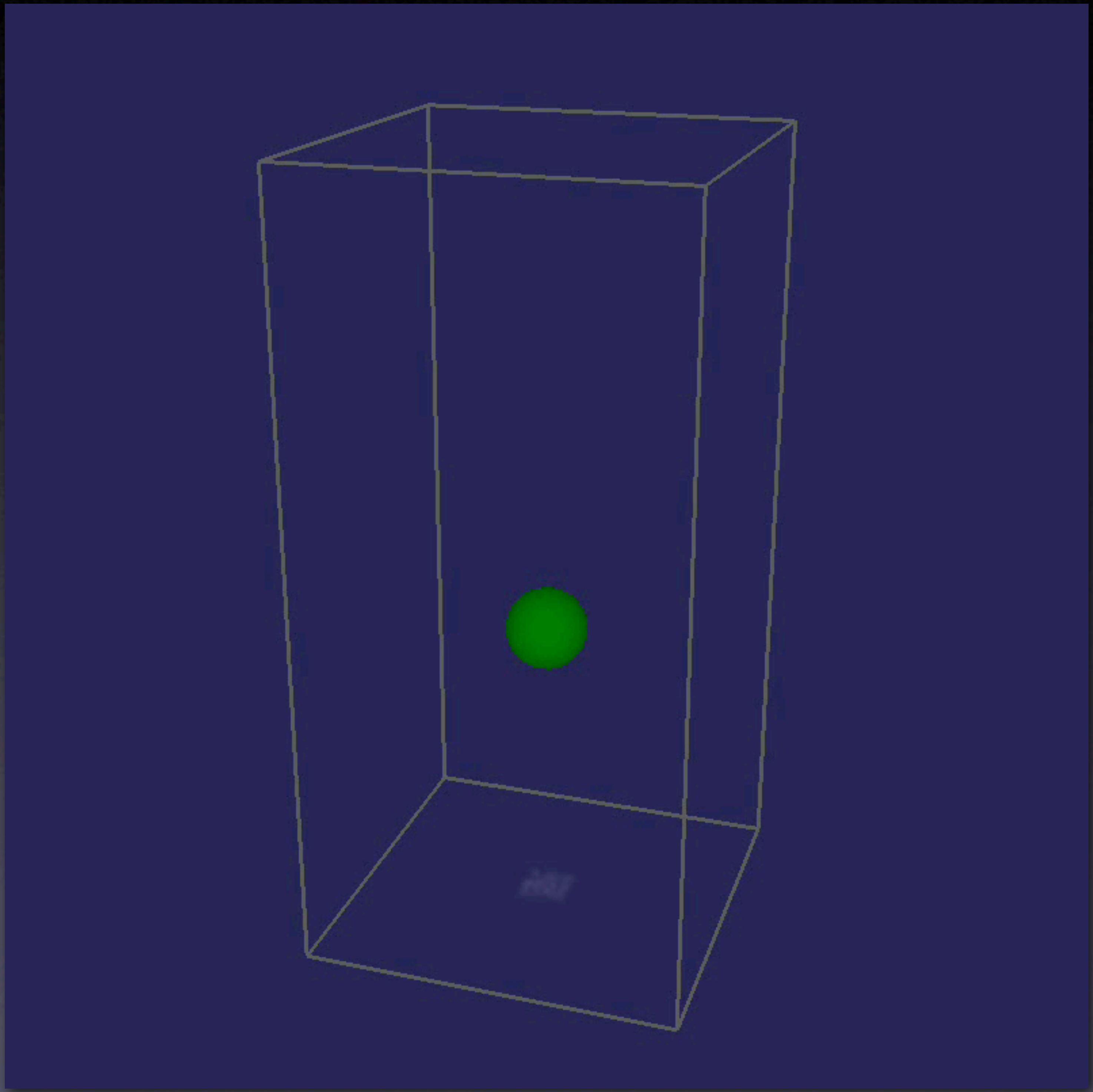
Forces:

$$\mathbf{r} \mathrel{+}= B^T\mathbf{f}$$

$$\dot{\mathbf{u}} = \underline{-(\mathbf{u}\cdot\nabla)\mathbf{u}} - \underline{\nu\nabla^2\mathbf{u}} + \underline{\nabla p} + \underline{\mathbf{f}}$$

Summary:	Full	Reduced
Advection	$O(n)$	$O(m^3)$
Diffusion	$O(n)$	$O(m^2)$
Projection	$O(n)$	$O(0)$
Forces	$O(n)$	$O(m)$
	minutes/ frame	~~8.8 ms/~~ frame

Example



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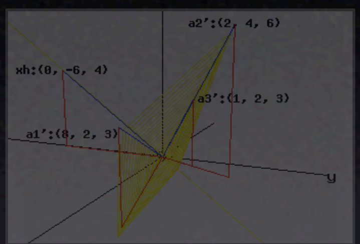
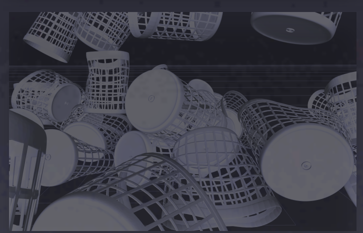


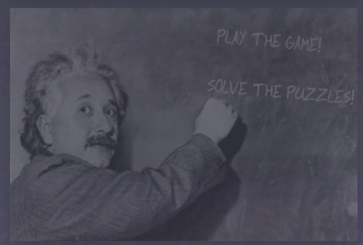
General Model Reduction

Model Reduction of Fluids



Model Reduction of Deformable Solids

Coupling



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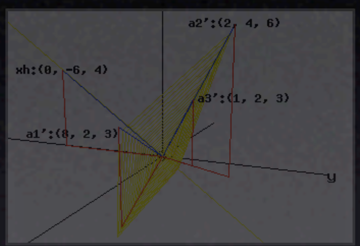

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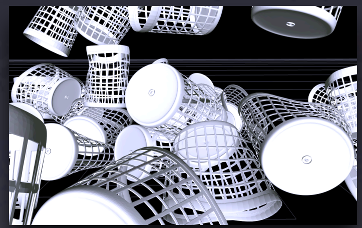

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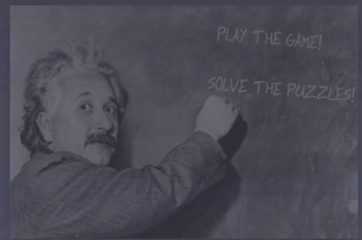


Questions

Reduced Deformation

Full System:

$$M\ddot{u} + D(u,\dot{u}) + R(u) = f.$$

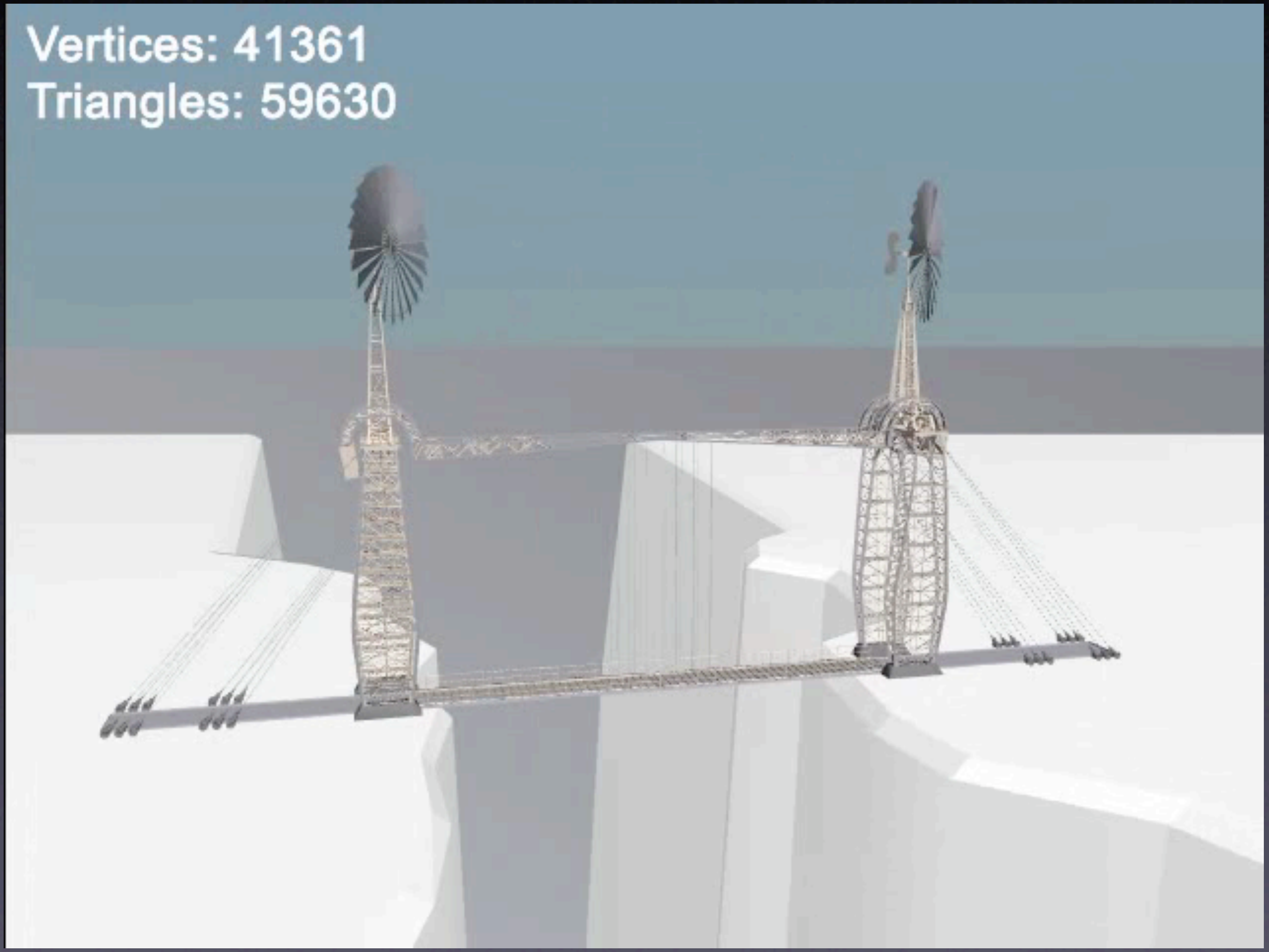
Reduced System:

$$\ddot{q} + \tilde{D}(q,\dot{q}) + \tilde{R}(q) = \tilde{f}$$

$$\begin{aligned}
\tilde{D} &= U^T D(Uq, U\dot{q}), \\
\tilde{R}(q) &= U^T R(Uq), \\
\tilde{f} &= U^T f.
\end{aligned}$$

Example

Example



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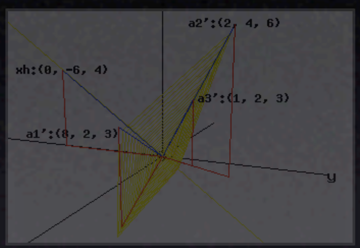

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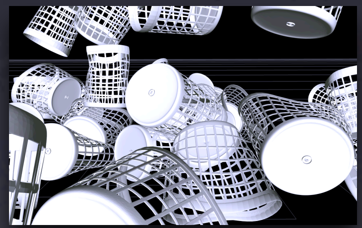

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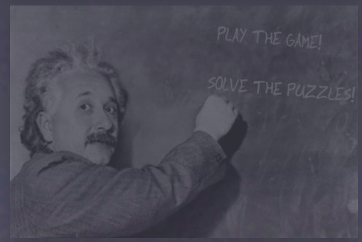

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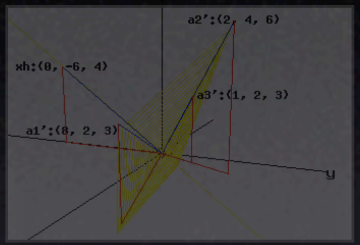

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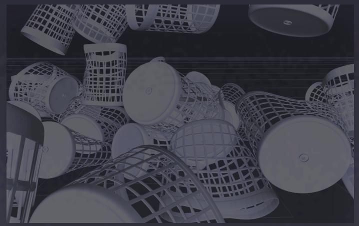

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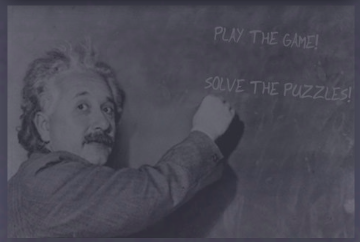

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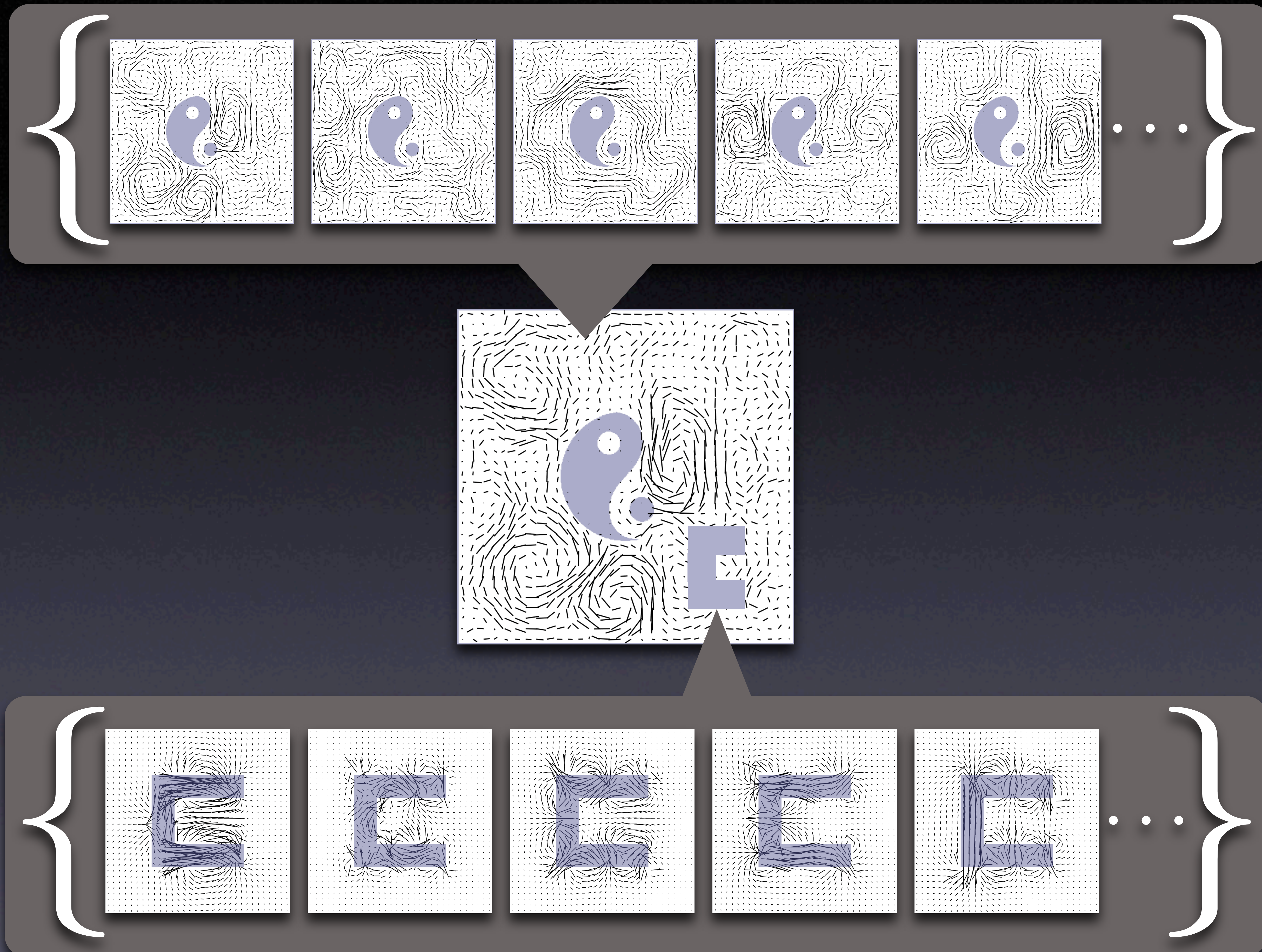
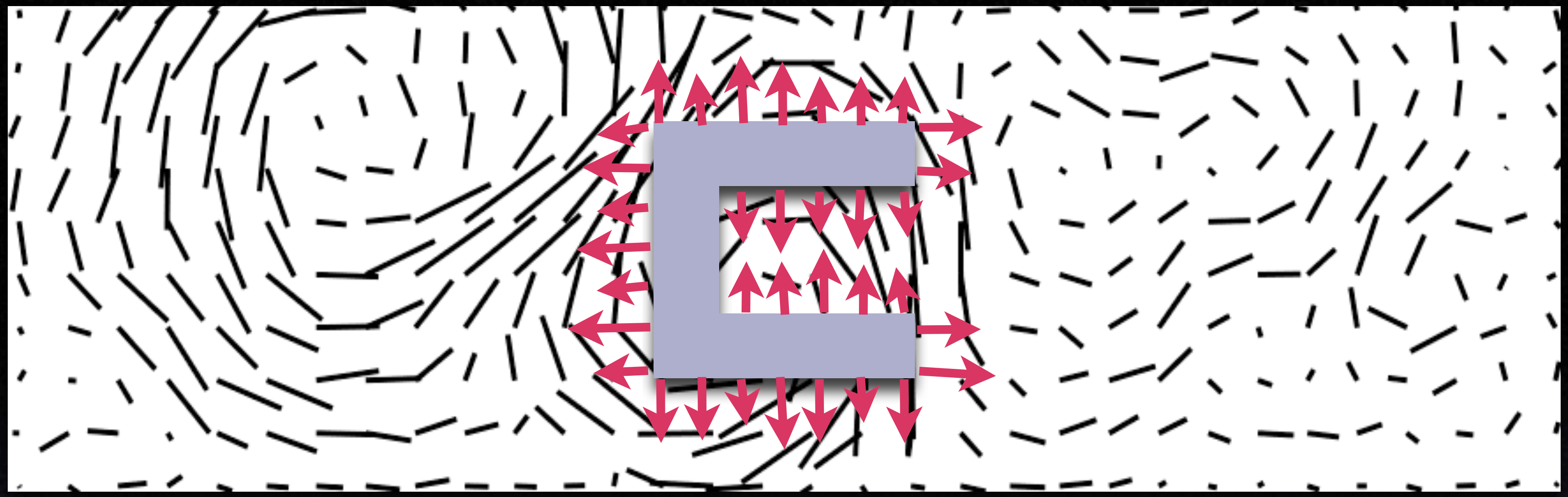




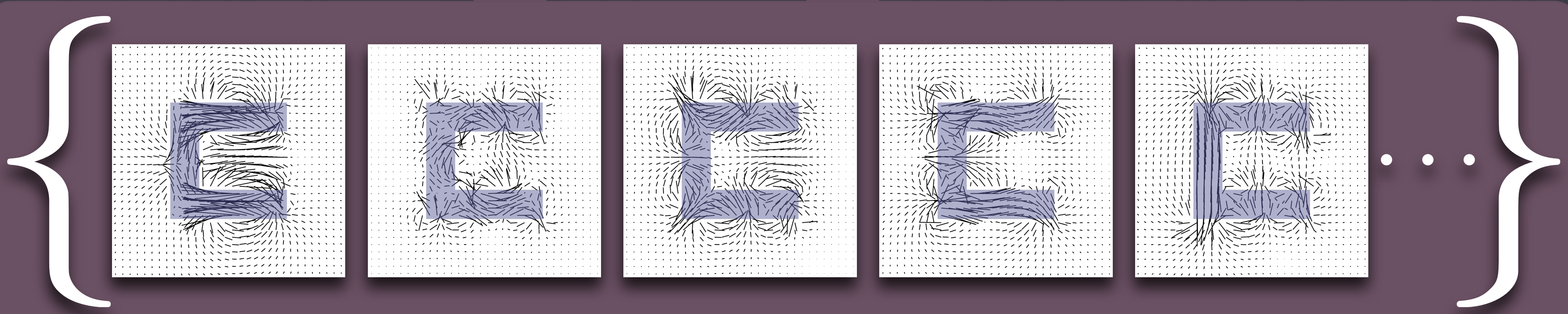
Background → Object

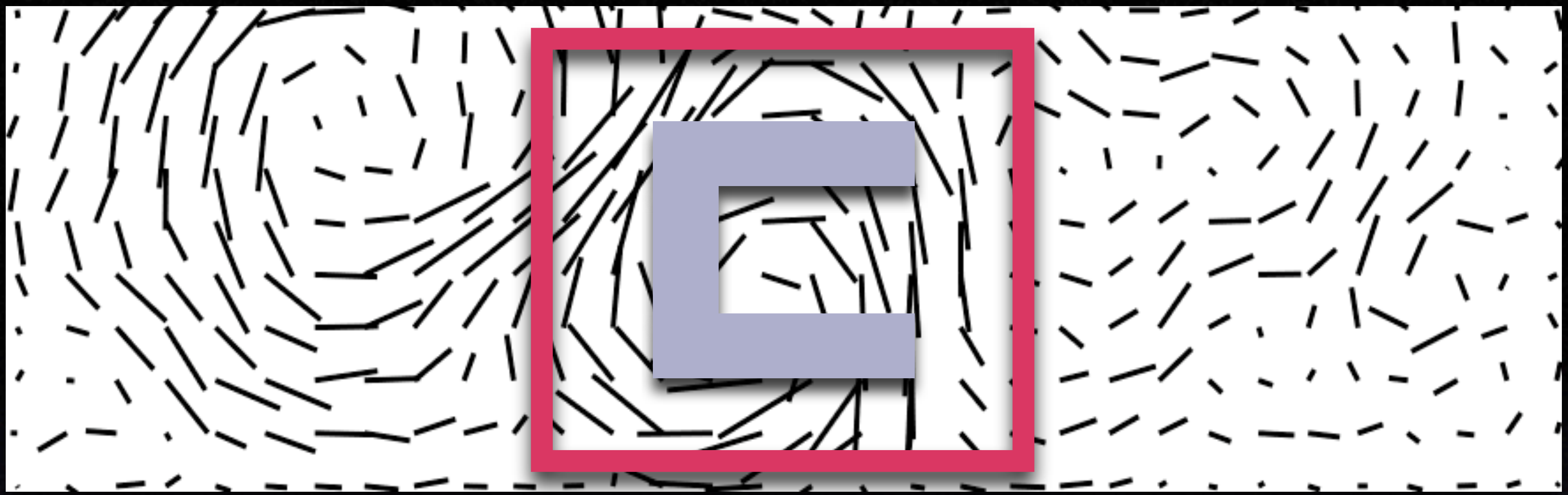
Full

$$\mathbf{0} = S\left(\hat{B}\hat{\mathbf{r}} + B\mathbf{r}\right)$$

Reduced

$$\hat{\mathbf{r}} = M\mathbf{r}$$





Background → Object

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$$\mathbf{0} = S\left(\hat{B}\hat{\mathbf{r}} + B\mathbf{r}\right)$$

Reduced

$$\hat{\mathbf{r}} = M\mathbf{r}$$

Object → Background

Reduced!

Full

$$\mathbf{r} {+}{=}\; B^T\hat{B}\hat{\mathbf{r}}$$

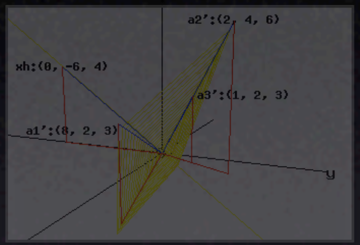
Reduced

$$\mathbf{r} {+}{=}\; N\hat{\mathbf{r}}$$

QuickTime™ and a
decompressor
are needed to see this picture.


Galerkin v1.01

Overview



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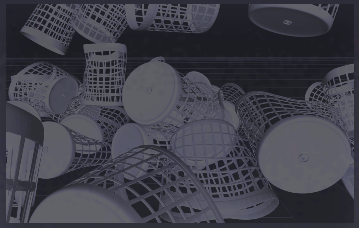
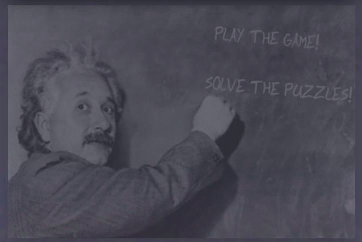


Model Reduction of Deformable Solids

Coupling



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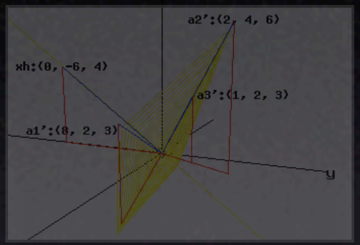

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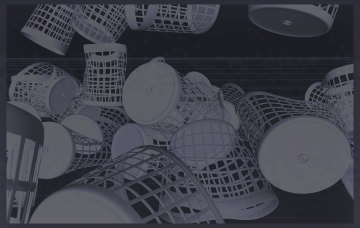

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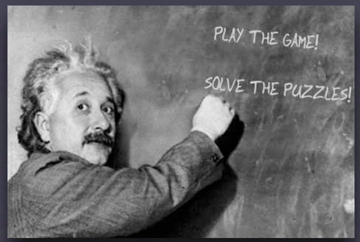


Questions

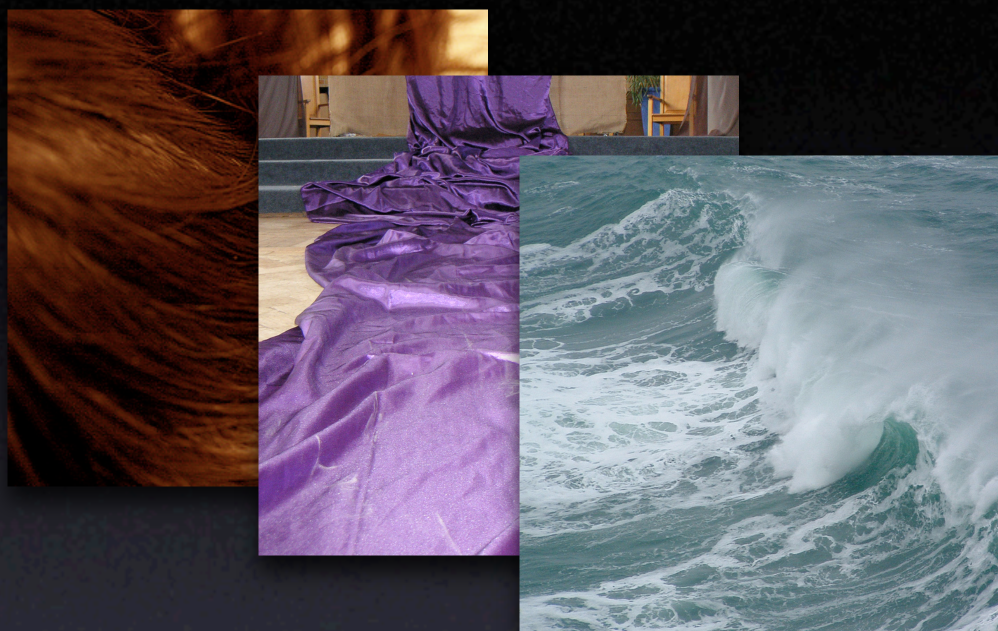
- Good Reviews :)

- What's the next big thing?
- Machine Learning + Graphics:
  - Learning physics?
- Networking + Graphics:
  - Distributed physics?
- Biology + Graphics:
  - Animal Morphology?
- Other disciplines:
  - Urban planning?
  - What else?

What's the next big thing?

- video tape an object under known lighting conditions to create a "data driven" model of its surface reflectance
- huge database of mocap data
- if we get the physics right, machine learning should be able to estimate parameters
    - how much force is being applied from a motion capture animation?
- distribute the calculation of massive fluid simulations across a network
- distributing computation between CPU and GPU
- once we solve biomechanics in graphics + then we can build robotic prothetic arms
- network-based graphics
    - separate the objects from the scene, render each separately
    - take advantage of the # of cores to do faster rendering
- computer graphics must wait for developments in machine learning for certain applications
- physics is parallel, computer cores are becoming parallel... how can we take advantage of this?
- sound synthesis
    - *model reduction for sound synthesis*
- haptics: glove with little actuators
- surface feeling "rendering" - "roughness rendering"
- wear and tear on an environment

Future Directions



- More Phenomena

- Coupling

- Control