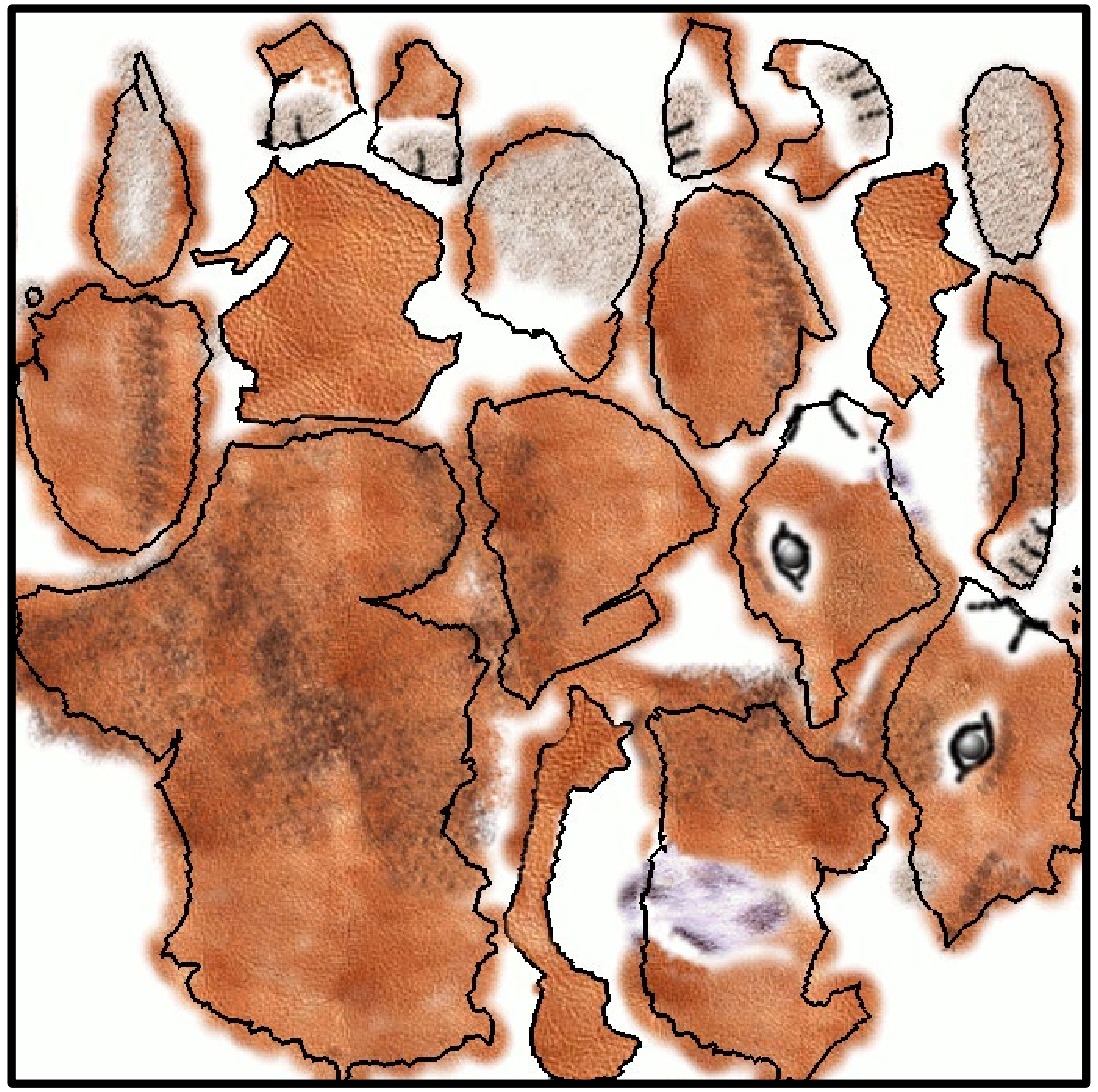
Texture mapping



15-468, 15-668, 15-868
Physics-based Rendering
Spring 2025, Lecture 4

http://graphics.cs.cmu.edu/courses/15-468

Course announcements

- Make sure to re-download TQ1 if you downloaded it before Tuesday.
- No recitation tomorrow (given TQ1 was shifted by a week).

Overview of today’s lecture

- Texture mapping.

Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

Why texture mapping?

Real objects have spatially varying details

Representing as geometry is correct, but tedious/expensive

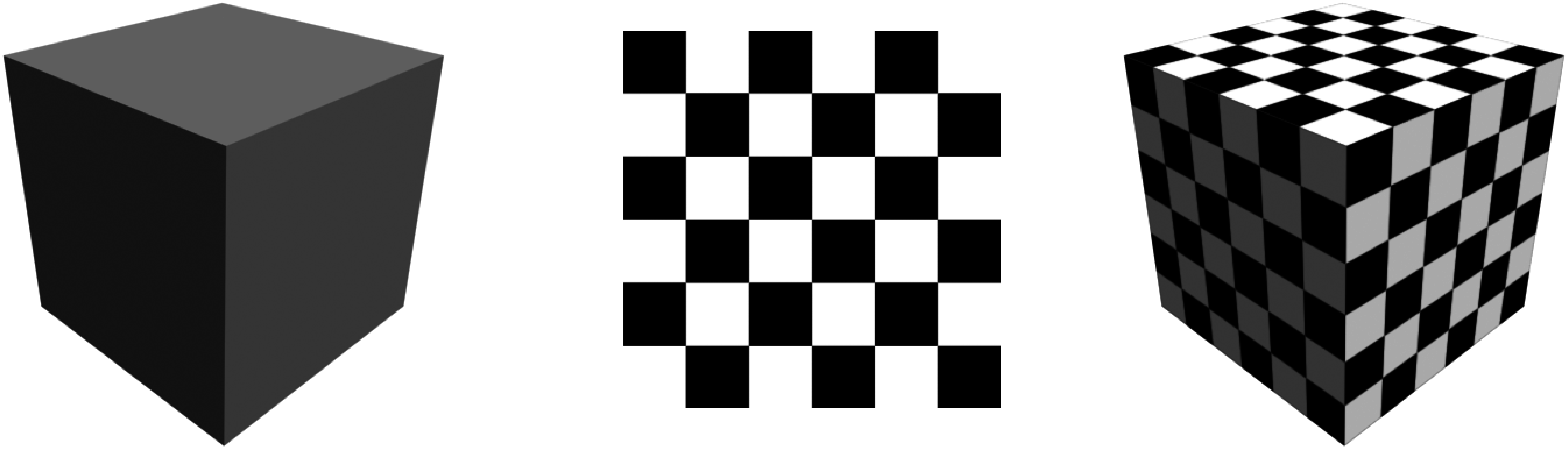
Why texture mapping?

Use simple geometry

Store varying properties in images

Map to objects

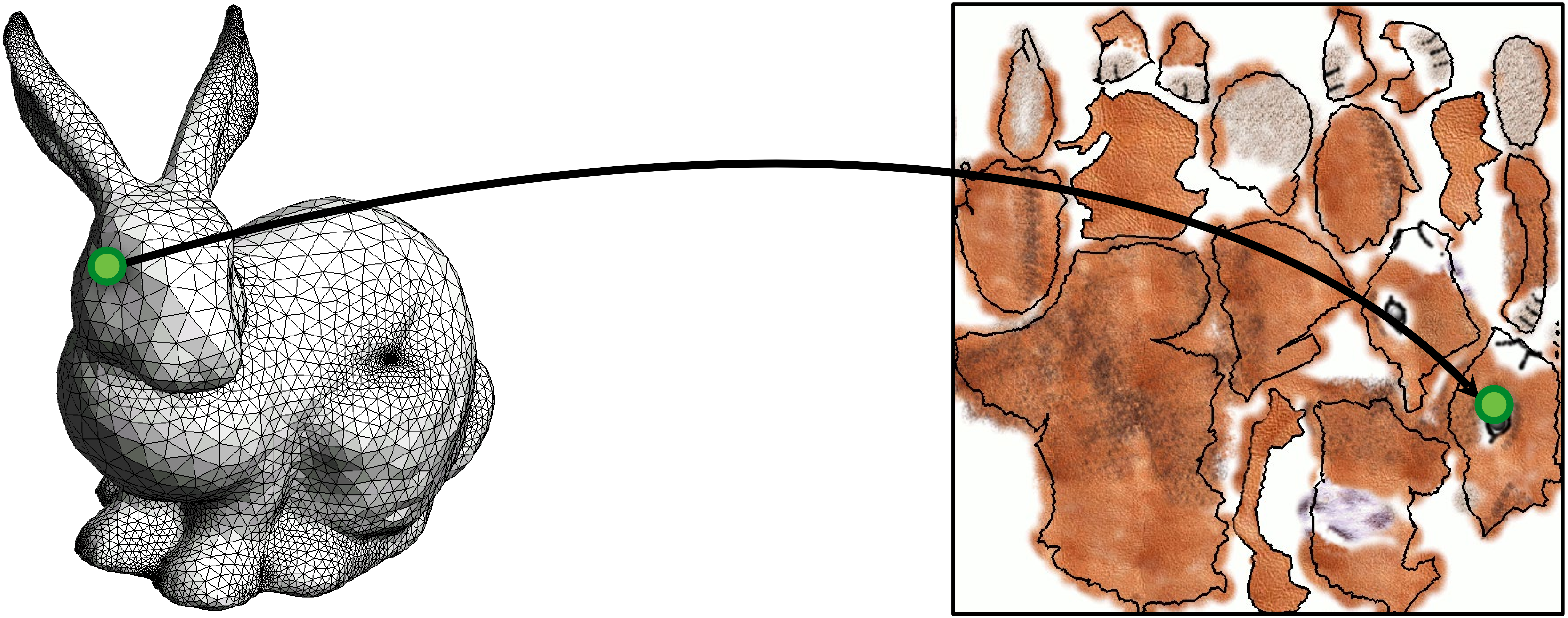
Proposed by
Ed Catmull in the 70s



After a slide by Fabio Pellacini

Texture mapping – definition

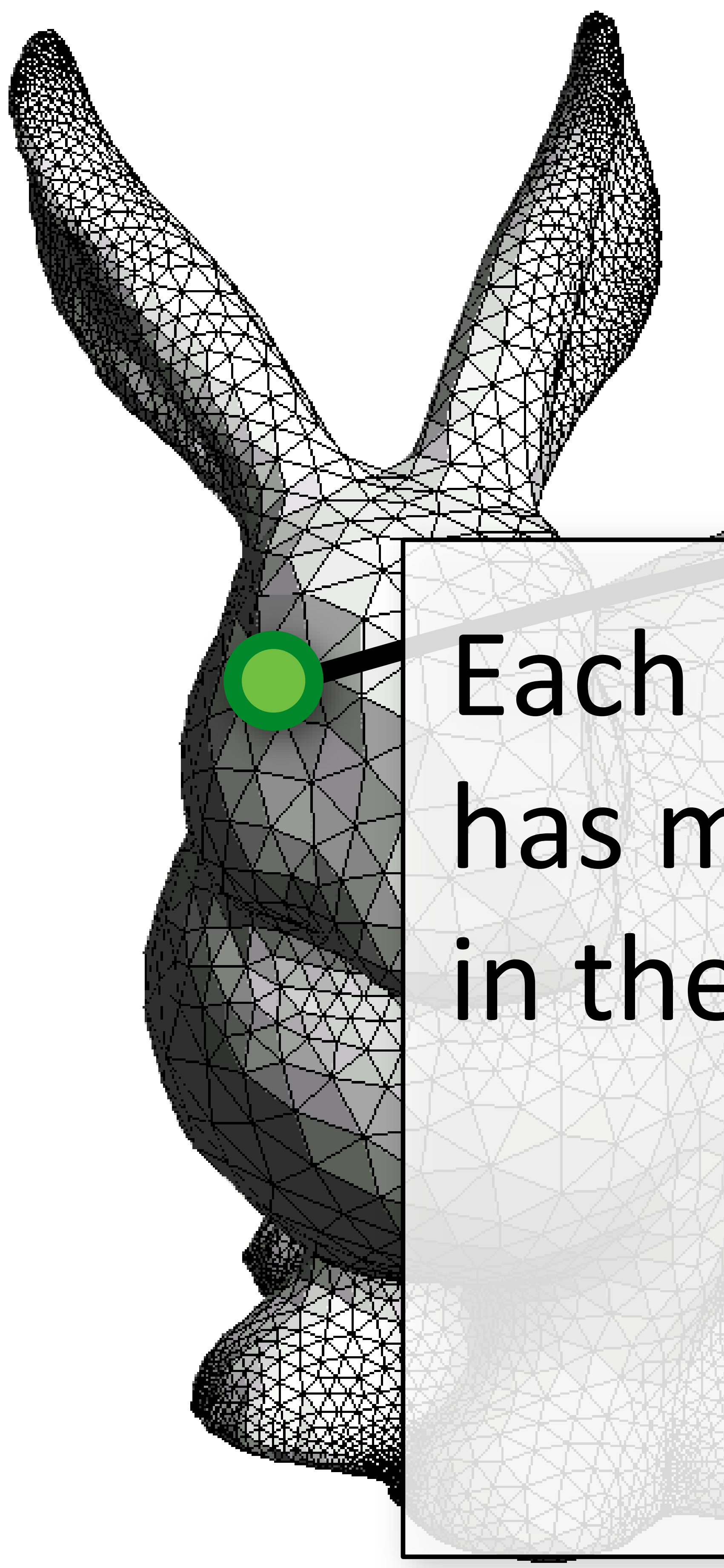
Mapping between the surface and the image



After a slide by Daniele Panozzo

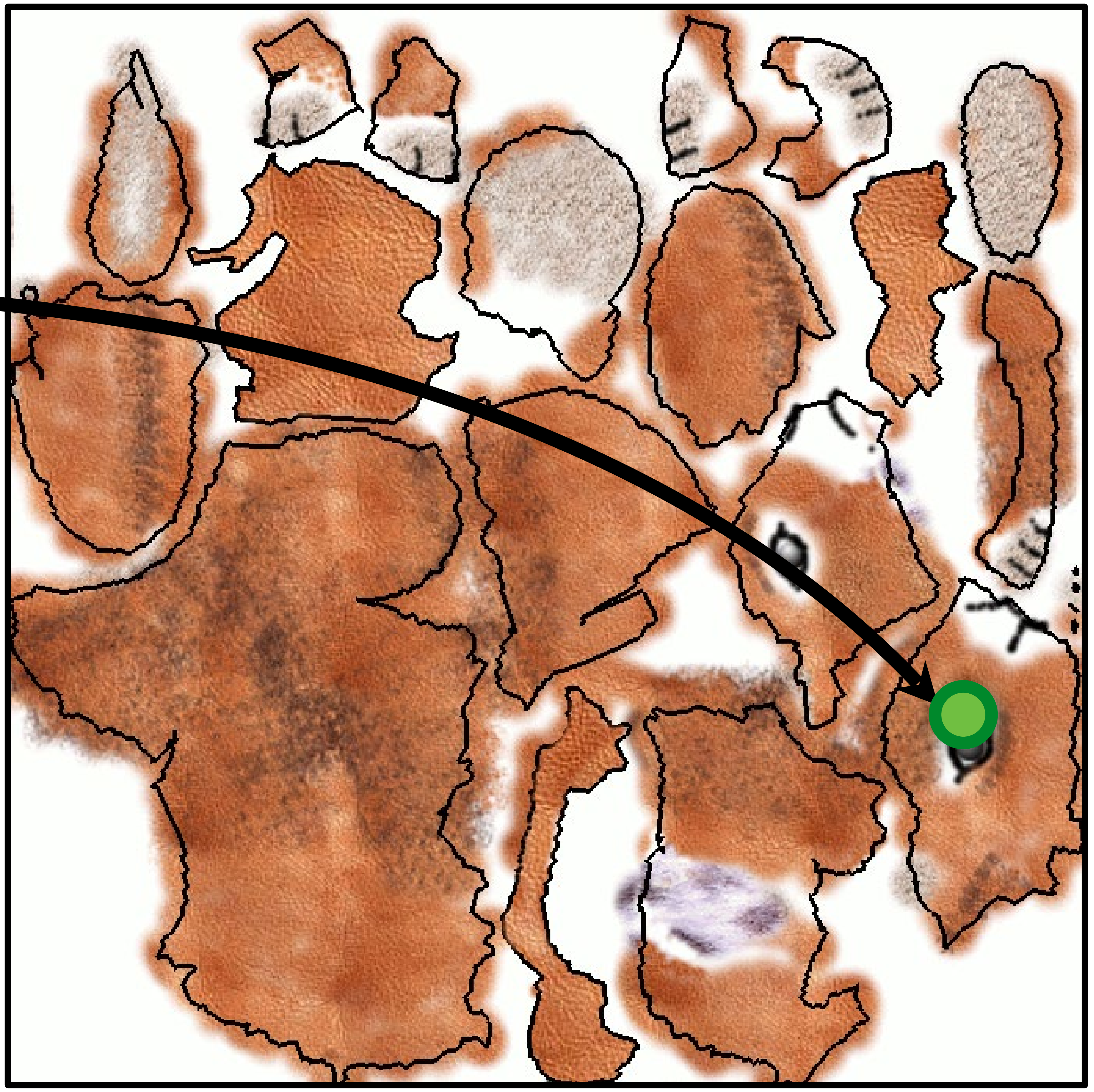
Texture mapping – definition

Mapping between the surface and the image



Each point (x,y,z) on the surface has mapped coordinates (u,v) in the texture image:

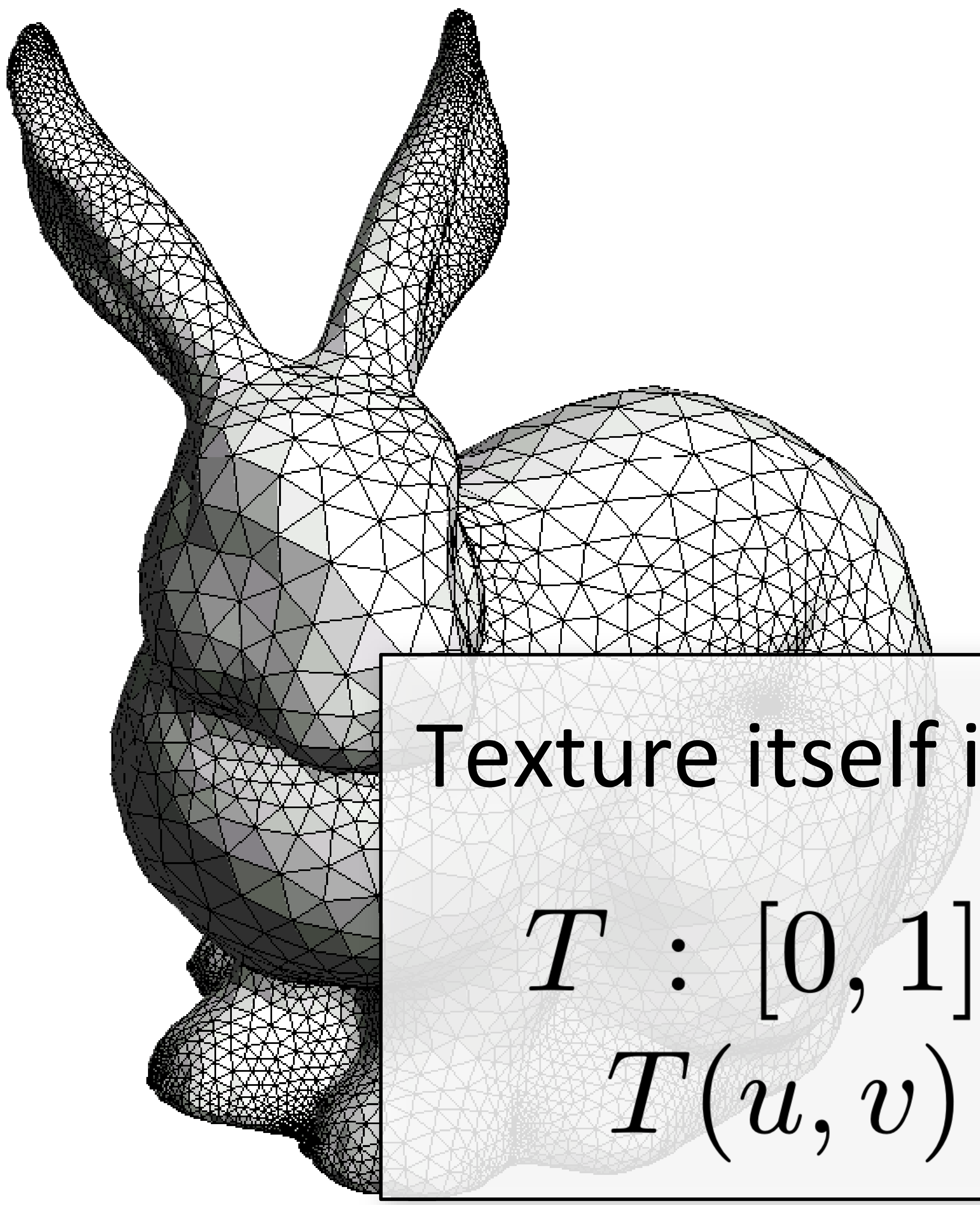
$$f : \mathbb{R}^3 \rightarrow [0,1]^2$$
$$f(x,y,z) = (u,v)$$



After a slide by Daniele Panozzo

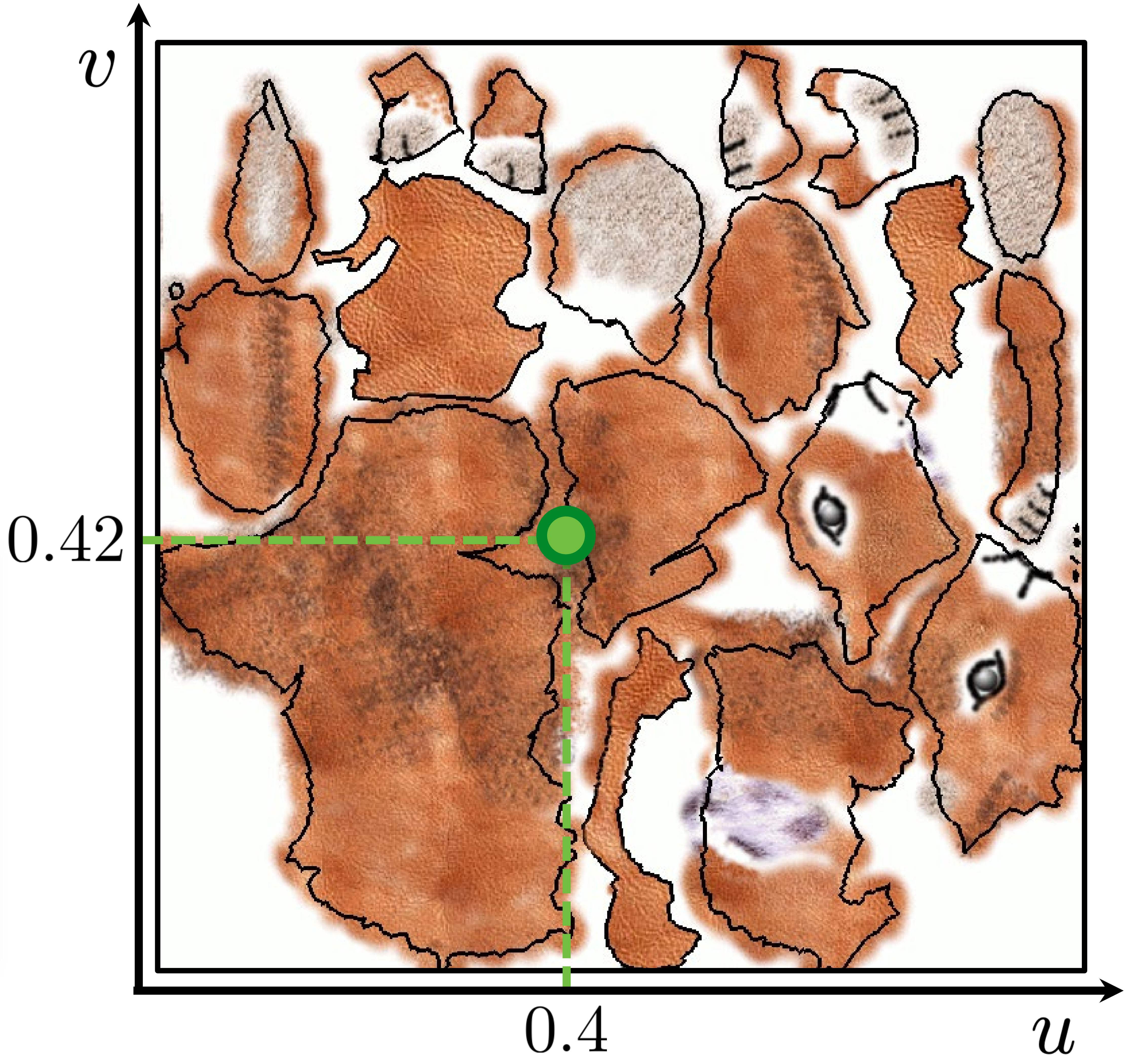
Texture mapping – definition

Mapping between the surface and the image



Texture itself is a function:

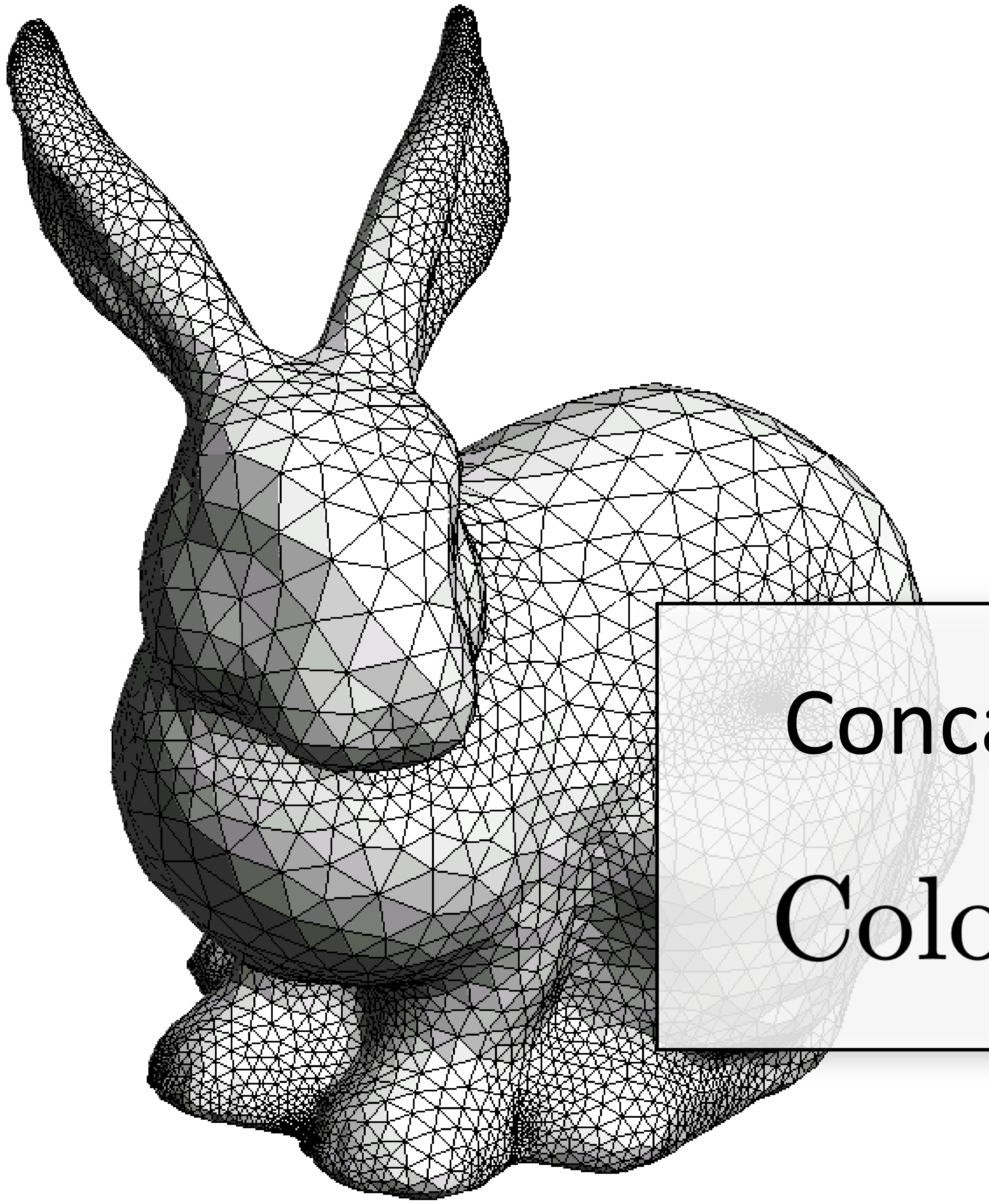
$$T\ :\ [0,1]^2 \rightarrow \mathrm{RGB}$$
$$T(u,v) = (r,g,b)$$

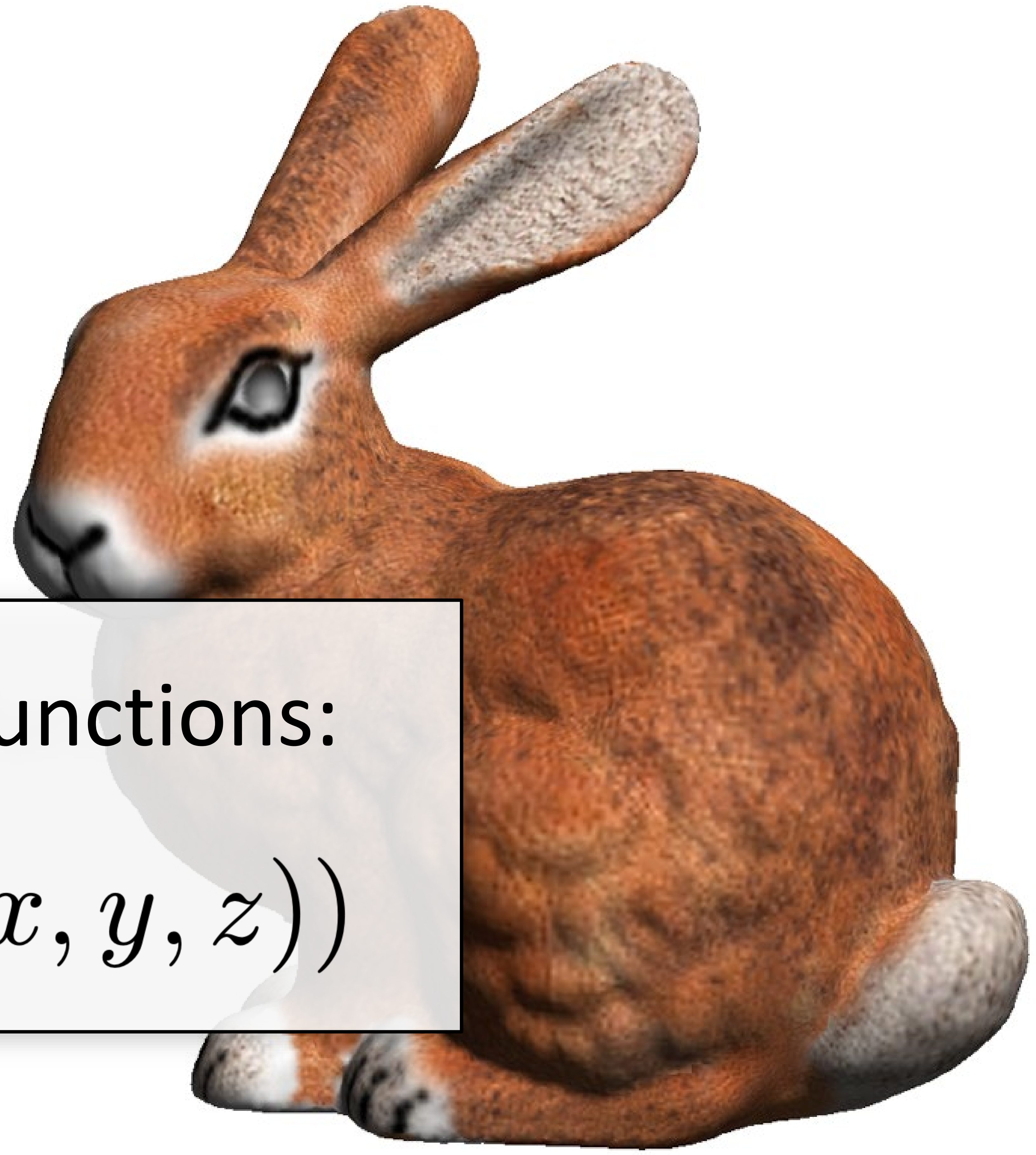


After a slide by Daniele Panozzo

Texture mapping – definition

Mapping between the surface and the image





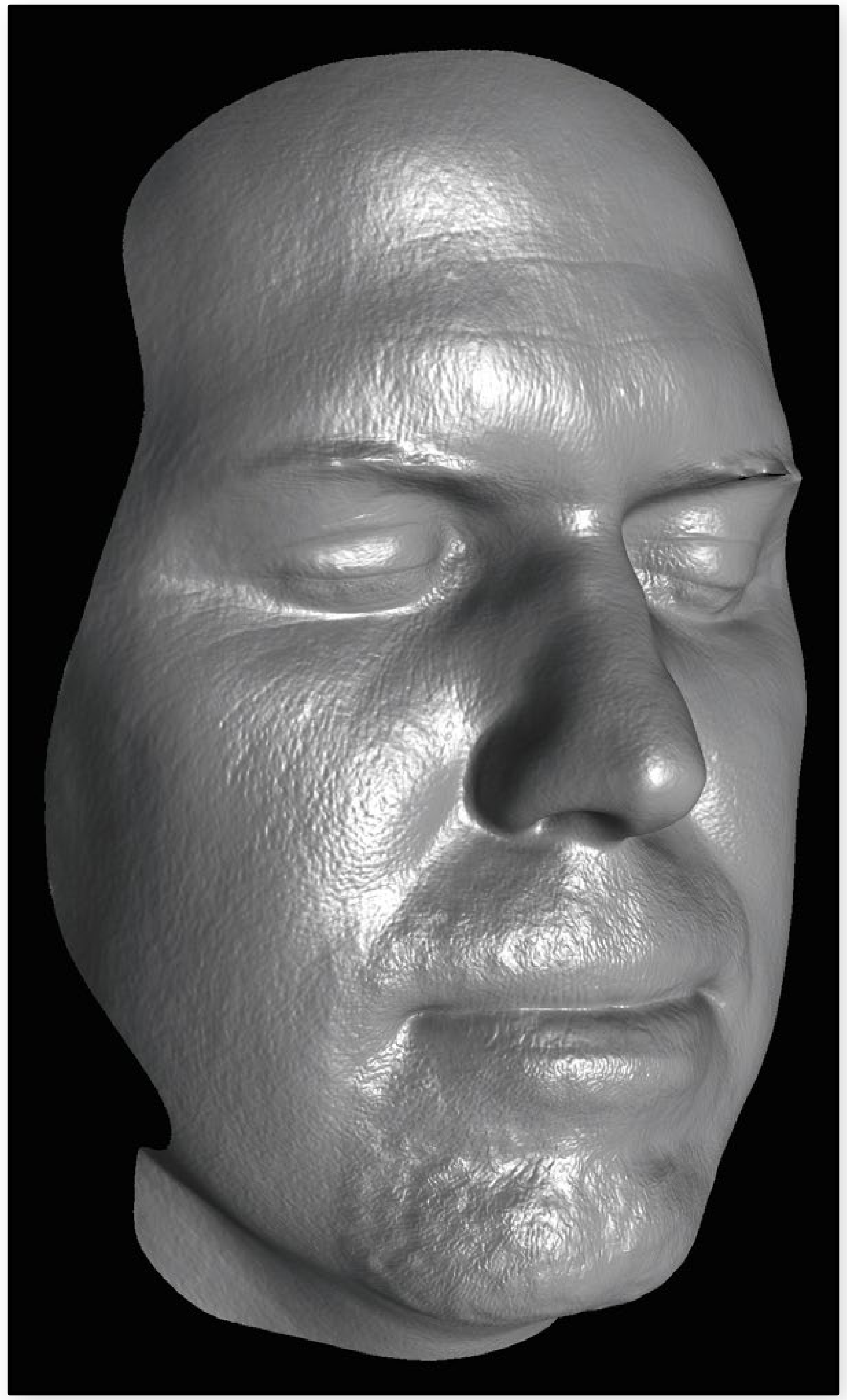
Concatenation of the two functions:

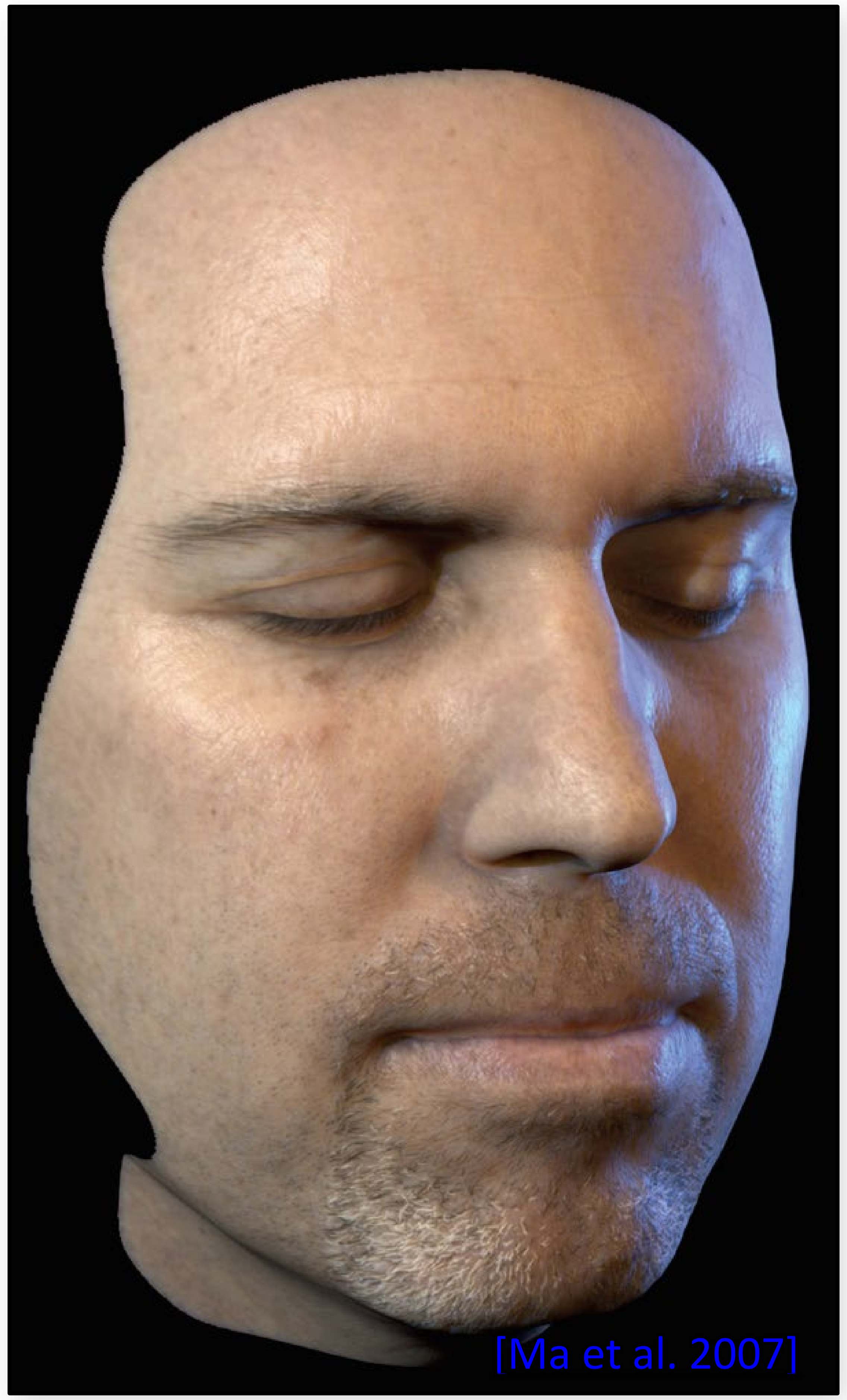
$$\mathrm{Color}(x, y, z) = T(f(x, y, z))$$

After a slide by Daniele Panozzo

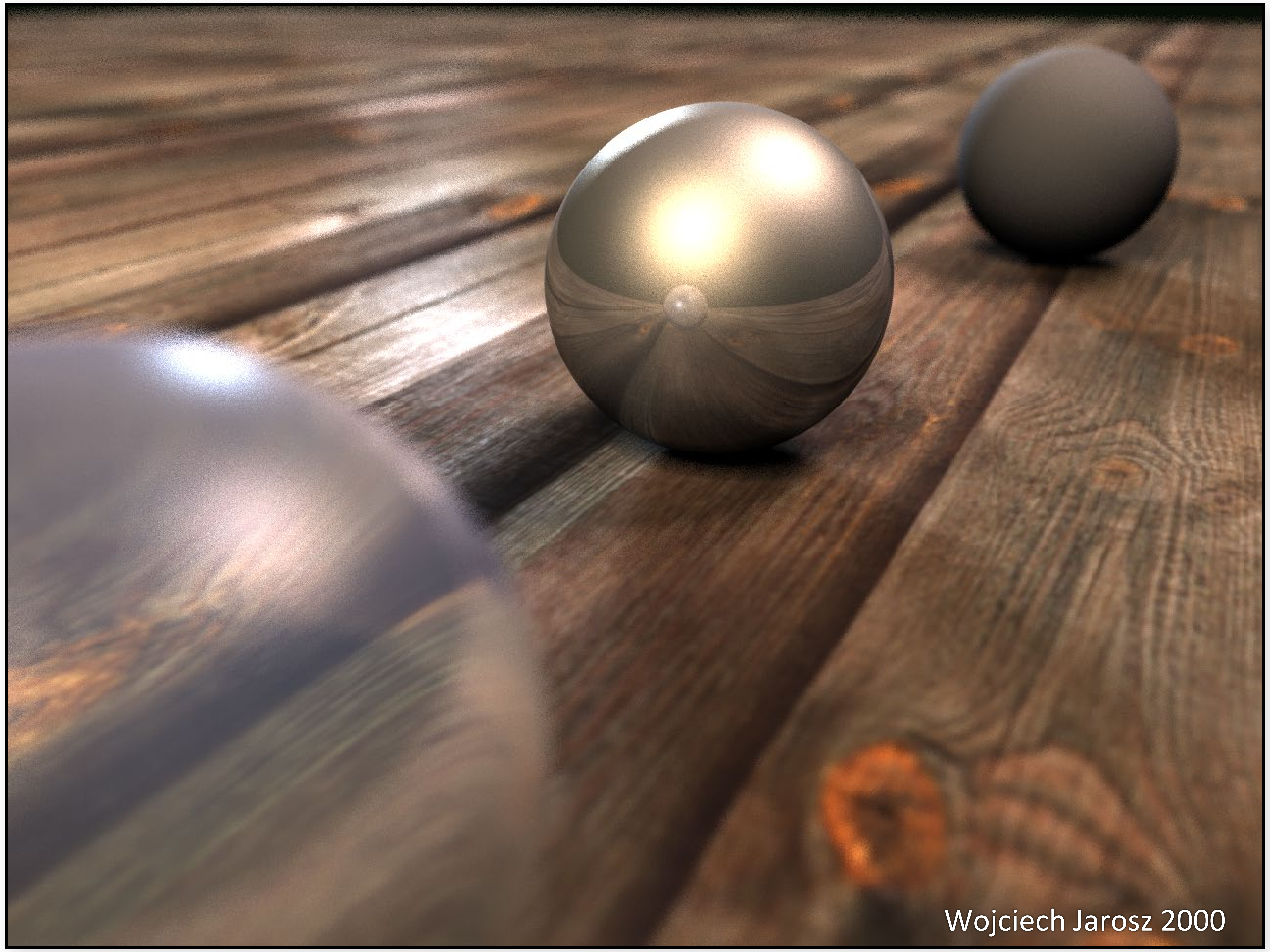
Why texture mapping?

Produces compelling results





[Ma et al. 2007]



Wojciech Jarosz 2000

Agenda

How do we map between surface and texture?

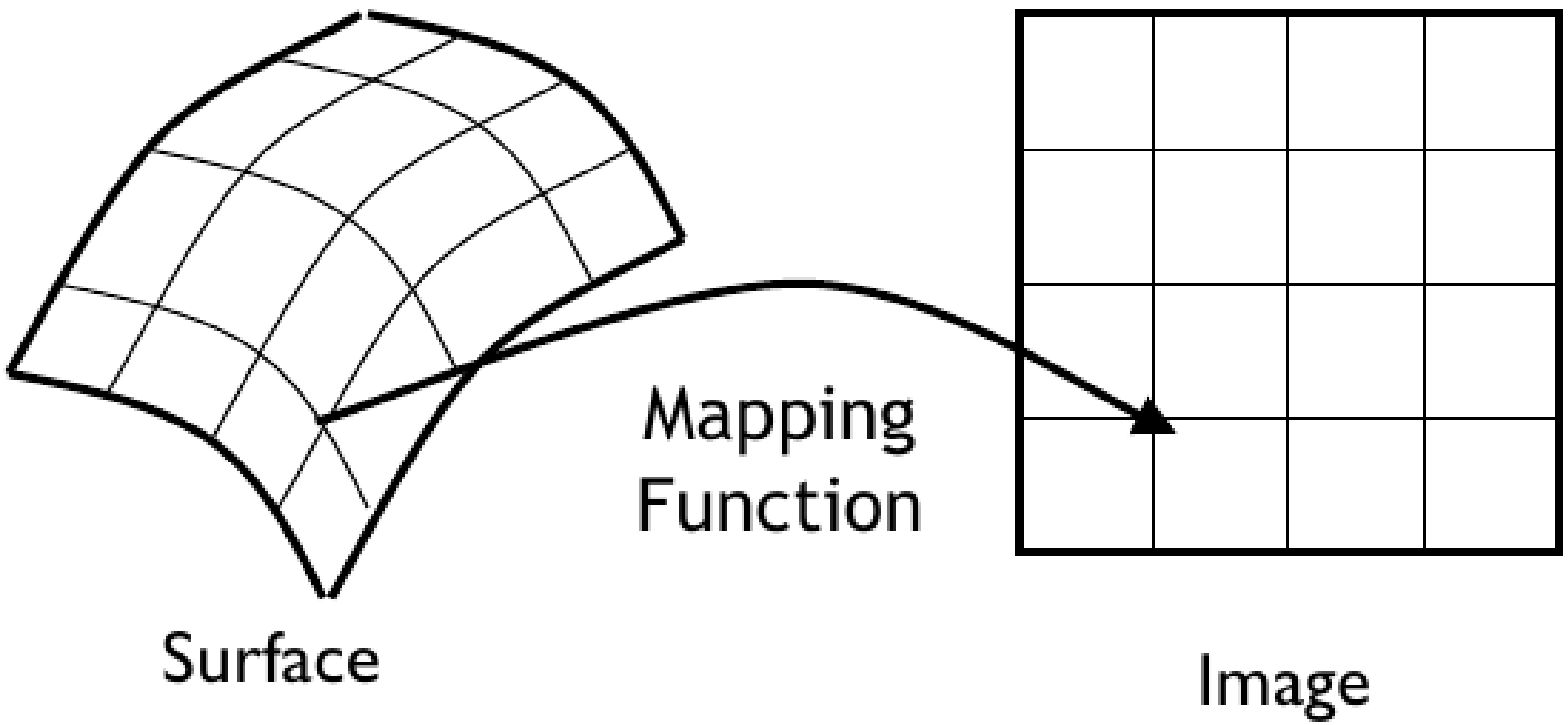
What do we map onto the surface?

Surface parameterization

A surface in 3D is a two-dimensional thing

How do we map a surface point to a point in a texture?

- Defining 2D coordinates is parameterizing the surface



After a slide by Fabio Pellacini

Surface parameterization

A surface in 3D is a two-dimensional thing

How do we map a surface point to a point in a texture?

- Defining 2D coordinates is parameterizing the surface

Examples:

- cartesian coordinates on a rectangle (or other planar shape)
- cylindrical coordinates (θ, y) on a cylinder
- latitude and longitude on the Earth's surface
- spherical coordinates (θ, ϕ) on a sphere

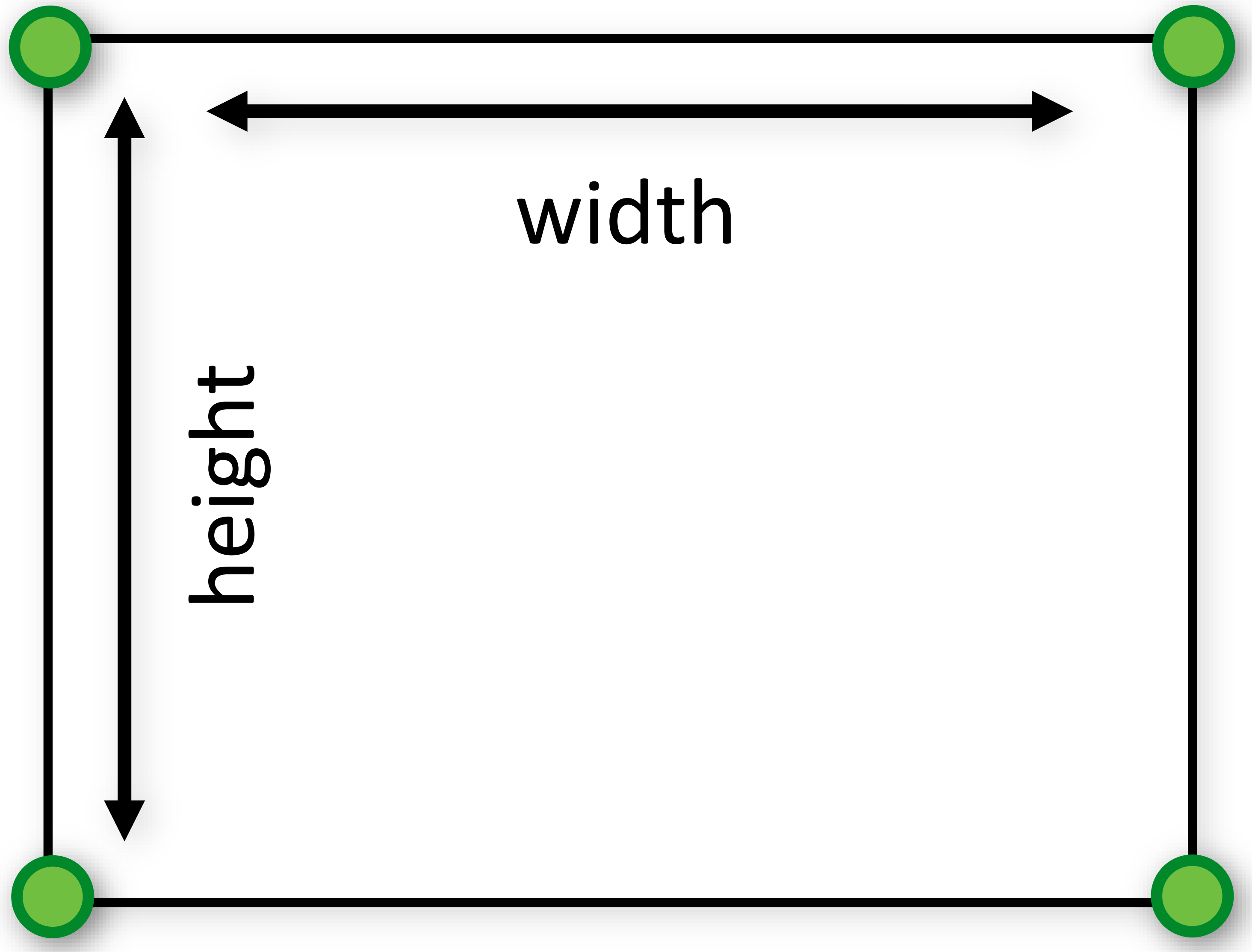
After a slide by Steve Marschner

A rectangle

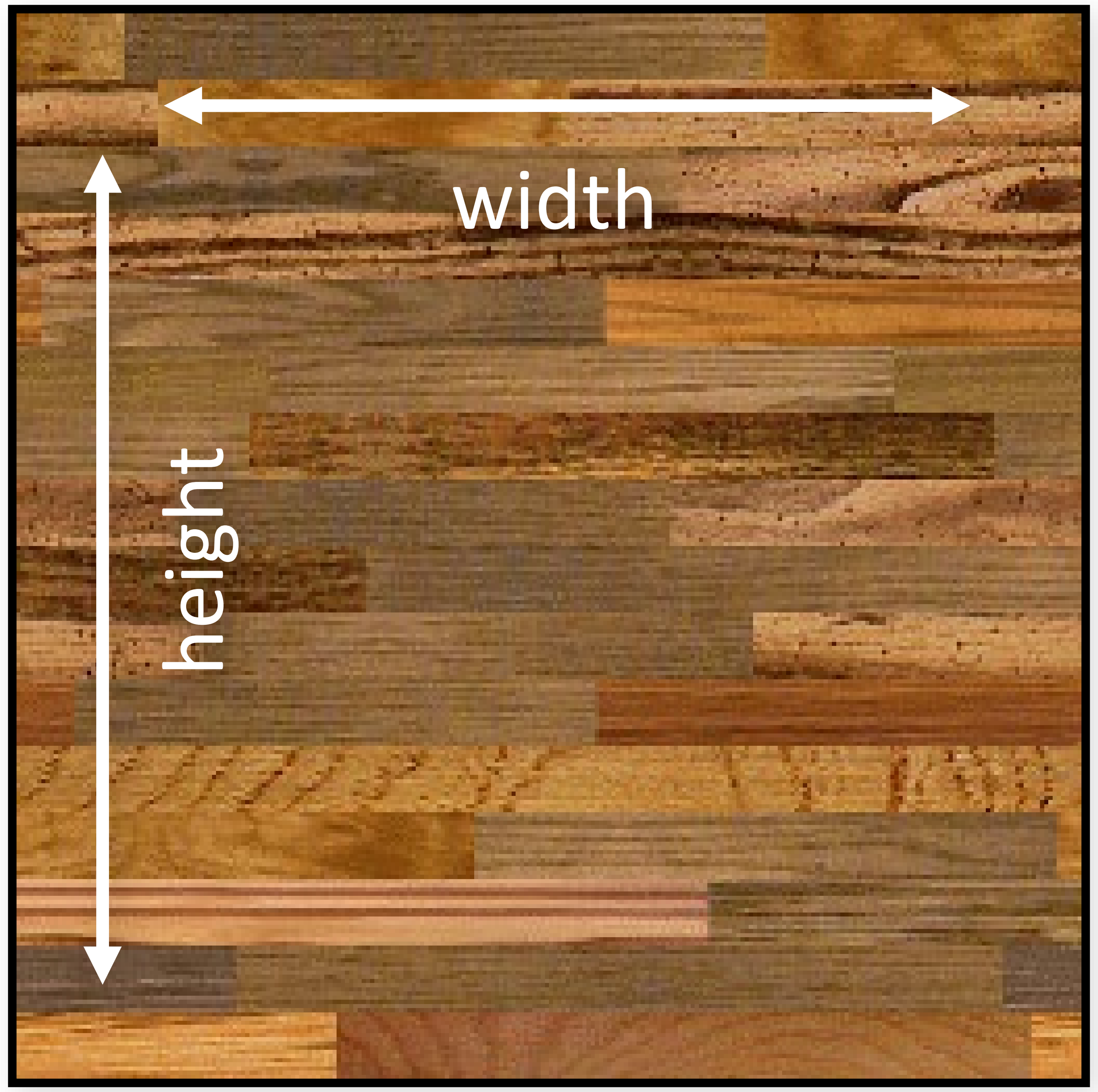
Image can be mapped directly, unchanged

Texturing a rectangle

Image can be mapped directly, unchanged



Object Space

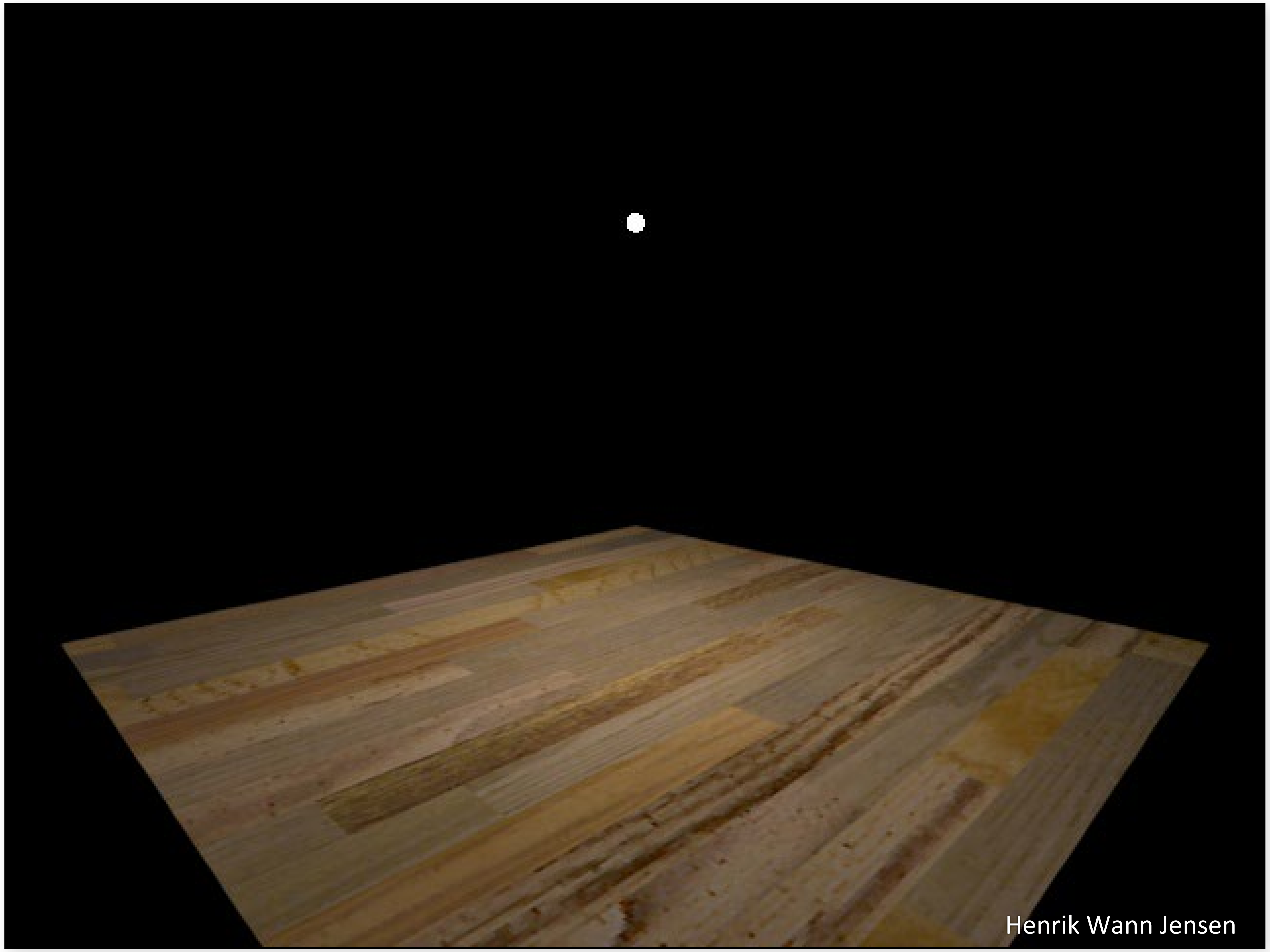


Texture Space

Object Space

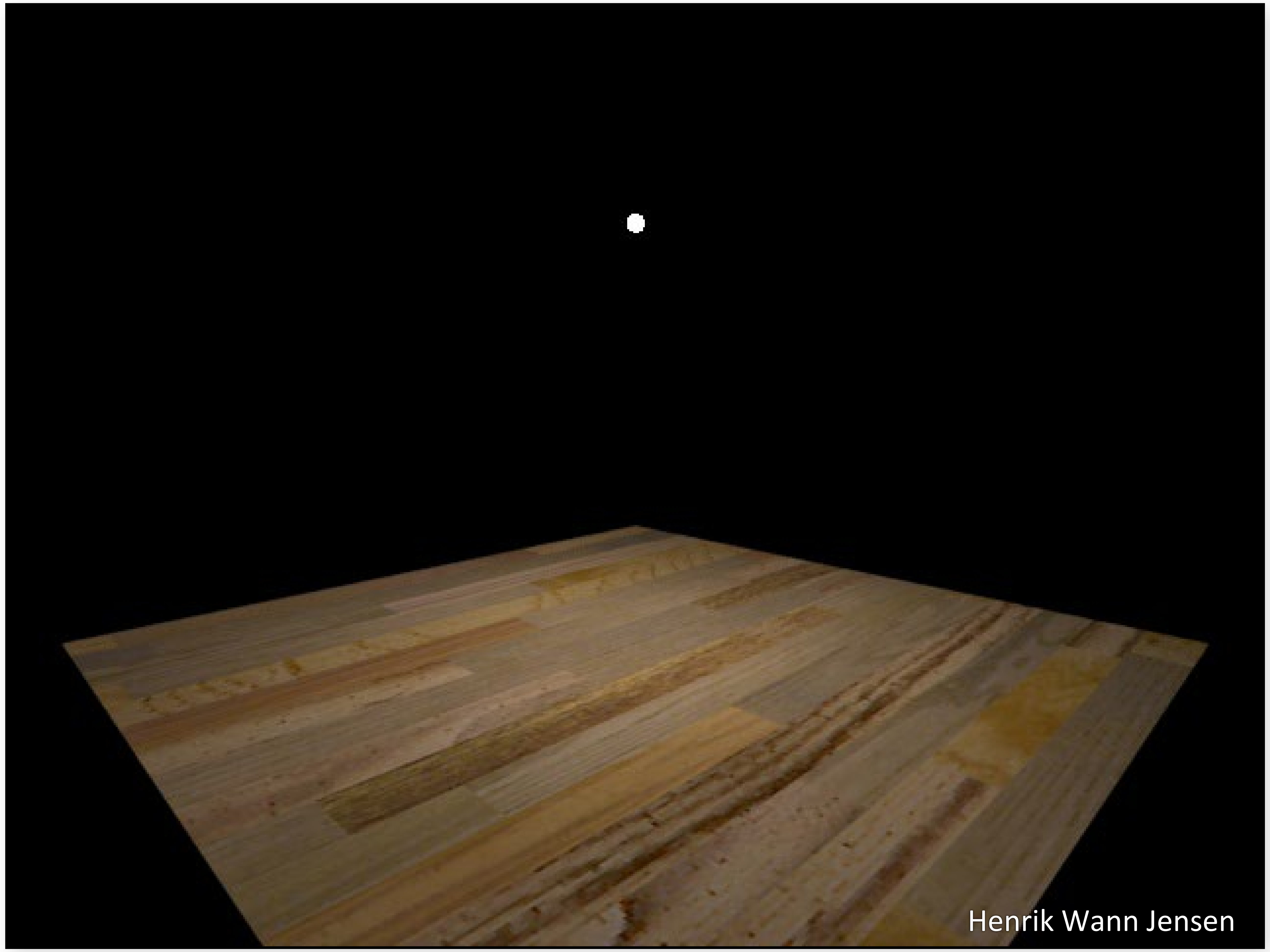
A textured rectangle

albedo = T(u,v)



Transformation of shape

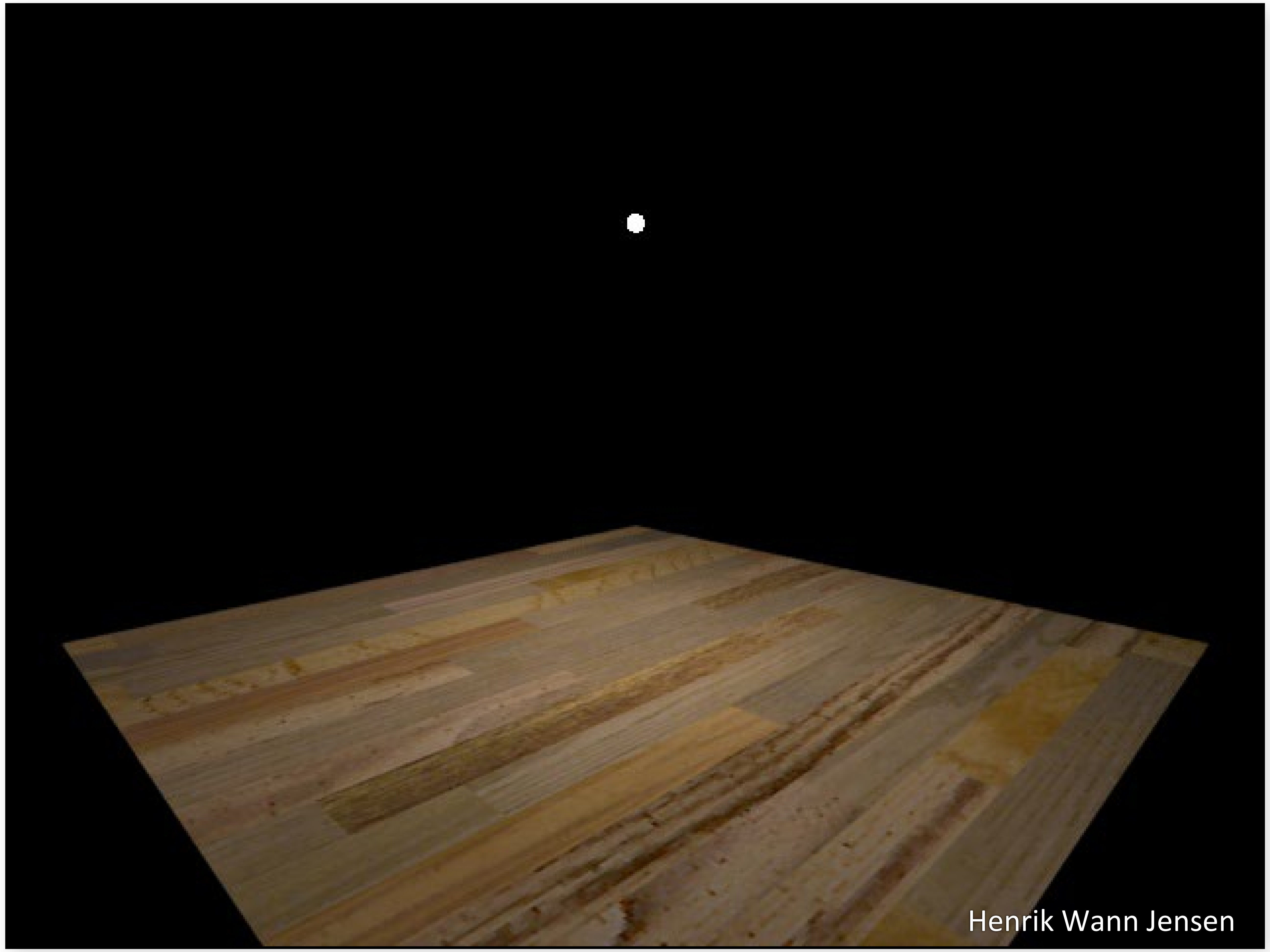
albedo=$T([u,v](x,y,z))$

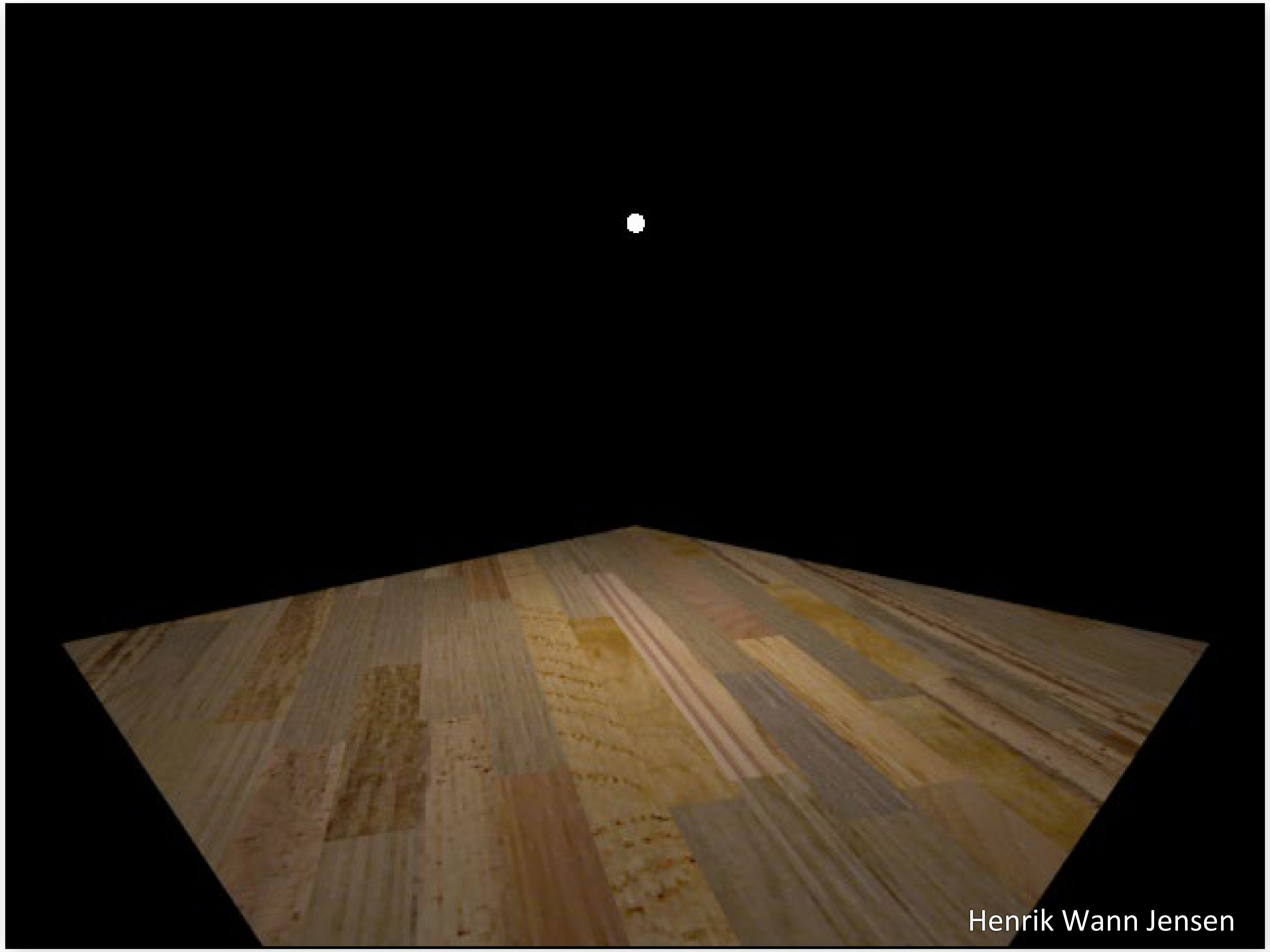


albedo=$T([u,v](\text{transform}^{-1}(x,y,z)))$

Transformation of texture

albedo=$T([u,v](x,y,z))$ albedo=$T(transform([u,v](x,y,z)))$



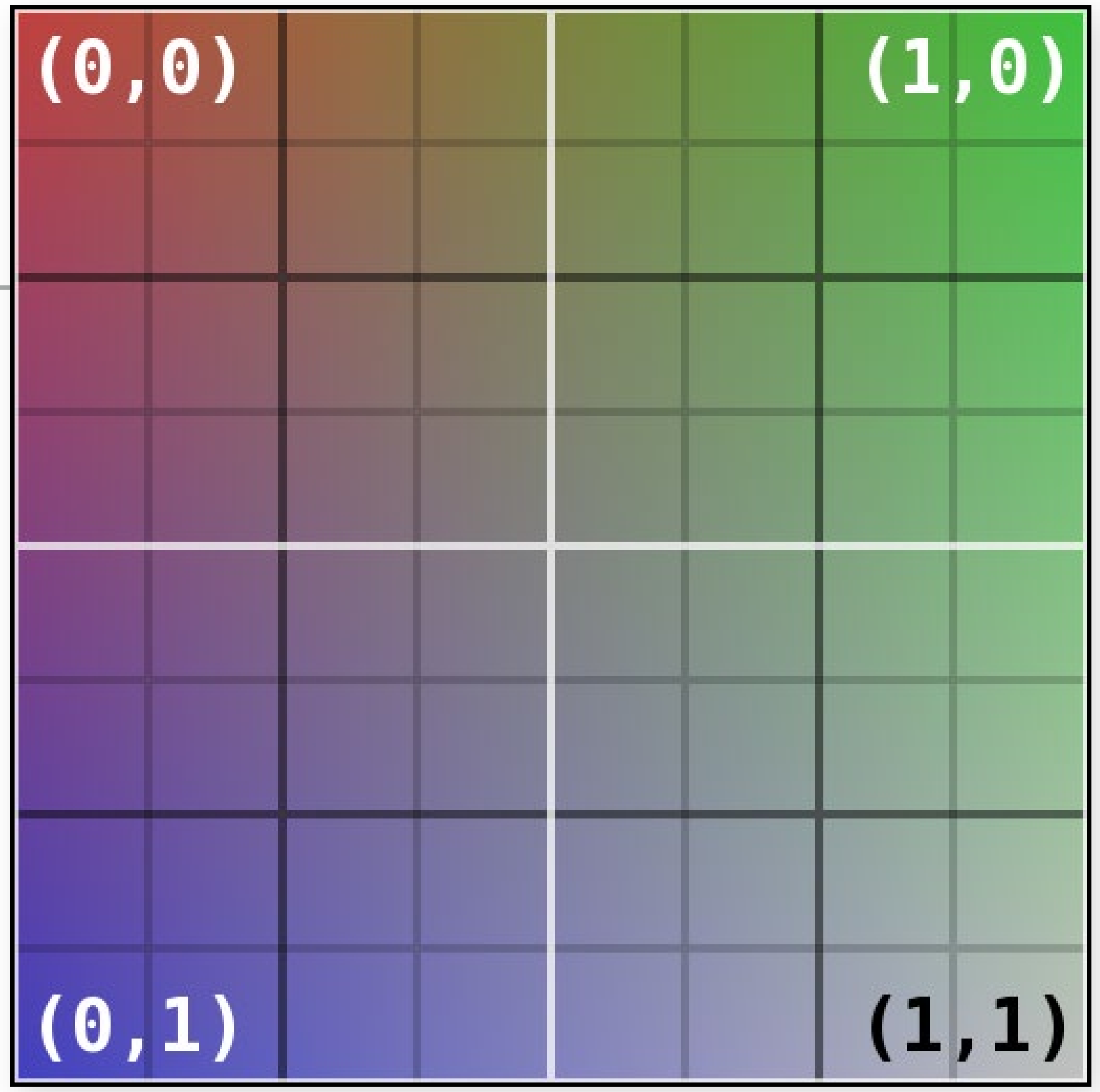


What if the object is not rectangular?

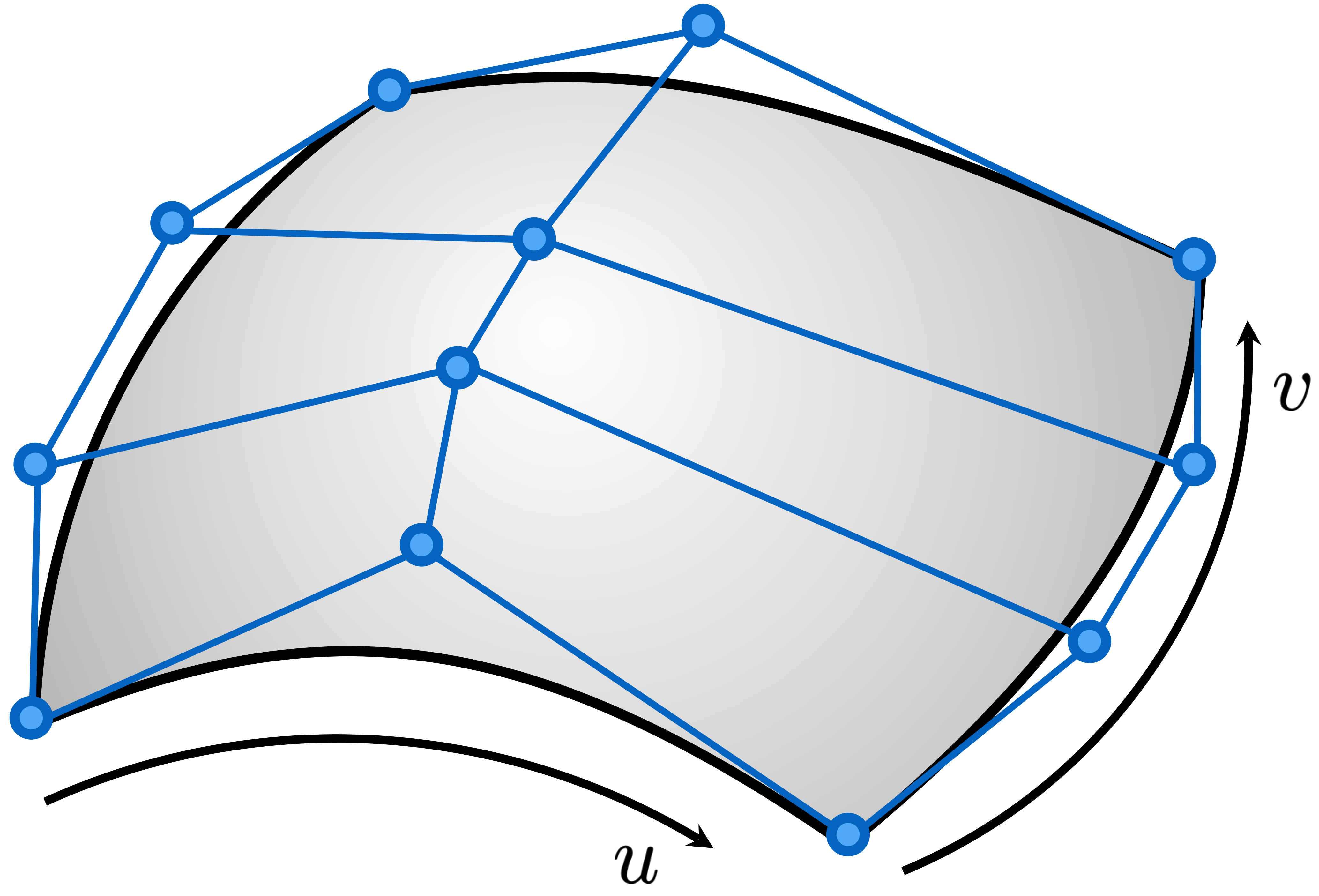
Need to determine mapping from 3D coordinates to 2D texture coordinates

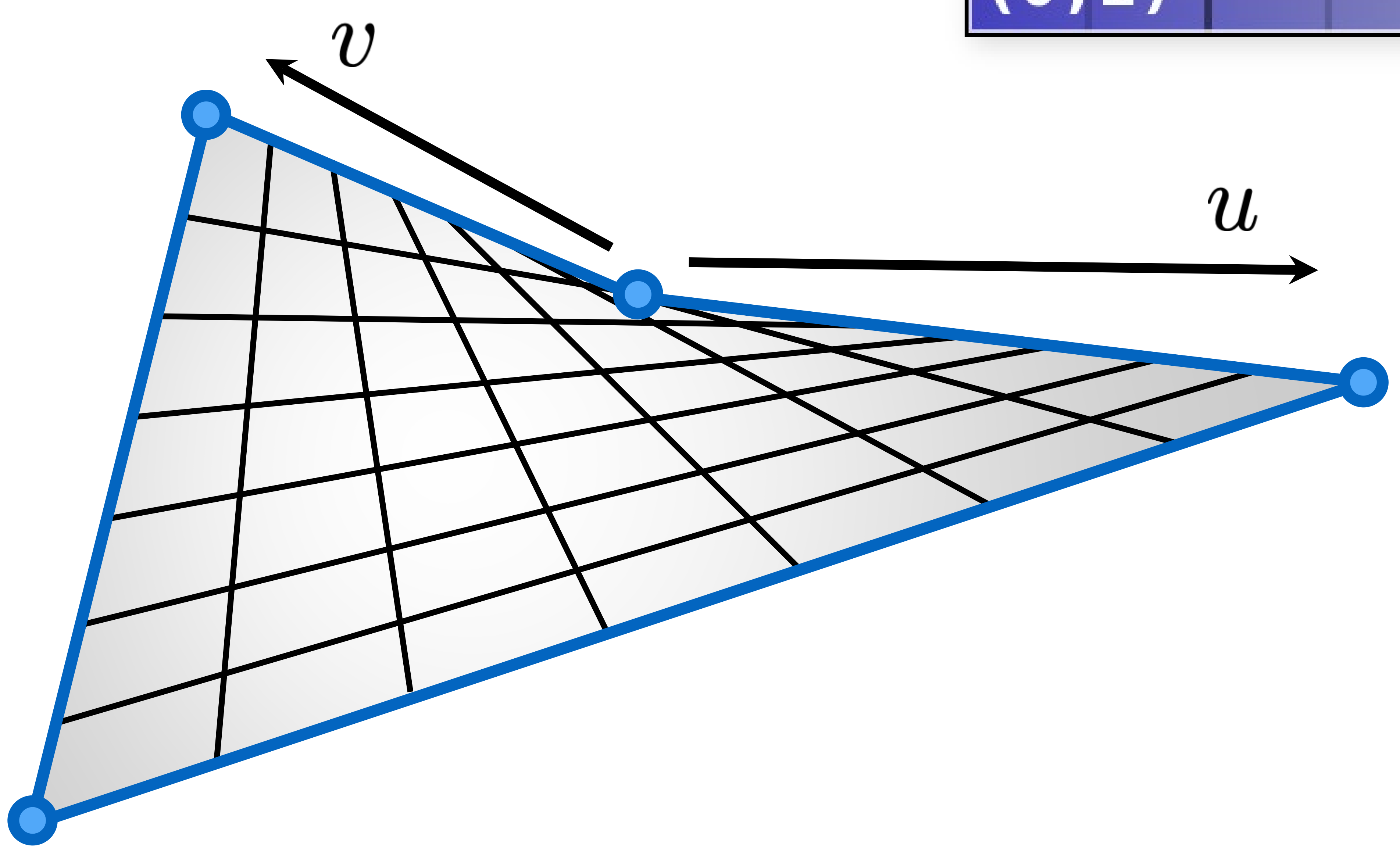
- Parametric surfaces
- Projection mapping
- UV mapping

Parametric surfaces



Parametric surfaces have a natural 2D coordinate system.





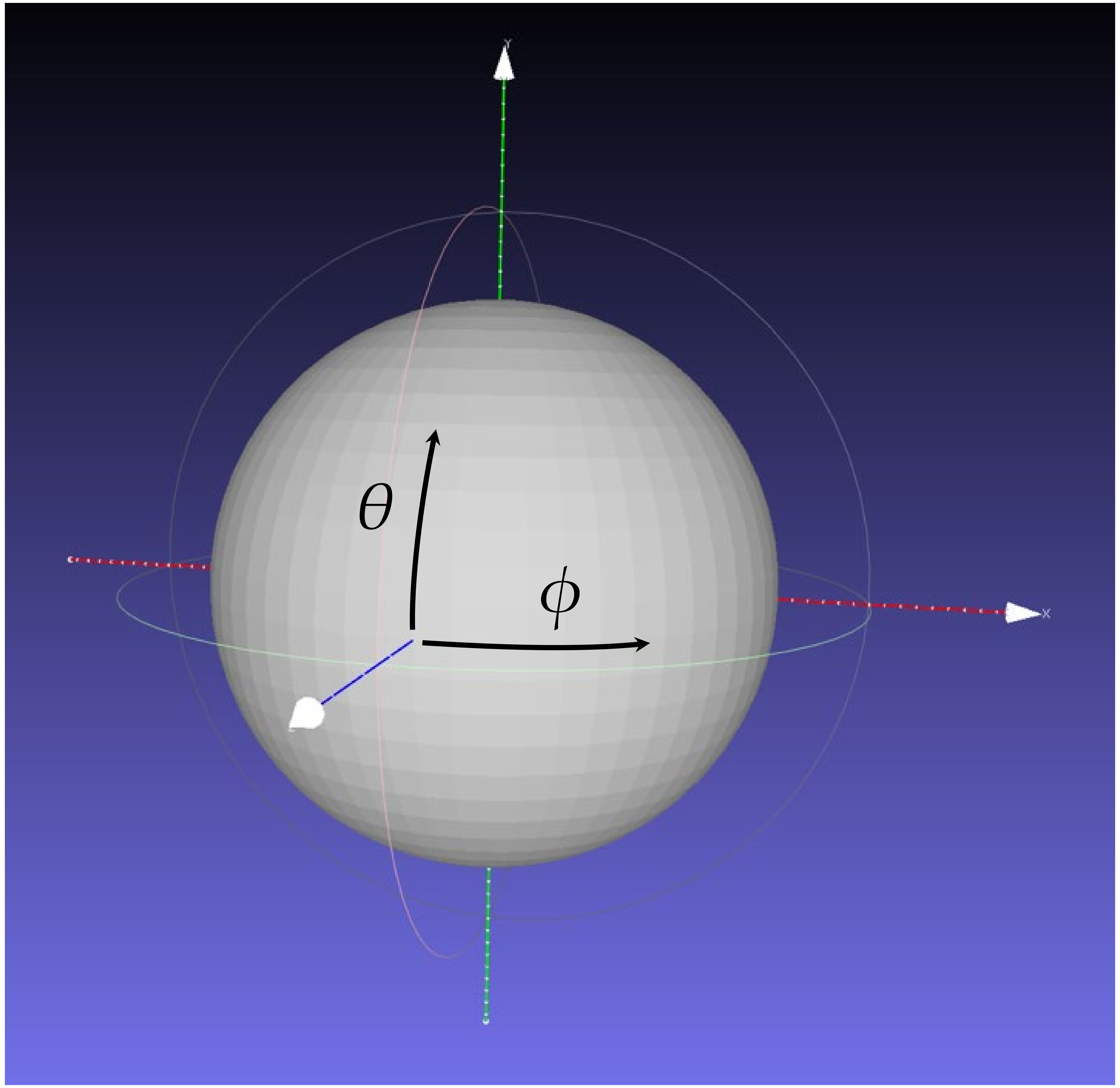
Can also define a sphere parametrically

Position:

$$x = \cos\theta \sin\phi$$

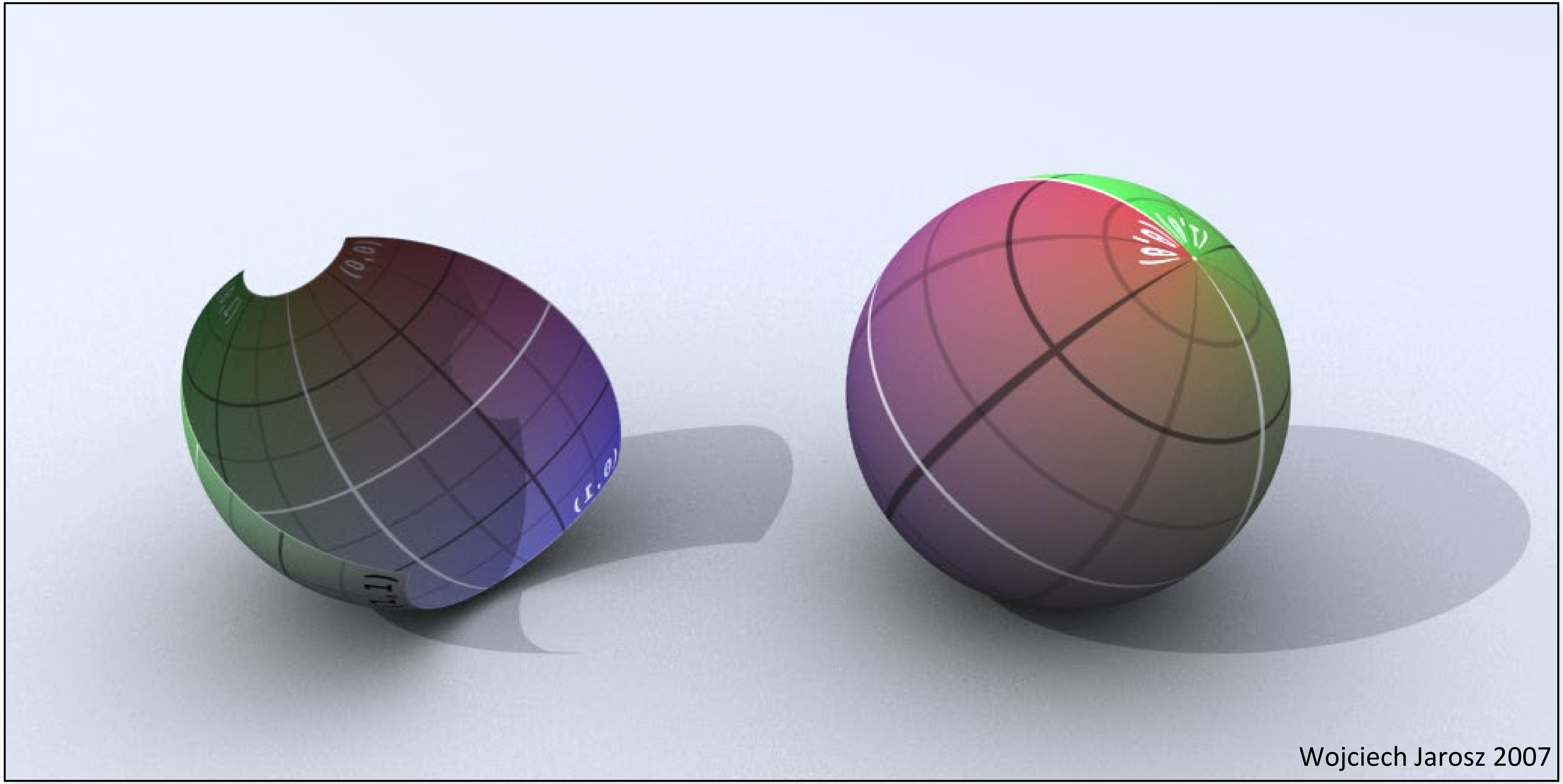
$$y = \sin\theta$$

$$z = \cos\theta \cos\phi$$



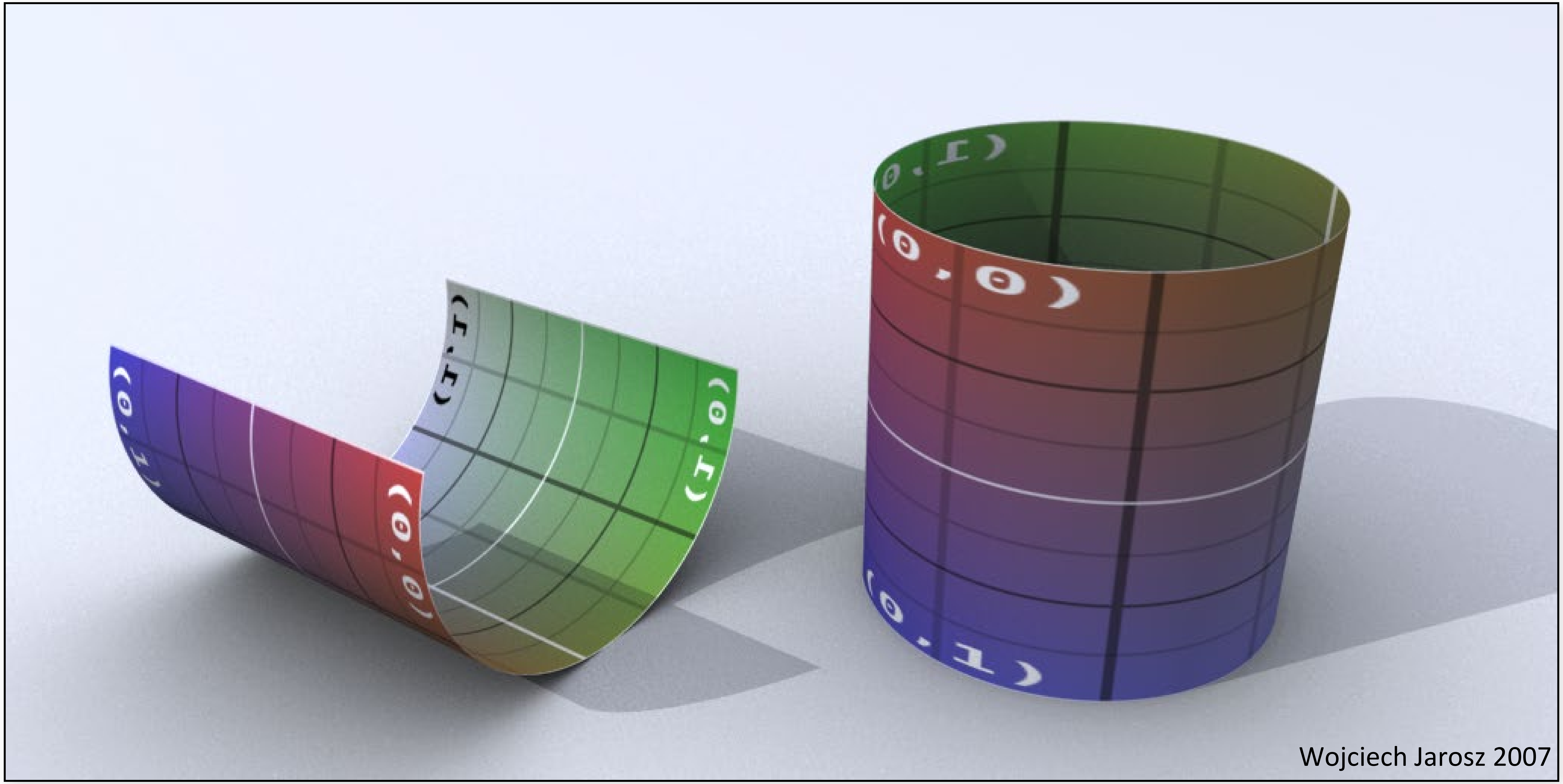
After a slide by Steve Marschner

Parametric spheres



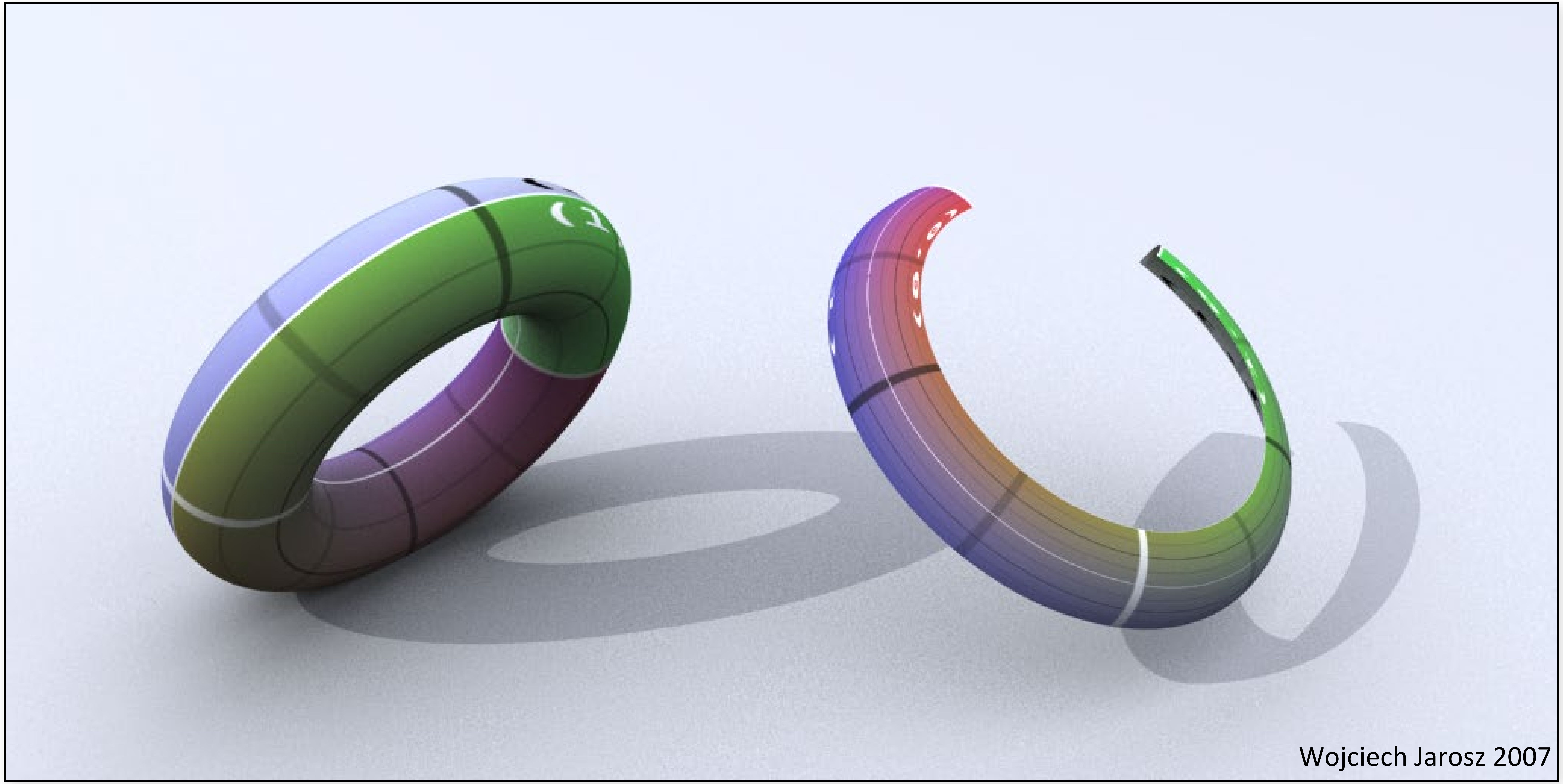
Wojciech Jarosz 2007

Parametric cylinders



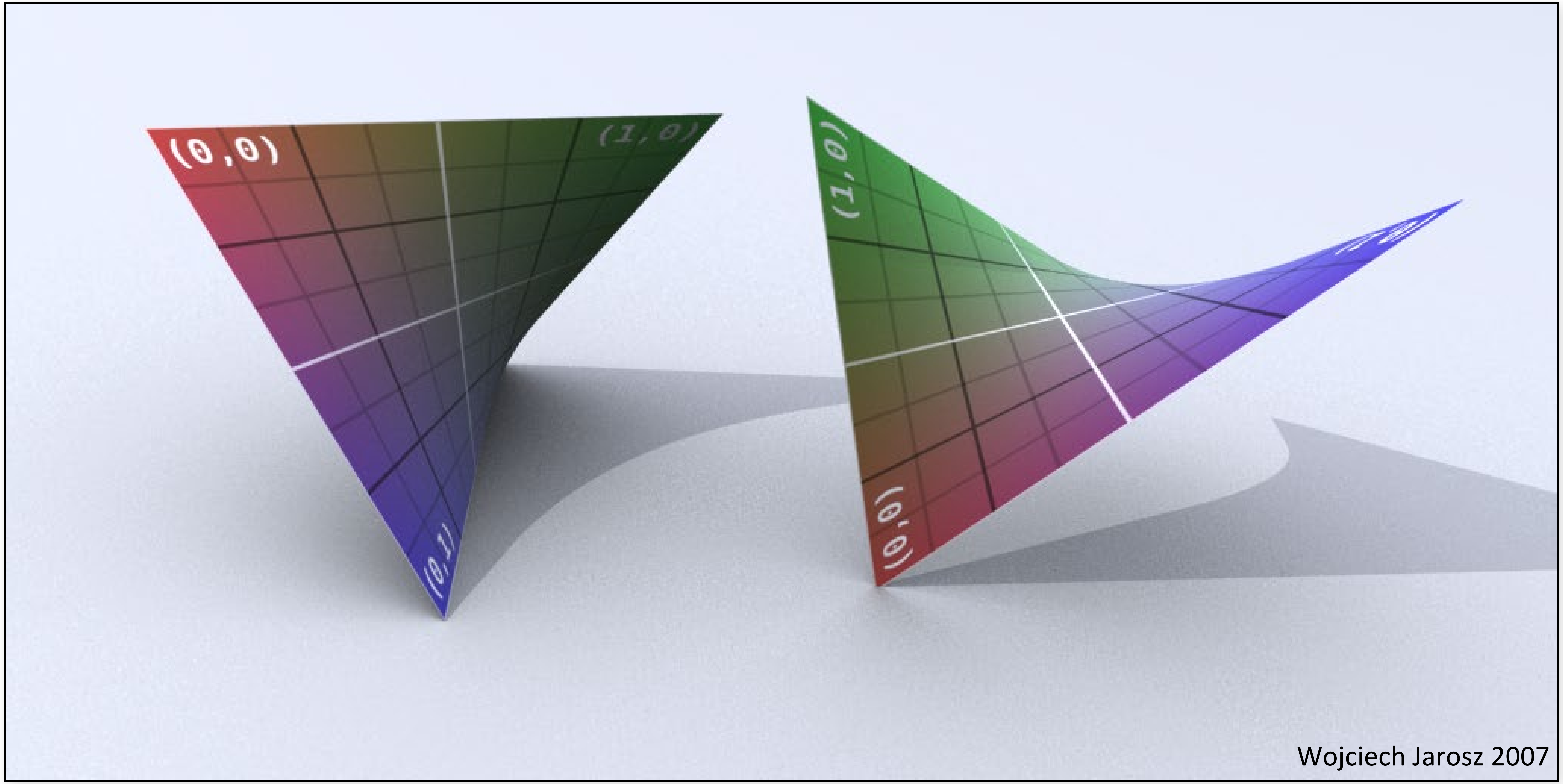
Wojciech Jarosz 2007

Parametric toruses



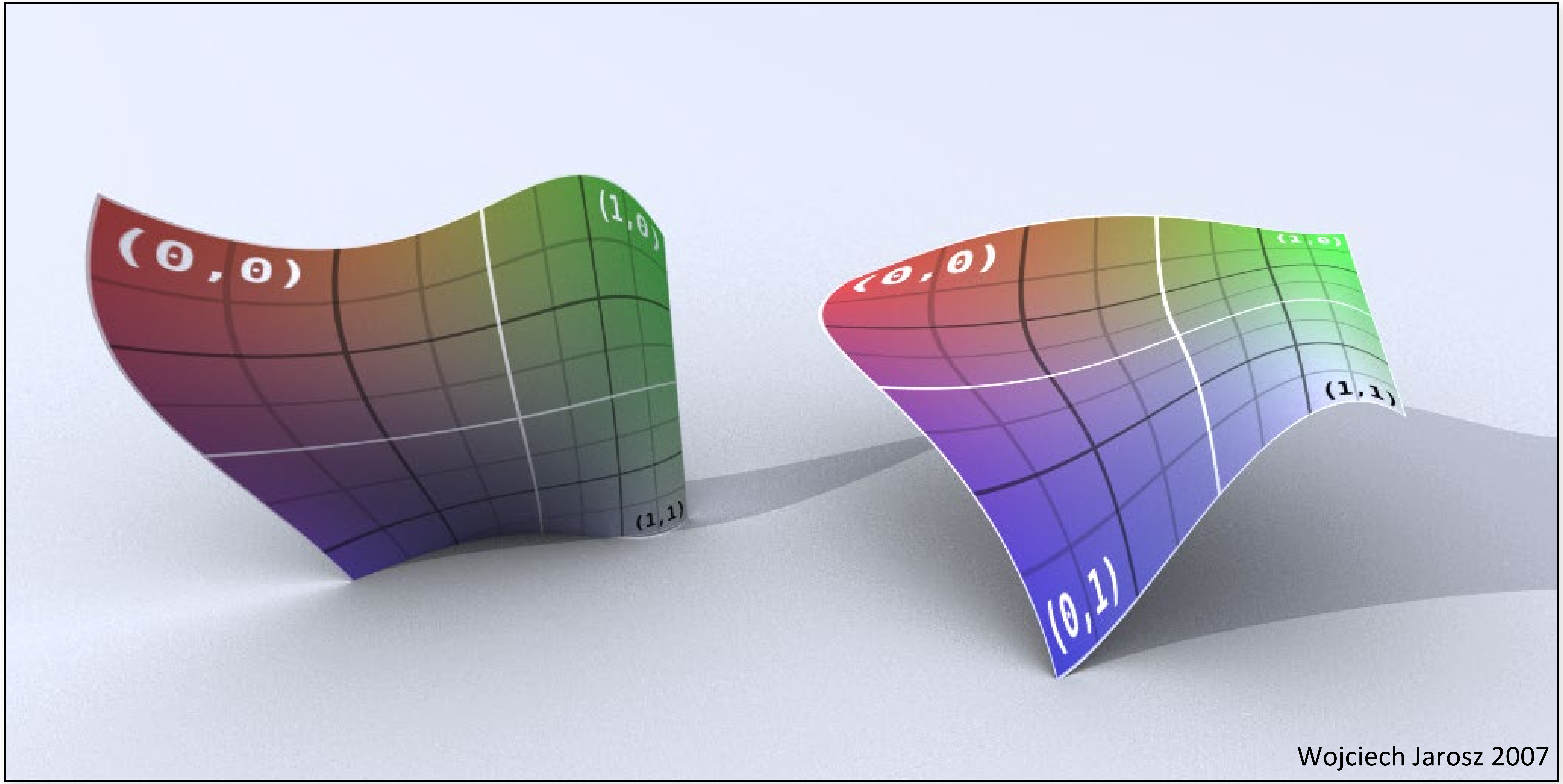
Wojciech Jarosz 2007

Bilinear patches



Wojciech Jarosz 2007

Bicubic patches



Creating parameterizations

For non-parametric surfaces it is trickier

Need to create a parametrization!

- Projection mapping
- UV mapping

[Wolfe / SG97]

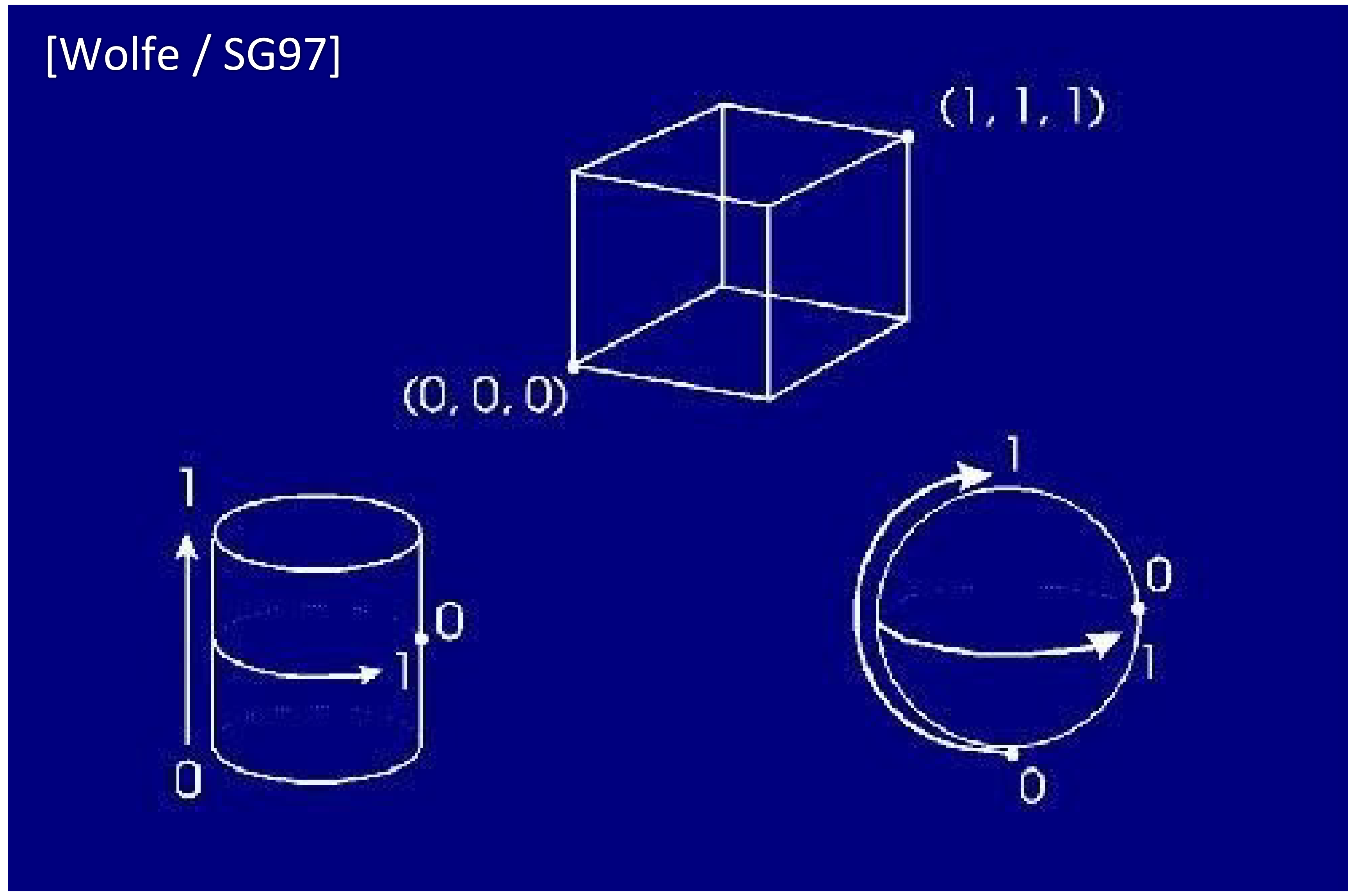
Projection mapping

Maps 3D surface points to 2D image coordinates

$$f \,:\, \mathbb{R}^3 \rightarrow [0,1]^2$$

Different types of projections

- often corresponding to simple shapes
- useful for simple objects

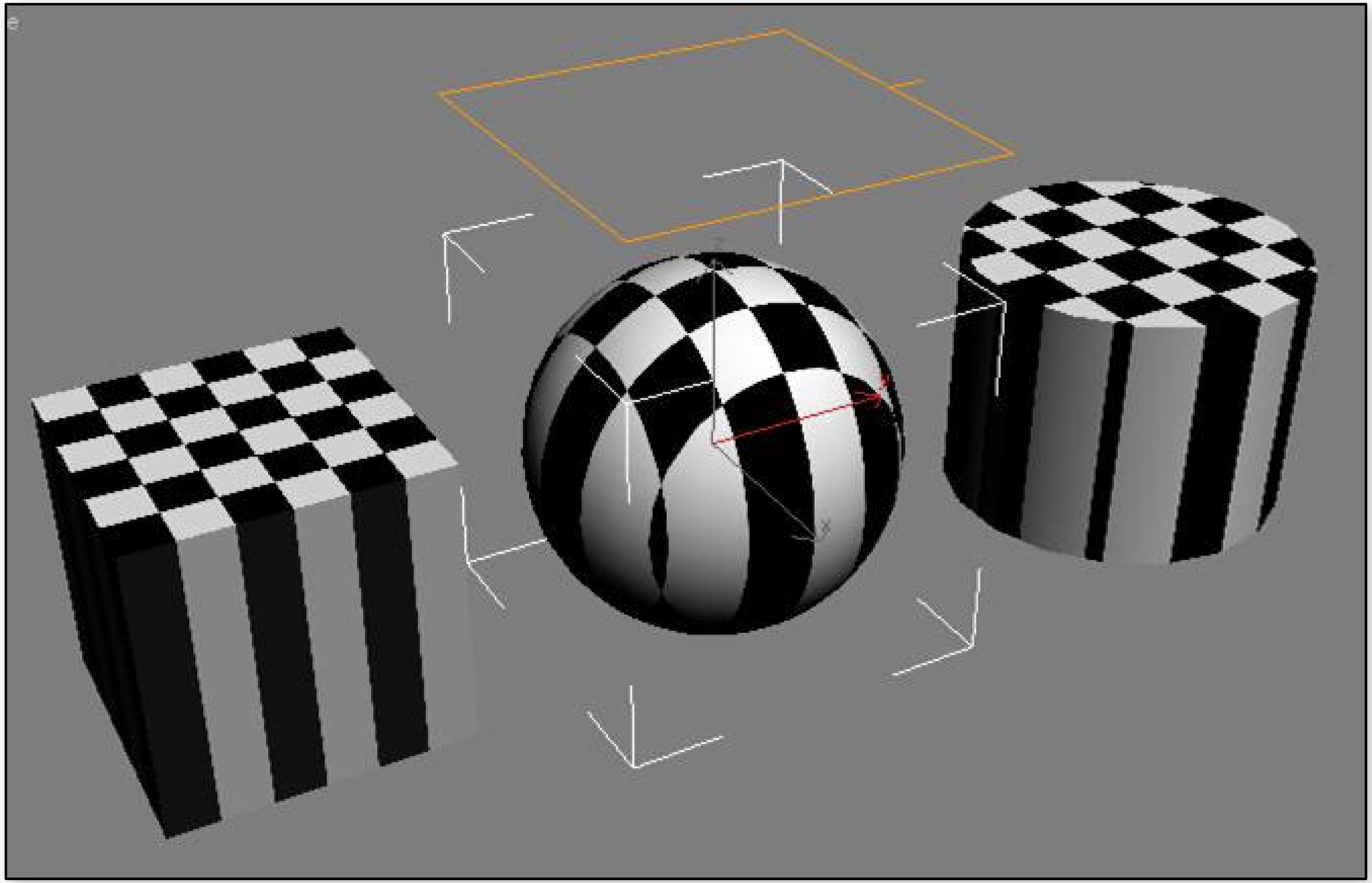


After a slide by Fabio Pellacini

Projections — planar

Planar projection along xy plane of size (w, h)

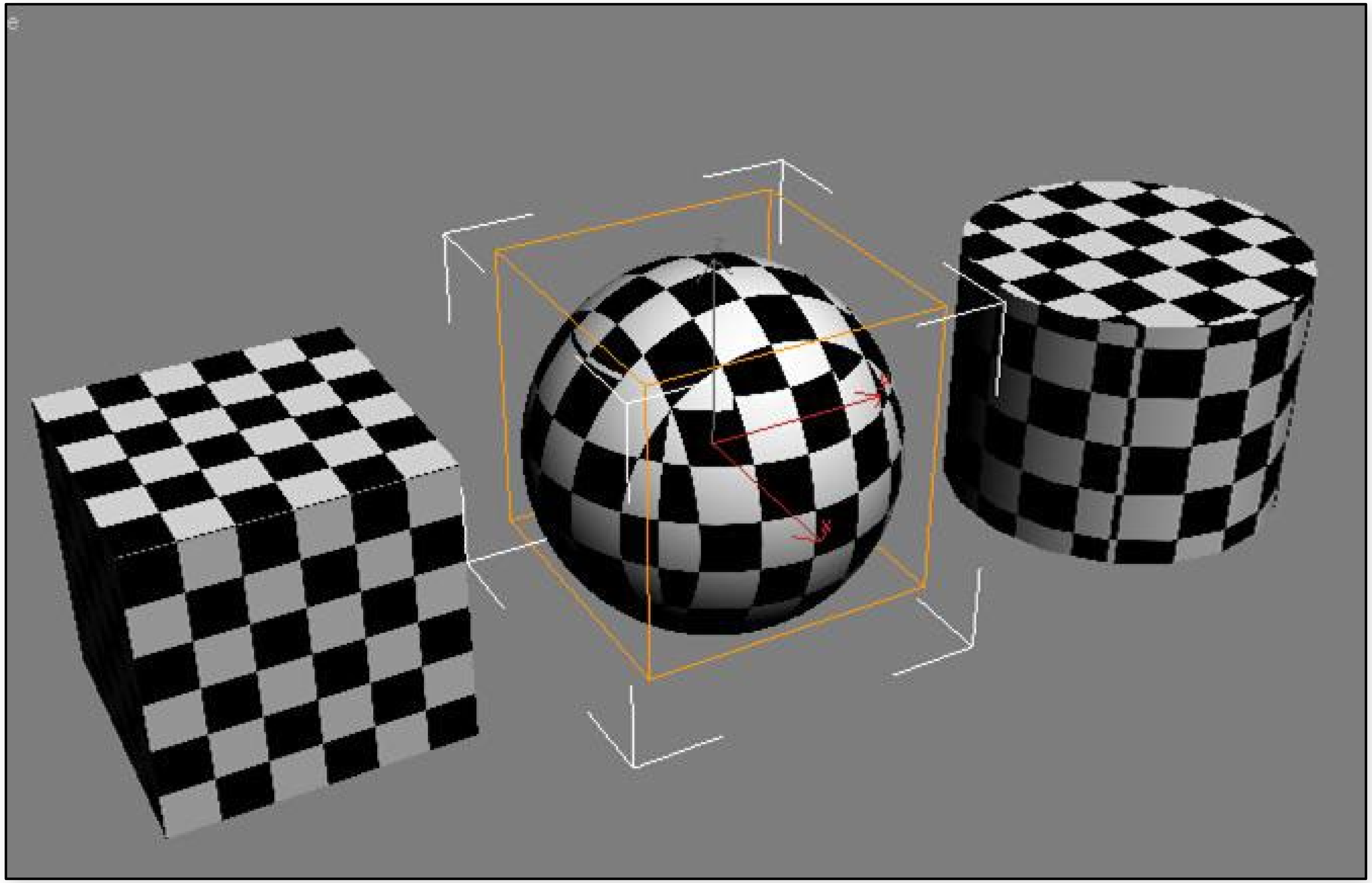
$$f(\mathbf{p}) = \begin{bmatrix} \frac{p_x}{w} \\ \frac{p_y}{h} \end{bmatrix}$$



After a slide by Fabio Pellacini

Projections — cubical

Planar projection onto one of 6 faces of a cube based on surface normal



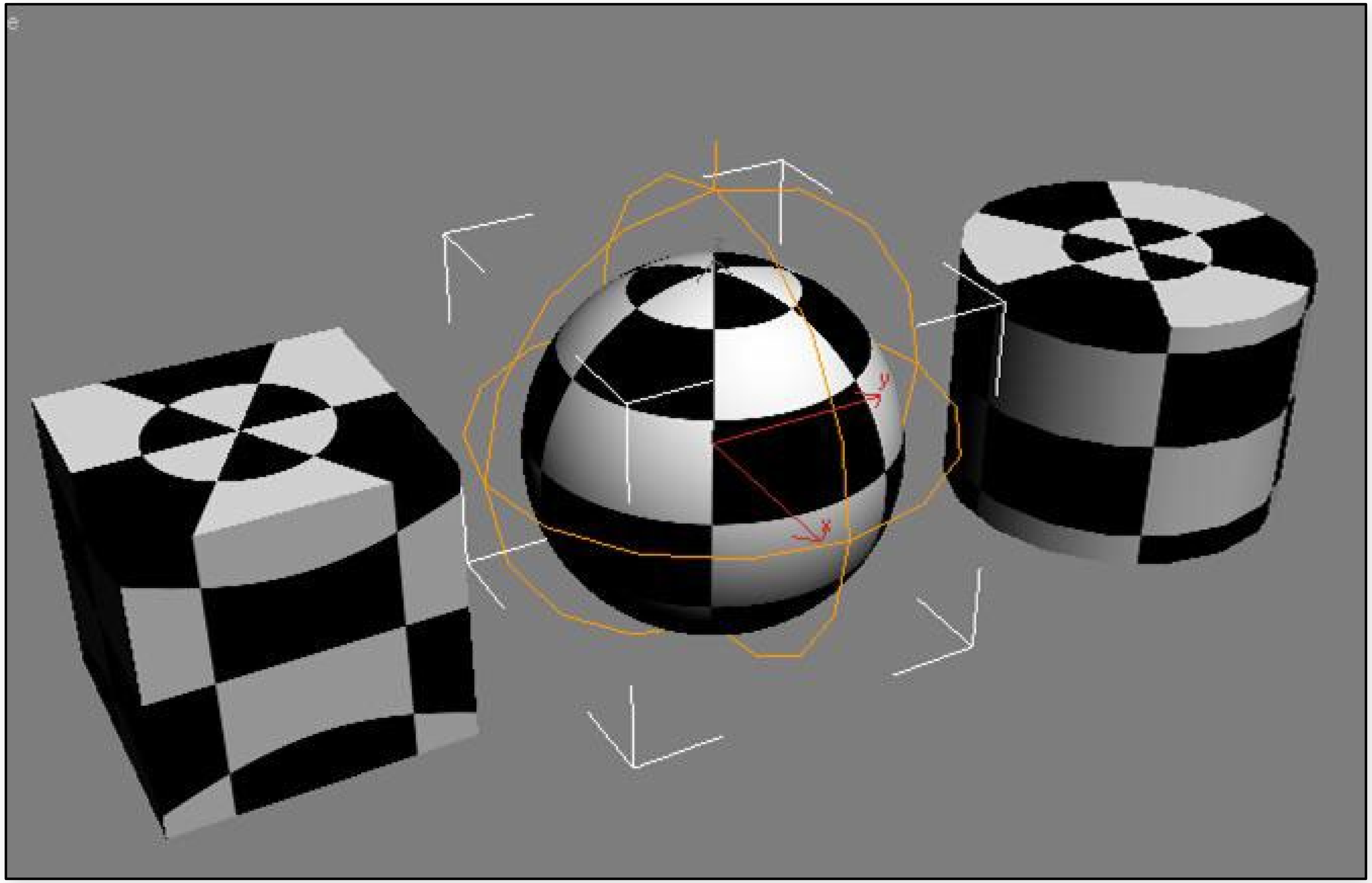
After a slide by Fabio Pellacini

Projections — spherical

Project point onto unit sphere

- compute spherical coordinates

$$f(\mathbf{p}) = \begin{bmatrix} \frac{\phi}{2\pi} \\ \frac{\theta}{\pi} \end{bmatrix}$$



After a slide by Fabio Pellacini

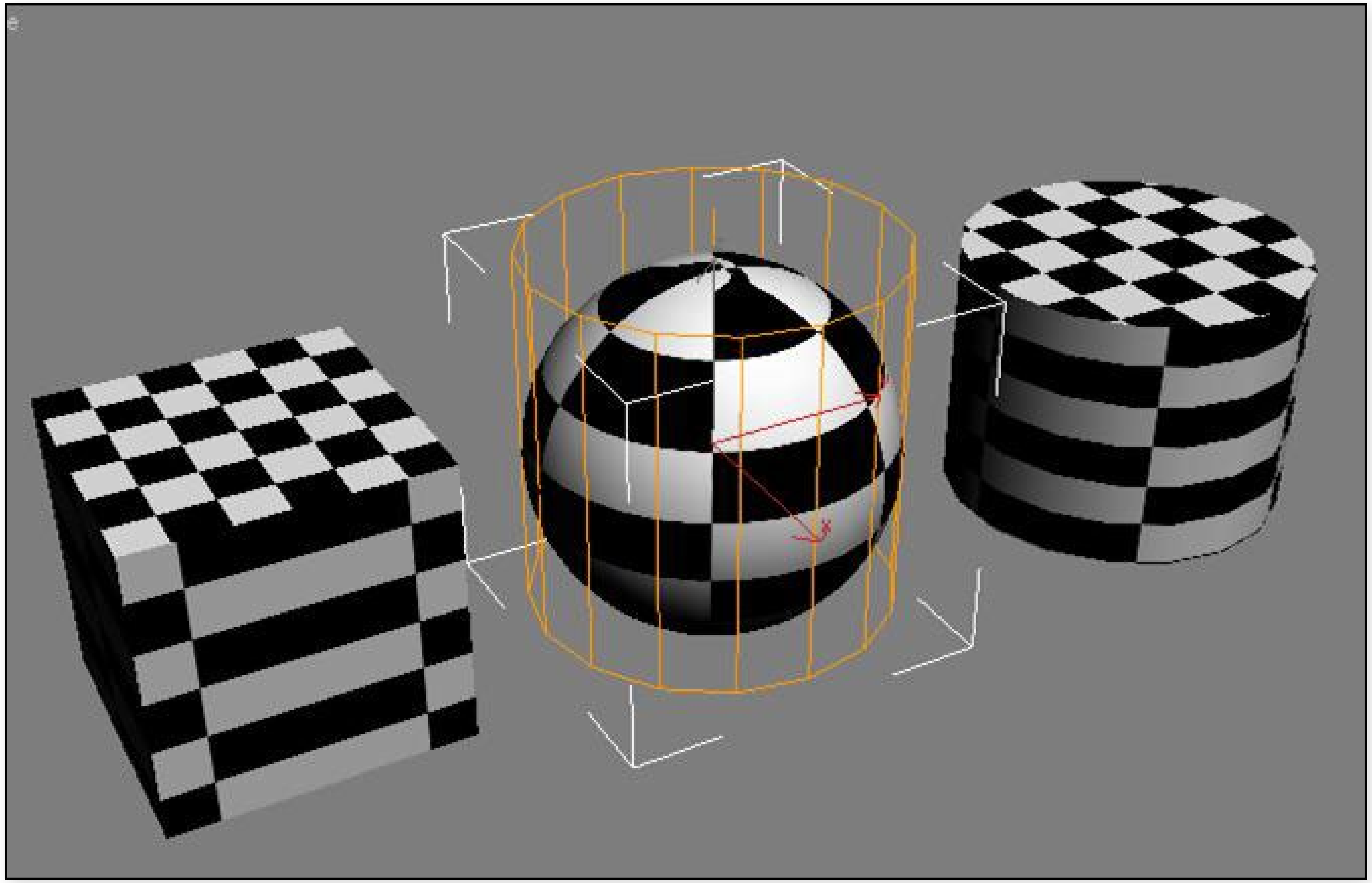
Projections — cylindrical

Project point onto cylinder of height h

- compute cylindrical coordinates

$$f(\mathbf{p}) = \begin{bmatrix} \frac{\phi}{2\pi} \\ \frac{p_y}{h} \end{bmatrix}$$

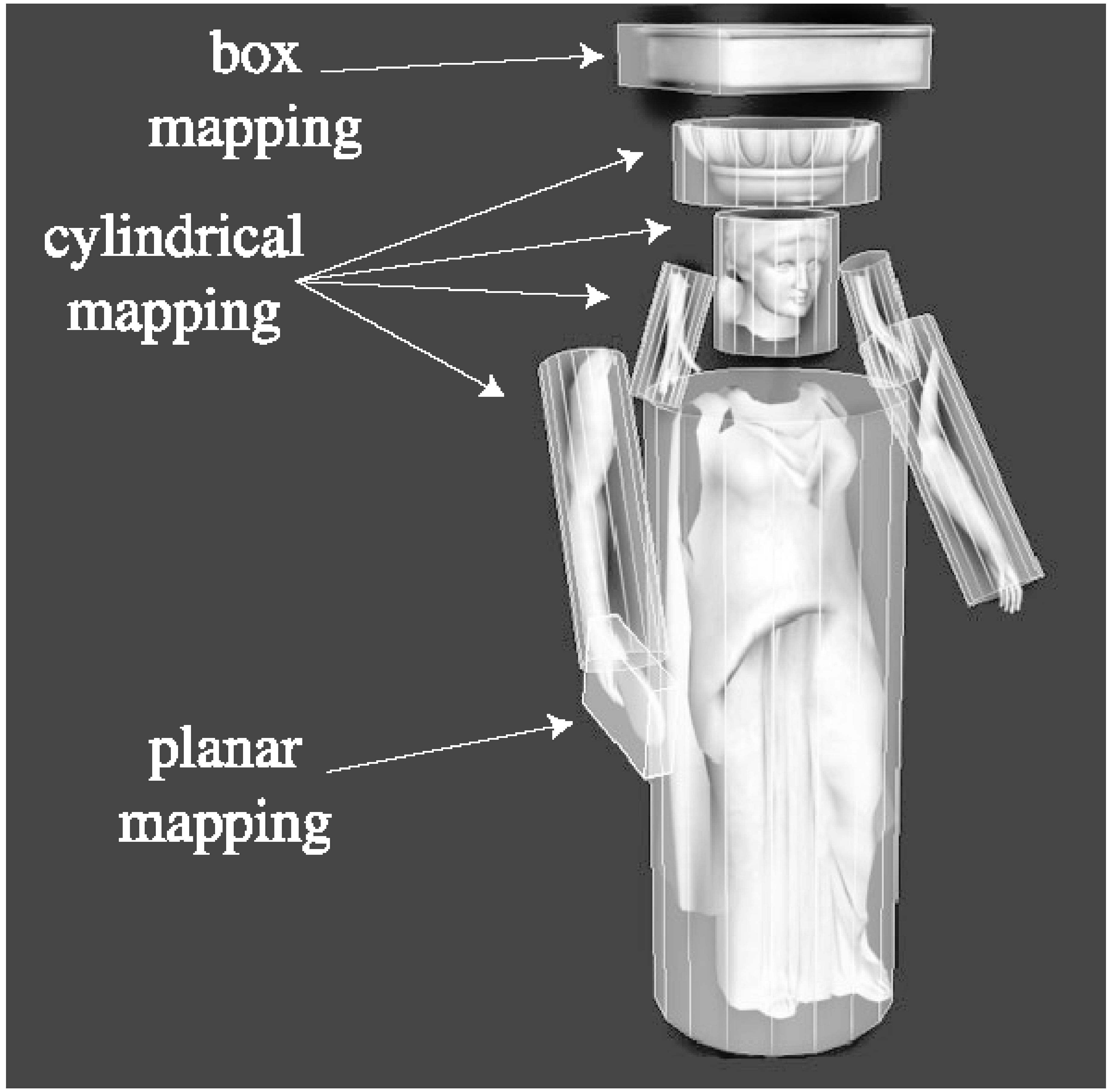
- treat caps separately



After a slide by Fabio Pellacini

Combining projections

Non-parametric surfaces: project pieces to parametric surface



[Moller & Haines 2002]

Combining projections

"Atlas" - break up model into single texture

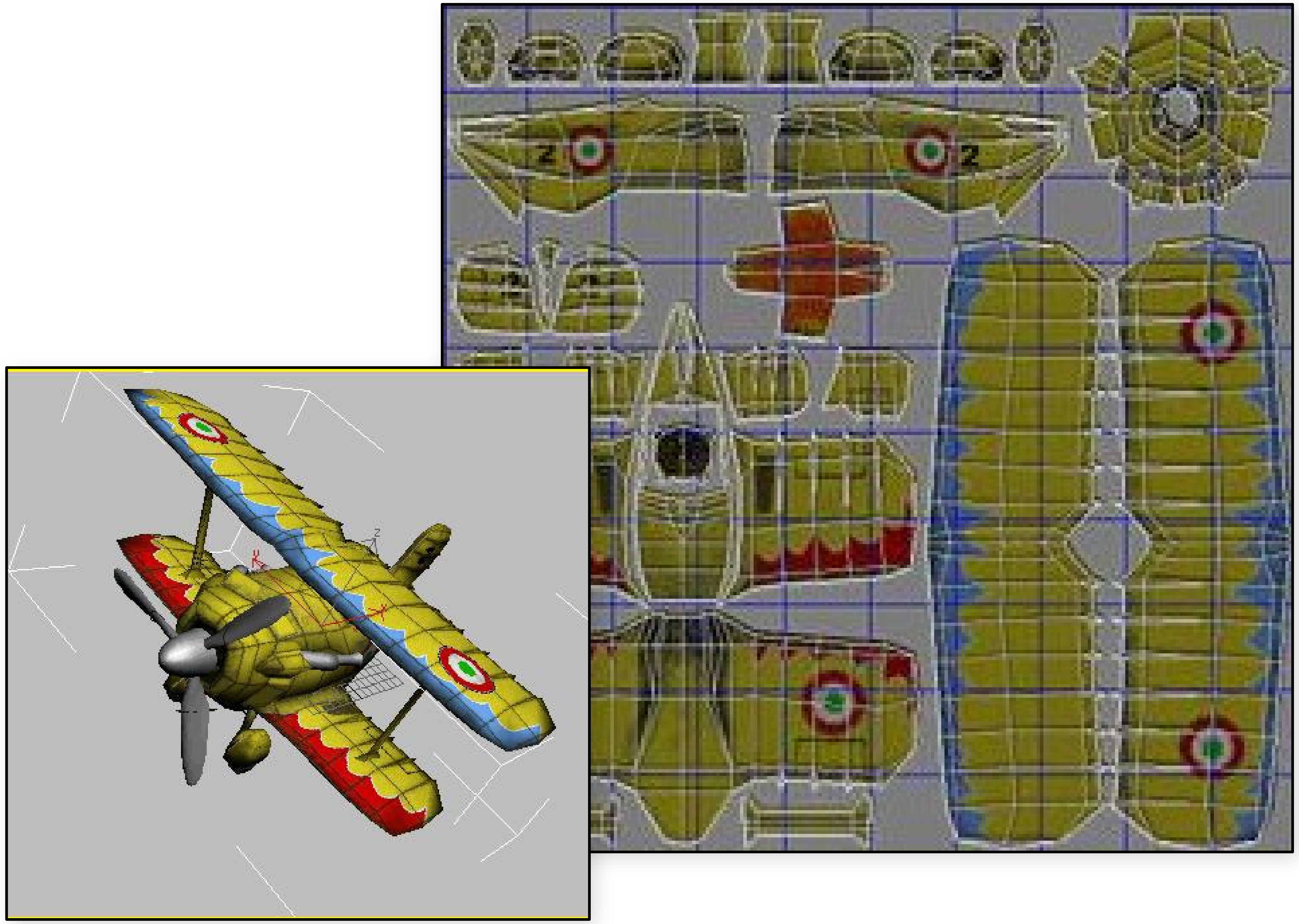
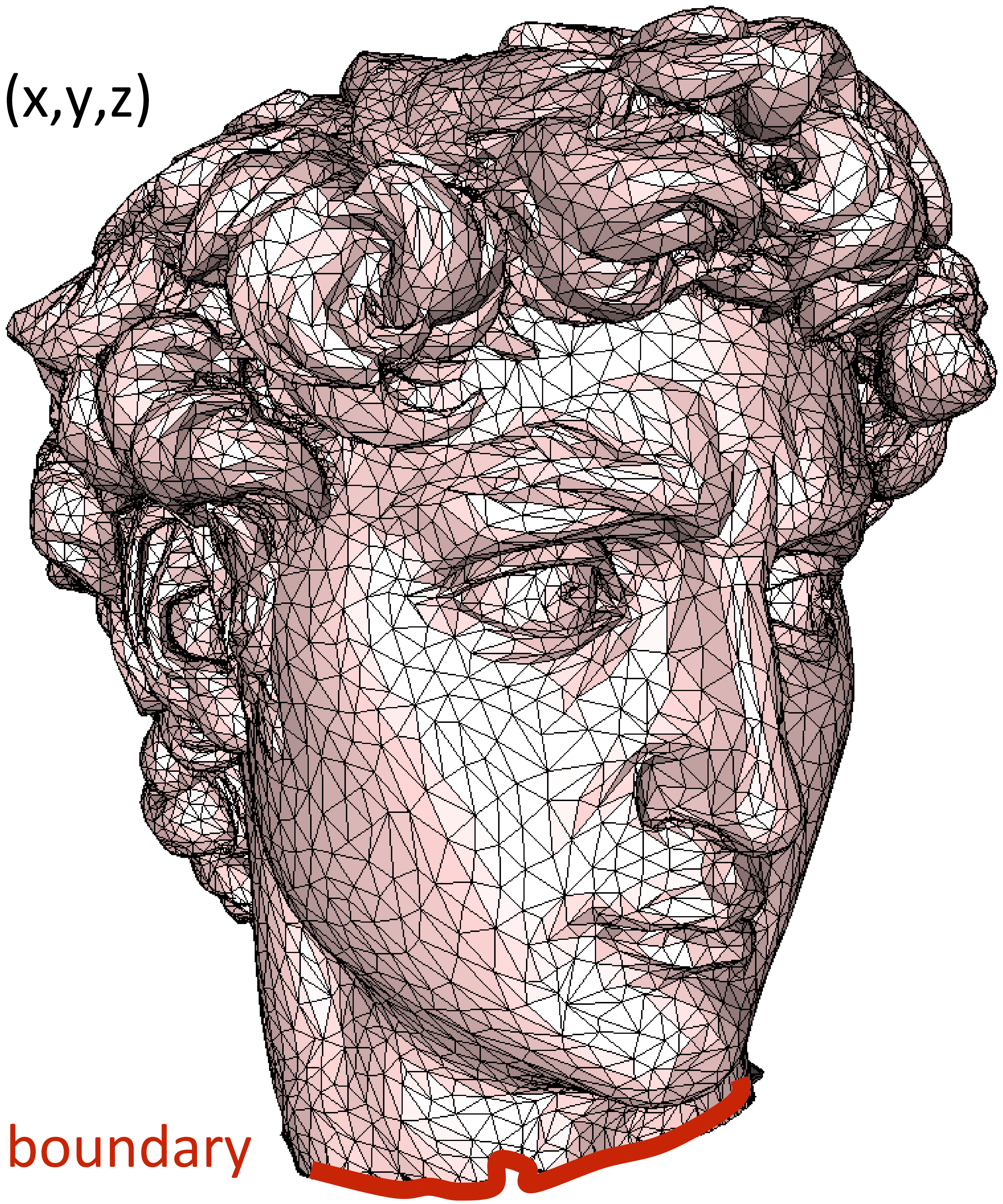


Image from Vallet and Levy, techreport INRIA

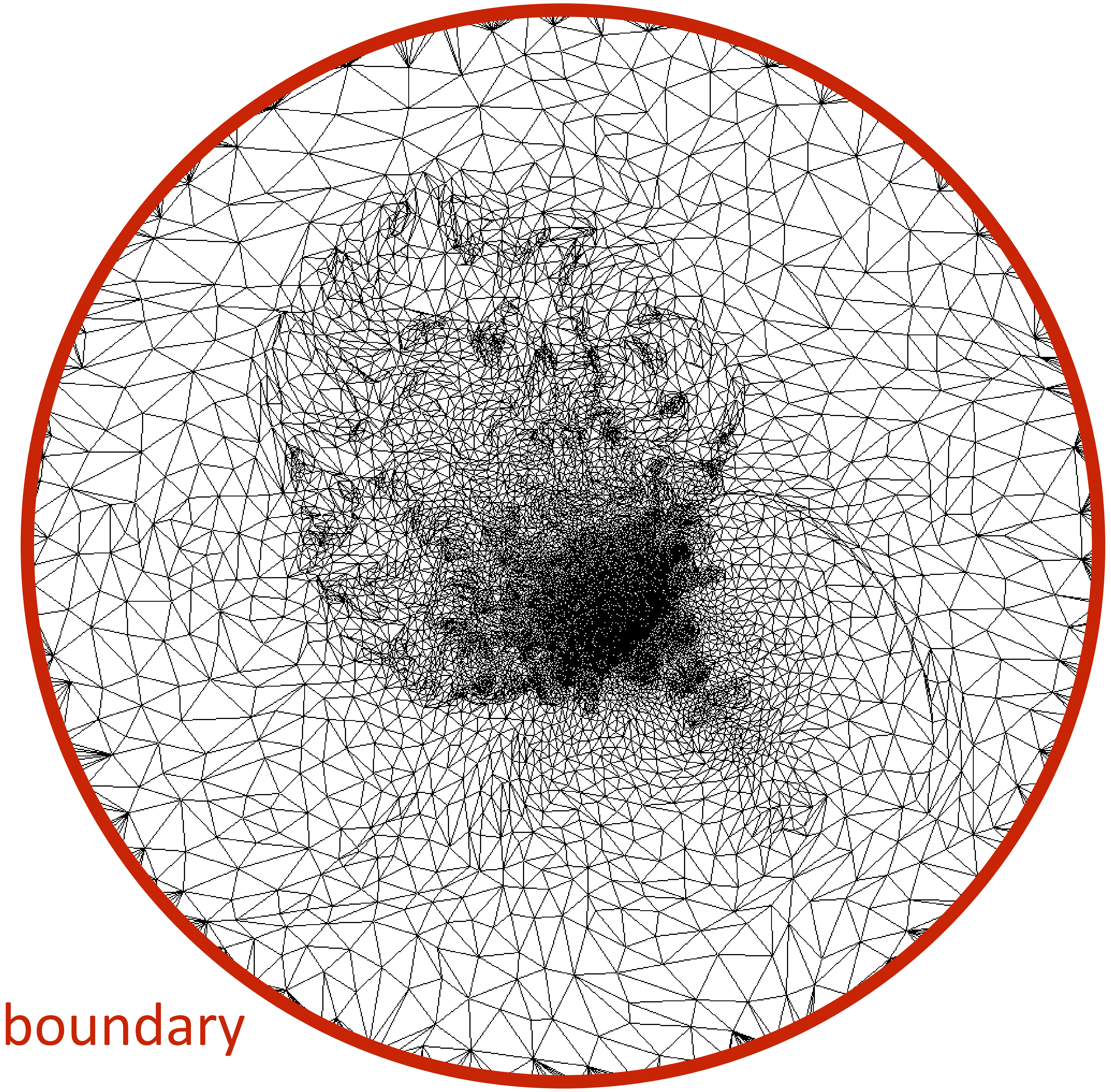
Creating UV parameterizations

3D space (x,y,z)



boundary

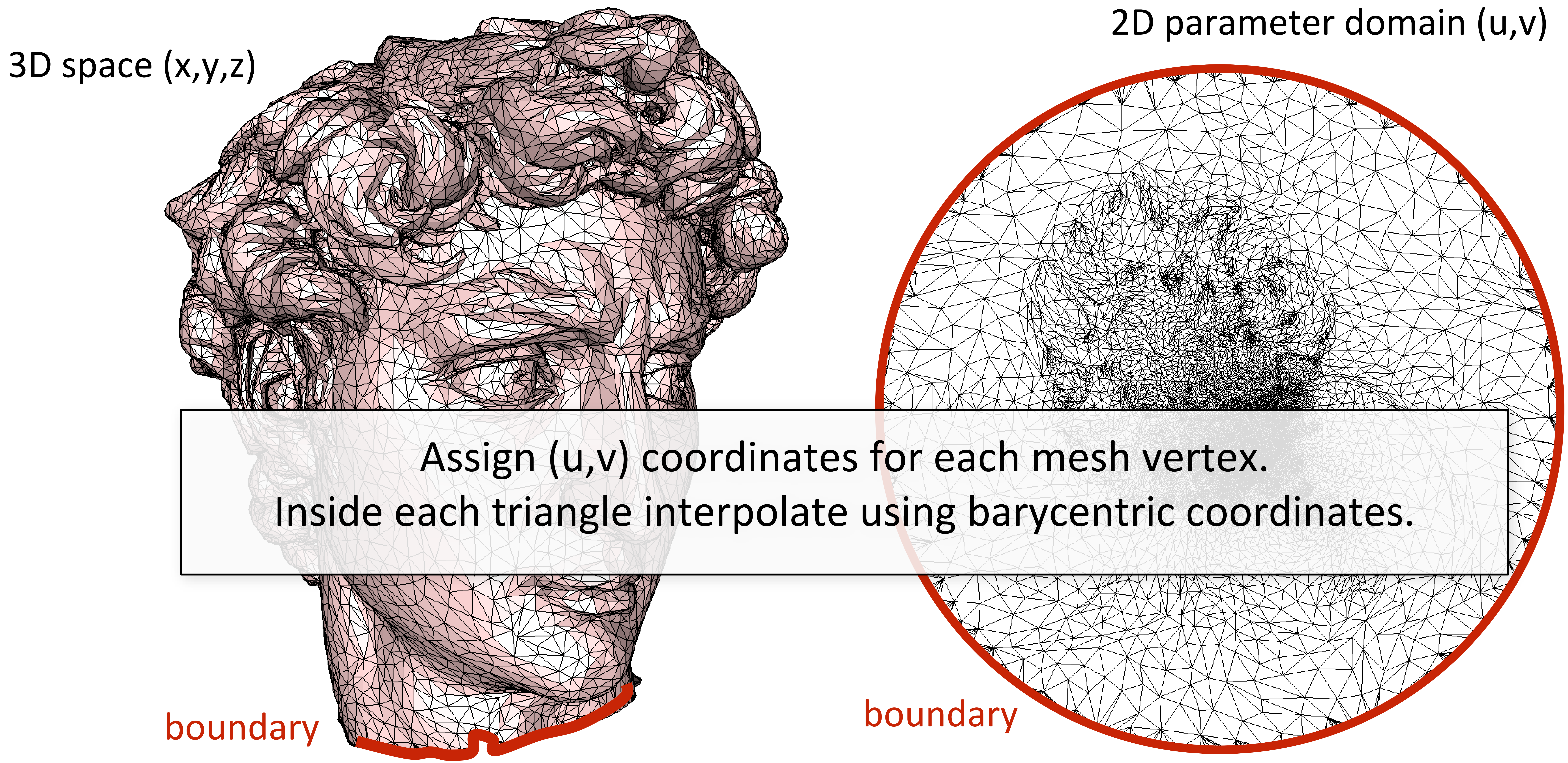
2D parameter domain (u,v)



boundary

After a slide by Daniele Panozzo

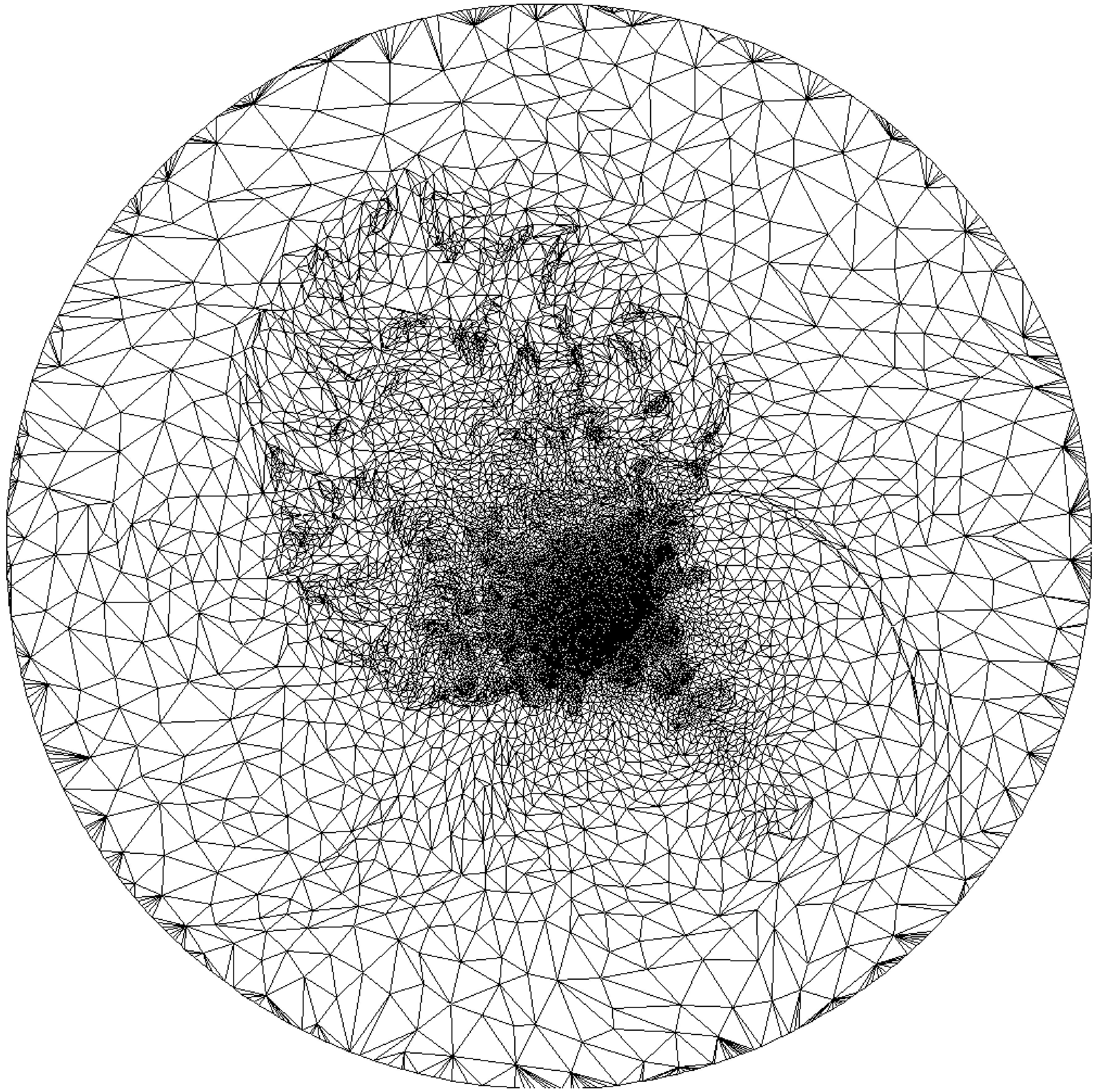
Creating UV parameterizations

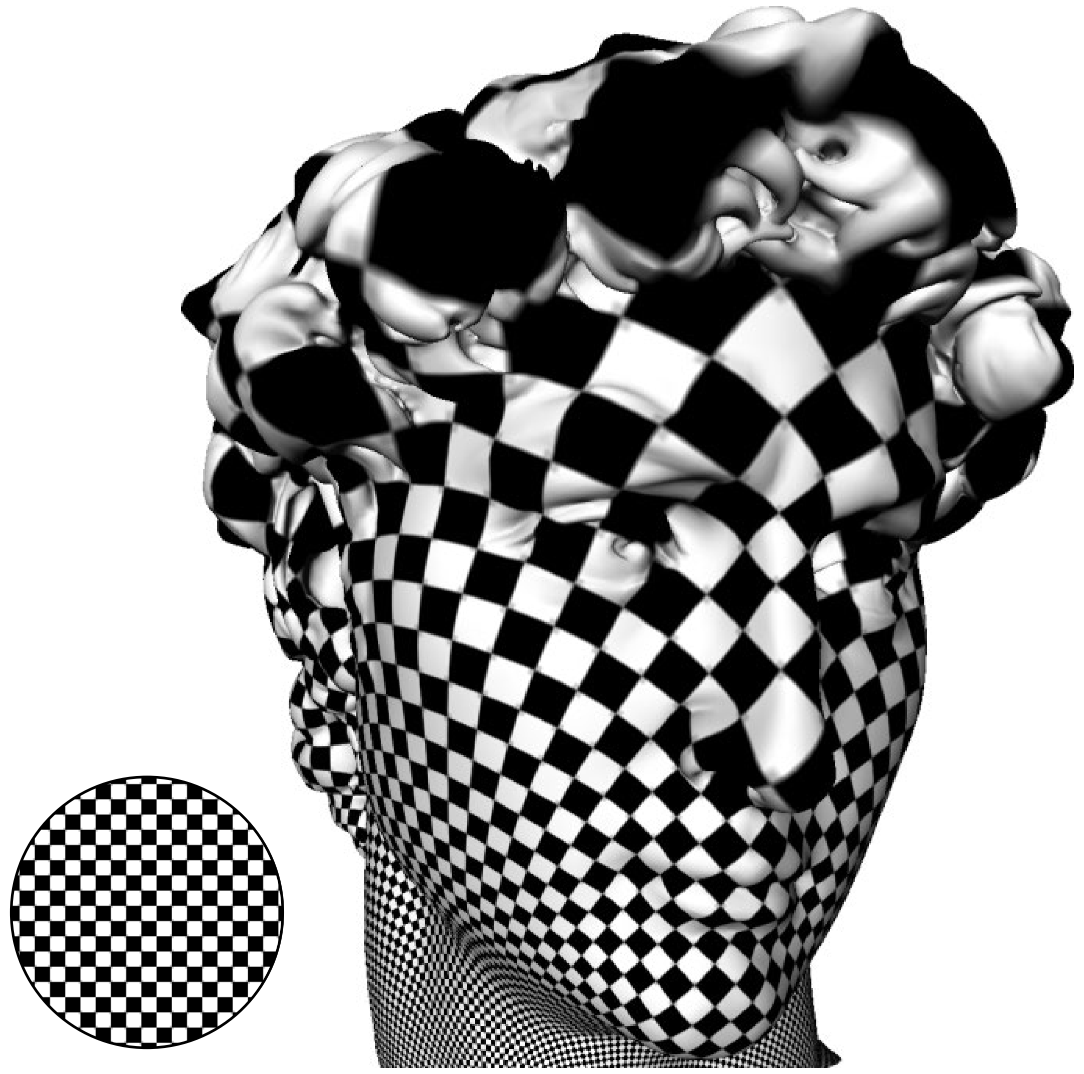


Assign (u,v) coordinates for each mesh vertex.
Inside each triangle interpolate using barycentric coordinates.

After a slide by Daniele Panozzo

Creating UV parameterizations

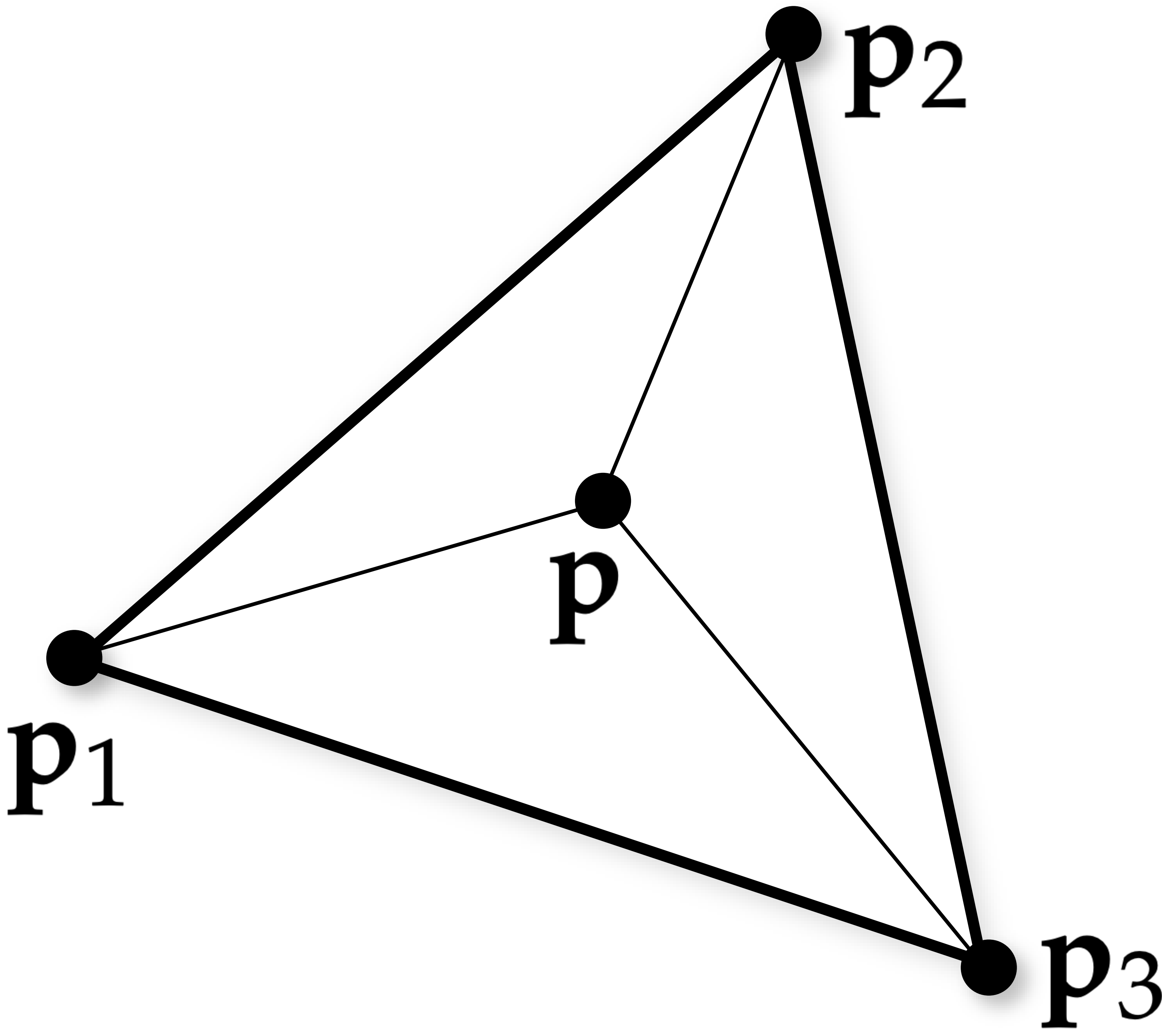




After a slide by Daniele Panozzo

Barycentric coordinates

Barycentric interpolation: $$\mathbf{p}(\alpha, \beta, \gamma) = \alpha\mathbf{p}_1 + \beta\mathbf{p}_2 + \gamma\mathbf{p}_3$$



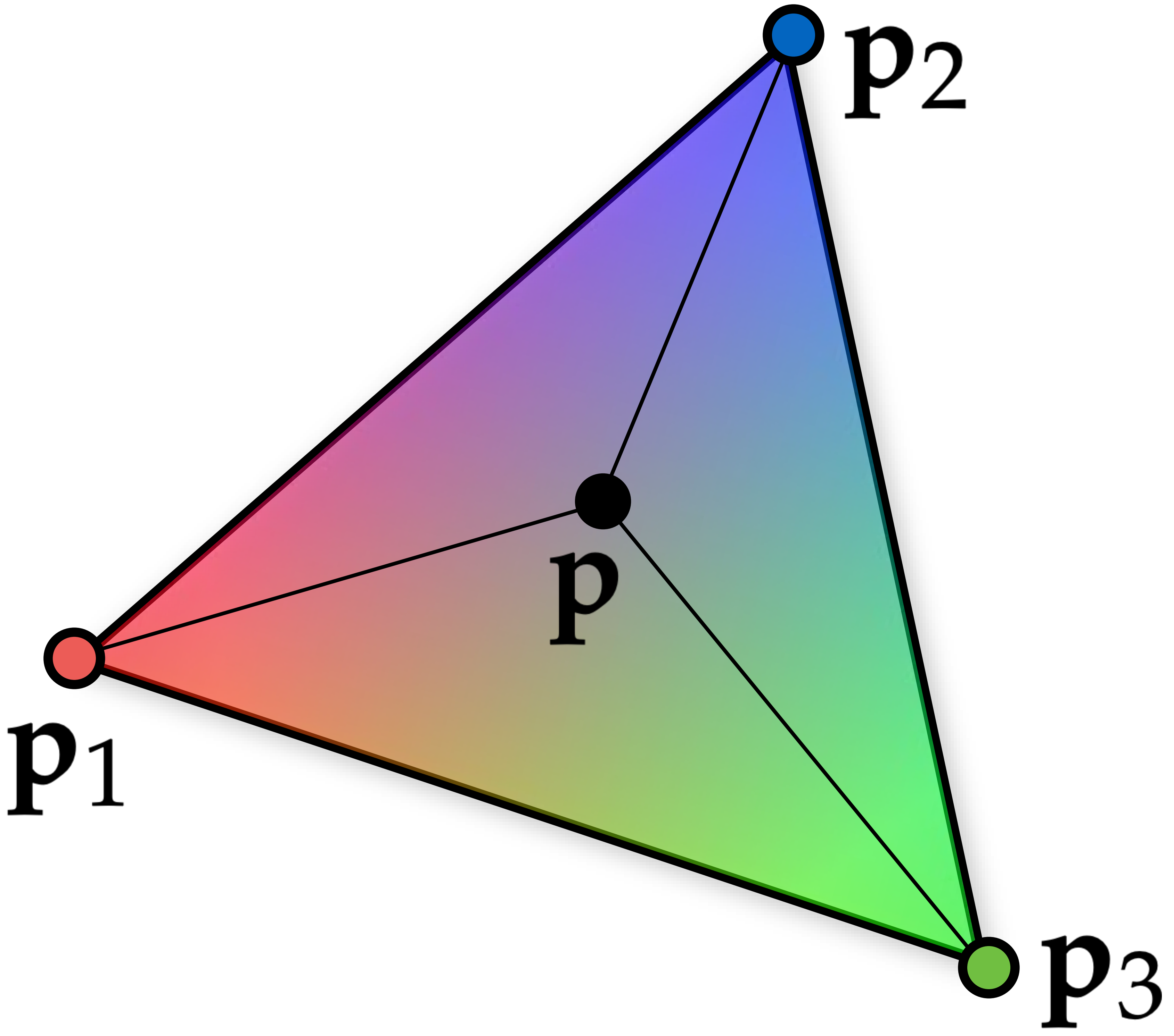
Can use this eqn. to interpolate any vertex quantity across triangle!

Barycentric coordinates

Barycentric interpolation:

$$\mathbf{p}(\alpha, \beta, \gamma) = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3$$

$$\mathbf{c}(\alpha, \beta, \gamma) = \alpha \mathbf{c}_1 + \beta \mathbf{c}_2 + \gamma \mathbf{c}_3$$

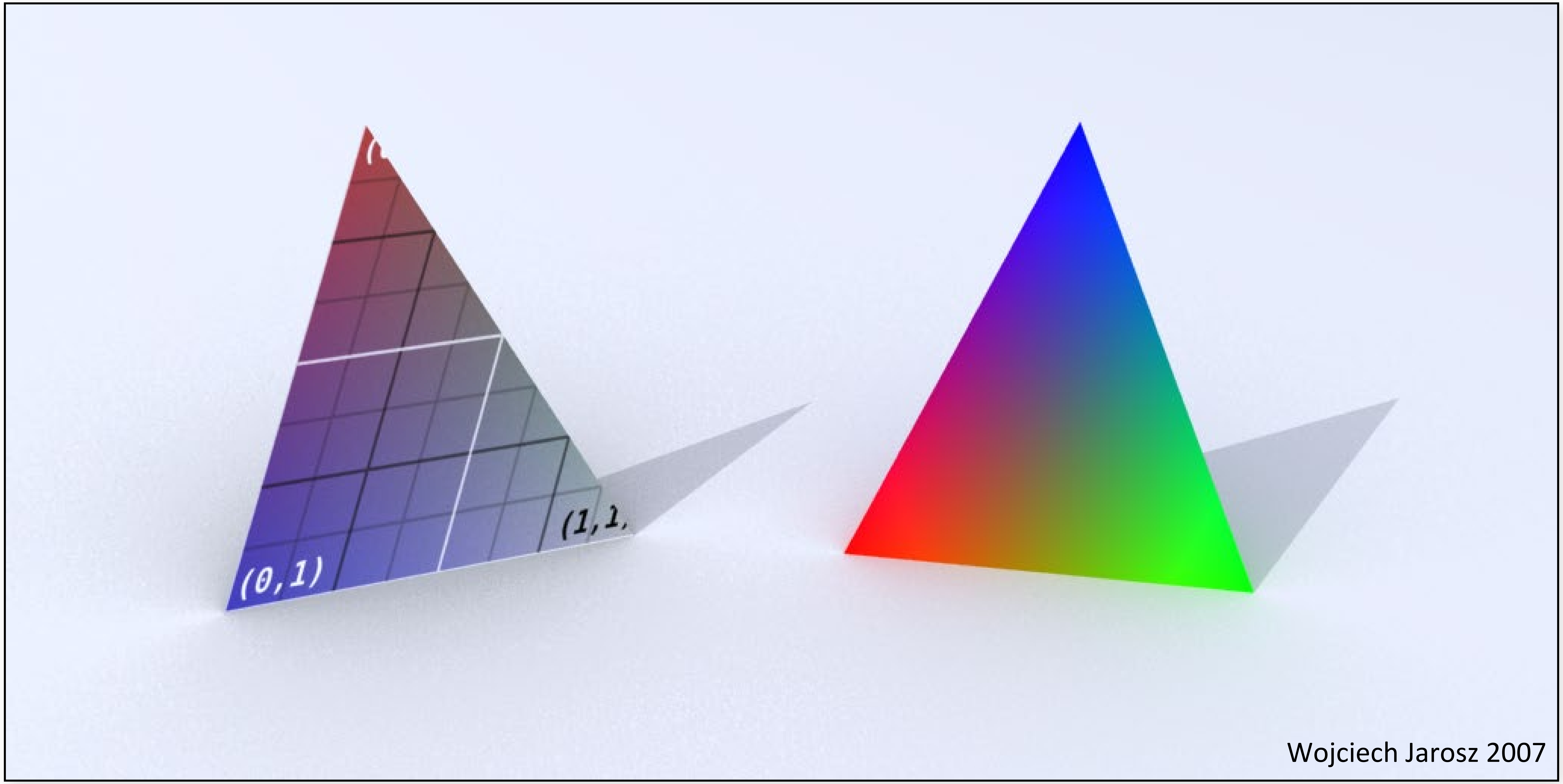


Can use this eqn. to interpolate any vertex quantity across triangle!

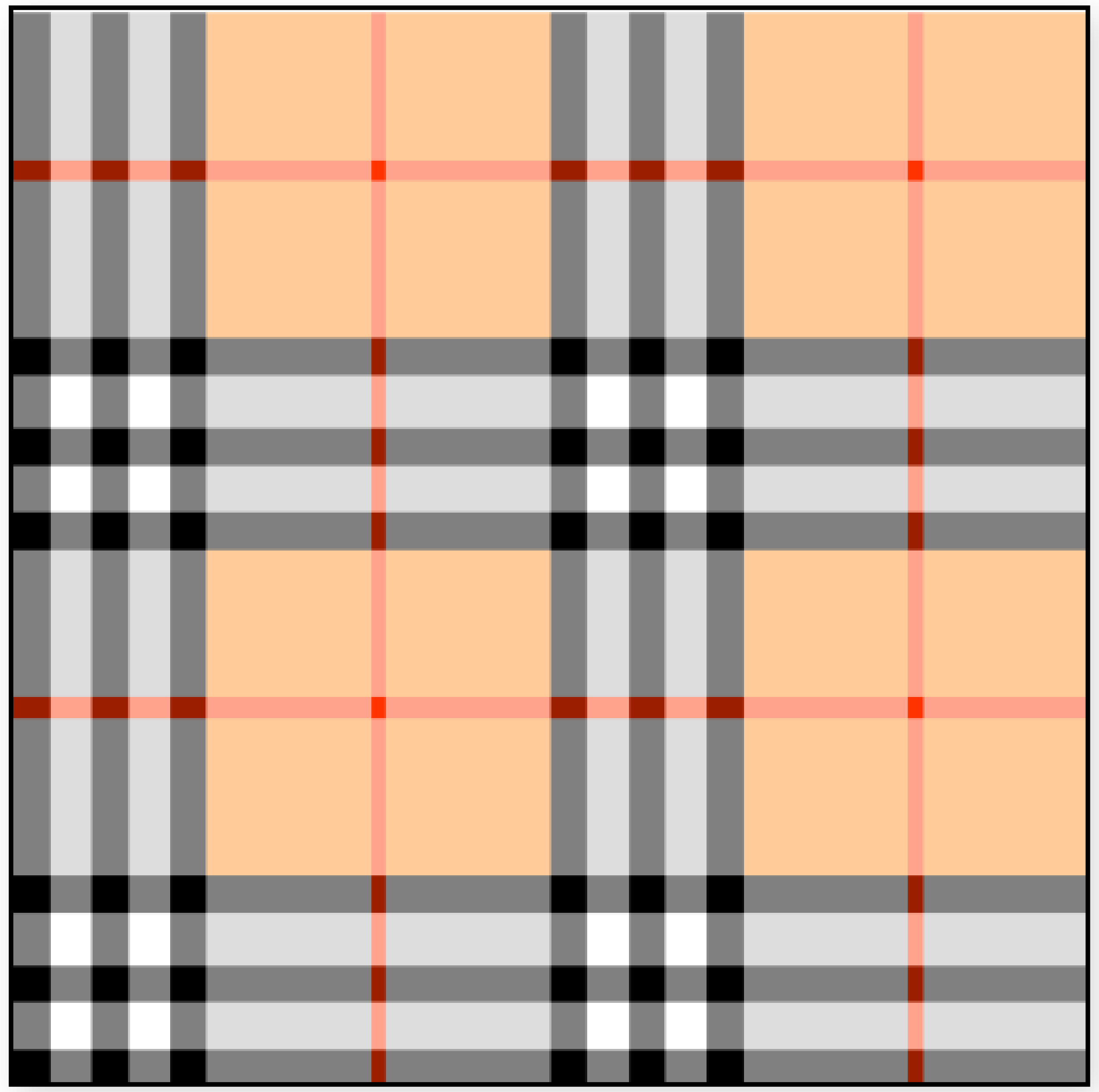
UV texturing

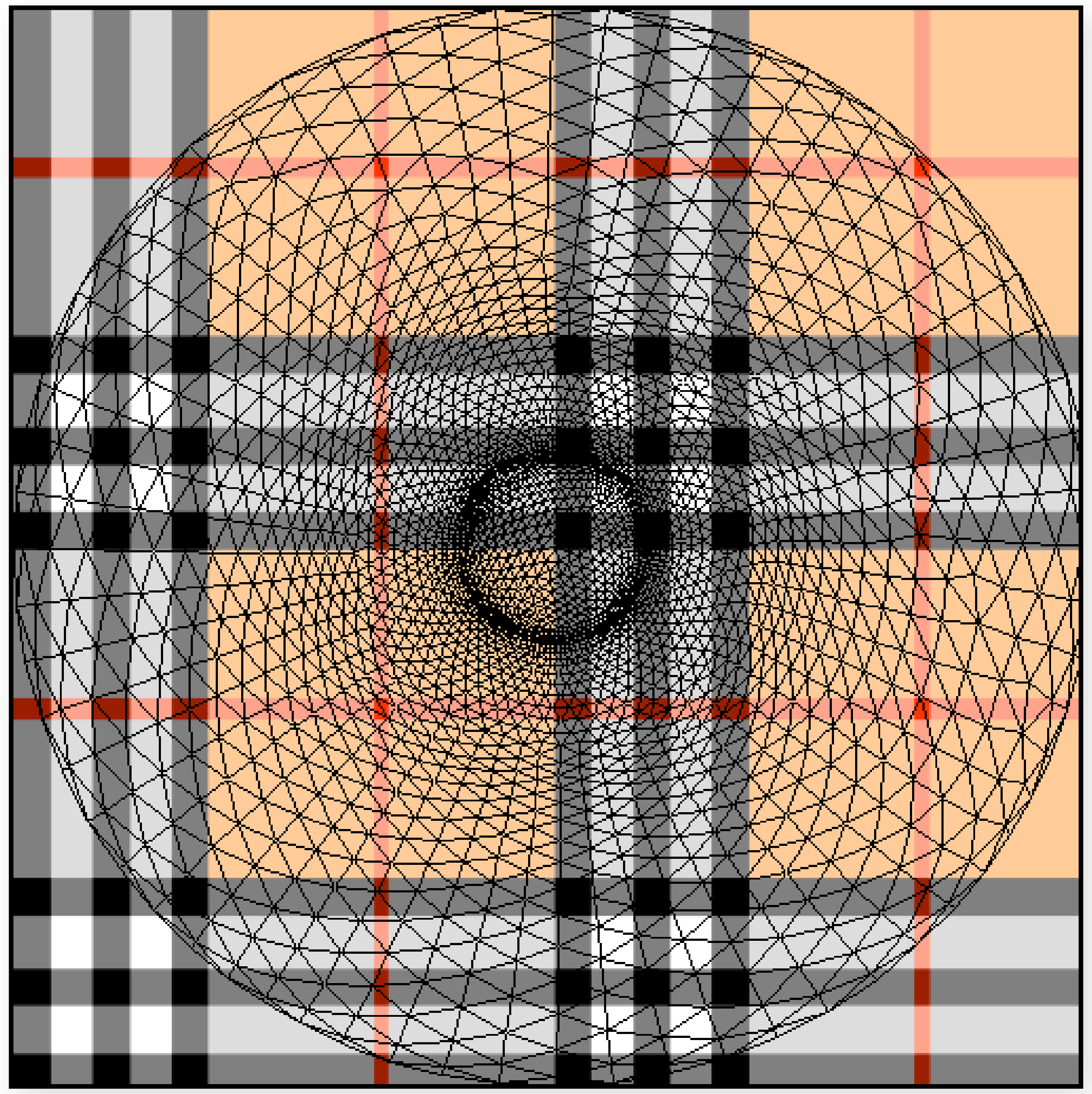
$$(u,v) = \alpha(u_1,v_1) + \beta(u_2,v_2) + \gamma(u_3,v_3)$$

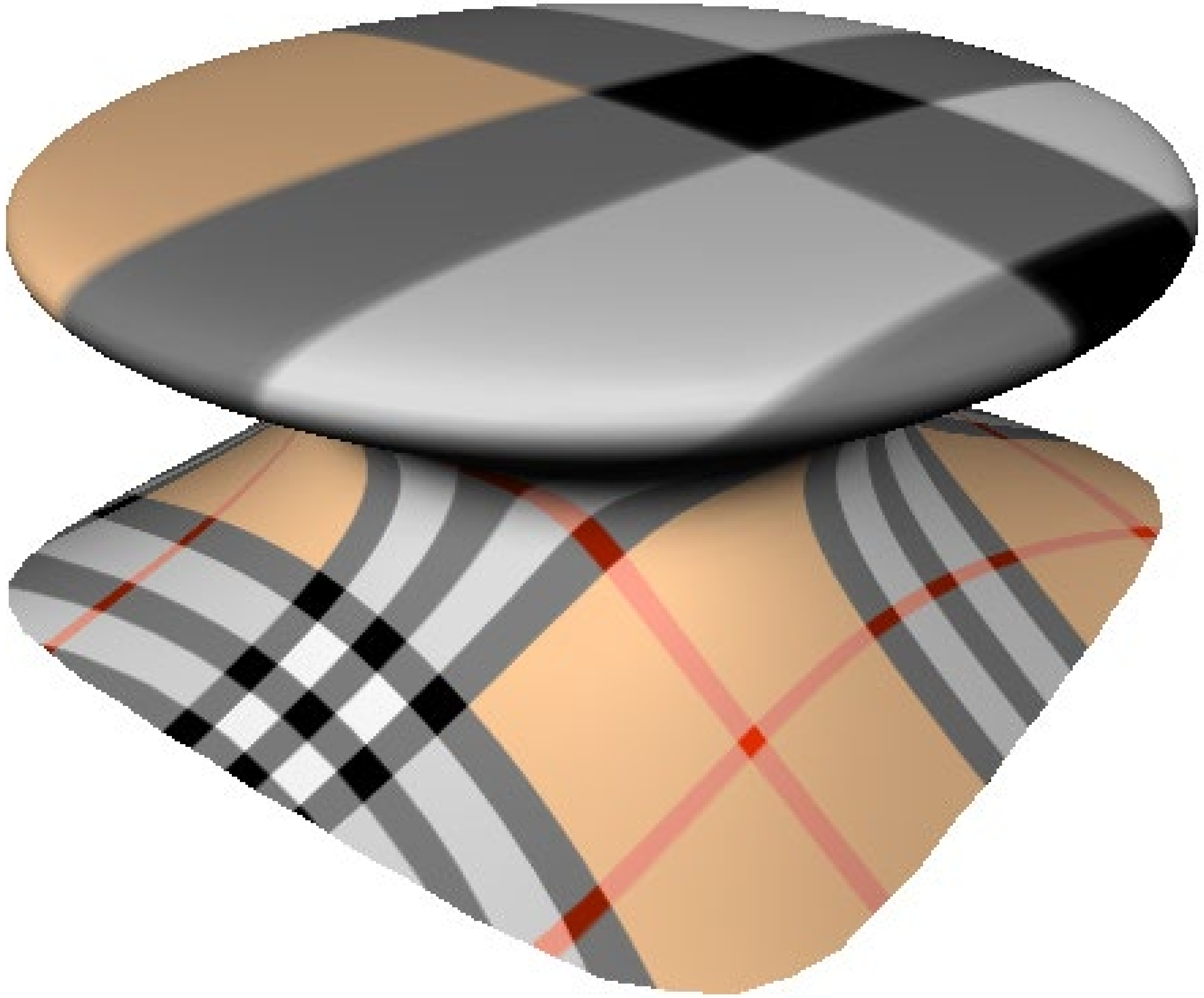
UV texturing



Area distortion vs. angle distortion







After a slide by Daniele Panozzo

Area distortion vs. angle distortion

How do we compute the areal and angular distortion at a surface point due to any given texture mapping function?

Texture lookup

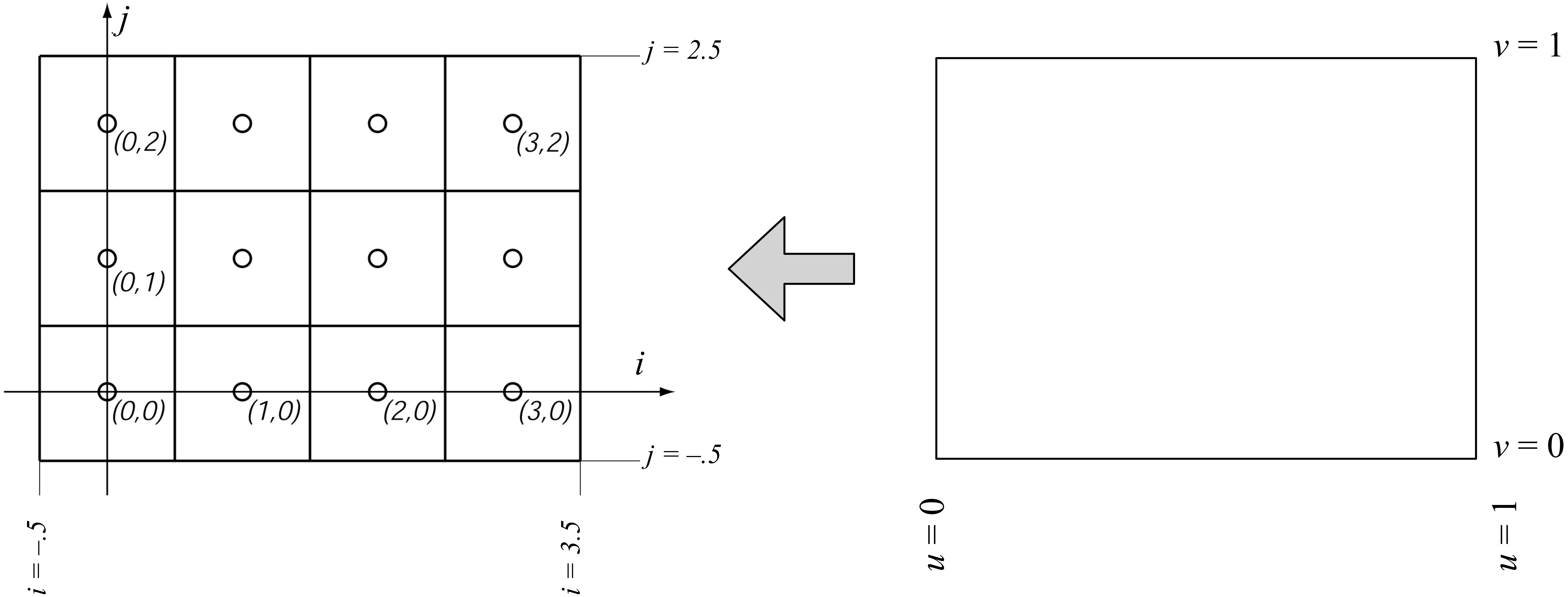
Texture lookups and wrapping

In shading calculation, when you need a texture value you perform a *texture lookup*

Convert (u, v) texture coordinates to (i, j) texel coordinates, and read a value from the image

After a slide by Steve Marschner

Obtaining (i,j) from (u,v)



$$i = u\,n_x - 0.5$$
$$j = v\,n_y - 0.5$$

for an image of
n_x by n_y pixels

Looking up texture values

Lookup locations will fall at fractional texel coordinates

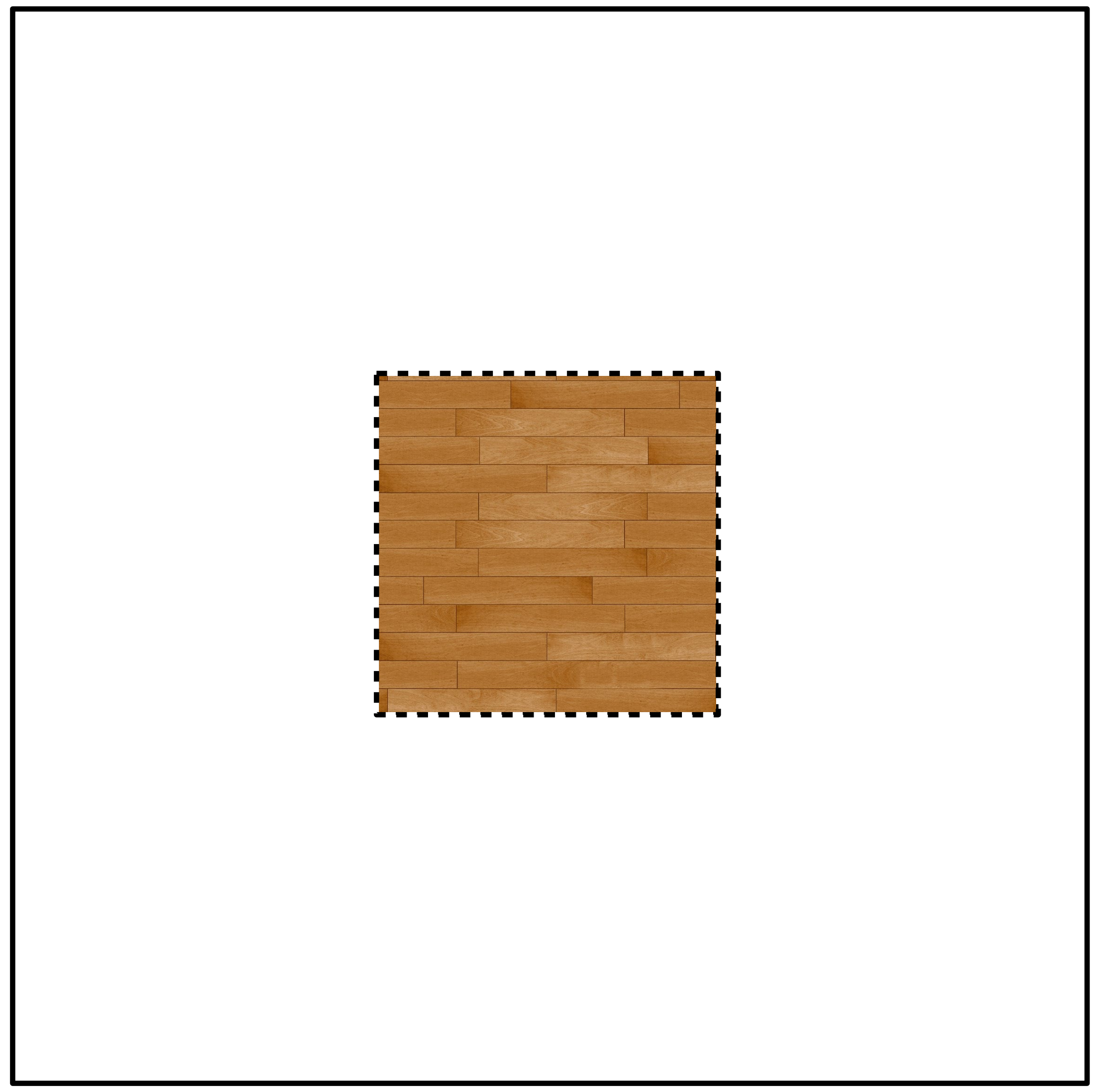
- simplest: round to nearest (nearest neighbor lookup)
- various ways to be smarter and get smoother results, at the cost of increased computation (e.g., bilinear or bicubic interpolation)

Texture lookups and wrapping

What if `i` and `j` are out of range?

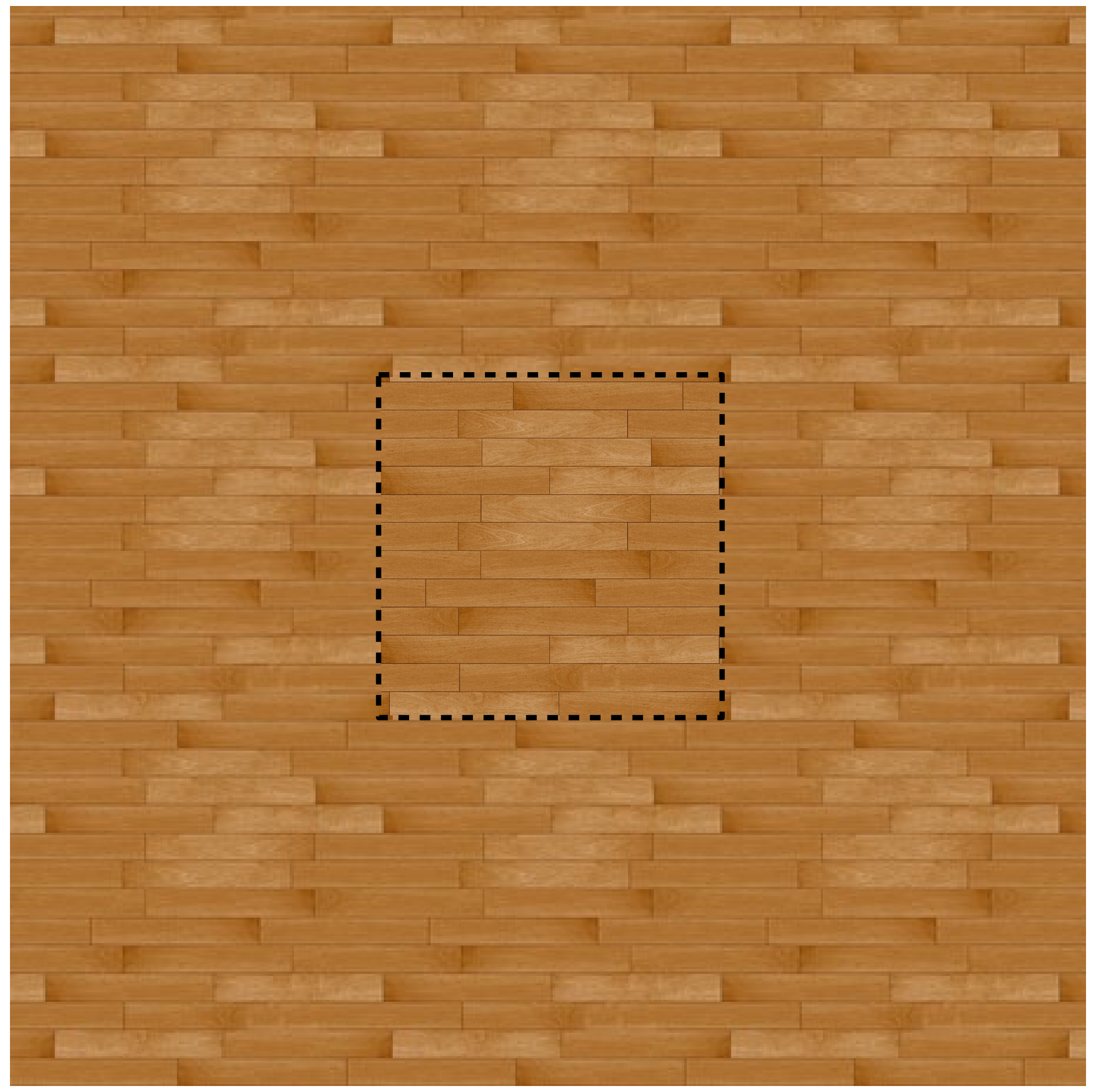
Option 1, clamp: take the nearest pixel that is in the image

$$i_{\text{pixel}} = \max(0, \min(n_x - 1, i_{\text{lookup}}))$$



Option 2, wrap: treat the texture as periodic, so that falling off the right side causes the look up to come in the left

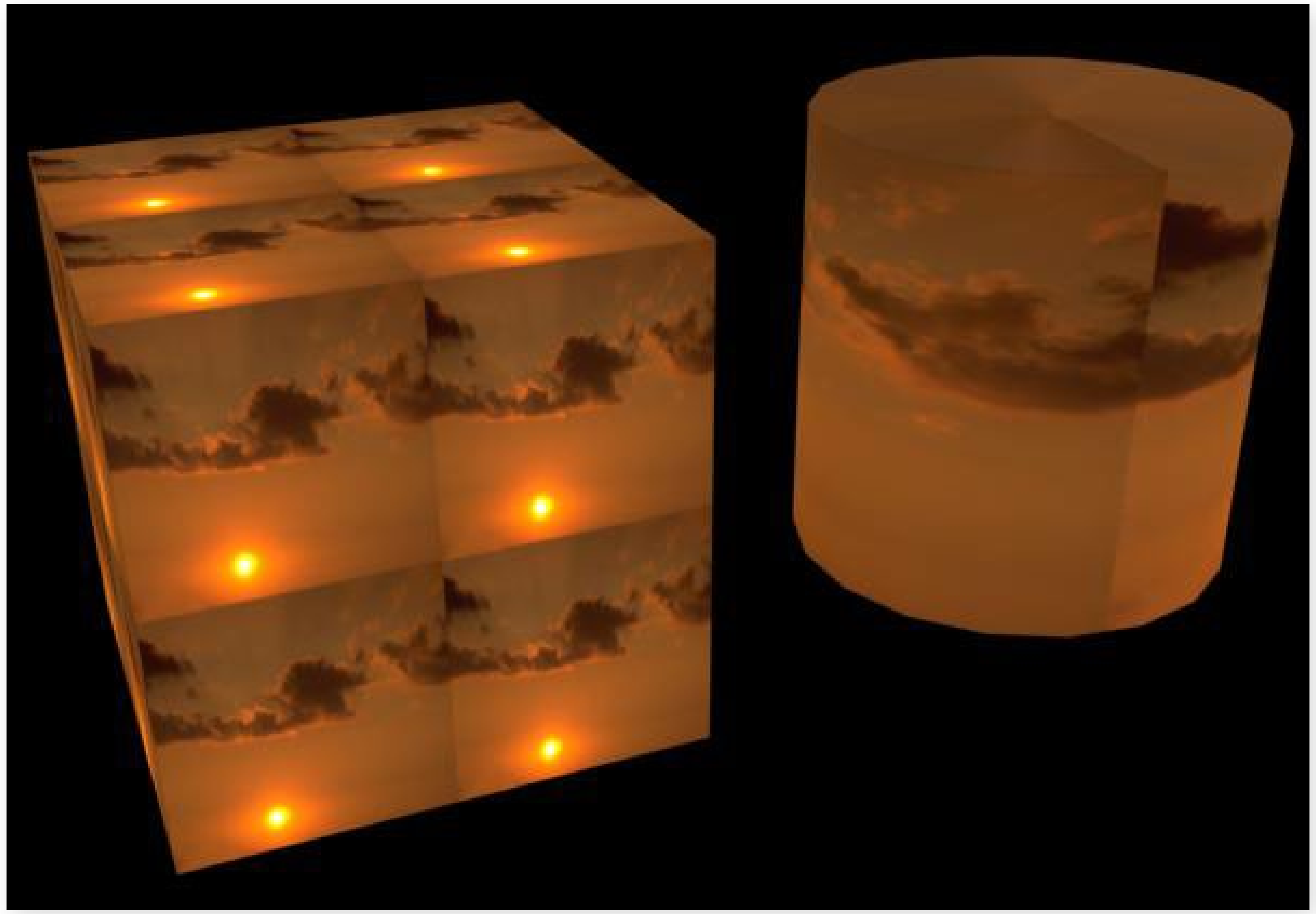
$$i_{\text{pixel}} = \text{remainder}(i_{\text{lookup}}, n_x)$$



Texture mapping artifacts

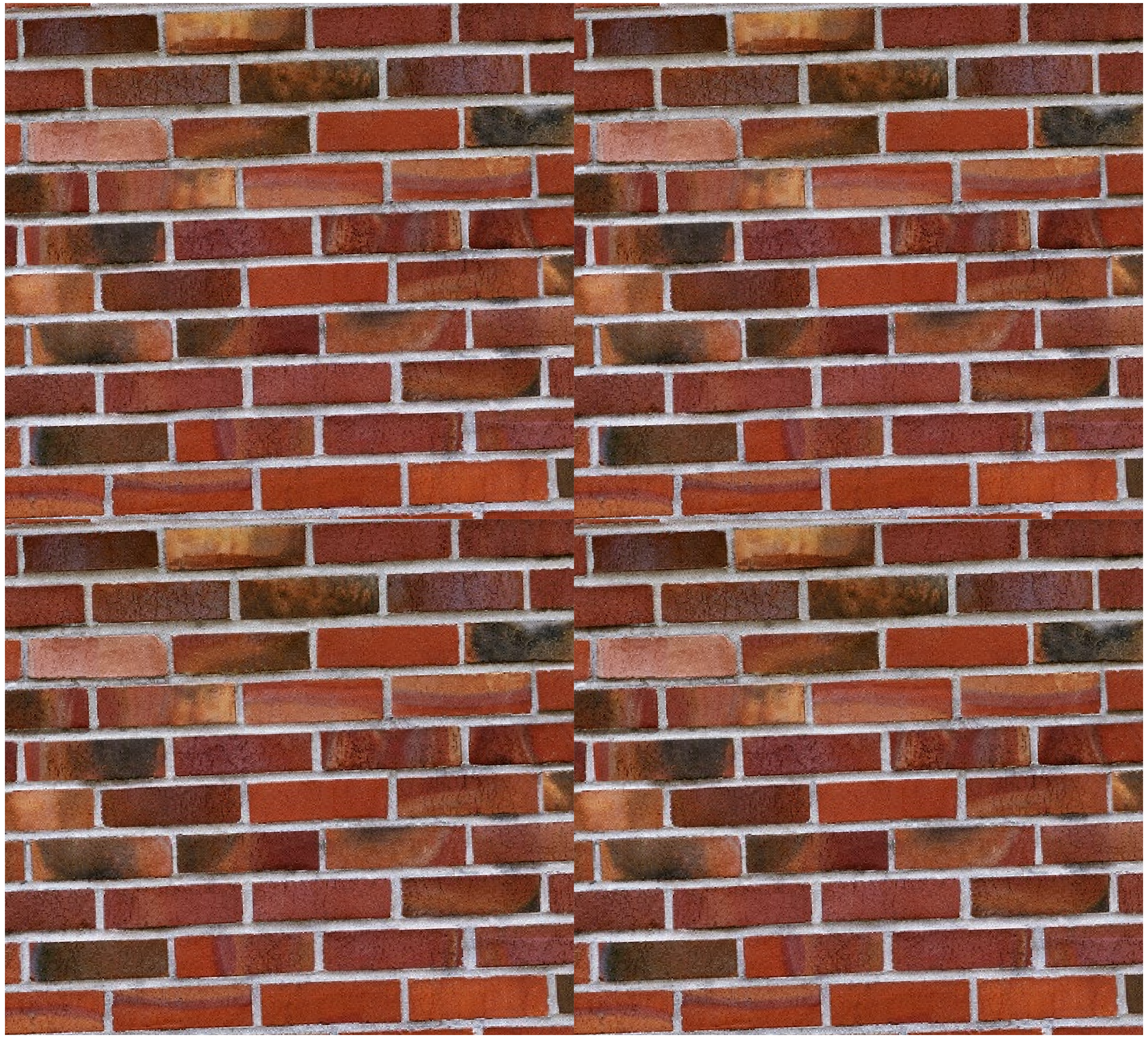
Tiling textures might introduce seams

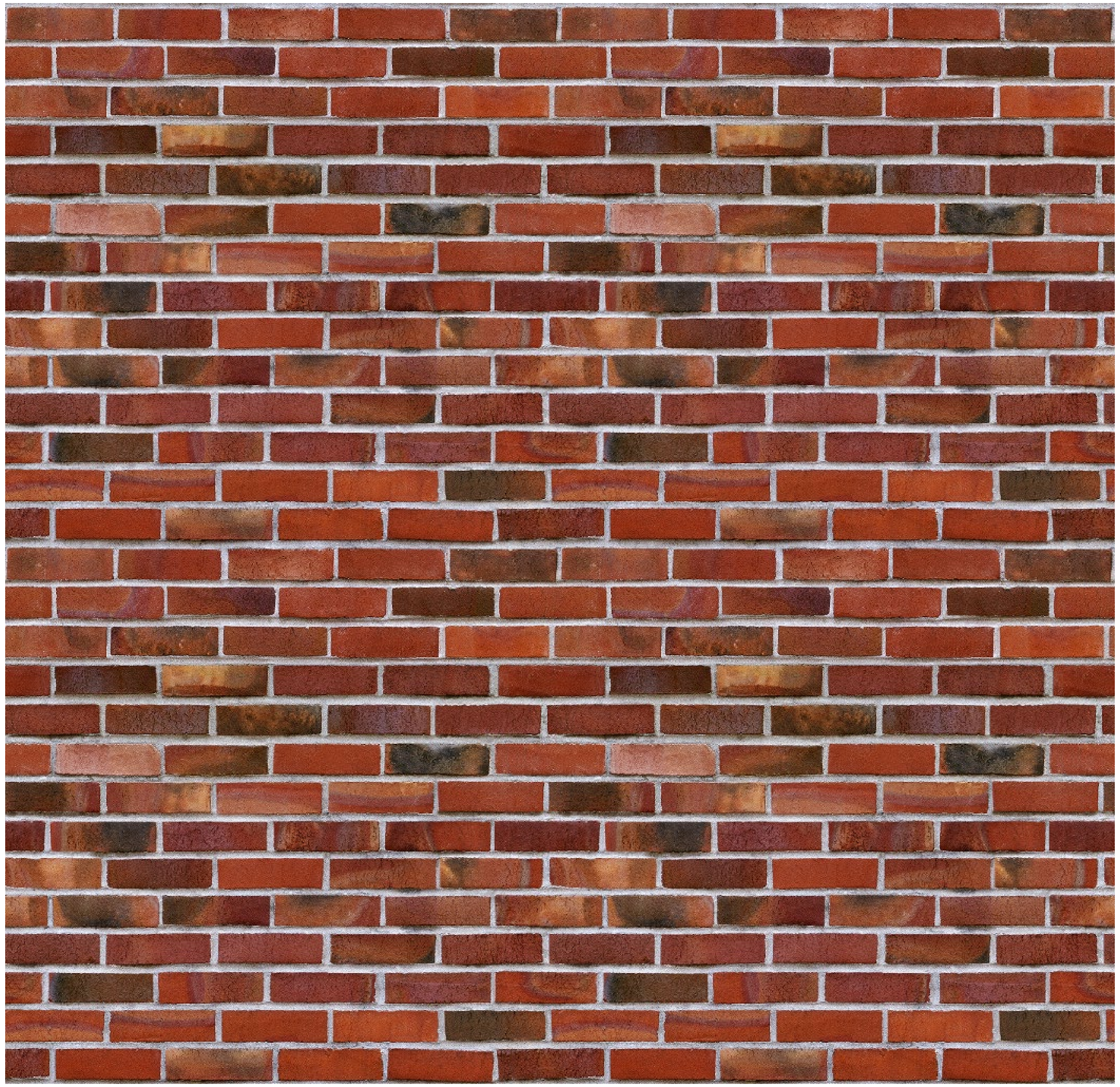
- discontinuities in the mapping function
- change textures to be "tileable" when possible



After a slide by Fabio Pellacini

Seamlessly “tileable" textures





So far

How do we map between surface and texture?

- parametric surfaces
- projection mapping
- uv texturing
- texture lookup

Agenda

What do we map onto the surface?

- reflectance (color, diffuse + specular coeffs., etc)
- surface normal (bump mapping)
- geometry (displacement mapping)
- illumination (environment, reflection, shadows)

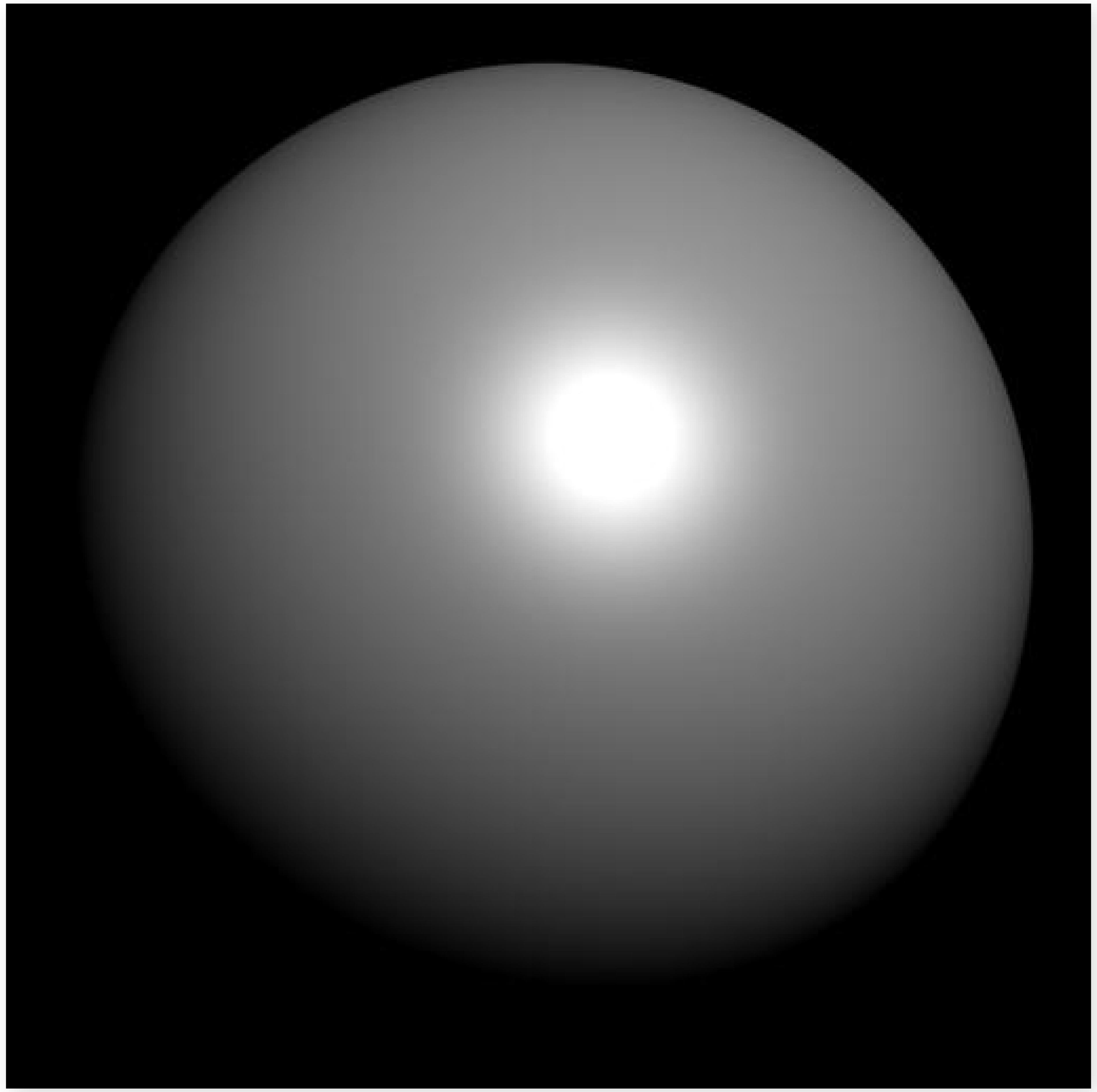
Texturing reflectance properties

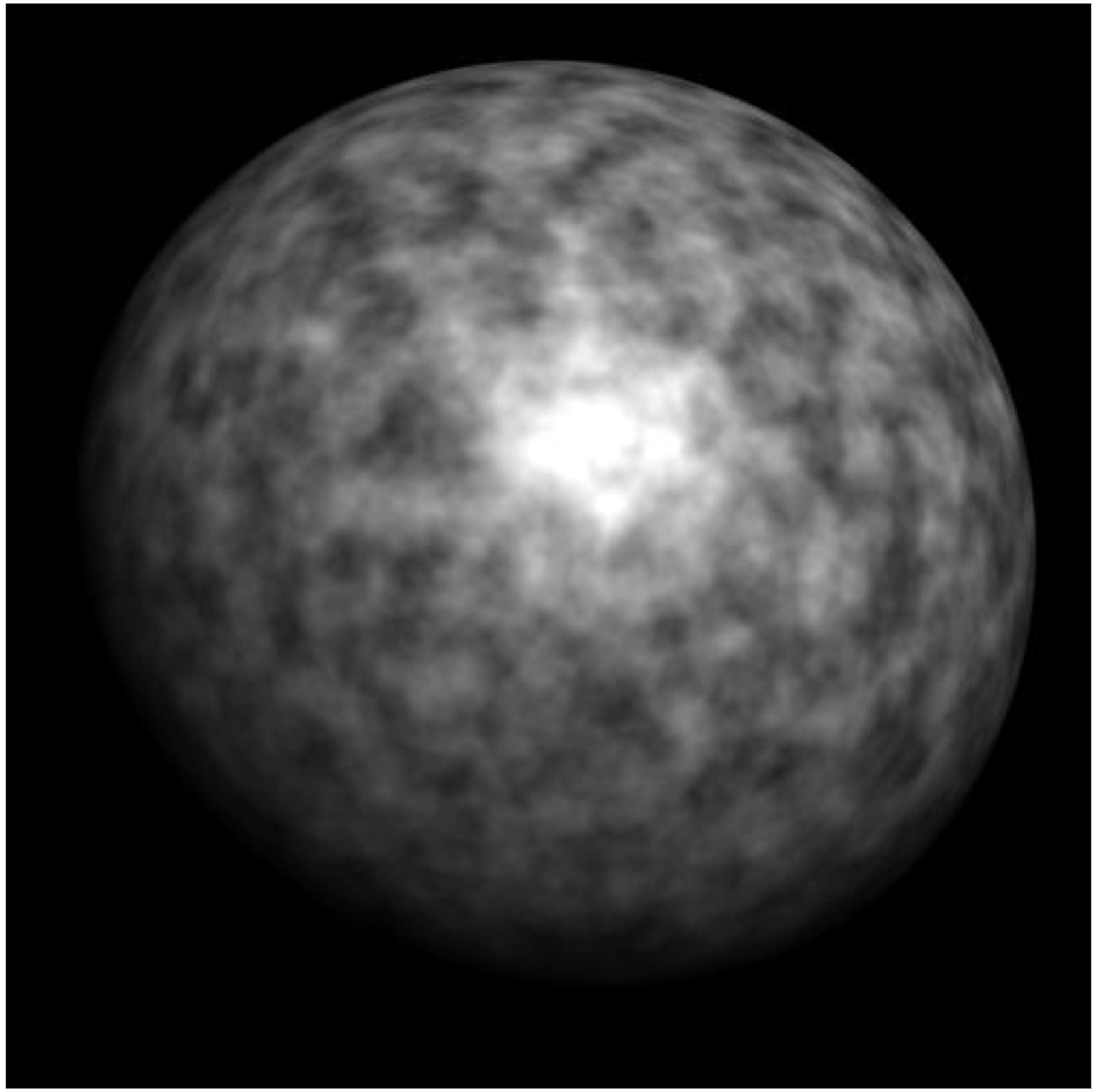
Texture mapping

Surface properties are not the same everywhere

- diffuse color varies due to changing pigmentation
- roughness varies due to changing roughness and surface contamination

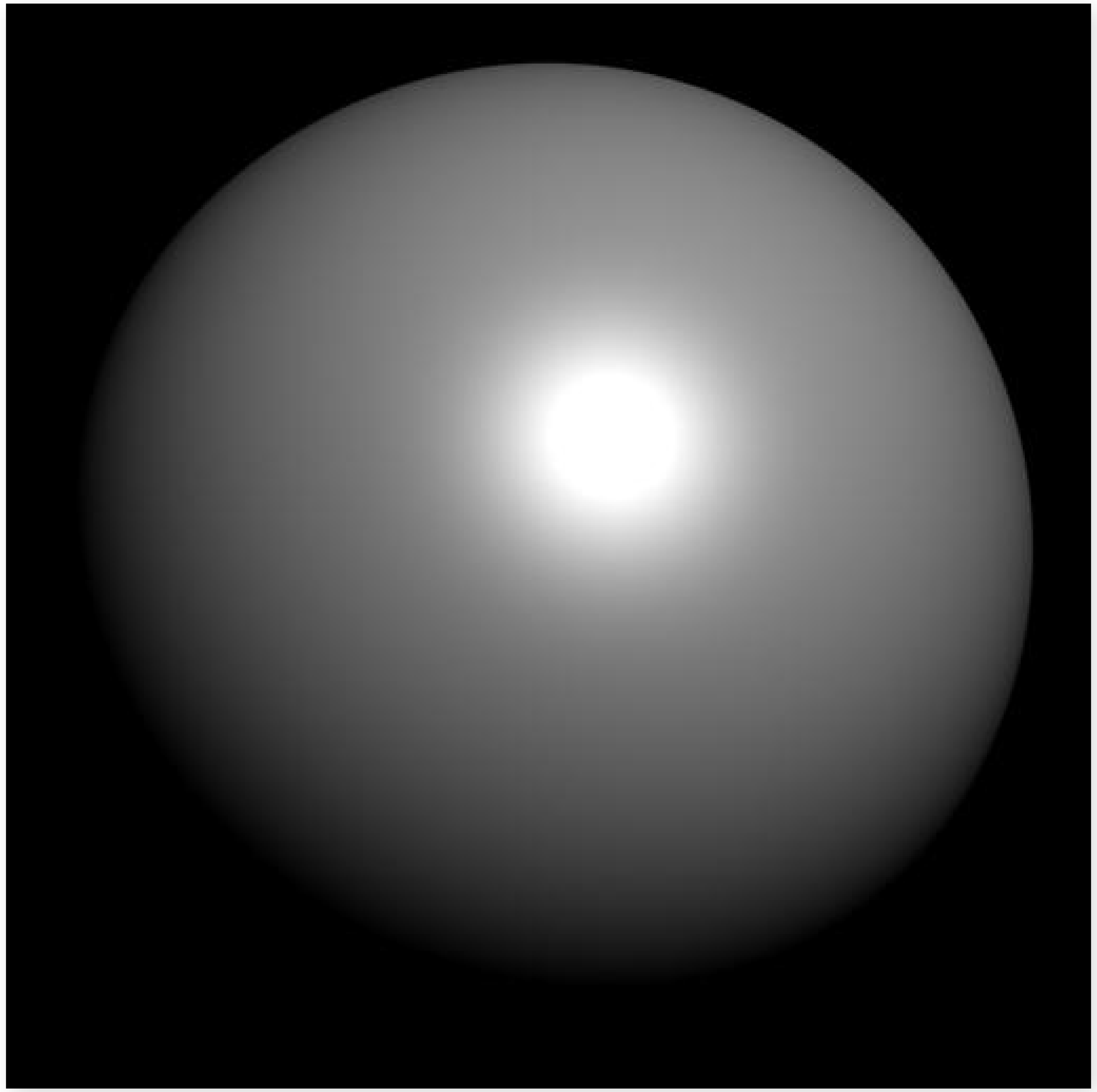
Diffuse coefficient

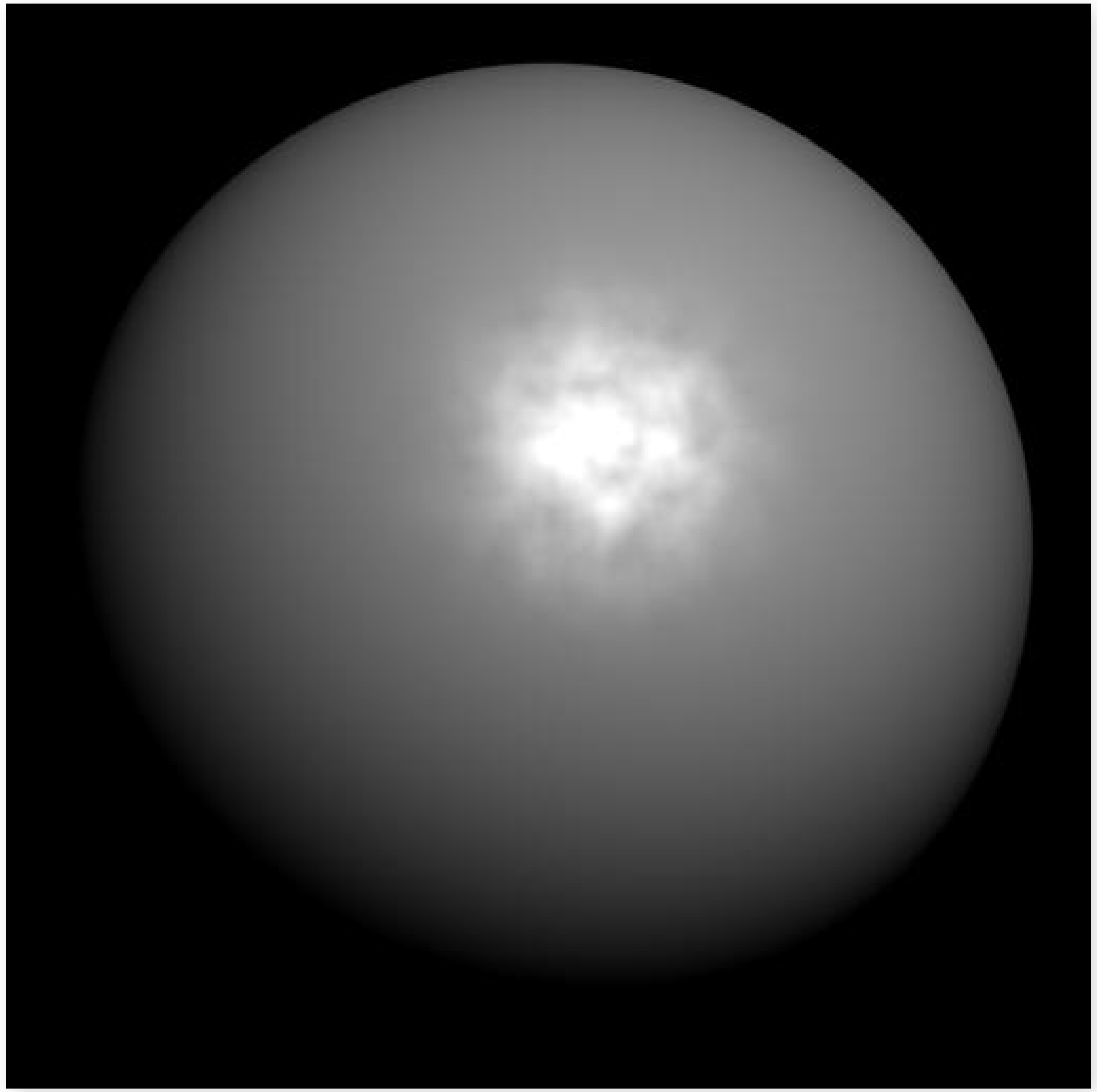




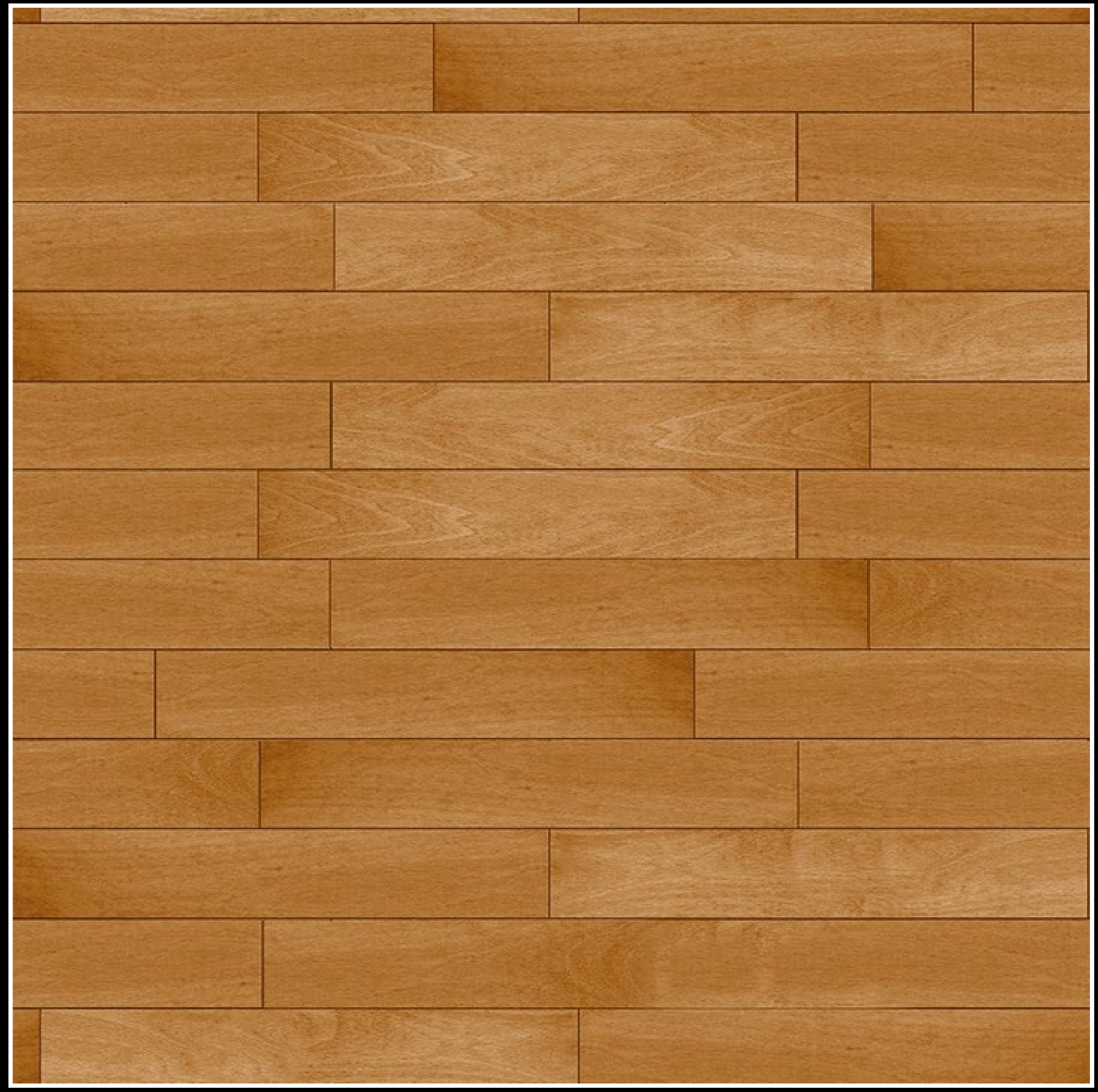
After a slide by Fabio Pellacini

Specular “shininess” coefficient

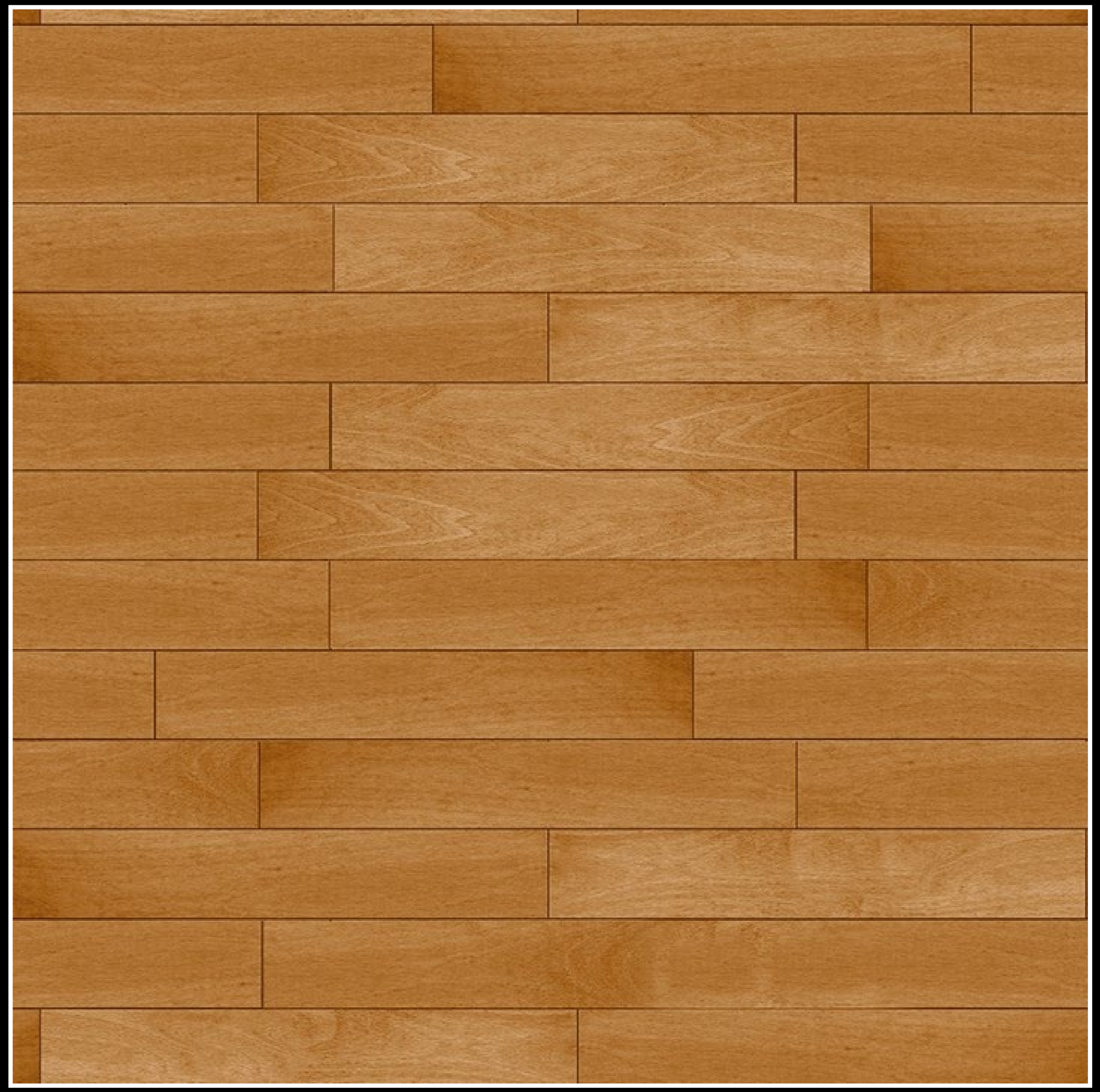




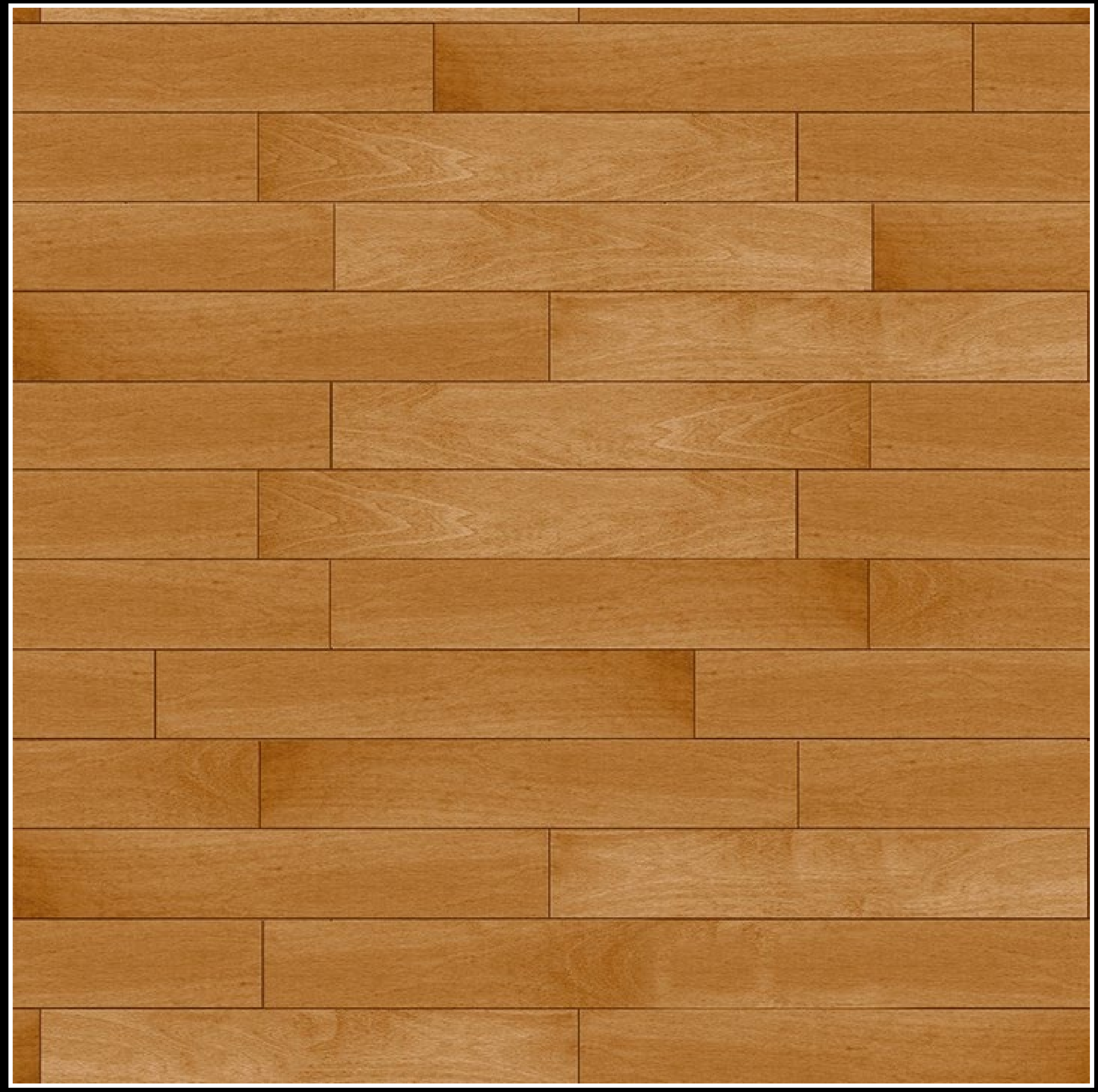
After a slide by Fabio Pellacini

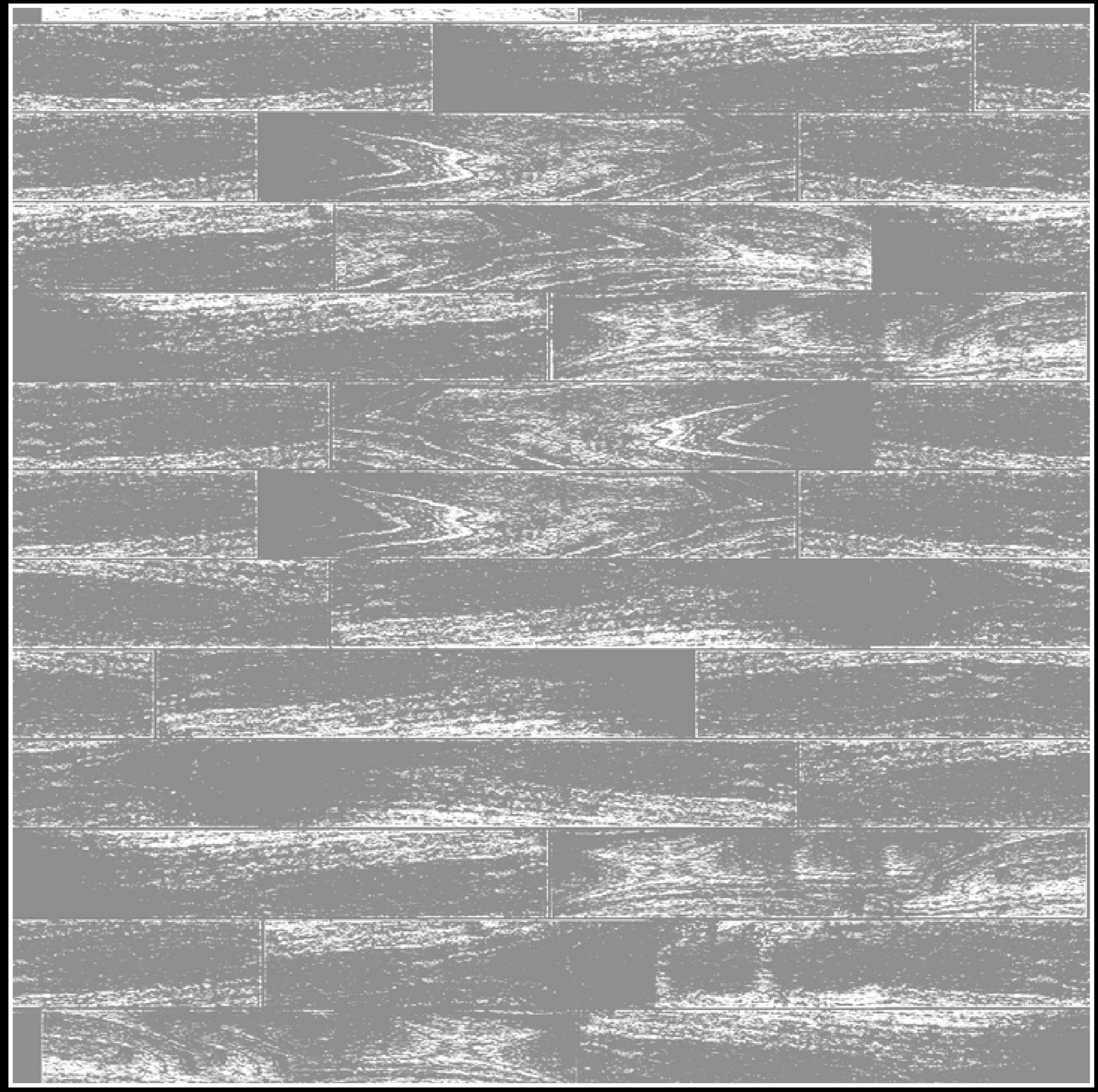


After a slide by Steve Marschner



After a slide by Steve Marschner





After a slide by Steve Marschner

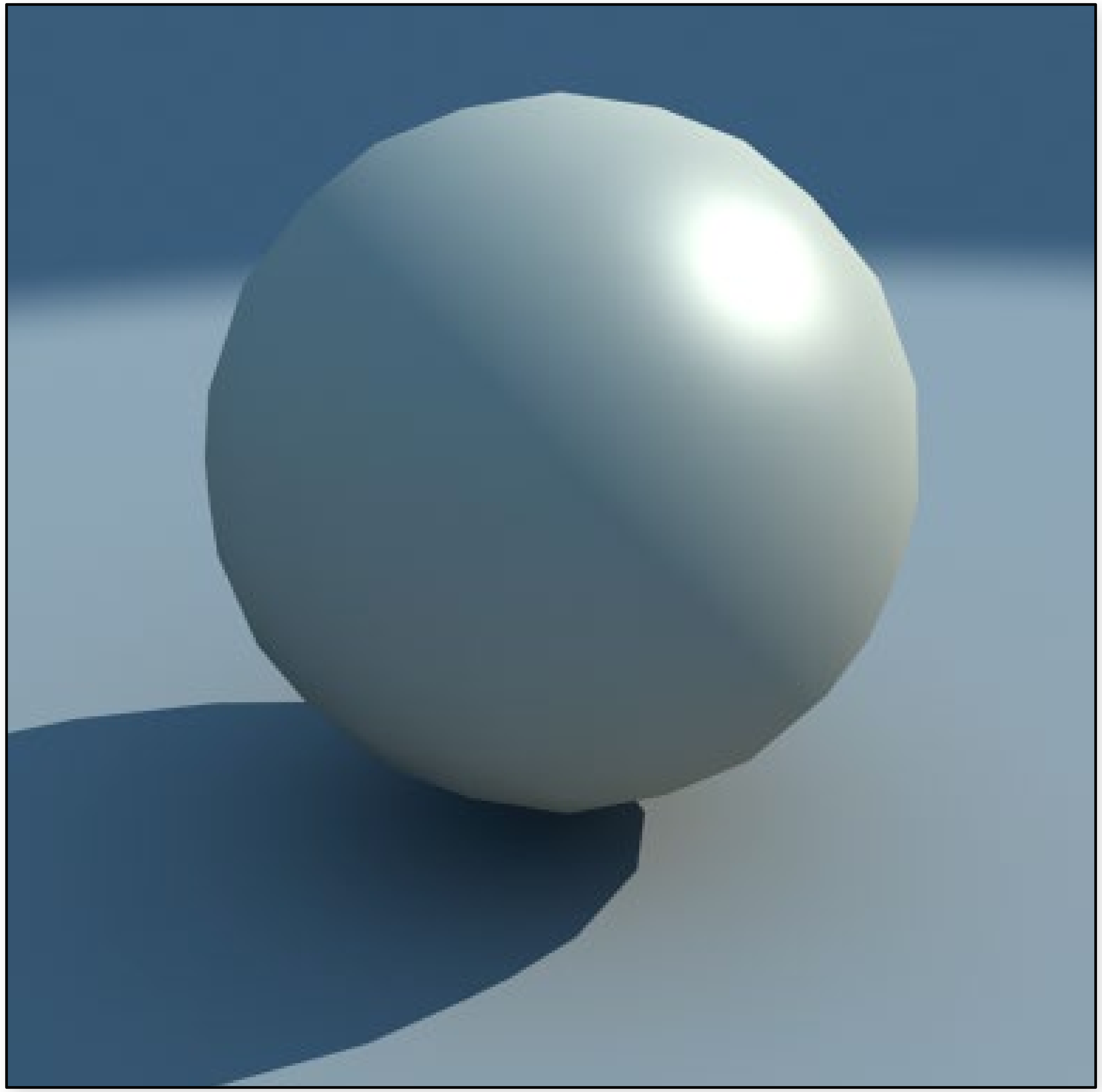
Texturing geometric detail

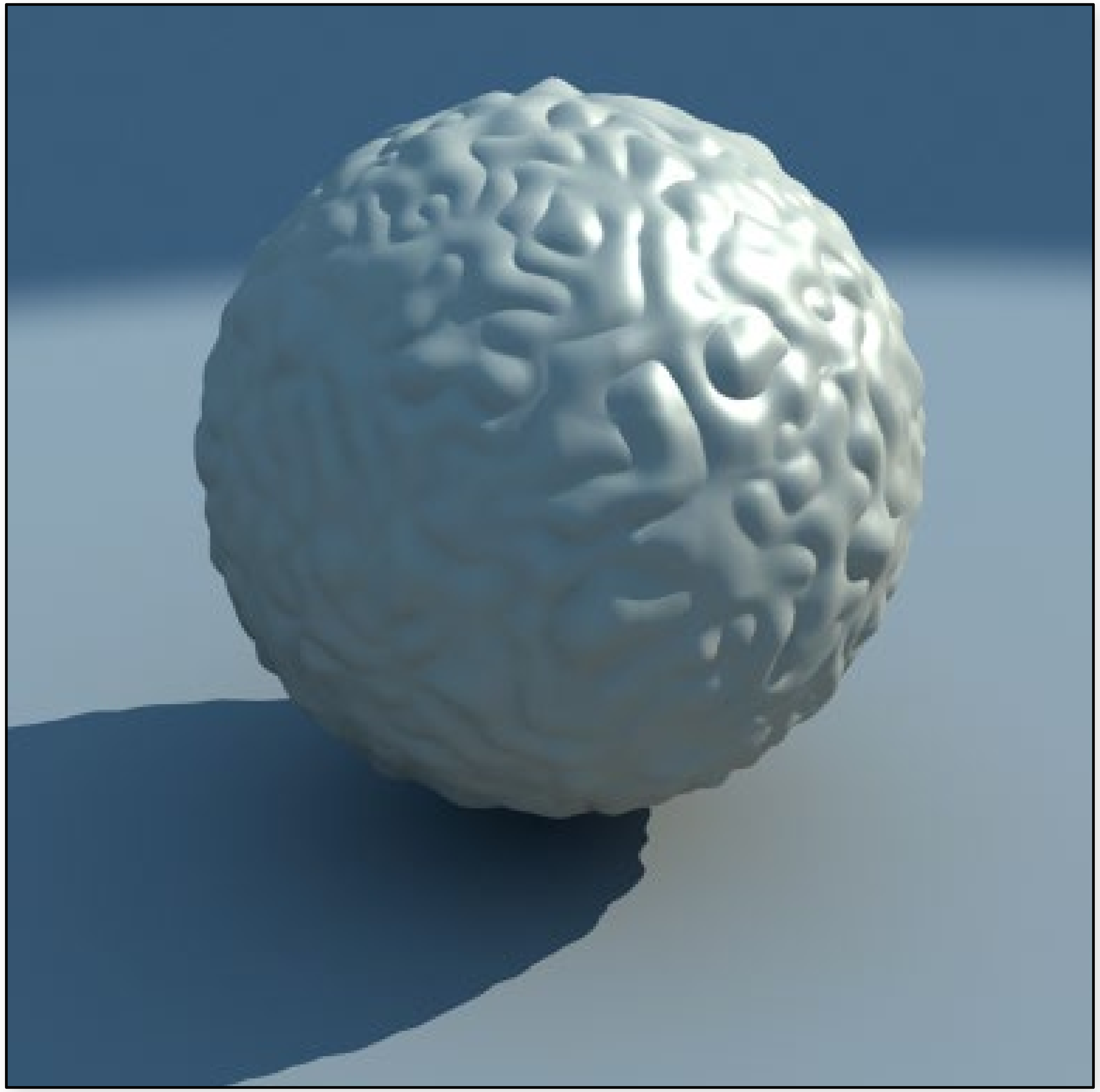
Texturing geometric detail

Modify geometric properties based on a texture

- normals
- surface positions

Displacement mapping



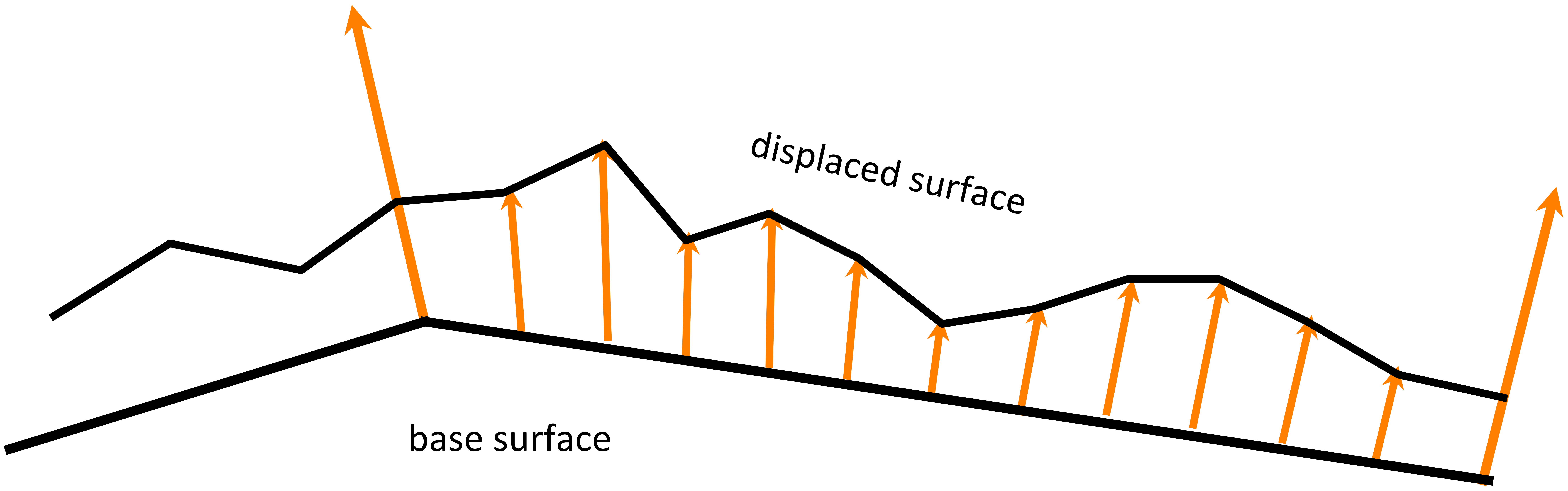


from: www.spot3d.com

Displacement mapping

Encode a displacement distance in the texture map

- Measured, e.g., along interpolated normal



After a slide by Daniele Panozzo

Displacement mapping

Update position by displacing points along normal

$$\mathbf{p}_d = \mathbf{p} + h\mathbf{n}$$

Recompute normals by evaluating derivatives

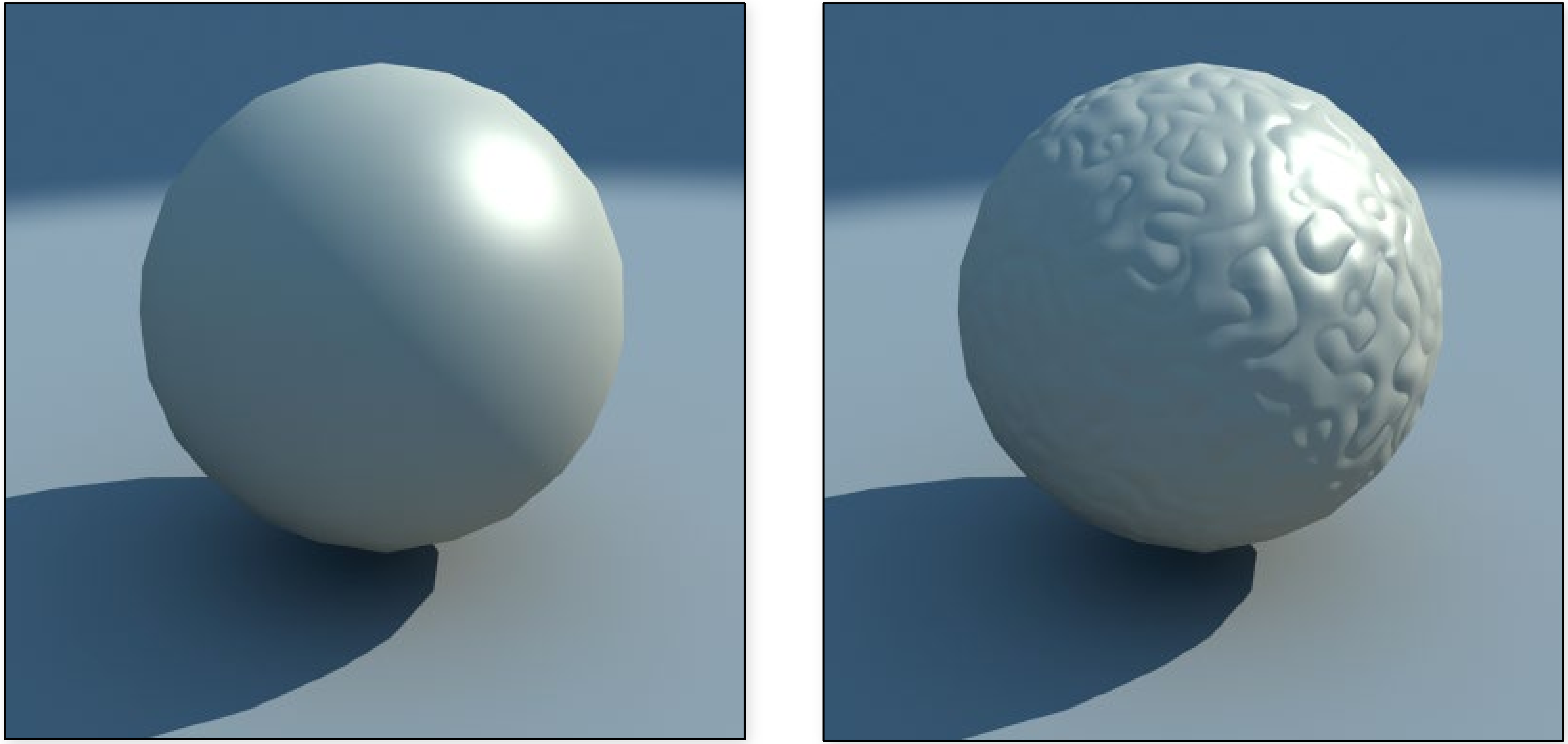
- often no closed form solution, so use finite differences

$$\mathbf{n}_d \propto \frac{\partial \mathbf{p}_d}{\partial u} \times \frac{\partial \mathbf{p}_d}{\partial v} \approx \frac{\Delta \mathbf{p}_d}{\Delta u} \times \frac{\Delta \mathbf{p}_d}{\Delta v}$$

After a slide by Fabio Pellacini

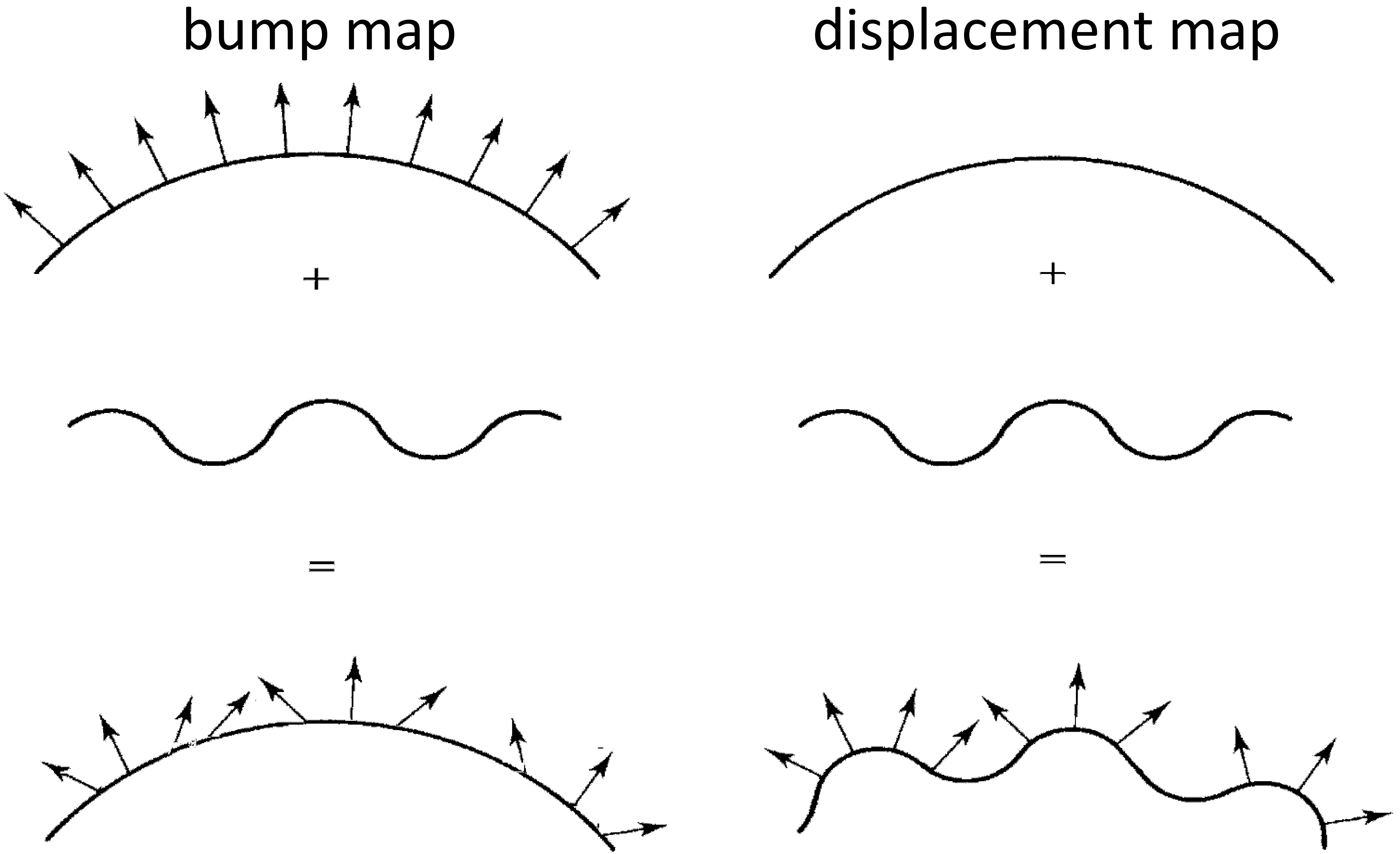
Bump mapping

Apply normal perturbation without updating positions



from: www.spot3d.com

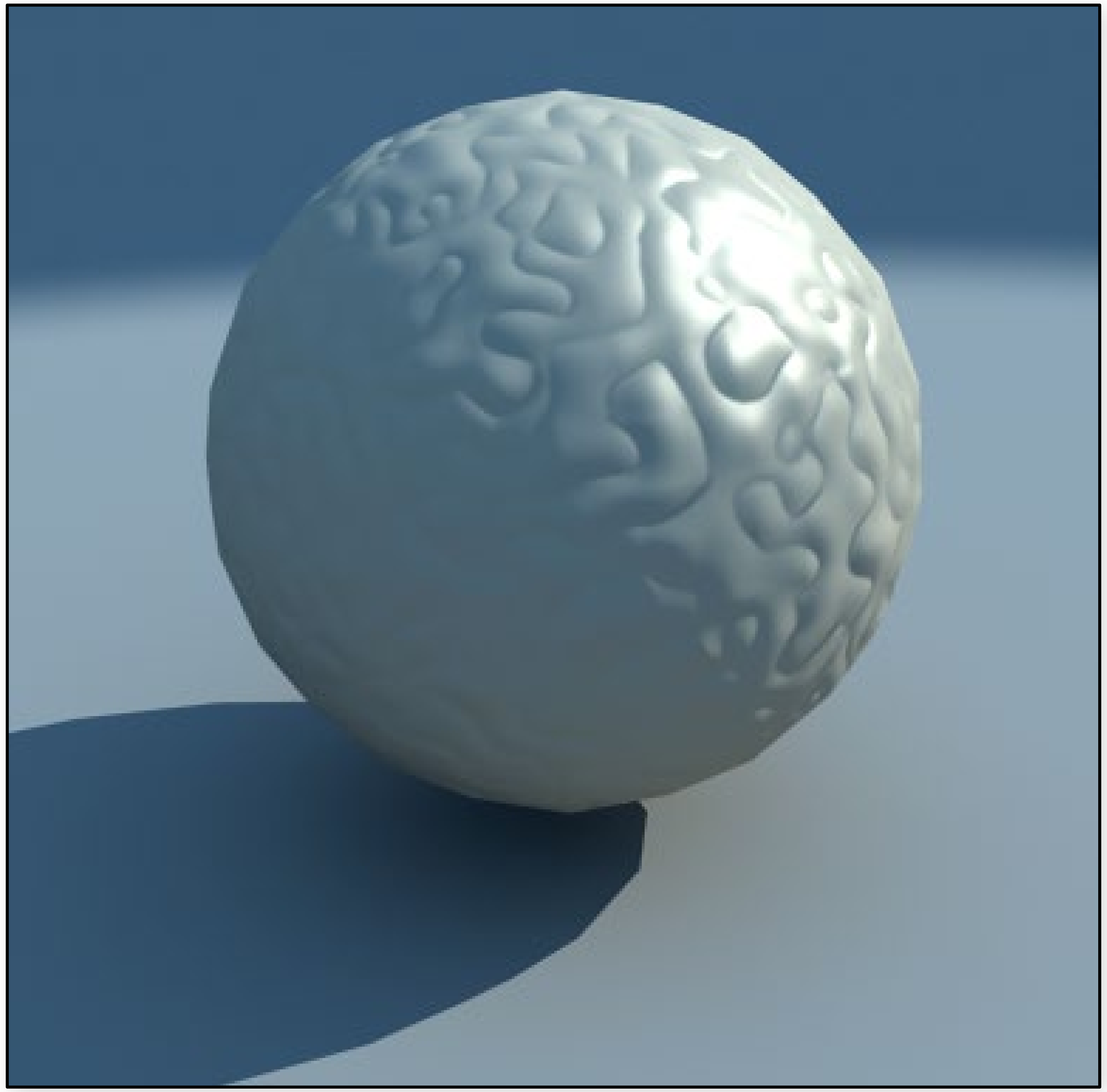
Bump vs. displacement mapping



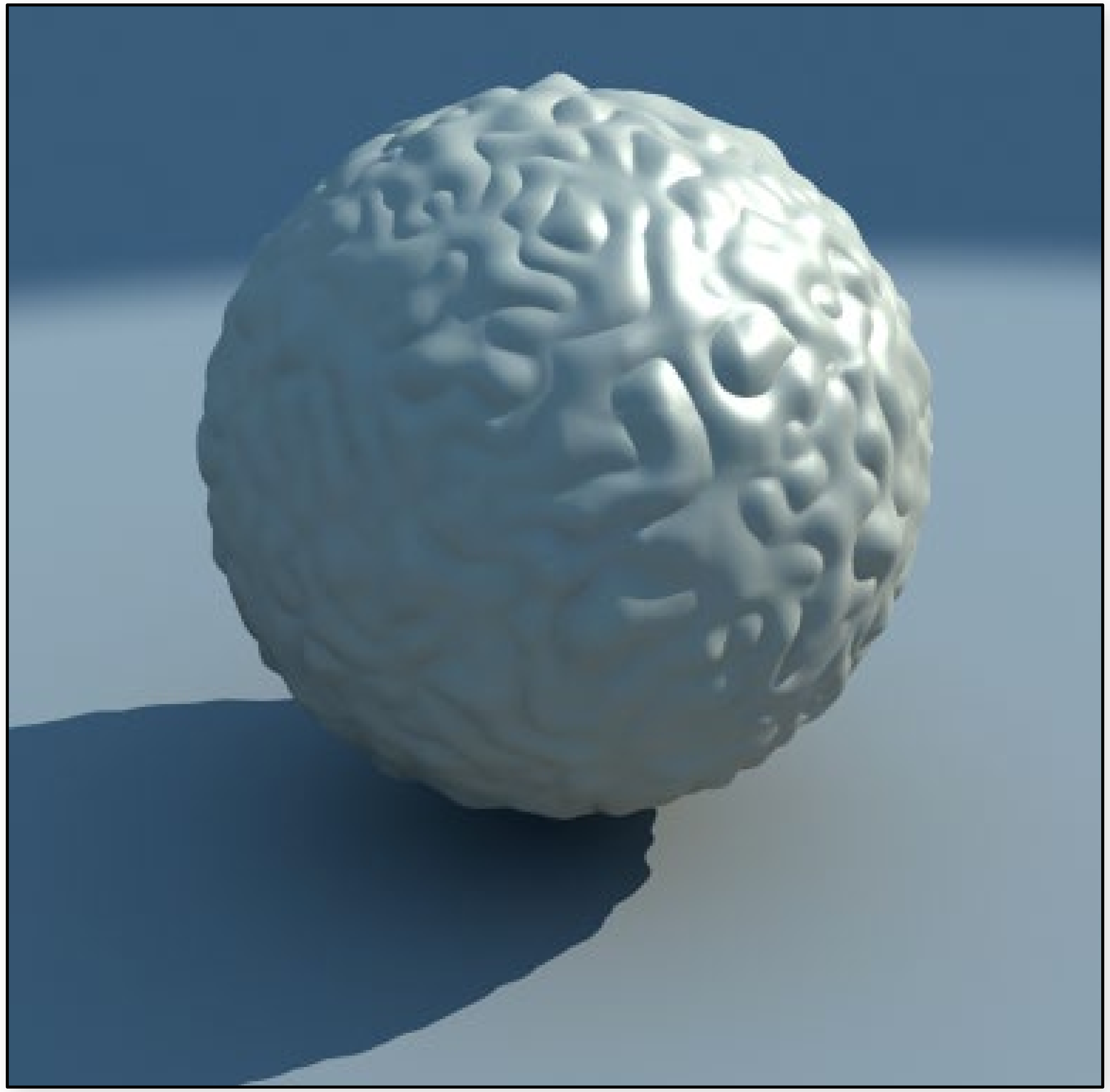
After a slide by Fabio Pellacini

Bump vs. displacement mapping

bump map

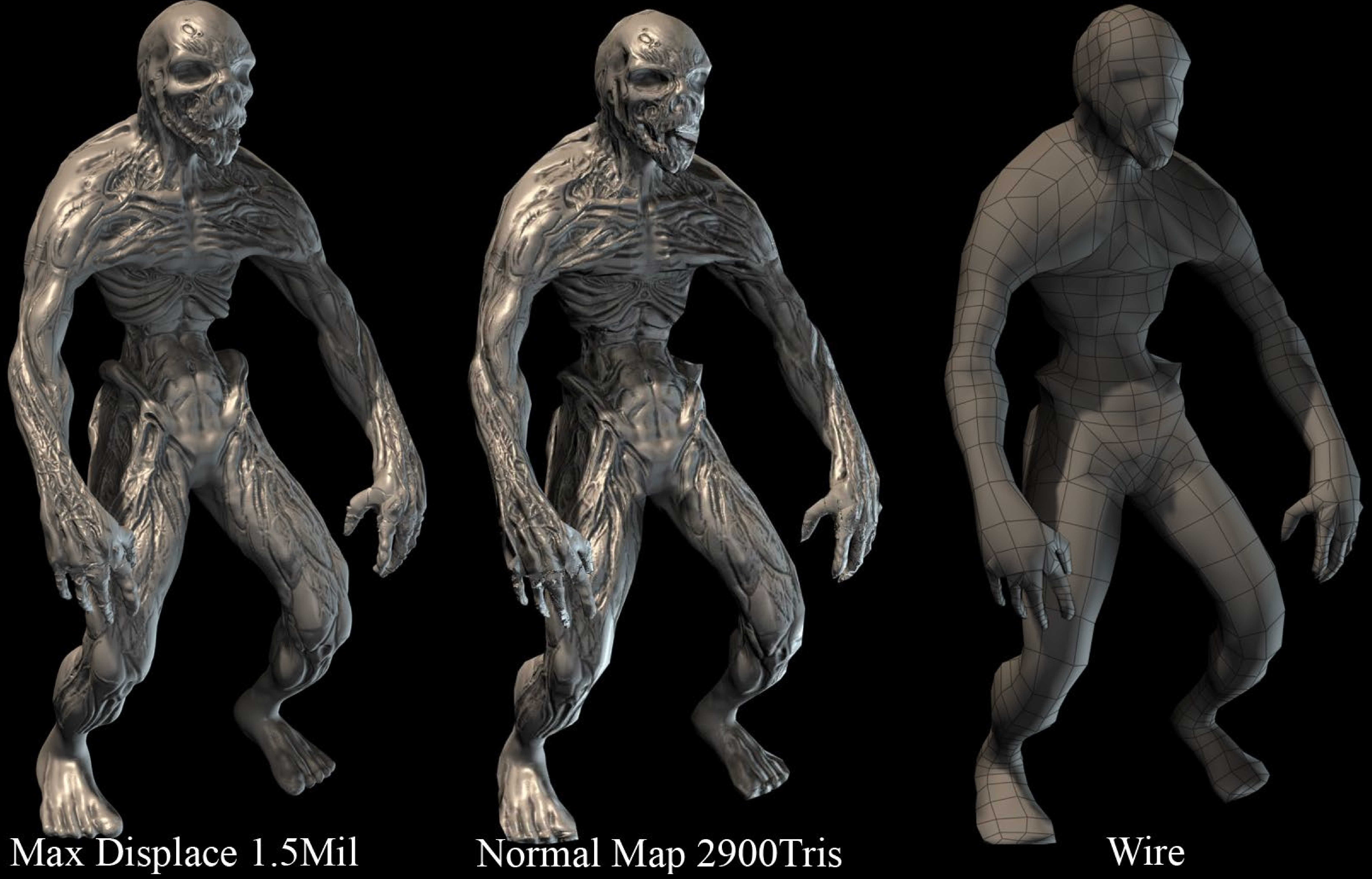


displacement map



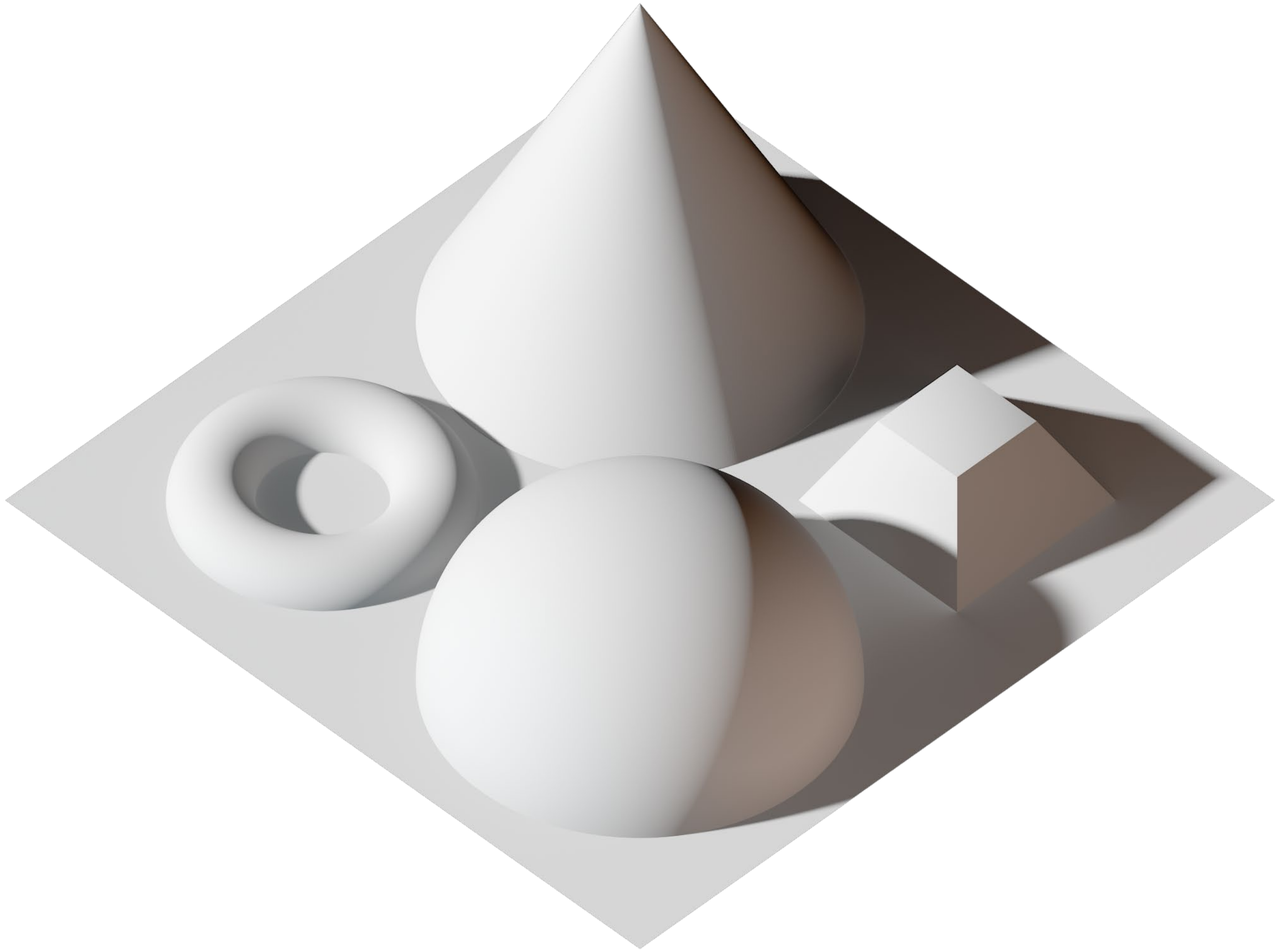
from: www.spot3d.com

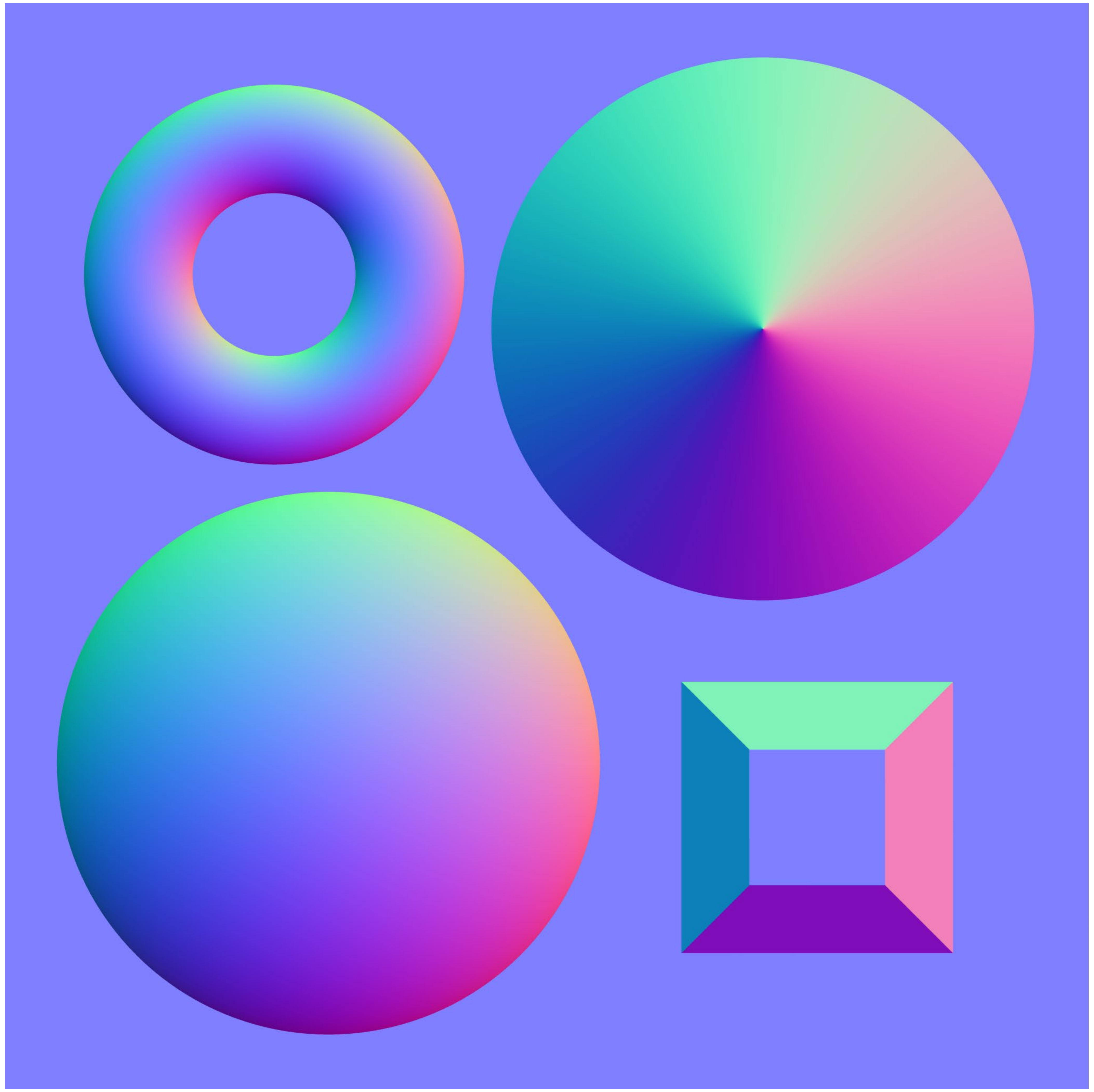
Displacement vs. bump mapping



After a slide by Kavita Bala

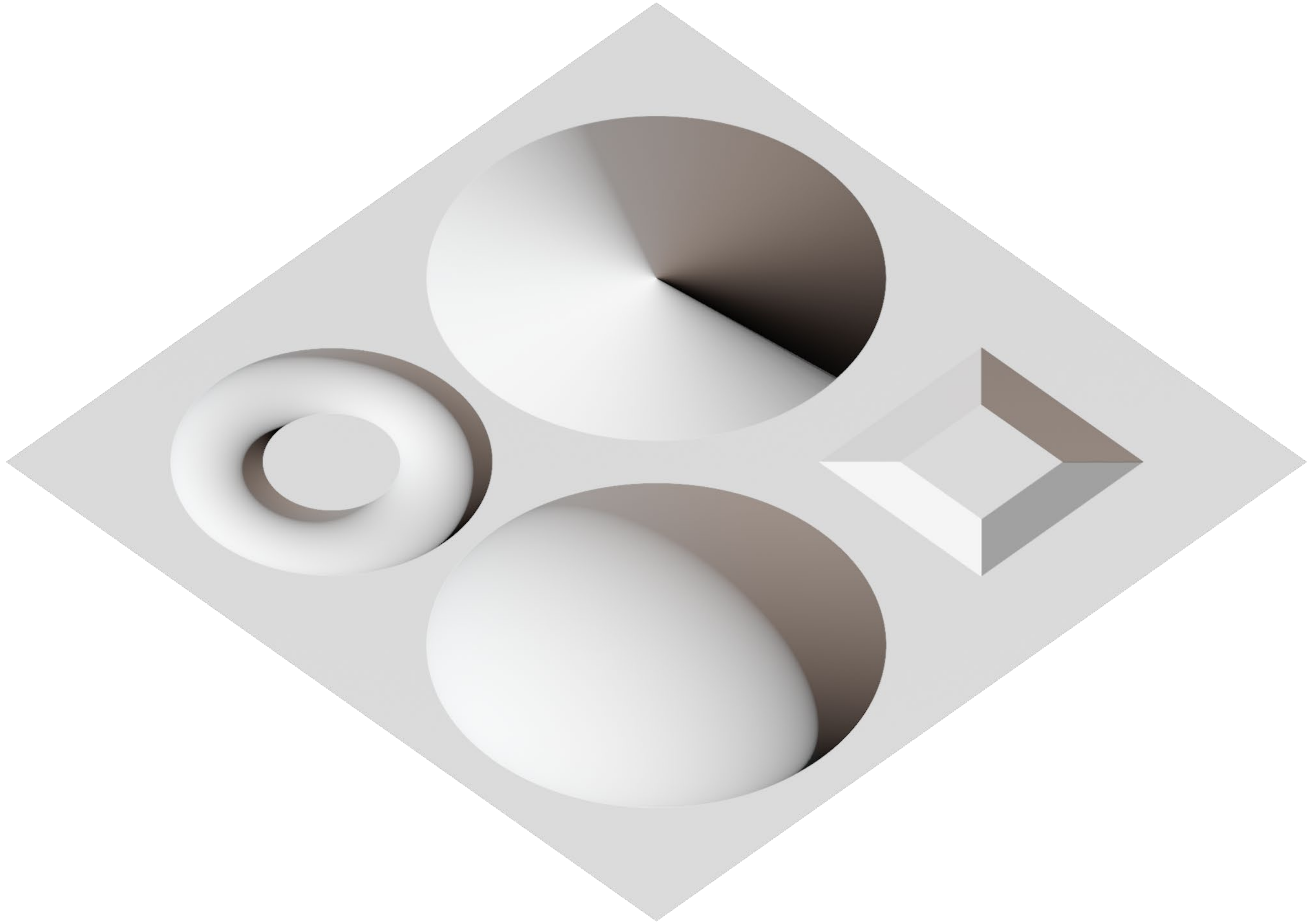
Surface normals

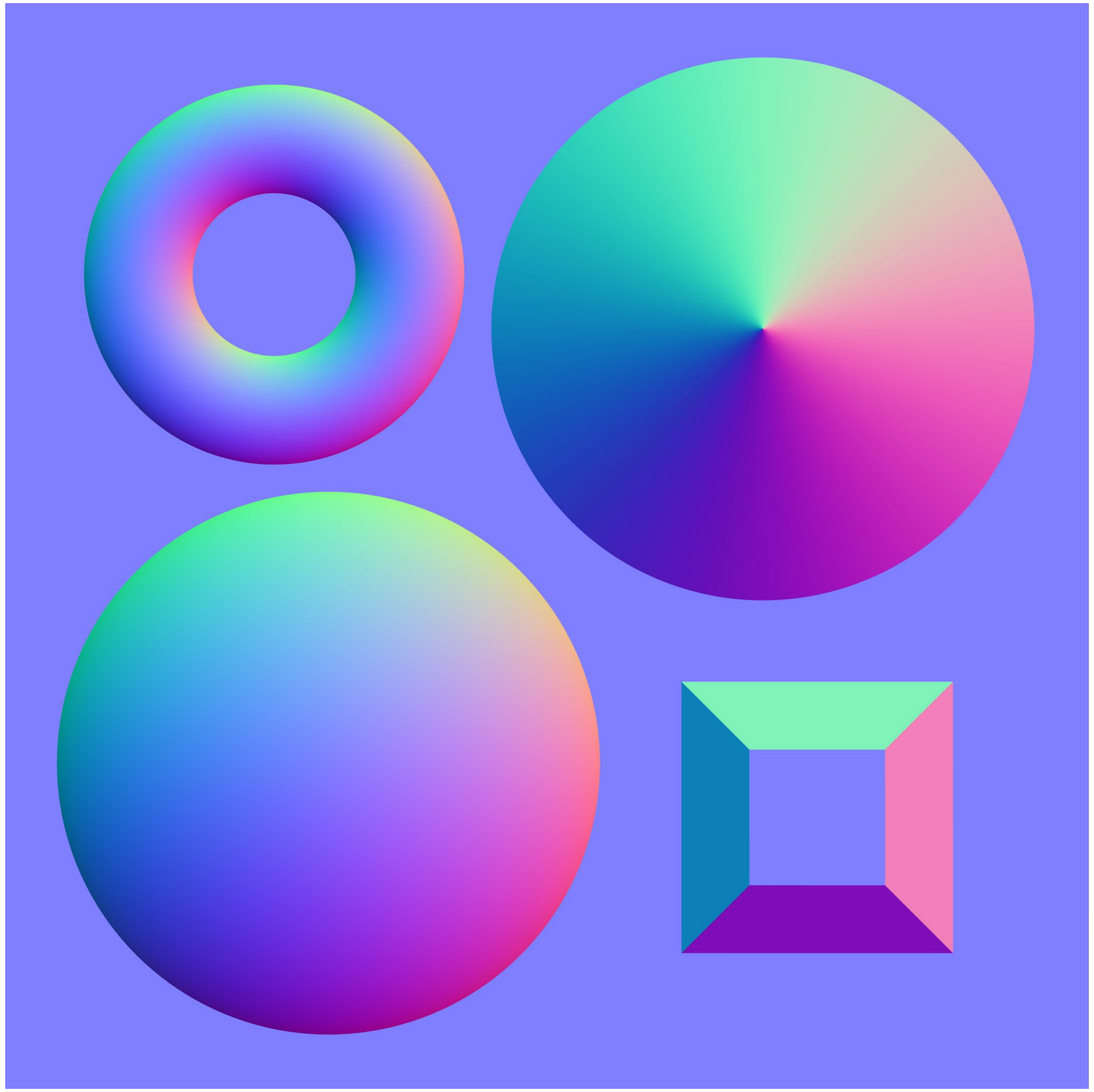




[Wikipedia]

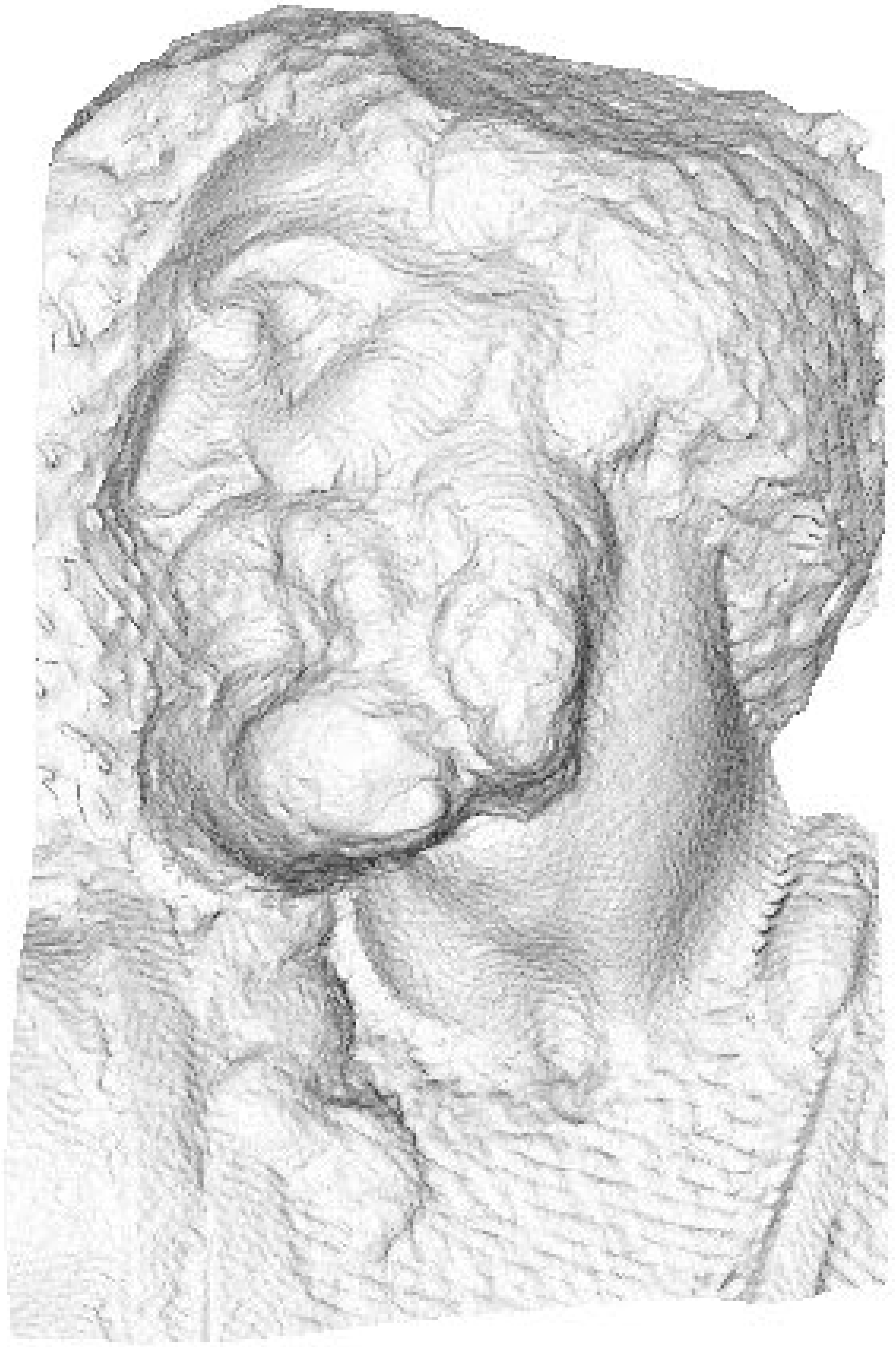
Normal mapping



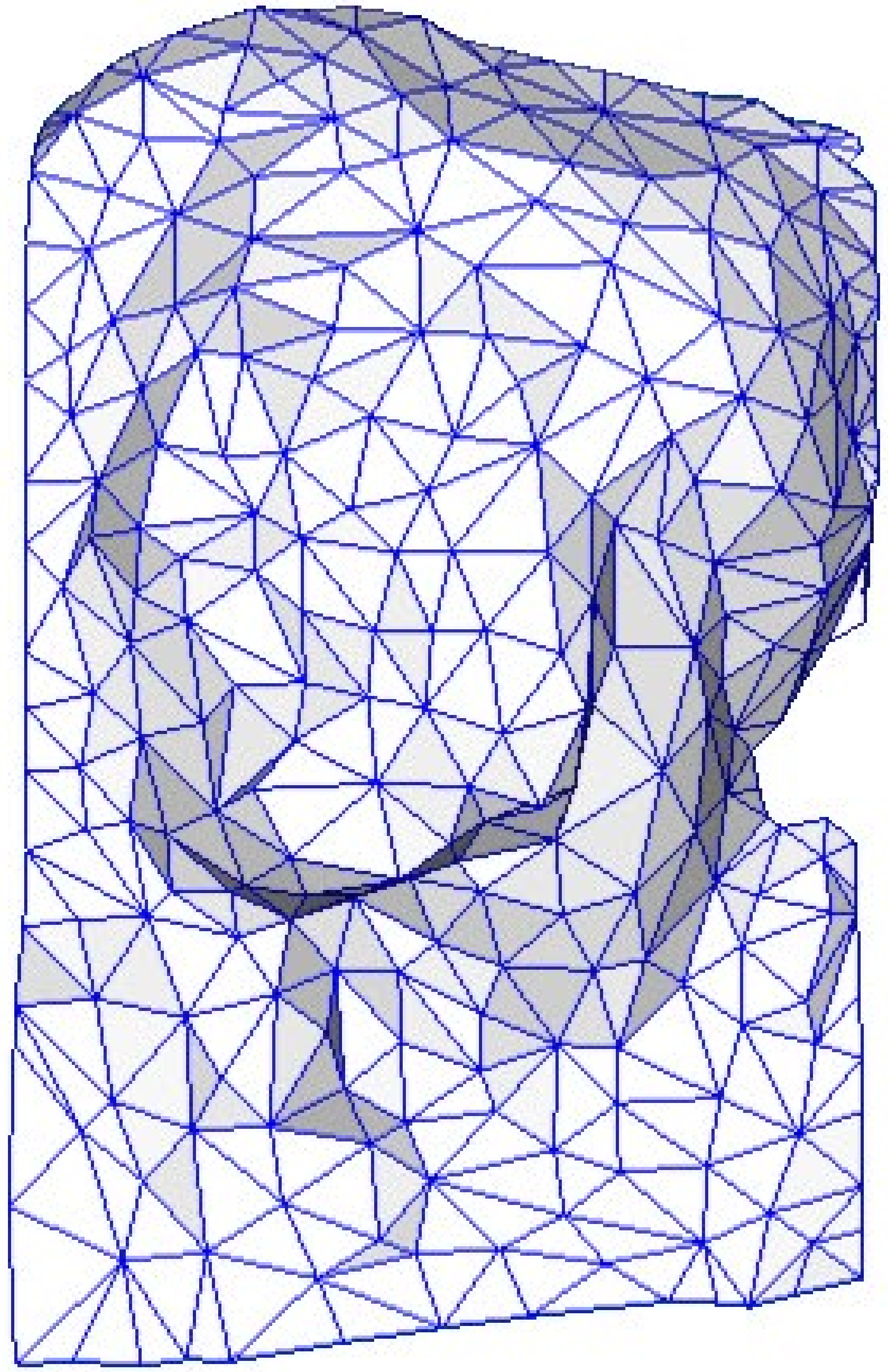


[Wikipedia]

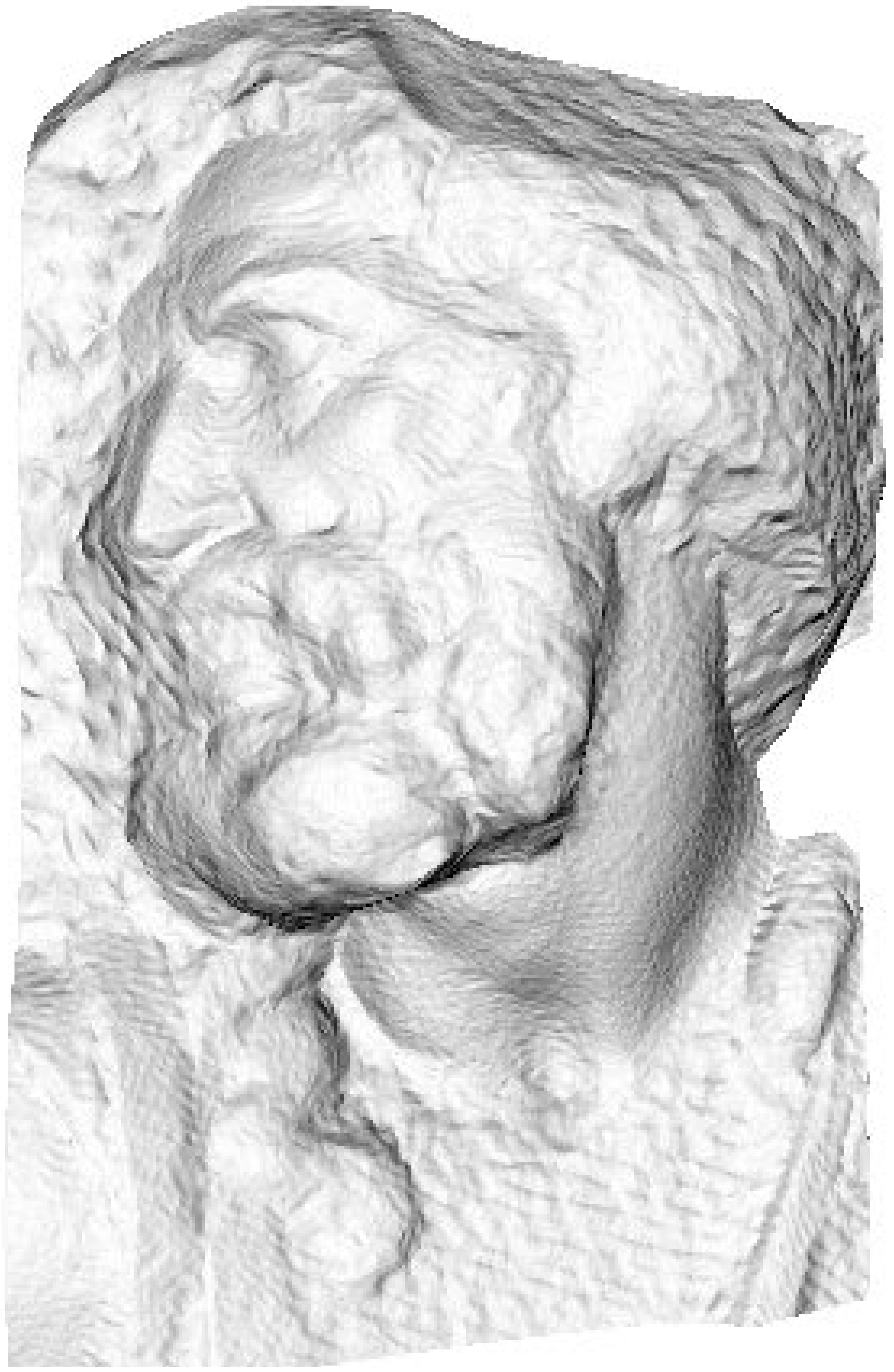
Normal mapping



original mesh
4M triangles



simplified mesh
500 triangles



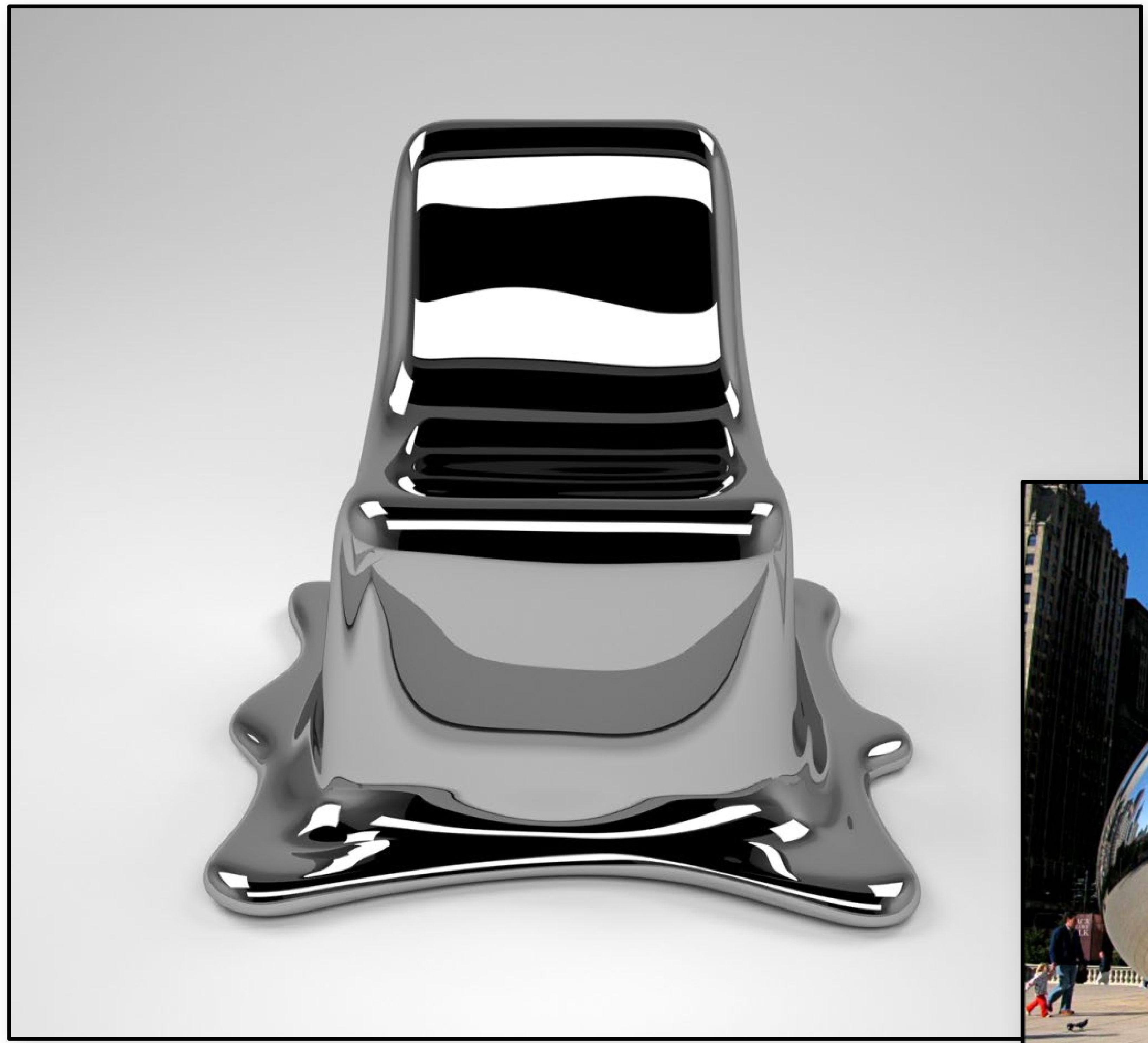
simplified mesh
and normal mapping
500 triangles

After a slide by Daniele Panozzo

Environment & Reflection mapping

Shiny objects

The key to creating a realistic shiny-looking material is providing something for it to reflect.



Environment/reflection mapping

Sidesteps tedious modeling of the environment by representing it using one or more images

The image "wraps" around the virtual scene, serving as a source for reflections

source: [3delight]

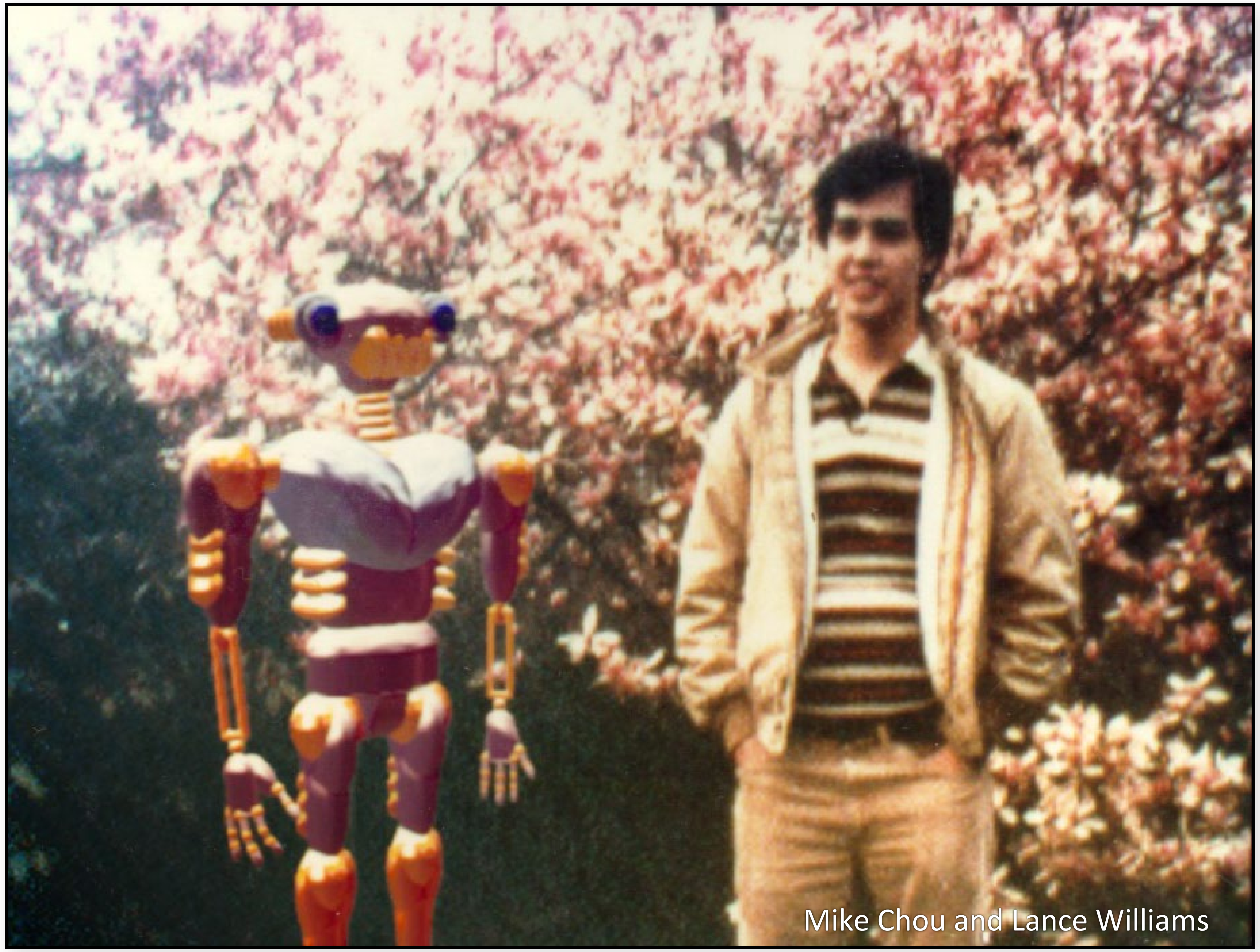
Environment map

A function from the sphere to colors, stored as a texture.

After a slide by Steve Marschner

[Blinn & Newell 1976]

Reflection mapping (1982)

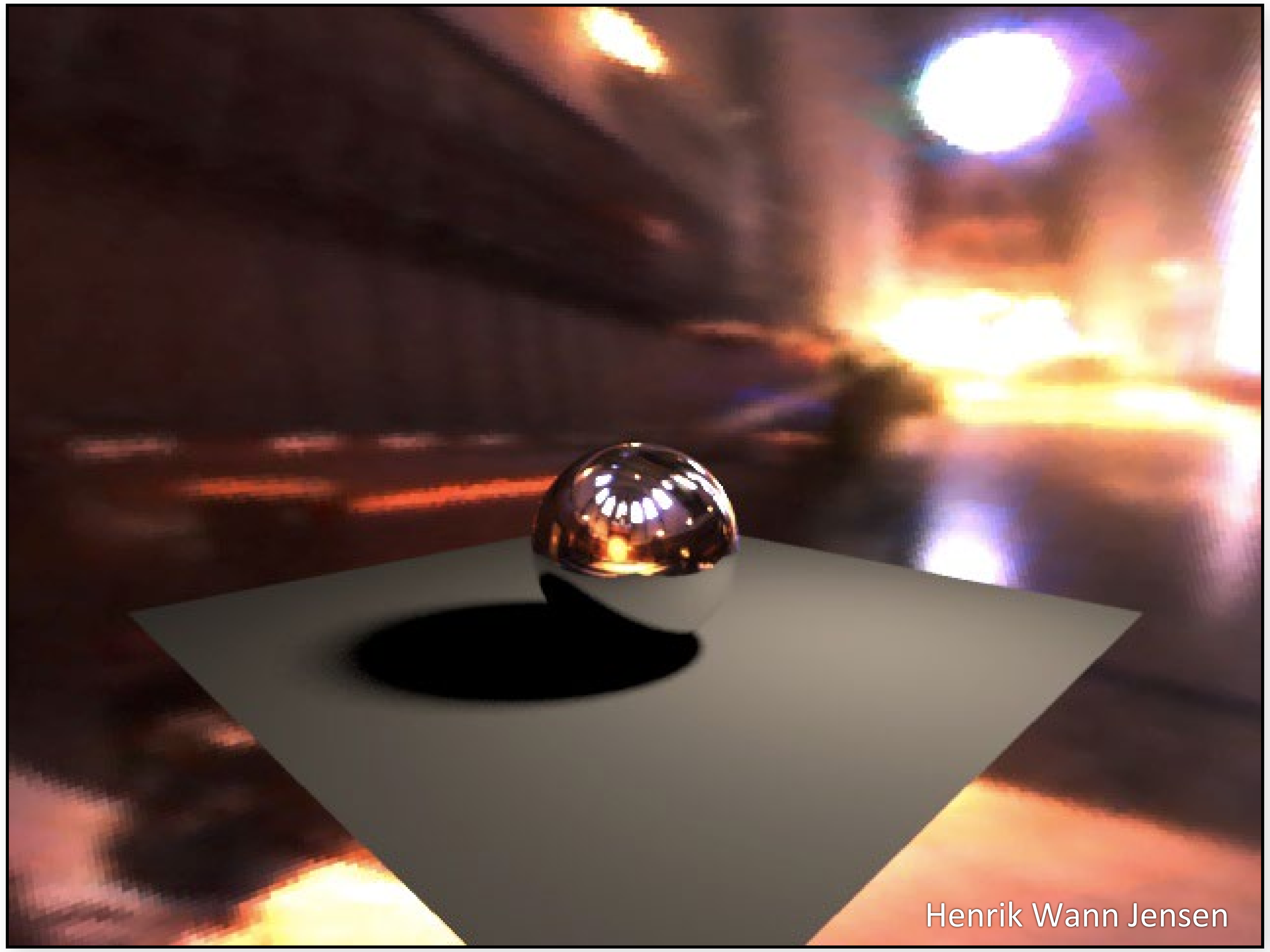

Mike Chou and Lance Williams


Gene Miller and Ken Perlin

Environment mapping

Ray tracing easy: rays that hit nothing look up in envmap

- How exactly do we map these rays to the envmap?



Acquiring environment maps

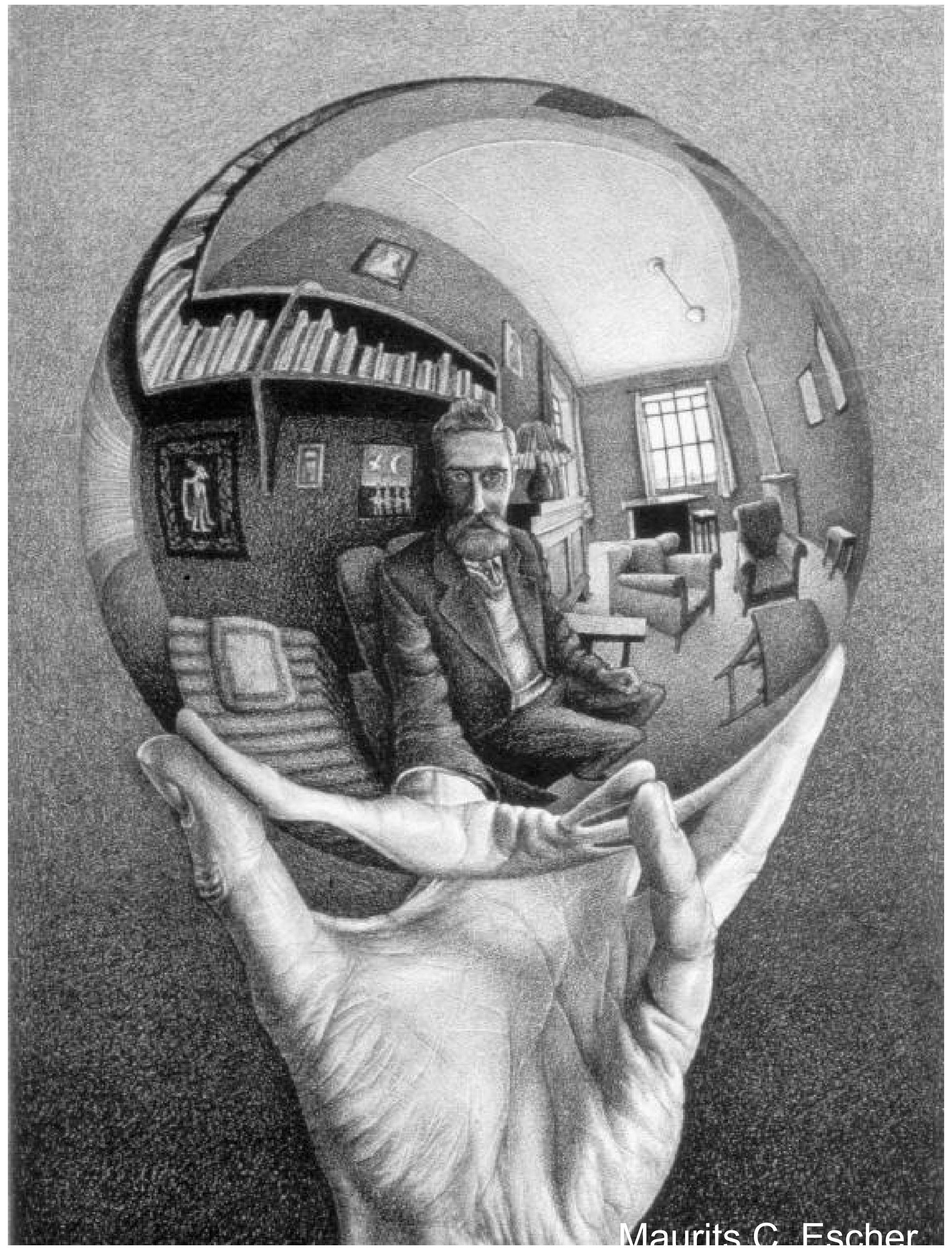
Mirrored ball + camera

Fisheye lens images

Stitching images together

Panoramic camera

Acquisition - Low Tech.



Lightprobe

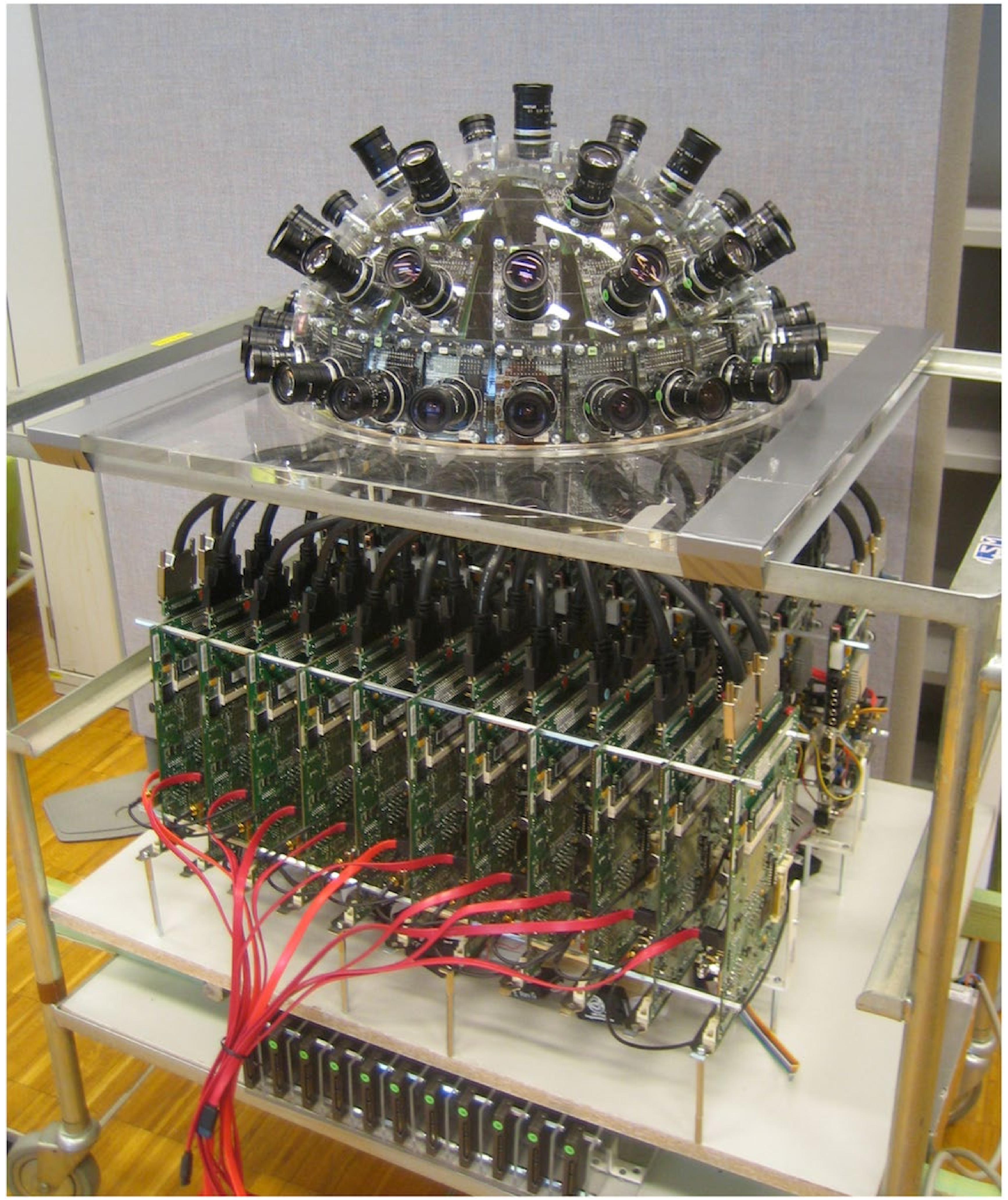
omnidirectional,
360° panoramic,
HDR image

Acquisition - Even Lower Tech.

Acquisition - Stitched Panorama

source: photoanswers.co.uk

Acquisition - High Tech.



lsm.epfl.ch

immersivemedia.com

Acquisition

Storing environment maps

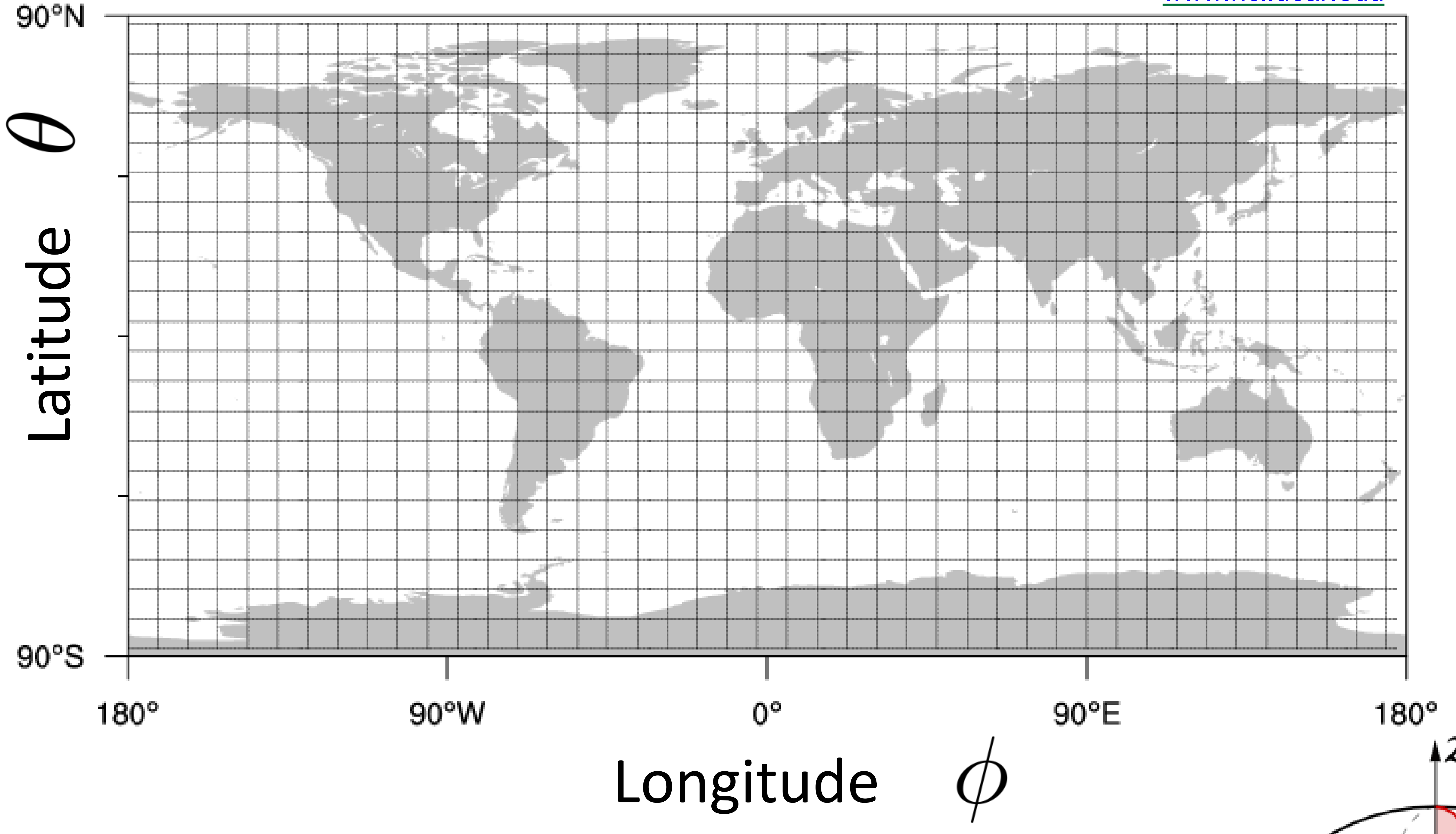
Various ways to parametrize environment maps

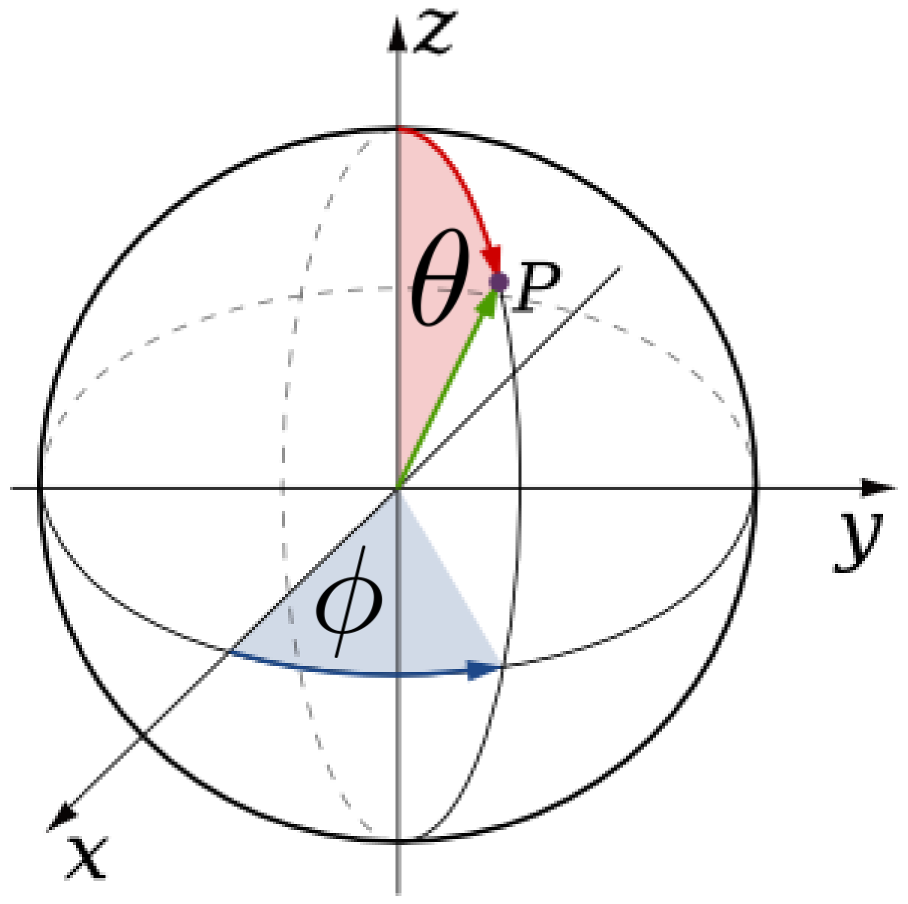
Related to cartography

- Projecting the earth (sphere) onto a plane

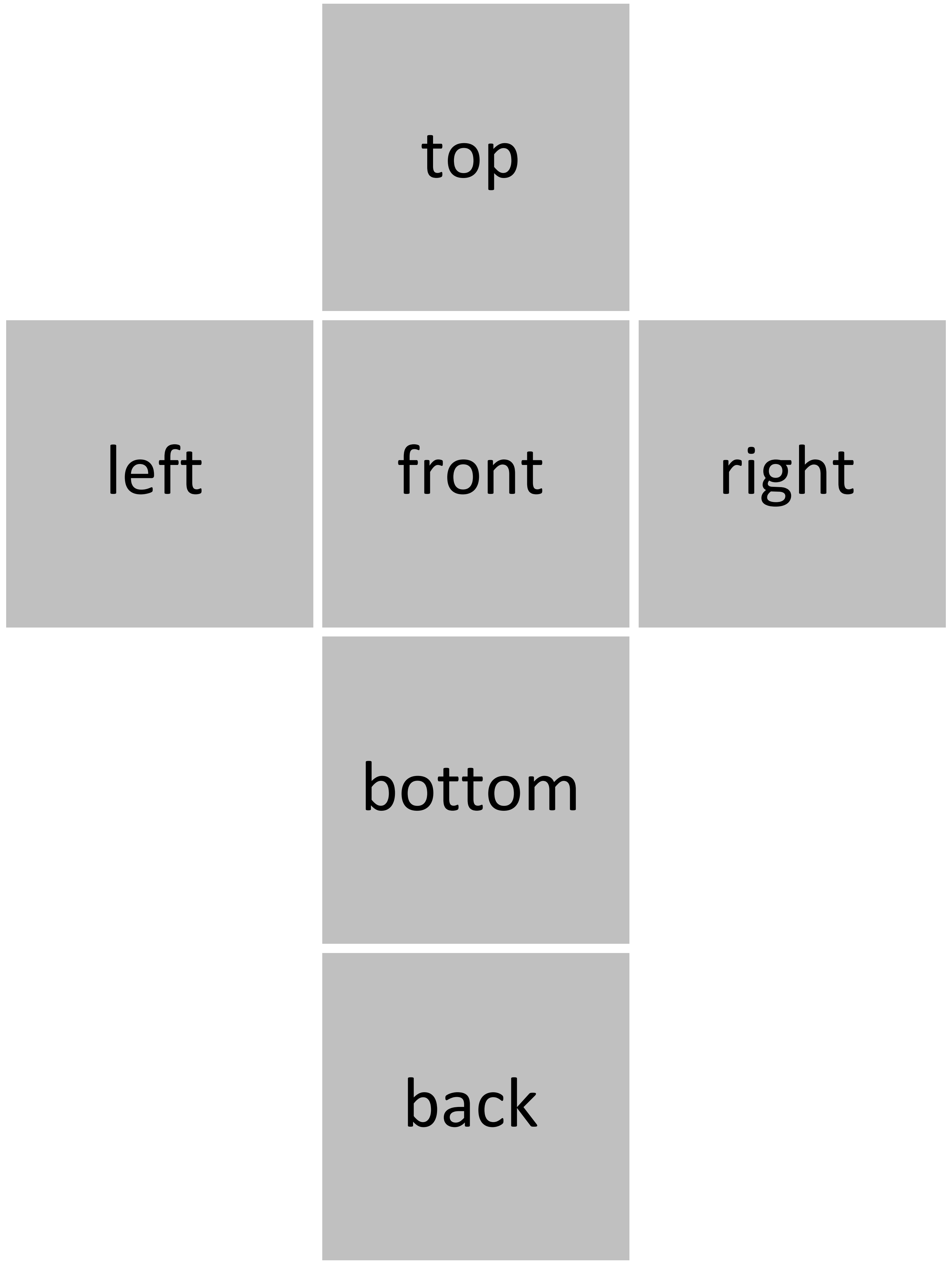
Latitude/Longitude Map

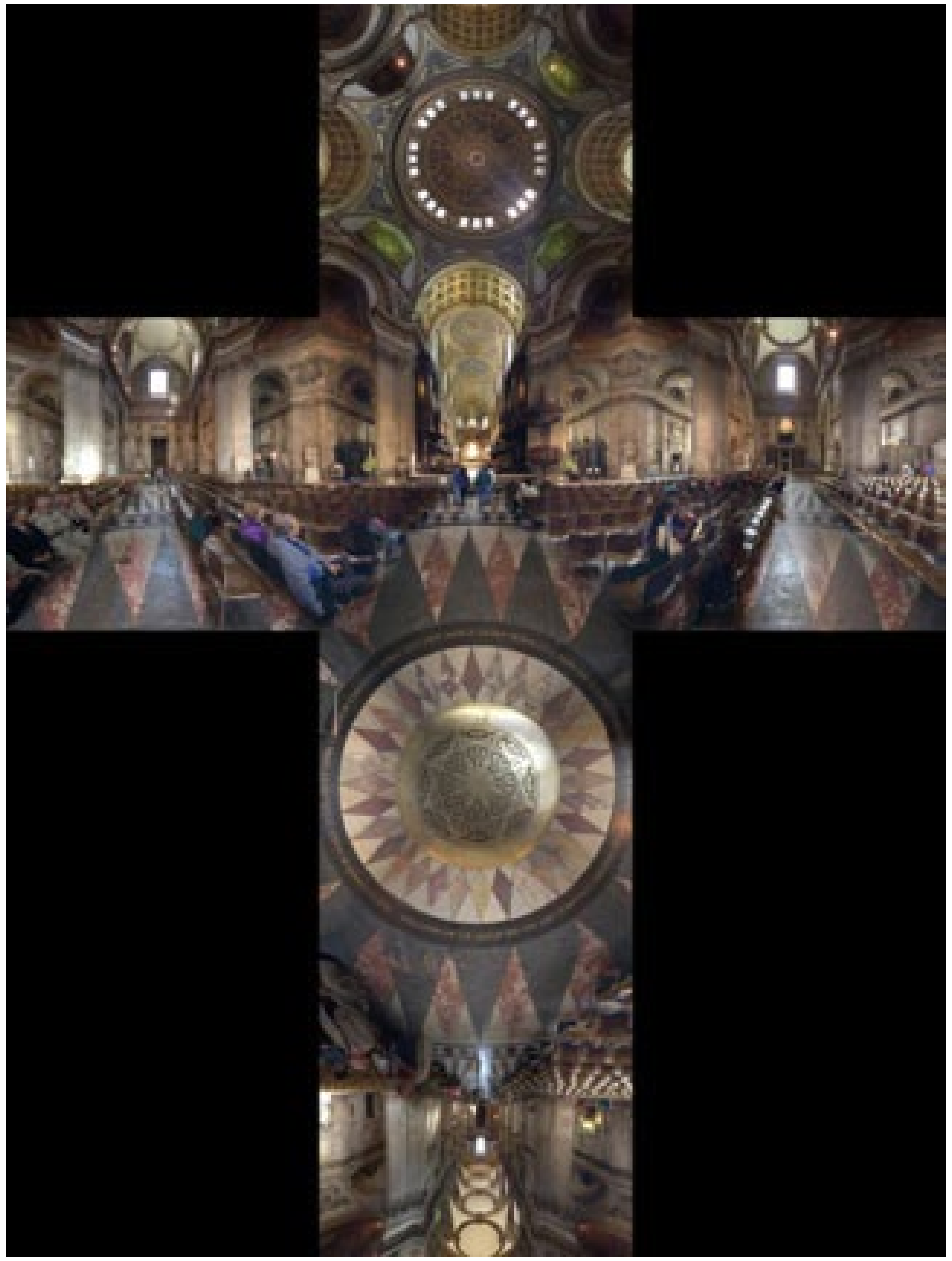
www.ncl.ucar.edu





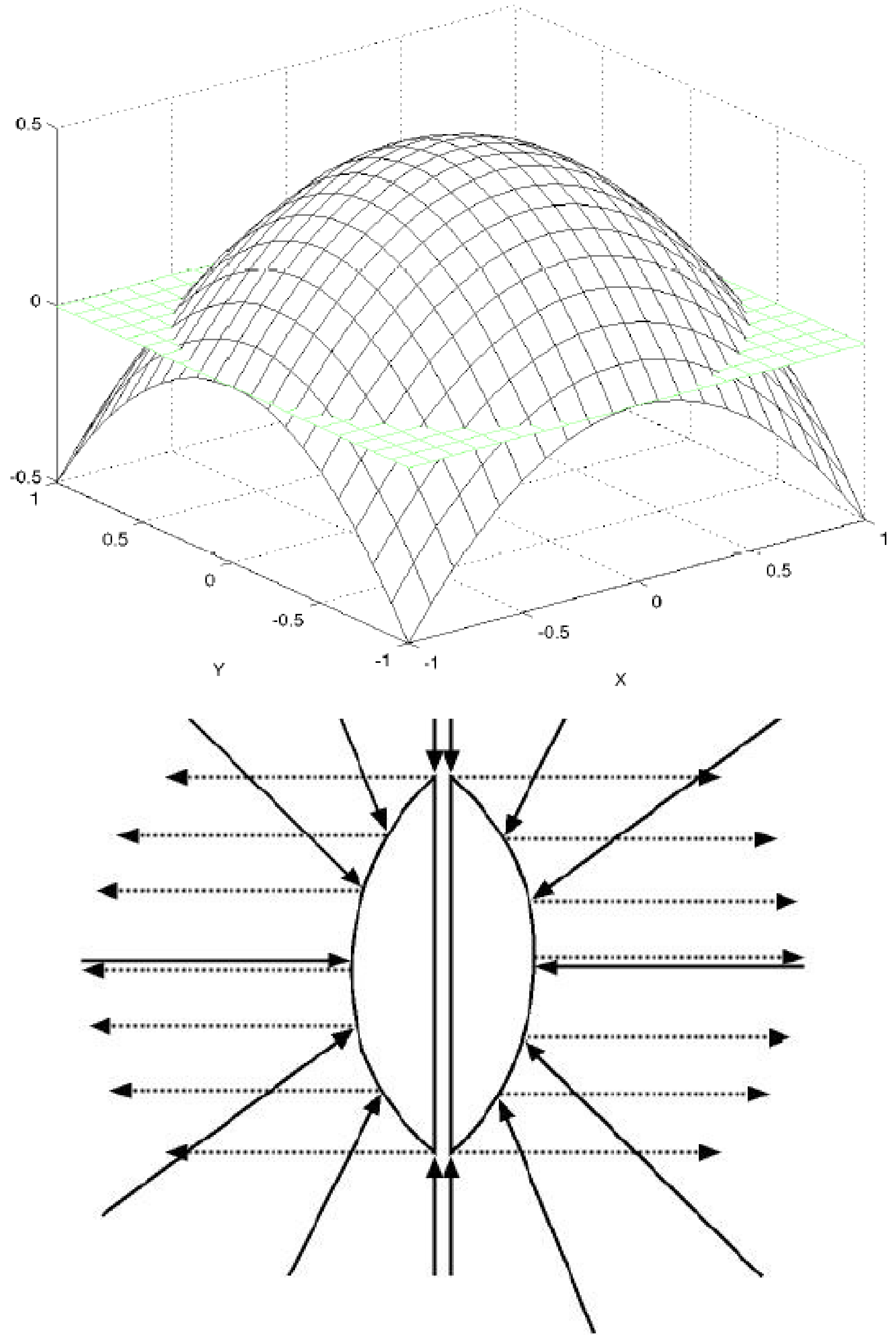
Cube Map (Skybox)





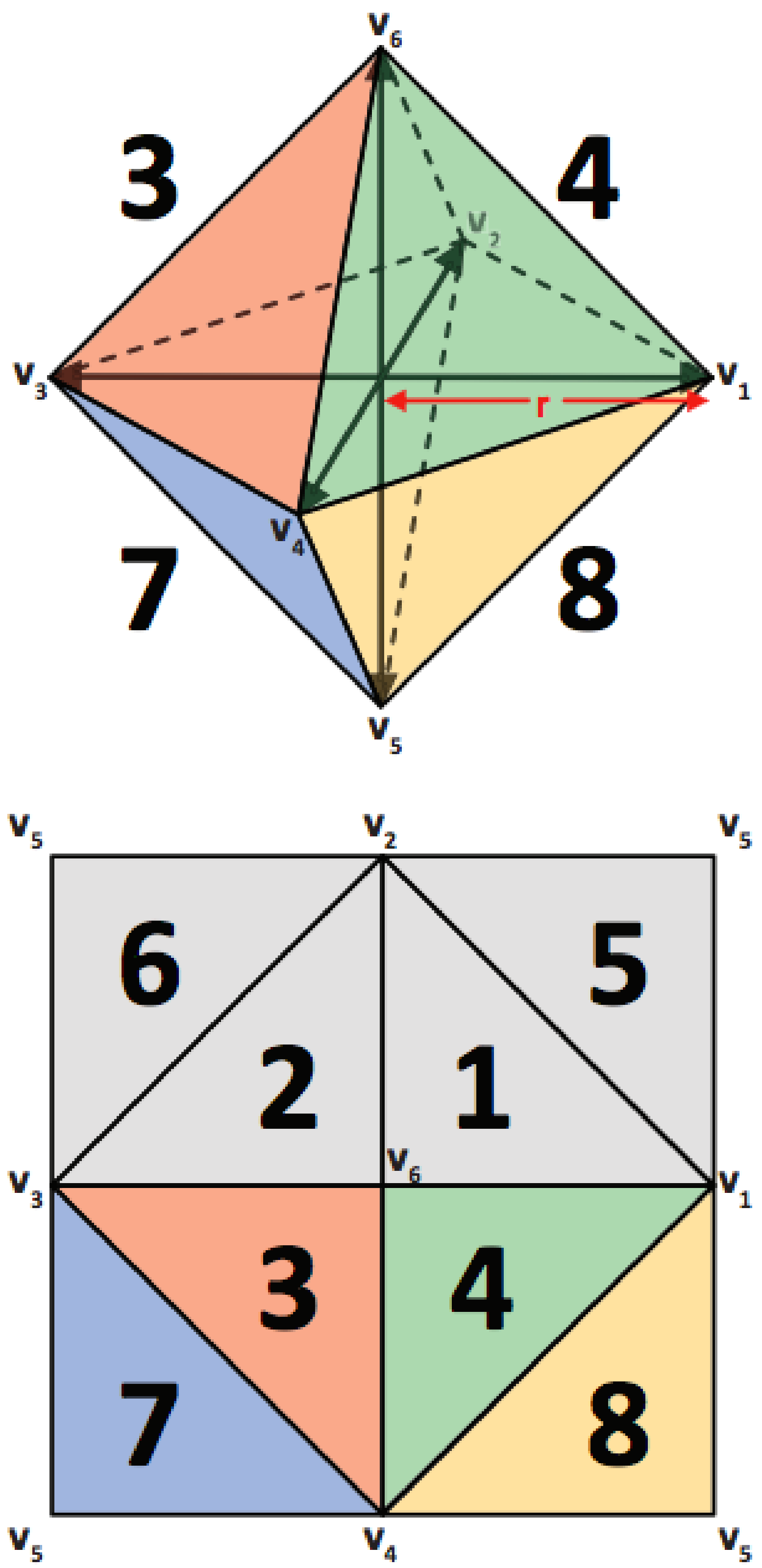
Other Parameterizations

Dual-paraboloid



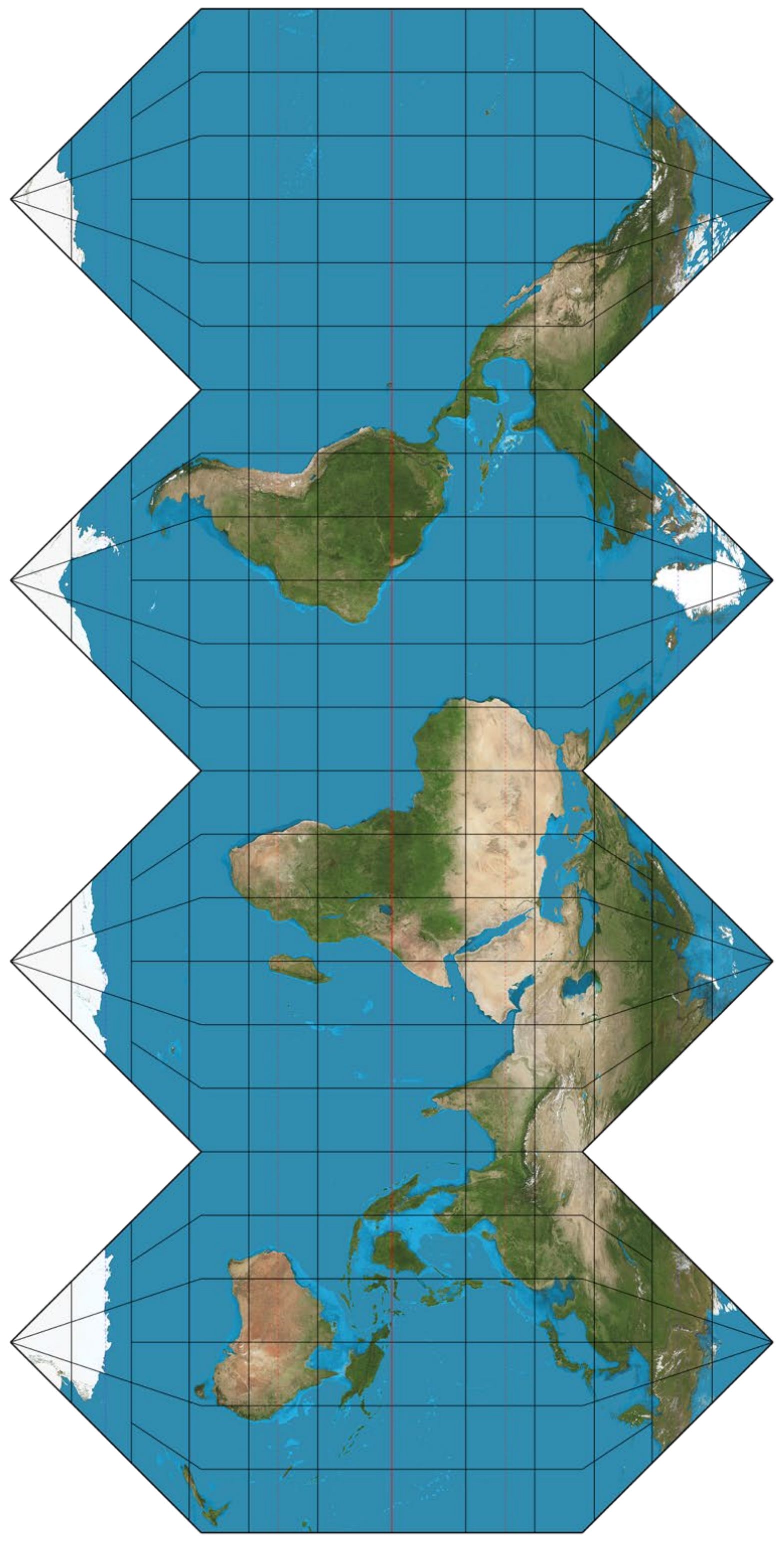
Courtesy of Brabec

Octahedron map



Courtesy of Engelhardt and Dachsbacher

HEALPix



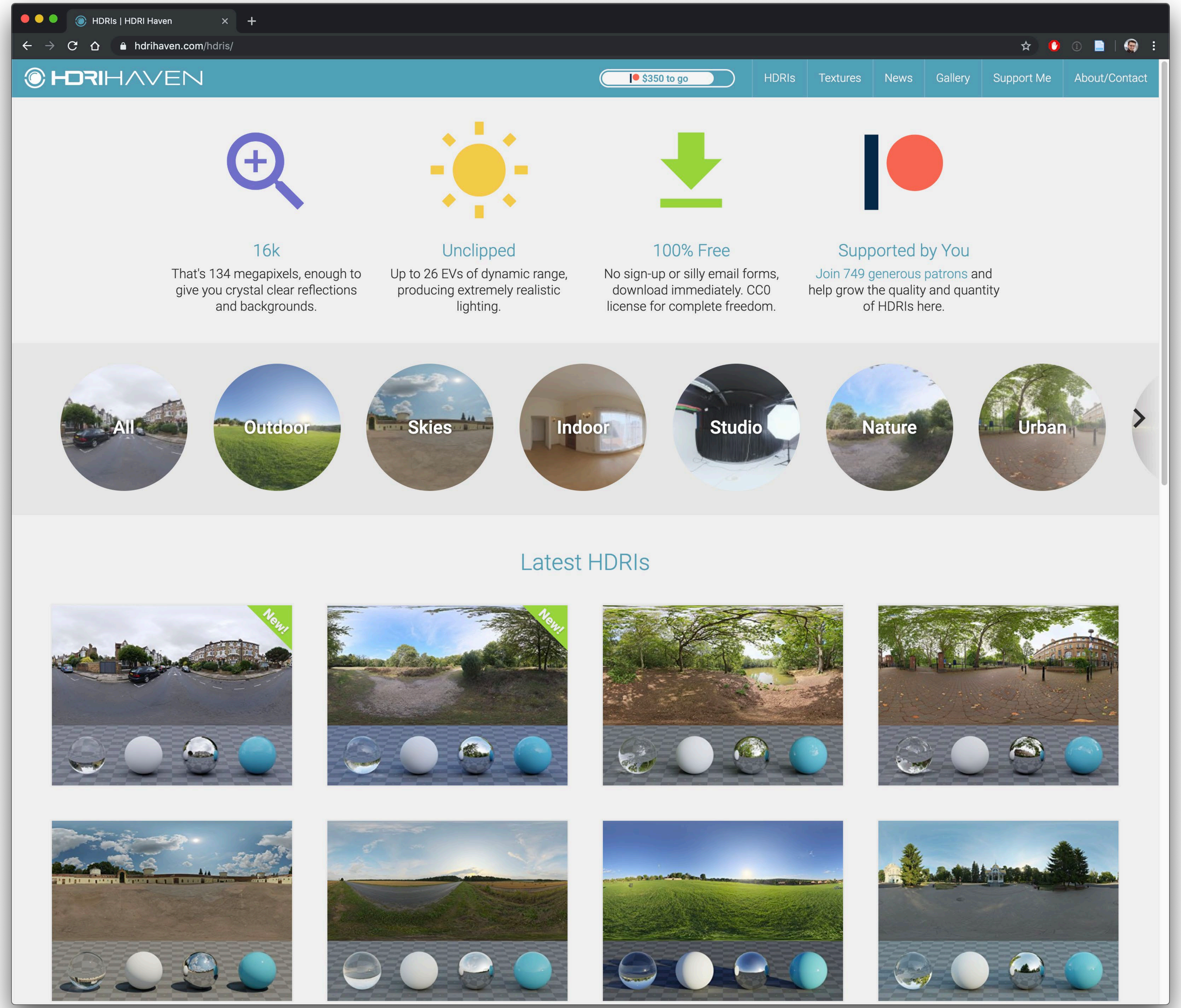
Courtesy of Ryazanov

Get them online

hdrihaven.com

The original:

- www.pauldebevec.com/Probes
- gl.ict.usc.edu/Data/HighResProbes



Exploit texturing!