Texture mapping
Course announcements

• Yannis’ office hours for this week: today, 4 – 5 pm.
Overview of today’s lecture

• Texture mapping.
Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).
Why texture mapping?
Real objects have spatially varying details
Representing as geometry is correct, but tedious/expensive
Why texture mapping?

Use simple geometry

Store varying properties in images

Map to objects

Proposed by Ed Catmull in the 70s
Texture mapping – definition

Mapping between the surface and the image
Texture mapping – definition

Mapping between the surface and the image

Each point \((x, y, z)\) on the surface has mapped coordinates \((u, v)\) in the texture image:

\[
f : \mathbb{R}^3 \rightarrow [0, 1]^2 \\
f(x, y, z) = (u, v)
\]
Texture mapping – definition

Mapping between the surface and the image

Texture itself is a function:

\[ T : [0, 1]^2 \rightarrow \text{RGB} \]

\[ T(u, v) = (r, g, b) \]
Texture mapping – definition

Mapping between the surface and the image

Concatenation of the two functions:

\[
\text{Color}(x, y, z) = T(f(x, y, z))
\]
Why texture mapping?

Produces compelling results

[Ma et al. 2007]

Wojciech Jarosz 2000
Agenda

How do we map between surface and texture?
What do we map onto the surface?
Surface parameterization

A surface in 3D is a two-dimensional thing

How do we map a surface point to a point in a texture?
- Defining 2D coordinates is parameterizing the surface
Surface parameterization

A surface in 3D is a two-dimensional thing

How do we map a surface point to a point in a texture?
- Defining 2D coordinates is parameterizing the surface

Examples:
- cartesian coordinates on a rectangle (or other planar shape)
- cylindrical coordinates \((\theta, y)\) on a cylinder
- latitude and longitude on the Earth’s surface
- spherical coordinates \((\theta, \phi)\) on a sphere
A rectangle

Image can be mapped directly, unchanged
Texturing a rectangle

Image can be mapped directly, unchanged

Object Space

Texture Space

Object Space
A textured rectangle

albedo = T(u,v)
Transformation of shape

\[ \text{albedo} = T([u, v](x, y, z)) \]  
\[ \text{albedo} = T([u, v](\text{transform}^{-1}(x, y, z))) \]
Transformation of texture

\[ \text{albedo} = T([u,v](x,y,z)) \]

\[ \text{albedo} = T(\text{transform}([u,v](x,y,z))) \]
What if the object is not rectangular?

Need to determine mapping from 3D coordinates to 2D texture coordinates

- Parametric surfaces
- Projection mapping
- UV mapping
Parametric surfaces

Parametric surfaces have a natural 2D coordinate system.
Can also define a sphere parametrically

Position:

\[ x = \cos \theta \sin \phi \]
\[ y = \sin \theta \]
\[ z = \cos \theta \cos \phi \]
Parametric spheres

Wojciech Jarosz 2007
Parametric cylinders

Wojciech Jarosz 2007
Parametric toruses

Wojciech Jarosz 2007
Bilinear patches

Wojciech Jarosz 2007
Bicubic patches

Wojciech Jarosz 2007
Creating parameterizations

For non-parametric surfaces it is trickier

Need to create a parametrization!

- Projection mapping
- UV mapping
Projection mapping

Maps 3D surface points to 2D image coordinates

\[ f : \mathbb{R}^3 \rightarrow [0, 1]^2 \]

Different types of projections
- often corresponding to simple shapes
- useful for simple objects
Projections — planar

Planar projection along xy plane of size \((w, h)\)

\[
f(p) = \begin{bmatrix} \frac{px}{w} \\ \frac{py}{h} \end{bmatrix}
\]
Projections — cubical

Planar projection onto one of 6 faces of a cube based on surface normal
Projections — spherical

Project point onto unit sphere

- compute spherical coordinates

\[ f(p) = \begin{bmatrix} \frac{\phi}{2\pi} \\ \frac{\theta}{\pi} \end{bmatrix} \]
Projections — cylindrical

Project point onto cylinder of height $h$

- compute cylindrical coordinates

$$f(p) = \begin{bmatrix} \frac{\phi}{2\pi} \\ \frac{p_y}{h} \end{bmatrix}$$

- treat caps separately
Combining projections

Non-parametric surfaces: project pieces to parametric surface

[Box mapping]
[cylindrical mapping]
[Planar mapping]

[Moller & Haines 2002]
Creating UV parameterizations

3D space (x,y,z)

2D parameter domain (u,v)

boundary
Creating UV parameterizations

Assign \((u,v)\) coordinates for each mesh vertex. Inside each triangle interpolate using barycentric coordinates.
Creating UV parameterizations

After a slide by Daniele Panozzo
Barycentric coordinates

Barycentric interpolation: \(p(\alpha, \beta, \gamma) = \alpha p_1 + \beta p_2 + \gamma p_3\)

Can use this eqn. to interpolate any vertex quantity across triangle!
Barycentric coordinates

Barycentric interpolation:

\[ p(\alpha, \beta, \gamma) = \alpha p_1 + \beta p_2 + \gamma p_3 \]

\[ c(\alpha, \beta, \gamma) = \alpha c_1 + \beta c_2 + \gamma c_3 \]

Can use this eqn. to interpolate any vertex quantity across triangle!
UV texturing

Object Space

Texture Space

Object Space

\[(u, v) = \alpha(u_1, v_1) + \beta(u_2, v_2) + \gamma(u_3, v_3)\]
UV texturing
Area distortion vs. angle distortion
Creating UV parameterizations

Can compute vertex UVs using projections

[Moller & Haines 2002]
Creating UV parameterizations

“Atlas” - break up model into single texture
Texture lookup
Texture lookups and wrapping

In shading calculation, when you need a texture value you perform a *texture lookup*

Convert \((u, v)\) texture coordinates to \((i, j)\) texel coordinates, and read a value from the image
Obtaining \((i,j)\) from \((u,v)\)

\[
i = u n_x - 0.5
\]

\[
j = v n_y - 0.5
\]

for an image of \(n_x\) by \(n_y\) pixels
Looking up texture values

Lookup locations will fall at fractional texel coordinates
- simplest: round to nearest (nearest neighbor lookup)
- various ways to be smarter and get smoother results
Texture lookups and wrapping

What if \( i \) and \( j \) are out of range?

Option 1, clamp: take the nearest pixel that is in the image

\[
i_{\text{pixel}} = \max(0, \min(n_x - 1, i_{\text{lookup}}))
\]

Option 2, wrap: treat the texture as periodic, so that falling off the right side causes the look up to come in the left

\[
i_{\text{pixel}} = \text{remainder}(i_{\text{lookup}}, n_x)
\]
Texture mapping artifacts

Tiling textures might introduce seams
- discontinuities in the mapping function
- change textures to be "tileable" when possible
Seamlessly “tileable" textures
Pixels versus texels

In general, we will not have a 1-to-1 mapping between image pixels and texture pixels, or “texels”

Two issues arise:

- **Magnification**: Texel size larger than pixel size
- **Minification**: Texel size smaller than pixel size (potential aliasing)
Texture Filtering - Magnification
Texture Filtering - Magnification

Nearest Neighbor

Bilinear
Texture minification (aliasing)

Point-sampling introduces artifacts (aliasing)
- need average of texture within area of a pixel
Texture minification

In ray tracing, in theory, you’re already sending many rays randomly through area of pixel

Minification artifacts will go away, eventually

In production, texture filtering techniques still useful for improved speed/quality
- mipmapping
- ripmapping
- summed area tables
Texture filtering - minification

point sample

area average
Texture filtering - minification

4×4 supersampling
expensive!

area average
Mipmapping

Store textures at different resolutions

Look up in appropriate image based on projected pixel size
Mipmapping

Bilinear: Look up in closest resolution and bilinearly interpolate
- transition artifacts

Trilinear: linearly interpolate between two closest levels
Mipmapping

Mipmaps average in squares, but pixel footprints are not square! ➔ overblurring
Ripmap

One possible improvement:
- downscale independently in x, y
So far

How do we map between surface and texture?
- parametric surfaces
- projection mapping
- uv texturing
- texture lookup
Agenda

What do we map onto the surface?
- reflectance (color, diffuse + specular coeffs., etc)
- surface normal (bump mapping)
- geometry (displacement mapping)
- illumination (environment, reflection, shadows)
Texturing reflectance properties
Texture mapping

Surface properties are not the same everywhere
- diffuse color varies due to changing pigmentation
- roughness varies due to changing roughness and surface contamination
Diffuse coefficient

After a slide by Fabio Pellacini
Specular “shininess” coefficient
After a slide by Steve Marschner
Texturing geometric detail
Texturing geometric detail

Modify geometric properties based on a texture
- normals
- surface positions
Displacement mapping
Displacement mapping

Encode a displacement distance in the texture map
- Measured, e.g., along interpolated normal
Displacement mapping

Update position by displacing points along normal

\[ \mathbf{p}_d = \mathbf{p} + h \mathbf{n} \]

Recompute normals by evaluating derivatives
- often no closed form solution, so use finite differences

\[ \mathbf{n}_d \propto \frac{\partial \mathbf{p}_d}{\partial u} \times \frac{\partial \mathbf{p}_d}{\partial v} \approx \frac{\Delta \mathbf{p}_d}{\Delta u} \times \frac{\Delta \mathbf{p}_d}{\Delta v} \]
Bump mapping

Apply normal perturbation without updating positions
Bump vs. displacement mapping

bump map

+ 

= 

+ 

= 

displacement map

After a slide by Fabio Pellacini
Bump vs. displacement mapping

bump map

displacement map

from: www.spot3d.com
Displacement vs. bump mapping

Max Displace 1.5Mil
Normal Map 2900Tris
Wire
Surface normals

[Wikipedia]
Normal mapping

[Wikipedia]
Normal mapping

original mesh
4M triangles

simplified mesh
500 triangles

simplified mesh and normal mapping
500 triangles
Environment & Reflection mapping
Shiny objects

The key to creating a realistic shiny-looking material is providing something for it to reflect.
Environment/reflection mapping

Sidesteps tedious modeling of the environment by representing it using one or more images.

The image “wraps” around the virtual scene, serving as a source for reflections.

source: 3delight
Environment map

A function from the sphere to colors, stored as a texture.

After a slide by Steve Marschner

[Blinn & Newell 1976]
Reflection mapping (1982)
Terminator 2 (1991)
Environment mapping

Ray tracing easy: rays that hit nothing look up in envmap
Acquiring environment maps

Mirrored ball + camera
Fisheye lens images
Stitching images together
Panoramic camera
Acquisition - Low Tech.

Maurits C. Escher

Lightprobe

omnidirectional, 360° panoramic, HDR image
Acquisition - Even Lower Tech.
Acquisition - Stitched Panorama
Acquisition - High Tech.

ism.epfl.ch

immersivemedia.com
Acquisition
Storing environment maps

Various ways to parametrize environment maps

Related to cartography
- Projecting the earth (sphere) onto a plane
Latitude/Longitude Map

www.ncl.ucar.edu
Cube Map (Skybox)
Other Parameterizations

Dual-paraboloid

Courtesy of Brabec

Octahedron map

Courtesy of Engelhardt and Dachsbacher

HEALPix

Courtesy of Ryazanov
Get them online

hdrihaven.com

The original:
- www.pauldebevec.com/Probes
- gl.ict.usc.edu/Data/HighResProbes
Exploit texturing!
Projective texture example

Modeling from photographs

Using input photos as textures

[Debevec et al. 1996]
More details

“Rendering with Natural Light”
- [http://www.pauldebevec.com/RNL/](http://www.pauldebevec.com/RNL/)

“Fiat Lux”
- [http://www.pauldebevec.com/FiatLux/](http://www.pauldebevec.com/FiatLux/)

History of reflection mapping