

Solid and procedural textures



15-468, 15-668, 15-868
Physics-based Rendering
Spring 2025, Lecture 5

Course announcements

- Programming assignment 1 is due on Friday 2/14.
 - Any issues with the homework?
- Take-home quiz 1 due tonight.
- Make-up lecture tomorrow, **Wednesday 2/5, 11 am, in GHC 6115.**

Overview of today's lecture

- 3D textures.
- Procedural textures.
- Generating “realistic noise”.

Slide credits

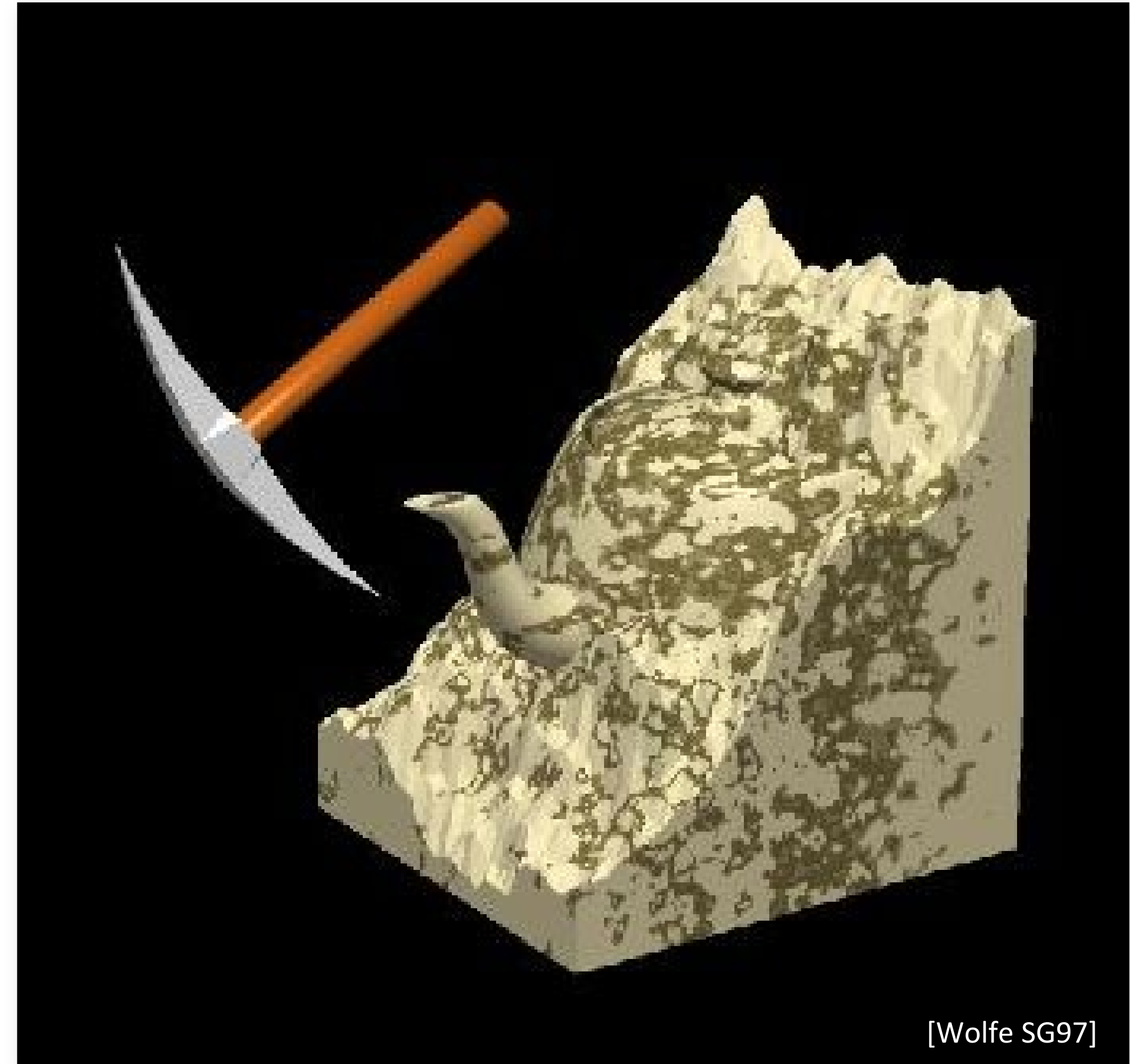
Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

3D textures

Texture is a function of (u, v, w)

- can evaluate texture at 3D point
- good for solid materials
- often defined procedurally



Procedural texturing

Instead of using rasterized image data, define texture procedurally

Simple example:

- $\text{color} = 0.5 * \sin(x) + 0.5$

Often called “solid texturing” because texture can easily vary in all 3 dimensions.

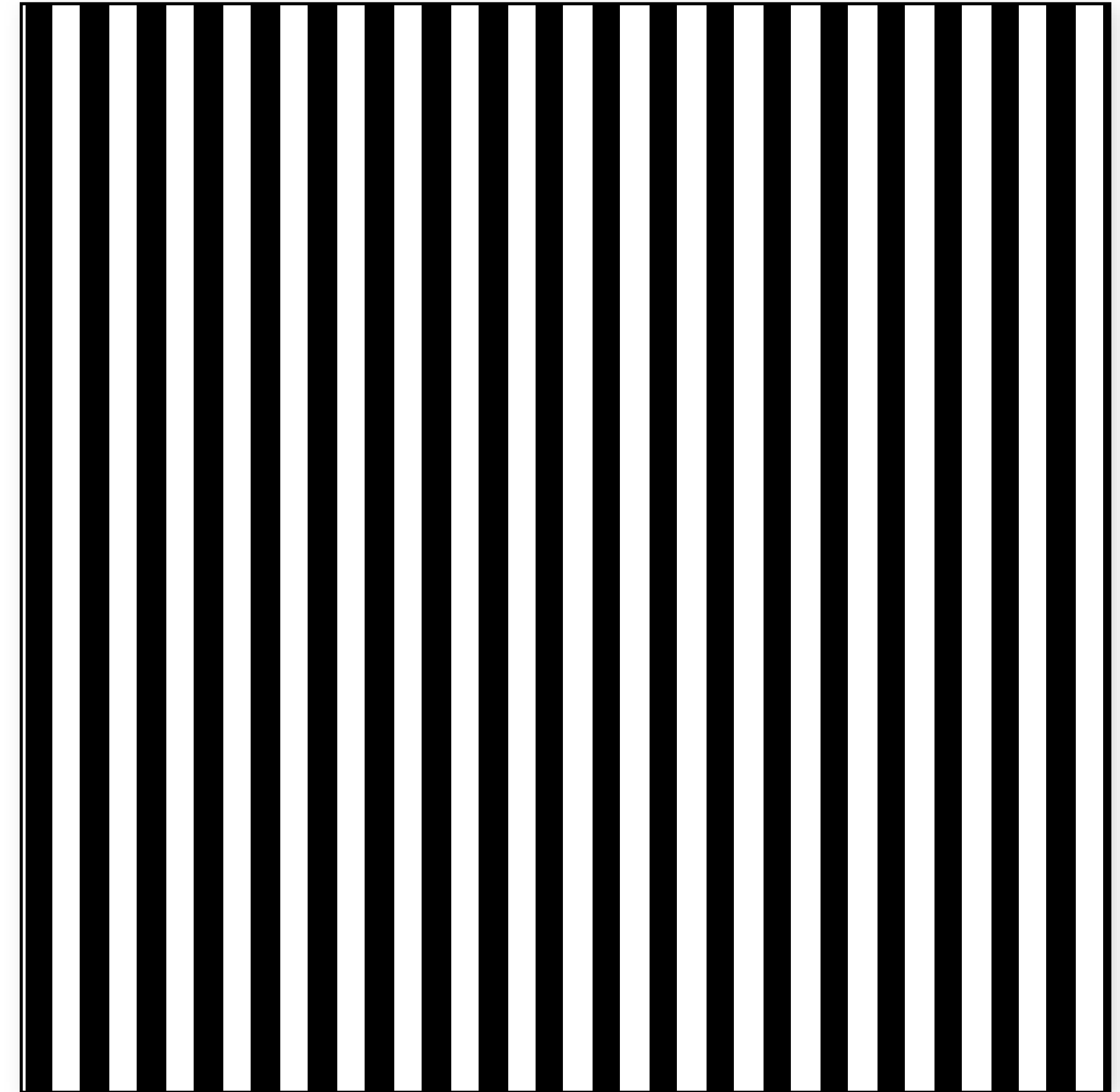
- but you can also do 2D or 1D procedural textures

Raster vs. procedural textures

Why use procedural textures?

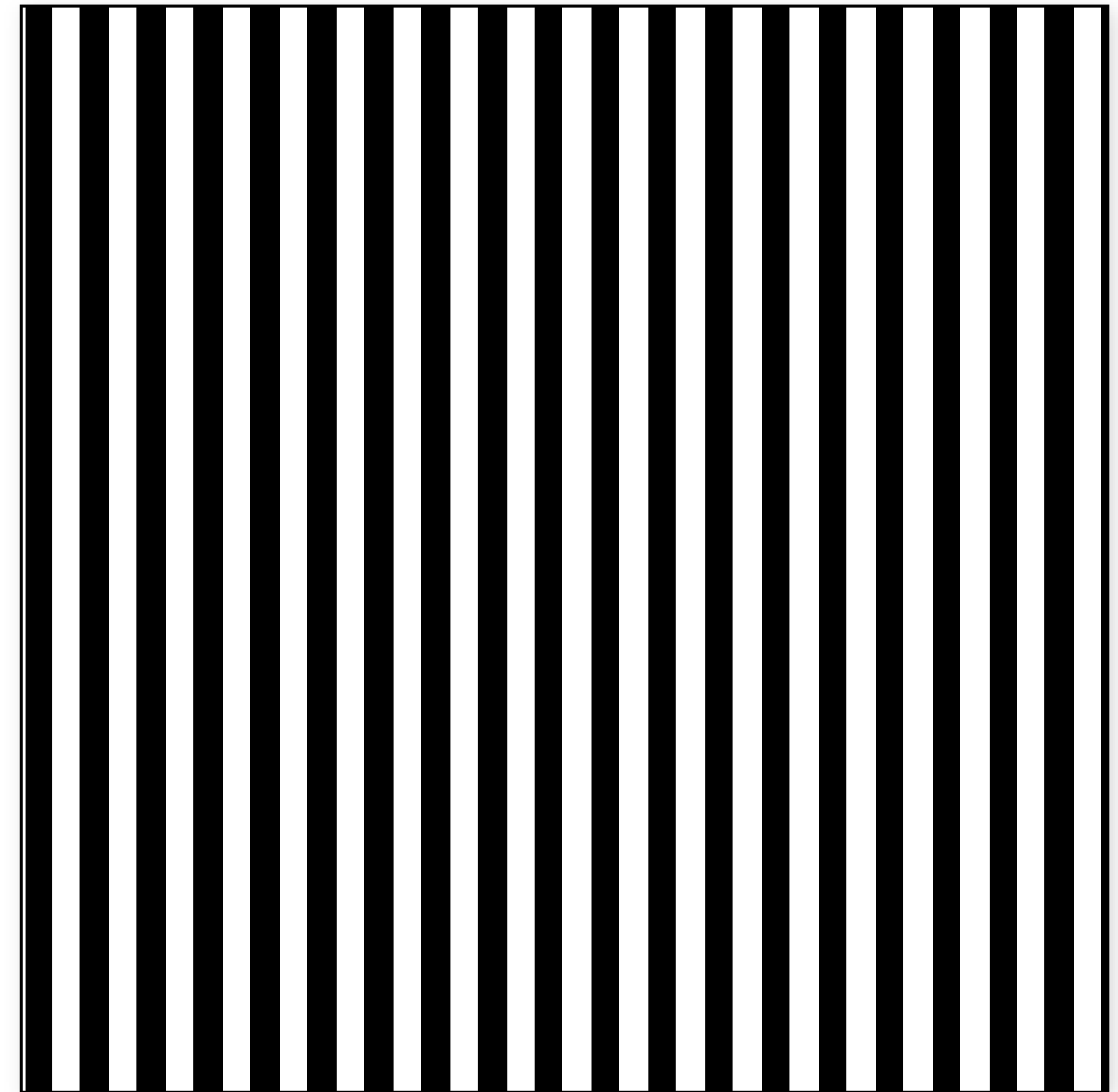
- low memory usage
- infinite resolution
- solid texture: no need to parametrize surface

3D stripe texture

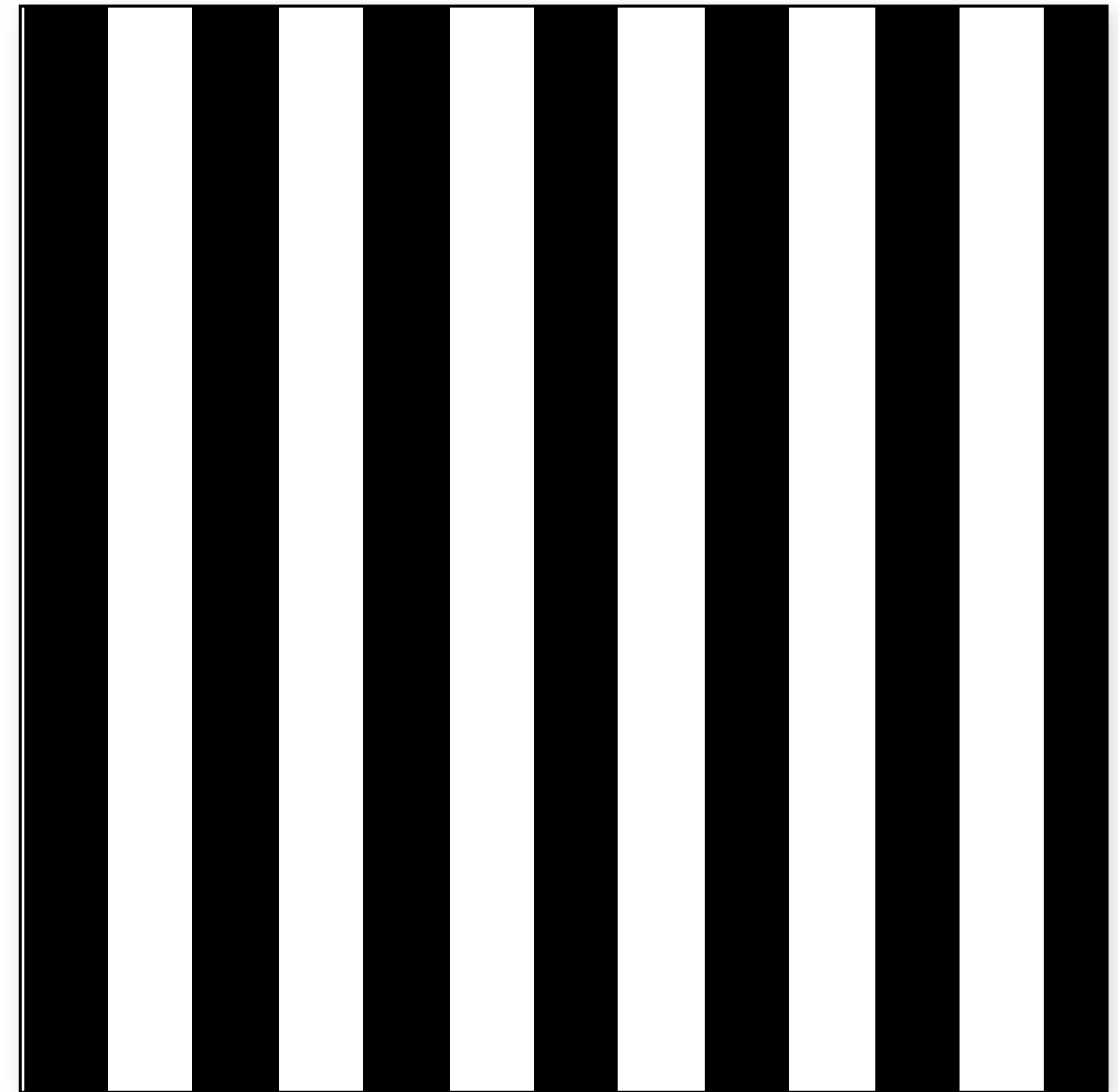


3D stripe texture

```
color stripe(point p):  
    if (sin(px) > 0)  
        return c0  
    else  
        return c1
```

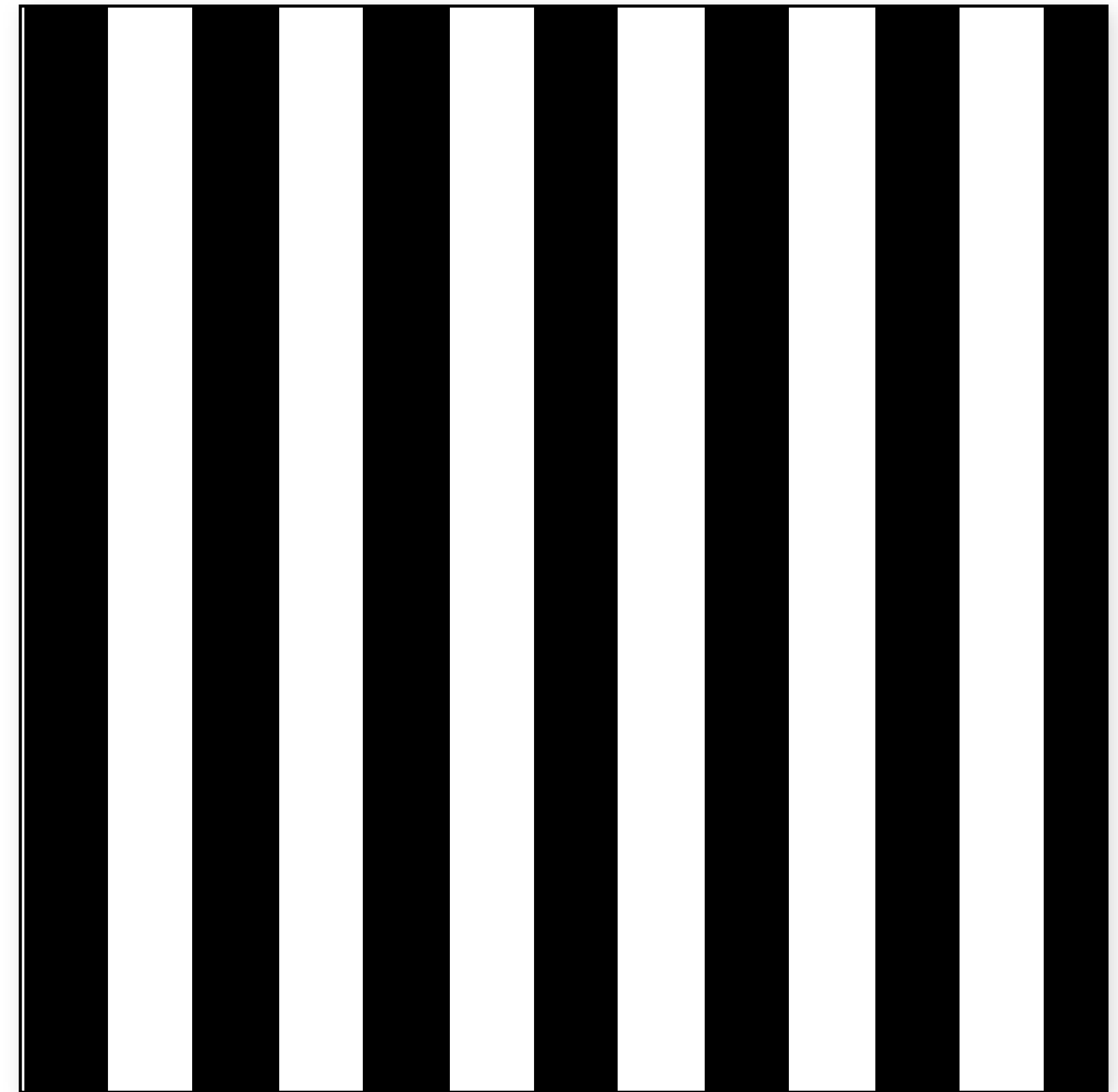


3D stripe texture



3D stripe texture

```
color stripe(point p, real w):  
    if ( $\sin(\pi \mathbf{p}_x / w) > 0$ )  
        return  $c_0$   
    else  
        return  $c_1$ 
```

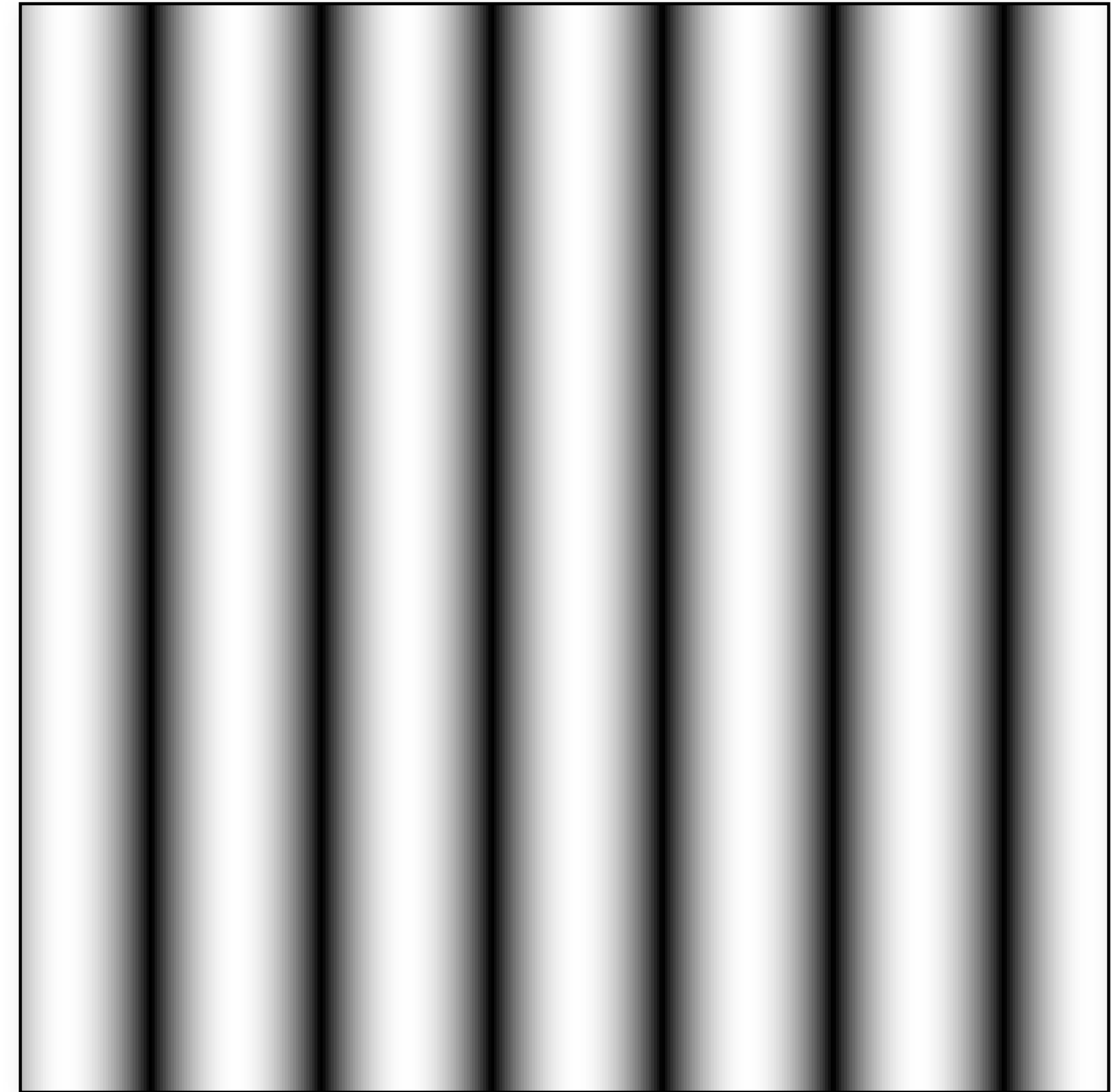


3D stripe texture

```
color stripe(point p, real w):
```

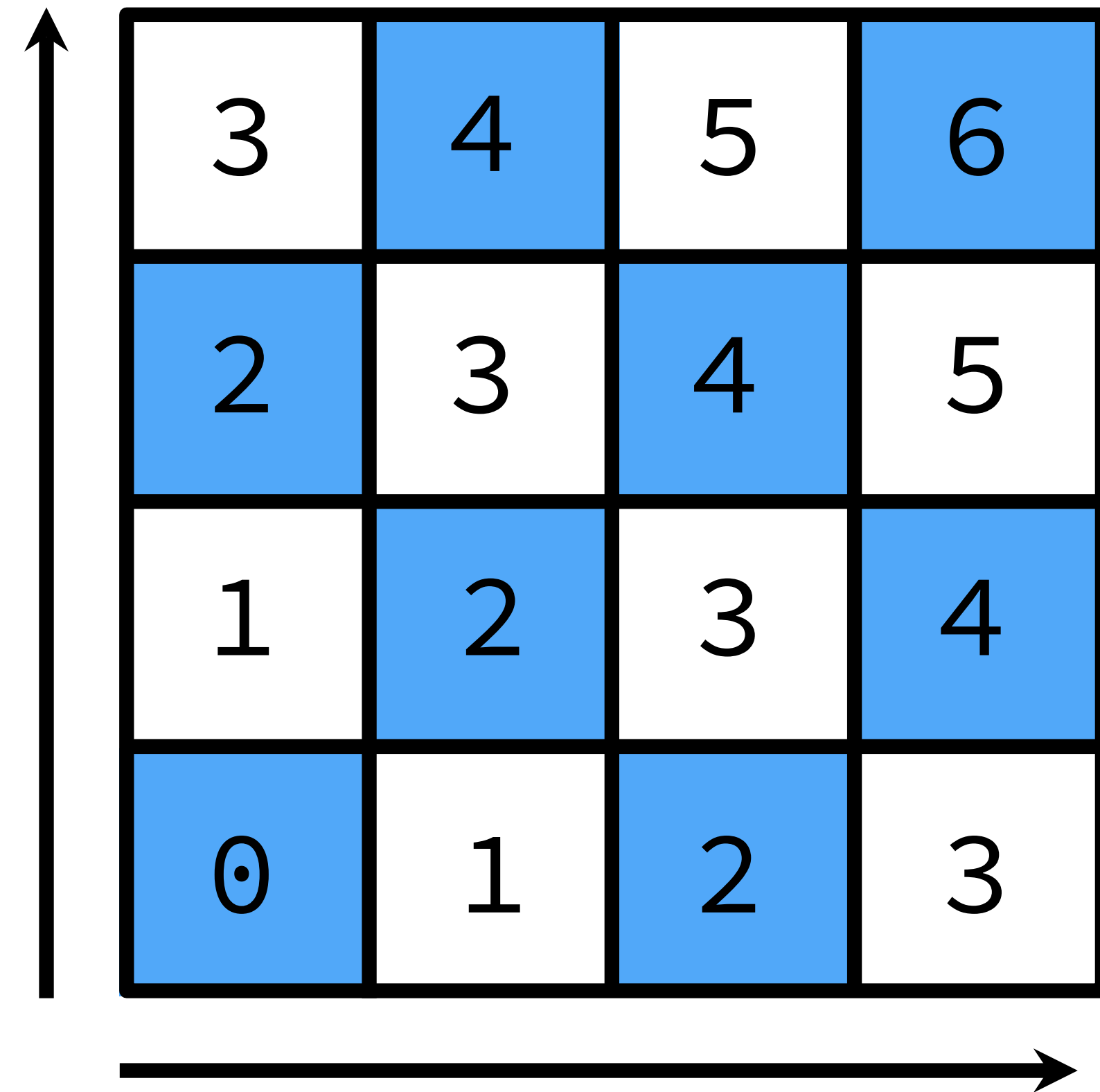
```
     $t = (1 + \sin(\pi \mathbf{p}_x / w)) / 2$ 
```

```
    return lerp( $c_0$ ,  $c_1$ ,  $t$ )
```



2D checkerboard texture

```
color checkerboard(point p):  
    real a = floor(px)  
    real b = floor(py)  
    real val = a+b  
    if (isEven(val))  
        return c0  
    else  
        return c1
```



3D checkerboard texture

```
color checkerboard(point p):
```

```
    real a = floor(px)
```

```
    real b = floor(py)
```

```
    real c = floor(pz)
```

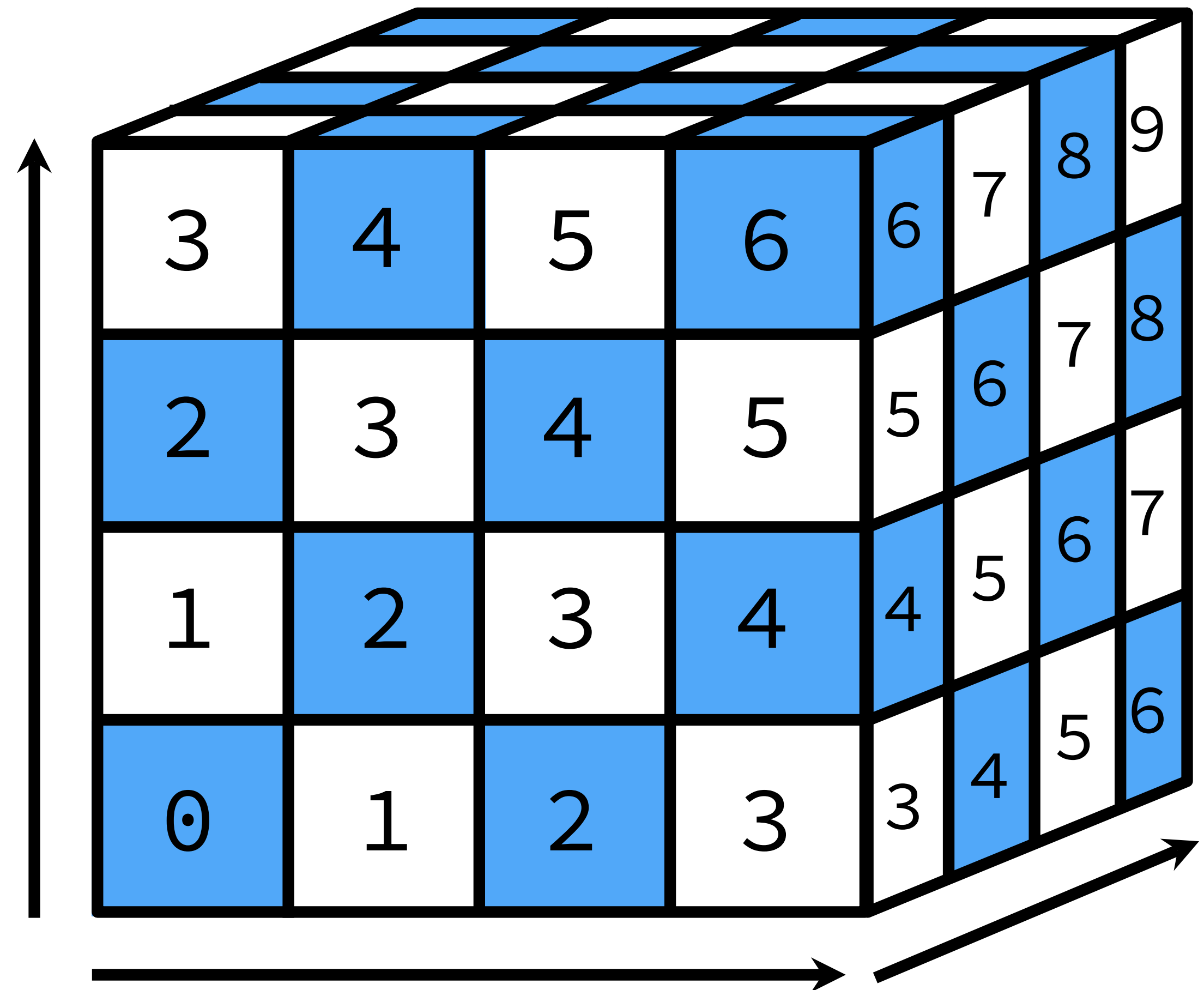
```
    real val = a+b+c
```

```
    if (isEven(val))
```

```
        return c0
```

```
    else
```

```
        return c1
```



Procedural synthesis



Procedural synthesis



Procedural synthesis



Industrial Light + Magic

Procedural synthesis



Procedural textures

Our procedurals are “too perfect”

Often want to add controlled variation to a texture

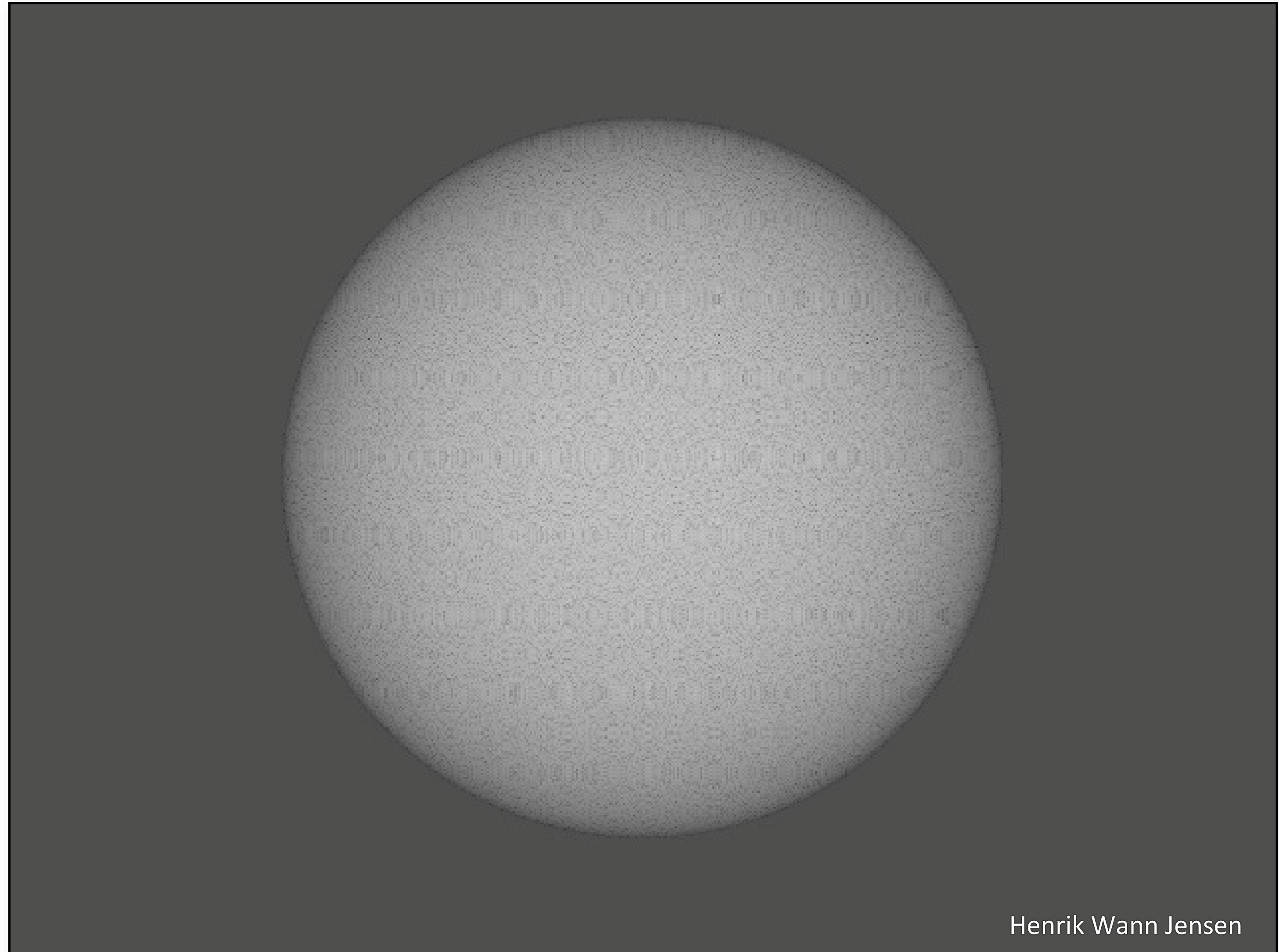
- Real textures have many imperfections

Just calling `rand()` is not that useful.

Random noise

albedo = randf();

Not band-limited,
white noise.



Noise functions

Function: $\mathbf{R}^n \rightarrow [-1, 1]$, where $n = 1, 2, 3, \dots$

Desirable properties:

- no obvious repetition
- rotation invariant
- band-limited (i.e., not scale-invariant)

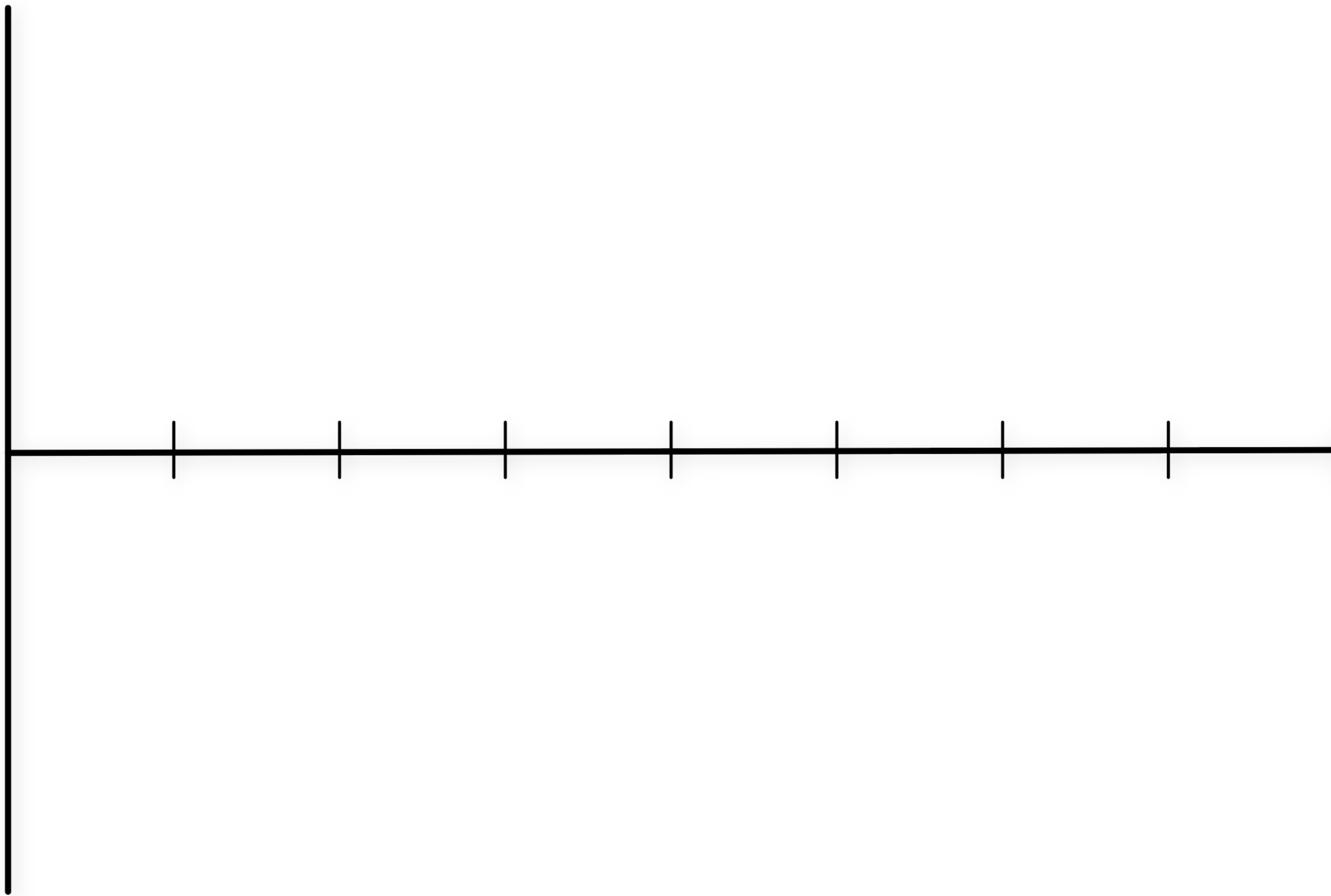
Fundamental building block of most procedural textures

Value noise

Values associated with integer lattice locations

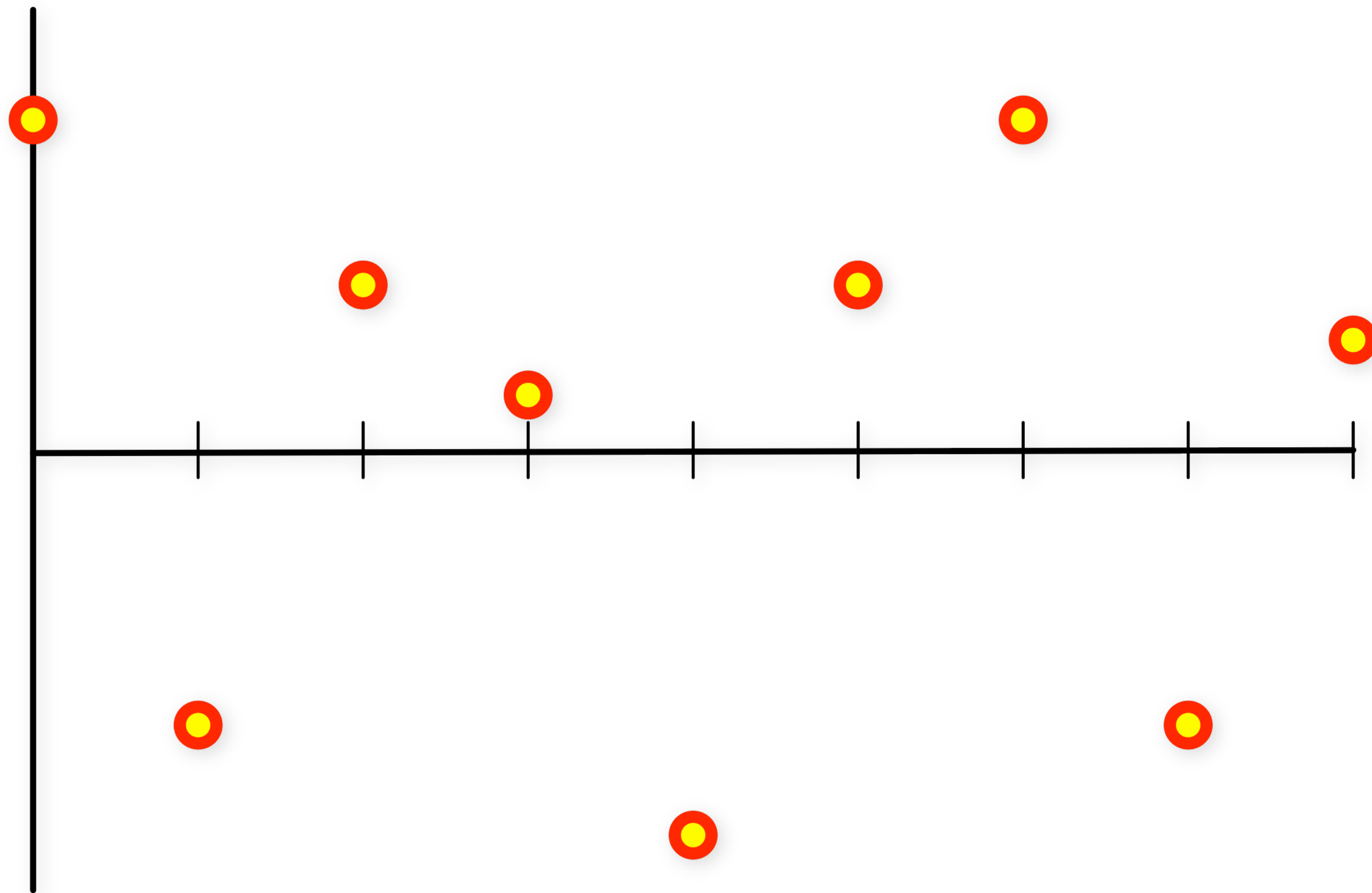
Given arbitrary position, interpolate value from neighboring lattice points

Value noise example



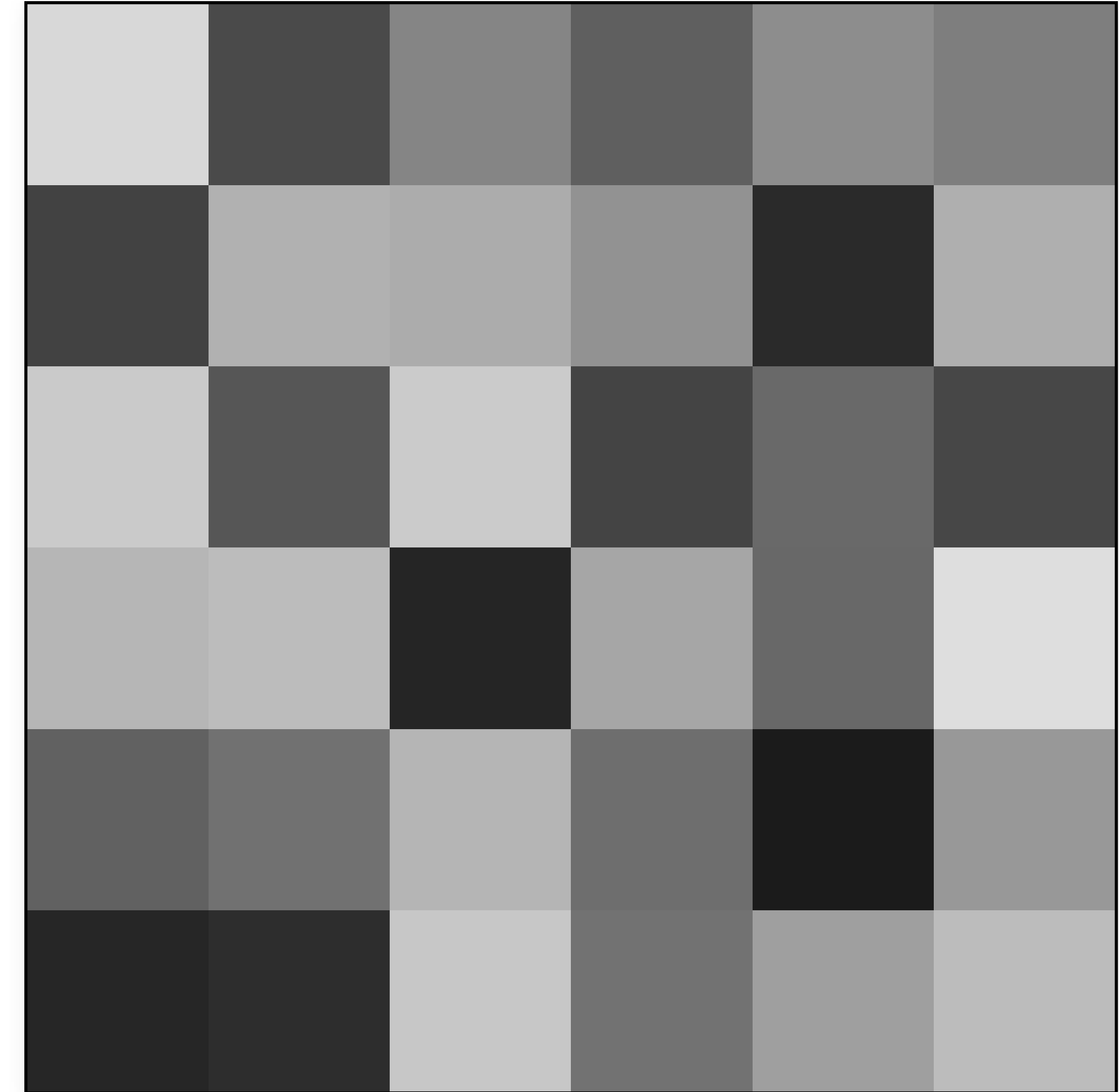
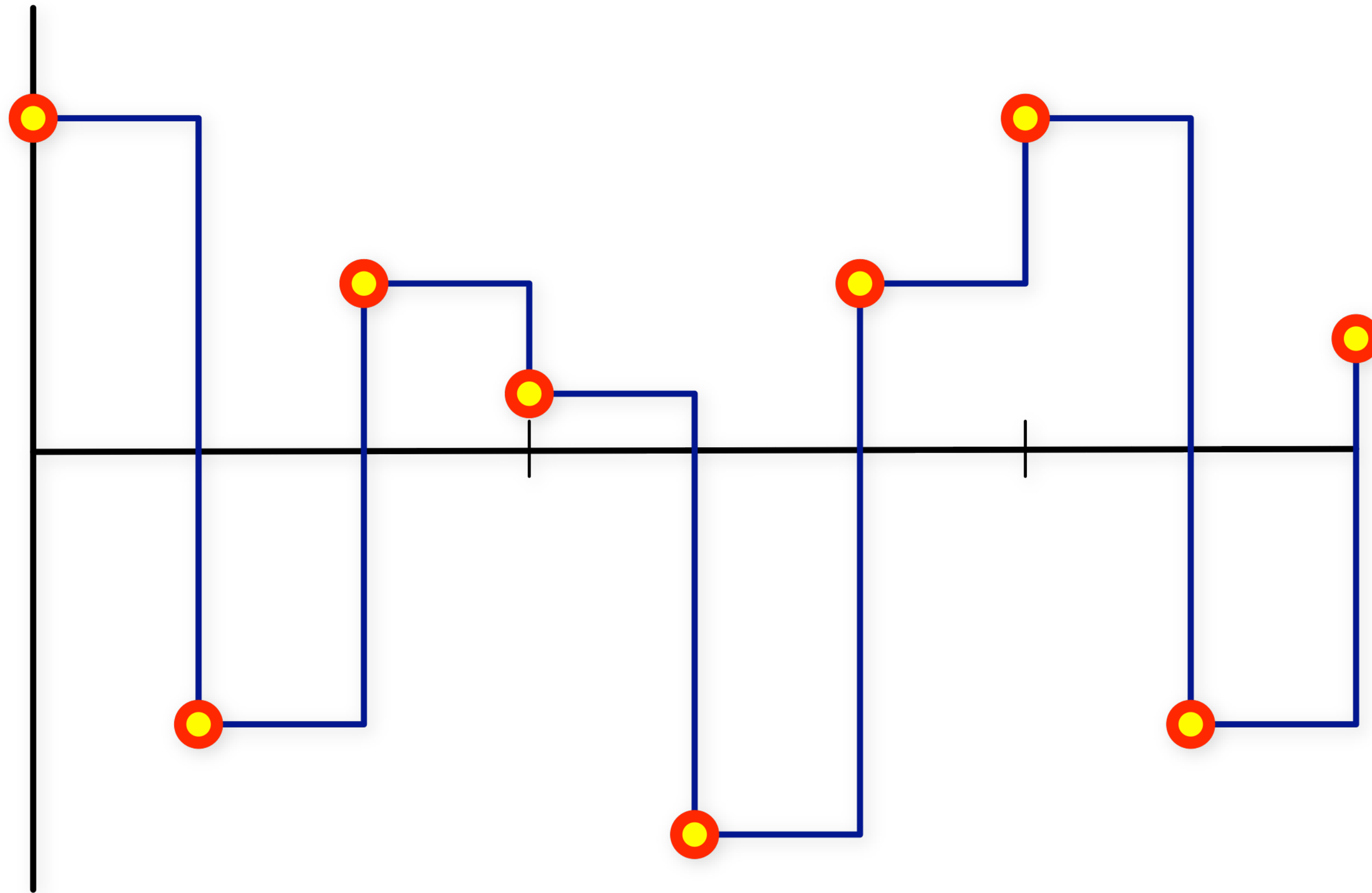
Value noise example

Random values on grid



Value noise example

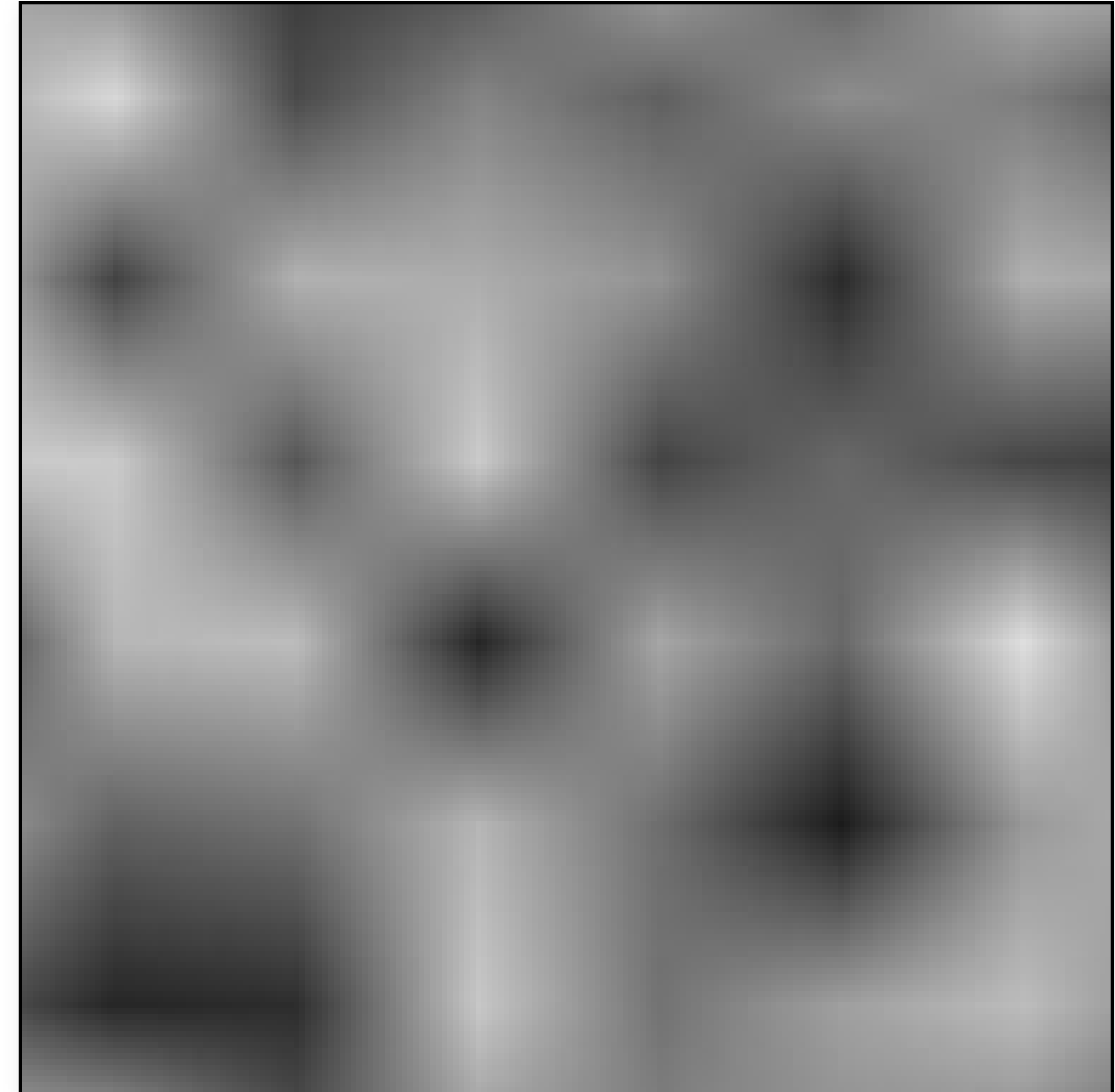
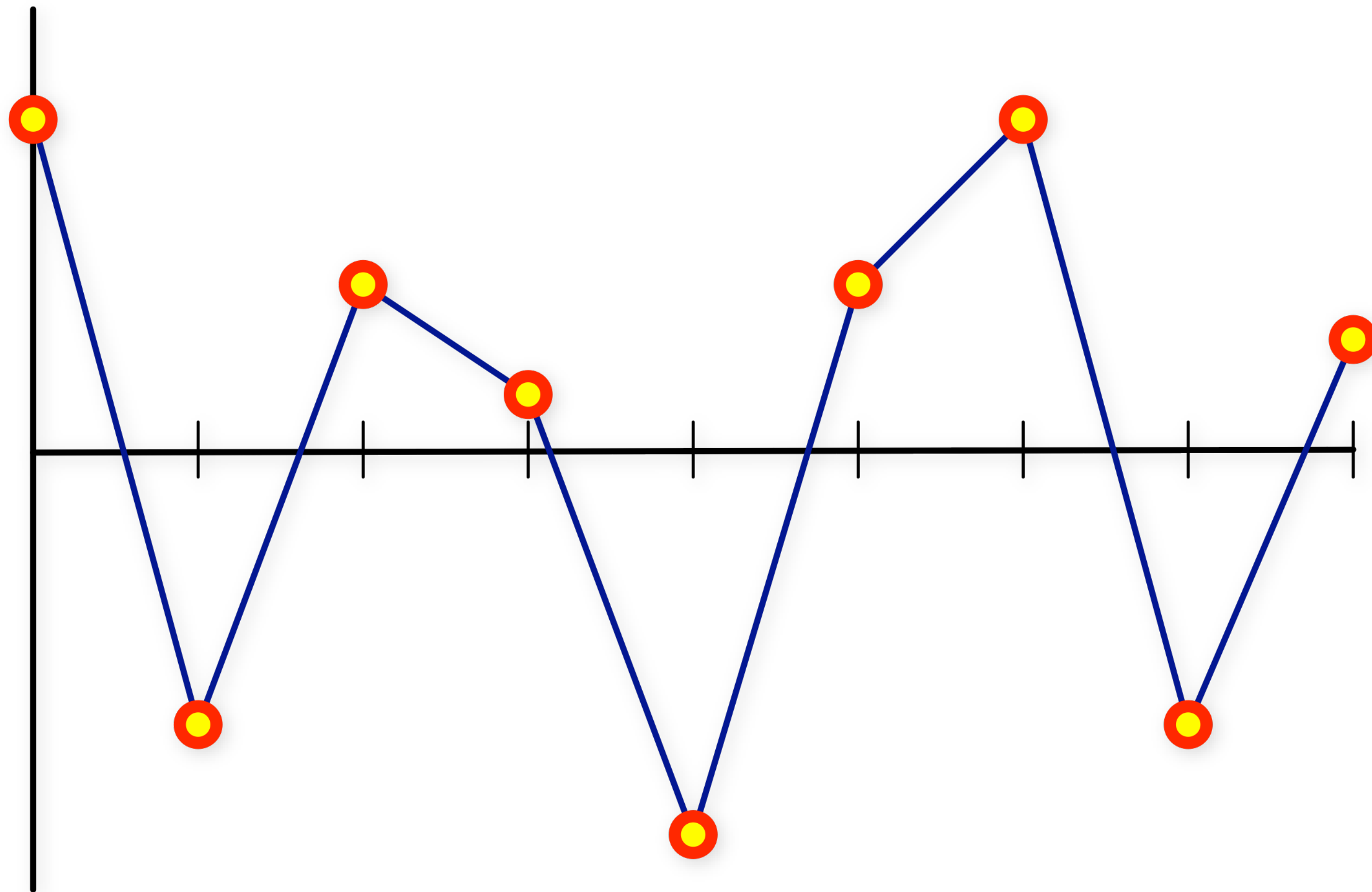
Random values on grid



Cell noise: use value of nearest point on grid

Value noise example

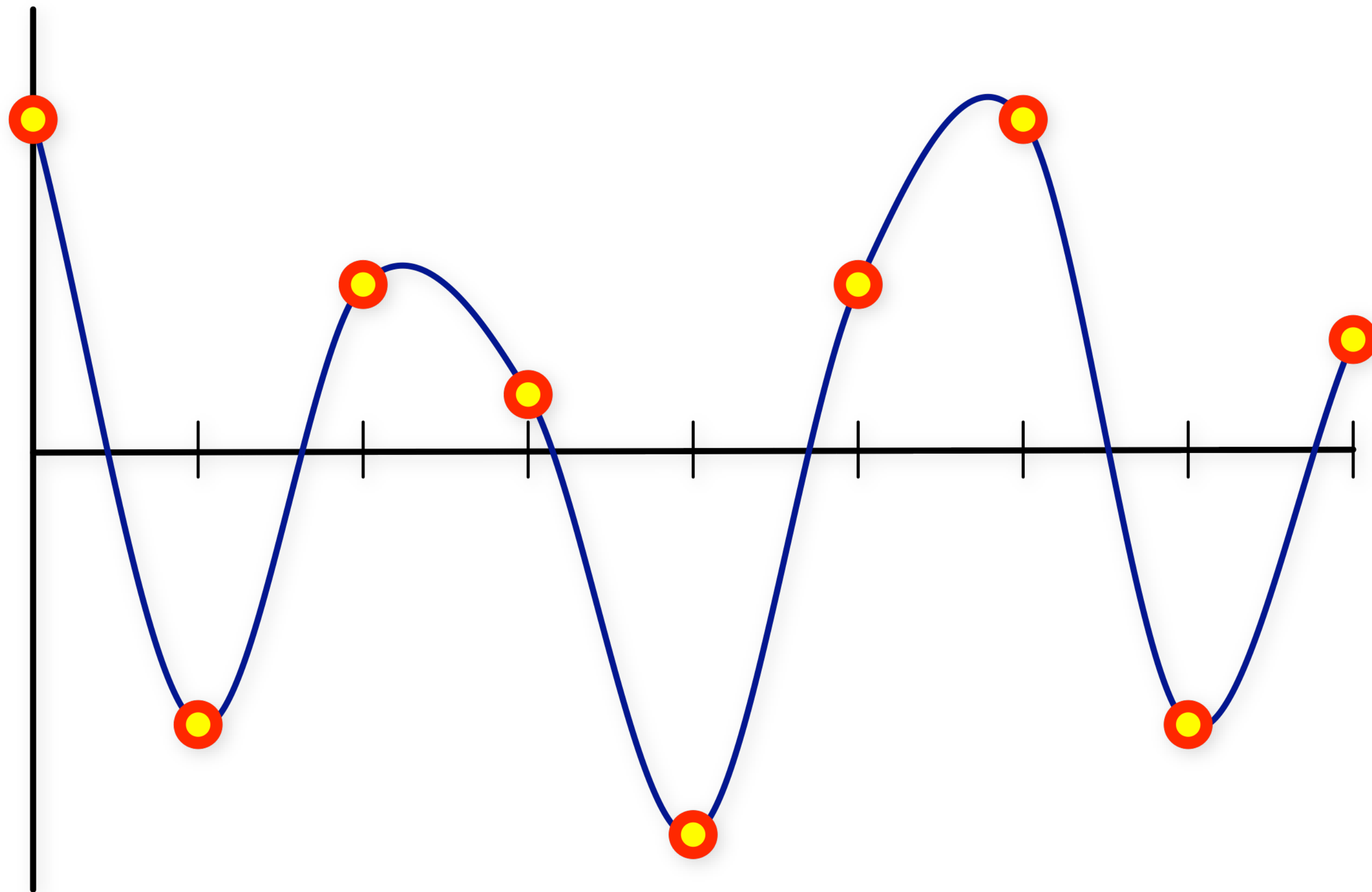
(Bi-) linearly interpolated values



Interpolate between 2^n nearest grid points

Value noise example

(Bi-) cubic interpolation



Interpolate between 4^n nearest grid points

Value noise - implementation issues

Not feasible to store values at all integer locations

- pre-compute an array of pseudo-random values
- use a randomized hash function to map lattice locations to pseudo-random values

Value noise - implementation details

```
// randomly permuted array of 0...255, duplicated
const unsigned char values[256*2] = [1, 234, ...];

float noise1D(float x)
{
    int xi = int(floor(x)) & 255;
    return lerp(values[xi], values[xi+1], x-xi)/128.0-1;
}

// 2D hashing:
// values[xi + values[yi]];
// 3D hashing:
// values[xi + values[yi + values[zi]]];
// etc.
```

Value noise - limitations



Value noise - limitations

Lattice structure apparent

- Minimal/maxima always on lattice

Slow/many lookups

- 8 values for trilinear
- 64 values for tricubic
 - 4^n for n dimensions



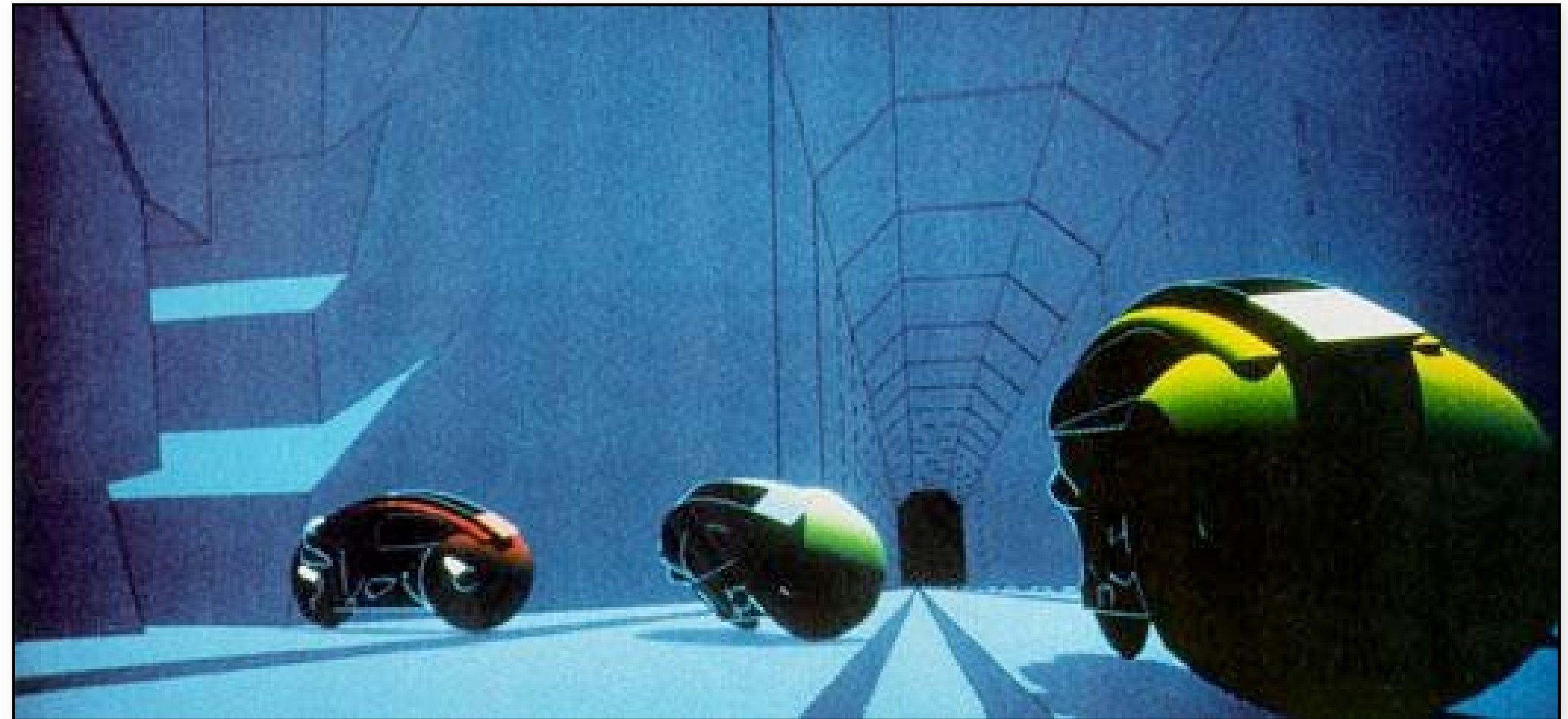
Perlin noise

Perlin noise, invented by Ken Perlin in 1982

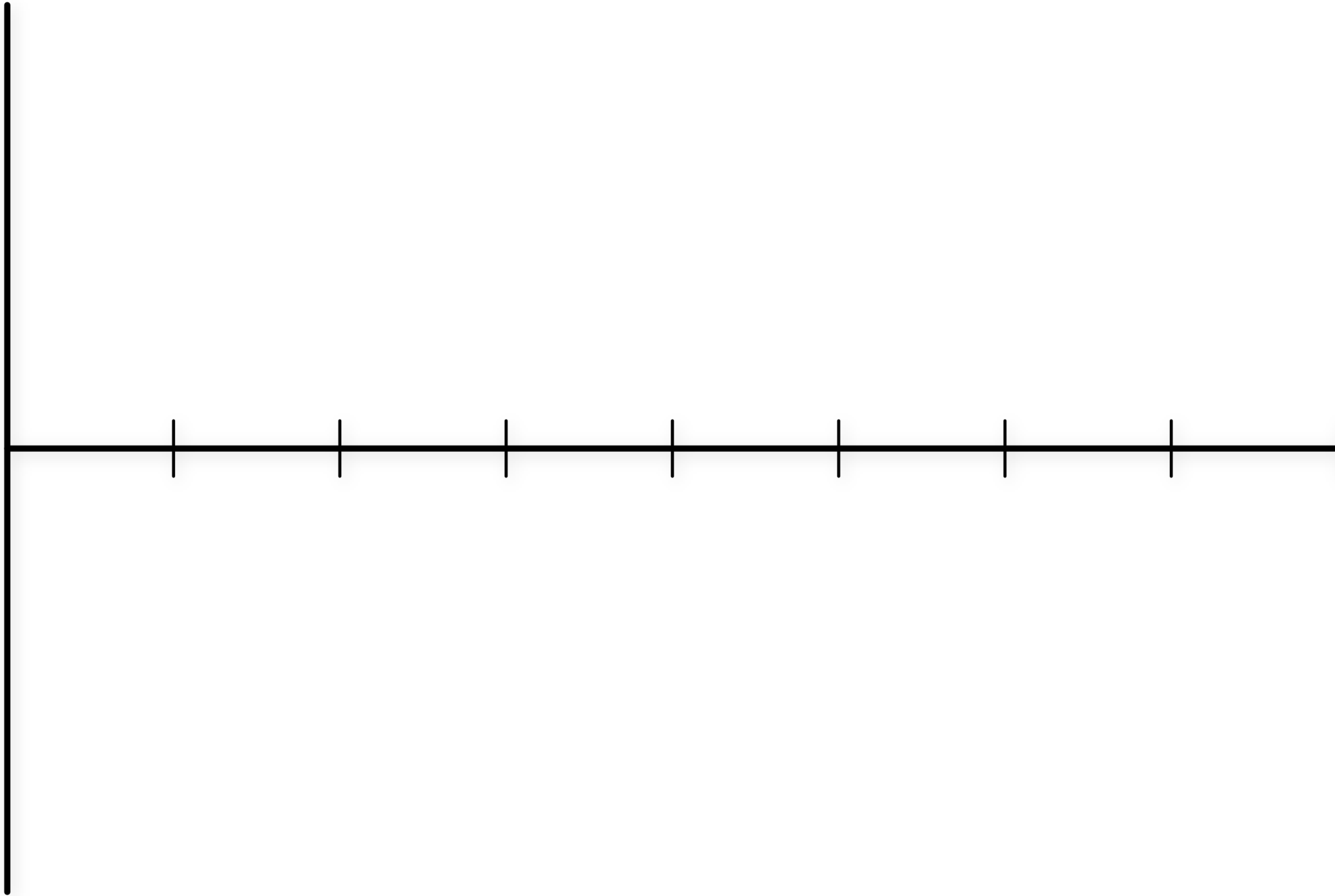
- First used in the movie Tron!

Store random vectors/gradients on lattice

- Use Hermite interp.
- a.k.a. “gradient noise”

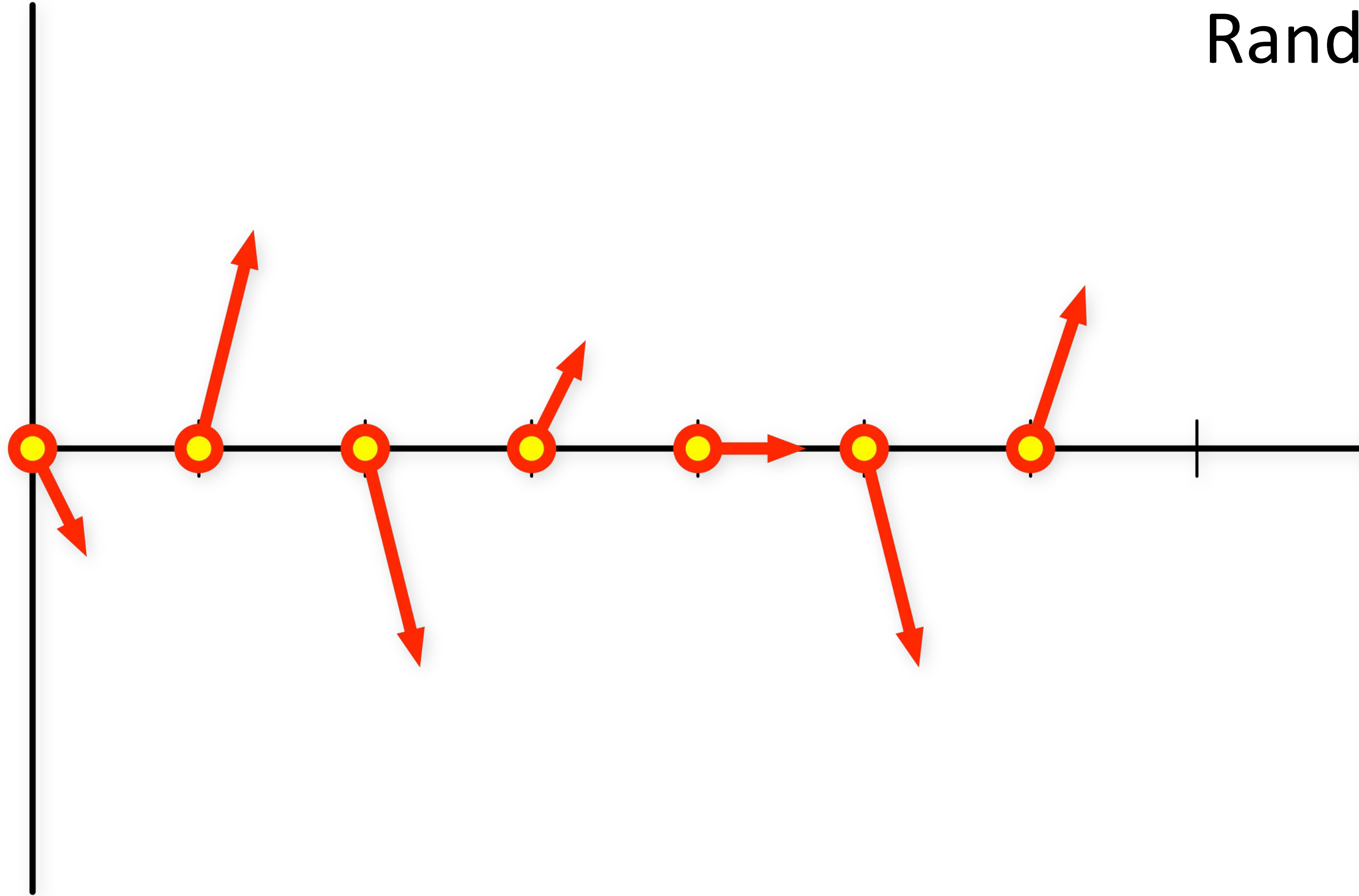


Classic Perlin noise

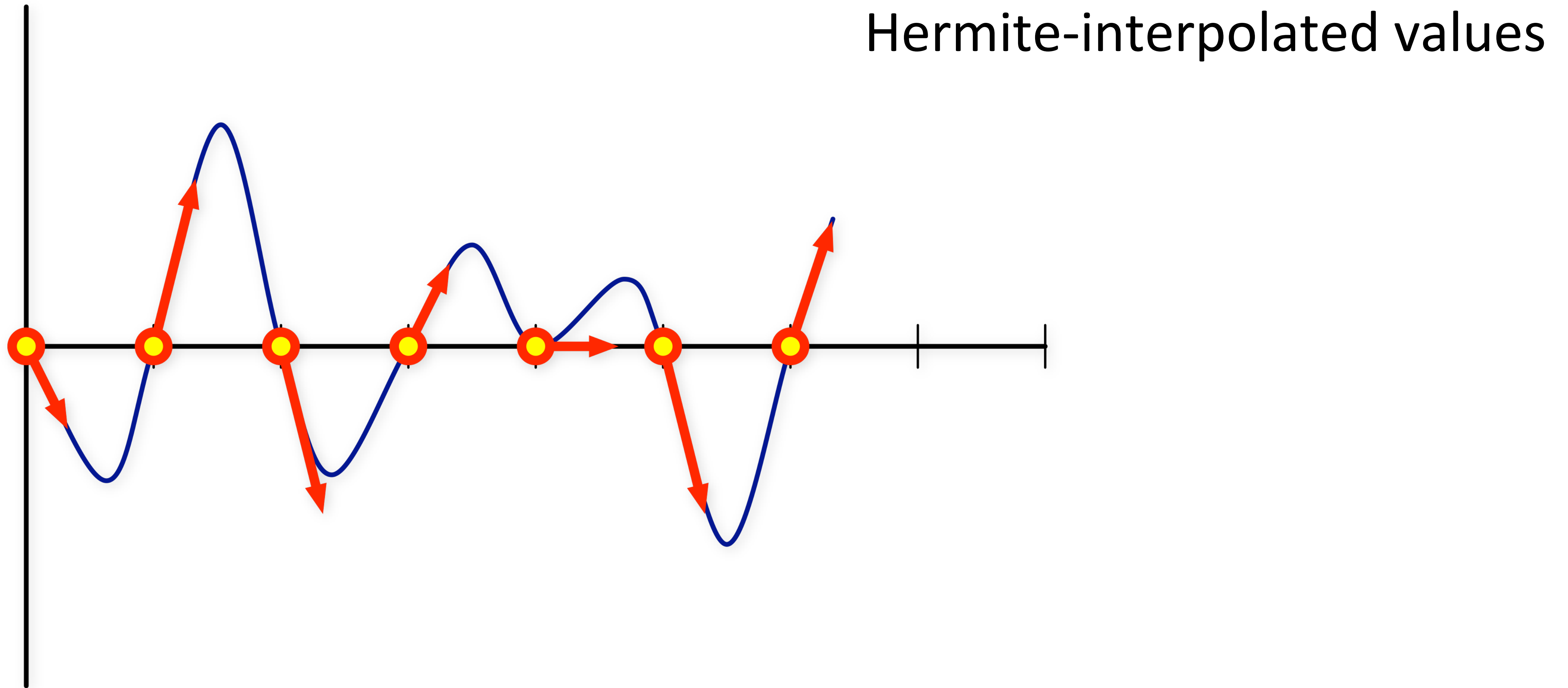


Classic Perlin noise

Random gradients on grid

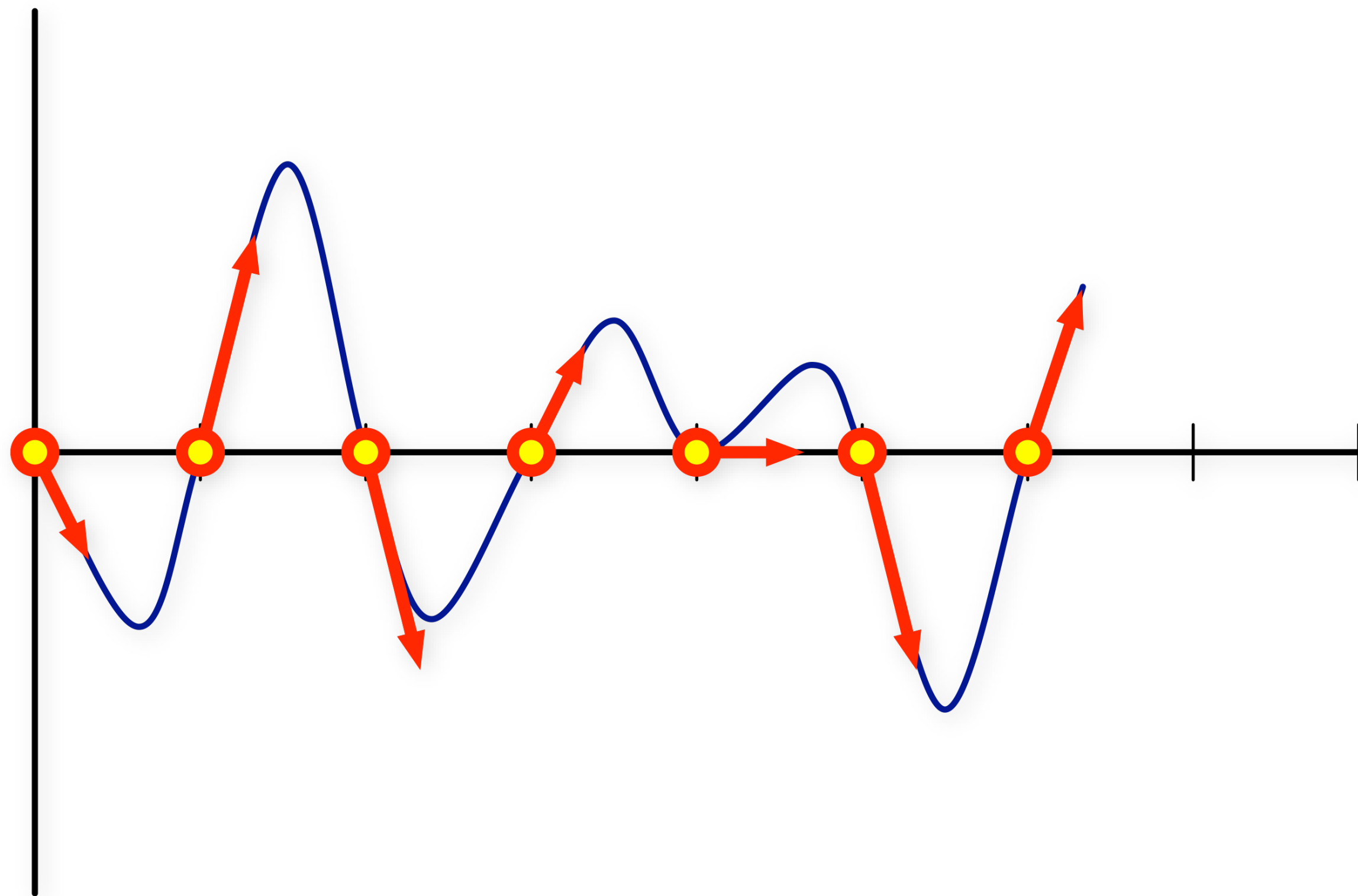


Classic Perlin noise

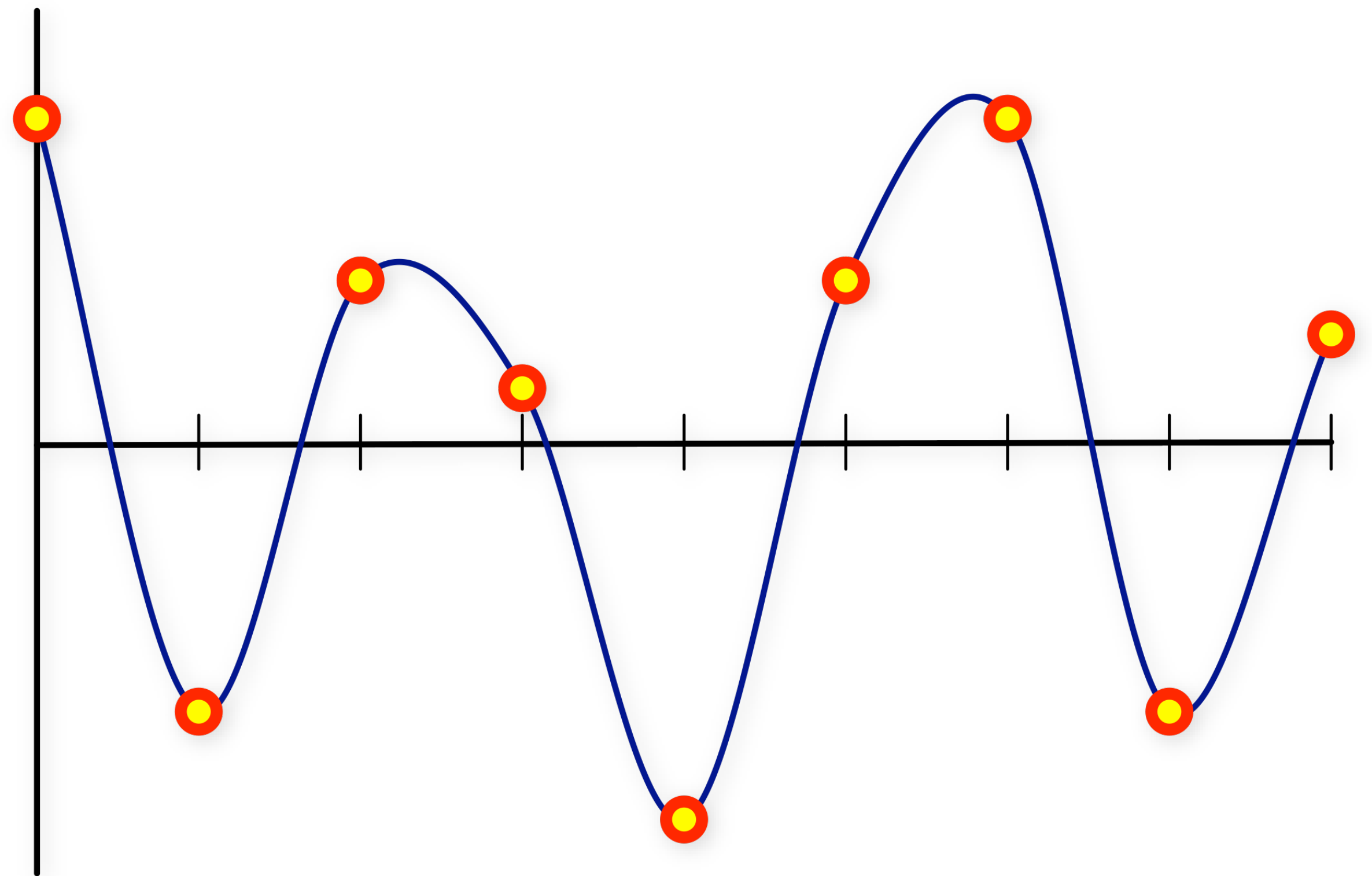


Perlin noise vs. value noise

Perlin Noise
(gradient noise)



Cubic Value Noise

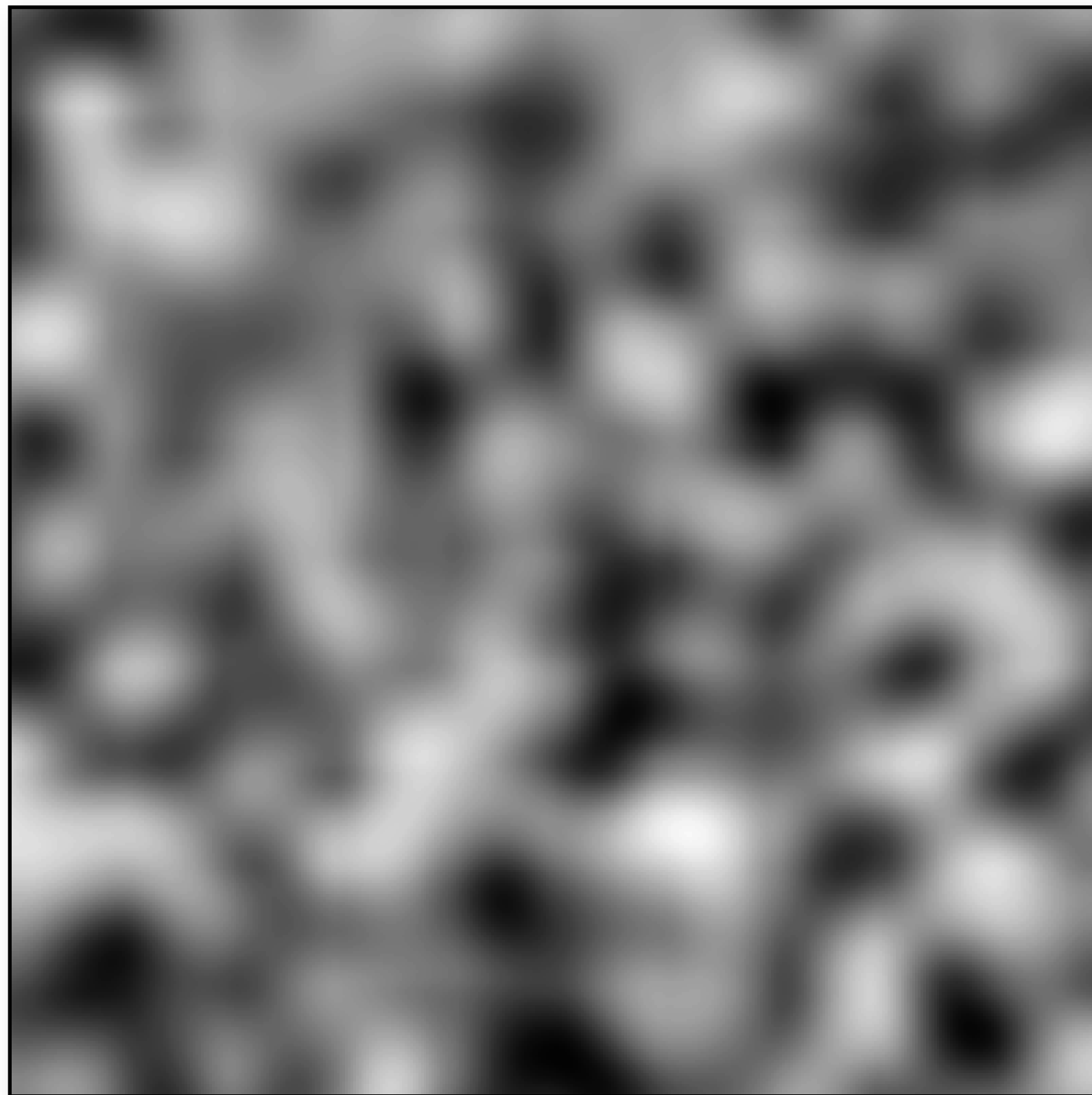


Why is Perlin noise better?

Perlin noise

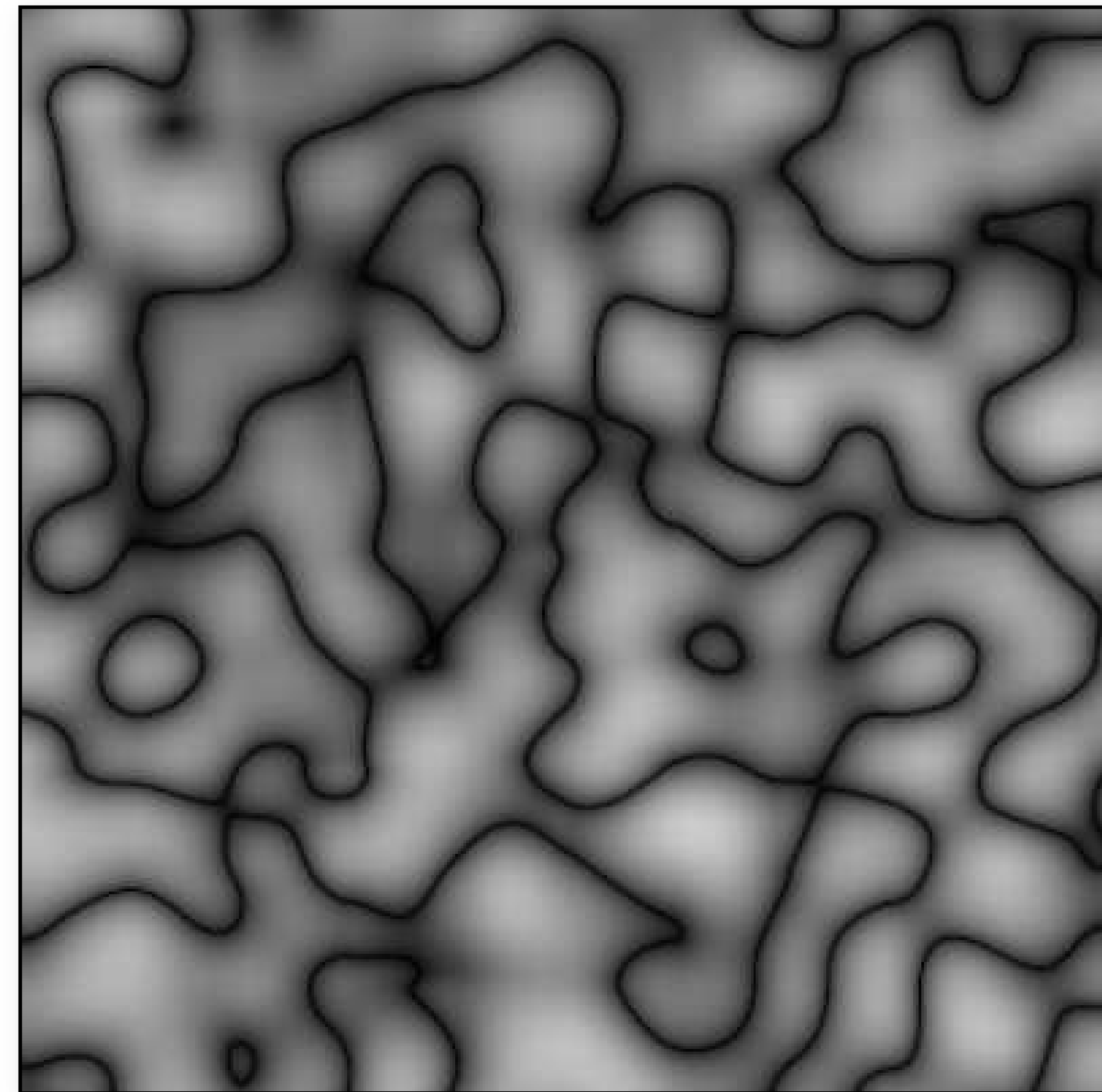
Typically signed by default, \sim in $[-1,1]$ with a mean of 0

offset/scale to put into $[0,1]$ range



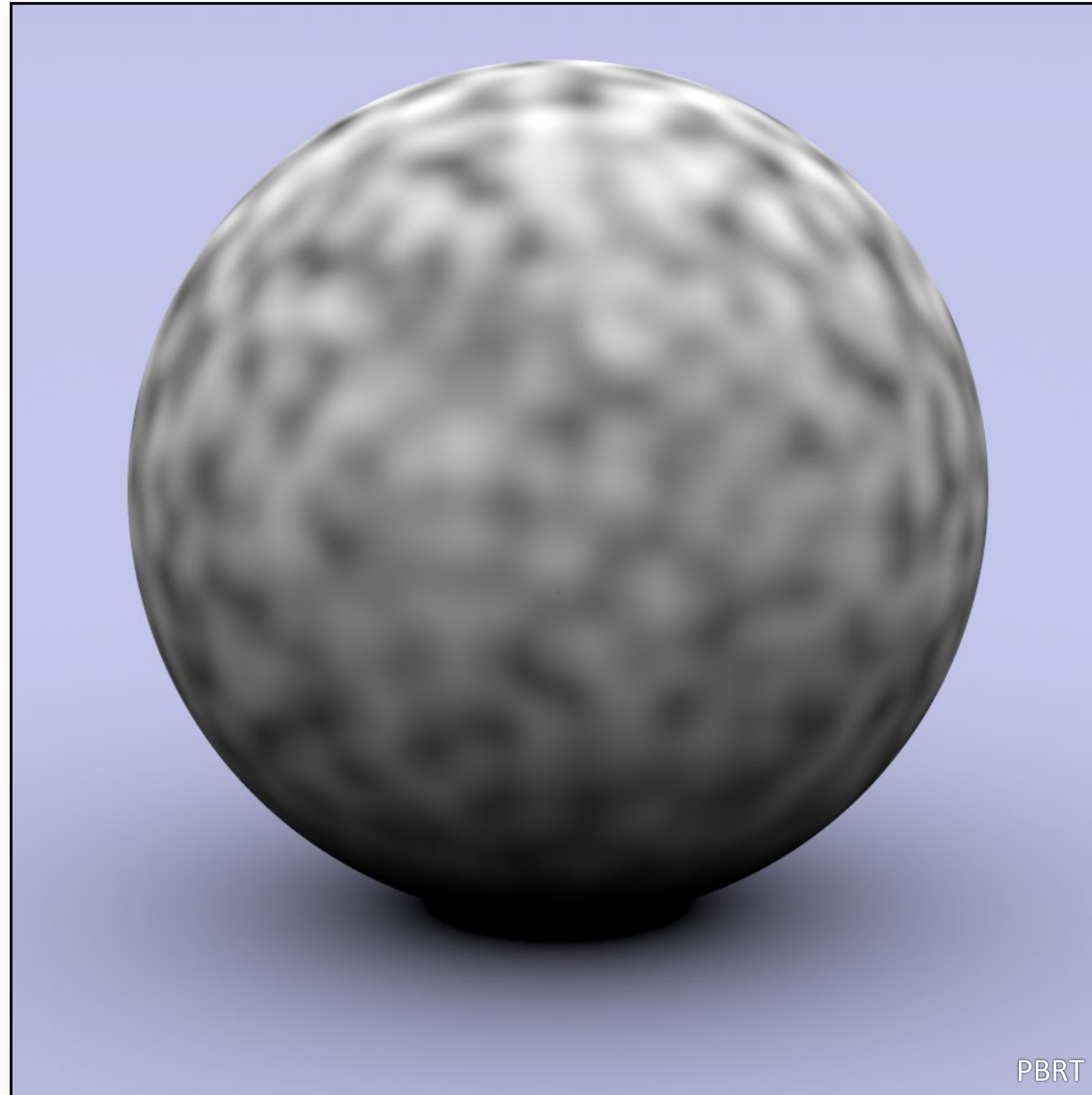
$$(\text{noise}(\mathbf{p}) + 1) / 2$$

take absolute value

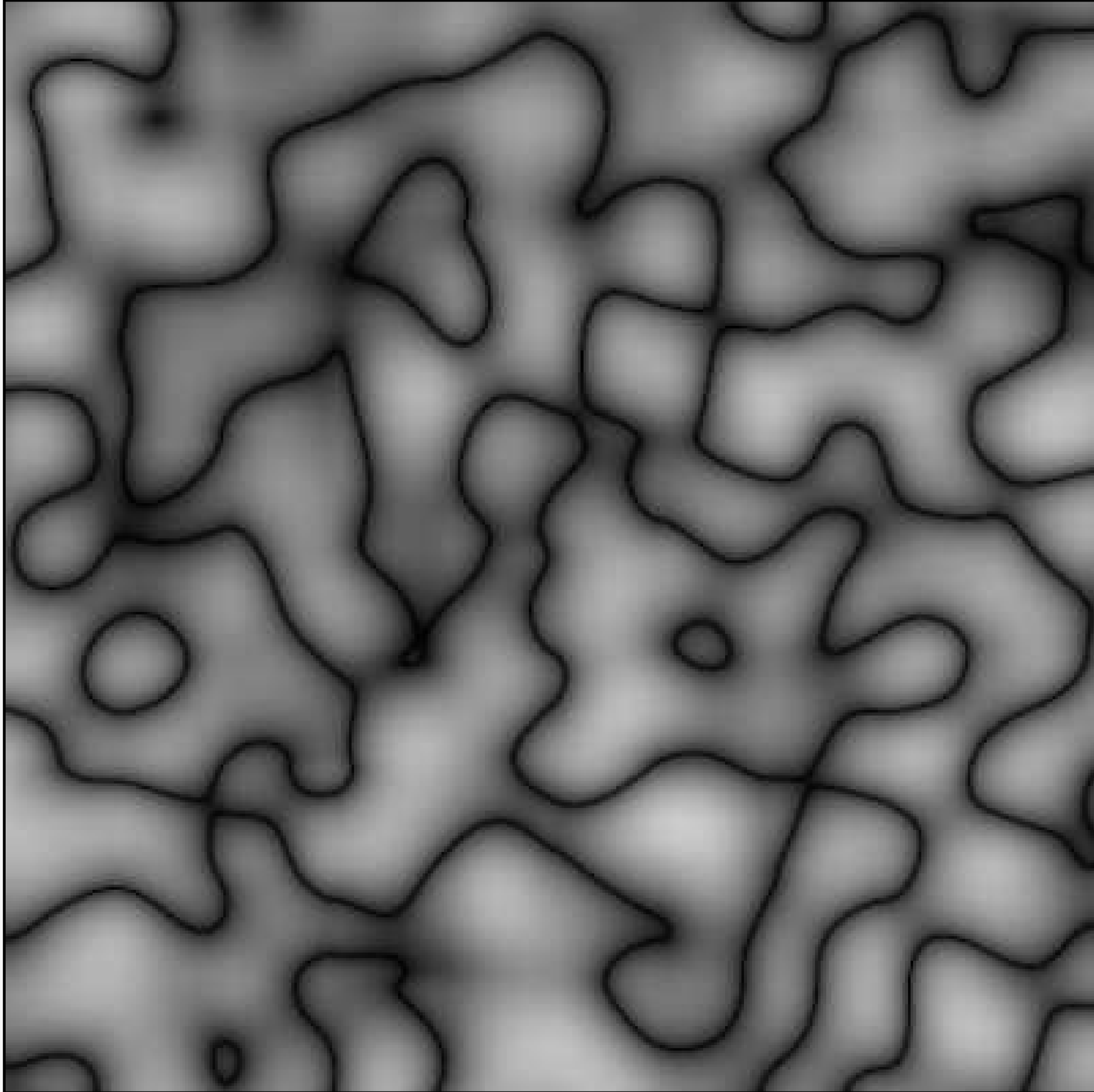


$$|\text{noise}(\mathbf{p})|$$

3D Perlin noise



Absolute value of noise



Perlin noise

Change frequency: ?

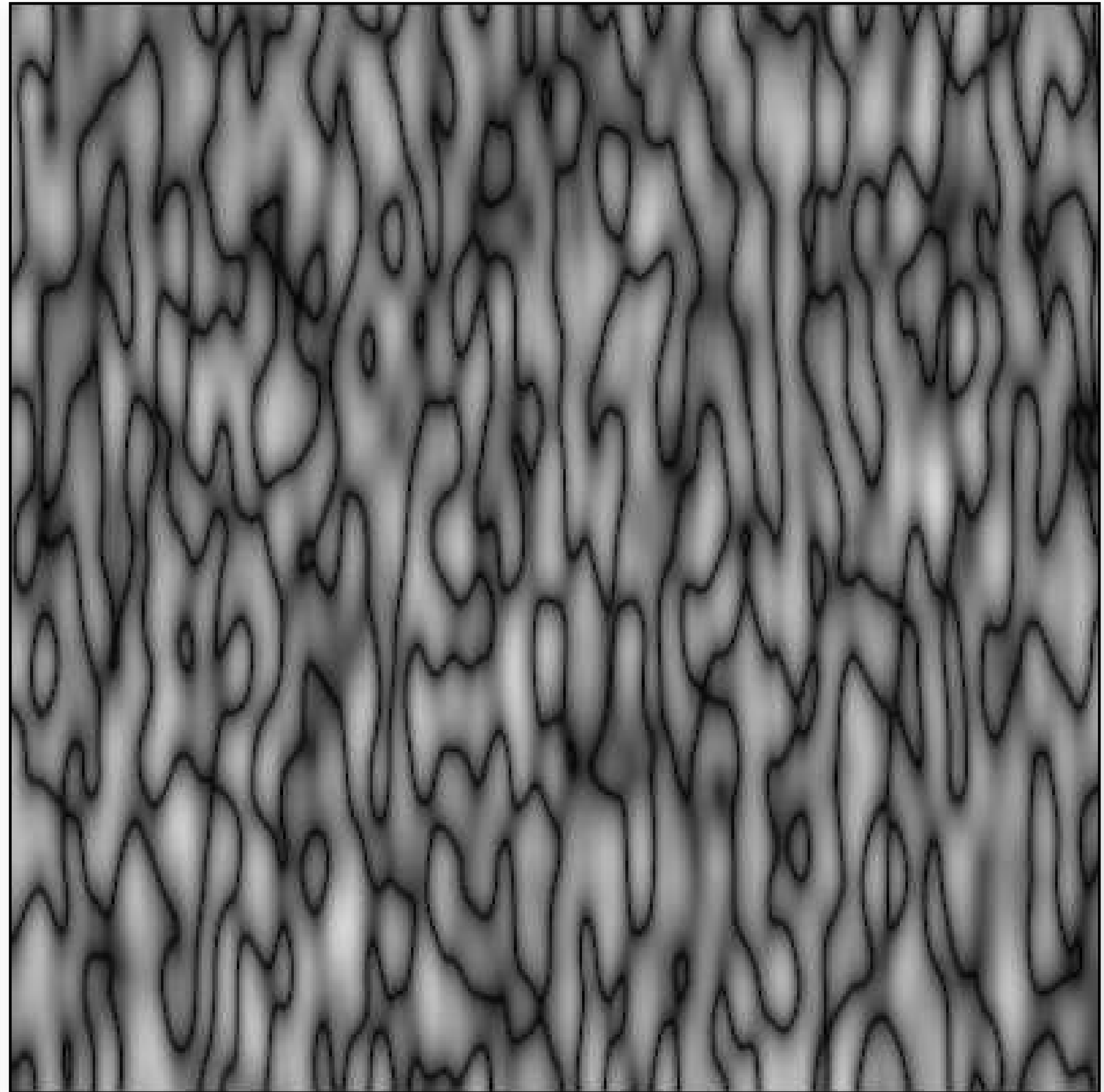
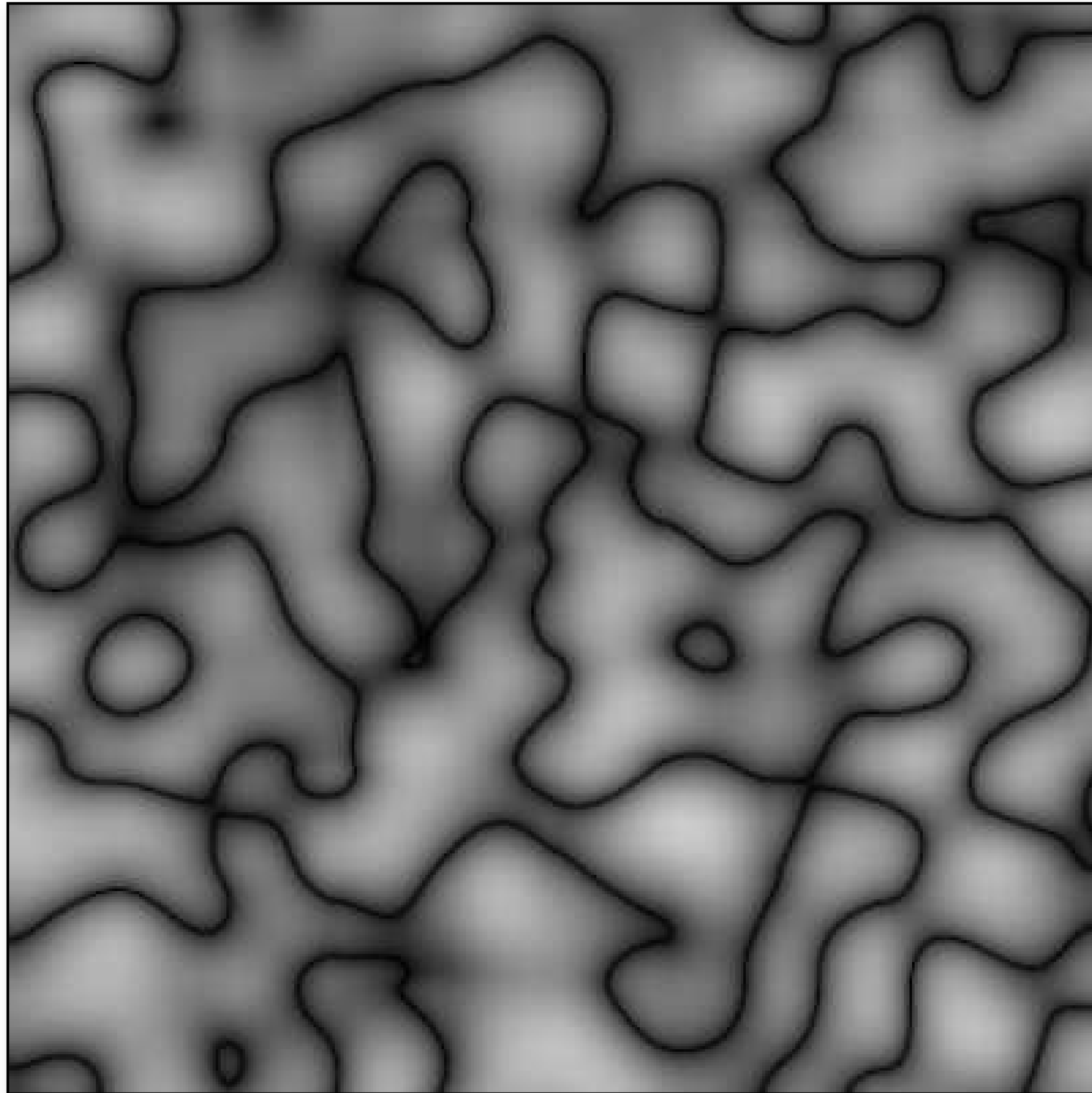
Change amplitude: ?

Perlin noise

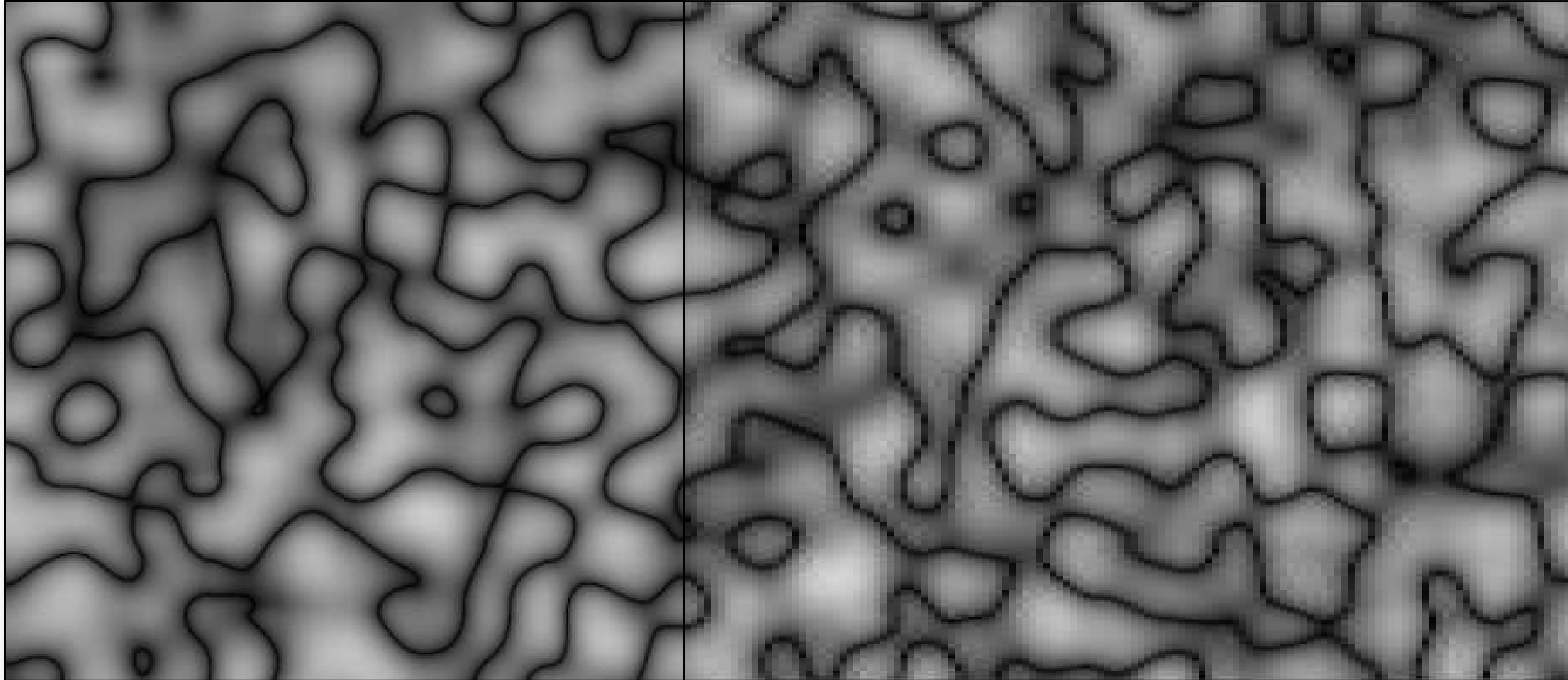
Change frequency: $\text{noise}(10 * x)$

Change amplitude: $10 * \text{noise}(x)$

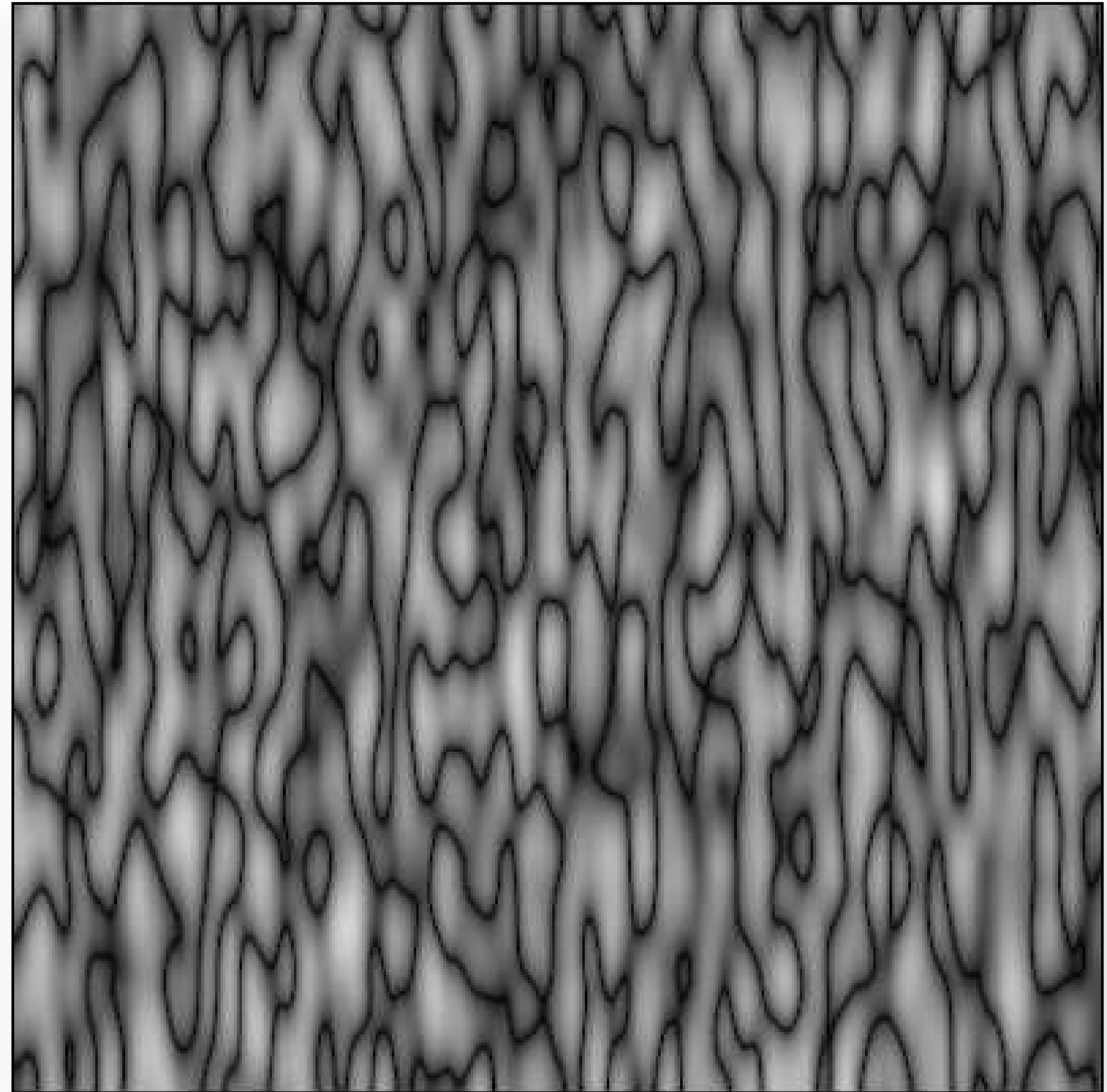
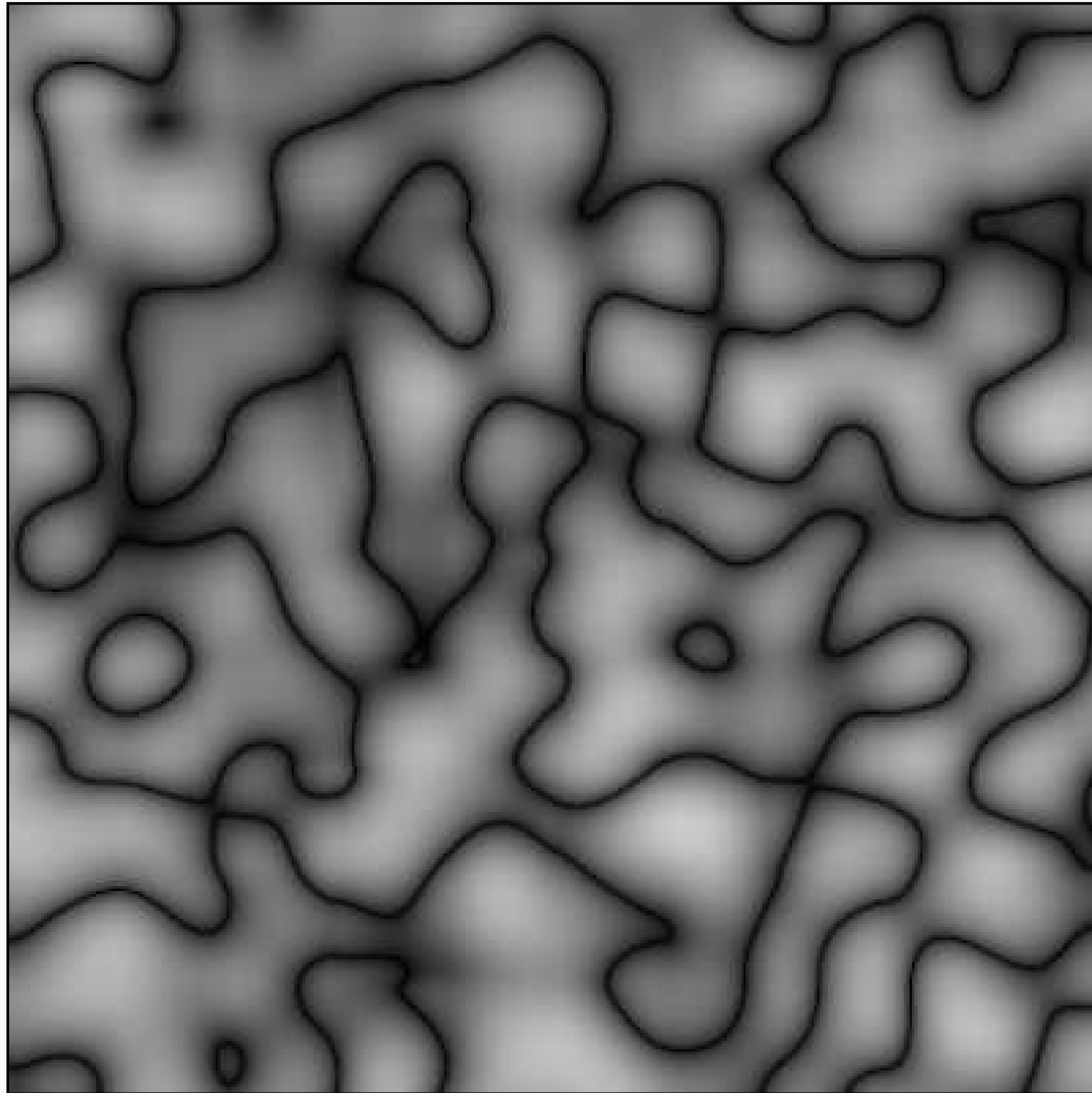
Absolute value of noise



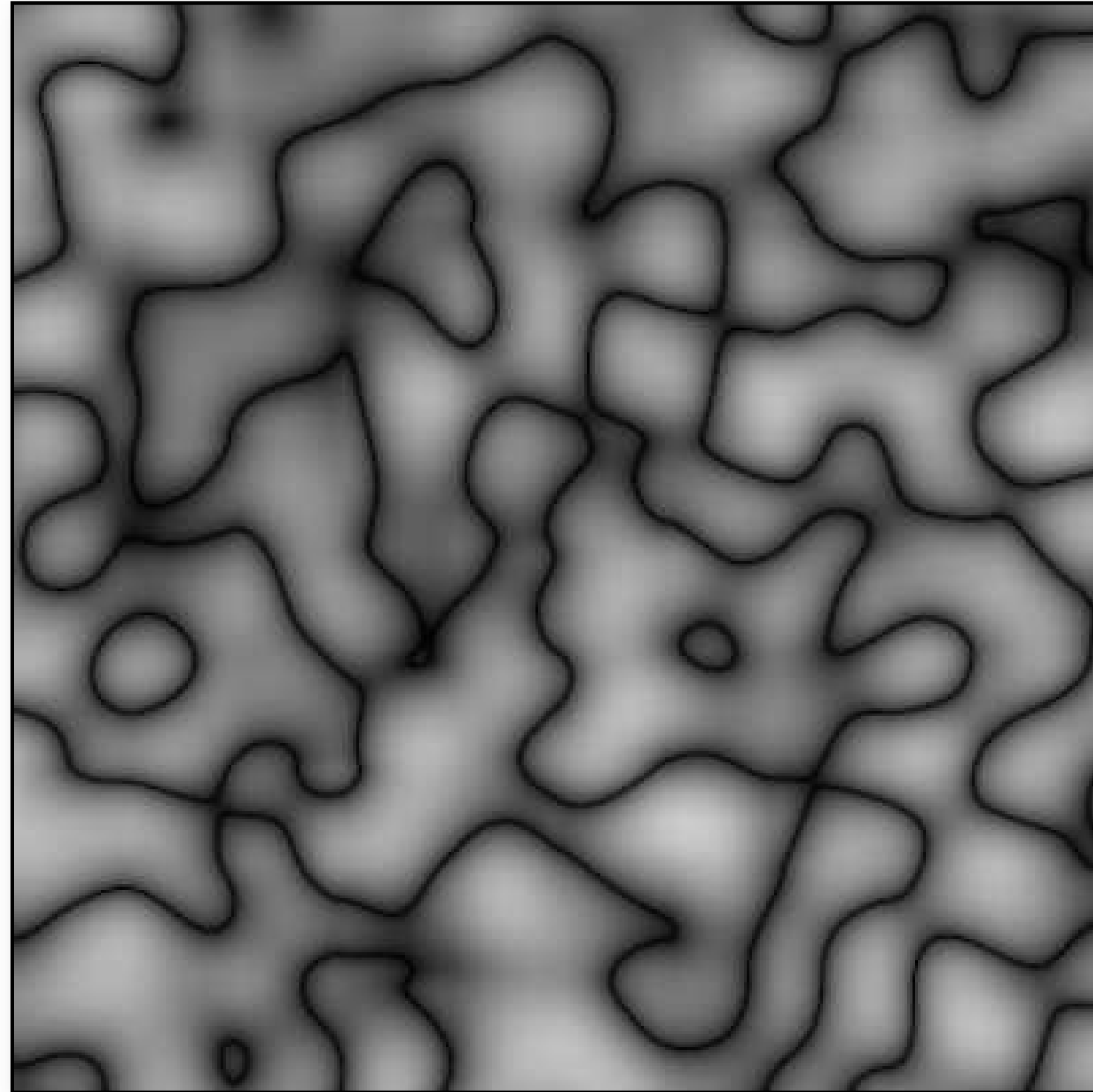
Absolute value of noise



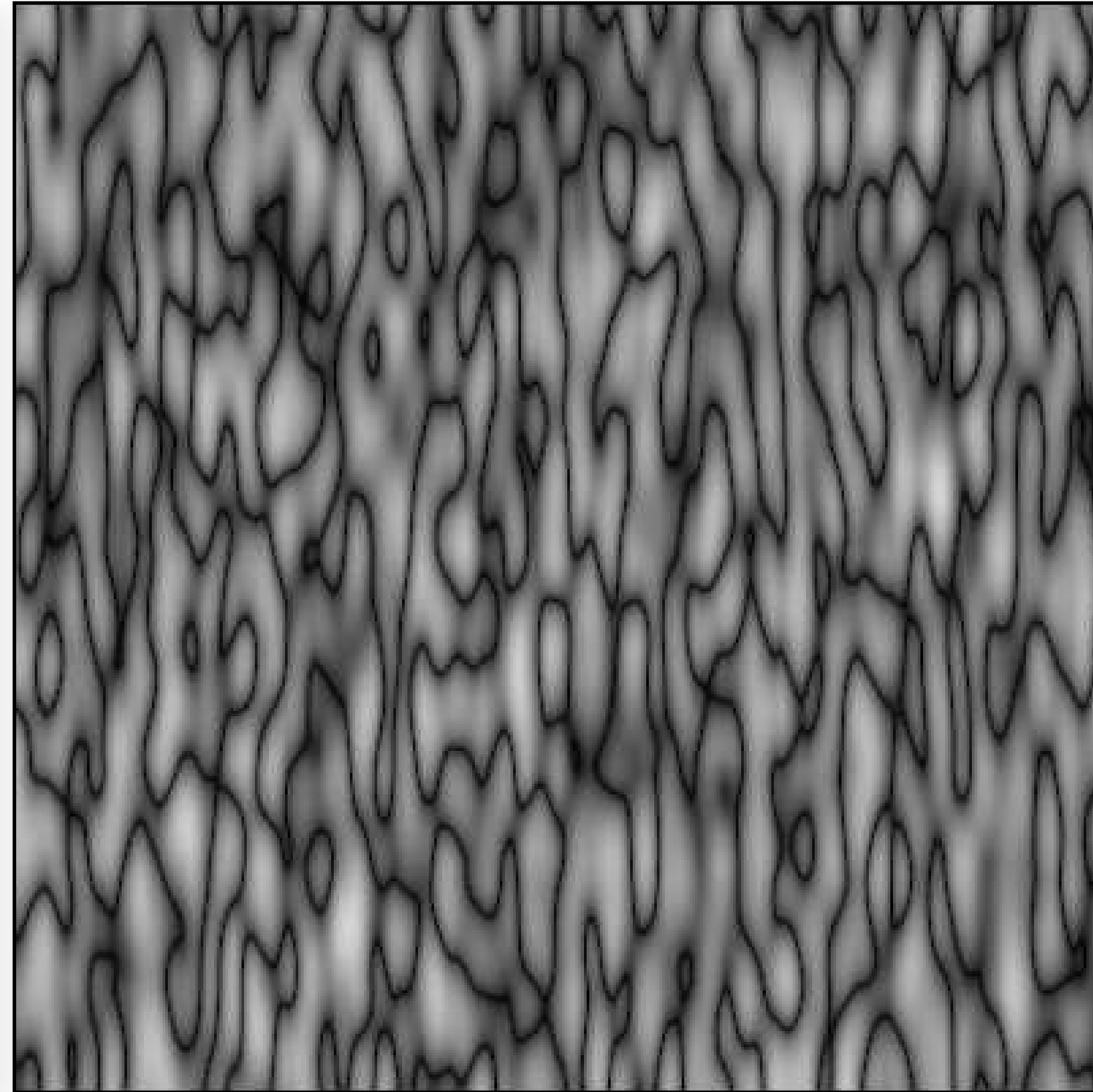
Absolute value of noise



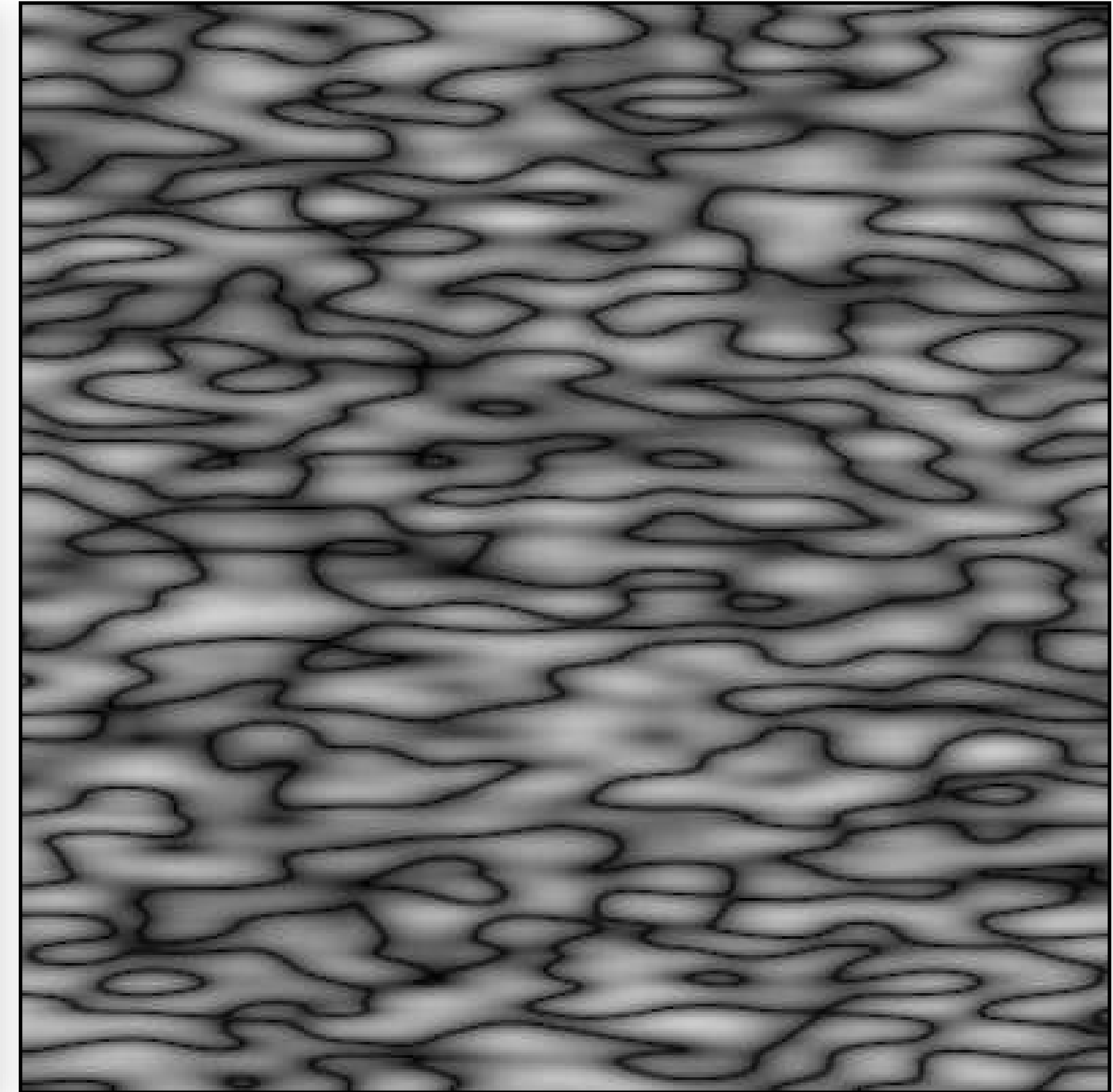
Absolute value of noise



$|\text{noise}(\mathbf{p})|$

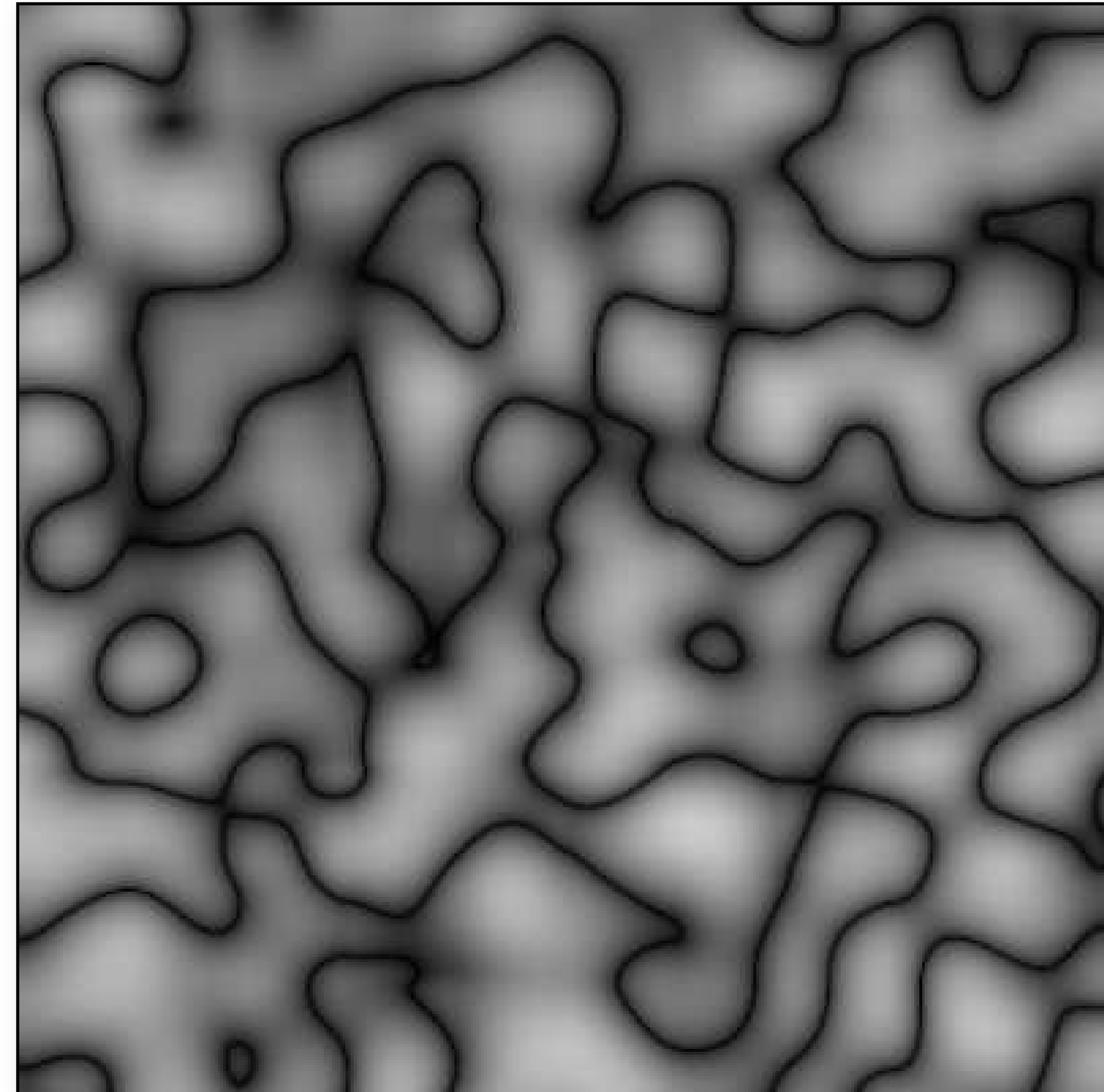


$|\text{noise}(4 \mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)|$



$|\text{noise}(\mathbf{p}_x, 4 \mathbf{p}_y, \mathbf{p}_z)|$

Perlin noise - limitations



Perlin noise - limitations

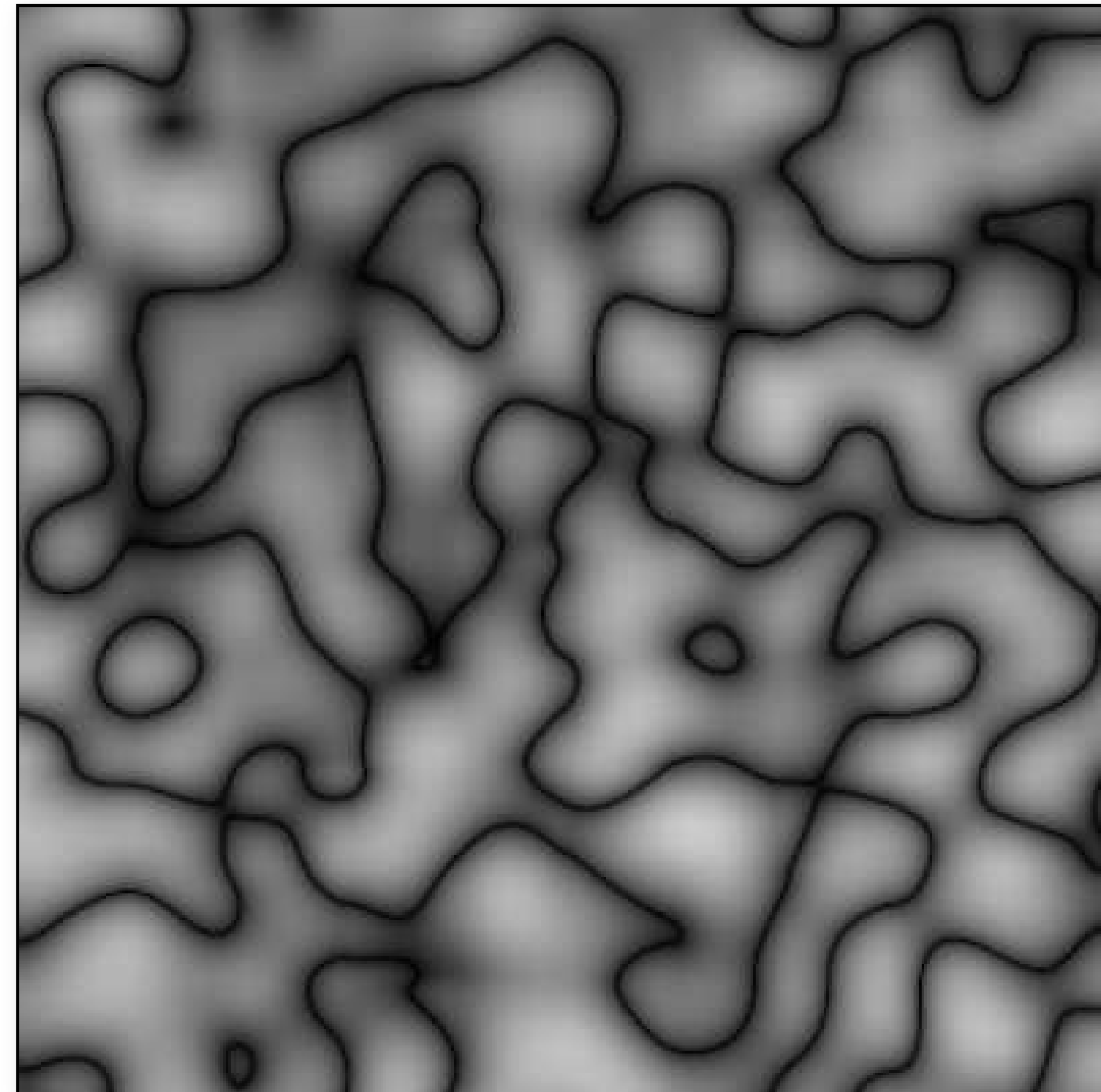
Lattice structure apparent for |noise|

- all lattice locations have value 0

Lookups faster, but still slow:

- Perlin is 2^n for n dimensions instead of 4^n for value noise
- other variations: simplex noise ($O(n)$)

Not quite rotation invariant



More reading

Fantastic explorable explanation by Andrew Kensler at Pixar

- eastfarthing.com/blog/2015-04-21-noise

Spectral synthesis

Representing a complex function $f_s(\mathbf{p})$ by a sum of weighted contributions from a scaled function $f(\mathbf{p})$:

$$f_s(\mathbf{p}) = \sum_i w_i f(s_i \mathbf{p})$$

Called a “fractal sum” if w_i and s_i are set so:

- increasing frequencies have decreasing amplitude,
e.g.: $w_i = 2^{-i}$, $s_i = 2^i$
- when $s_i = 2^i$, each term in summation is called an “octave”

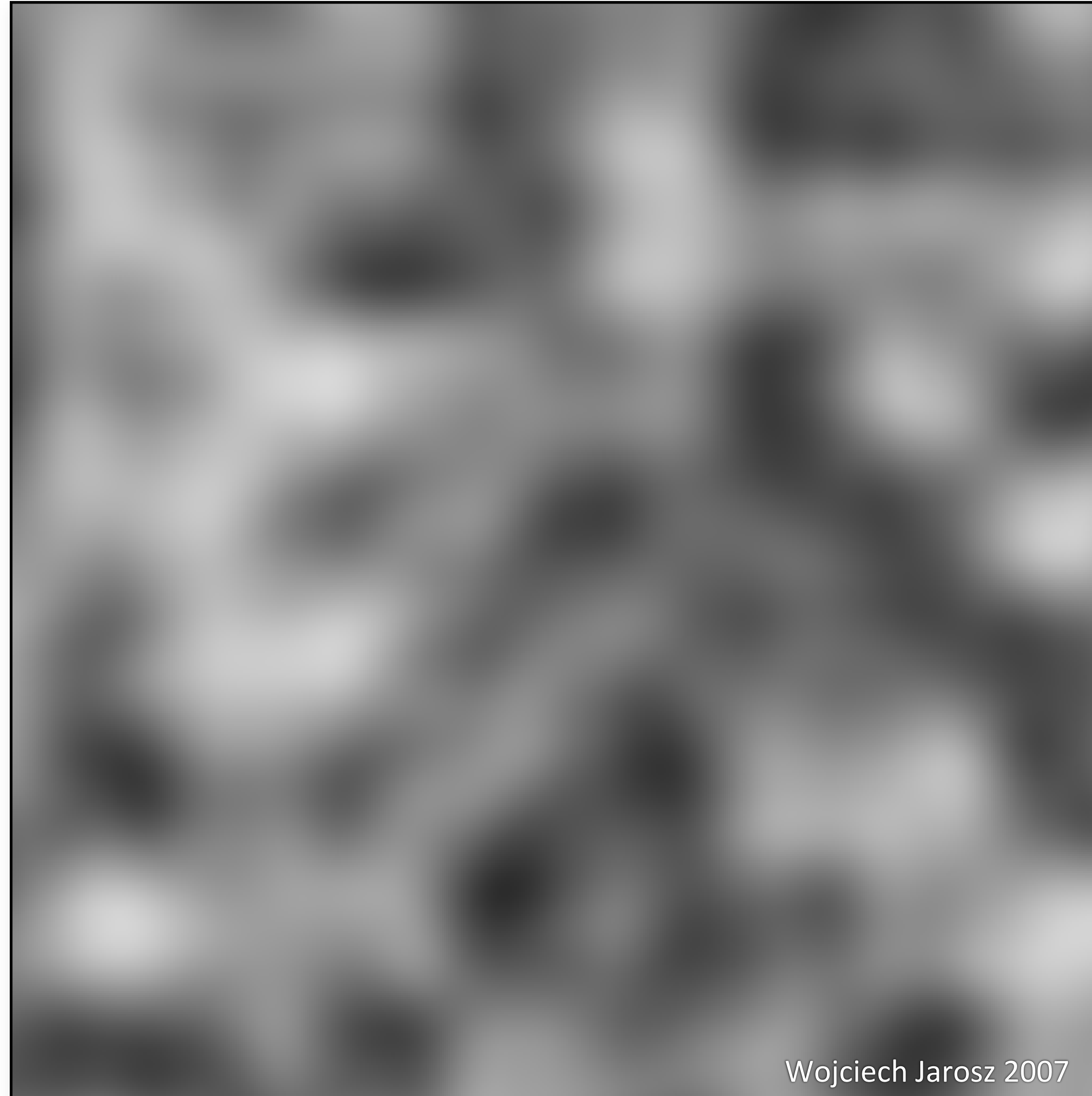
What function $f(\mathbf{p})$ should we use?

fBm - fractional Brownian motion

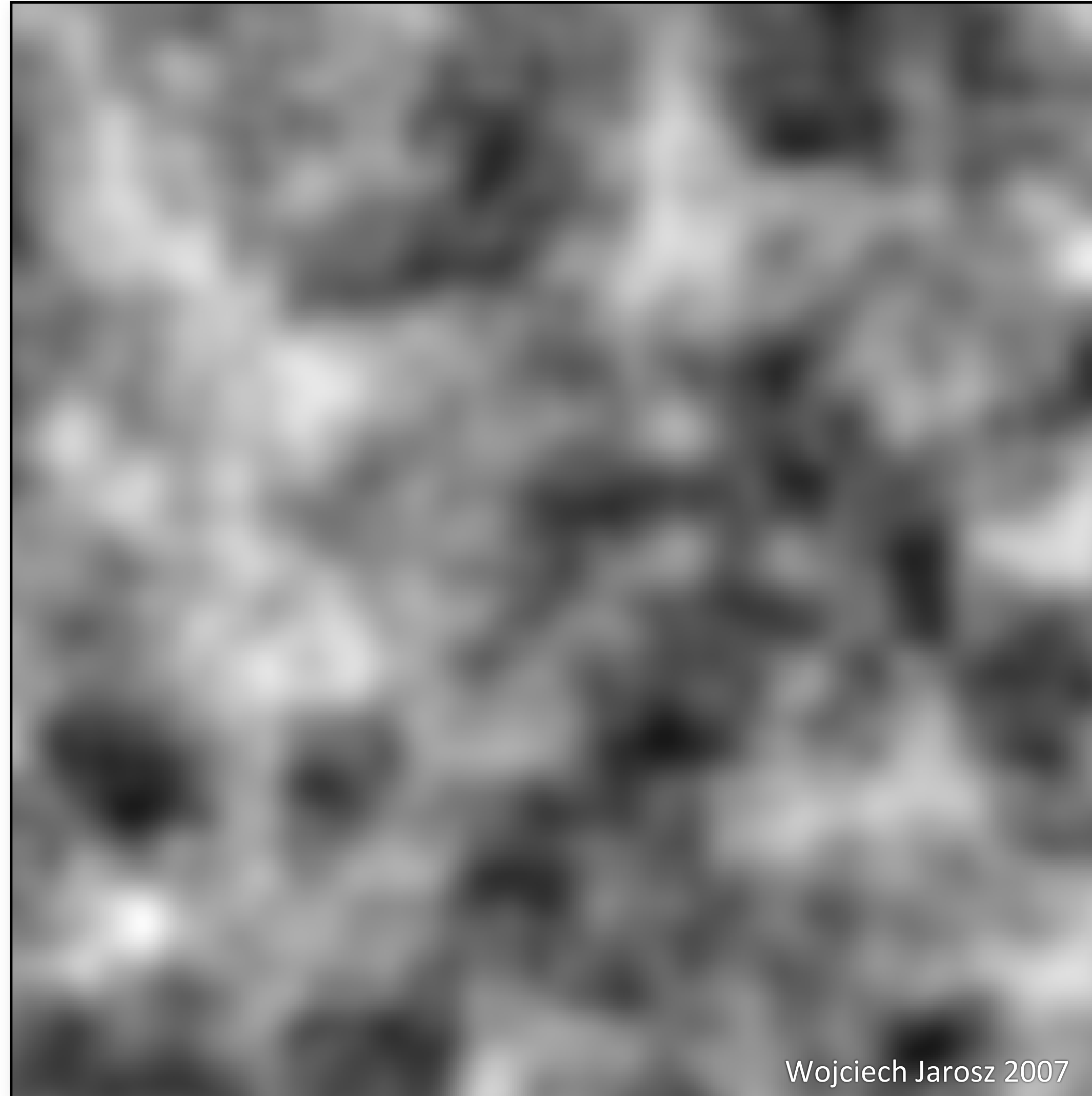
In graphics:

- Fractal sum of Perlin noise functions
- “Fractal noise”

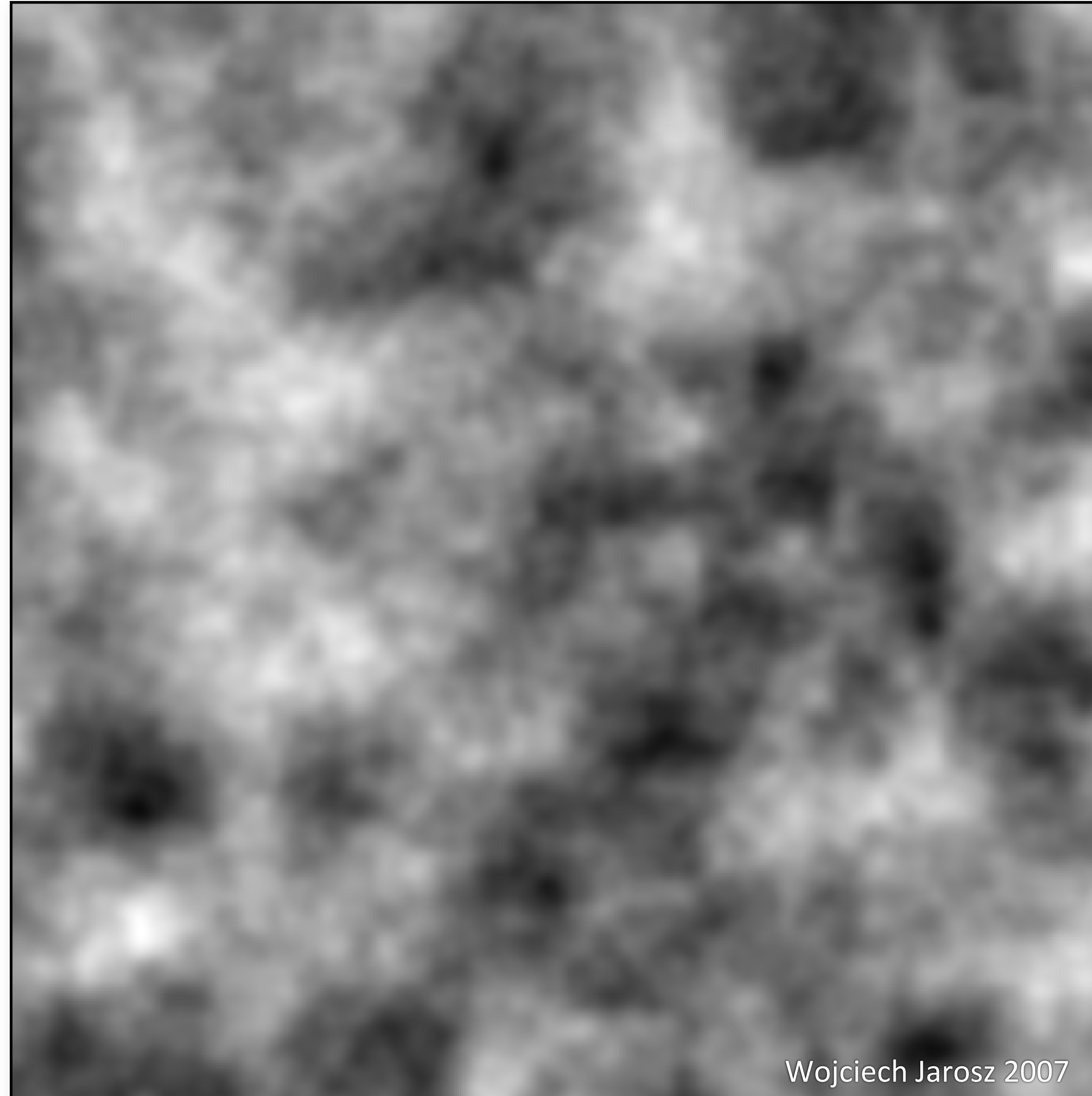
fBm - 1 octave



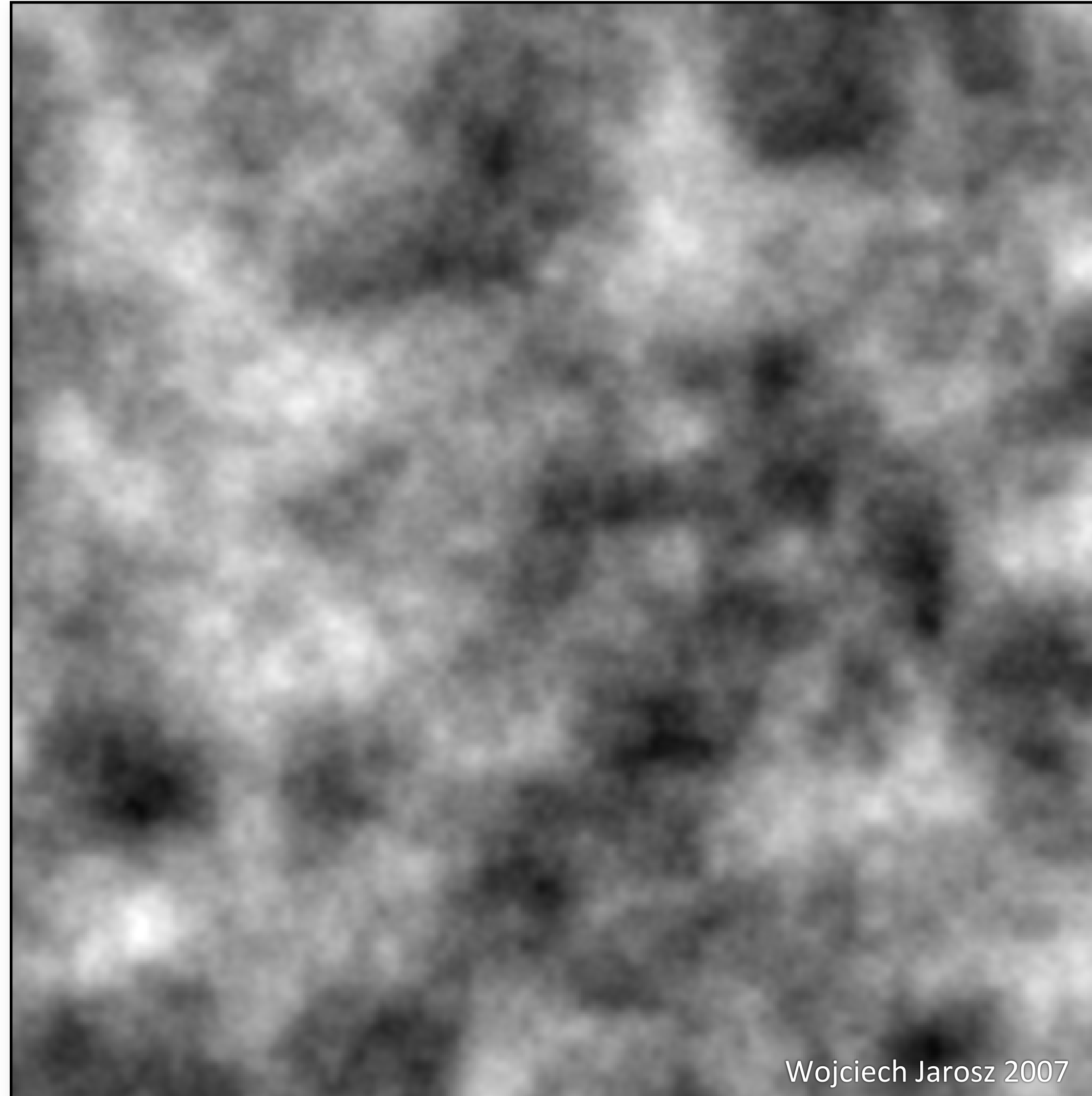
fBm - 2 octaves



fBm - 3 octaves



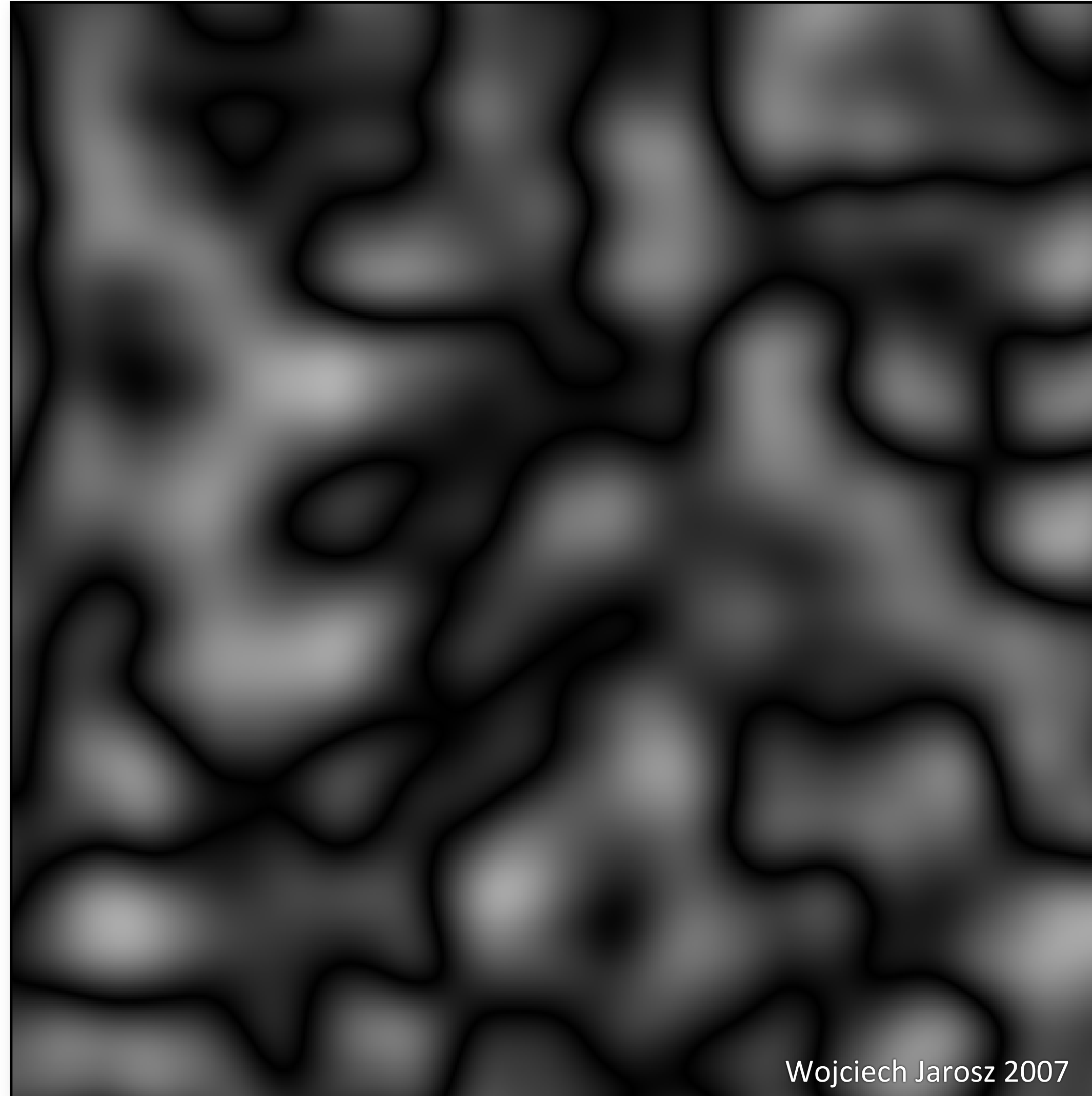
fBm - 4 octaves



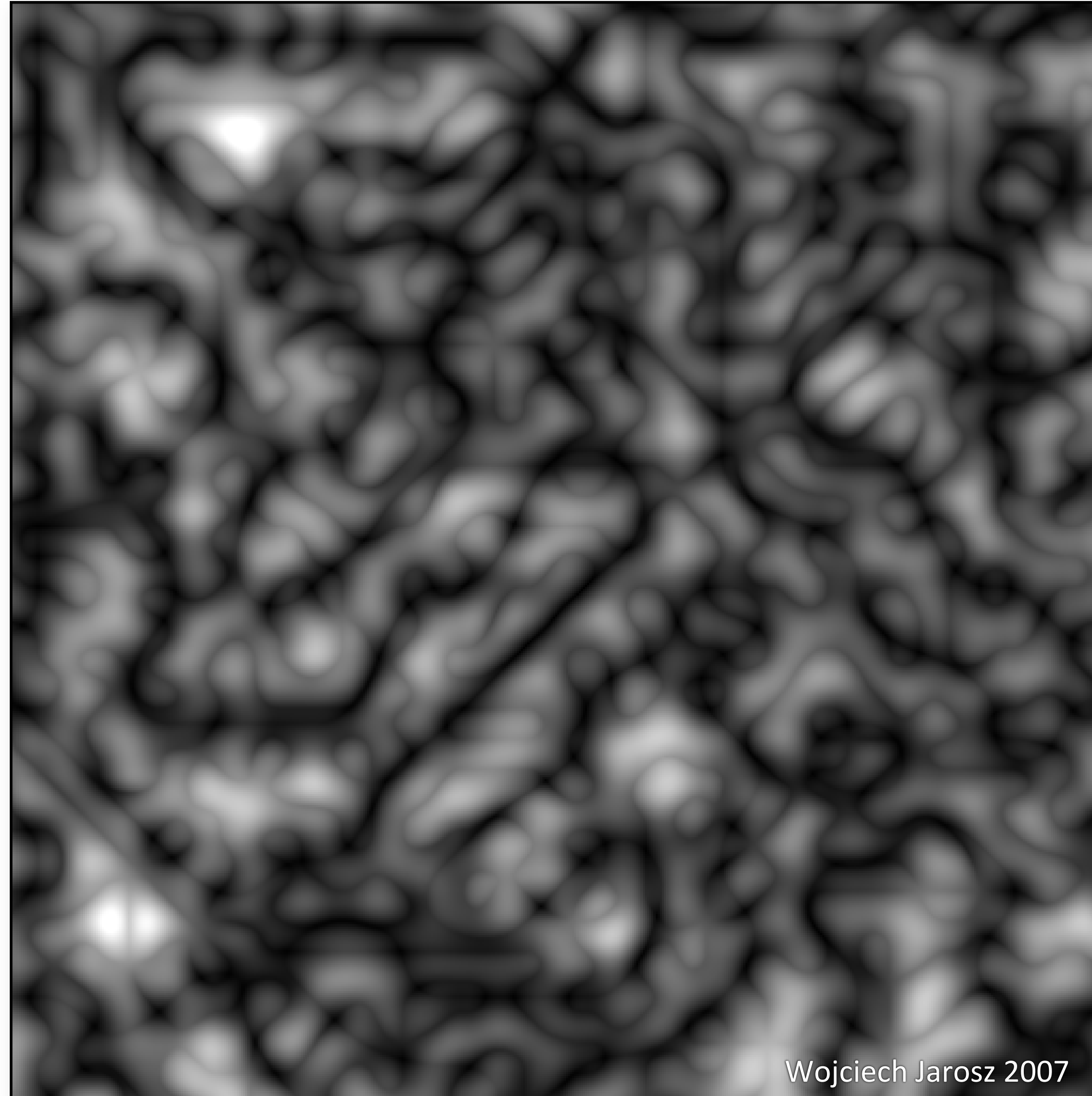
Turbulence

Same as fBm, but sum absolute value of noise function

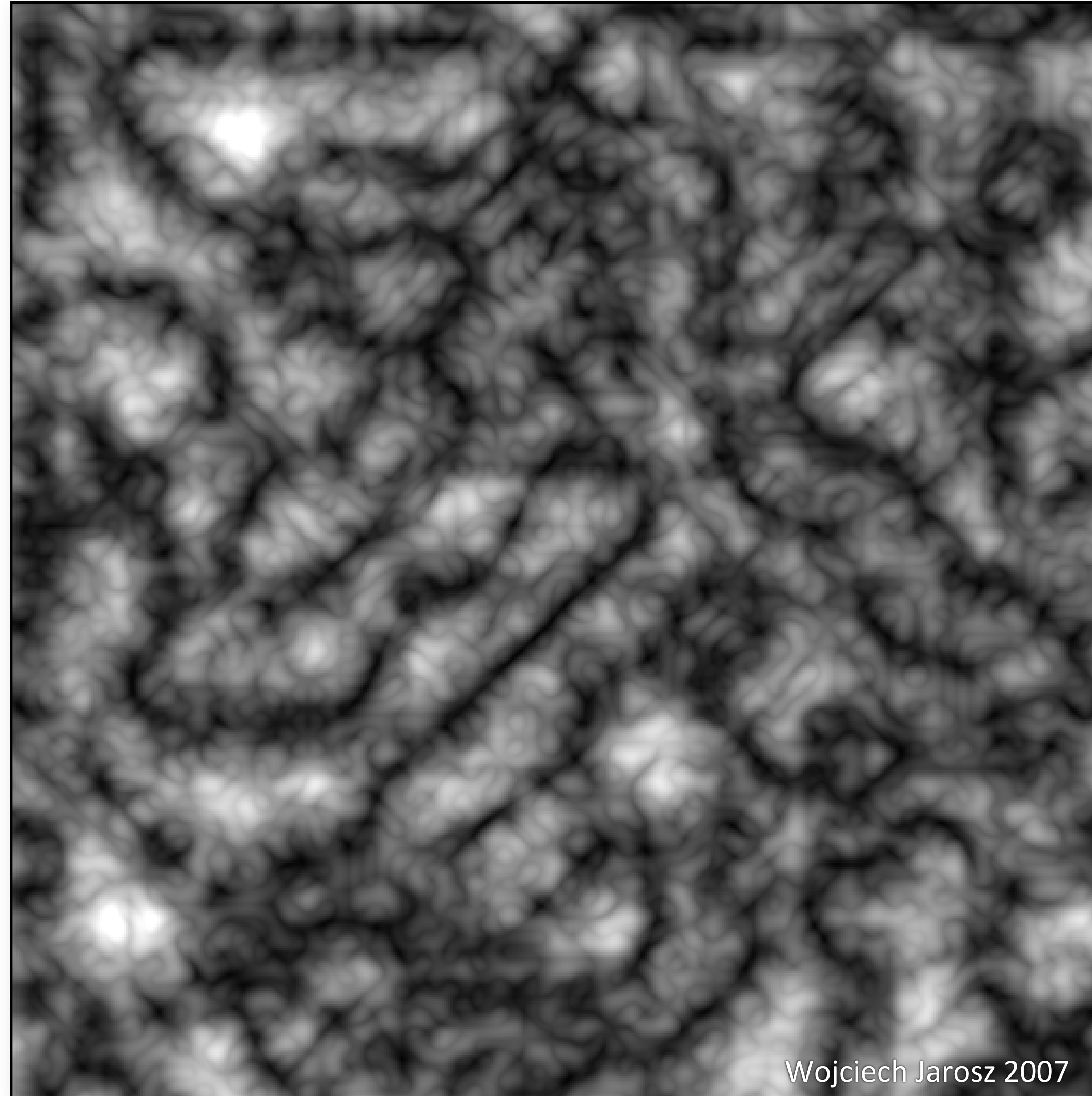
Turbulence - 1 octave



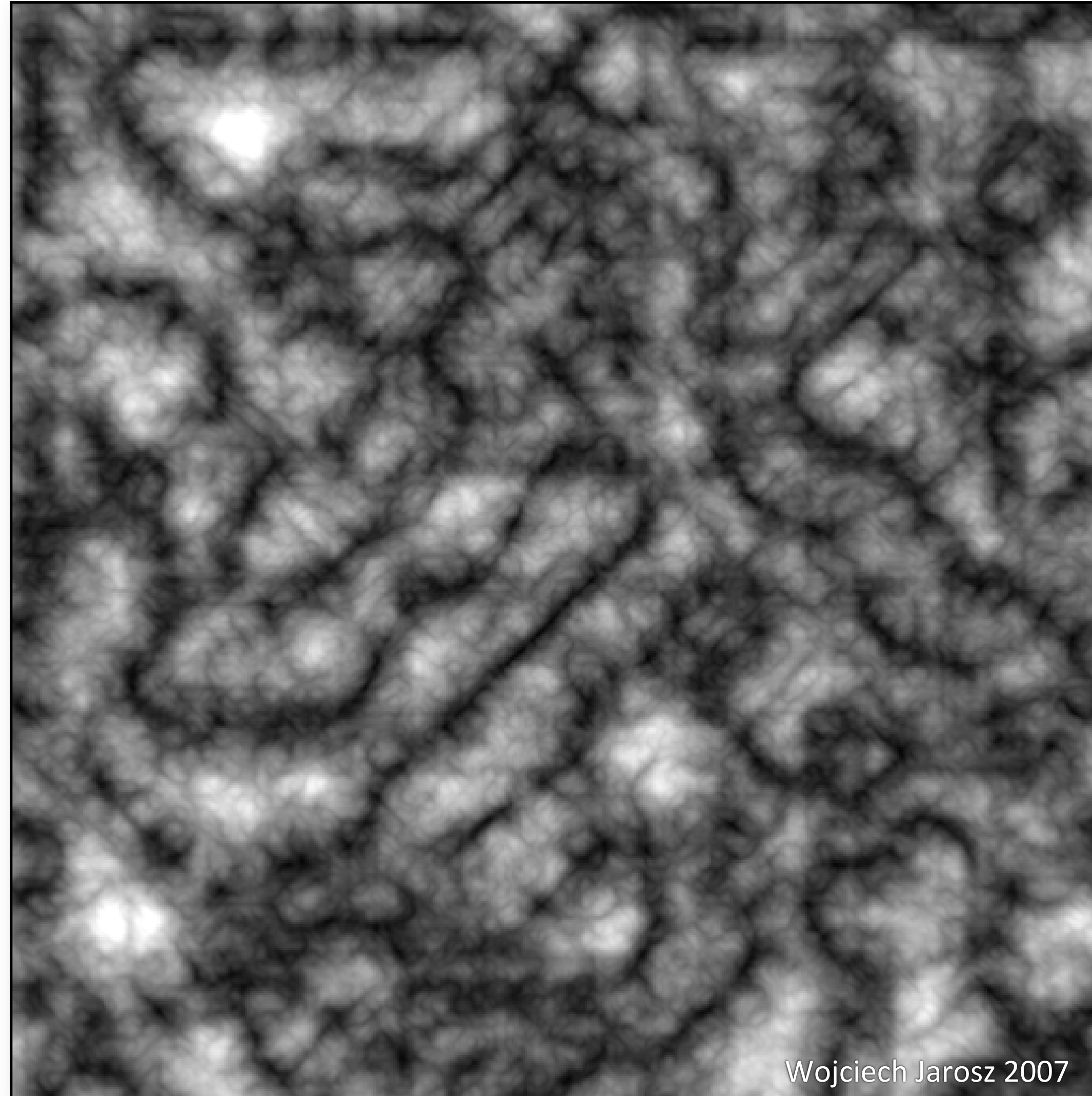
Turbulence - 2 octaves



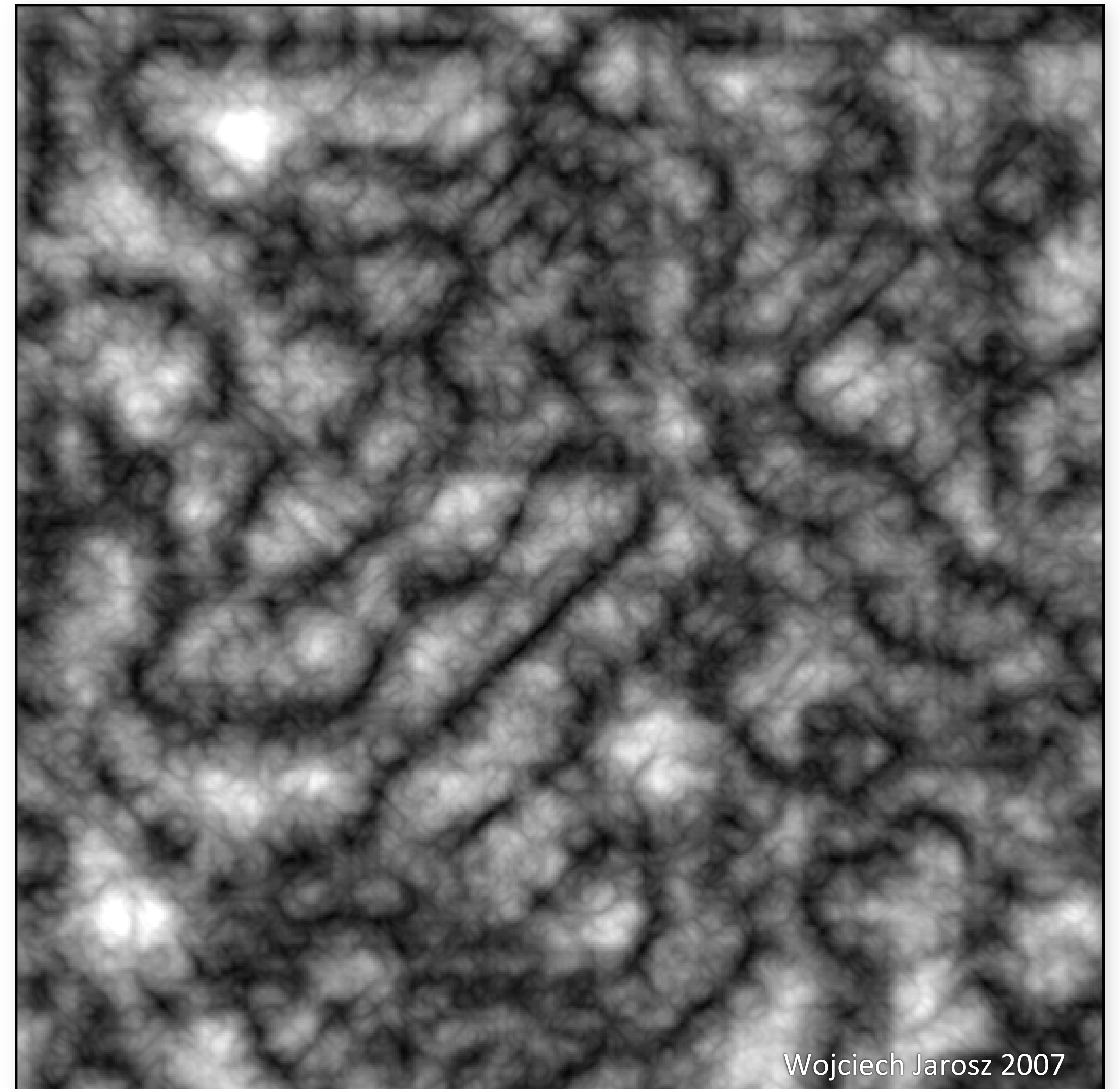
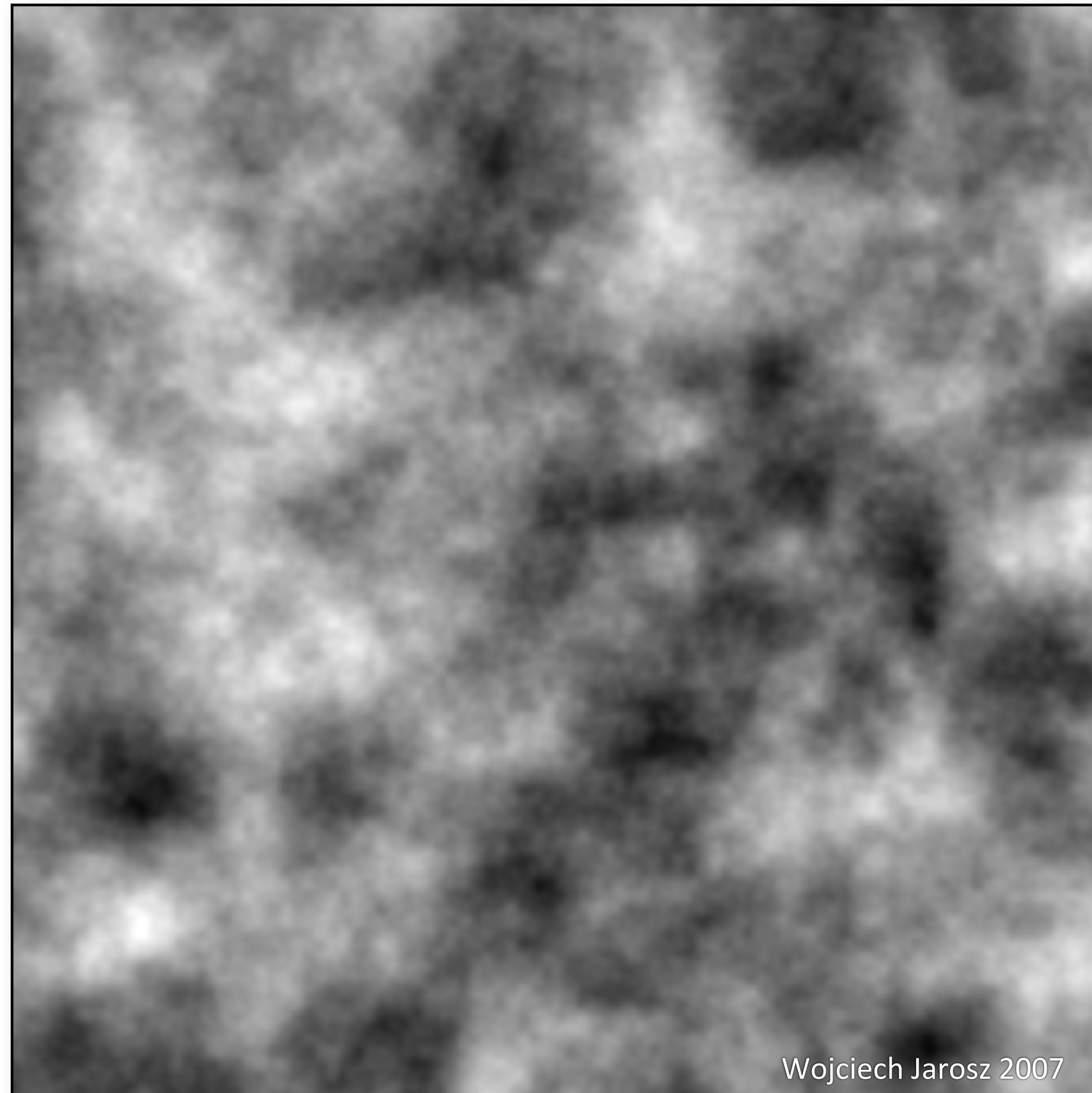
Turbulence - 3 octaves



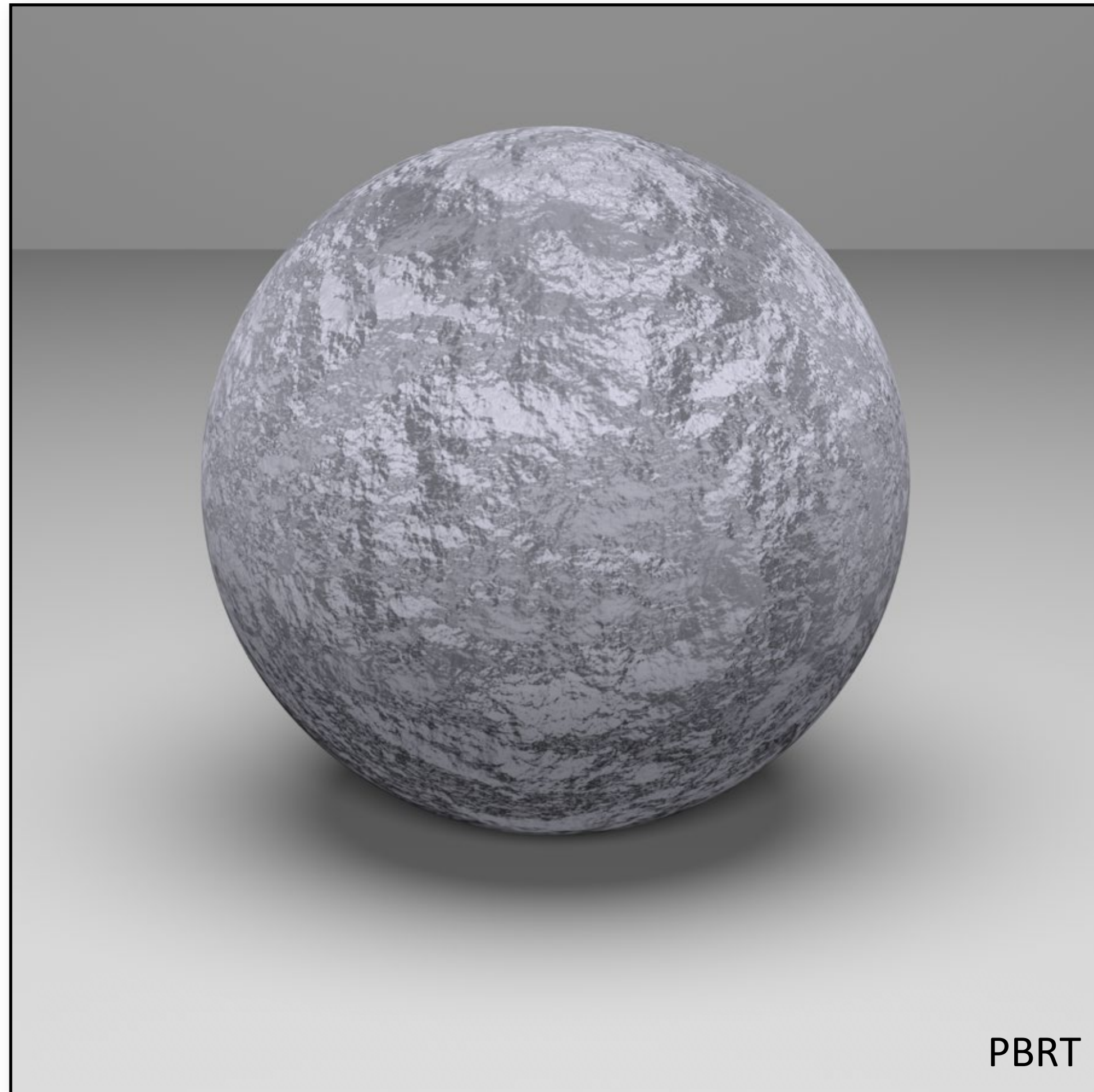
Turbulence - 4 octaves



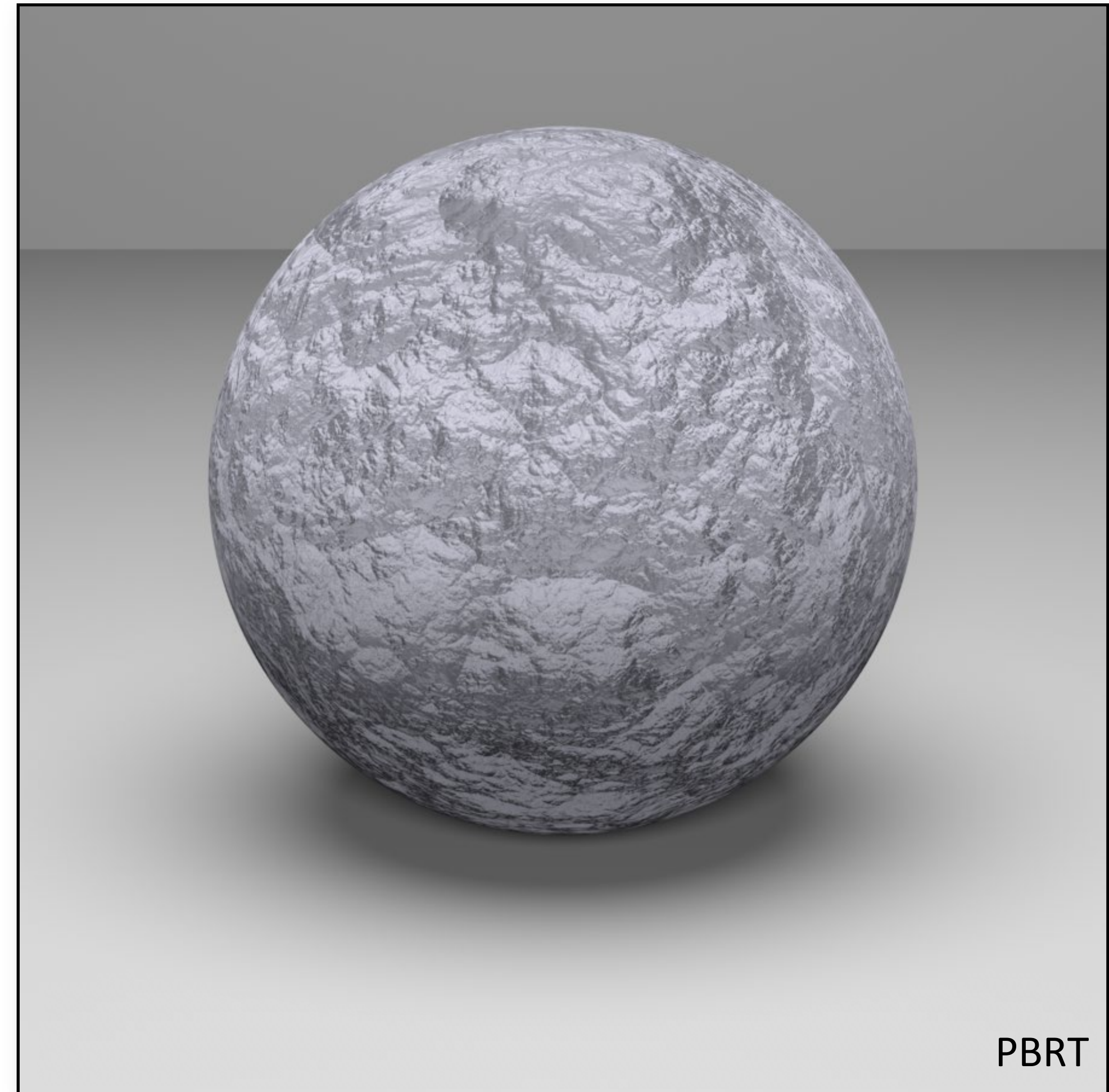
fBm vs Turbulence



Bump mapping

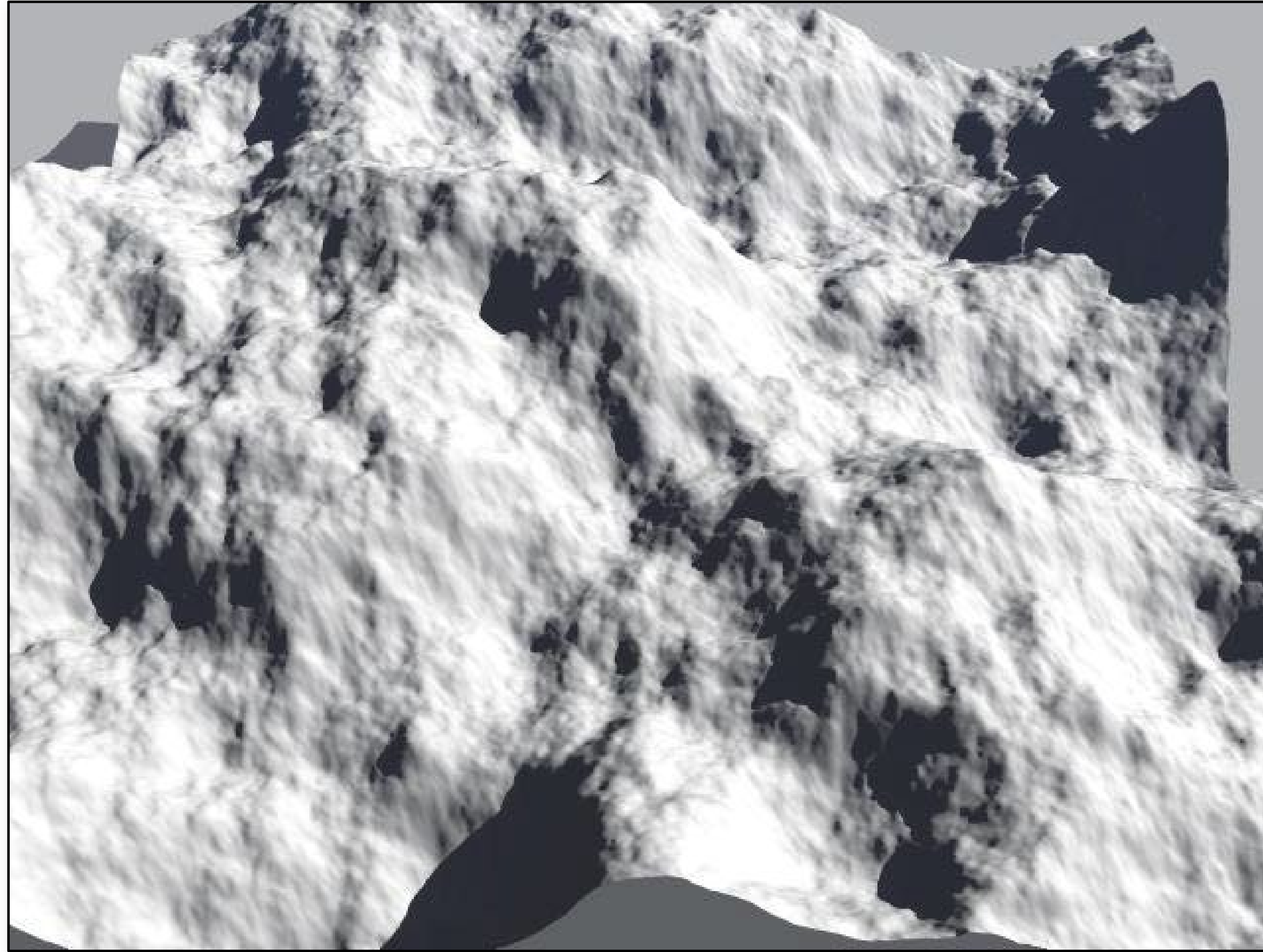


fBm



Turbulence

2D fBm



A fractional Brownian motion (fBm) terrain patch of fractal dimension ~ 2.1 .

Fractal dimension

Fractals have *fractional* dimension, e.g. $D = 1.2$.

- under some appropriate definition of dimension...

Integer component indicates the underlying Euclidean dimension of the fractal, in this case a line ("1" in 1.2).

Fractional part is called the fractal increment (".2" in 1.2).

Fractal increment varies from .0 to .999...

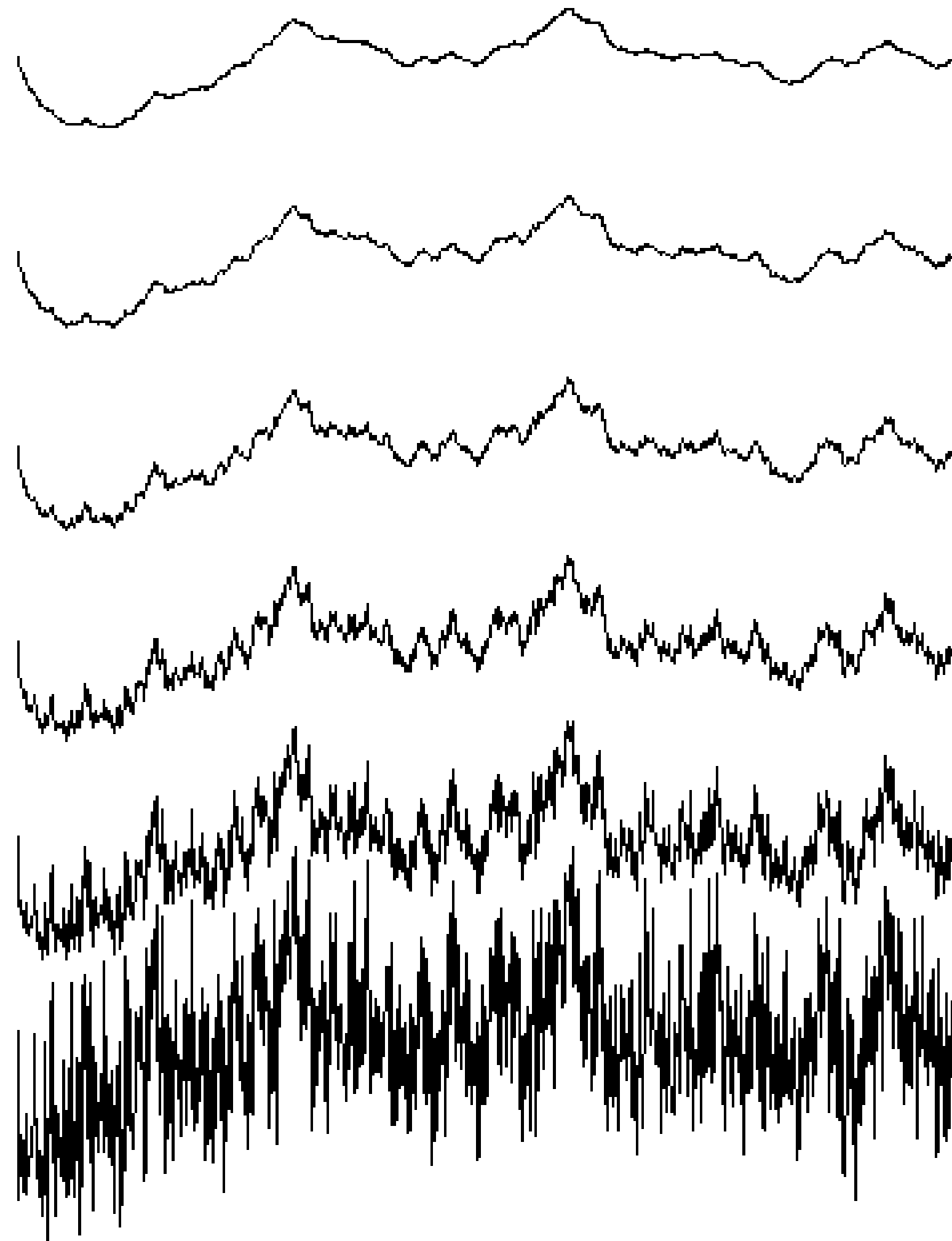
- fractal goes from (locally) occupying only its underlying Euclidean dimension (the line), to filling some part of the next higher dimension (the plane).

Continuous "slider" for the visual complexity of a fractal

- "smoother" \Leftrightarrow "rougher"

What determines the dimension of fBm?

Fractal dimension of fBm



Traces of fBm for H varying from 1.0 to 0.0 in increments of 0.2

fBm

fBm is statistically homogeneous and isotropic.

- Homogeneous means "the same everywhere"
- Isotropic means "the same in all directions"

Fractal phenomena in nature are rarely so simple and well-behaved.

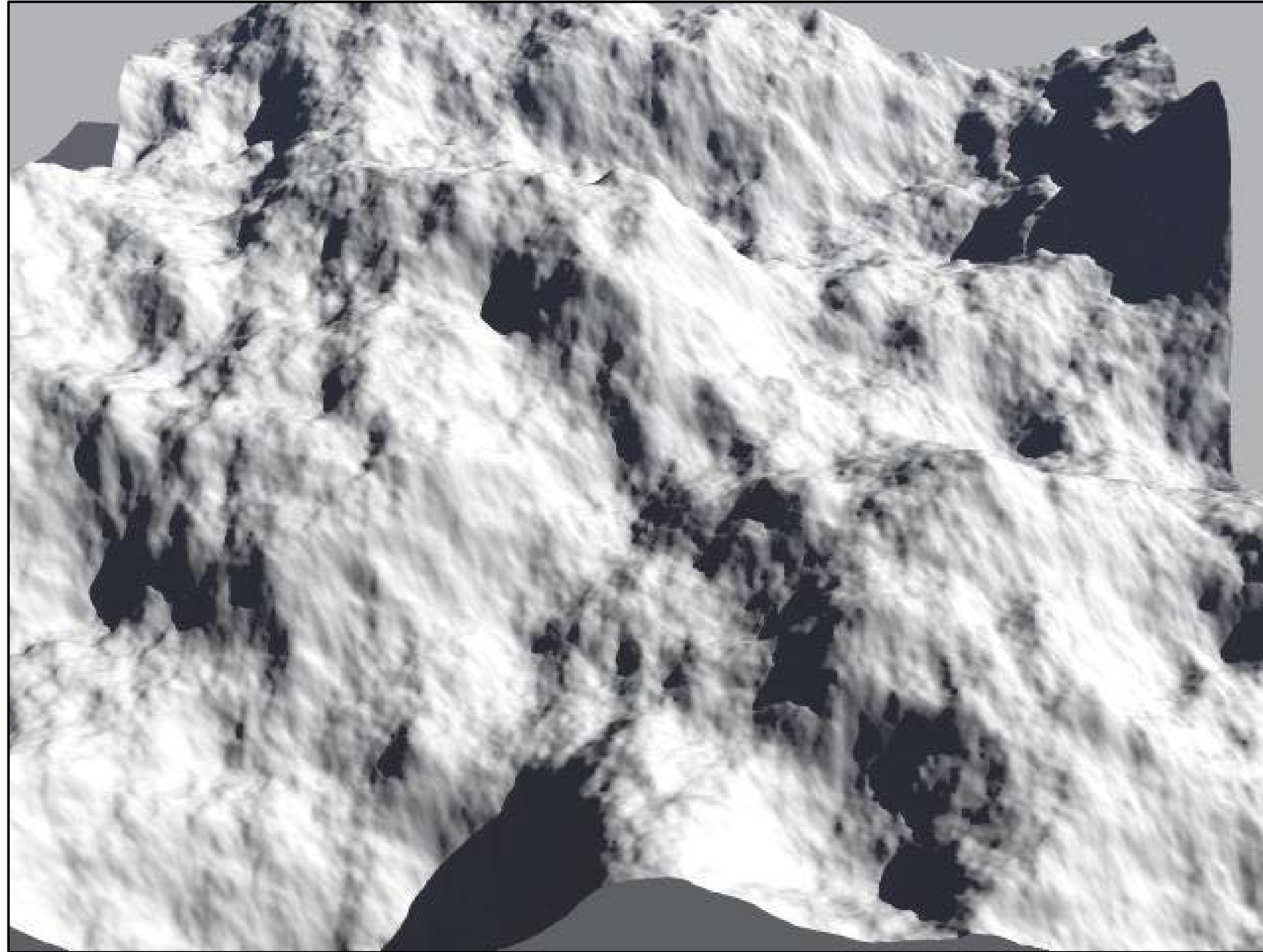
Multifractals

Fractal system which has a different fractal dimension in different regions

Heterogeneous fBm

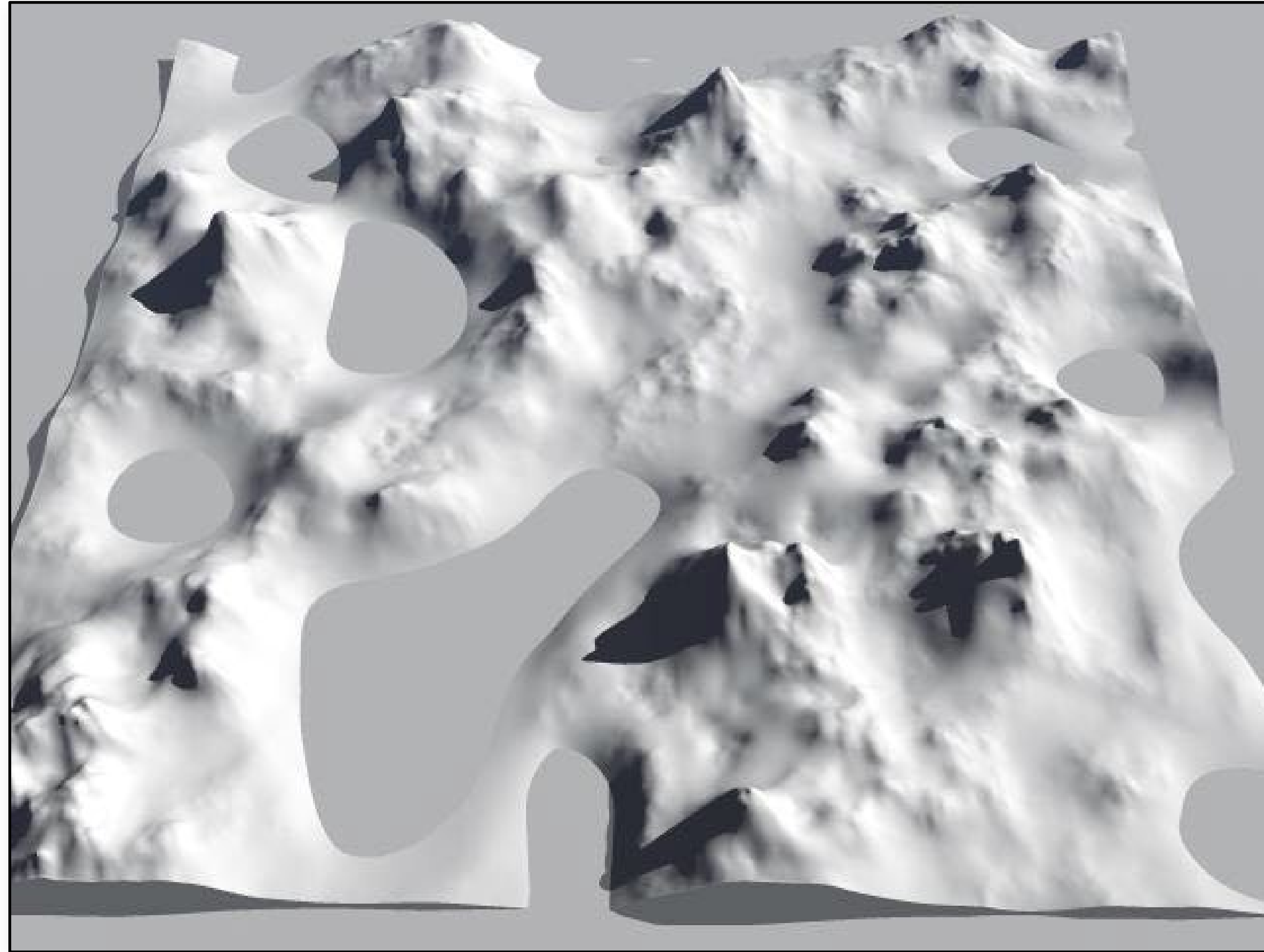
- Scale higher frequencies in the summation by the value of the previous frequency.
- Many possibilities: hetero terrain, hybrid multifractal, ridged multifractal

2D fBm



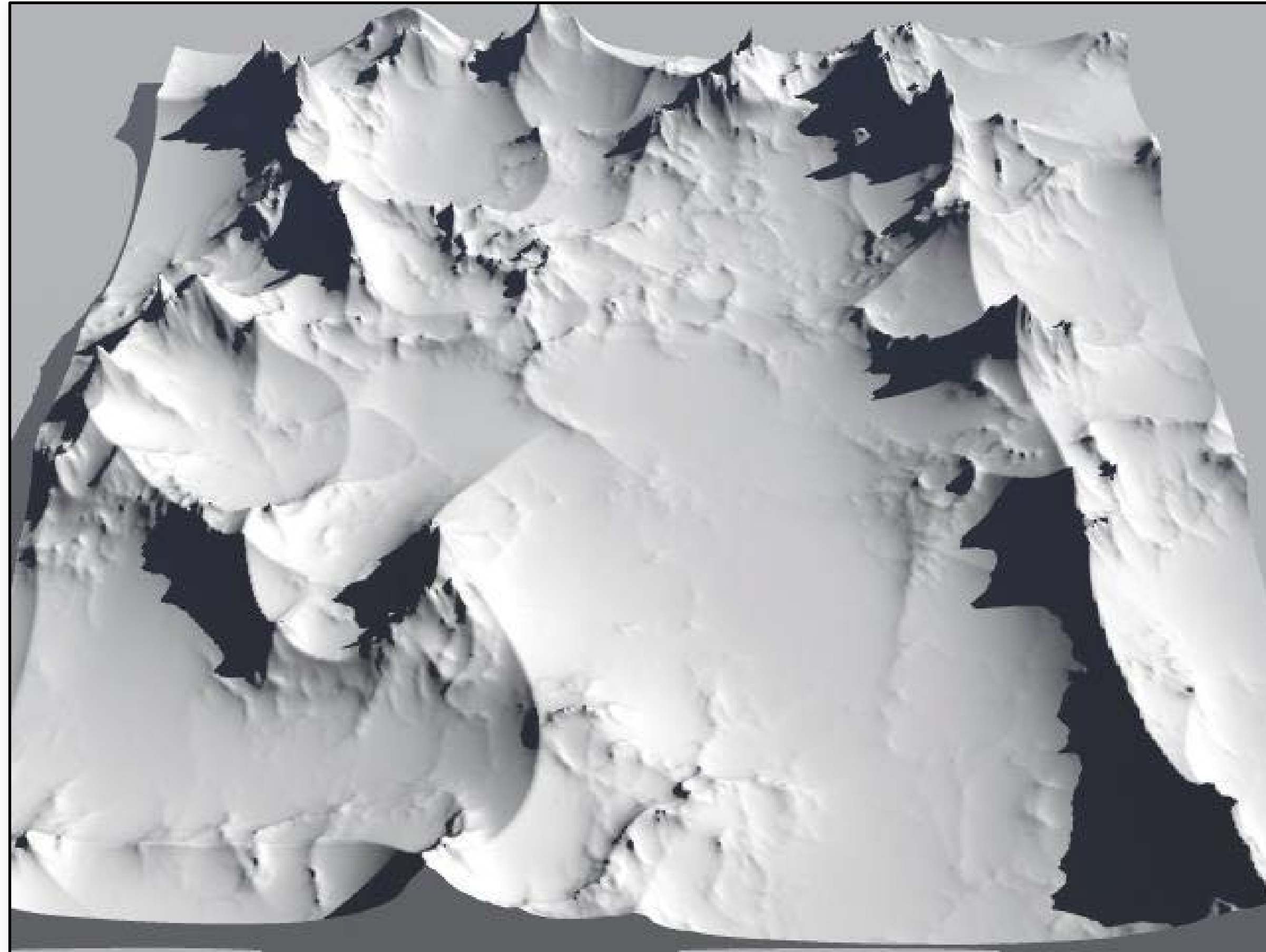
A fractional Brownian motion (fBm) terrain patch of fractal dimension ~ 2.1 .

Heterogeneous fBm



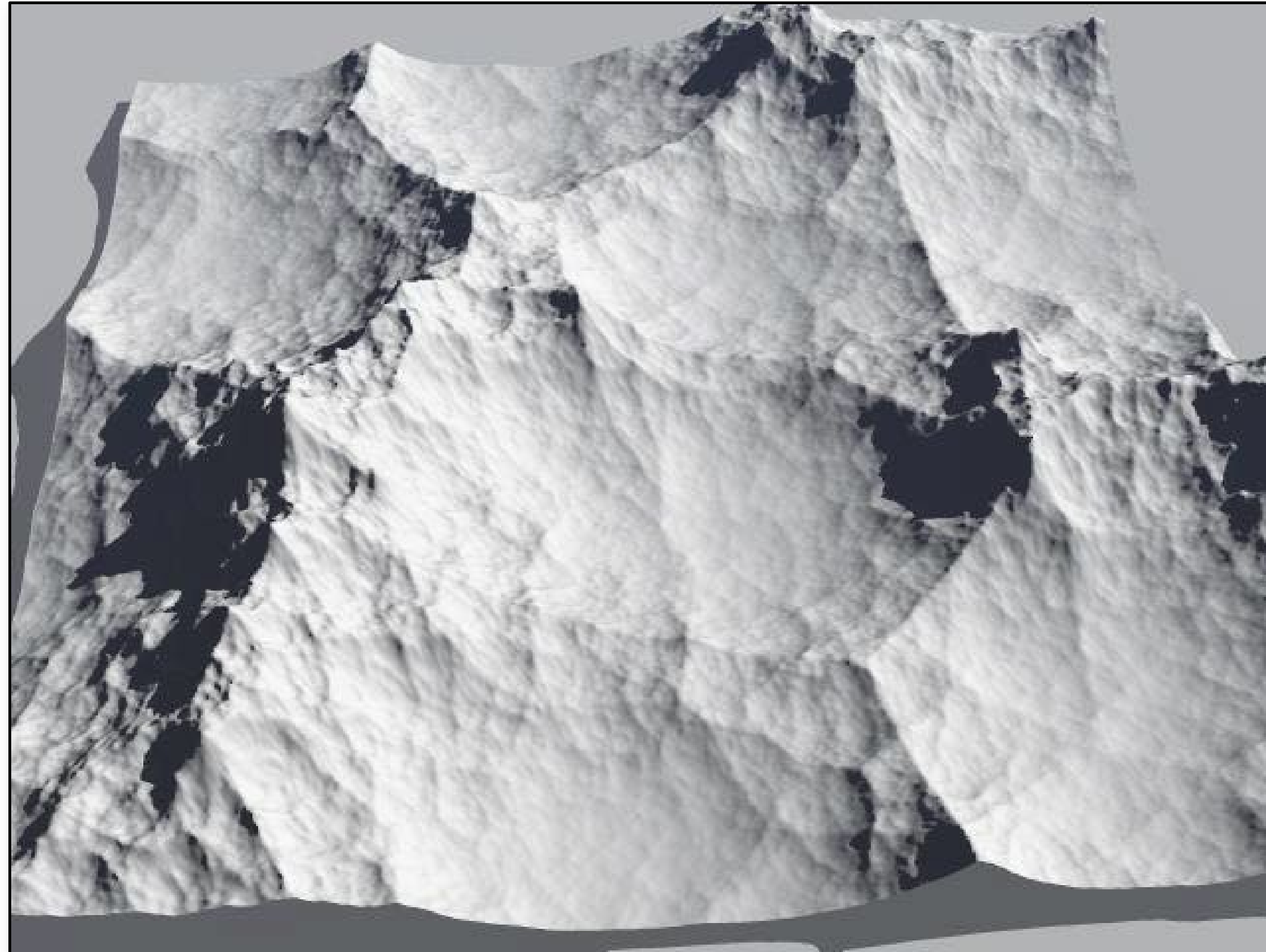
A hybrid multifractal terrain patch made with a Perlin noise basis: the “alpine hills” Bryce 4 terrain model.

Heterogeneous fBm



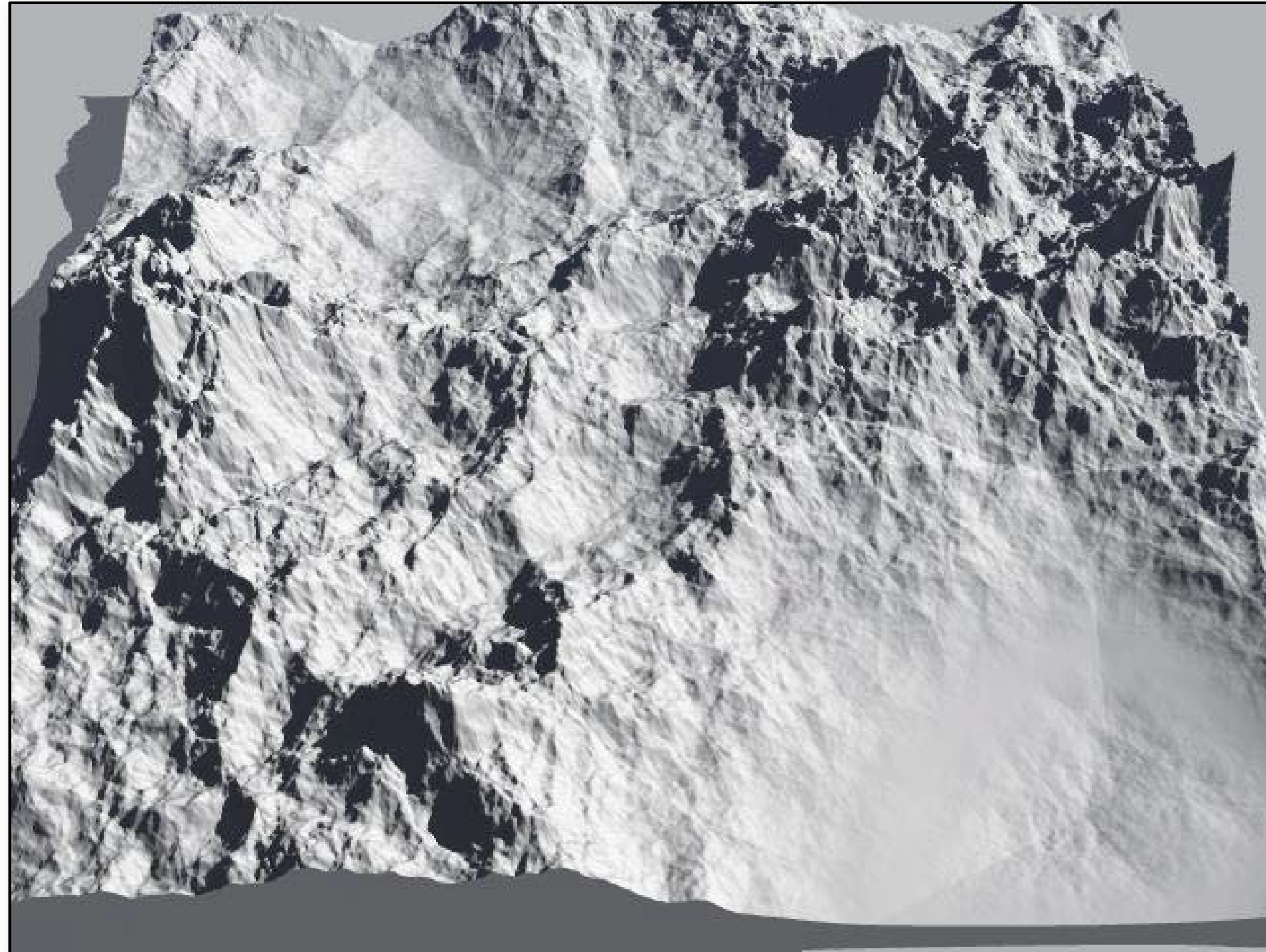
The “ridges” terrain model from Bryce 4: a hybrid multifractal made from one minus the absolute value of Perlin noise.

Heterogeneous fBm



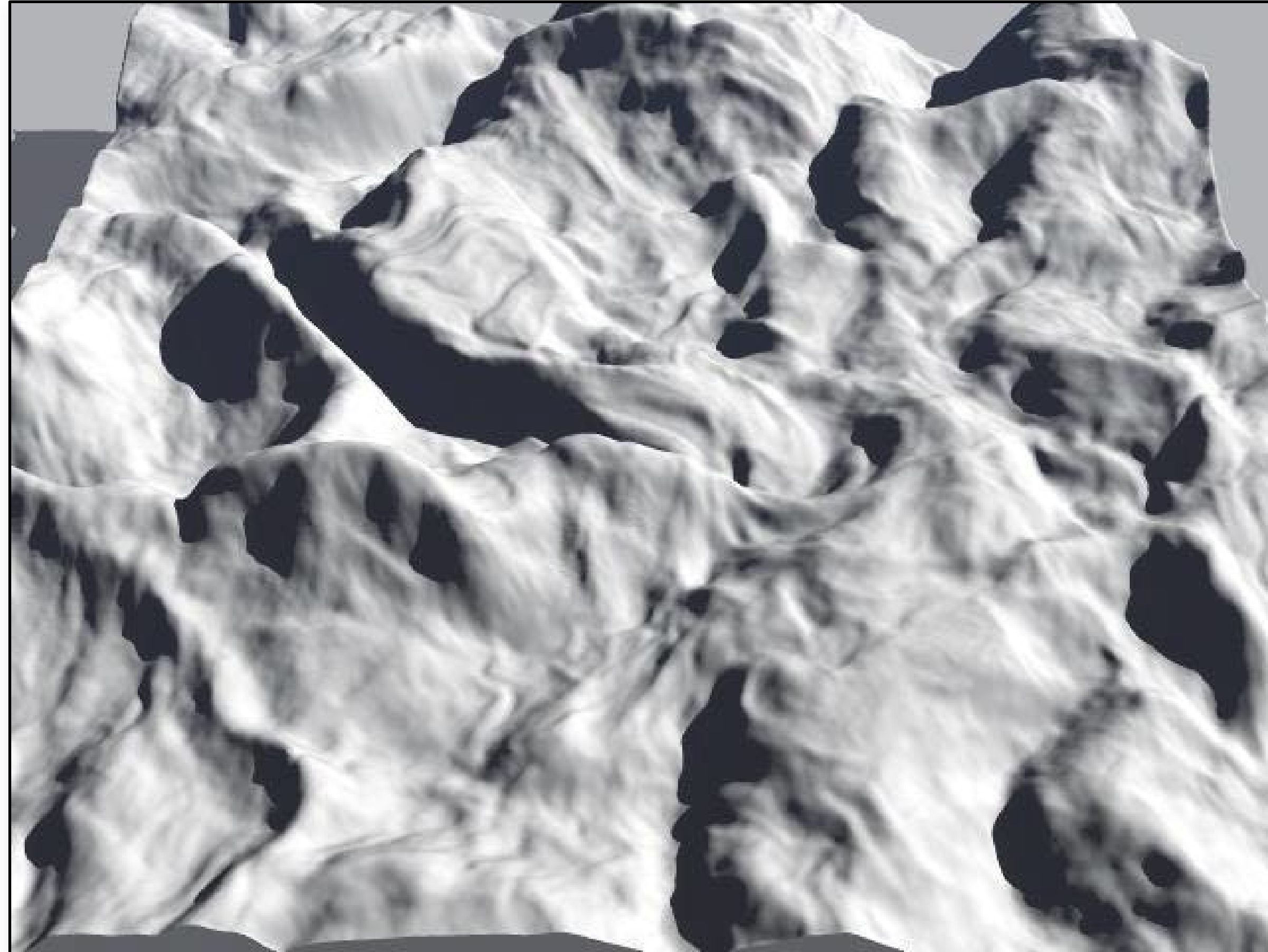
A hybrid multifractal made from Worley's Voronoi distance-squared basis

Heterogeneous fBm



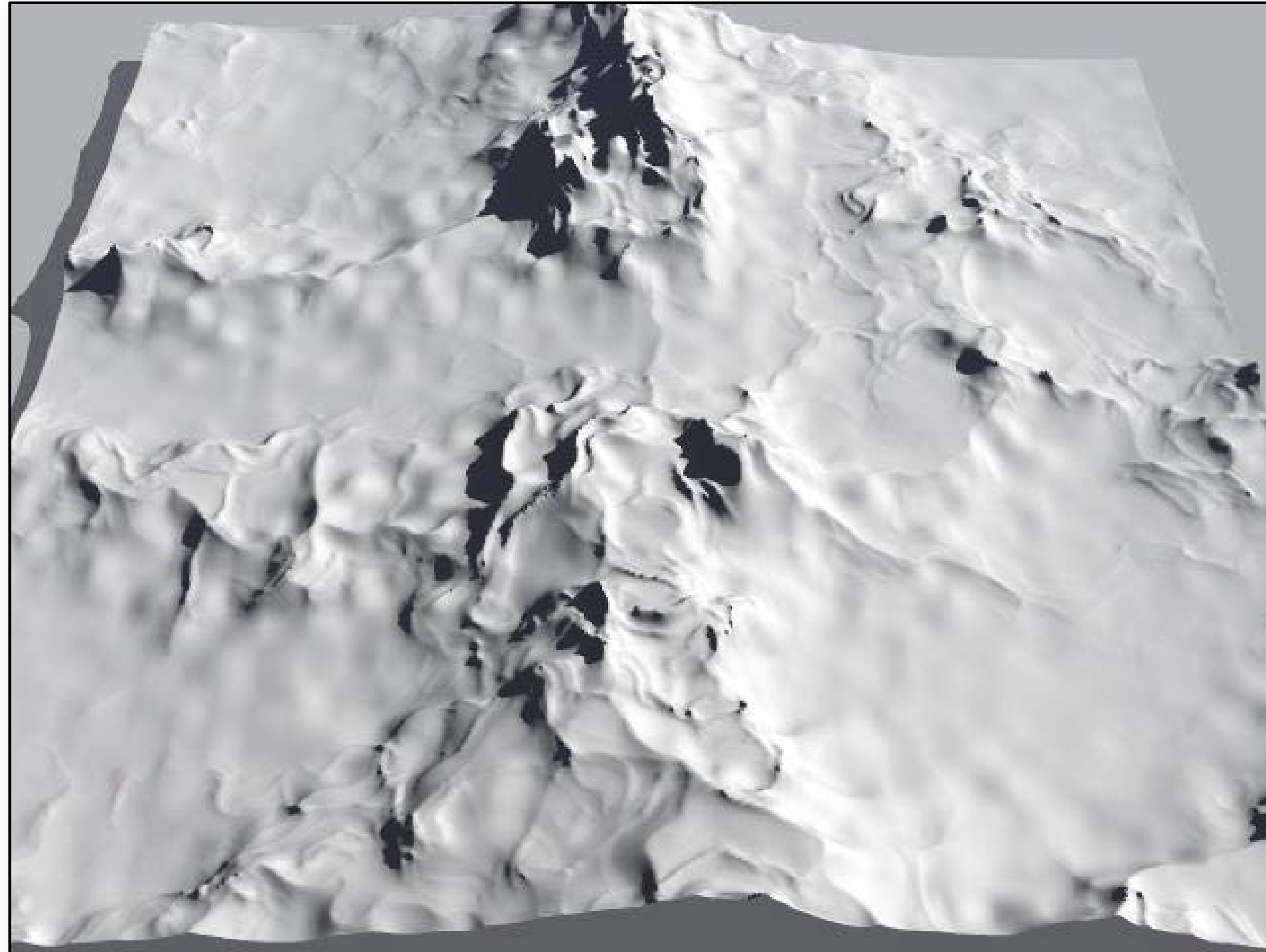
A hybrid multifractal made from Worley's Voronoi distance basis

Domain Distortion



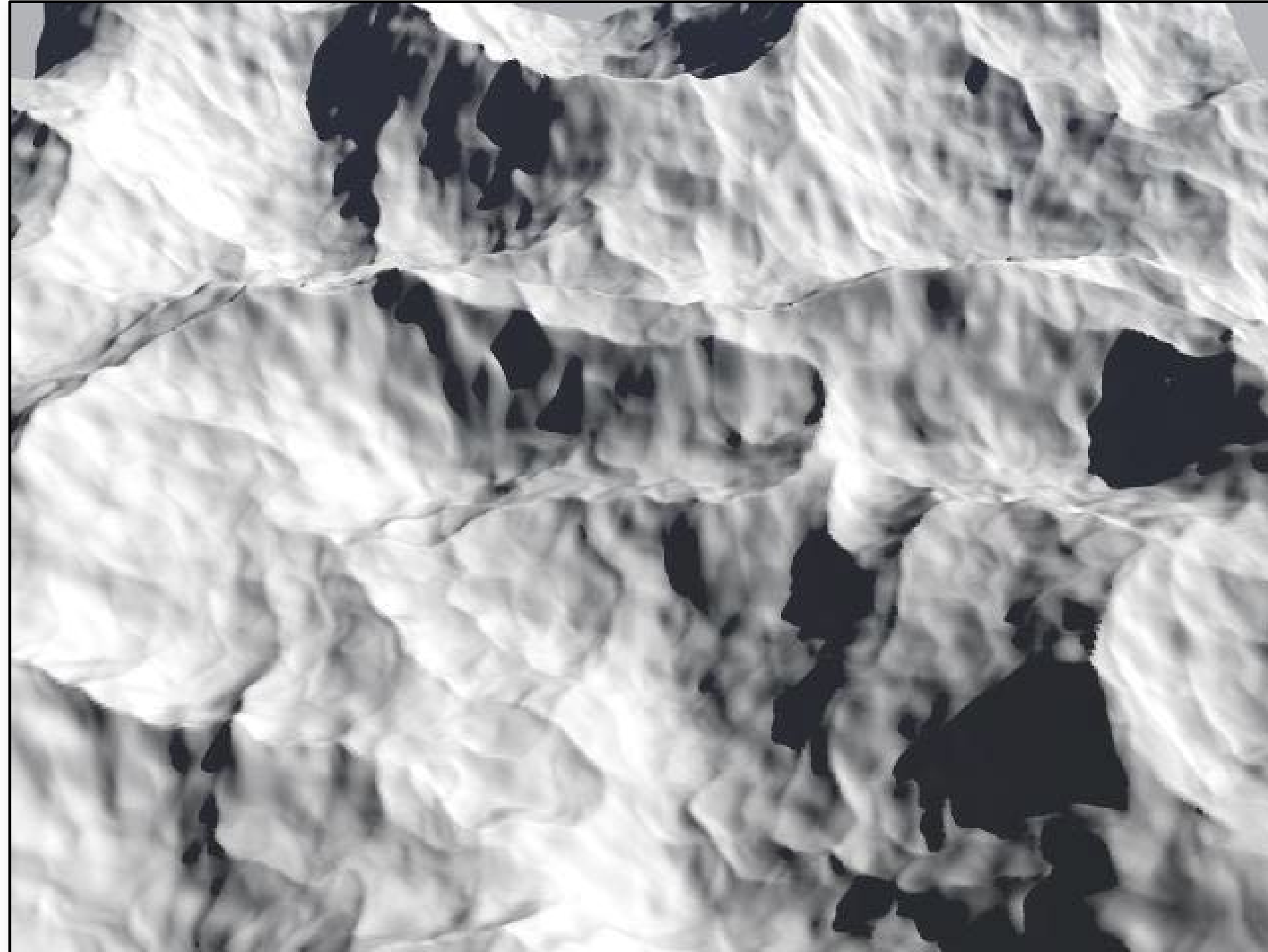
fBm distorted with fBm

Domain Distortion



A sample of the “warped ridges” terrain model in Bryce 4: the “ridges” model distorted with fBm.

Domain Distortion

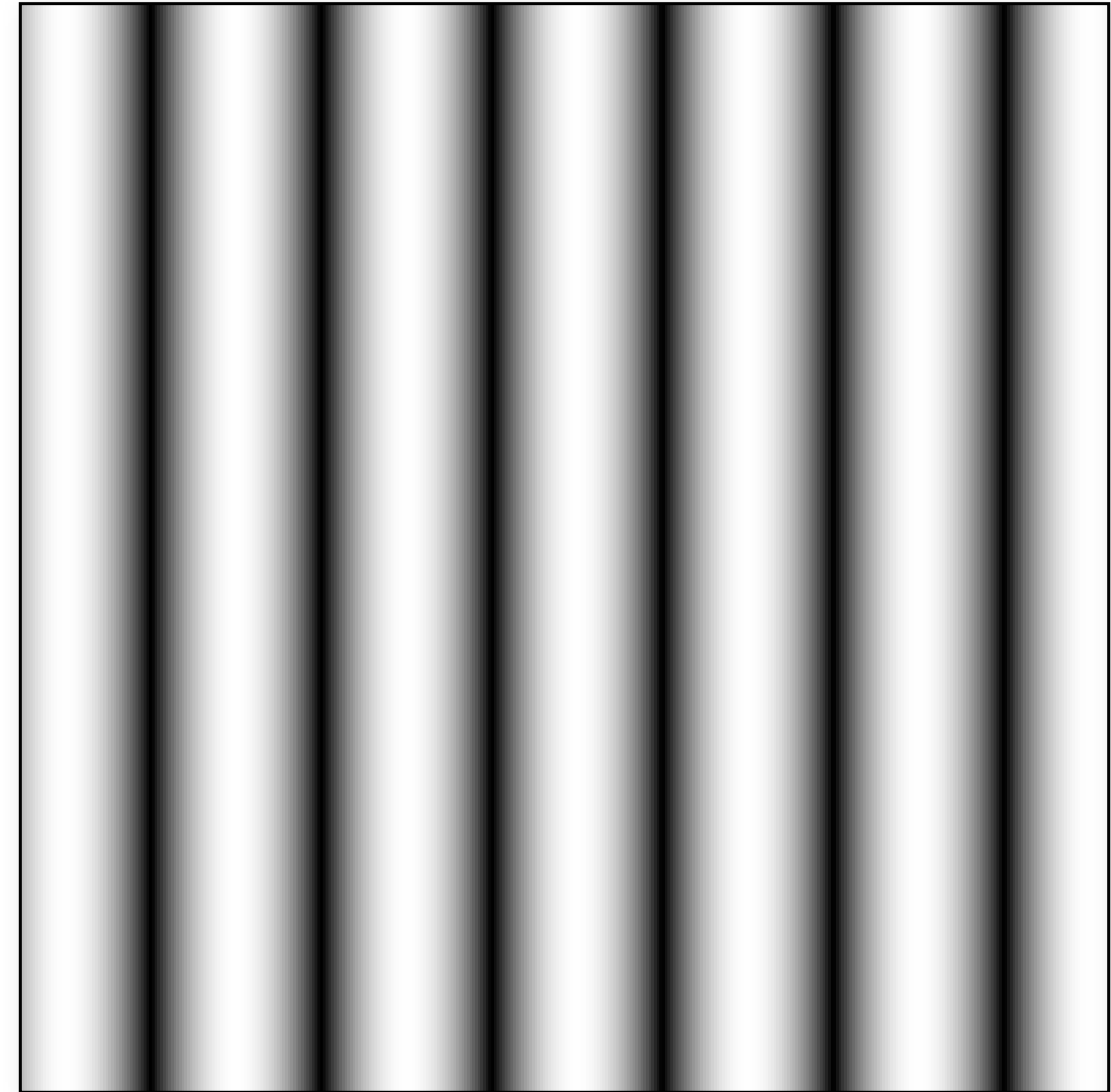


A sample of the “warped slickrock” terrain model in Bryce 4: fBm constructed from one minus the absolute value of Perlin noise, distorted with fBm.

Recall: 3D stripe texture

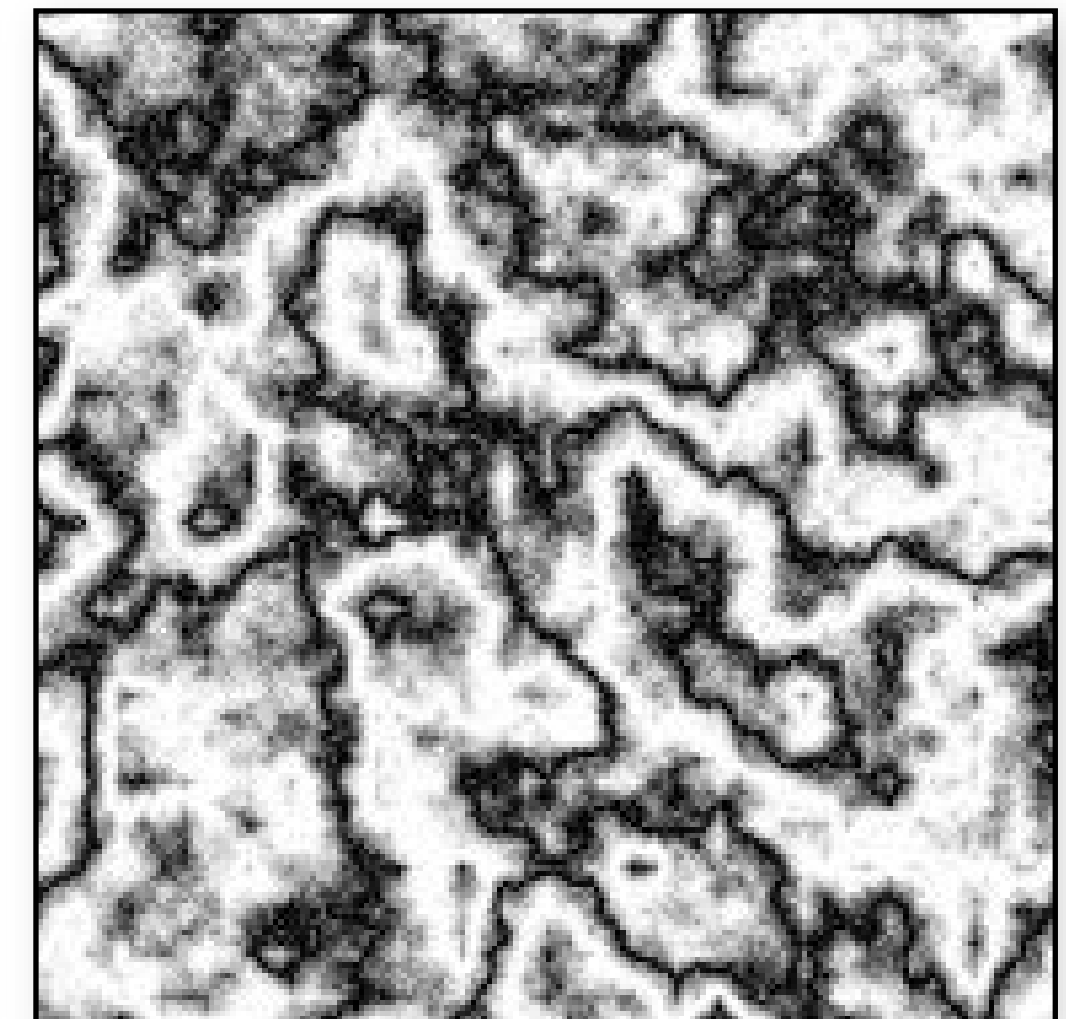
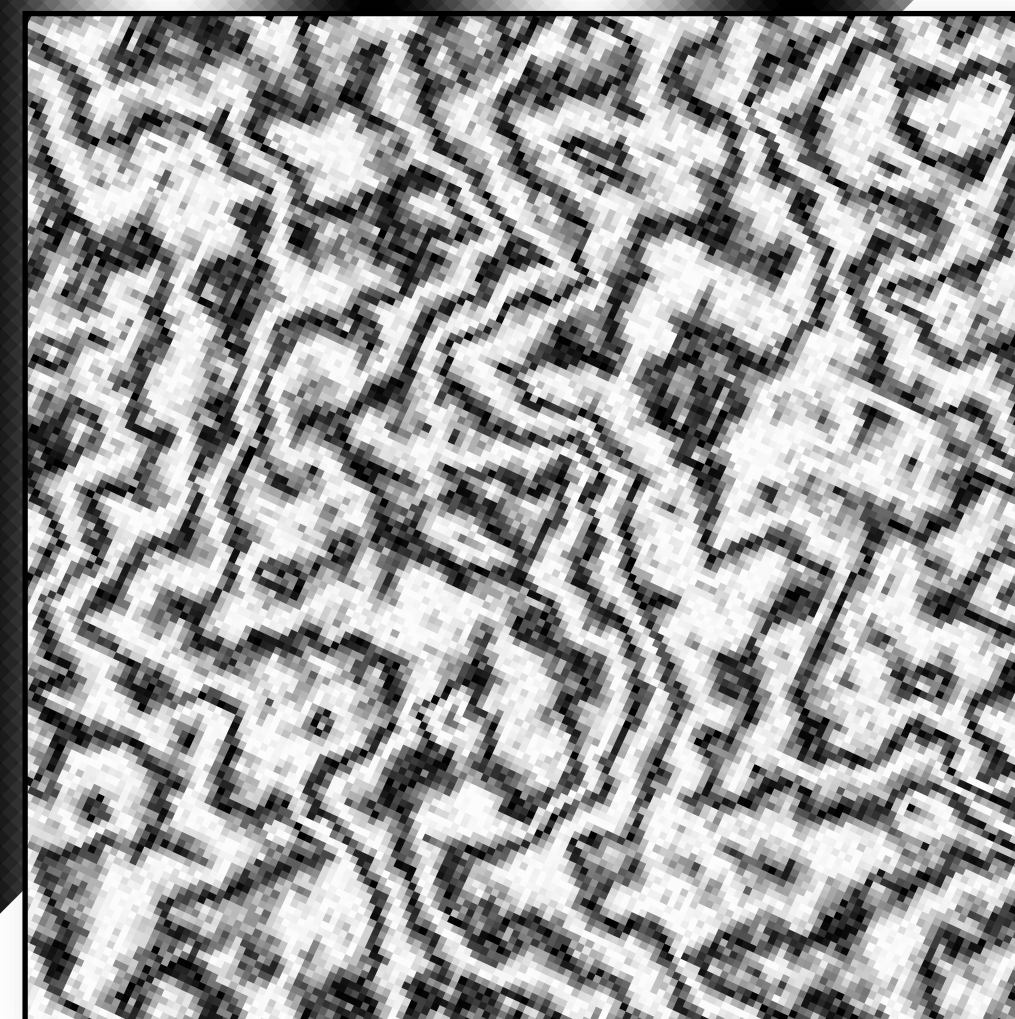
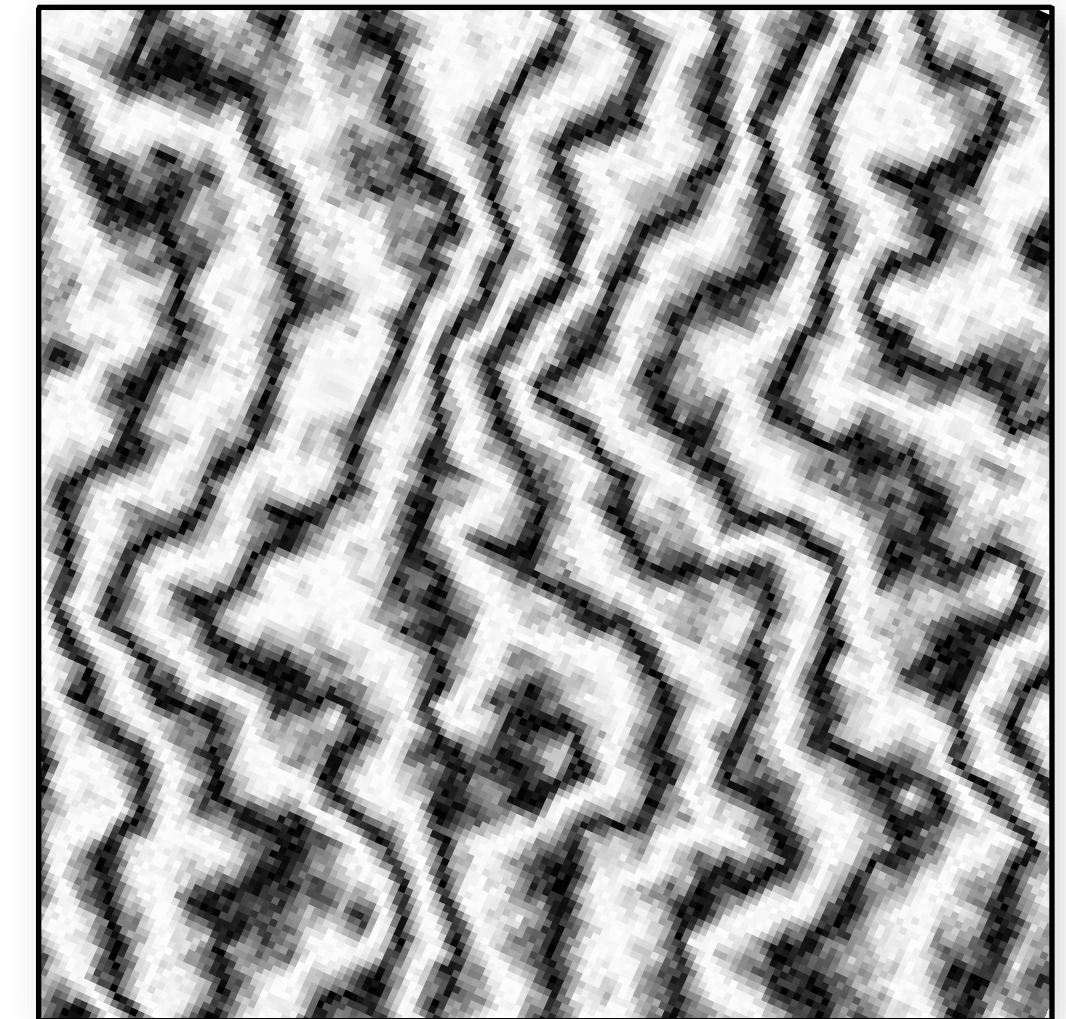
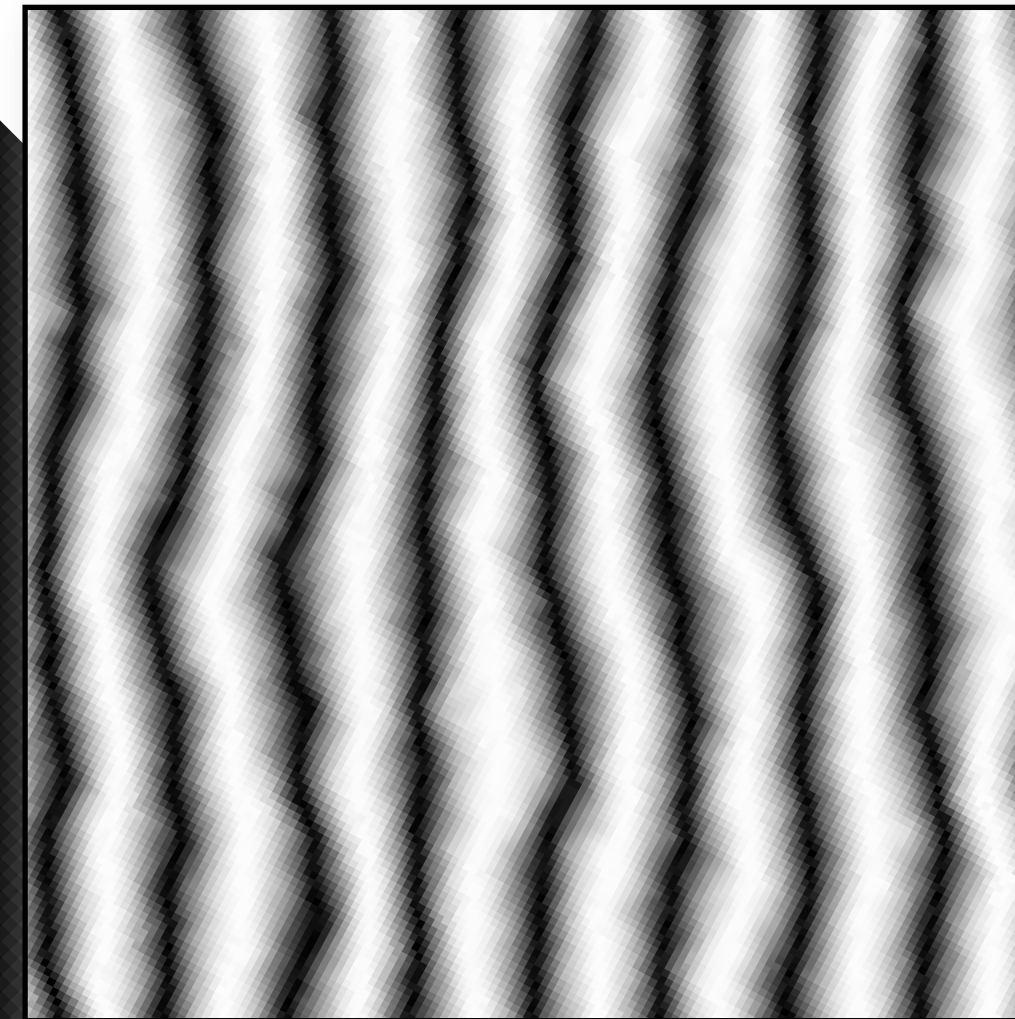
```
color stripe(point p, real w):  
     $t = (1 + \sin(\pi \mathbf{p}_x / w)) / 2$   
    return lerp( $c_0$ ,  $c_1$ ,  $t$ )
```

How can we make this less structured
(less “boring”)?



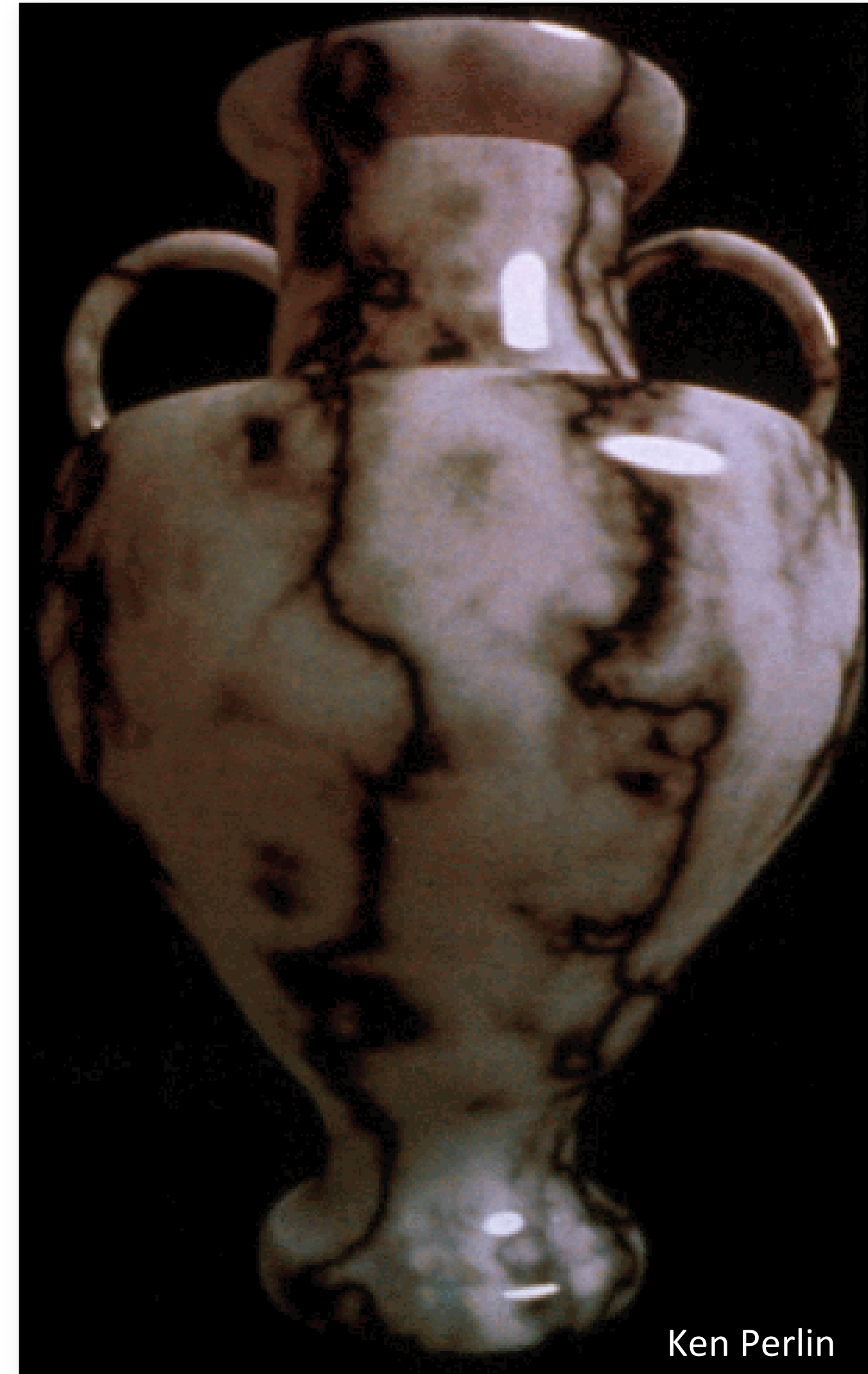
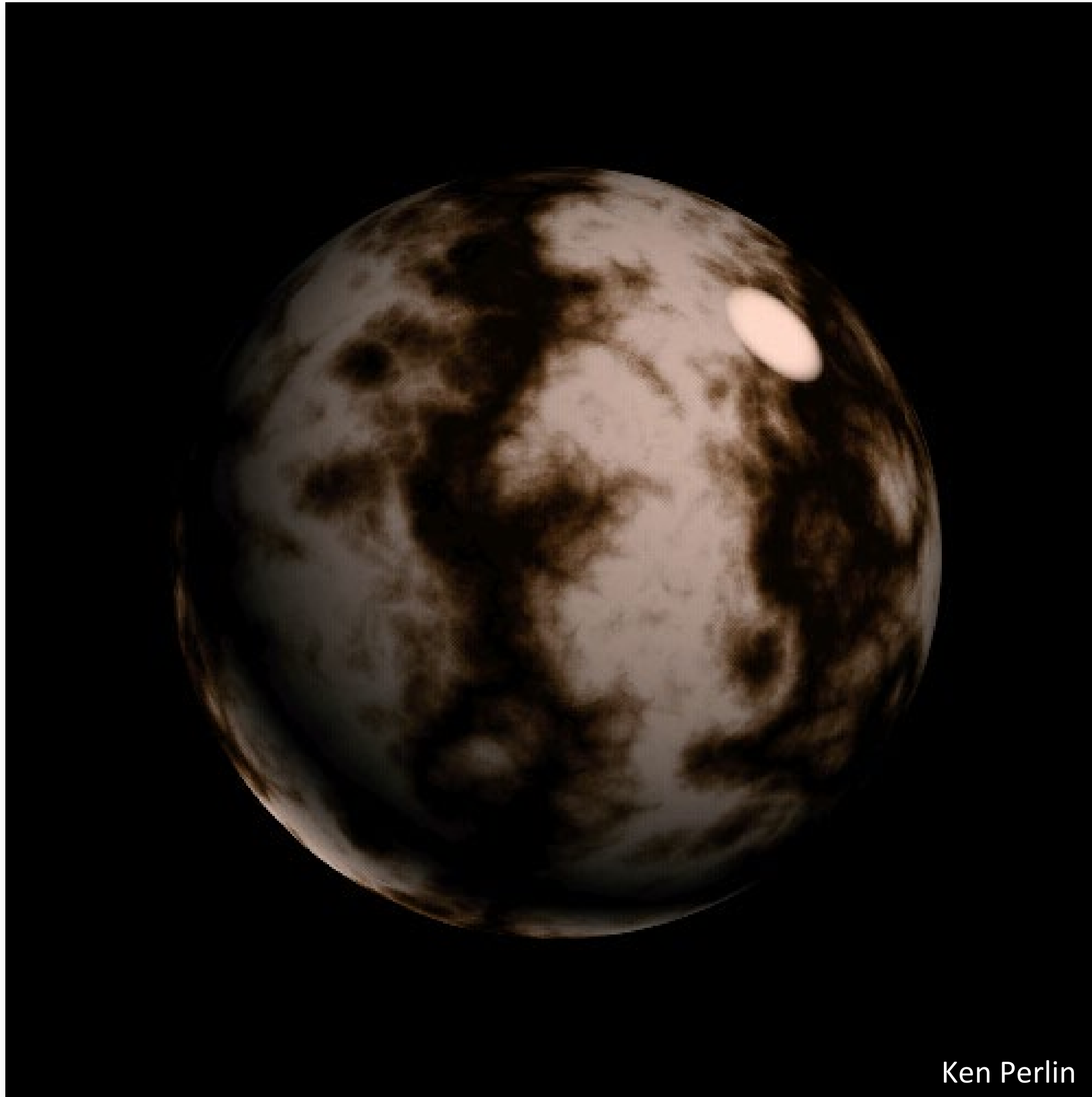
Mar

$$(1 + \sin(k_1 \mathbf{p}_x + \text{turbulence}(k_2 \mathbf{p})) / w) / 2$$



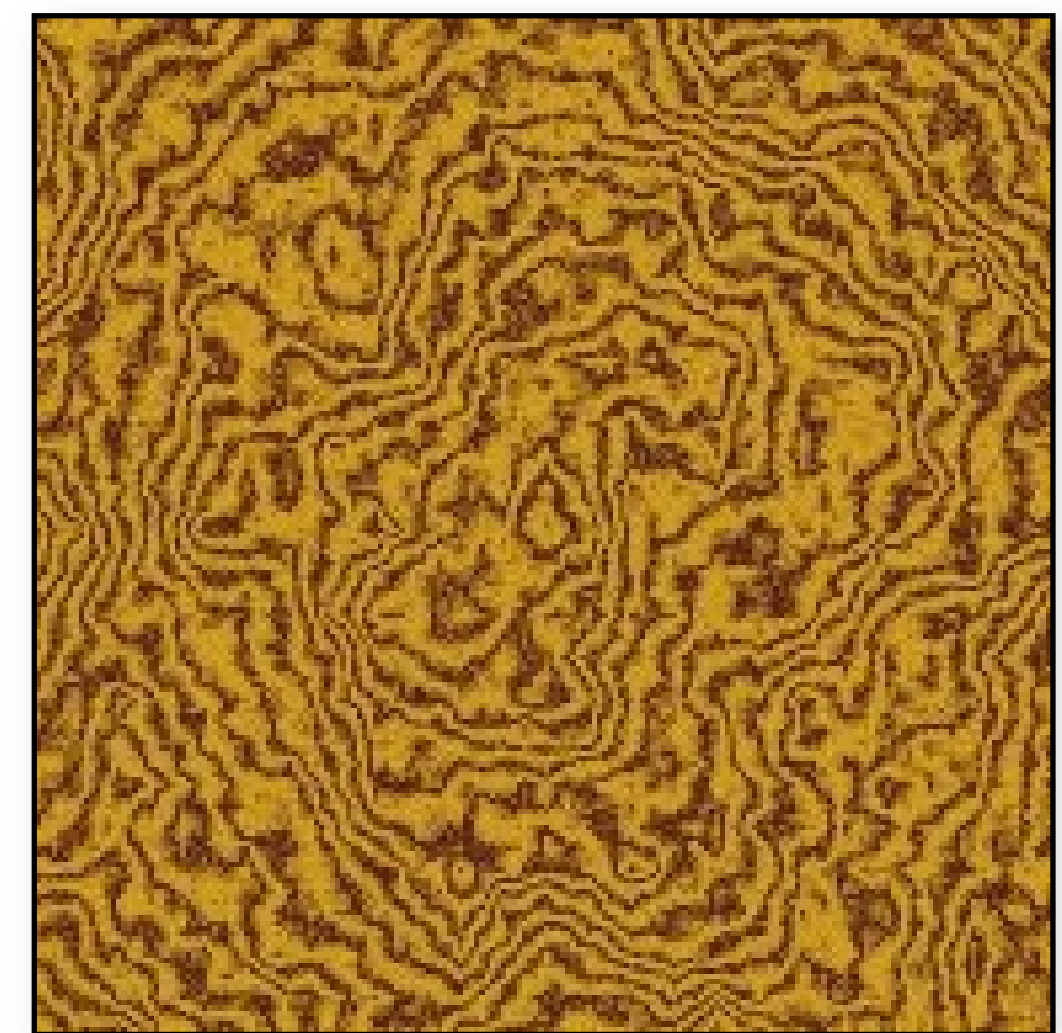
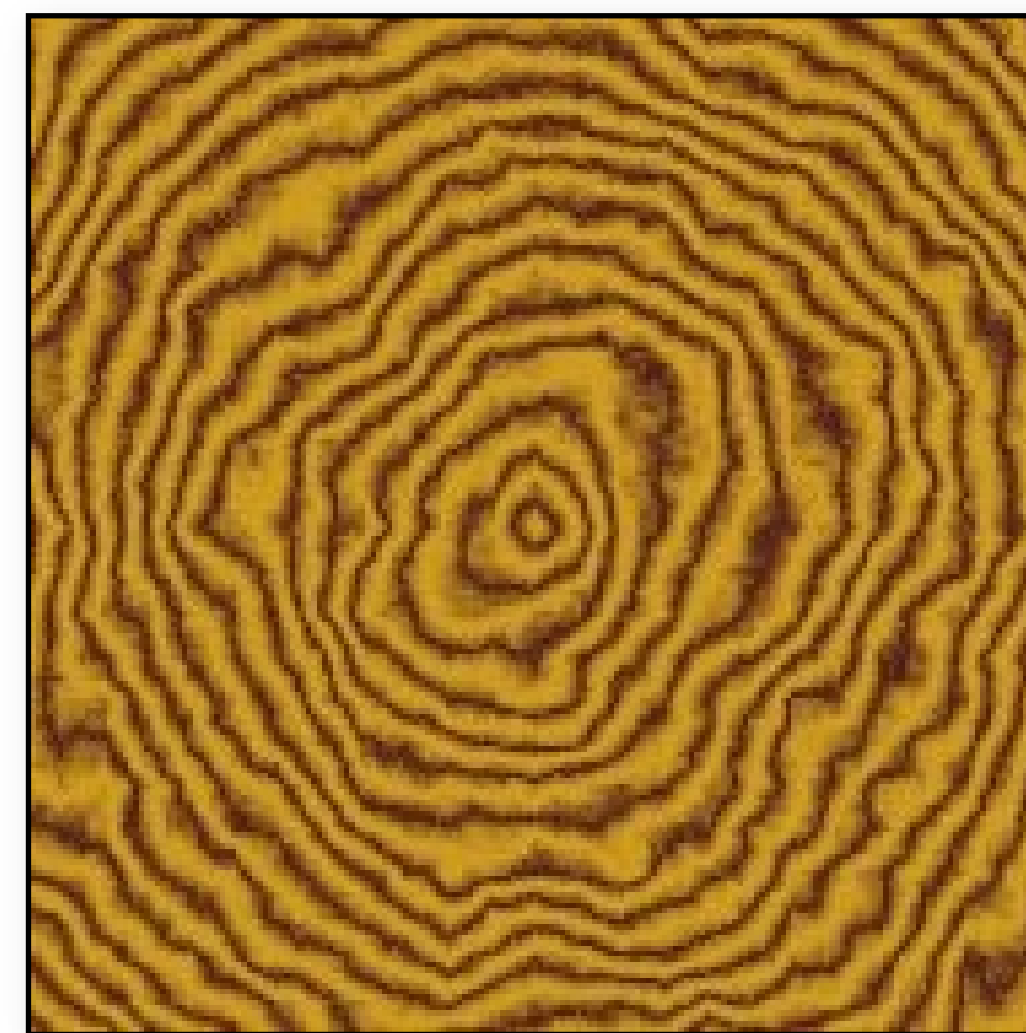
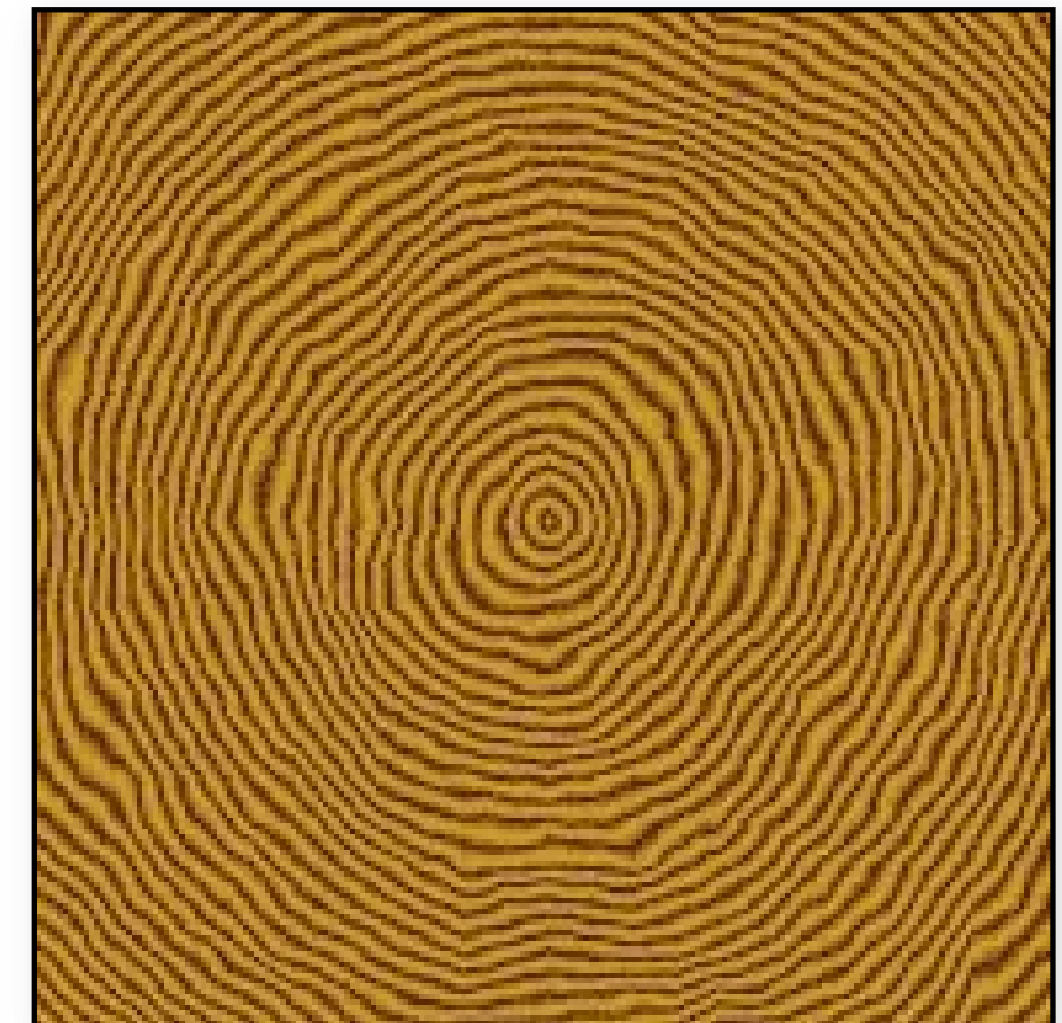
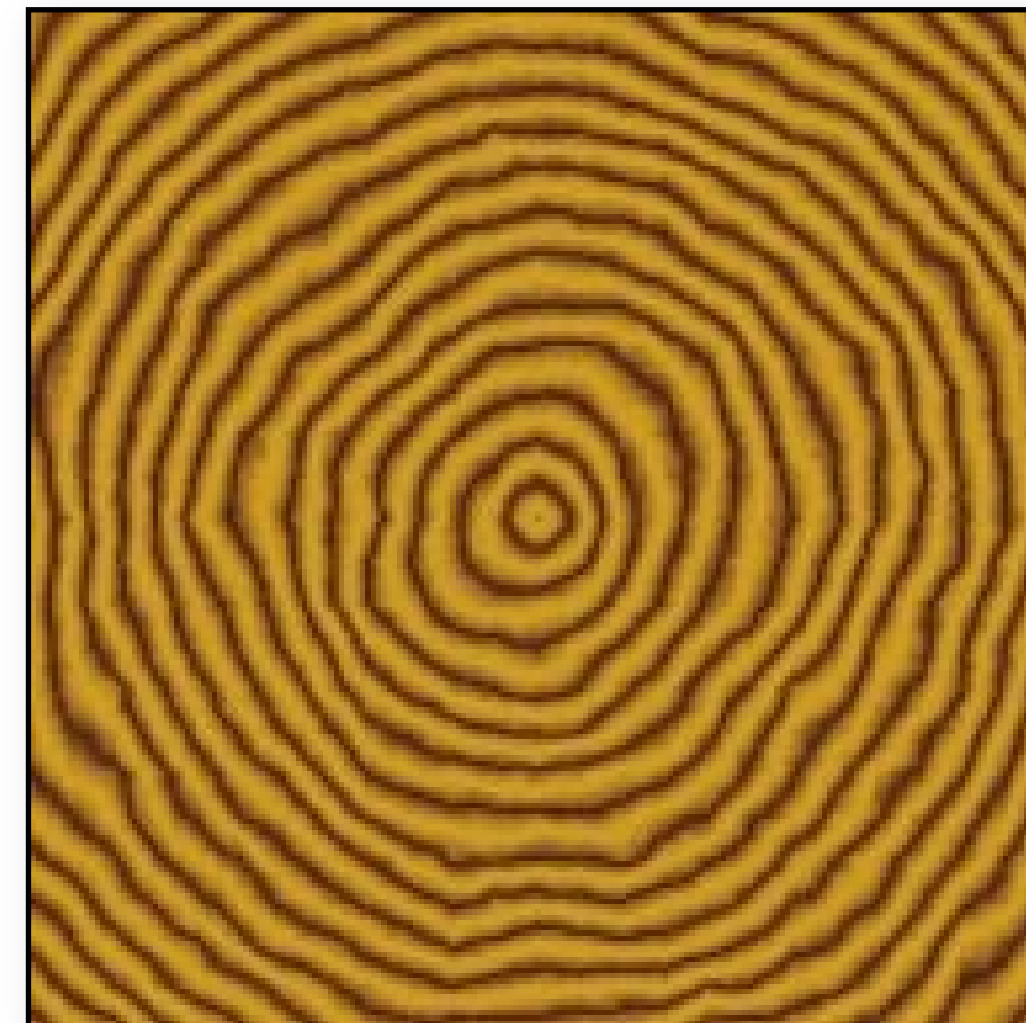
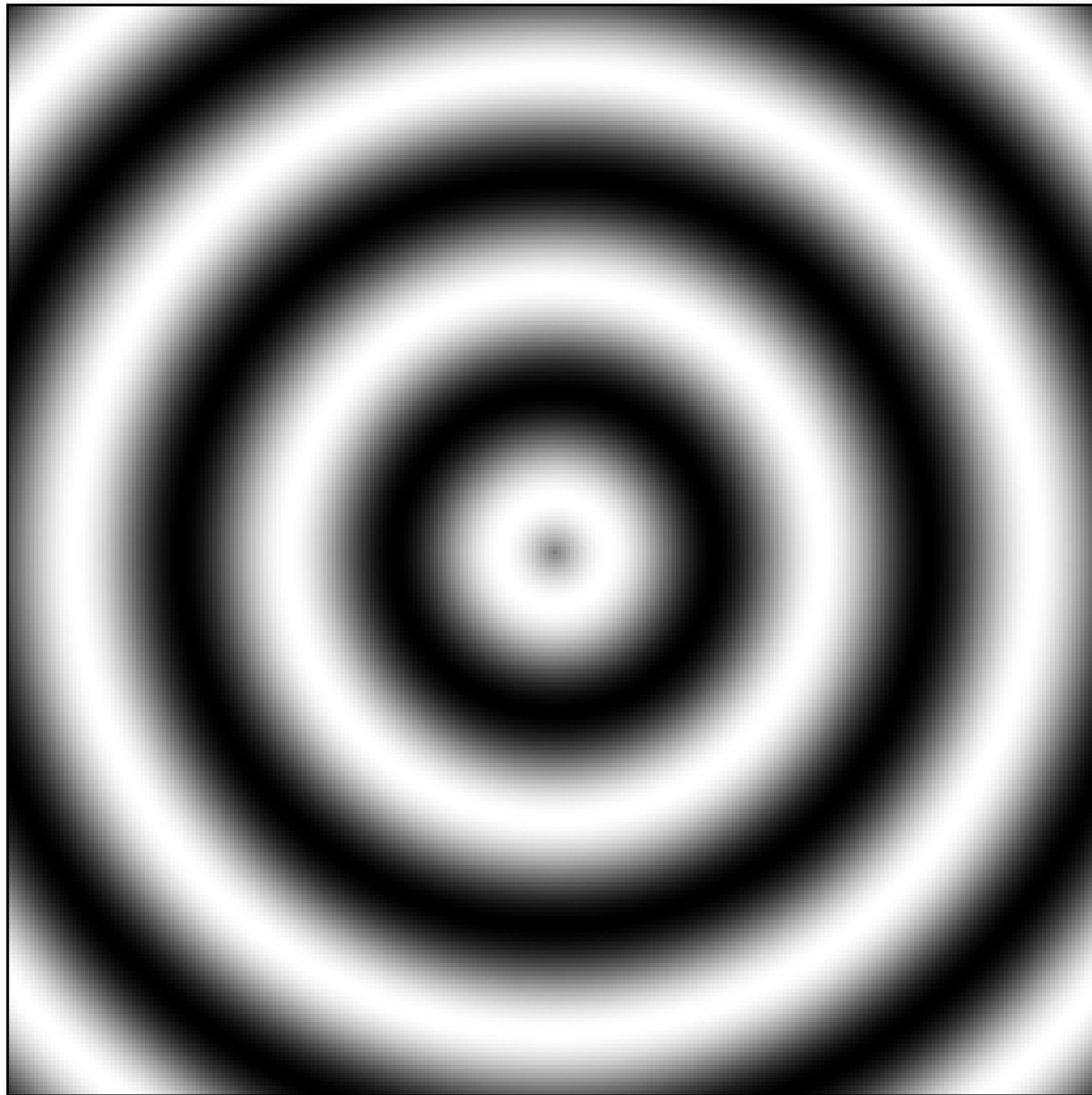
Marble

$$(1 + \sin(k_1 \mathbf{p}_x + \text{turbulence}(k_2 \mathbf{p})) / w) / 2$$



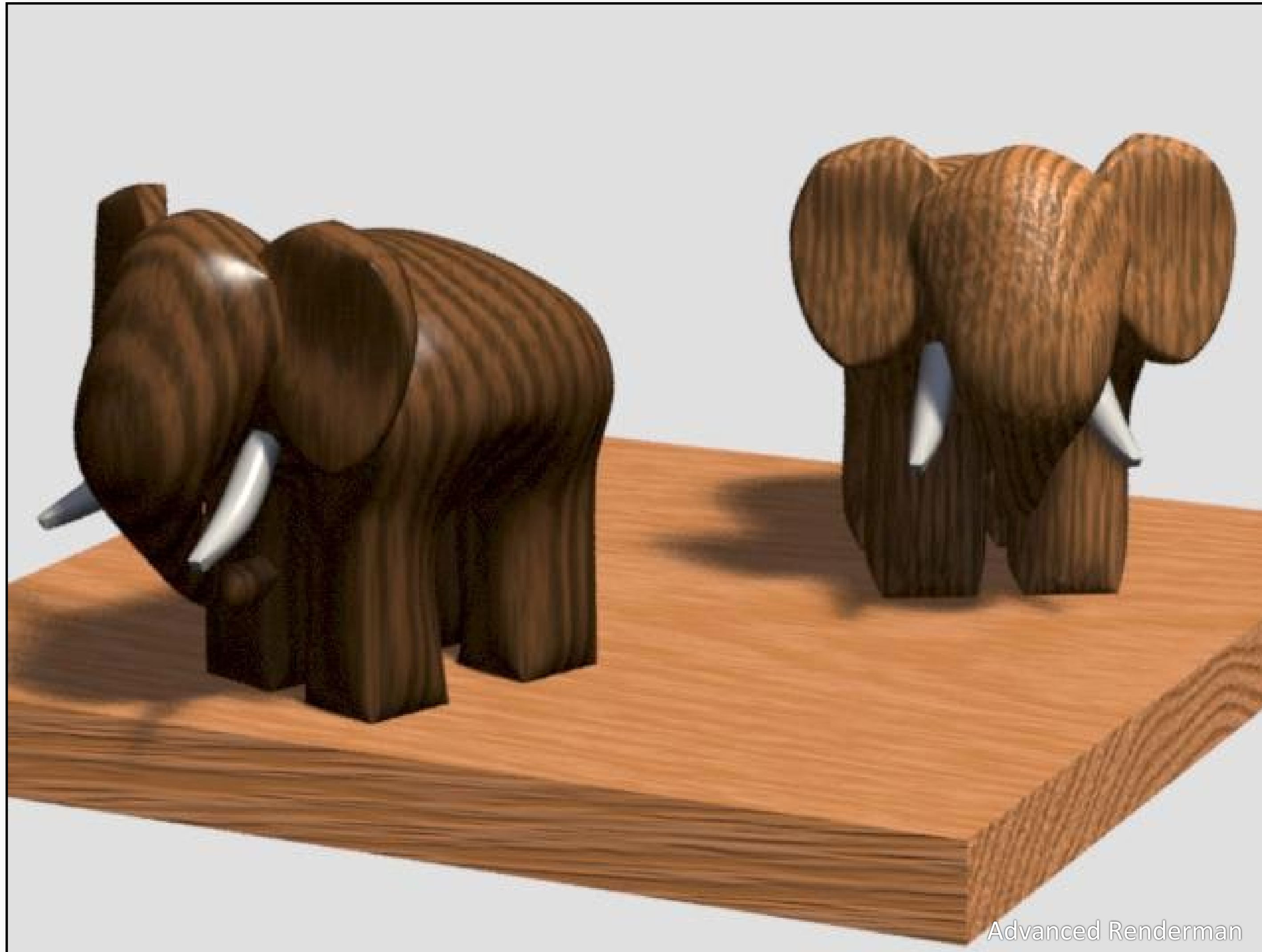
Wood

$$(1 + \sin(\sqrt{\mathbf{p}_x^2 + \mathbf{p}_y^2}) + \text{fBm}(\mathbf{p})) / 2$$

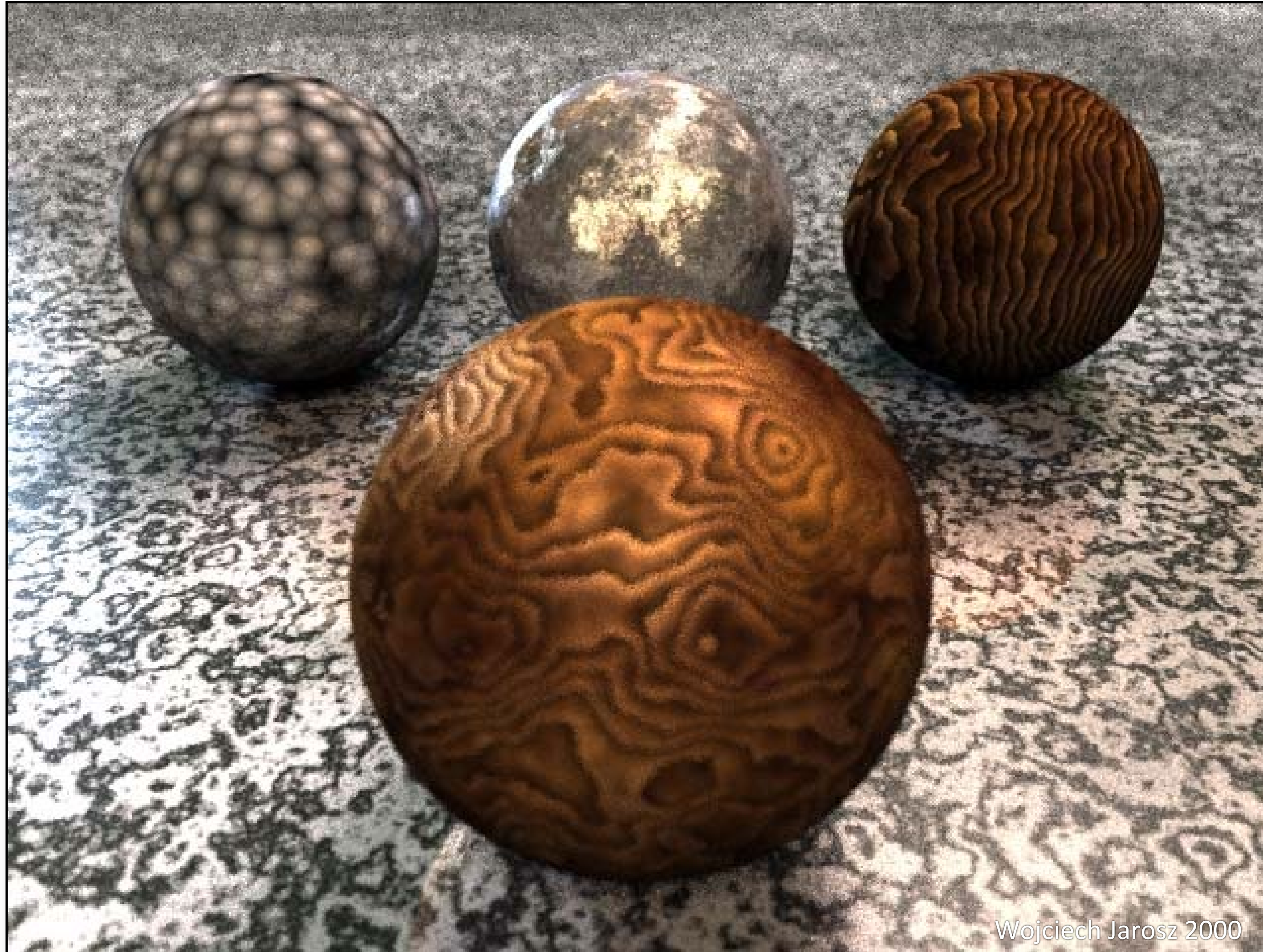


Wood

$$(1 + \sin(\sqrt{\mathbf{p}_x^2 + \mathbf{p}_y^2}) + \text{fBm}(\mathbf{p})) / 2$$



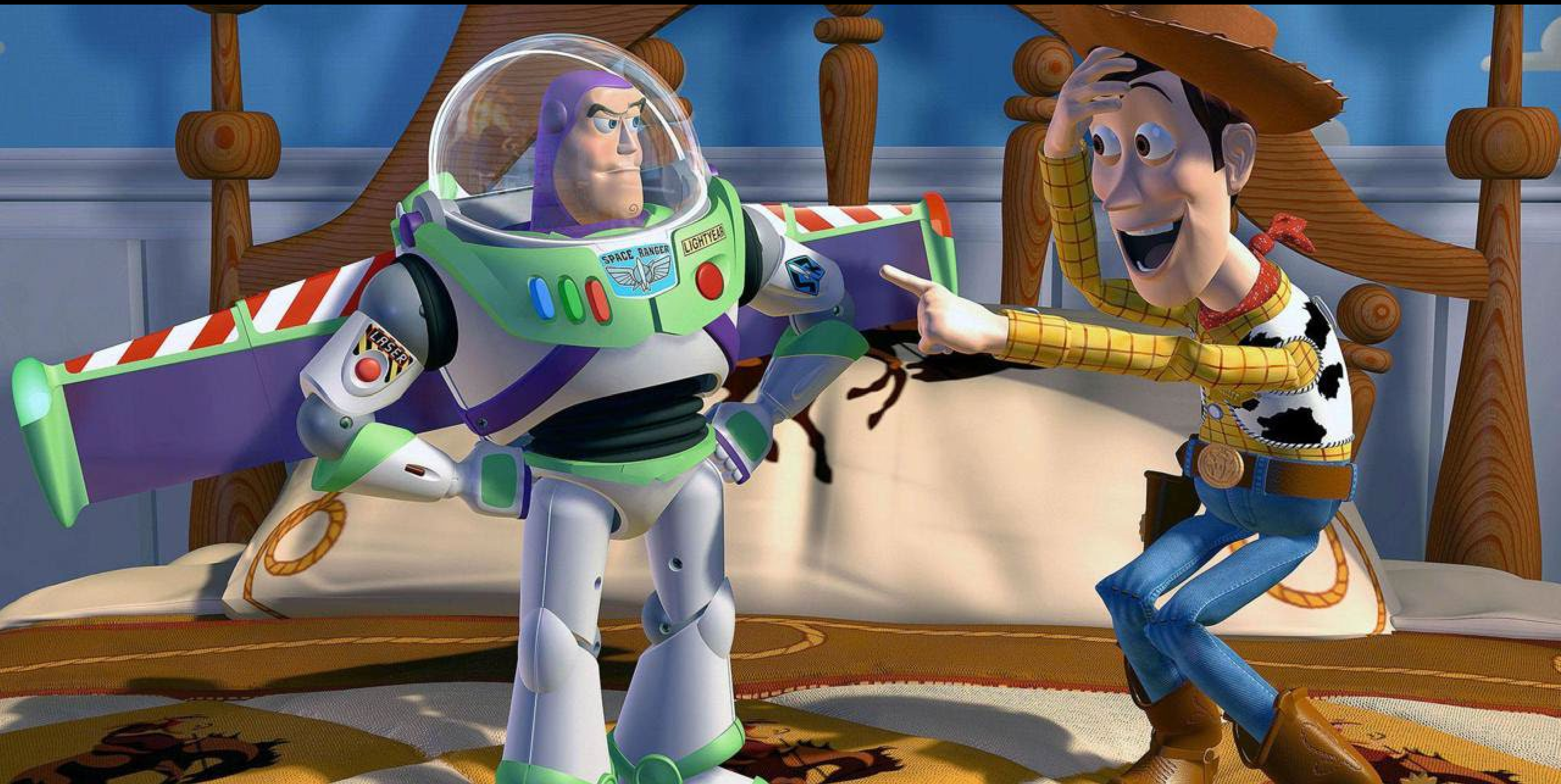
and more...



and more...



Wojciech Jarosz 2000







Worley noise

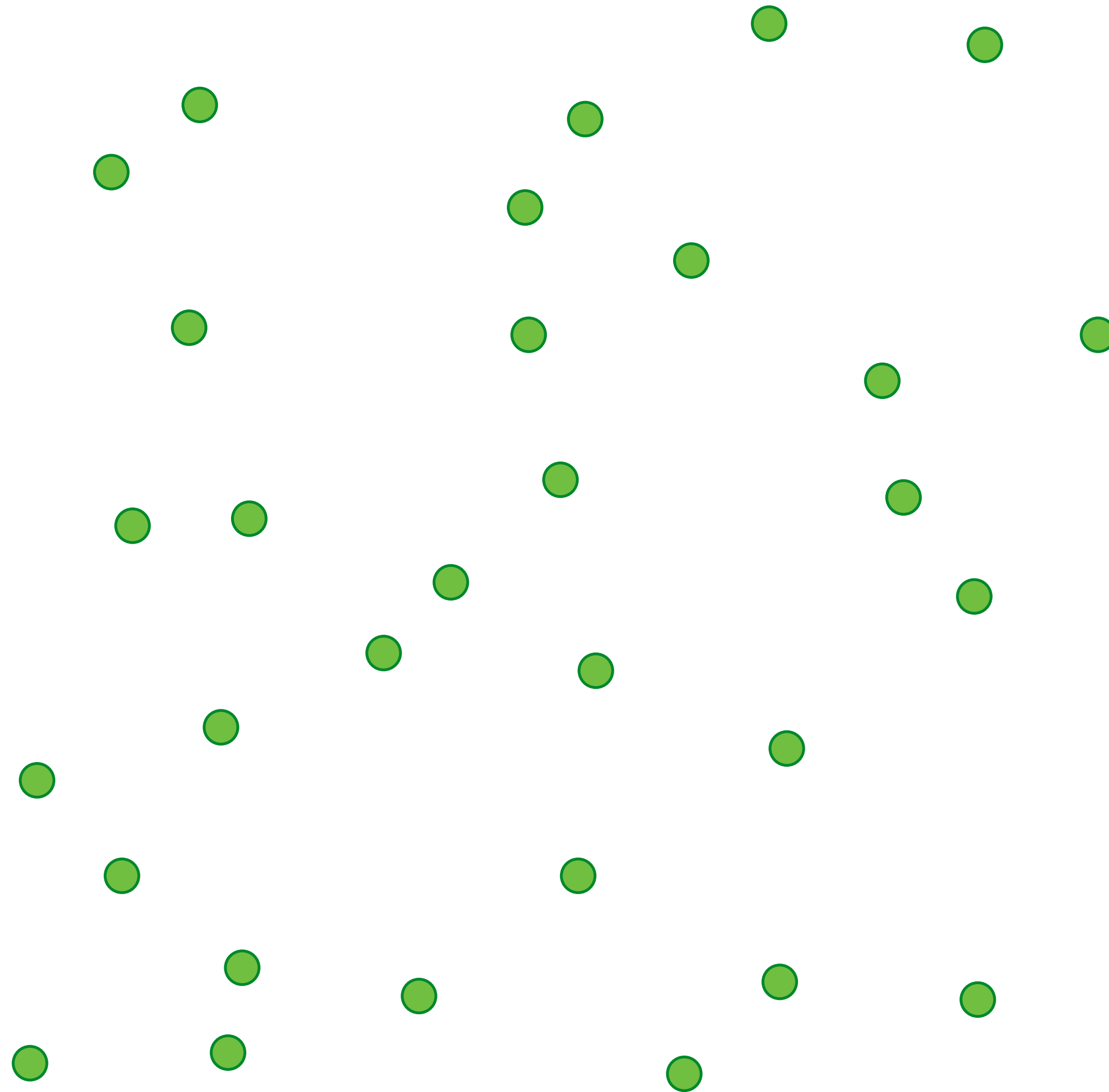
“Cellular texture” function

- Introduced in 1996 by Steve Worley
- Different from *cell* texture!

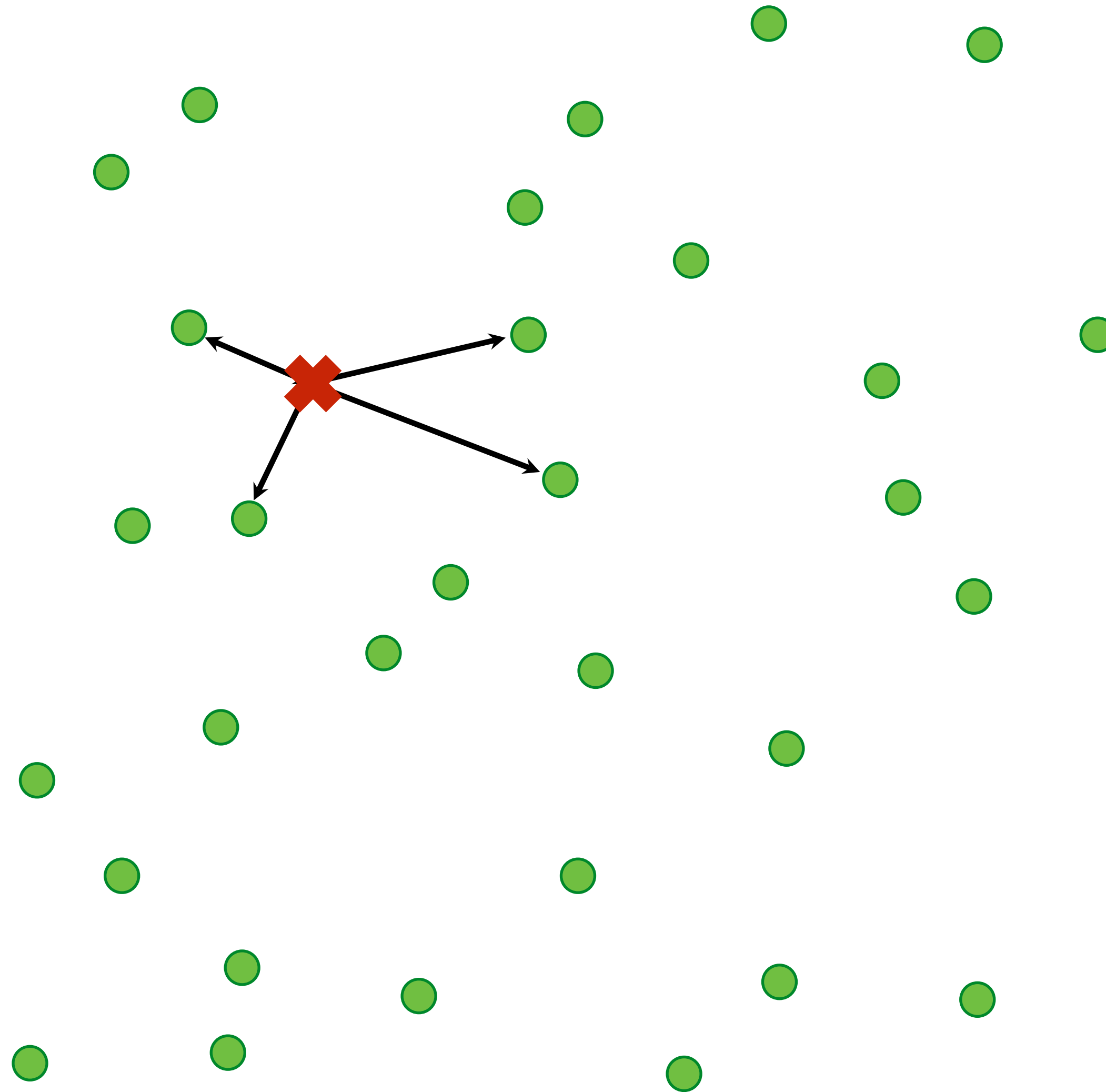
Randomly distribute “feature points” in space

- $f_n(x)$ = distance to n^{th} closest point to x

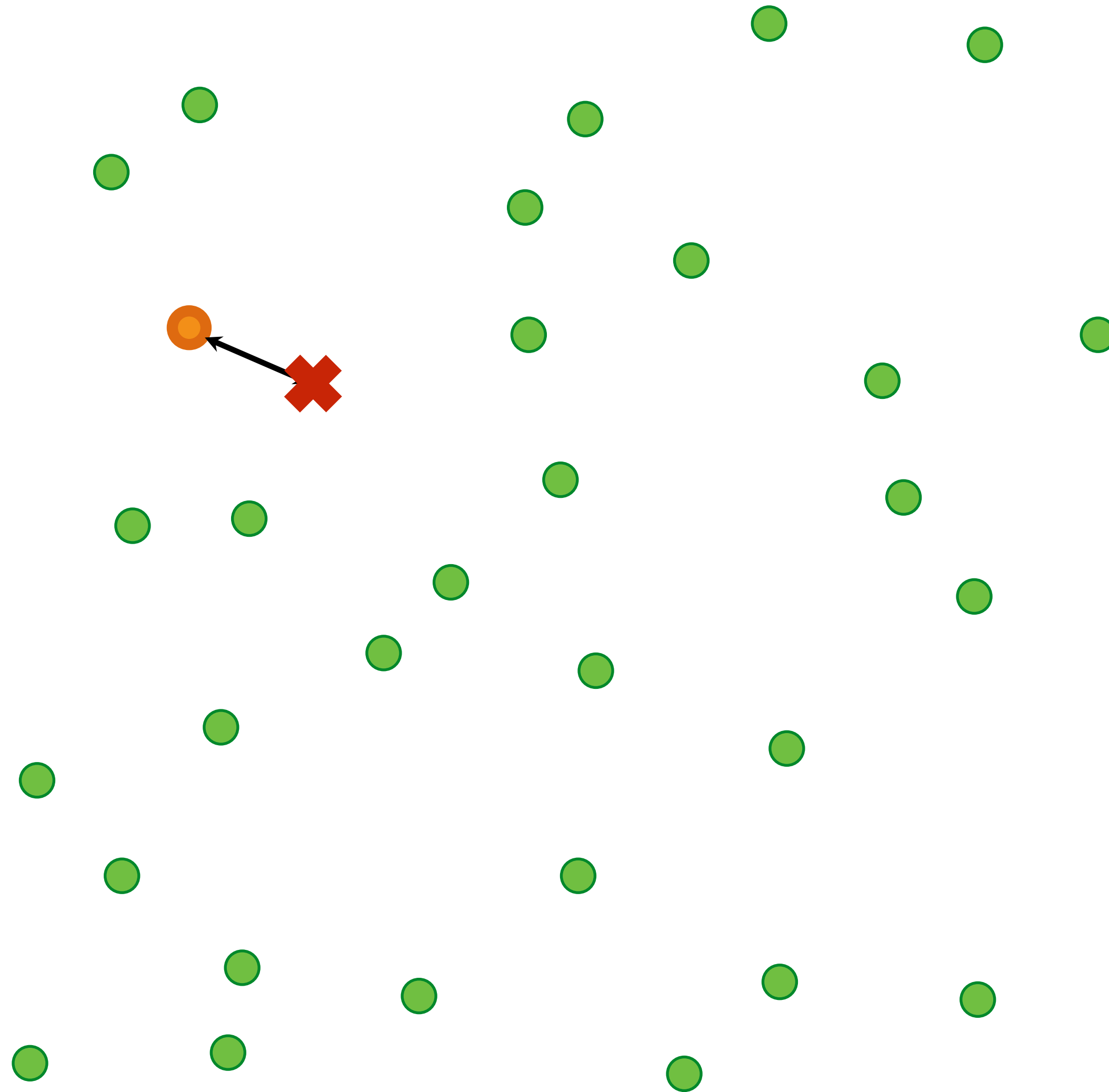
2D Worley noise: f1



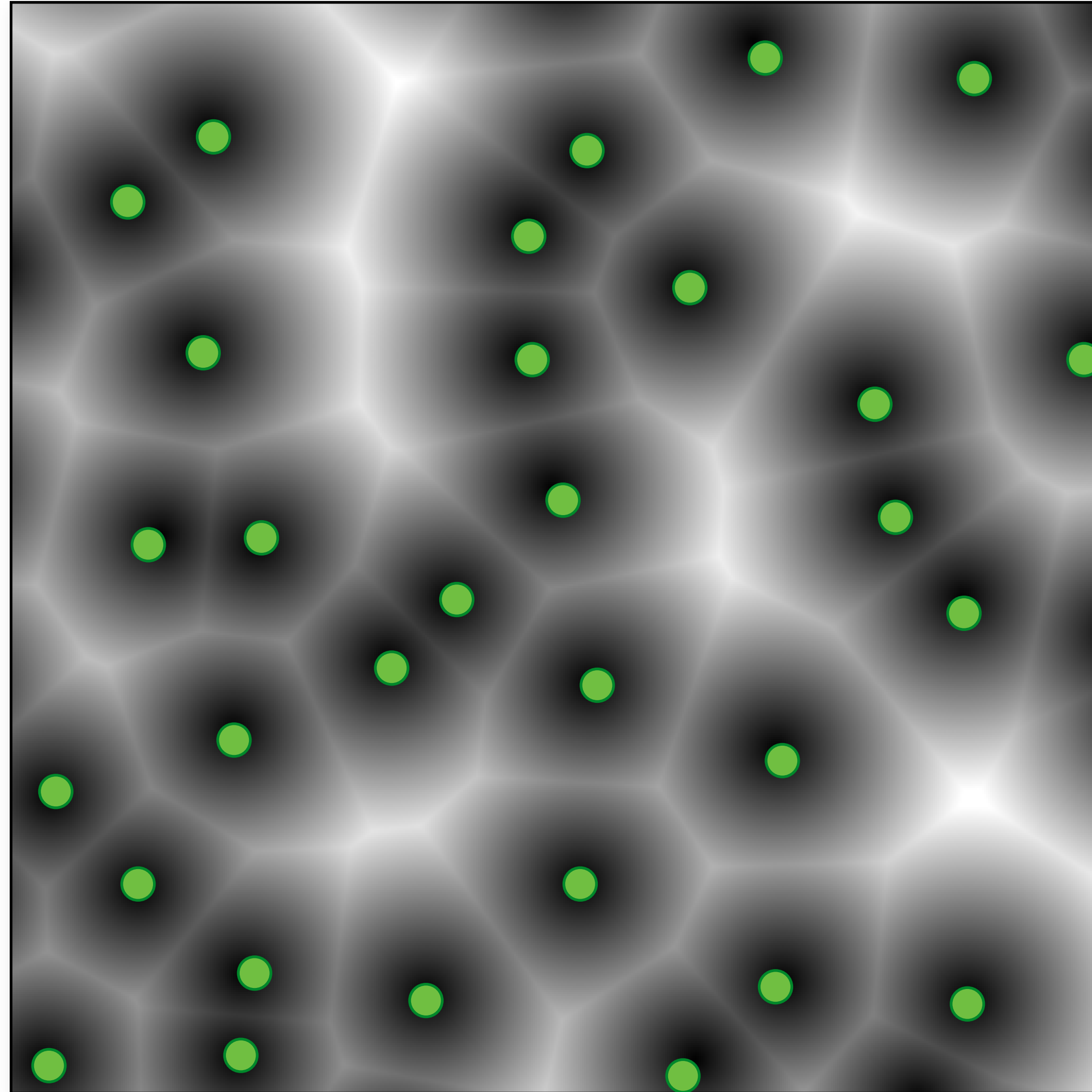
2D Worley noise: f1



2D Worley noise: f1

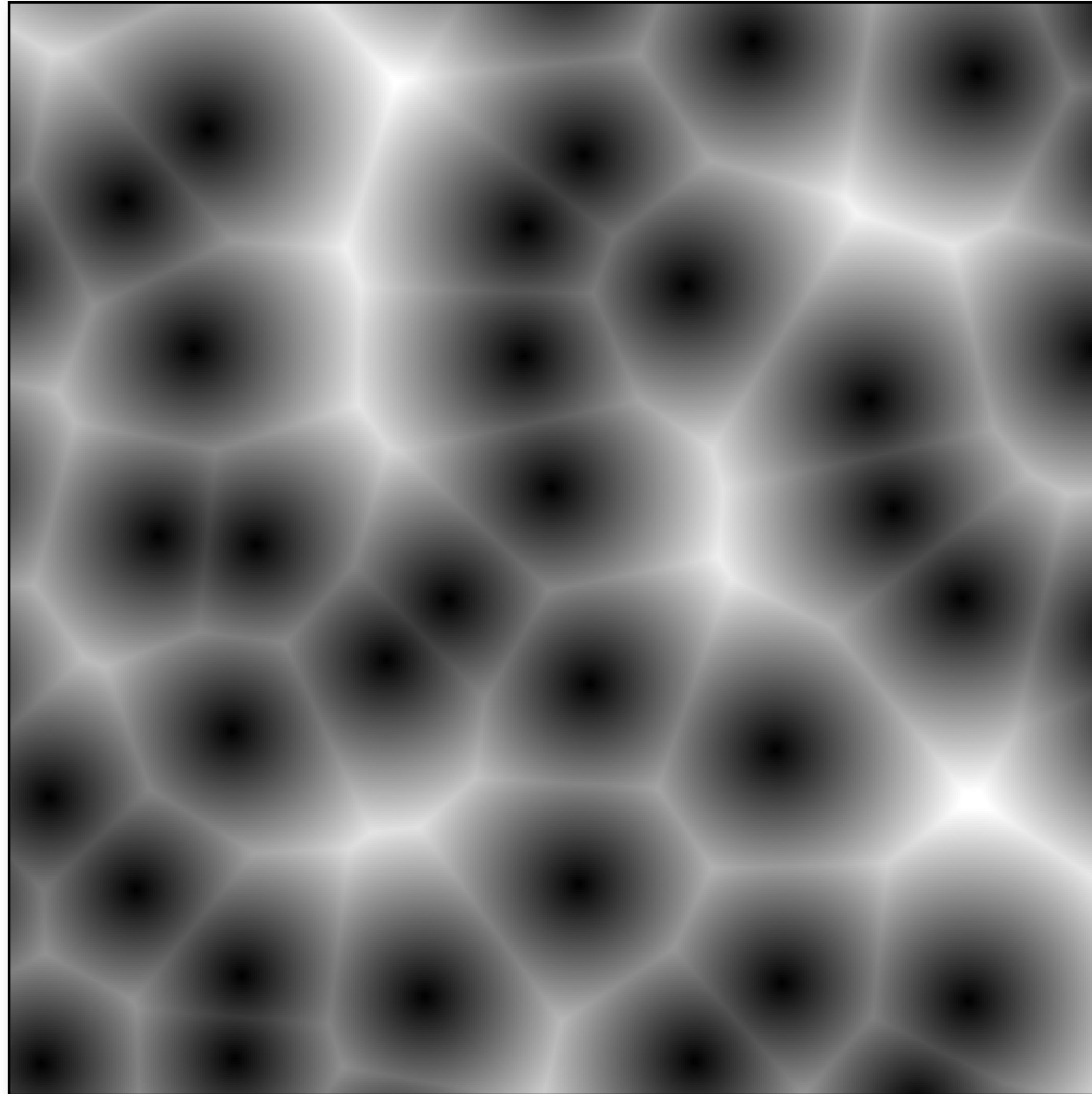


2D Worley noise: f_1



What do we call this image in geometry?

2D Worley noise: f_1



Worley Noise



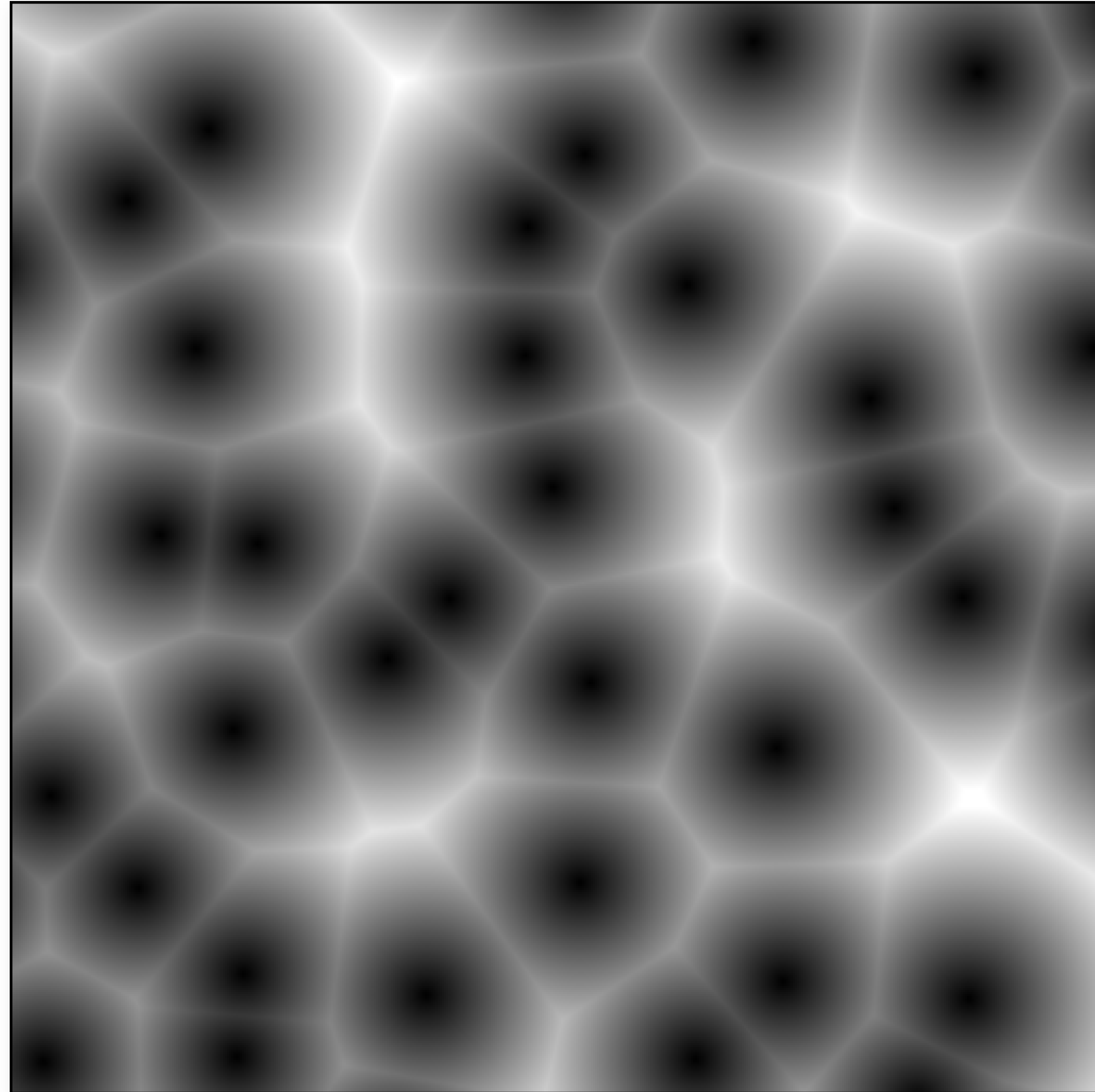
fractal F1, bump map

Worley Noise



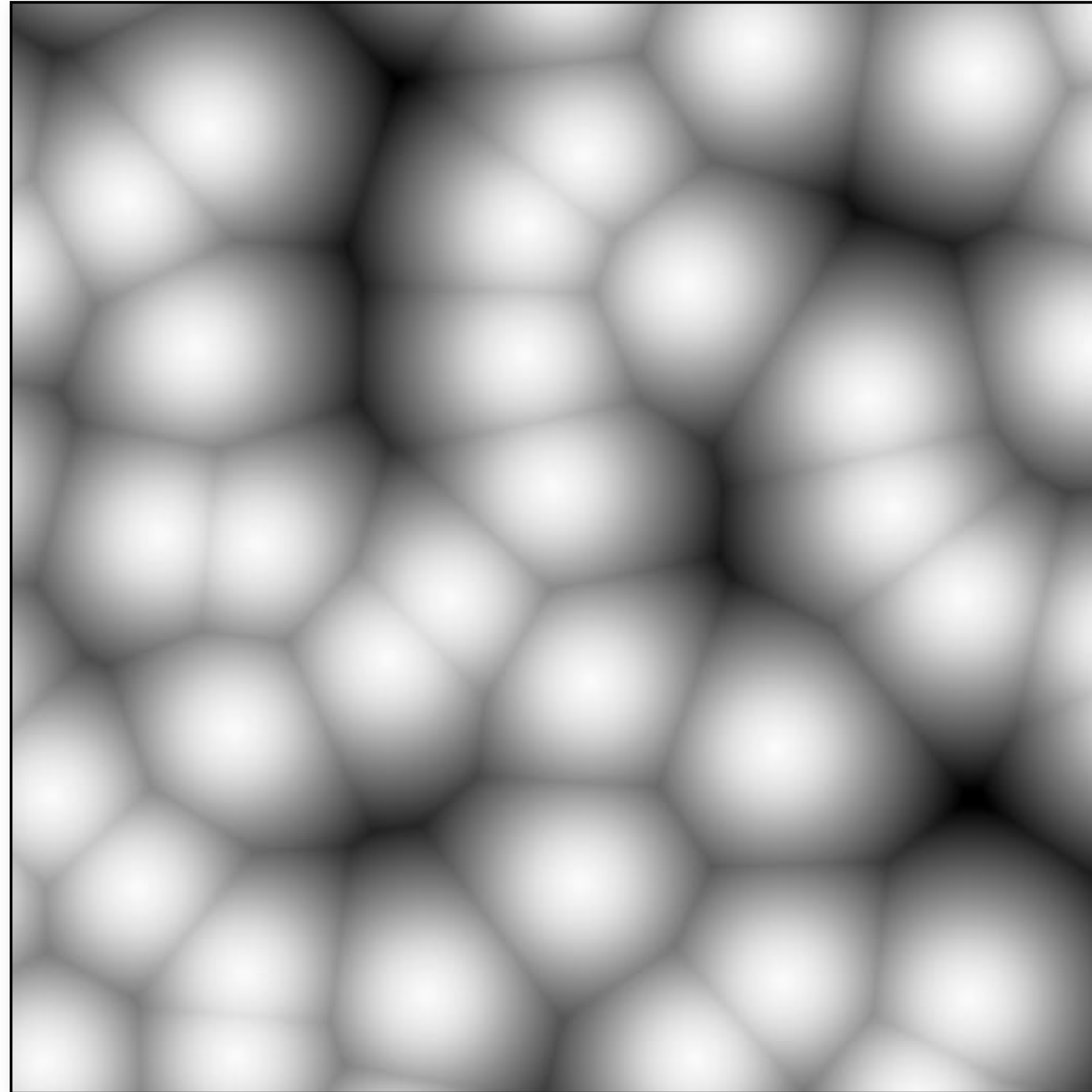
fractal F1, bump map

2D Worley noise: f_1



Wojciech Jarosz 2007

2D Worley noise: $1-f_1$



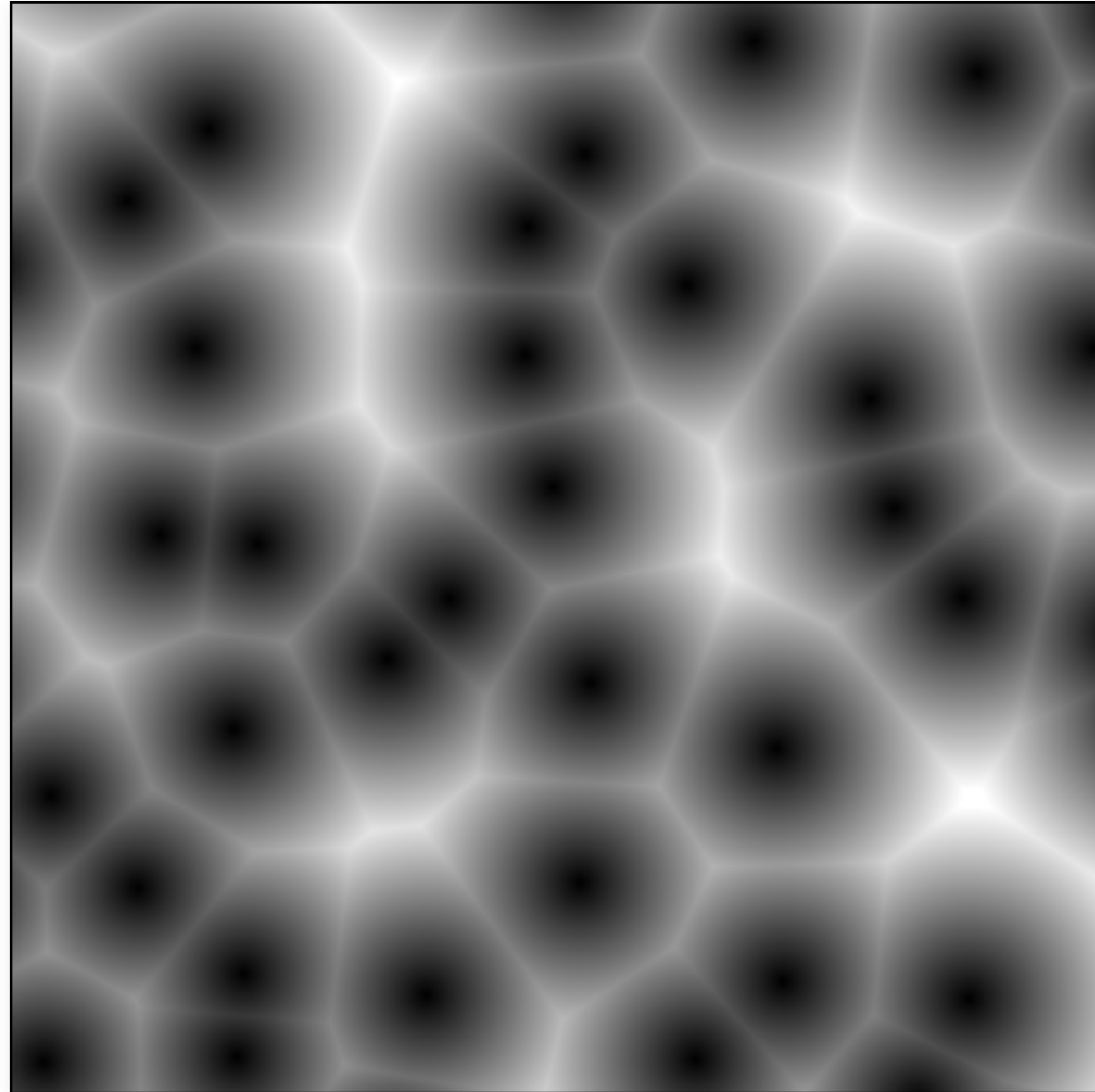
Wojciech Jarosz 2007

Worley Noise



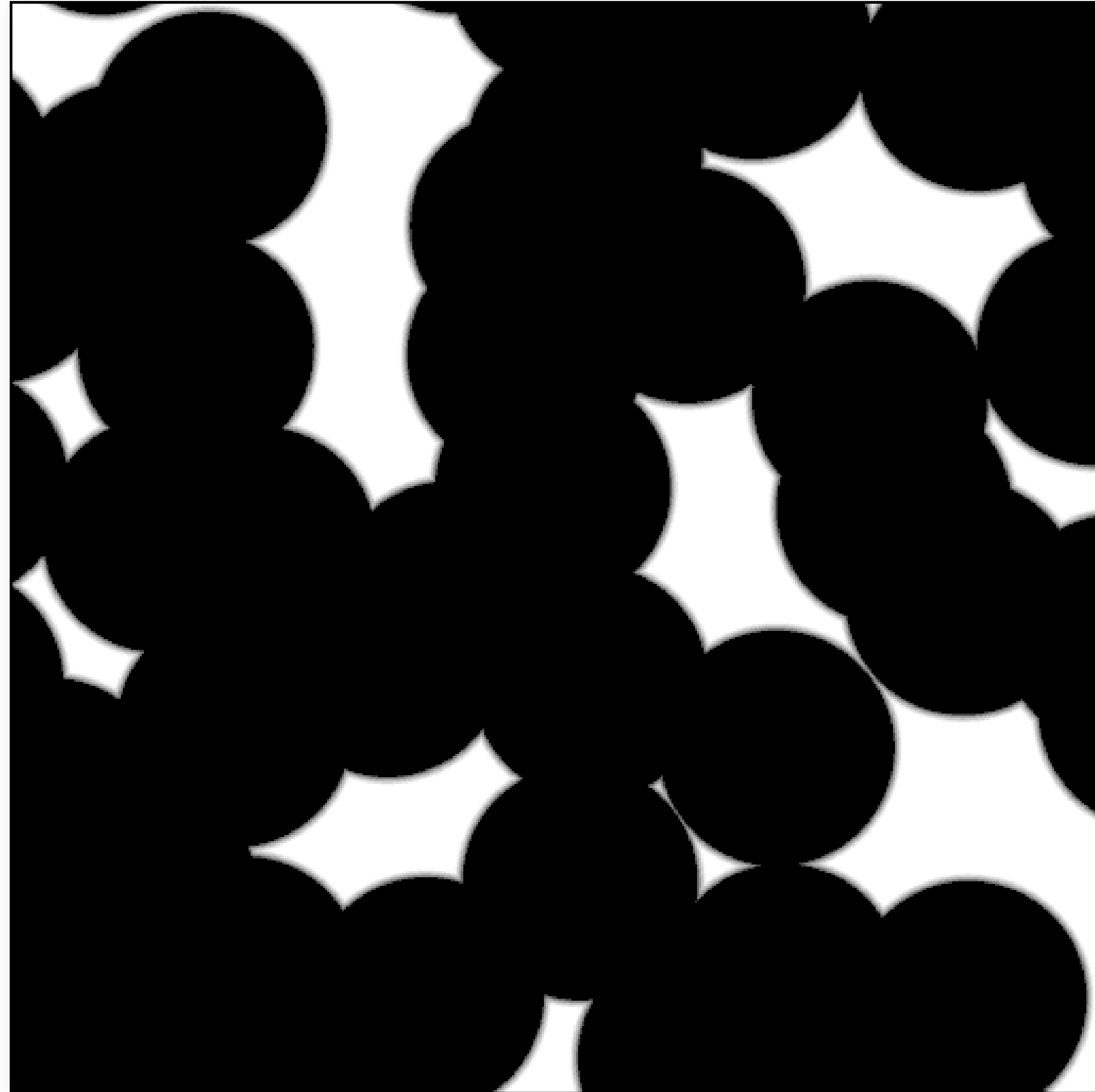
fractal $1-f_1$, color and bump map

2D Worley noise: f_1



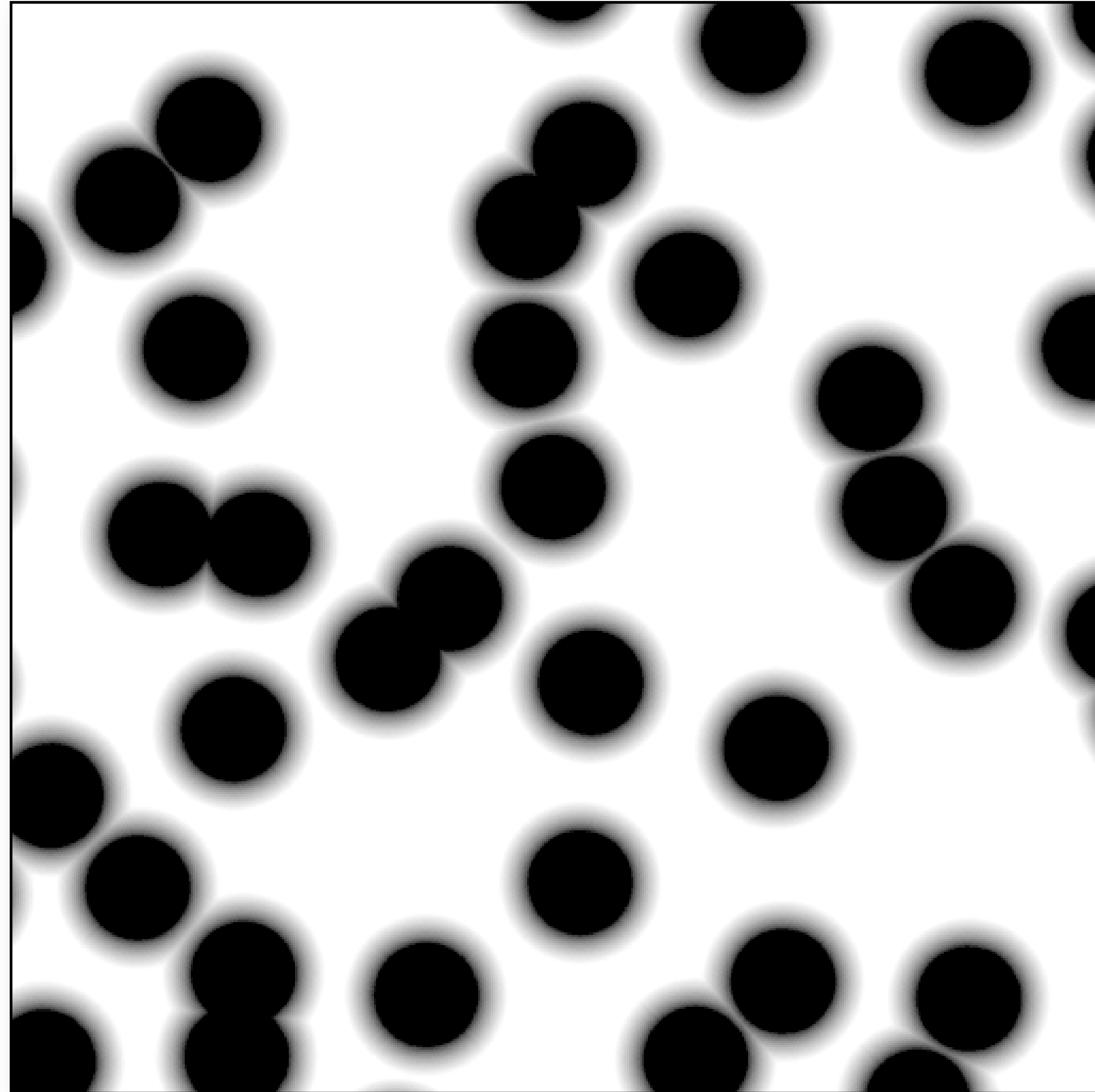
Wojciech Jarosz 2007

2D Worley noise: f_1 , thresholded



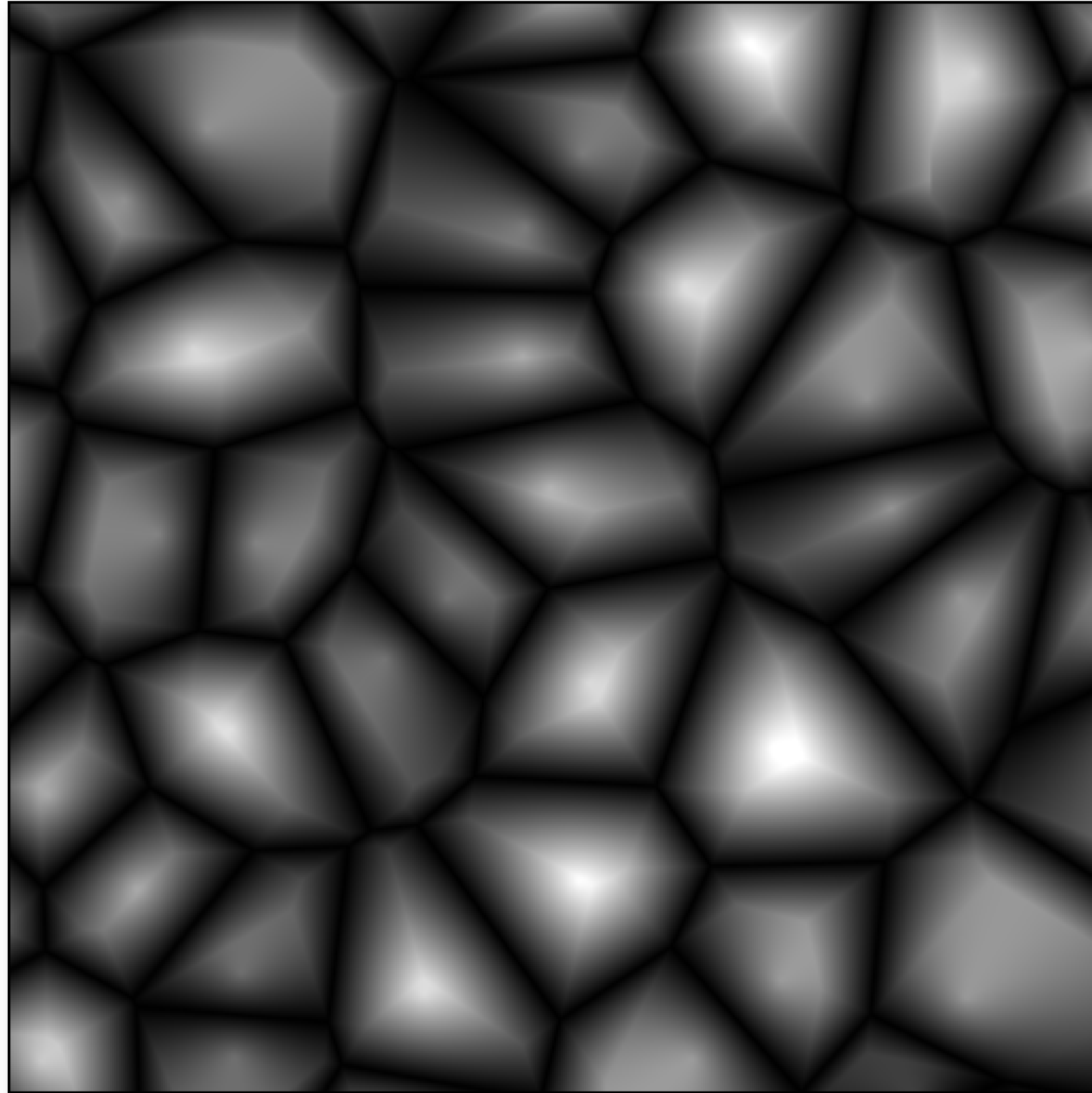
Wojciech Jarosz 2007

2D Worley noise: f_1 , thresholded



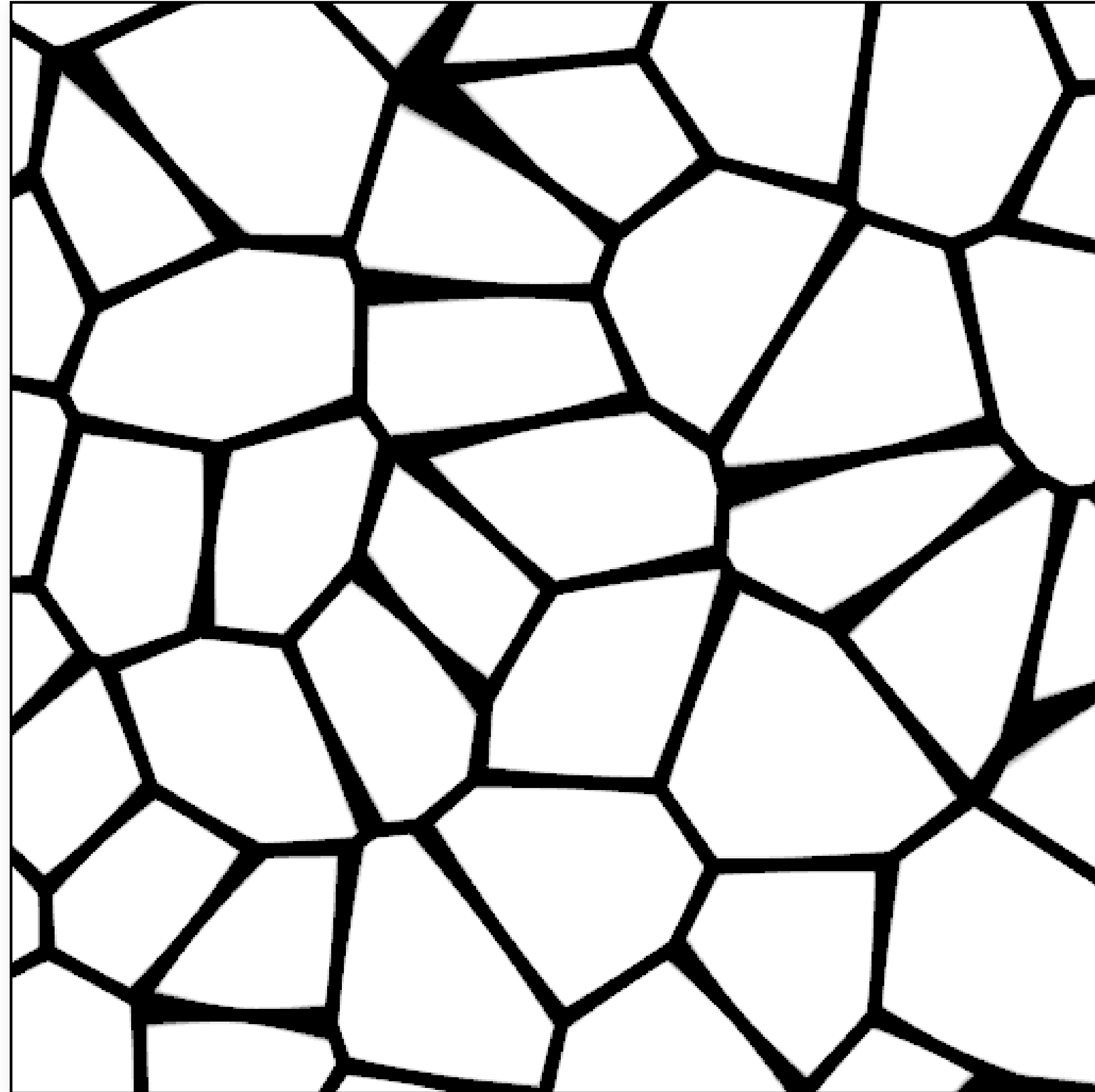
Wojciech Jarosz 2007

2D Worley noise: $f_2 - f_1$



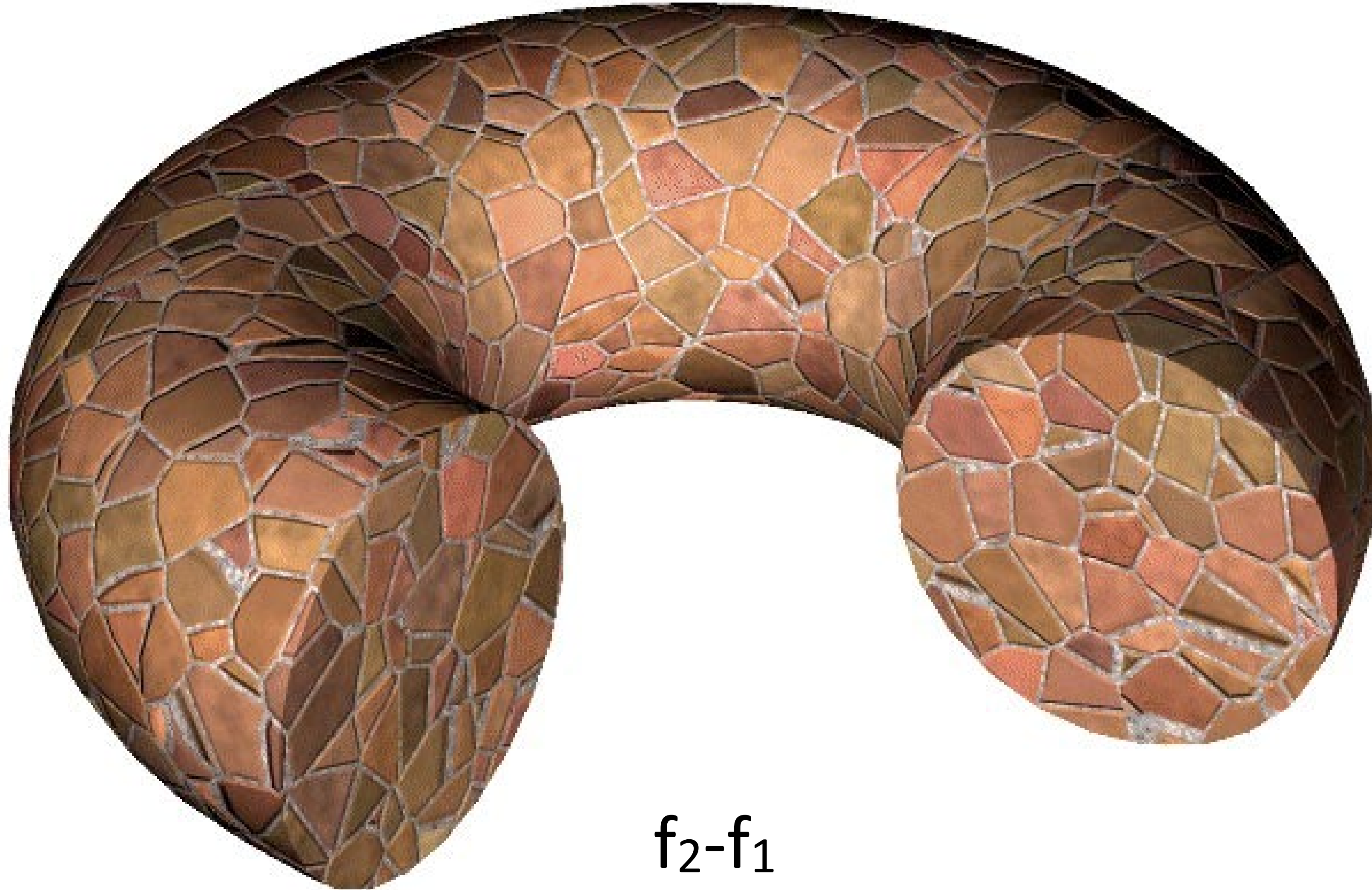
Wojciech Jarosz 2007

2D Worley noise: $f_2 - f_1$, thresholded



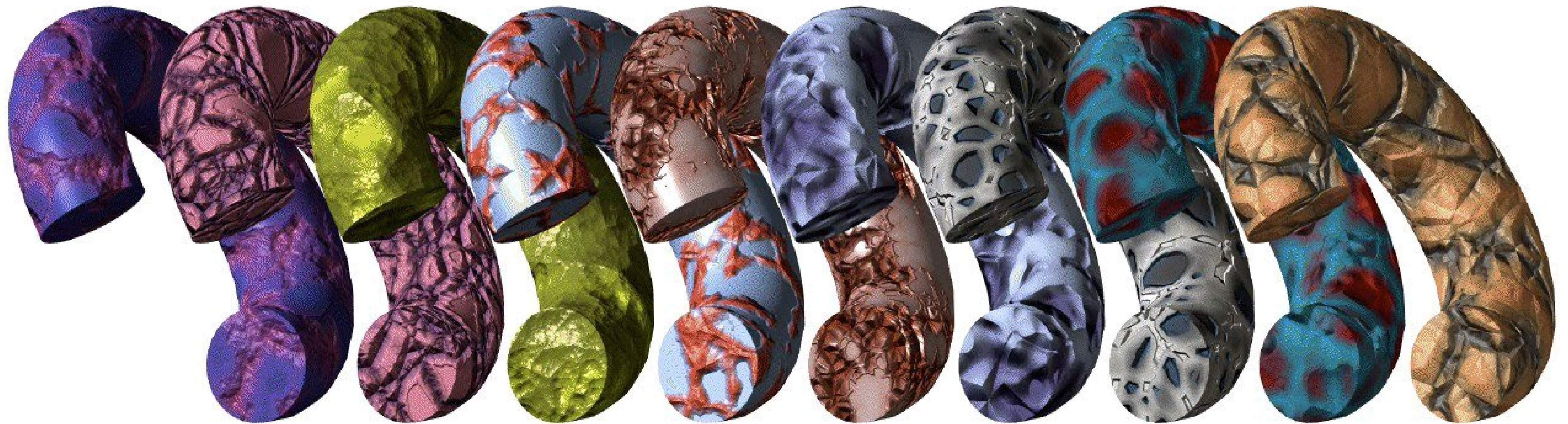
Wojciech Jarosz 2007

3D Worley noise



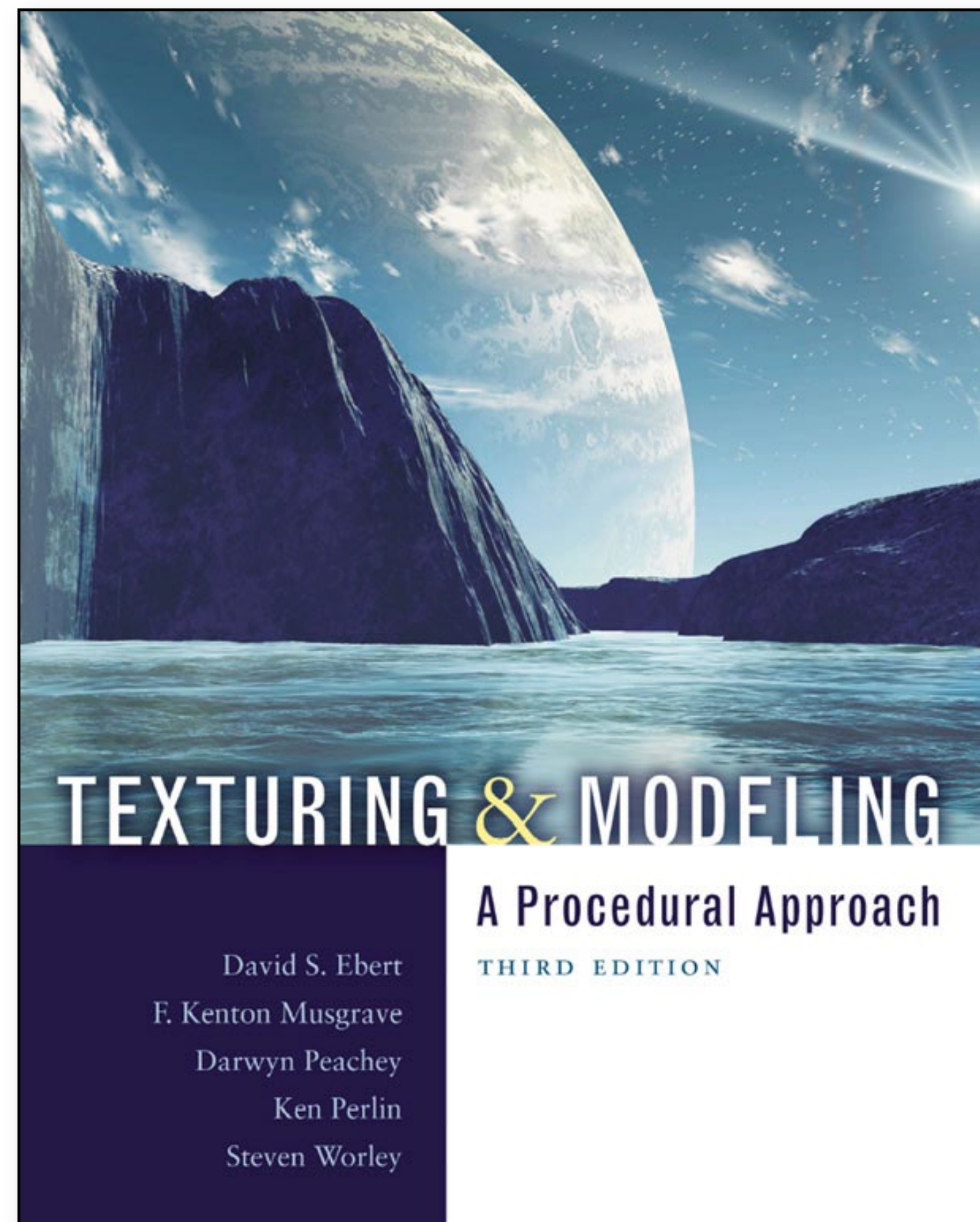
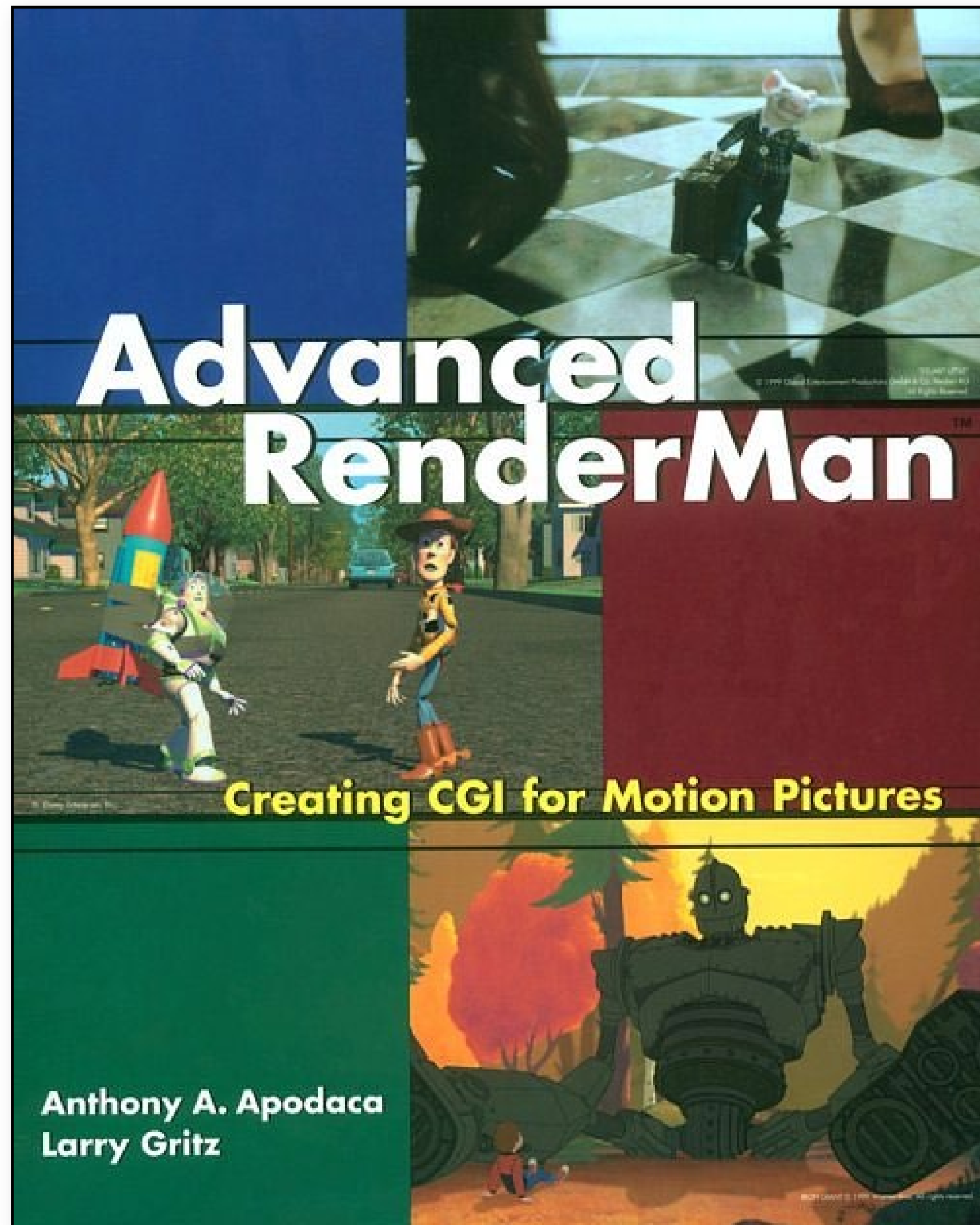
$f_2 - f_1$

Worley Noise



fractal f1-f4 combinations

Other Resources



Demos

Amazing realtime demos using fractal noise:

- <http://www.iquilezles.org/www/articles/morenoise/morenoise.htm>
- <https://www.shadertoy.com/view/4ttSWf>
- <https://www.shadertoy.com/view/XttSz2>