Modeling BRDFs

15-468, 15-668, 15-868
Physics-based Rendering
Spring 2025, Lecture 7

http://graphics.cs.cmu.edu/courses/15-468

Course announcements

- Take-home quiz 3 posted tonight, due next Tuesday.
- Programming assignment 1 posted, due this Friday.
- Recitations are in the graphics lounge (Smith Hall 236), ignore the room mentioned in the registar’s system!

Overview of today's lecture

- BRDF modeling.
- Microfacet BRDFs.
- Data-driven BRDFs.

Slide credits

Most of these slides were directly adapted from:

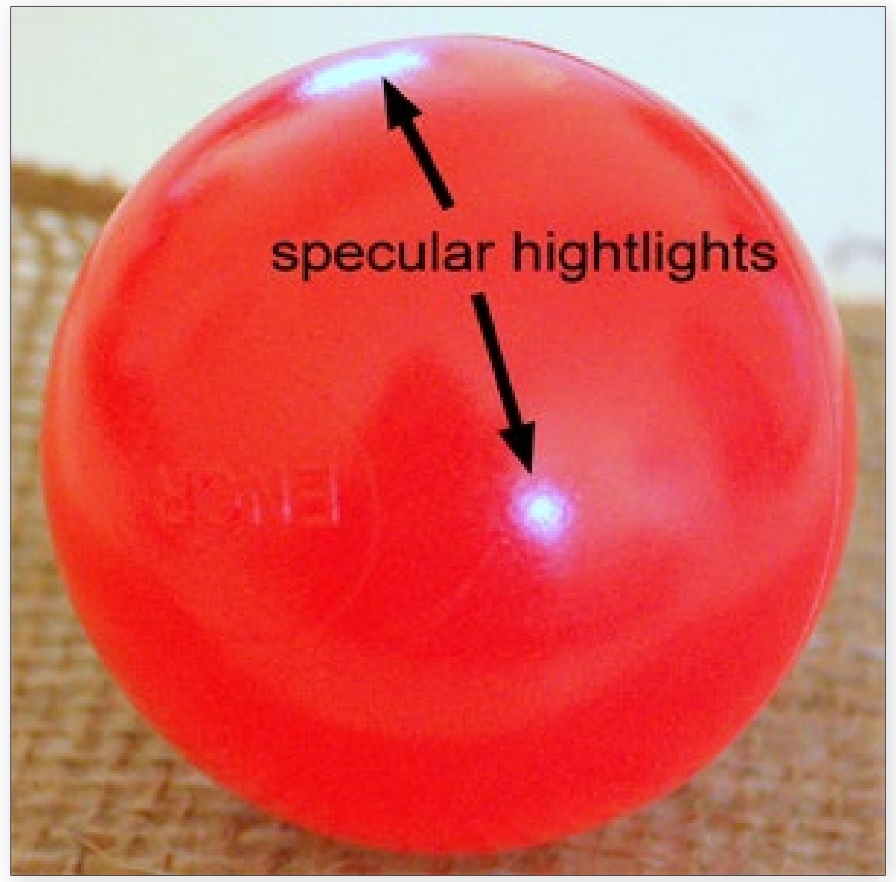
- Wojciech Jarosz (Dartmouth).

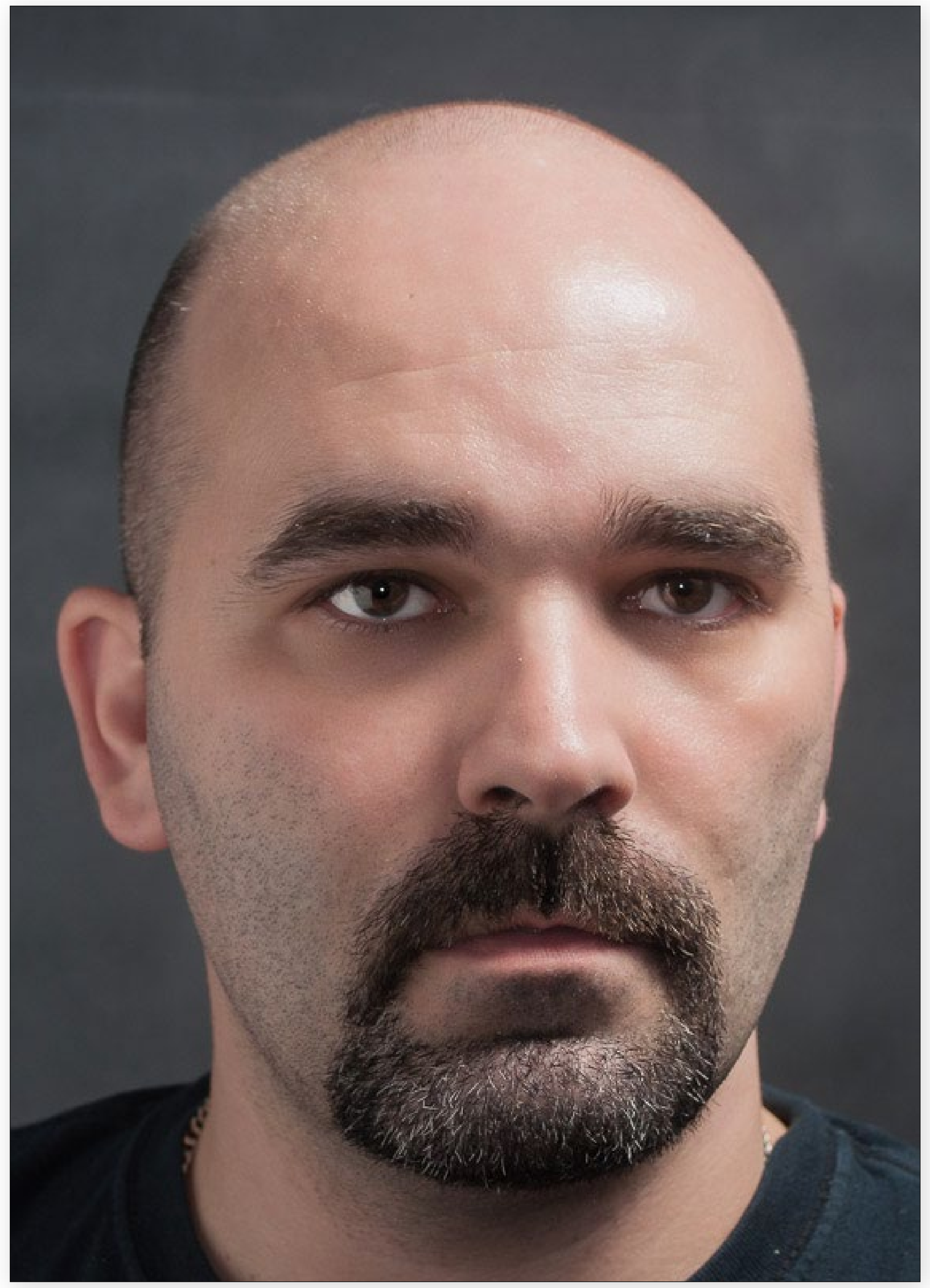

Spivak
CALCULUS ON MANIFOLDS
Westview
Real materials are complex

Rough materials

In reality, most materials are neither perfectly diffuse nor specular, but somewhere in between

- Imagine a shiny surface scratched up at a microscopic level
- “Blurry” reflections of the light source



Conductors vs. Dielectrics

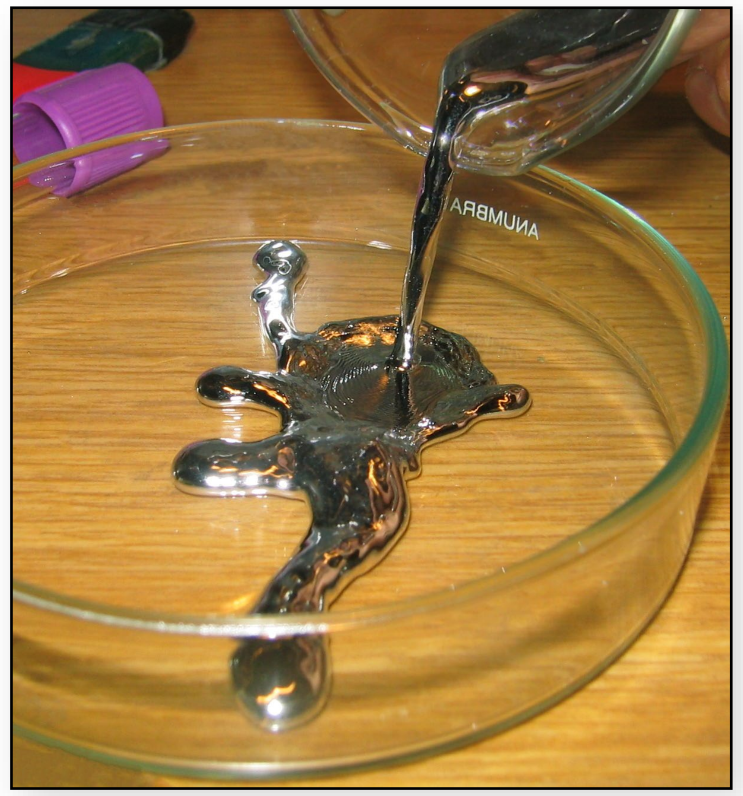
Copper

Iron

Glass

Ethanol

Gold



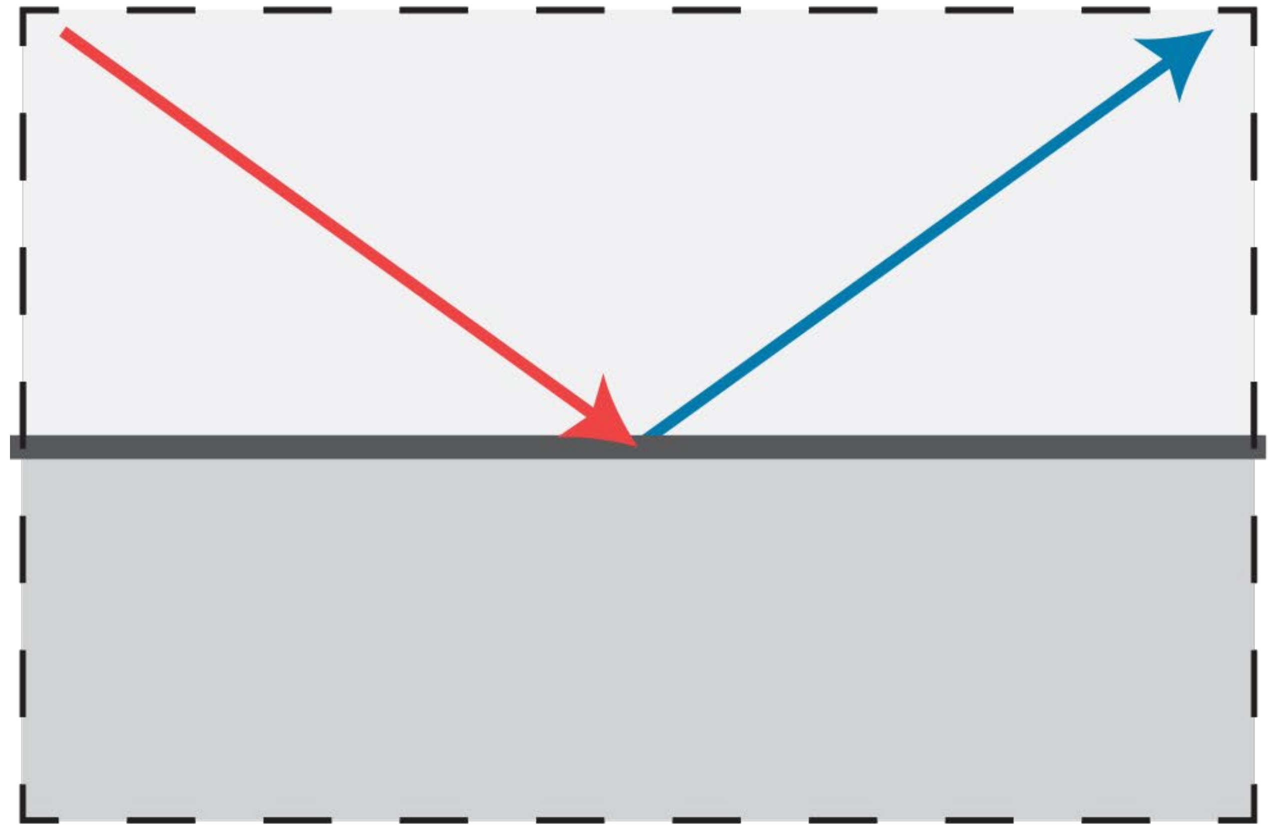
Mercury

Water

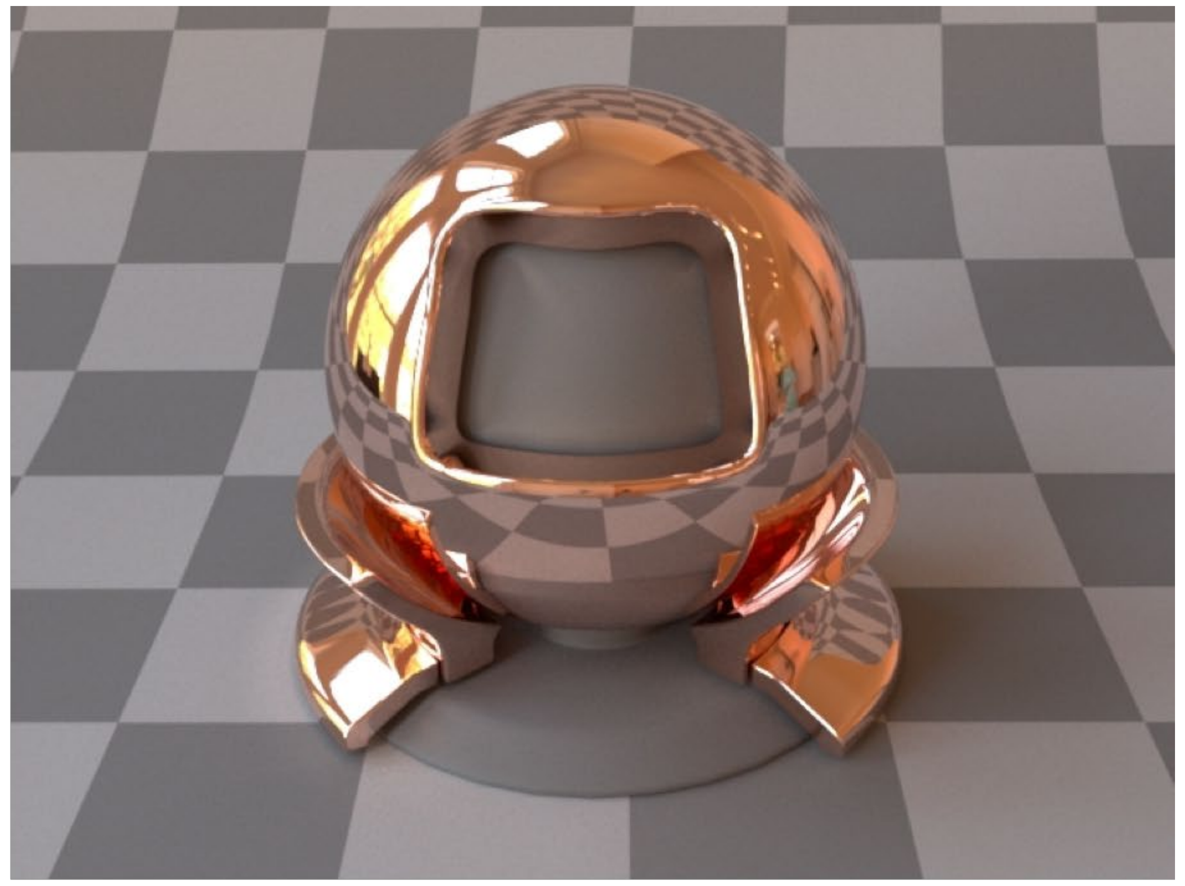
Air

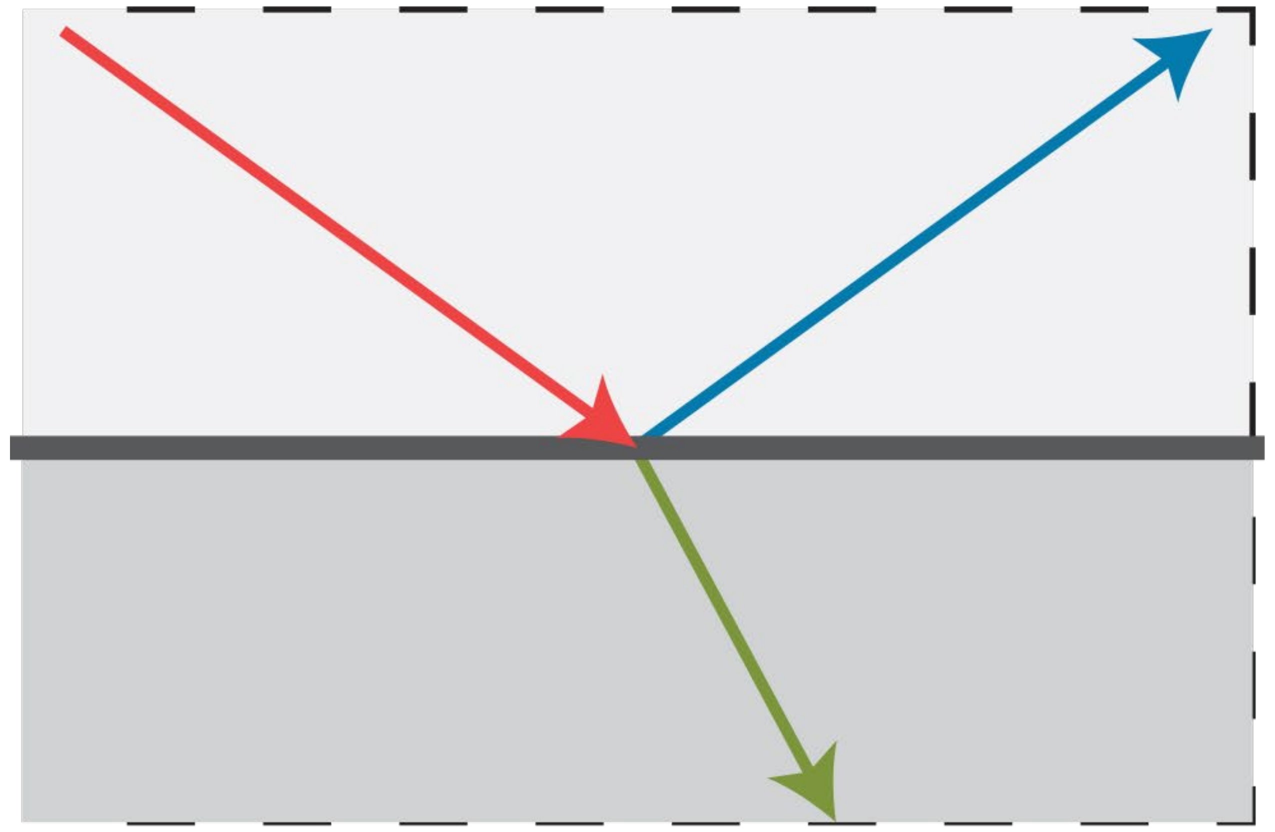
Image credits: Wikipedia Commons

Conductors vs. Dielectrics

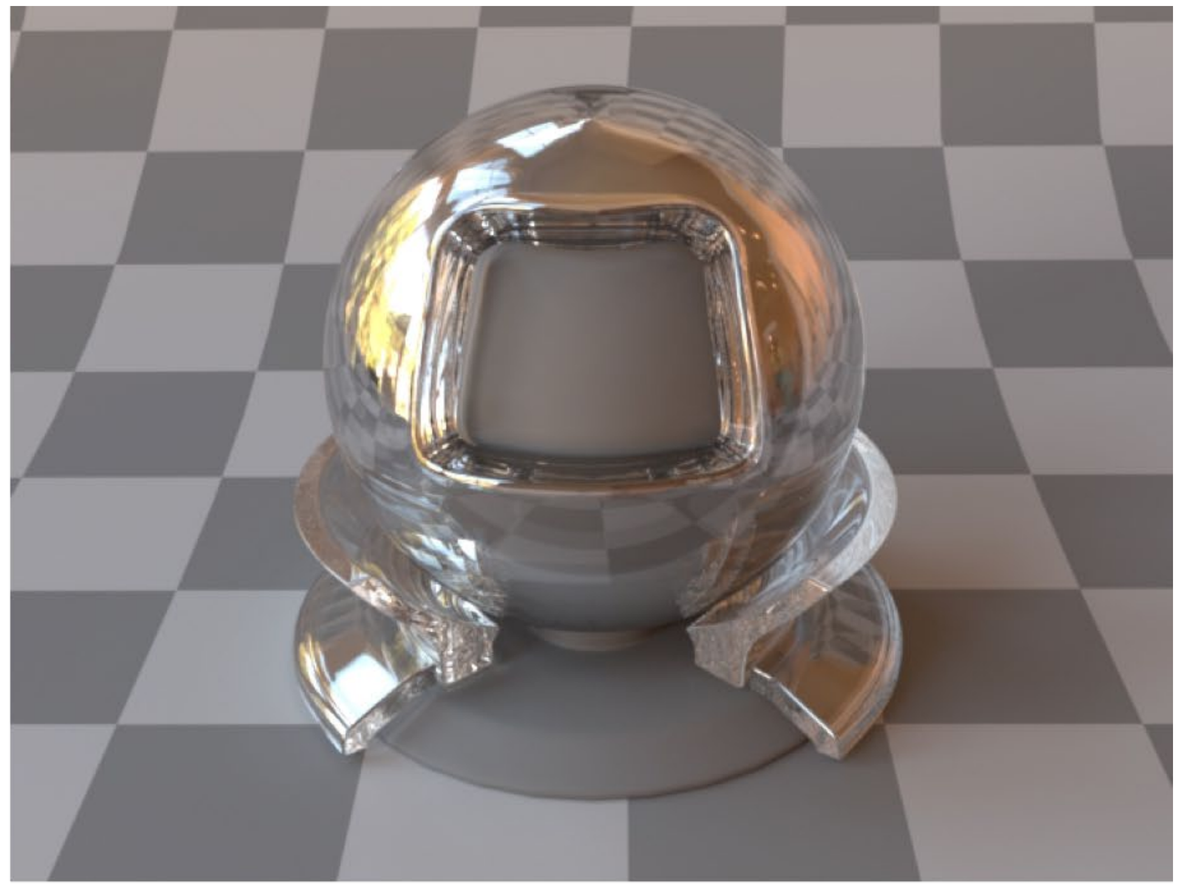


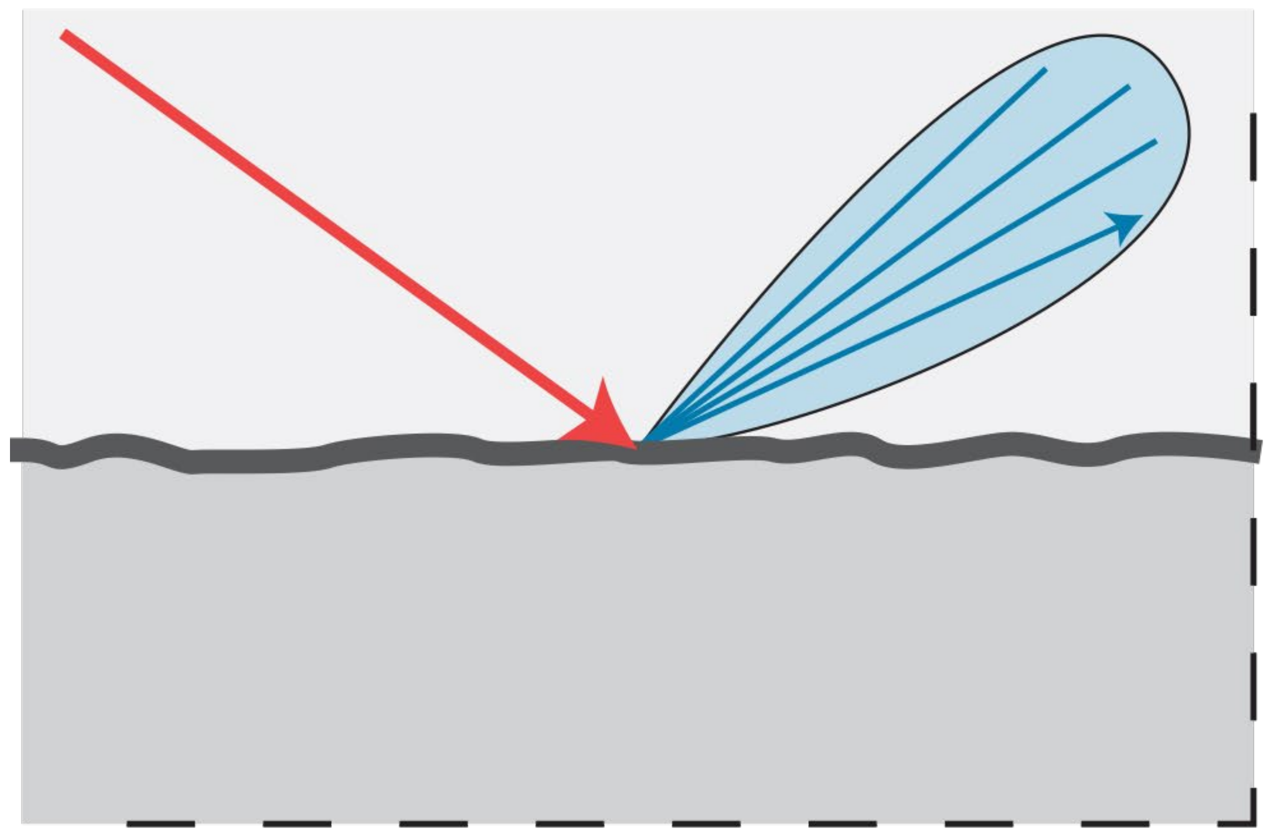
Smooth conducting material



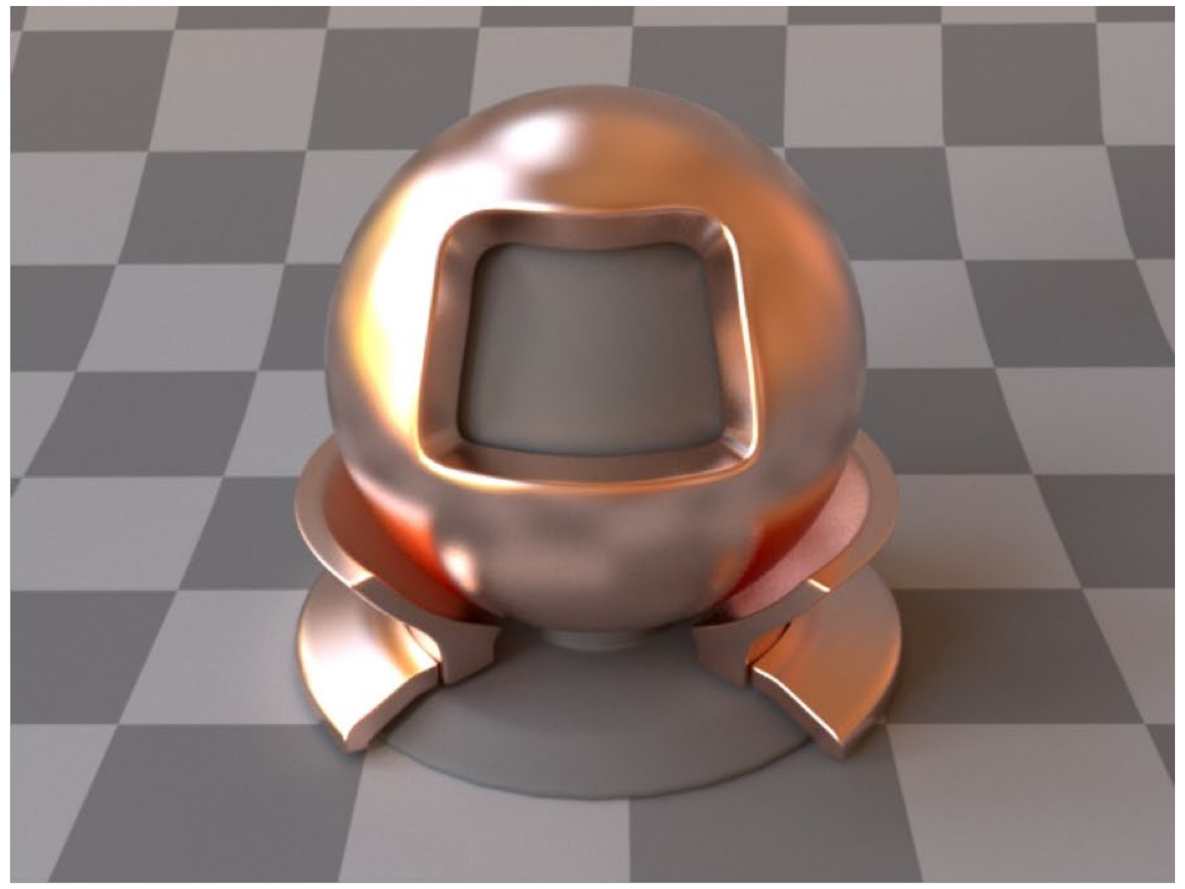


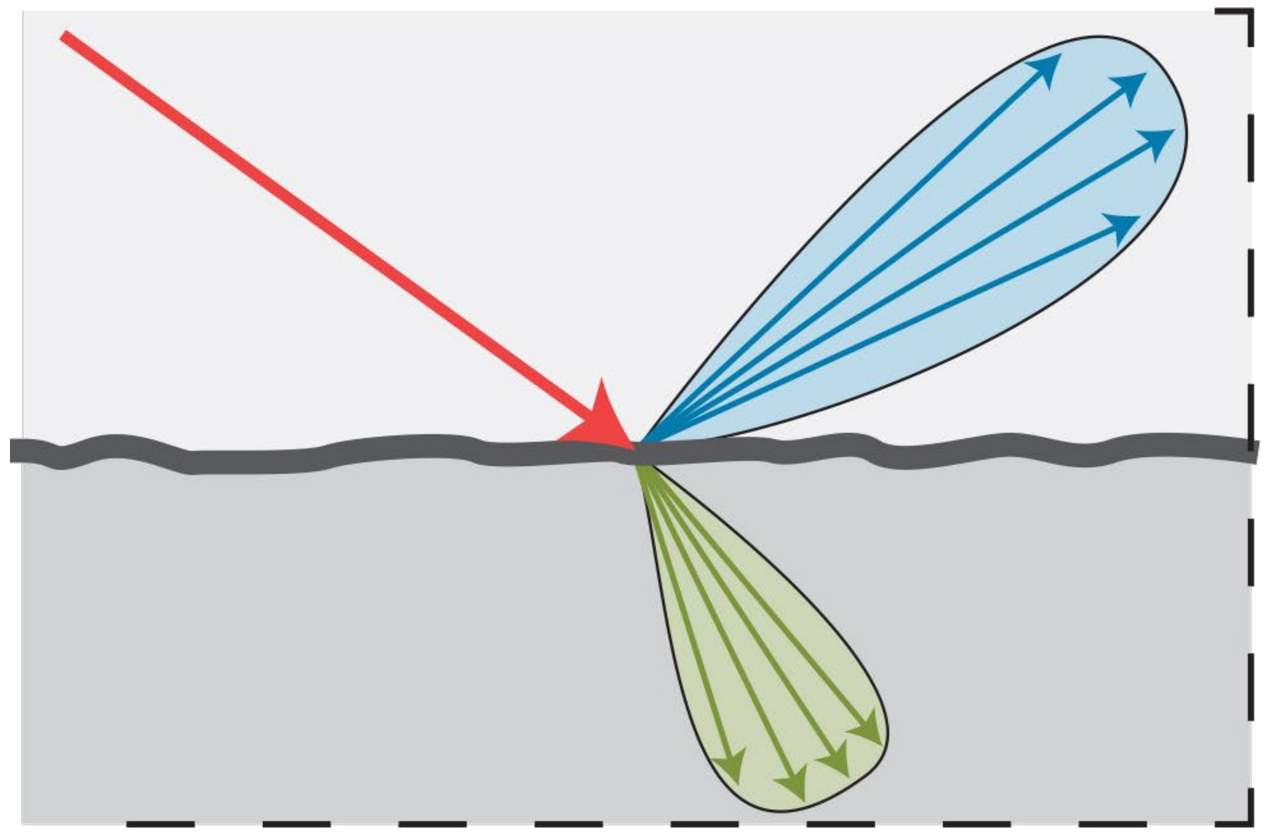
Smooth dielectric material



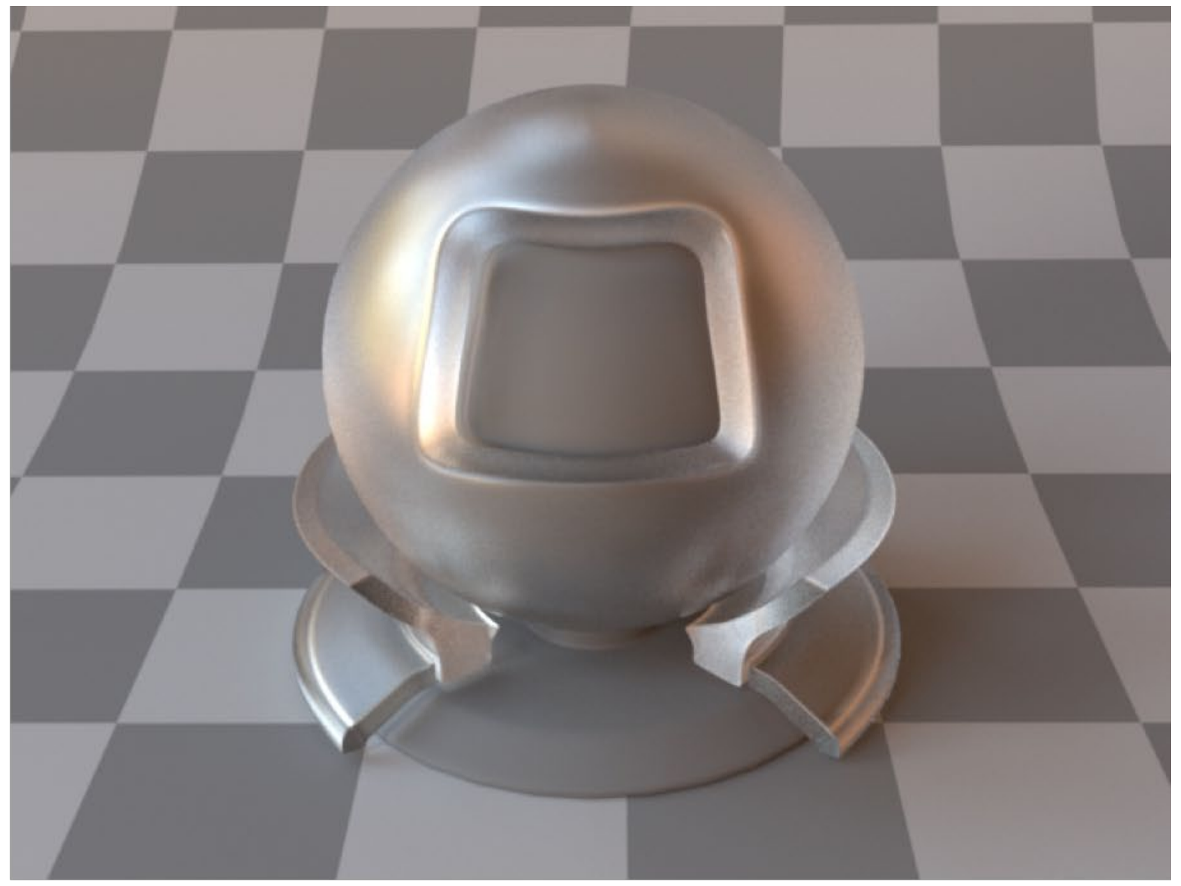


Rough conducting material





Rough dielectric material



BRDF History

1970s: Empirical models

- Phong’s illumination model

1980s:

- Physically based models
- Microfacet models (e.g. Cook-Torrance model)

1990s:

- Physically-based appearance models of specific effects (materials, weathering, dust, etc)

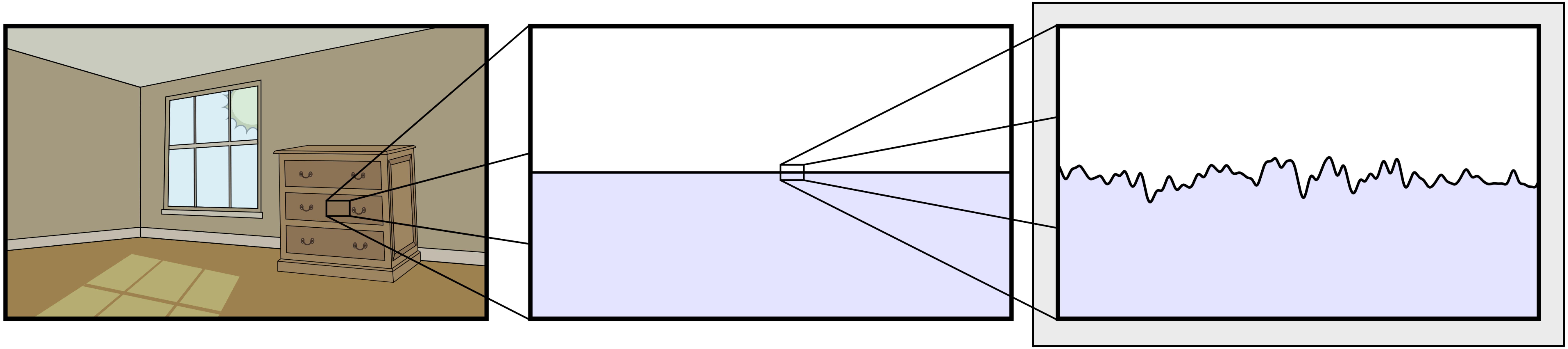
2000s:

- Measurement & acquisition of static materials/lights (wood, translucence, etc)

Three Levels of Detail

Key idea:

- transition from individual interactions to statistical averages



Macro scale

Scene geometry

Meso scale

Detail at intermediate scales

(can have variations here too)

Micro scale

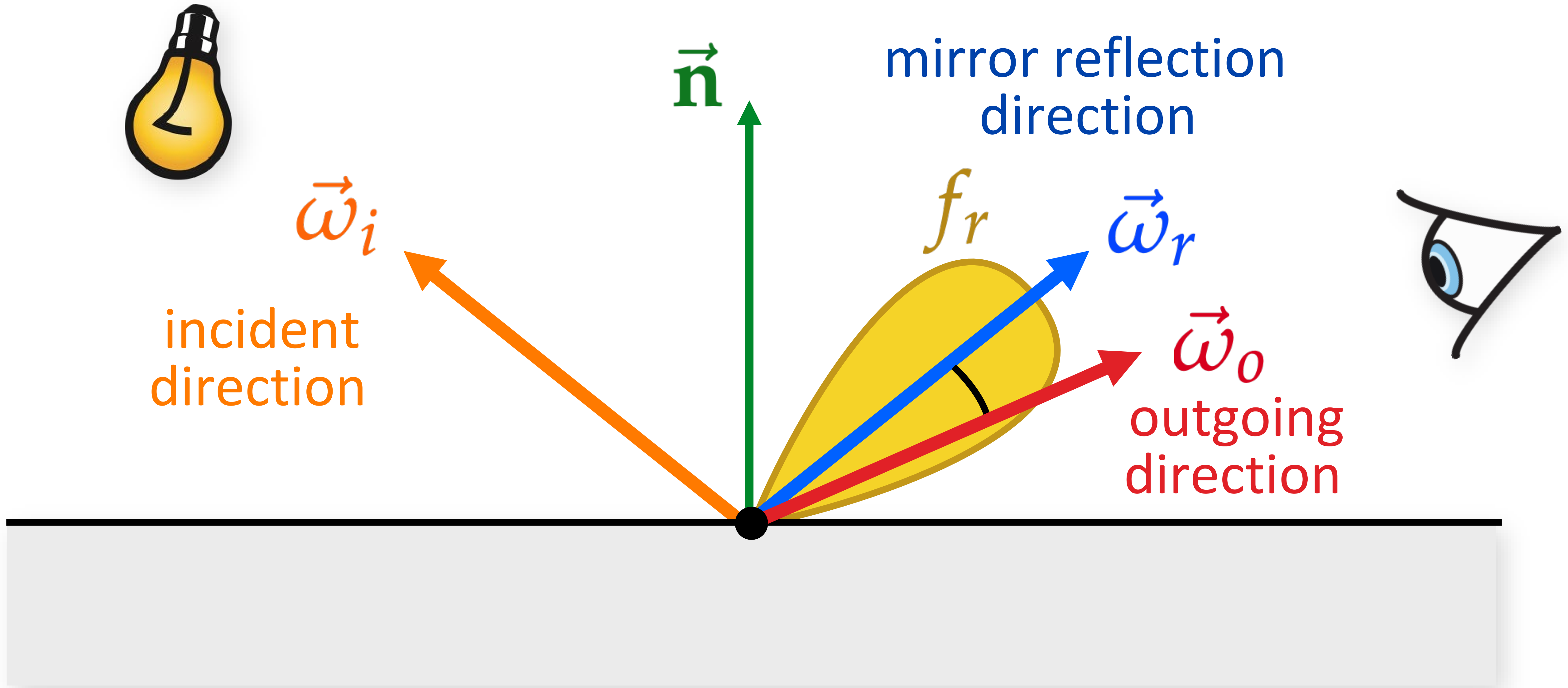
Roughness

Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_r \cdot \vec{\omega}_o)^e$$

$$\vec{\omega}_r = (2\vec{\mathbf{n}}(\vec{\mathbf{n}} \cdot \vec{\omega}_i) - \vec{\omega}_i)$$



Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_r \cdot \vec{\omega}_o)^e$$

$$\vec{\omega}_r = (2\vec{\mathbf{n}}(\vec{\mathbf{n}} \cdot \vec{\omega}_i) - \vec{\omega}_i)$$

Interpretation

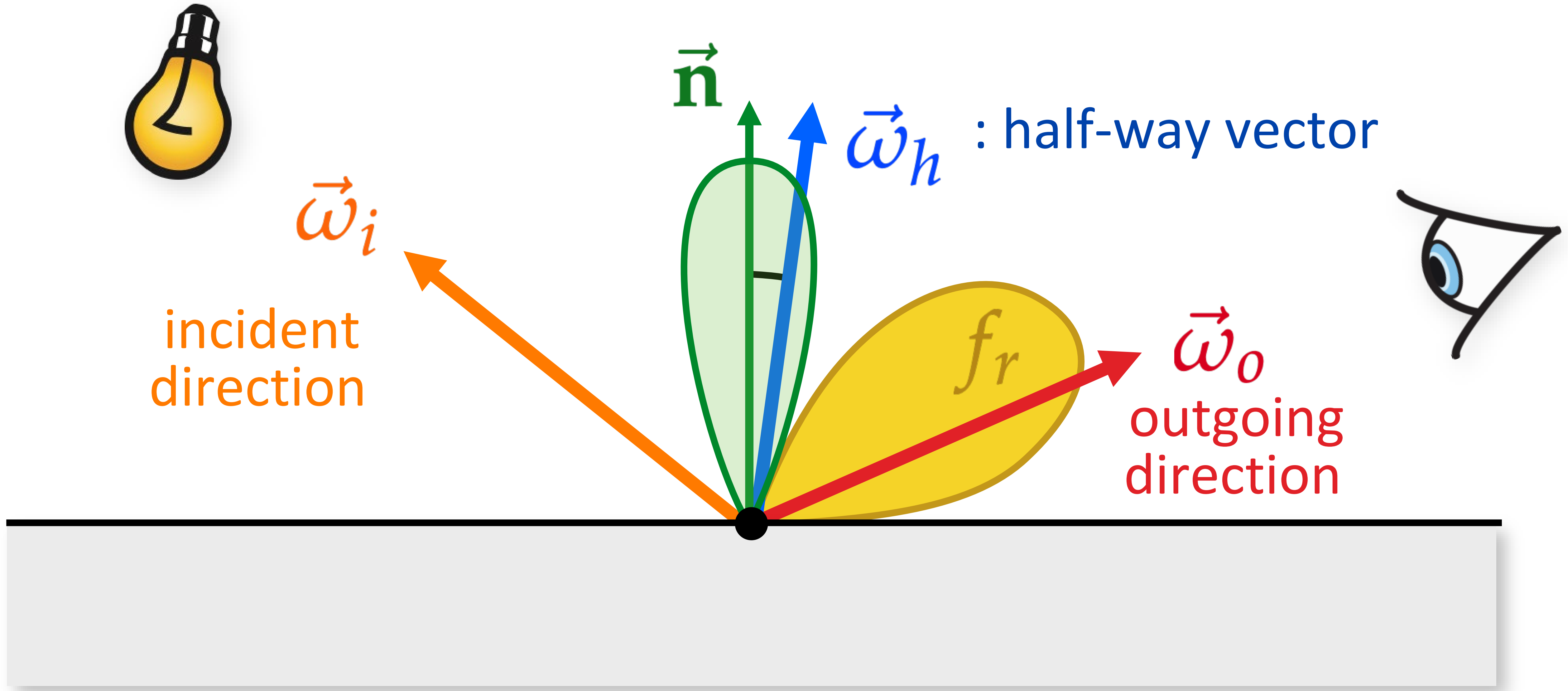
- randomize reflection rays in a lobe about mirror direction
- perfect mirror reflection of a blurred light

Blinn-Phong BRDF

Distribution of normals instead of reflection directions

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_h \cdot \vec{\mathbf{n}})^e$$

$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$



Phong BRDF

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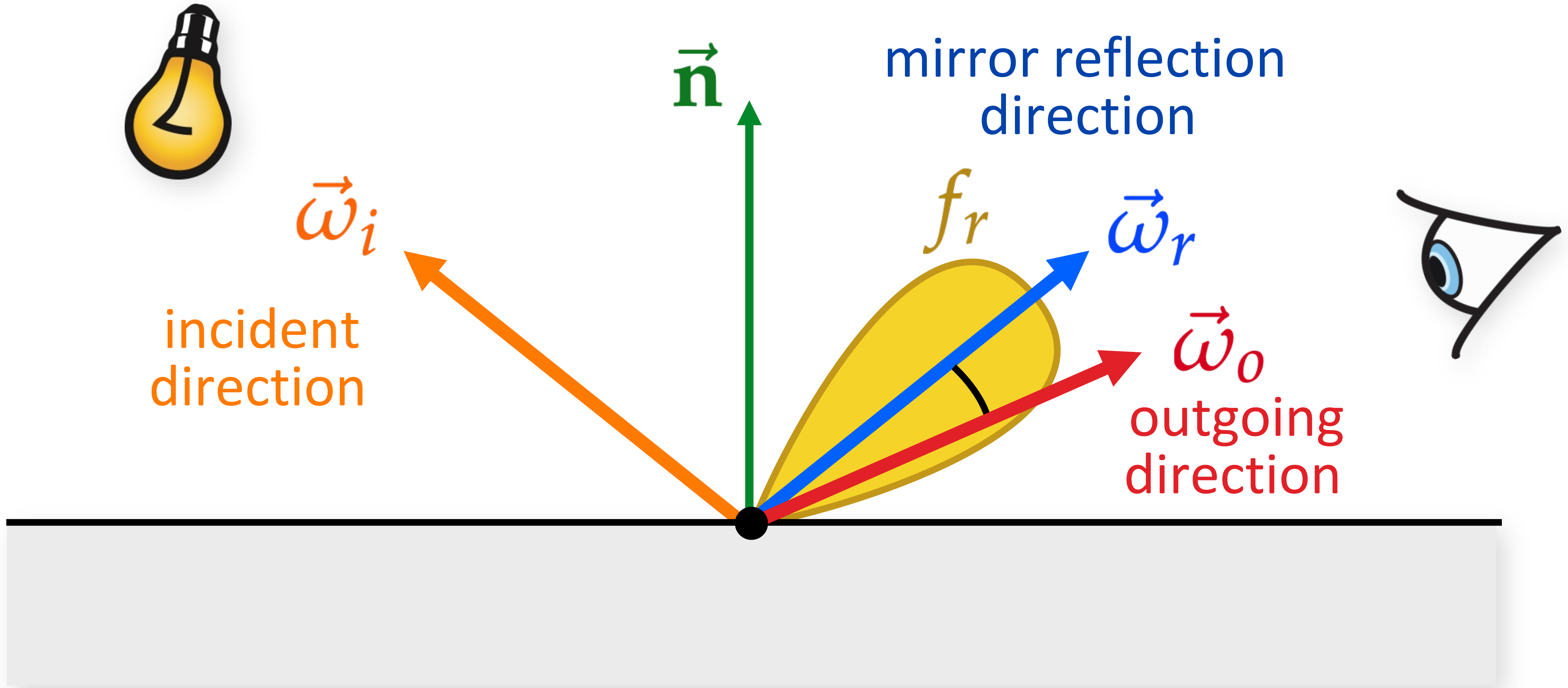


Halfway vector vs. mirror direction BRDFs

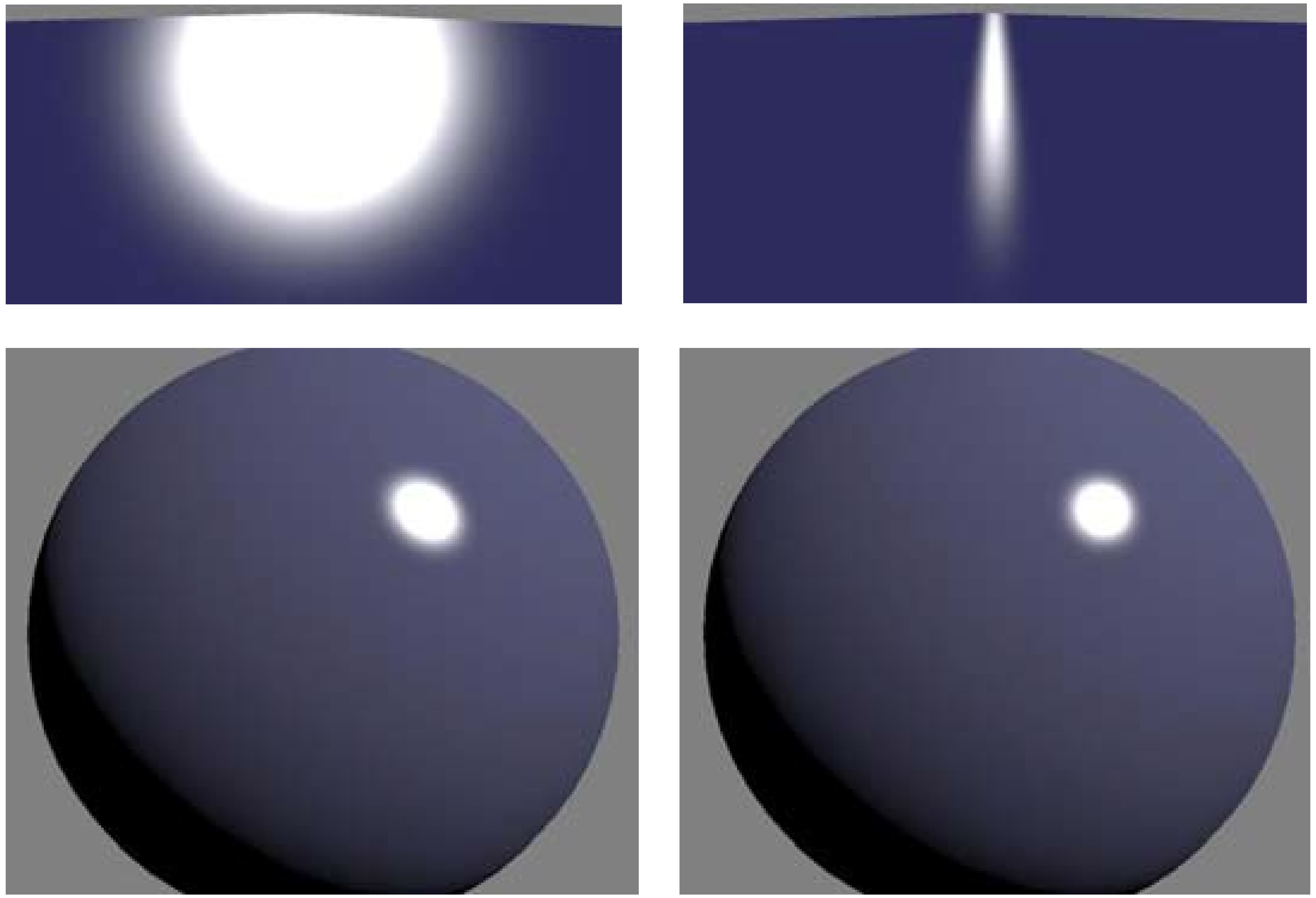
BRDFs based on mirror reflection direction have round highlights

Highlights of BRDFs based on halfway vector get increasingly narrow at glancing angles

Halfway vector vs. mirror direction BRDFs

Amount of difference depends on circumstance

- Significant for floors, walls, etc. at grazing angles
- Less for highly curvy surfaces and moderate angle



After a slide by Naty Hoffman, SIGGRAPH 2006

Ward model

Gaussian blur distribution over half vector slopes

Original version had issues with energy conservation and singularities; several modified variants exist

Rough Surfaces

Empirical glossy models have limitations:

- not physically-based
- (often) not reciprocal
- not energy-preserving (can be normalized)
  - many conflicting normalizations in the literature
- (often) no Fresnel effects
- cannot accurately model appearance of many glossy surfaces

Blinn-Phong was first step in the right direction

Can do better

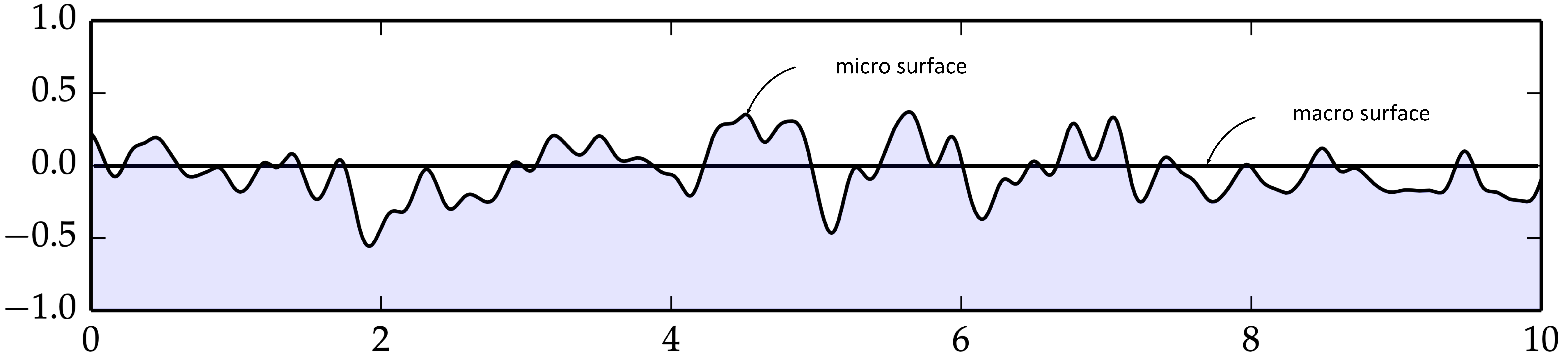
Microfacet Theory

Microfacet Theory

Assume surface consists of tiny facets

Assume that the differential area being viewed/illuminated is relatively large compared to the size of microfacets

A facet can be perfectly specular or diffuse



Microfacet Distribution

Microfacet Distribution

What fraction of the surface participates in the reflection?

- Answer 1: difficult to say (need an actual microsurface to compute this, tedious...)
- Answer 2: solve using principles of statistical physics
  - Is there something general we can say about the surface when there are many bumps?

Torrance-Sparrow Model

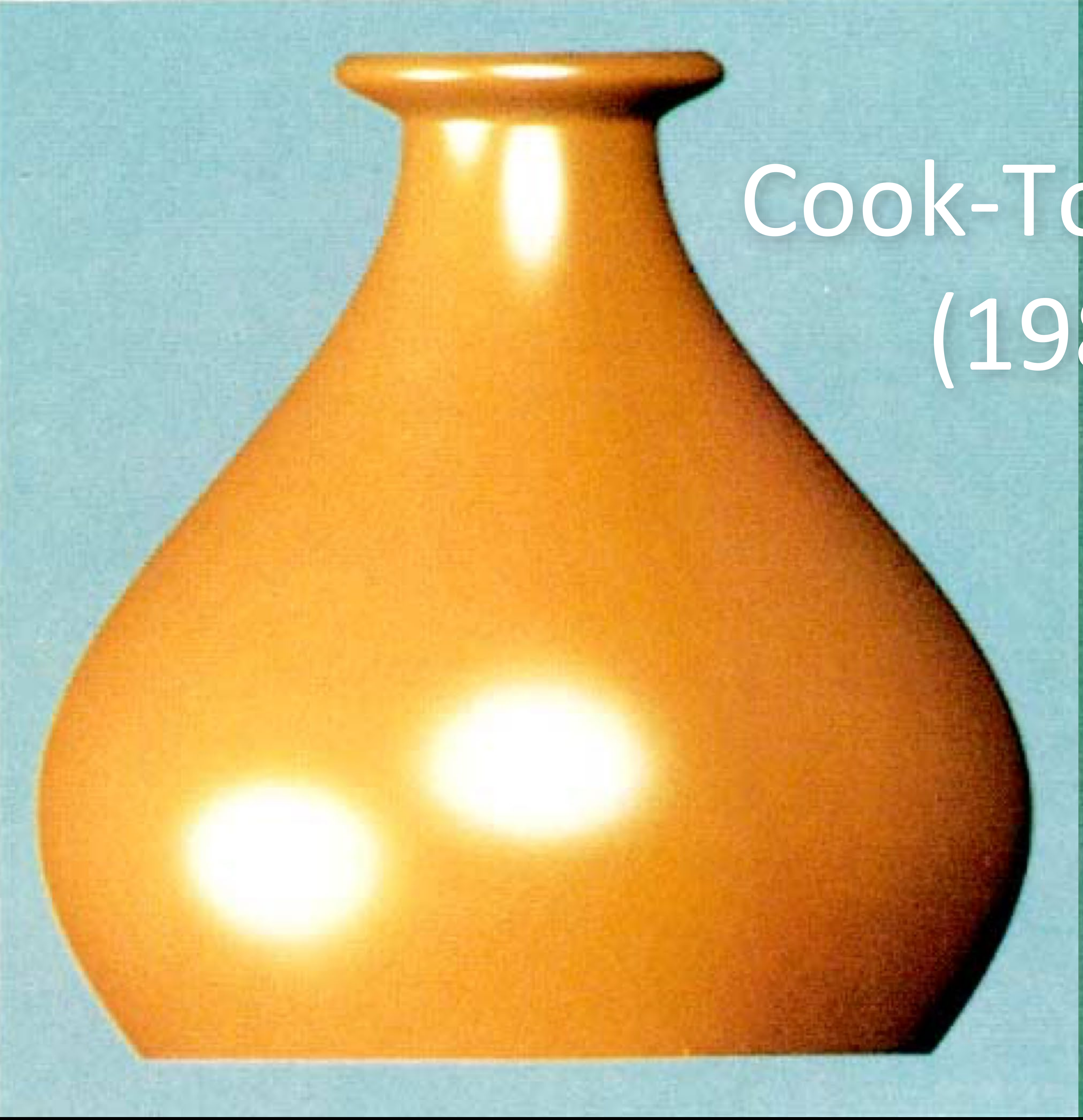
Developed by Torrance & Sparrow in 1967

- Originally used in the physics community
- Adapted by Cook & Torrance and Blinn for graphics
  - added ambient and diffuse terms

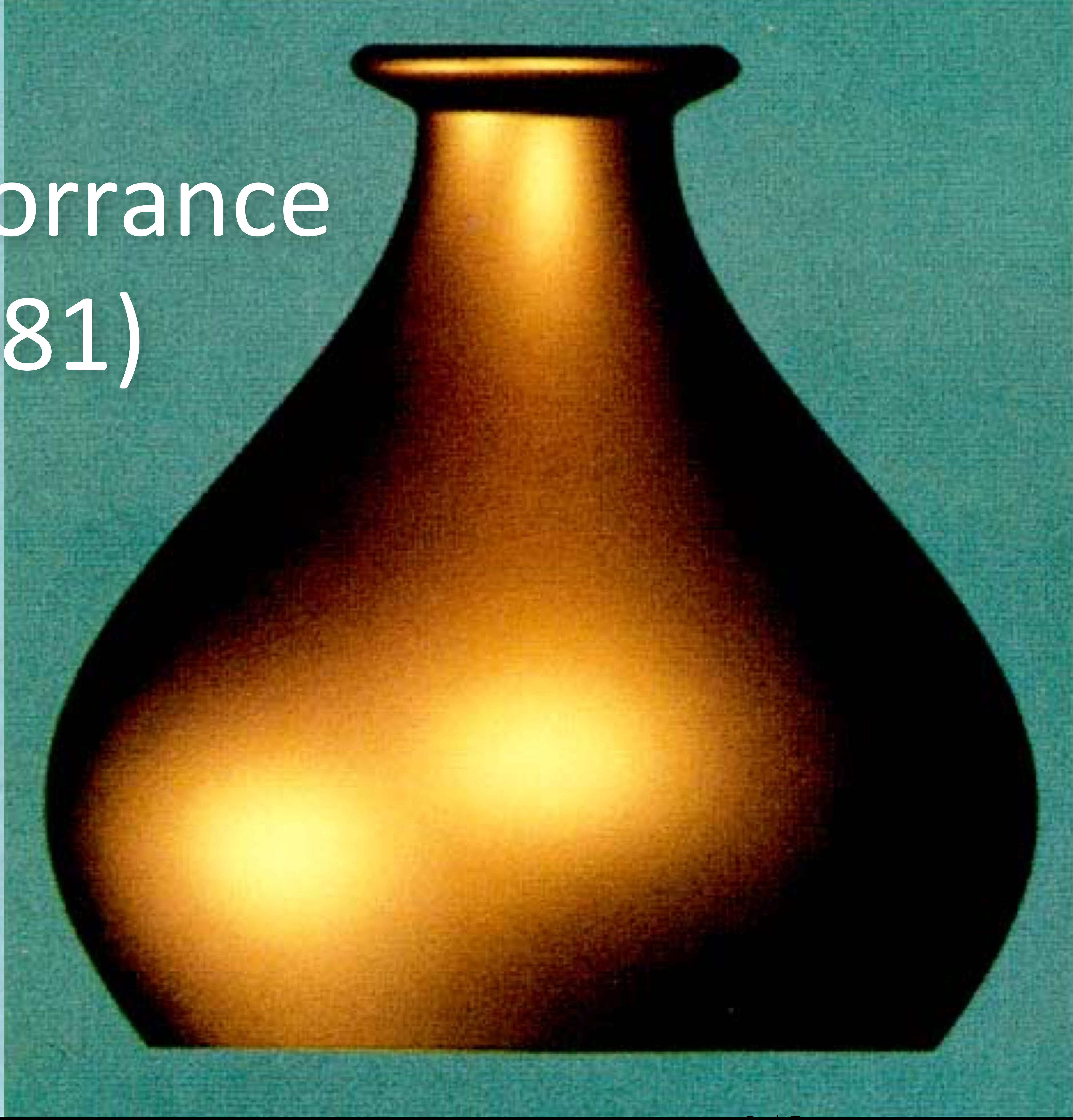
Explains off-specular peaks

Assumes surface is composed of many micro-grooves, each of which is a perfect mirror.

Cook-Torrance (1981)



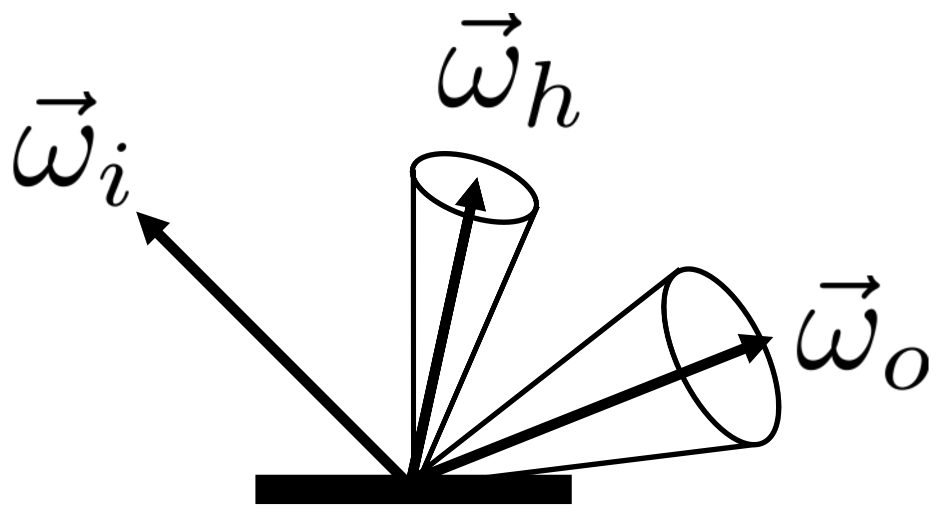
Copper-colored plastic



Copper

General Microfacet Model

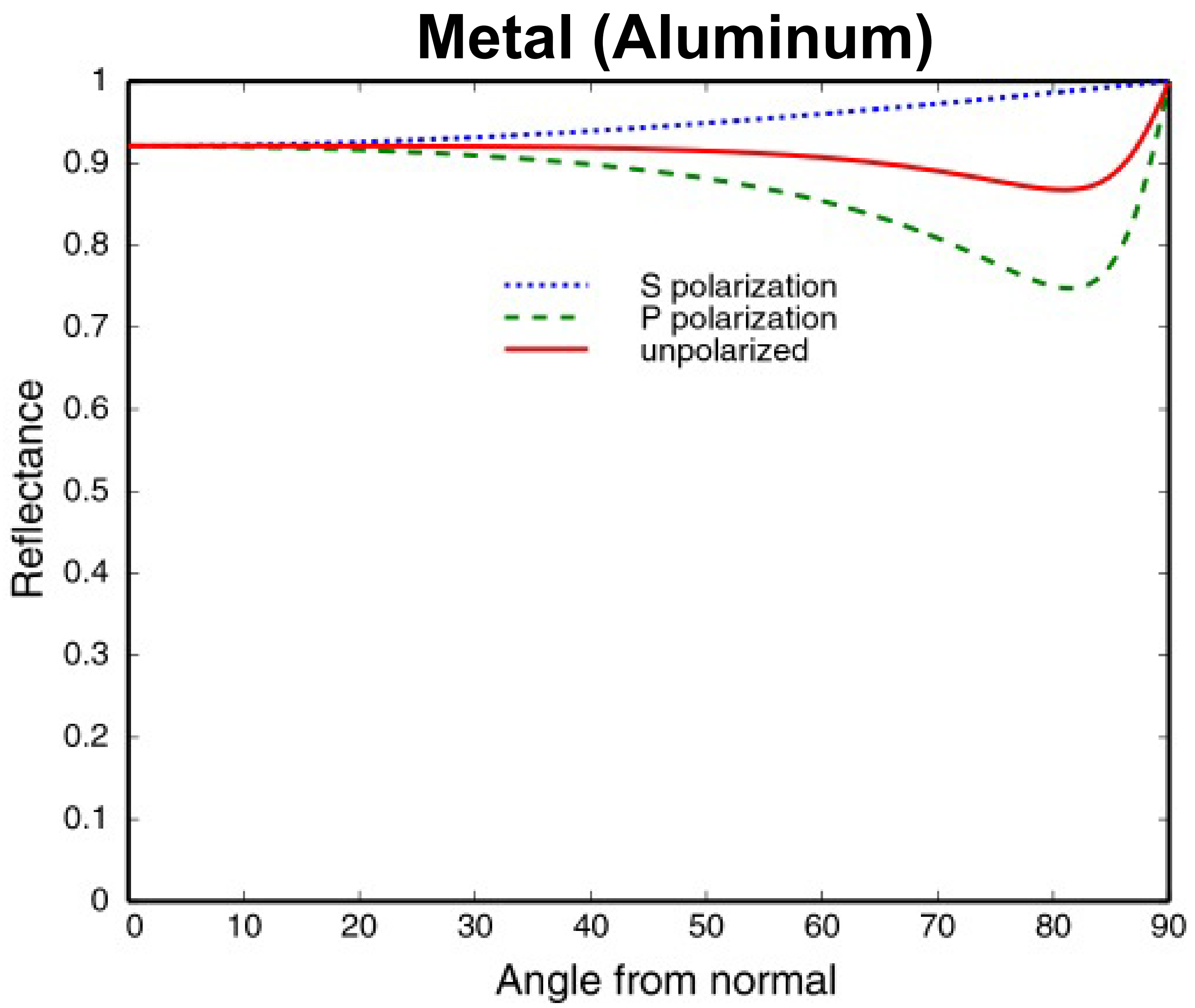
$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{\overbrace{F(\vec{\omega}_h, \vec{\omega}_o)}^{\text{Fresnel coefficient}} \cdot \overbrace{D(\vec{\omega}_h)}^{\text{Microfacet distribution}} \cdot \overbrace{G(\vec{\omega}_i, \vec{\omega}_o)}^{\text{Shadowing/ masking}}}{4|(\vec{\omega}_i \cdot \mathbf{n})(\vec{\omega}_o \cdot \mathbf{n})|}$$

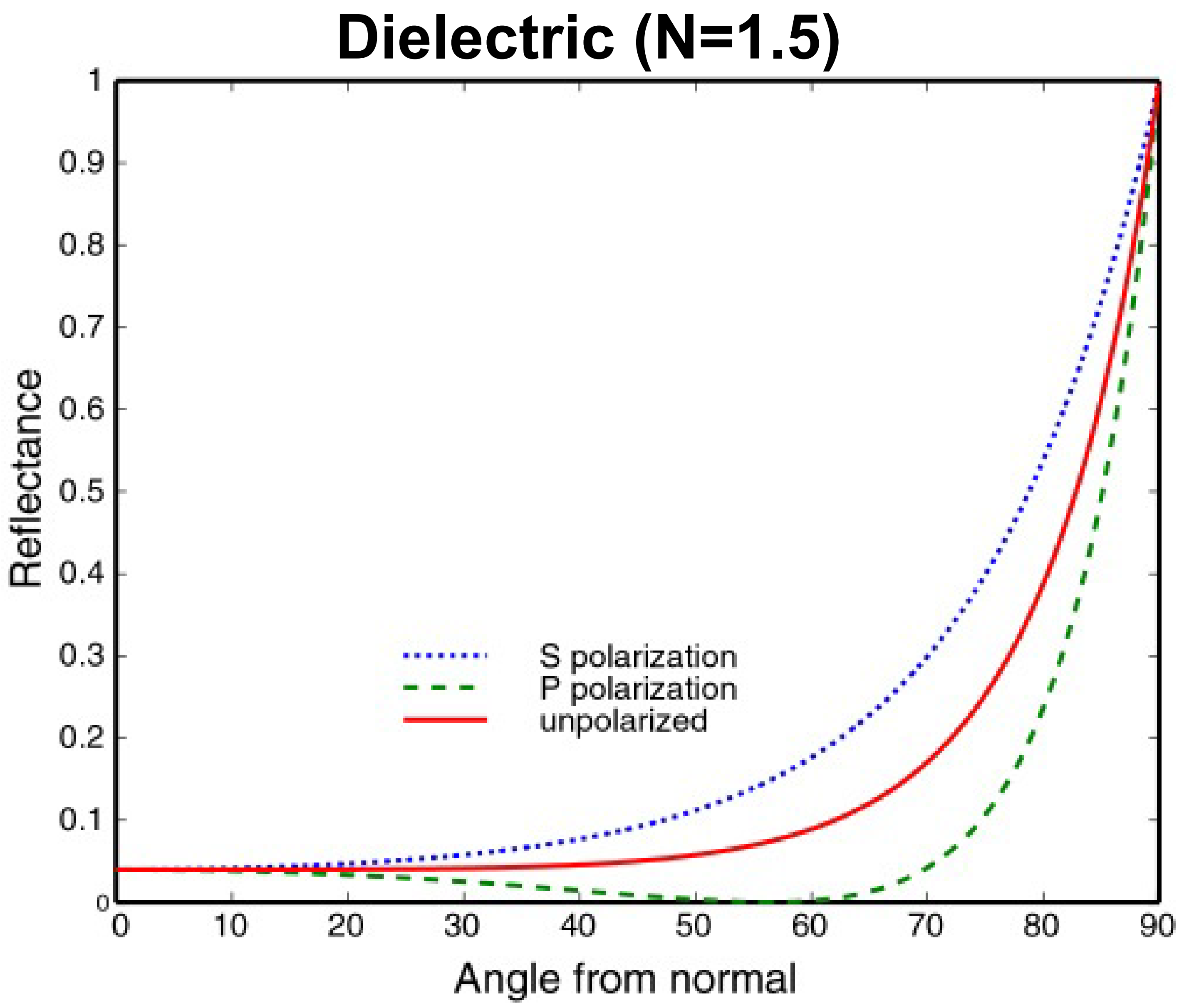


$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$

Fresnel Term

Gold F(0)=0.82
Silver F(0)=0.95





Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

General Microfacet Model

Microfacet
distribution

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|(\vec{\omega}_i \cdot \vec{\mathbf{n}})(\vec{\omega}_o \cdot \vec{\mathbf{n}})|}$$

Microfacet Distribution

Fraction of microfacets facing each direction

Probability density function over *projected* solid angle (must be normalized):

$$\int_{H^2} D(\vec{\omega}_h) \cos\theta_h \, \mathrm{d}\vec{\omega}_h = 1$$

The Beckmann Distribution

The slopes follow a Gaussian distribution

Let's express slope distribution wrt. directions

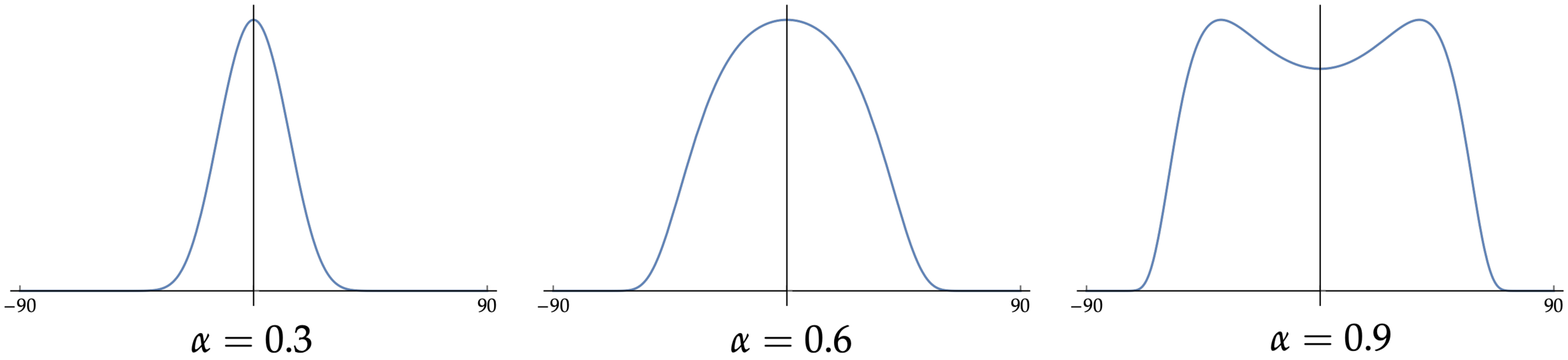
– Slope of θ_h is $\tan\theta_h$

$$D(\vec{\omega}_h) = \frac{1}{\pi\alpha^2\cos^4\theta_h}\, e^{-\frac{\tan^2\theta_h}{\alpha^2}}$$

The Beckmann Distribution

The slopes follow a Gaussian distribution

Let's express slope distribution wrt. directions



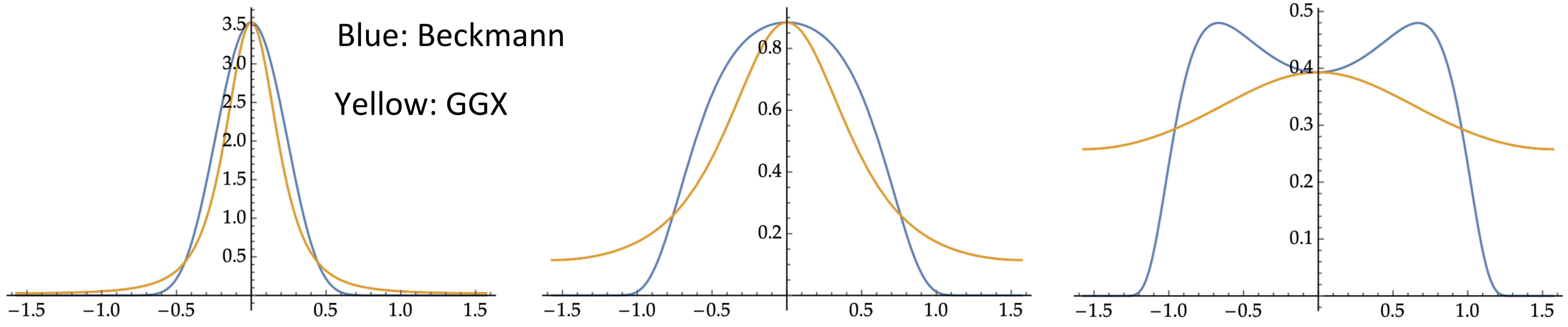
Other Distributions

The Blinn distribution:

$$D(\vec{\omega}_h) = \frac{e+2}{2\pi}(\vec{\omega}_h \cdot \vec{\mathbf{n}})^e$$

GGX distribution, see [Walter et al., EGSR 2007]

Anisotropic distributions, see [PBRTv2, Ch. 8]



General Microfacet Model

Shadowing/
masking

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot \boxed{G(\vec{\omega}_i, \vec{\omega}_o)}}{4|(\vec{\omega}_i \cdot \vec{\mathbf{n}})(\vec{\omega}_o \cdot \vec{\mathbf{n}})|}$$

Shadowing and Masking

Microfacets can be *shadowed* and/or *masked* by other microfacets

Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Beckman distribution:

$$G(\vec{\omega}) = \frac{2}{1 + \operatorname{erf}(s) + \frac{1}{s\sqrt{\pi}} e^{-s^2}} \qquad s = \frac{1}{\alpha \tan \theta}$$

$$G(\vec{\omega}_i, \vec{\omega}_o) = G(\vec{\omega}_i) \cdot G(\vec{\omega}_o)$$

Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Beckman distribution (approximated):

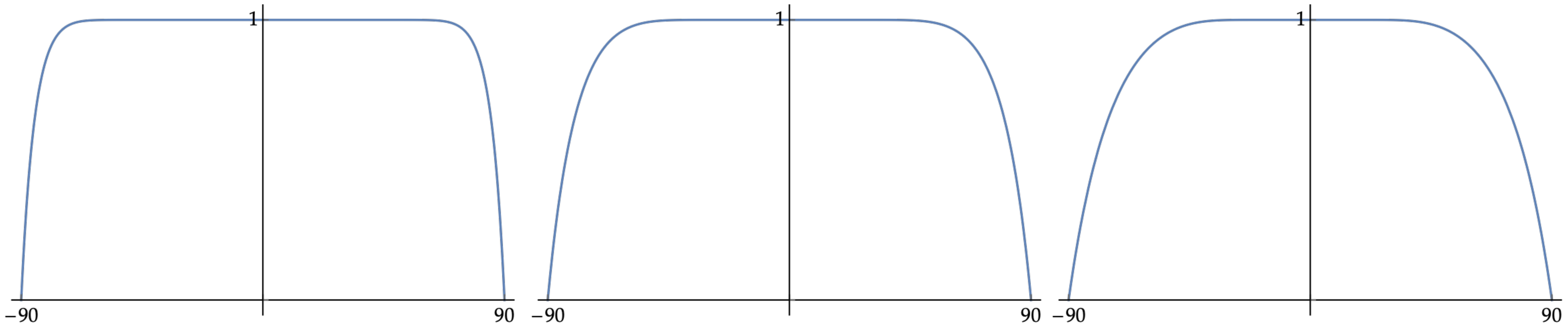
$$G(\vec{\omega}) \approx \begin{cases} \frac{3.535s+2.181s^2}{1+2.276s+2.577s^2}, & s < 1.6 \\ 1, & \text{otherwise} \end{cases}$$

$$G(\vec{\omega}_i, \vec{\omega}_o) = G(\vec{\omega}_i) \cdot G(\vec{\omega}_o)$$

Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Beckman distribution (approximated):



$\alpha = 0.3$ $\alpha = 0.6$ $\alpha = 0.9$

Shadowing and Masking

Each microfacet distribution typically has its respective shadowing and masking term

Torrance-Sparrow (Blinn):

$$G(\vec{\omega}_i, \vec{\omega}_o) = \min\left(1, \frac{2(\vec{\mathbf{n}} \cdot \vec{\omega}_h)(\vec{\mathbf{n}} \cdot \vec{\omega}_i)}{(\vec{\omega}_h \cdot \vec{\omega}_i)}, \frac{2(\vec{\mathbf{n}} \cdot \vec{\omega}_h)(\vec{\mathbf{n}} \cdot \vec{\omega}_o)}{(\vec{\omega}_h \cdot \vec{\omega}_o)}\right)$$

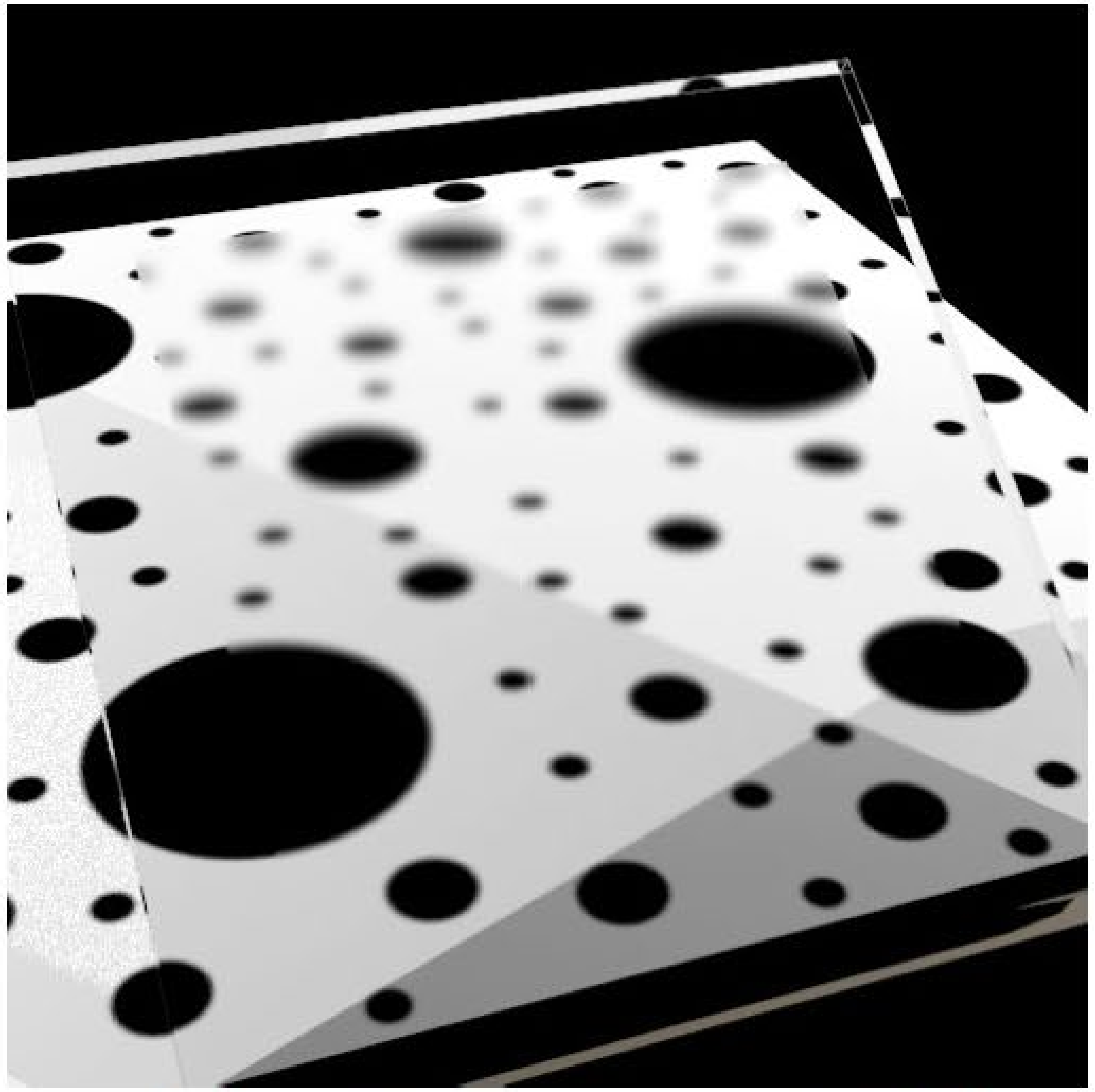
General Microfacet Model

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4\left|(\vec{\omega}_i \cdot \vec{\mathbf{n}})(\vec{\omega}_o \cdot \vec{\mathbf{n}})\right|}$$

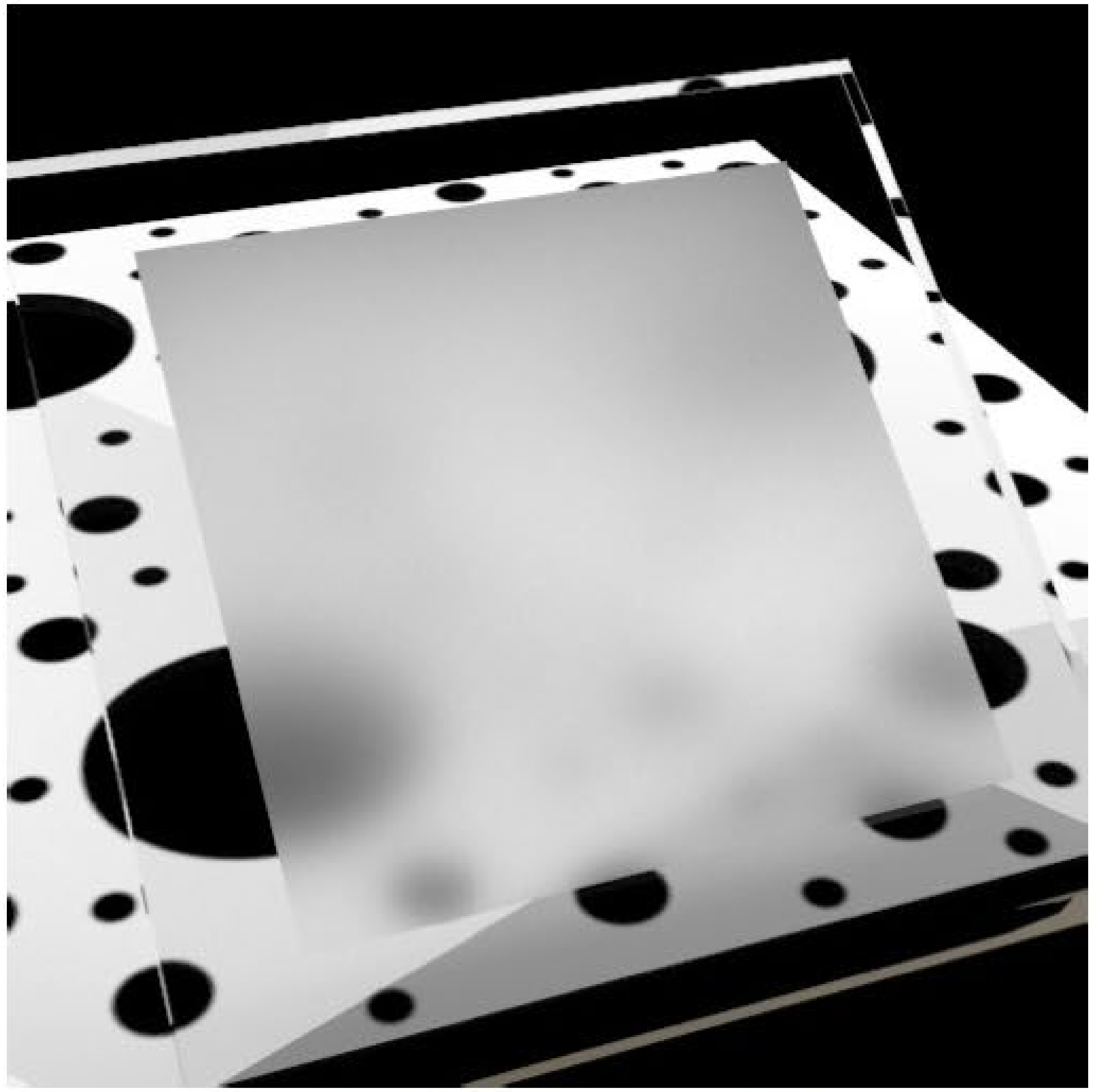
Denominator: correction term coming from energy conservation, Jacobians, etc.

- see PBR book and Walter et al. [EGSR 2007] for more detail

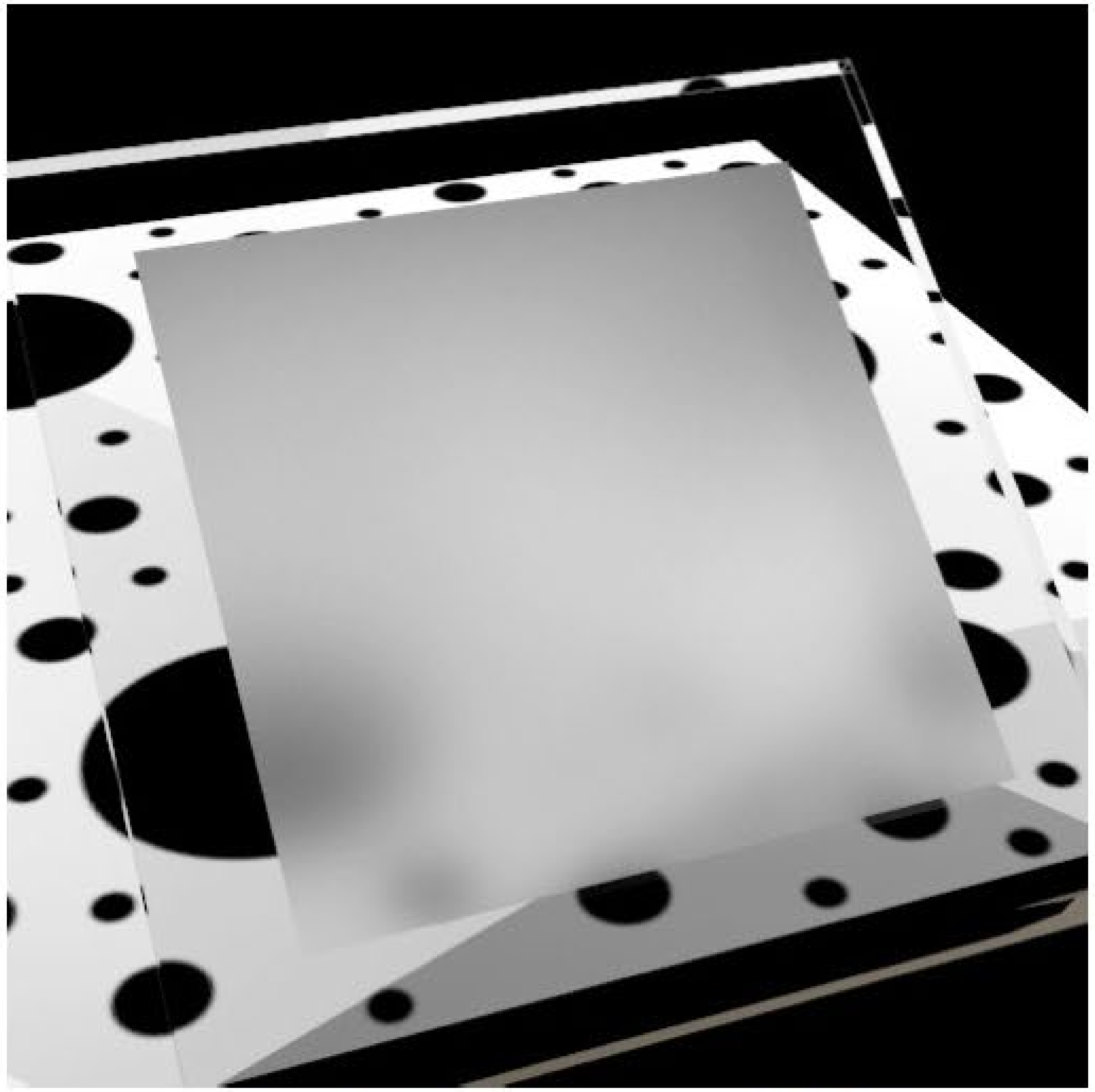
GGX and Beckmann



anti-glare (Beckman, $\alpha_b = 0.023$)



ground (GGX, $\alpha_g = 0.394$)



etched (GGX, $\alpha_g = 0.553$)

Walter et al. 07

Energy Loss Issue

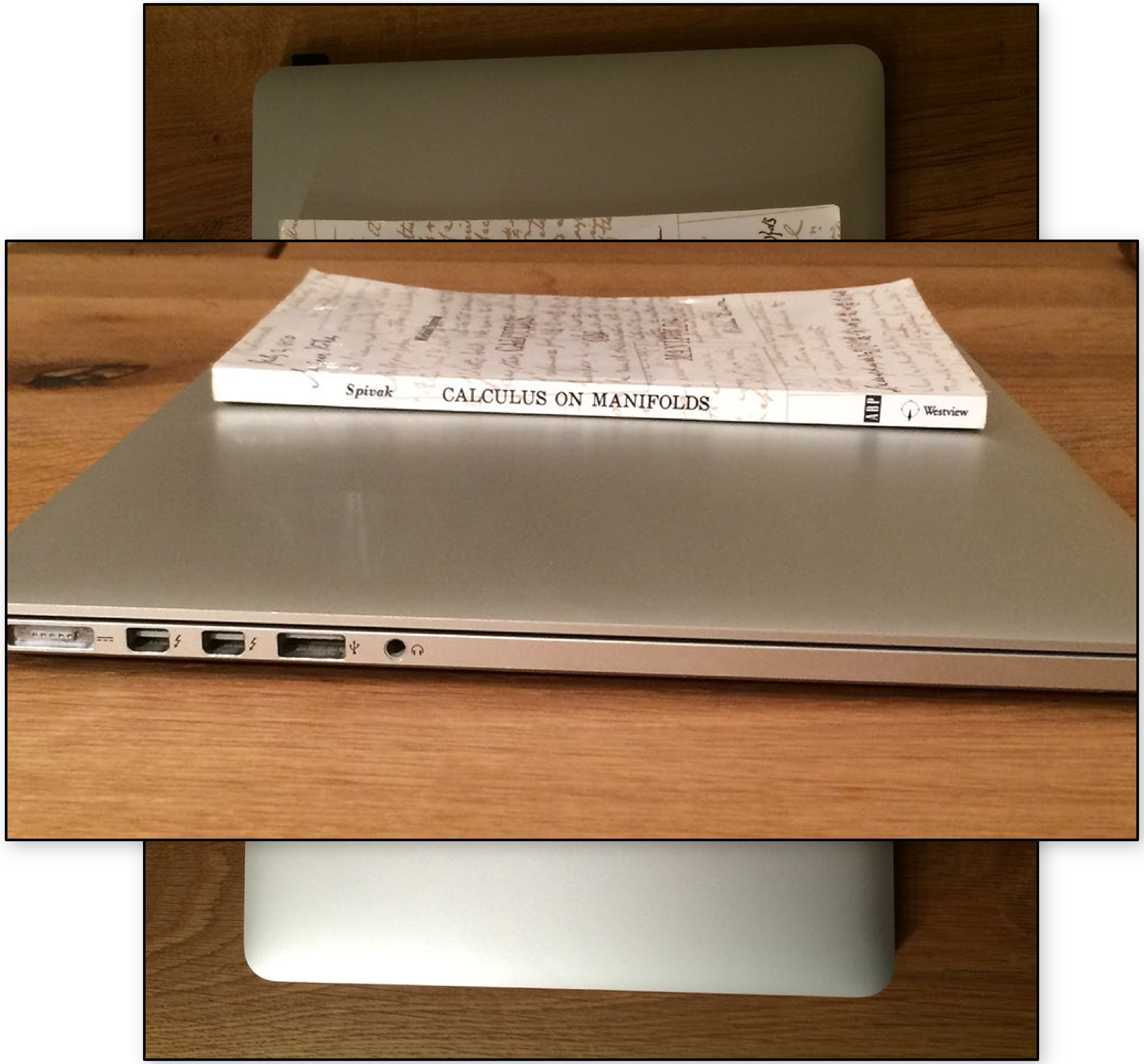
Energy Loss Issue - Conductor

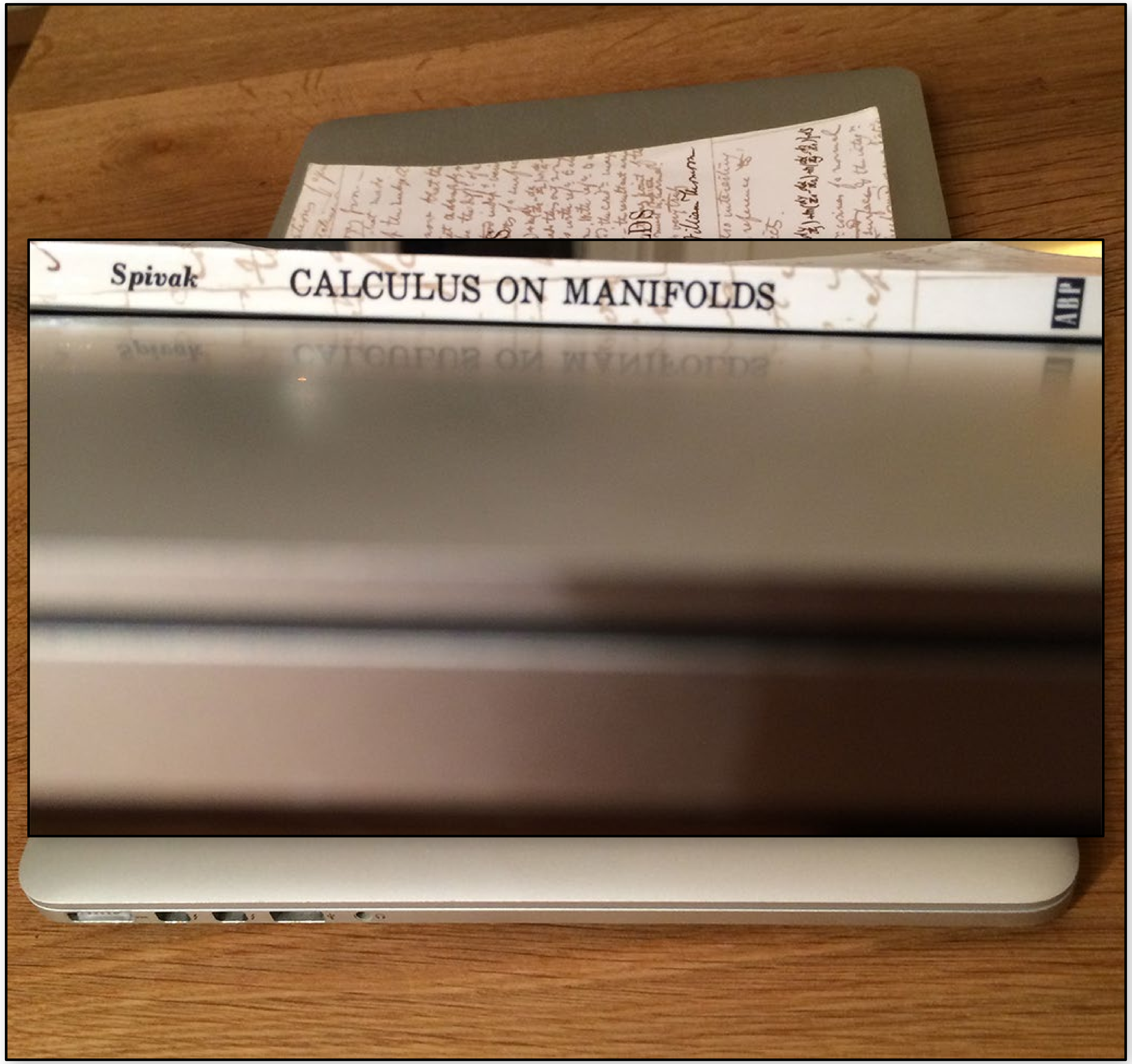
Increasing roughness $\alpha = 0.01 .. 2.0$

Energy Loss Issue - Dielectric

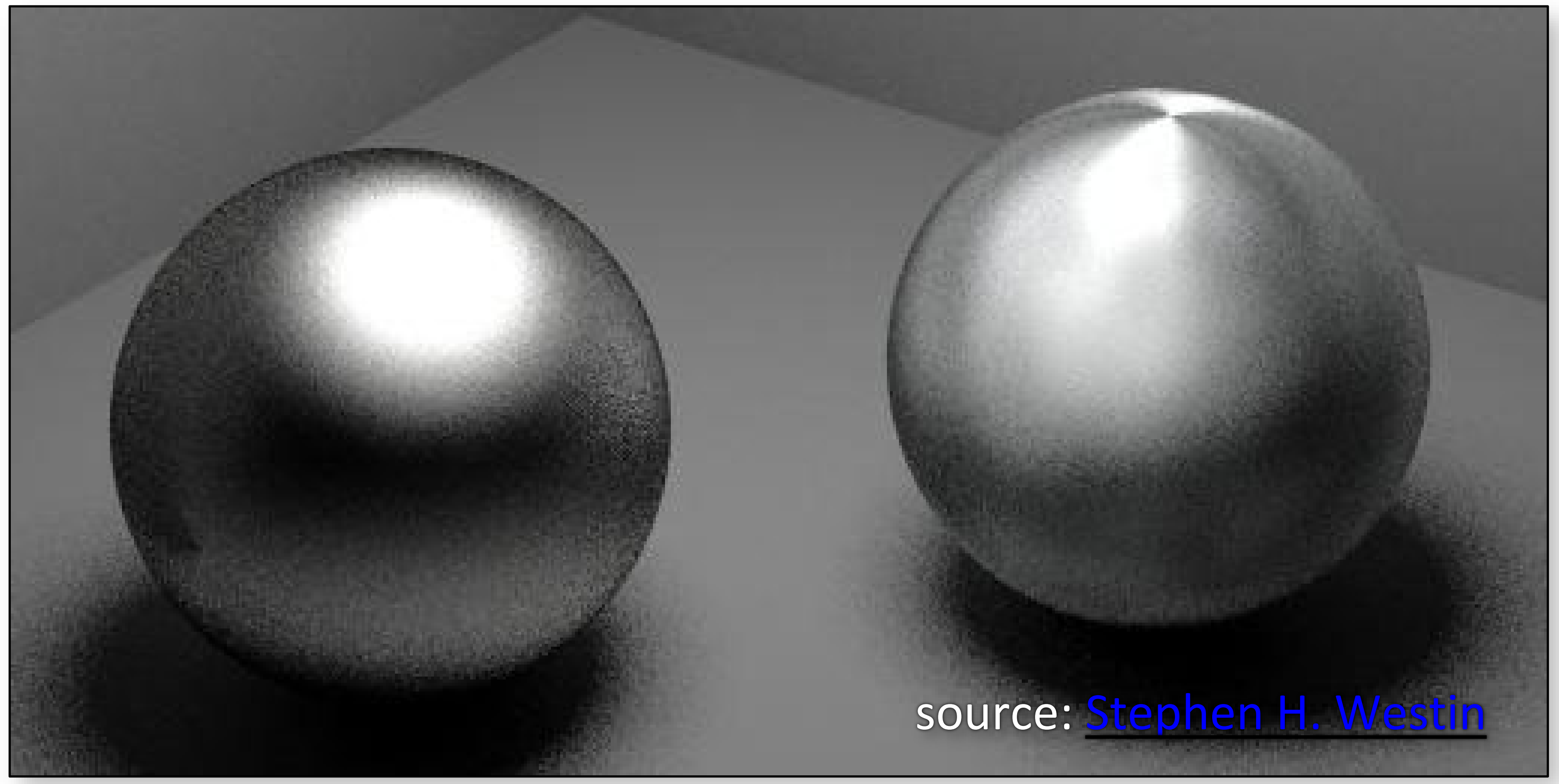
Increasing roughness α = 0.01 .. 2.0

Interesting grazing angle behavior





Extension: Anisotropic Reflection



BRDF of the moon

What BRDF does the moon have?

BRDF of the moon

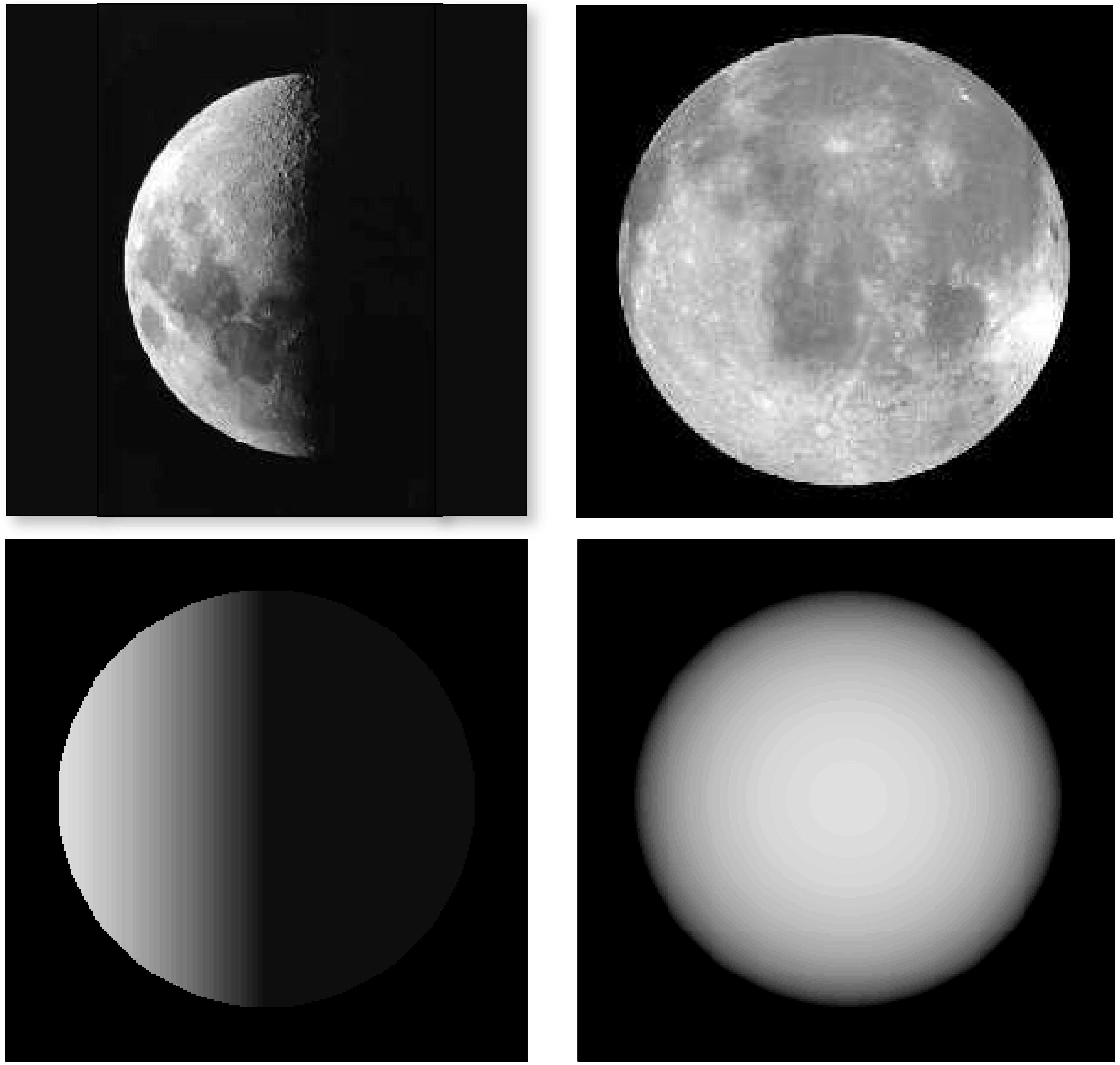
What BRDF does the moon have?

- Can it be diffuse?

BRDF of the moon

What BRDF does the moon have?

- Can it be diffuse?



Even though the moon appears matte, its edges remain bright.

The Oren-Nayar Model

Same concept as the microfacet models, but assumes that the facets are diffuse

Shadowing/masking + interreflections

No analytic solution; fitted approximation

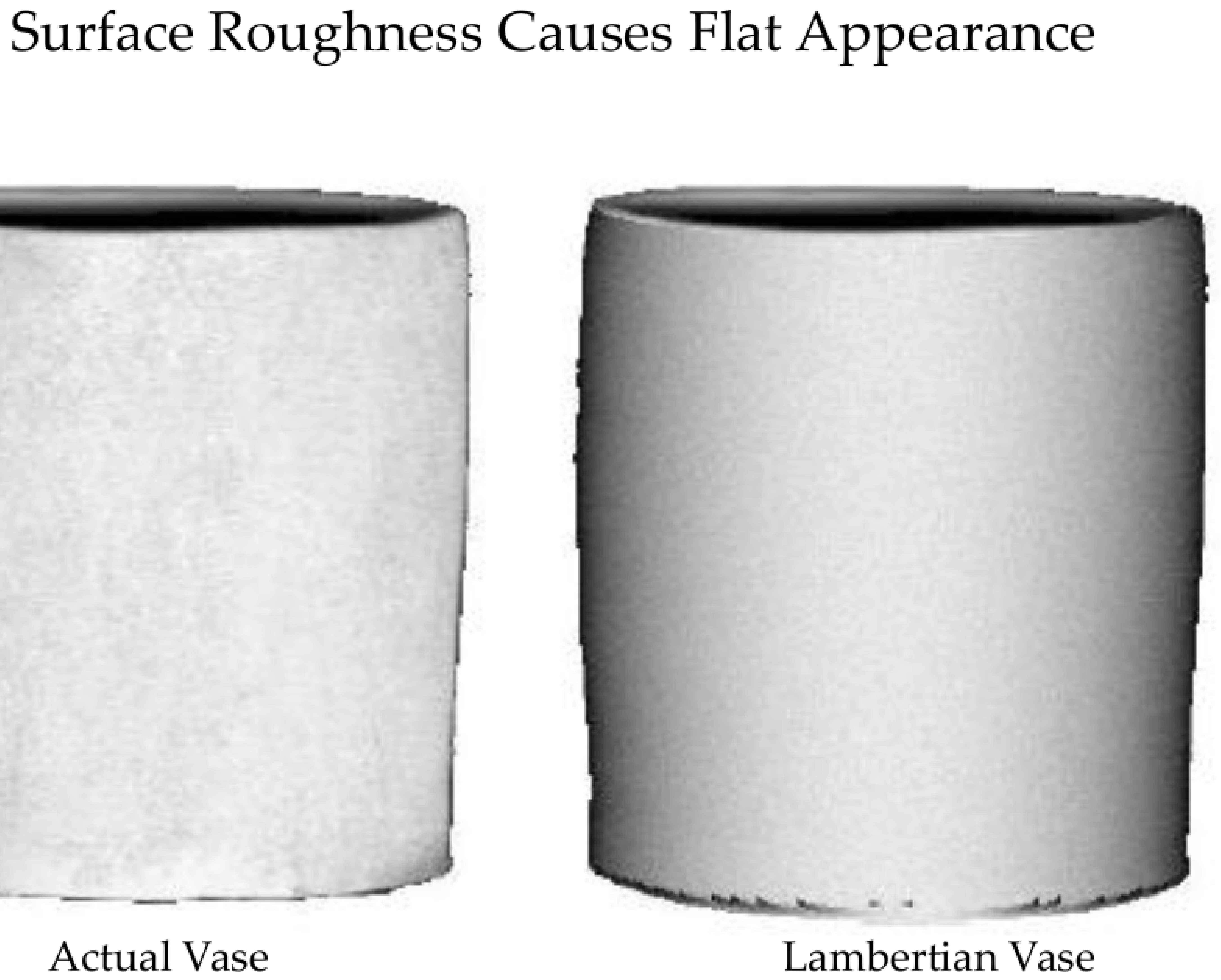
$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{\rho}{\pi}\left(A + B\ \max(0, \cos(\phi_i - \phi_o)) \sin\alpha \tan\beta\right)$$

$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \qquad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09}$$

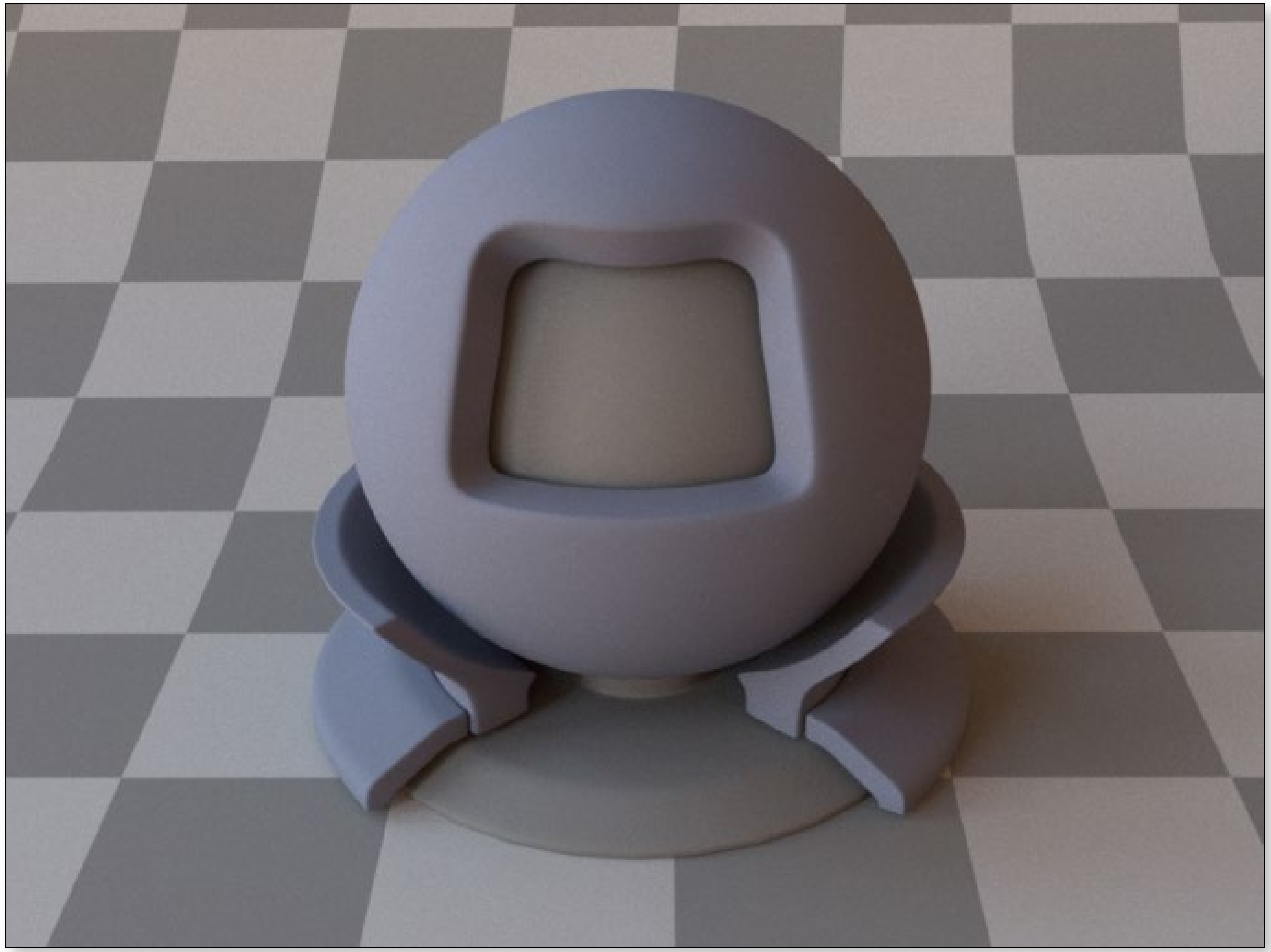
$$\alpha = \max(\theta_i, \theta_o) \qquad \beta = \min(\theta_i, \theta_o)$$

Ideal Lambertian is just a special case (σ = 0)

Rough diffuse appearance

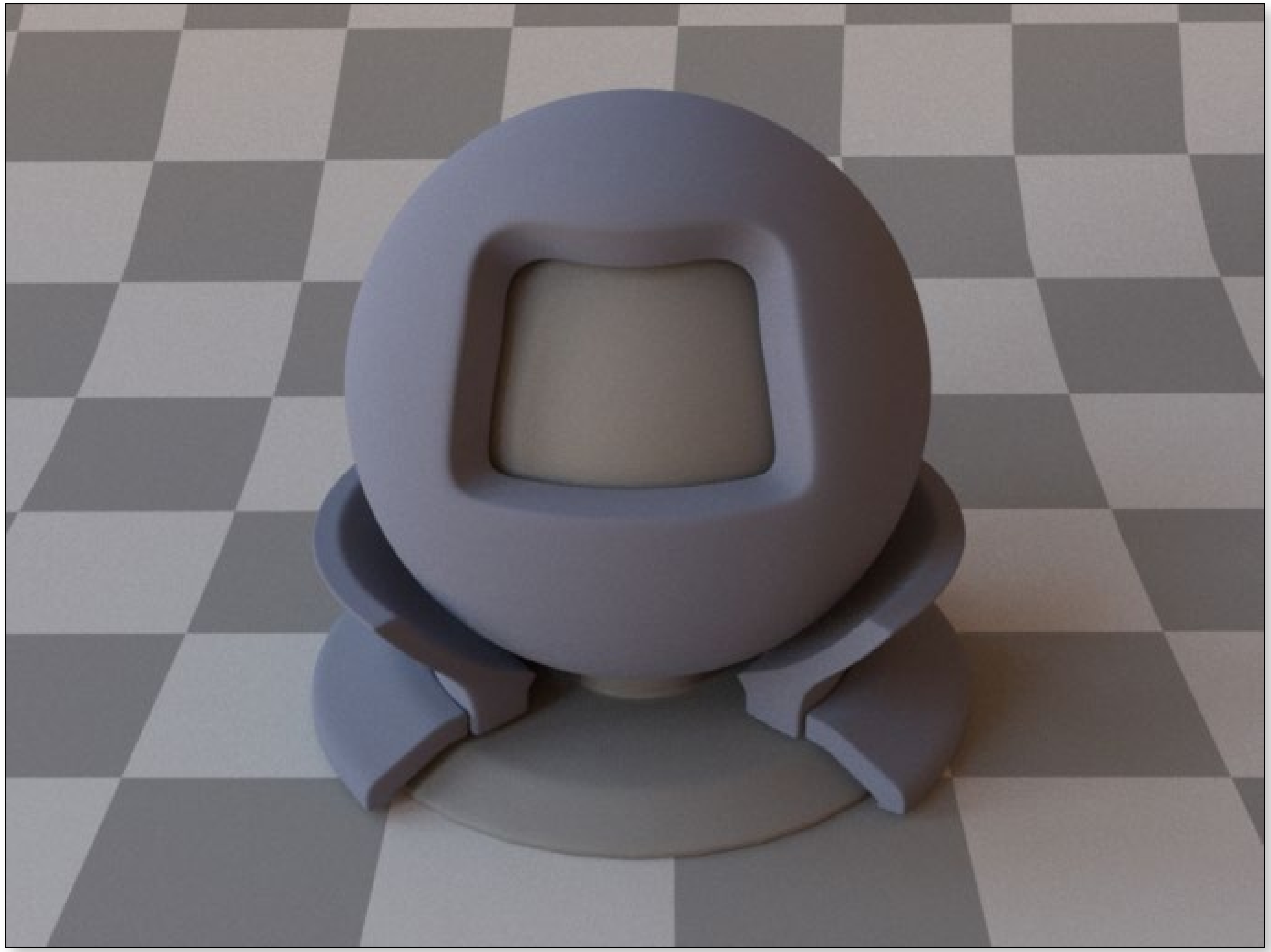


Smooth Diffuse



source: Wenzel Jakob

Rough Diffuse

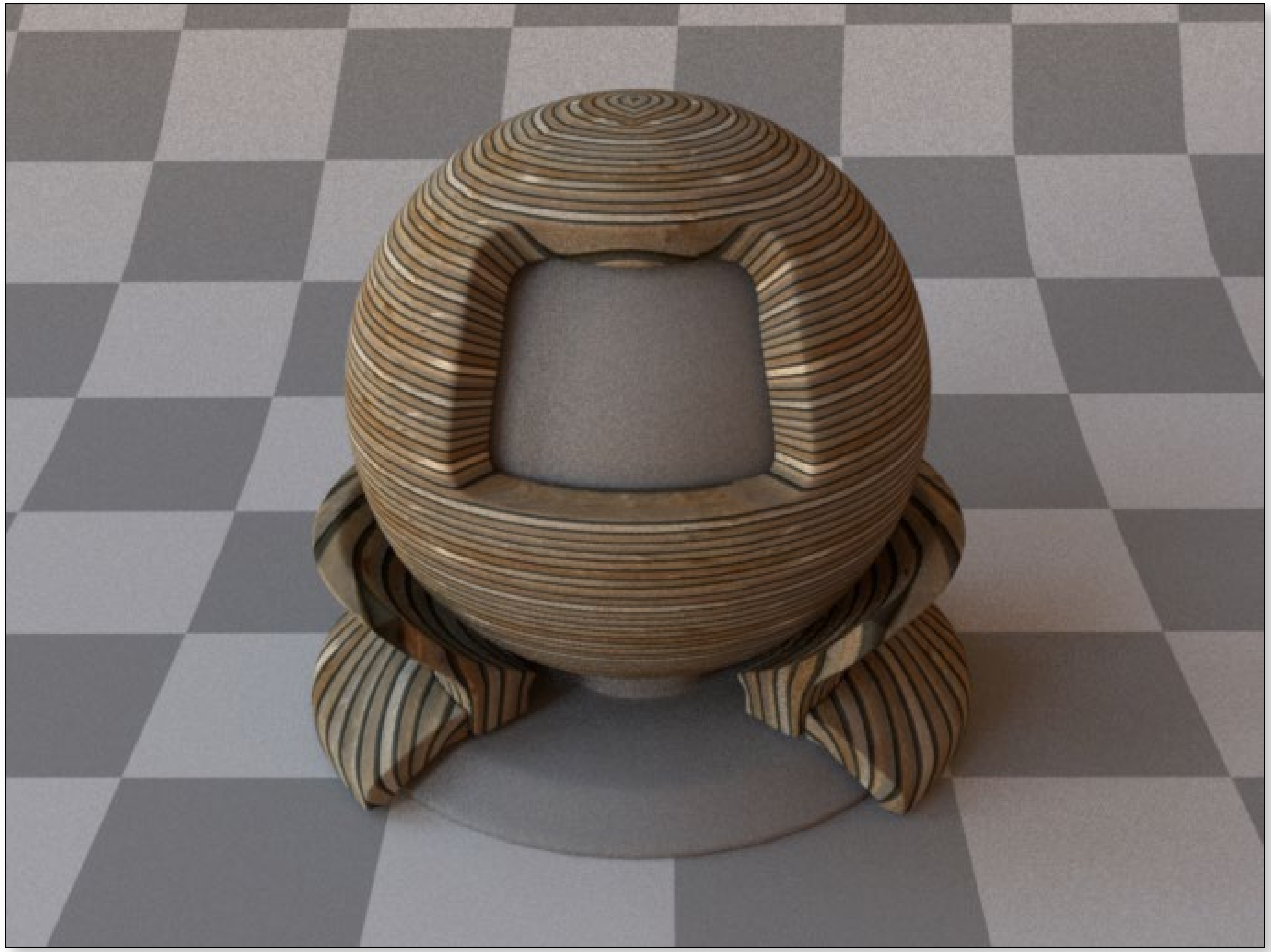


source: Wenzel Jakob

Extension: layered materials

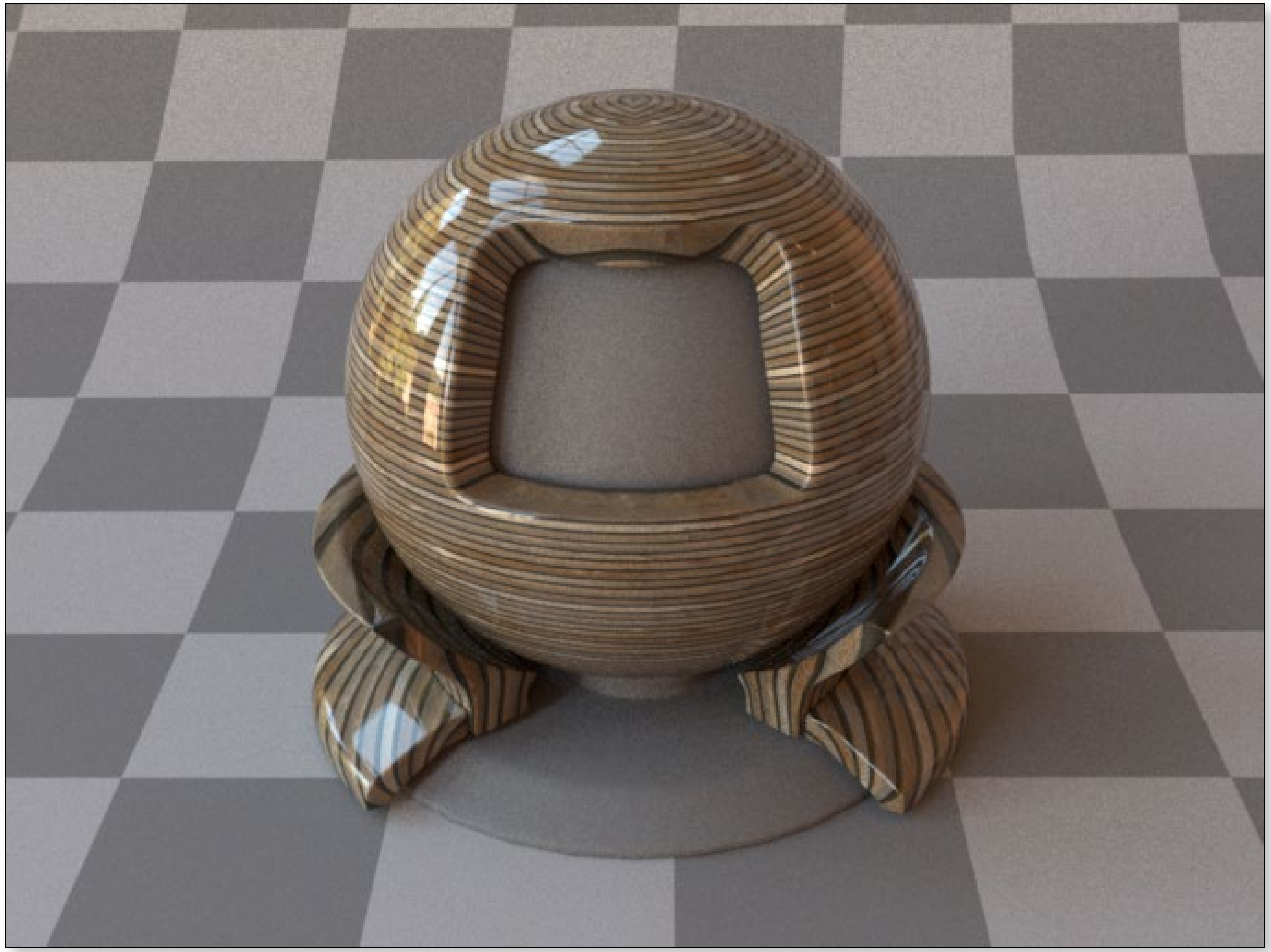
Diffuse base layer coated using a perfectly smooth dielectric
(can do something similar with microfacets)

Smooth Diffuse



source: Wenzel Jakob

Smooth Plastic

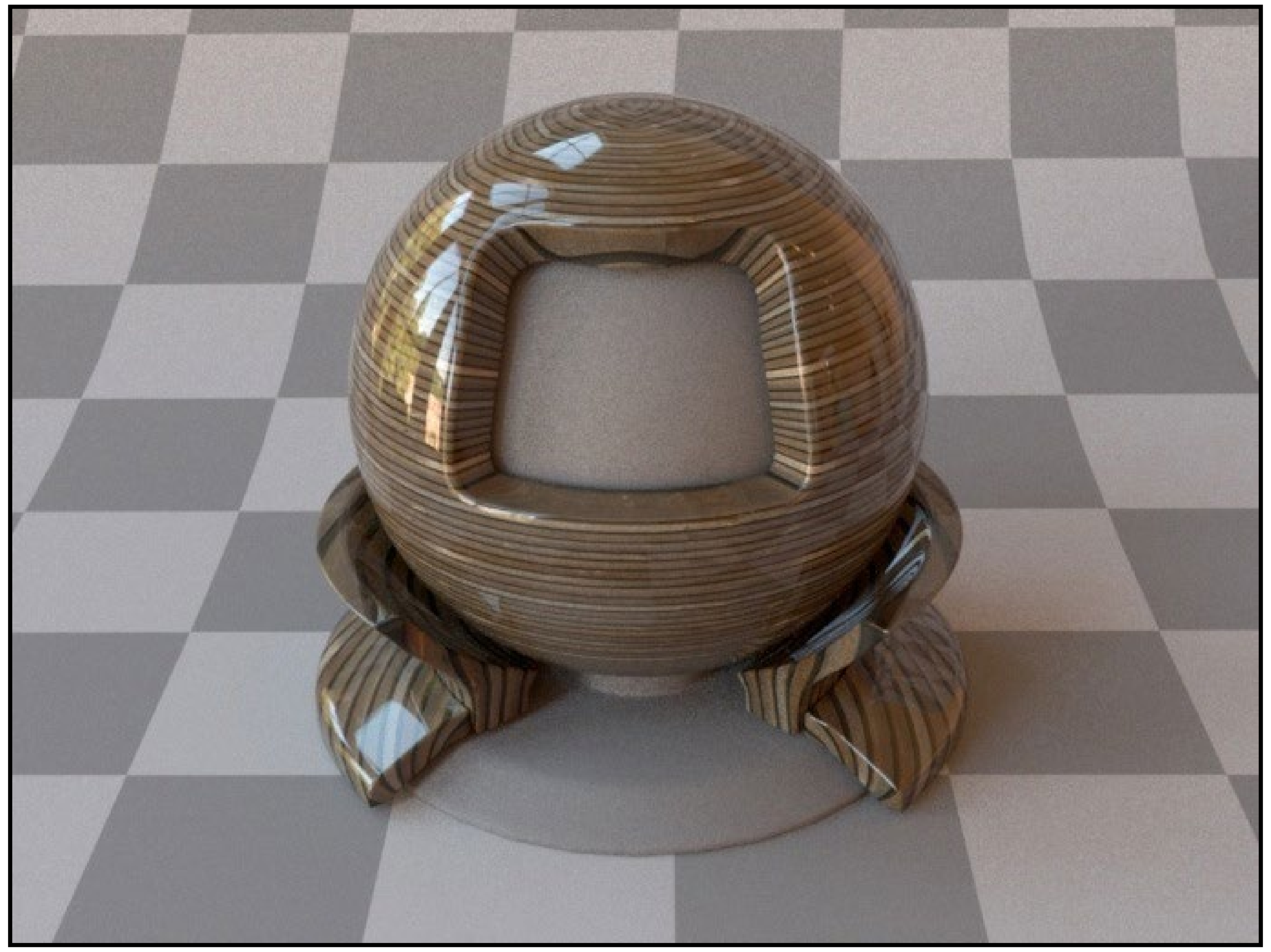


source: Wenzel Jakob

Smooth Plastic

Plain diffuse material

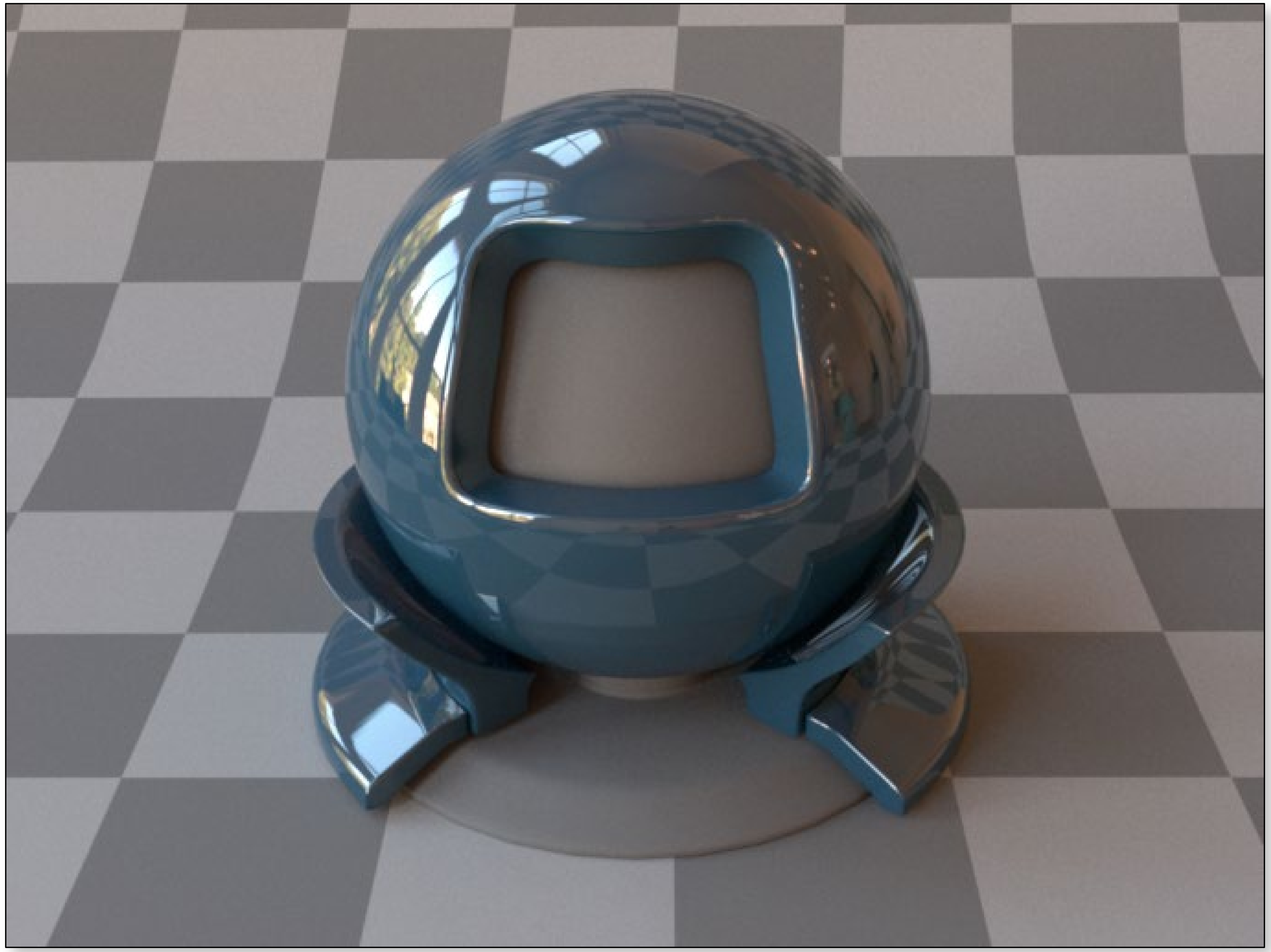
Naïve blend of diffuse + specular
(*incorrect*)



Specular-matte
(**correct**)

source: Wenzel Jakob

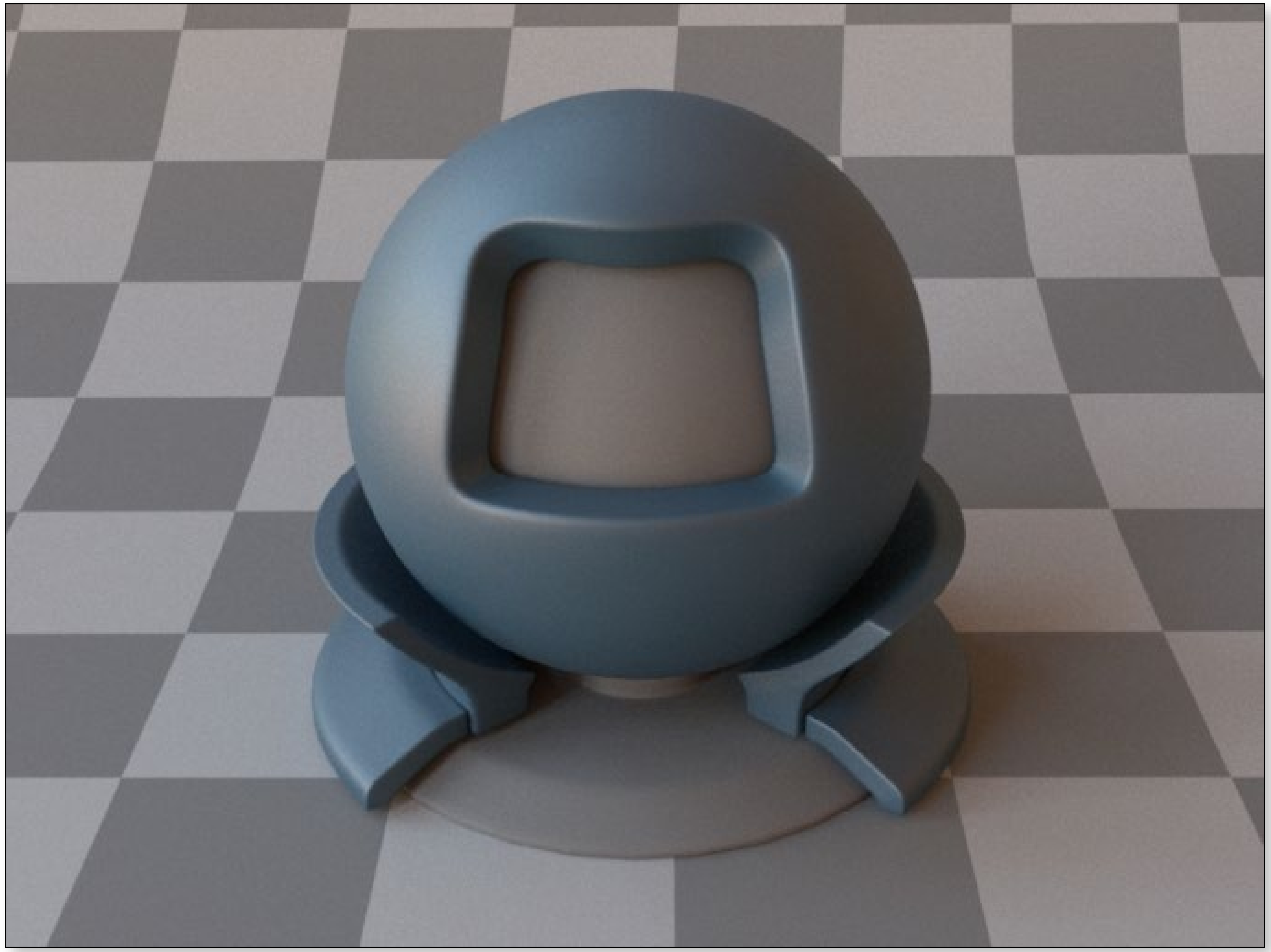
Smooth Plastic



Smooth dielectric varnish on top of diffuse surface

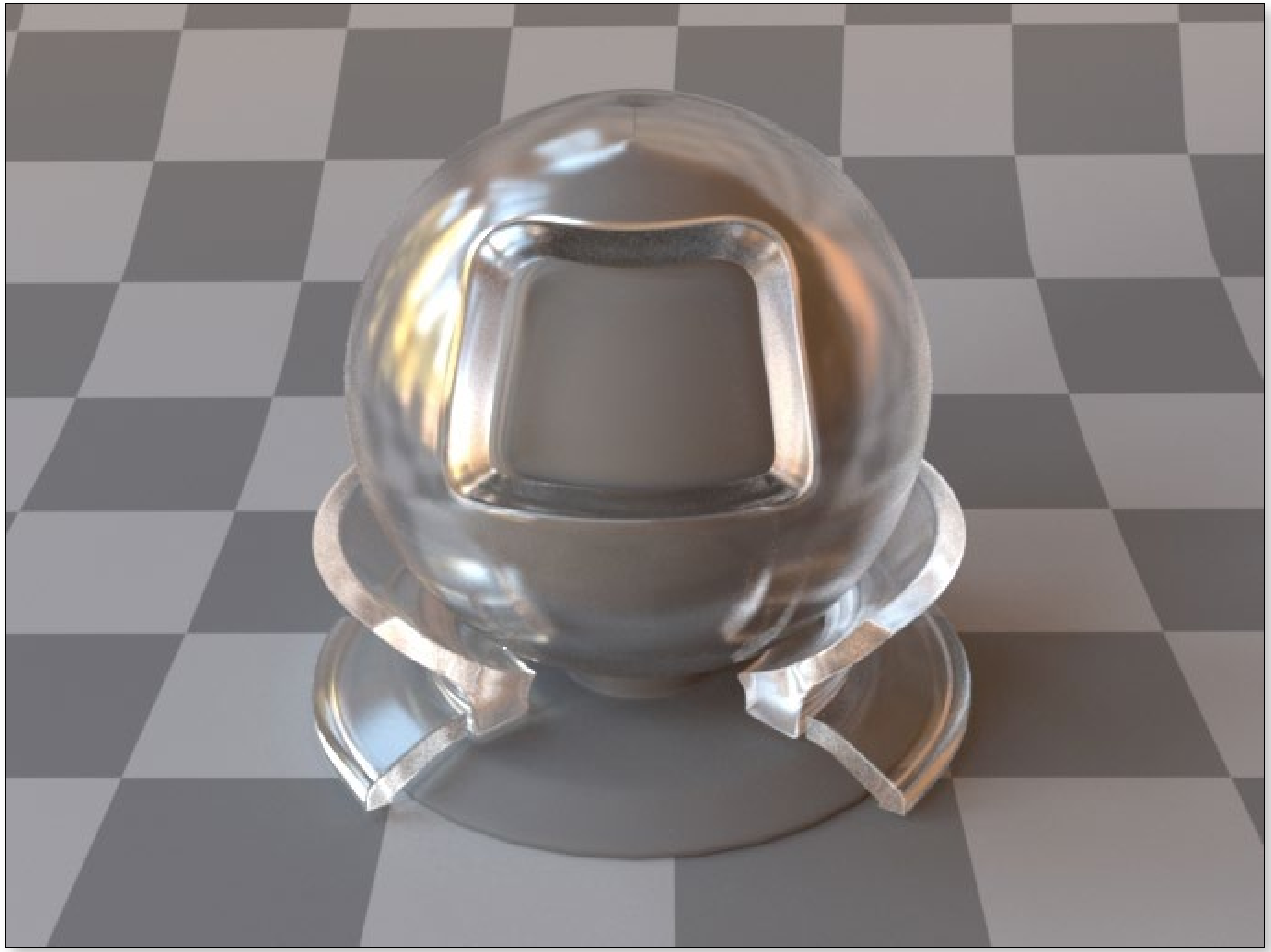
source: Wenzel Jakob

Rough Plastic



Rough dielectric varnish on top of diffuse surface

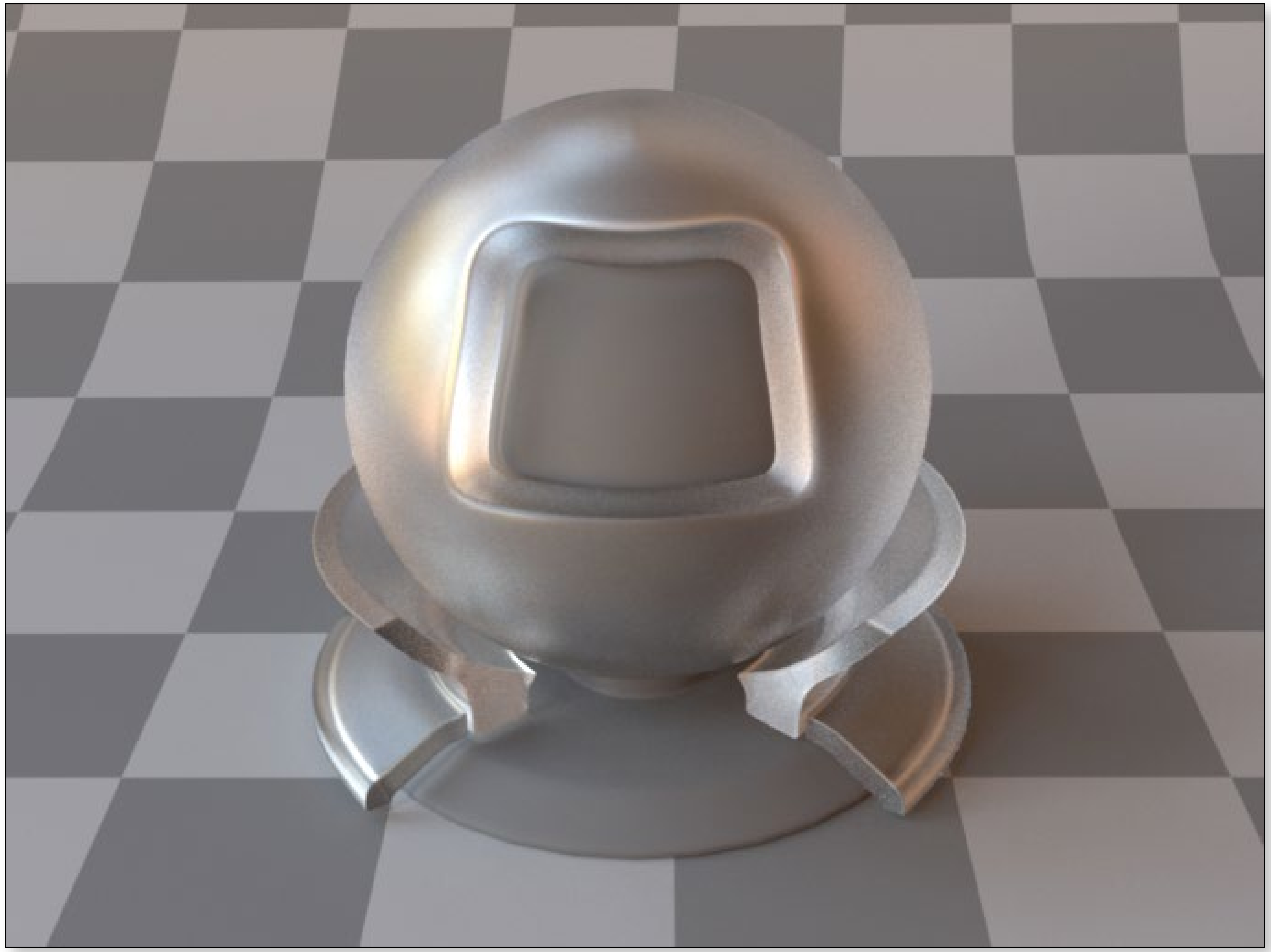
Rough Dielectric



Anti-glare glass ($m = 0.02$)

source: Wenzel Jakob

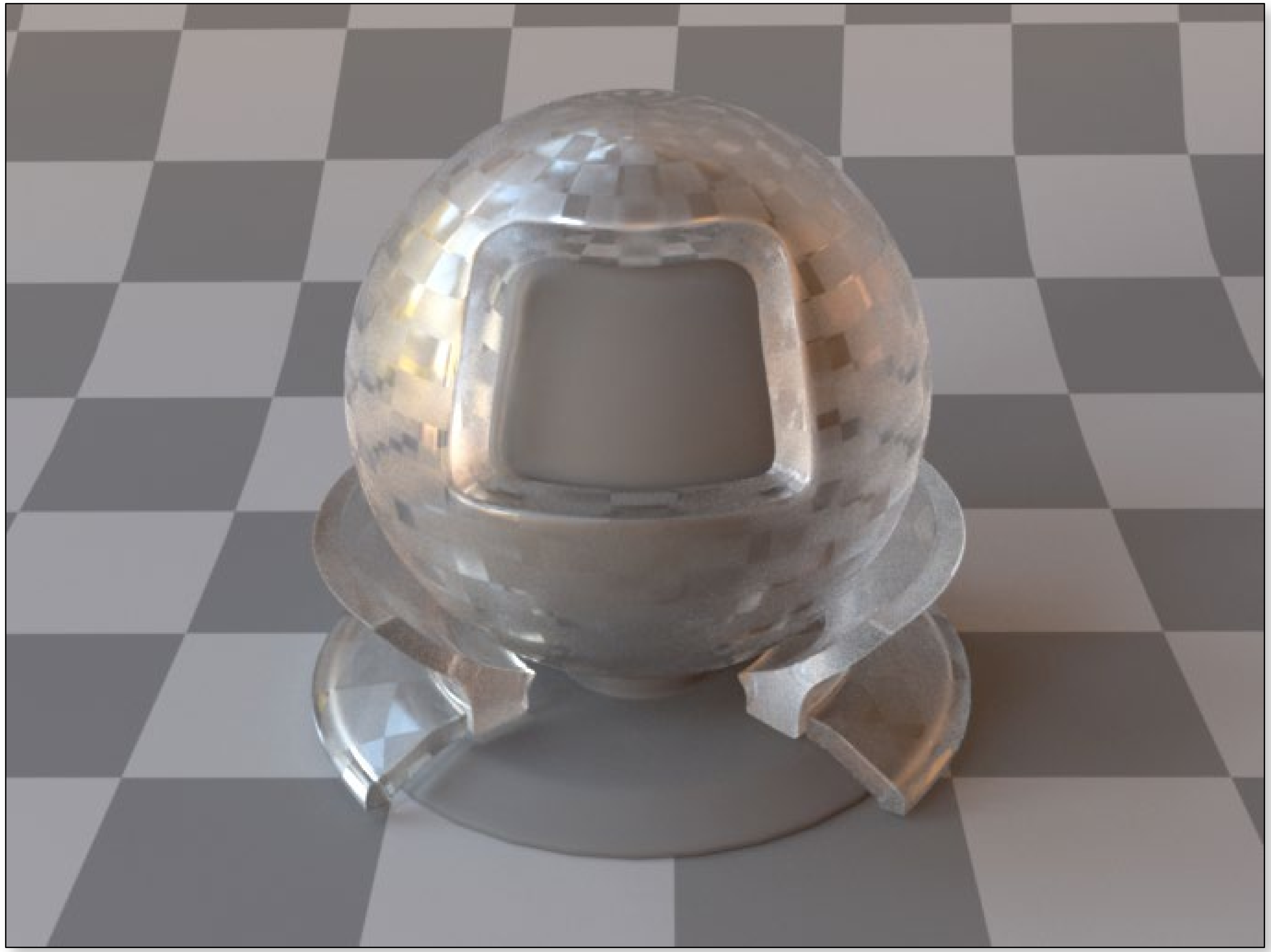
Rough Dielectric



Rough glass ($m = 0.1$)

source: Wenzel Jakob

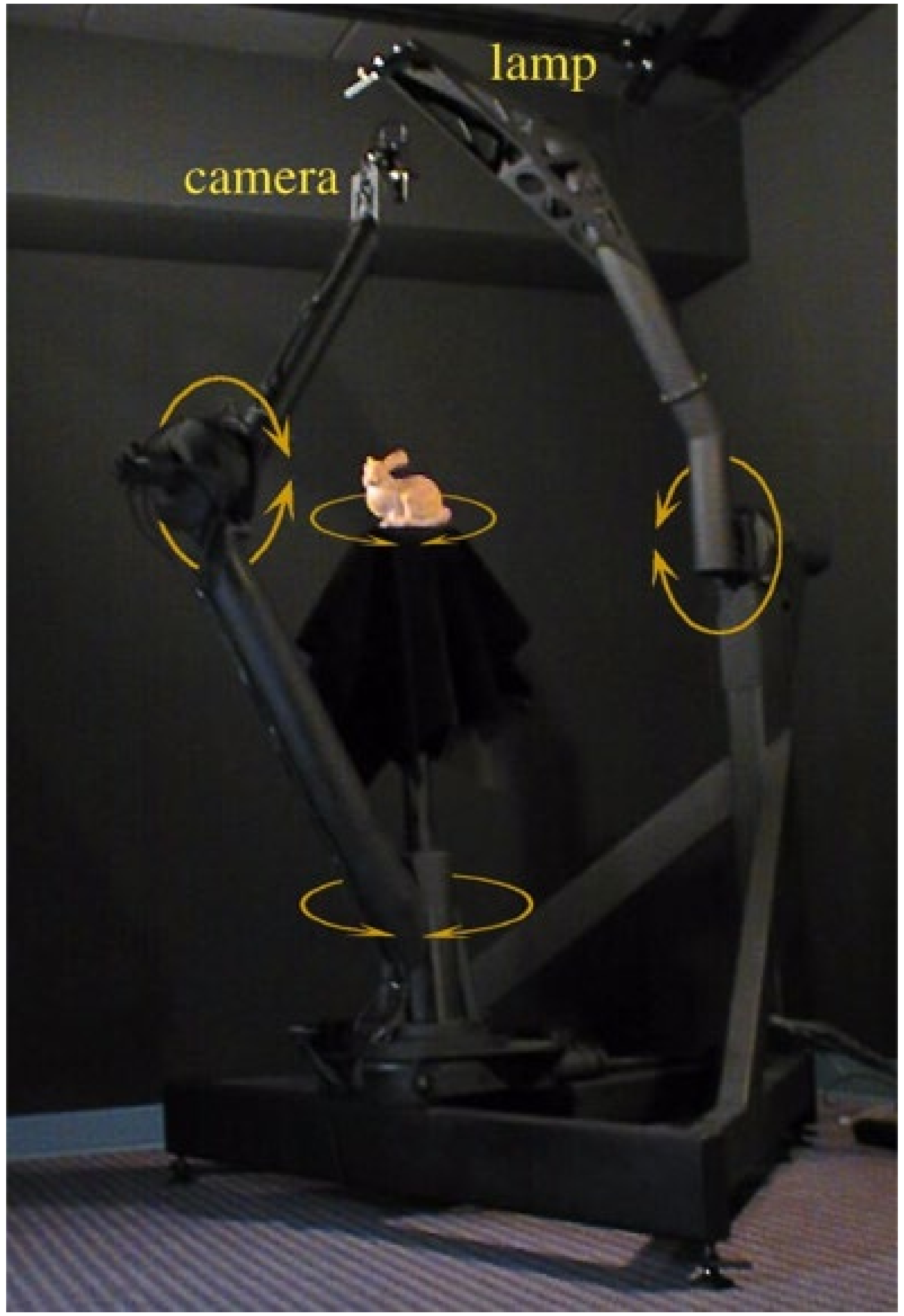
Rough Dielectric



Textured roughness

Data-Driven BRDFs

Spherical gantry



Measuring BRDFs

source: Matusik et al. 2003

Nickel

source: Matusik et al. 2003

Hematite

source: Matusik et al. 2003

Gold Paint

source: Matusik et al. 2003

Pink Fabric

source: Matusik et al. 2003

BRDF Editing/Navigation

Given a large database, can mix/match and interpolate between BRDFs

The MERL Database

"A Data-Driven Reflectance Model"
Wojciech Matusik, Hanspeter Pfister, Matt Brand and Leonard McMillan.
ACM Transactions on Graphics 22, 3(2003), 759-769.

Download them and use them in your own renderer!

- http://www.merl.com/brdf/

Measuring and Modeling the Appearance of Wood

Stephen R. Marschner, Stephen H. Westin,
Adam Arbree, and Jonathan T. Moon

Cornell University

Reading

PBRTv3 Chapter 8, and 14.1