Modeling BRDFs
Course announcements

• Take-home quiz 3 will be posted tonight, due next Tuesday.

• Programming assignment 1 posted, due this Friday.
  - How many of you have looked at/started/finished it?
  - Any questions?
Overview of today’s lecture

- BRDF modeling.
- Microfacet BRDFs.
- Data-driven BRDFs.
Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).
Real materials are complex
Rough materials

In reality, most materials are neither perfectly diffuse nor specular, but somewhere in between

- Imagine a shiny surface scratched up at a microscopic level
- “Blurry” reflections of the light source
Conductors vs. Dielectrics

- Copper
- Iron
- Glass
- Ethanol
- Gold
- Mercury
- Water
- Air

Image credits: Wikipedia Commons
Conductors vs. Dielectrics

Smooth conducting material

Smooth dielectric material

Rough conducting material

Rough dielectric material
BRDF History

1970s: Empirical models
- Phong’s illumination model

1980s:
- Physically based models
- Microfacet models (e.g. Cook-Torrance model)

1990s:
- Physically-based appearance models of specific effects (materials, weathering, dust, etc)

2000s:
- Measurement & acquisition of static materials/lights (wood, translucence, etc)
Three Levels of Detail

Key idea:
- transition from individual interactions to statistical averages

Macro scale
Scene geometry

Meso scale
Detail at intermediate scales
(can have variations here too)

Micro scale
Roughness
Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

\[
fr(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_r \cdot \vec{\omega}_o)^e
\]

\[
\vec{\omega}_r = (2\vec{n}(\vec{n} \cdot \vec{\omega}_i) - \vec{\omega}_i)
\]
Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

\[
f_r(\hat{\omega}_o, \hat{\omega}_i) = \frac{e + 2}{2\pi} (\hat{\omega}_r \cdot \hat{\omega}_o)^e
\]

\[
\hat{\omega}_r = (2\hat{n}(\hat{n} \cdot \hat{\omega}_i) - \hat{\omega}_i)
\]

Interpretation

- randomize reflection rays in a lobe about mirror direction
- perfect mirror reflection of a blurred light
Blinn-Phong BRDF

Distribution of normals instead of reflection directions

\[ f_r(\tilde{\omega}_o, \tilde{\omega}_i) = \frac{e + 2}{2\pi} (\tilde{\omega}_h \cdot \tilde{n})^e \]

\[ \tilde{\omega}_h = \frac{\tilde{\omega}_i + \tilde{\omega}_o}{\|\tilde{\omega}_i + \tilde{\omega}_o\|} \]

\( \tilde{\omega}_i \): incident direction

\( \tilde{n} \): normal vector

\( \tilde{\omega}_h \): half-way vector

\( \tilde{\omega}_o \): outgoing direction
Phong BRDF

\[ f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_r \cdot \vec{\omega}_o)^e \]

\[ \vec{\omega}_r = (2\vec{n}(\vec{n} \cdot \vec{\omega}_i) - \vec{\omega}_i) \]
Halfway vector vs. mirror direction BRDFs

BRDFs based on mirror reflection direction have round highlights

Highlights of BRDFs based on halfway vector get increasingly narrow at glancing angles
Halfway vector vs. mirror direction BRDFs

Amount of difference depends on circumstance

- Significant for floors, walls, etc. at grazing angles
- Less for highly curvy surfaces and moderate angle

After a slide by Naty Hoffman, SIGGRAPH 2006
Ward model

Gaussian blur distribution over half vector slopes

Original version had issues with energy conservation and singularities; several modified variants exist
Rough Surfaces

Empirical glossy models have limitations:
- not physically-based
- (often) not reciprocal
- not energy-preserving (can be normalized)
  • many conflicting normalizations in the literature
- (often) no Fresnel effects
- cannot accurately model appearance of many glossy surfaces

Blinn-Phong was first step in the right direction

Can do better
Microfacet Theory
Microfacet Theory

Assume surface consists of tiny facets

Assume that the differential area being viewed/illuminated is relatively large compared to the size of microfacets

A facet can be perfectly specular or diffuse
Microfacet Distribution

How much of the surface reflects?

\( \vec{\omega_i} \) \( \vec{\omega_o} \)
Microfacet Distribution

What fraction of the surface participates in the reflection?

- Answer 1: difficult to say (need an actual microsurface to compute this, tedious...)

- Answer 2: solve using principles of statistical physics
  
  • Is there something general we can say about the surface when there are many bumps?
Torrance-Sparrow Model

Developed by Torrance & Sparrow in 1967

- Originally used in the physics community
- Adapted by Cook & Torrance and Blinn for graphics
  - added ambient and diffuse terms

Explains off-specular peaks

Assumes surface is composed of many micro-grooves, each of which is a perfect mirror.
Copper-colored plastic

Copper

Cook-Torrance (1981)
General Microfacet Model

\[ f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|\vec{\omega}_i \cdot \vec{n}|} \]

where \(\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{||\vec{\omega}_i + \vec{\omega}_o||}\)
Fresnel Term

<table>
<thead>
<tr>
<th>Material</th>
<th>n</th>
<th>F(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal (Aluminum)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>1.5</td>
<td>0.82</td>
</tr>
<tr>
<td>Silver</td>
<td>1.5</td>
<td>0.95</td>
</tr>
<tr>
<td>Dielectric (N=1.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td>1.5</td>
<td>0.04</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.4</td>
<td>0.15</td>
</tr>
</tbody>
</table>
General Microfacet Model

\[ f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|\vec{\omega}_i \cdot \vec{n})(\vec{\omega}_o \cdot \vec{n})|} \]
Microfacet Distribution

Fraction of microfacets facing each direction

Probability density function over *projected* solid angle (must be normalized):

\[
\int_{H^2} D(\hat{\omega}_h) \cos \theta_h \, d\hat{\omega}_h = 1
\]
The Beckmann Distribution

The slopes follow a Gaussian distribution

Let’s express slope distribution wrt. directions

– Slope of $\theta_h$ is $\tan \theta_h$

$$D(\tilde{\omega}_h) = \frac{1}{\pi \alpha^2 \cos^4 \theta_h} e^{-\frac{\tan^2 \theta_h}{\alpha^2}}$$
The Beckmann Distribution

The slopes follow a Gaussian distribution

Let’s express slope distribution wrt. directions
Other Distributions

The Blinn distribution:

\[ D(\vec{\omega}_h) = \frac{e + 2}{2\pi} (\vec{\omega}_h \cdot \vec{n})^e \]

GGX distribution, see [Walter et al., EGSR 2007]

Anisotropic distributions, see [PBRTv2, Ch. 8]
General Microfacet Model

\[
f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|\vec{\omega}_i \cdot \vec{n}|(\vec{\omega}_o \cdot \vec{n})|}
\]
Shadowing and Masking

Microfacets can be *shadowed* and/or *masked* by other microfacets

\[ \vec{\omega} \]

Angle = 85 degrees
Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Beckman distribution:

\[ G(\vec{\omega}) = \frac{2}{1 + \text{erf}(s) + \frac{1}{s\sqrt{\pi}}e^{-s^2}} \]

\[ s = \frac{1}{\alpha \tan \theta} \]

\[ G(\vec{\omega}_i, \vec{\omega}_o) = G(\vec{\omega}_i) \cdot G(\vec{\omega}_o) \]
Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Beckman distribution (approximated):

\[
G(\tilde{\omega}) \approx \begin{cases} 
\frac{3.535s + 2.181s^2}{1 + 2.276s + 2.577s^2}, & s < 1.6 \\
1, & \text{otherwise}
\end{cases}
\]

\[
G(\tilde{\omega}_i, \tilde{\omega}_o) = G(\tilde{\omega}_i) \cdot G(\tilde{\omega}_o)
\]
Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Beckman distribution (approximated):

\[
\begin{align*}
\alpha &= 0.3 \\
\alpha &= 0.6 \\
\alpha &= 0.9
\end{align*}
\]
Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Torrance-Sparrow (Blinn):

\[
G(\vec{\omega}_i, \vec{\omega}_o) = \min \left( 1, \frac{2(\vec{n} \cdot \vec{\omega}_h)(\vec{n} \cdot \vec{\omega}_i)}{\vec{\omega}_h \cdot \vec{\omega}_i}, \frac{2(\vec{n} \cdot \vec{\omega}_h)(\vec{n} \cdot \vec{\omega}_o)}{\vec{\omega}_h \cdot \vec{\omega}_o} \right)
\]
General Microfacet Model

\[ f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4 |(\vec{\omega}_i \cdot \vec{n})(\vec{\omega}_o \cdot \vec{n})|} \]

Denominator: correction term coming from energy conservation, Jacobians, etc.

- see PBR book and Walter et al. [EGSR 2007] for more detail
GGX and Beckmann

anti-glare (Beckman, $\alpha_b = 0.023$)

ground (GGX, $\alpha_g = 0.394$)

etched (GGX, $\alpha_g = 0.553$)
Energy Loss Issue
Energy Loss Issue - Conductor

Increasing roughness $\alpha = 0.01 \ldots 2.0$
Energy Loss Issue - Dielectric

Increasing roughness $\alpha = 0.01 .. 2.0$
Interesting grazing angle behavior
Extension: Anisotropic Reflection

source: luxology.com

source: Stephen H. Westin
BRDF of the moon

What BRDF does the moon have?
BRDF of the moon

What BRDF does the moon have?
• Can it be diffuse?
BRDF of the moon

What BRDF does the moon have?
- Can it be diffuse?

Even though the moon appears matte, its edges remain bright.
The Oren-Nayar Model

Same concept as the microfacet models, but assumes that the facets are diffuse

Shadowing/masking + interreflections

No analytic solution; fitted approximation

\[ f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{\rho}{\pi} \left( A + B \max(0, \cos(\phi_i - \phi_o)) \sin \alpha \tan \beta \right) \]

\[ A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \quad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09} \]

\[ \alpha = \max(\theta_i, \theta_o) \quad \beta = \min(\theta_i, \theta_o) \]

Ideal Lambertian is just a special case (\(\sigma = 0\))
Rough diffuse appearance

Surface Roughness Causes Flat Appearance

Actual Vase

Lambertian Vase
Smooth Diffuse

source: Wenzel Jakob
Rough Diffuse

source: Wenzel Jakob
Extension: layered materials

Diffuse base layer coated using a perfectly smooth dielectric (can do something similar with microfacets)
Smooth Diffuse
Smooth Plastic

source: Wenzel Jakob
Smooth Plastic

Plain diffuse material

Naïve blend of diffuse + specular
*(incorrect)*

Specular-matte
*(correct)*

source: Wenzel Jakob
Smooth Plastic

Smooth dielectric varnish on top of diffuse surface

source: Wenzel Jakob
Rough Plastic

Rough dielectric varnish on top of diffuse surface
Rough Dielectric

Anti-glare glass ($m = 0.02$)

source: Wenzel Jakob
Rough Dielectric

Rough glass ($m = 0.1$)

source: Wenzel Jakob
Rough Dielectric

Textured roughness

source: Wenzel Jakob
Data-Driven BRDFs
Spherical gantry
Measuring BRDFs

source: Matusik et al. 2003
Nickel

source: Matusik et al. 2003
Hematite

source: Matusik et al. 2003
Gold Paint

source: Matusik et al. 2003
Pink Fabric

source: Matusik et al. 2003
BRDF Editing/Navigation

Given a large database, can mix/match and interpolate between BRDFs
The MERL Database

"A Data-Driven Reflectance Model"
Wojciech Matusik, Hanspeter Pfister, Matt Brand and Leonard McMillan.

Download them and use them in your own renderer!
Measuring and Modeling the Appearance of Wood

Stephen R. Marschner, Stephen H. Westin, Adam Arbree, and Jonathan T. Moon

Cornell University
Reading

PBRTv3 Chapter 8, and 14.1