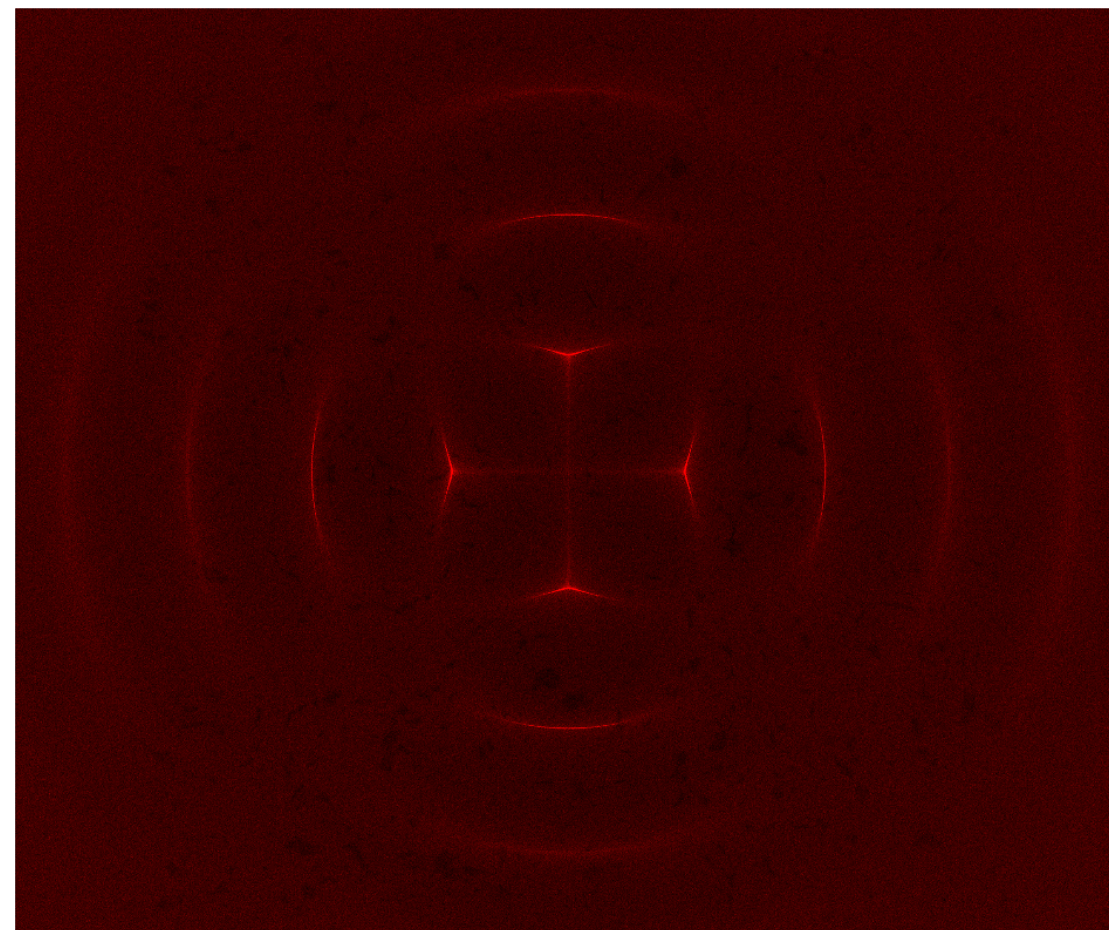
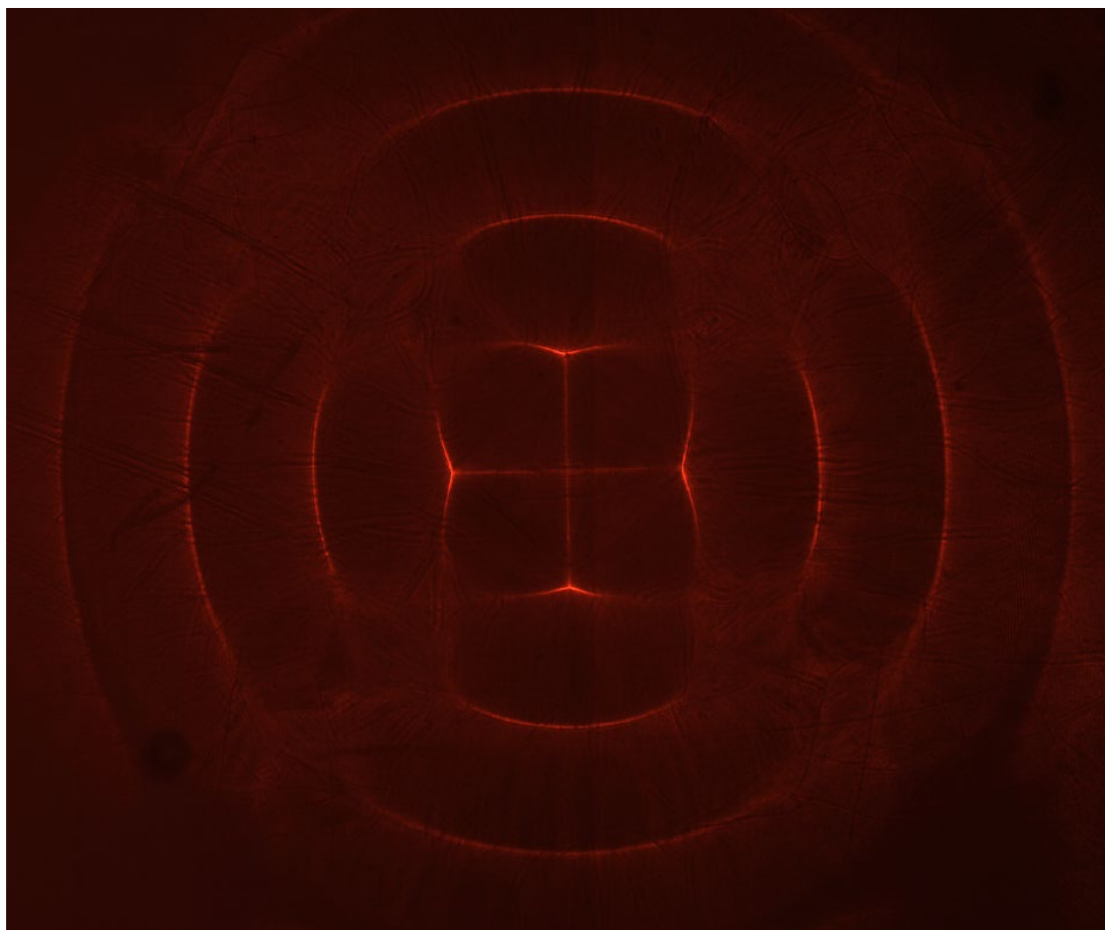


Rendering for scientific imaging applications



15-468, 15-668, 15-868
Physics-based Rendering
Spring 2025, Lecture 16

Course announcements

- We're all done with homework!

Overview of today's lecture

- Rendering continuous refraction.
- GRIN optics.
- Rendering the refractive radiative transfer equation.
- Acousto-optics.
- Rendering speckle.
- Fluorescence microscopy.

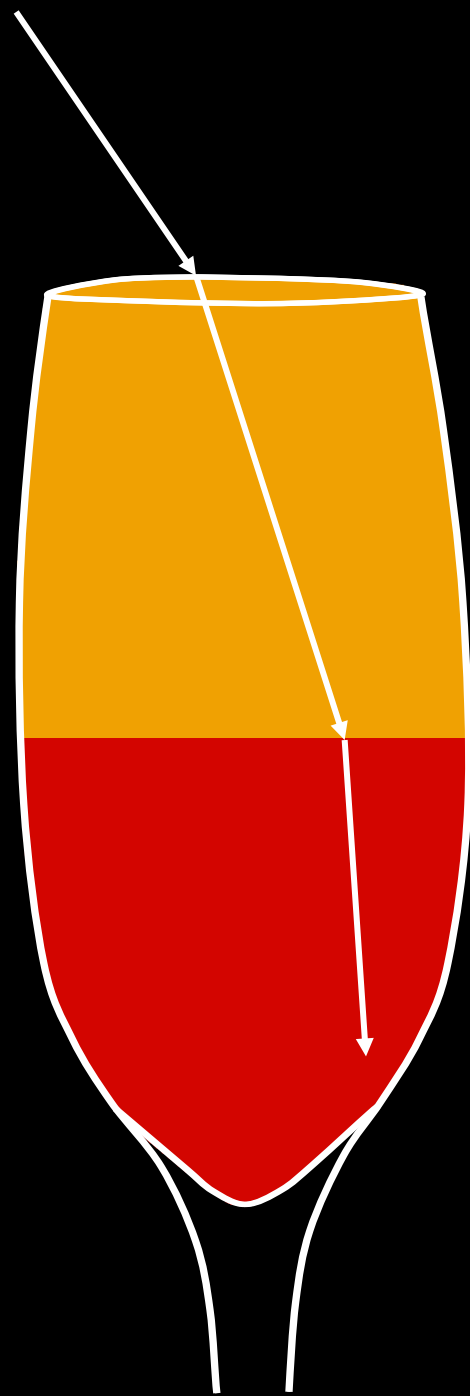
Slide credits

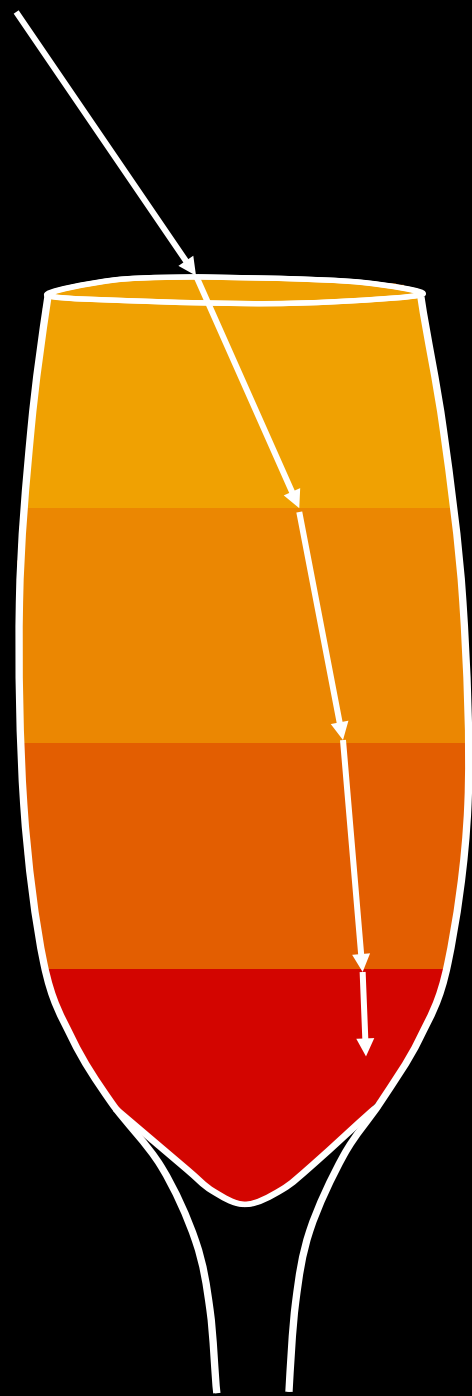
Many of these slides were directly adapted from:

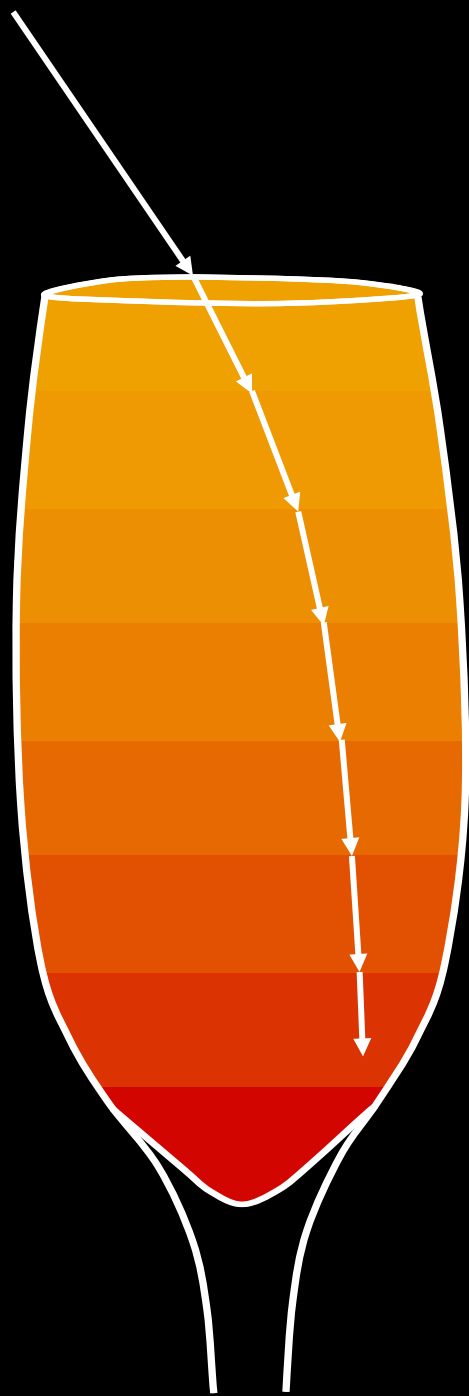
- Adithya Pediredla (CMU).
- Arjun Teh (CMU).
- Chen Bar (Technion).

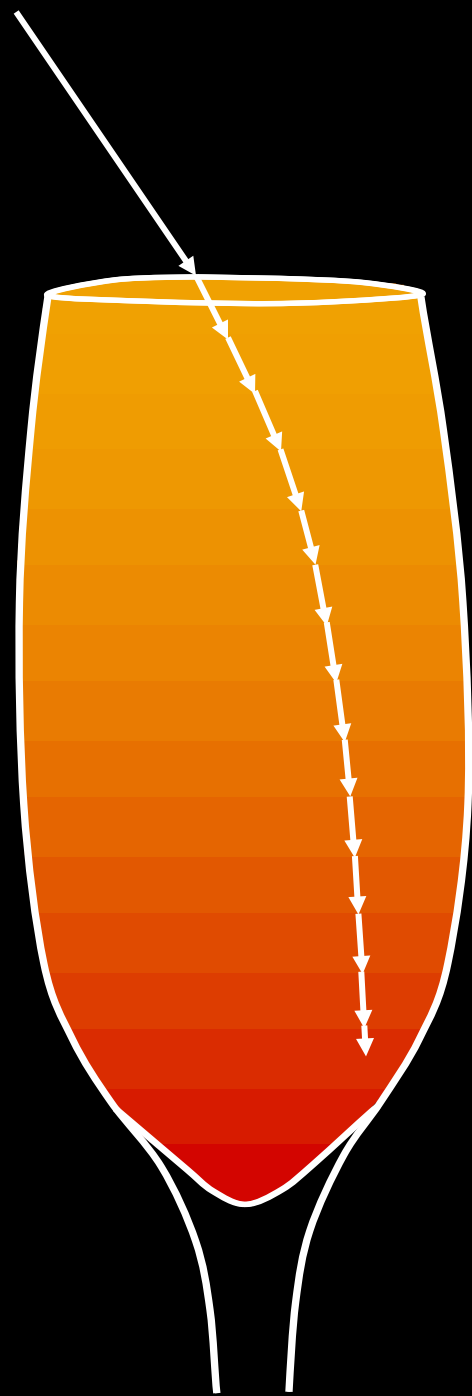


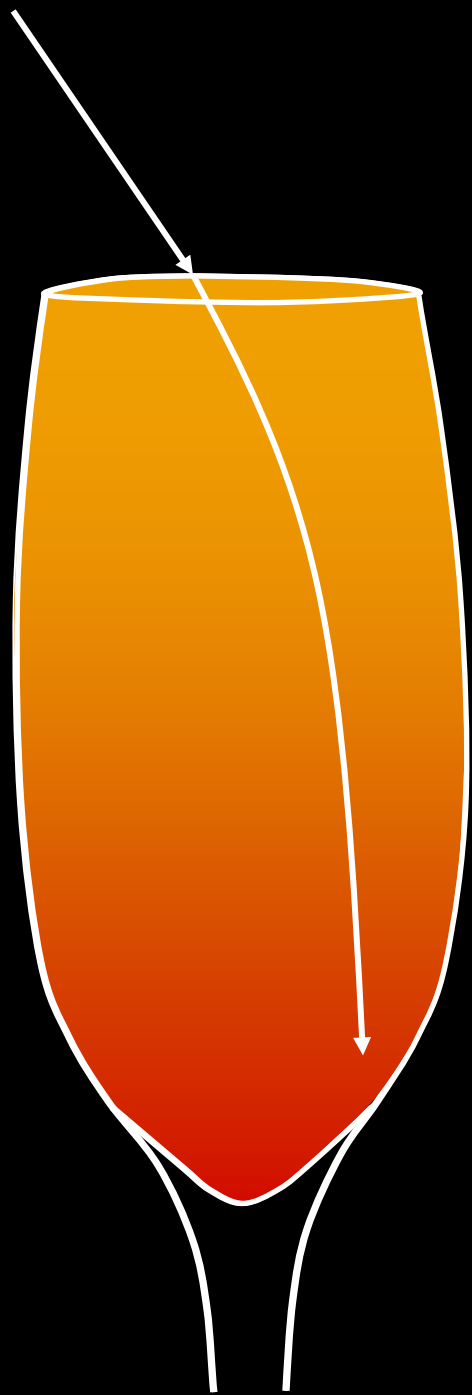
Media with continuously varying refractive index and scattering



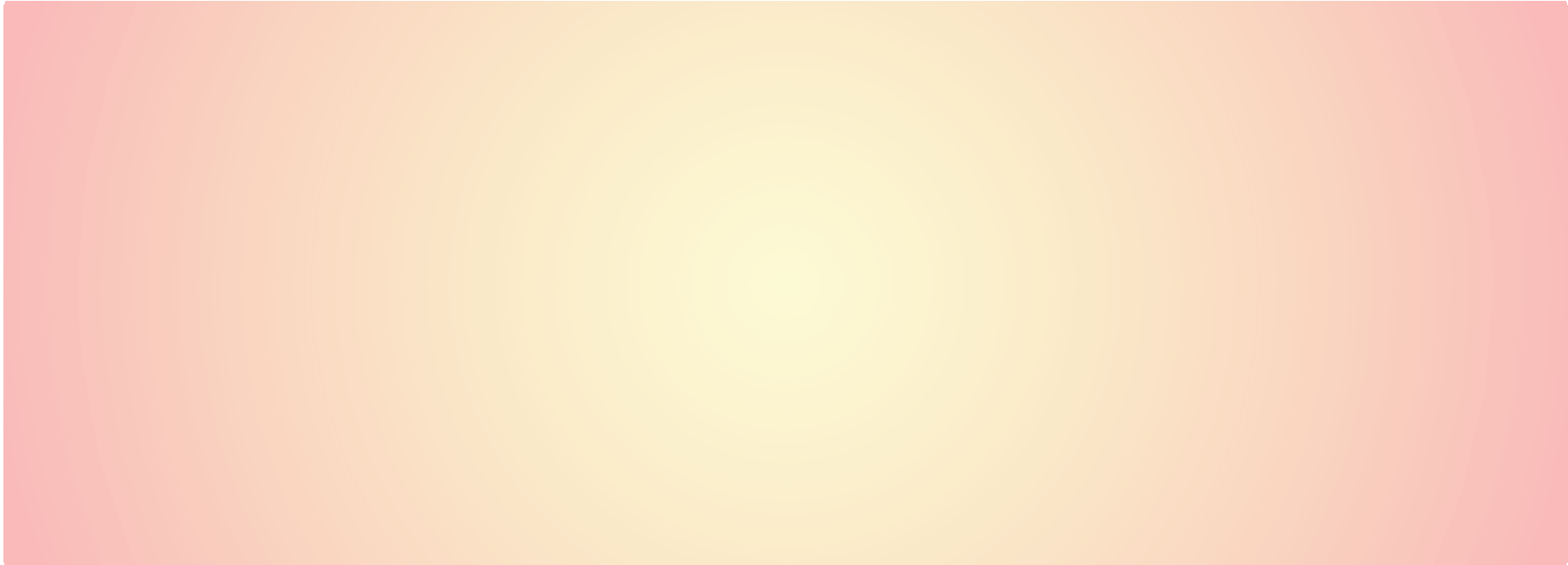






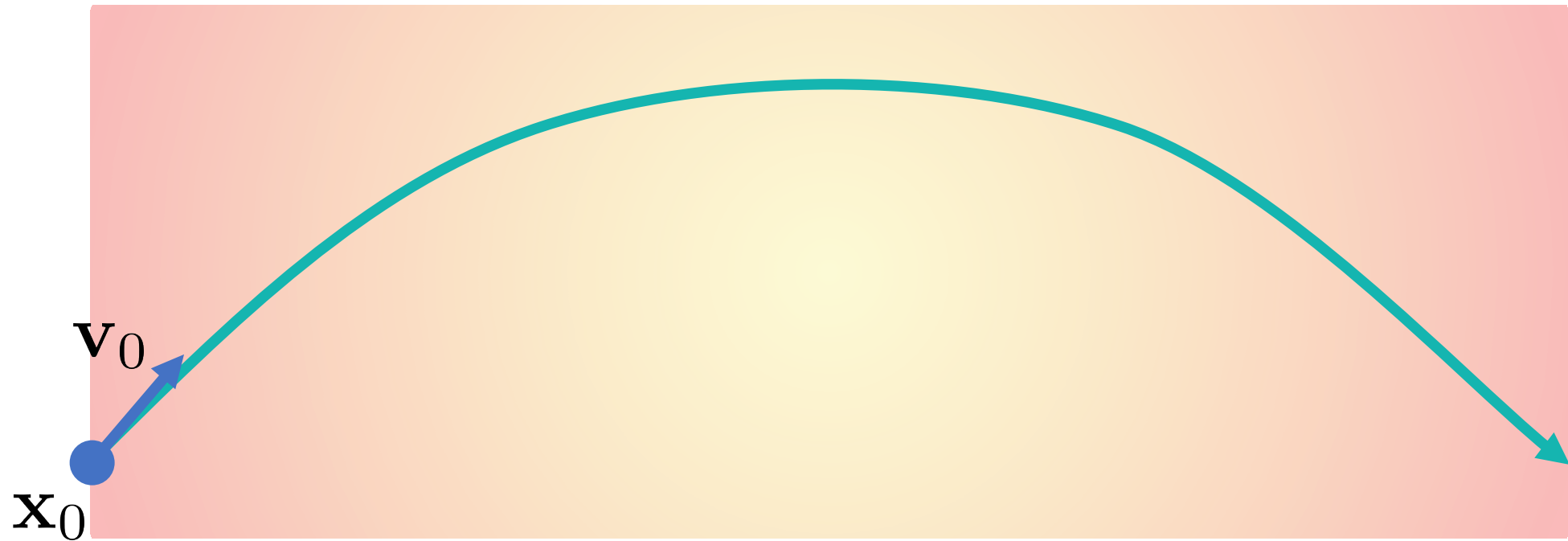


Nonlinear Ray Tracing



$\eta(\mathbf{x})$: refractive index of the volume at location, \mathbf{x}

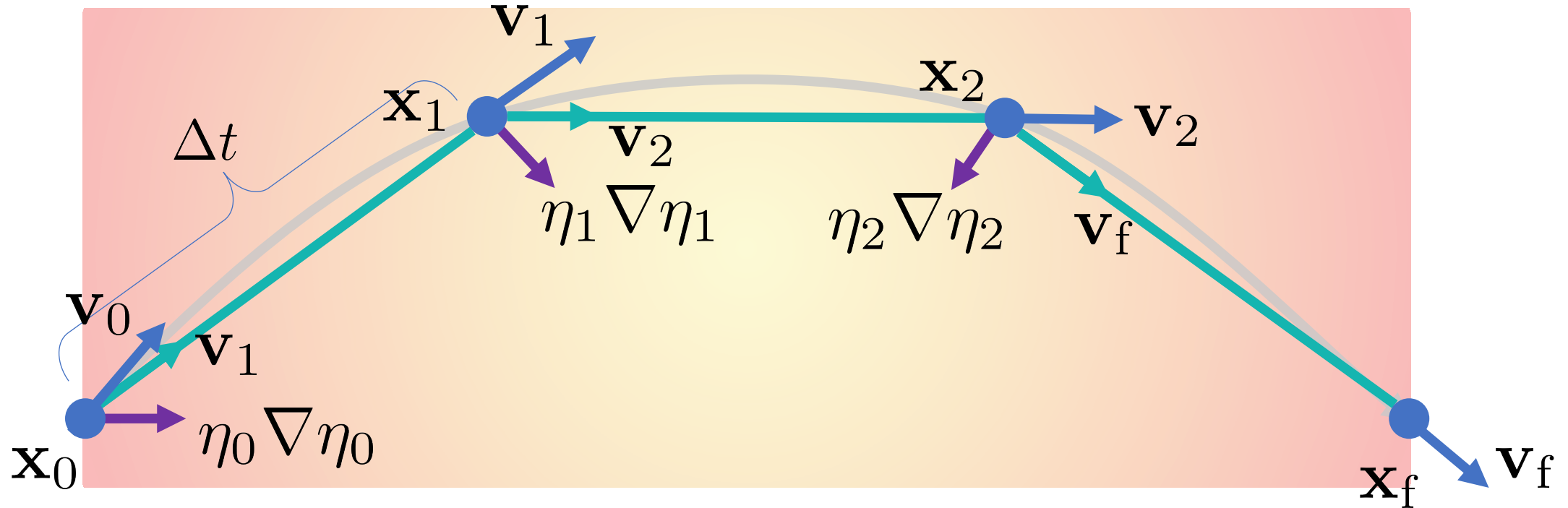
Nonlinear Ray Tracing



$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \eta \nabla \eta$$

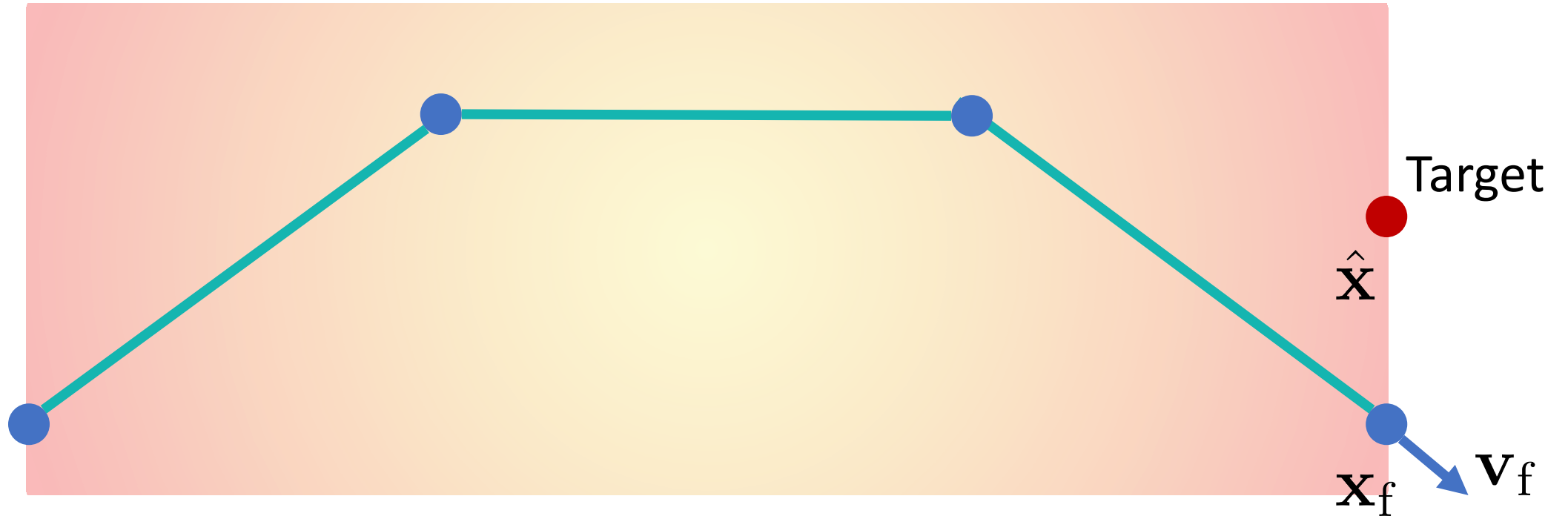
Nonlinear Ray Tracing



$$\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t$$

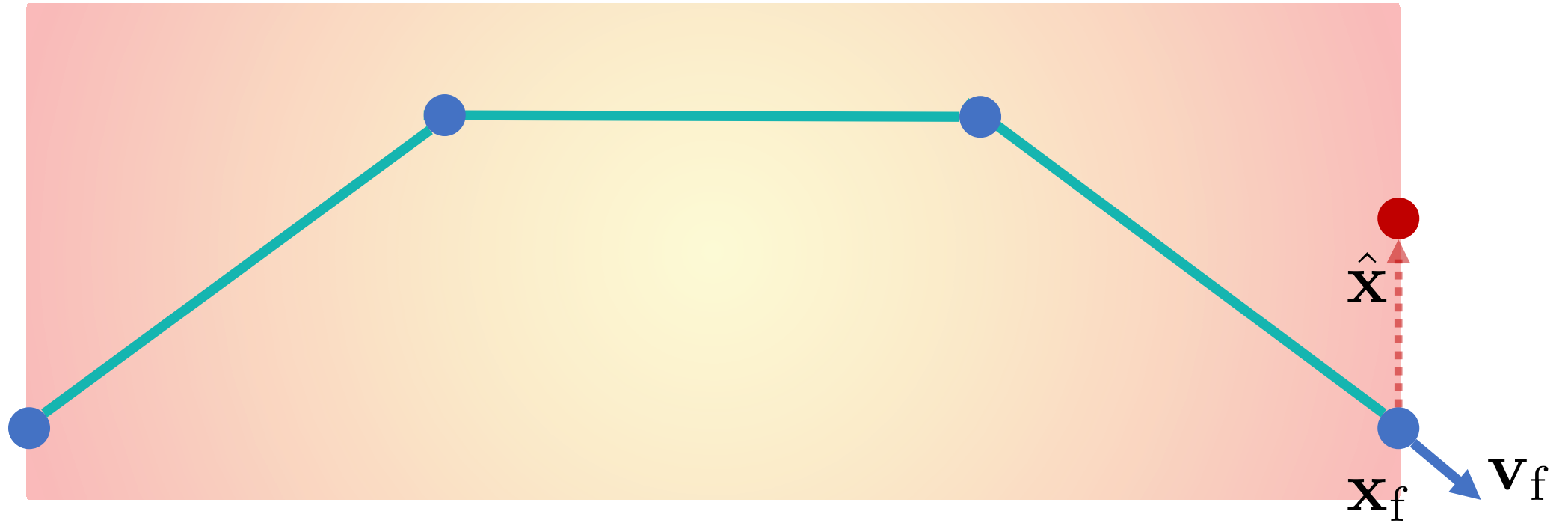
$$\mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t$$

Nonlinear Ray Tracing



$$\min_{\eta} \|\hat{\mathbf{x}} - \mathbf{x}_f\|^2$$

Nonlinear Ray Tracing



$$\frac{d}{d\eta} \|\hat{\mathbf{x}} - \mathbf{x}_f\|^2$$

Nonlinear Ray Tracing in reverse

$$\begin{aligned} \min_{\eta} \sum_{i=1}^N \mathcal{F}_i & \left[\iint_{(\mathbf{x}_0, \mathbf{v}_0) \in \Omega} C_i \left(\mathbf{x} \left(\sigma_f; \eta, \mathbf{x}_0, \mathbf{v}_0 \right), \mathbf{v} \left(\sigma_f; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) \right) d\mathbf{x}_0 d\mathbf{v}_0 \right] \\ \text{s.t. } \dot{\mathbf{x}} \left(\sigma; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \mathbf{v}, \quad \forall \sigma \in [0, \sigma_f], \\ \dot{\mathbf{v}} \left(\sigma; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \eta \nabla \eta, \quad \forall \sigma \in [0, \sigma_f], \\ \mathbf{x} \left(0; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \mathbf{x}_0, \\ \mathbf{v} \left(0; \eta, \mathbf{x}_0, \mathbf{v}_0 \right) &= \mathbf{v}_0, \end{aligned} \quad (15)$$

\mathbf{x}_0

$\hat{\mathbf{x}}$

$$\dot{\lambda} = - \left(\nabla \eta \left(\nabla \eta \right)^{\top} + \eta \text{Hess} \left(\eta \right) \right) \mu, \quad \forall \sigma \in [0, \sigma_f] \quad (19)$$

$$\dot{\mu} = -\lambda, \quad \forall \sigma \in [0, \sigma_f] \quad (20)$$

$$\lambda \left(\sigma_f \right) = \frac{\partial \mathcal{C}}{\partial \mathbf{x}}, \quad (21)$$

$$\mu \left(\sigma_f \right) = \frac{\partial \mathcal{C}}{\partial \mathbf{v}}. \quad (22)$$

$\eta \nabla \eta$

$\eta \nabla \eta$

$$\mathbf{x}_{i-1} = \mathbf{x}_i - \mathbf{v}_i \Delta \sigma, \quad (28)$$

$$\mathbf{v}_{i-1} = \mathbf{v}_i - \eta \left(\mathbf{x}_{i-1} \right) \nabla \eta \left(\mathbf{x}_{i-1} \right) \Delta \sigma, \quad (29)$$

$$\begin{aligned} \lambda_{i-1} &= \lambda_i \\ &\quad + \left(\nabla \eta \left(\mathbf{x}_{i-1} \right) \left(\nabla \eta \left(\mathbf{x}_{i-1} \right) \right)^{\top} + \eta \left(\mathbf{x}_{i-1} \right) \text{Hess} \left(\eta \left(\mathbf{x}_{i-1} \right) \right) \right) \mu_i \Delta \sigma, \end{aligned} \quad (30)$$

$$\mu_{i-1} = \mu_i + \lambda_{i-1} \Delta \sigma. \quad (31)$$

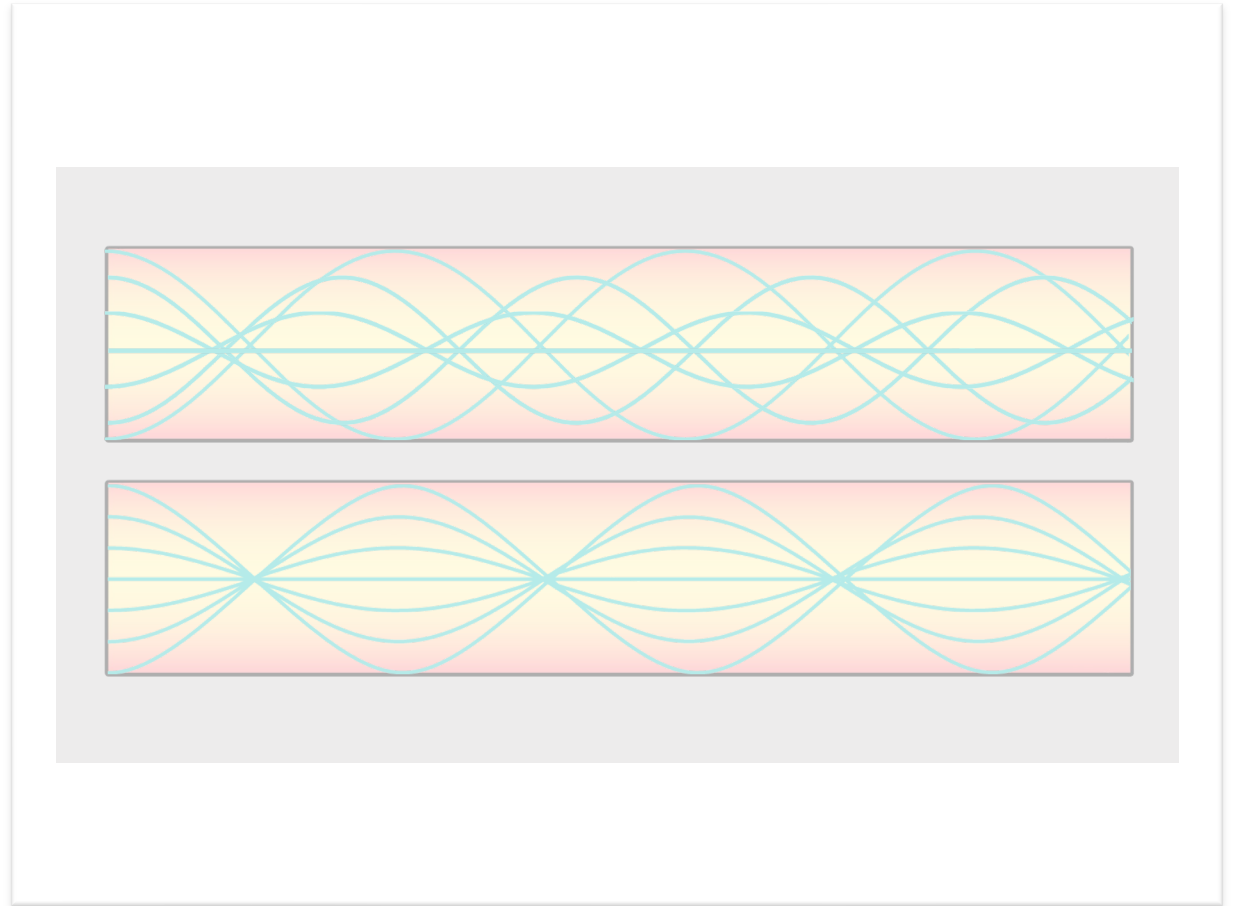
$\eta \nabla \eta$

$\eta \nabla \eta$

Optimizing Gradient-Index (GRIN) Optics

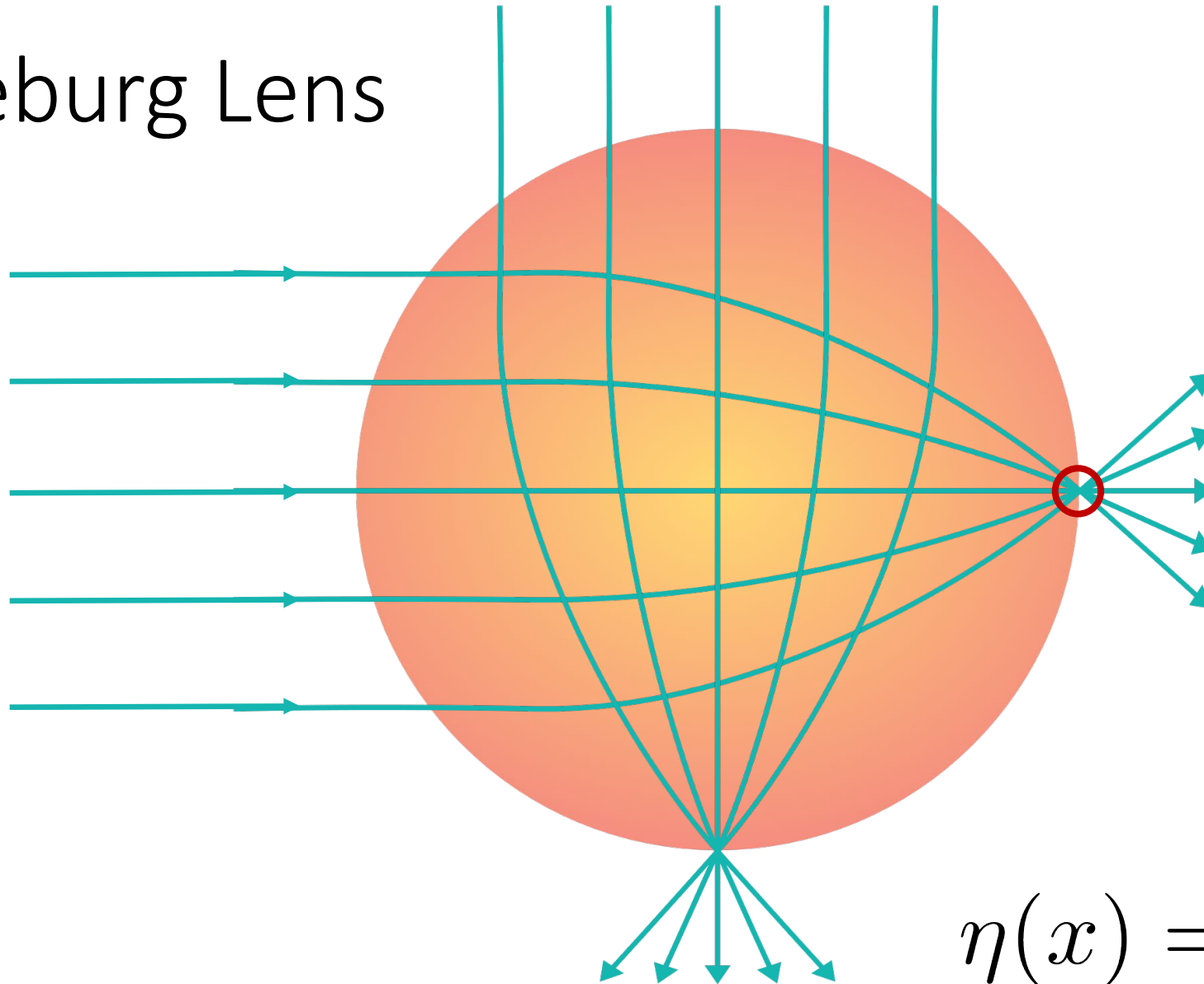


Luneburg Lens



GRIN Fiber

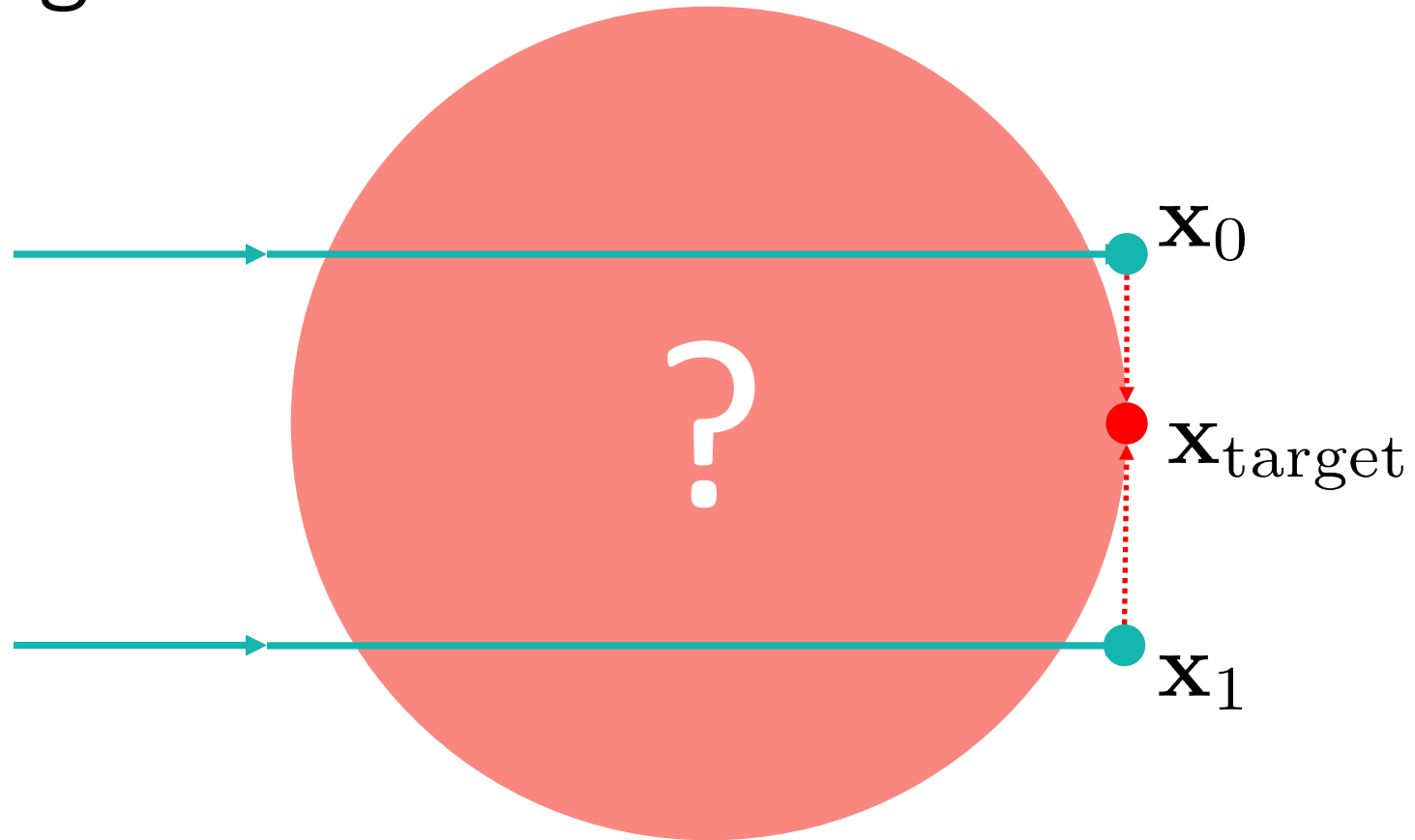
Luneburg Lens



$$\eta(x) = \sqrt{2 - \|x\|^2}$$

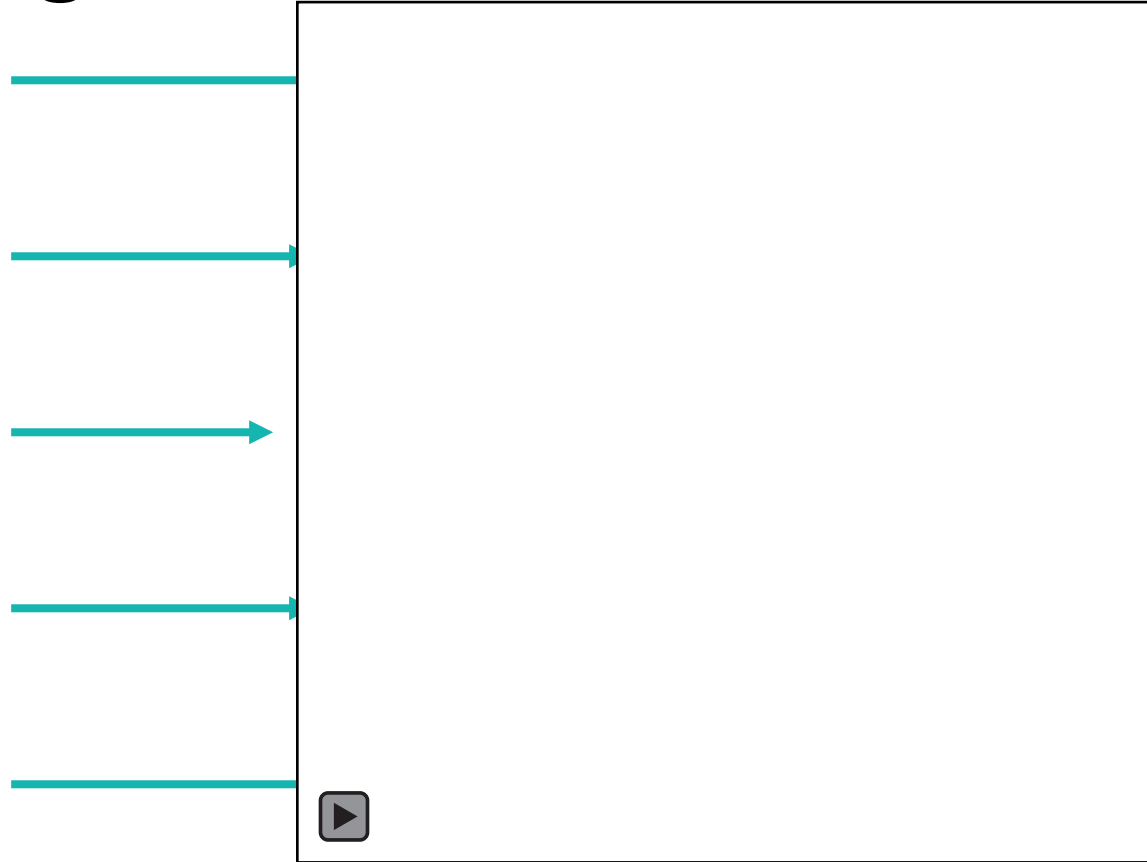
[Luneburg, R. K. 1944]

Luneburg Lens

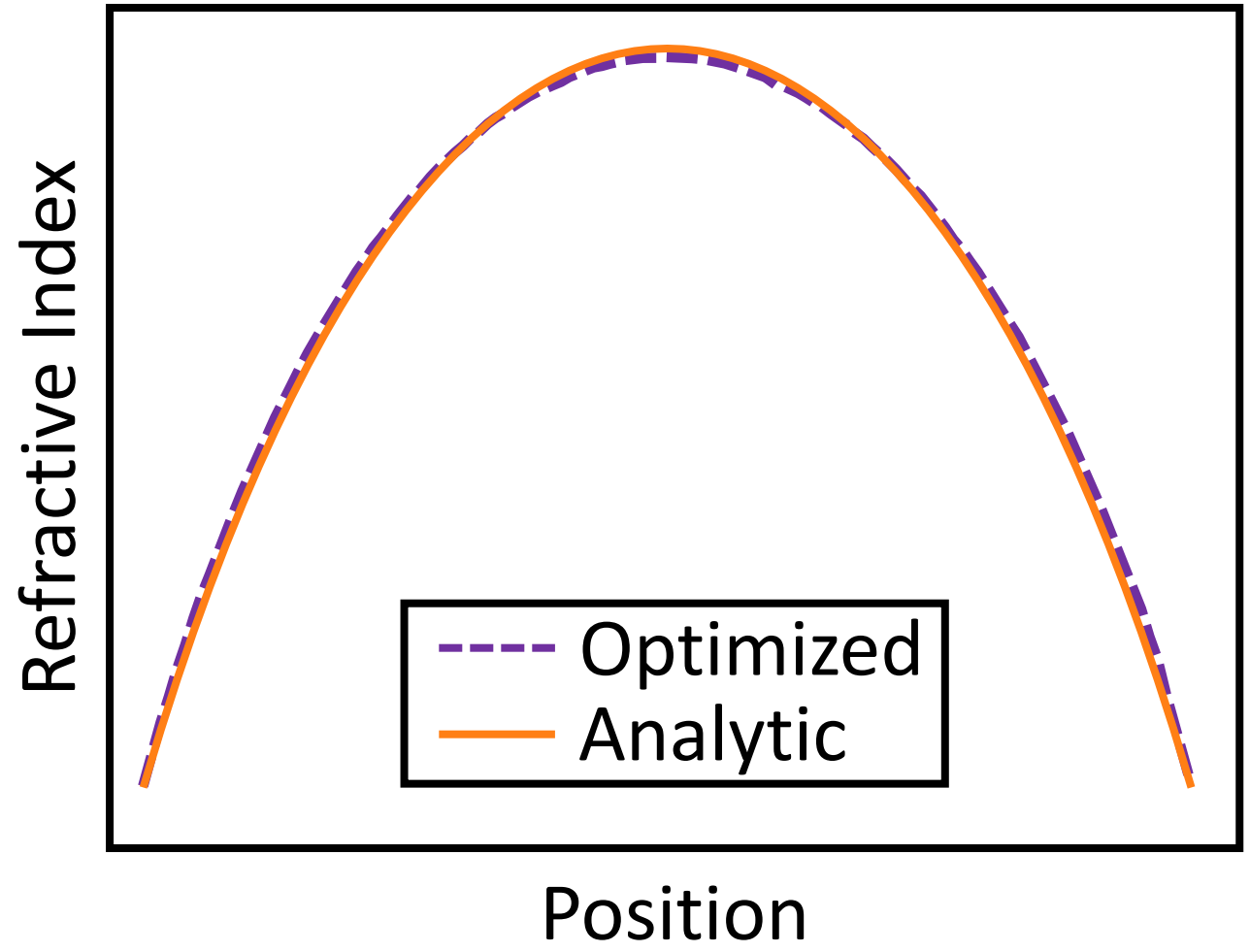
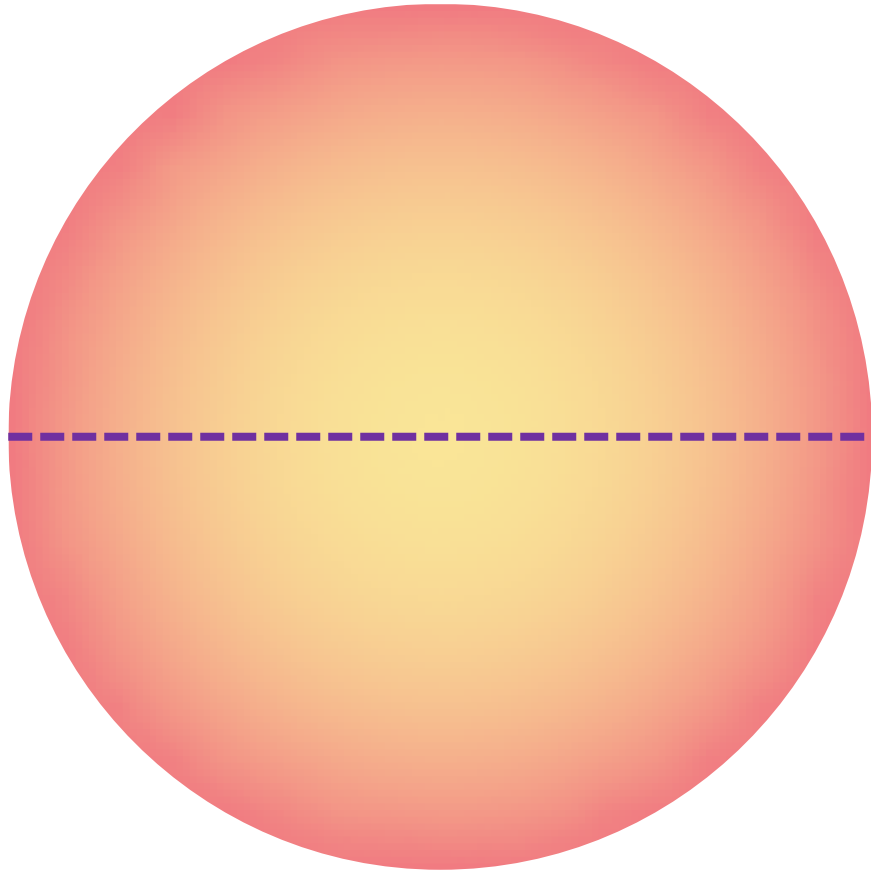


$$\min_{\eta} \sum_{i \in \text{rays}} \|\mathbf{x}_{\text{target}} - \mathbf{x}_i\|^2$$

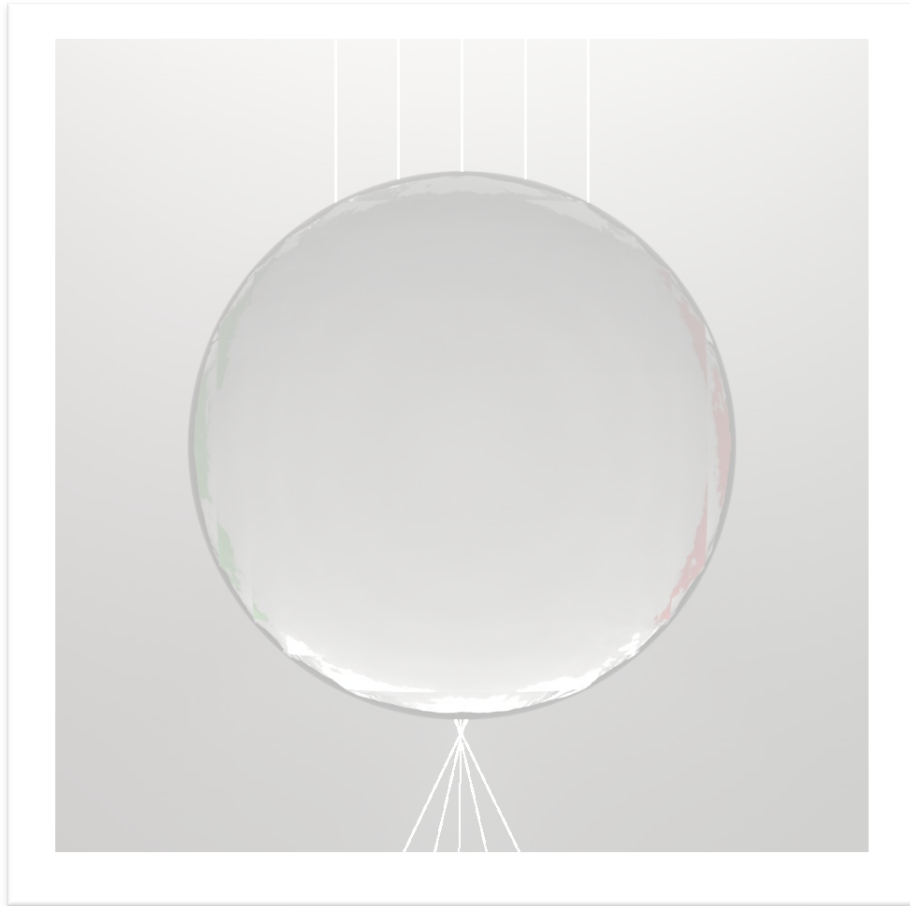
Luneburg Lens



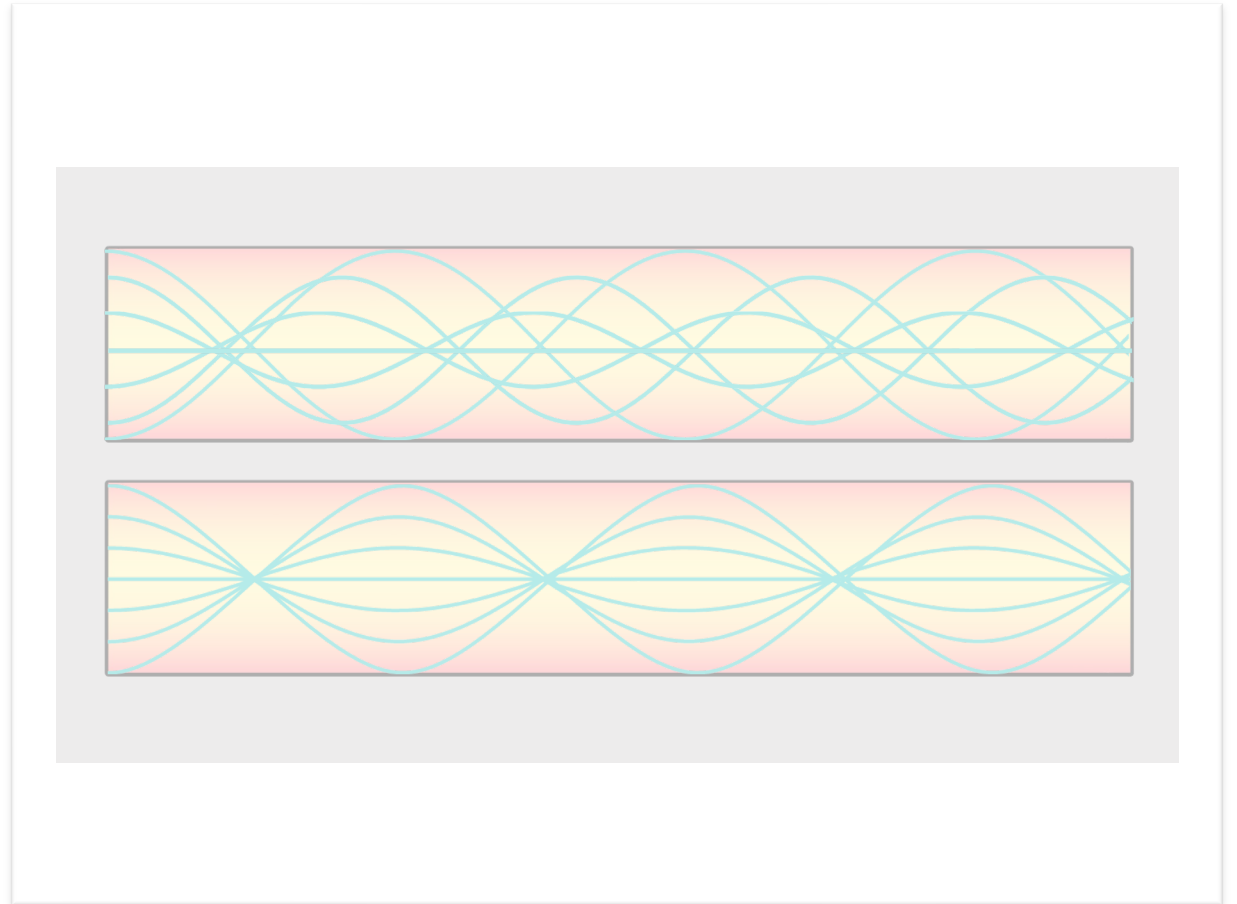
Luneburg Lens



Optimizing Gradient-Index (GRIN) Optics

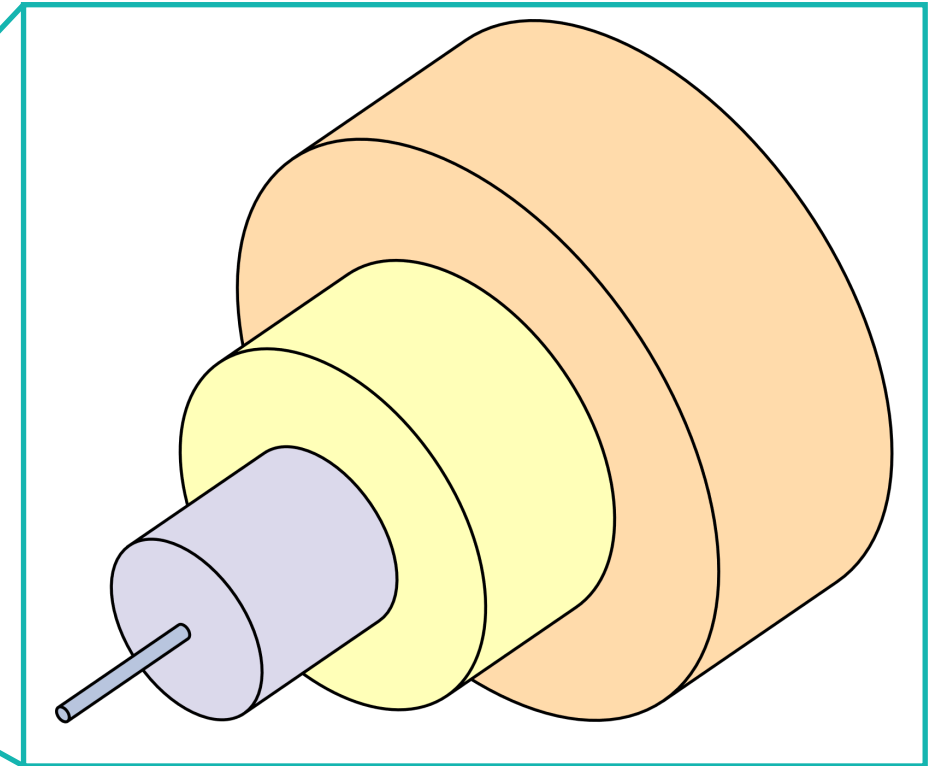
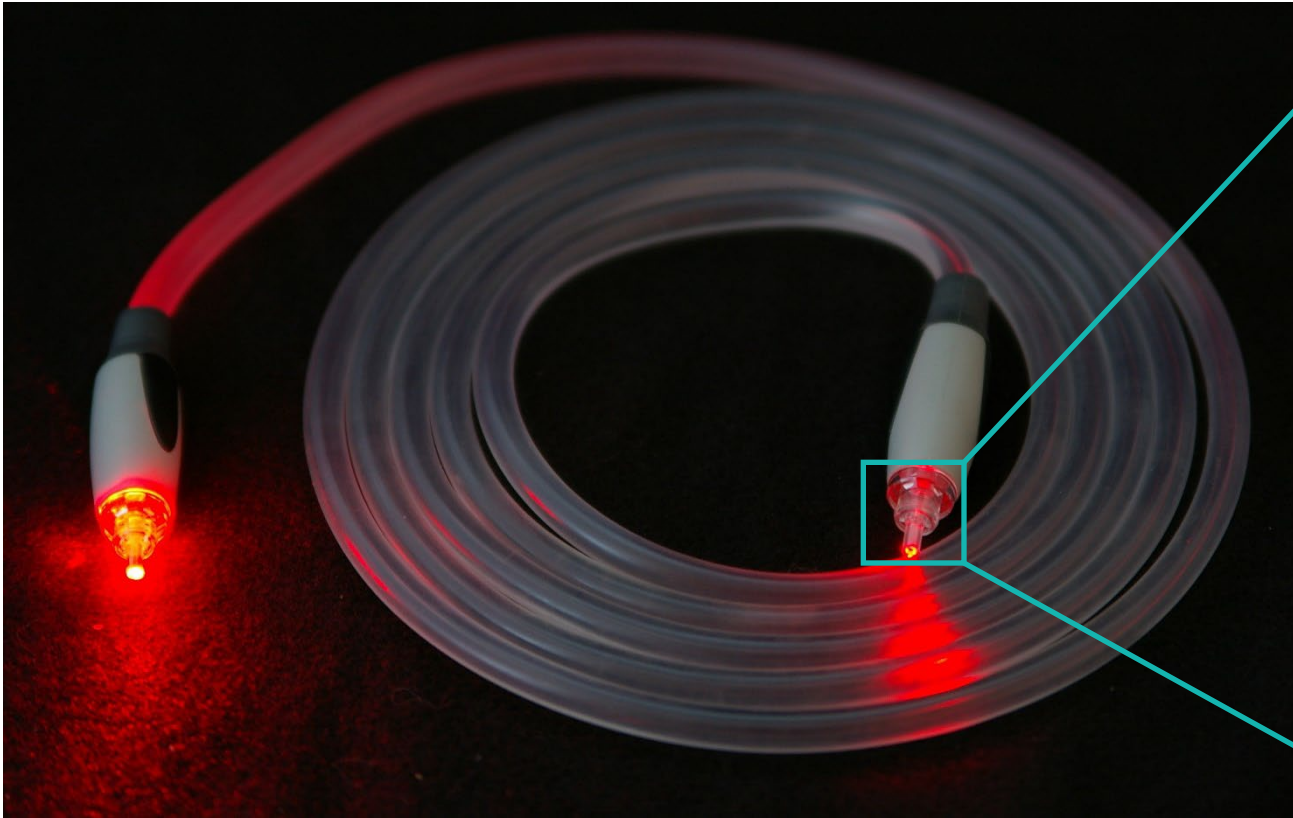


Luneburg Lens



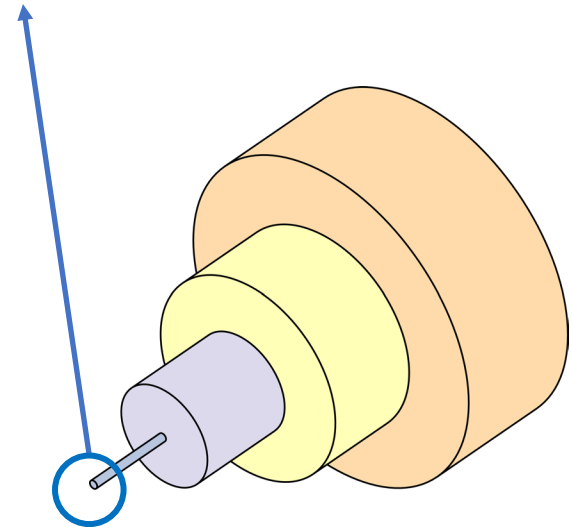
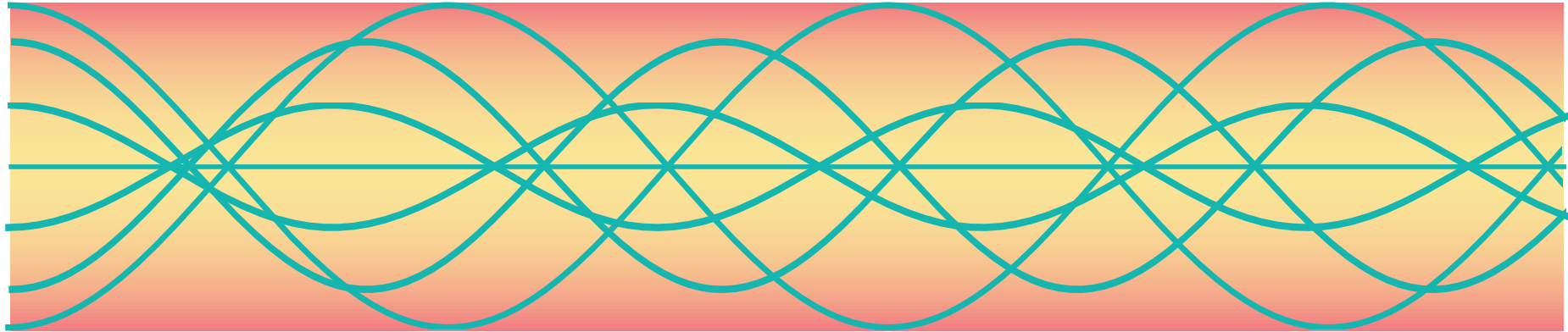
GRIN Fiber

GRIN Fiber



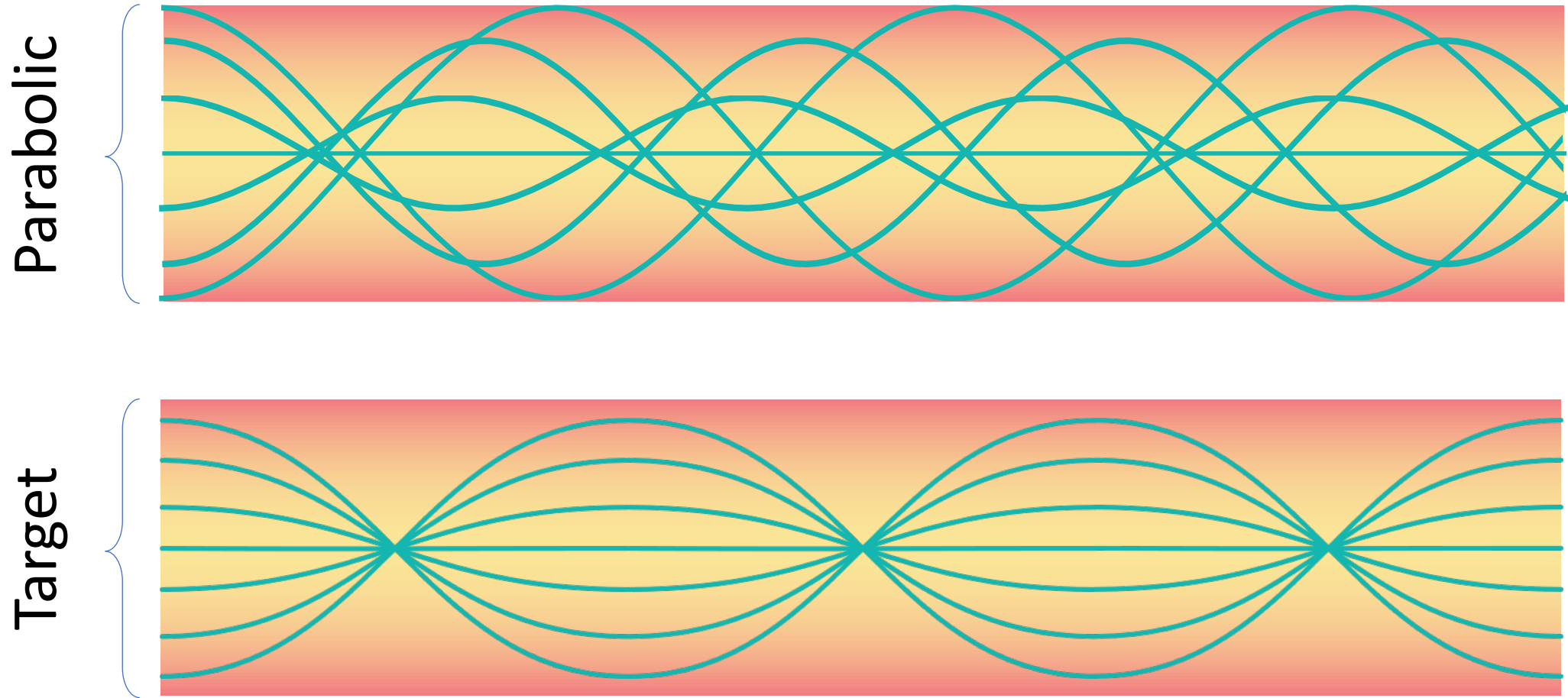
GRIN Fiber

Modal dispersion

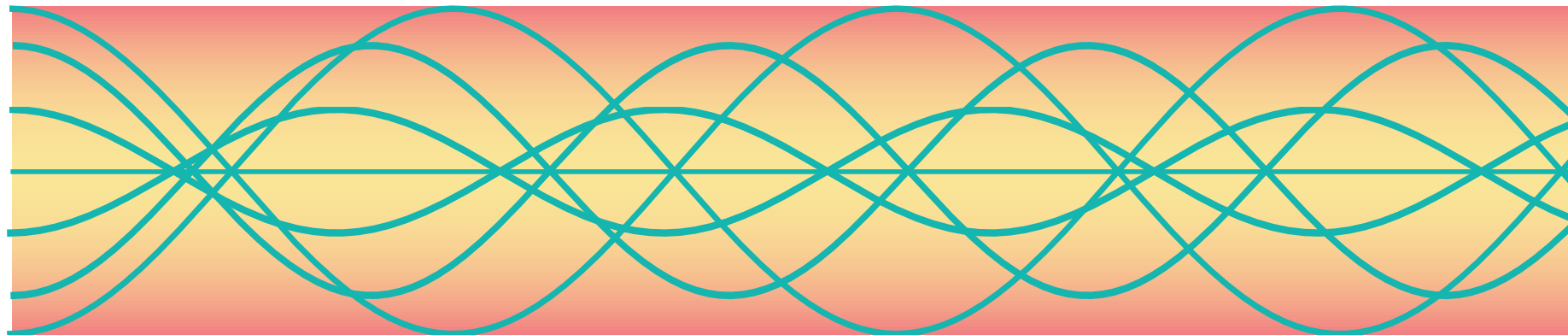


GRIN Fiber

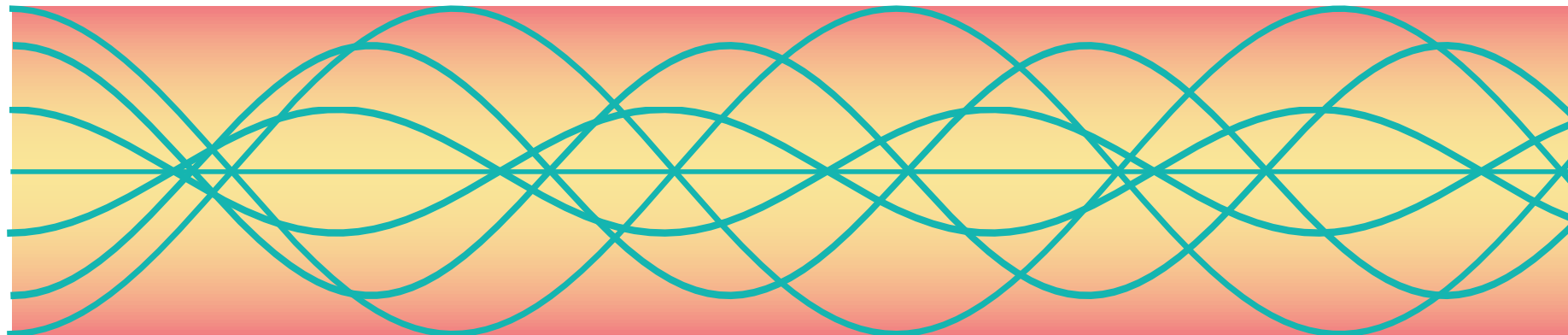
Modal dispersion



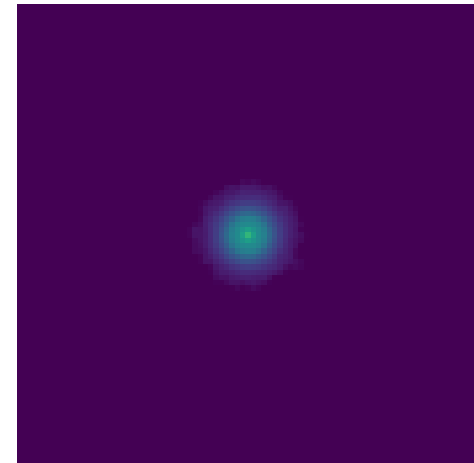
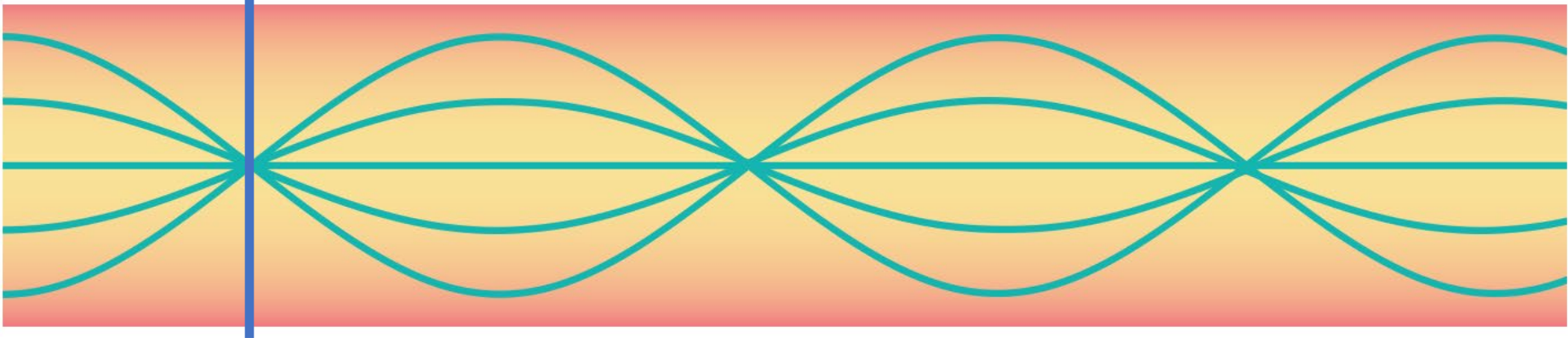
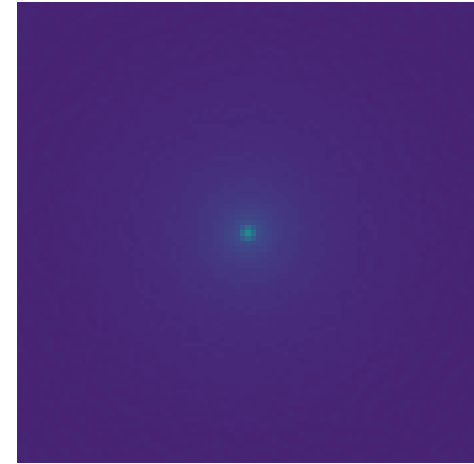
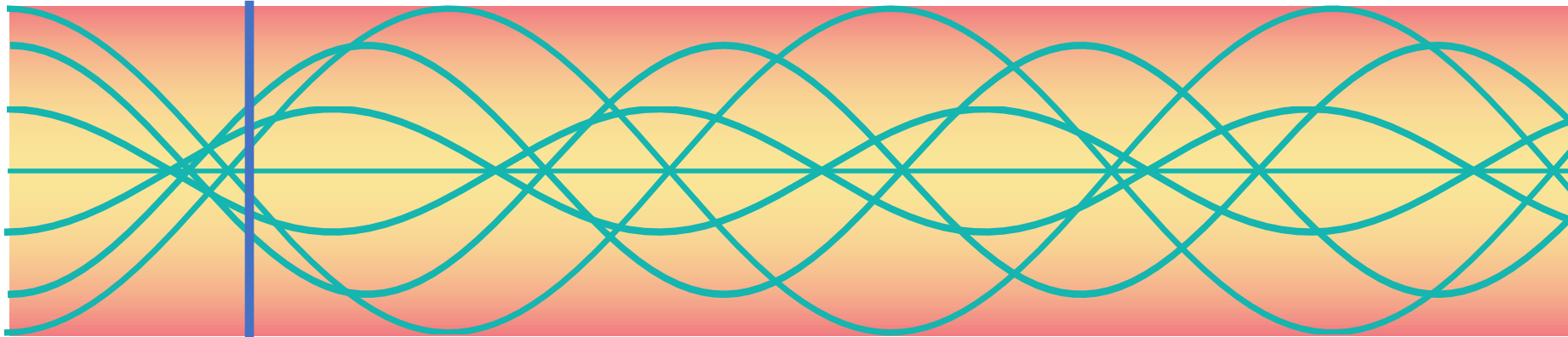
GRIN Fiber



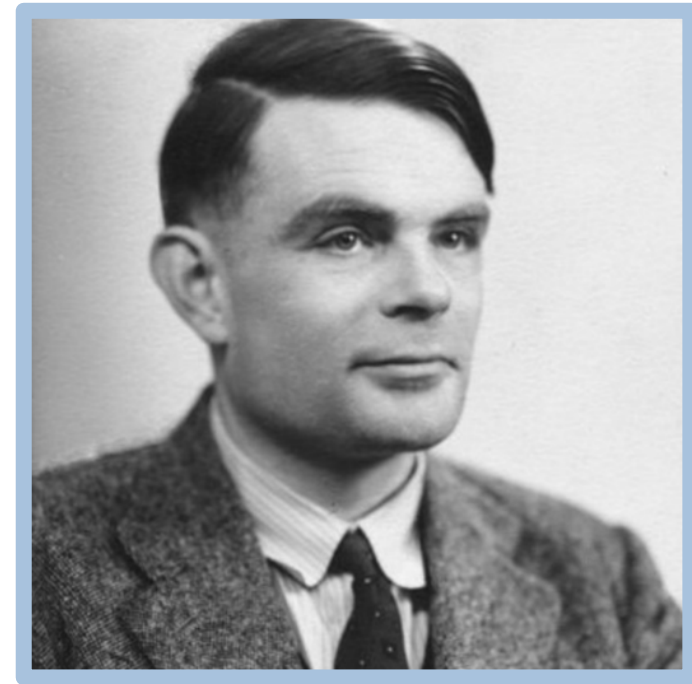
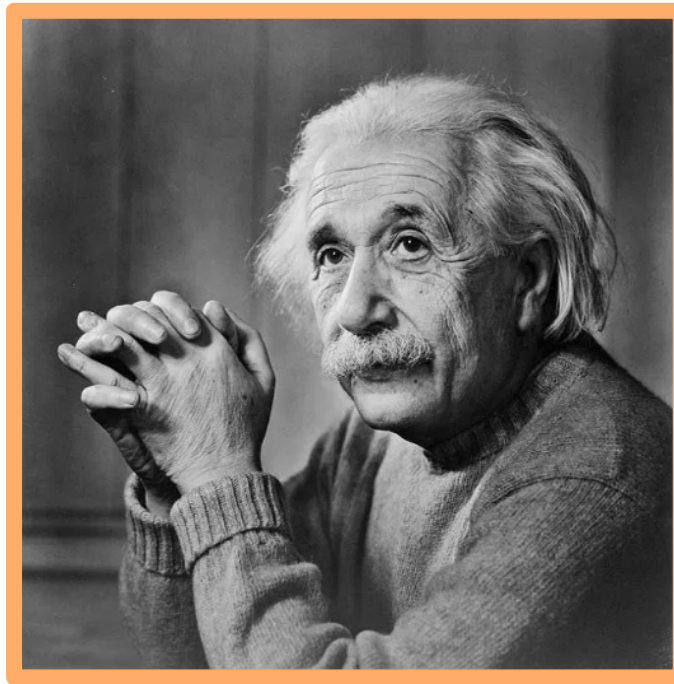
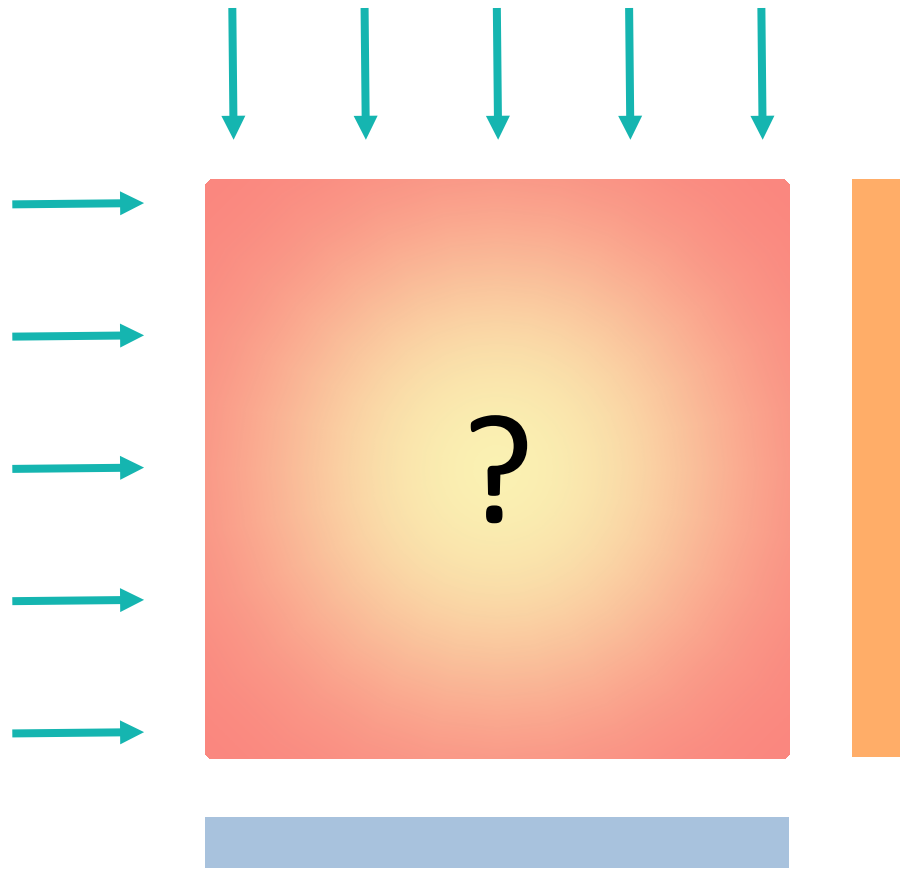
GRIN Fiber



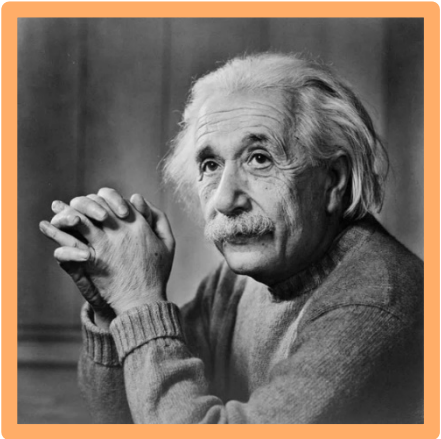
GRIN Fiber



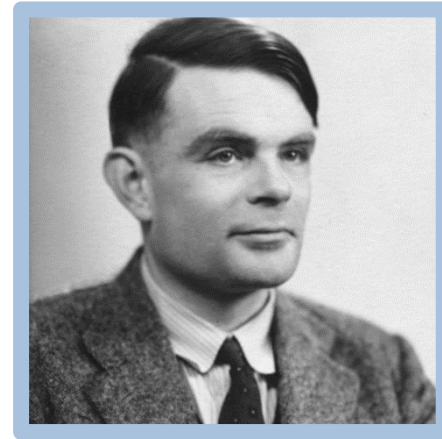
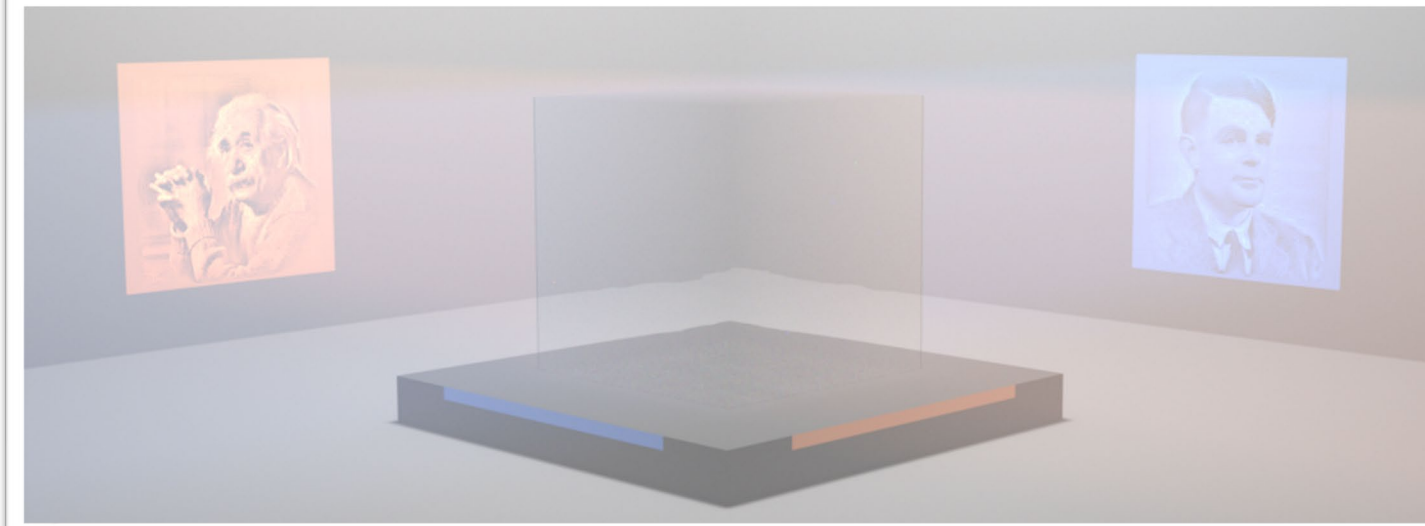
Multiview Display



Multiview Display



Target



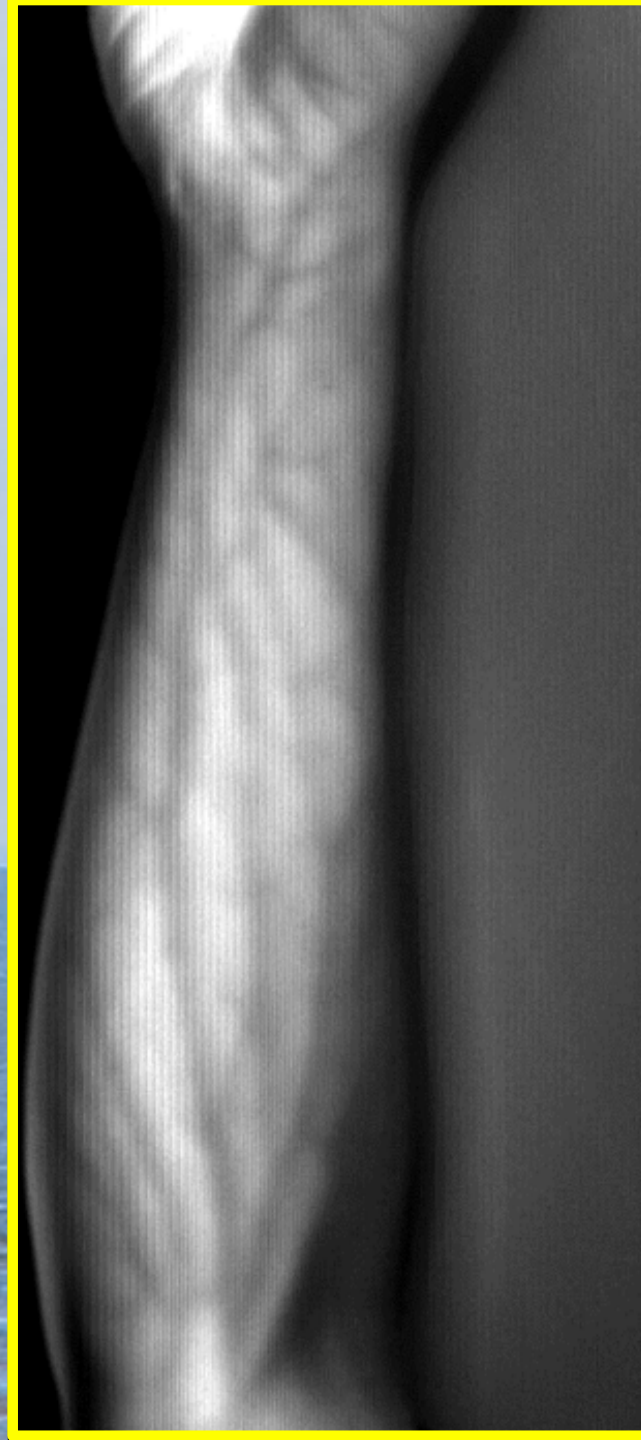
Target



optimization
results

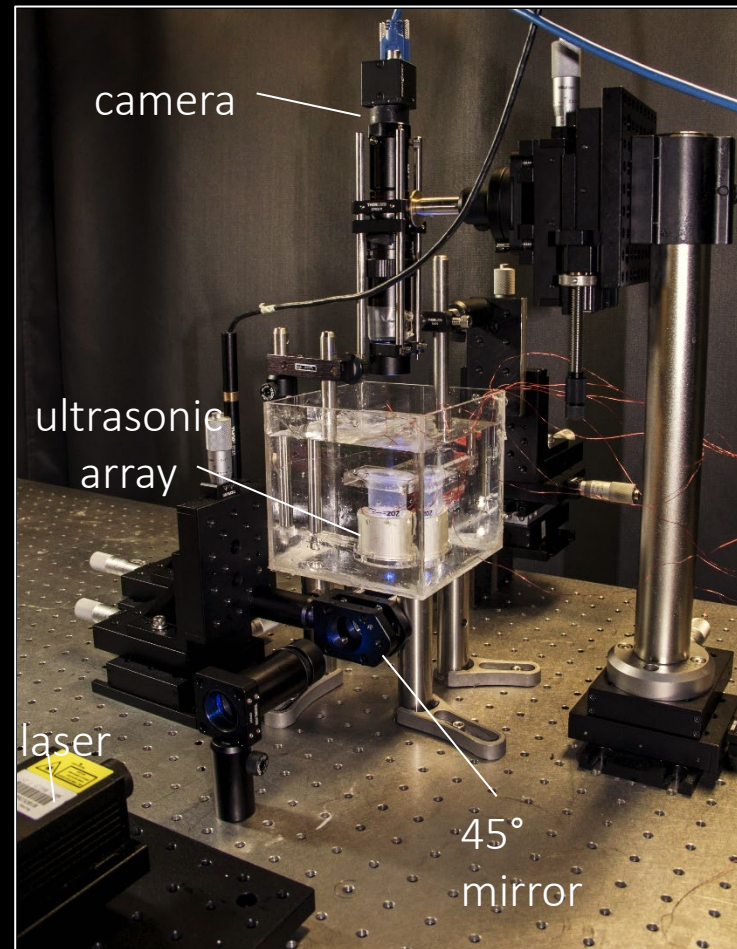


Media with continuously varying refractive index and scattering

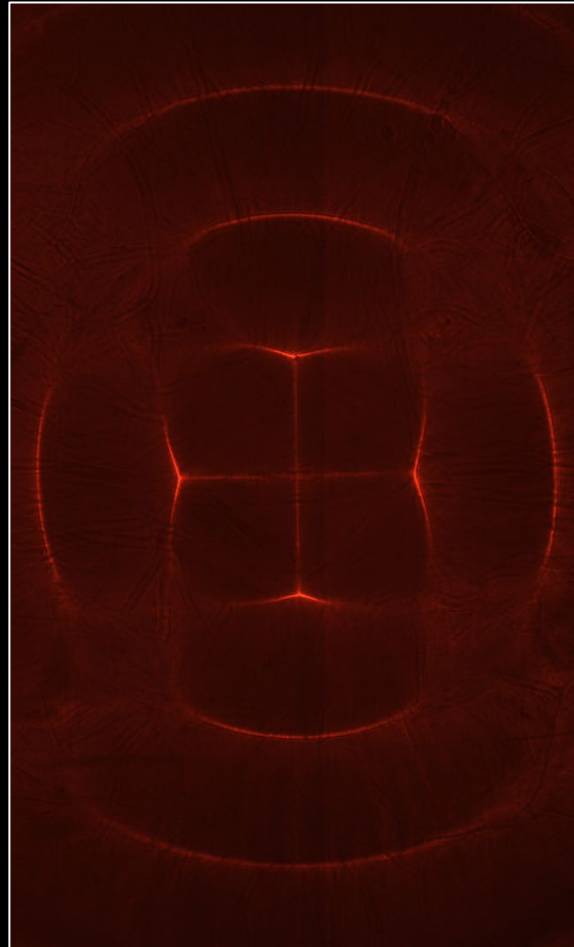


unbiased techniques for scientific imaging

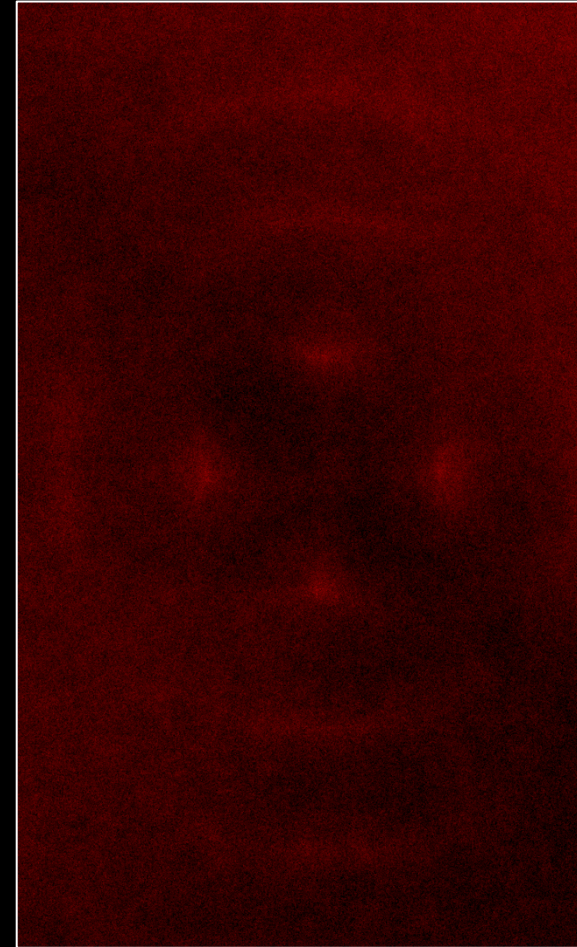
experimental
hardware



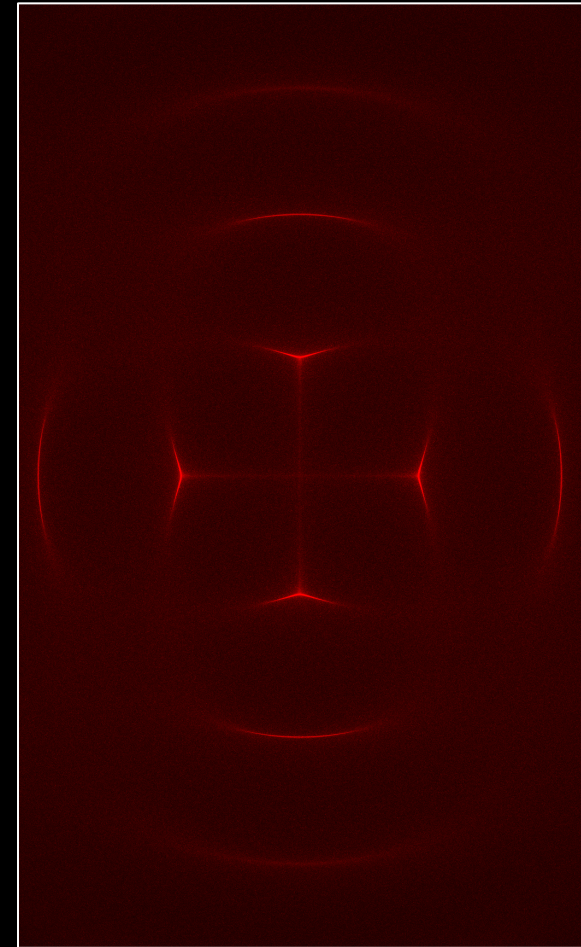
experimental
capture

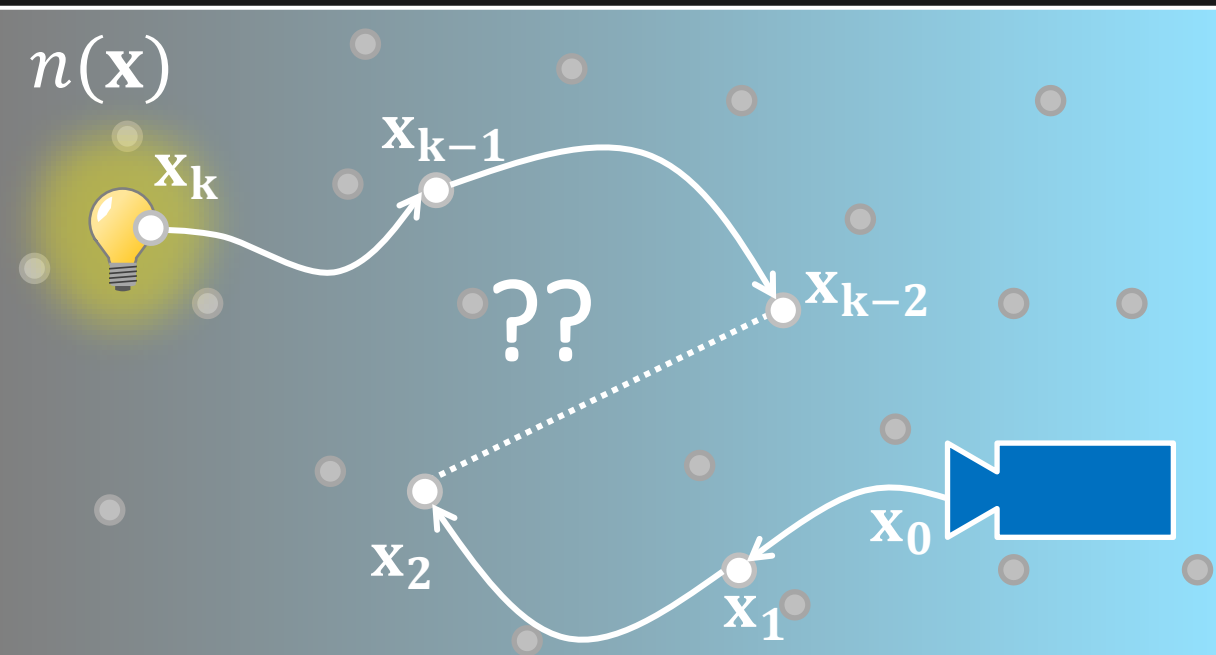


photon mapping

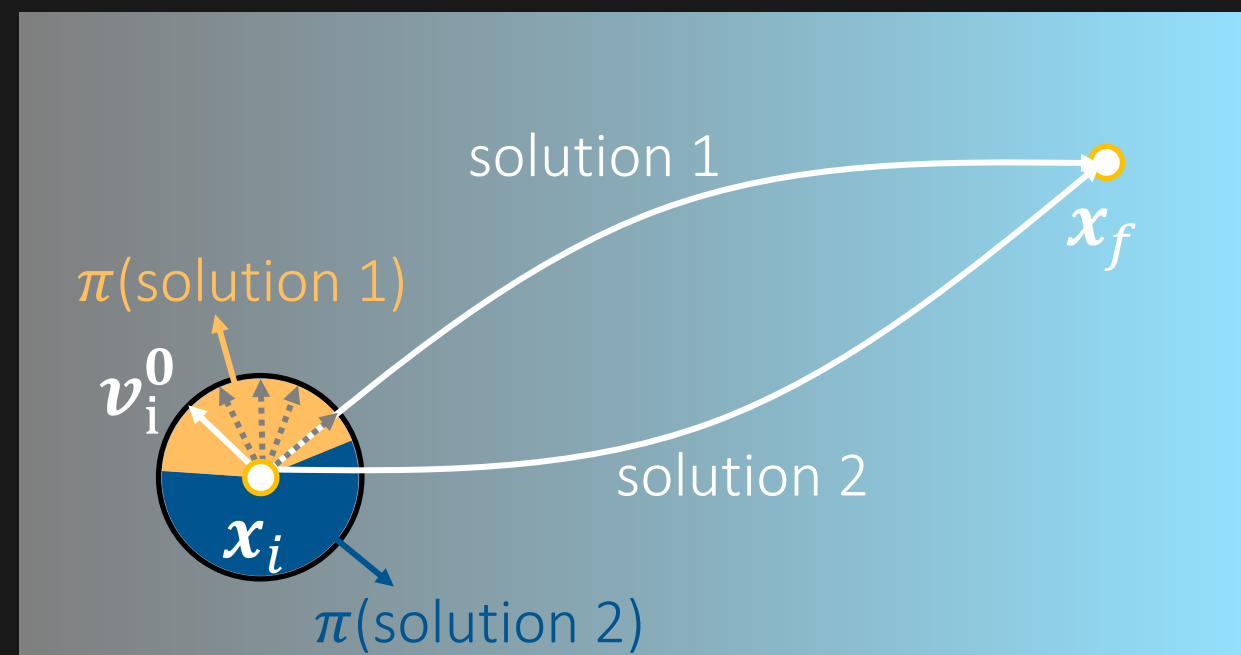


unbiased (ours)

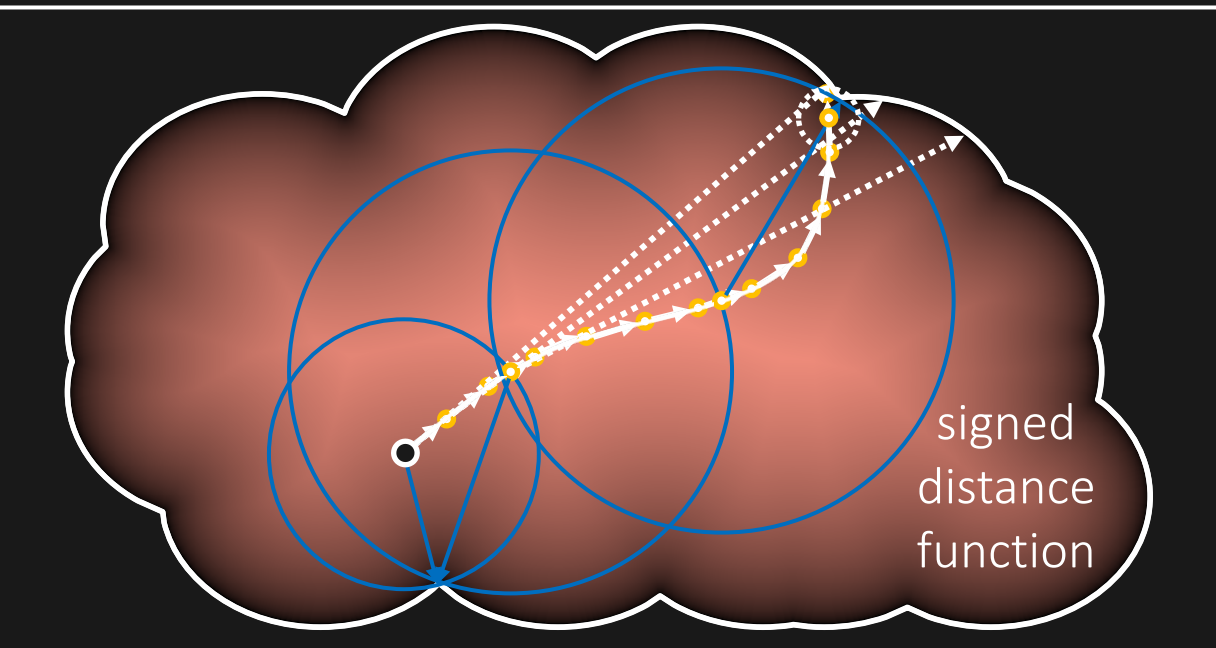




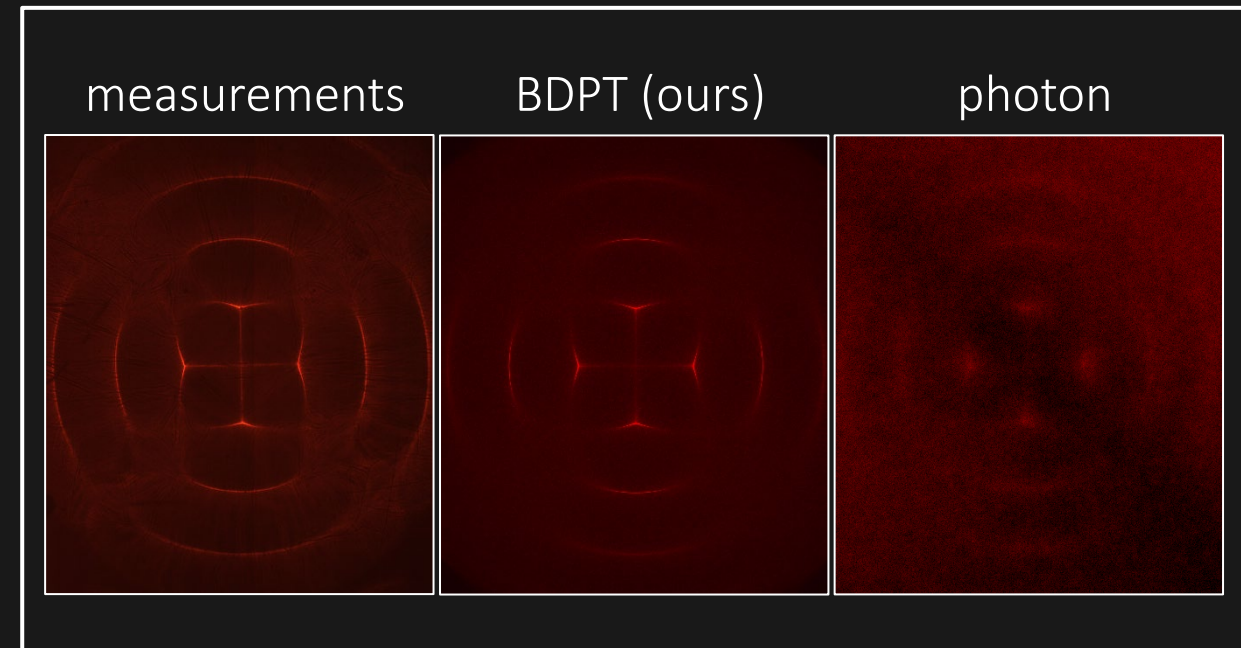
1. background on refractive radiative transfer equation



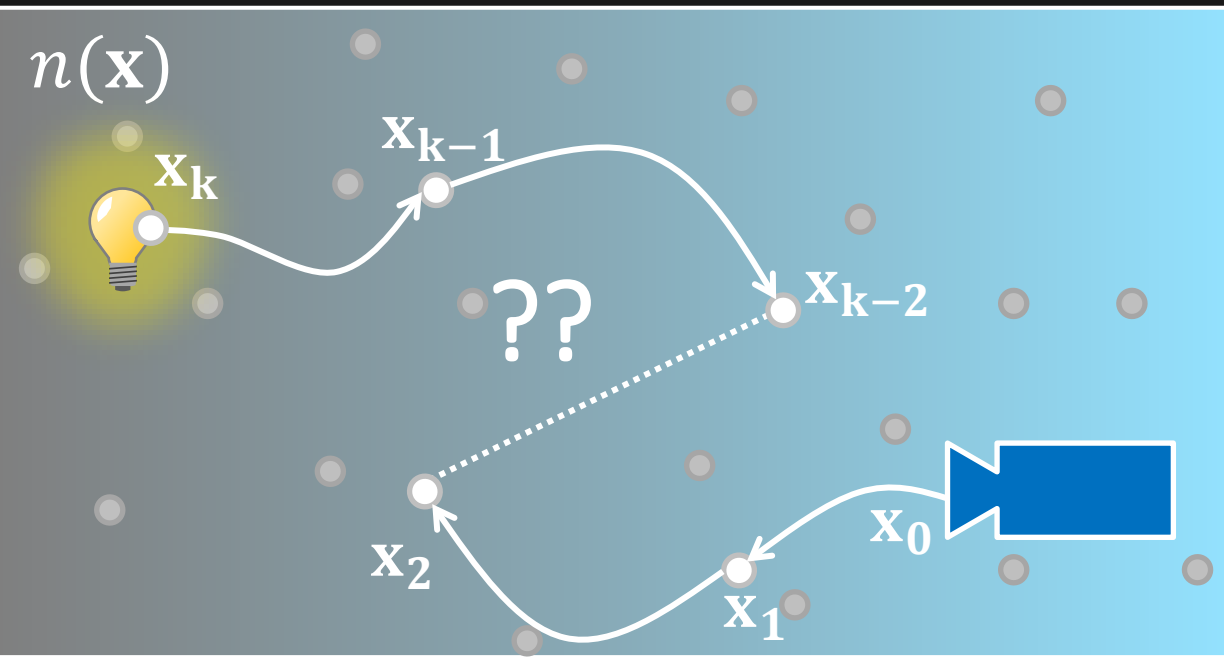
2. direct connections: our solution to unbiased rendering



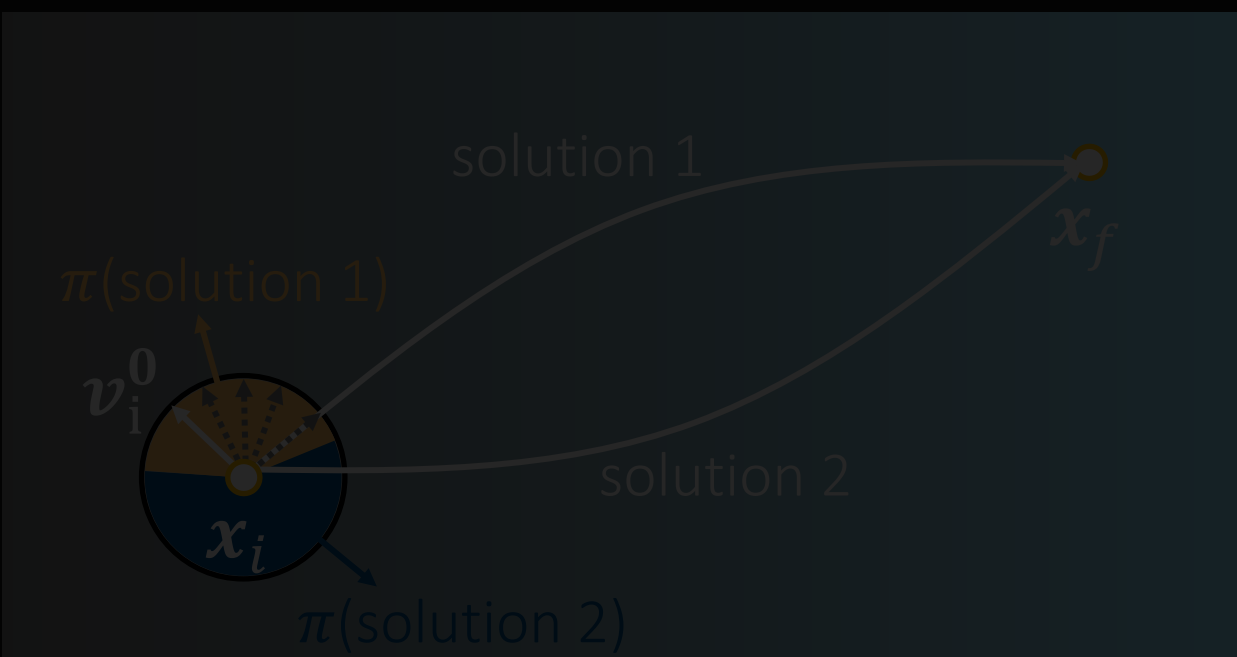
3. acceleration techniques



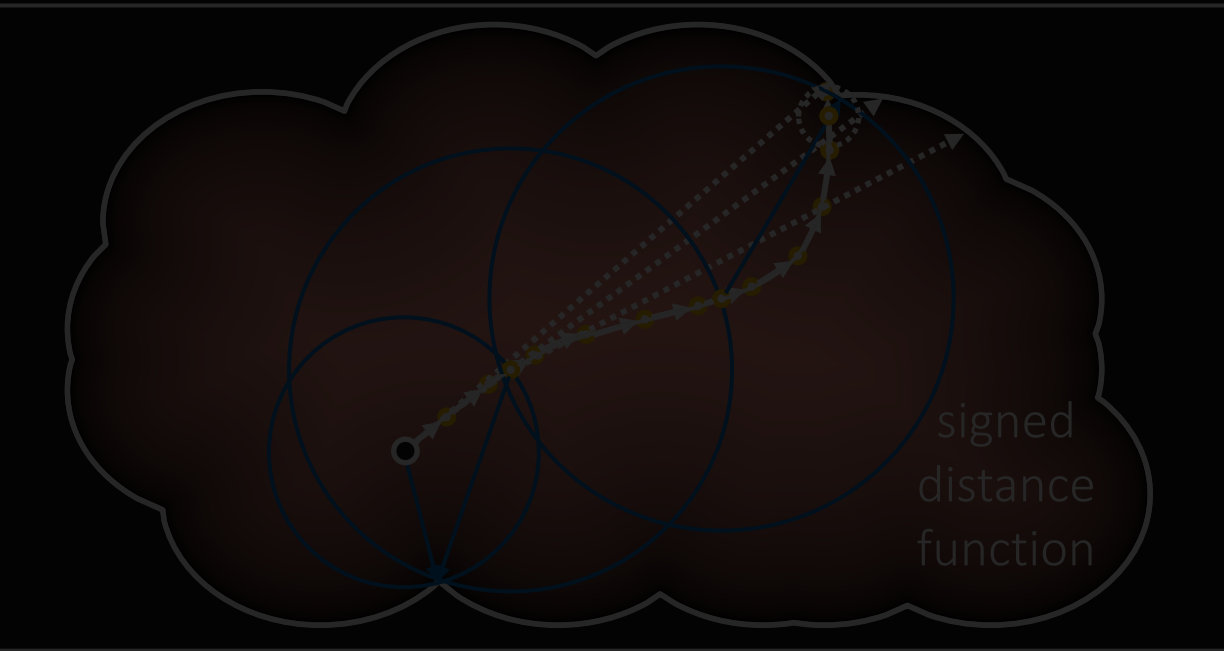
4. experiments



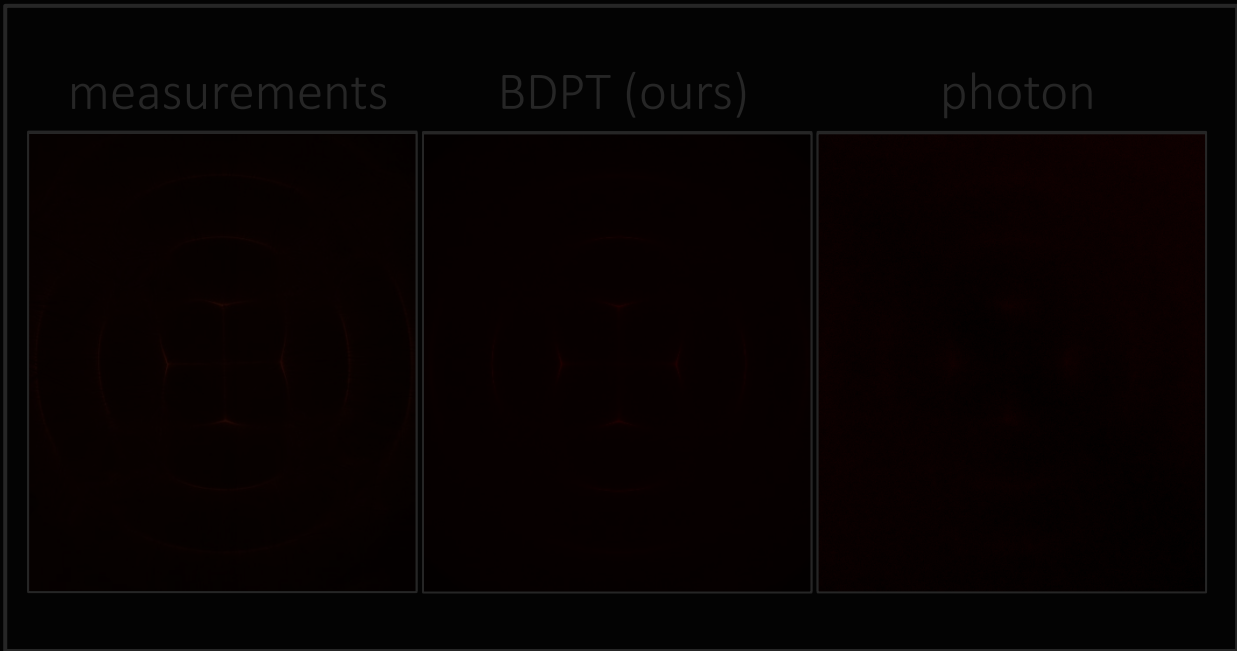
1. background on refractive radiative transfer equation



2. direct connections: our solution to unbiased rendering

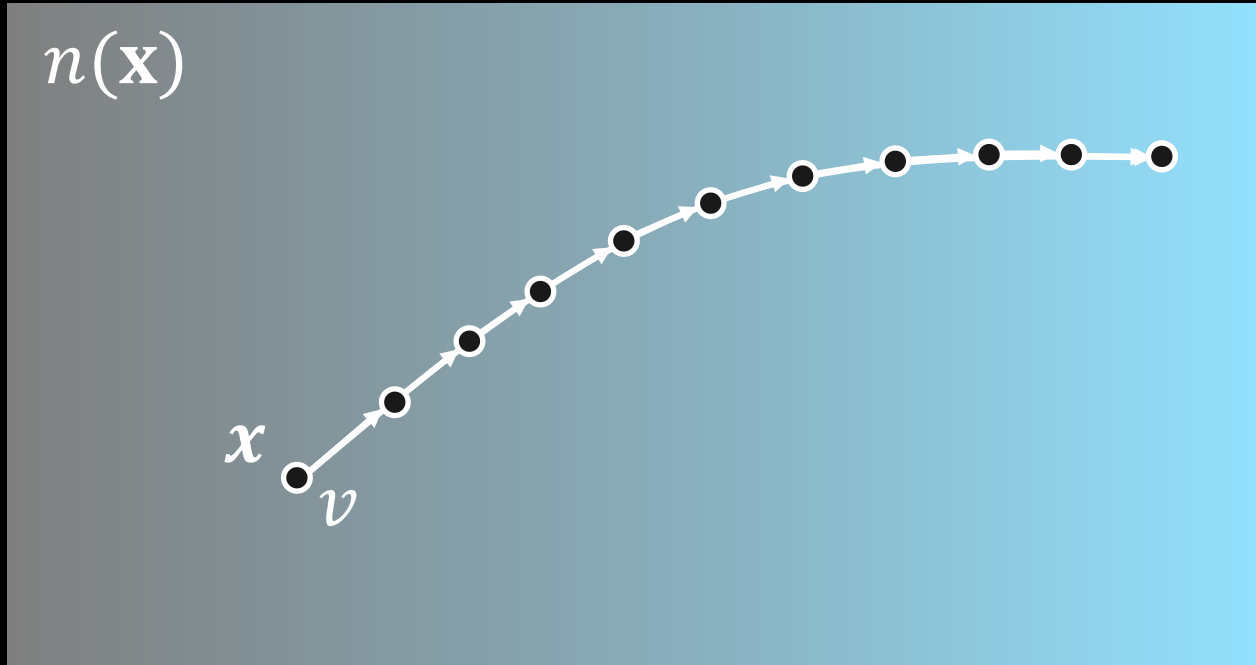


3. acceleration techniques



4. experiments

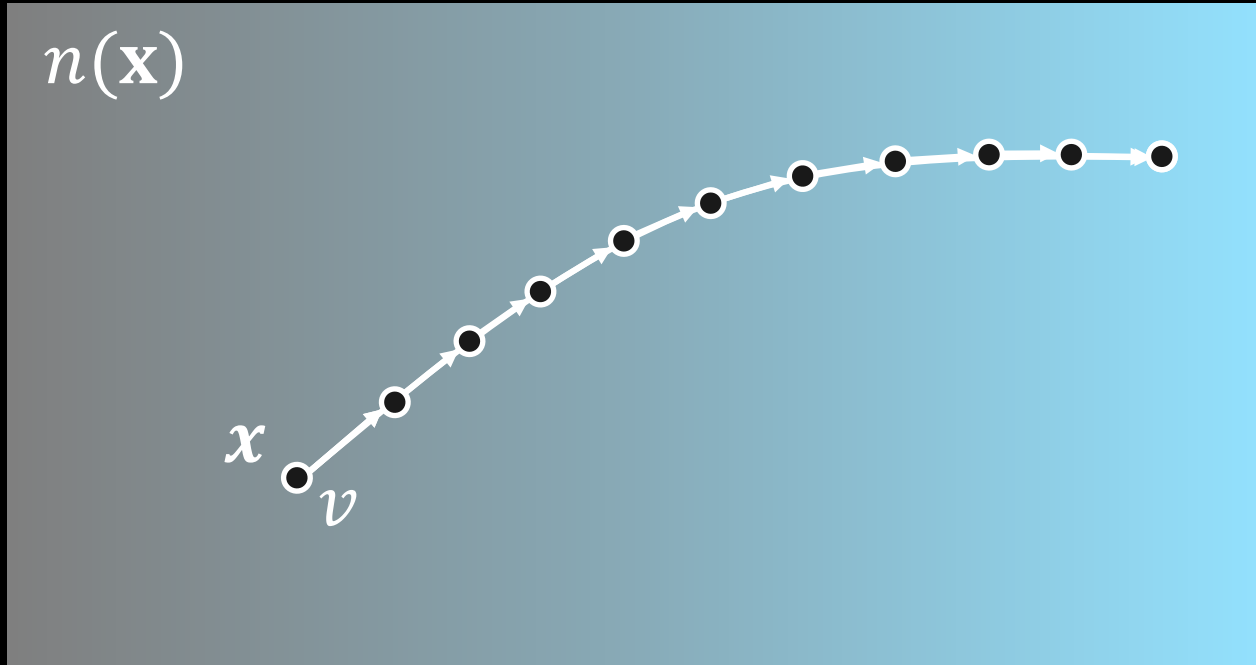
continuous refraction and no scattering



Hamilton's equations for refractive ray tracing

$$\frac{d\mathbf{v}}{ds} = \nabla_{\mathbf{x}} n(\mathbf{x})$$
$$\frac{d\mathbf{x}}{ds} = \frac{\mathbf{v}}{n(\mathbf{x})}$$

continuous refraction and no scattering



Hamilton's equations for refractive ray tracing

$$\frac{d\mathbf{v}}{ds} = \nabla_{\mathbf{x}} n(\mathbf{x})$$

solved using symplectic integration

$$\frac{d\mathbf{x}}{ds} = \frac{\mathbf{v}}{n(\mathbf{x})}$$

scattering and no continuous refraction



radiative transfer equation (RTE)

$$\frac{dL}{ds} = \sigma_a L_e - (\sigma_a + \sigma_s) L + \frac{\sigma_s}{4\pi} \int f_s(\omega', \omega) L d\omega'$$

scattering and no continuous refraction

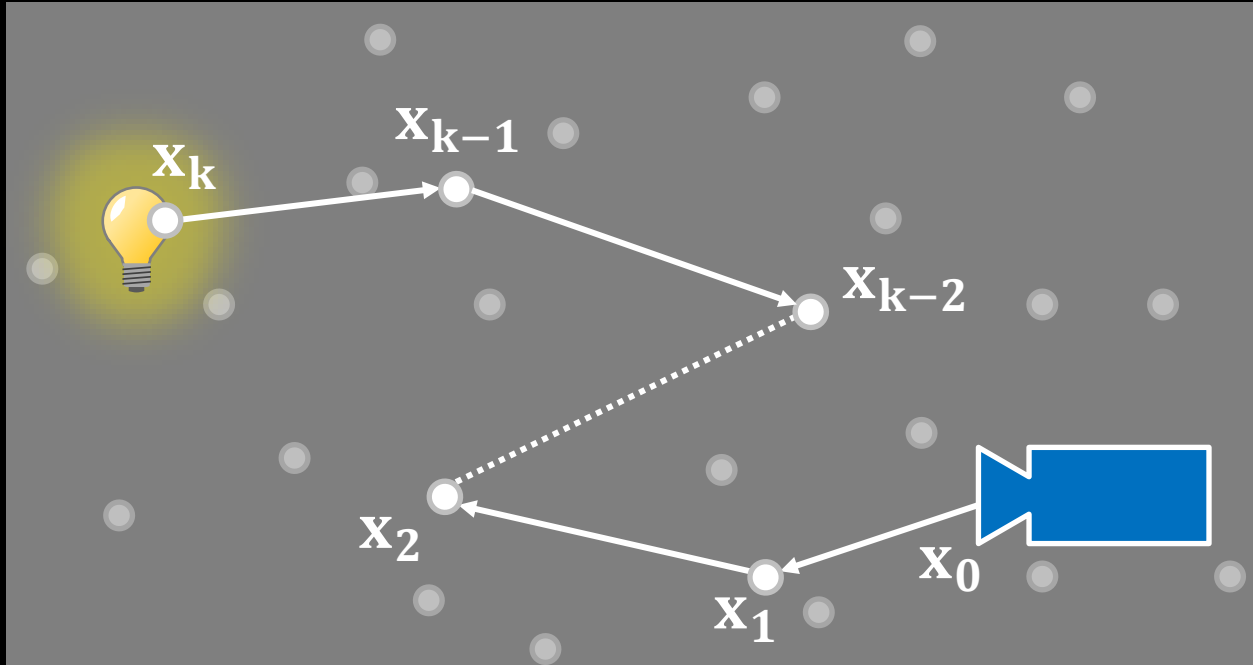


radiative transfer equation (RTE)

$$\frac{dL}{ds} = \sigma_a L_e - (\sigma_a + \sigma_s)L + \frac{\sigma_s}{4\pi} \int f_s(\omega', \omega) L d\omega'$$

solved using Monte Carlo integration

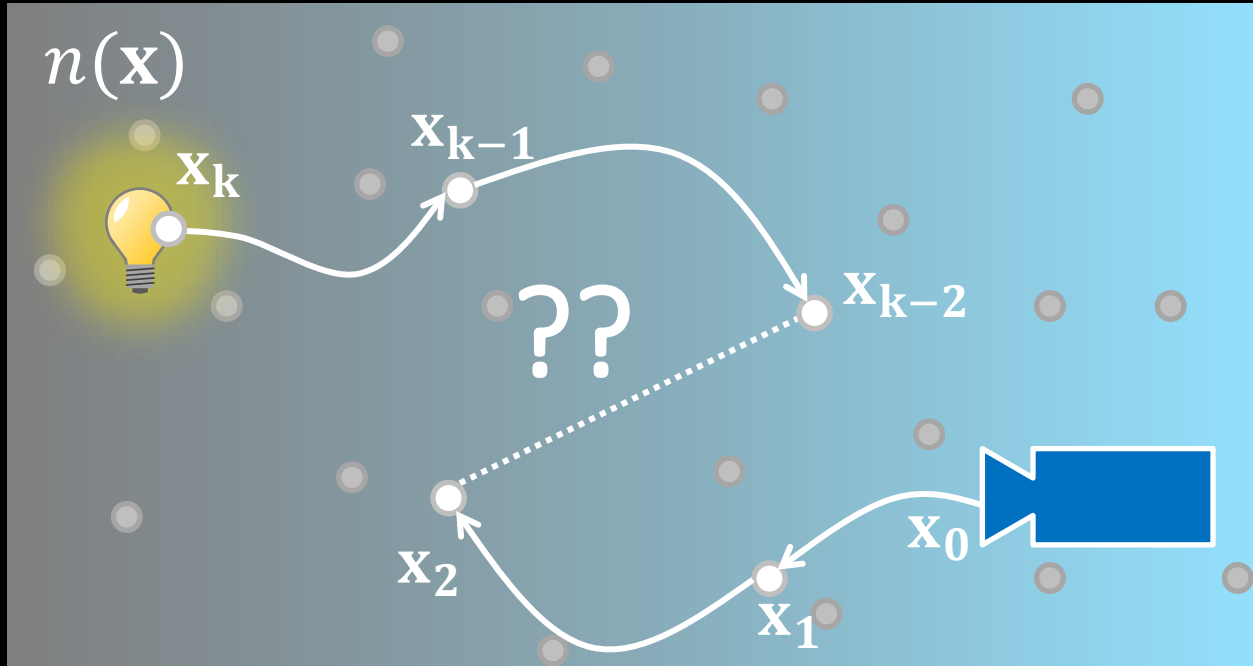
scattering and no continuous refraction



bidirectional path tracing (BDPT):

- 1.trace a random sensor subpath
- 2.trace a random emitter subpath
- 3.join vertices with a straight line

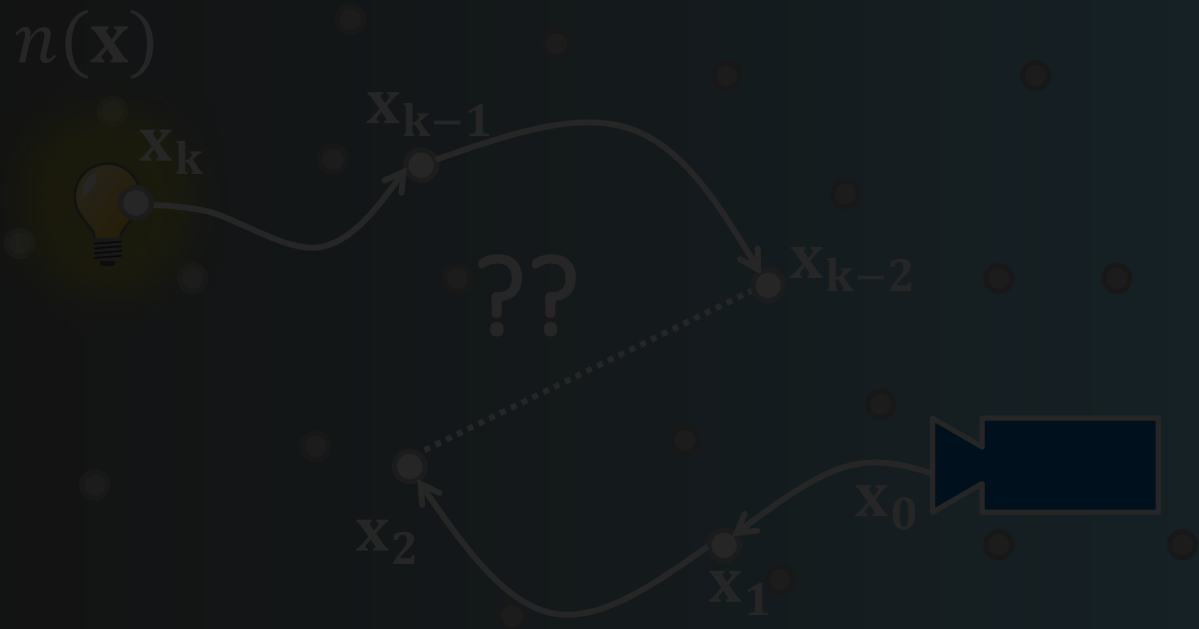
continuous refraction and scattering



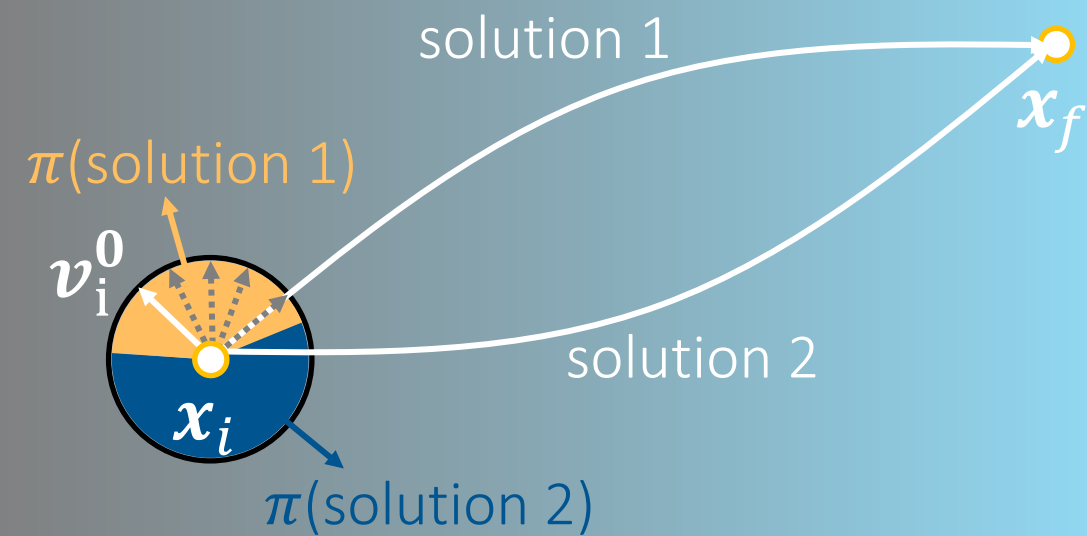
bidirectional path tracing (BDPT):

1. trace a random sensor subpath
use refractive ray tracing
2. trace a random emitter subpath

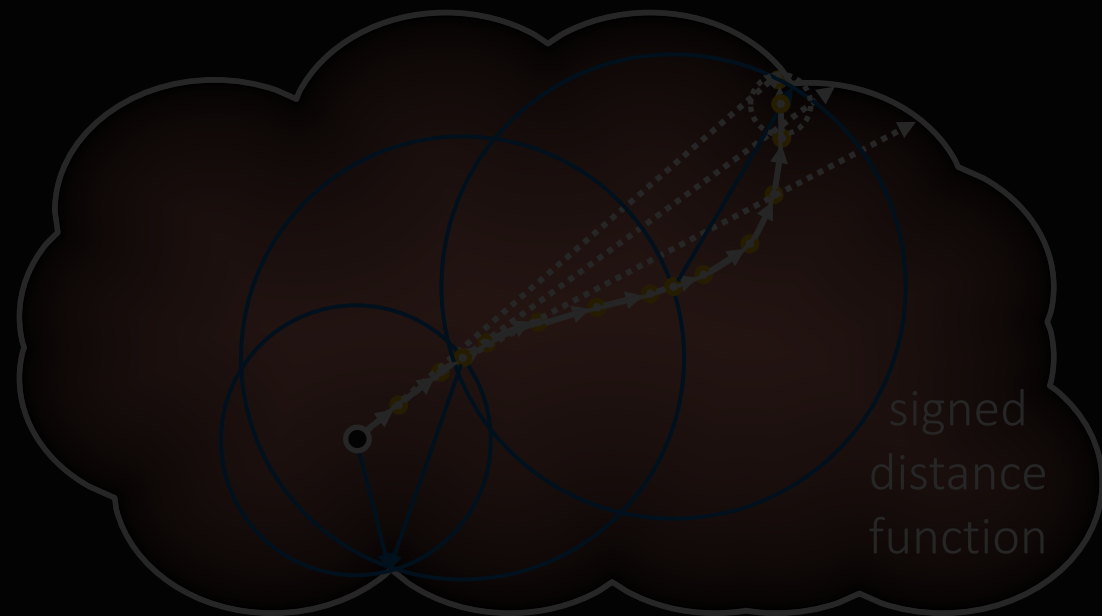
3. join vertices with a ~~straight line~~ curve



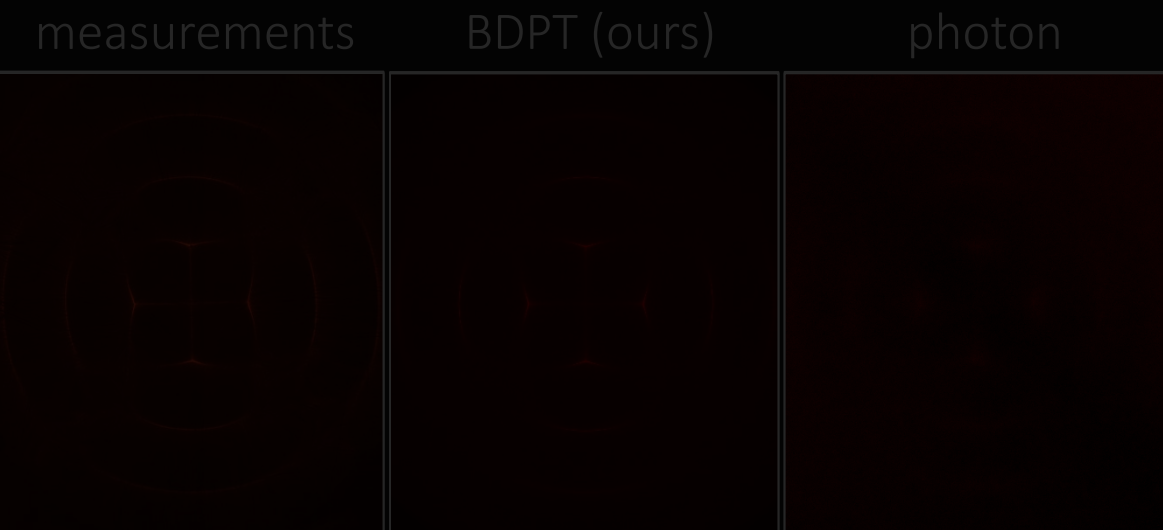
1. background on refractive radiative transfer equation



2. direct connections: our solution to unbiased rendering



3. acceleration techniques



4. experiments

direct connections



we have to solve this:

$$\frac{d\mathbf{v}}{ds} = \nabla_{\mathbf{x}} n(\mathbf{x}), \quad \frac{d\mathbf{x}}{ds} = \frac{\mathbf{v}}{n(\mathbf{x})}$$

boundary conditions: $\mathbf{x}_i, \mathbf{x}_f$

boundary value problem (BVP)

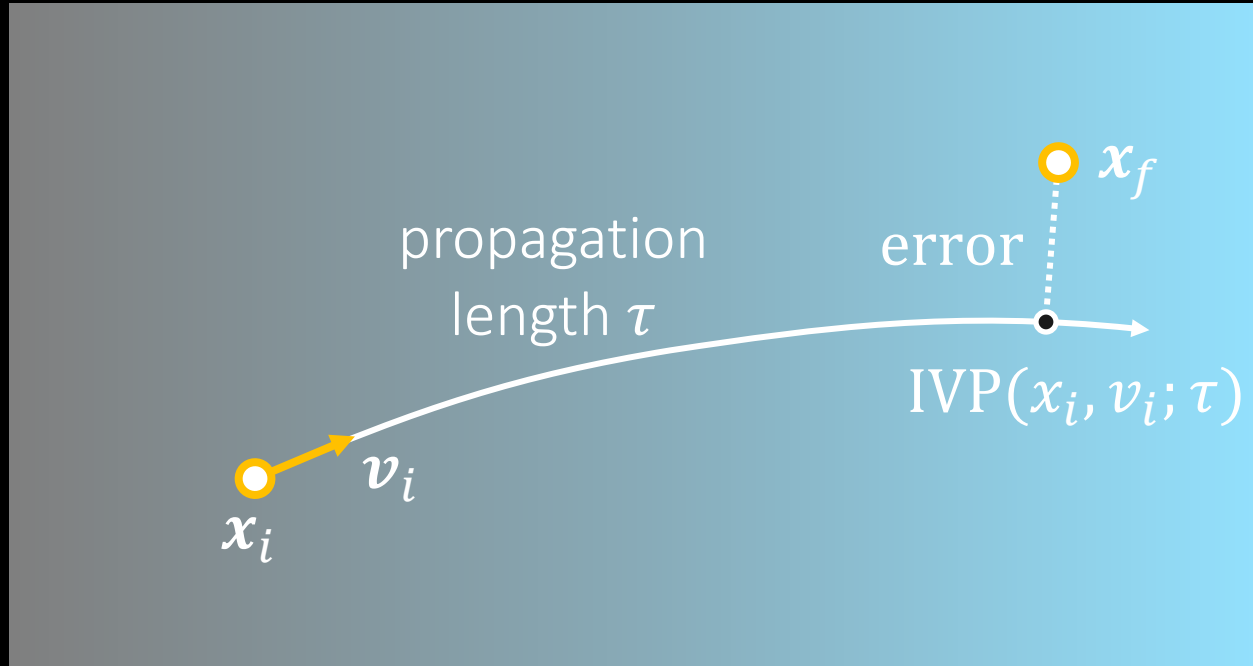
we know how to solve this:

$$\frac{d\mathbf{v}}{ds} = \nabla_{\mathbf{x}} n(\mathbf{x}), \quad \frac{d\mathbf{x}}{ds} = \frac{\mathbf{v}}{n(\mathbf{x})}$$

boundary conditions: $\mathbf{x}_i, \mathbf{v}_i$

initial value problem (IVP),
a.k.a. refractive ray tracing

direct connections



$$\text{error}(x_f, x_i, v_i) \equiv \min_{\tau} \|x_f - \text{IVP}(x_i, v_i; \tau)\|^2$$

we have to solve this:

$$\frac{d\mathbf{v}}{ds} = \nabla_{\mathbf{x}} n(\mathbf{x}), \quad \frac{d\mathbf{x}}{ds} = \frac{\mathbf{v}}{n(\mathbf{x})}$$

boundary conditions: $\mathbf{x}_i, \mathbf{x}_f$

boundary value problem (BVP)

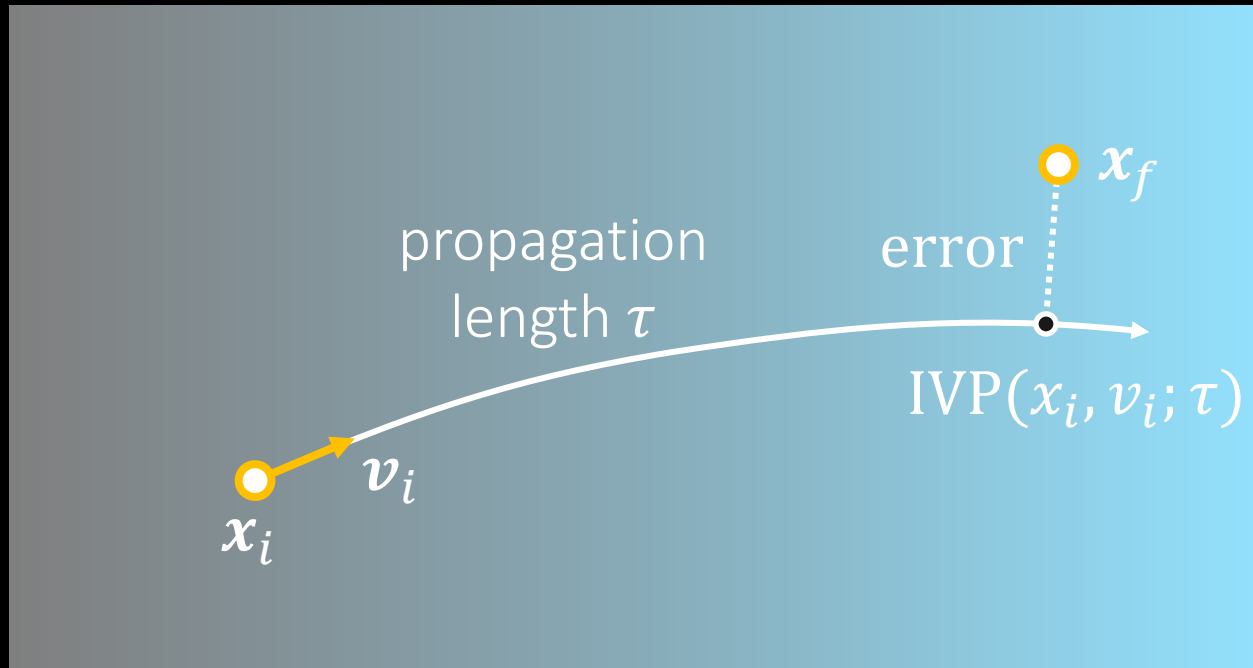
we know how to solve this:

$$\frac{d\mathbf{v}}{ds} = \nabla_{\mathbf{x}} n(\mathbf{x}), \quad \frac{d\mathbf{x}}{ds} = \frac{\mathbf{v}}{n(\mathbf{x})}$$

boundary conditions: $\mathbf{x}_i, \mathbf{v}_i$

initial value problem (IVP),
a.k.a. refractive ray tracing

direct connections



$$\text{error}(x_f, x_i, v_i) \equiv \min_{\tau} \|x_f - \text{IVP}(x_i, v_i; \tau)\|^2$$

we have to solve this:

$$\min_{v_i} \text{error}(x_f, x_i, v_i)$$

boundary conditions: x_i, x_f

boundary value problem (BVP)

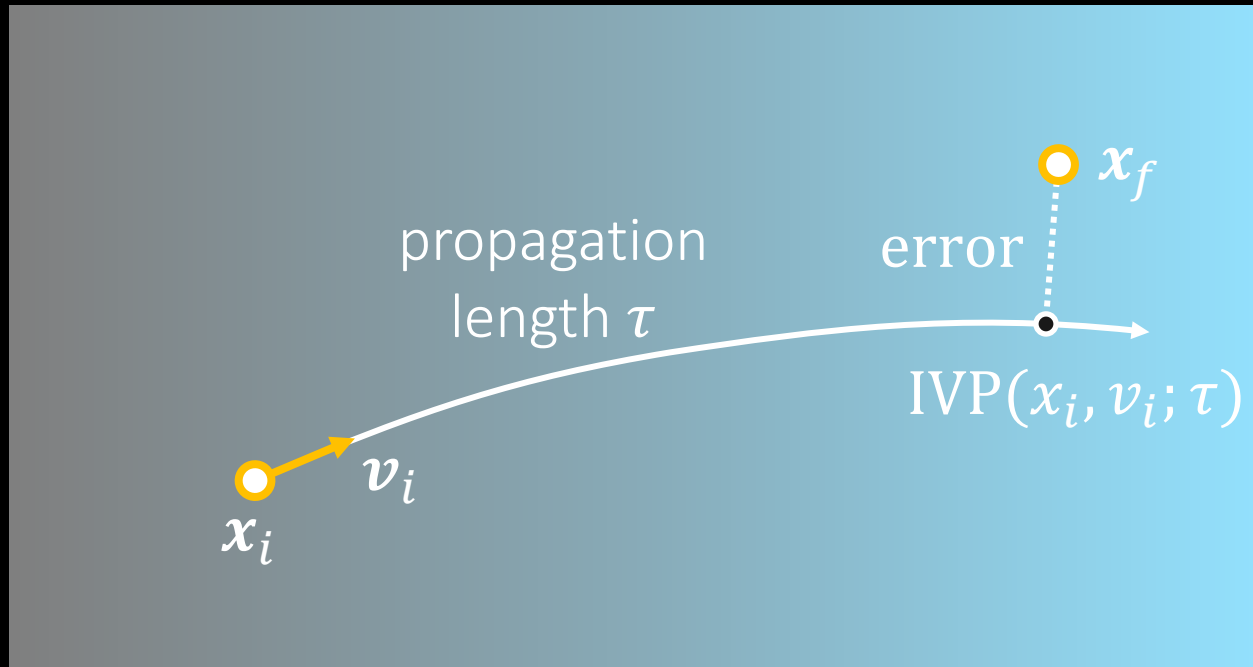
we know how to solve this:

$$\frac{dv}{ds} = \nabla_x n(x), \quad \frac{dx}{ds} = \frac{v}{n(x)}$$

boundary conditions: x_i, v_i

initial value problem (IVP),
a.k.a. refractive ray tracing

direct connections



differentiable

$$\text{error}(x_f, x_i, v_i) \equiv \min_{\tau} \|x_f - \text{IVP}(x_i, v_i; \tau)\|^2$$

we have to solve this:

$$\min_{v_i} \text{error}(x_f, x_i, v_i)$$

boundary conditions: x_i, x_f

boundary value problem (BVP)

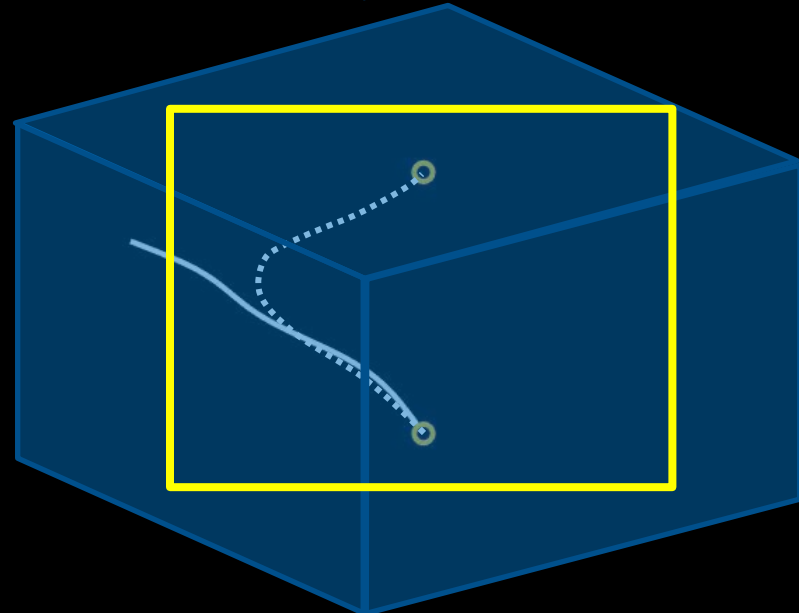
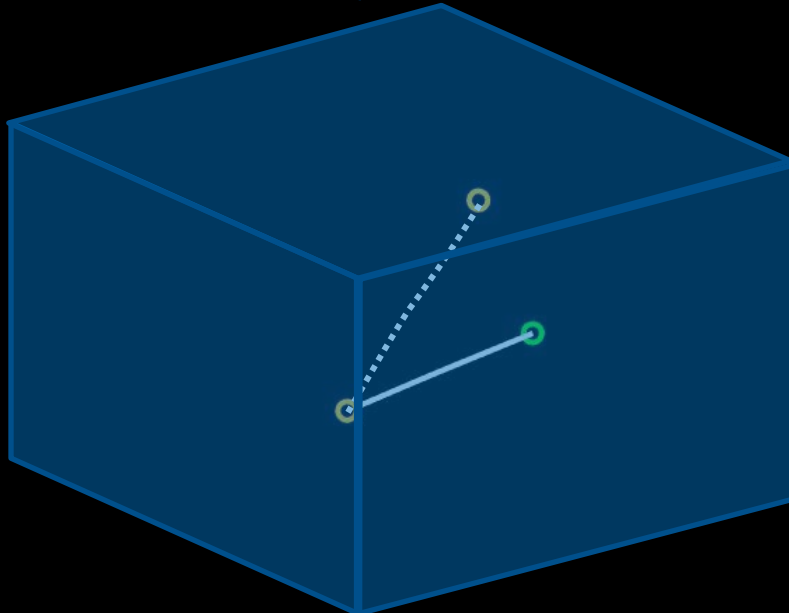
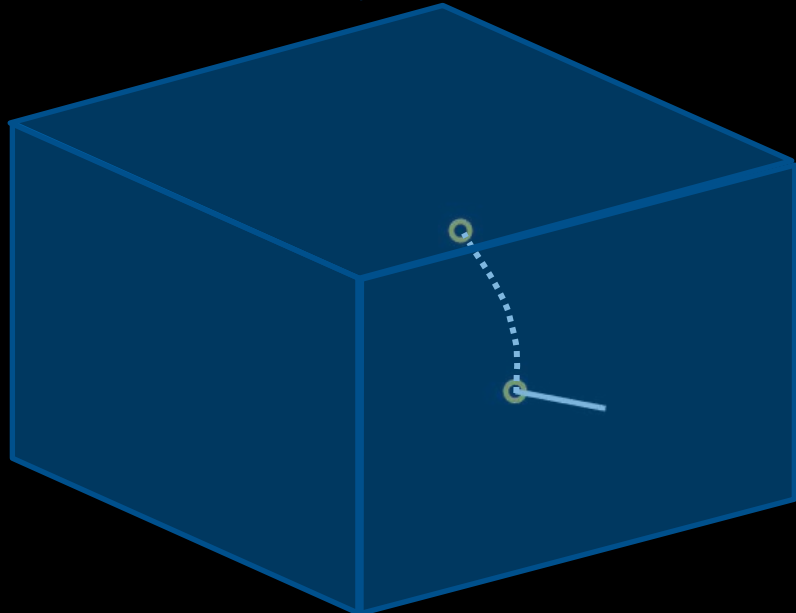
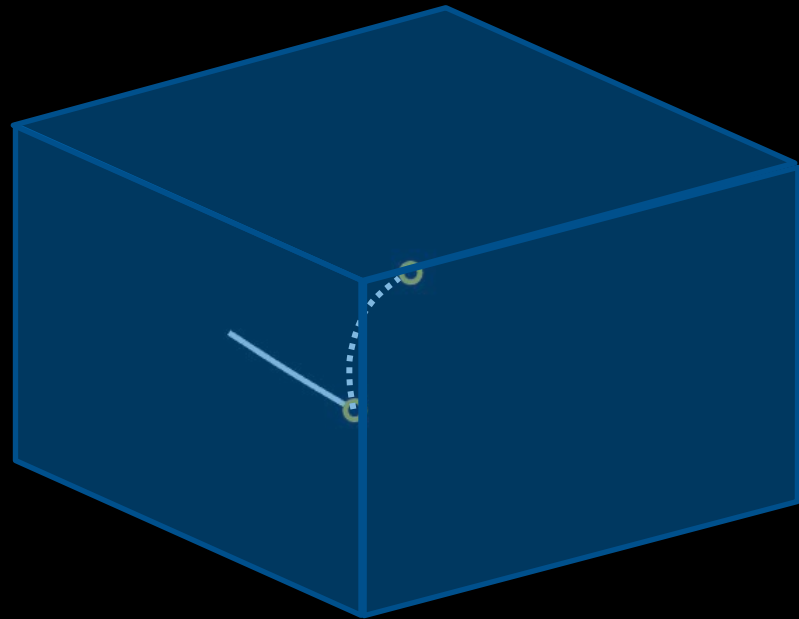
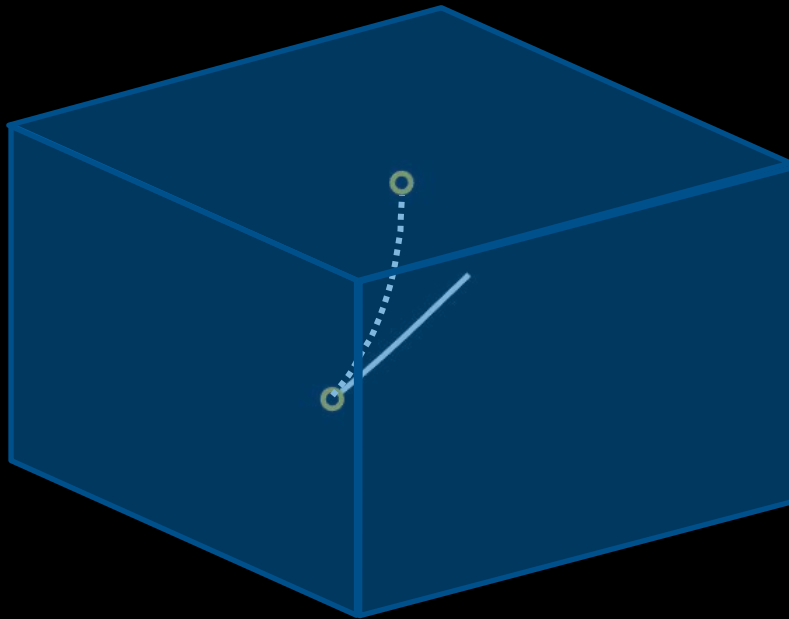
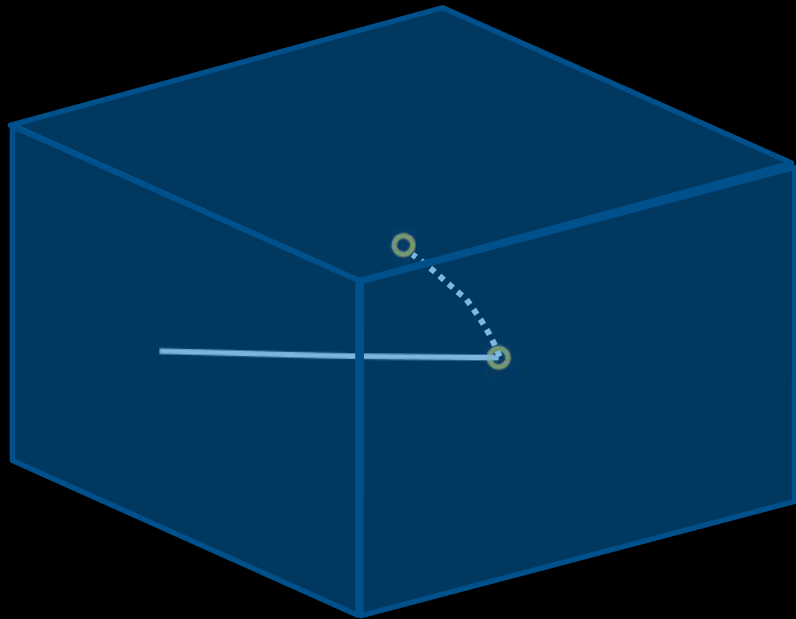
differentiable

$$\frac{dv}{ds} = \nabla_x n(x), \quad \frac{dx}{ds} = \frac{v}{n(x)}$$

boundary conditions: x_i, v_i

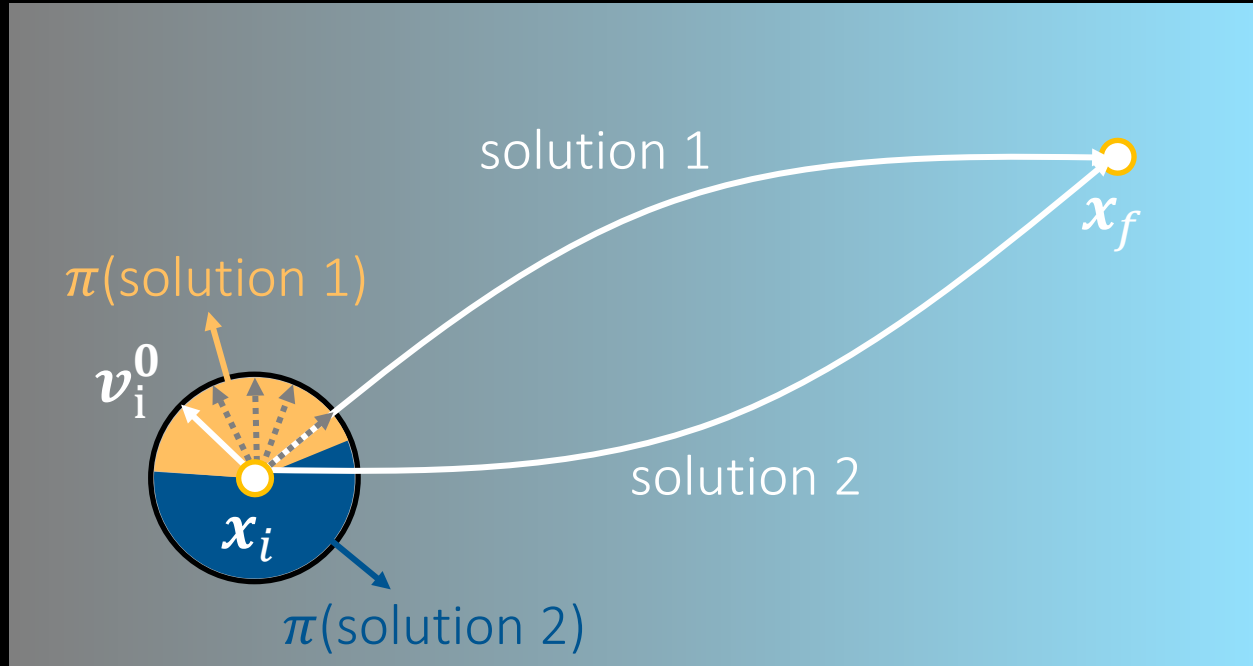
initial value problem (IVP),
a.k.a. refractive ray tracing

direct connections



multiple direct connections

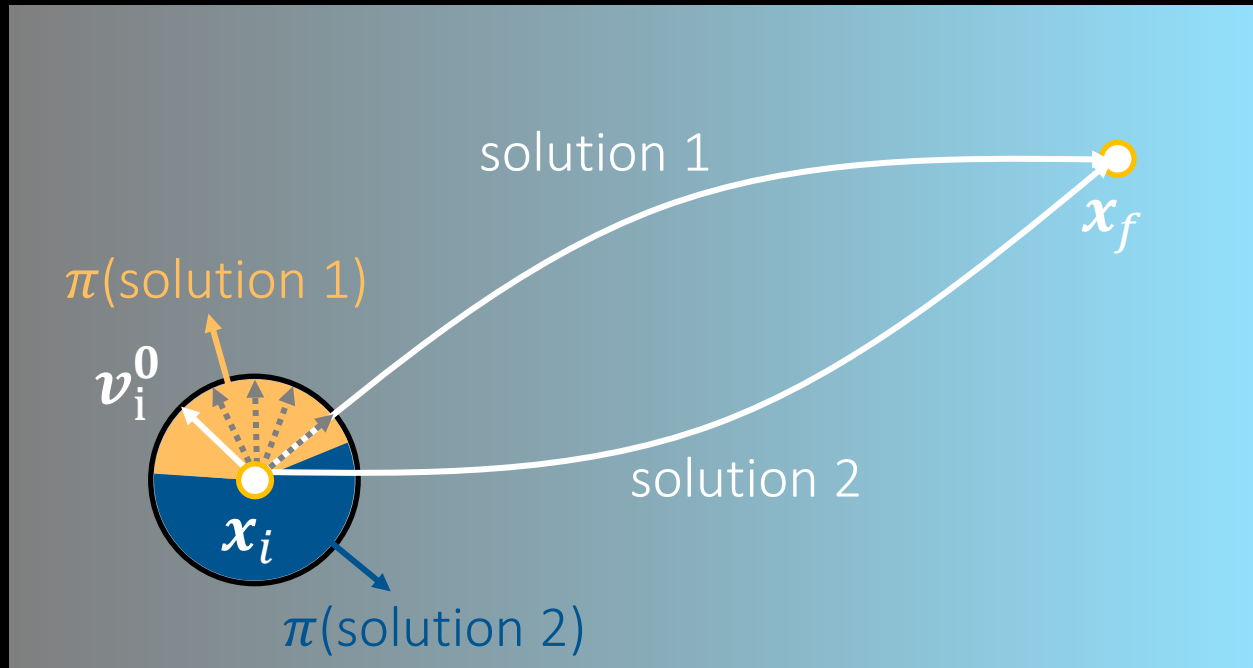
$$\text{total throughput} = \sum_{\text{all solutions}} \text{throughput}(\text{solution})$$



approach 1:
exhaustively enumerate all solutions

multiple direct connections

$$\text{total throughput} = \sum_{\text{all solutions}} \text{throughput}(\text{solution})$$

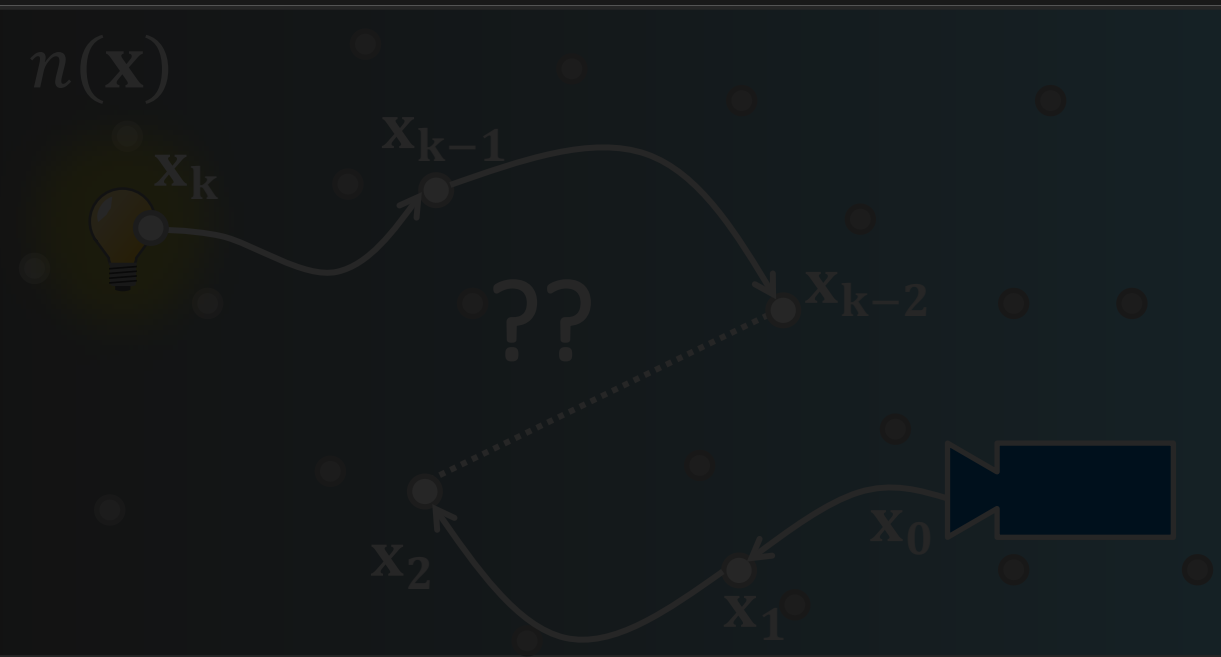


approach 1: **impractical**
exhaustively enumerate all solutions

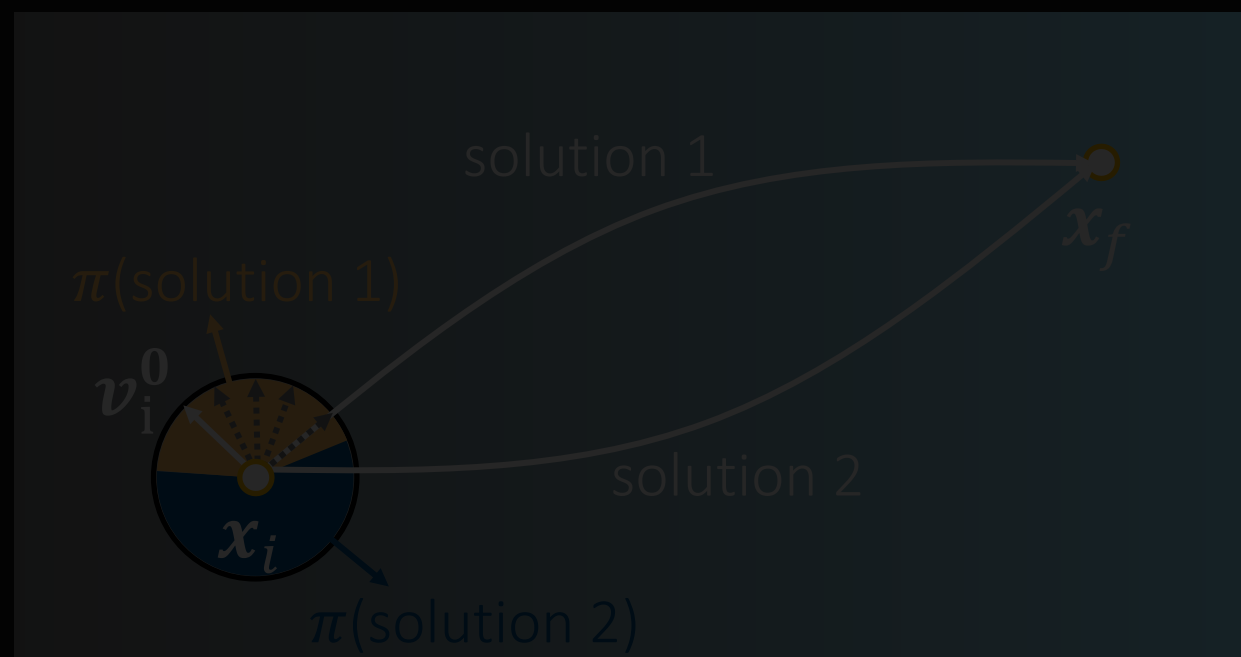
approach 2:
unbiased single-sample Monte Carlo
1. randomly sample initial direction
2. solve BVP
3. form estimate

$$\text{total throughput} \approx \frac{\text{throughput}(\text{solution})}{\text{probability}(\text{solution})}$$

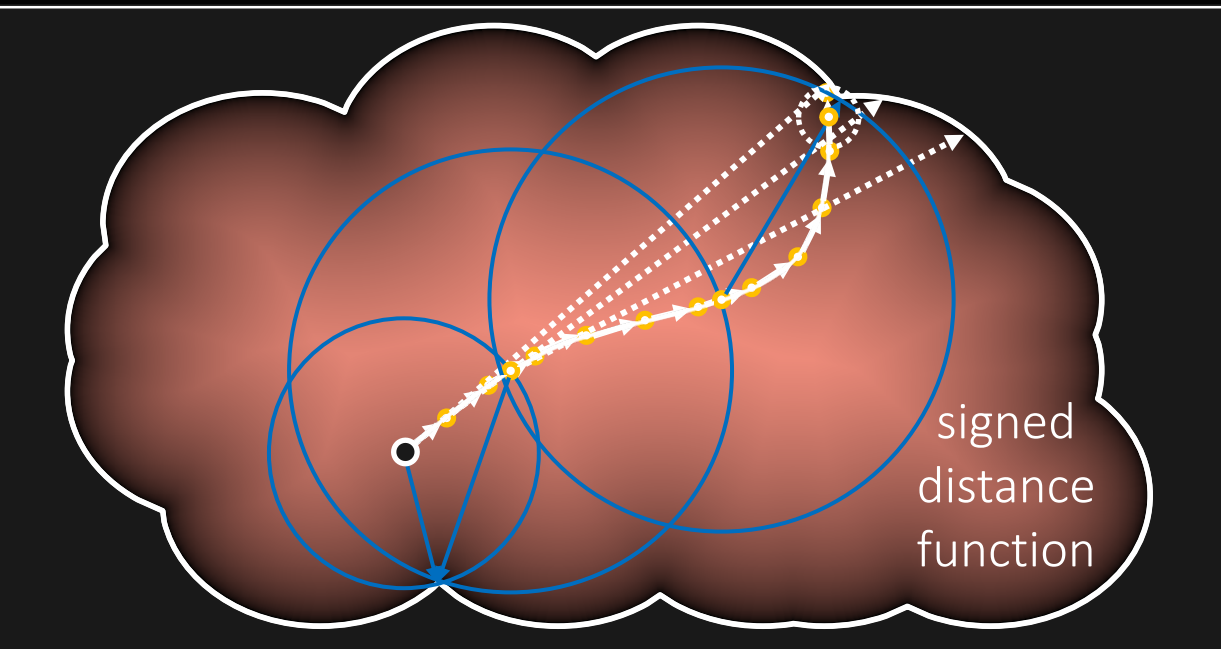
set of initial directions that converge to the solution



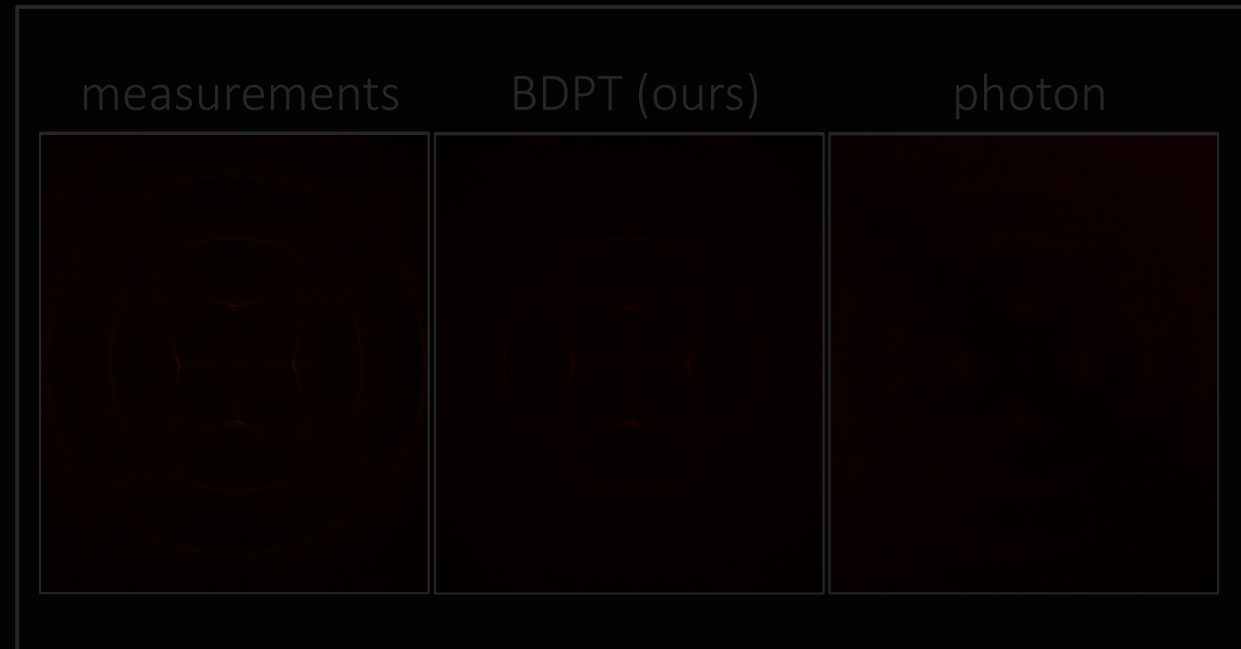
1. background on refractive radiative transfer equation



2. direct connections: our solution to unbiased rendering



3. acceleration techniques

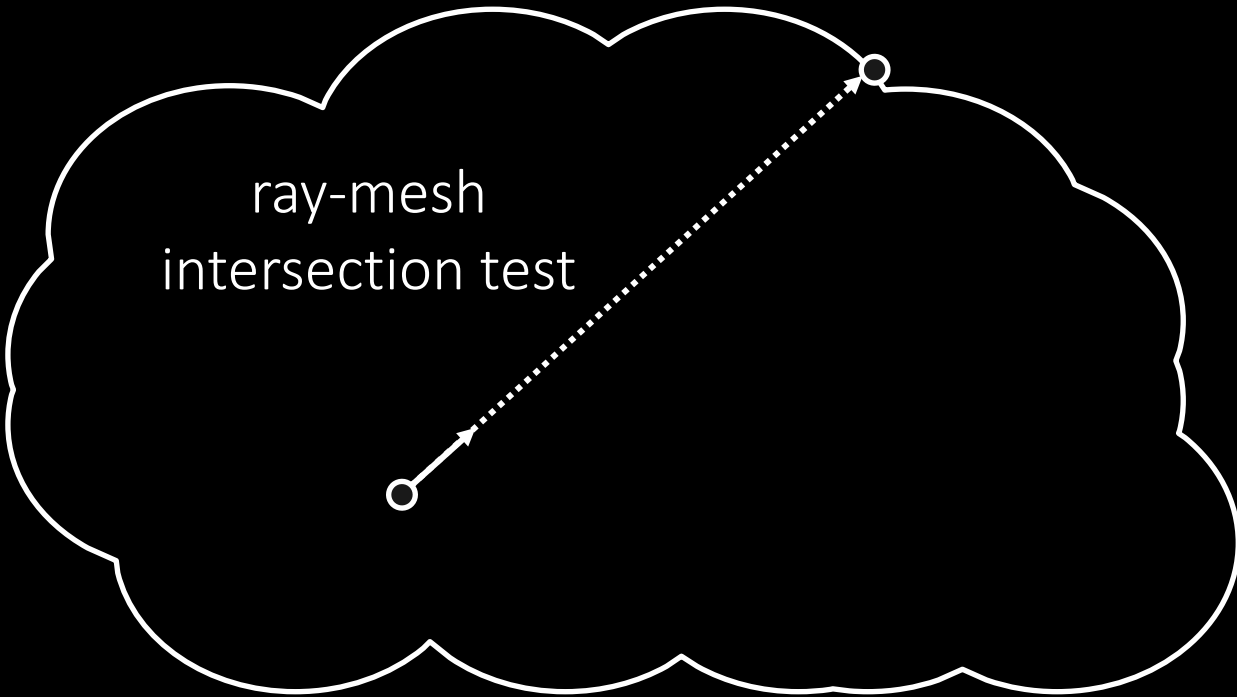


4. experiments

acceleration: sphere tracing

standard ray tracing

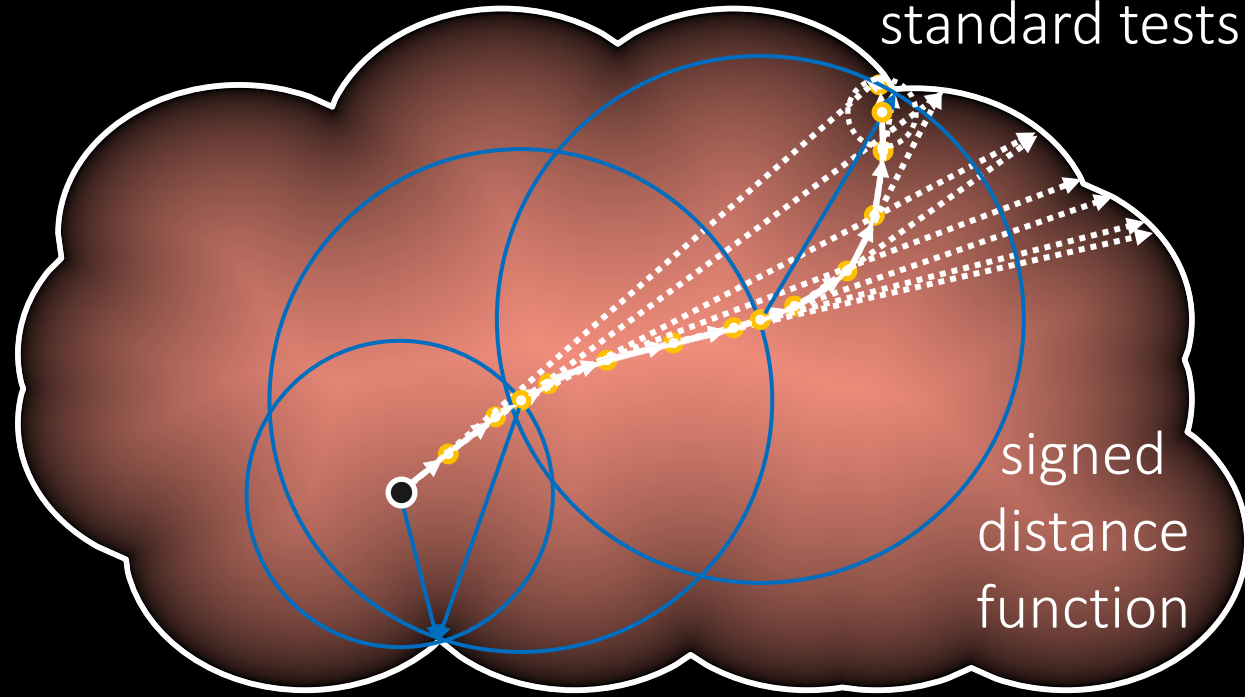
ray-mesh
intersection test



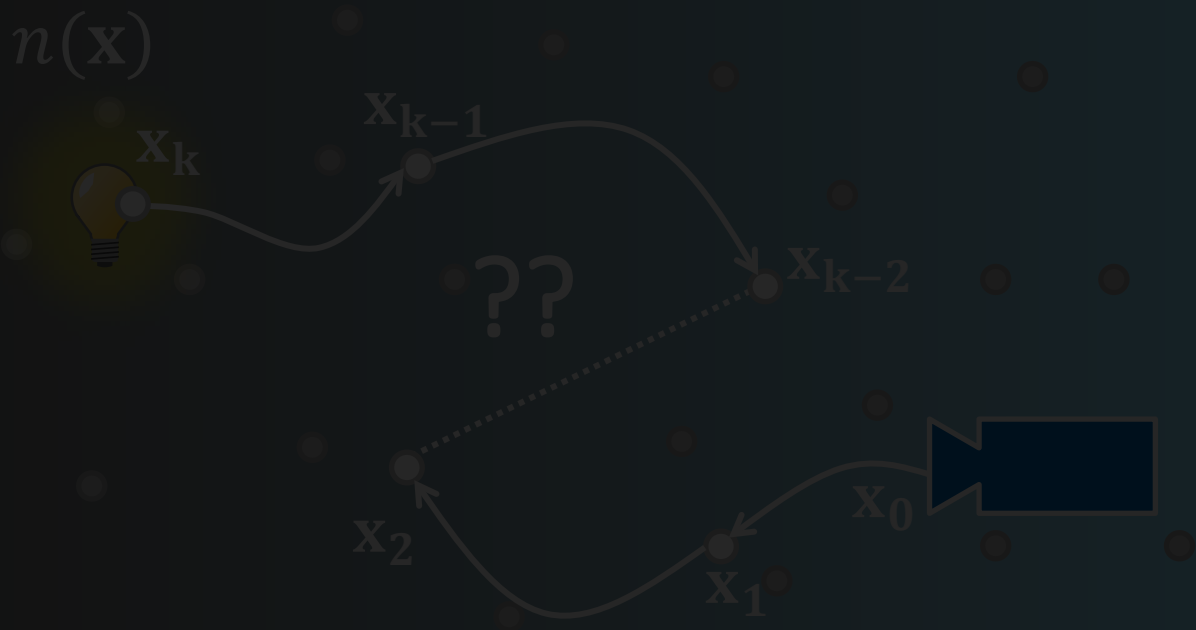
refractive ray tracing

switch to
standard tests

signed
distance
function



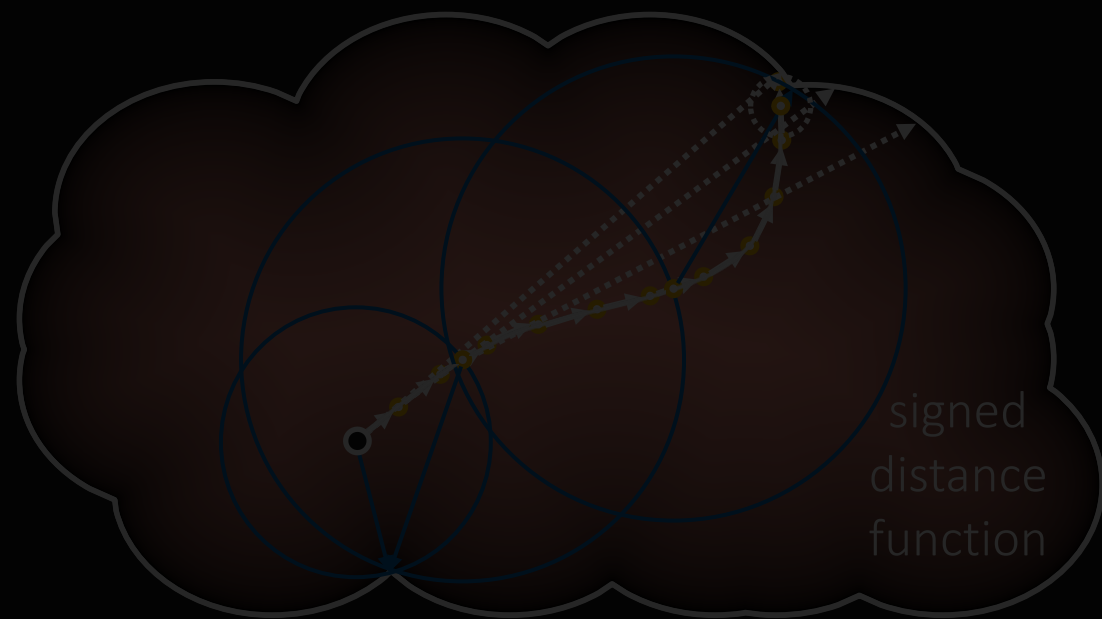
does not introduce bias



1. background on refractive radiative transfer equation



2. direct connections: our solution to unbiased rendering

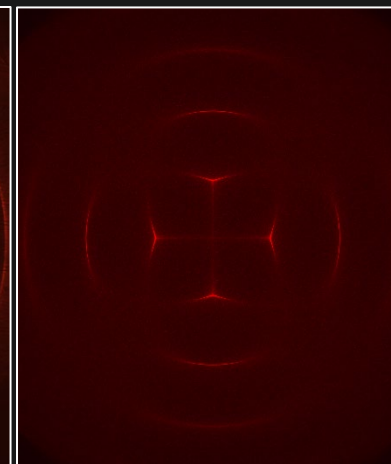
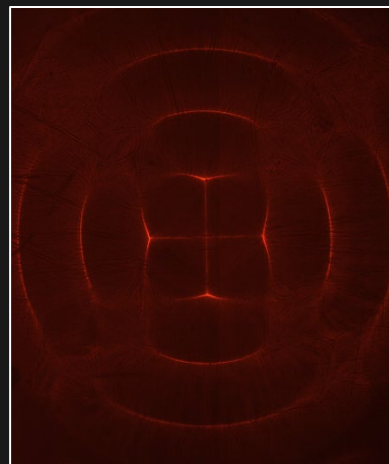


3. acceleration techniques

measurements

BDPT (ours)

photon

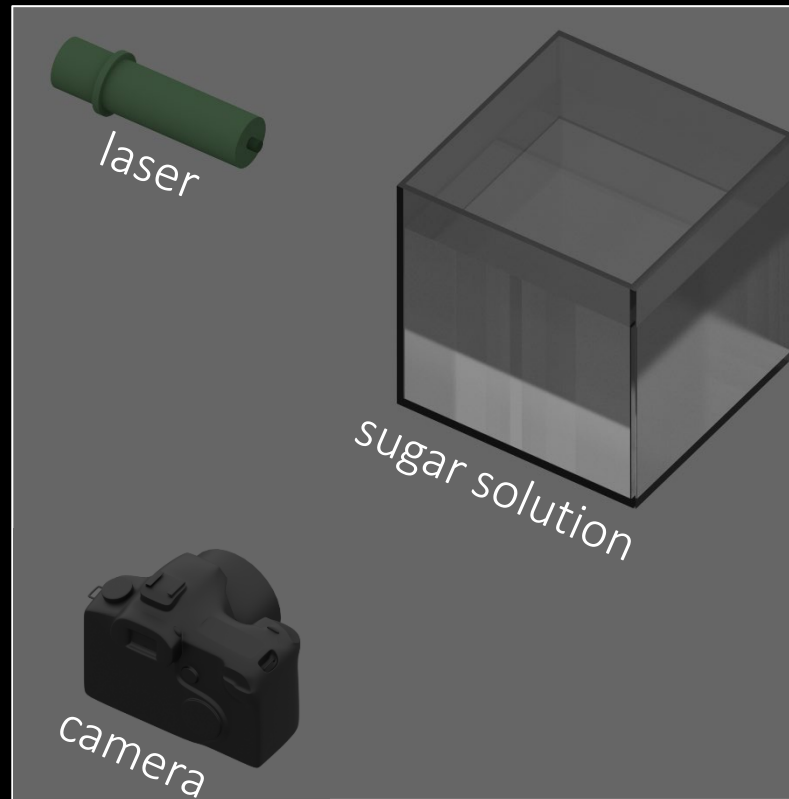


4. experiments

continuously refractive media and scattering



real scene

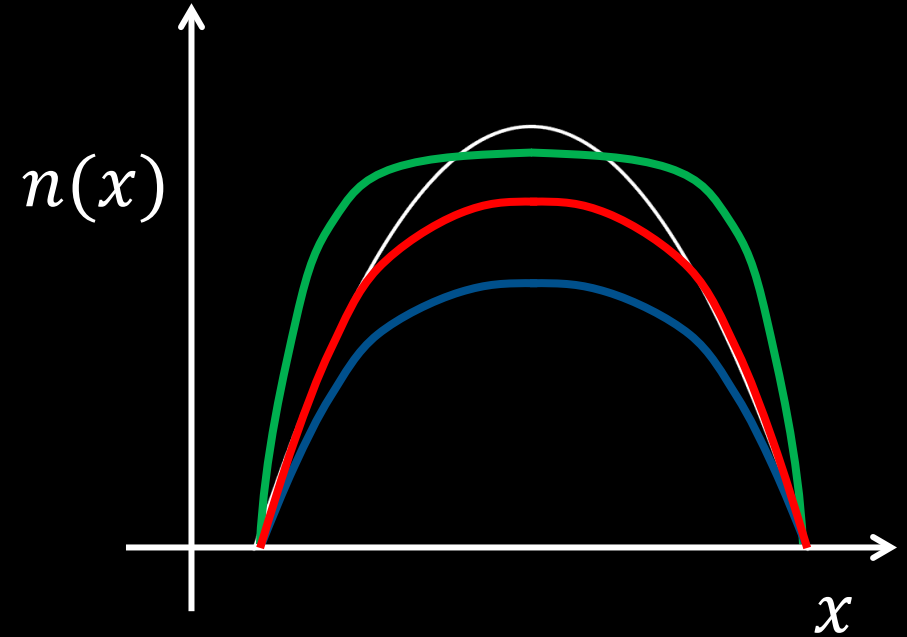
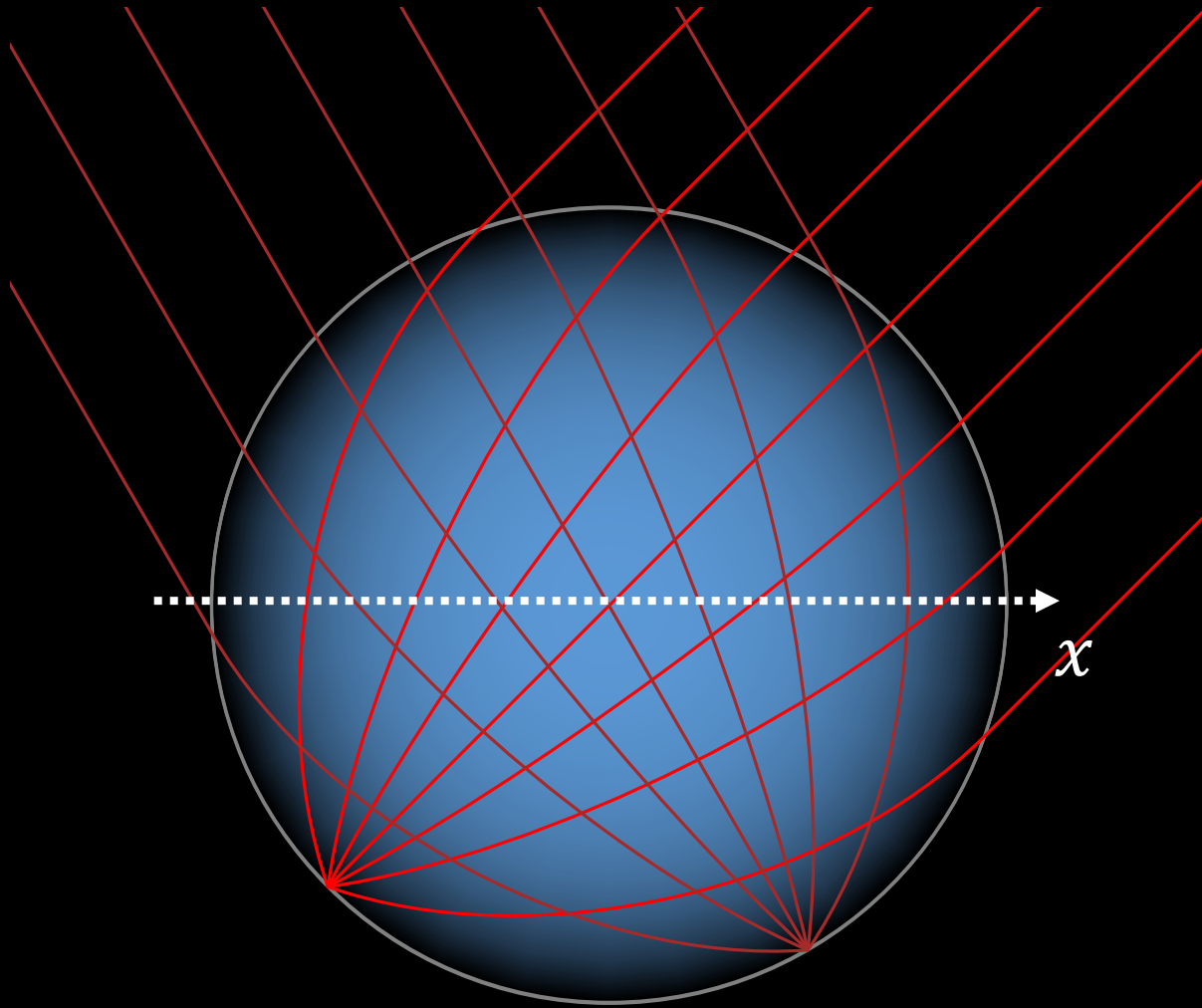


set up

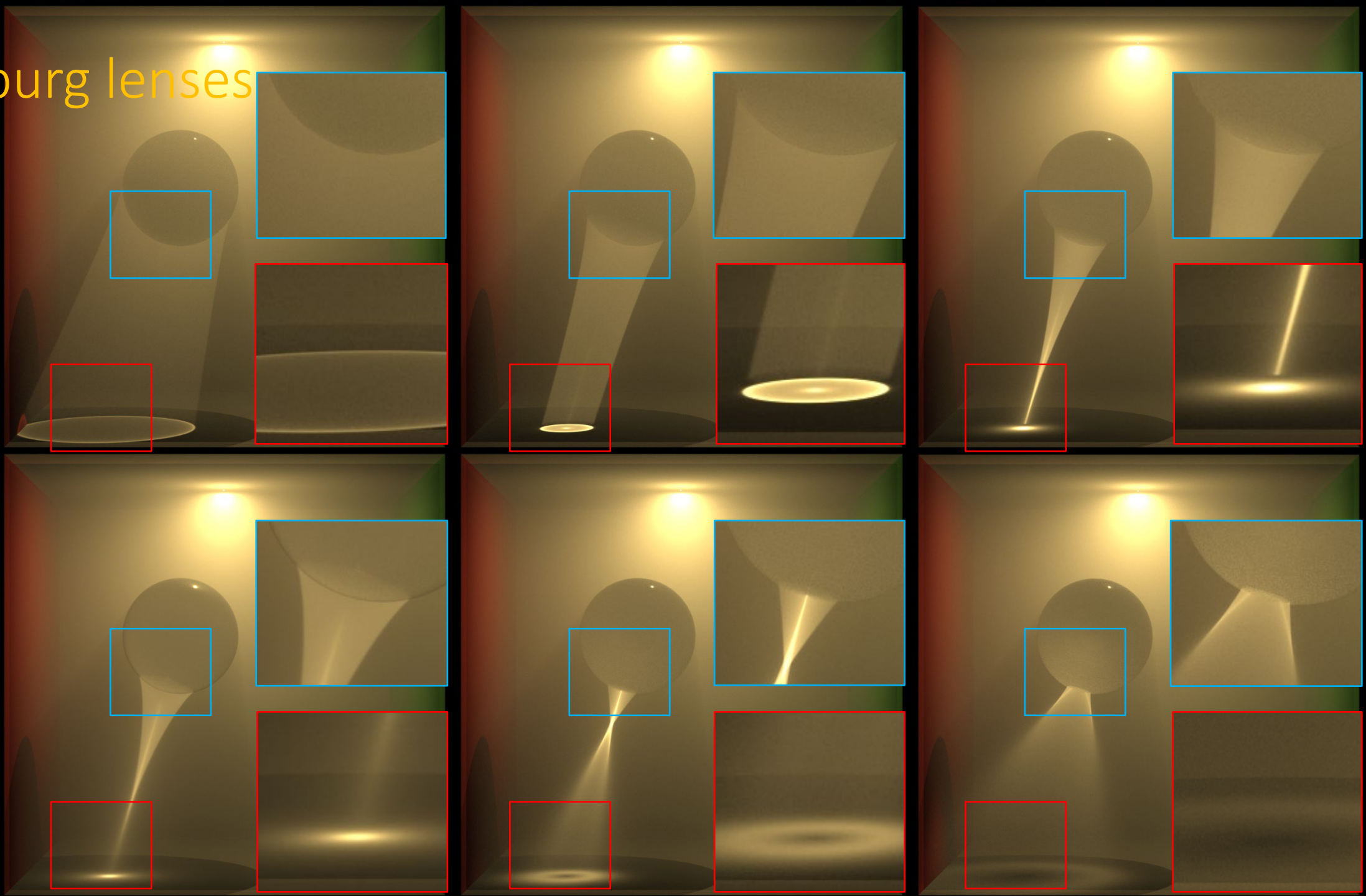


light propagation

Luneburg lenses



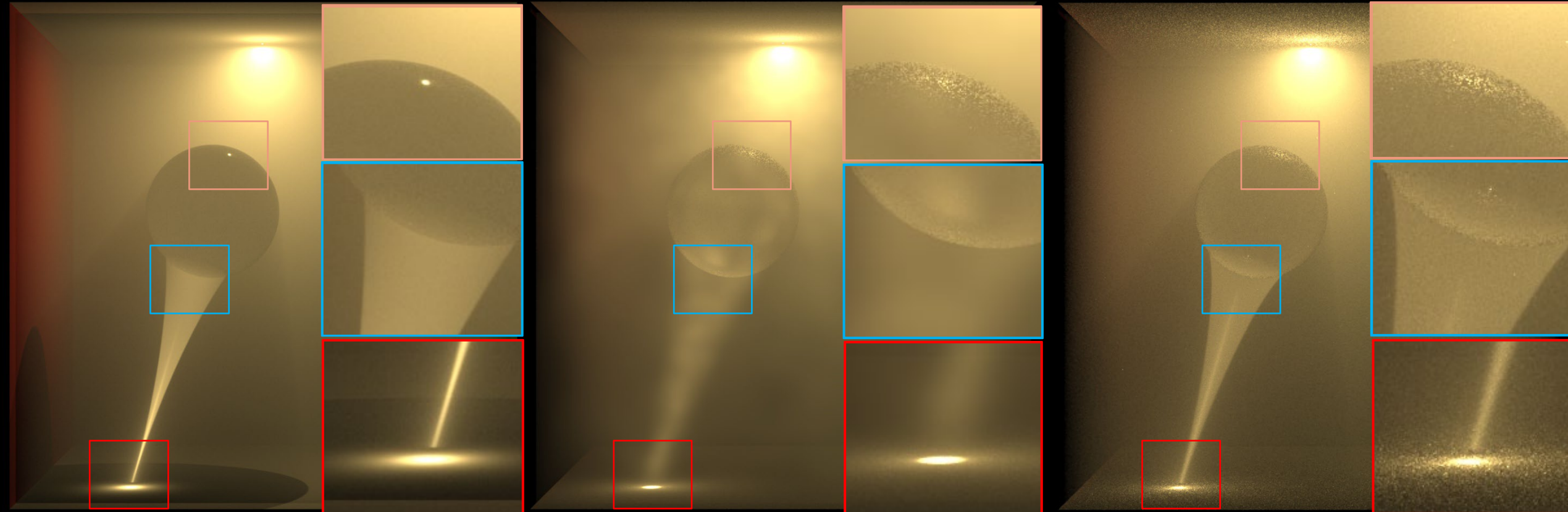
Luneburg lenses



power
equal to
standard
lens

rendering
time: 10 mins

comparison with photon mapping



BDPT (ours)

photon mapping
(default parameters)

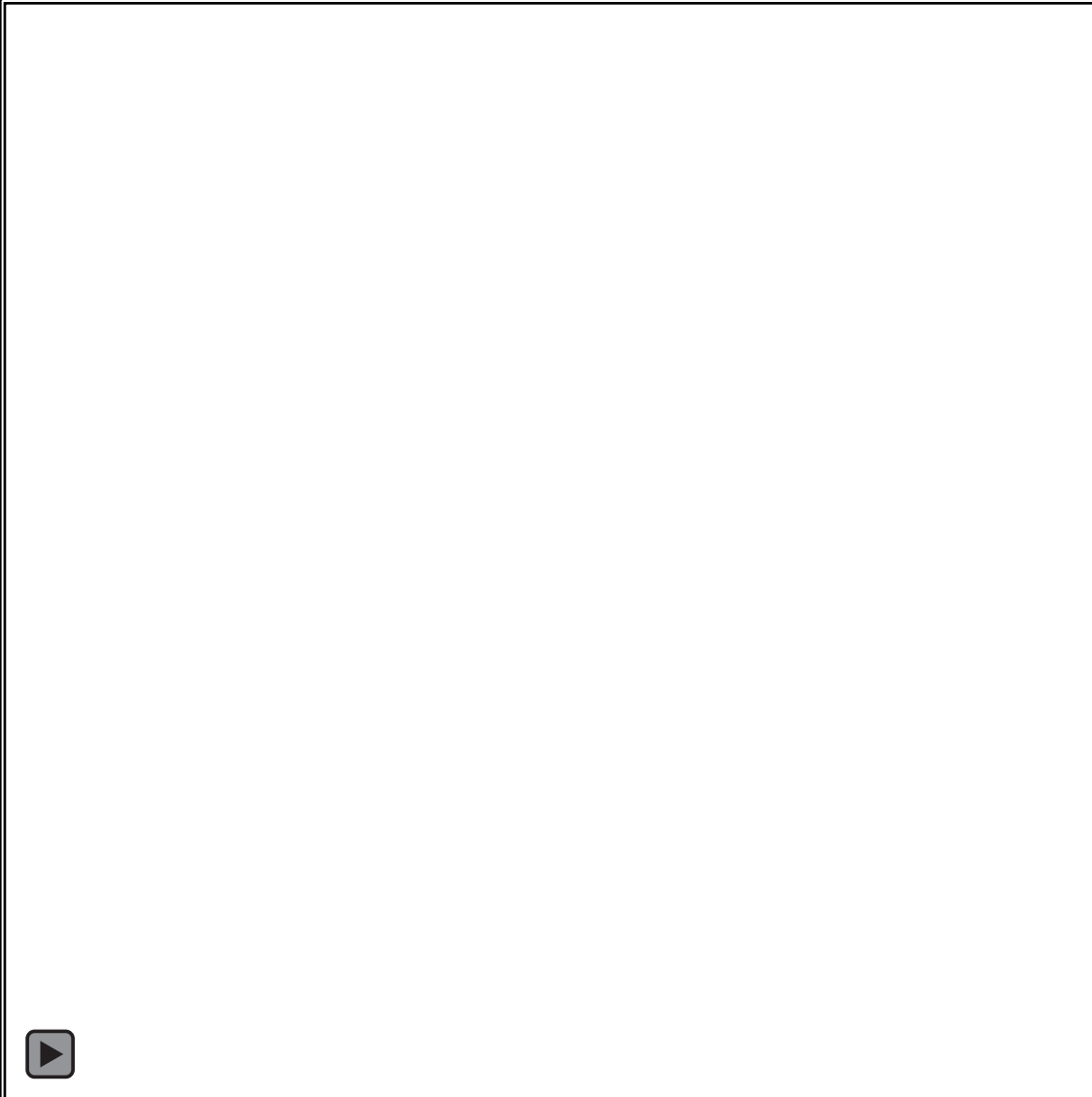
photon mapping
(optimized parameters)

rendering time: 10 min

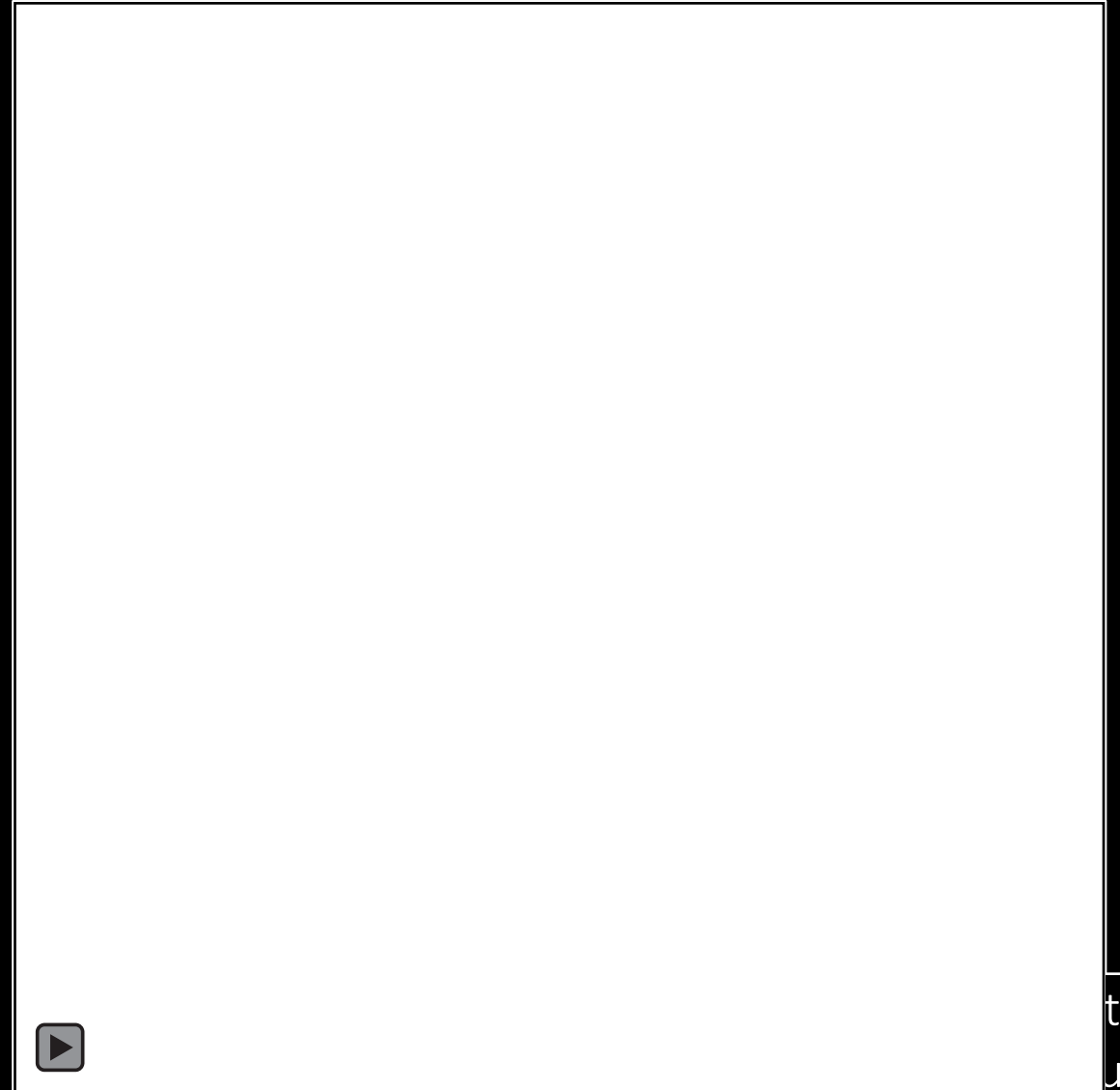
BDPT is 5x faster than photon mapping

transient rendering (videos)

constant refractive index



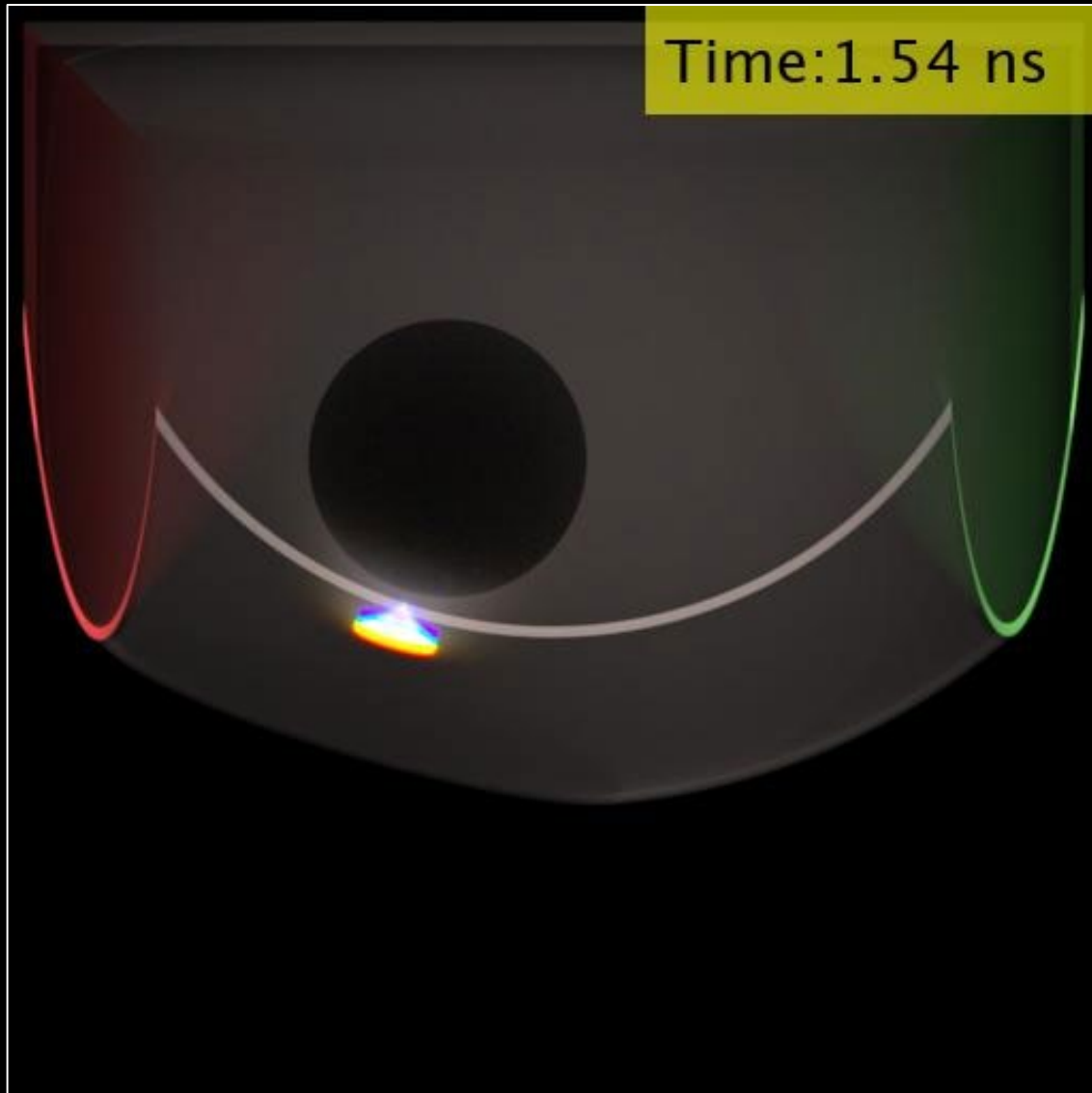
continuous refractive index



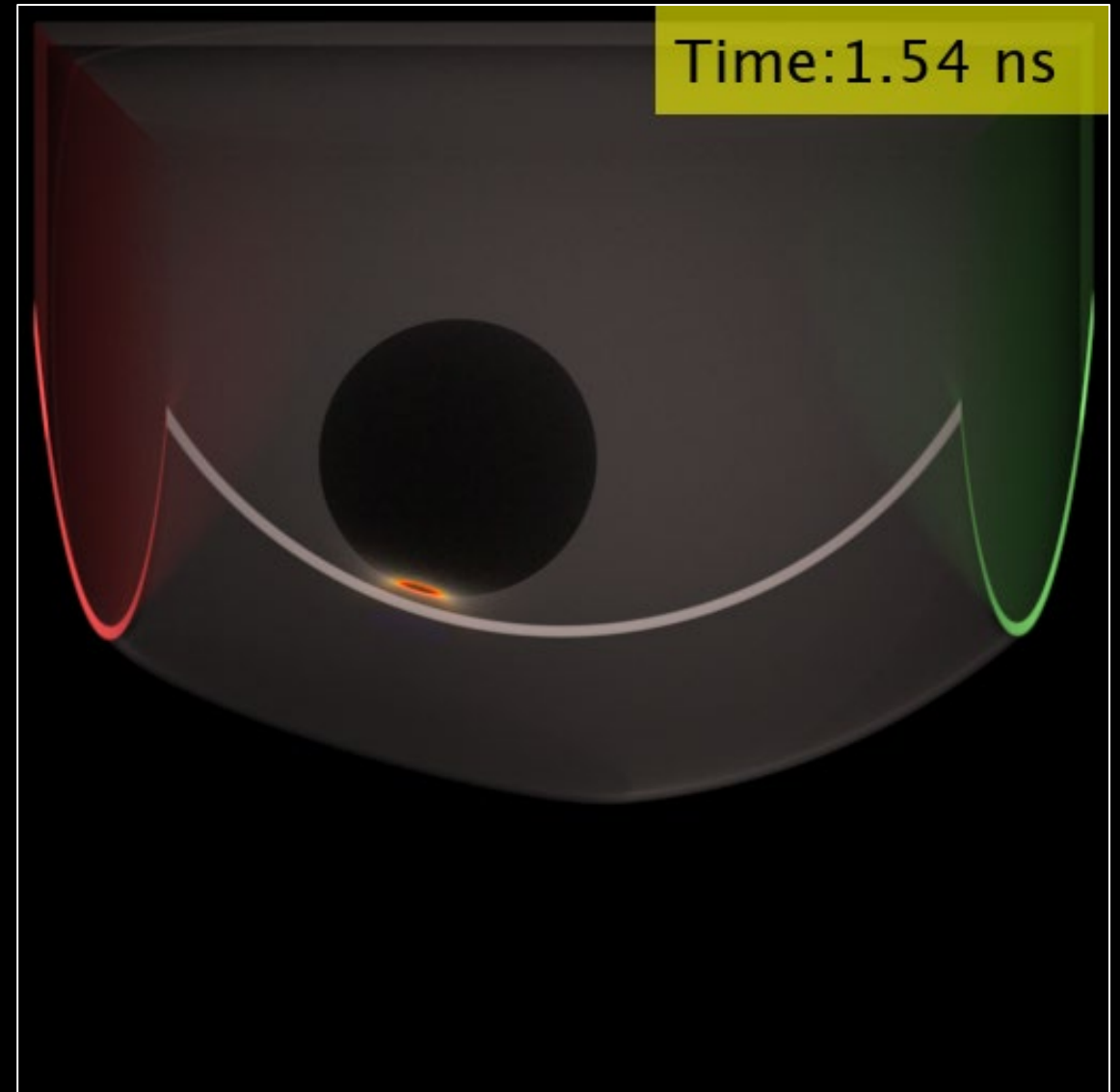
time:
urs

transient rendering

constant refractive index



continuous refractive index

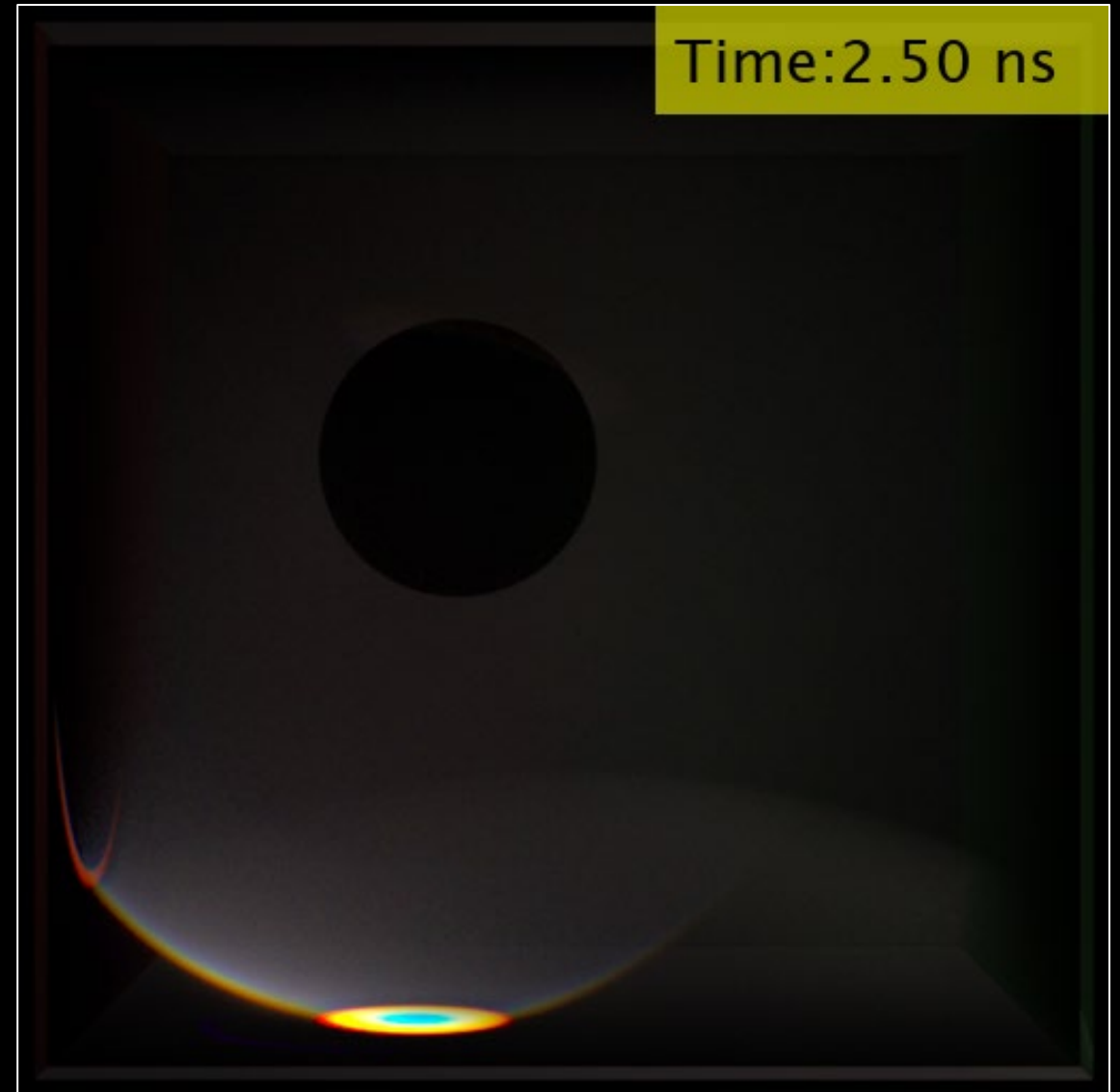


transient rendering

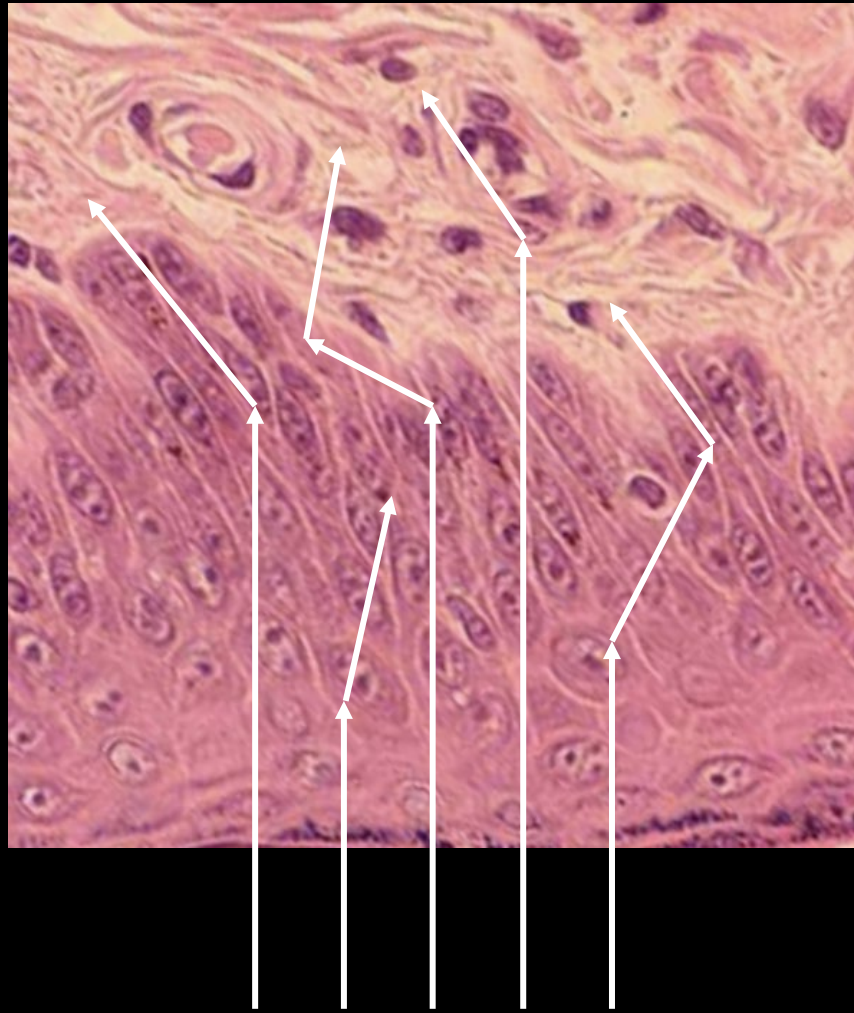
constant refractive index



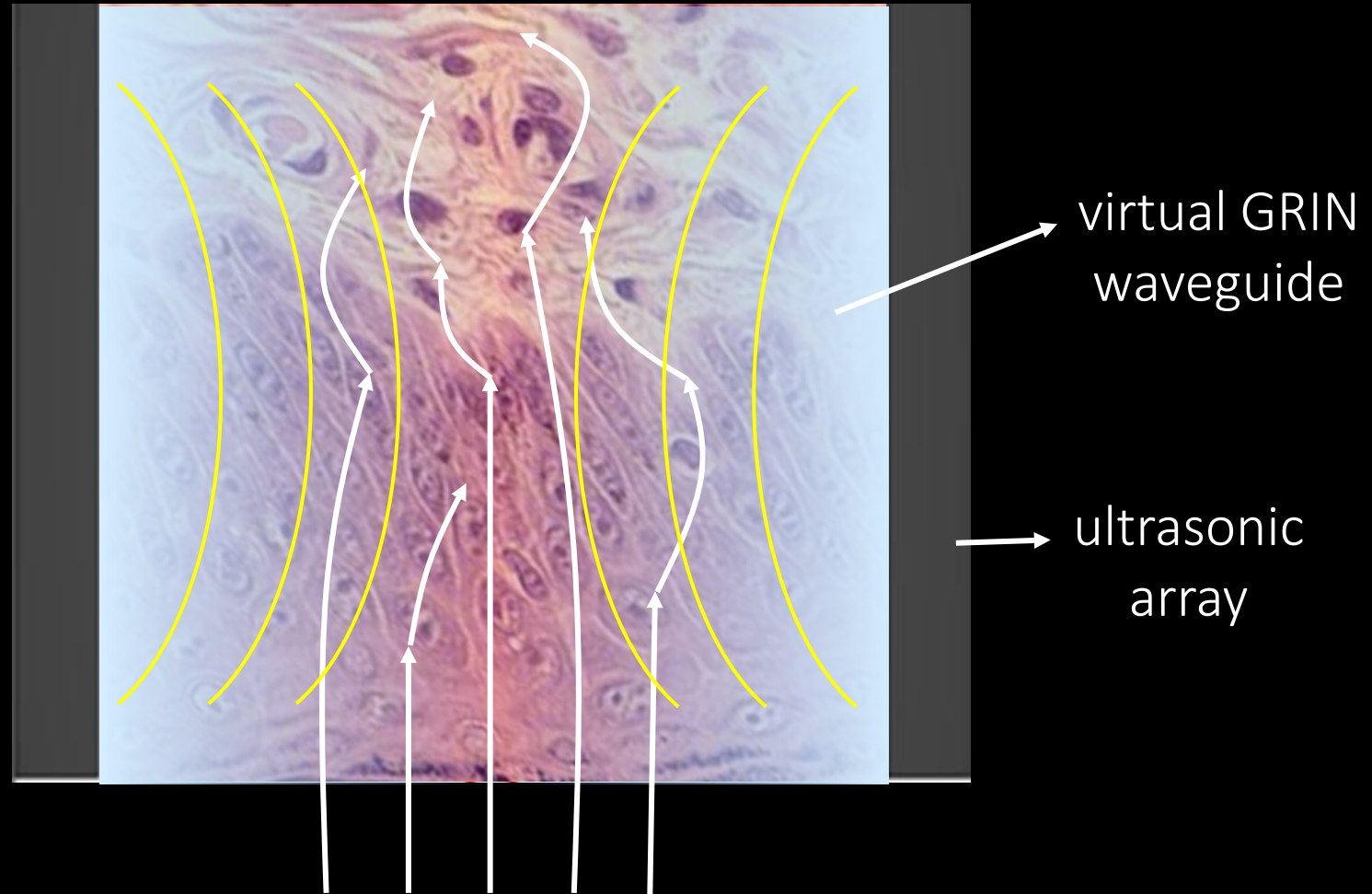
continuous refractive index



virtual ultrasonic waveguides

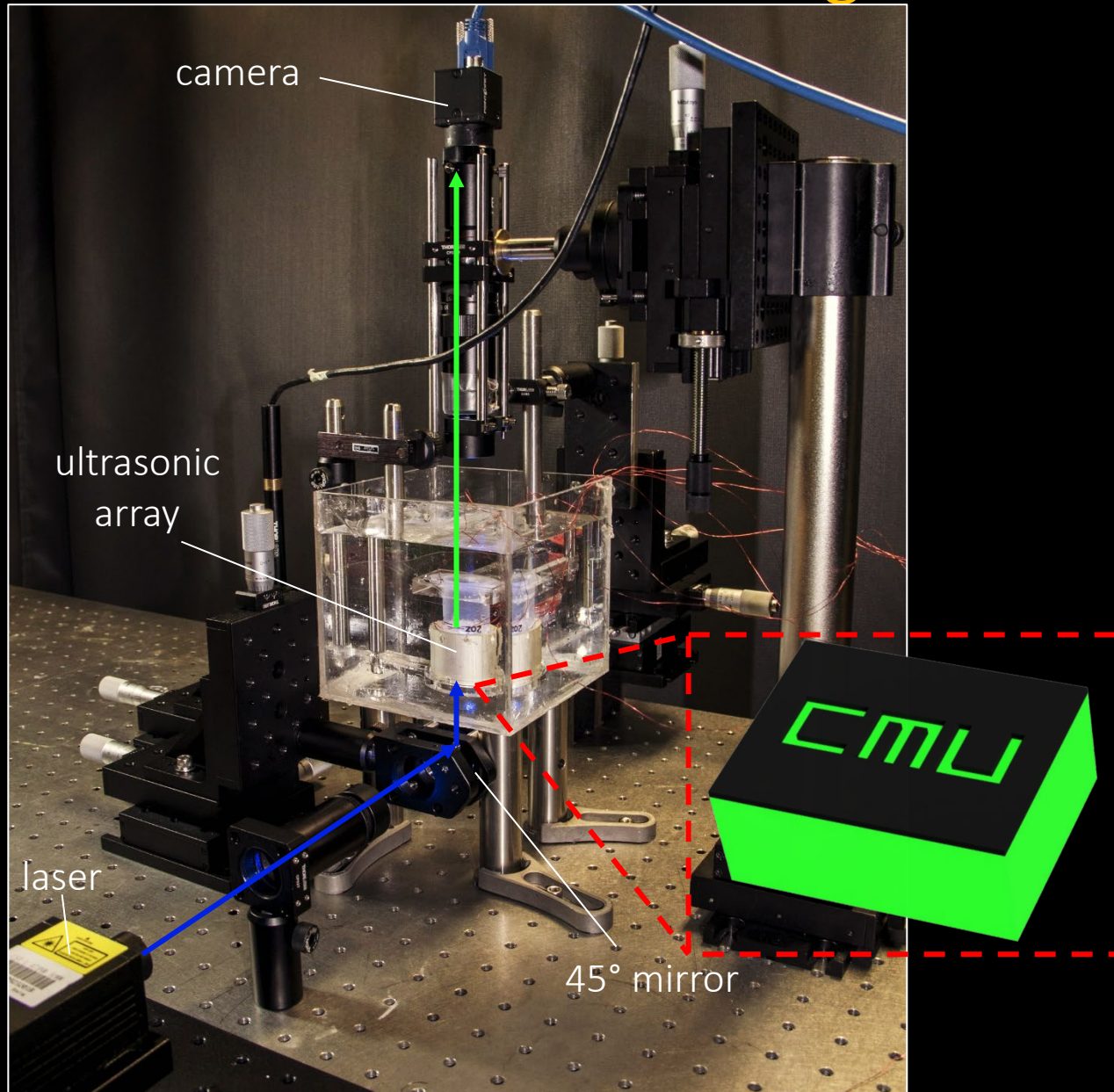


virtual ultrasonic waveguides

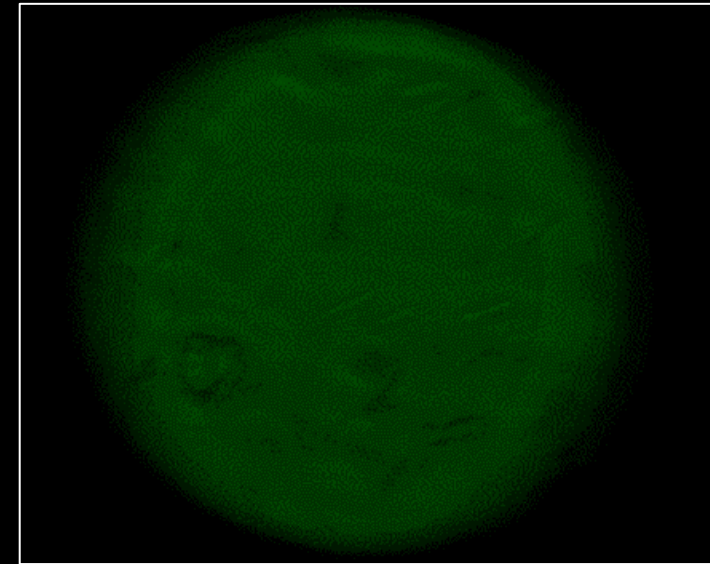


Chamanzar et al. "Ultrasonic sculpting of virtual optical waveguides in tissue". Nature communications, 2019
Scopelliti et al. "Ultrasonically sculpted virtual relay lens for in situ microimaging". Light: Science and Applications, 2019
Karimi et al. "In situ 3D reconfigurable ultrasonically sculpted optical beam paths". Optics express, 2019

virtual ultrasonic waveguides



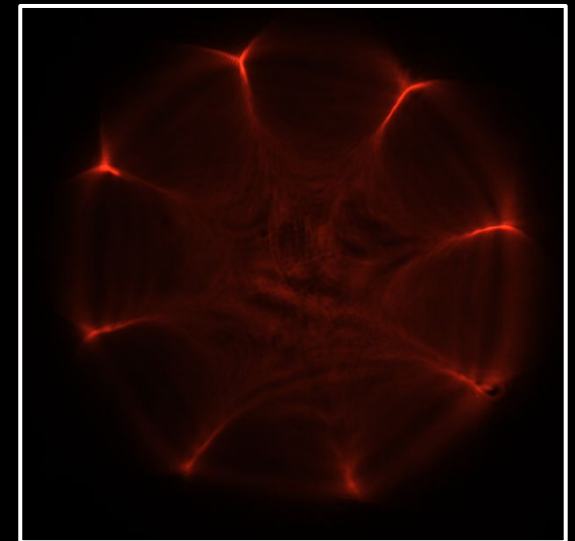
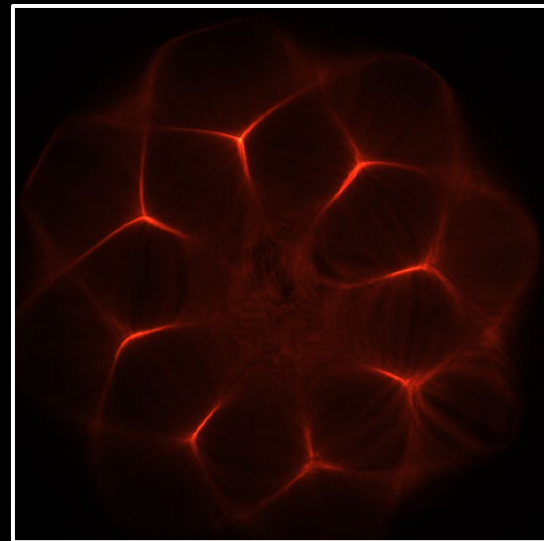
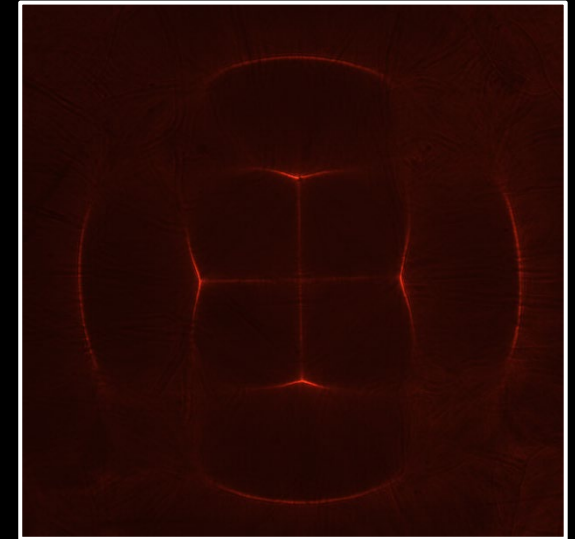
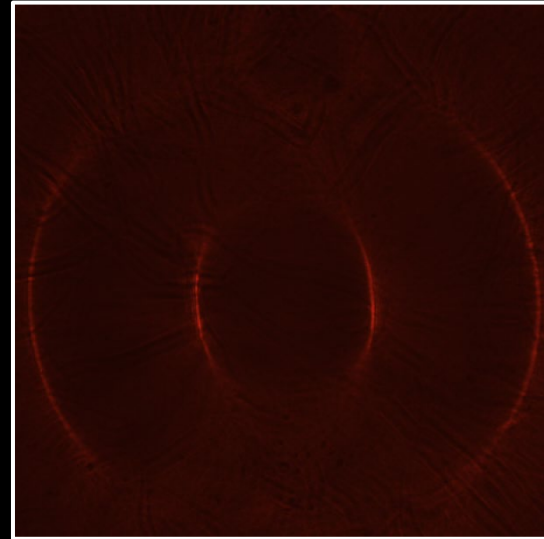
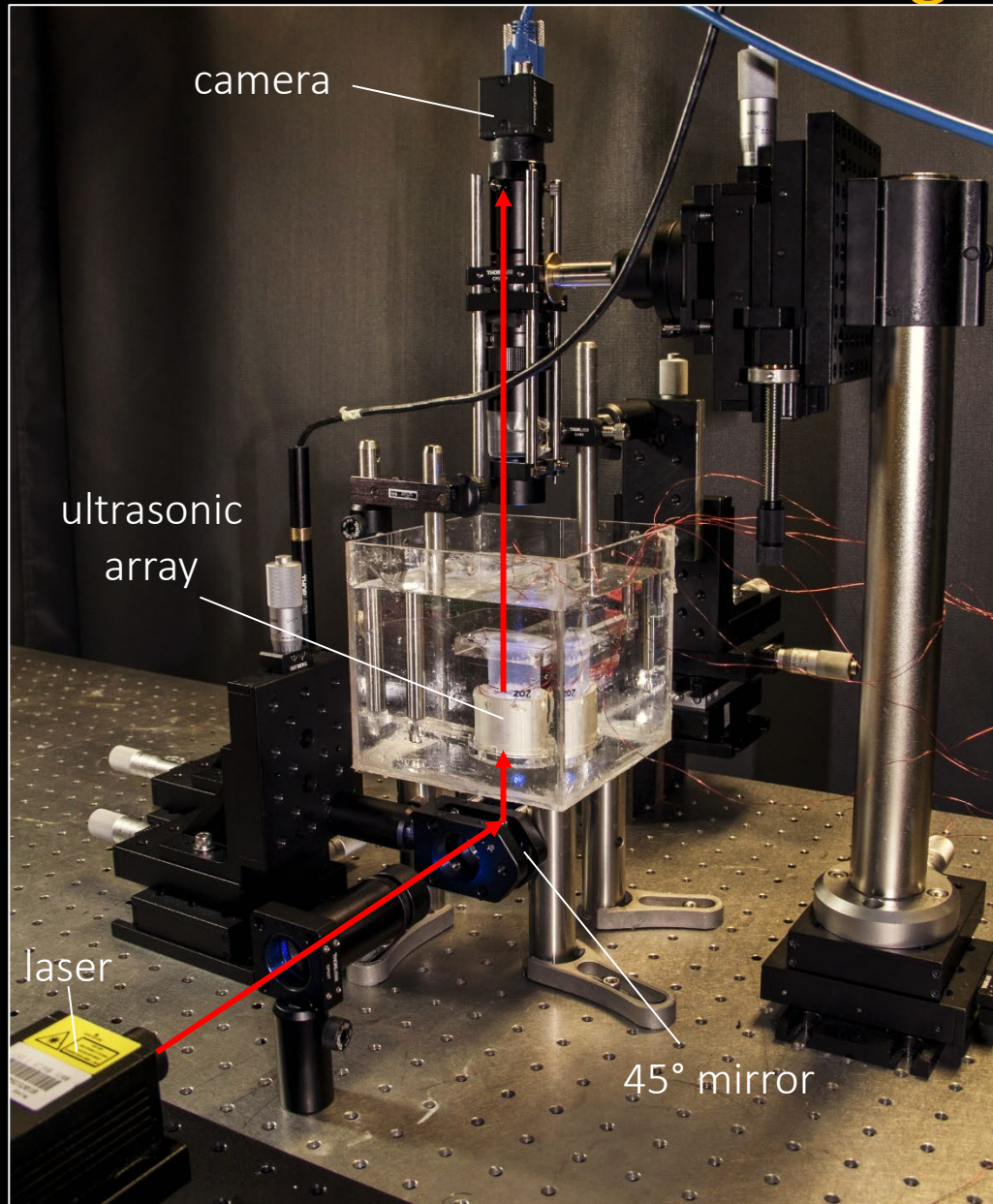
no waveguide



virtual waveguide

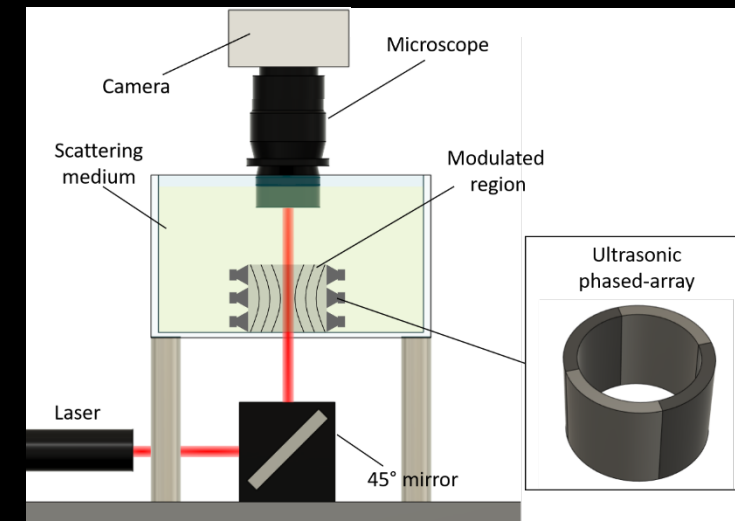


virtual ultrasonic waveguides



Rendering acousto-optics

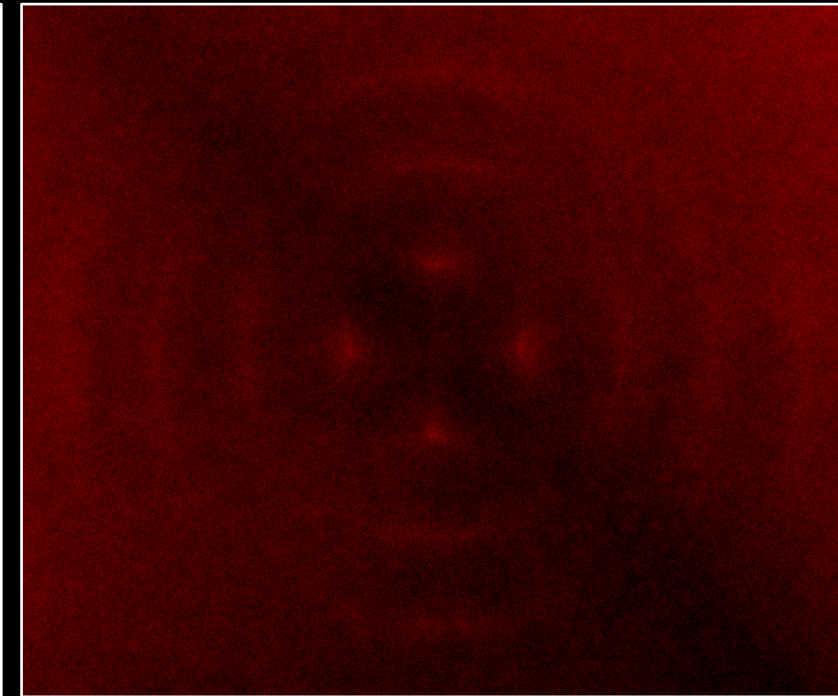
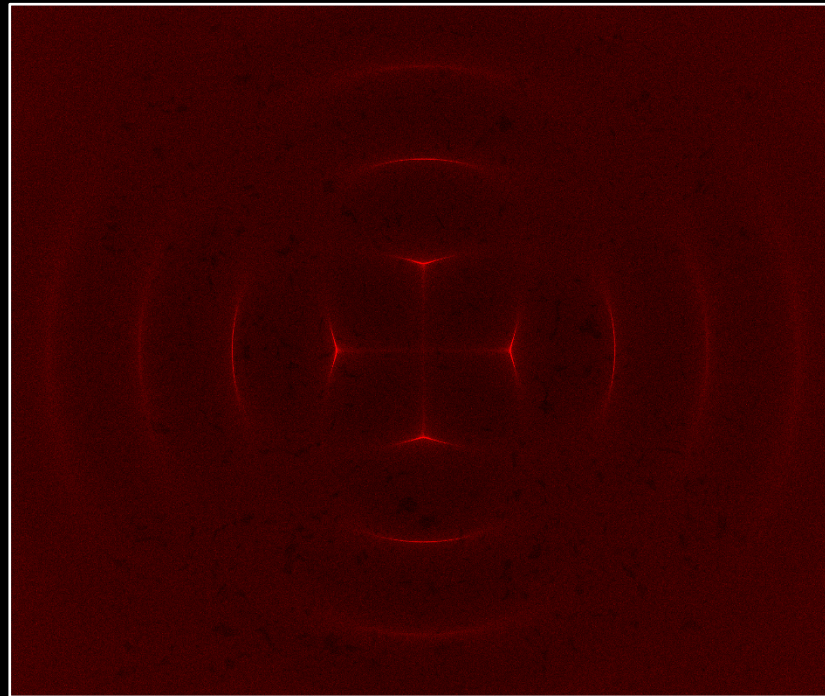
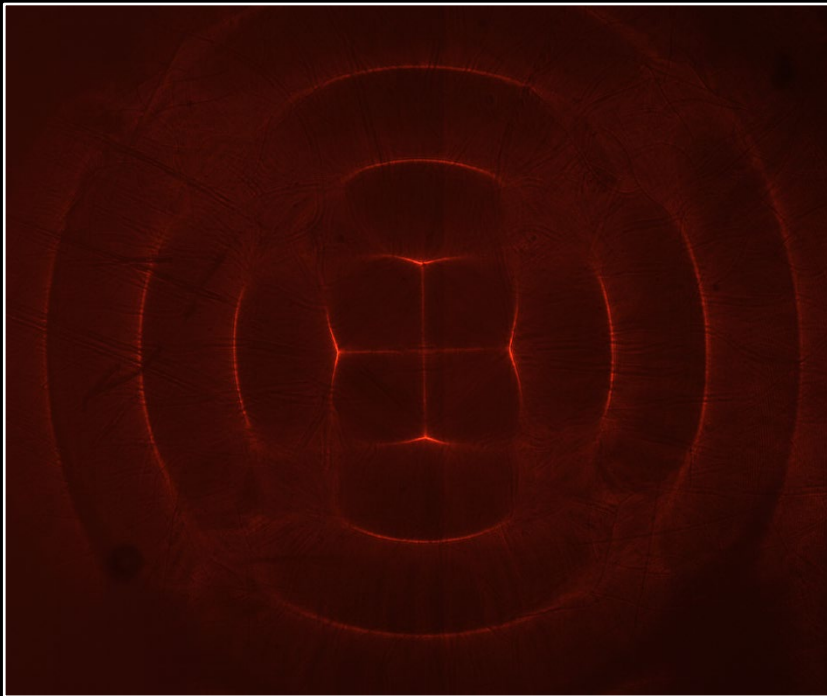
setup for ultrasonic
lensing in scattering



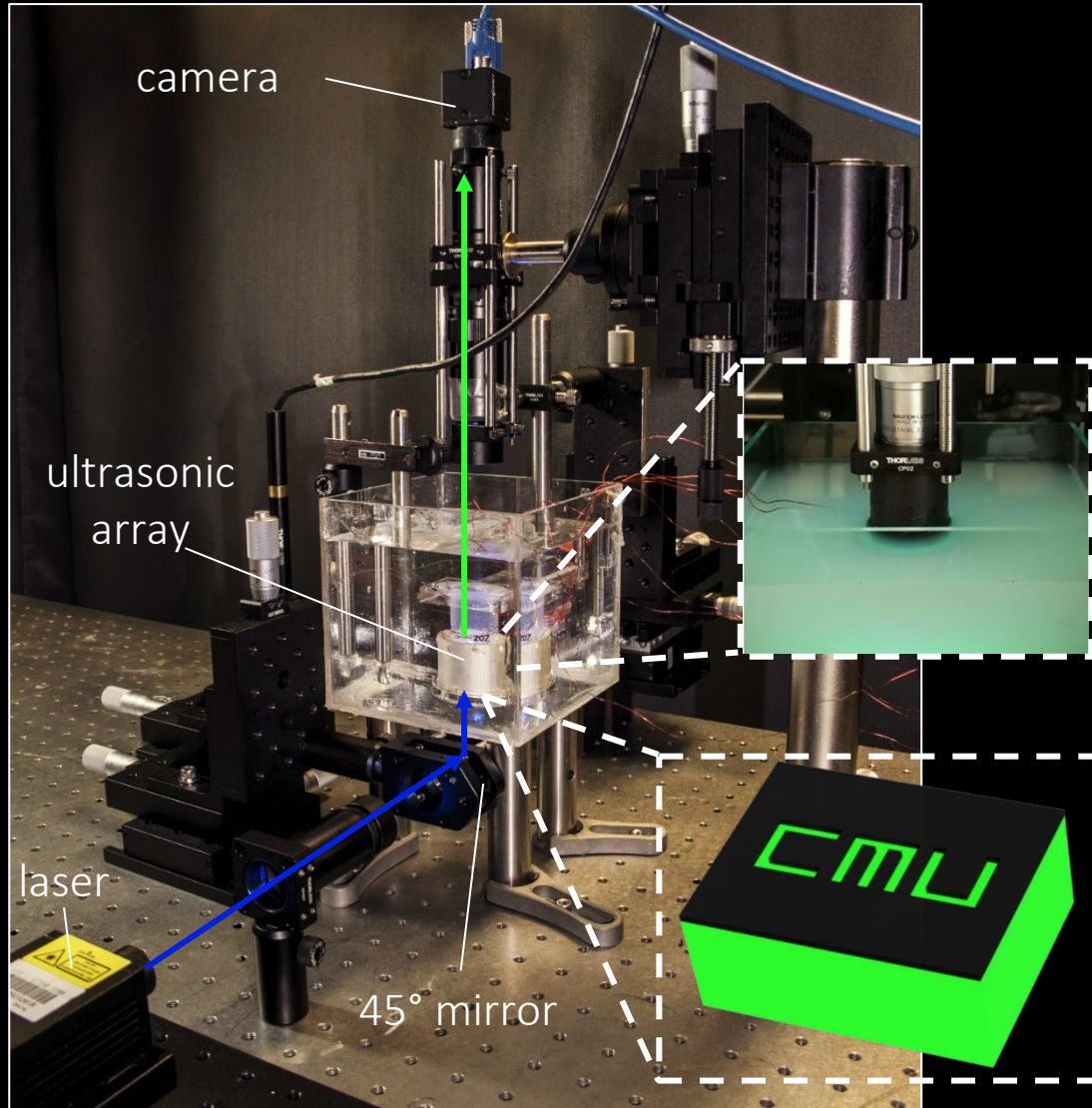
real capture

our algorithm

previous algorithm



Ultrasonic light guiding inside tissue



High-dimensional, highly-non-linear design problem:

- ultrasound frequency
- ultrasound voltage
- shape of waveguides
- placement of transducers
- sensor size
- and more...

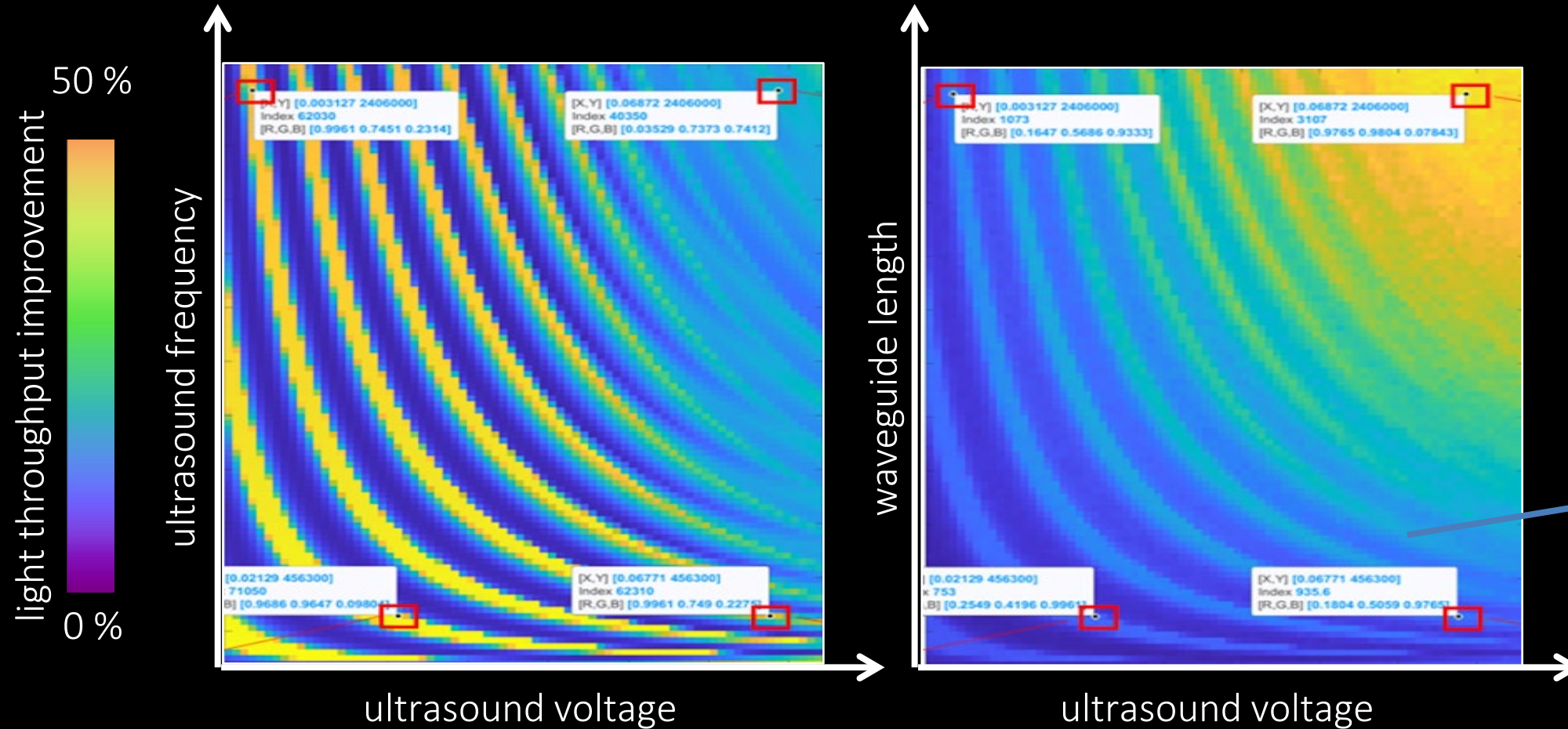
Guiding performance strongly affected by different parameter values

Painstaking experiments:

- several hours of work to test one set of parameter values

Optimizing ultrasonic GRIN waveguides

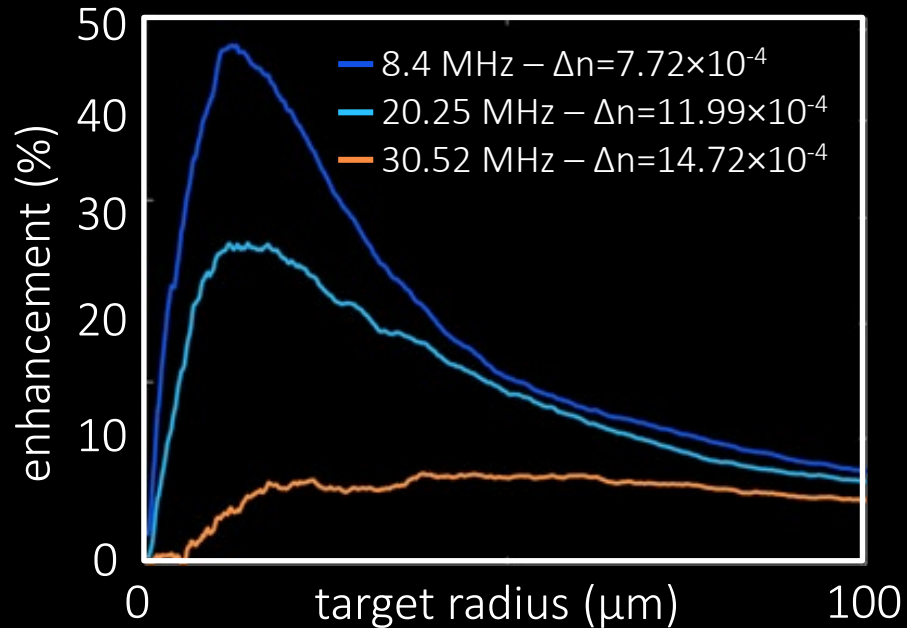
- Hundreds of thousands of virtual experiments.



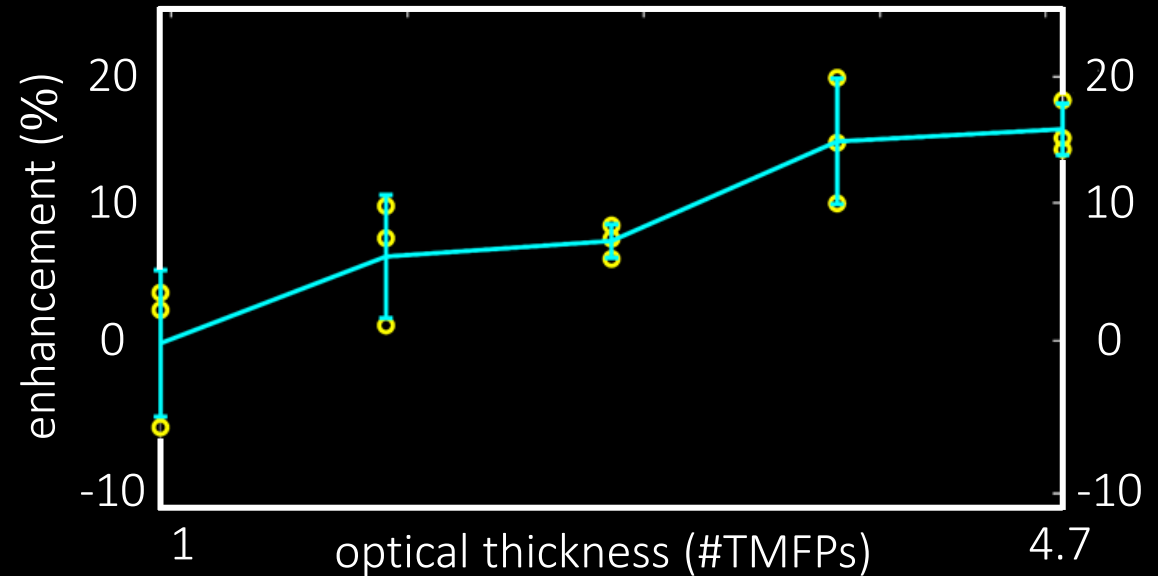
Each dot on these graphs would have been a real experiment taking a PhD student a full day's work

Improved light guiding in human bladder

simulations

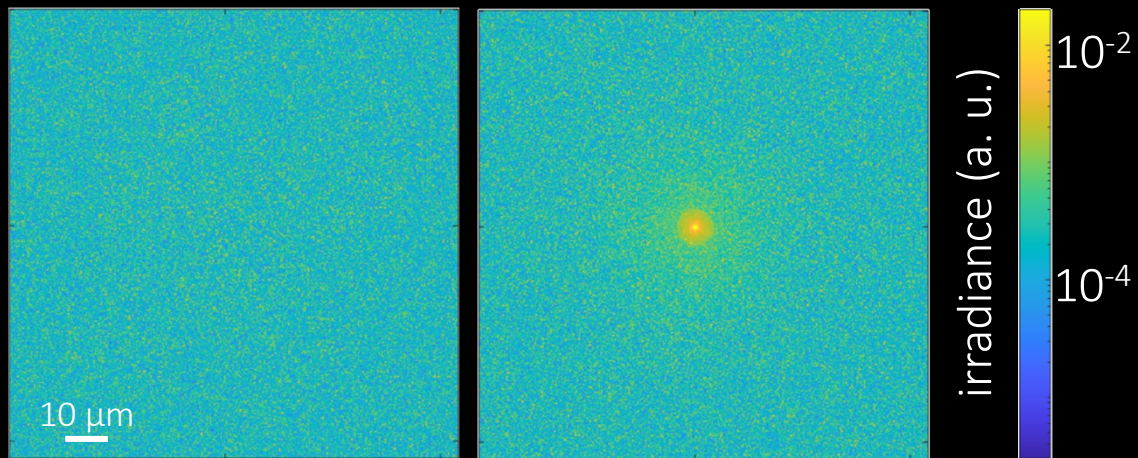


real data



ideal lens

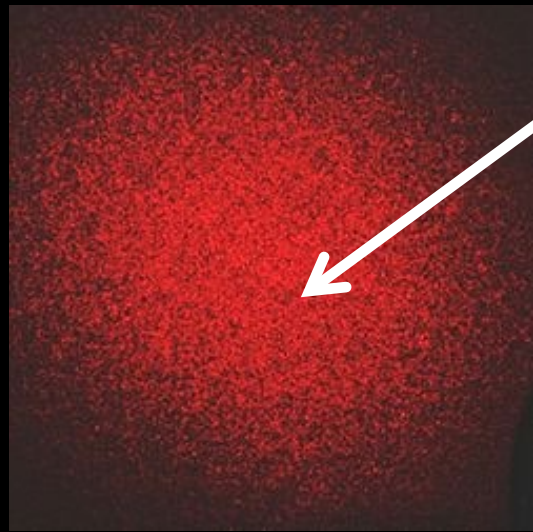
GRIN waveguide



- Improved light guiding performance by
- 200% compared to unoptimized waveguides
 - 50% compared to external optics
- Simulation predictions verified experimentally

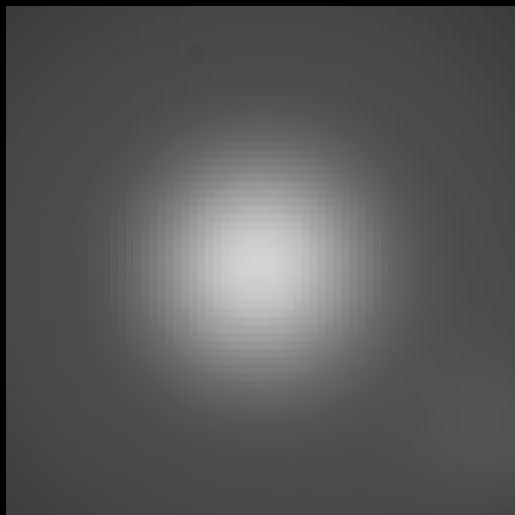
[Pediredla et al., submitted to Nature Communications 2021]

Speckle and memory effect

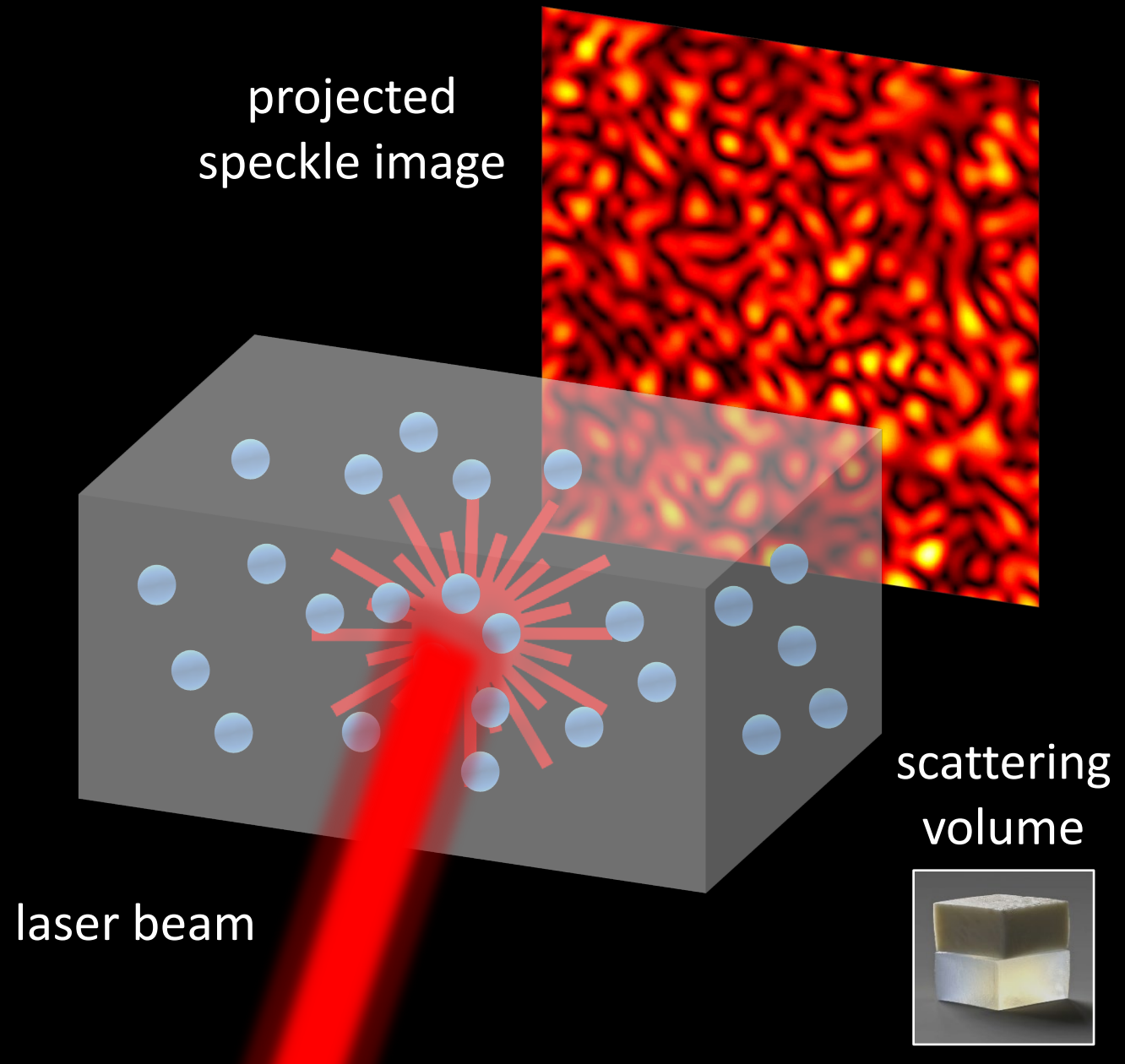


speckle: noise-
like pattern

what real laser
images look like



what standard
rendered images
look like



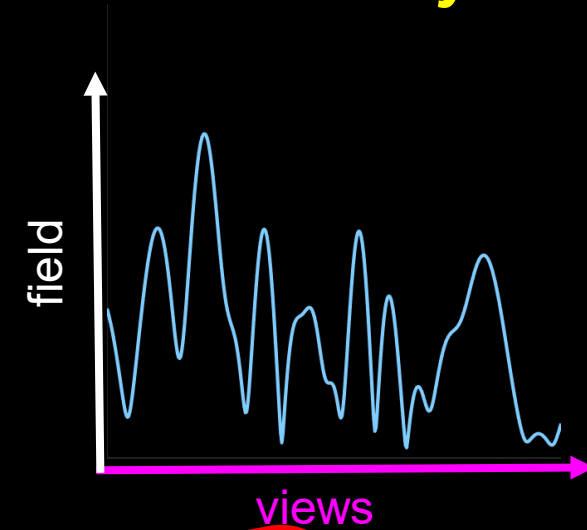
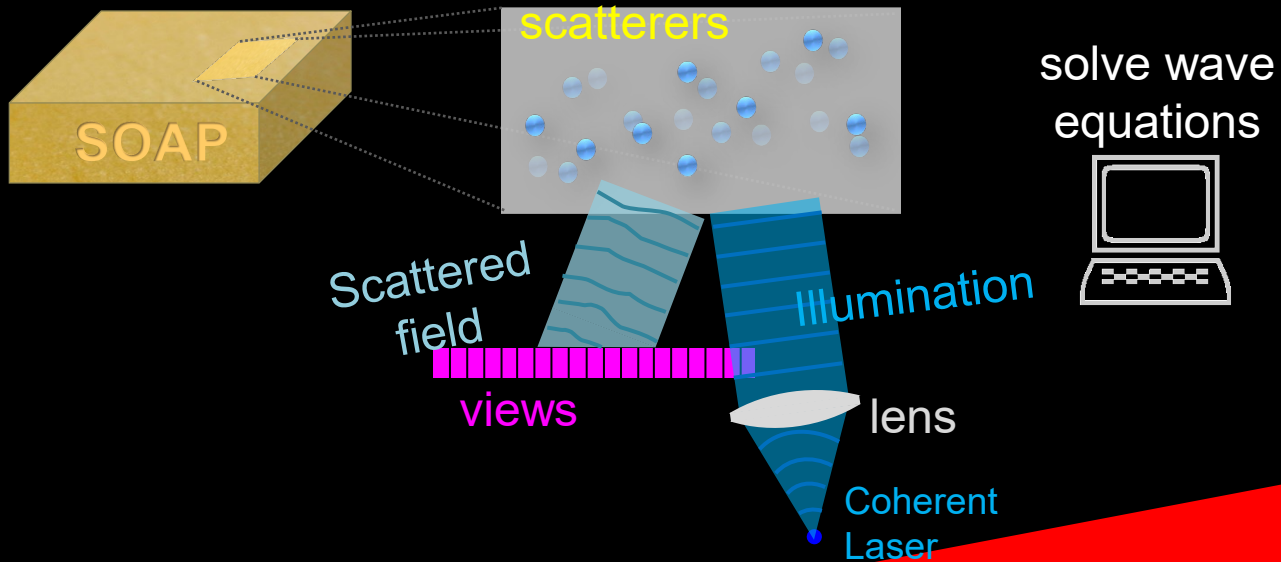
Simulating speckles

inefficient

Specify exact (sub-wavelength) position of scatterers



In graphics we describe materials by **statistical** bulk parameters, as the **density** of scatterers



Wave equation solvers

- Differential equation F
- Integral equation (e.g.,

Slow
Practical only for tiny
or optically thin media

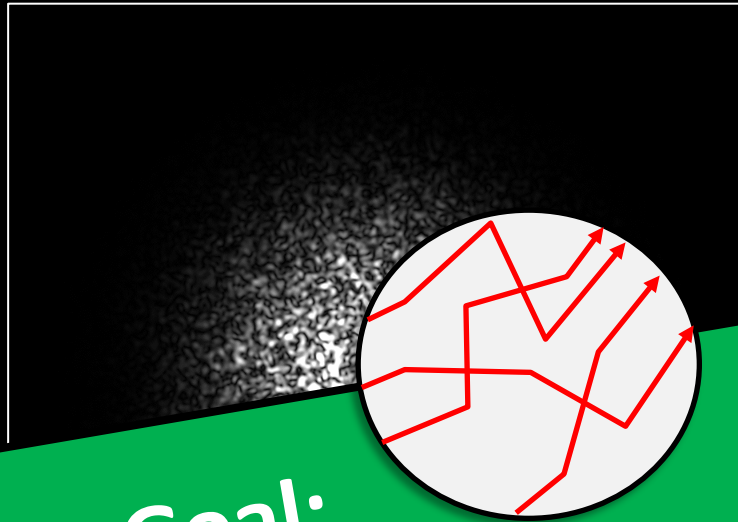


For simplicity: **Flatland**
Scattering medium is 2D
Sensor is 1D
Speckle pattern is 1D

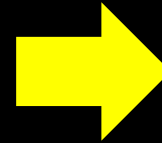
Monte Carlo (MC) Simulation of Speckles

MC Advantage:

1. **Fast**
2. input is scatterer **density** rather than exact scatterer locations

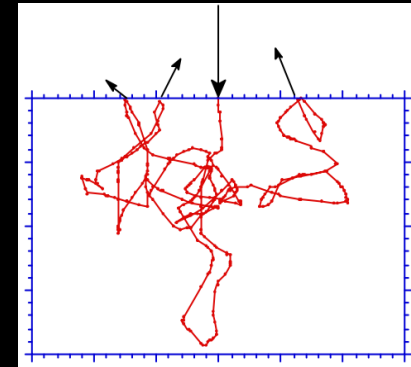
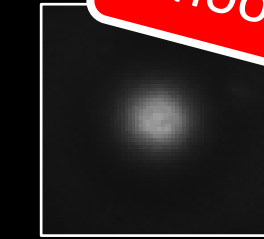


Our Goal:
Extend efficient MC tools
to evaluate speckles and
their coherent statistics



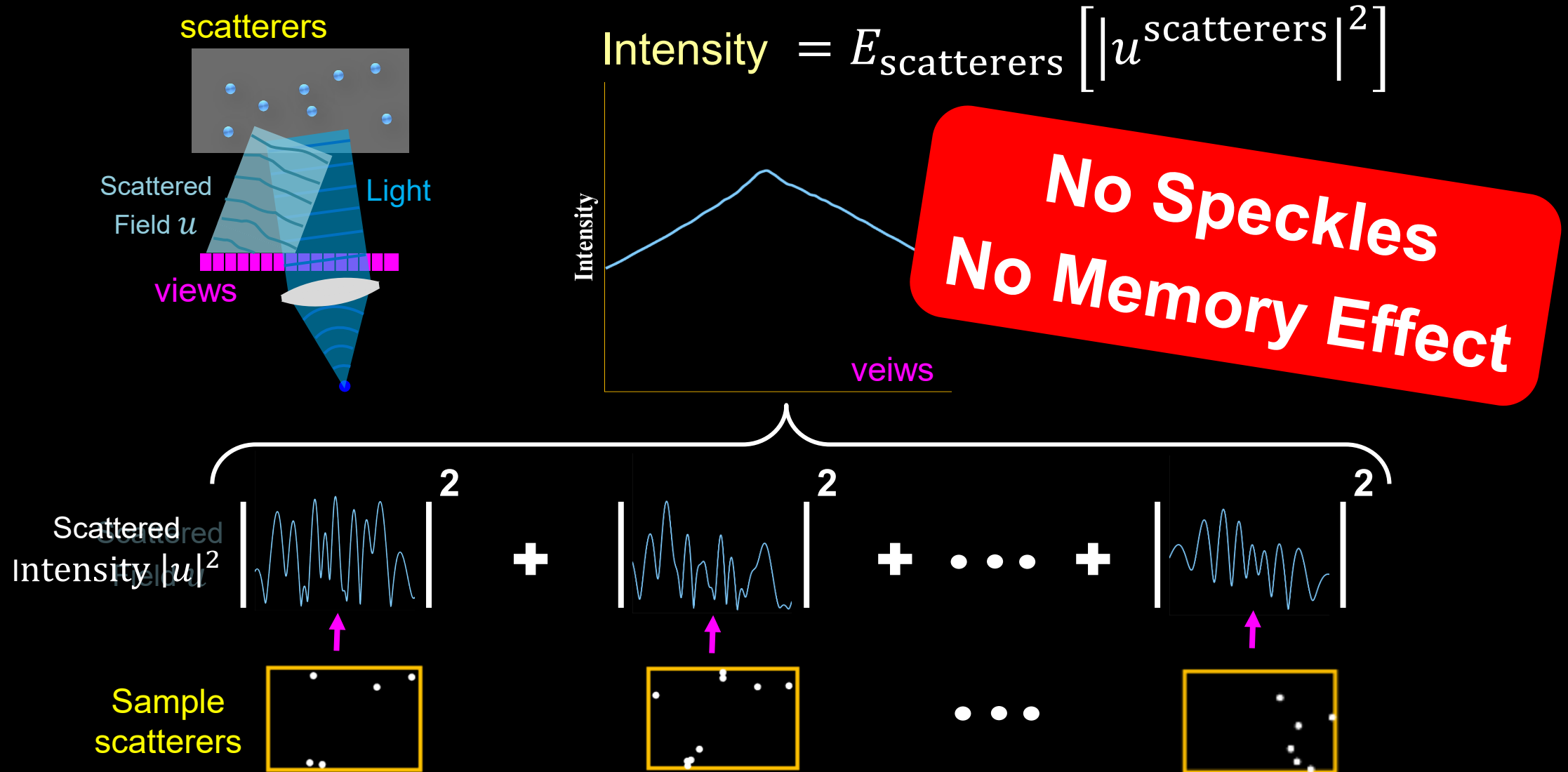
Standard intensity MC

Smooth, no speckles



Monte Carlo Modeling of Light
Transport in Multi-layered Tissues,
Wang & Jacques, 1992

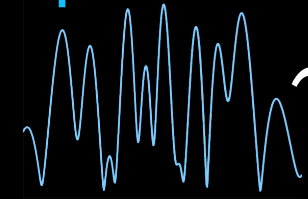
Wave Solution v.s. Monte Carlo



MC requires the scatterers density – no need for exact positions

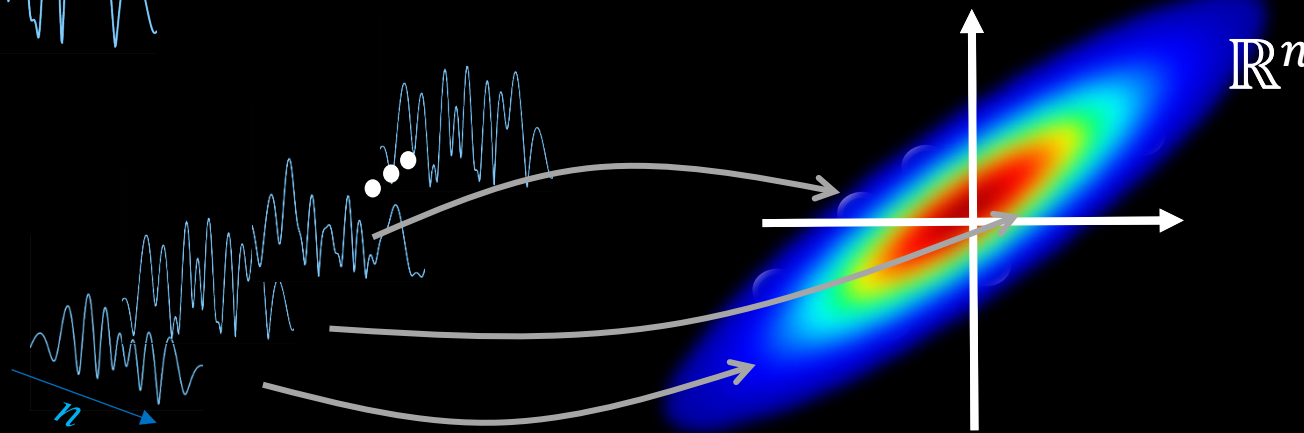
Speckle Statistics

Speckles



$\sim \mathcal{N}(\text{Mean, Covariance})$

Sufficient Statistics

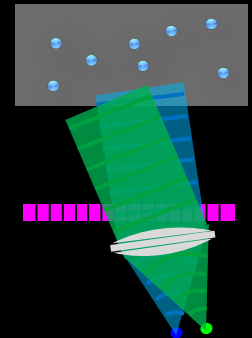
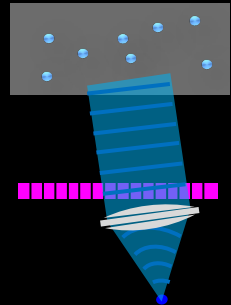


1st moment

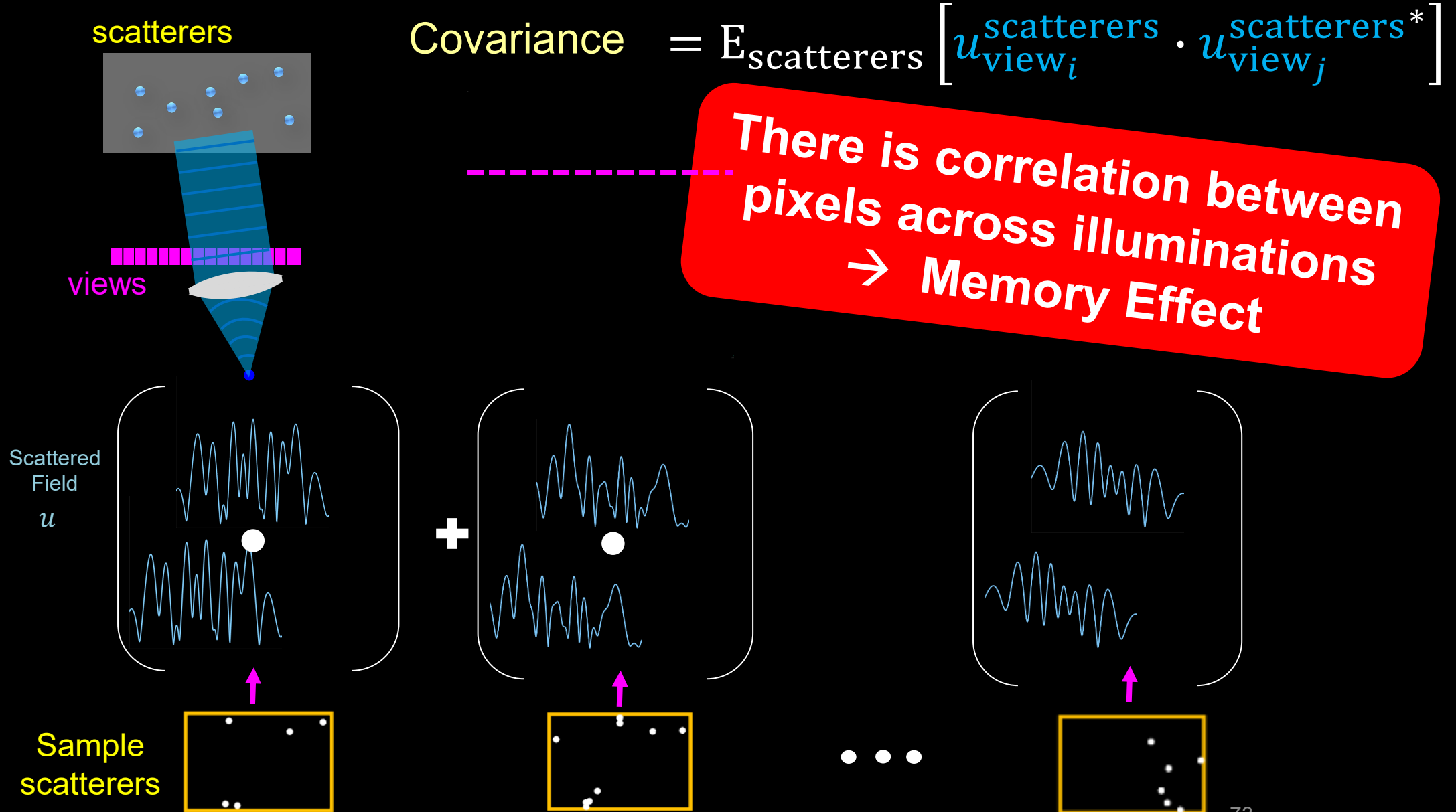
~~$$\text{Intensity Mean} = E_{\text{scatterers}} [|u_{\text{scatterers}}|^2] \Rightarrow \text{Incoherent Summation}$$~~

Cross-Illumination

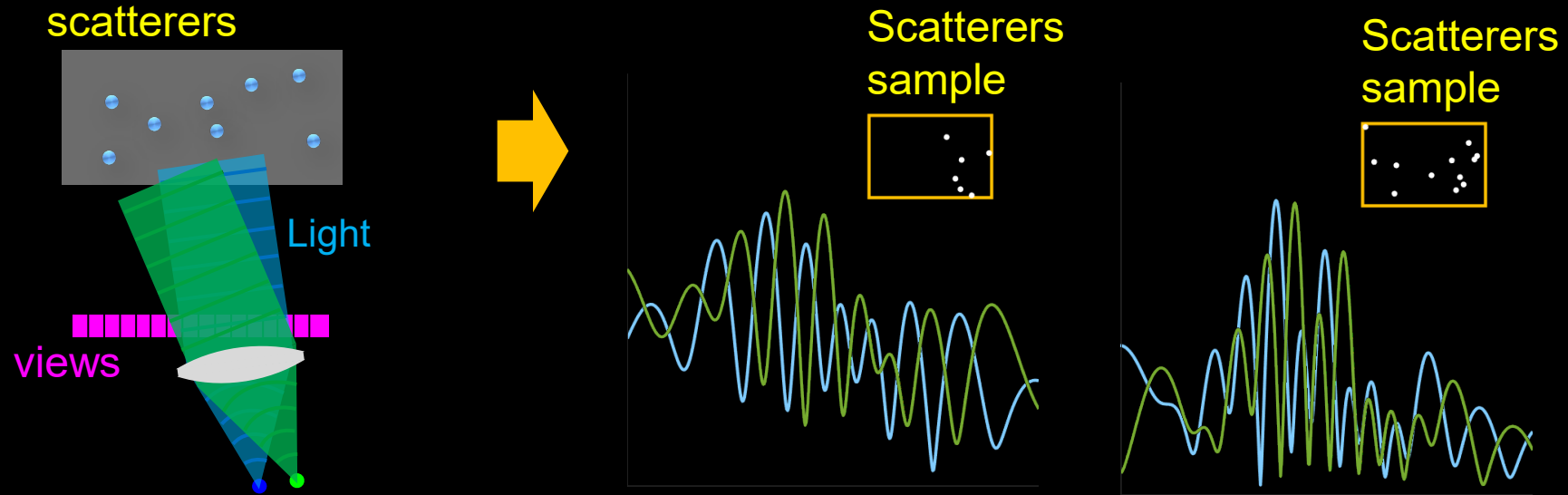
Field Covariance = $E_{\text{scatterers}} \left[u_{\text{view}_i}^{\text{light}_1, \text{scatterers}} \cdot u_{\text{view}_j}^{\text{light}_2, \text{scatterers}*} \right]$



2nd Moment - Covariance



Cross –illumination statistics



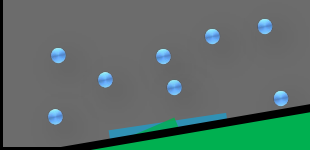
Memory Effect:

tilting illumination results in highly correlated shifted speckles

Next: Cross Illumination Covariance

Cross-illumination statistics

scatterers



Covariance = $E_{\text{scatterers}}$



al shift =
ift

We developed:
Efficient MC
for Speckle Covariance

$$\left[u_{\text{view}_i}^{\text{light}_1, \text{scatterers}} \cdot u_{\text{view}_j}^{\text{light}_2, \text{scatterers}*} \right]$$

Speckles are Gaussian:
Mean + Covariance
are sufficient statistics

This is all we need to describe speckles

Scattered
Field
 u^{light_2}

Sample
scatterers



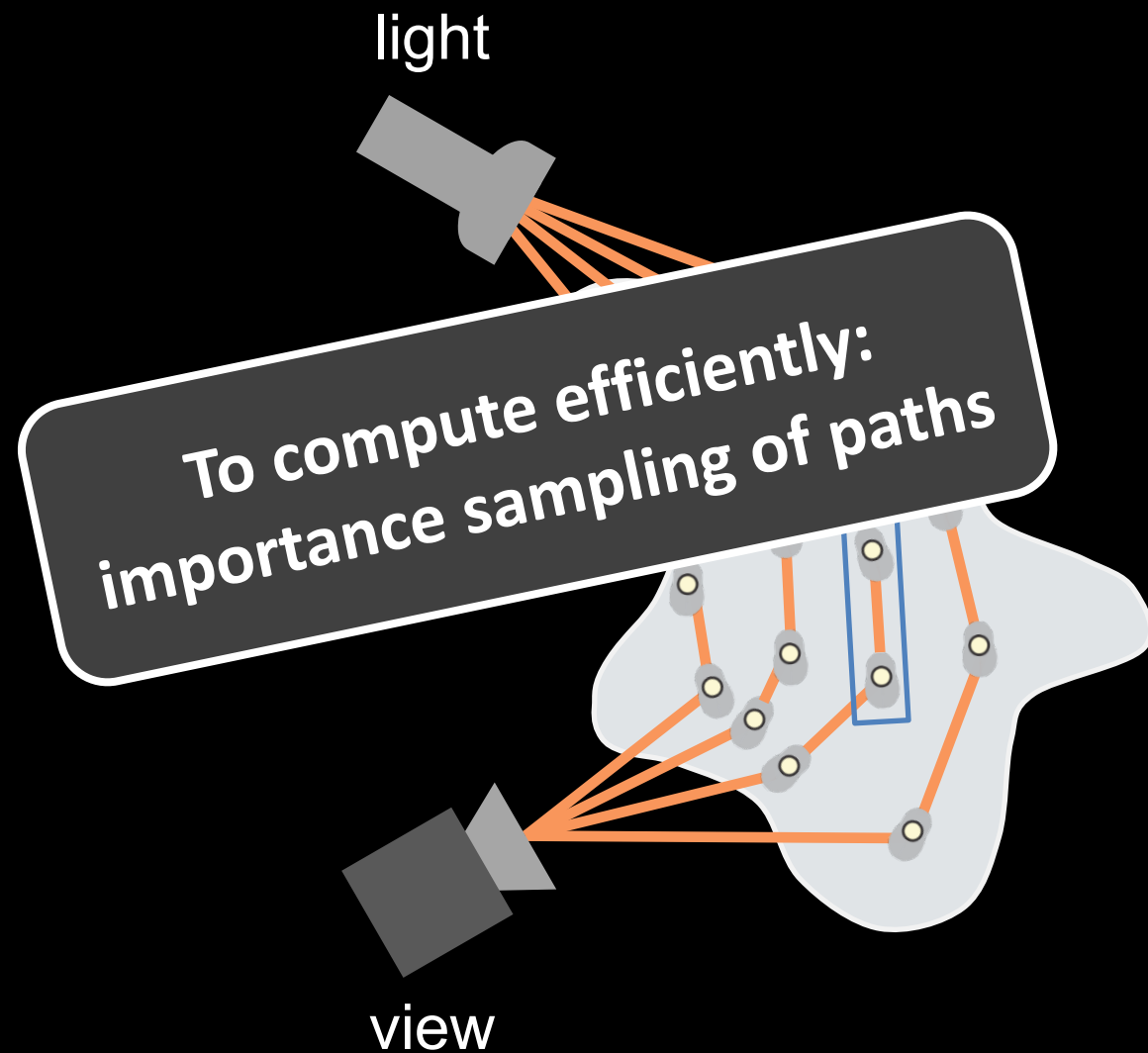
+



...

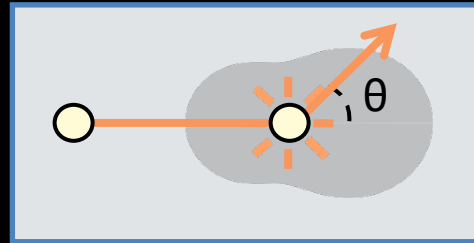


Monte Carlo Rendering 101



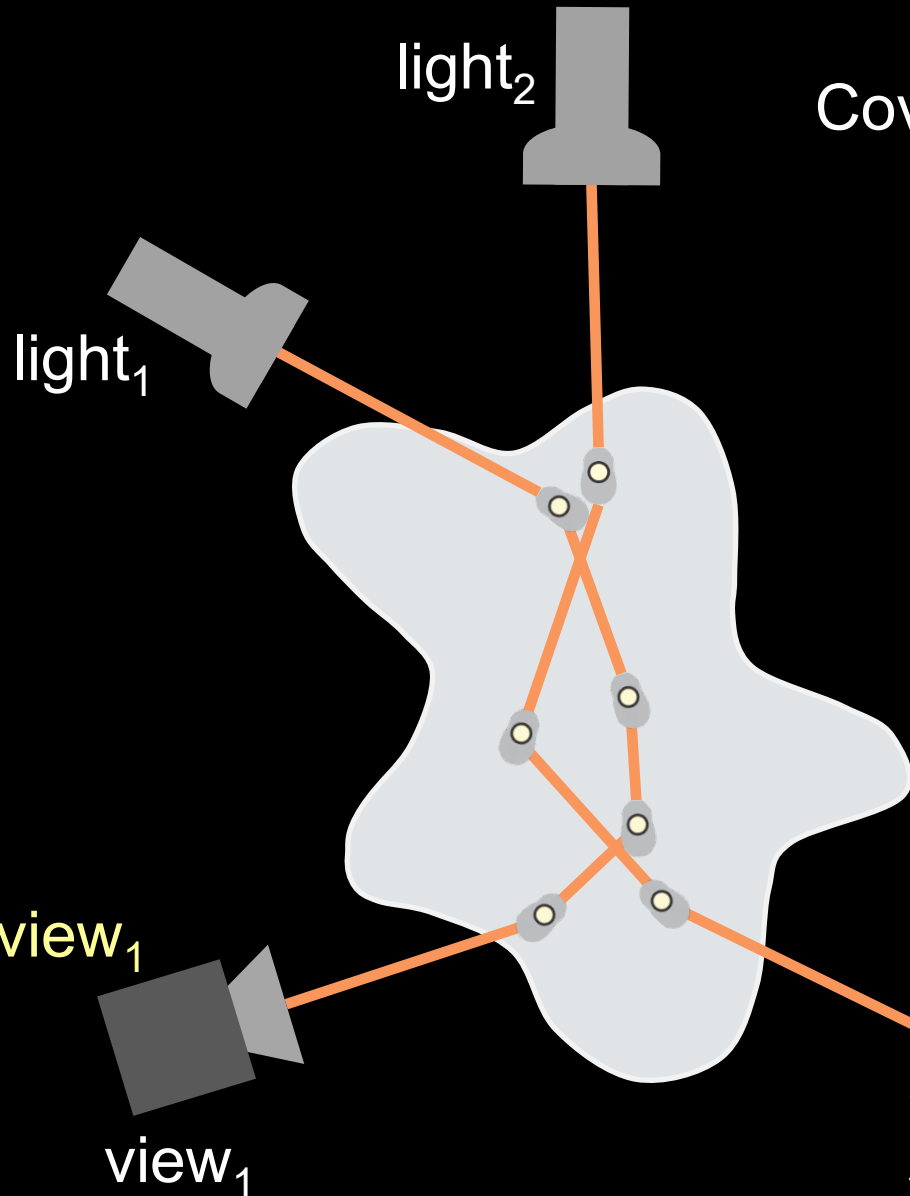
$$\text{Image} = \int_{\text{paths}} f(\text{path})$$

Throughput that acts on each path, depends on the scattering material



volumetric density
(extinction coefficient) σ
scattering albedo a
phase function p_{θ}

Covariance Rendering



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

$$u = |u| e^{i \cdot \text{phase}}$$

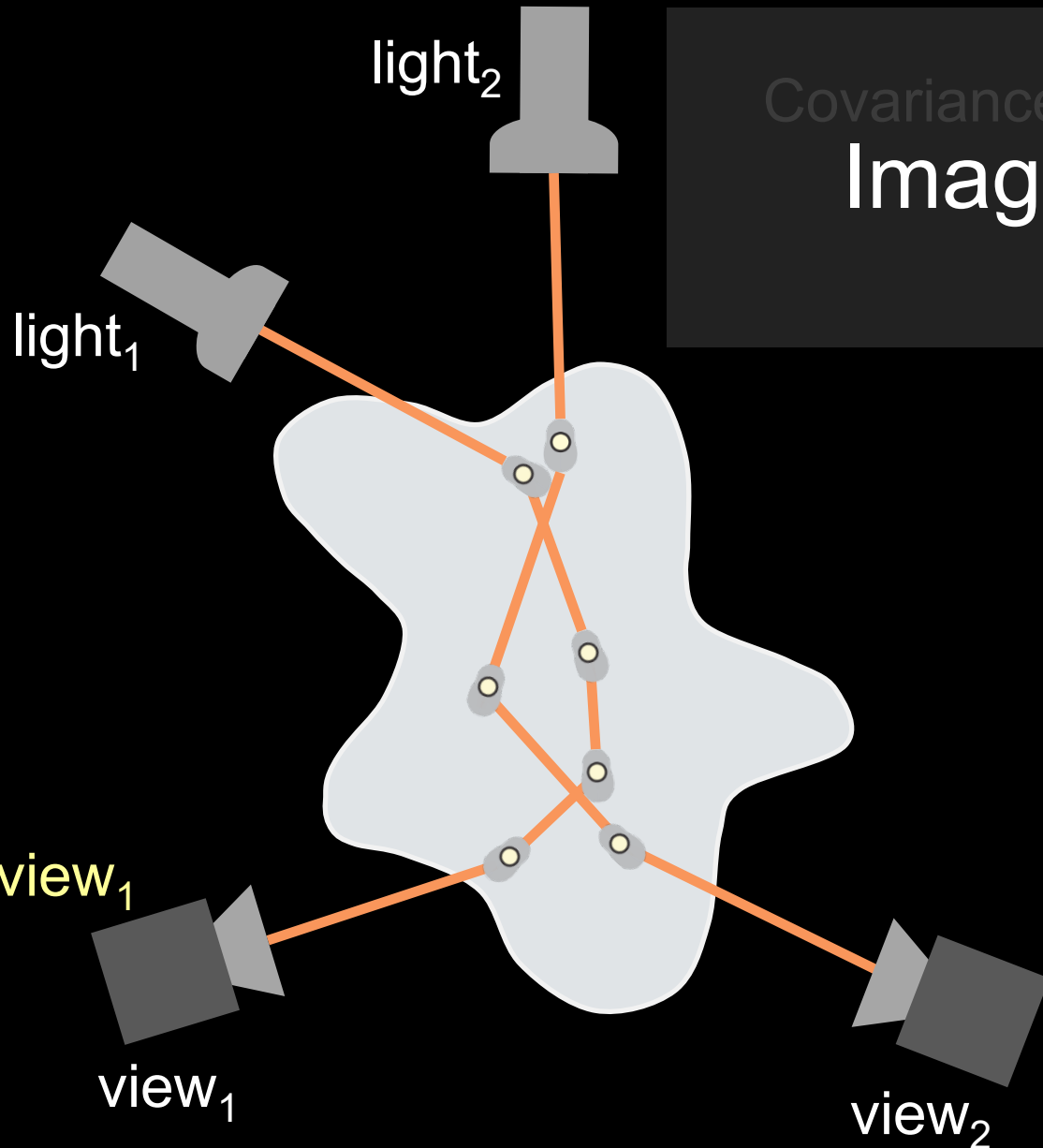
Need to consider products of **pairs** of paths

Each path contributes a complex number u

$\text{phase} \propto \text{Length}(\text{path})$

$\Delta \text{phase} \propto \text{Length}(\text{path}_1) - \text{Length}(\text{path}_2)$

Covariance Rendering



$$\text{Image} = \int_{\text{paths}} f(\text{path})$$

Covariance = $\int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$

Real throughout $u = |u| e^{i \cdot \text{phase}}$

$$\text{path}_1 = \text{path}_2$$



Same complex contribution

$$u(\text{path}_1) = u(\text{path}_2)$$



$$\Delta \text{phase} = 0$$



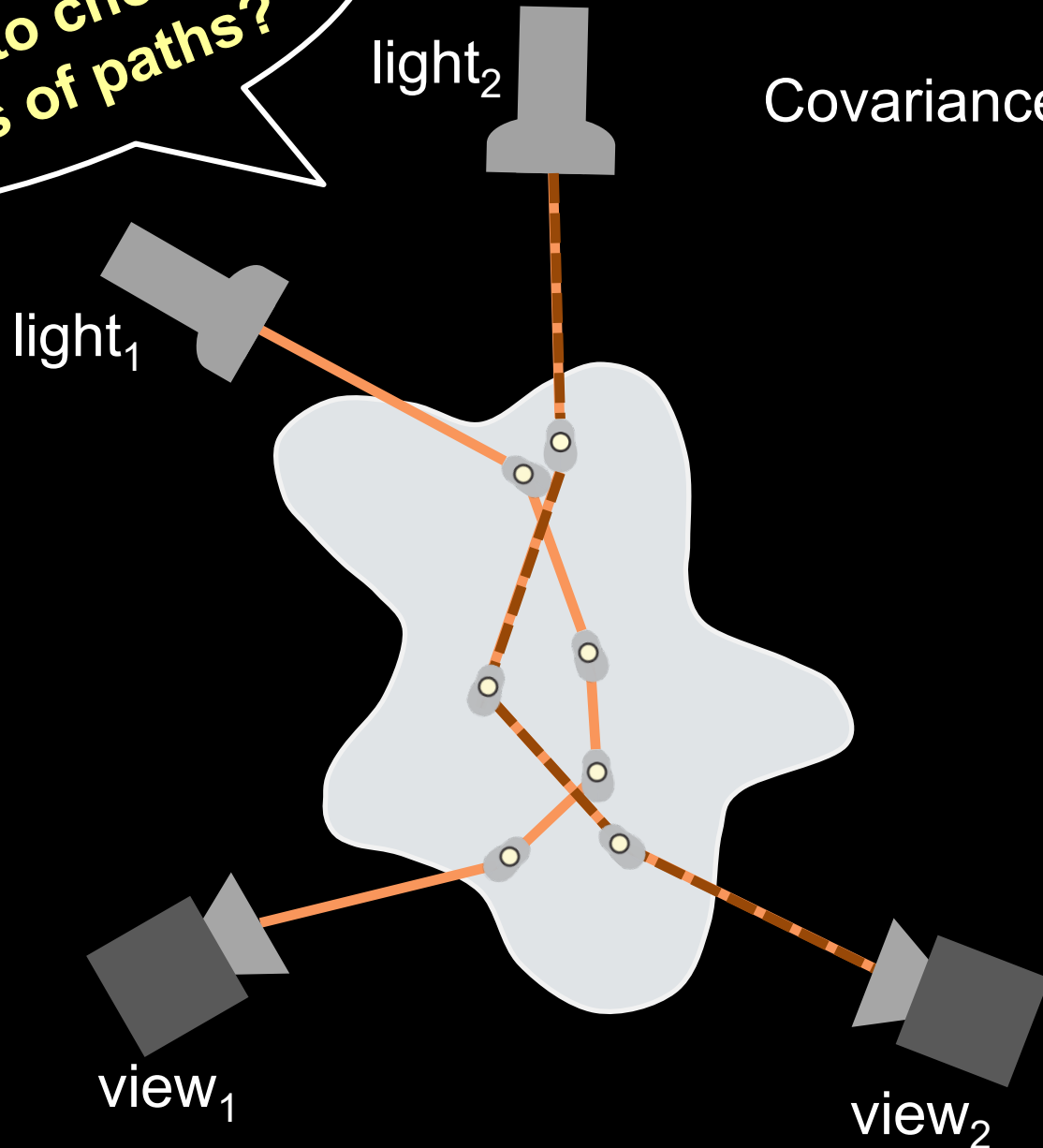
$$\text{path}_2 : u(\text{path}) \cdot u^*(\text{path}) = f(\text{path})$$

$\text{light}_2 \rightarrow \text{view}_2$

Real

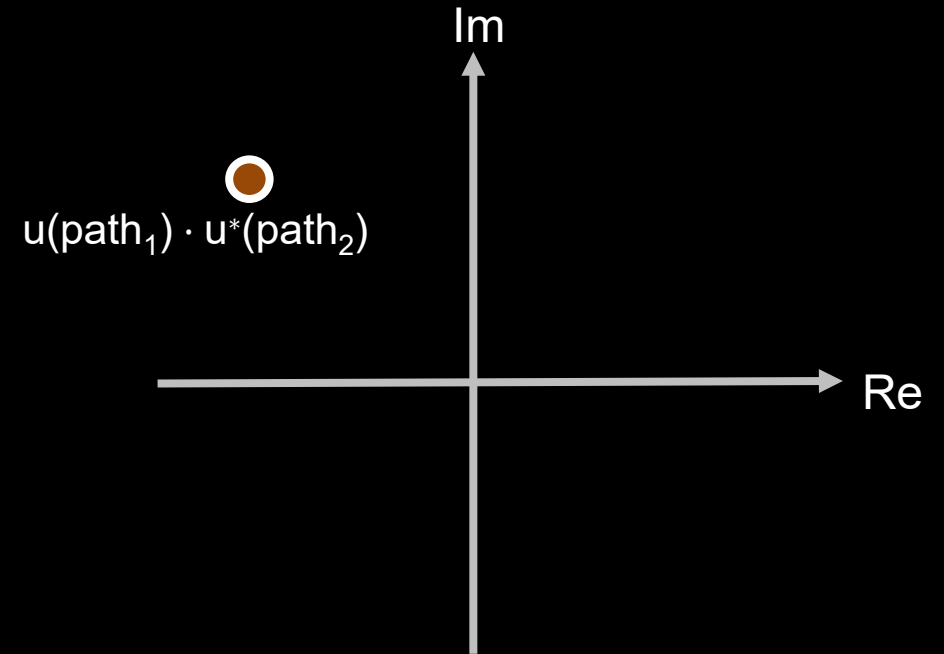
Covariance rendering

How to choose
pairs of paths?

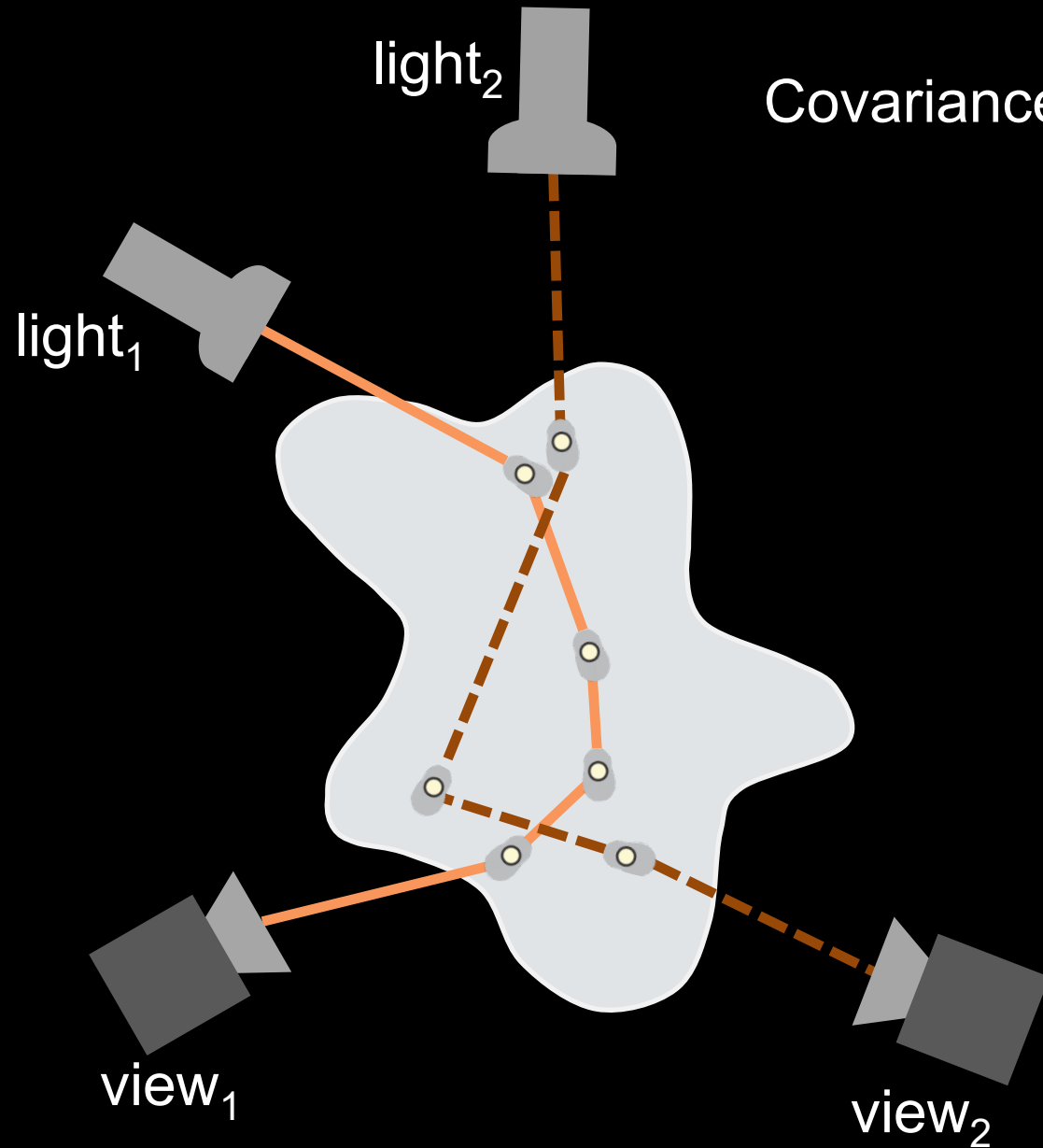


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

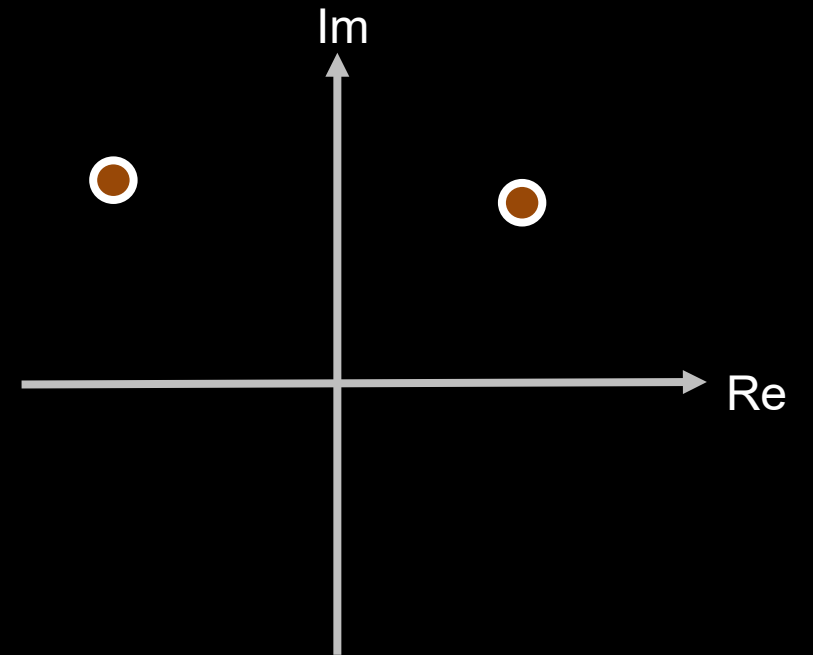
$$u = |u| e^{i \cdot \text{phase}}$$



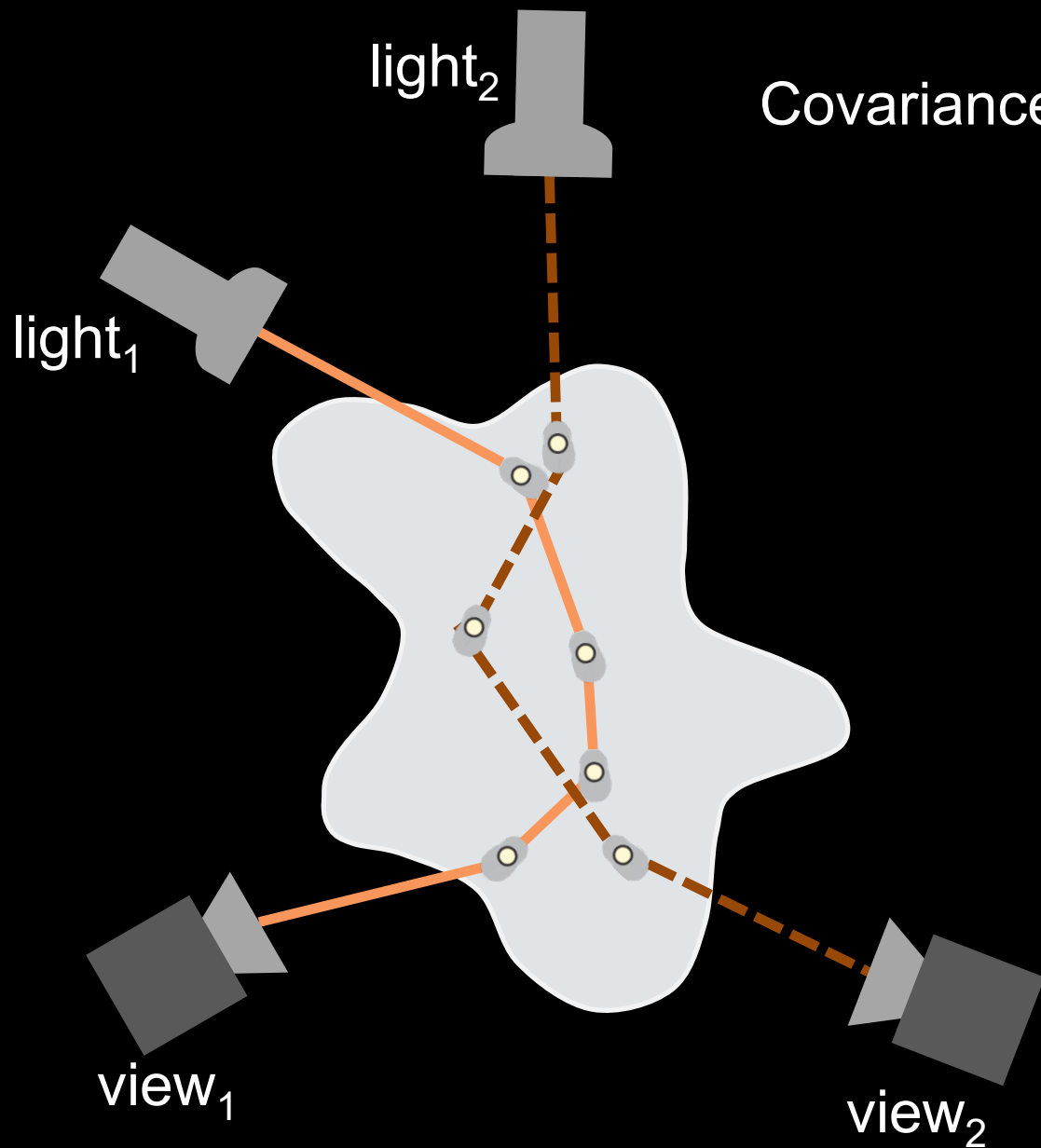
Covariance rendering



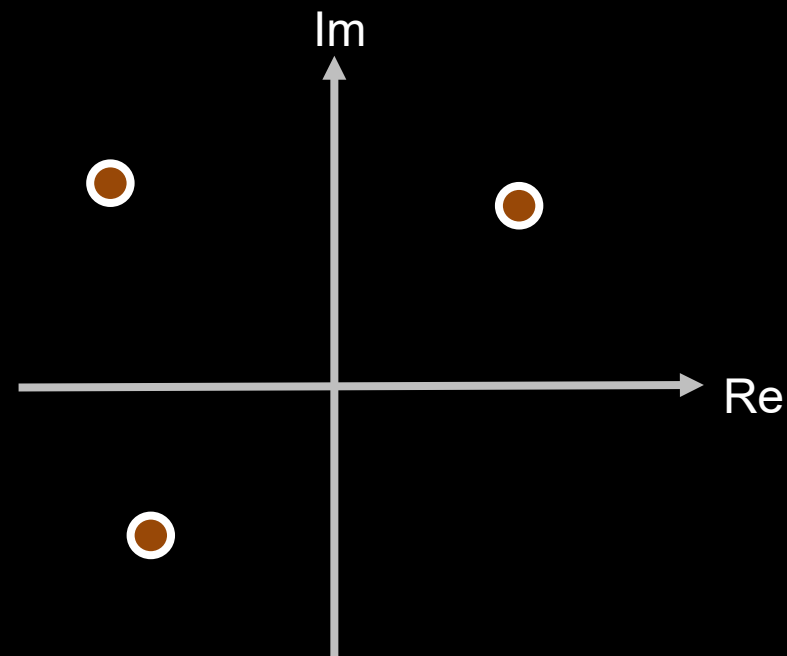
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



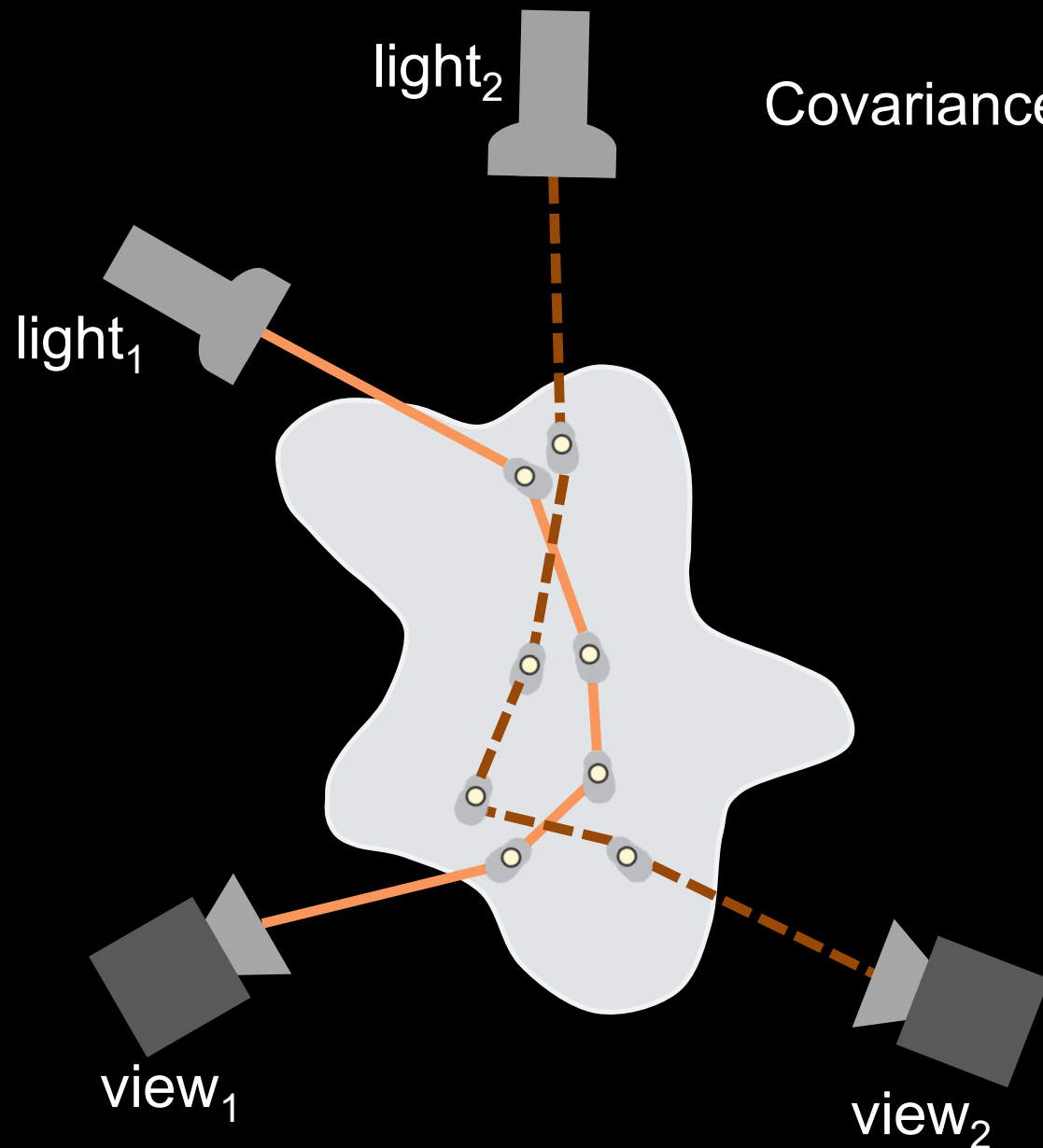
Covariance rendering



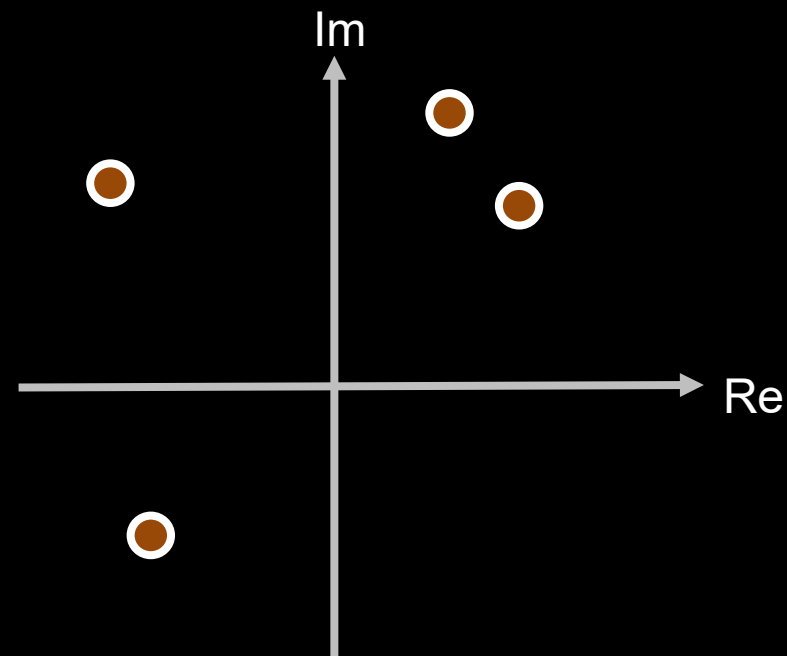
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



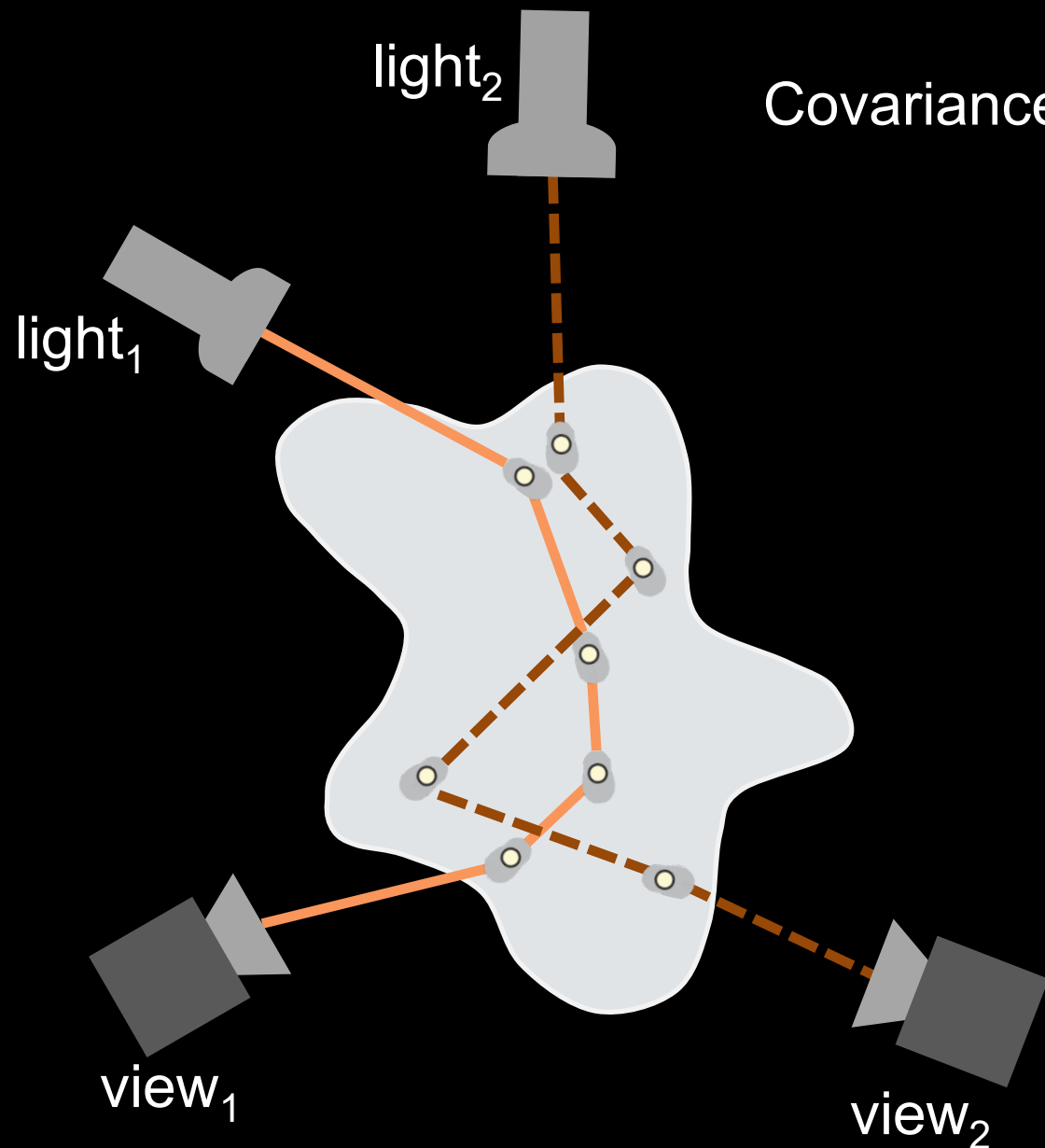
Covariance rendering



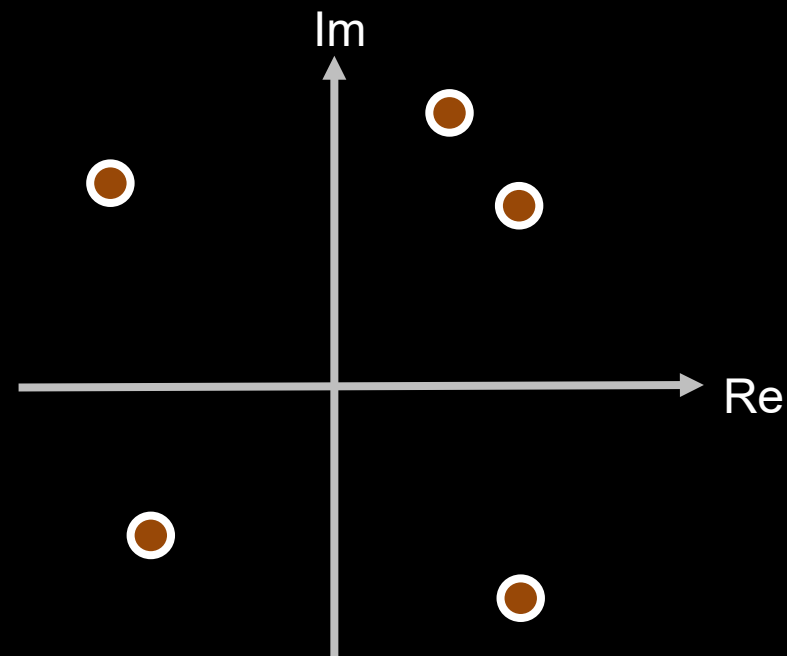
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



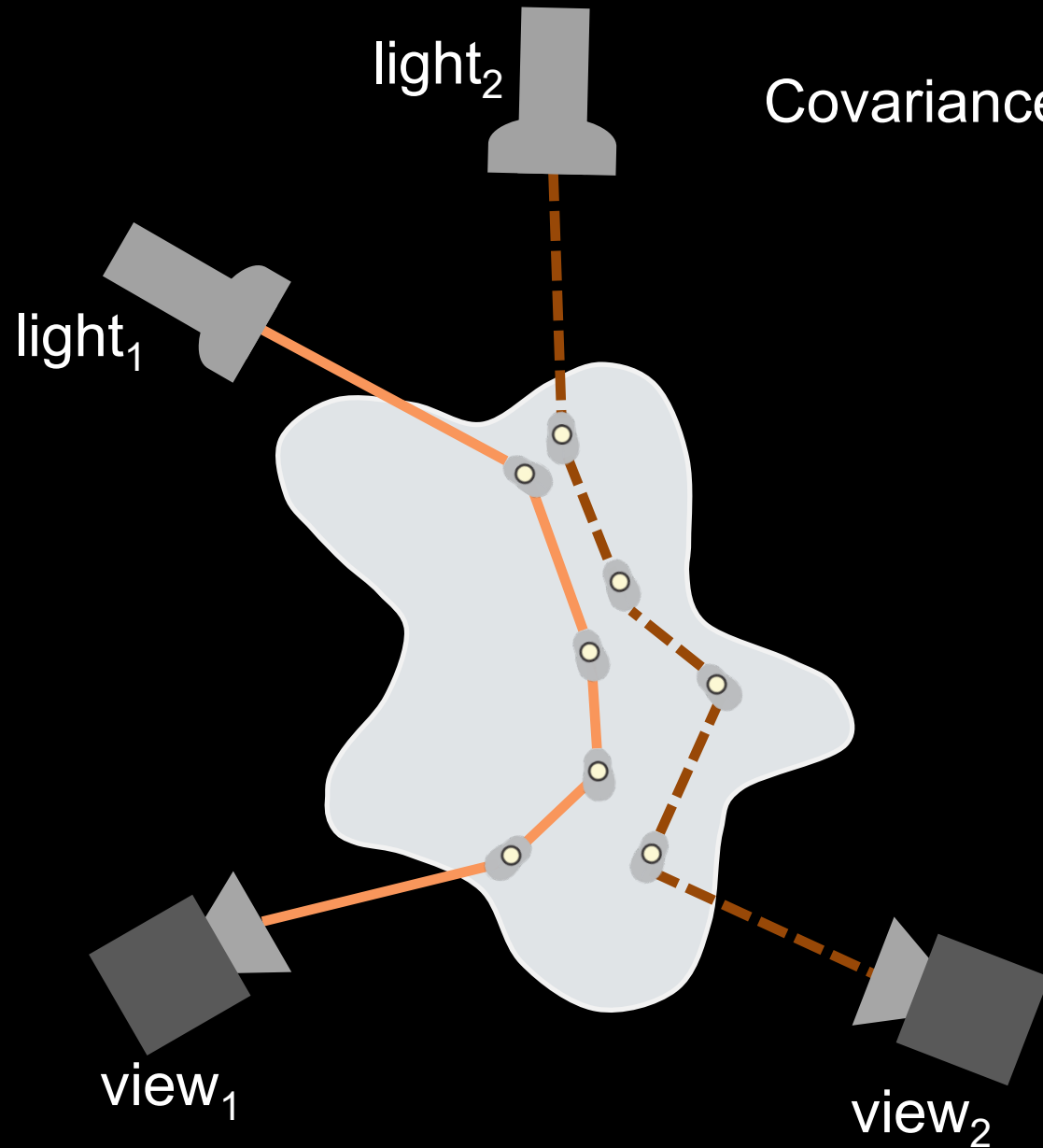
Covariance rendering



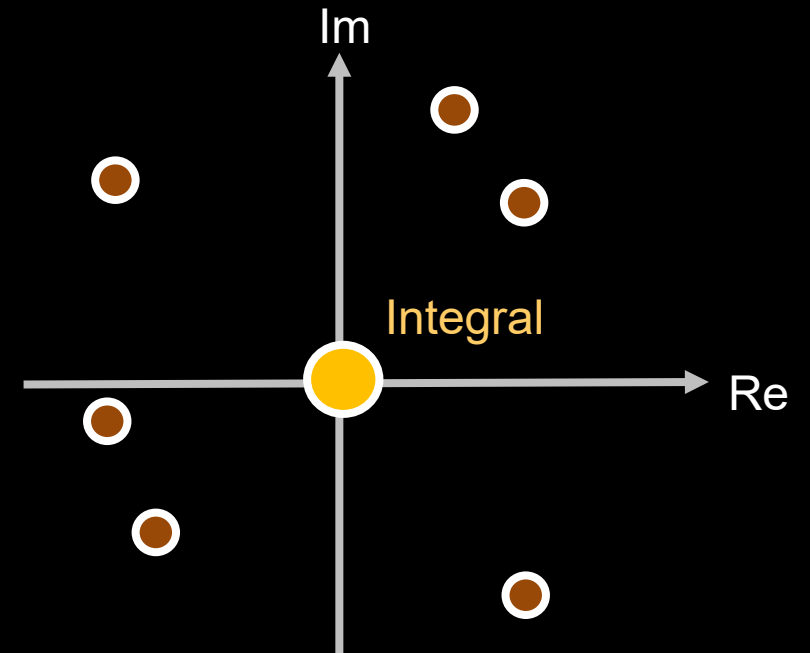
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



Covariance rendering

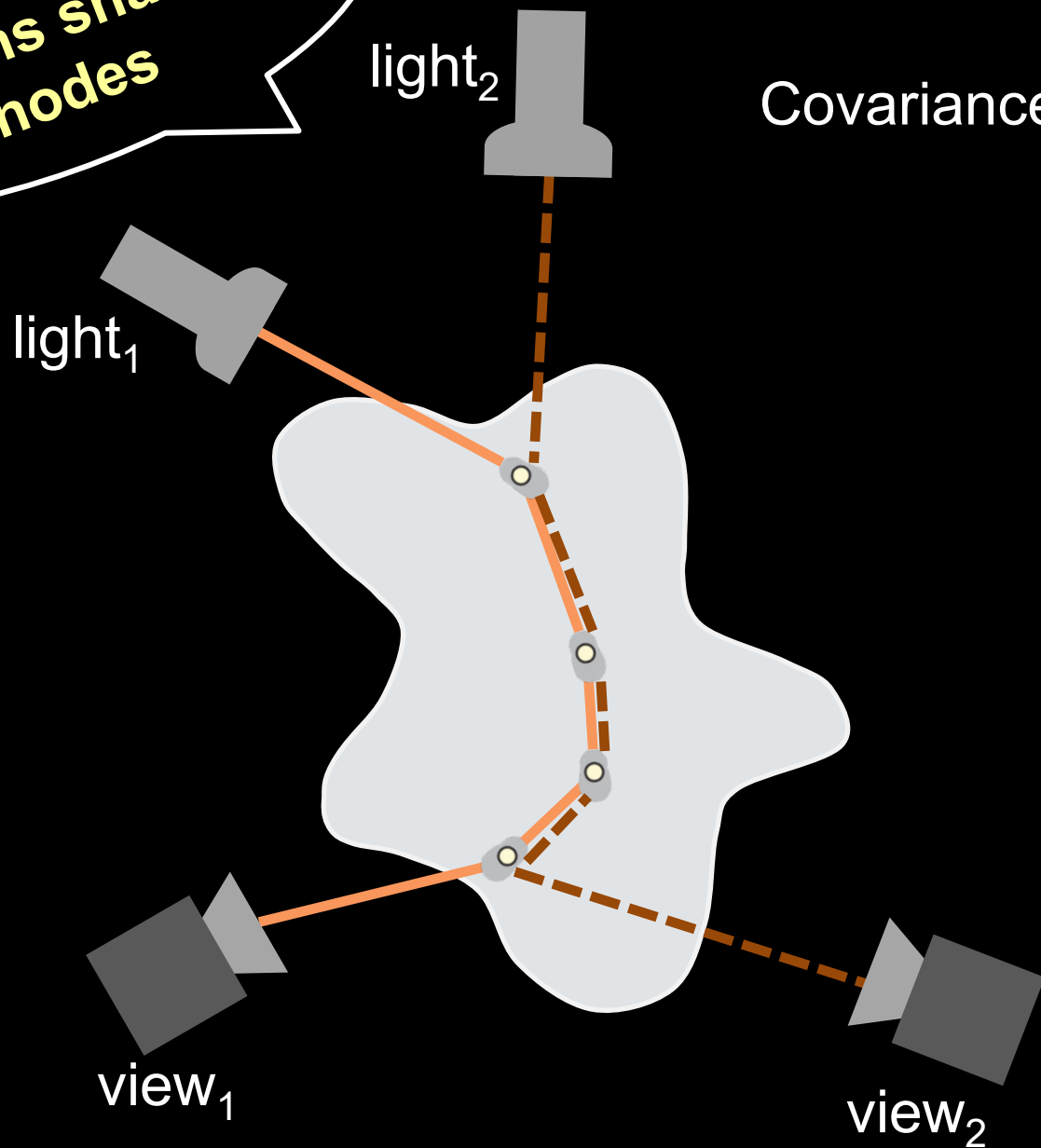


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

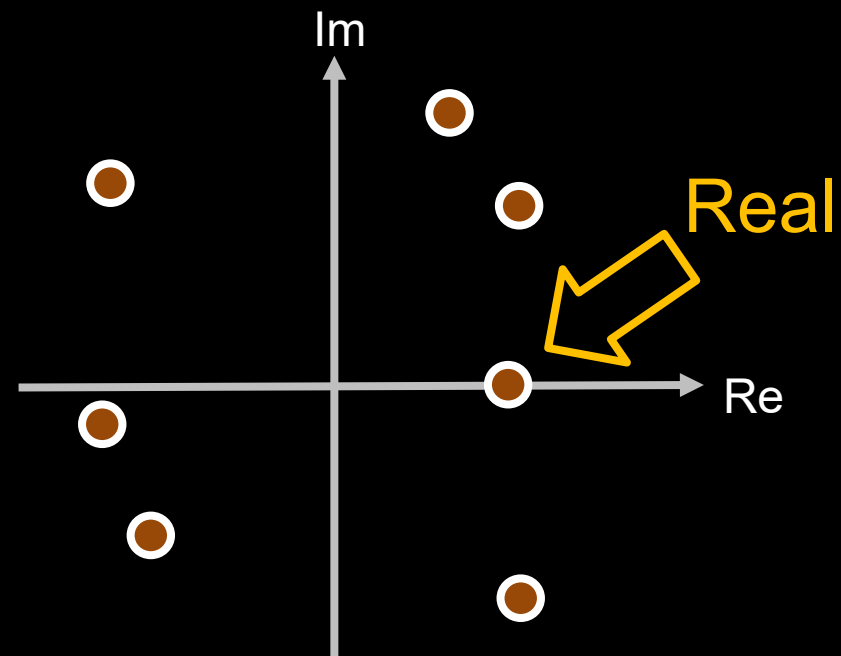


Covariance rendering

Paths share
nodes

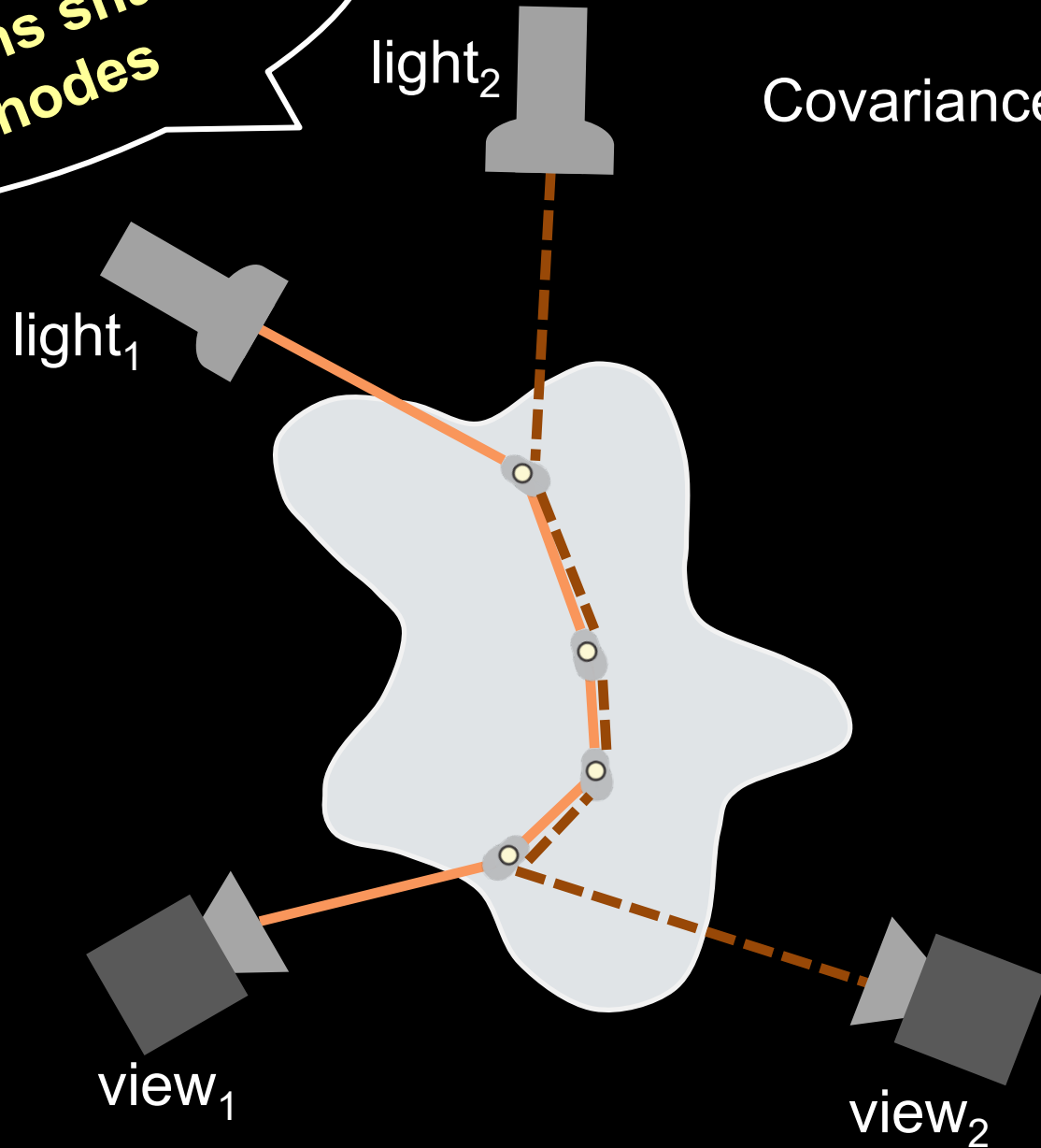


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

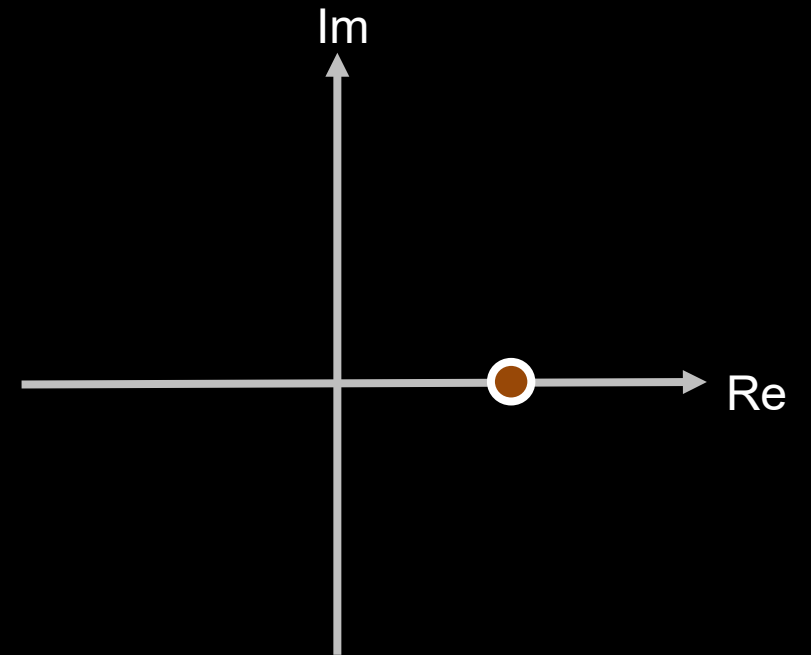


Covariance rendering

Paths share
nodes

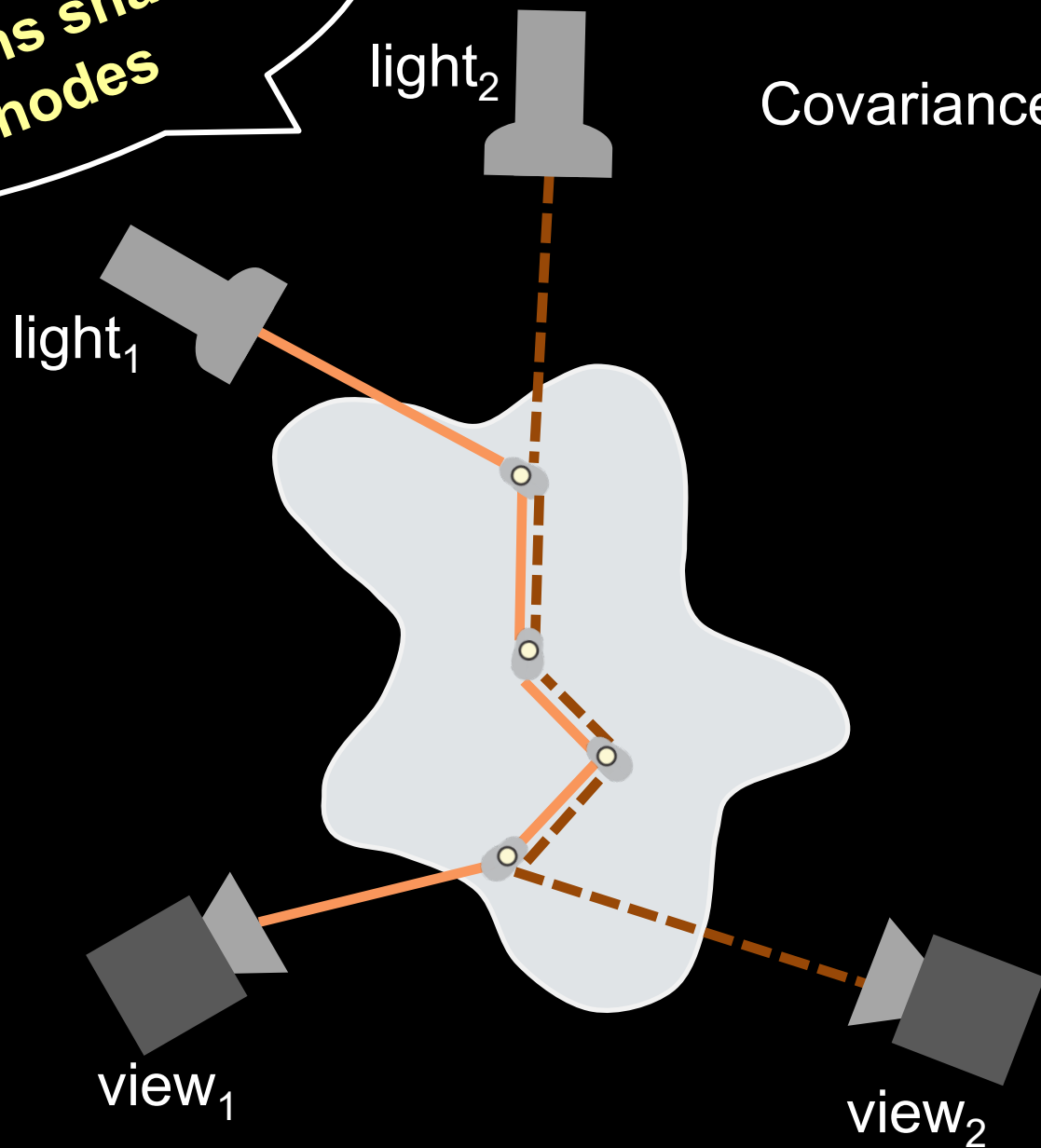


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

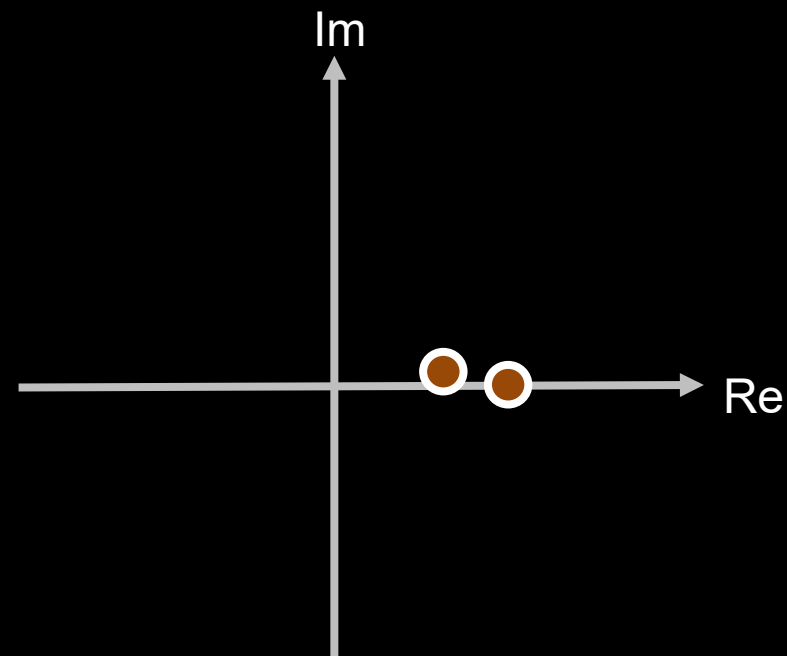


Covariance rendering

Paths share
nodes

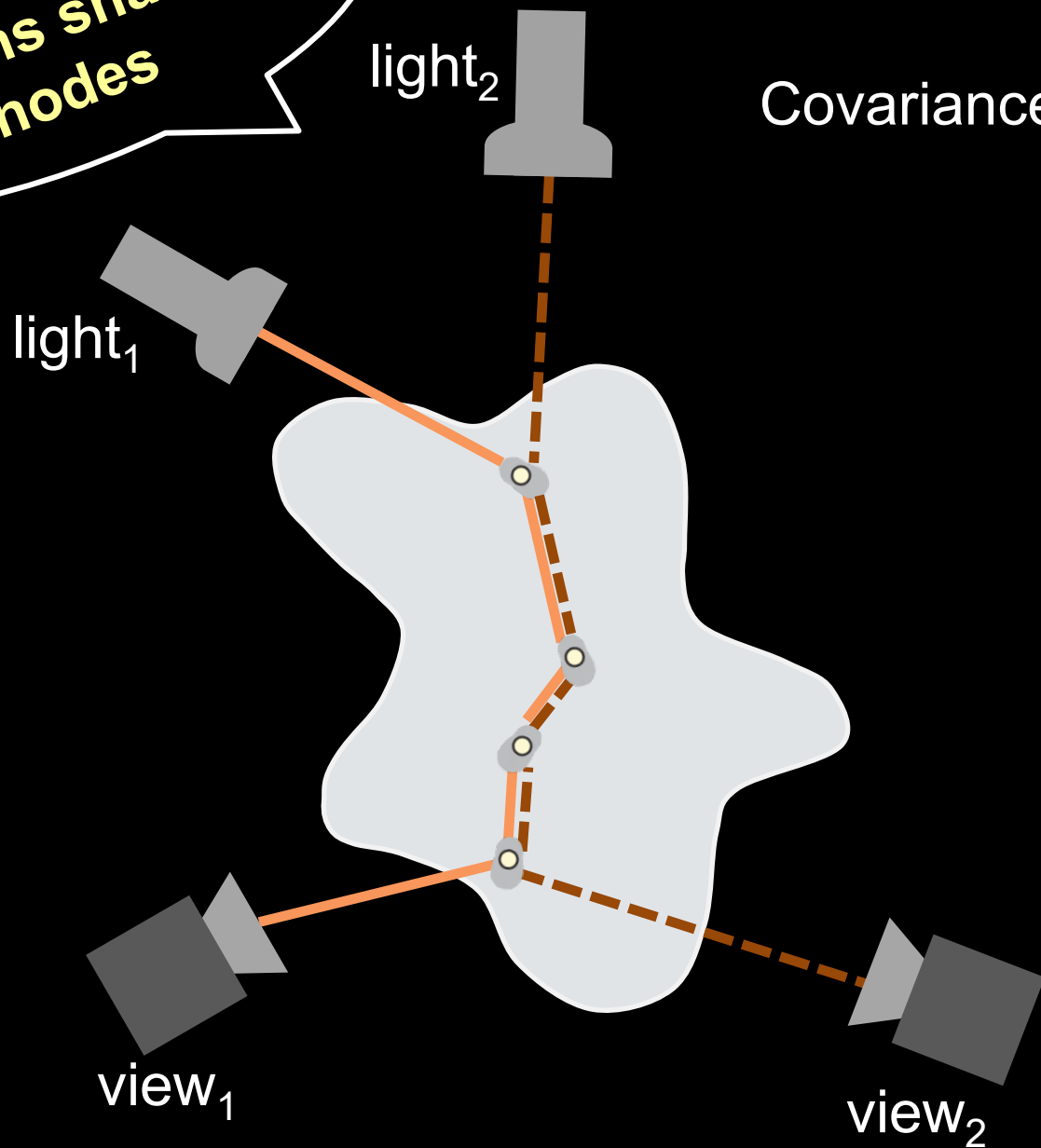


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

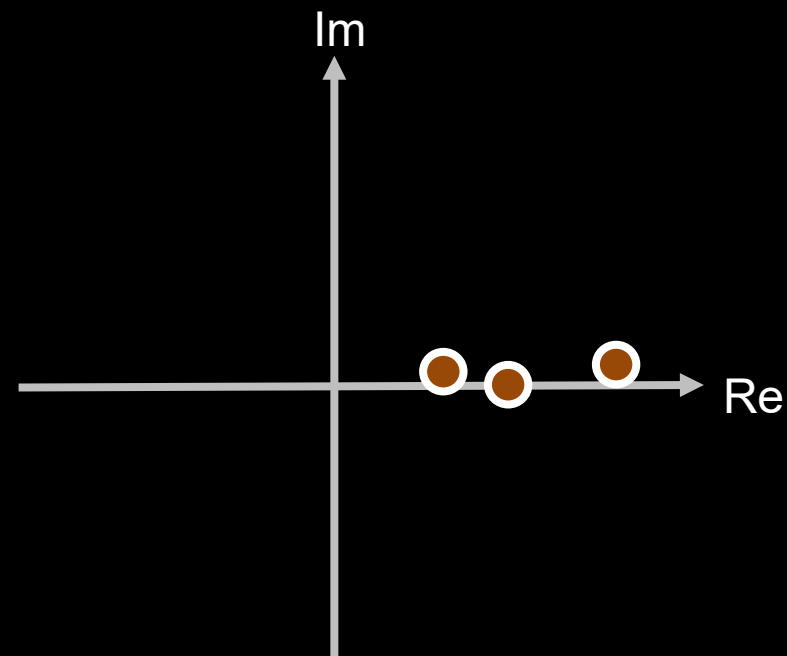


Covariance rendering

Paths share
nodes

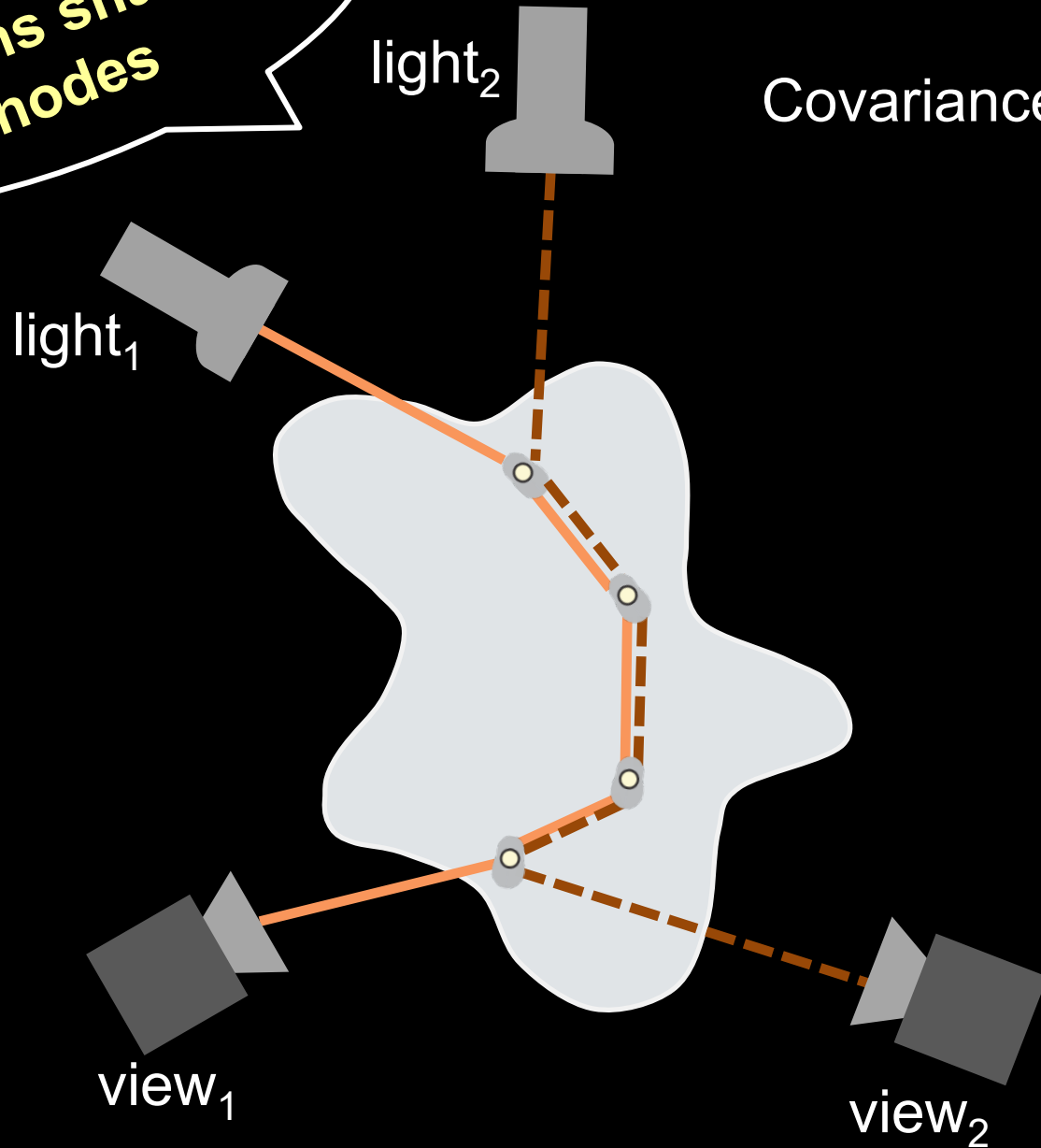


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

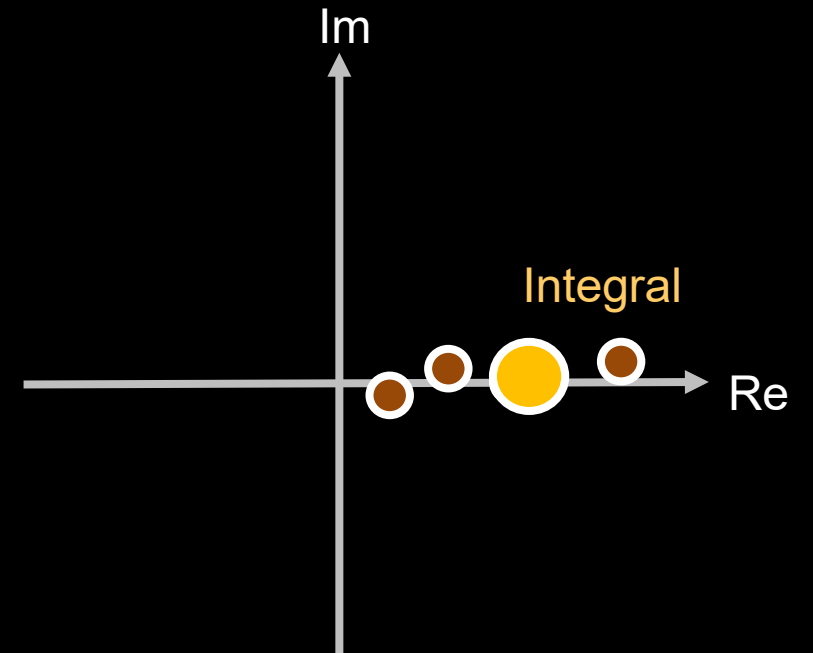


Covariance rendering

Paths share
nodes



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



Covariance rendering

$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

Observation: need to consider only path pairs that share the same nodes (except at the end)

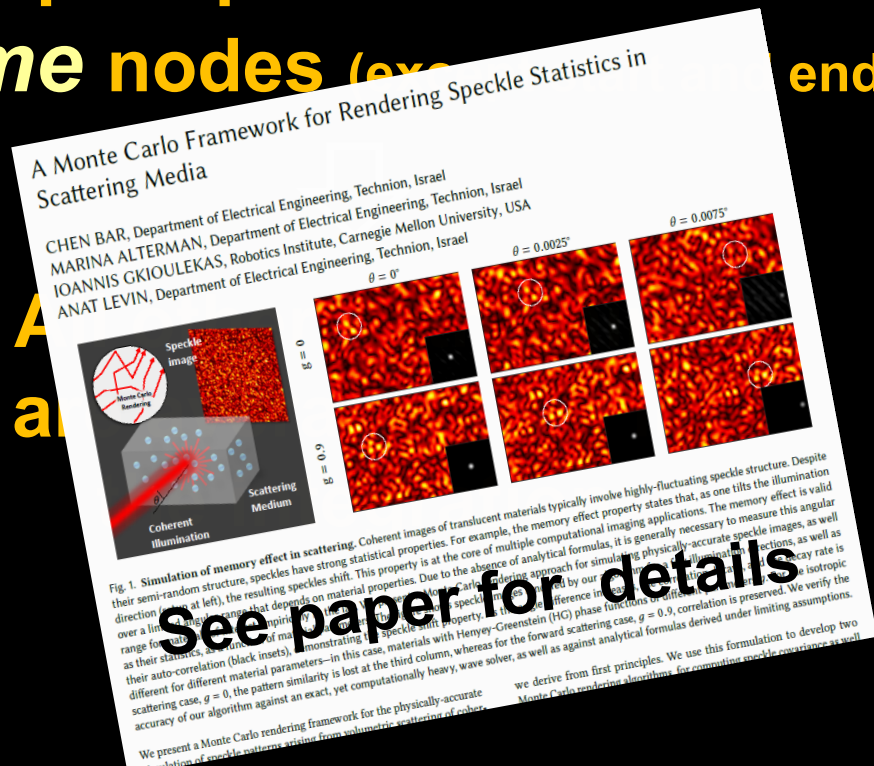
Paths share nodes

Efficiency of our algorithm comes from the ability to neglect all other paths pairs

Classical MC

view₁

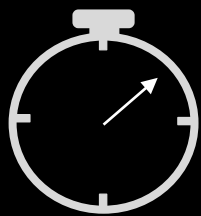
view₂



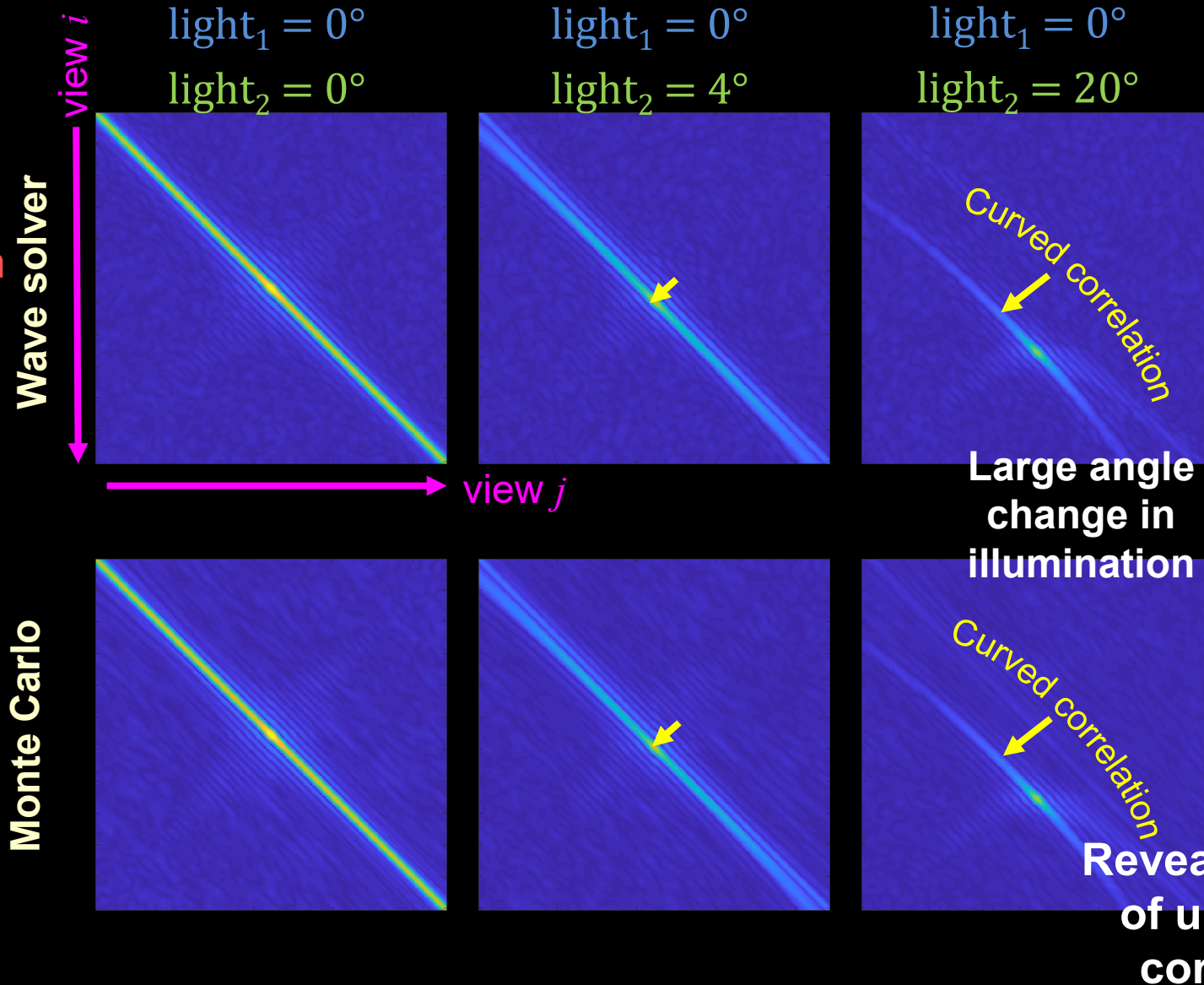
See paper for details

Validation: Wave Equation Covariances v.s. MC

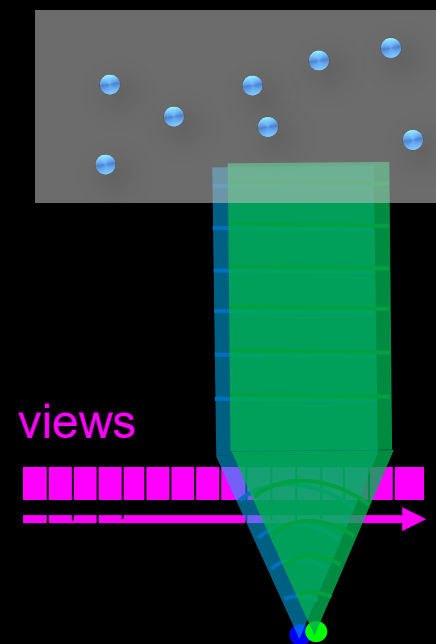
Computation
takes days



Several
minutes

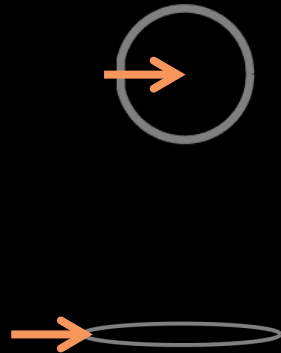


Classical ME
holds for
relatively
small angles
Setup

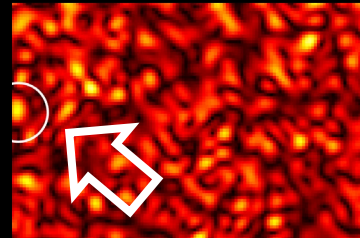


Rendering Speckles

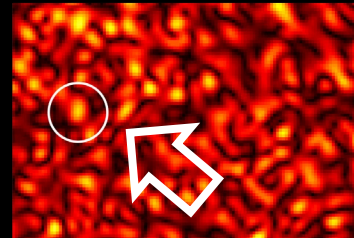
Phase Function



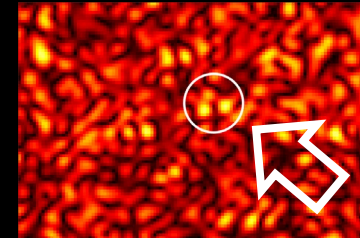
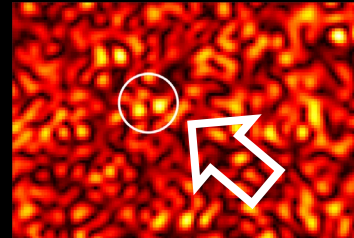
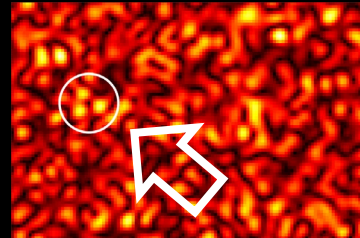
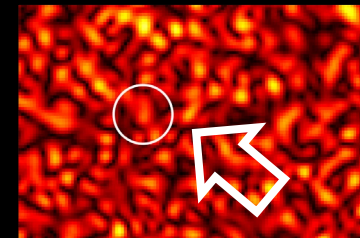
$\theta = 0^\circ$



$\theta = 0.0025^\circ$



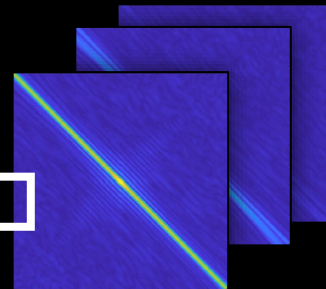
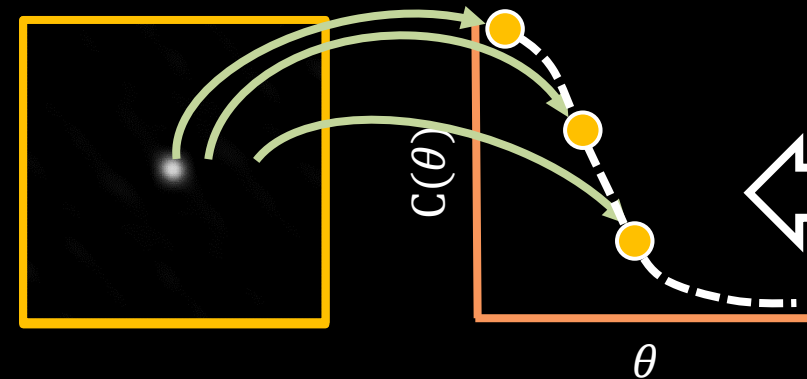
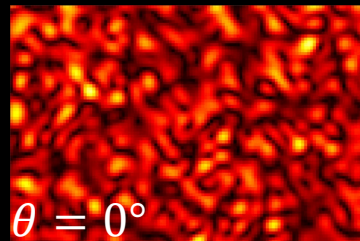
$\theta = 0.005^\circ$



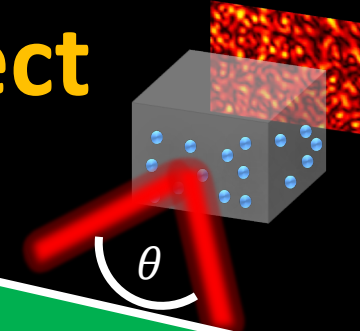
Classical ME holds for relatively small angles

Exact ME extent is different for different materials.

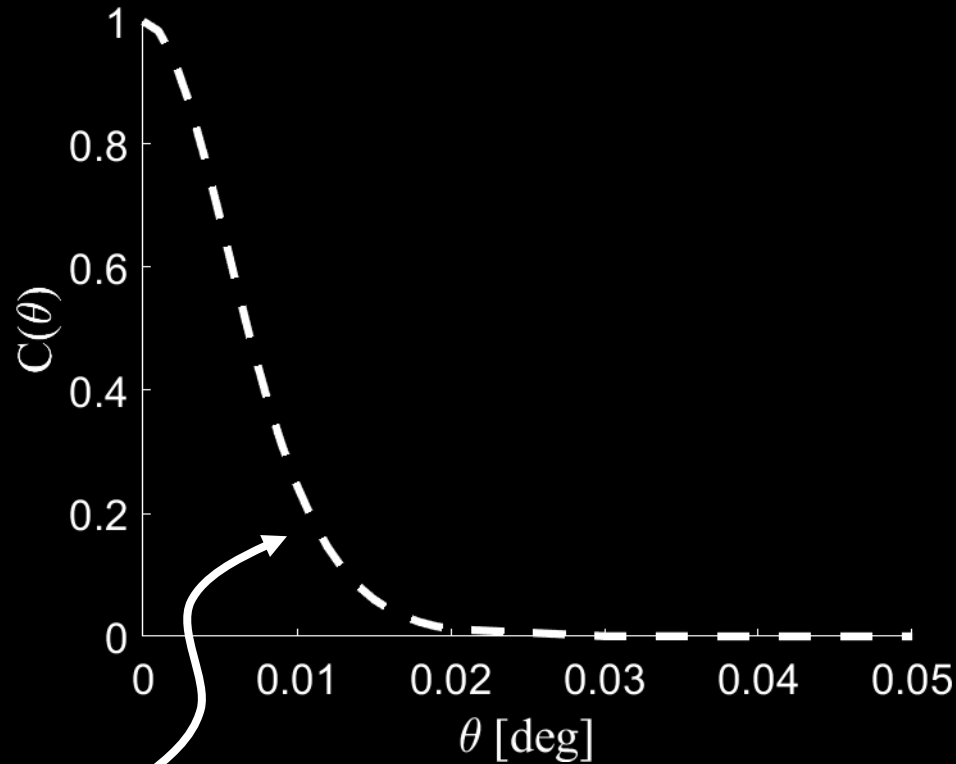
Computing ME extent as a function of θ :



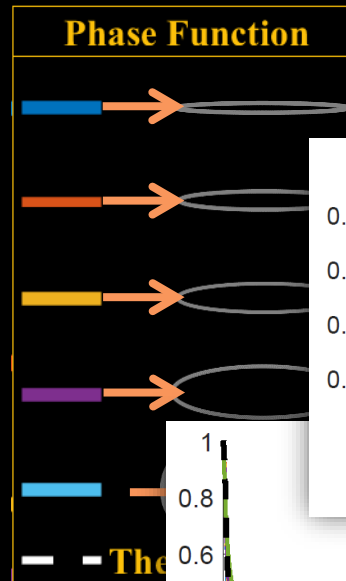
Evaluating the Memory Effect



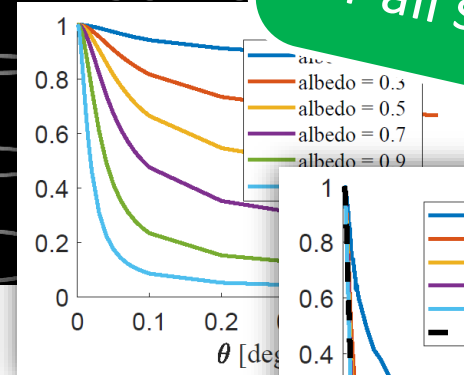
Now we can efficiently compute ME curves for all scattering parameters



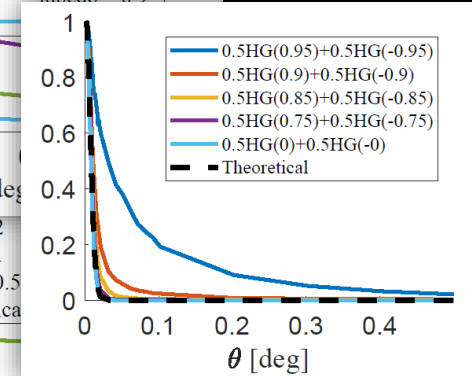
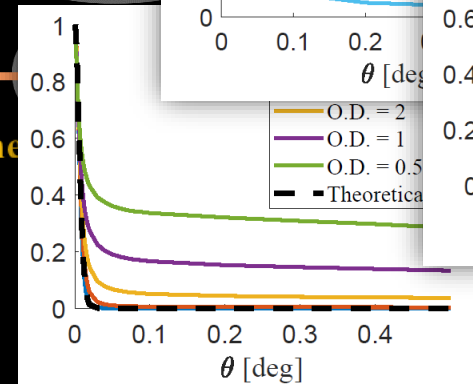
Analytical solution based on diffusion approximation



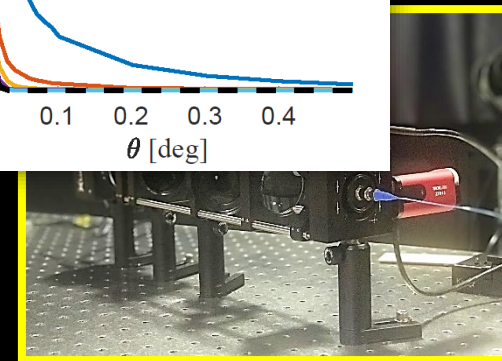
This a some



E extent

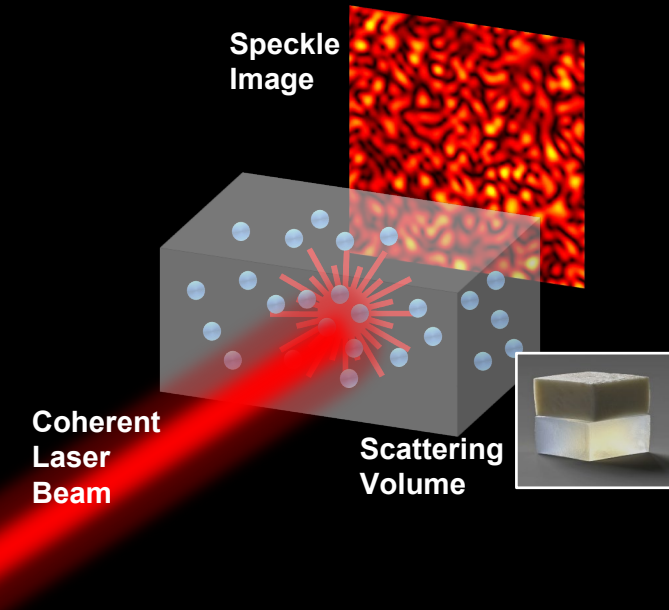


empirically in



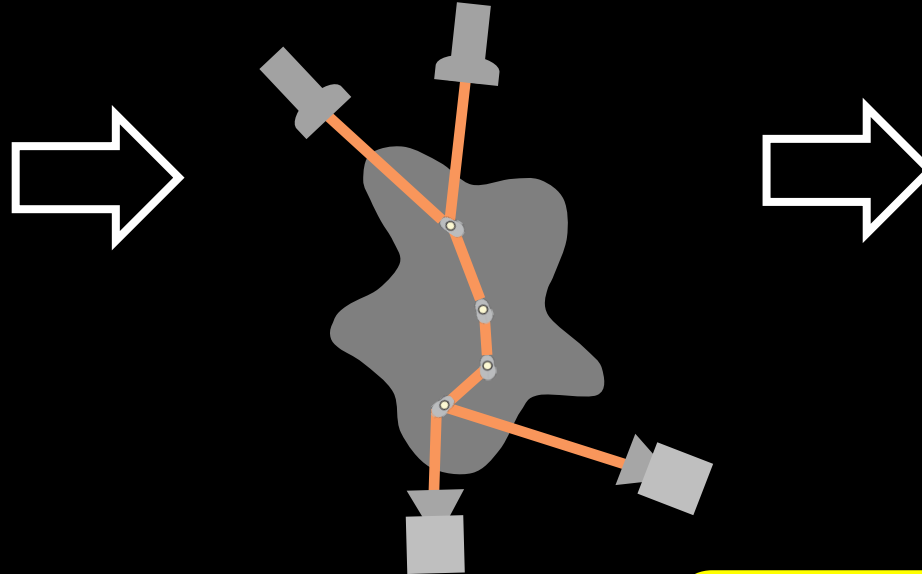
Summary

Problem:
Coherent Scattering

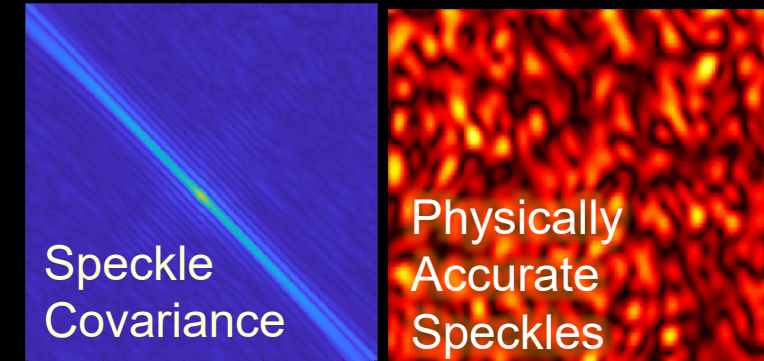


Path-integral formulation
for speckle covariance

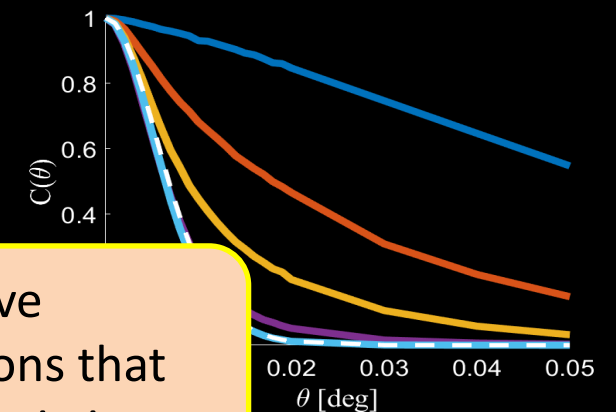
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



Efficient MC Rendering

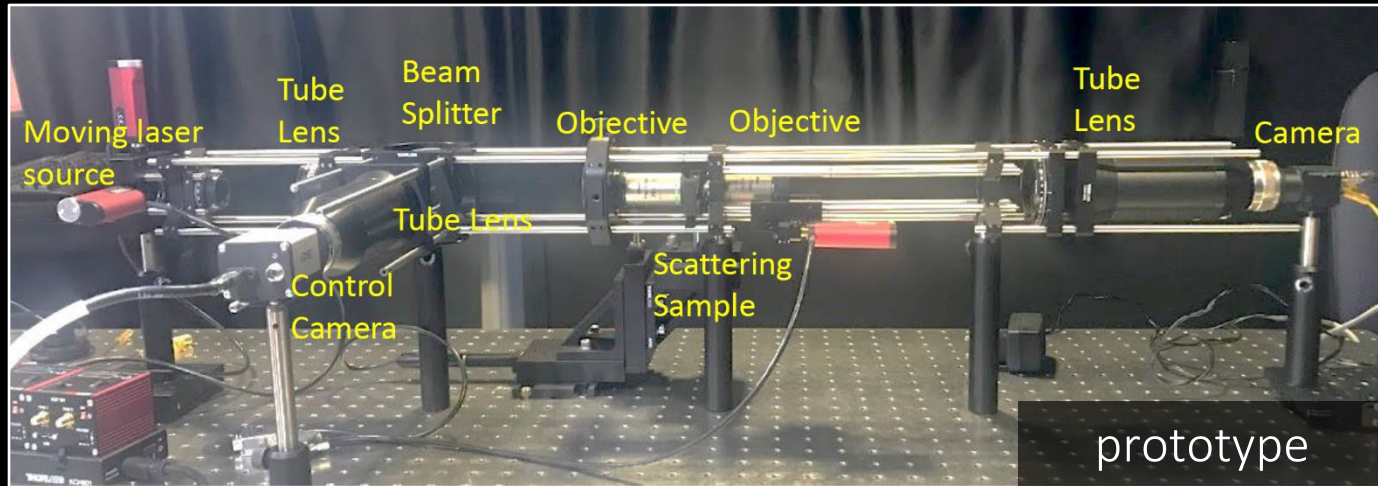
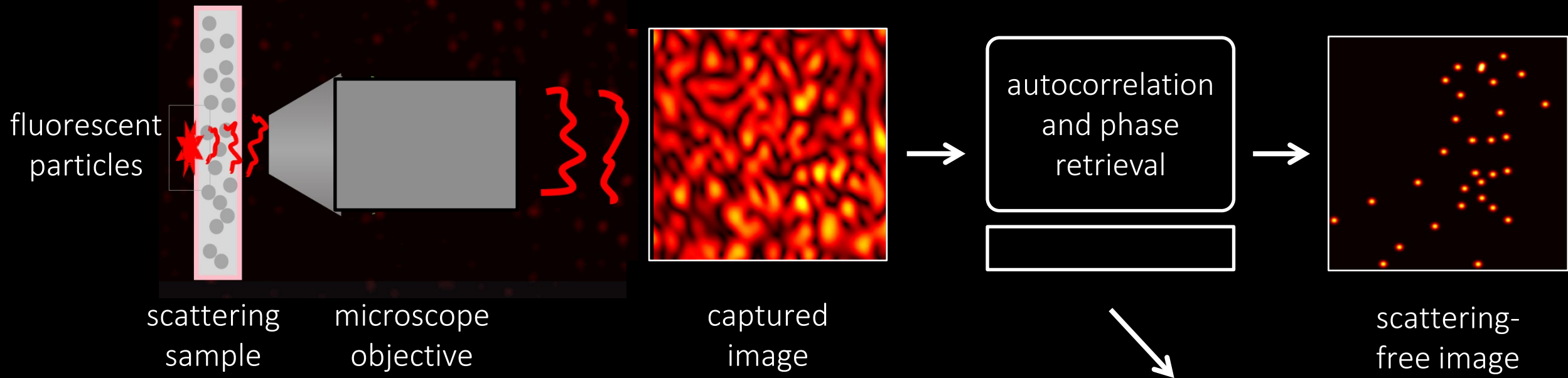


Memory Effect Evaluation



Potentially improve
imaging applications that
rely on speckle statistics

Speckle-based fluorescence microscopy

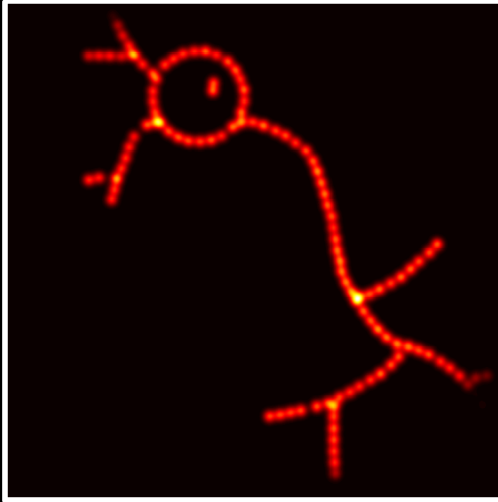


Performance strongly depends on:

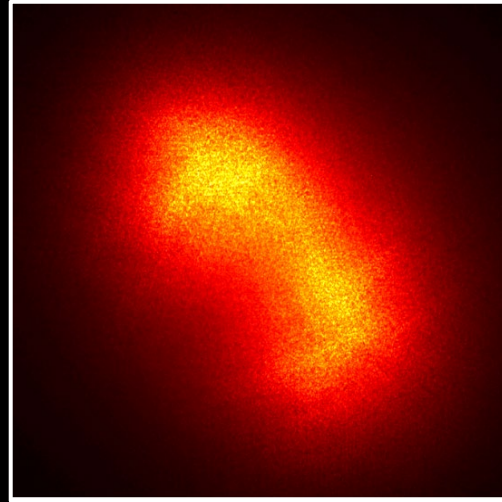
- speckle statistics
- image priors
- tissue parameters

Better algorithms for fluorescence microscopy

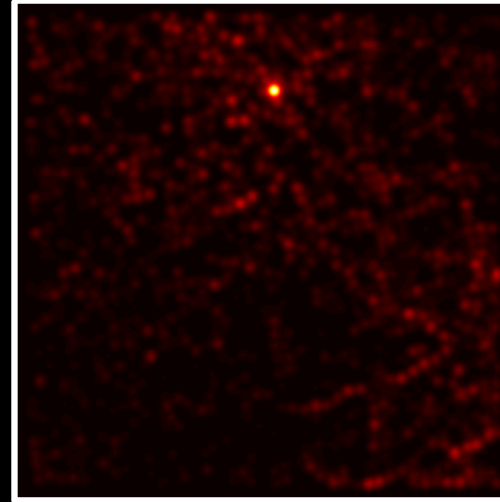
groundtruth



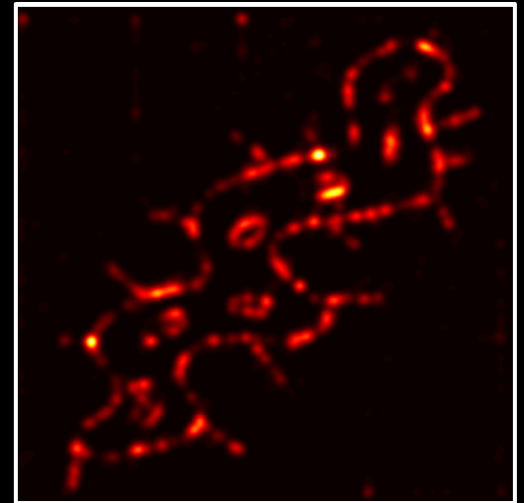
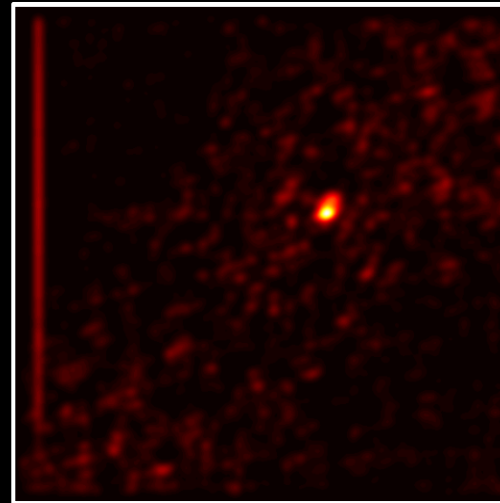
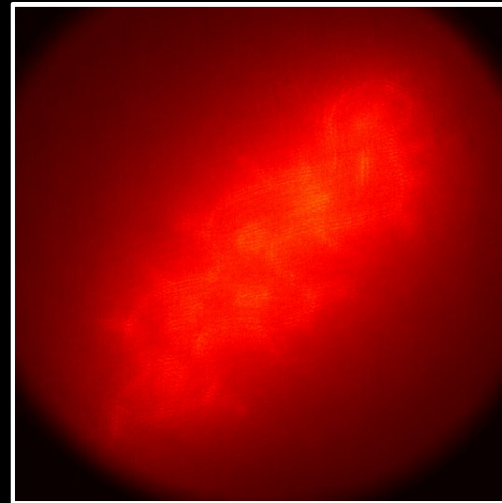
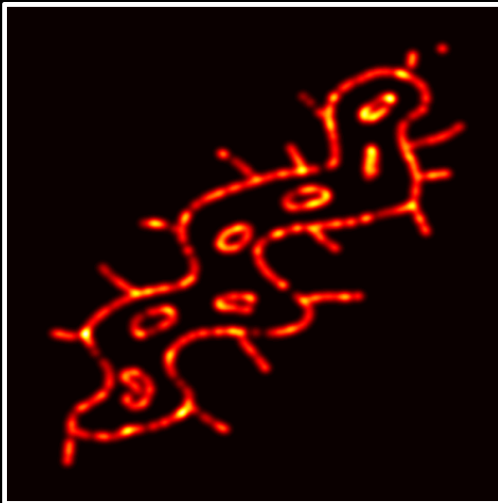
input image



prior algorithm



our algorithm



Acquisition of scattering materials

Use differentiable speckle rendering to recover material parameters from speckle images

