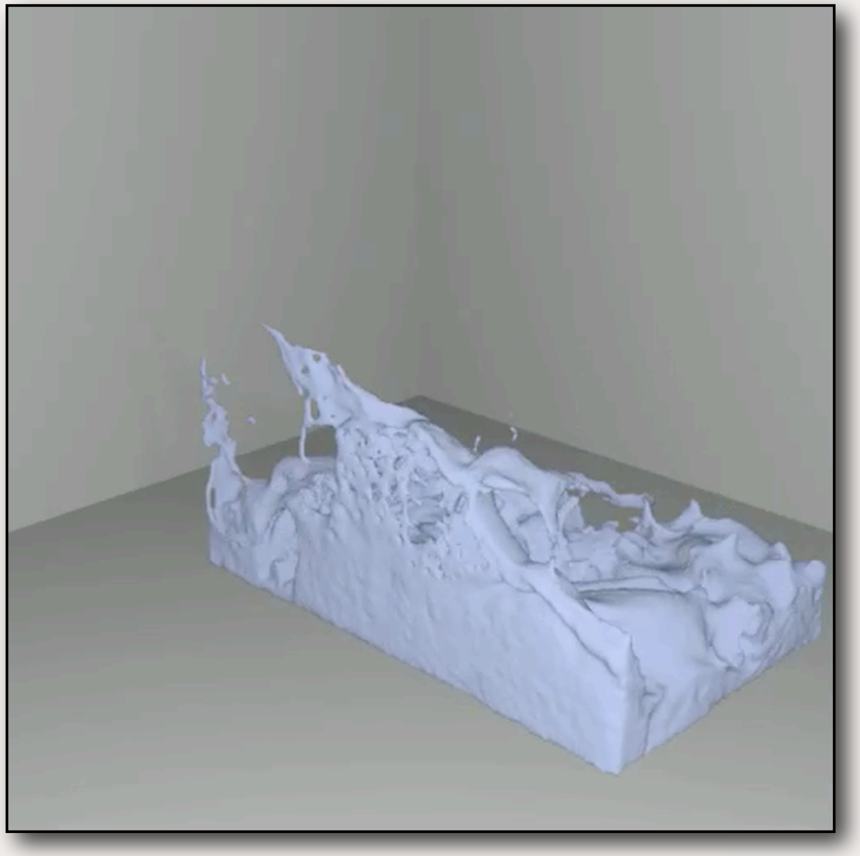
Particle-based Liquids



Adrien Treuille

Reference:
Matthias Müller, David Charypar and Markus Gross. *Particle-Based Fluid Simulation for Interactive Applications*. Eurographics/SIGGRAPH Symposium on Computer Animation (2003).

Adminstrative

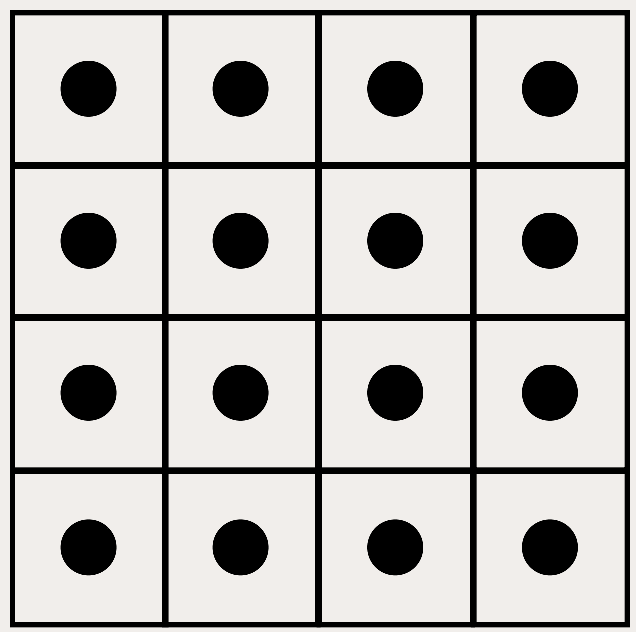
- Will update the syllabus.
- Questions about project 2..?

A New Perspective

- **Continuous Fields:**

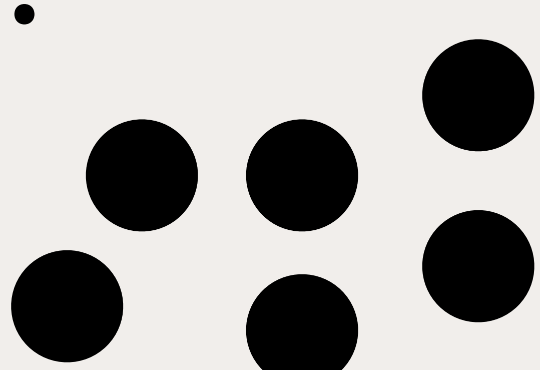
$$\rho(x, y, z)$$

- **Grids:**



(Eulerian)

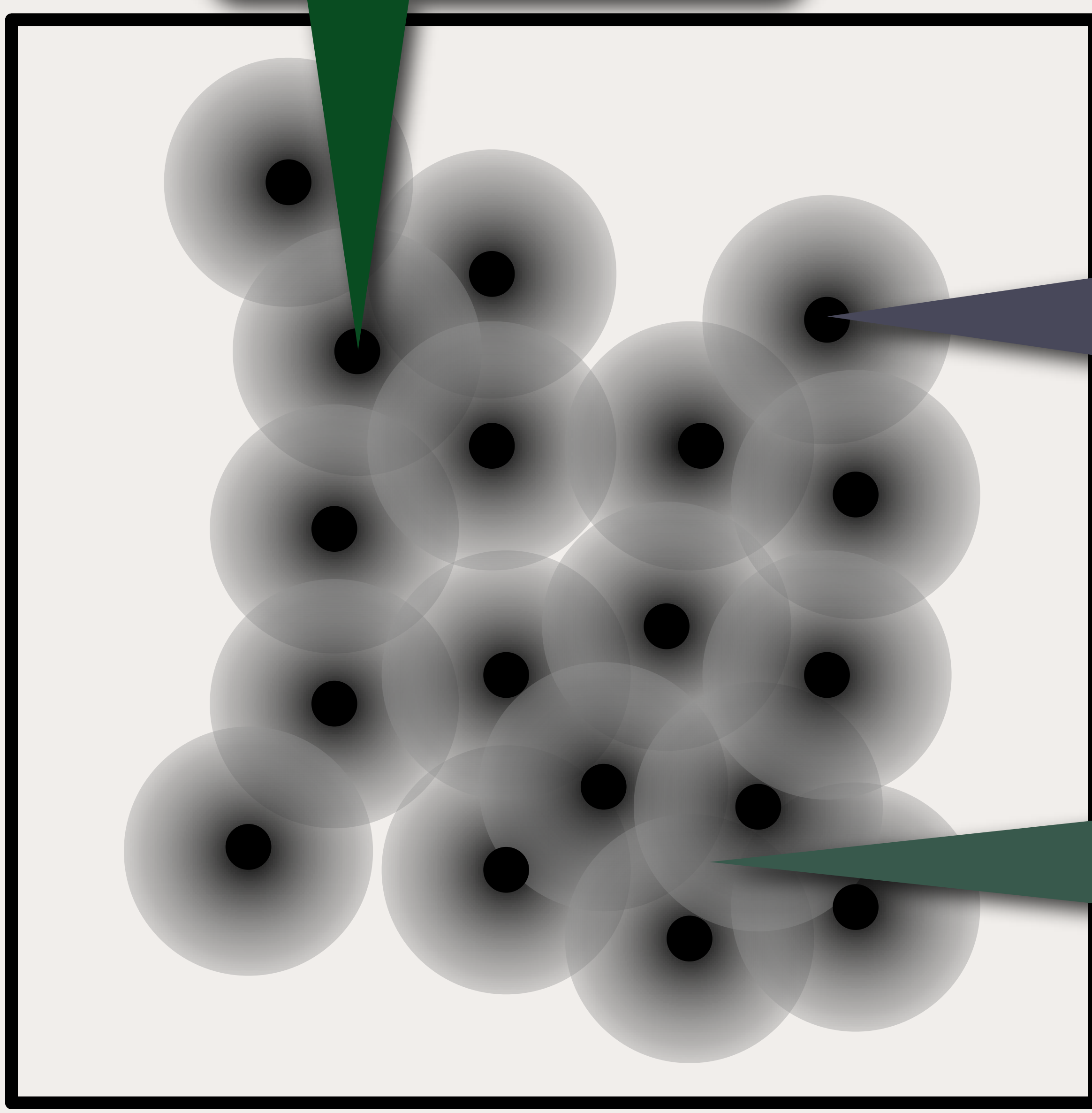
- **Particles?**



(Lagrangian)

Smoothed Particle Hydrodynamics

```
struct particle {
   double mass;
   double position[3];
   double velocity[3];
}
```

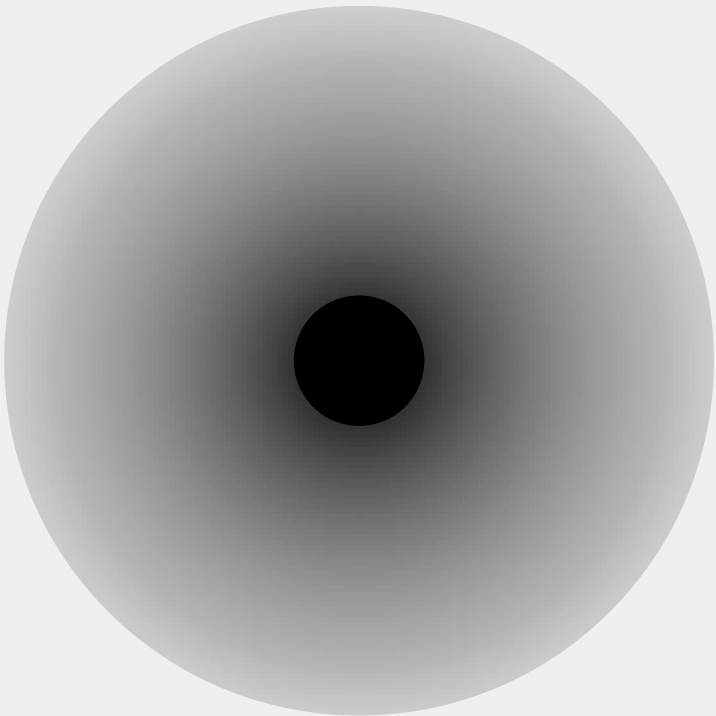


compute physical quantities at these points: p,ρ

what about in between?

sum of kernel function to define everywhere else

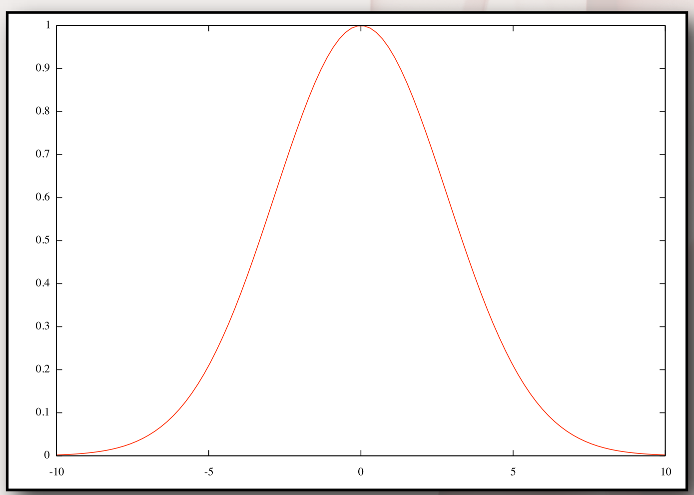
Kernel Functions



$$W(\mathbf{r})$$

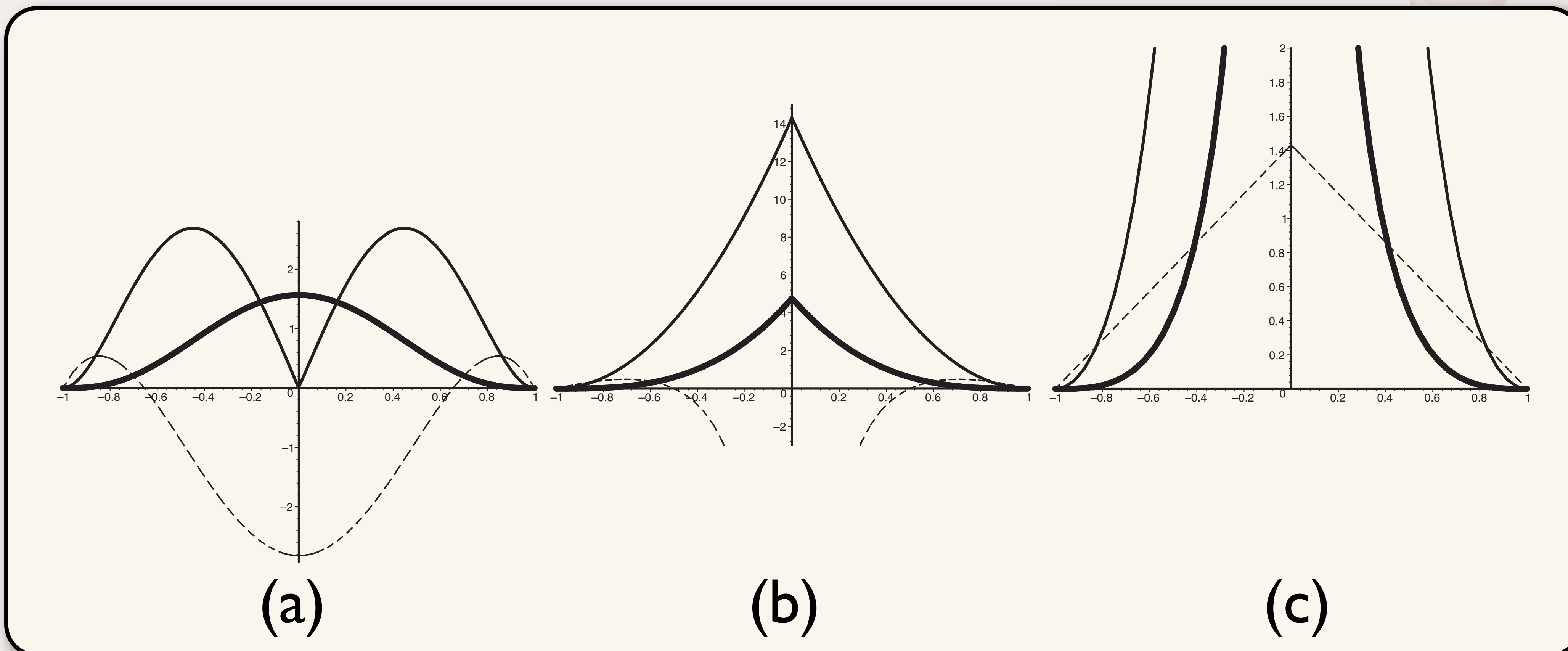
or

$$W(\mathbf{r}, h)$$



- **Properties:**
  - **Symmetric:** $W(\mathbf{x}) = W(-\mathbf{x})$
  - **Finite Support:** $W(\mathbf{x}) = 0 \ \forall ||x|| > h$
  - **Flat at center:** $\nabla W(0) = 0$
  - **Normalized:** $\int W(\mathbf{x}) d\mathbf{x} = 1$

Kernel Function Examples

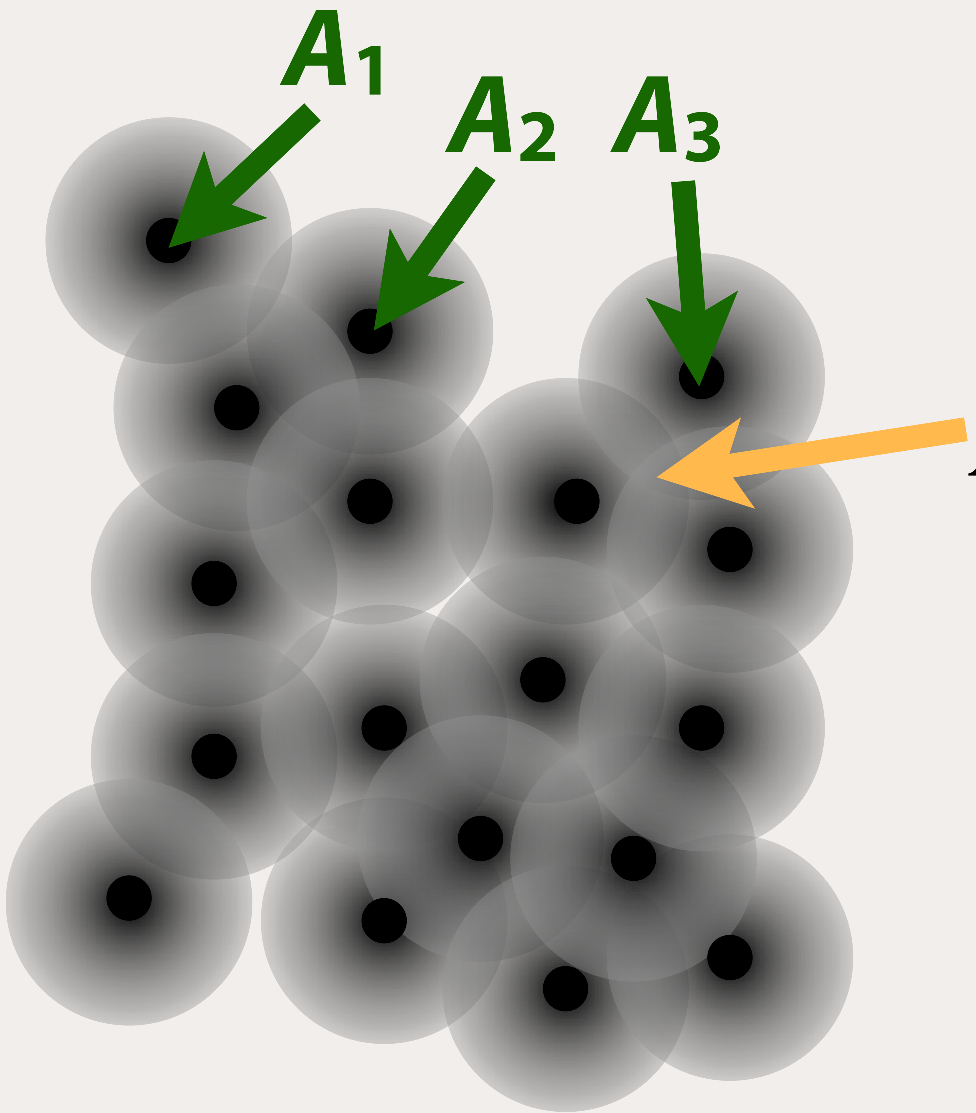


Kernel

Gradient Length

Laplacian

Interpolation



For a physical quantity A:

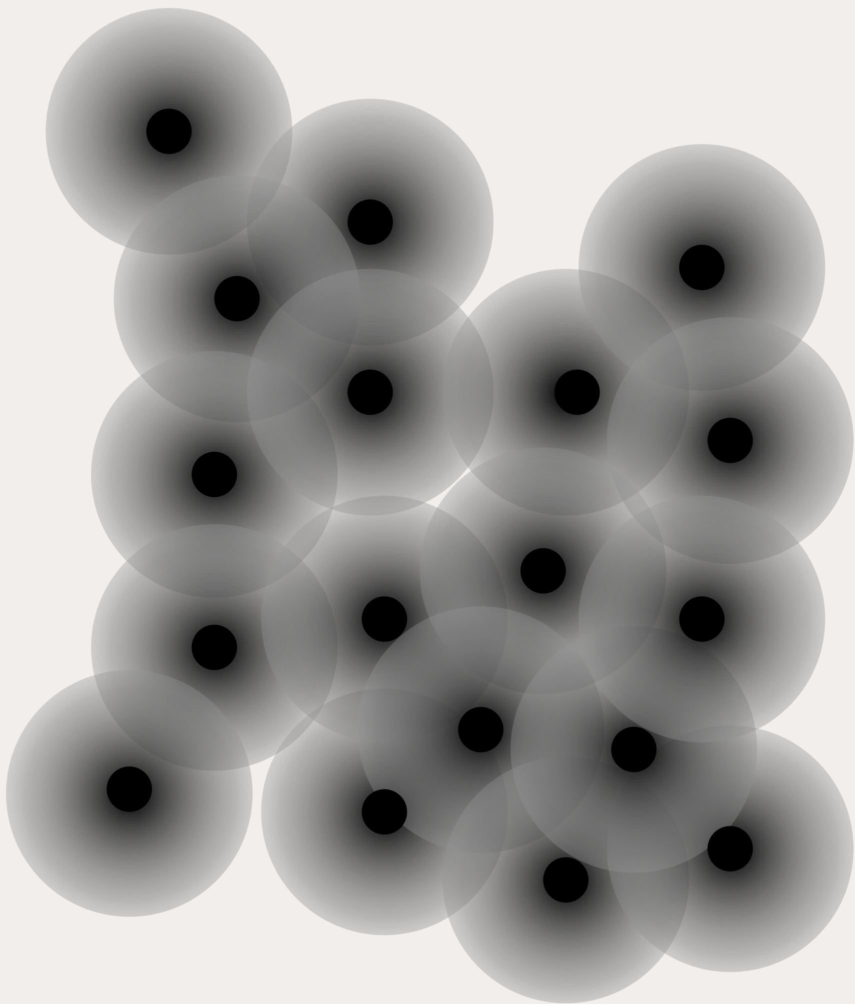
$$A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h),$$

mass
quantity
position
density
kernel
kernel width

Example, density:

$$\rho_S(\mathbf{r}) = \sum_j m_j \frac{\rho_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h)$$

What About Derivatives?



Function:

$$A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h),$$

Gradient:

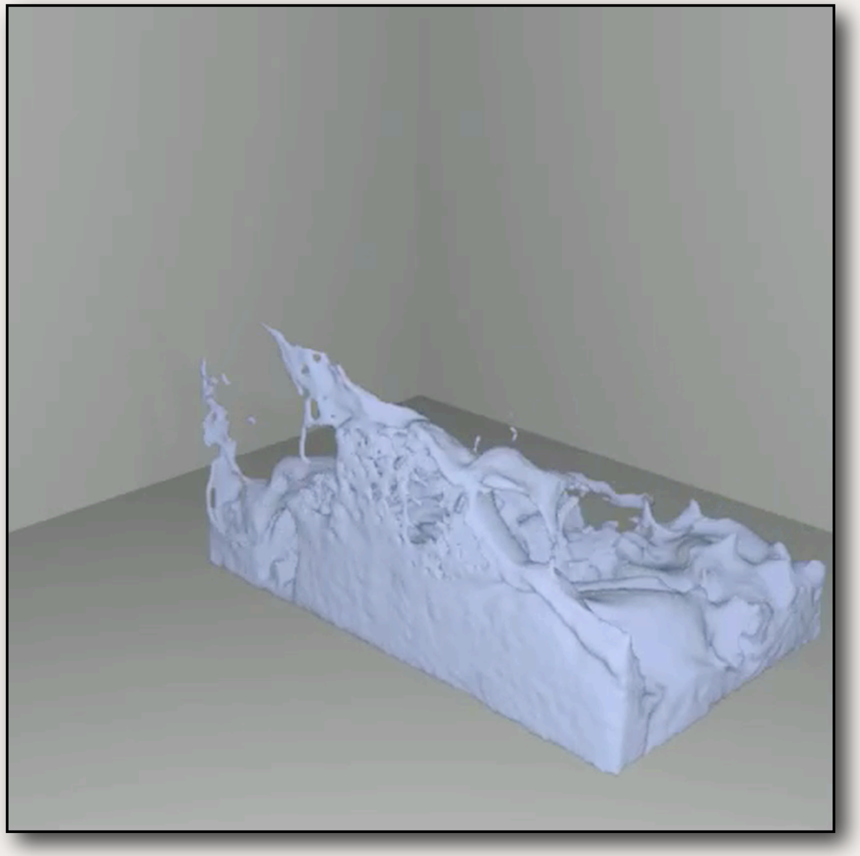
$$\nabla A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(\mathbf{r} - \mathbf{r}_j, h)$$

Laplacian:

$$\nabla^2 A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla^2 W(\mathbf{r} - \mathbf{r}_j, h).$$

But we're going to play tricks. ;-)

SPH Liquids



$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{v} - \nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v},$$

mass acceleration force

$$\frac{D\mathbf{v}}{Dt}$$

Therefore: **Treat it like a ordinary particle system with forces:**

$$-\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v},$$

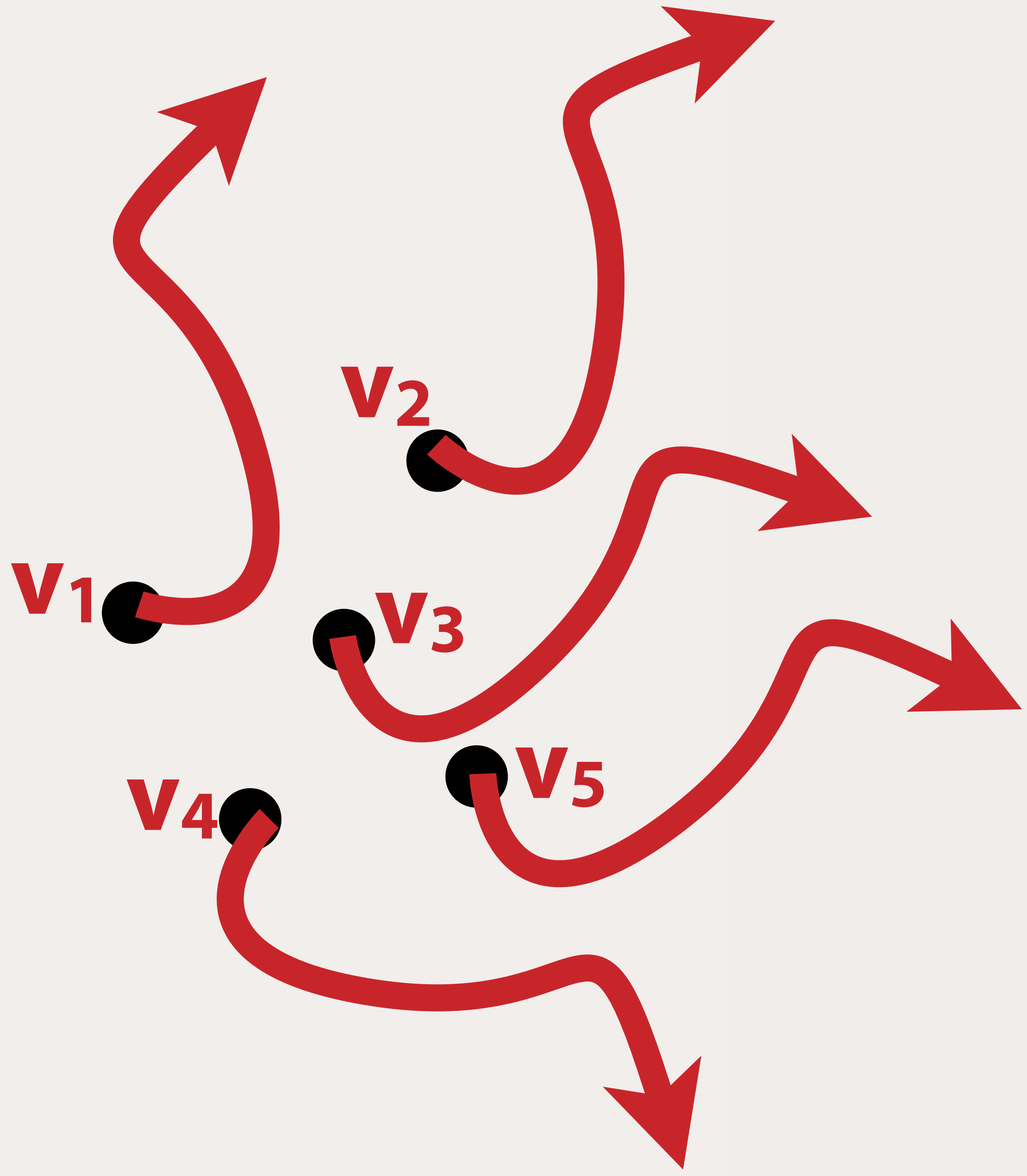
SPH Liquids

Treat it like a ordinary particle system with forces:

$$-\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v},$$

- **Fluid Steps:**
  - **Advection**
  - **Projection (Pressure)**
  - **Diffusion**
  - **External Forces**

Advection



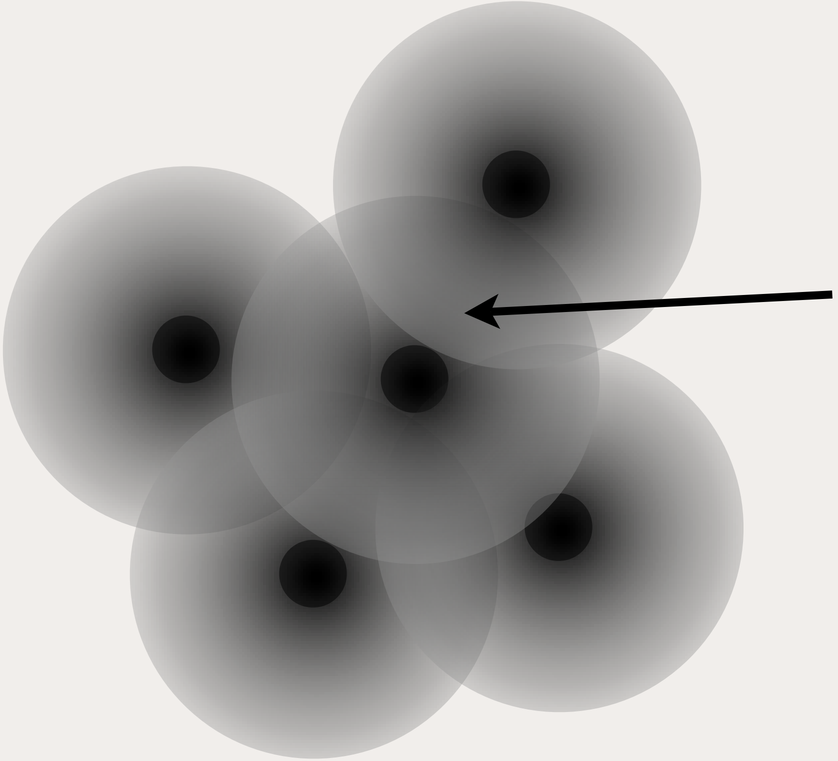
Pressure

For One Particle:

$$p_j = \kappa(\rho_j - \overline{\rho})$$

$$\text{pressure force} = -\nabla p(\mathbf{r})$$

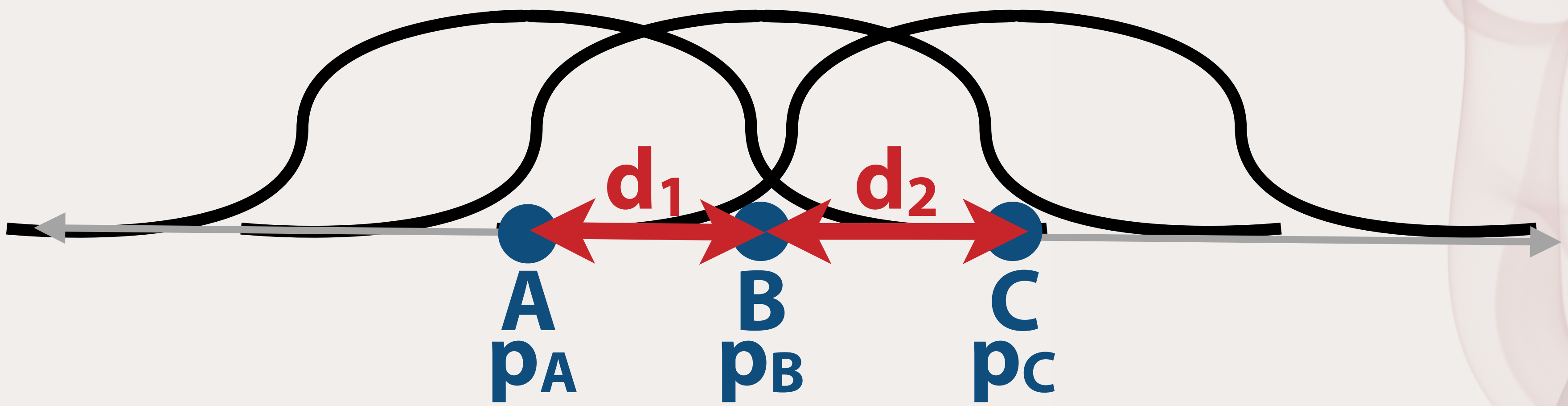
Spatial Pressure Evaluation:



$$\nabla p(\mathbf{r}) = \sum_j m_j \frac{p_j}{\rho_j} \nabla W(\mathbf{r} - \mathbf{r}_j, h)$$

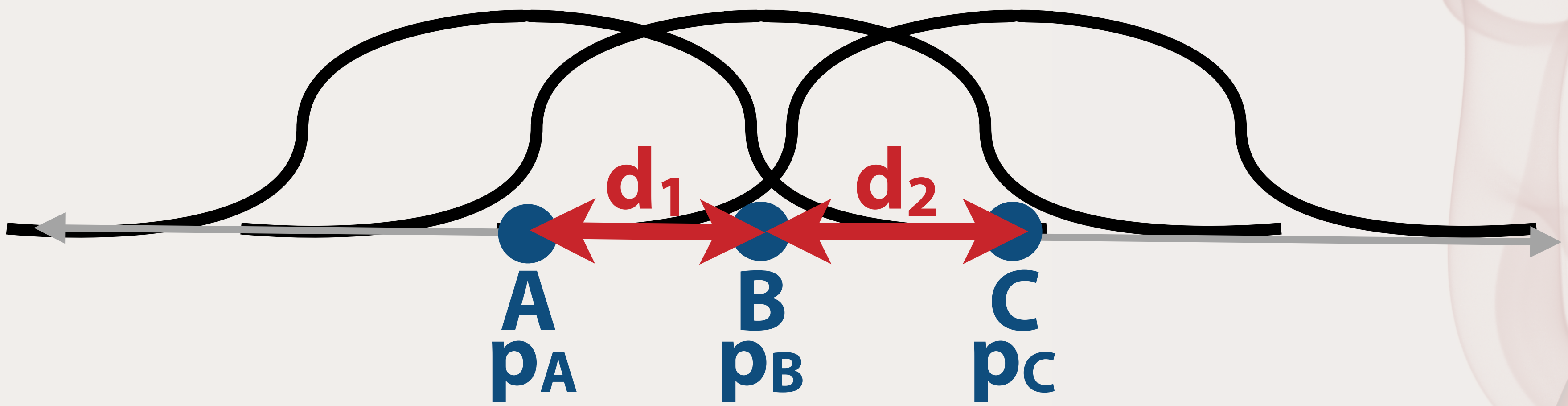
But wait... is this symmetric?

Pressure Symmetry



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

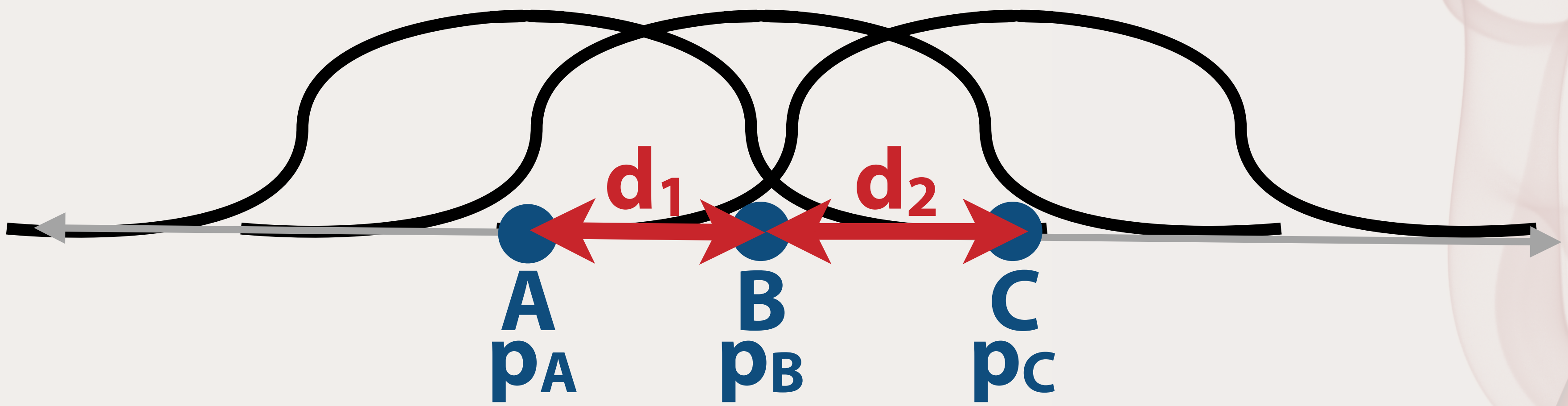
Pressure Symmetry



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$\mathbf{f}_A =$

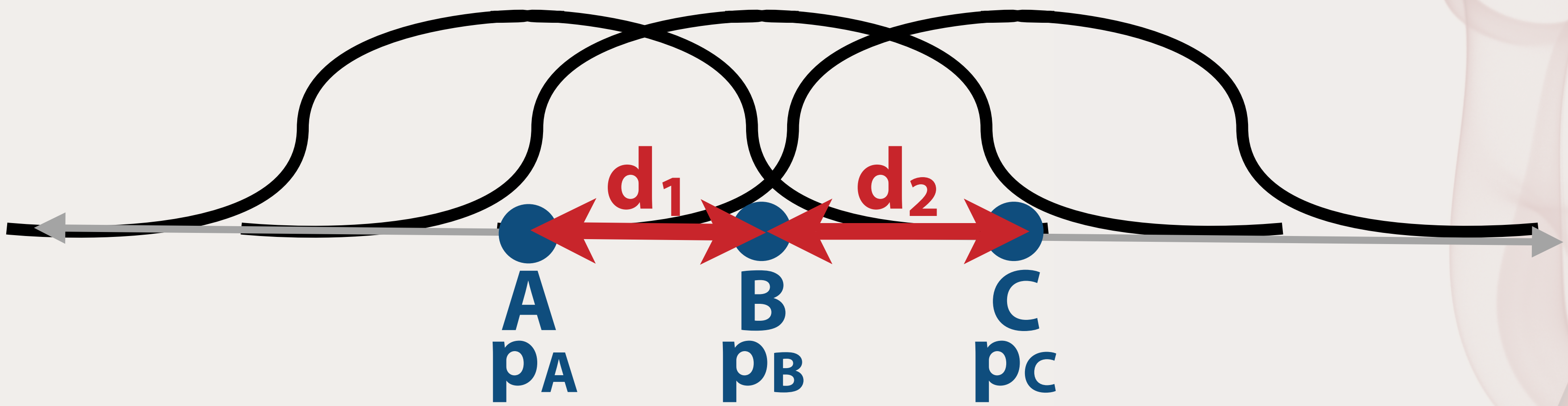
Pressure Symmetry



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\mathbf{f_A = -p_A \nabla W(0)}$$

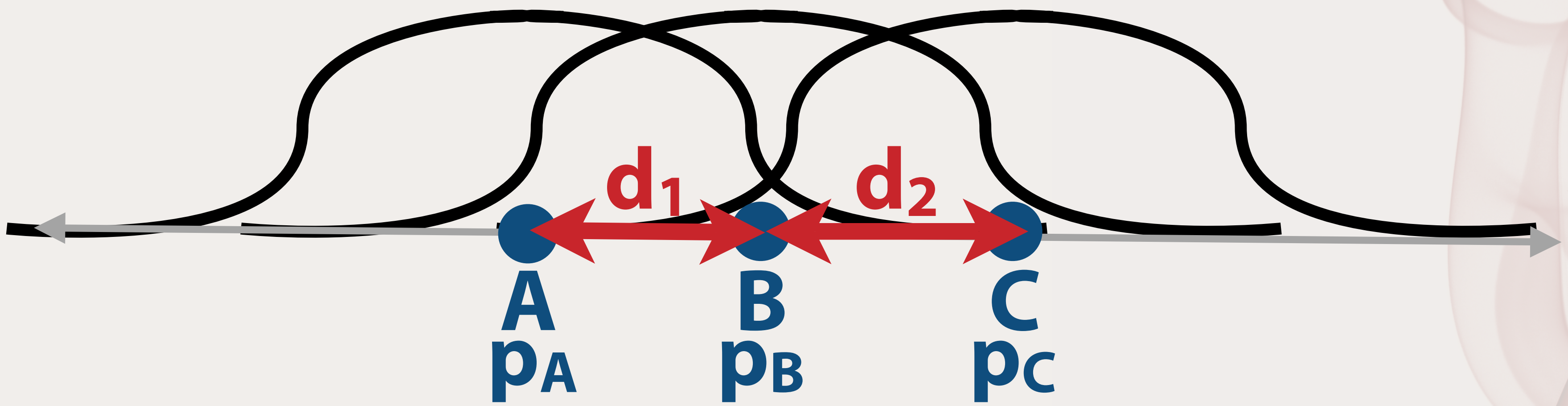
Pressure Symmetry



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\mathbf{f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1)}$$

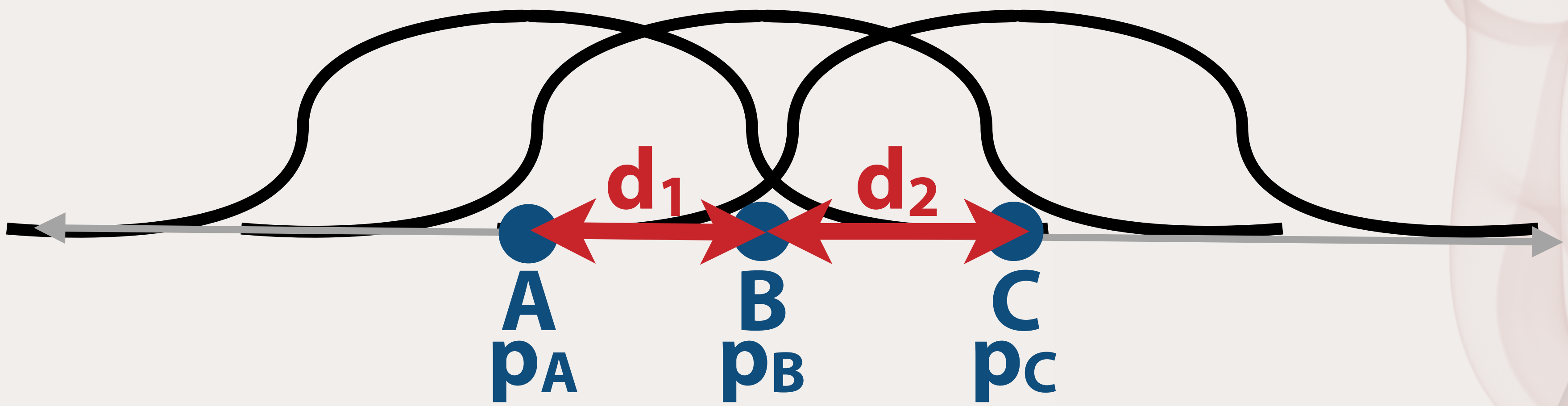
Pressure Symmetry



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\mathbf{f}_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1 - d_2)$$

Pressure Symmetry



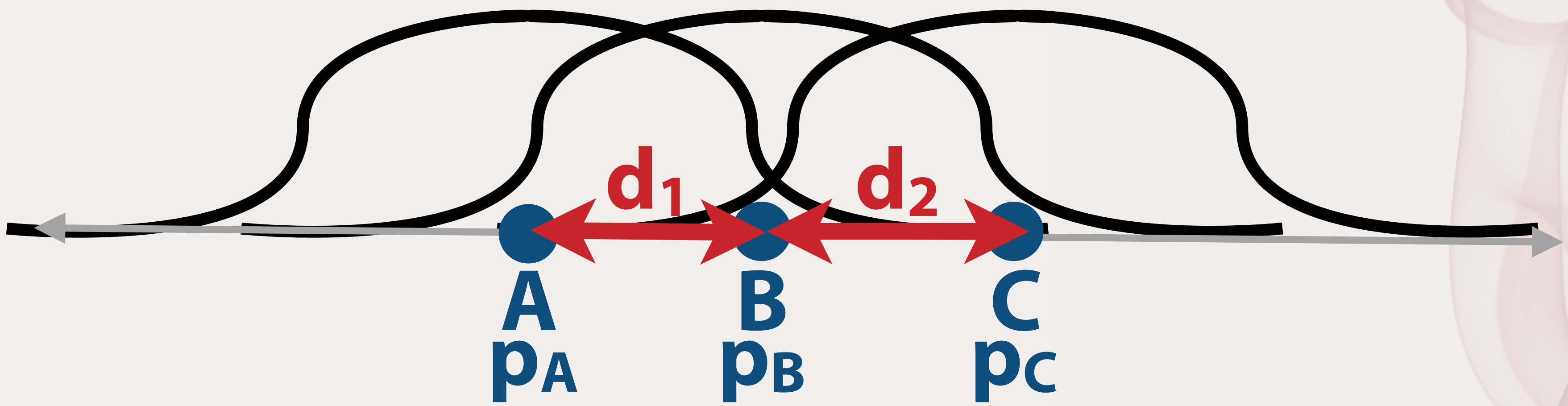
$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1 - d_2)$$

$$f_B = -p_A \nabla W(d_1) - p_B \nabla W(0) - p_C \nabla W(-d_2)$$

$$f_C = -p_A \nabla W(d_1 + d_2) - p_B \nabla W(d_2) - p_C \nabla W(0)$$

Pressure Symmetry



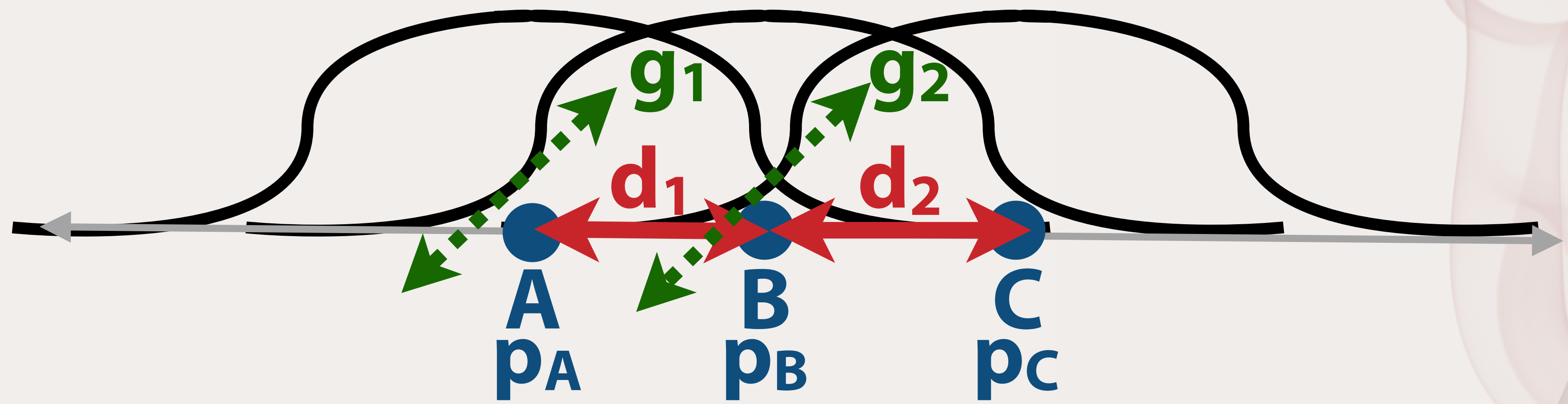
$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1 - d_2)$$

$$f_B = -p_A \nabla W(d_1) - p_B \nabla W(0) - p_C \nabla W(-d_2)$$

$$f_C = -p_A \nabla W(d_1 + d_2) - p_B \nabla W(d_2) - p_C \nabla W(0)$$

Pressure Symmetry



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

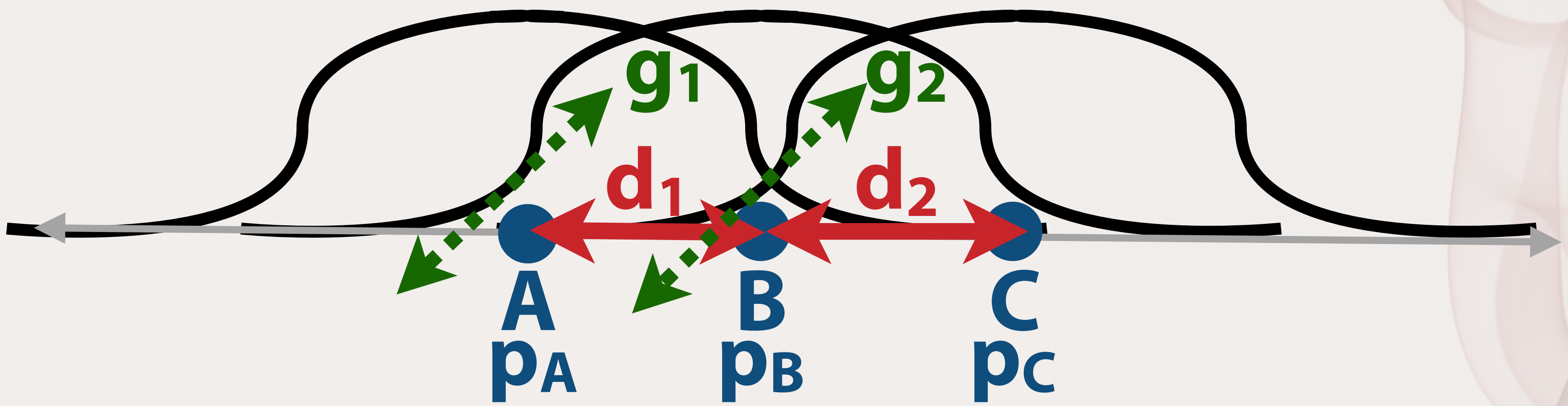
$f_A = -\cancel{p_A \nabla W(0)} - p_B \nabla W(-d_1) - p_C \cancel{\nabla W(-d_1 - d_2)}$

$f_B = -p_A \nabla W(d_1) - \cancel{p_B \nabla W(0)} - p_C \nabla W(-d_2)$

$f_C = -p_A \cancel{\nabla W(d_1 + d_2)} - p_B \nabla W(d_2) - \cancel{p_C \nabla W(0)}$

$g_1 = -\nabla W(-d_1) \quad g_2 = -\nabla W(-d_2)$

Pressure Symmetry

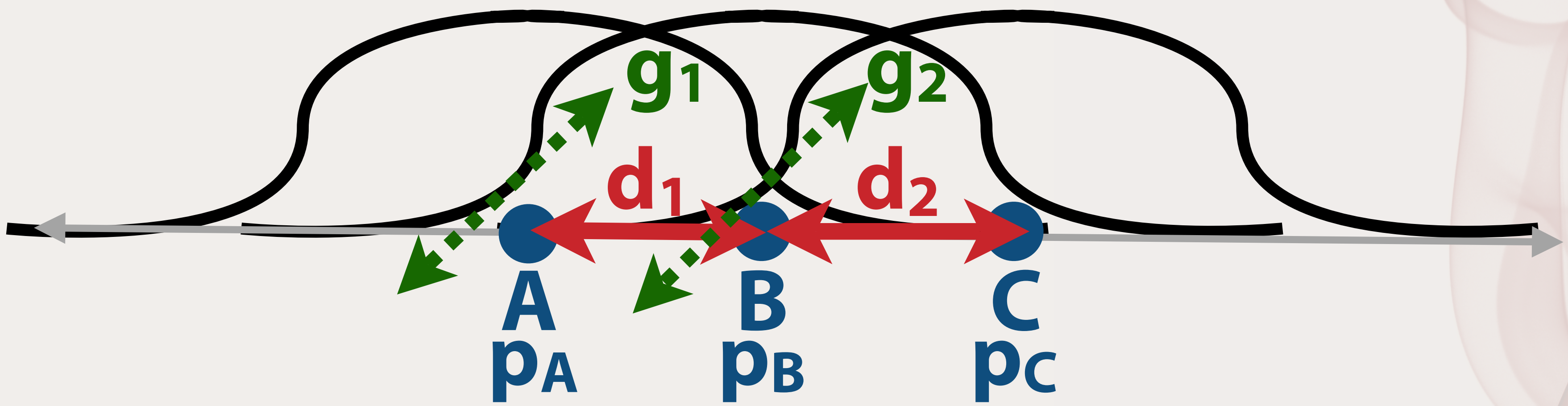


$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\left.\begin{aligned} f_A &= p_B\, g_1 \\ f_B &= -p_A\, g_1 + p_C\, g_2 \\ f_C &= -\, p_B\, g_2 \end{aligned}\right\} \neq 0$$

$$g_1 = -\,\nabla W(-d_1) \quad g_2 = -\,\nabla W(-d_2)$$

Pressure Symmetry



$$-\nabla p(\mathbf{r}_i) = \sum_j \frac{p_i + p_j}{2} \nabla W(\mathbf{r}_i - \mathbf{r}_j)$$

$$\left.\begin{aligned} f_A &= \tfrac{1}{2}(p_A + p_B) g_1 \\ f_B &= -\tfrac{1}{2}(p_A + p_B) g_1 + \tfrac{1}{2}(p_B + p_C) g_2 \\ f_C &= -\tfrac{1}{2}(p_B + p_C) g_2 \end{aligned}\right\} = 0$$

$$g_1 = -\nabla W(-d_1) \quad g_2 = -\nabla W(-d_2)$$

Viscosity

$$\mathbf{f}_i^{\text{viscosity}} = \mu \nabla^2 \mathbf{v}(\mathbf{r}_a) = \mu \sum_j m_j \frac{\mathbf{v}_j}{\rho_j} \nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h).$$

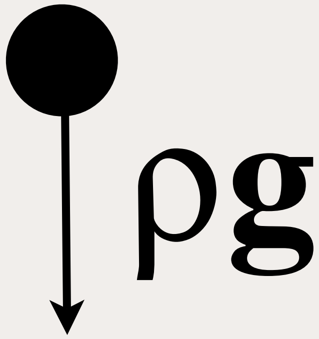
Symmetrization:

$$\mathbf{f}_i^{\text{viscosity}} = \mu \sum_j m_j \frac{\mathbf{v}_j - \mathbf{v}_i}{\rho_j} \nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h).$$

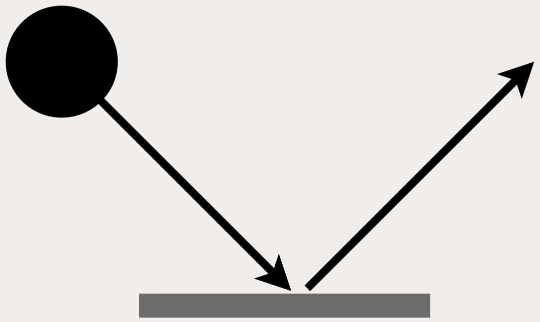
Spring that pulls the particle towards the velocity of it's neighbors.

External Forces

Gravity



Collisions



Simulation

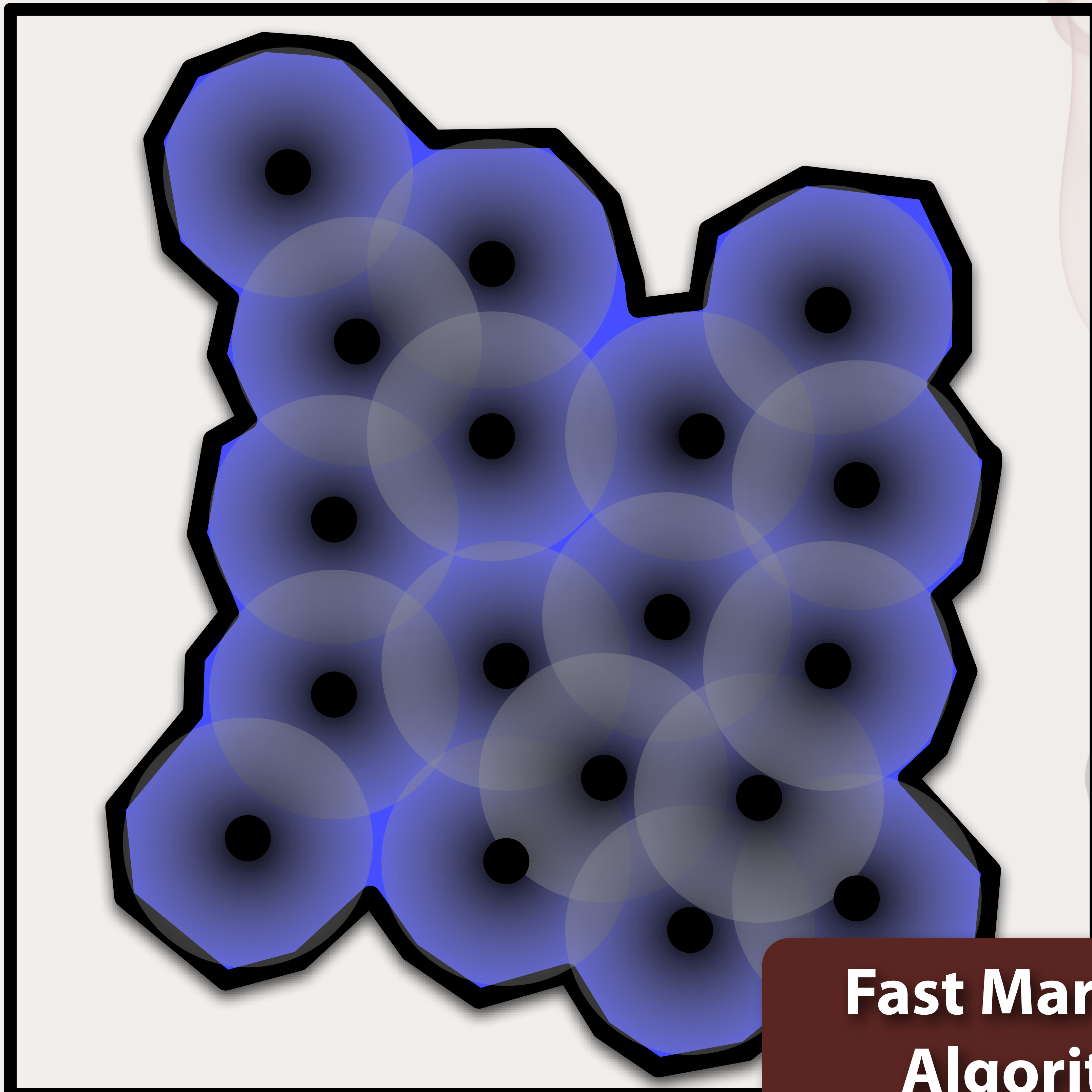
- **Fluid Steps:**
  - **Advection**
  - **Projection (Pressure)**
  - **Diffusion**
  - **External Forces**

A standard particle system:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v}, \end{cases}$$

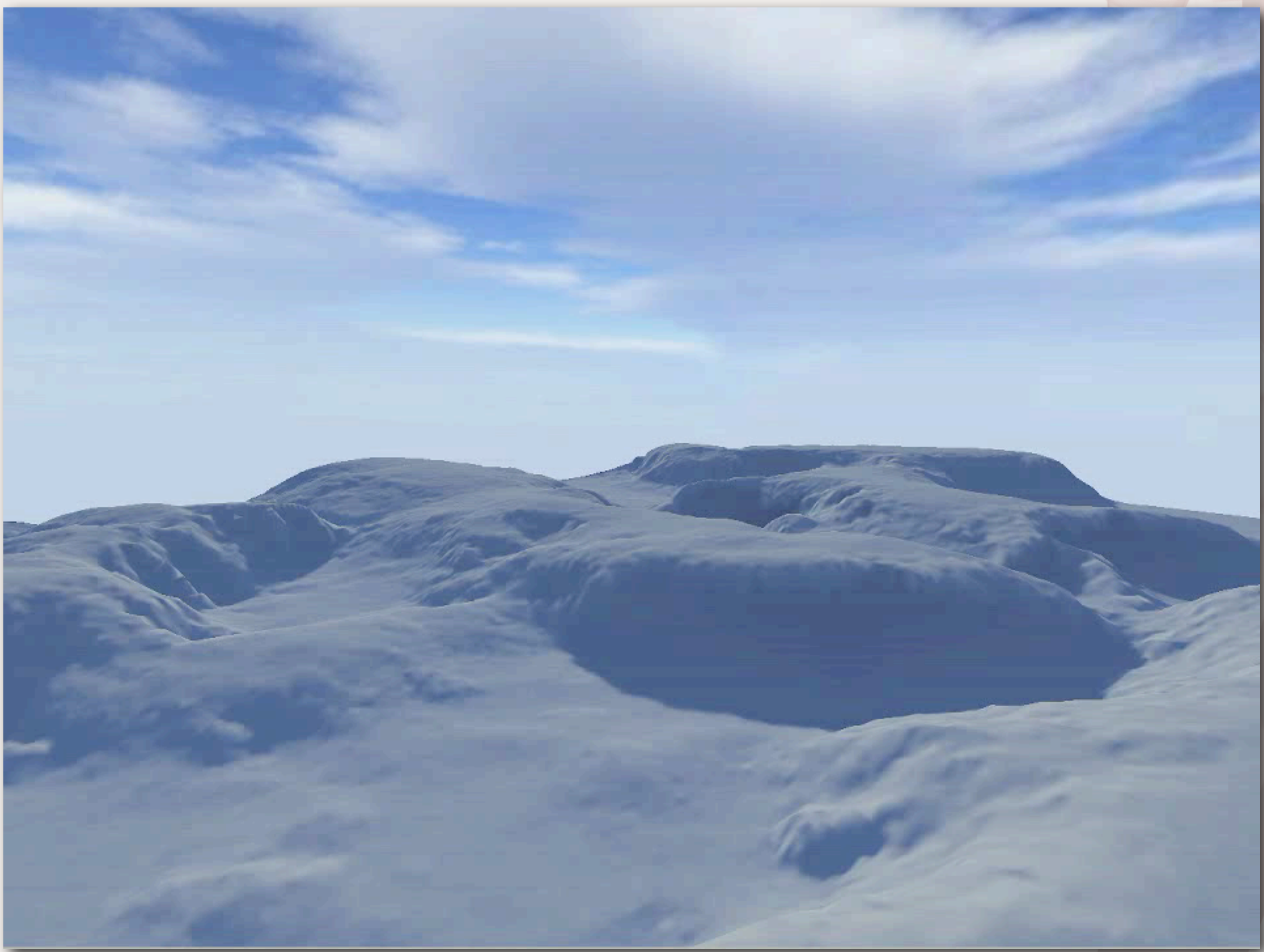
Use your favorite integrator!

Rendering

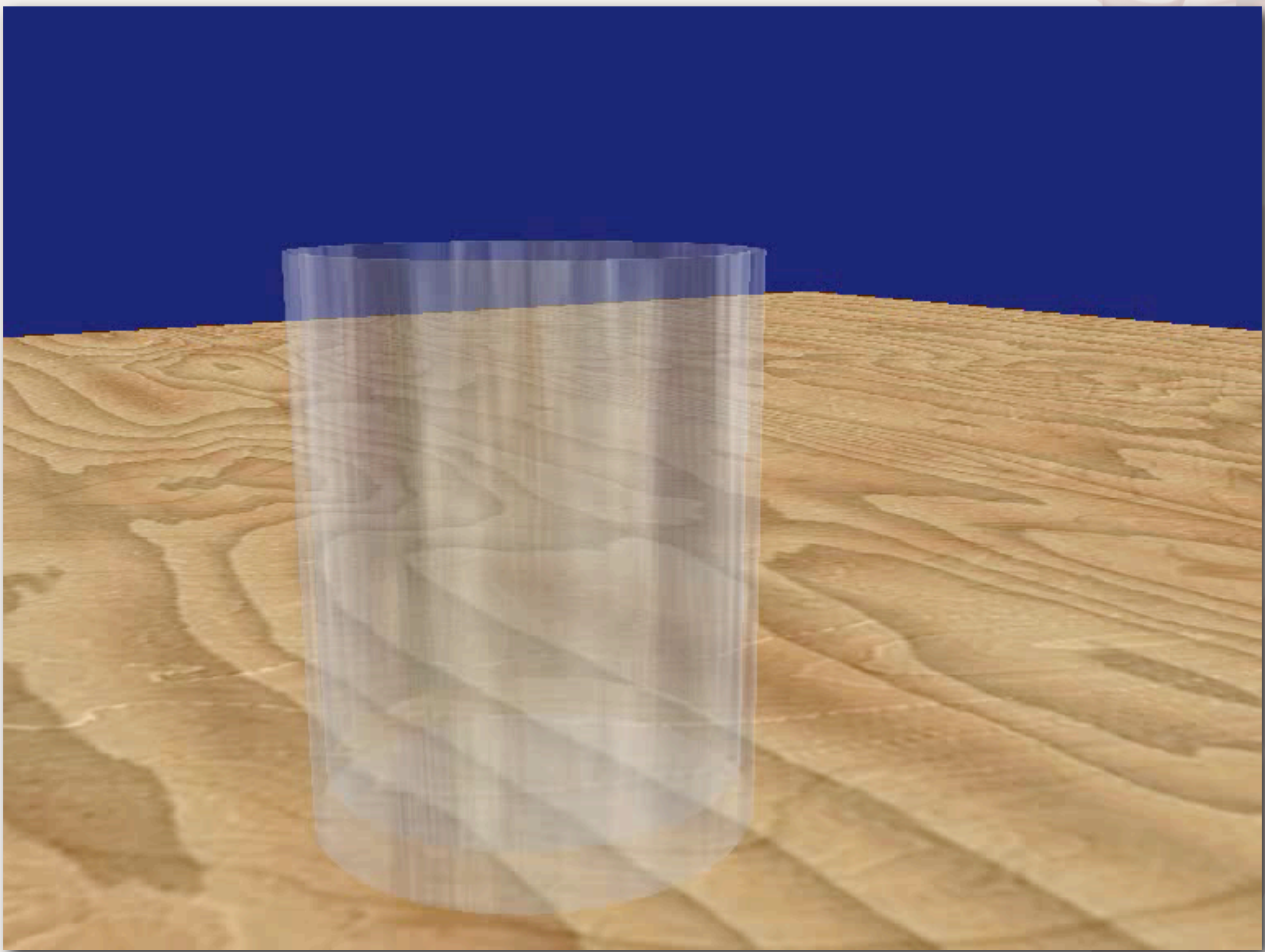


Fast Marching Algorithm

Examples



Examples

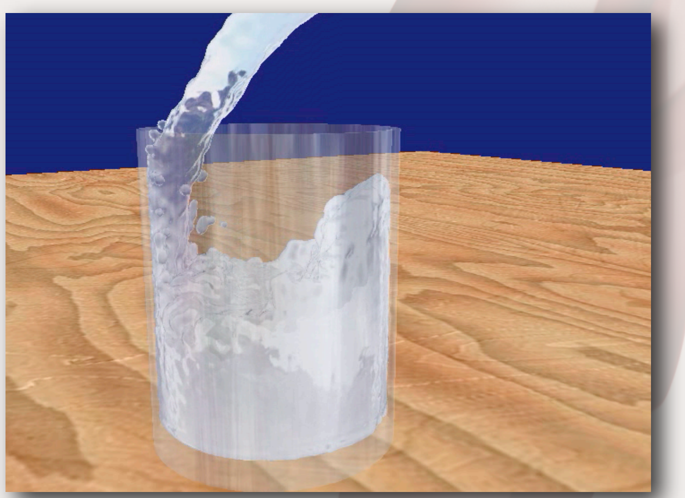


Comparison

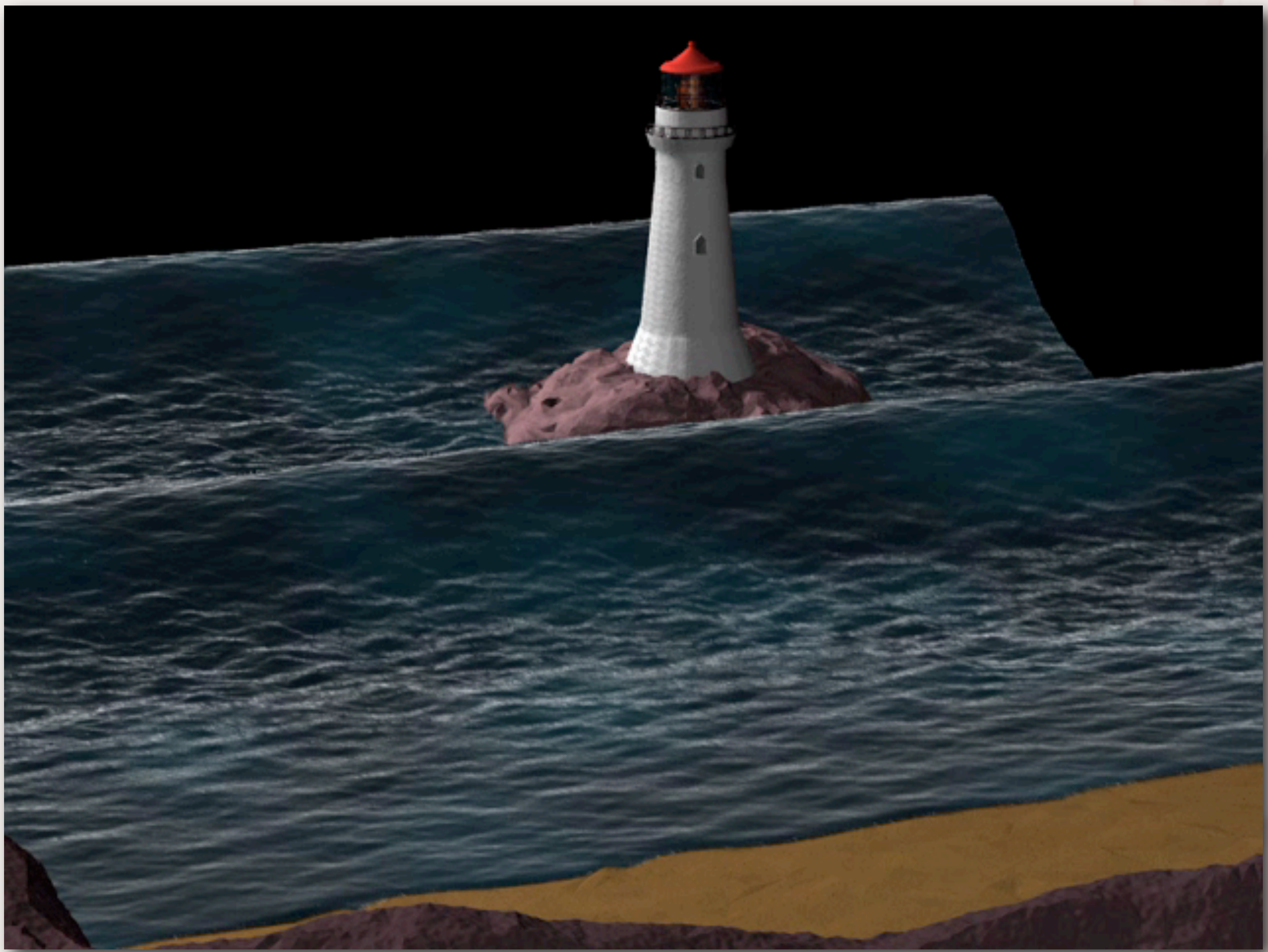
Remarks

- Grid-based (Eulerian)
  - good surfaces
  - bad splashes
  - stable

- Particle-based (Lagrangian)
  - bumpy surfaces
  - good splashes
  - efficient

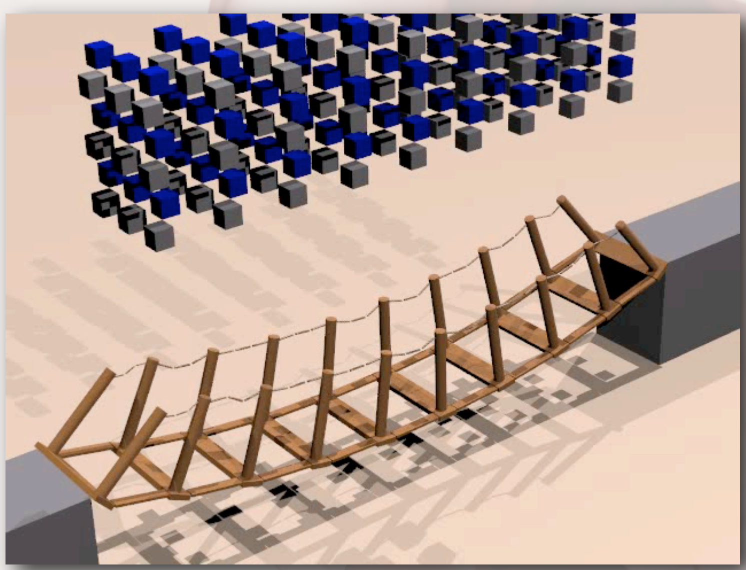


Hybrid Methods

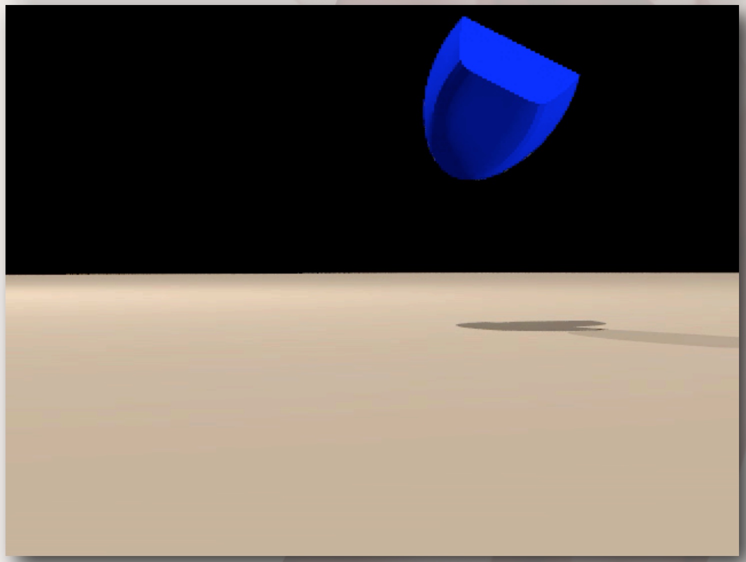


Question

- How do you represent a rigid body?
- Collisions...
  - What kinds are there?
  - Detection?
  - Simulation?



- What about thin objects?



- Constraints?