Photon mapping

15-468, 15-668, 15-868
Physics-based Rendering
Spring 2025, Lecture 14

http://graphics.cs.cmu.edu/courses/15-468

Course announcements

- PA4 has been posted.

Overview of today’s lecture

- Photon mapping.

Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

Today’s Menu

Difficult light paths

Photon Mapping

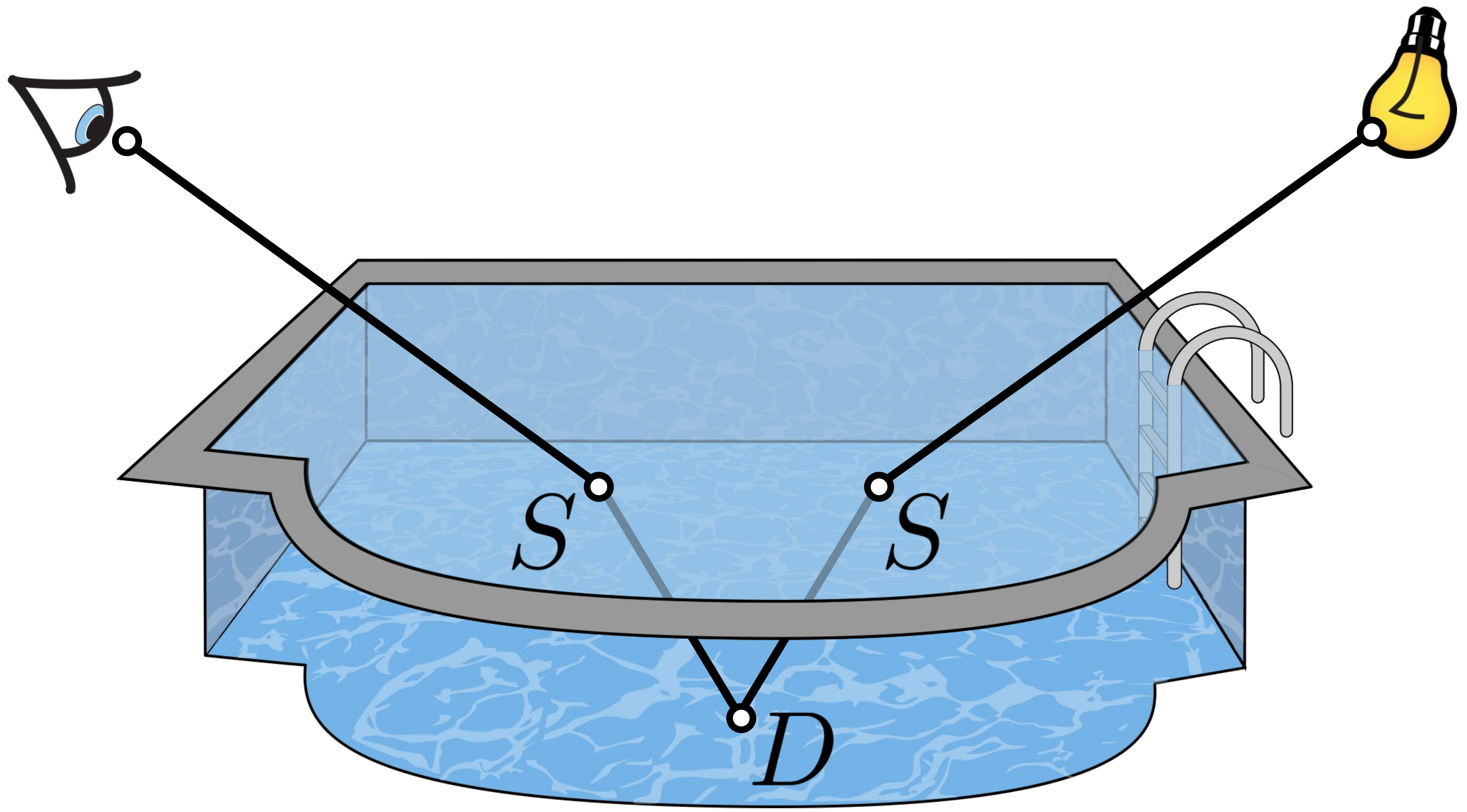
Specular-Diffuse-Specular Paths

Henrik Wann Jensen

Specular-Diffuse-Specular Paths

Images courtesy of J. Křivánek

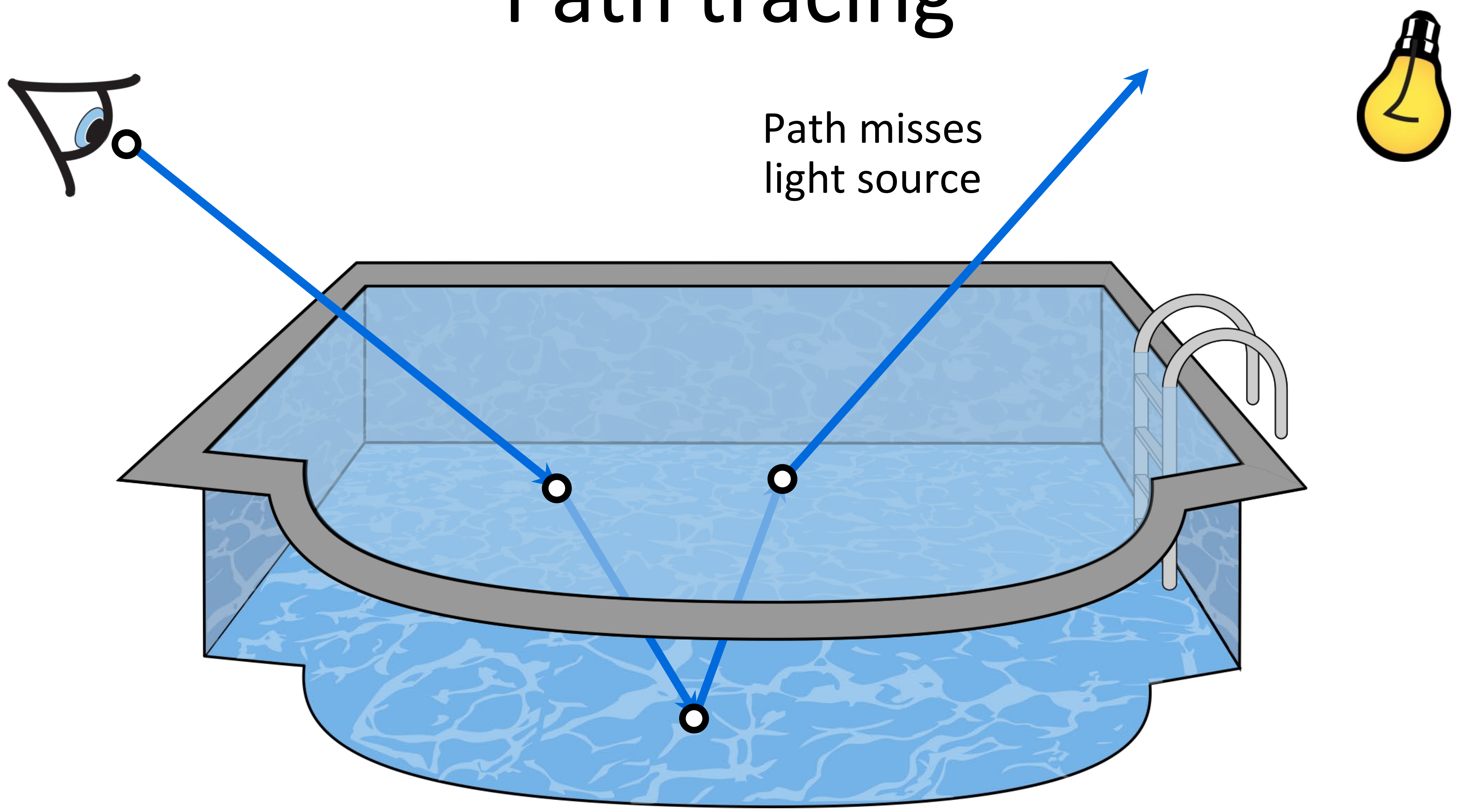
Specular-Diffuse-Specular Paths



$LSDSE$ paths are difficult for unbiased techniques

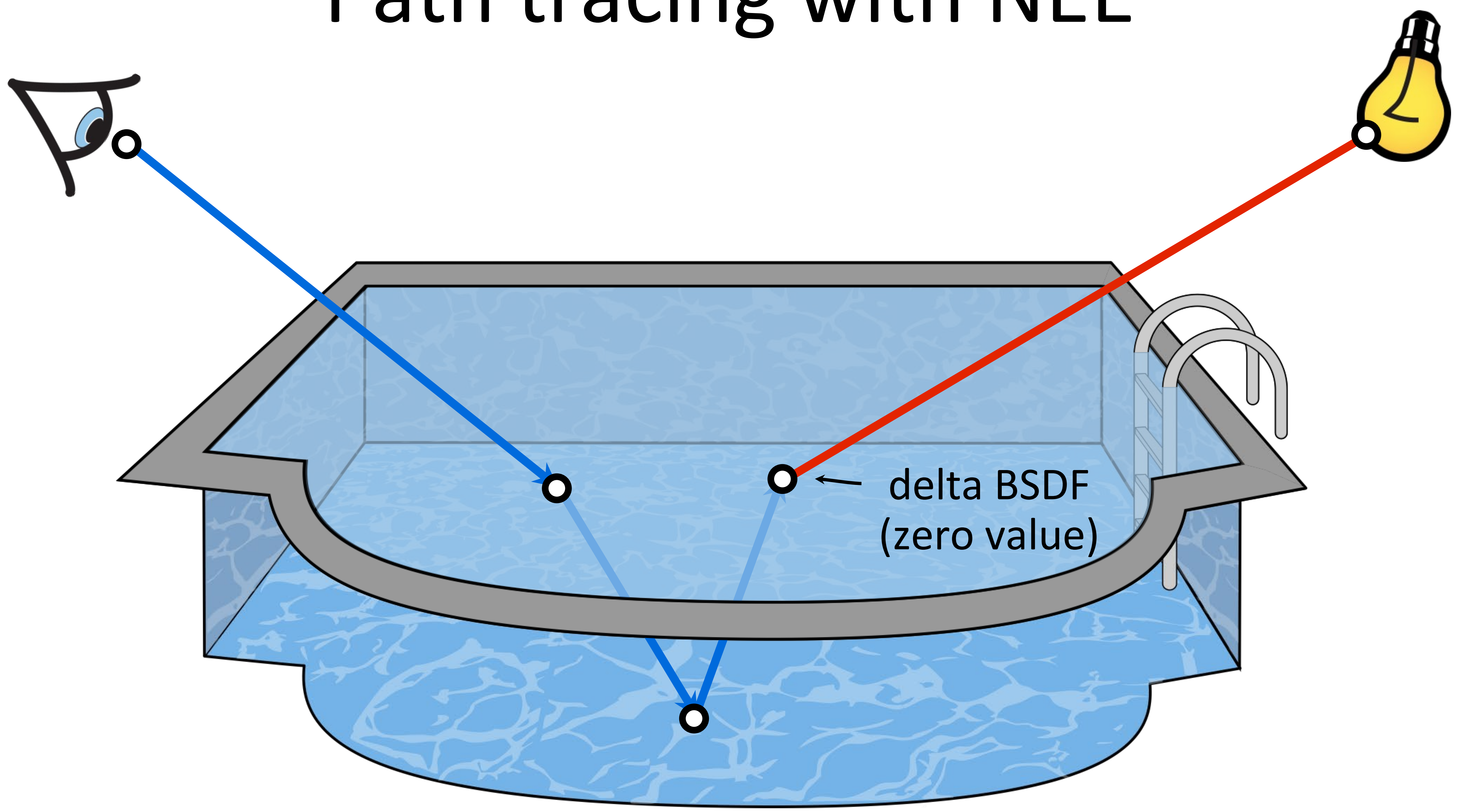
Specular-Diffuse-Specular Paths

Path tracing



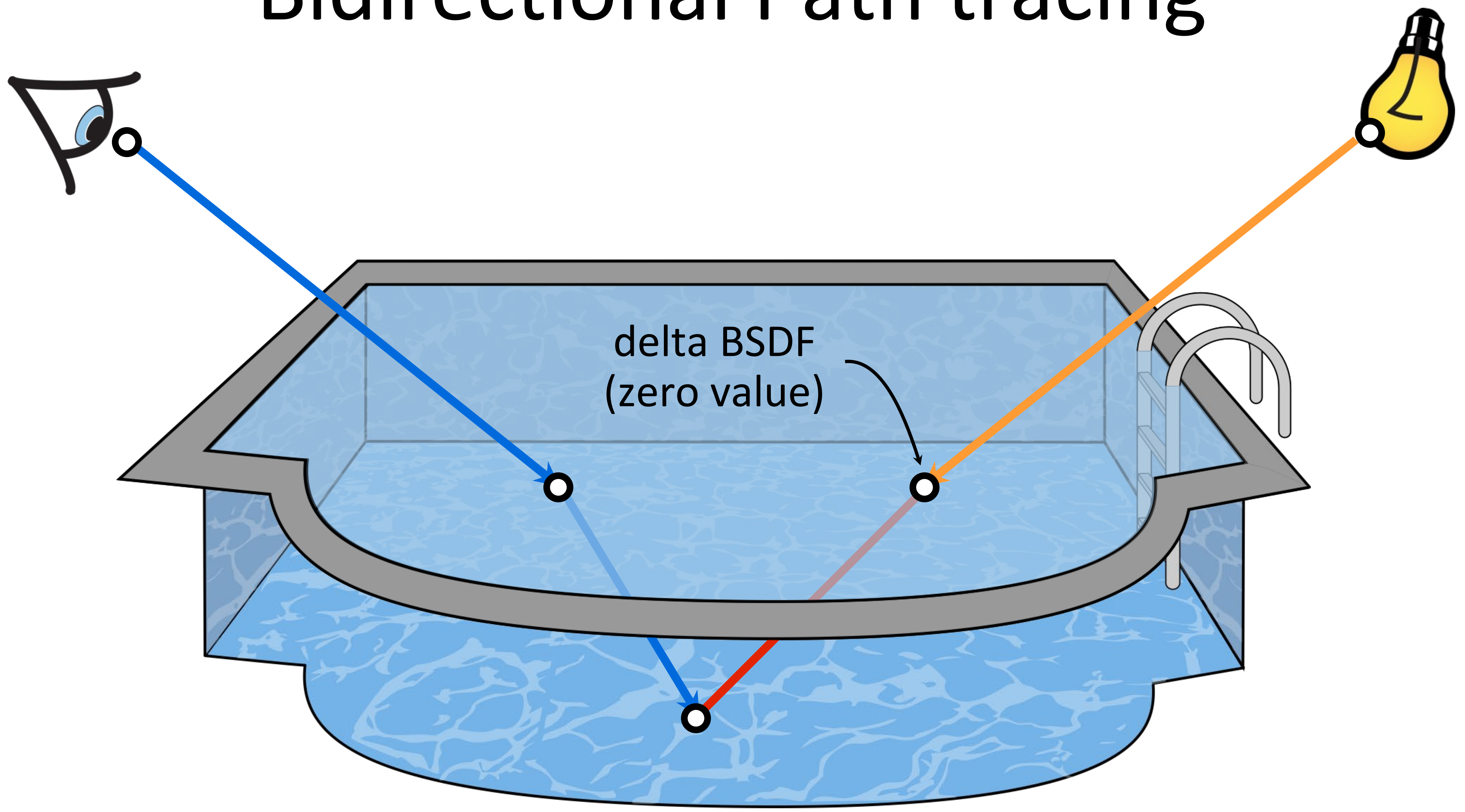
Specular-Diffuse-Specular Paths

Path tracing with NEE



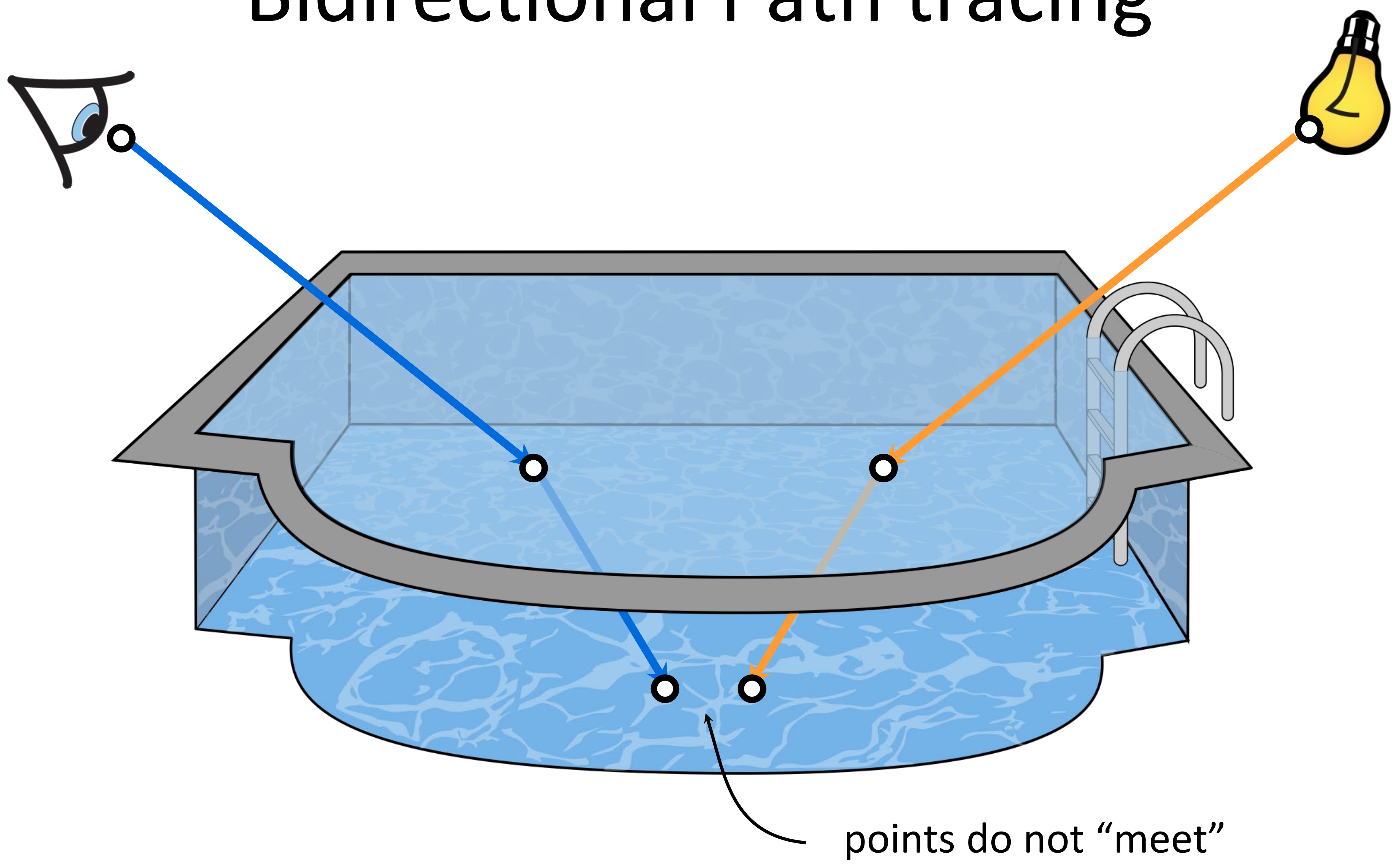
Specular-Diffuse-Specular Paths

Bidirectional Path tracing



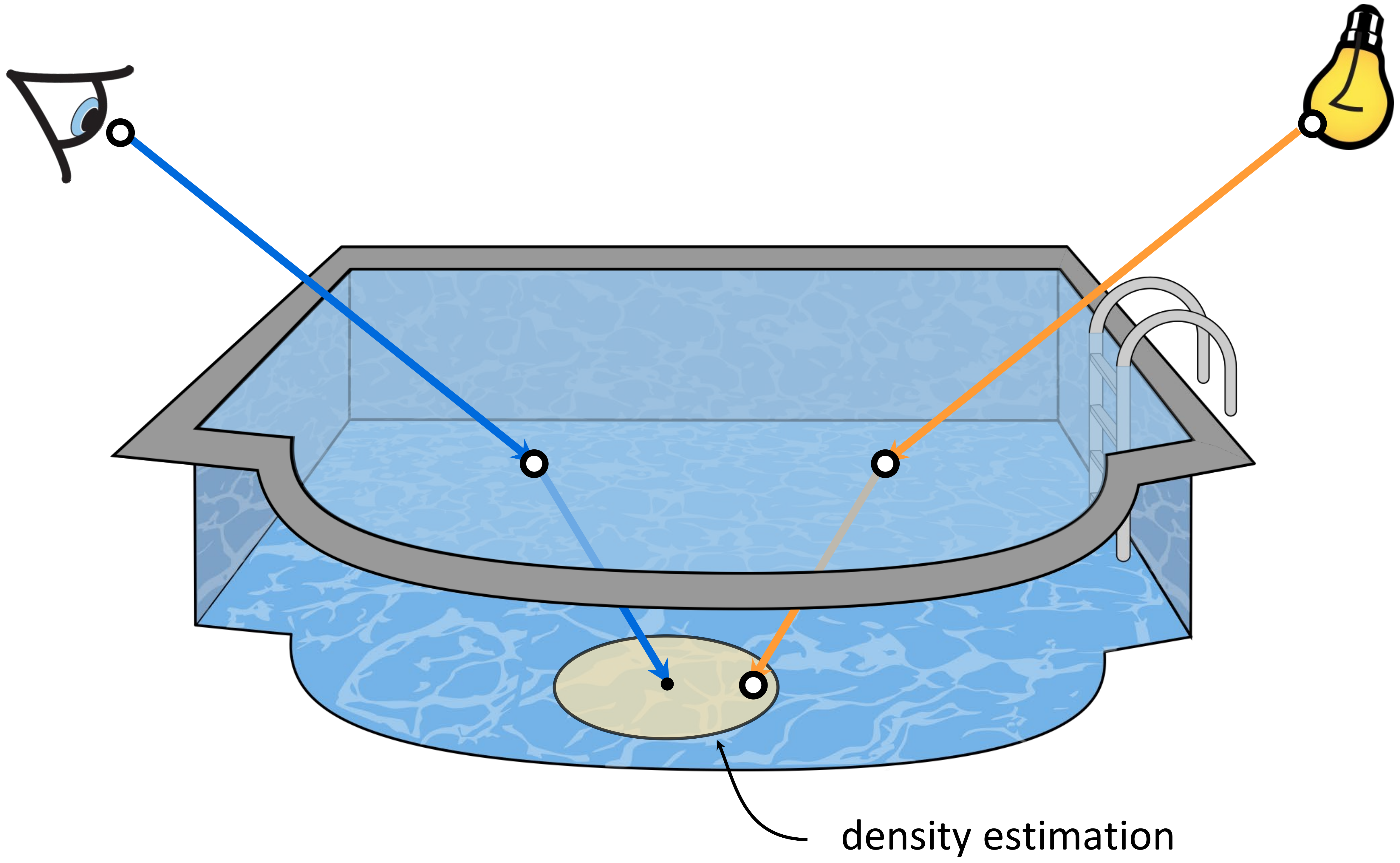
Specular-Diffuse-Specular Paths

Bidirectional Path tracing



What now?

Specular-Diffuse-Specular Paths



Regularize delta functions (path points):
e.g., by employing kernel density estimation (blurring in space)

“Backward” Ray Tracing

(predecessor of photon mapping)

“Backward” Ray Tracing

James Arvo. In *Developments in Ray Tracing*, SIGGRAPH ‘**86** Course Notes

Start paths from light sources and store energy in *illumination maps*

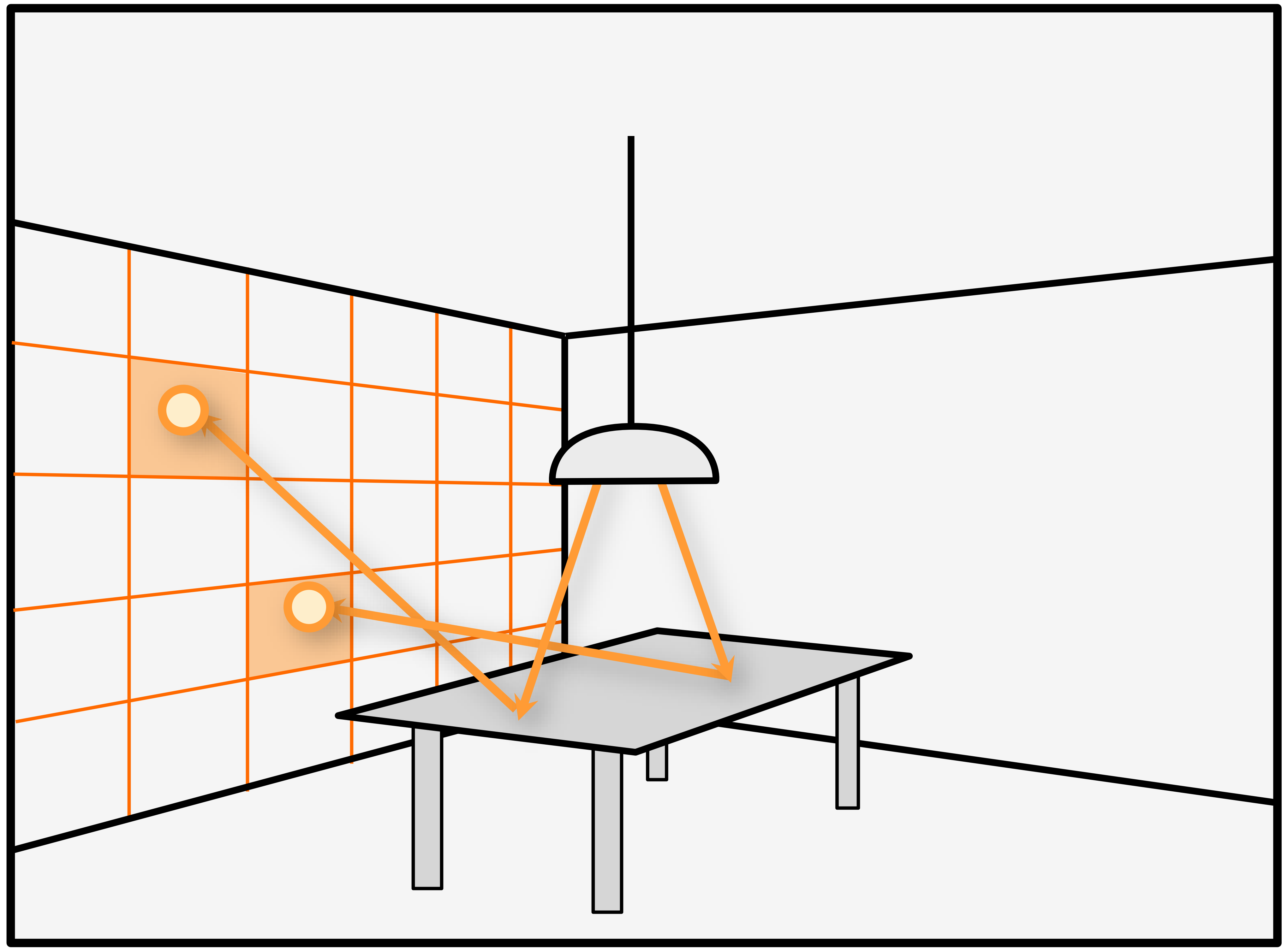
Illumination map = texture for accumulating irradiance

Note on the name of the technique: In retrospect, Arvo regretted using the term “backward” to refer to tracing light paths since many later publications use it in the opposite sense, i.e. tracing eye paths. To avoid confusion, he recommends terms such as *light tracing* and *eye tracing* as they are unambiguous.

“Backward” Ray Tracing

Preprocess:

- shoot light from light sources
- deposit photon energy in illumination maps

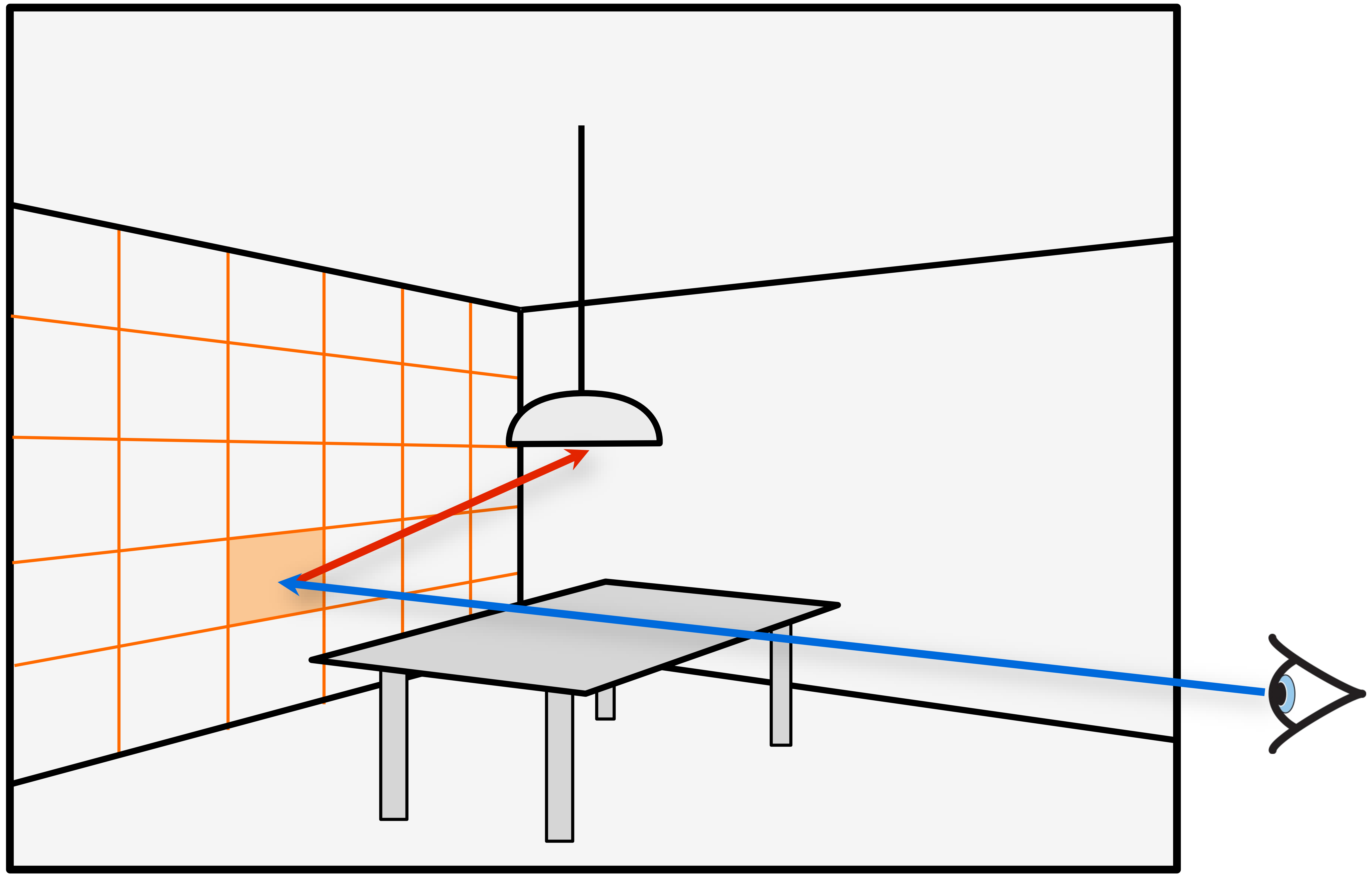


Irradiance: “number of photons hitting a small patch of a wall per second, divided by size of patch”

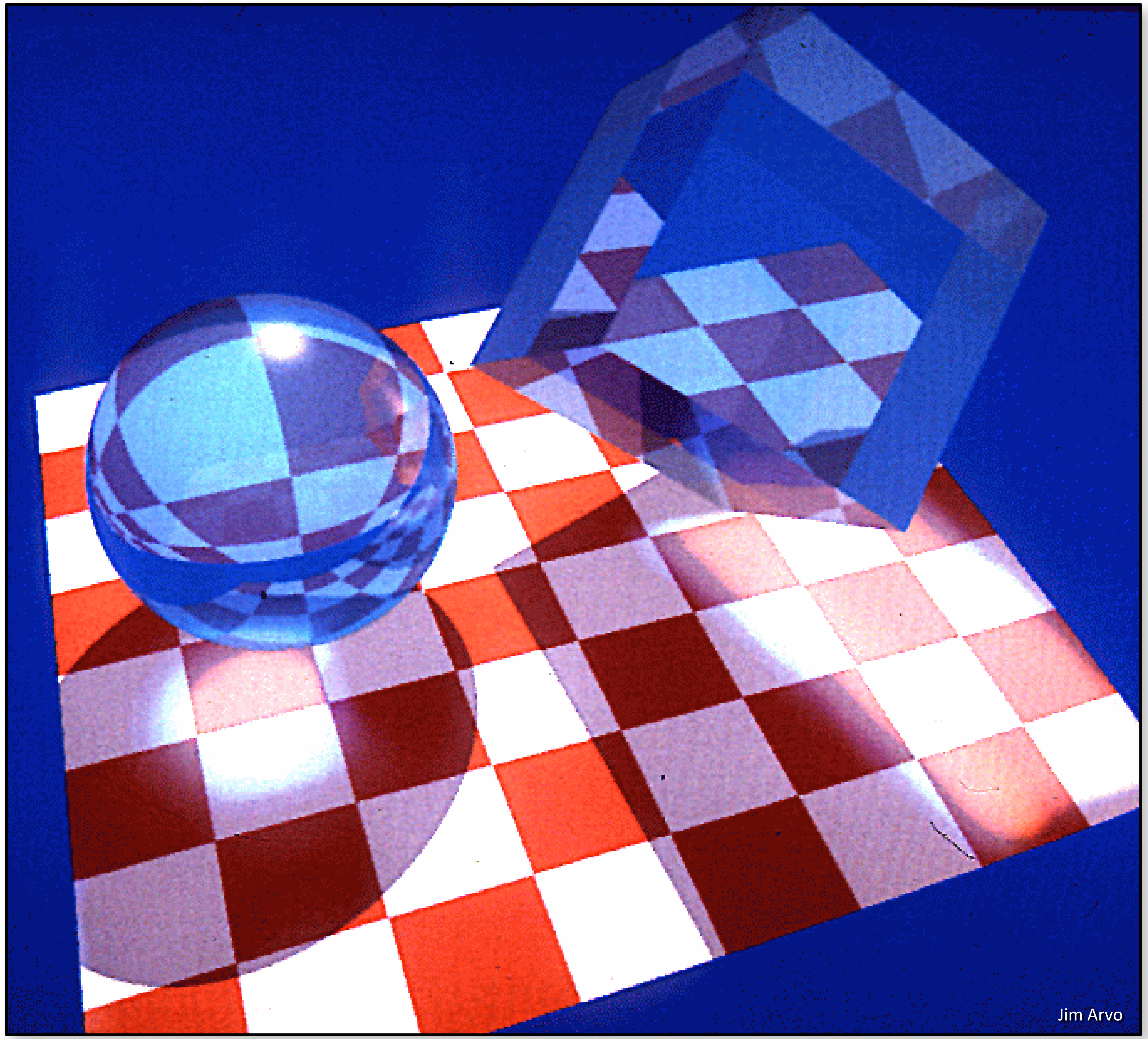
“Backward” Ray Tracing

For each shading point

- compute direct lighting
- lookup indirect lighting from illumination maps



“Backward” Ray Tracing



"Backward Ray Tracing", by James Arvo. In *Developments in Ray Tracing*, SIGGRAPH `86 Course Notes, Volume 12.

“Backward” Ray Tracing

✓ One of the first techniques to simulate caustics!

✗ Requires parametrizing surfaces or meshing

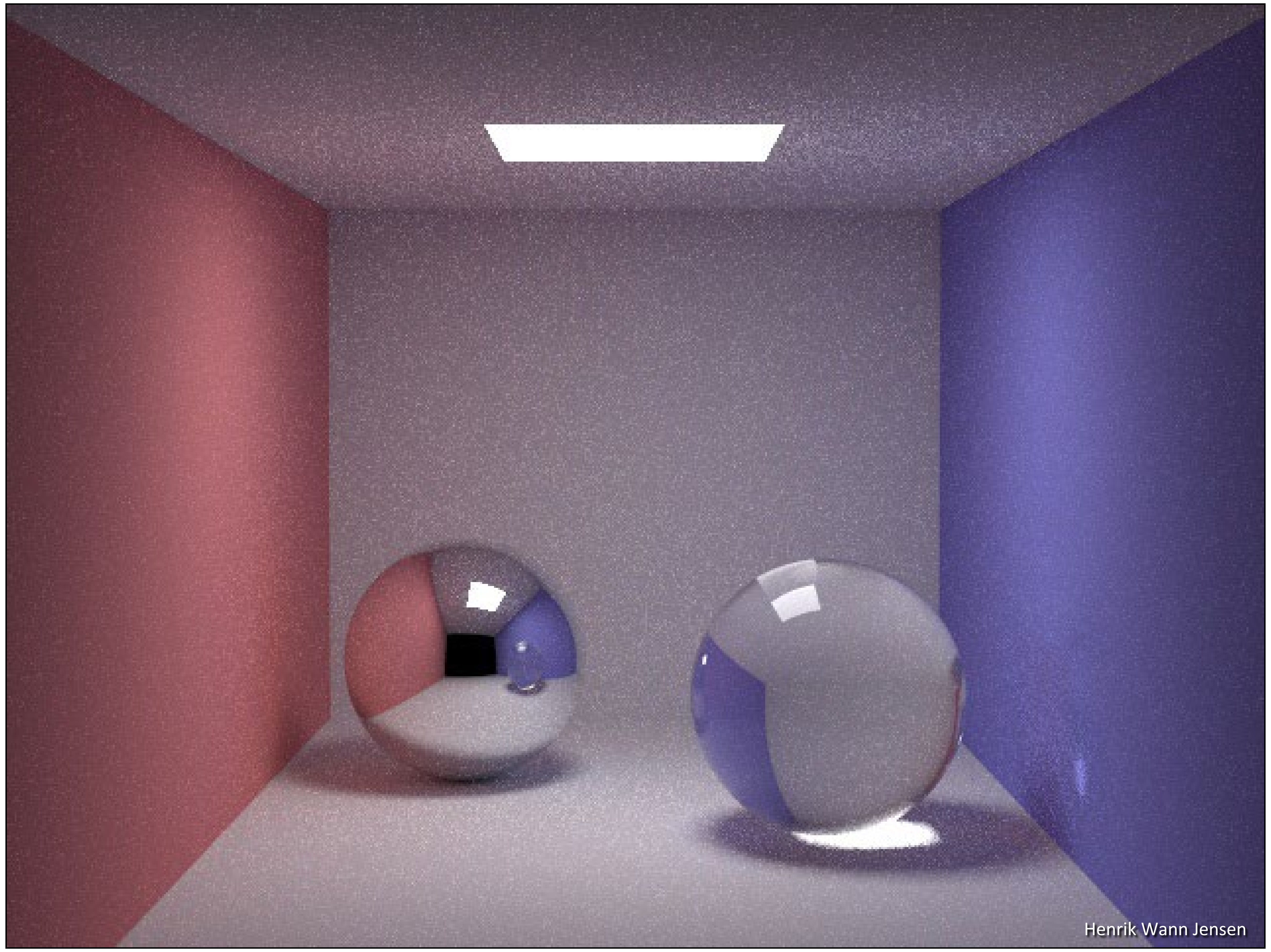
- Difficult to handle complex or procedural geometry

✗ Hard to choose illumination map resolution

- high resolution with few photons: high-frequency noise
- low resolution with many photons: blurred illumination

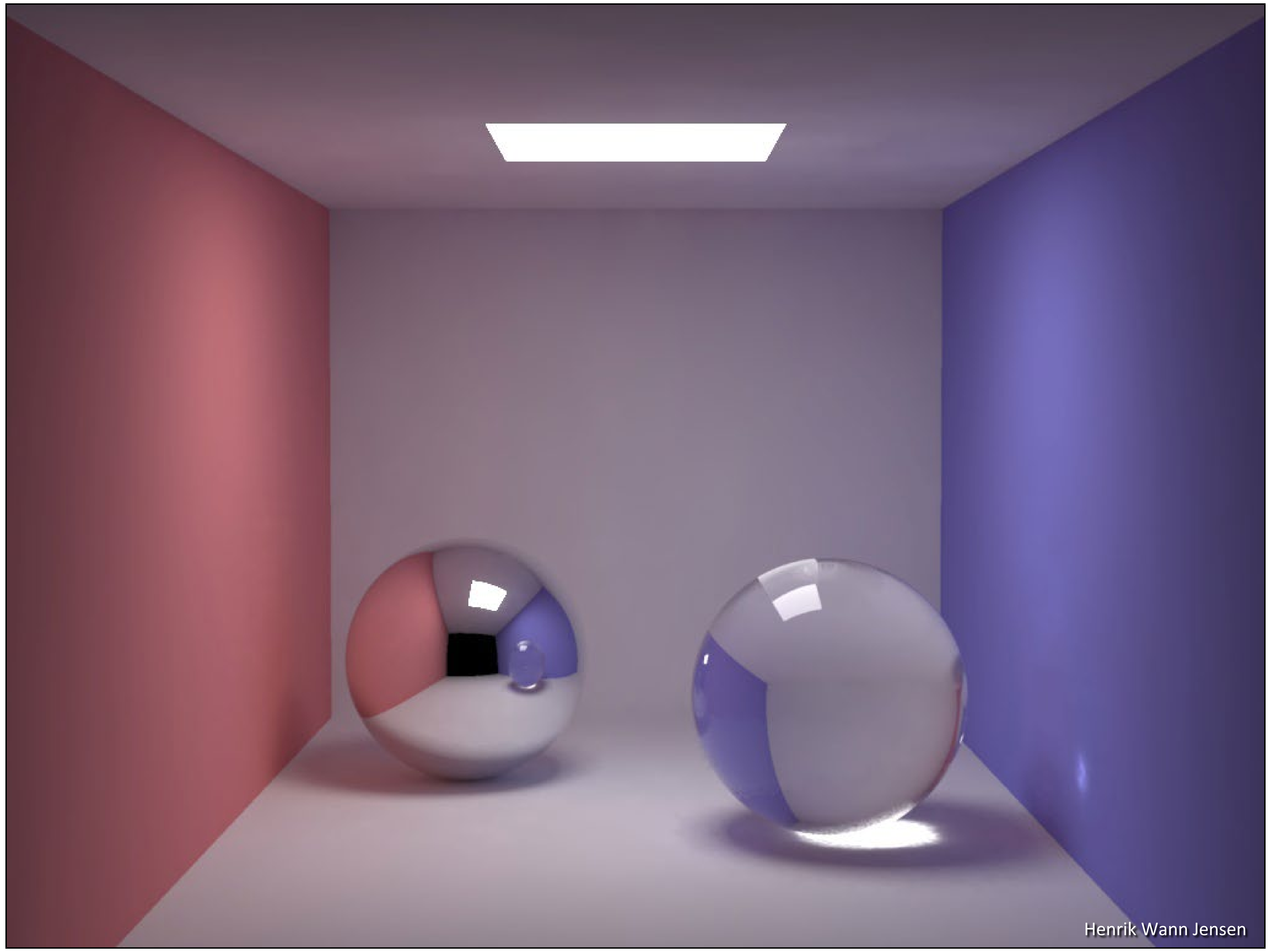
Photon Mapping

Path Tracing



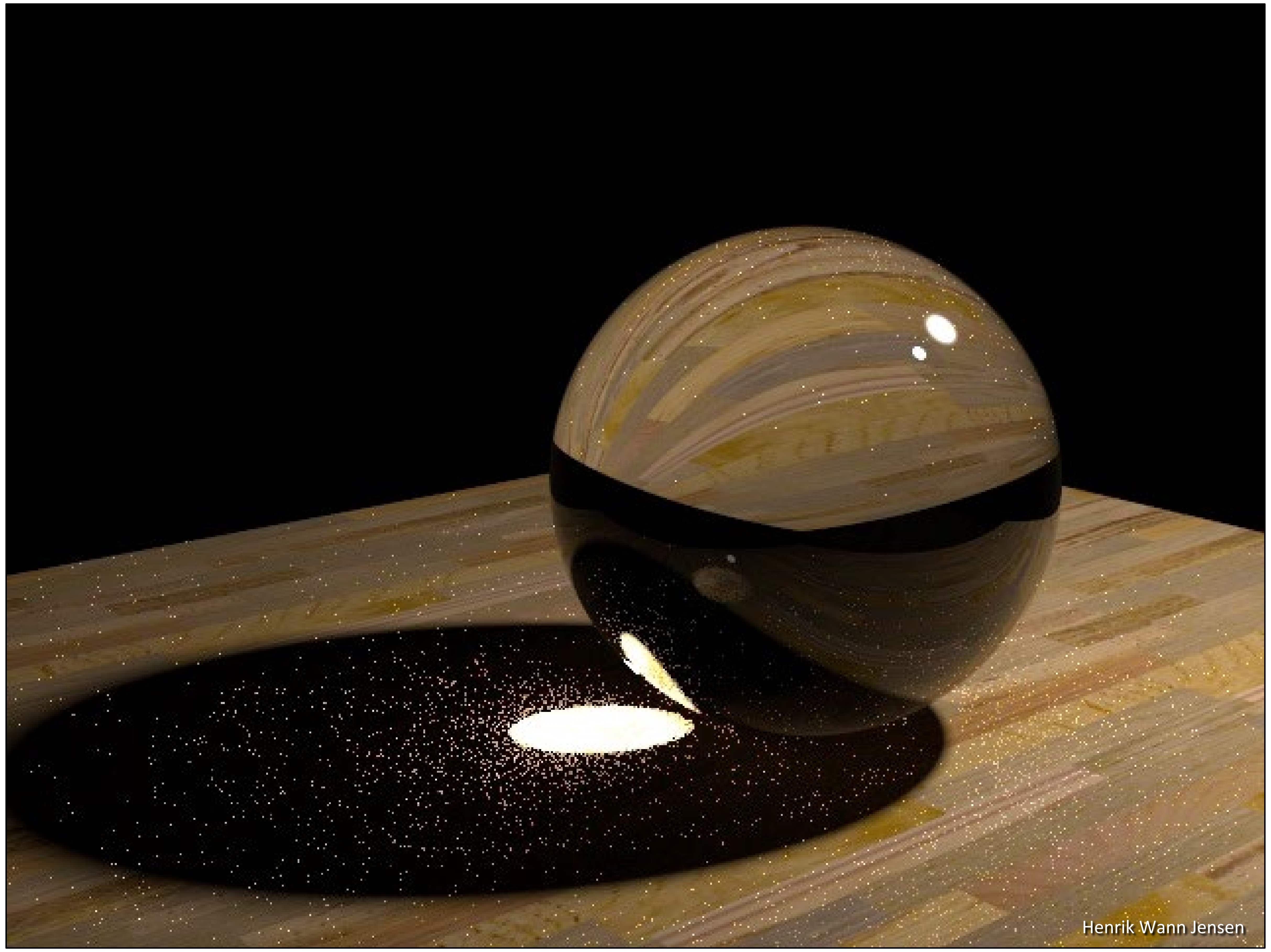
100 paths/pixel (5 minutes)

Photon Mapping



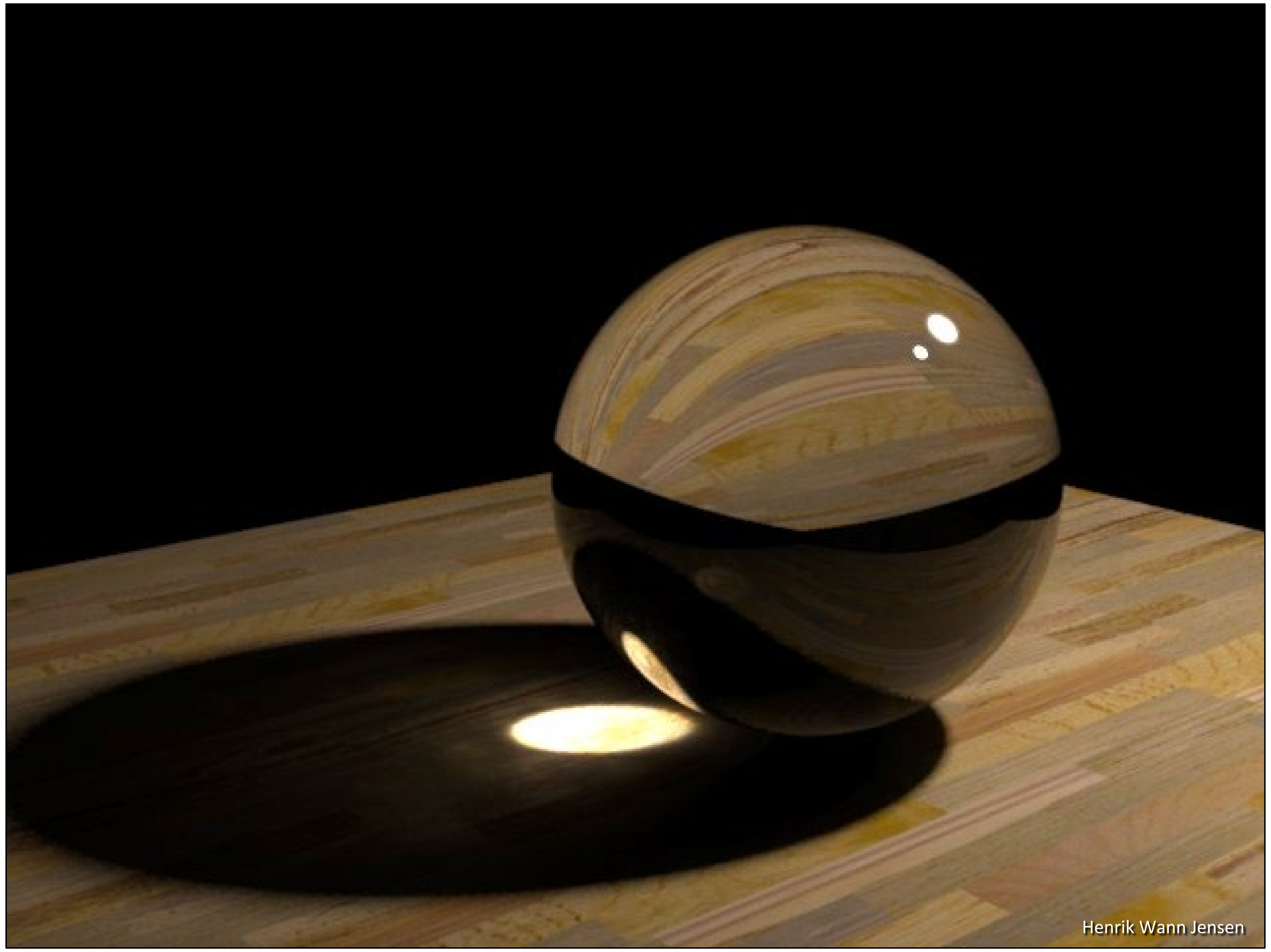
10 rays/pixel (5 seconds)

Path Tracing



1000 paths/pixel

Photon Mapping



Photon Mapping

A two-pass algorithm:

- Pass 1: Tracing photons from light sources, and caching them in a *photon map*
- Pass 2: Tracing from the eye and approximating indirect illumination using the photons

Similar to “backward” ray tracing, but different way of storing photons & computing density

Photon Mapping

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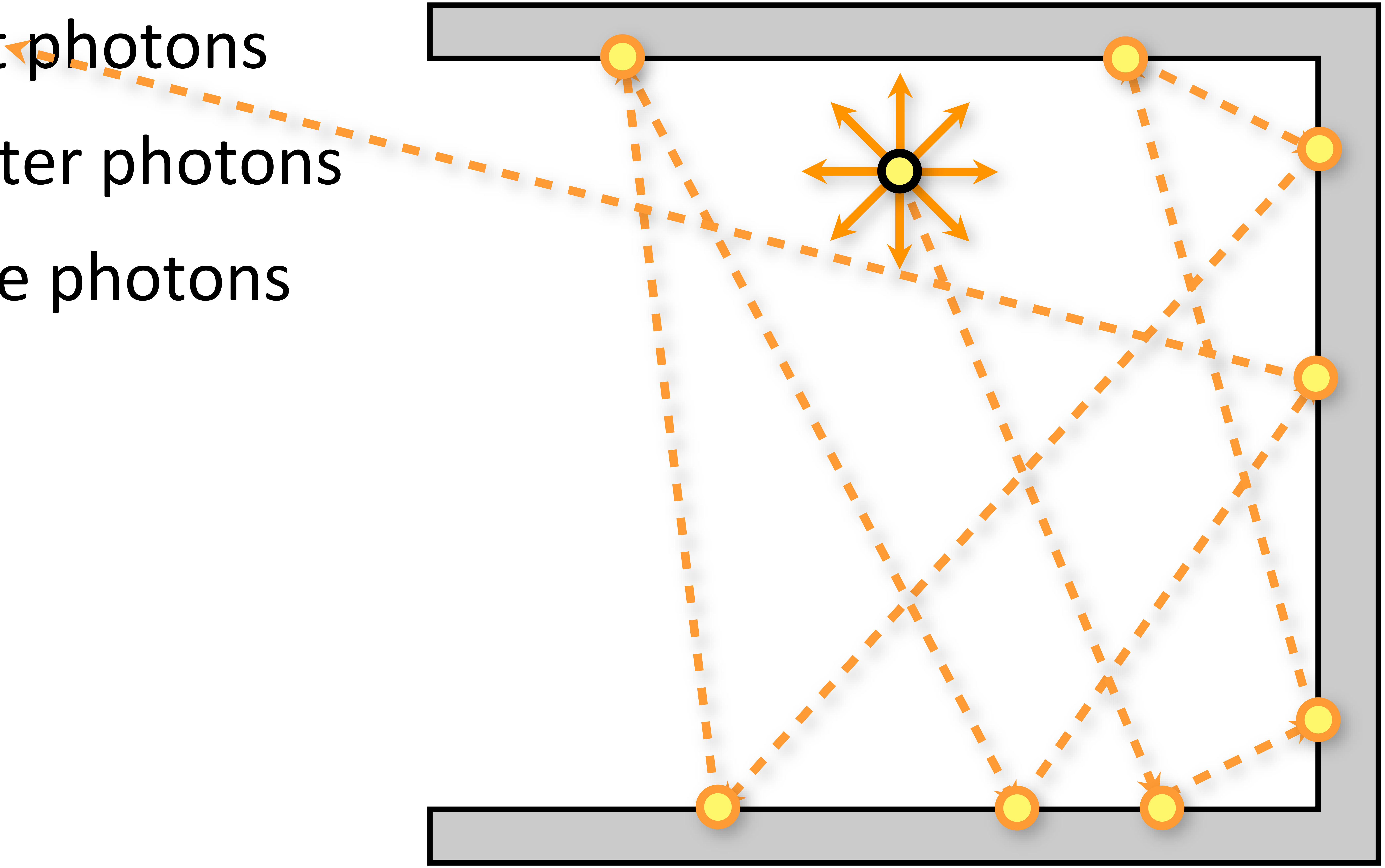
Similar to "backward" ray tracing, but different way of storing photons & computing density

Photon Tracing

1) Emit photons
2) Scatter photons
3) Store photons



Photon Tracing

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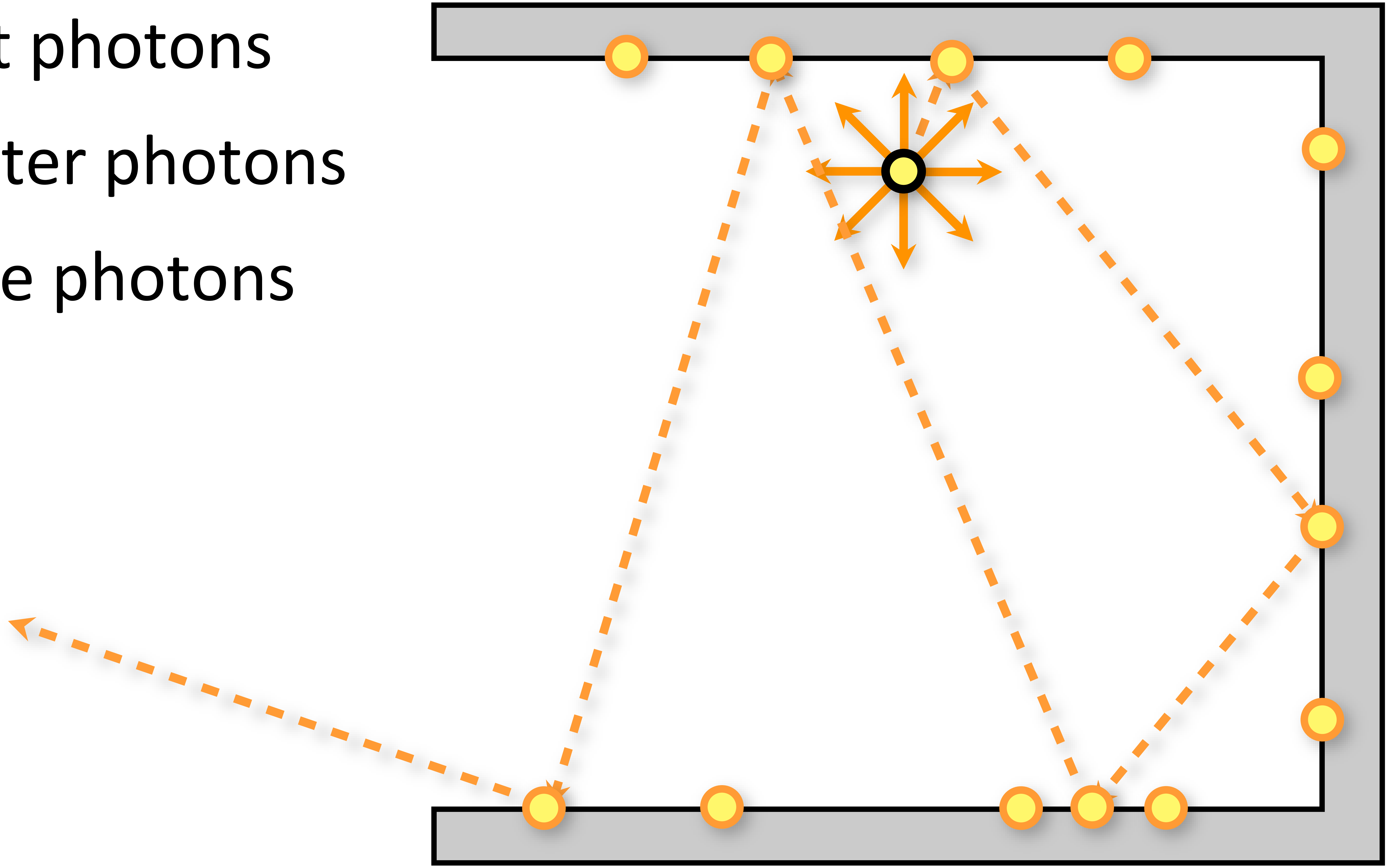

Photon Tracing

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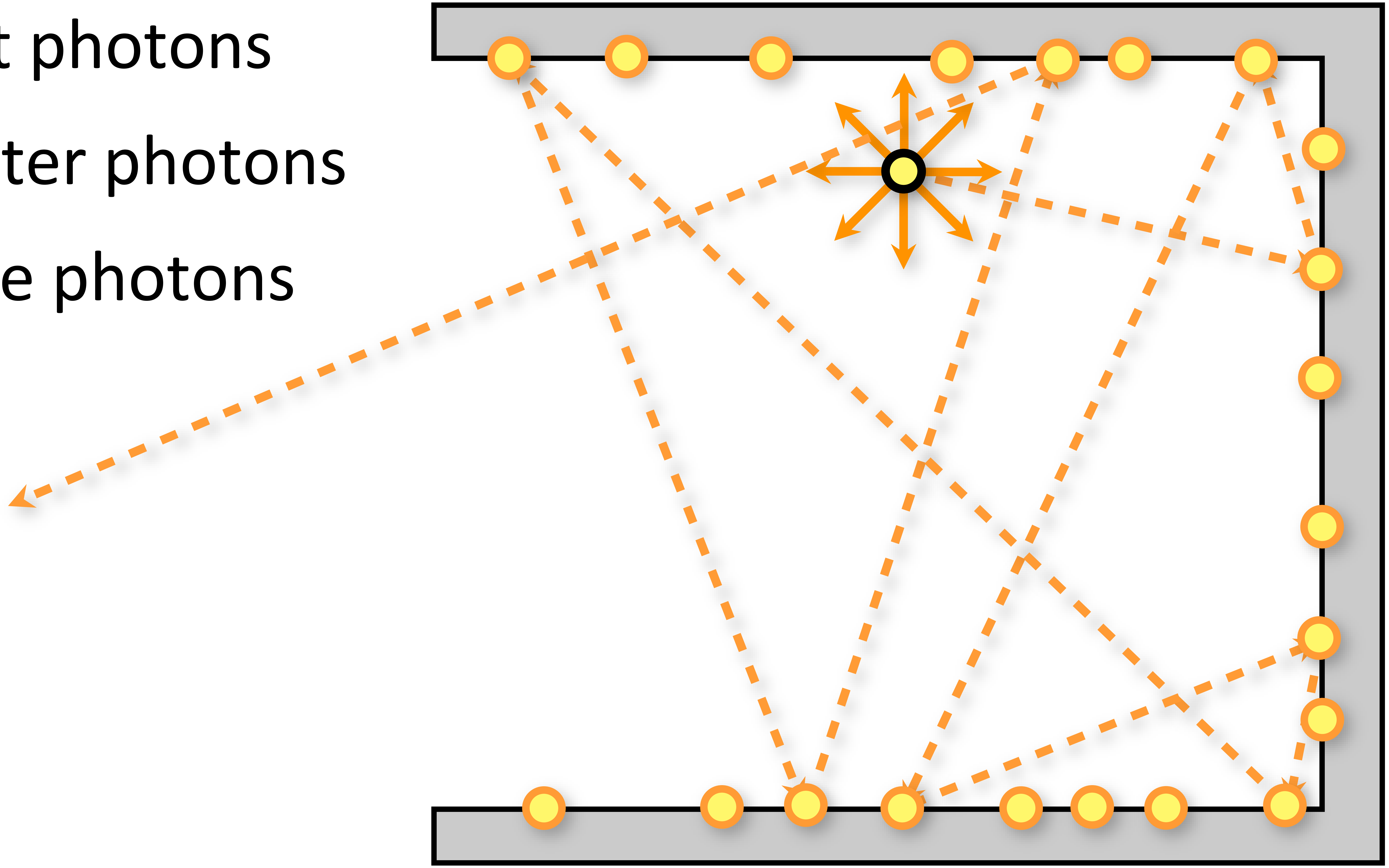
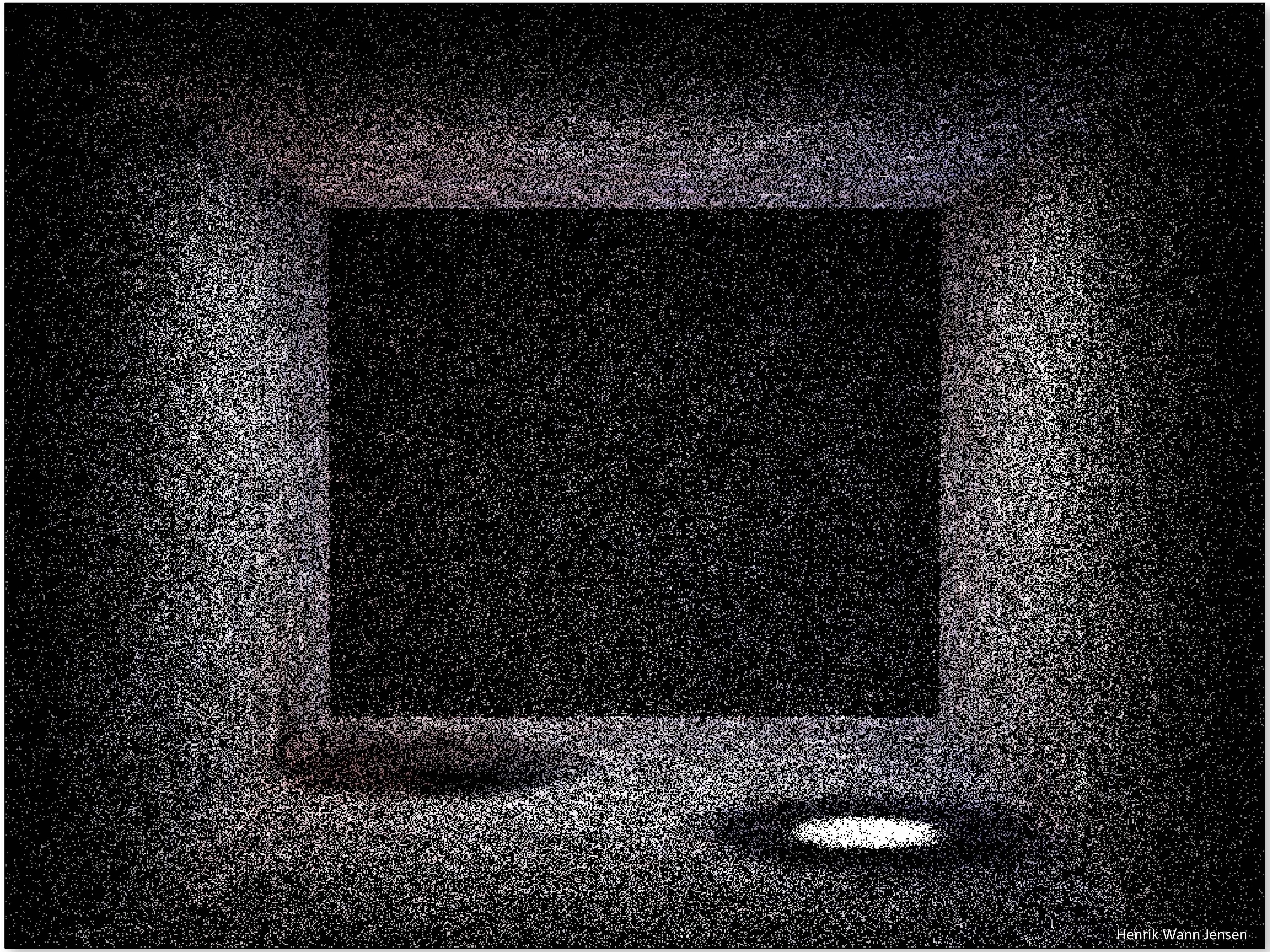


Visualization of the Photon Map



Photon Emission

Photons carry power (flux) not radiance!

- not a physical photon
- just a fraction of the light source power
- in most practical implementations, each photon carries multiple wavelengths (e.g. RGB)

Photon Emission

Define initial:

- $\mathbf{x}_p$: position
- $\vec{\omega}_p$: direction
- Φ_p: photon power

General recipe:

- Sample position on surface area of light with $p(\mathbf{x}_p)$
- Sample direction with $p(\vec{\omega}_p \mid \mathbf{x}_p)$

$$\Phi_p = \frac{1}{M} \frac{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos\theta_p}{p(\mathbf{x}_p) p(\vec{\omega}_p | \mathbf{x}_p)}$$

# of emitted photons

Aside: Can think of it as *one* term of an M-sample MC estimate of total flux

$$\Phi = \int_A \int_{H^2} L_e(\mathbf{x}, \vec{\omega}) \cos\theta \mathrm{d}\vec{\omega} \mathrm{d}\mathbf{x}$$

Photon Emission

Interesting derivation:

- if PDFs are proportional to the emission:

$$p(\mathbf{x}_p) = \frac{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos\theta \, d\vec{\omega}}{\int_A \int_{H^2} L_e(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} d\mathbf{x}} \qquad p(\vec{\omega}_p | \mathbf{x}_p) = \frac{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos\theta_p}{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos\theta \, d\vec{\omega}}$$

- then:

Total power of the light source

$$\Phi_p = \frac{1}{M} \frac{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos\theta_p}{p(\mathbf{x}_p) p(\vec{\omega}_p | \mathbf{x}_p)}$$

$$= \frac{1}{M} \frac{\cancel{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos\theta_p}}{\frac{\cancel{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos\theta \, d\vec{\omega}}}{\int_A \int_{H^2} L_e(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} d\mathbf{x}} \frac{\cancel{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos\theta_p}}{\cancel{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos\theta \, d\vec{\omega}}}} = \frac{\Phi}{M}$$

If you *perfectly importance sample* the emitted radiance,
just take the *total power* and divide by # of emitted photons.

Photon Emission Examples

Isotropic point light:

- Generate uniform random direction over sphere

Spotlight:

- Generate uniform random direction within spherical cap

Diffuse area light:

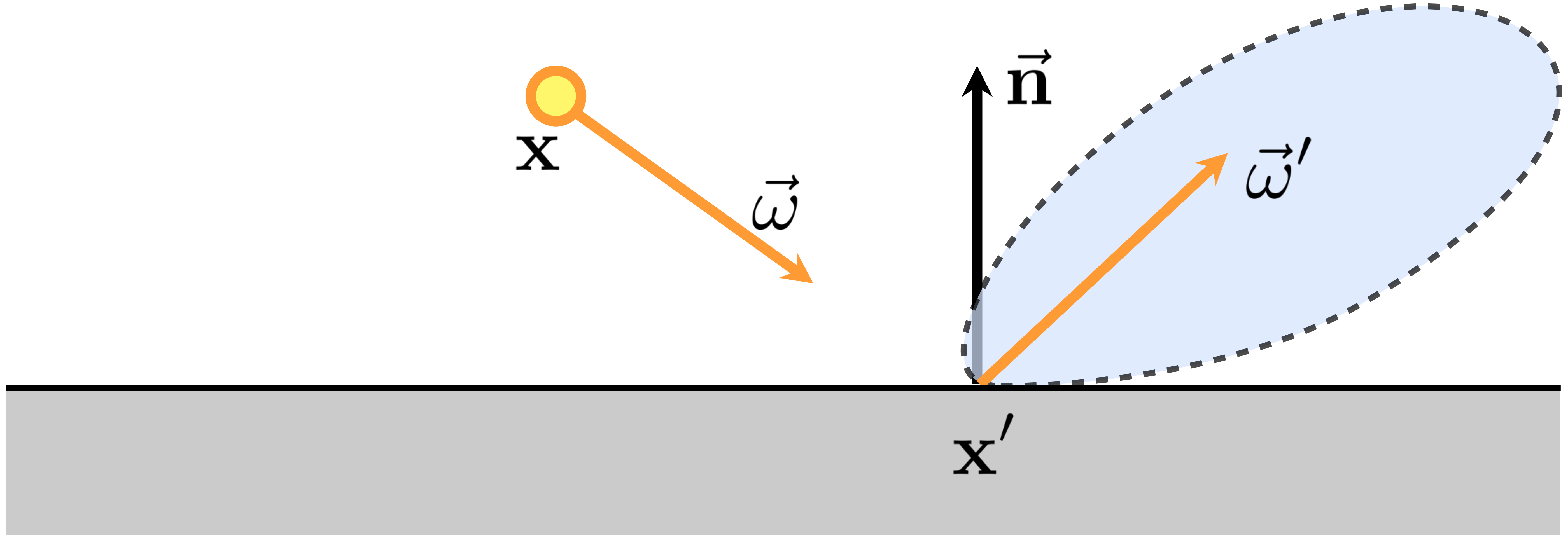
- Generate uniform random position on surface
- Generate cosine-weighted direction over hemisphere

Pseudocode

```
void generatePhotonMap()
    repeat:
    (l, Probₗ) = chooseRandomLight()
    (x, ω, Φ) = emitPhotonFromLight(l)
    tracePhoton(x, ω, Φ / Probₗ)
    until we have enough photons;
    divide all photon powers by number of emitted photons

void tracePhoton(x, ω, Φ)
```

Pseudocode




```
void tracePhoton(x, ω, Φ)
      (x’, n) = nearestSurfaceHit(x, ω)
      possiblyStorePhoton(x’, ω, Φ)
      (ω’, pdf) = sampleBSDF(x’, -ω)
      Φ’ = Φ * absDot(n, ω’) * BSDF(x’, -ω, ω’) / pdf
      tracePhoton(x’, ω’, Φ’)
```

Storing Photons

Store only on diffuse (or moderately glossy) surfaces

- Specular surfaces need to be handled using path tracing from the camera

Stored data: [36 bytes]

```
 struct Photon
{
   float position[3];
   float power[3];
   float direction[3];
 };
```

Storing Photons

Store only on diffuse (or moderately glossy) surfaces

- Specular surfaces need to be handled using path tracing from the camera

Stored data:

```
 struct Photon
{
   float position[3];
   char power[4];      // Packed RGBE format
   char phi, theta; // Packed direction
 };
```

Scattering of Photons

Photons can be:

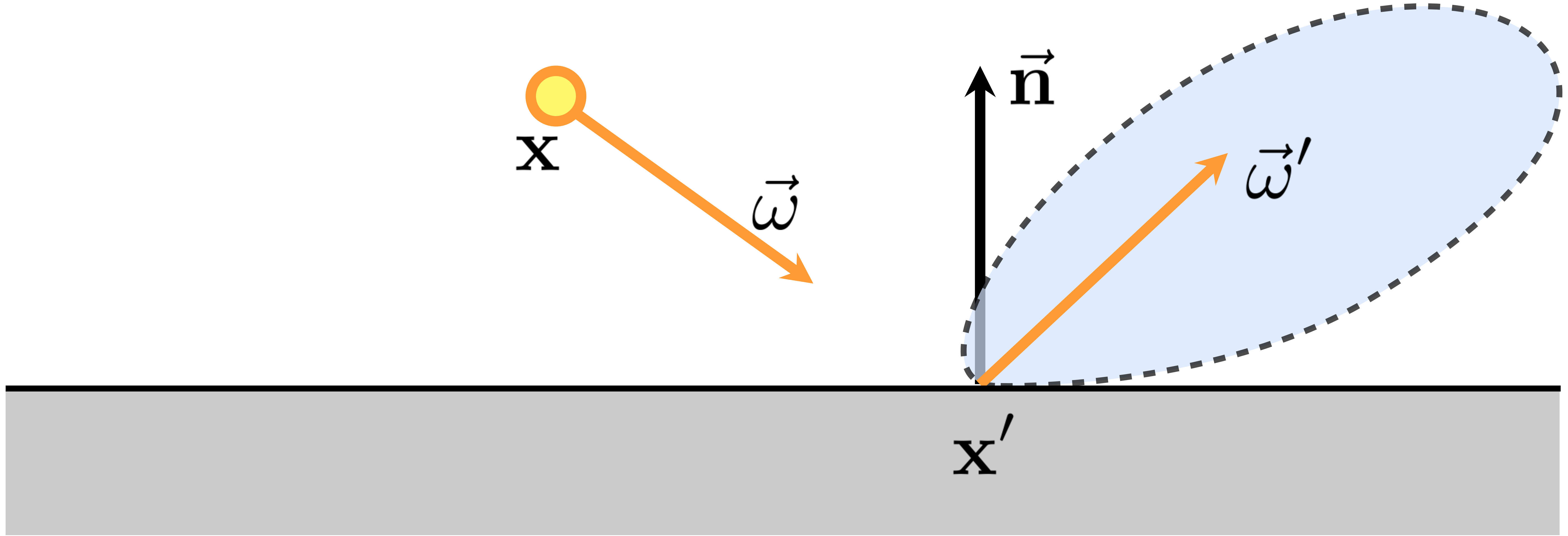
- absorbed, or scattered (reflected or refracted)
- BSDF sampling chooses either reflection or refraction
- the power of the scattered photon is lowered to account for absorption

Problem:

- as photons bounce they carry less and less power
- ideally all stored photons would have the same power
- also, when should we terminate the recursion?

Solution: Russian roulette

Pseudocode

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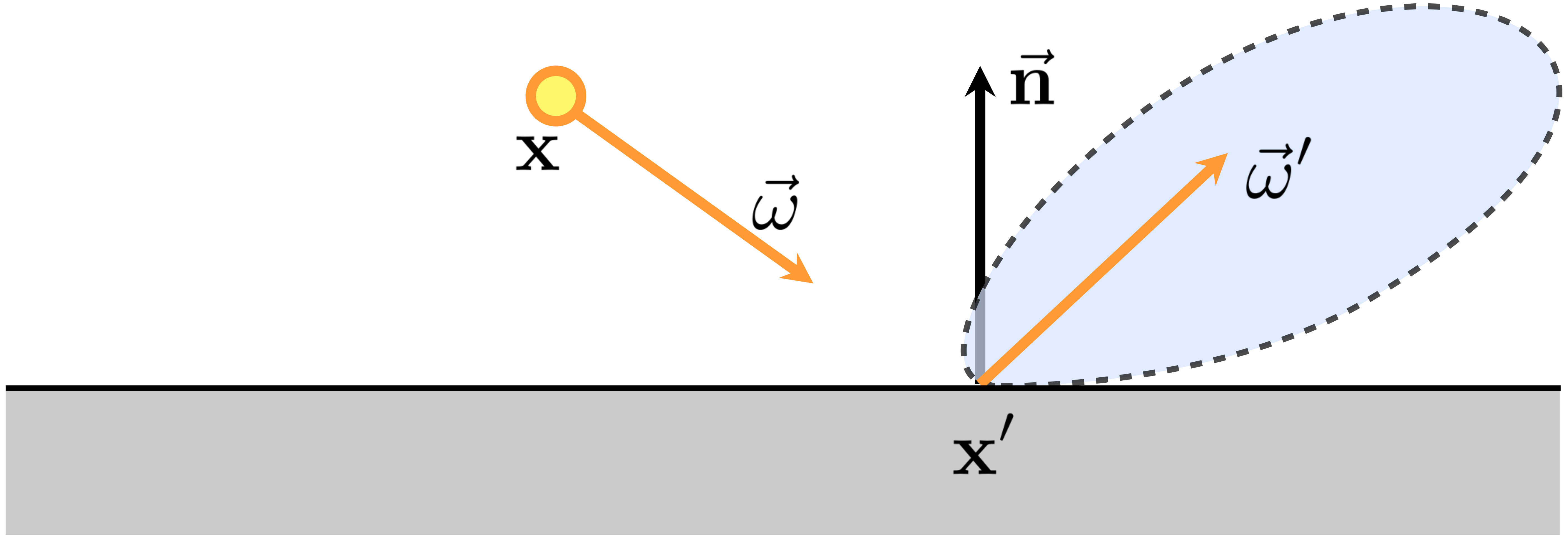
    Φ' = Φ * absDot(n, ω') * BSDF(x', -ω, ω') / pdf
    tracePhoton(x', ω', Φ')
```

Pseudocode

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    (ω', pdf) = sampleBSDF(x', -ω)
    Φ' = Φ * absDot(n, ω') * BSDF(x', -ω, ω') / pdf
    if survivedRussianRoulette(Φ, Φ')
      tracePhoton(x', ω', Φ')
```

Photon Path Termination

Probabilistically terminate the photon walk using Russian roulette (continue with prob. p)

$$E[F'] = (1 - p) \cdot 0 + p \cdot \frac{E[F]}{p} = E[F]$$

Option 1: local termination probability:

$$p = \min\left(1, \frac{\Phi'}{\Phi}\right)$$

Photon Path Termination

```
bool survivedRussianRoulette(Φ, Φ’)
    p = min(1, Φ’/Φ)
    if rand() > p:
        // terminate
        return false
    else:
        // continue with re-weighted power
        Φ’/= p
        return true
```

if Φ'/Φ is smaller than 1, then $\Phi' = \Phi'/p = \Phi$

i.e., the scattered photon has the same power!

Photon Path Termination

Probabilistically terminate the photon walk using Russian roulette (continue with prob. p)

$$E[F'] = (1-p)\cdot 0 + p \cdot \frac{E[F]}{p} = E[F]$$

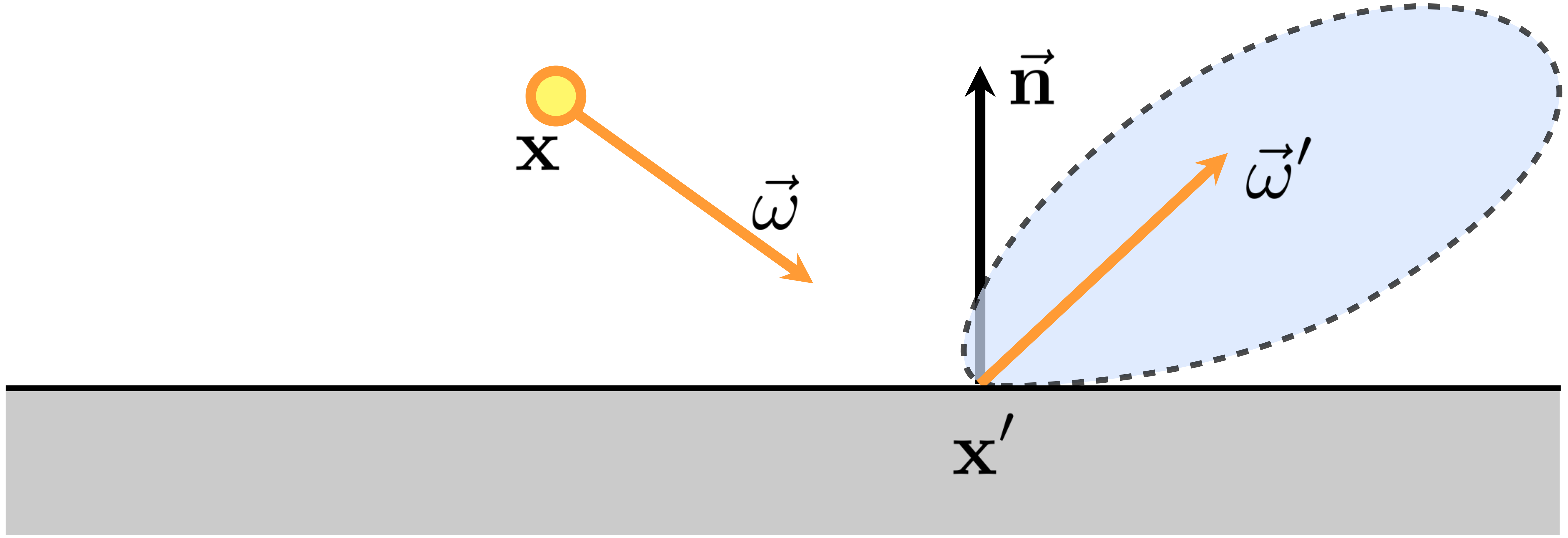
Option 1: local termination probability:

$$p = \min\left(1, \frac{\Phi'}{\Phi}\right)$$

Option 2: history-aware termination probability:

- try to keep each photon same power

Pseudocode

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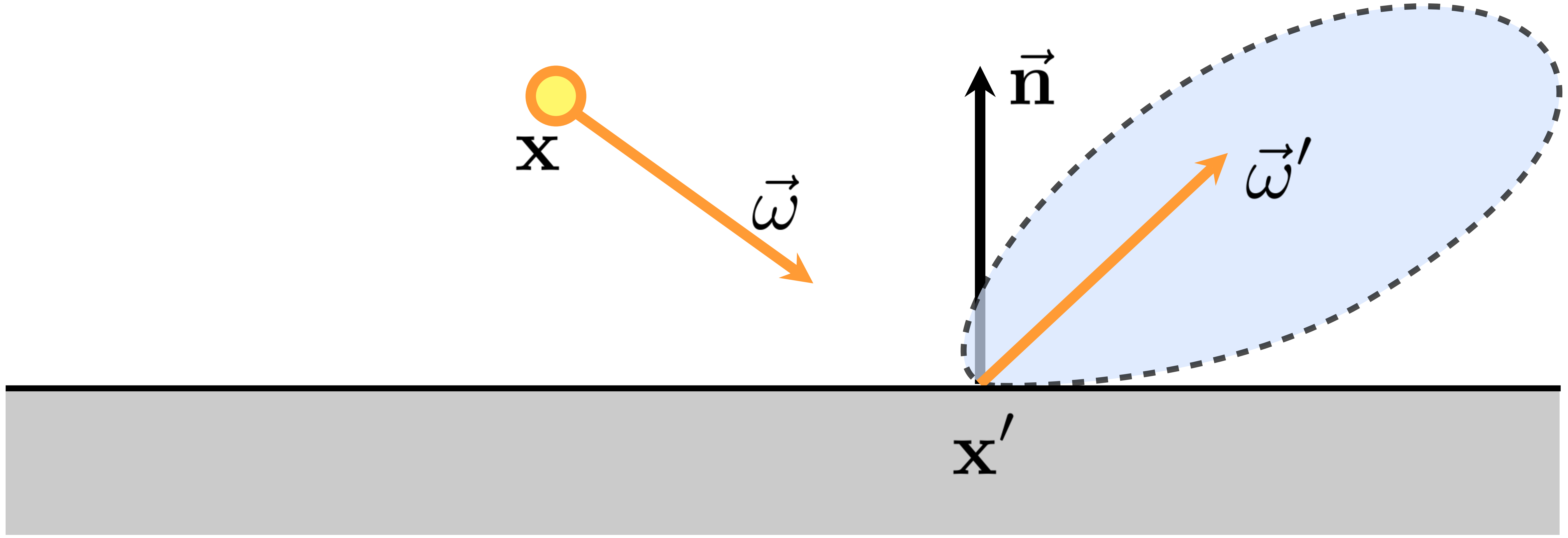
    if survivedRussianRoulette(Φ, Φ’)
      tracePhoton(x’, ω’, Φ’, Φ)
```

Pseudocode




```
void tracePhoton(x, ω, Φ, Φorig)
    (x', n) = nearestSurfaceHit(x, ω)
    possiblyStorePhoton(x', ω, Φ)
    (ω', pdf) = sampleBSDF(x', -ω)
    Φ' = Φ * absDot(n, ω') * BSDF(x', -ω, ω') / pdf
    if survivedRussianRoulette(Φorig, Φ')
      tracePhoton(x', ω', Φ', Φorig)
```

Russian Roulette Example

300 photons with power 1.0 W hit a surface with reflectance 50%

Instead of reflecting 300 photons with power 0.5 W, RR will make ~150 photons continue with power 1.0 W

Very important!

Photon Mapping

A two-pass algorithm:

- Pass 1: Tracing of photons from light sources, and caching them in a photon map

- Pass 2: Tracing from the eye and approximating indirect illumination using the photons

Photon Mapping

A two-pass algorithm:

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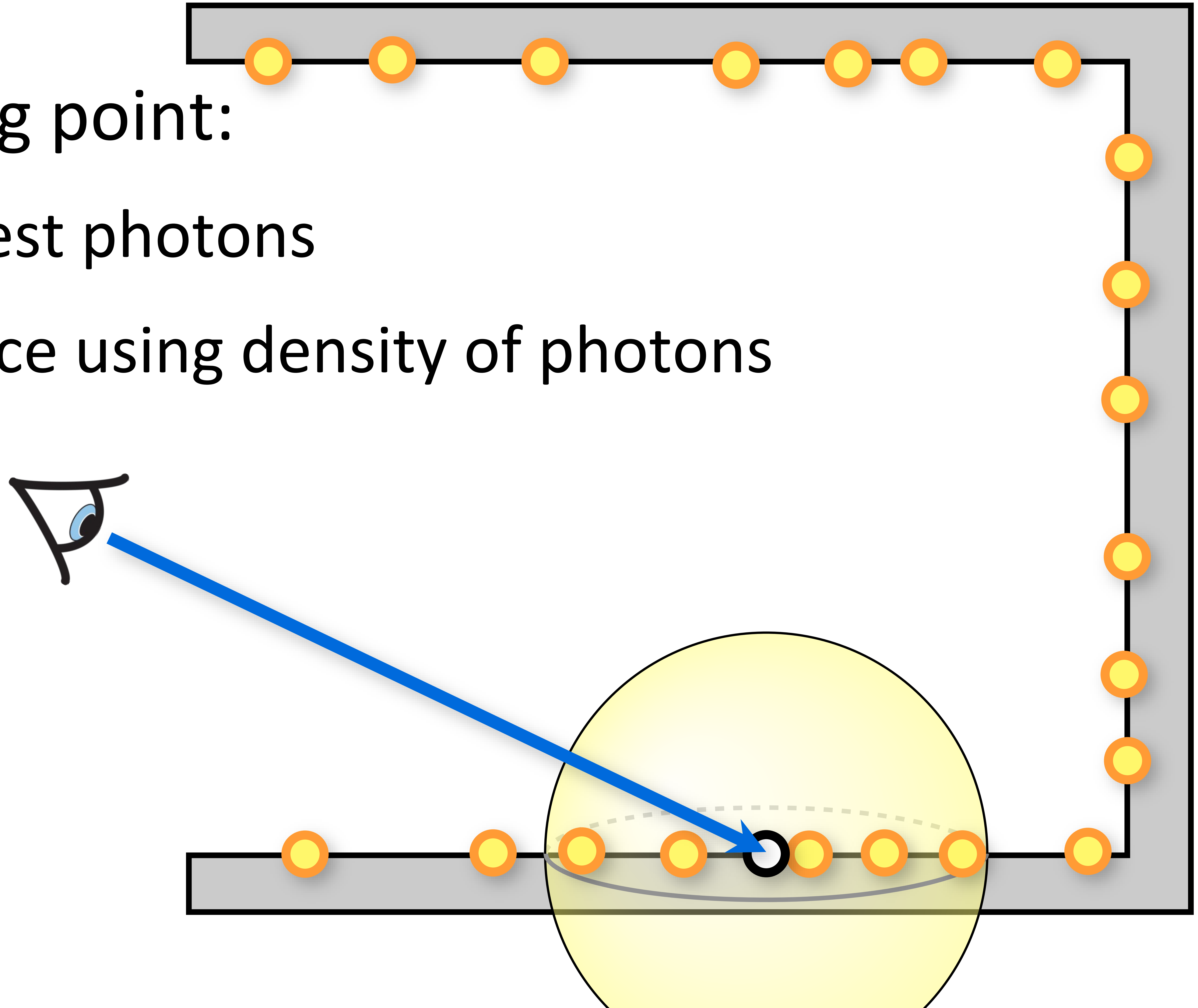
Rendering

For each shading point:

- Find the k closest photons
- Approx. radiance using density of photons



Rendering

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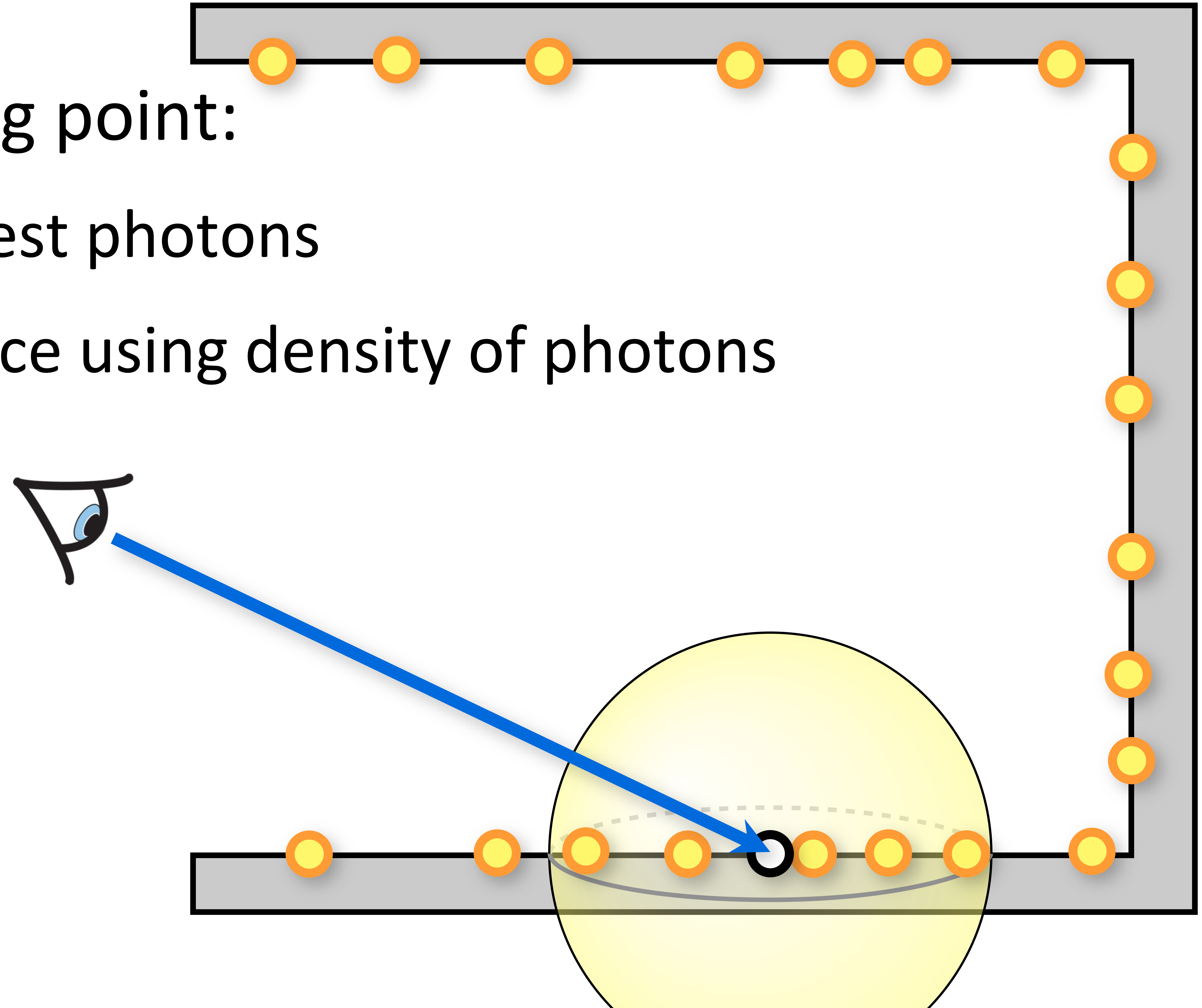
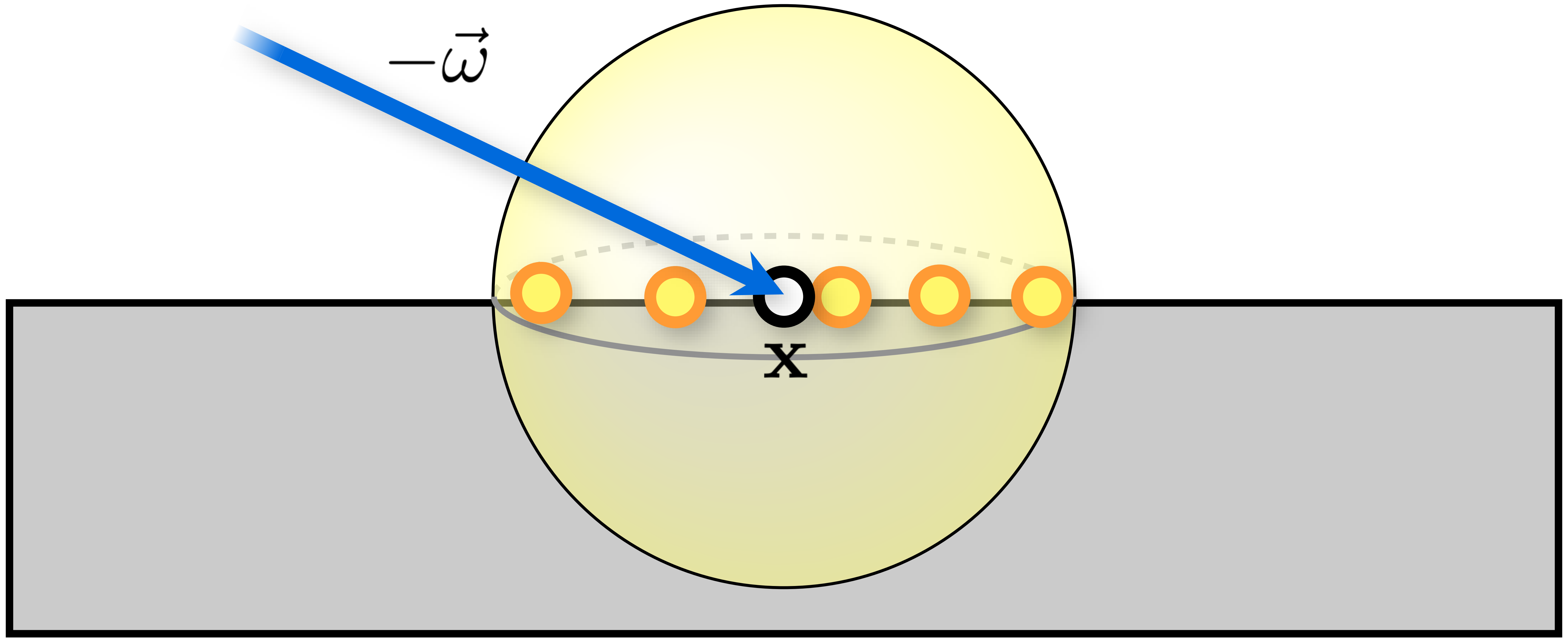


The Radiance Estimate

Based on kernel density estimation

- Non-parametric way of estimating the probability density of a random variable (photon density)



The Radiance Estimate

Based on kernel density estimation

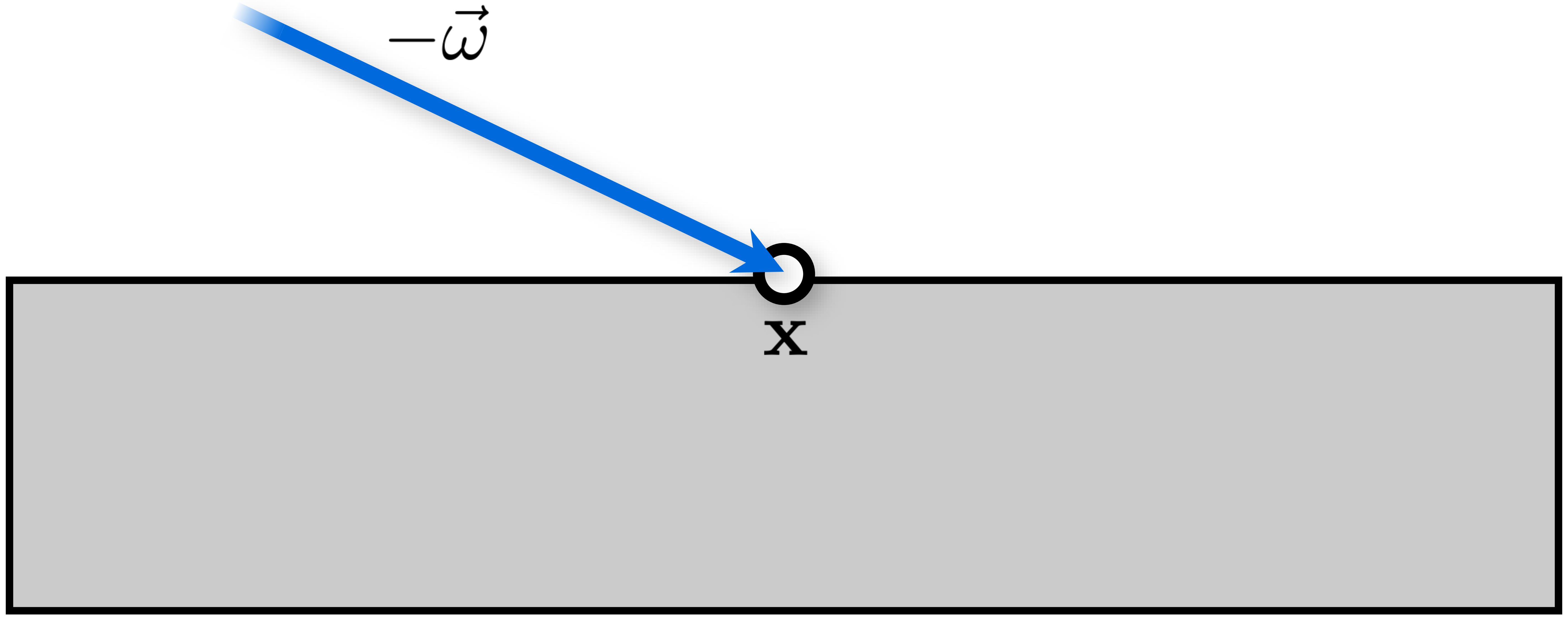
- Non-parametric way of estimating the probability density of a random variable (photon density)

$$
\begin{aligned}
L_r(\mathbf{x}, \vec{\omega}) &= \int_{H^2} f_r(\mathbf{x}, \vec{\omega}', \vec{\omega}) L_i(\mathbf{x}, \vec{\omega}') \cos\theta' d\vec{\omega}' \\
&= \int_{H^2} f_r(\mathbf{x}, \vec{\omega}', \vec{\omega}) \frac{d\Phi^2(\mathbf{x}, \vec{\omega}')}{\cos\theta' d\vec{\omega}' dA} \cos\theta' d\vec{\omega}' \\
&= \int_{H^2} f_r(\mathbf{x}, \vec{\omega}', \vec{\omega}) \frac{d\Phi^2(\mathbf{x}, \vec{\omega}')}{dA} \\
&\approx \sum_{p=1}^{n} f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Delta\Phi_p(\mathbf{x}, \vec{\omega}_p)}{\Delta A}
\end{aligned}
$$

The Radiance Estimate

Approach 1: first define area, then find photons

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^{k} f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p}{A}$$



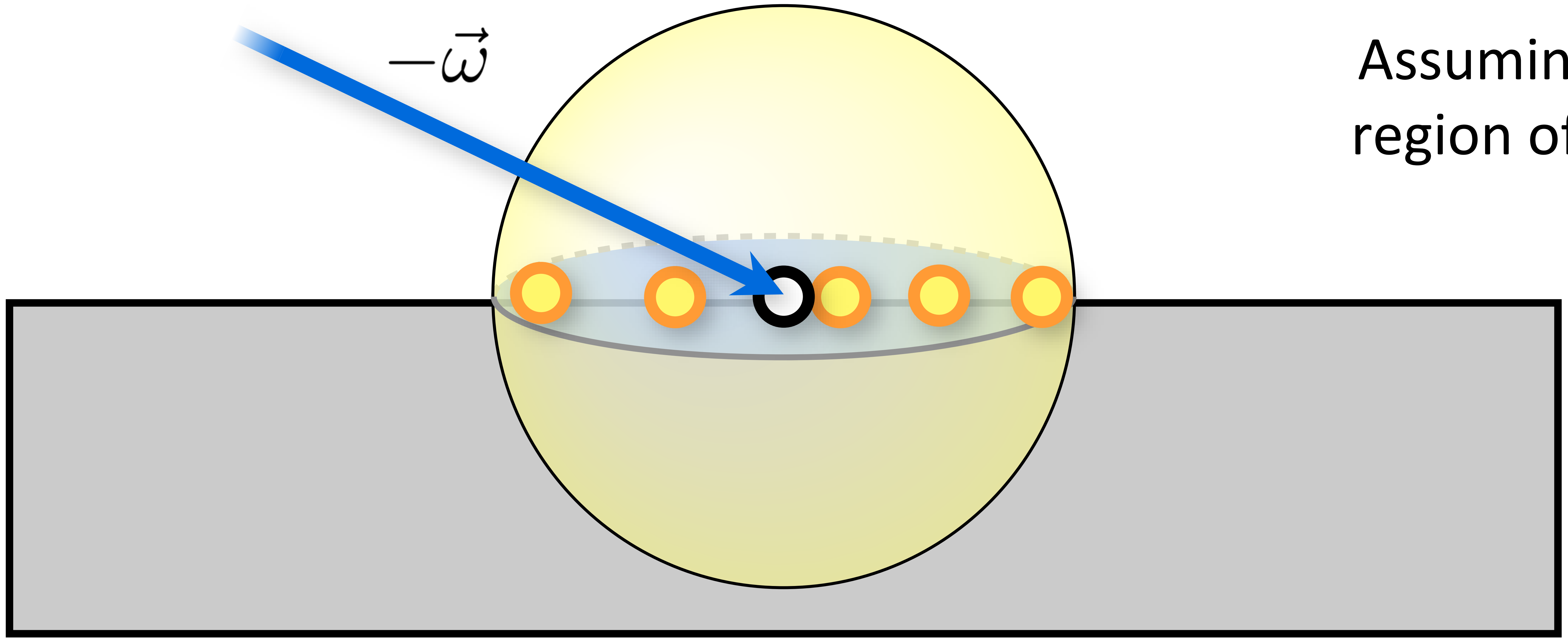
The Radiance Estimate

Approach 1: first define area, then find photons

$$L_r(\mathbf{x},\vec{\omega}) \approx \sum_{p=1}^{k} f_r(\mathbf{x},\vec{\omega}_p,\vec{\omega})\frac{\Phi_p}{\pi r^2}$$

# of photons within disk

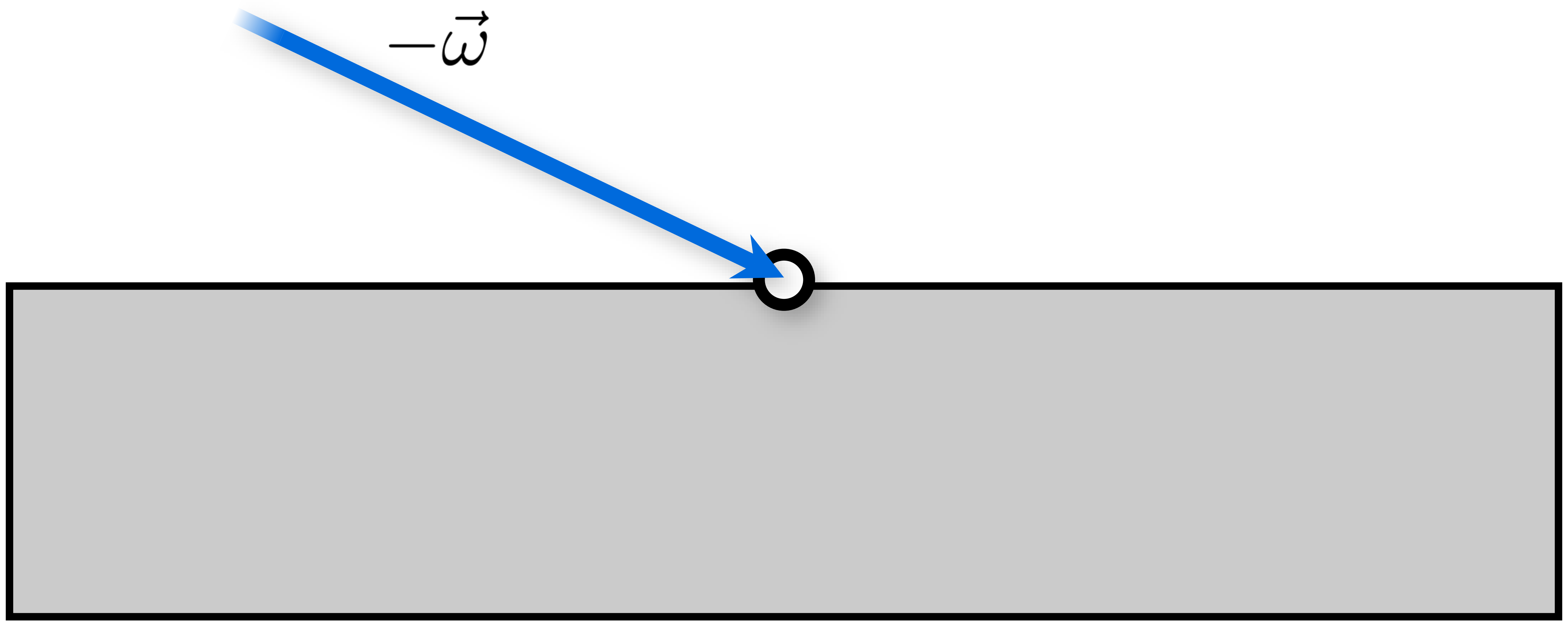
Assuming a disk region of radius r



The Radiance Estimate

Approach 2: first find k nearest photons, then define area

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^{k} f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p}{A}$$



The Radiance Estimate

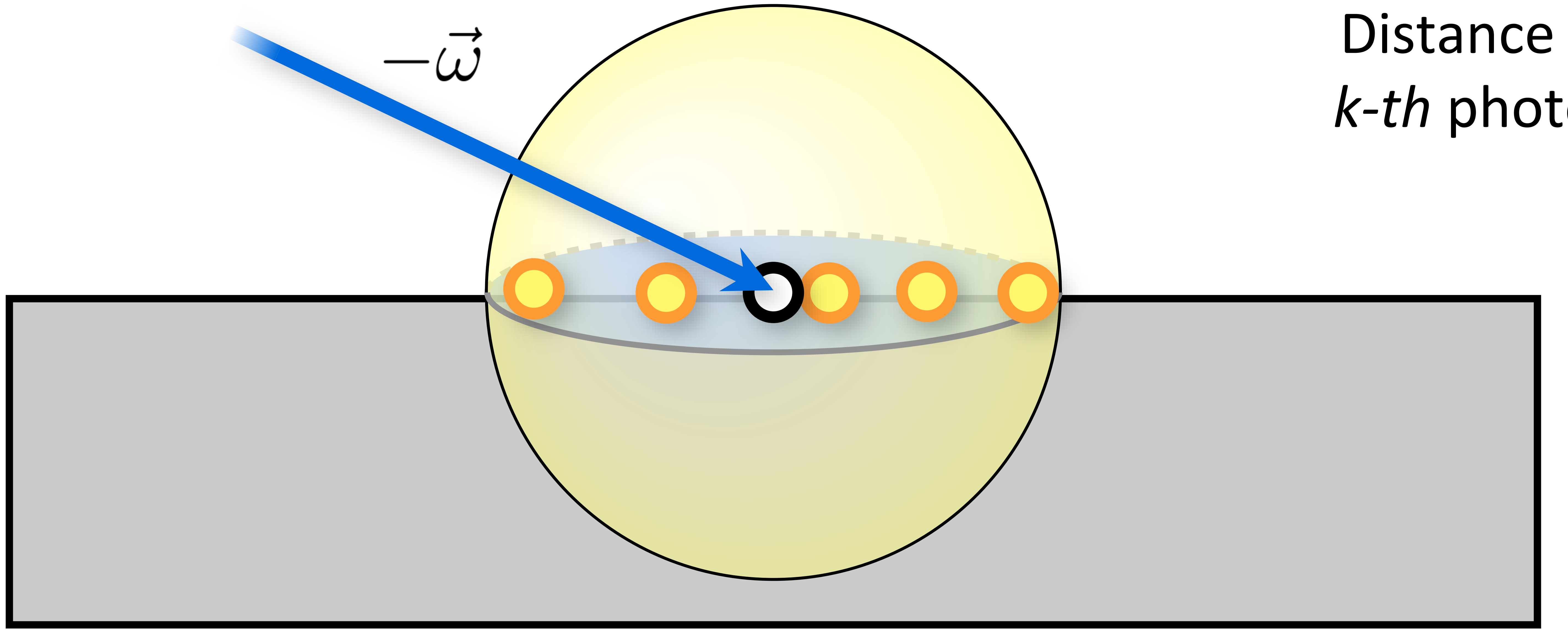
Approach 2: first find k nearest photons, then define area

Ignore the *k-th* photon

[García et al. 2012]

$$L_r(\mathbf{x},\vec{\omega}) \approx \sum_{p=1}^{k-1} f_r(\mathbf{x},\vec{\omega}_p,\vec{\omega})\frac{\Phi_p}{\pi r_k^2}$$

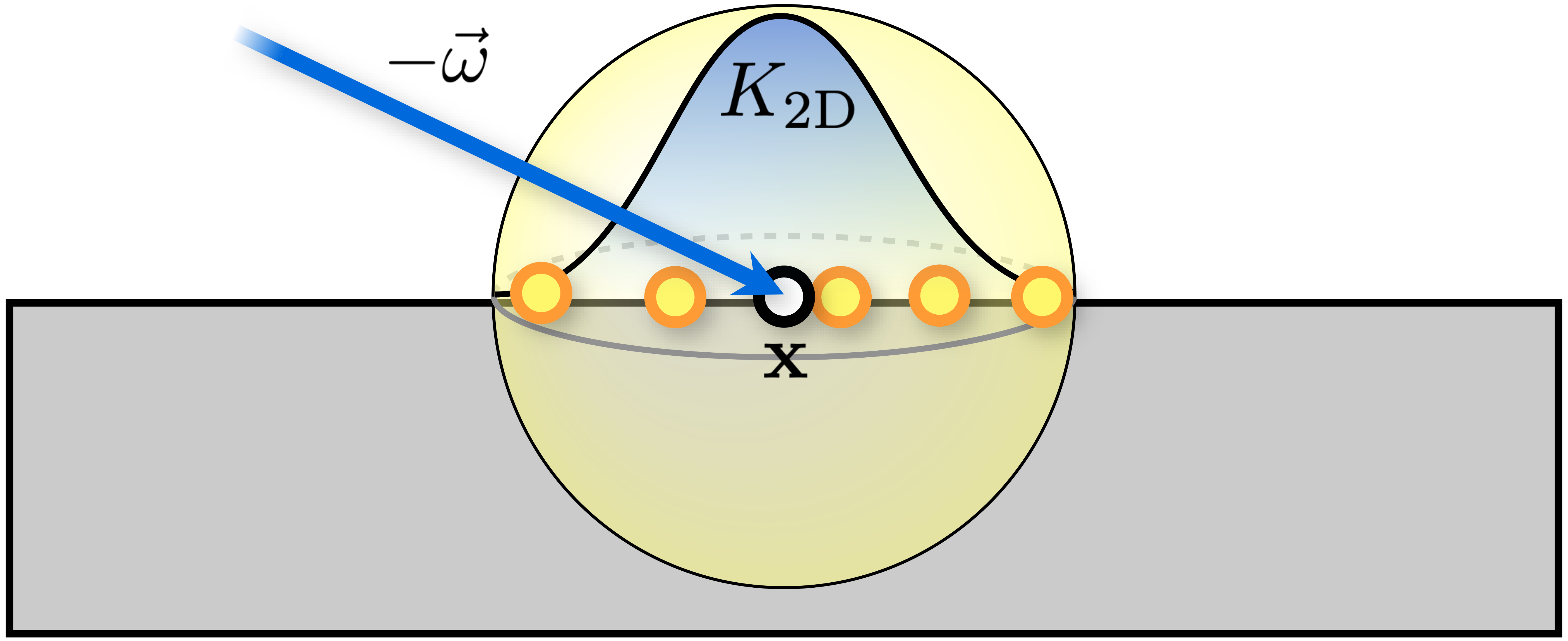
Distance to the *k-th* photon



The Radiance Estimate

Using a non-constant kernel:

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^{k-1} f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \Phi_p K_{2D}(r_p, r_k)$$

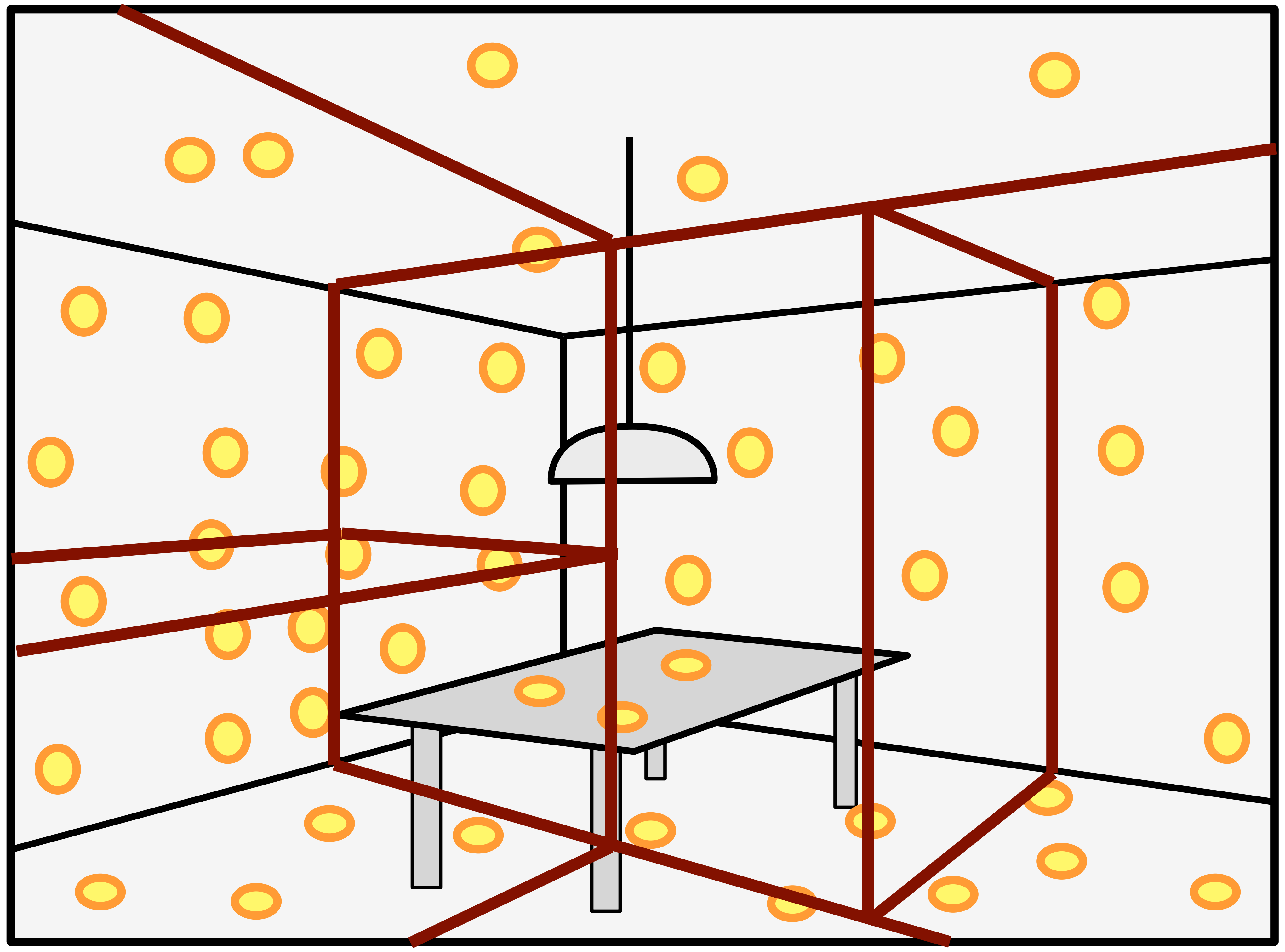


The Photon Map Data Structure

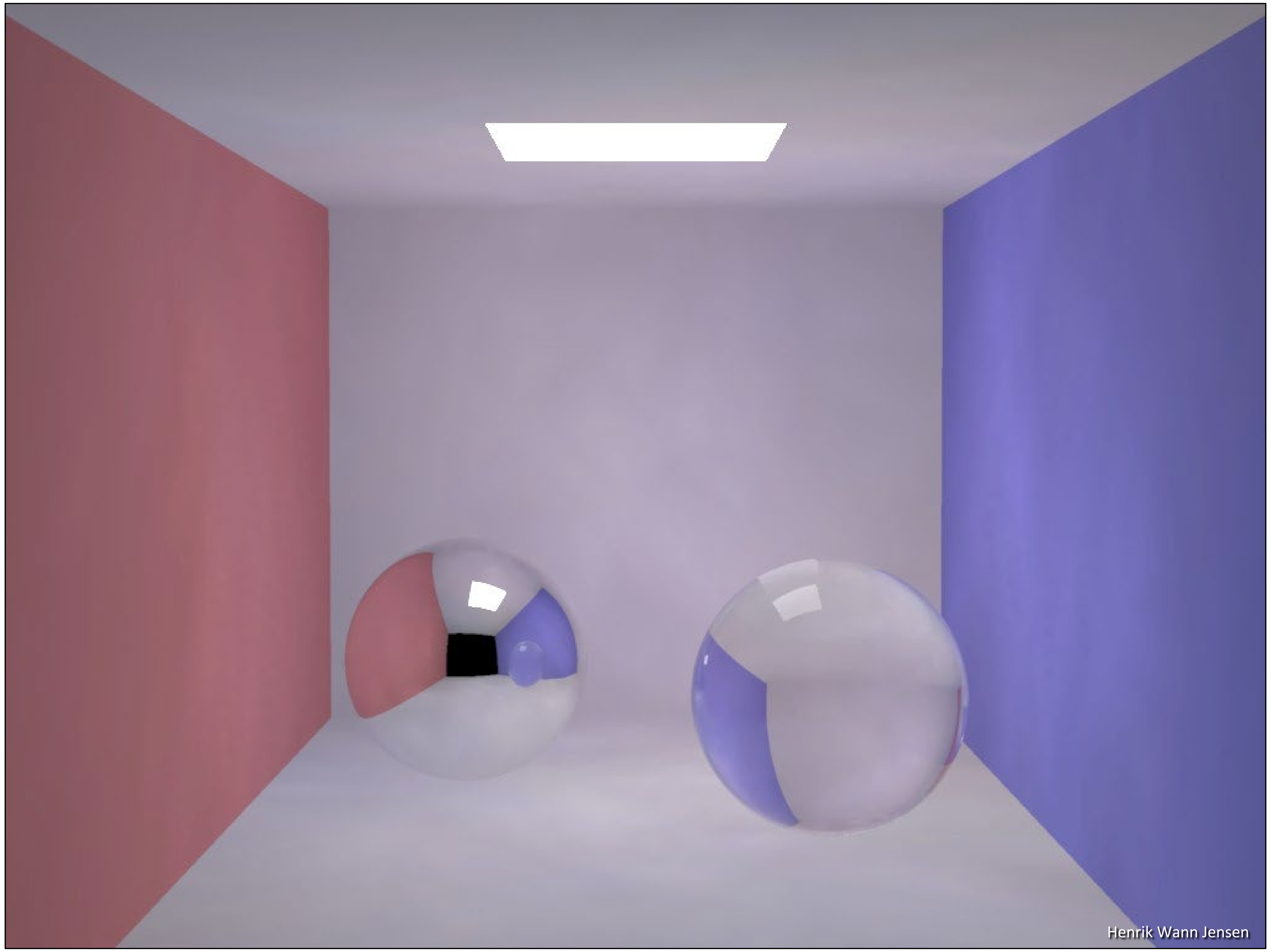
Requirements:

- Compact (we want many photons)
- Fast nearest neighbor search

KD-tree

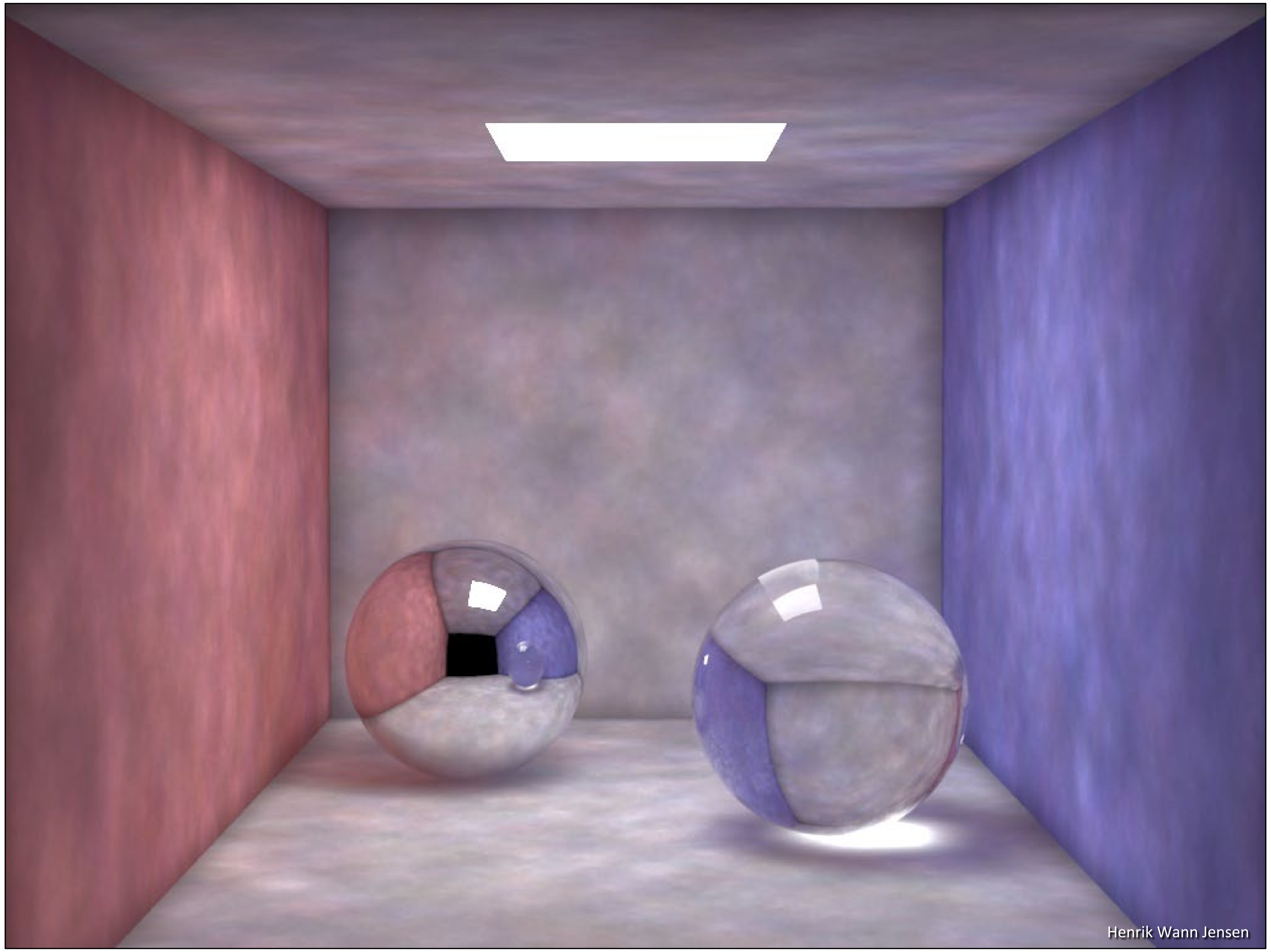


Photon Mapping



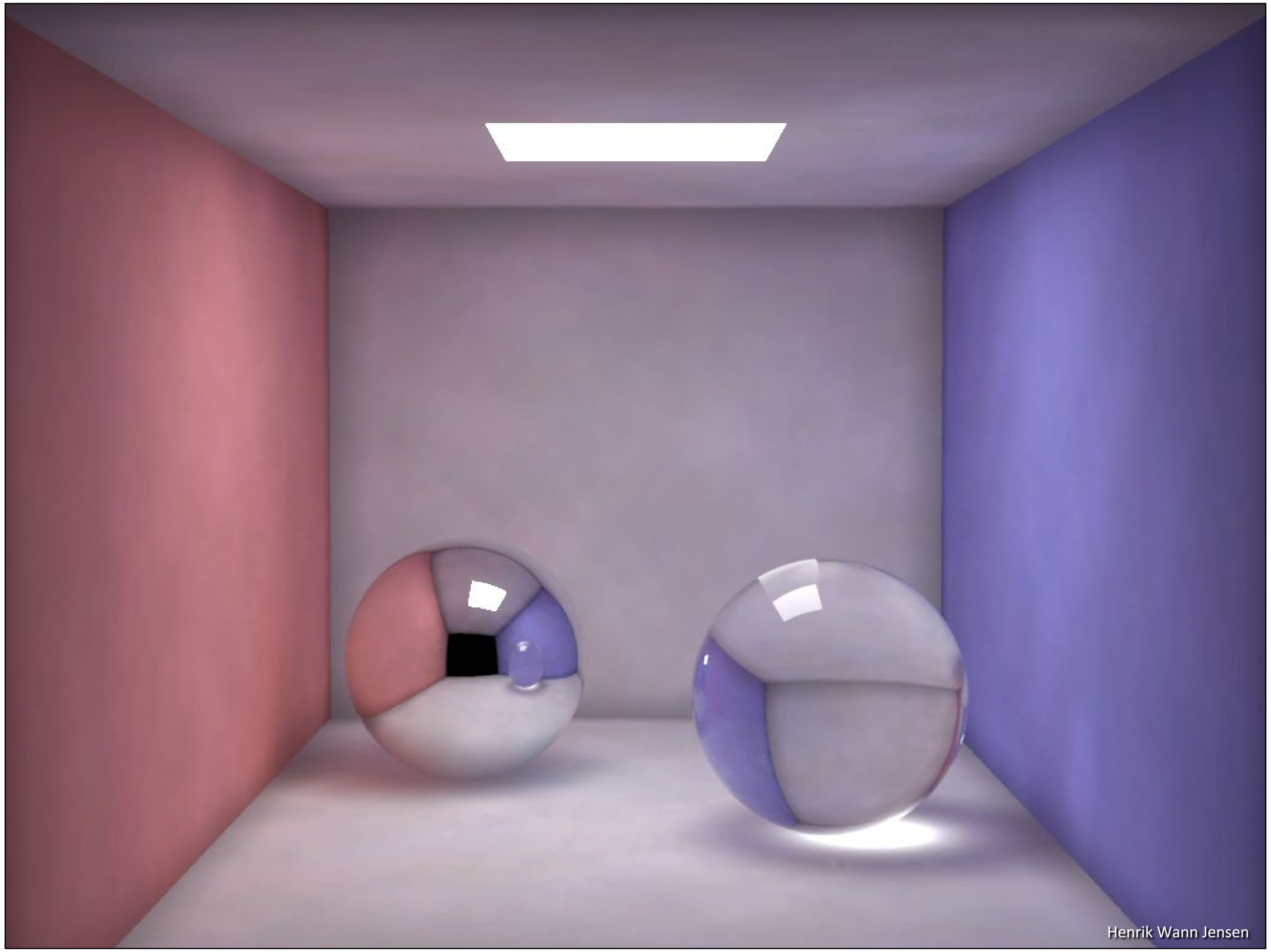
200 photons / 50 photons in radiance estimate

Photon Mapping



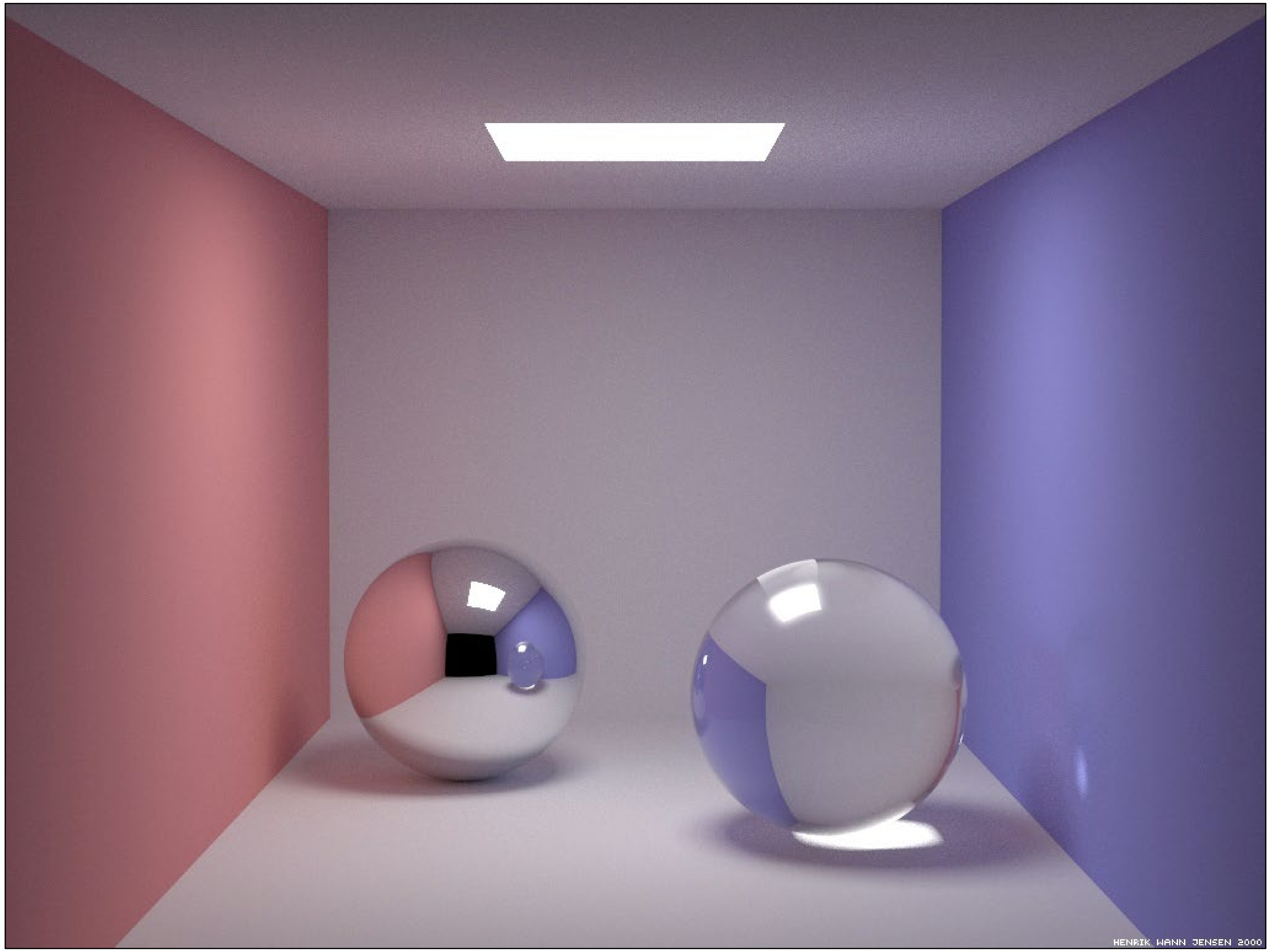
100,000 photons / 50 photons in radiance estimate

Photon Mapping

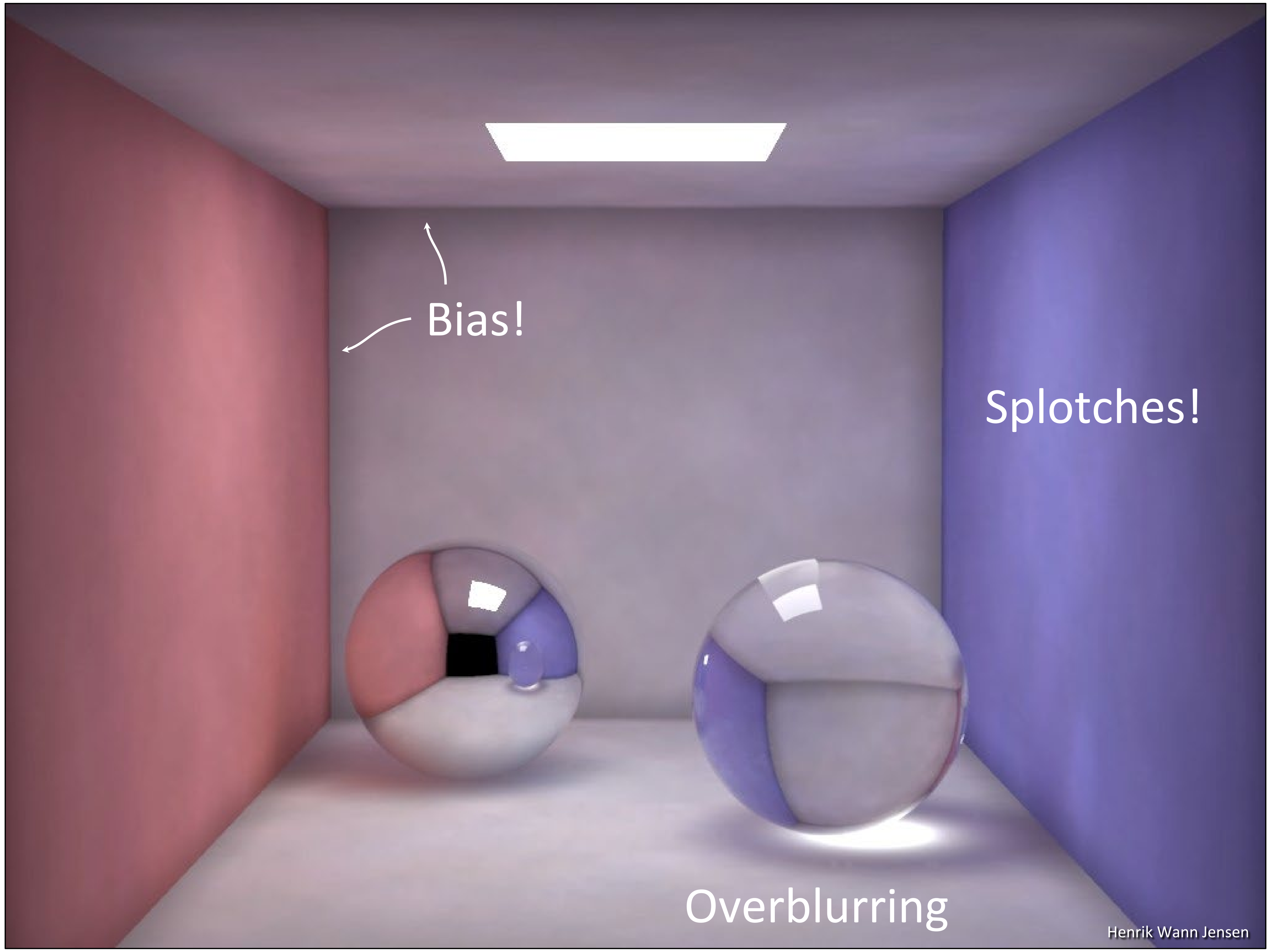


500,000 photons / 500 photons in radiance estimate

Path Tracing



Photon Mapping



500,000 photons / 500 photons in radiance estimate

Photon Mapping

Radiance estimate contains error/bias

- Produces darker/brighter, blotchy, blurry appearance
- Requires *many* photons for high quality

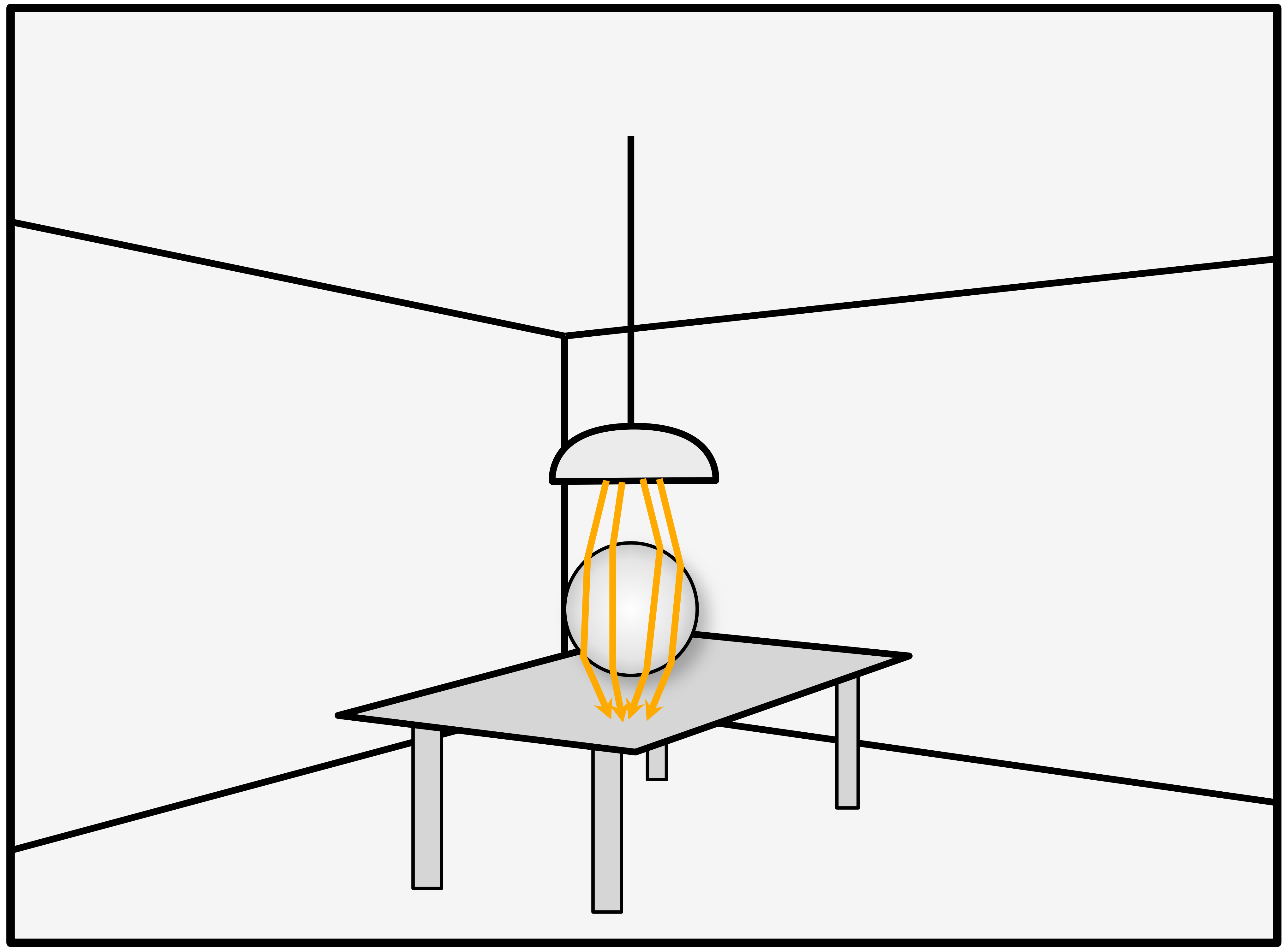
Split up lighting computation into components:

- Direct lighting
- Caustics (caustic photon map)
- Remaining indirect illumination (global photon map)

Improving Caustics

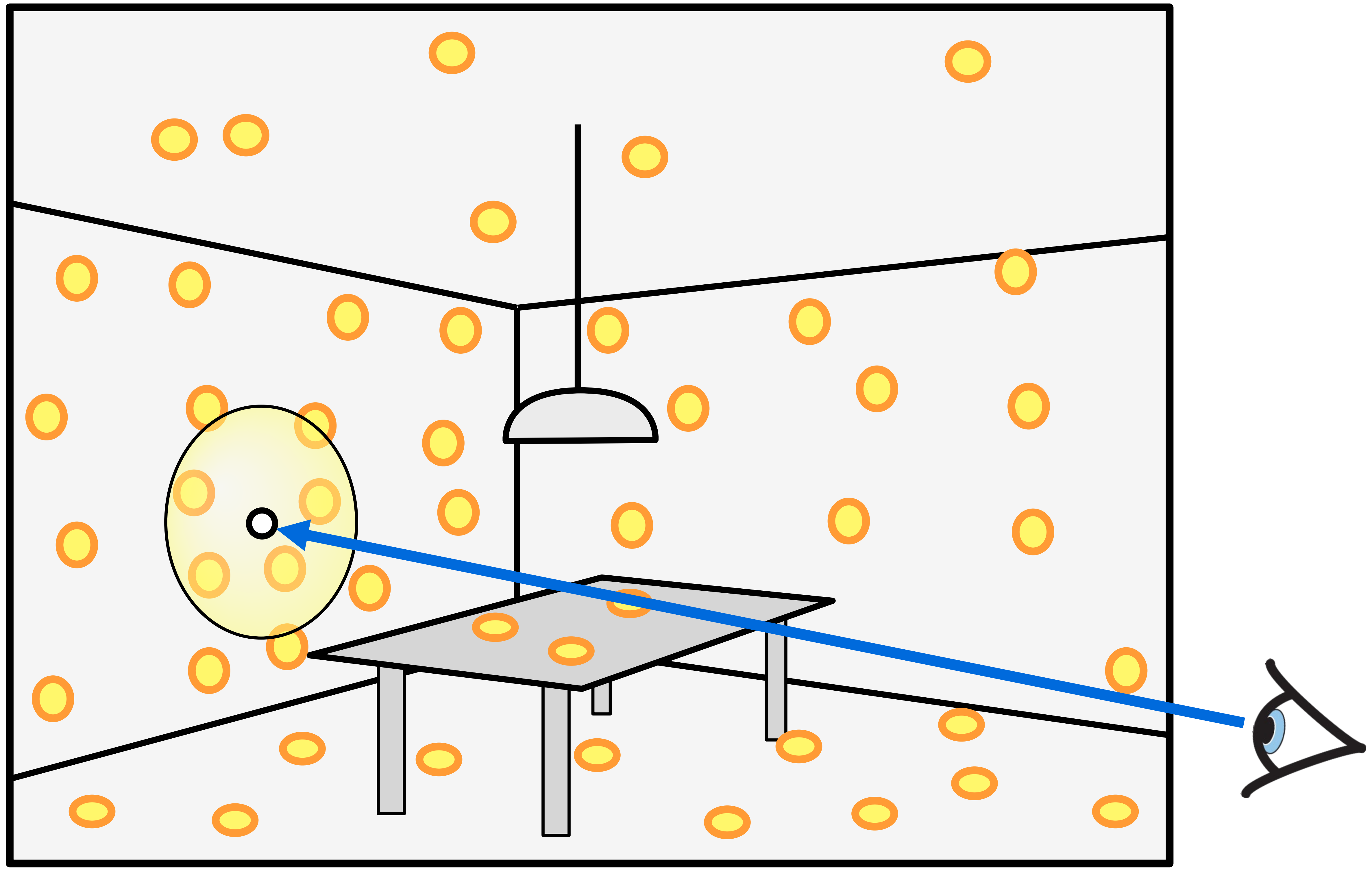
Higher quality photon map for caustics

- Only stores $LS^{+}D$ paths
- Many photons shot directly at specular objects



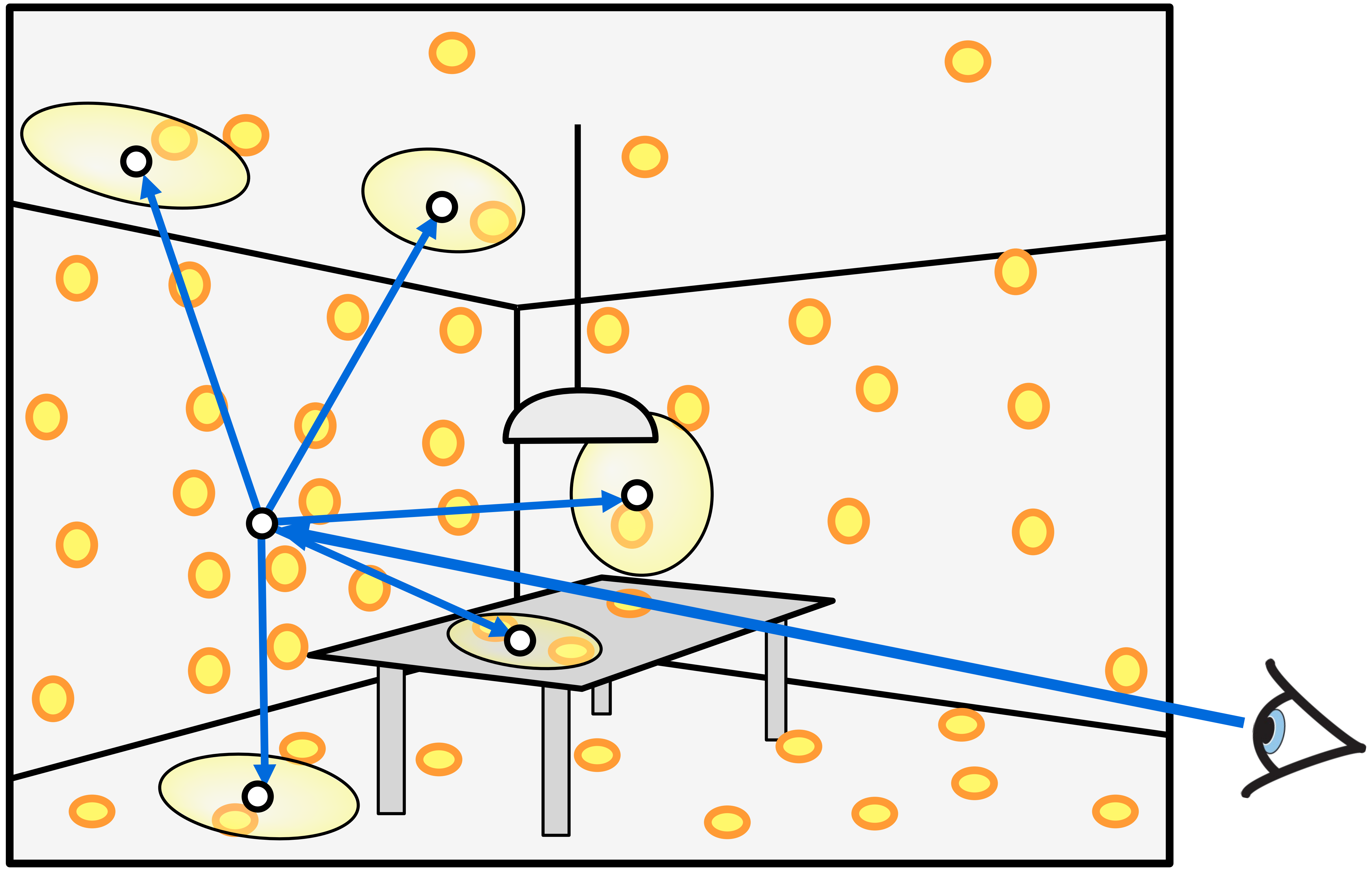
Improving Remaining Indirect

Original approach: direct density estimation



Improving Remaining Indirect

Improved approach: using *final gather* (i.e., path trace until second non-specular surface from camera)



Improved Photon Mapping

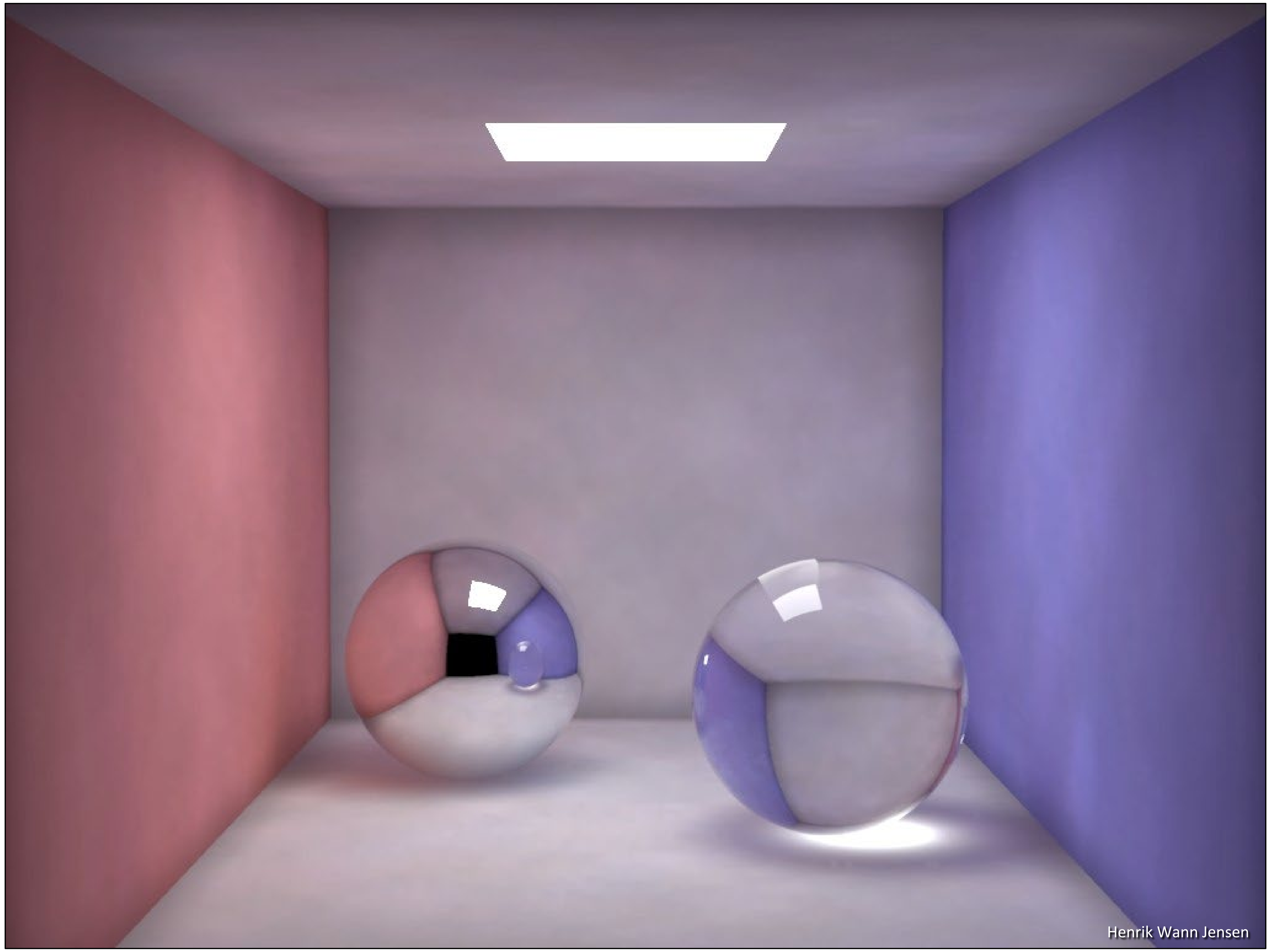
Camera tracing

- Trace camera paths until they hit the first non-specular surface point $\mathbf{x}$

At $\mathbf{x}$ we sum:

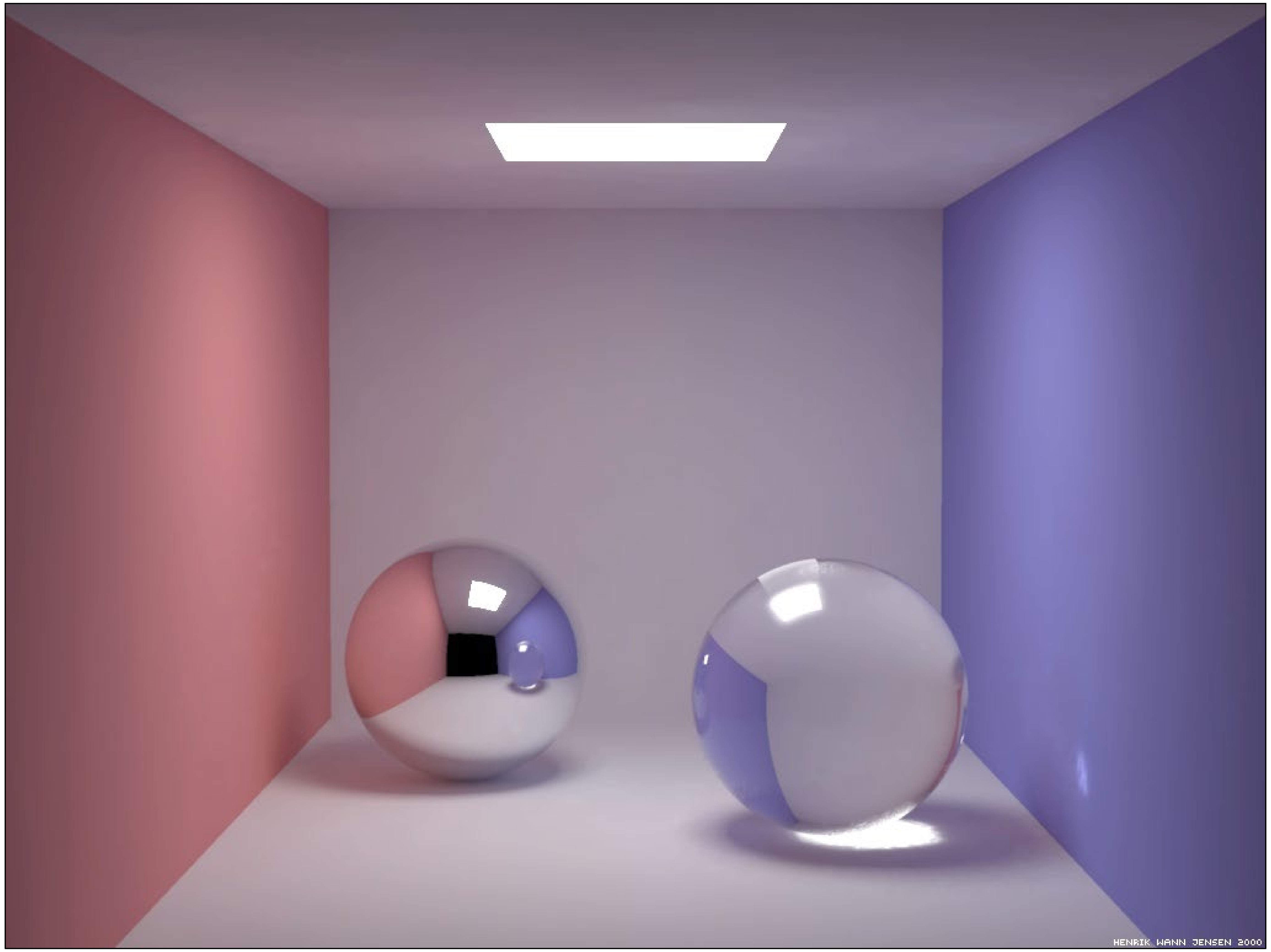
- Emission
- Direct illumination: trace shadow rays to lights
- Caustics: density estimation at $\mathbf{x}$ using only the *caustic* photon map
- Remaining indirect: continue path tracing until next non-specular vertex $\mathbf{y}$, perform density estimation from global photon map at $\mathbf{y}$

Photon Mapping



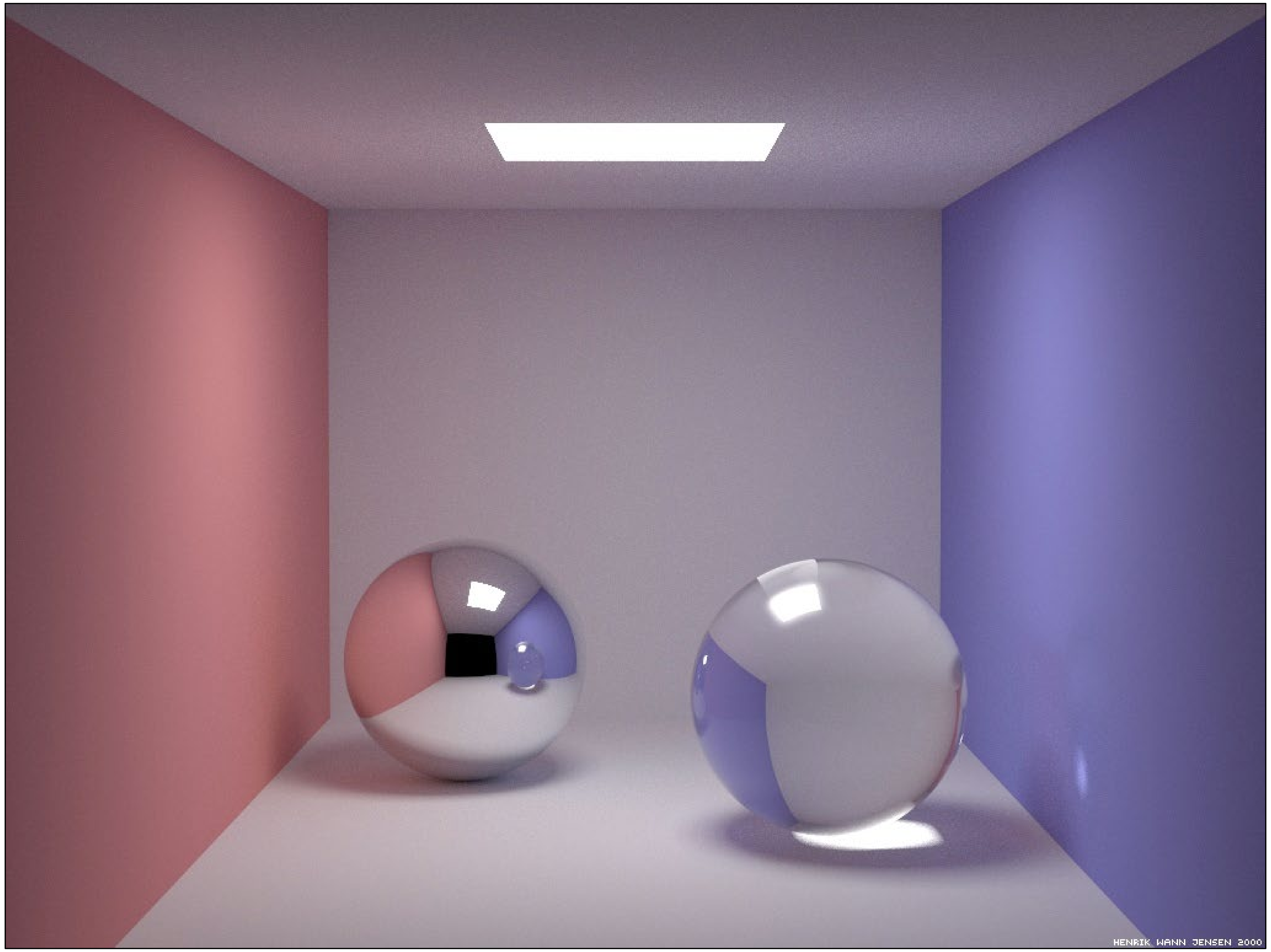
500000 photons / 500 photons in radiance estimate

Photon Mapping (Improved)



final gather + global photon map (200000) + caustic photon map (50000)

Path Tracing



Validation Tests

Test idea 1:

- store only direct photons
- visualize photon map directly
- compare to standard direct illumination
- should look identical with many photons

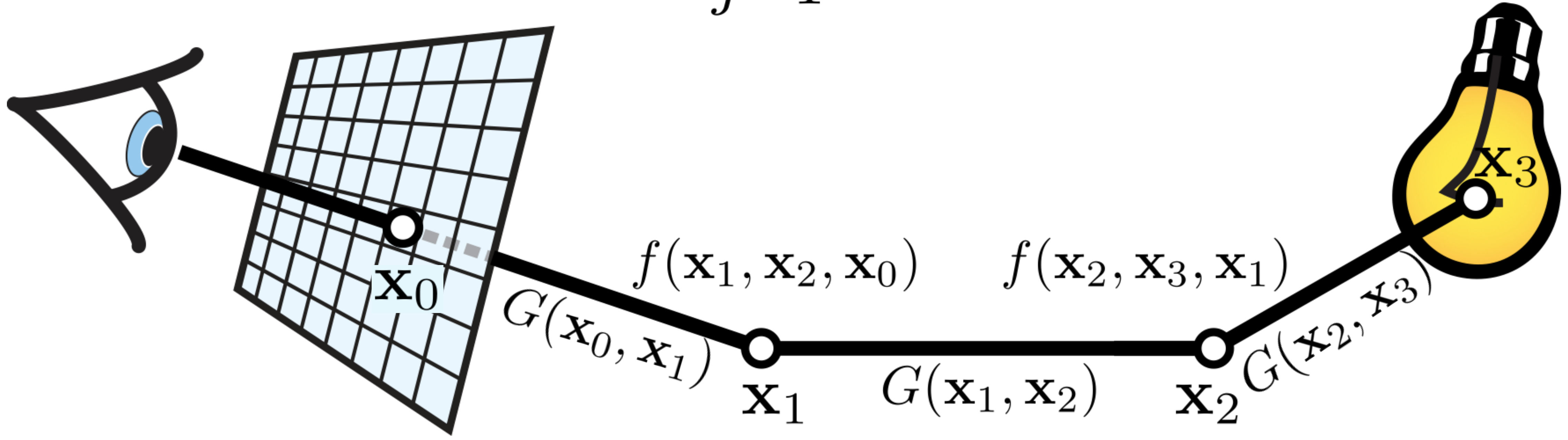
Test idea 2:

- create a perfectly transparent sphere (IOR = 1.0)
- store only caustic photons
- render direct illumination + caustics
- shadow should disappear

Recall: Path Integral Measurement Eq.

$$I_j = \int_{\mathcal{P}} W_e(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) T(\bar{\mathbf{x}})\, d\bar{\mathbf{x}}$$

path throughput $$T(\bar{\mathbf{x}}) = G(\mathbf{x}_0, \mathbf{x}_1) \prod_{j=1}^{k-1} f(\mathbf{x}_j, \mathbf{x}_{j+1}, \mathbf{x}_{j-1}) G(\mathbf{x}_j, \mathbf{x}_{j+1})$$



- Monte Carlo estimator:

$$I \approx \frac{W_e(\mathbf{x}_0, \mathbf{x}_1) T(\mathbf{x}_0, \cdots, \mathbf{x}_k) L_e(\mathbf{x}_k, \mathbf{x}_{k-1})}{\underbrace{p(\mathbf{x}_0, \cdots, \mathbf{x}_k)}_{\text{joint PDF of path vertices}}}$$

Photon Mapping

$$I \approx \frac{W_e(\mathbf{x}_0, \mathbf{x}_1) T(\mathbf{x}_0, \cdots, \mathbf{x}_k) L_e(\mathbf{x}_k, \mathbf{x}_{k-1})}{p(\mathbf{x}_0, \cdots, \mathbf{x}_k)}$$

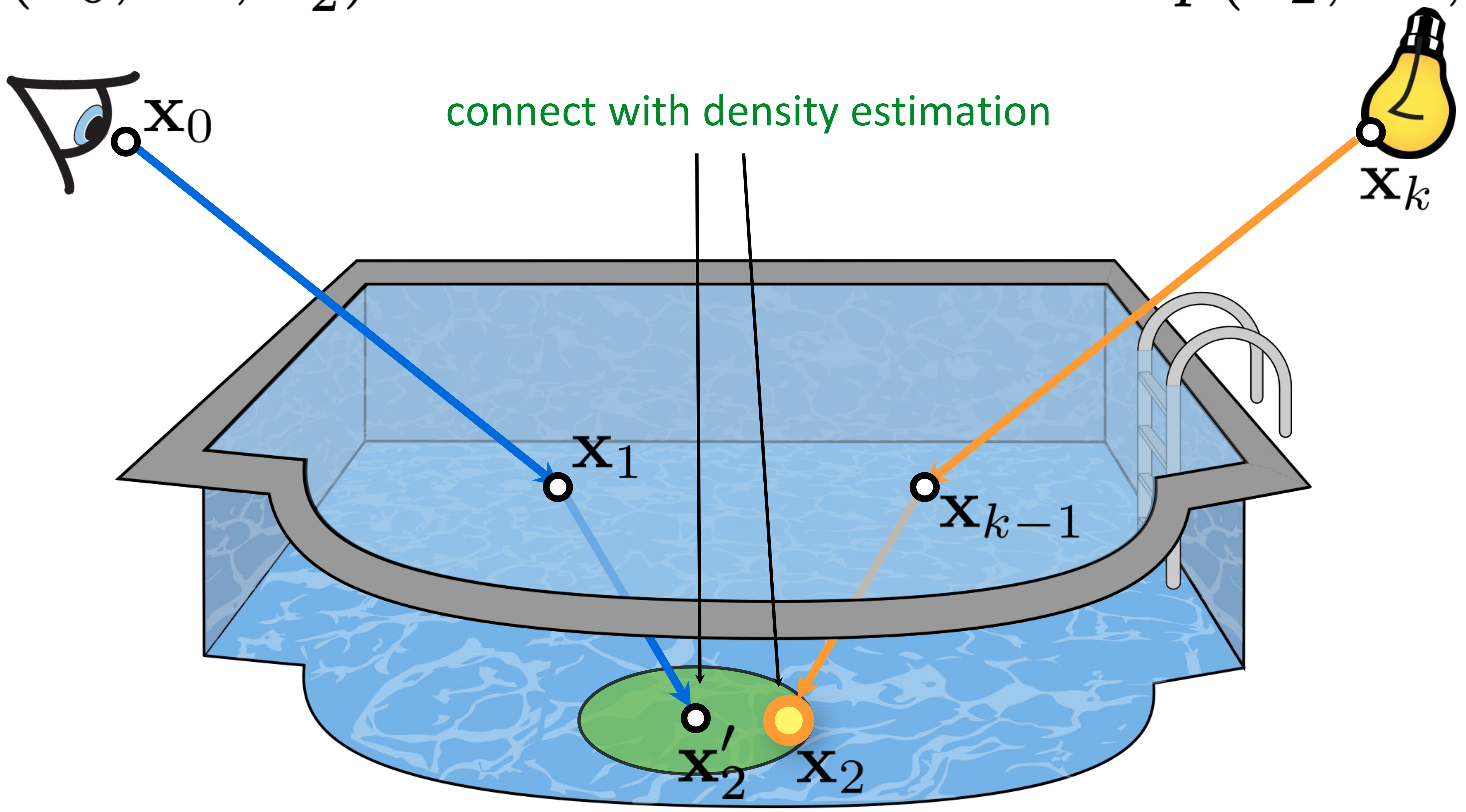
split path contribution into two parts

eye subpath

$$\frac{W_e(\mathbf{x}_0, \mathbf{x}_1) T(\mathbf{x}_0, \cdots, \mathbf{x}'_2)}{p(\mathbf{x}_0, \cdots, \mathbf{x}'_2)}$$

light subpath/photon "power"

$$\Phi_p = \frac{T(\mathbf{x}_2, \cdots, \mathbf{x}_k) L_e(\mathbf{x}_k, \mathbf{x}_{k-1})}{p(\mathbf{x}_2, \cdots, \mathbf{x}_k)}$$



Light Sources in the Real World

Complex shape

Covered with transparent materials

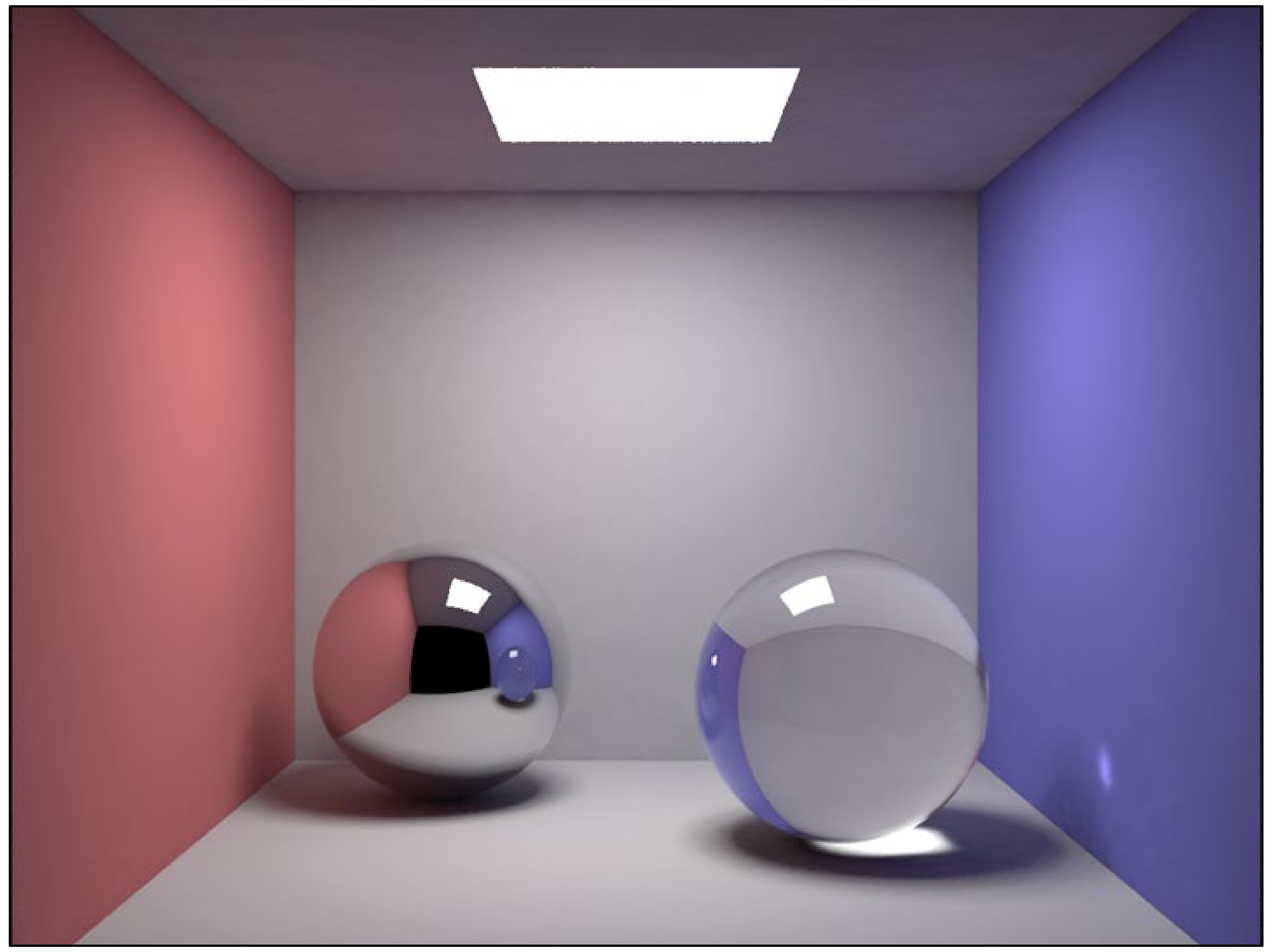
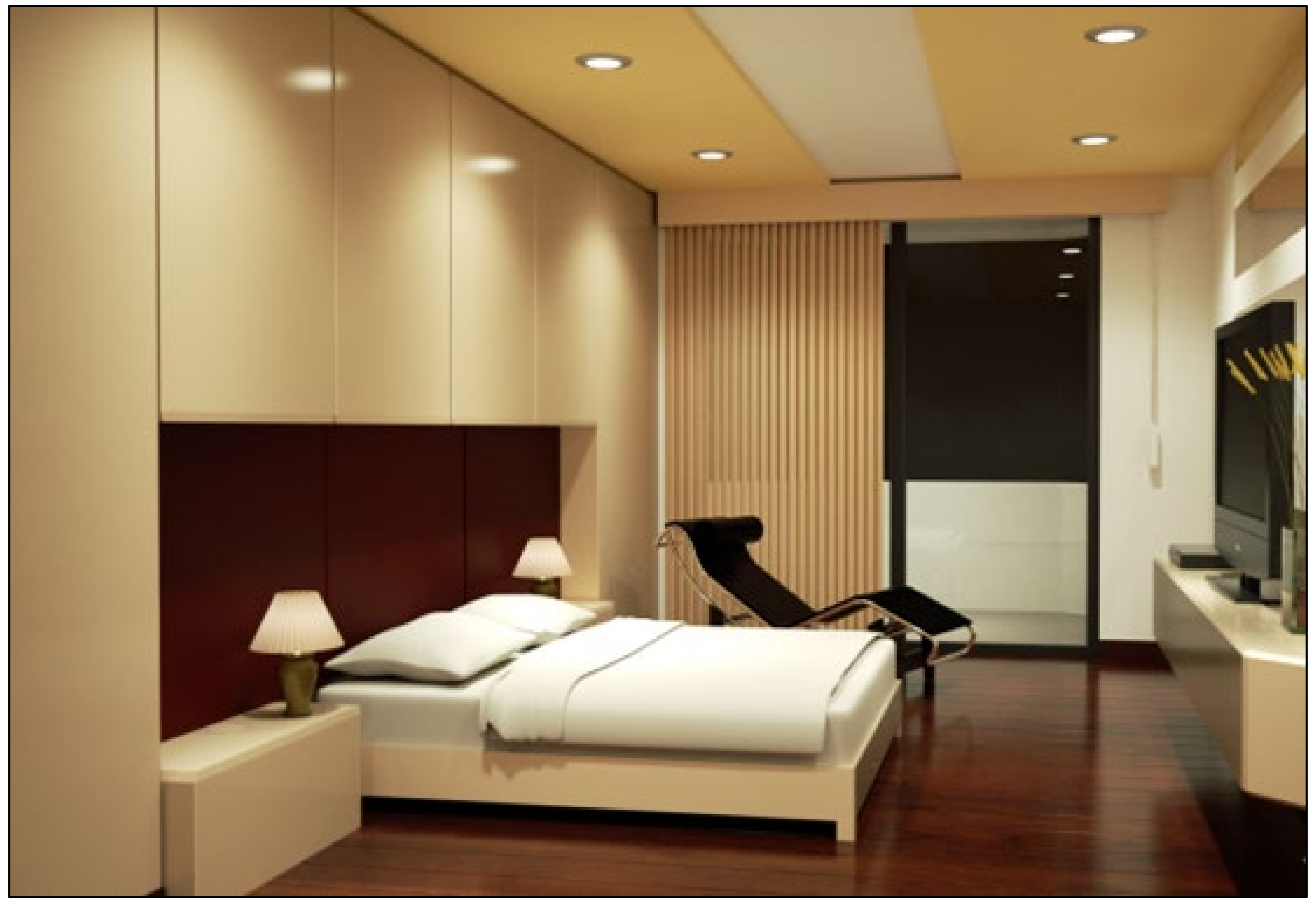
Only a small part emits light

Light Sources in CG

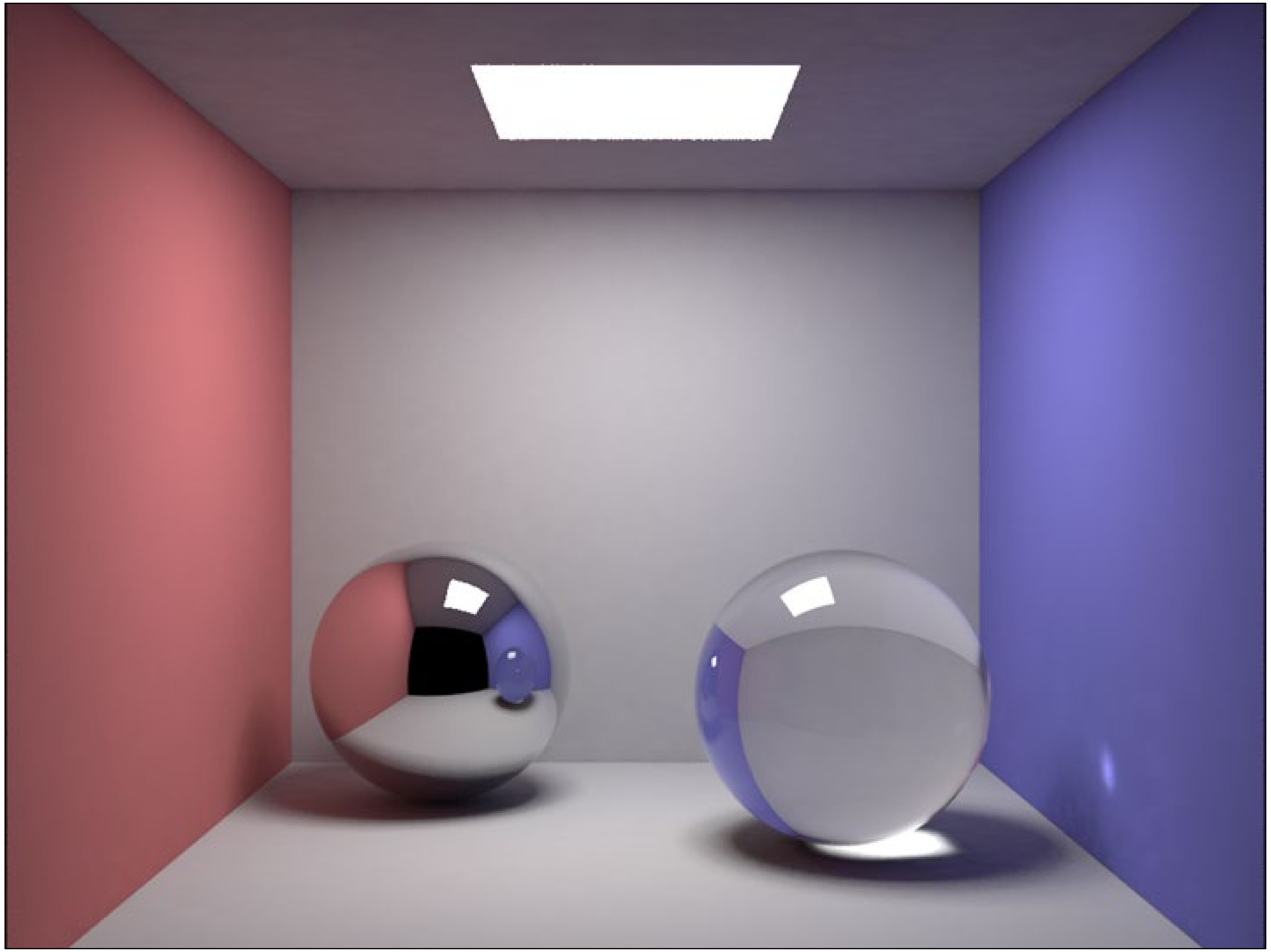
Simple shape

Bare light source

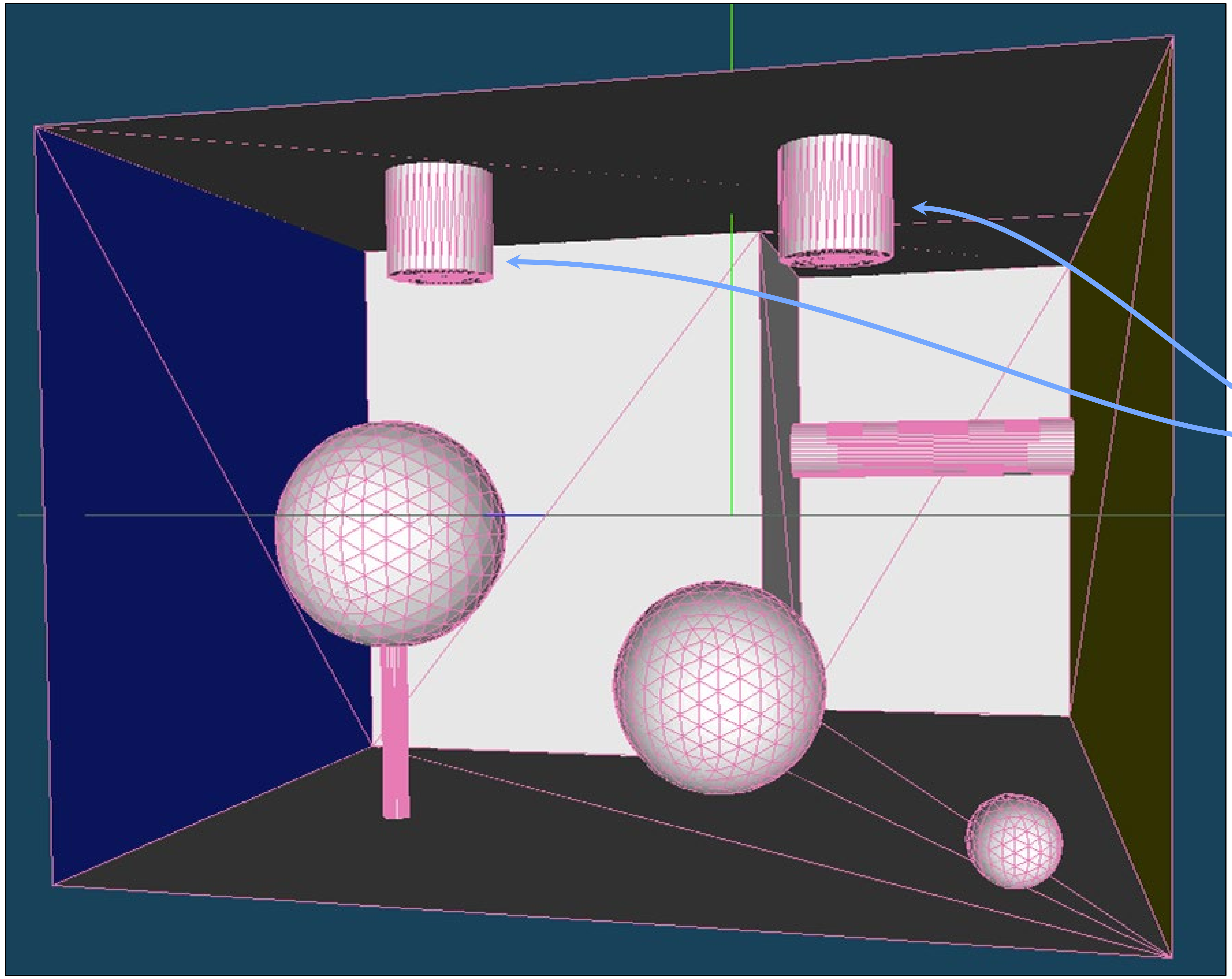
Entire part emits light

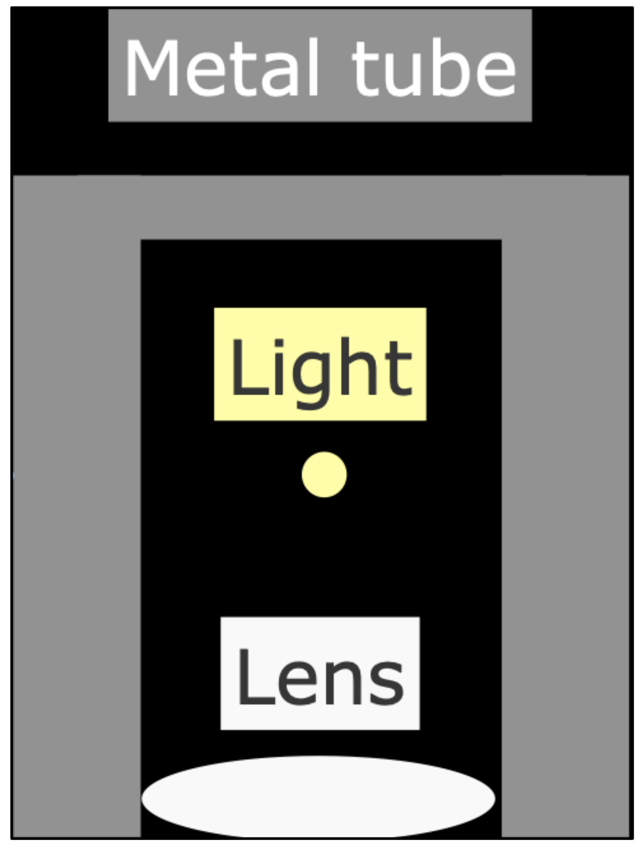
  

Why?



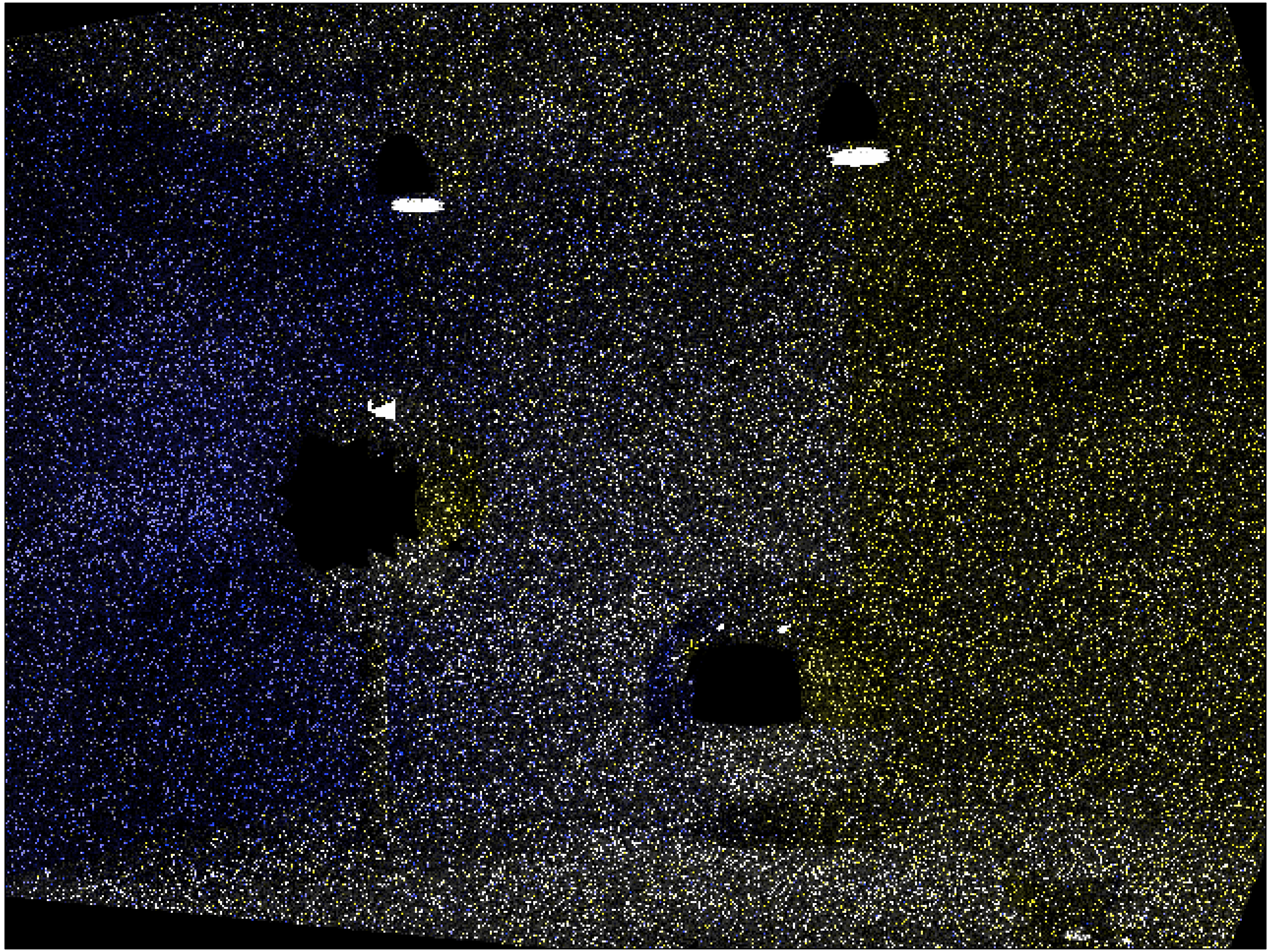
Scene with “Realistic” Lights





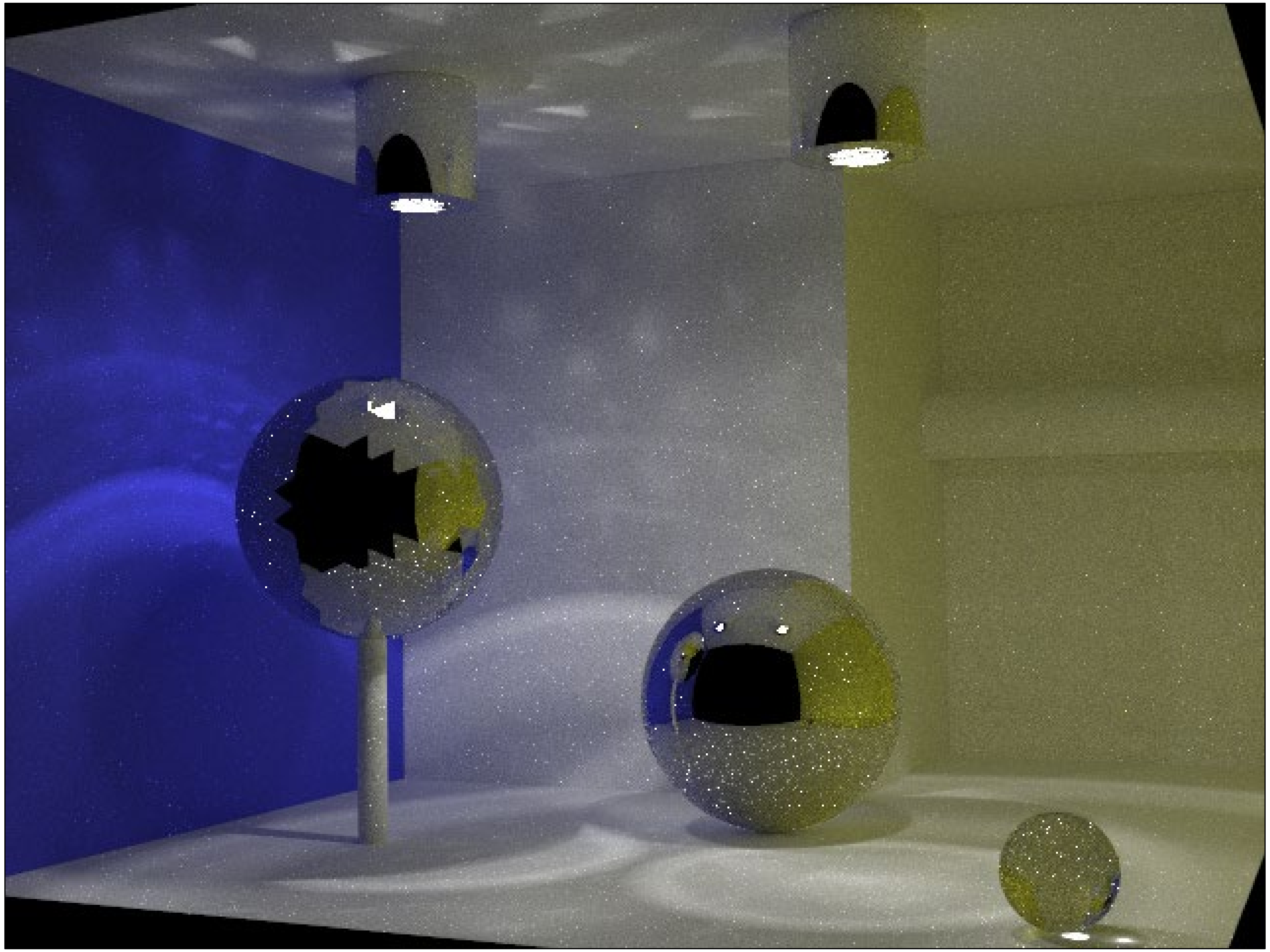
Images courtesy of T. Hachisuka

Path Tracing



Images courtesy of T. Hachisuka

Bidirectional Path Tracing



Images courtesy of T. Hachisuka

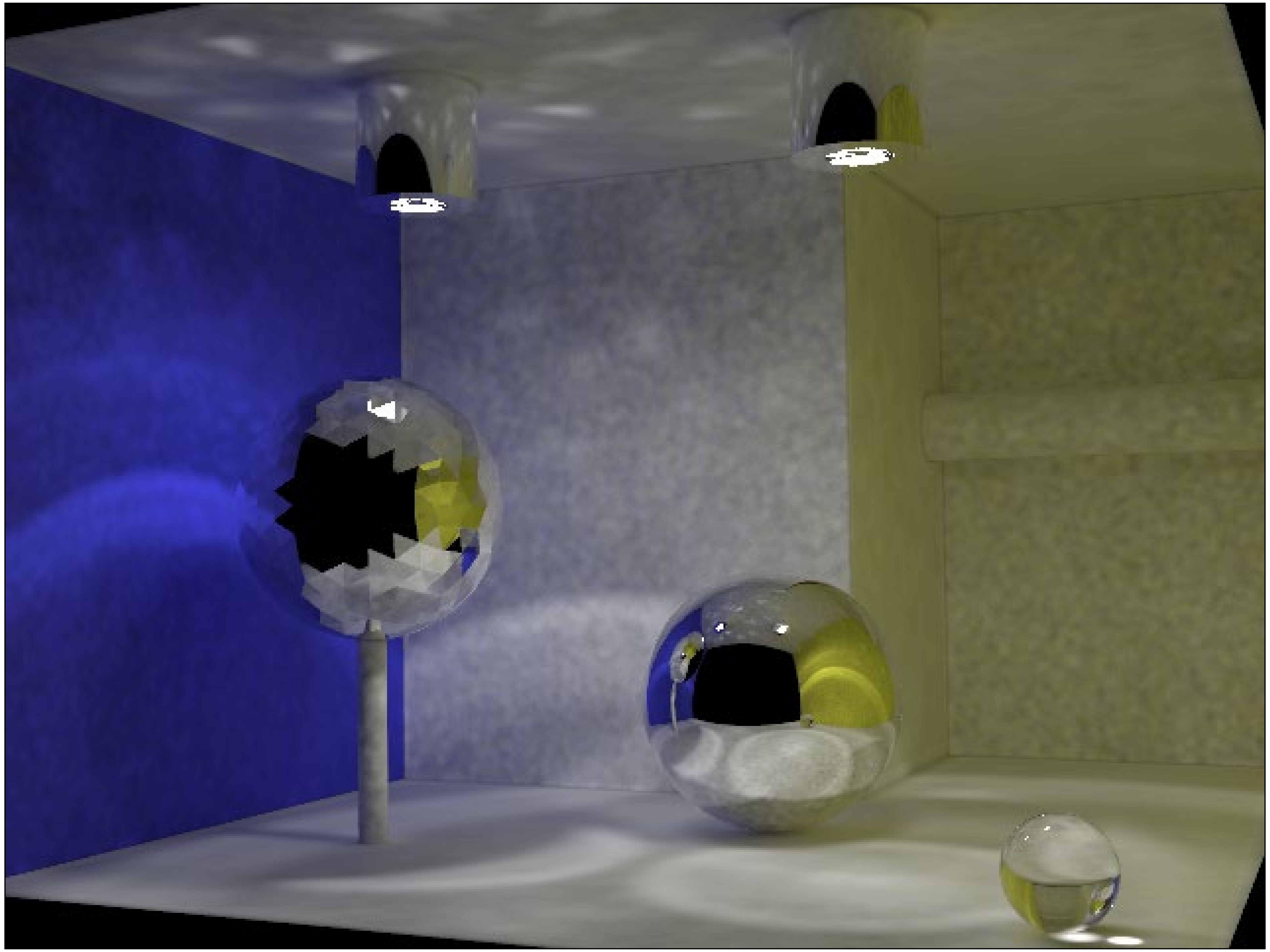
Robustness of Rendering Methods

None of these unbiased methods can handle real light sources well:

- Path Tracing
- Bidirectional Path Tracing

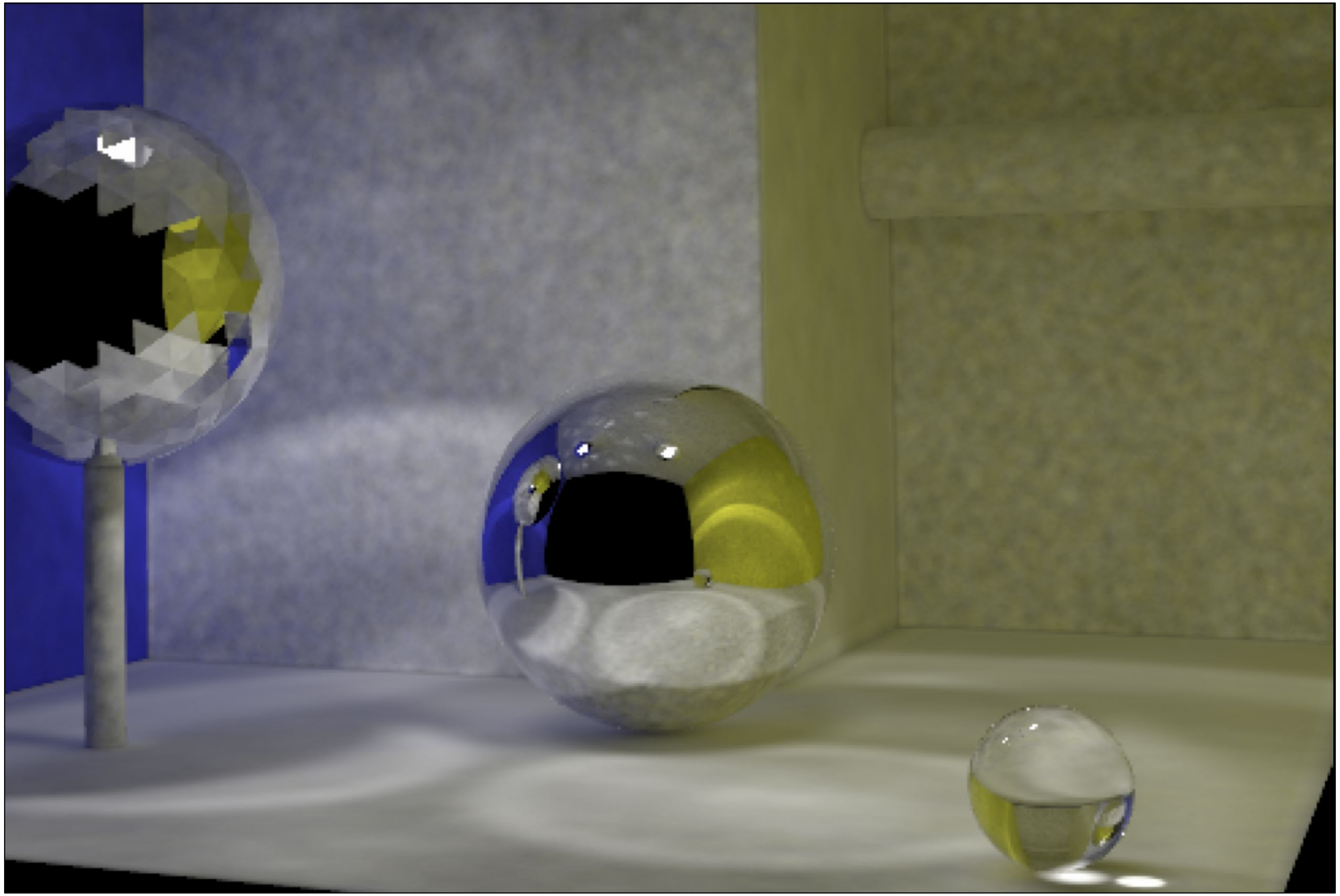
Photon Mapping?

Photon Mapping



Images courtesy of T. Hachisuka

Photon Mapping



Images courtesy of T. Hachisuka

Photon Mapping - Summary

Advantages

- Handles difficult paths more robustly than unbiased algorithms
- Consistent estimator
- Reuse of computation (photons)

Disadvantages

- Bias shows up in many different forms
- Requires additional data structure (KD-tree)
- No progressive rendering
- Large memory footprint
- Non-intuitive hyperparameter fine-tuning

Characteristics of Estimators

Unbiased estimator

- expected value equals the true value being estimated

$$E[F] = \int f(x)\,dx$$

- variance (noise) is the only error
- averaging infinitely many estimates (each with finite number of samples) also yields the correct answer

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \langle F^k \rangle = \int f(x)dx$$

Characteristics of Estimators

Bias of an estimator

- difference between the expected value of the estimator and the true value being estimated

$$\beta = E[F] - \int f(x)\, dx$$

- expected average difference

- averaging infinitely many estimates yields the correct answer plus the bias

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \langle F^k \rangle = \int f(x)dx + \beta$$

Characteristics of Estimators

Consistent estimator

- bias disappears in the limit

$$\lim_{N \to \infty} E[F] = \int f(x)\, dx$$

Consistent estimators and *unbiased* estimators are asymptotically equivalent

- both need an infinite number of samples to reduce the error to zero

Characteristics of Estimators

Mean Squared Error (MSE) of an estimator

- combines variance and squared bias

$$\mathrm{MSE}[F] = \mathrm{Var}[F] + \mathrm{Bias}[F]^2$$

Root Mean Squared Error (RMSE)

- has the same units as the quantity being estimated
- for unbiased estimators equal to std. deviation

$$\mathrm{RMSE}[F] = \sqrt{\mathrm{MSE}[F]}$$

Rendering Techniques

Examples of unbiased methods

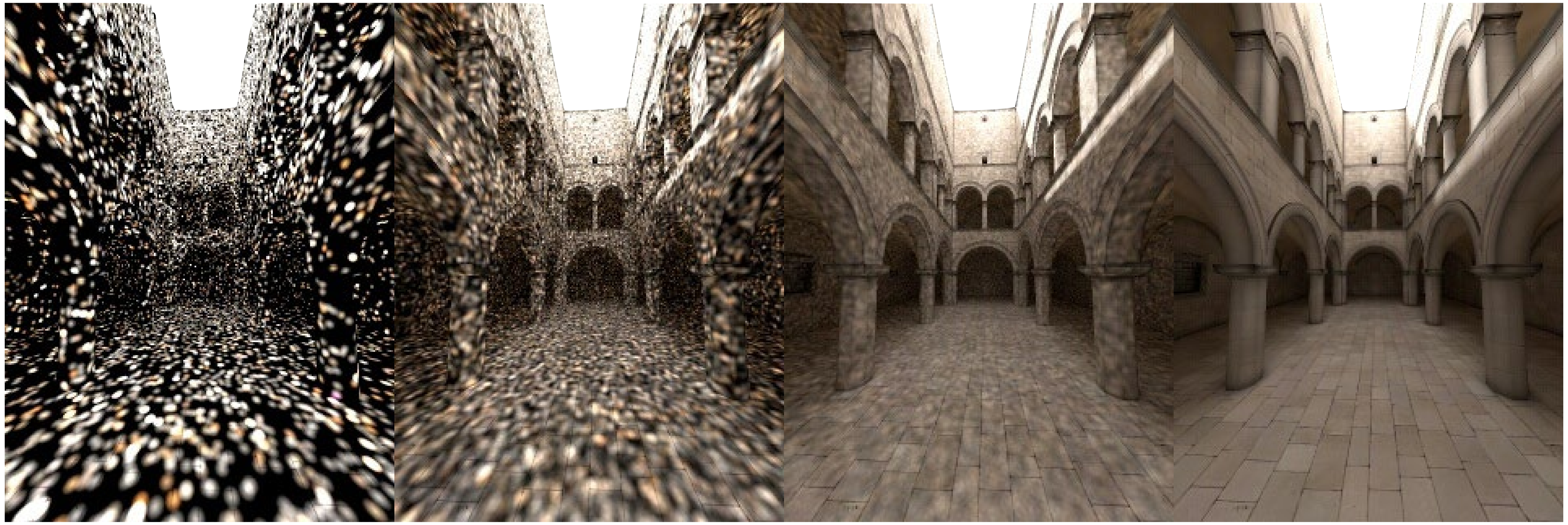
- Path tracing
- Light tracing
- Bidirectional path tracing

Examples of biased/consistent methods

- (Progressive) photon mapping
- Many-light methods

Consistency of Photon Mapping

Result converges to the correct solution



Conditions for convergence:

- Infinitesimally small radius
- Infinite number of nearby photons
  - **Infinite storage requirement!**

Images courtesy of T. Hachisuka

Progressive Photon Mapping

Key Idea

Progressively shrink the density estimation kernel

Hachisuka et al. 2008, 2009, …

- store/update statistics at each camera ray hitpoint

Knaus & Zwicker 2011

- no statistics, just render independent images with smaller and smaller radius, and average

Different kernel radii

Images courtesy of C. Knaus and M. Zwicker

Progressive Radius Reduction

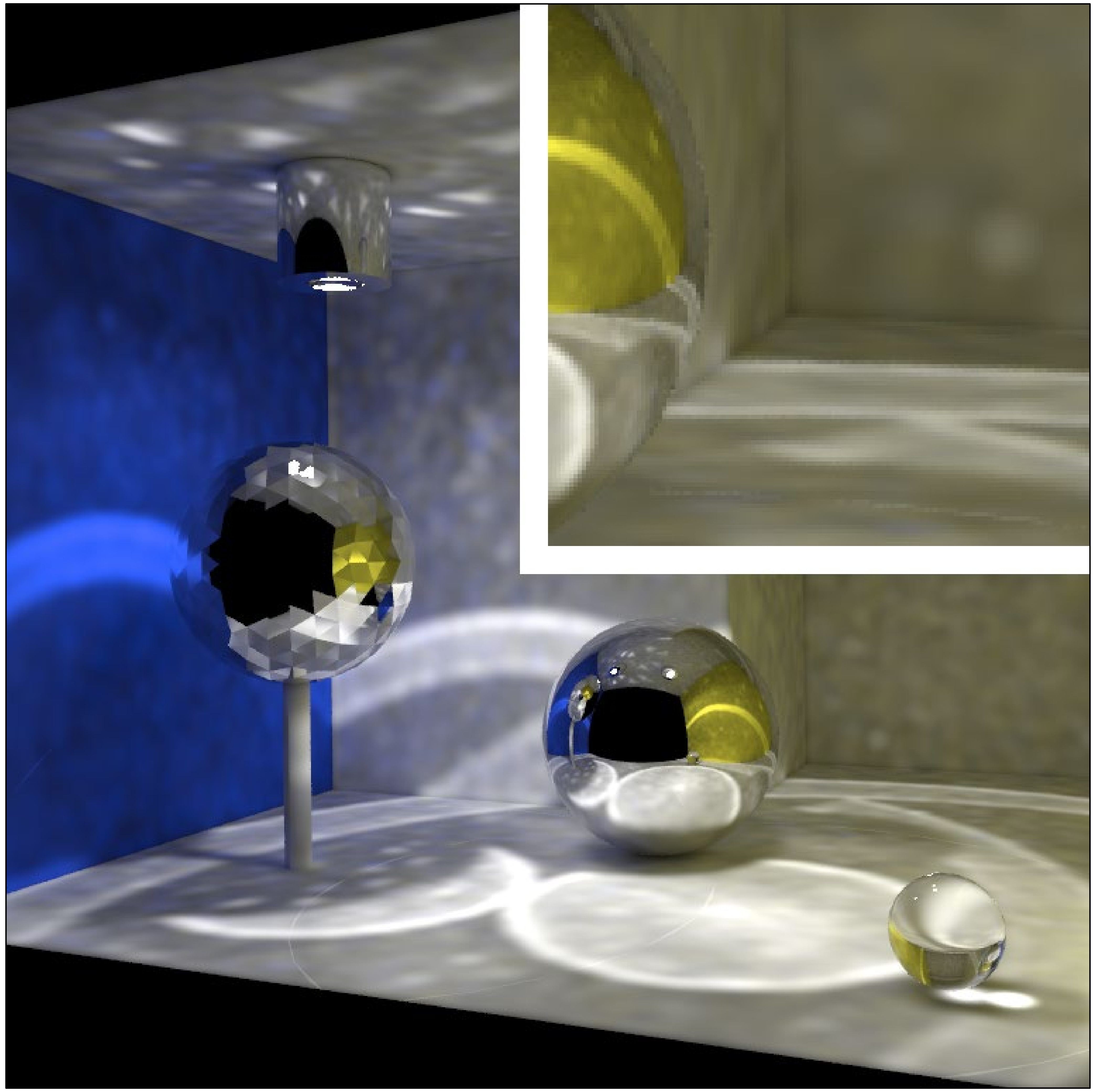


Image 1, $r = 20$

Images courtesy of C. Knaus and M. Zwicker

Progressive Radius Reduction

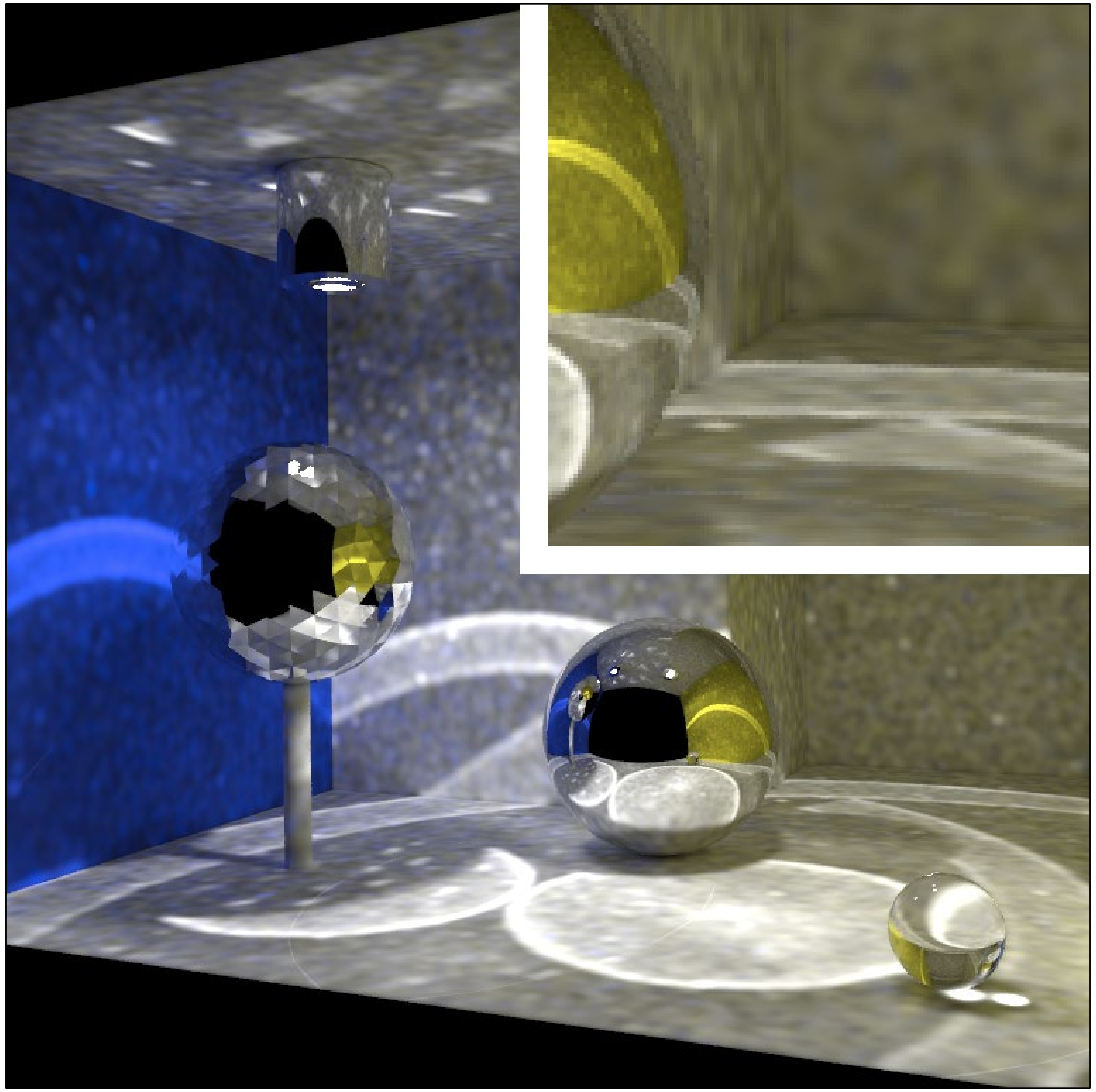


Image 10, $r = 11.87$

Images courtesy of C. Knaus and M. Zwicker

Progressive Radius Reduction

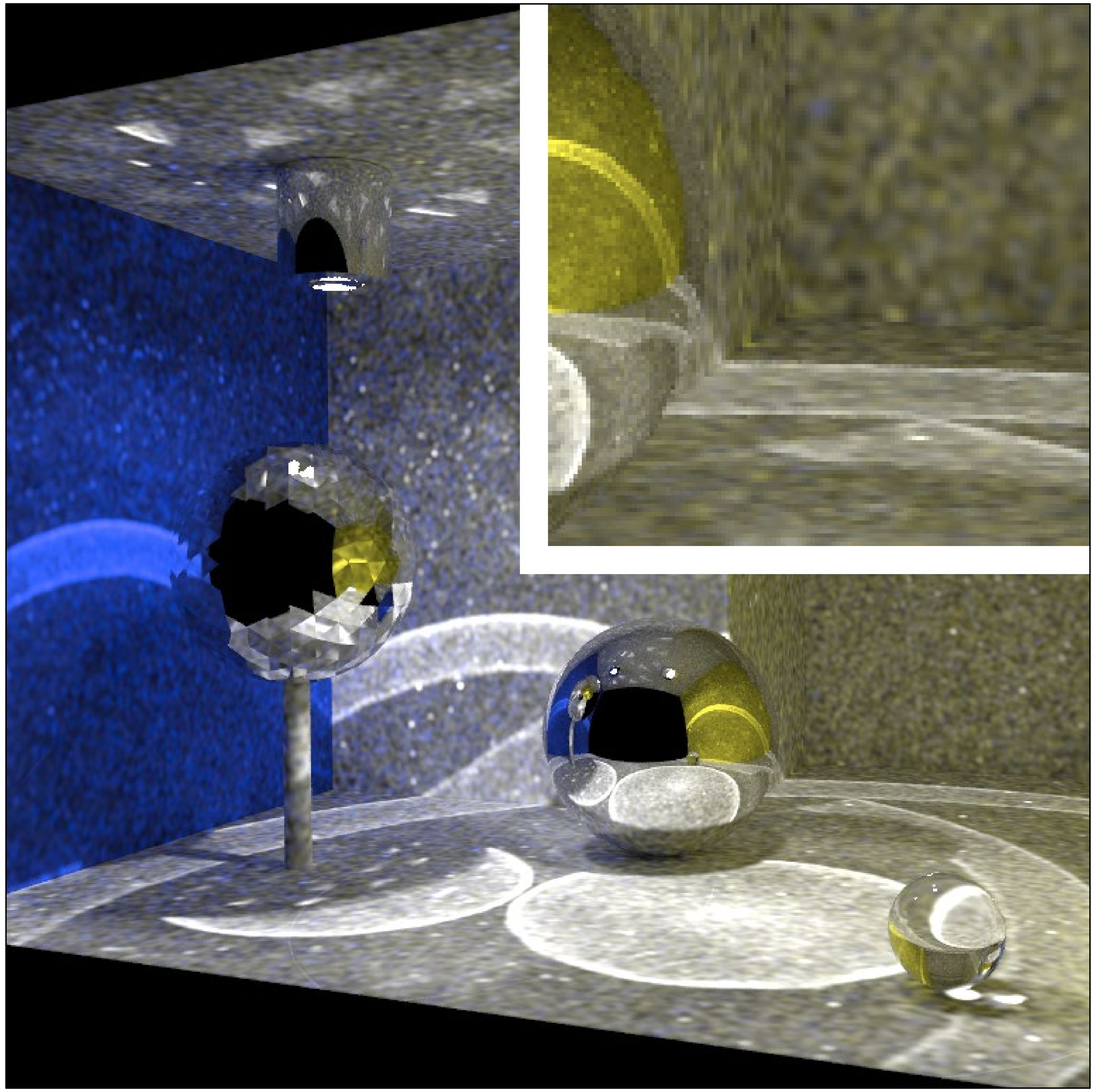
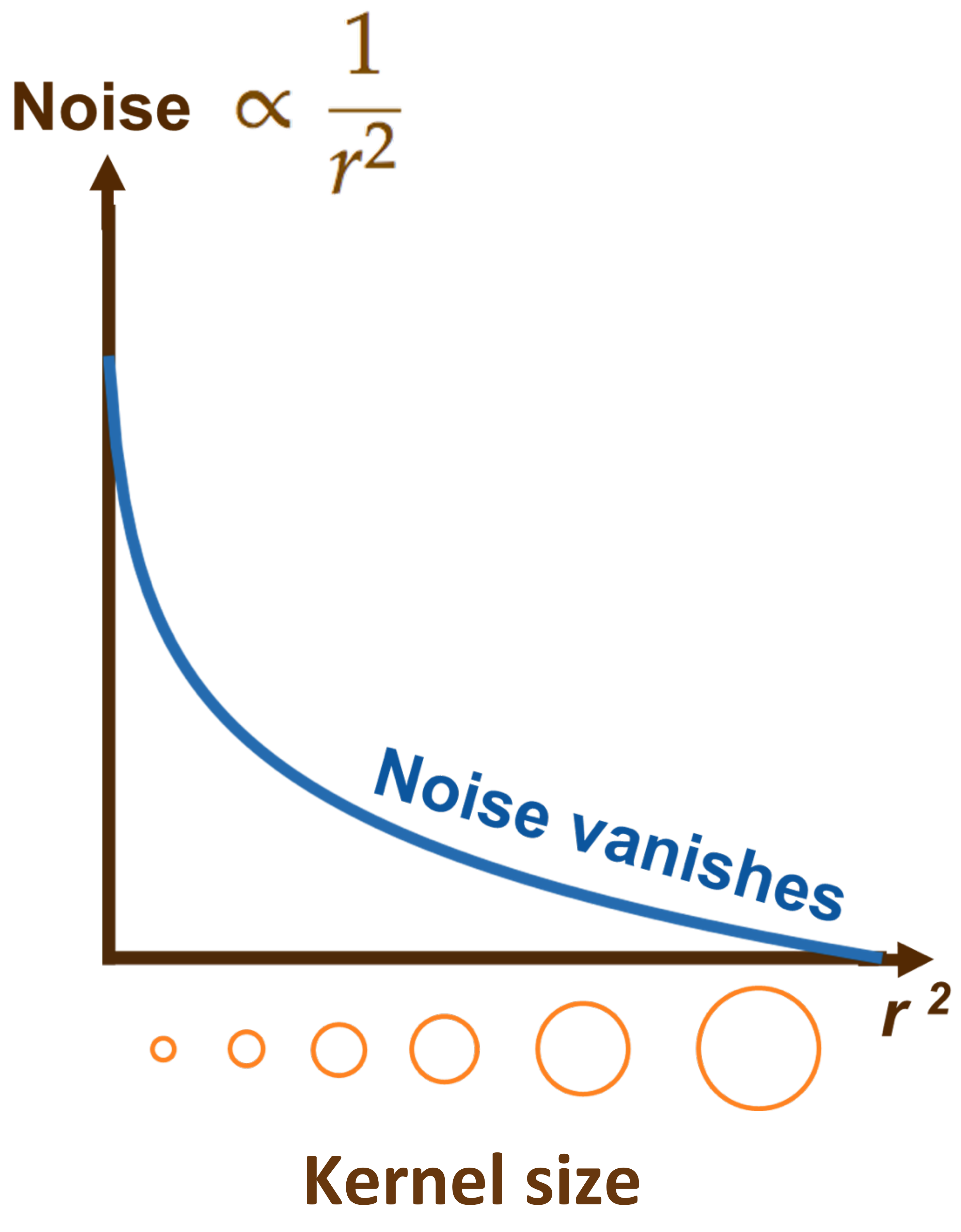


Image 100, $r = 6.71$

Images courtesy of C. Knaus and M. Zwicker

Progressive Radius Reduction



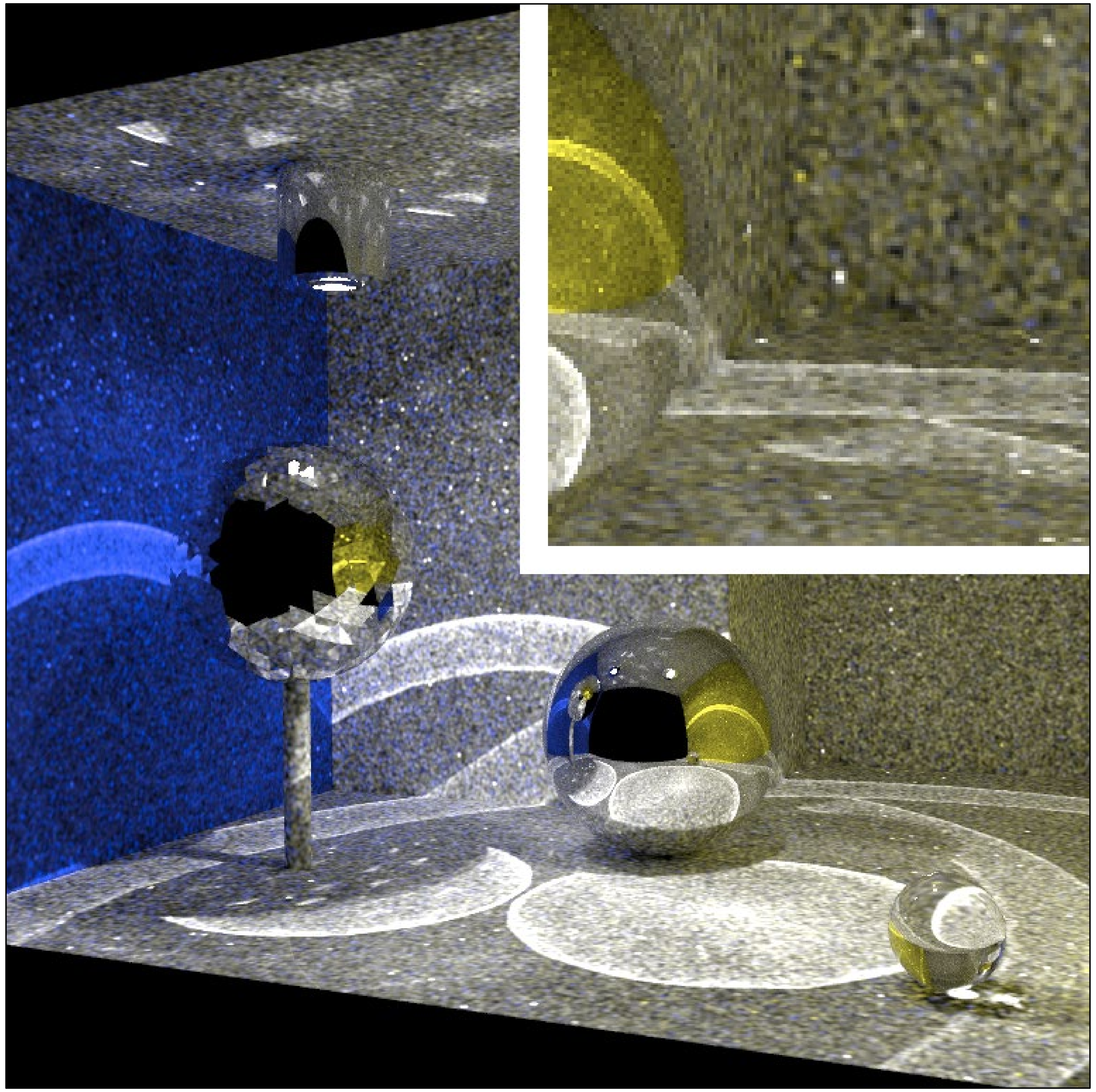
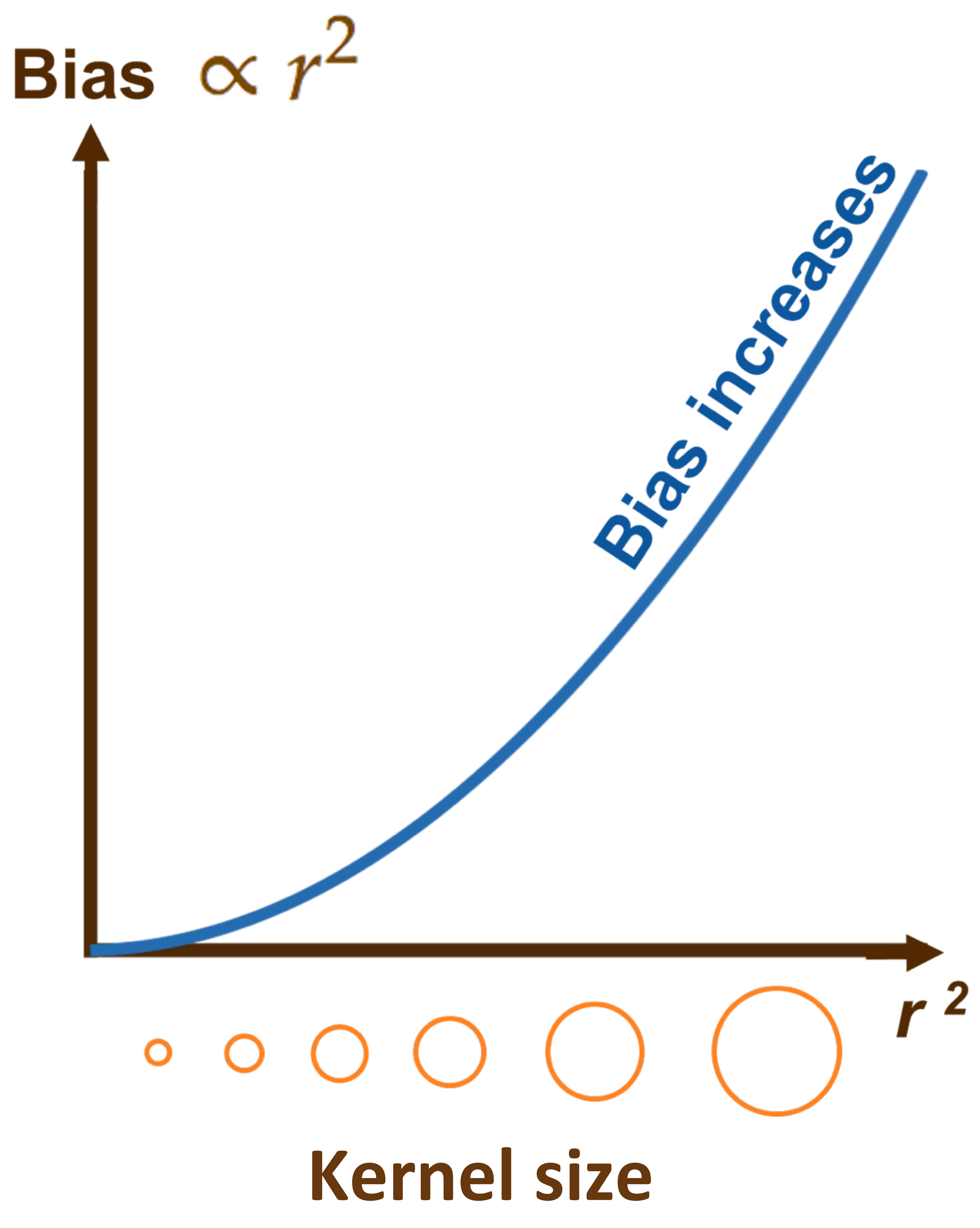


Image 1000, $r = 3.78$



Images courtesy of C. Knaus and M. Zwicker

Running Average

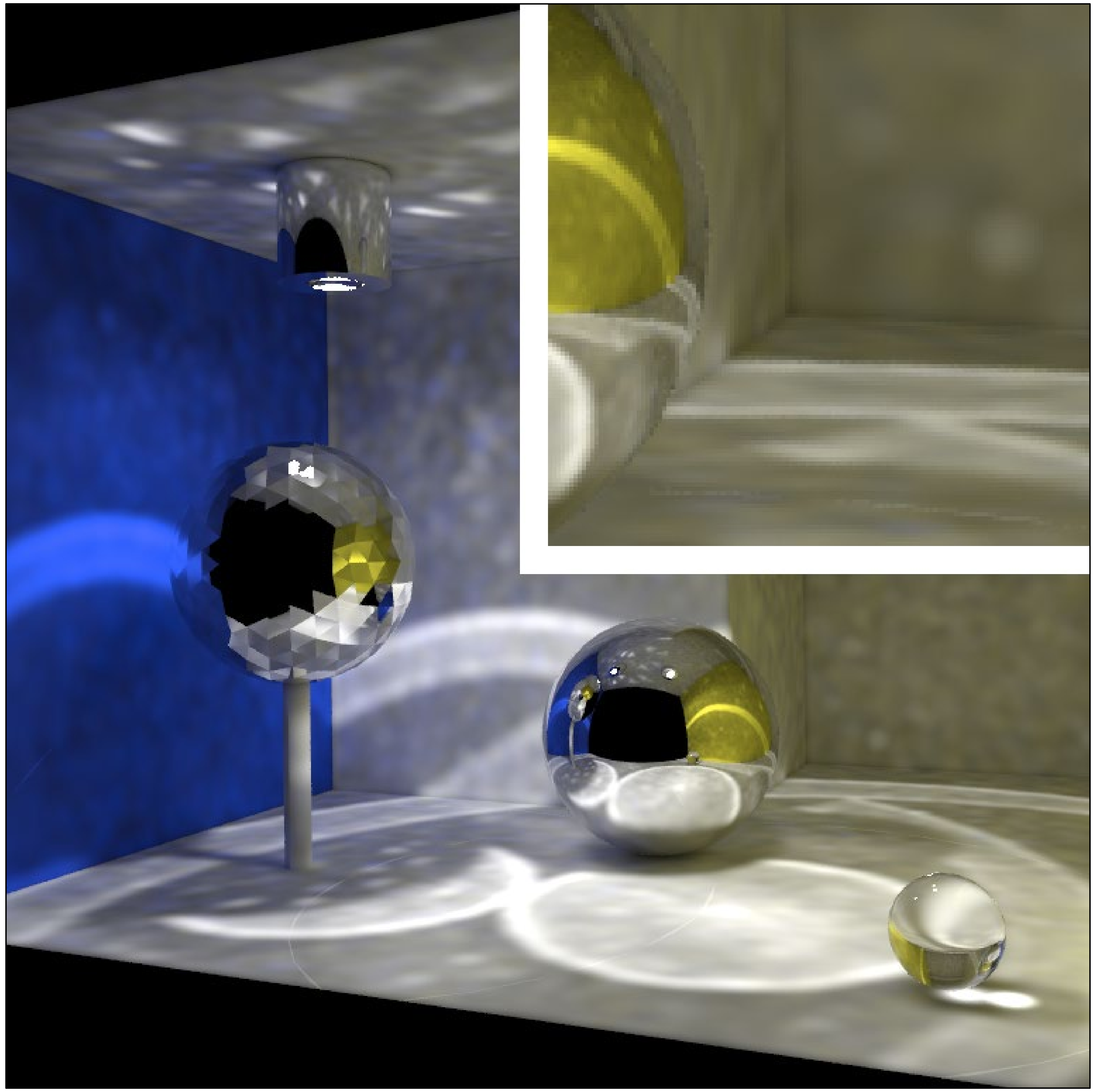
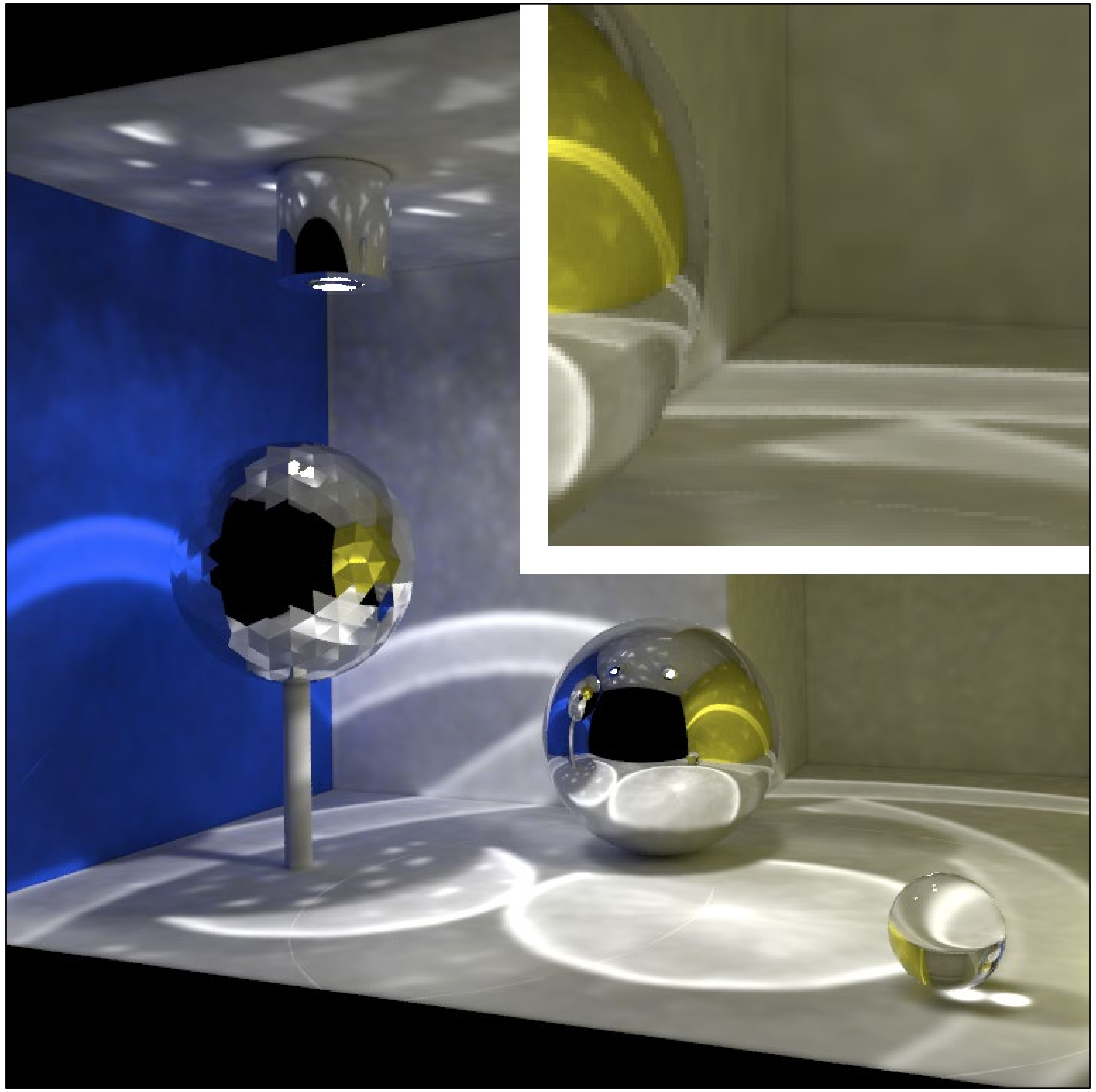


Image 1

Images courtesy of C. Knaus and M. Zwicker

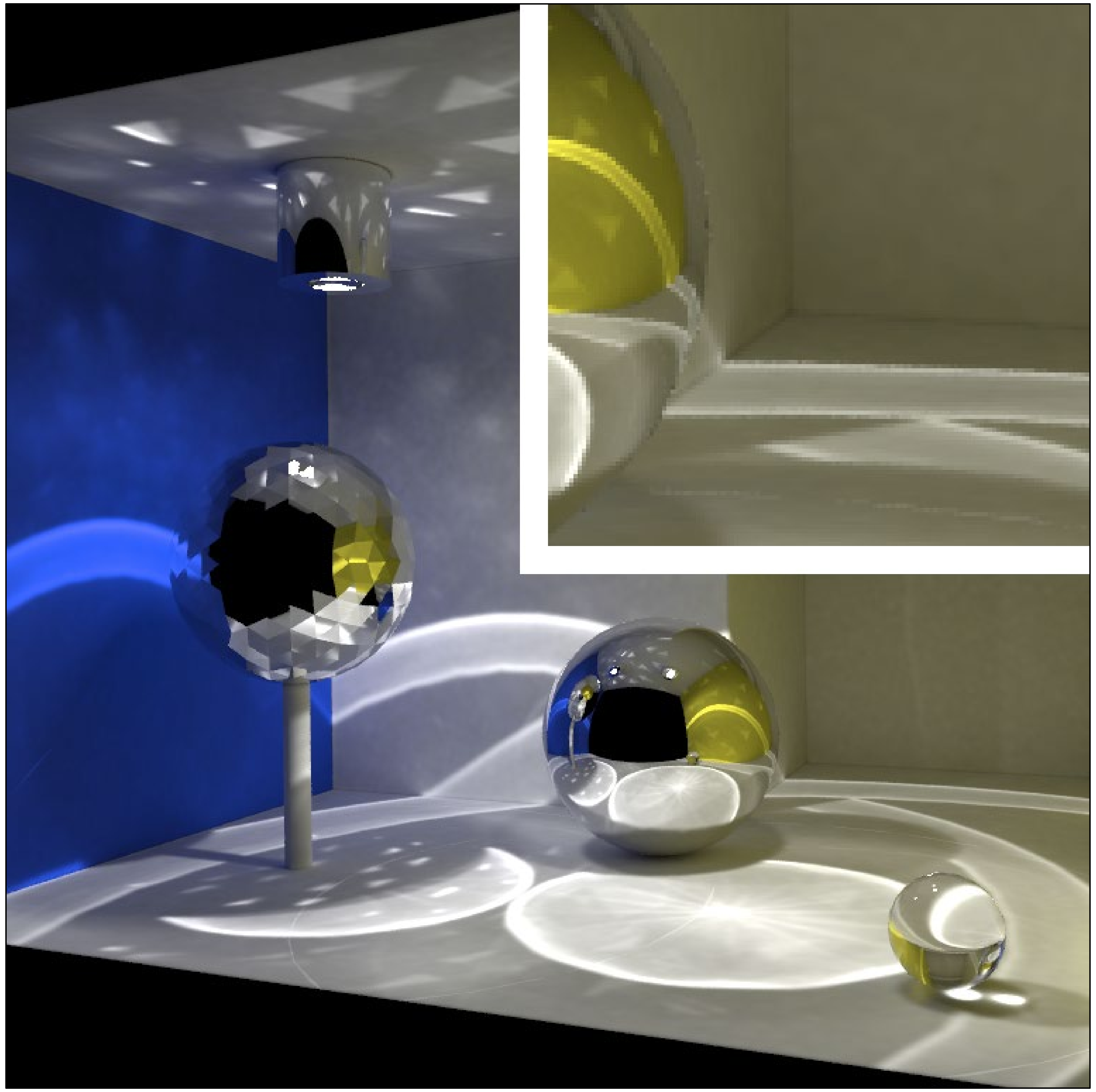
Running Average



Average of Images 1-10

Images courtesy of C. Knaus and M. Zwicker

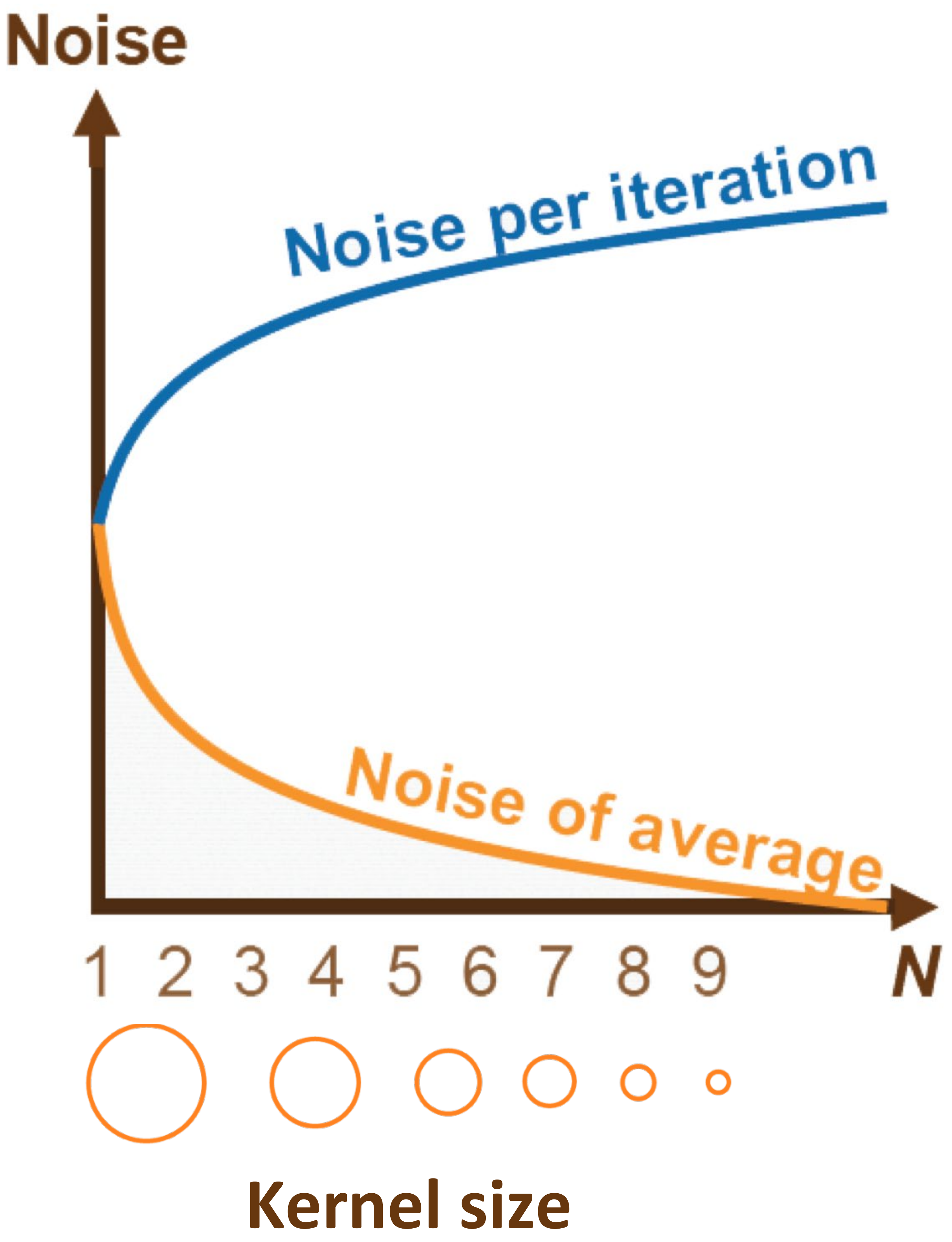
Running Average

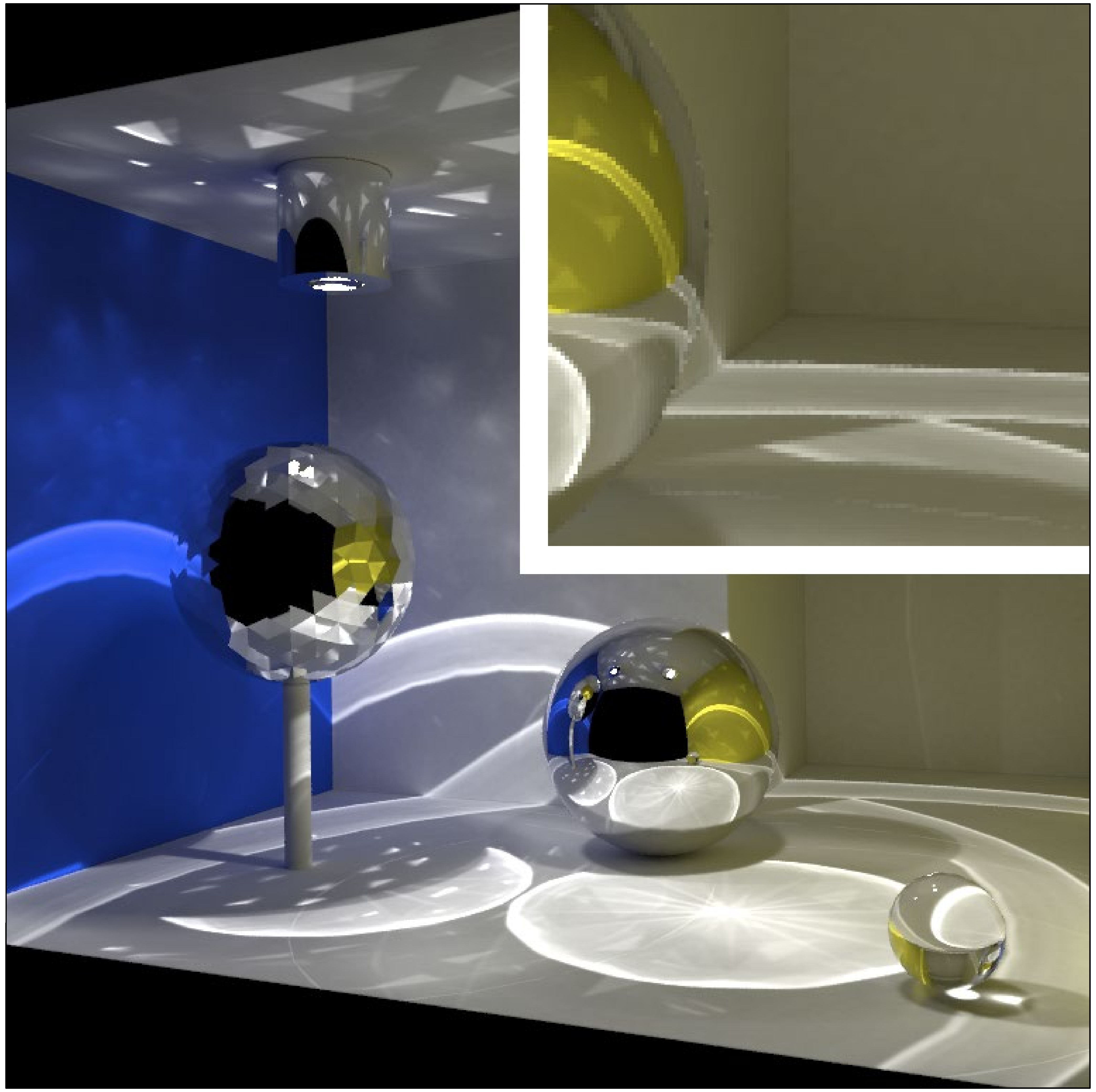


Average of Images 1-100

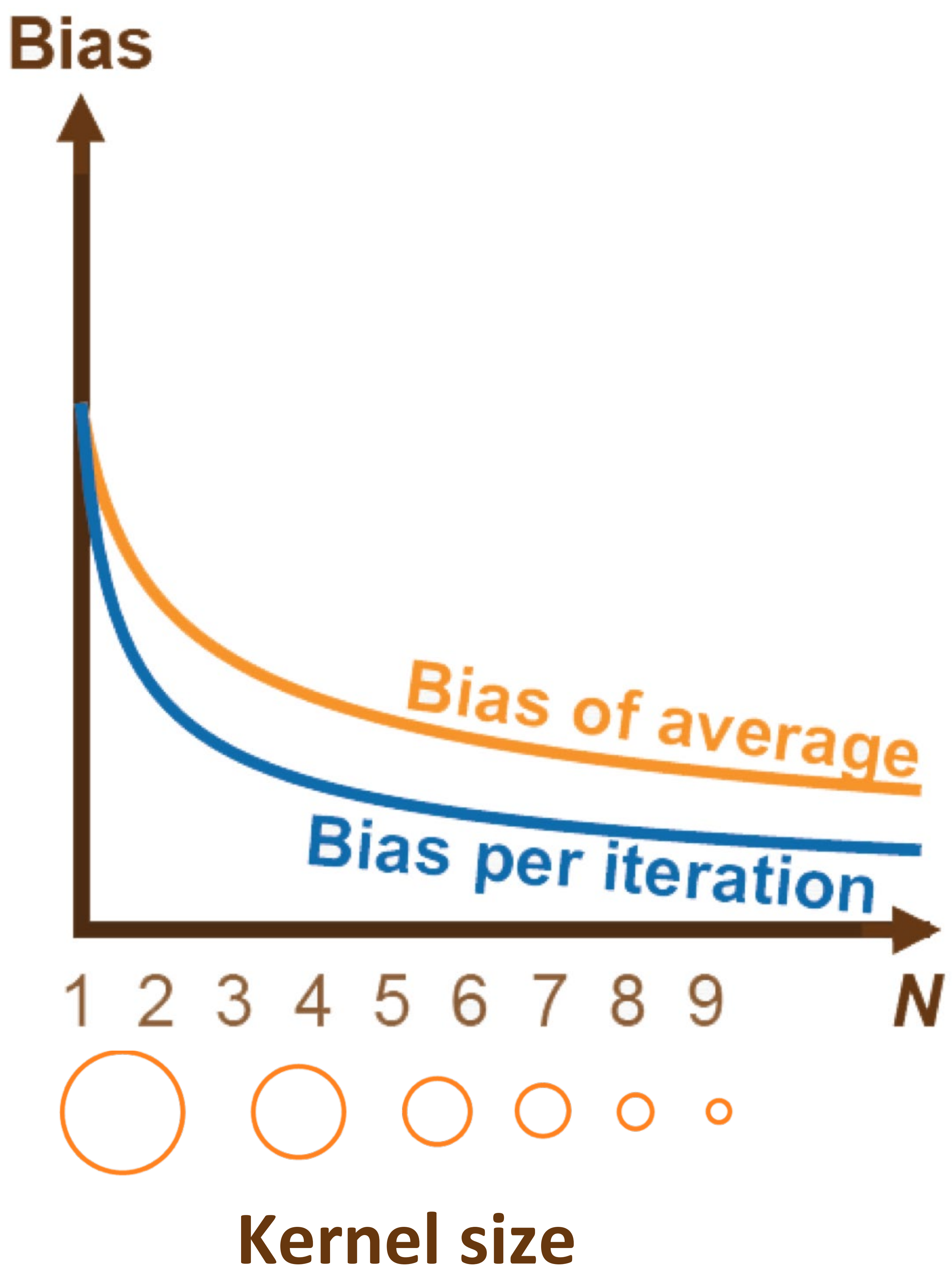
Images courtesy of C. Knaus and M. Zwicker

Running Average





Average of Images 1-1000



Images courtesy of C. Knaus and M. Zwicker

Individual iterations

Running average

Images courtesy of C. Knaus and M. Zwicker

Radius Reduction

Given:

- Iteration i
- Kernel radius r_i
- Parameter $\alpha \in (0, 1)$ for controlling the shrinking

The radius for the next iteration is: $r_{i+1}^2 = \frac{i + \alpha}{i + 1} r_i^2$

See [Knaus & Zwicker 2011] for derivation

Algorithm

Step 1:

- Photon tracing: emit, scatter, store photons

Step 2:

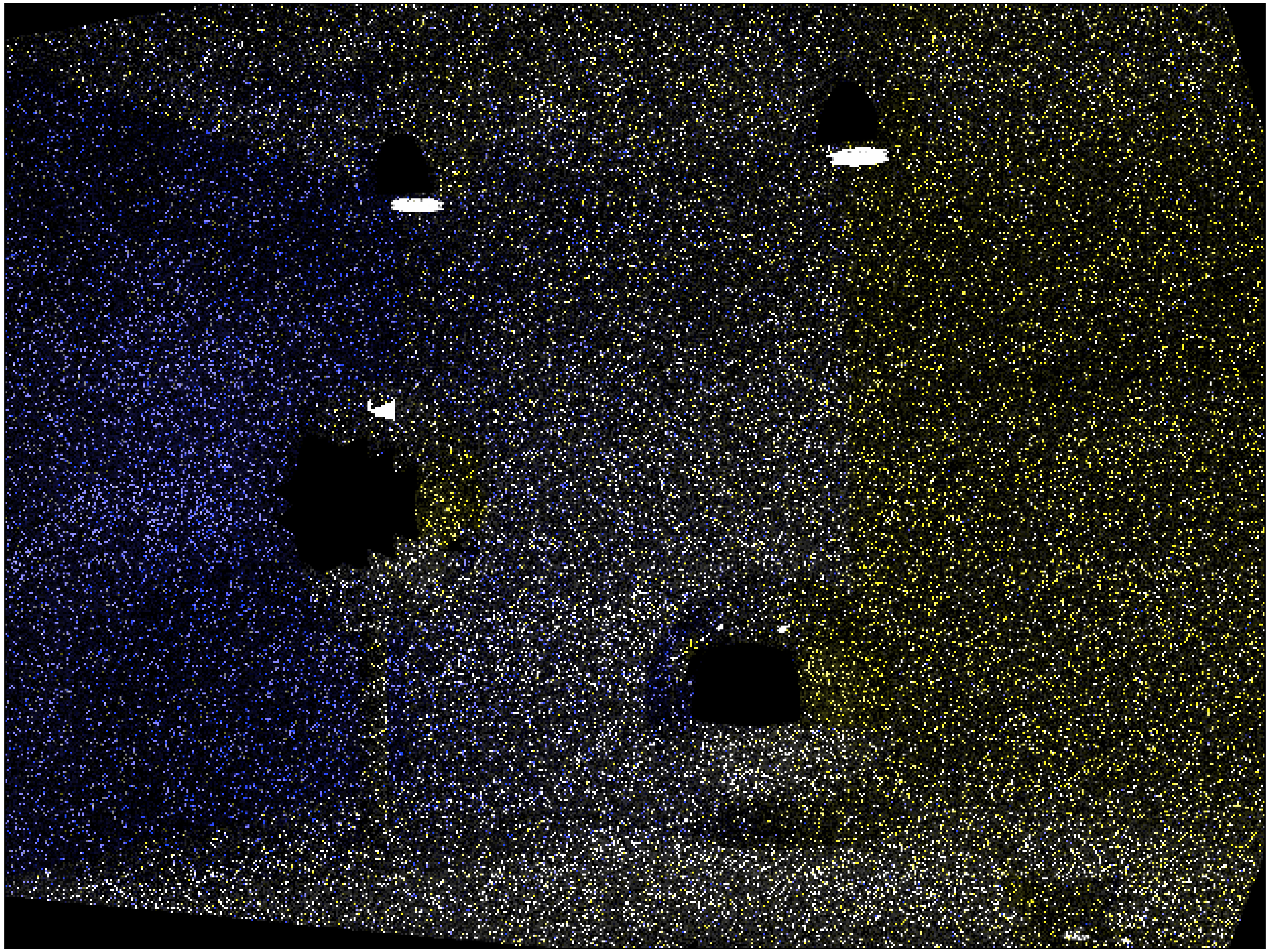
- Trace camera paths
- Evaluate radiance estimate using radius r_i

Display running average

Compute new radius $$r_{i+1}^2 = \frac{i+\alpha}{i+1} r_i^2$$ and repeat...

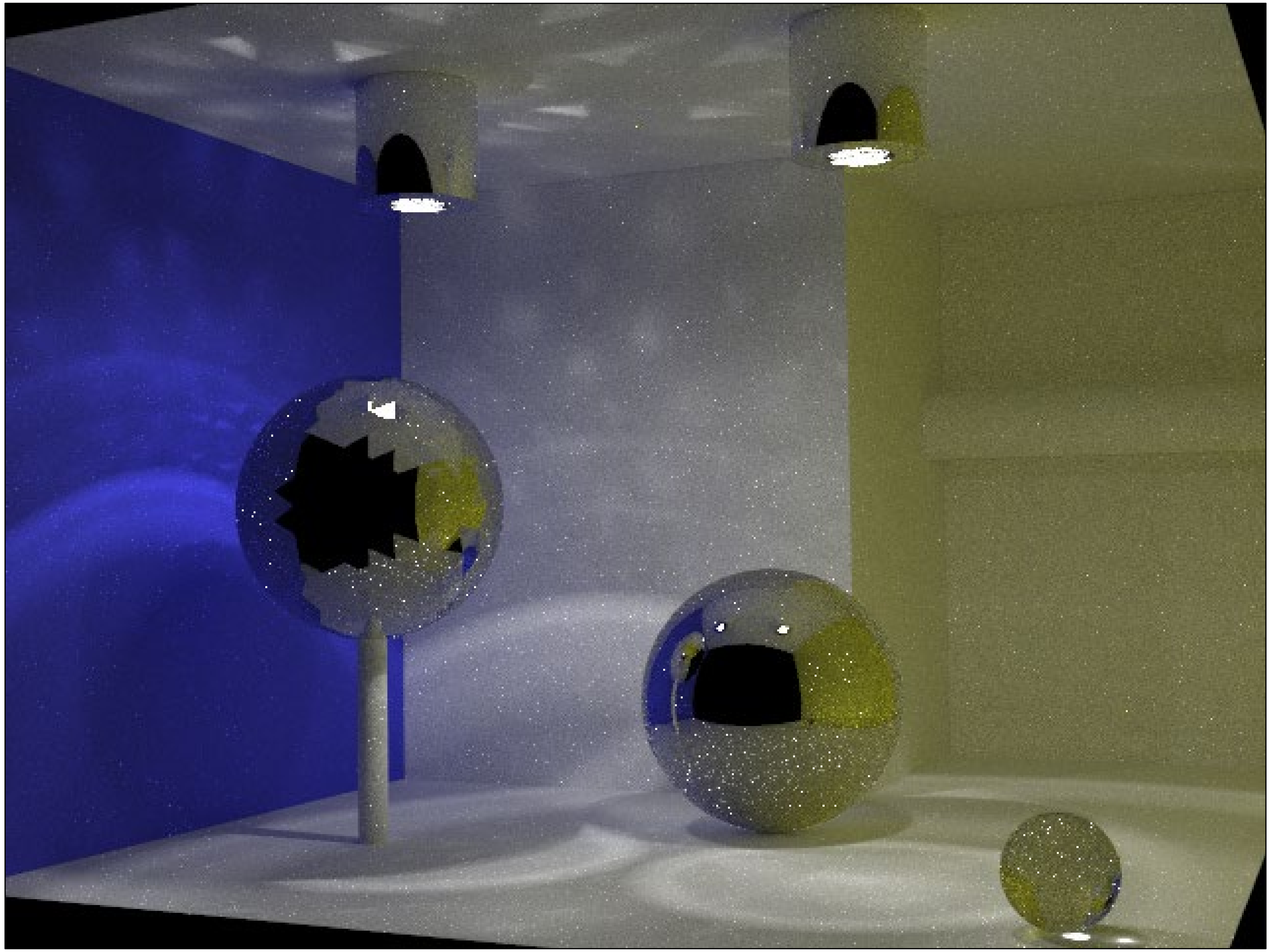
Trivially parallelizable by iteration

Path Tracing



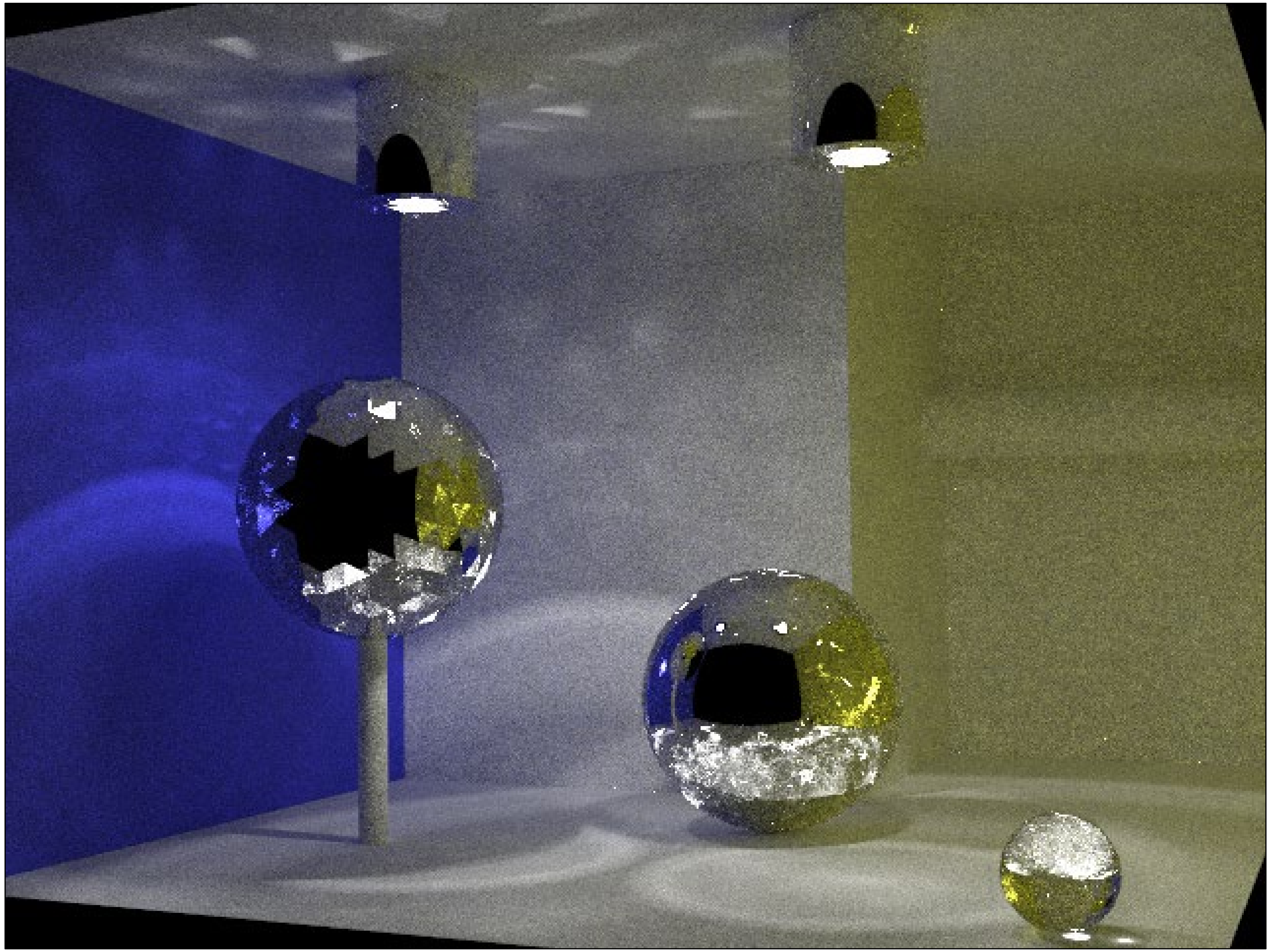
Images courtesy of C. Knaus and M. Zwicker

Bidirectional Path Tracing



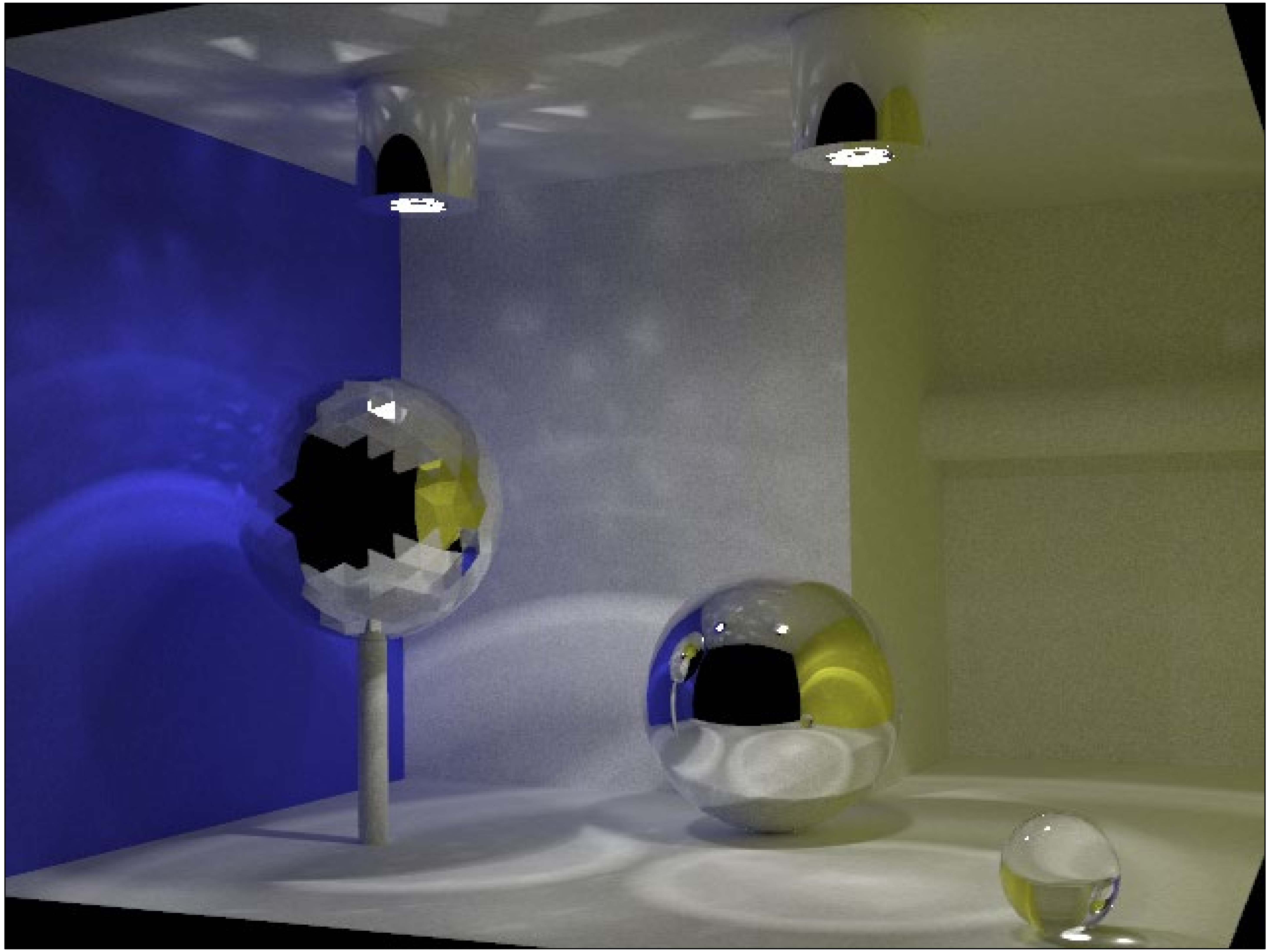
Images courtesy of T. Hachisuka

Metropolis Light Transport



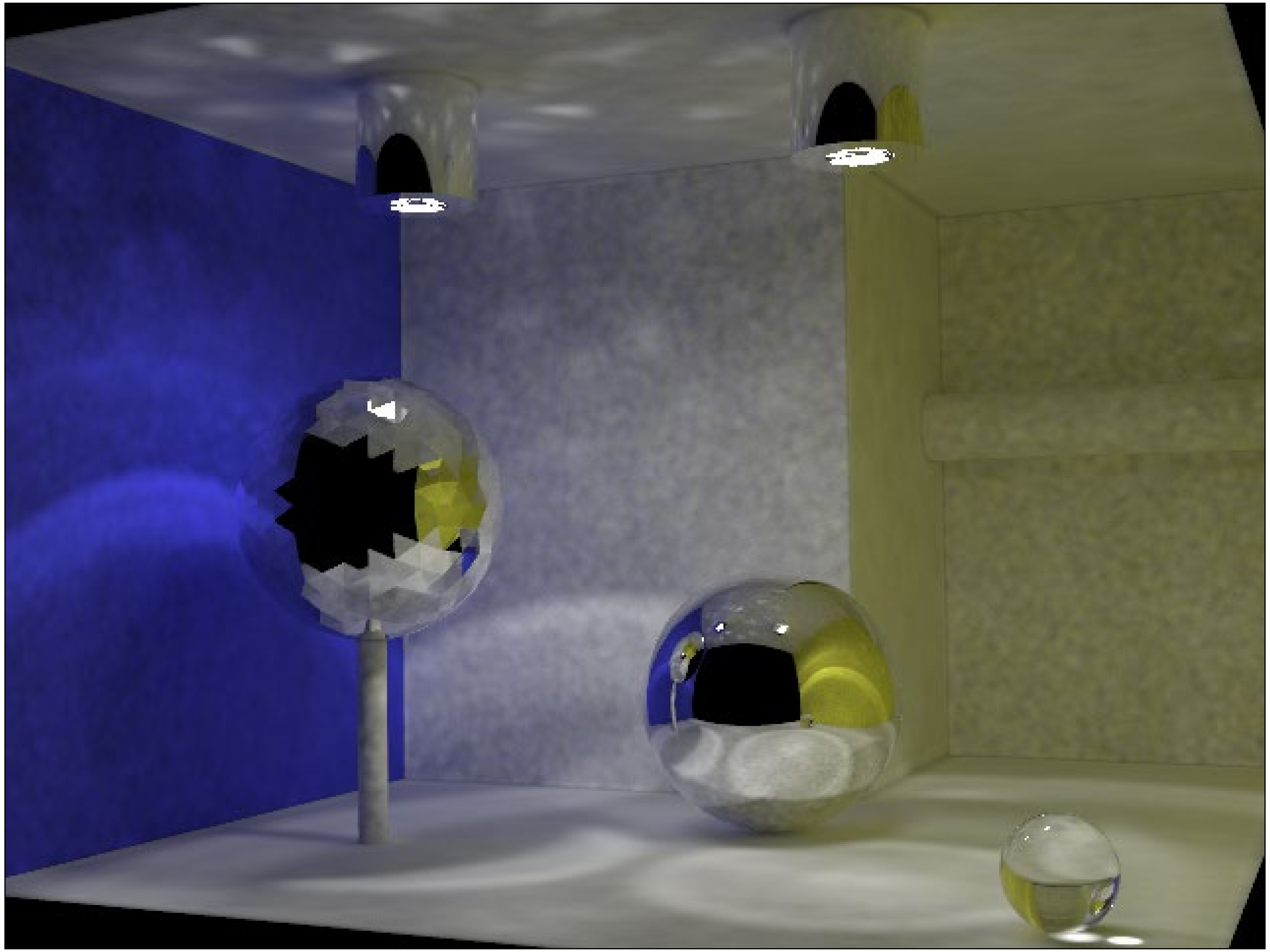
Images courtesy of T. Hachisuka

Progressive Photon Mapping



Images courtesy of T. Hachisuka

Photon Mapping



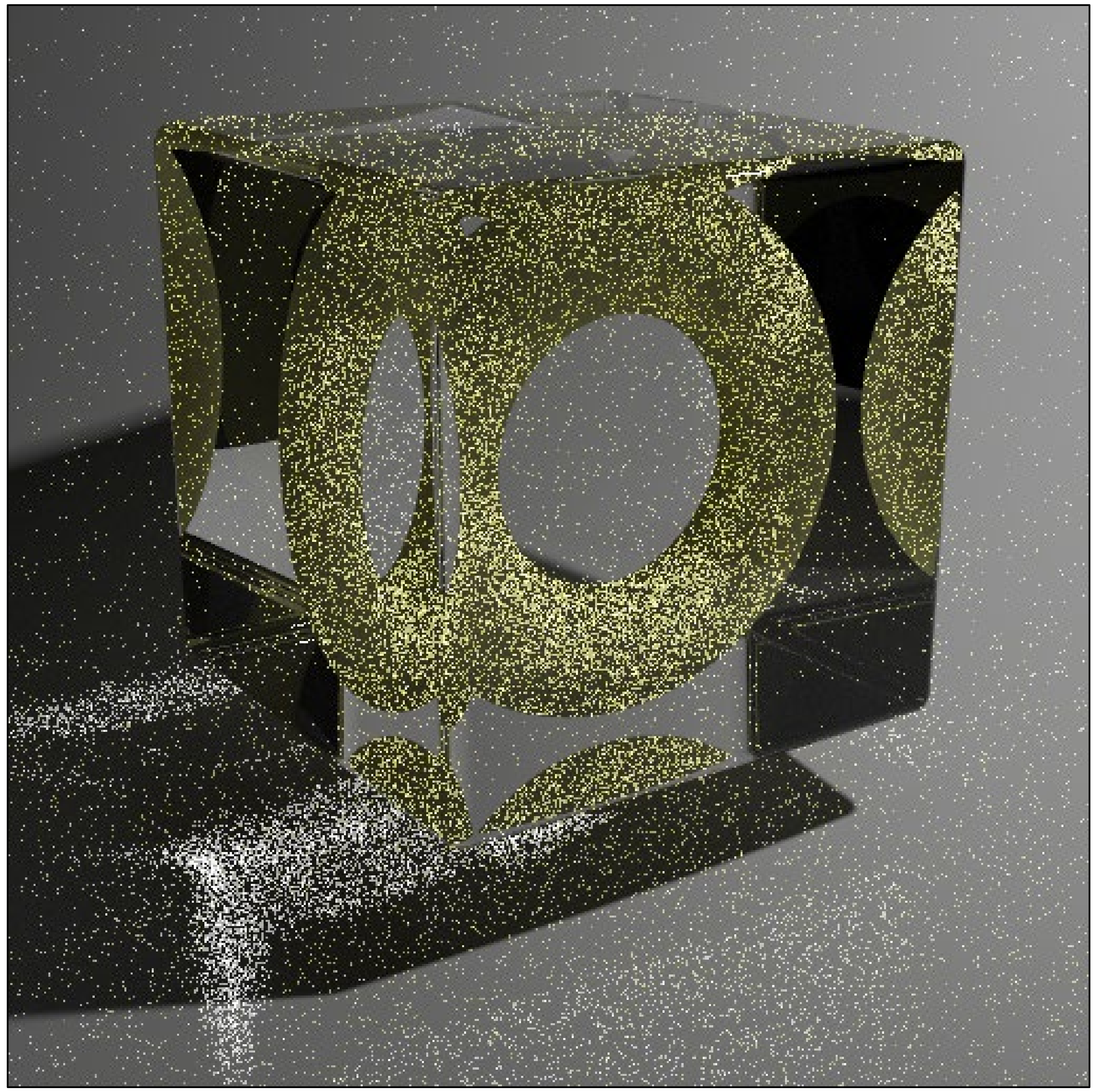
Images courtesy of T. Hachisuka

Glass Lantern

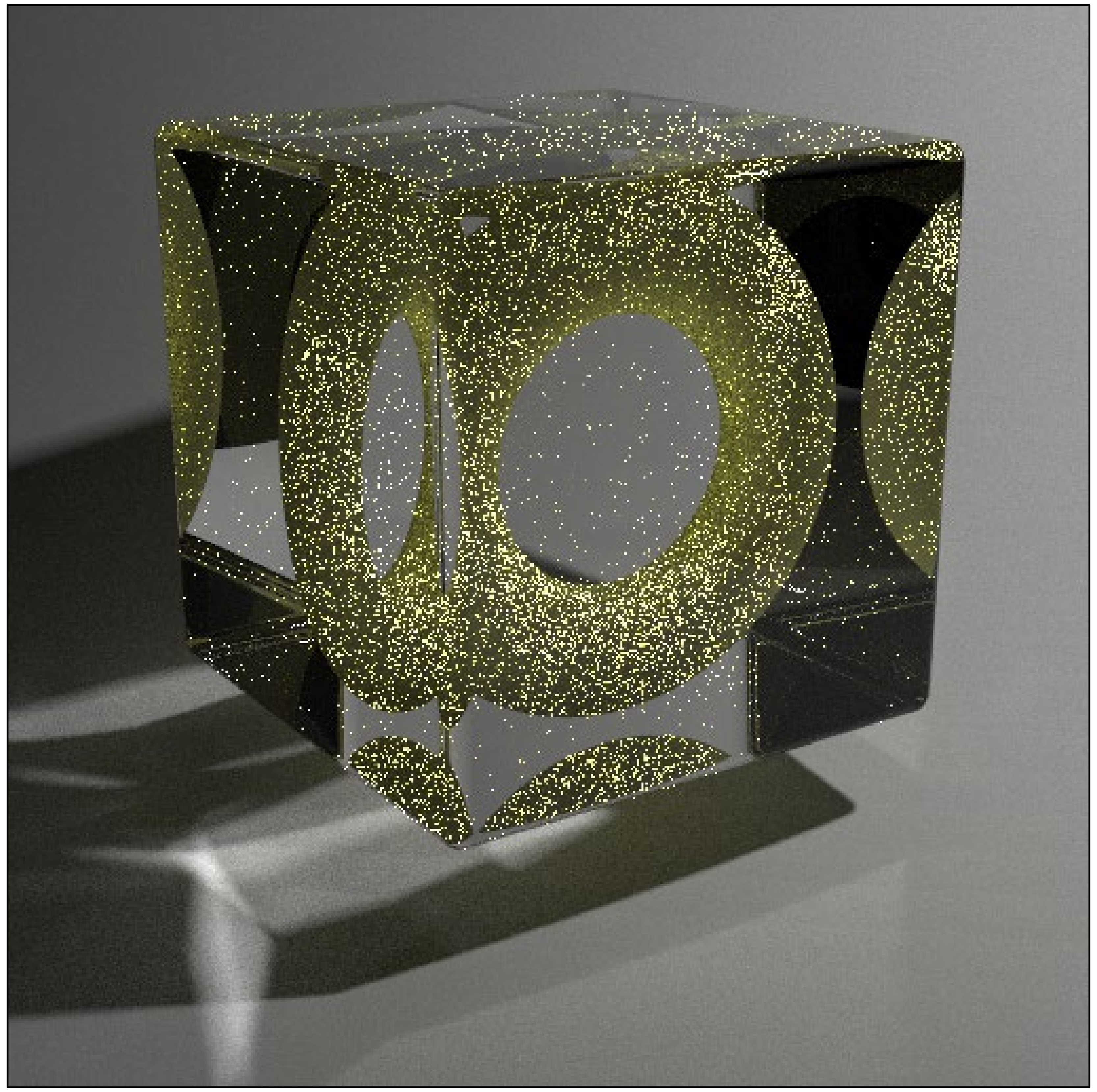
Path tracing | Bidirectional path tracing | Metropolis light transport | Photon mapping | Progressive photon mapping

Images courtesy of T. Hachisuka

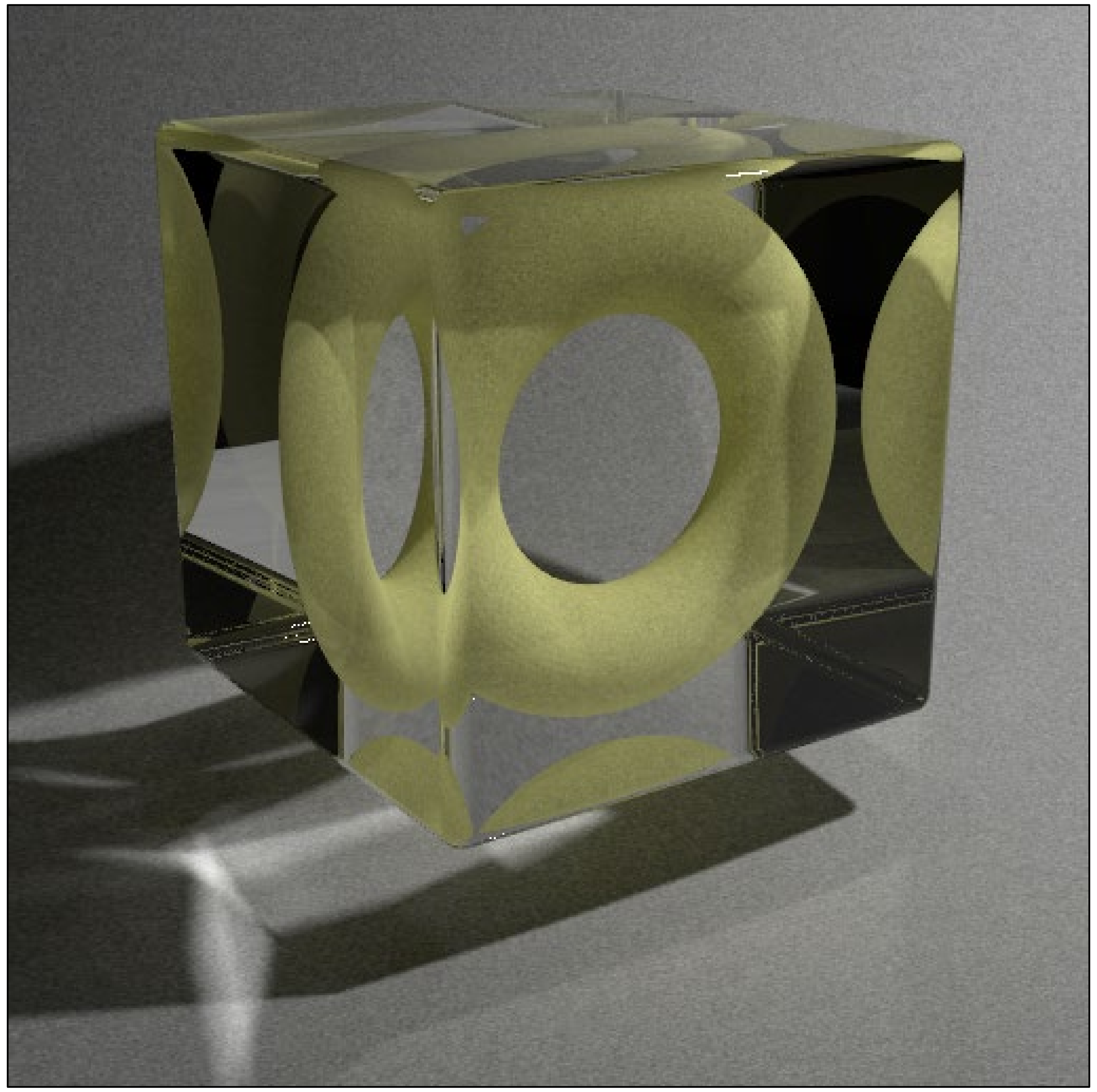
Torus in Cube ($LS^+D^*S^+E$)



Path Tracing



Bidirectional Path Tracing



Progressive Photon Mapping

Images courtesy of T. Hachisuka

Progressive PM - Summary

Reduces memory footprint

- Converges without requiring infinite memory

Renders progressively (user-friendly)

Data structure does not need to be as sophisticated

No need to bother using a caustic map, just use a single photon map for everything

More On Photon Mapping