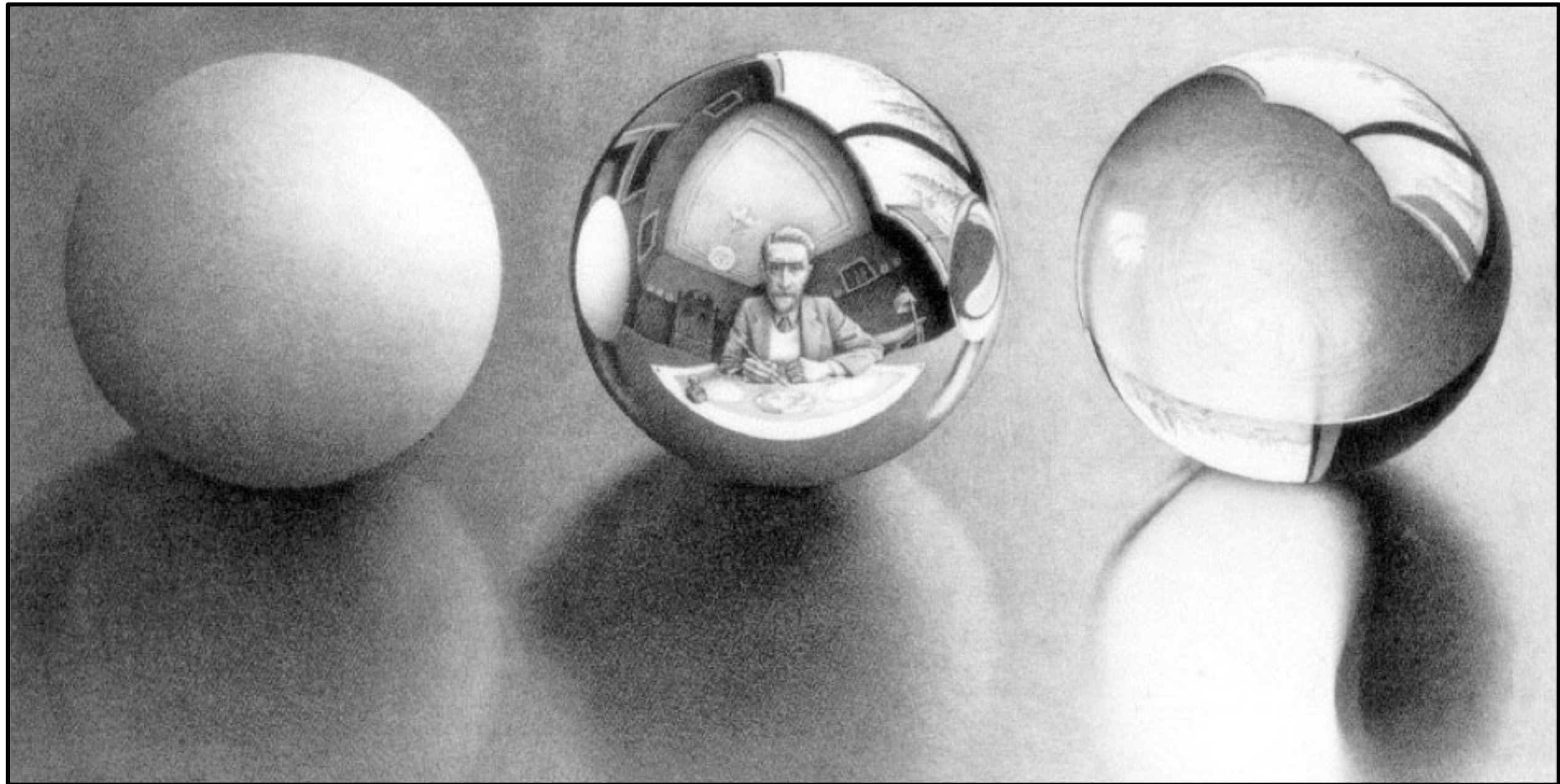


# Ray tracing and simple shading



15-468, 15-668, 15-868  
Physics-based Rendering  
Spring 2024, Lecture 3

# Course announcements

- Programming assignment 0 available on Canvas.
  - Ungraded, no due date.
  - Used to set up rendering environment and github-based submission system.
  - Should take no more than 1-2 hours max.
- Programming assignment 1 will be posted on Friday 1/26, will be due two weeks later.
- Take-home quiz 1 will be posted tonight, will be due Tuesday 1/30.

# Overview of today's lecture

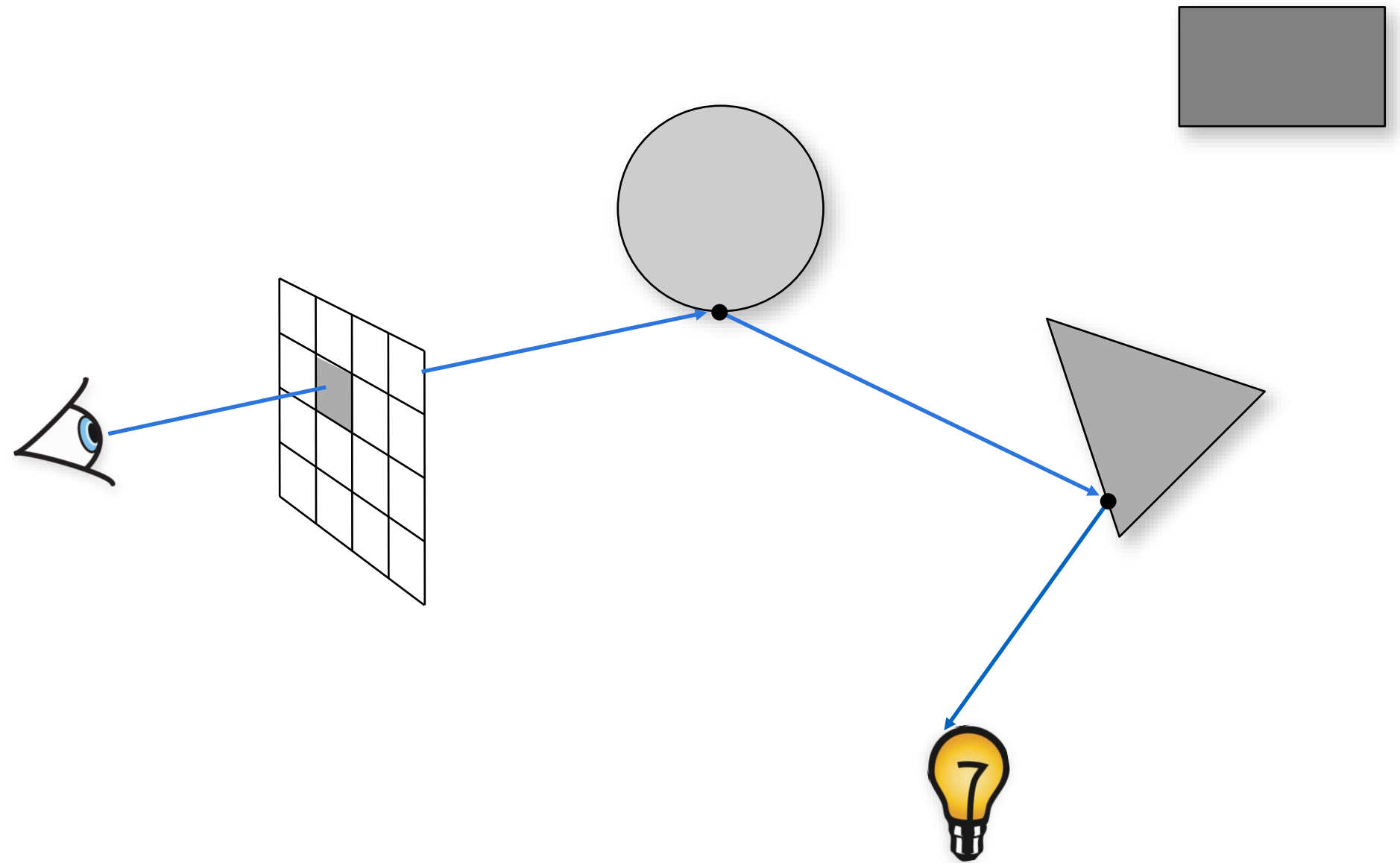
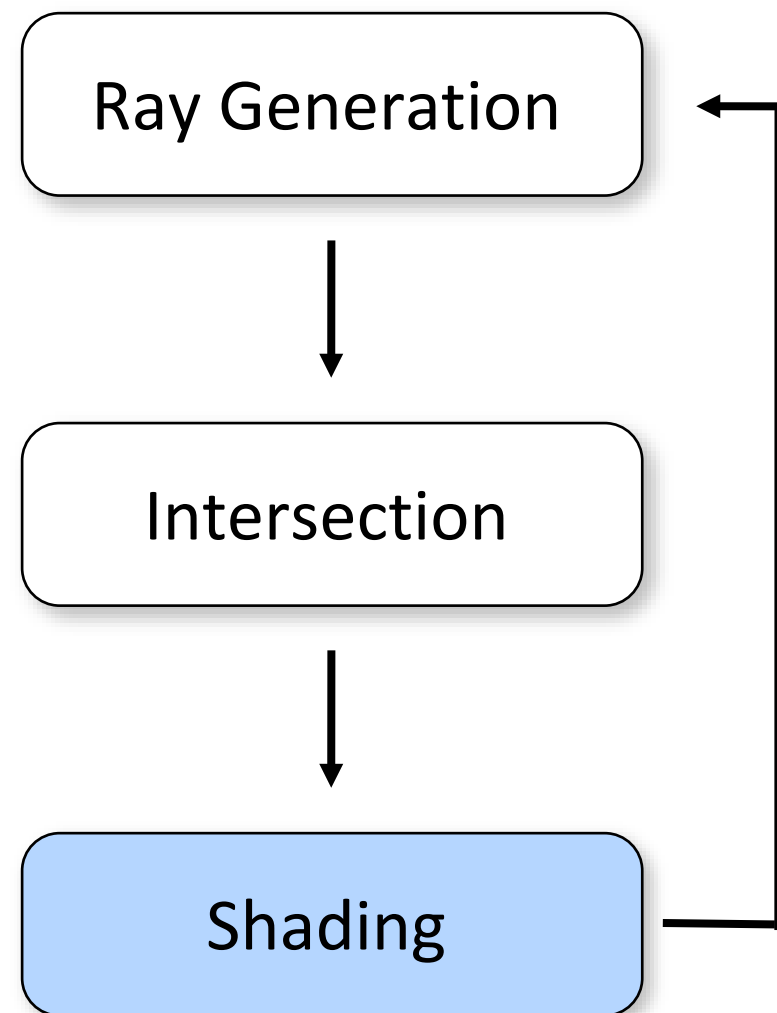
- Leftover from previous lecture: intersections, meshes, acceleration structures.
- Basics of shading.
- Basic reflection models.

# Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

# Recap: Raytracing



# Shading

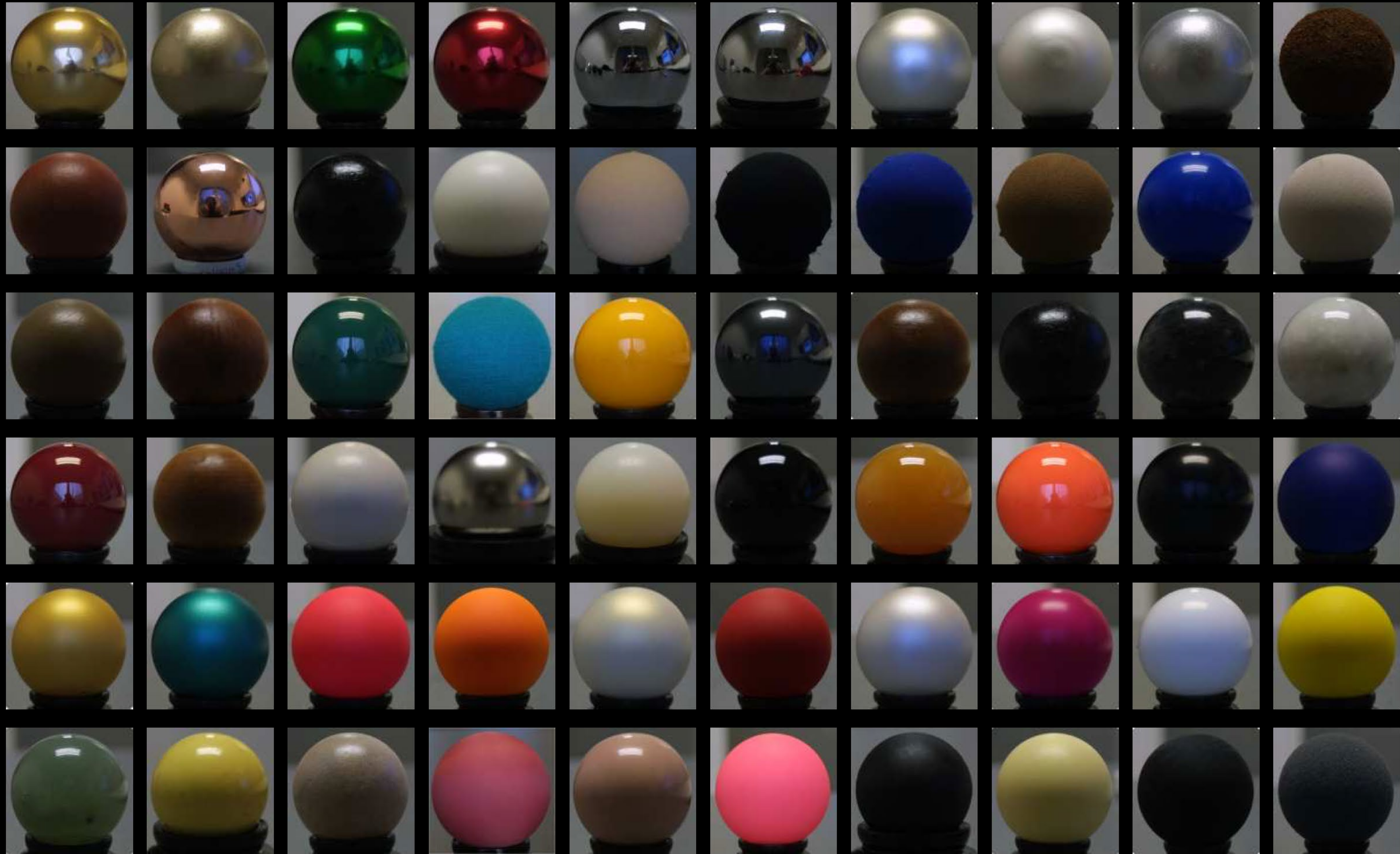
---

When ray hits a surface we perform *lighting/shading*

Determine “what color/light should we see at this location?”

Surfaces can scatter and/or emit light

- Surface emits light? just return emitted color  
(determined by the material)
- Surface scatters/reflects/refracts light?  
(recursively) trace a ray in a scattering direction  
(determined by the underlying material)



# Overview

---

Diffuse shading

Specular reflection

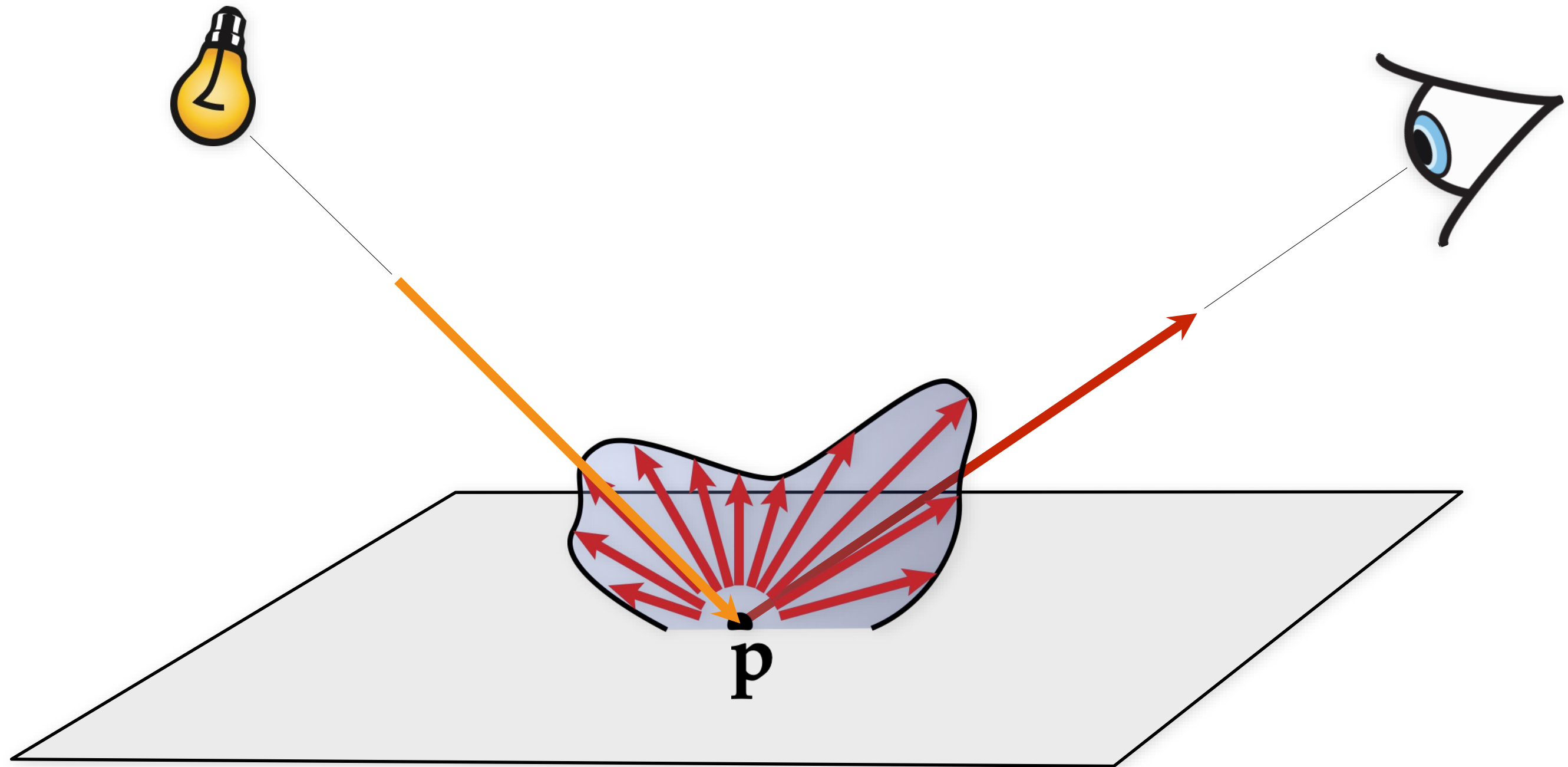
Refraction

Diffuse emission



# Light-material interactions

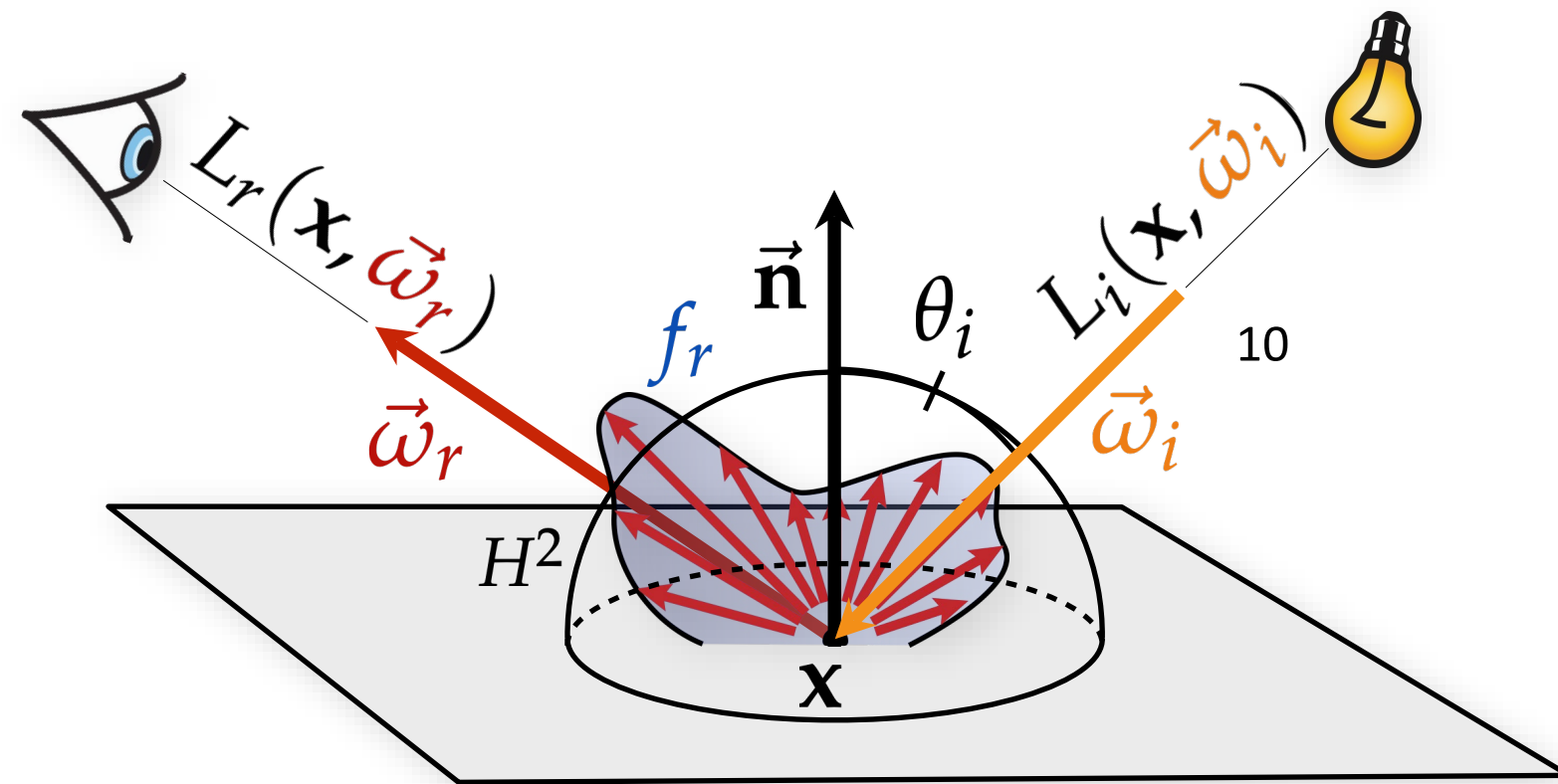
---



# The reflection equation

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

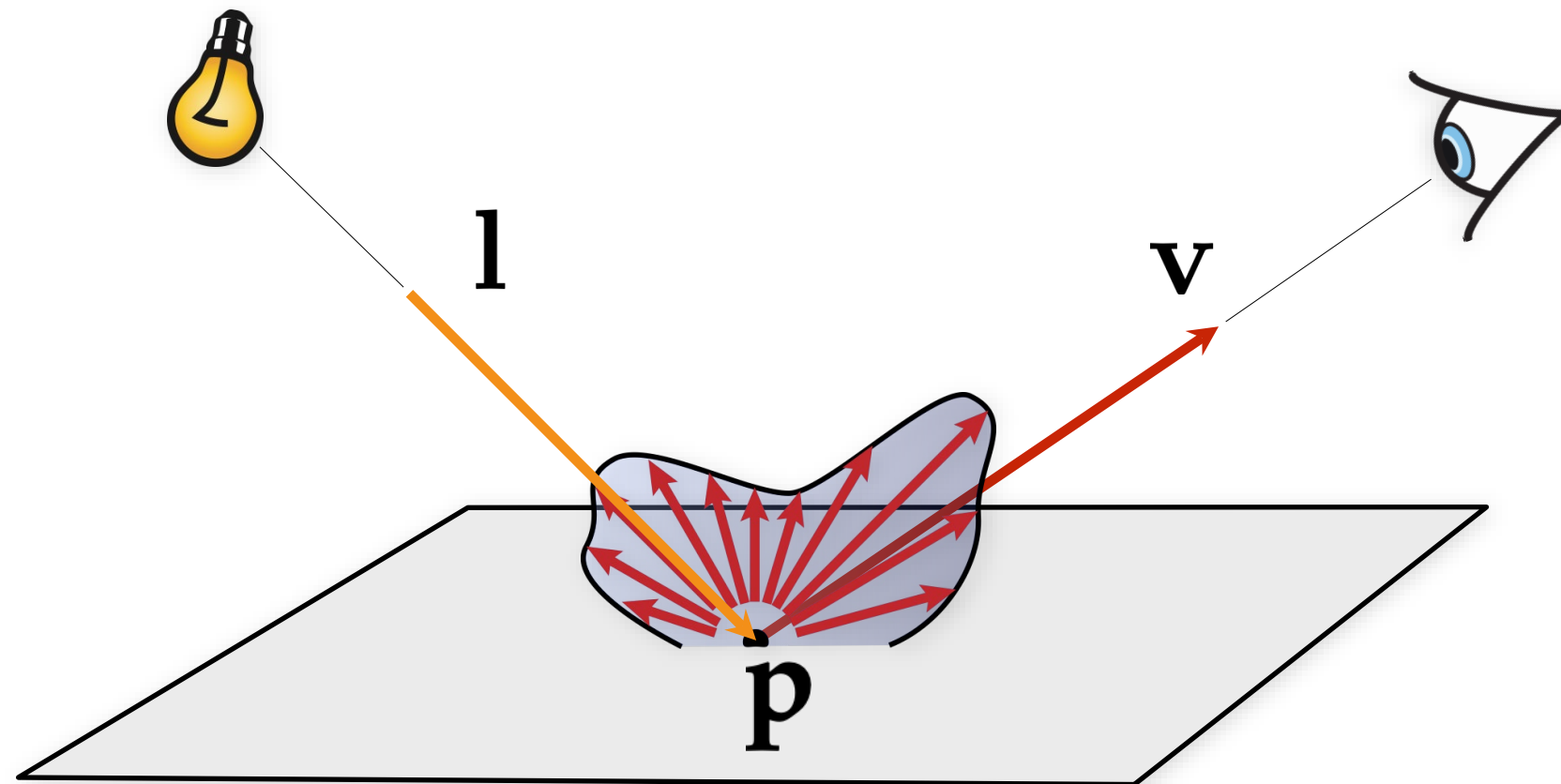


This describes a local illumination model

# The BSDF

## Bidirectional Scattering Distribution Function

- informally: how much the material scatters light coming from one direction  $\mathbf{l}$  into some other direction  $\mathbf{v}$ , at each point  $\mathbf{p}$

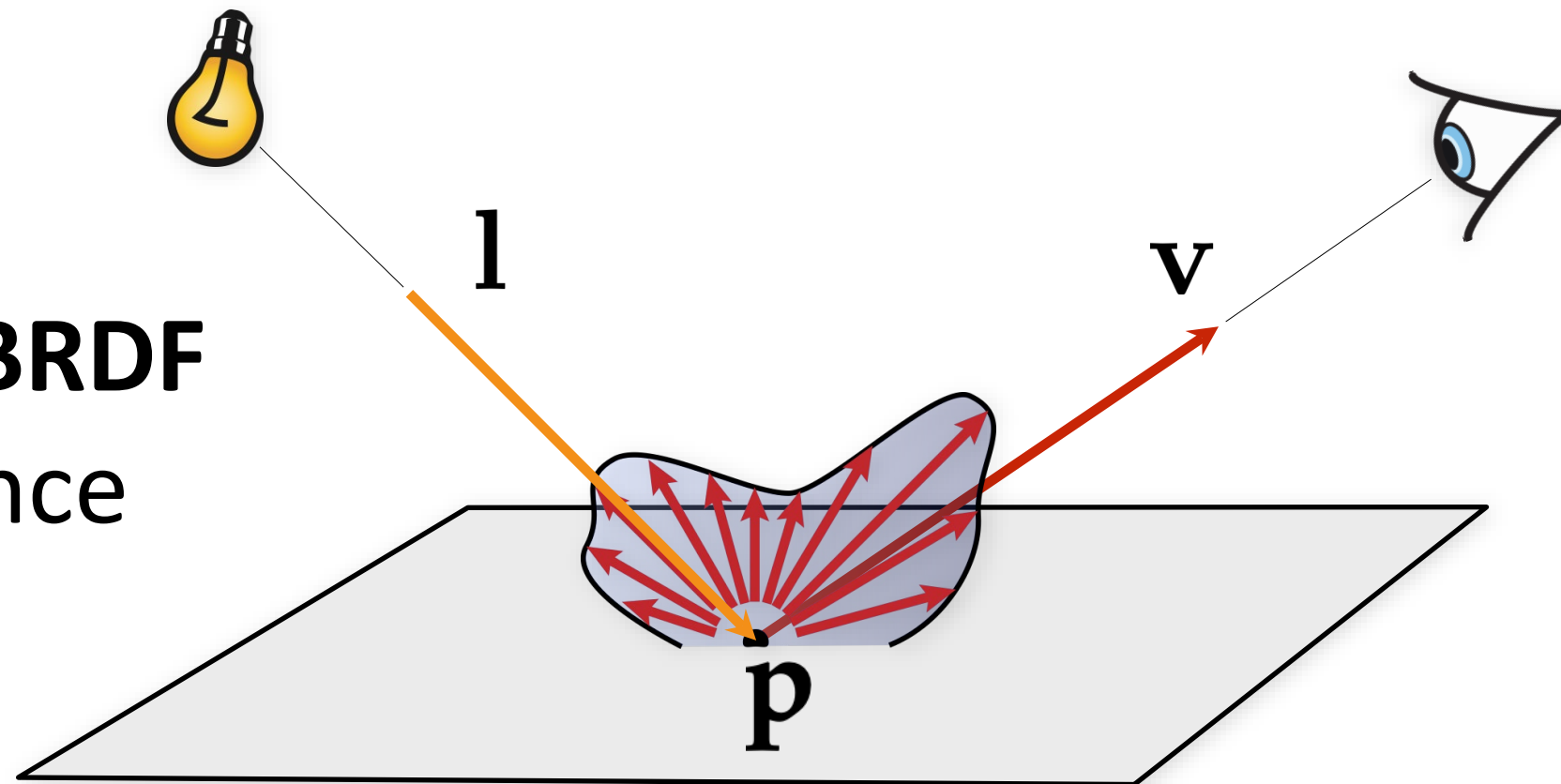


# The BSDF

## Bidirectional Scattering Distribution Function

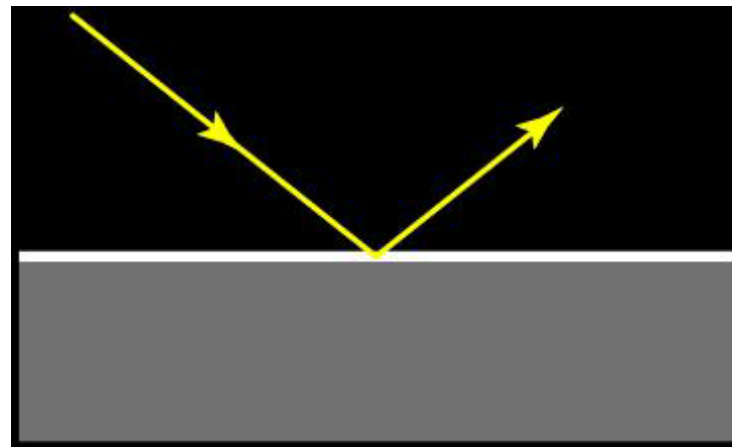
- informally: how much the material scatters light coming from one direction  $\mathbf{l}$  into some other direction  $\mathbf{v}$ , at each point  $\mathbf{p}$

You'll also see **BRDF**  
**R** for reflectance

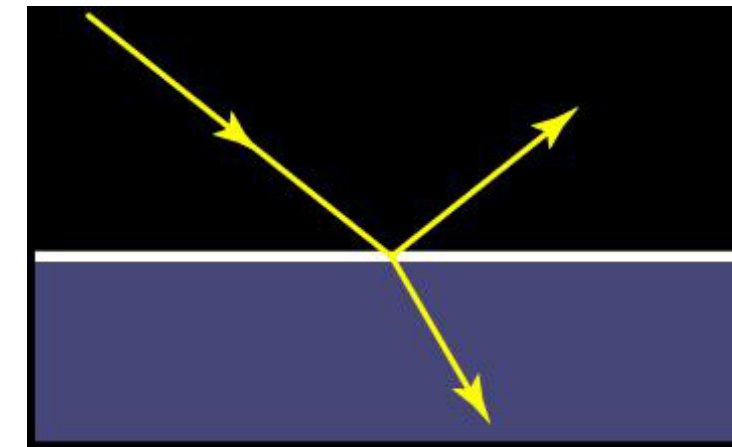


# Real-world materials

Metals

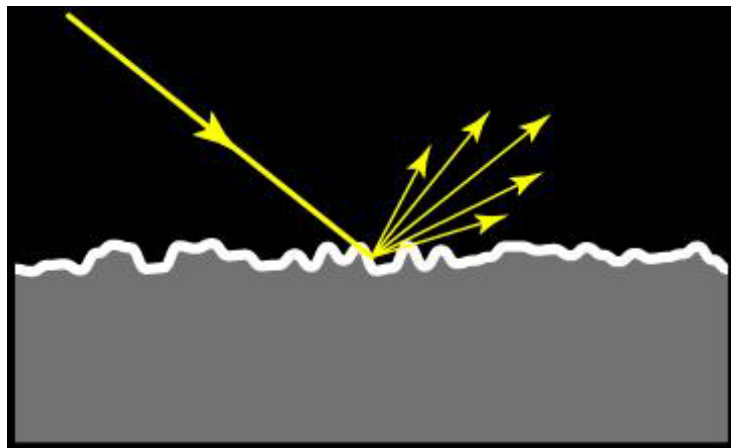


Dielectric

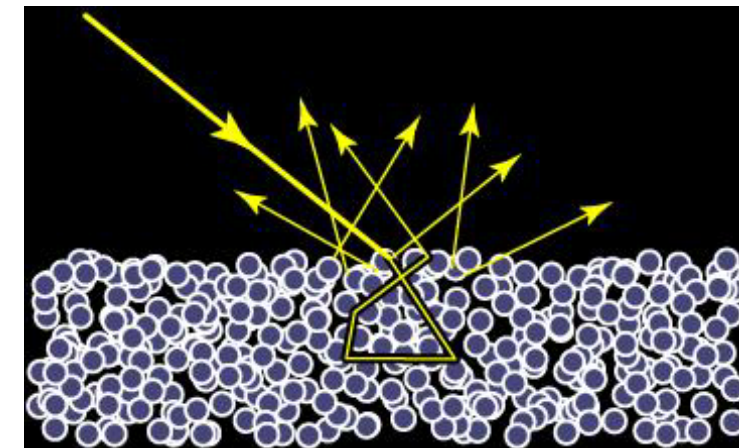


# Real-world materials

Metals



Dielectric



# Idealized material models

---

## Diffuse reflection

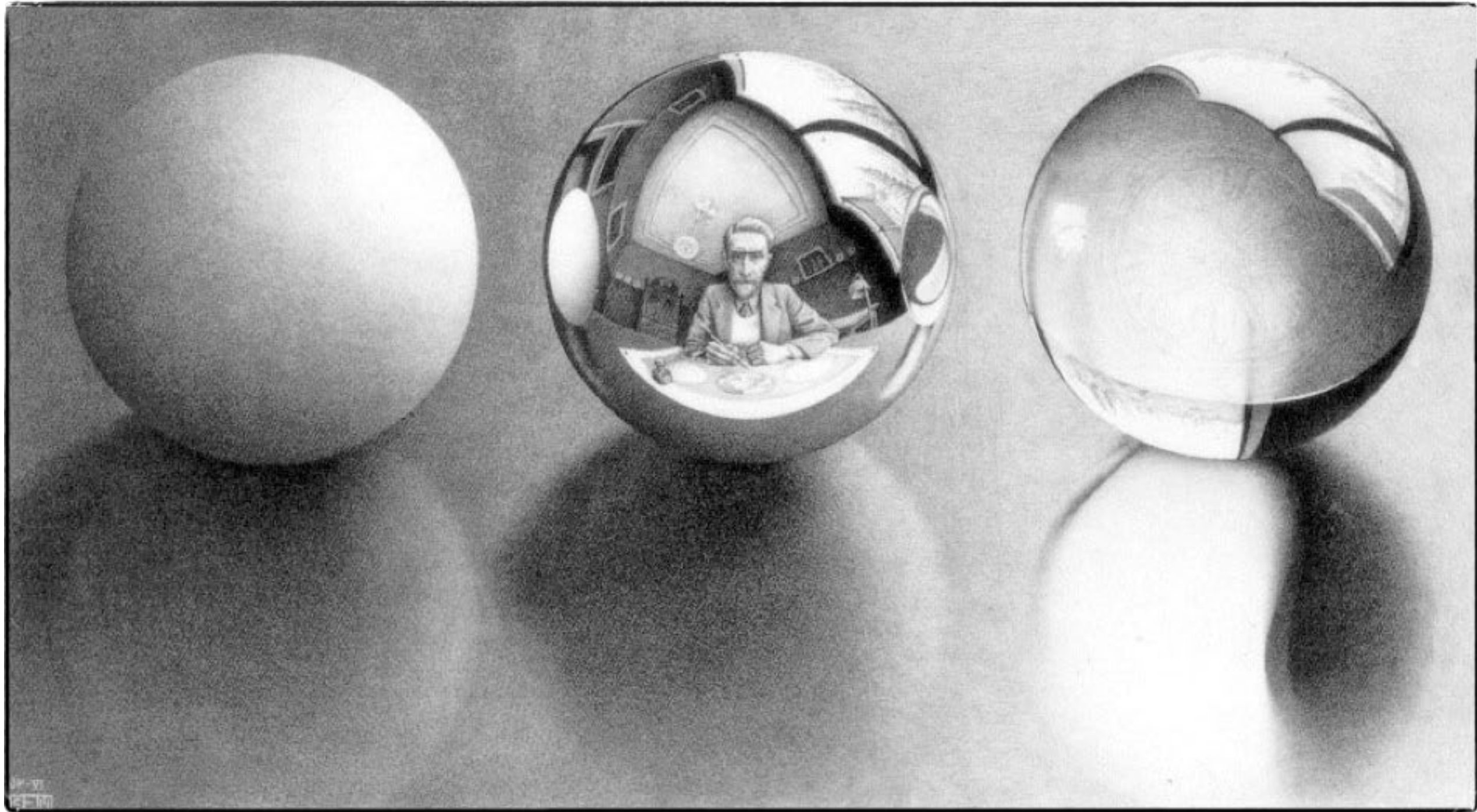
- light is reflected in all directions
- colored by surface color

## Smooth specular reflection/refraction (e.g., chrome, glass, glaze/varnish)

- light reflected/refracted only in a single direction
- colored by source color

# Idealized materials

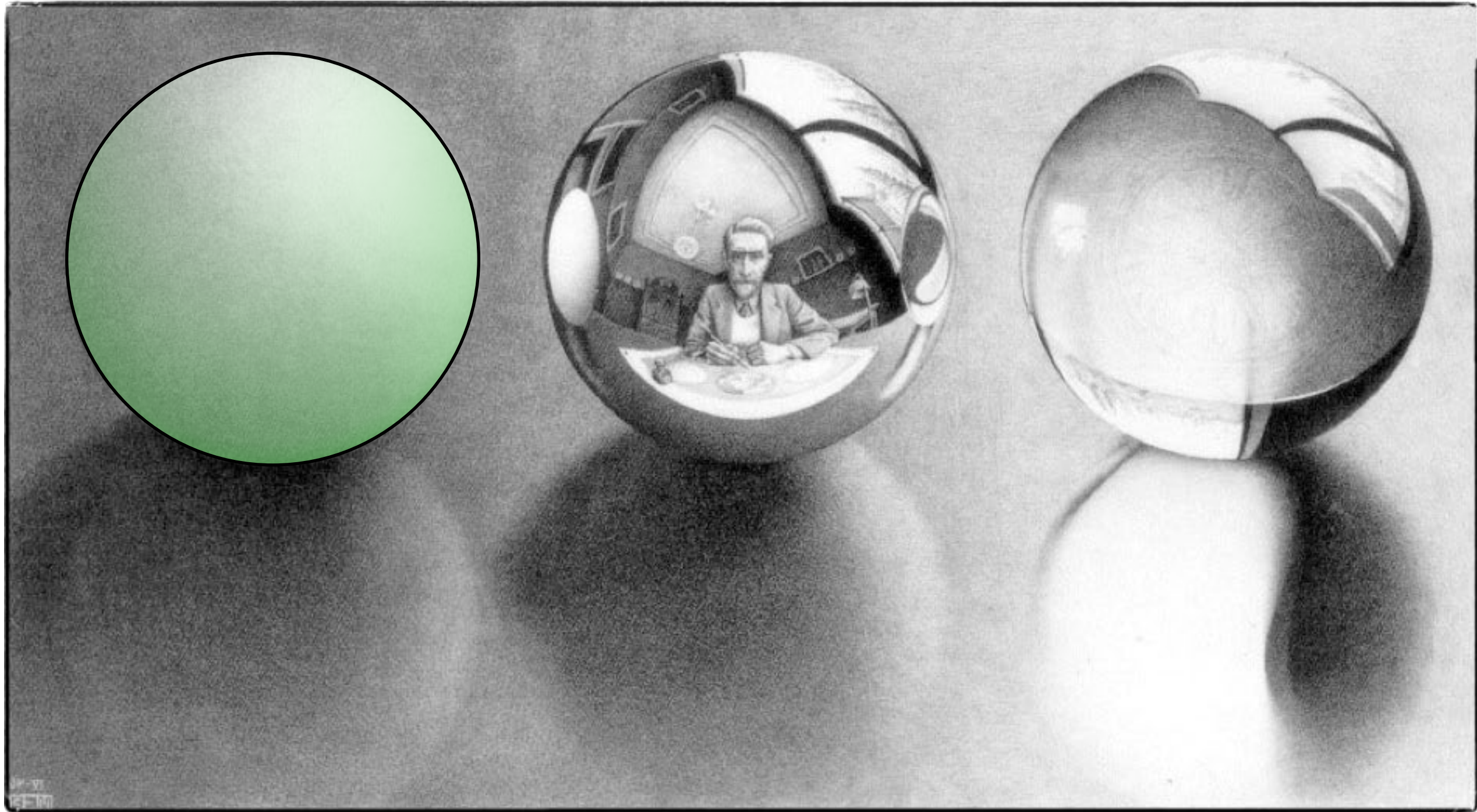
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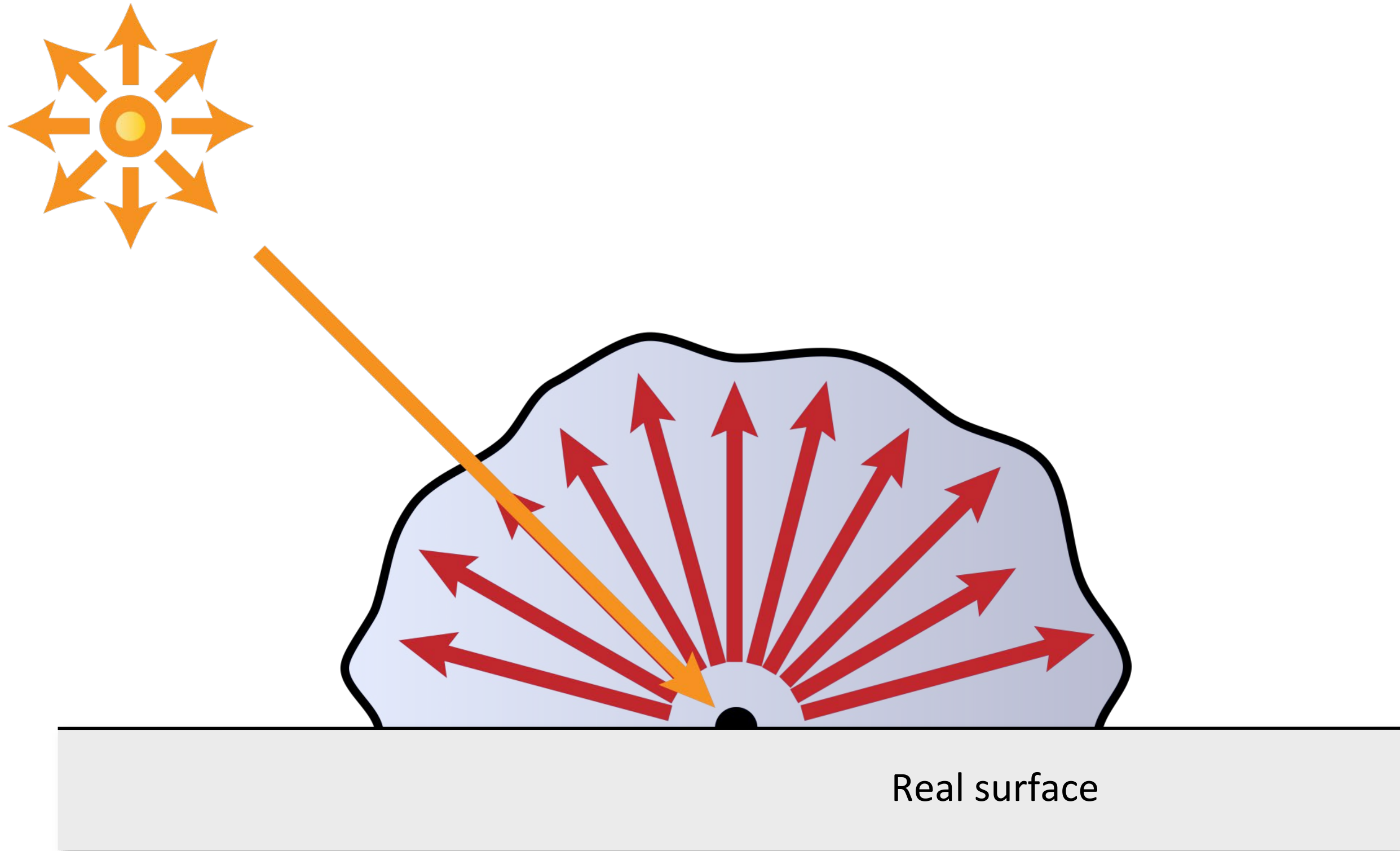
# Diffuse reflection

---



# Diffuse reflection

---

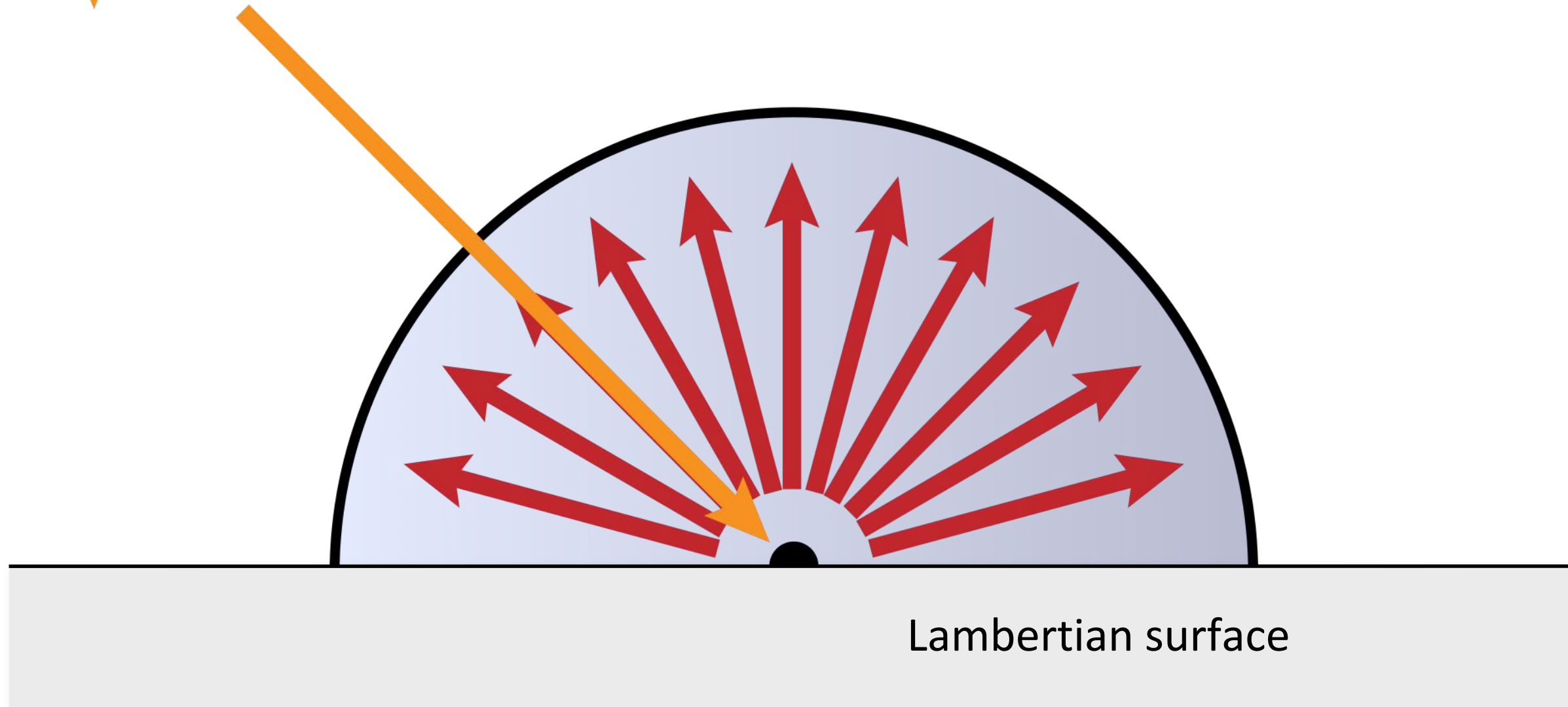


# Lambertian reflection

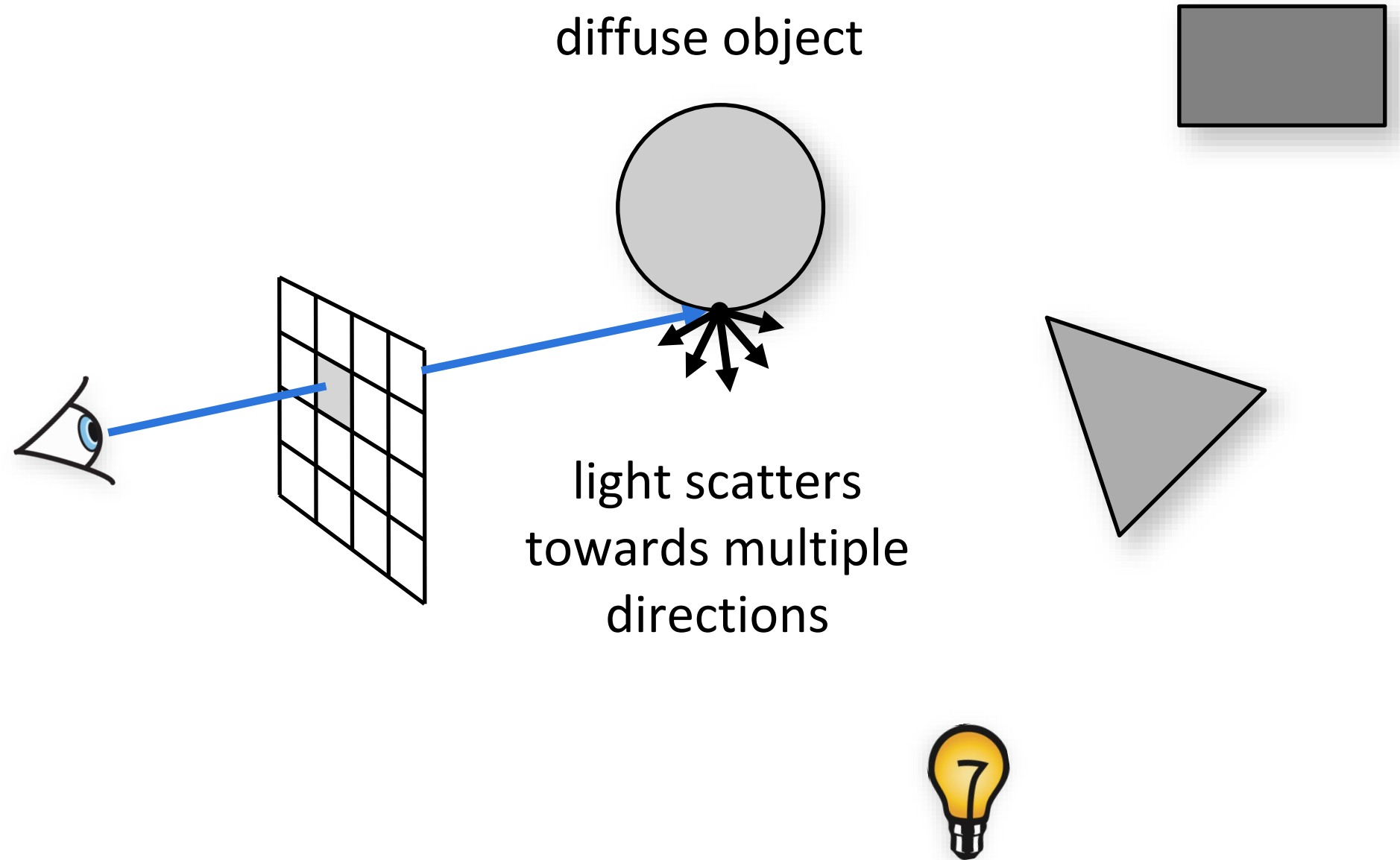
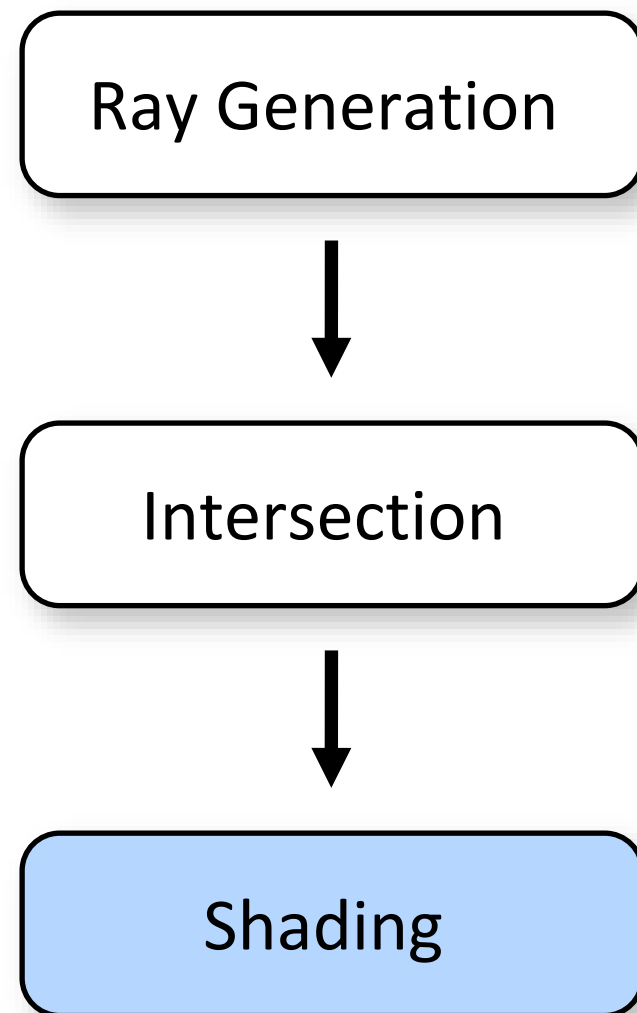
---



Also called ideal diffuse reflection

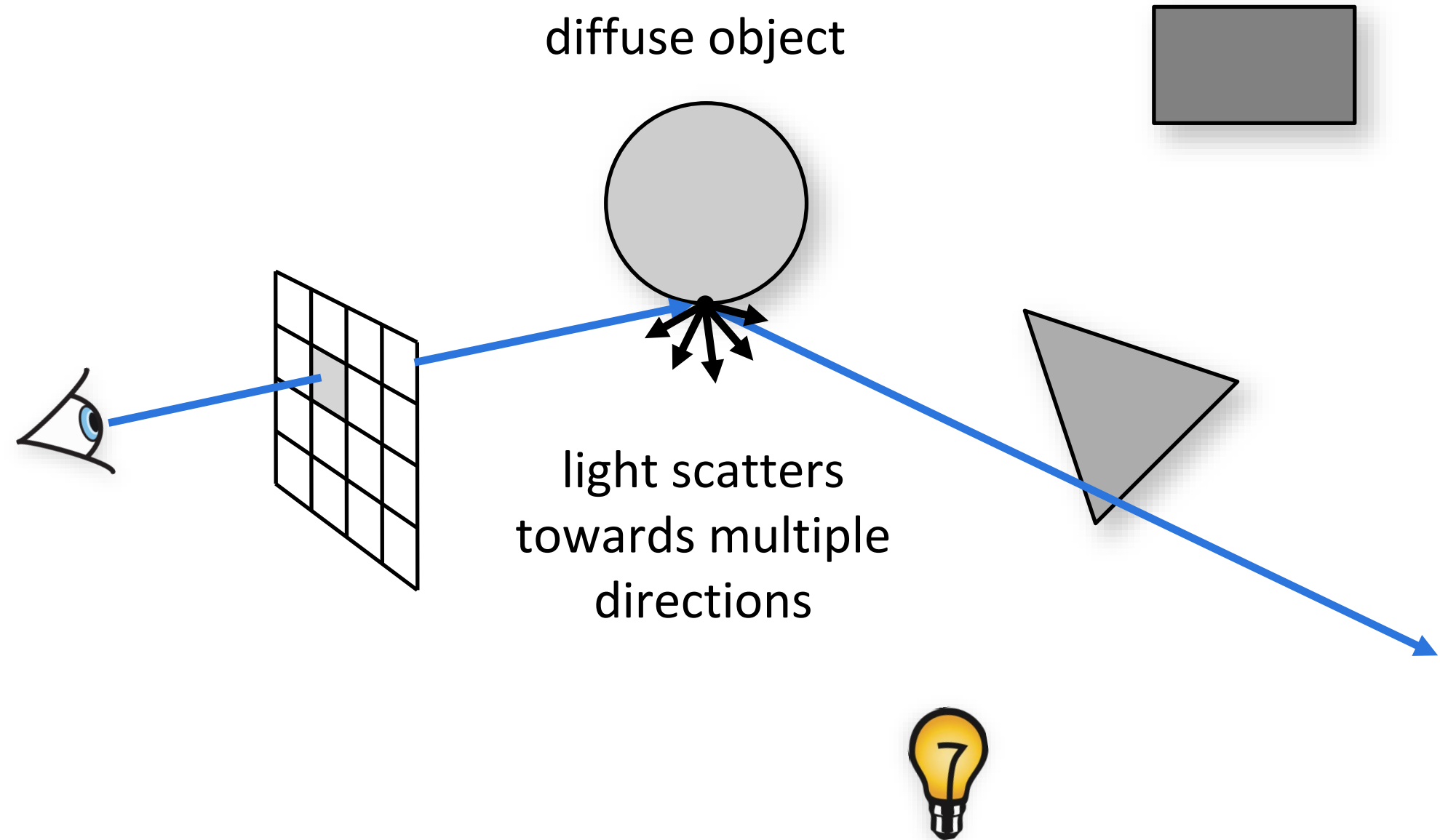
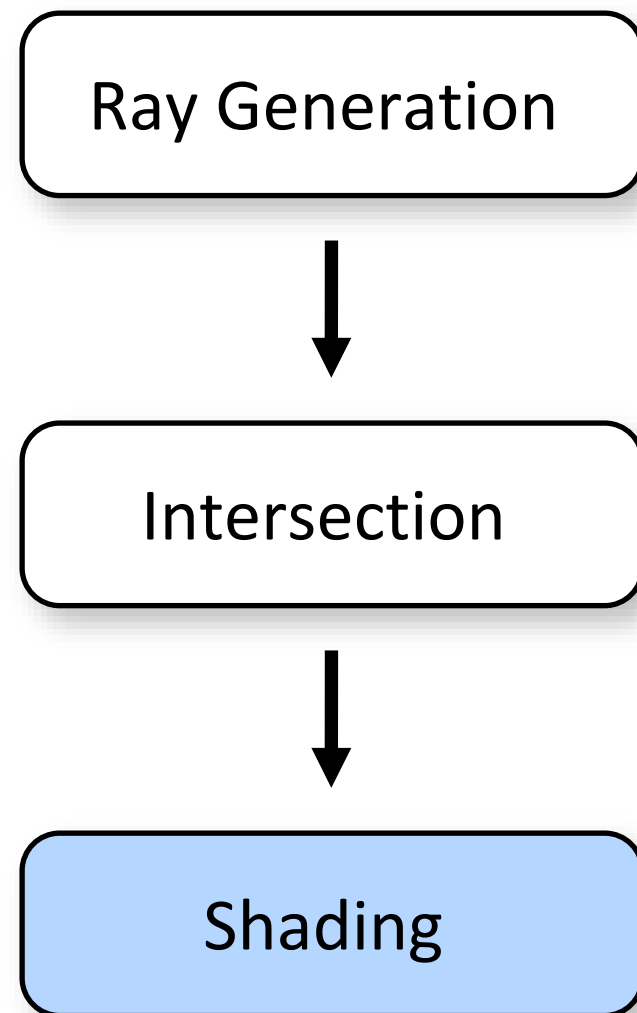


# Basic Ray Tracing Pipeline



What direction should we trace a new ray towards?

# Basic Ray Tracing Pipeline



What direction should we trace a new ray towards?

- Pick a direction at random!

# Sampling Lambertian scattering

---

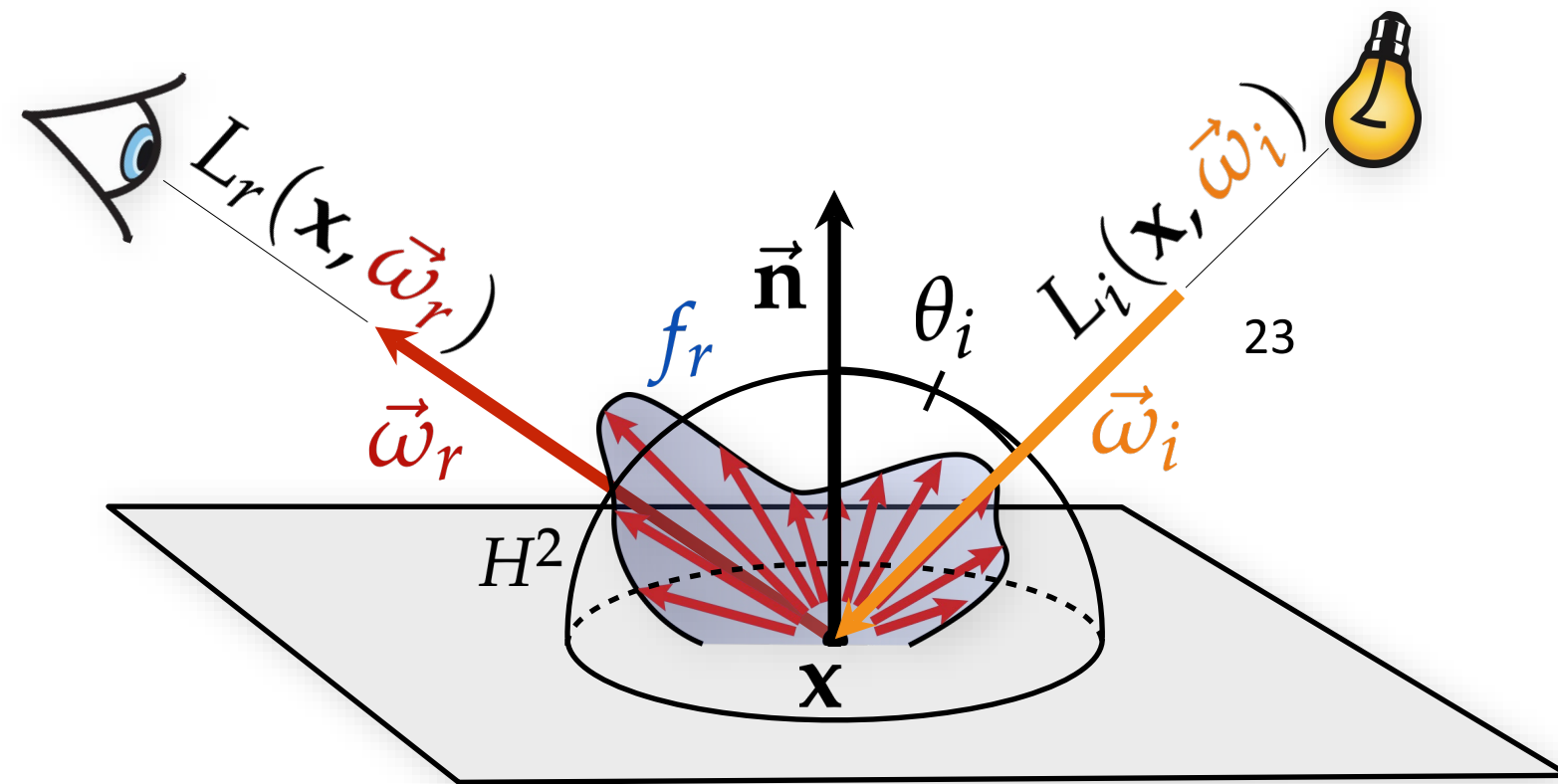
From what distribution should we sample directions?

# The reflection equation

---

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$



This describes a local illumination model

# Sampling Lambertian scattering

---

From what distribution should we sample directions?

- Probability proportional to  $\cos(\hat{n} \cdot \hat{\omega})$ .
- Even though BSDF scatters to all directions uniformly, we need to account for foreshortening.

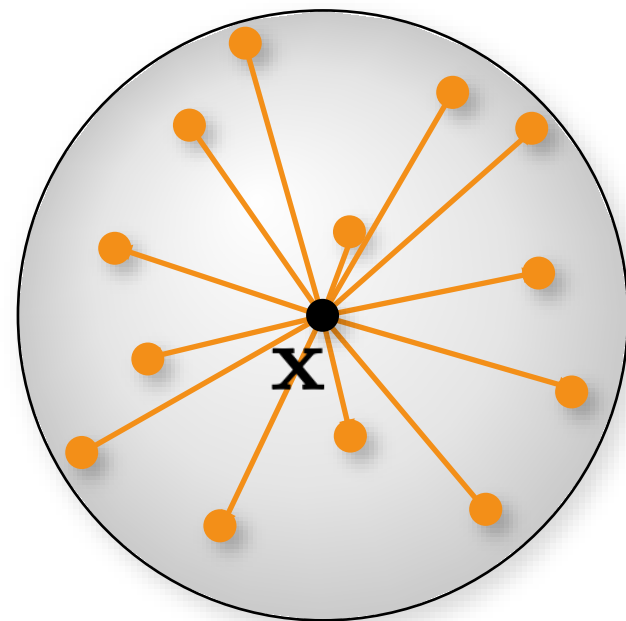
How do we sample directions based on this *cosine-weighted* distribution?



# Sampling Lambertian scattering

---

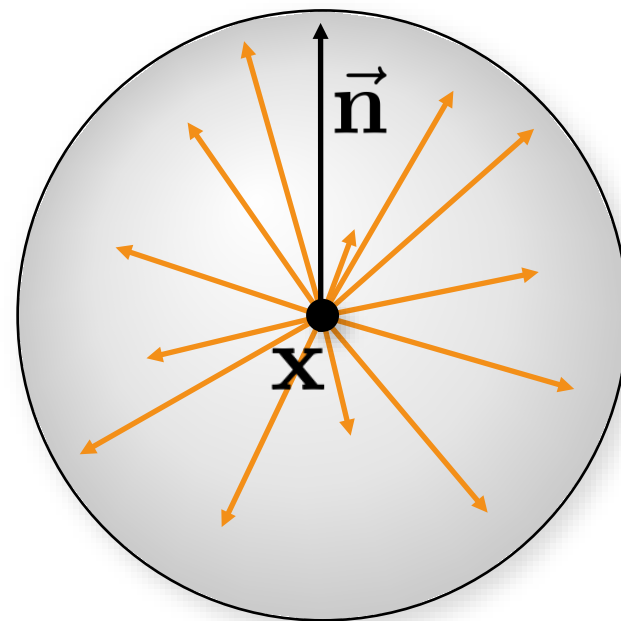
Generate points  
uniformly on sphere  
(unit directions)



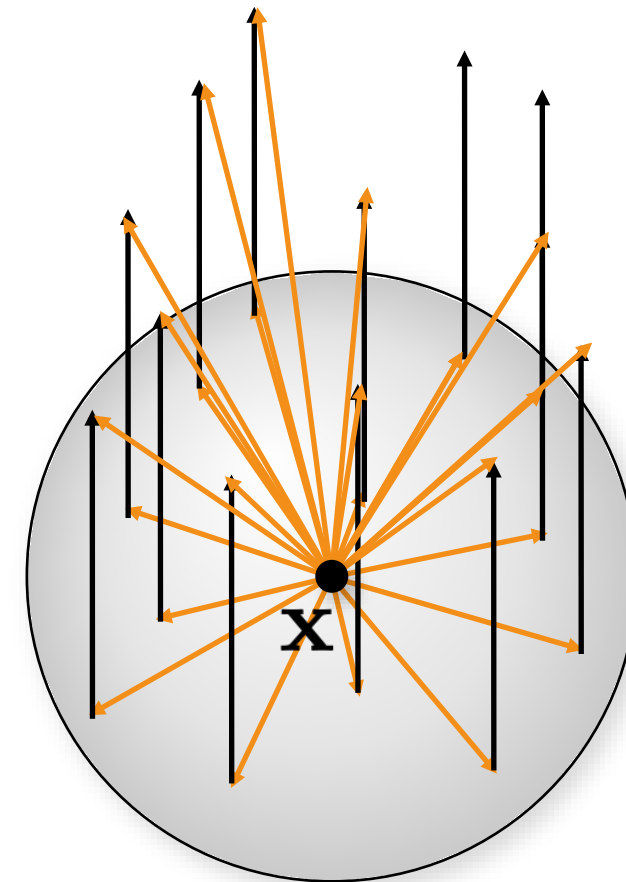
# Sampling Lambertian scattering

Generate points  
uniformly on sphere  
(unit directions)

unit normal



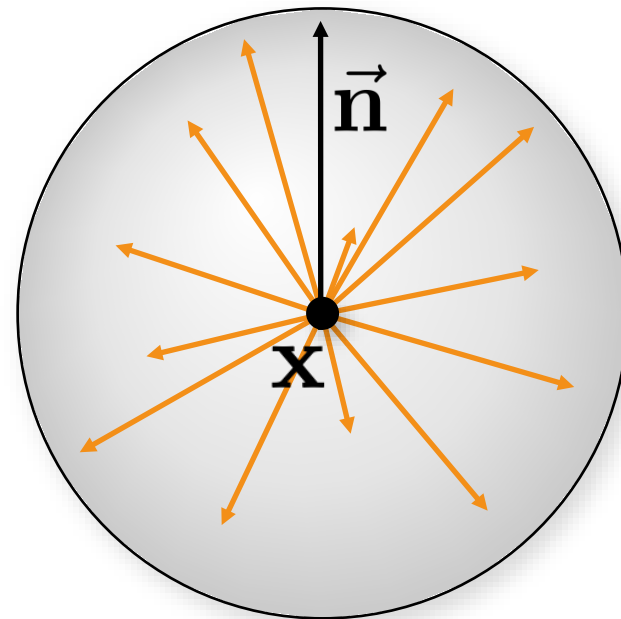
*Add unit normal*



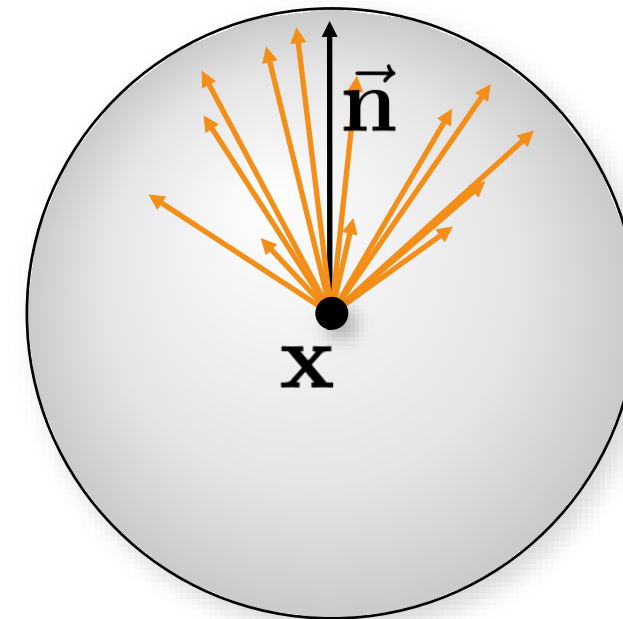
# Sampling Lambertian scattering

Generate points  
uniformly on sphere  
(unit directions)

unit normal

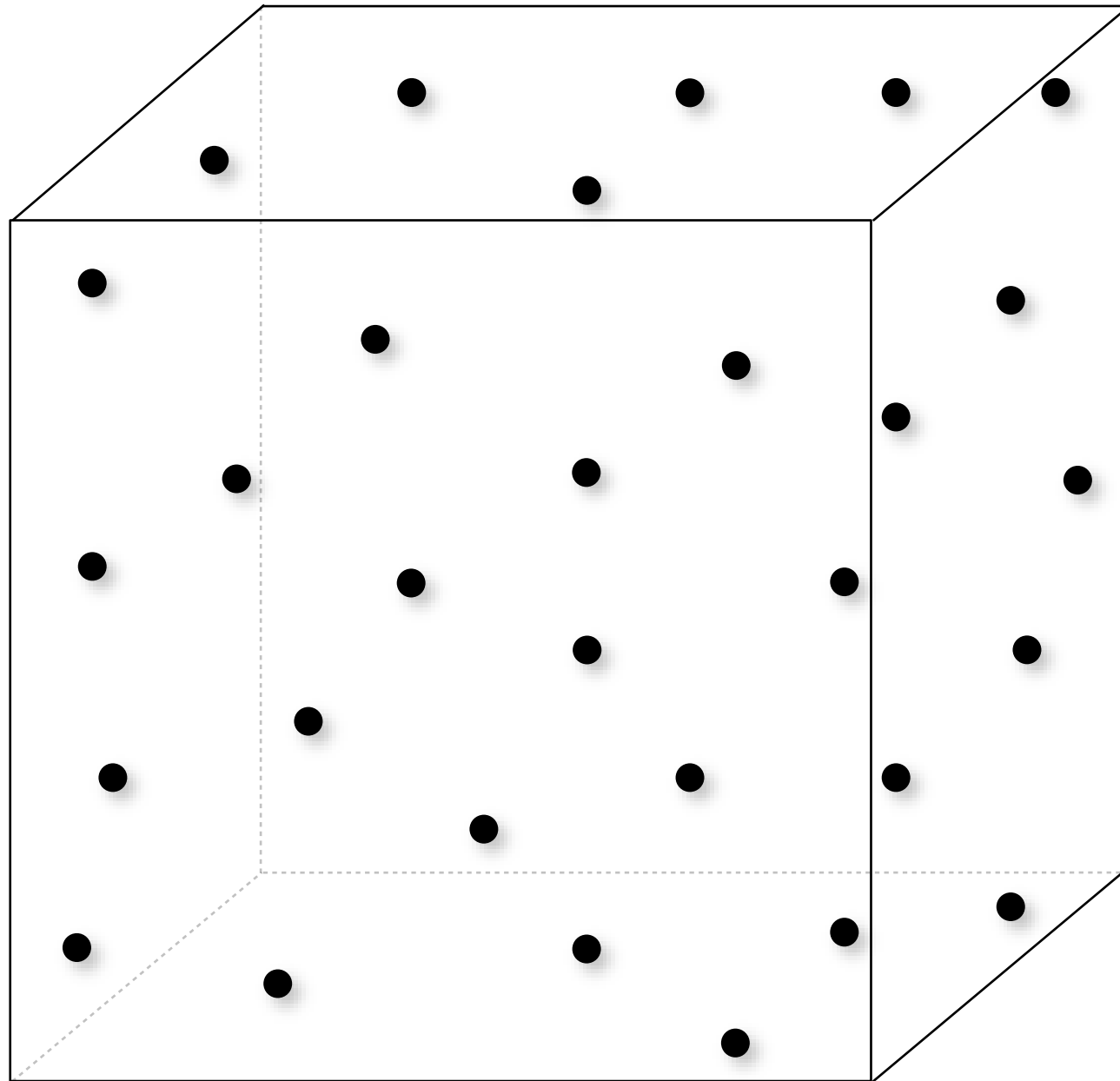


*Add* unit normal  
normalize



# Rejection Sampling a Sphere

---



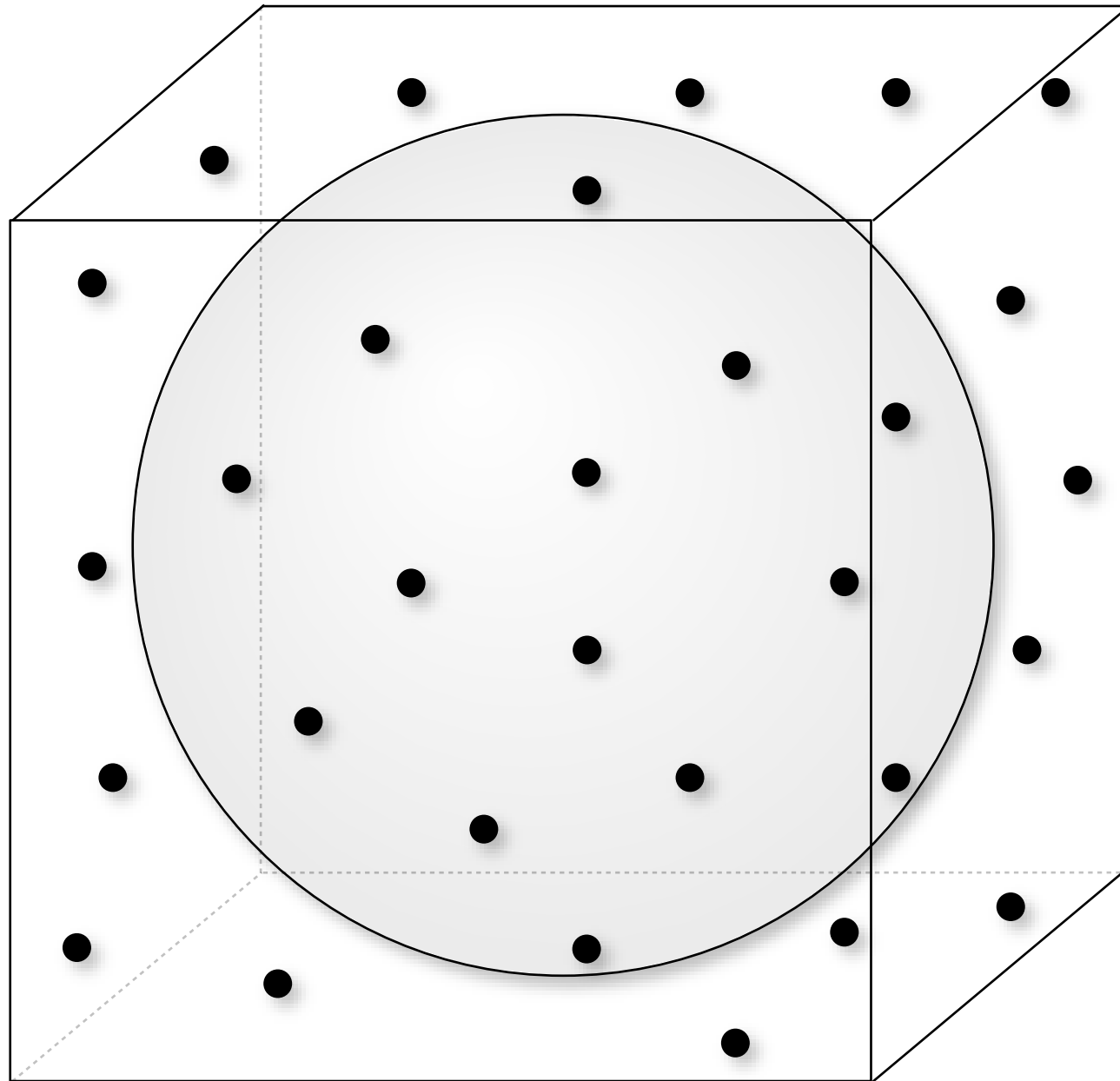
```
Vector3D v;
```

```
v.x = 1-2*randf();
```

```
v.y = 1-2*randf();
```

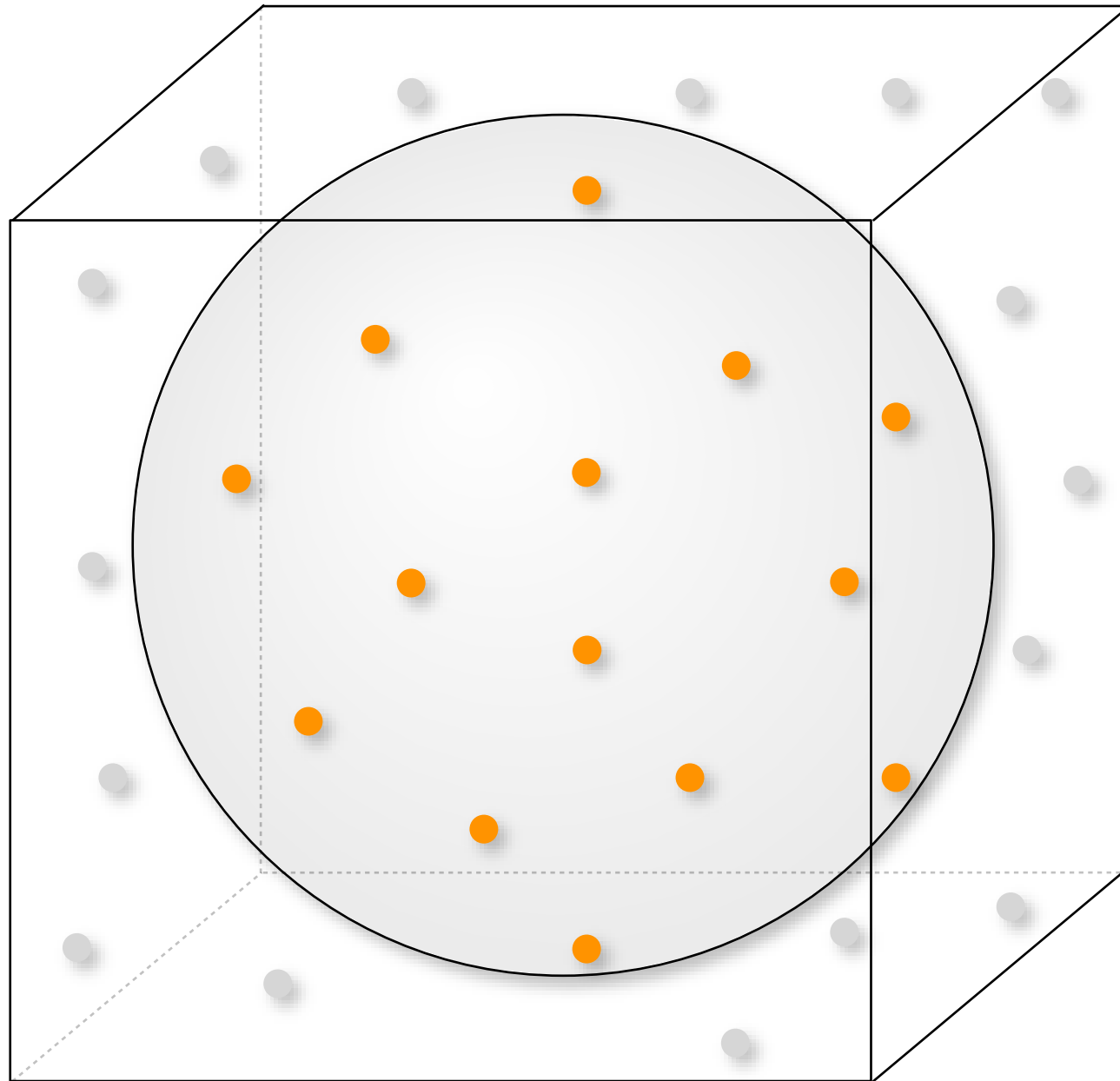
```
v.z = 1-2*randf();
```

# Rejection Sampling a Sphere



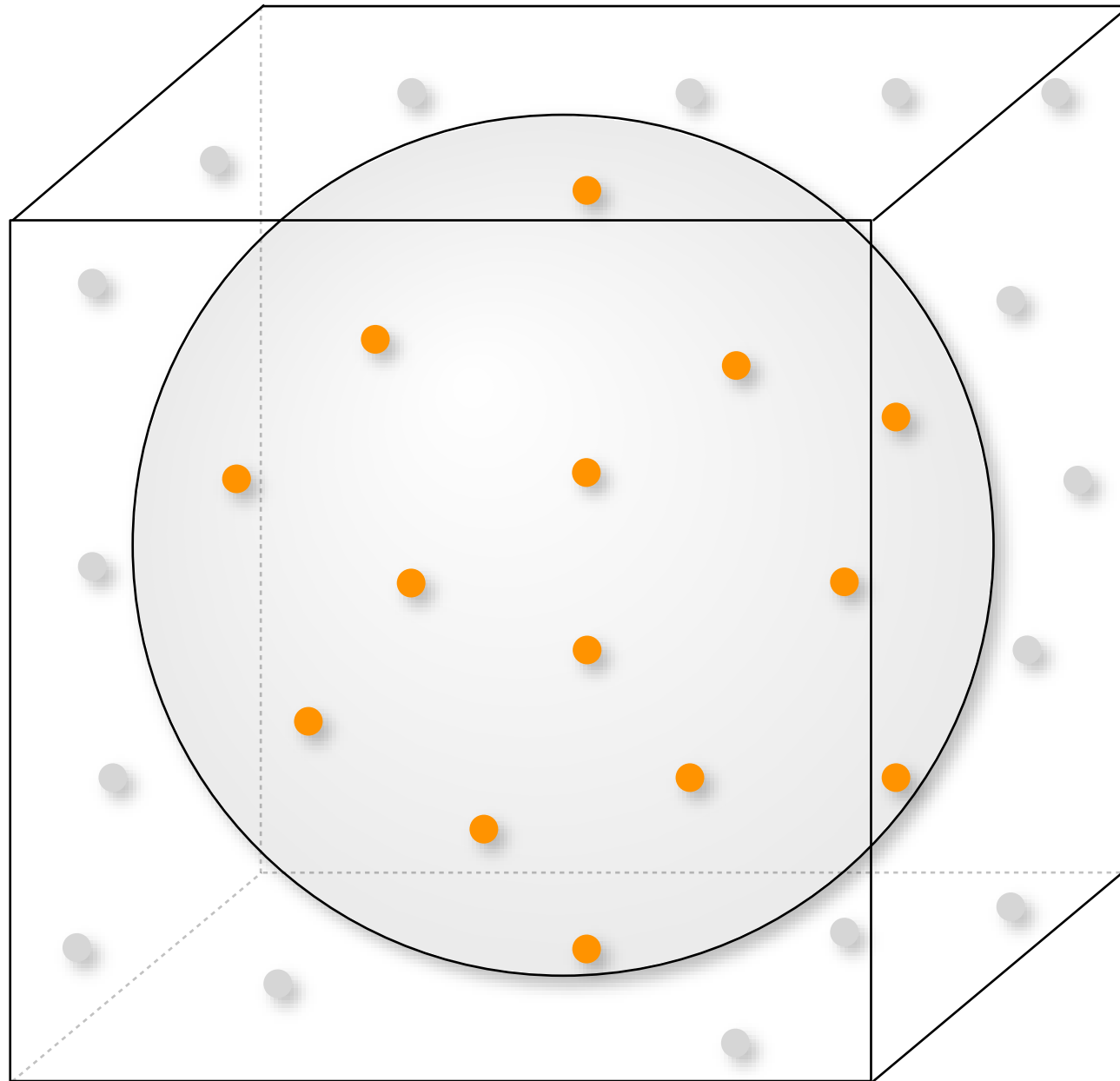
```
Vector3D v;  
do  
{  
    v.x = 1-2*randf();  
    v.y = 1-2*randf();  
    v.z = 1-2*randf();  
} while(v.length2() > 1)
```

# Rejection Sampling a Sphere



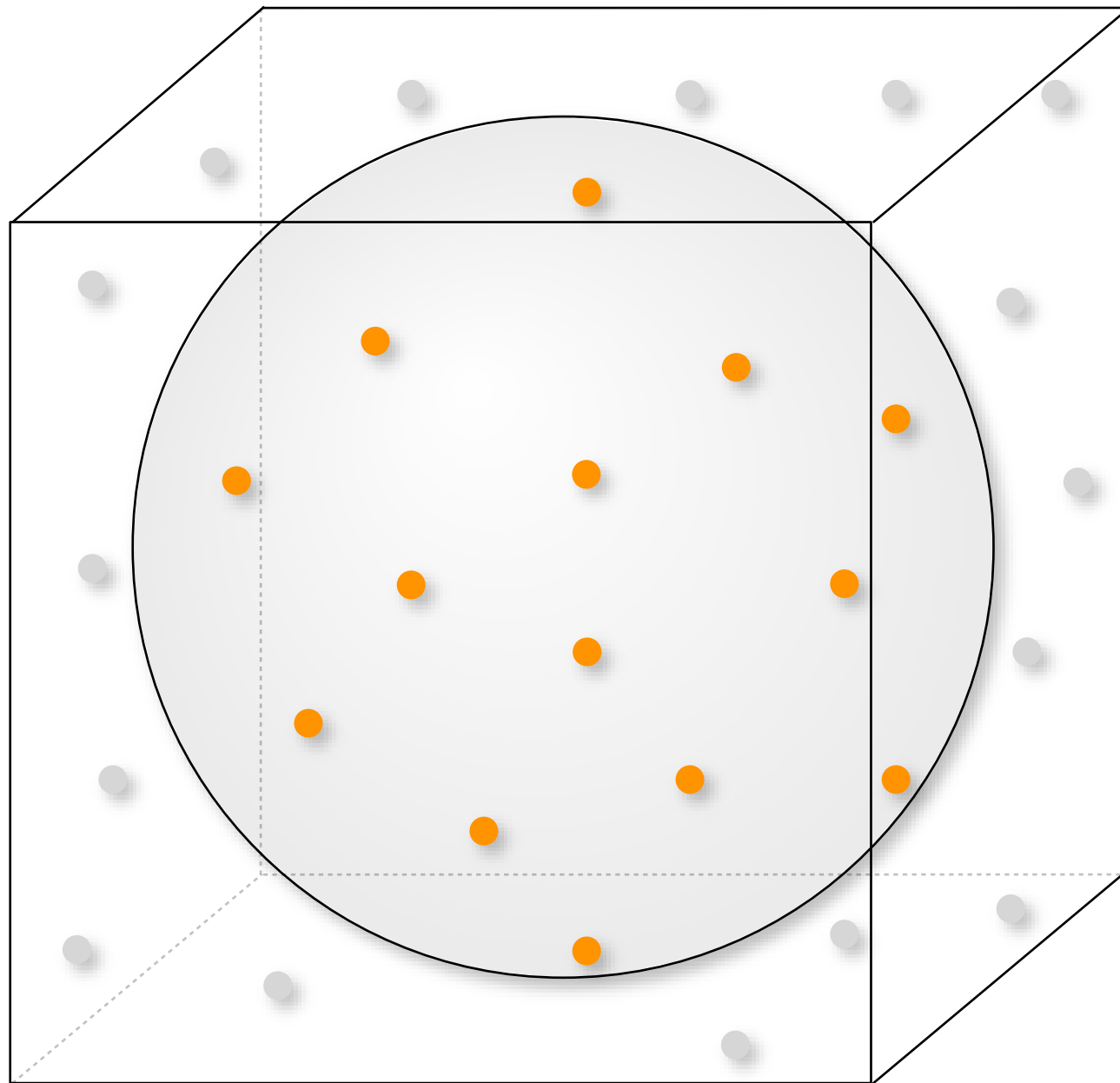
```
Vector3D v;  
do  
{  
    v.x = 1-2*randf();  
    v.y = 1-2*randf();  
    v.z = 1-2*randf();  
} while(v.length2() > 1)
```

# Rejection Sampling a Sphere



```
Vector3D v;  
do  
{  
    v.x = 1-2*randf();  
    v.y = 1-2*randf();  
    v.z = 1-2*randf();  
} while(v.length2() > 1)  
// Project onto sphere  
v /= v.length();
```

# Sampling a Sphere using normal samples



```
Vector3D v;  
v.x = randnf();  
v.y = randnf();  
v.z = randnf();  
// Project onto sphere  
v /= v.length();
```

- No rejection sampling required (no `while` loop).
- Need to use normal, rather than uniform, samples.



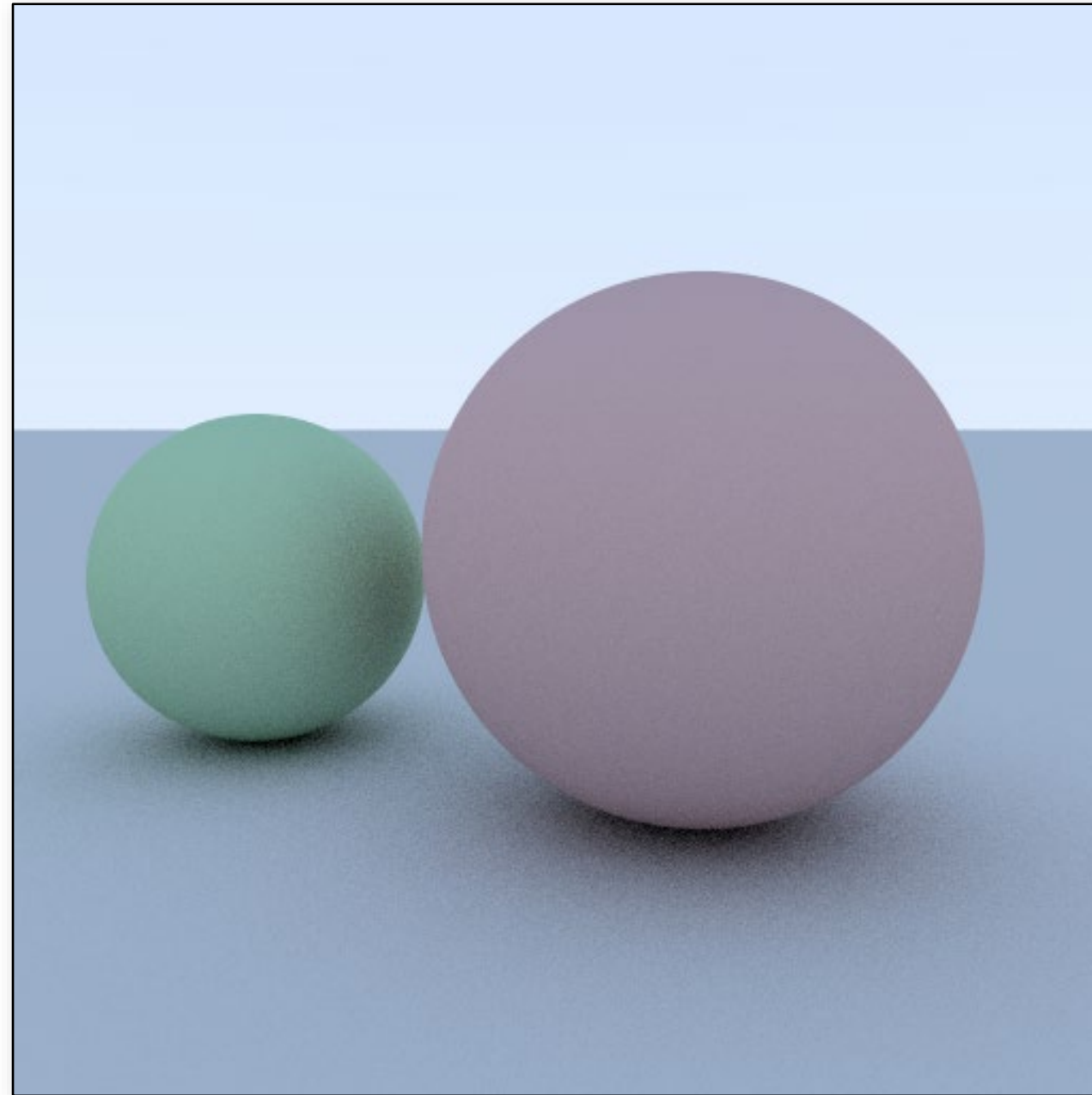
# Accounting for Lambertian albedo

---

At each diffuse shading event, you also need to multiply by the diffuse albedo (between 0 and 1).

# Diffuse shading

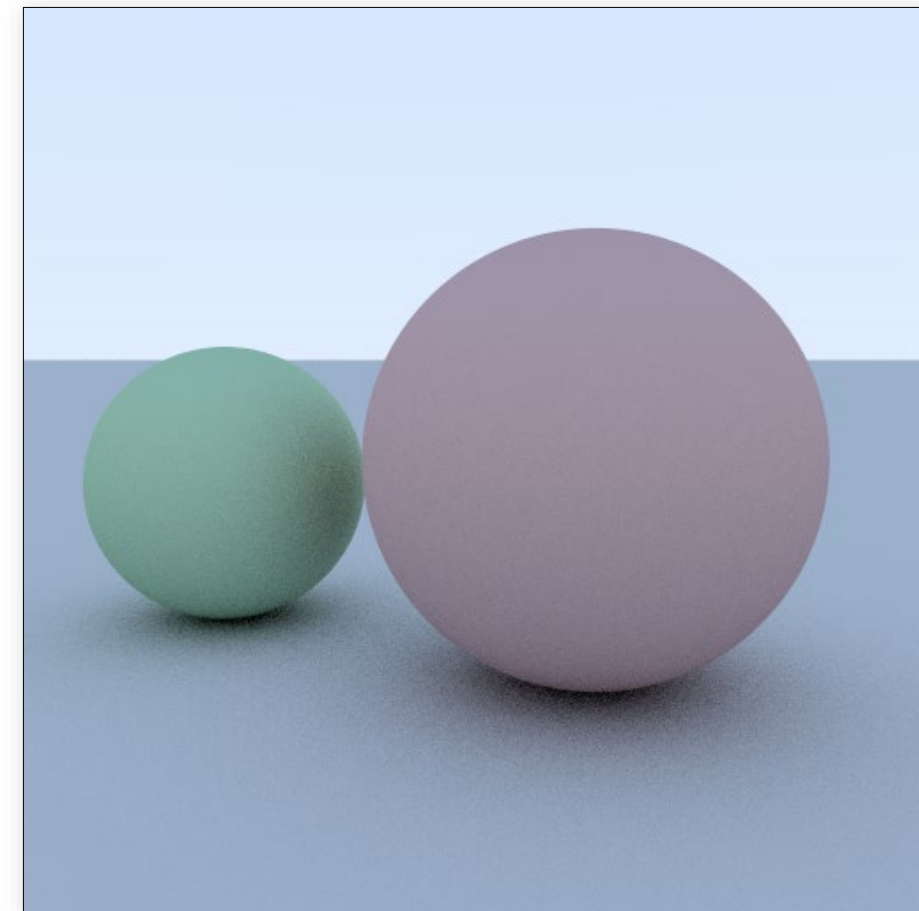
---



# Image so far

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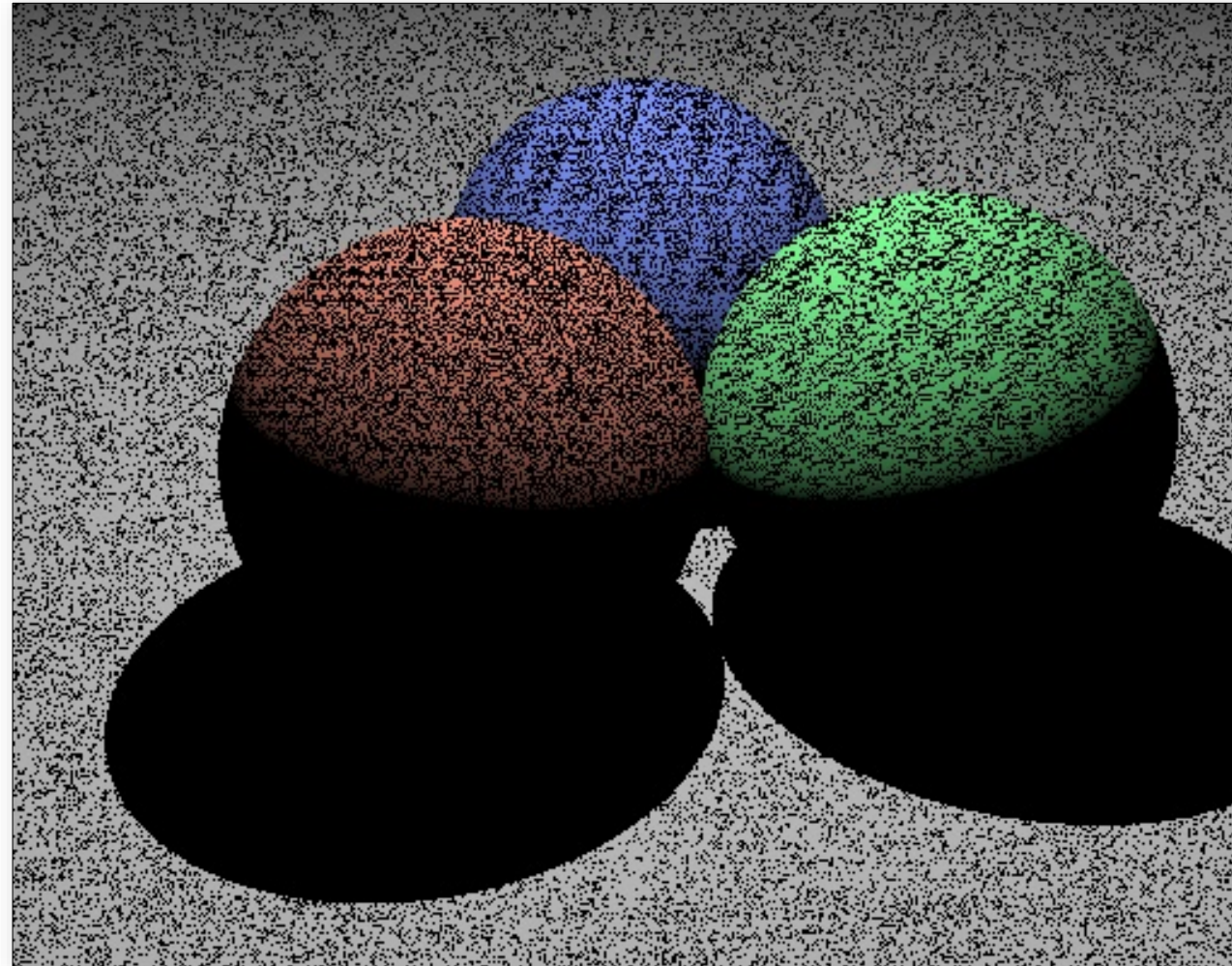
```
Scene::trace(Ray ray)
    hit = surfaces.intersect(ray);
    if hit
        [col, sRay] = hit->mat->scatter(ray)
        return col * trace(sRay);
    else
        return backgroundColor;
```



# Rounding errors

---

Don't fall victim to one of the classic blunders:

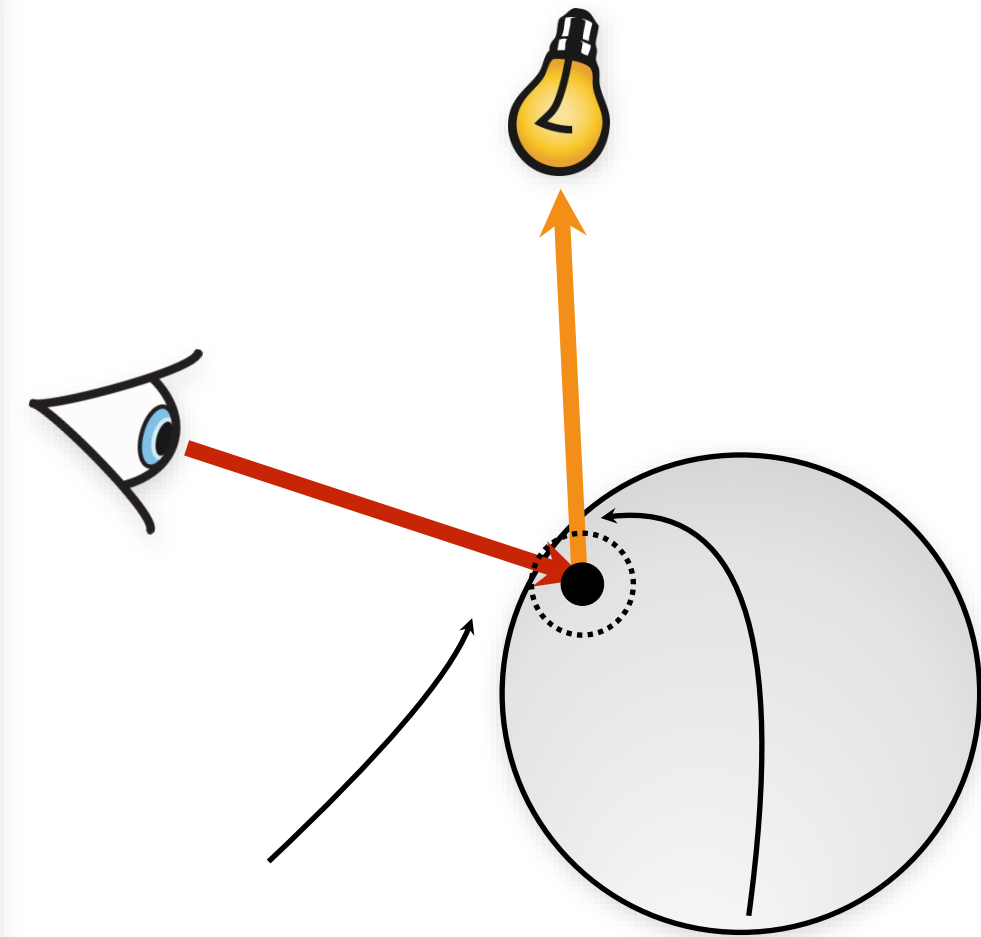
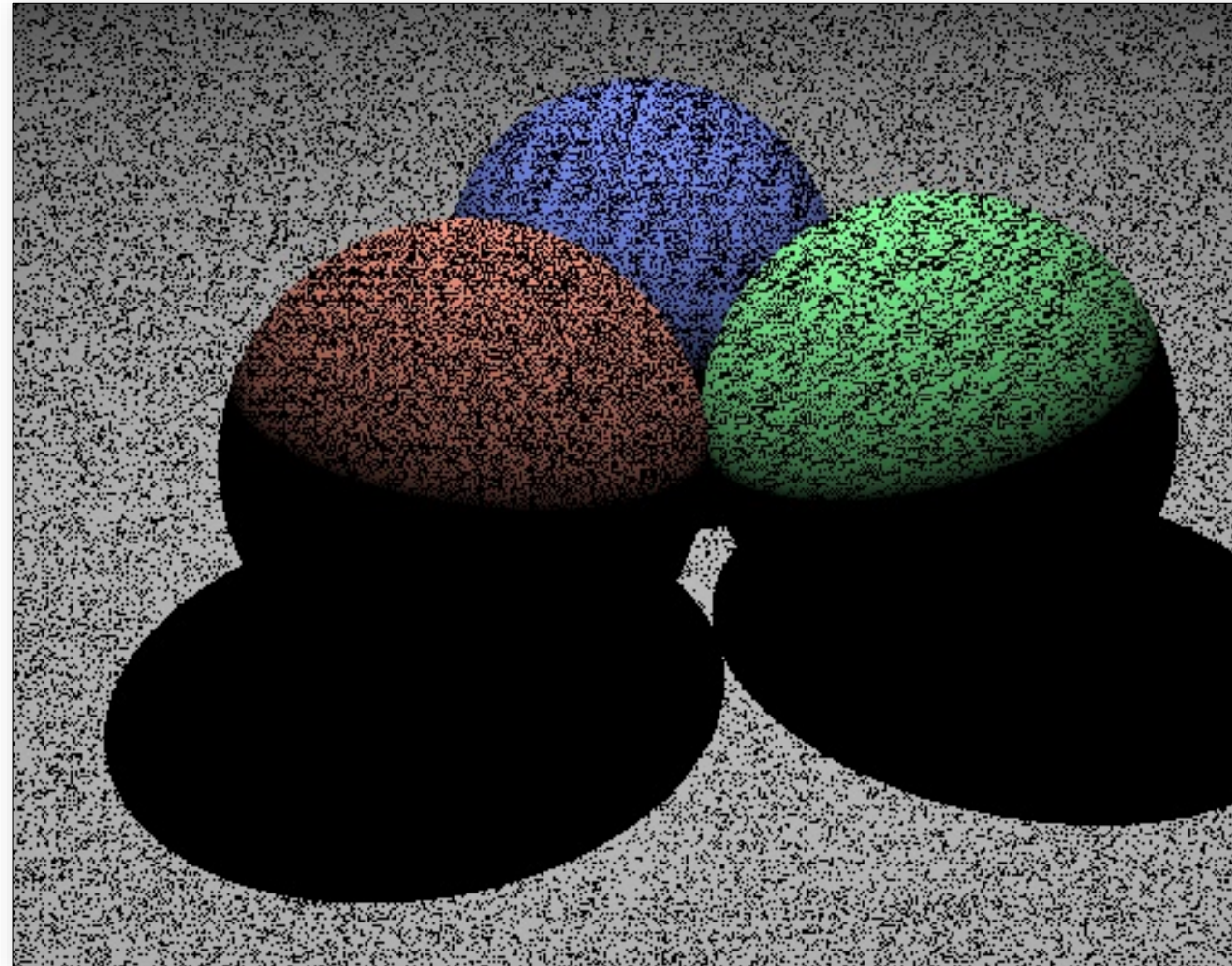
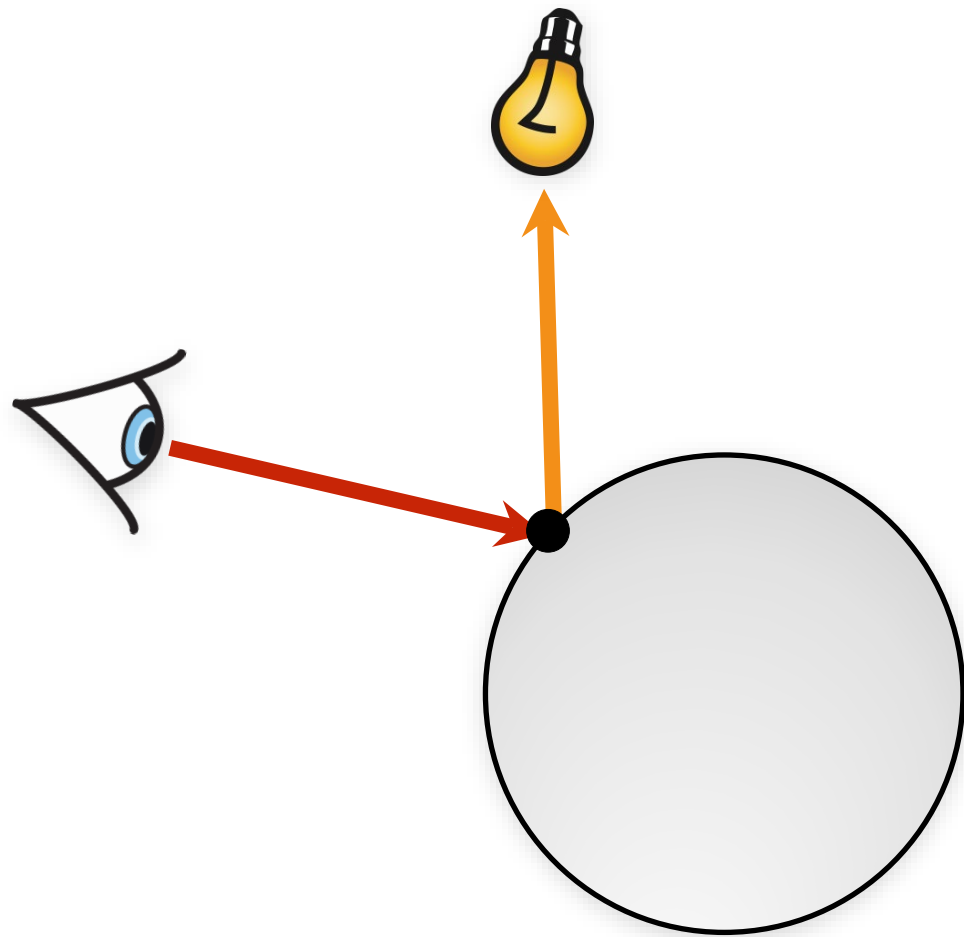


affectionately called  
“shadow acne”

What's going on?

# Rounding errors

Don't fall victim to one of the classic blunders:

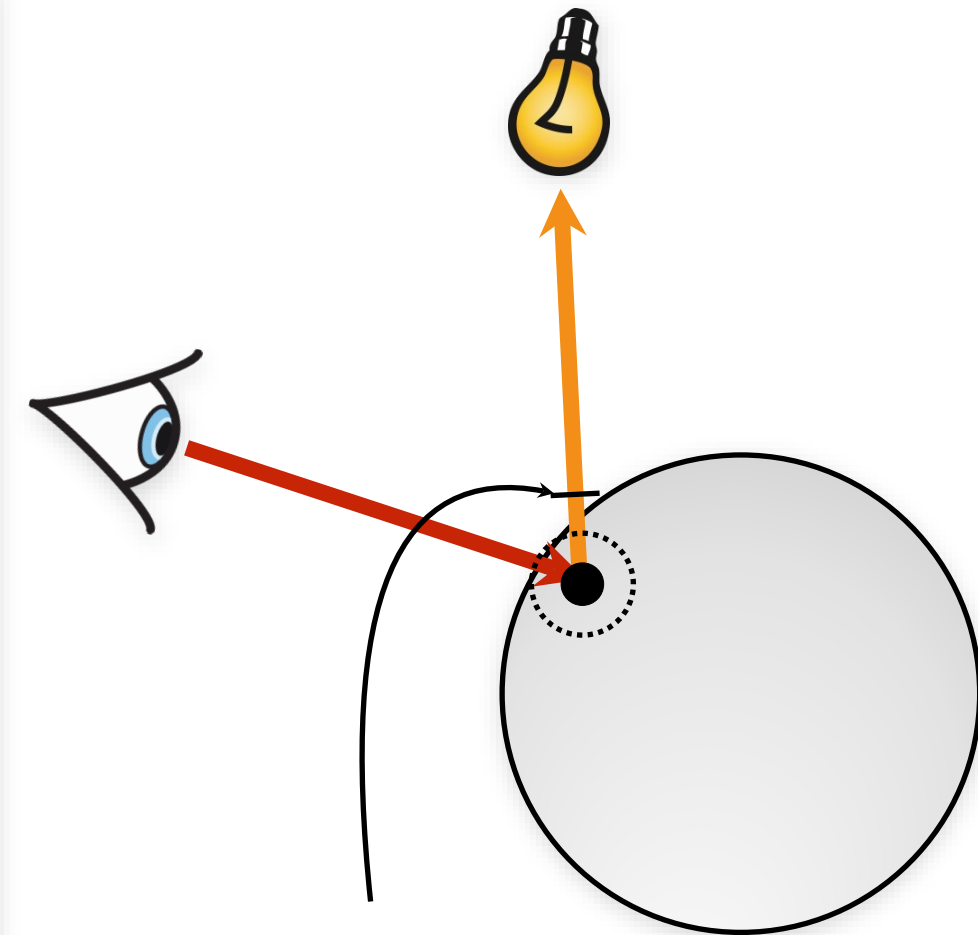
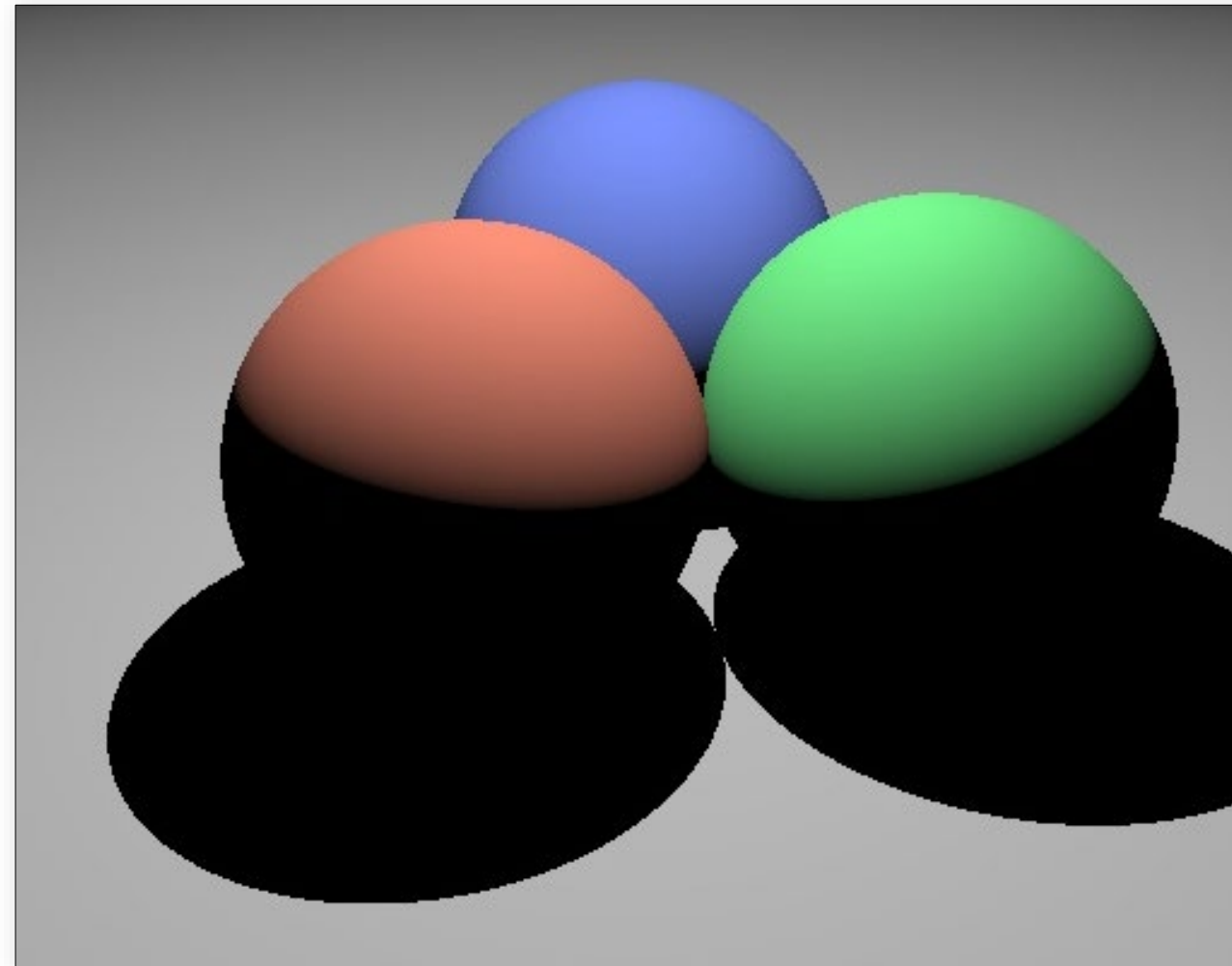


intersection inside  
due to floating point  
error

ray blocked by  
self-intersection

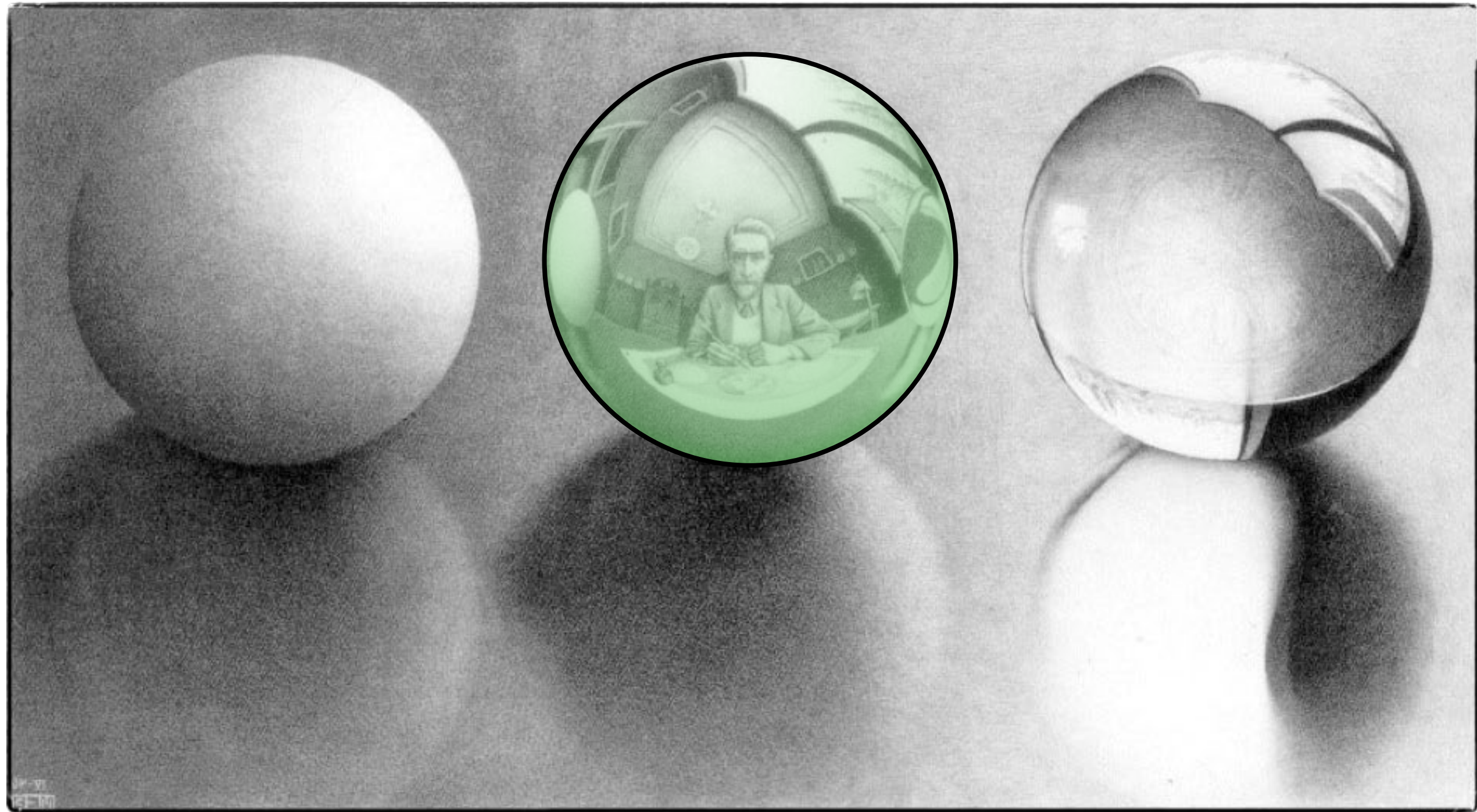
# Shadow rounding errors

Solution: recursive rays start a tiny distance from the surface

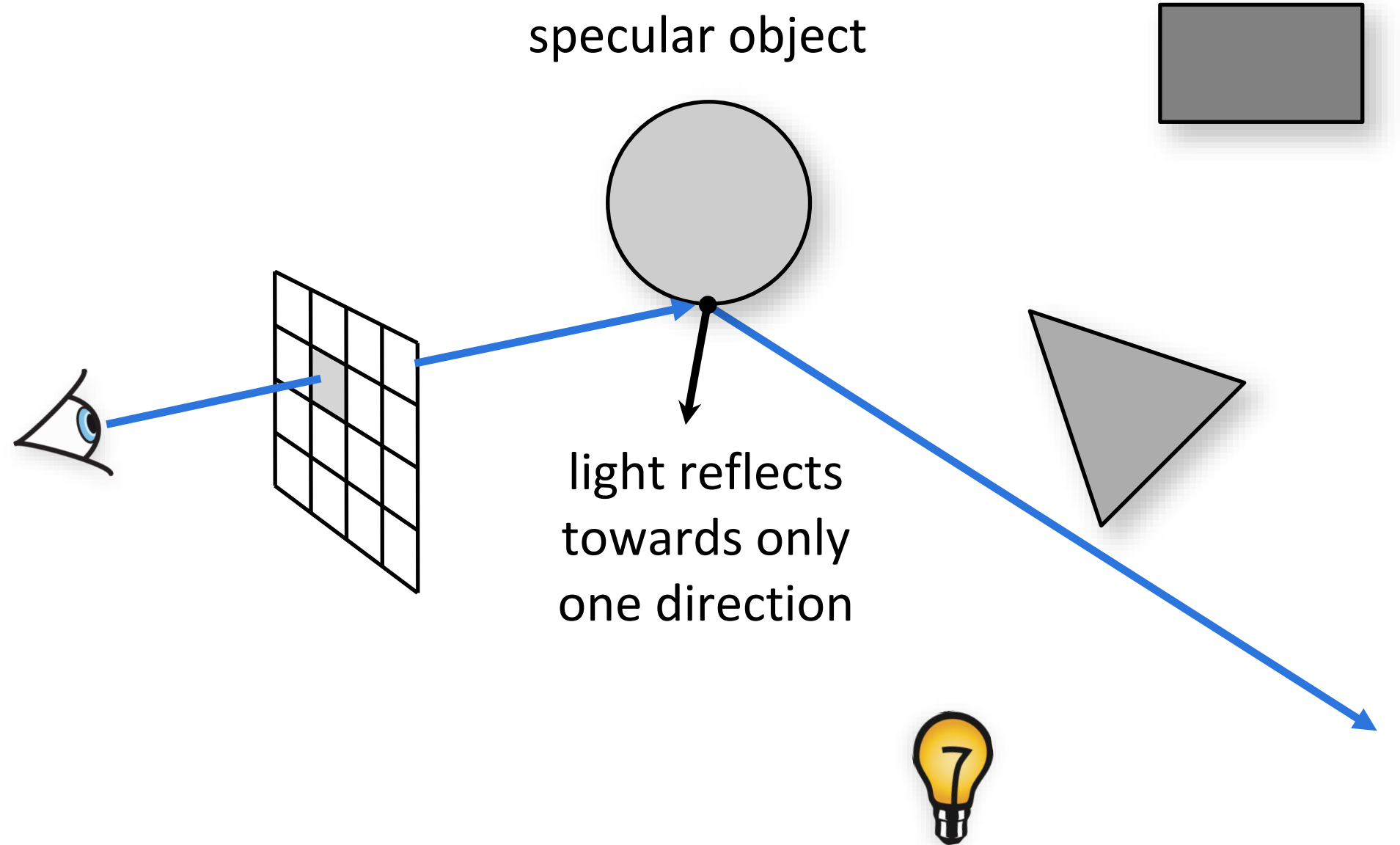
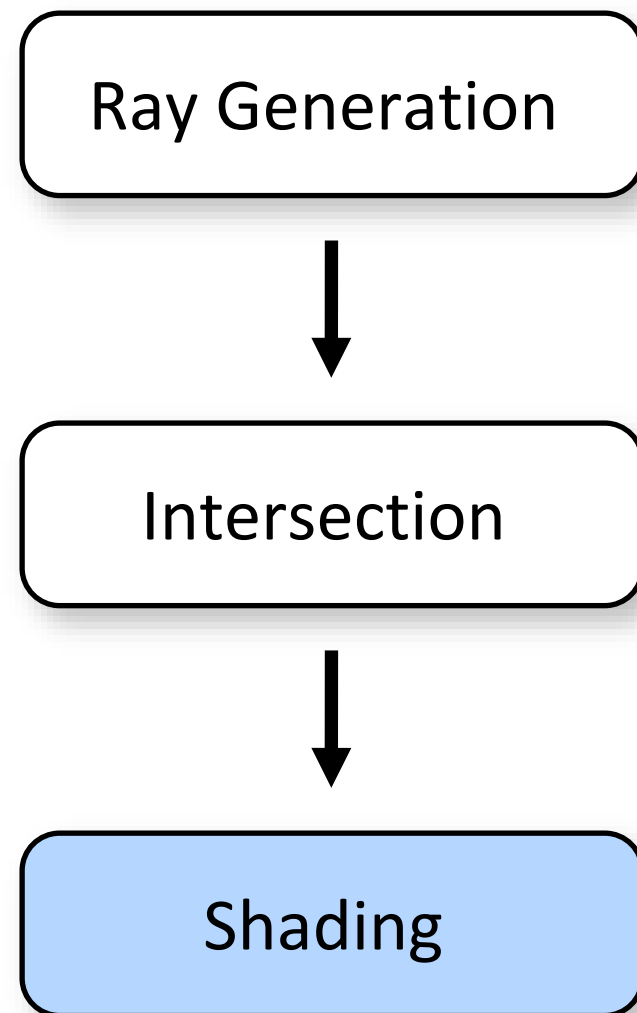


Do this by limiting  
the  $t$  range

# Specular/Mirror reflection



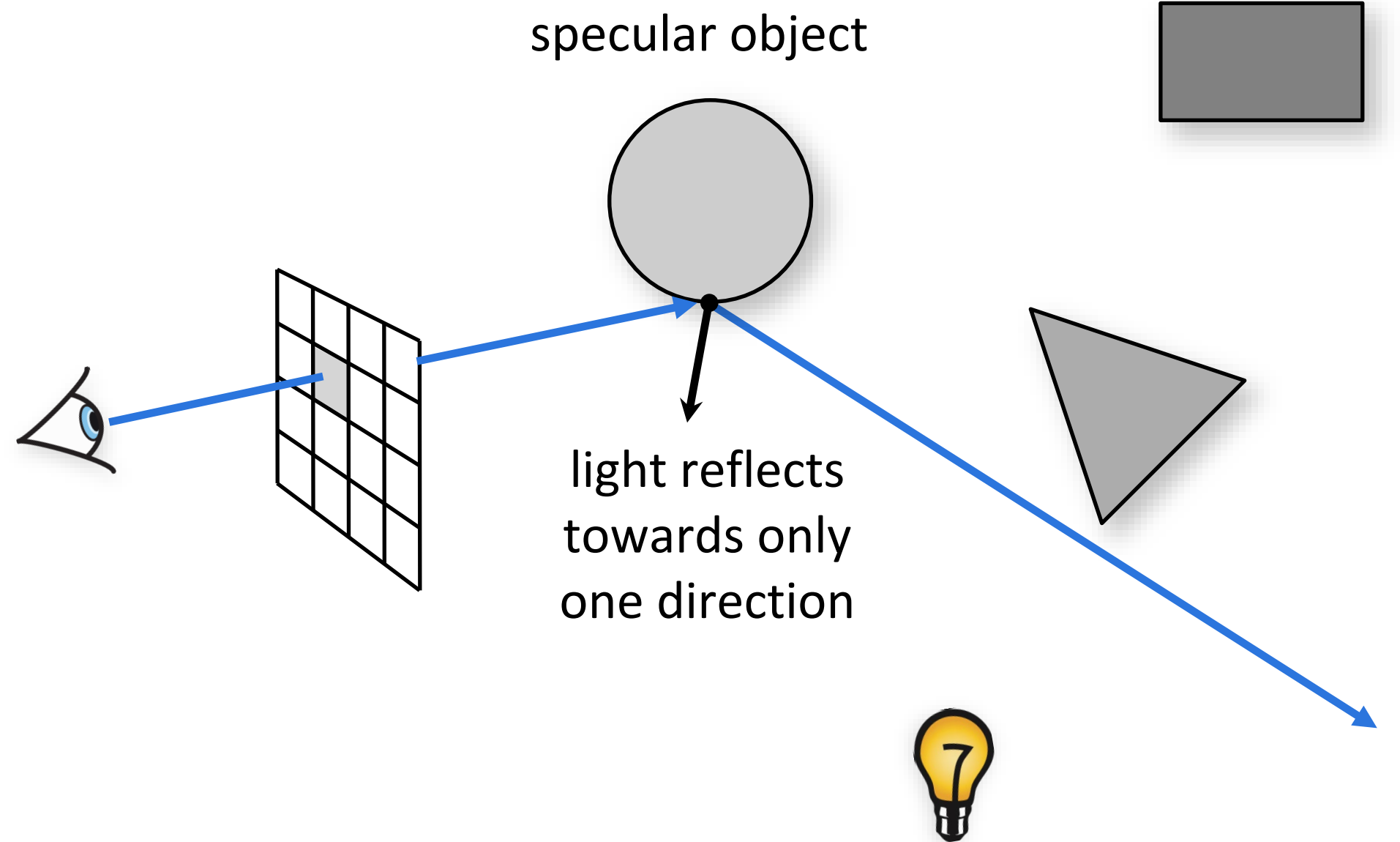
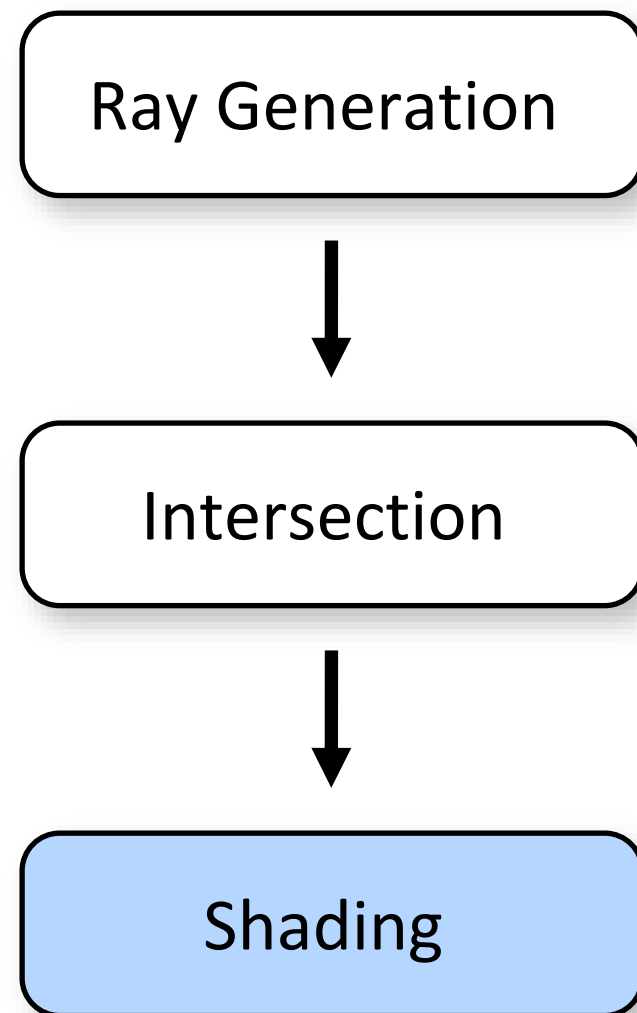
# Basic Ray Tracing Pipeline



What direction should we trace a new ray towards?



# Basic Ray Tracing Pipeline



What direction should we trace a new ray towards?

- Just use law of mirror reflection, no need for random selection!

# Mirror reflection

---

Consider perfectly shiny surface

- there's a reflection of other objects

Can render this using recursive ray tracing

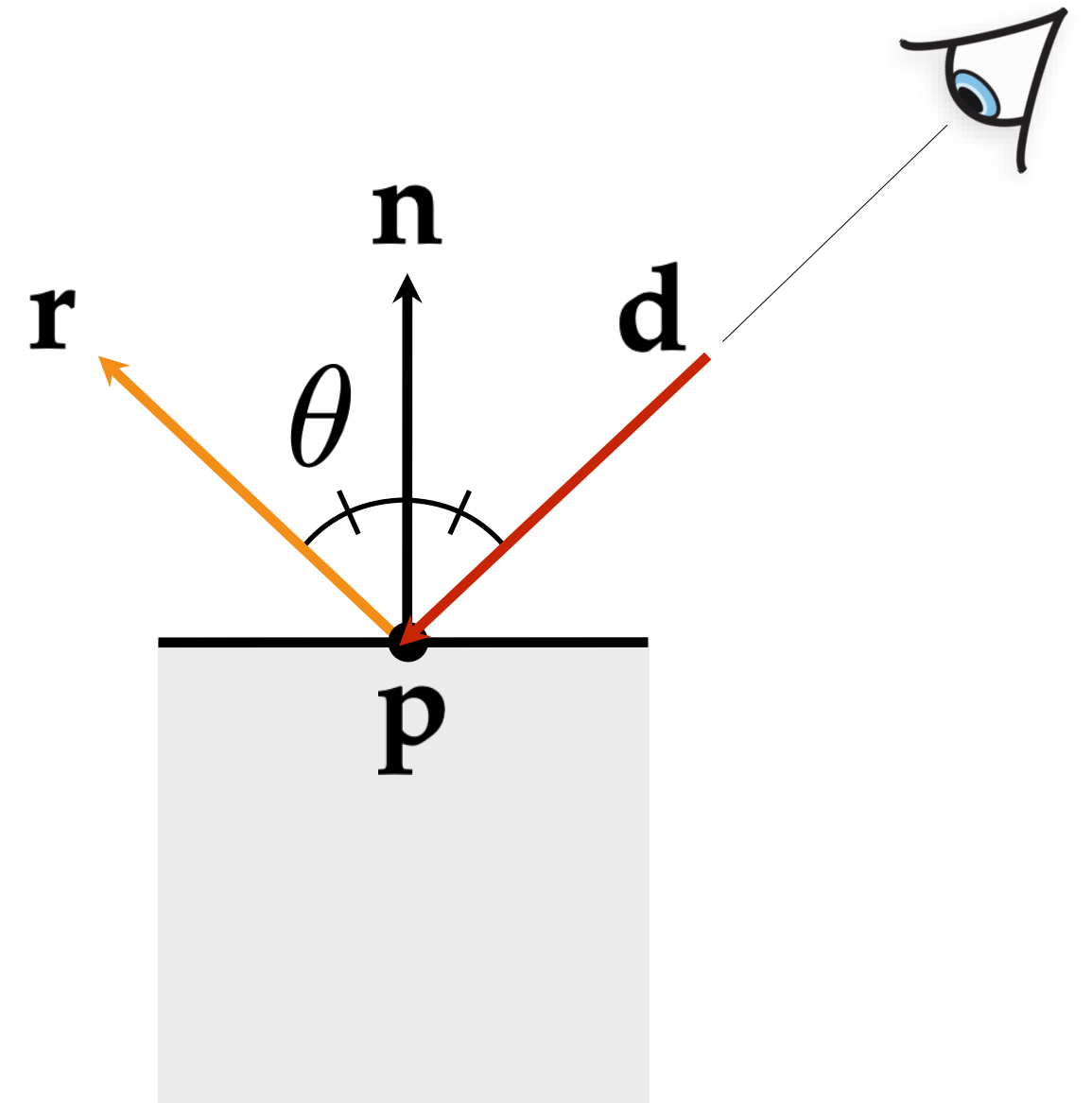
- to find out mirror reflection color ask:  
“what color is seen from surface point in reflection direction?”

# Mirror reflection

Evaluated by tracing a new ray:

$$L_r = k_r \text{trace}(\mathbf{p}, \mathbf{r})$$

reflected light      reflection scaled down/tinted      recursive call



# Mirror reflection

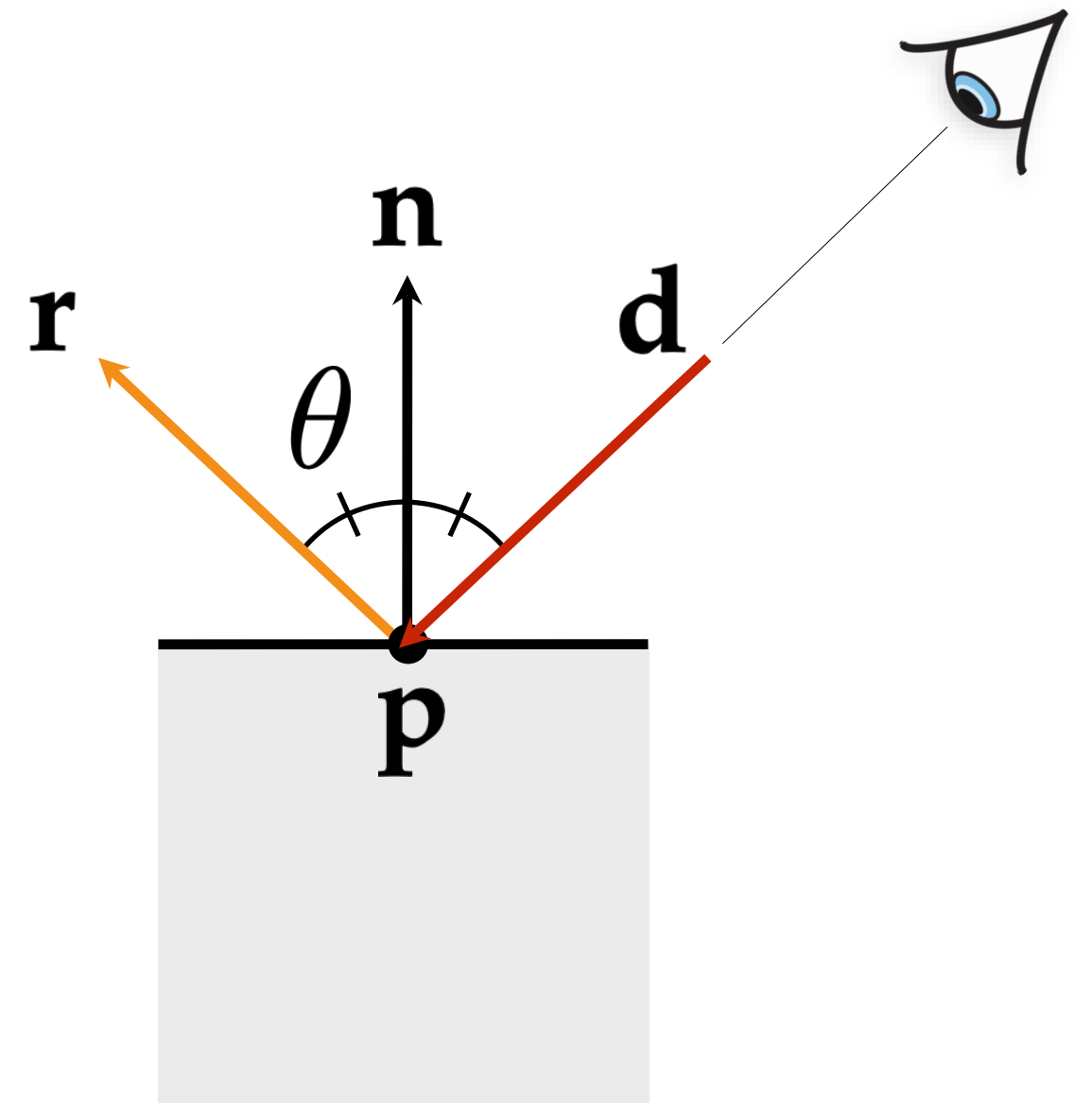
Evaluated by tracing a new ray:

$$L_r = k_r \text{trace}(\mathbf{p}, \mathbf{r})$$

reflected light      reflection scaled down/tinted      recursive call

Implementation details:

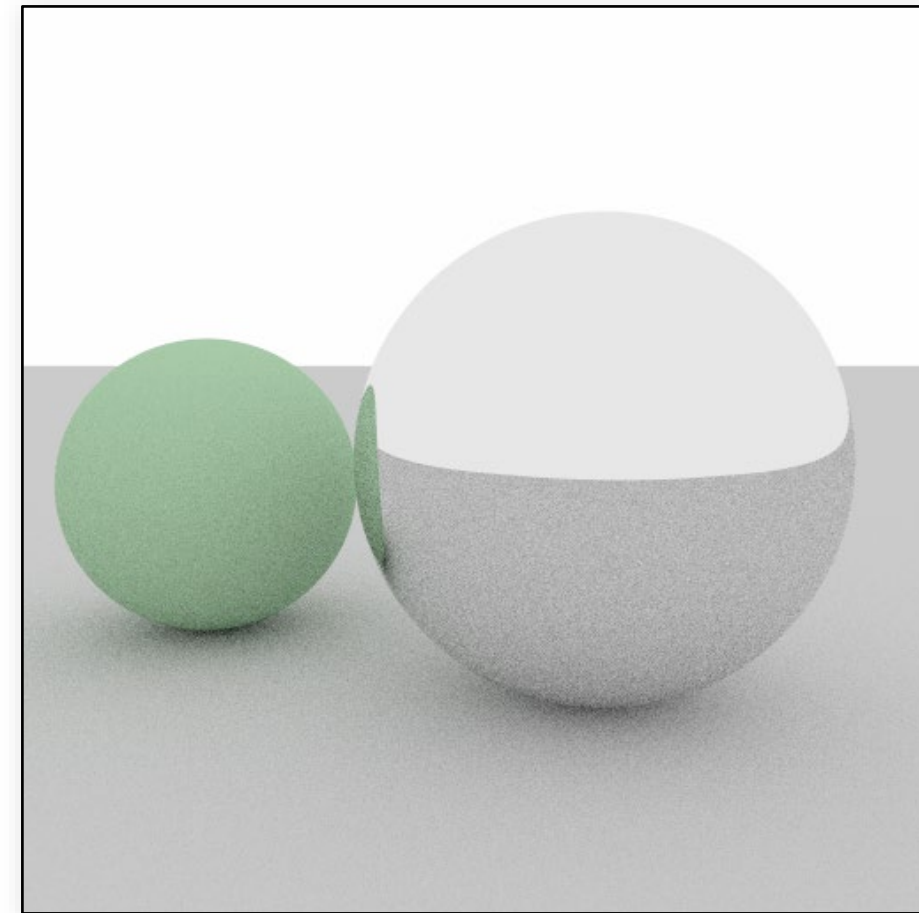
- don't self-intersect ( $t_{\min} > \epsilon$ )
- don't recurse indefinitely



# Same pseudo-code

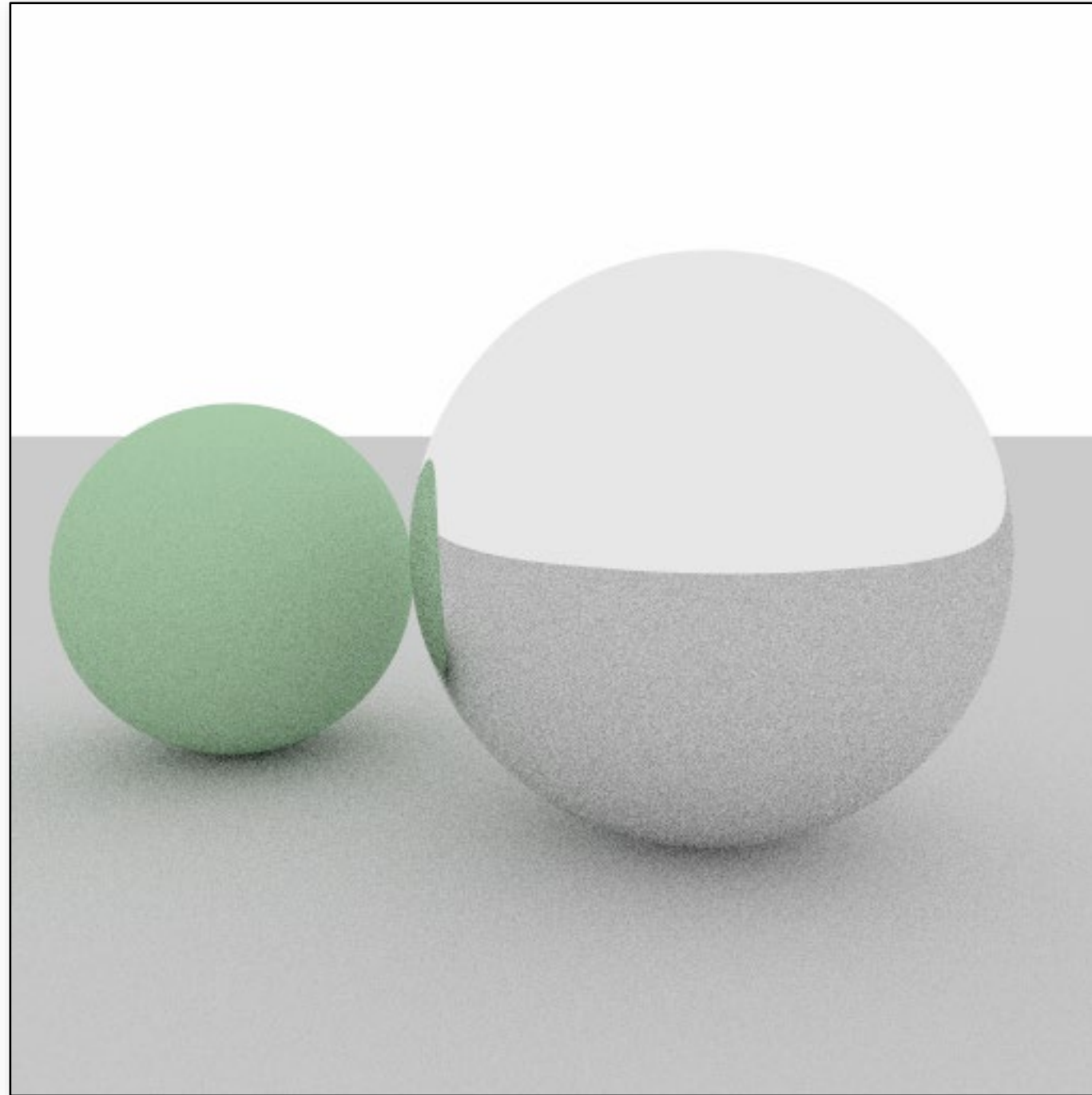
---

```
Scene::trace(Ray ray)
  hit = surfaces.intersect(ray);
  if hit
    [col, sRay] = hit->mat->scatter(ray)
    return col * trace(sRay);
  else
    return backgroundColor;
```



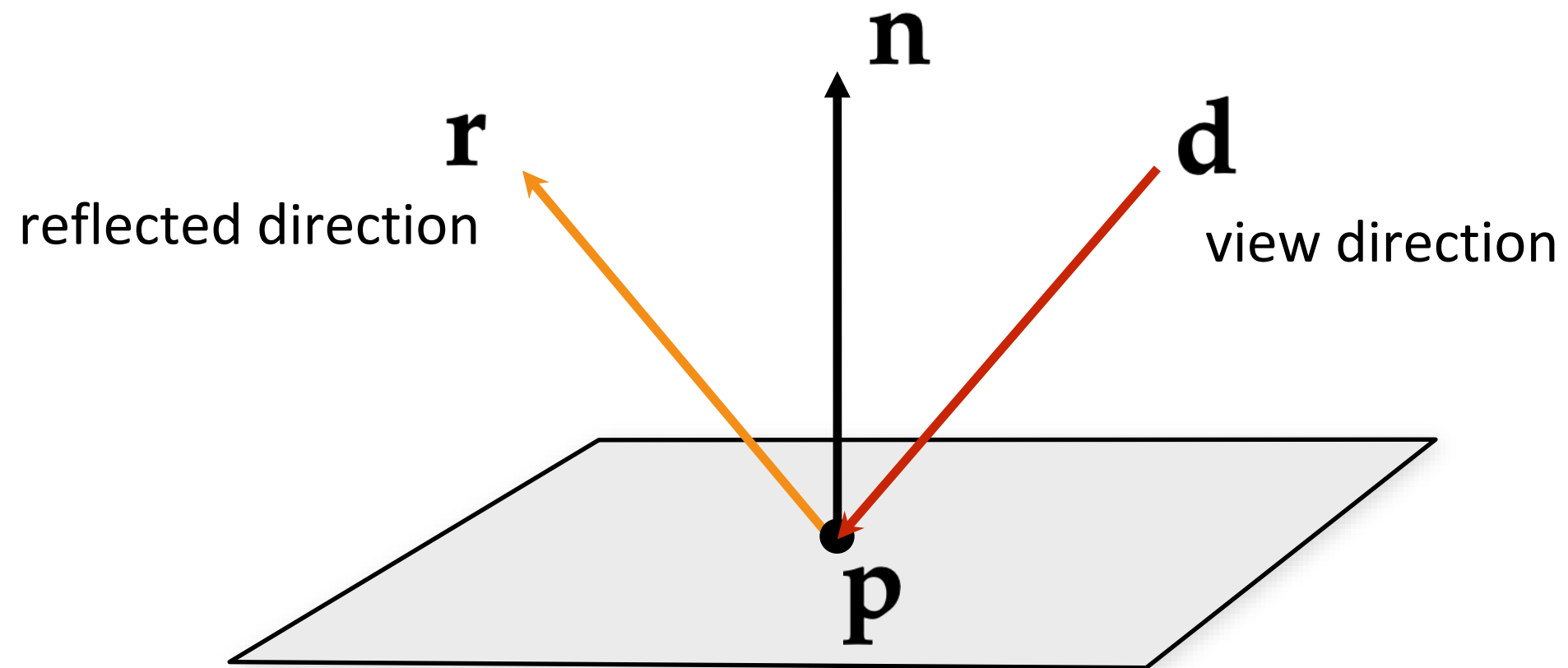
# Diffuse & mirror spheres

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# Mirror reflection

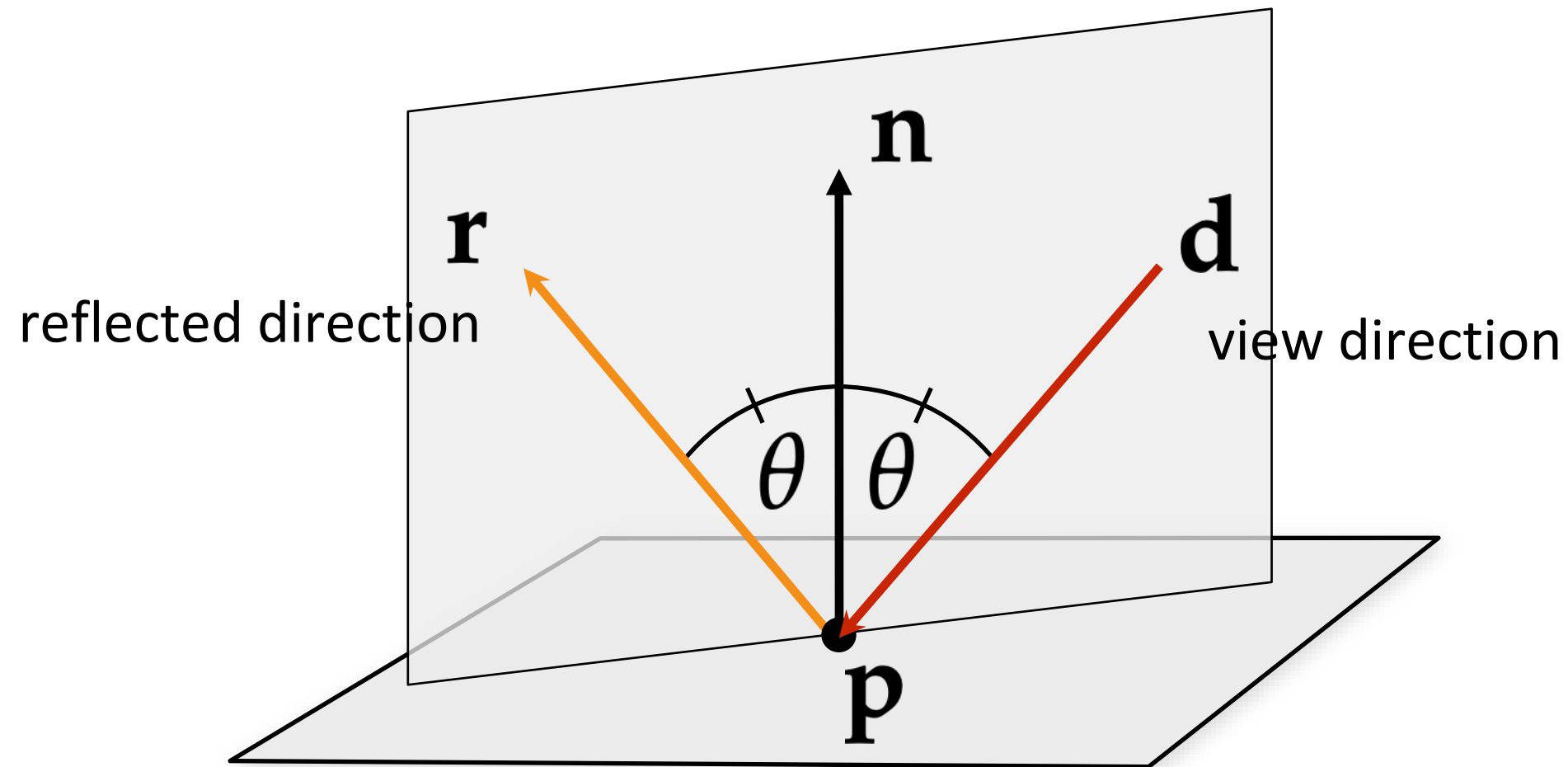
Assume  $\mathbf{n}$  is unit length



What two properties defined reflection direction?

# Mirror reflection

Assume  $\mathbf{n}$  is unit length



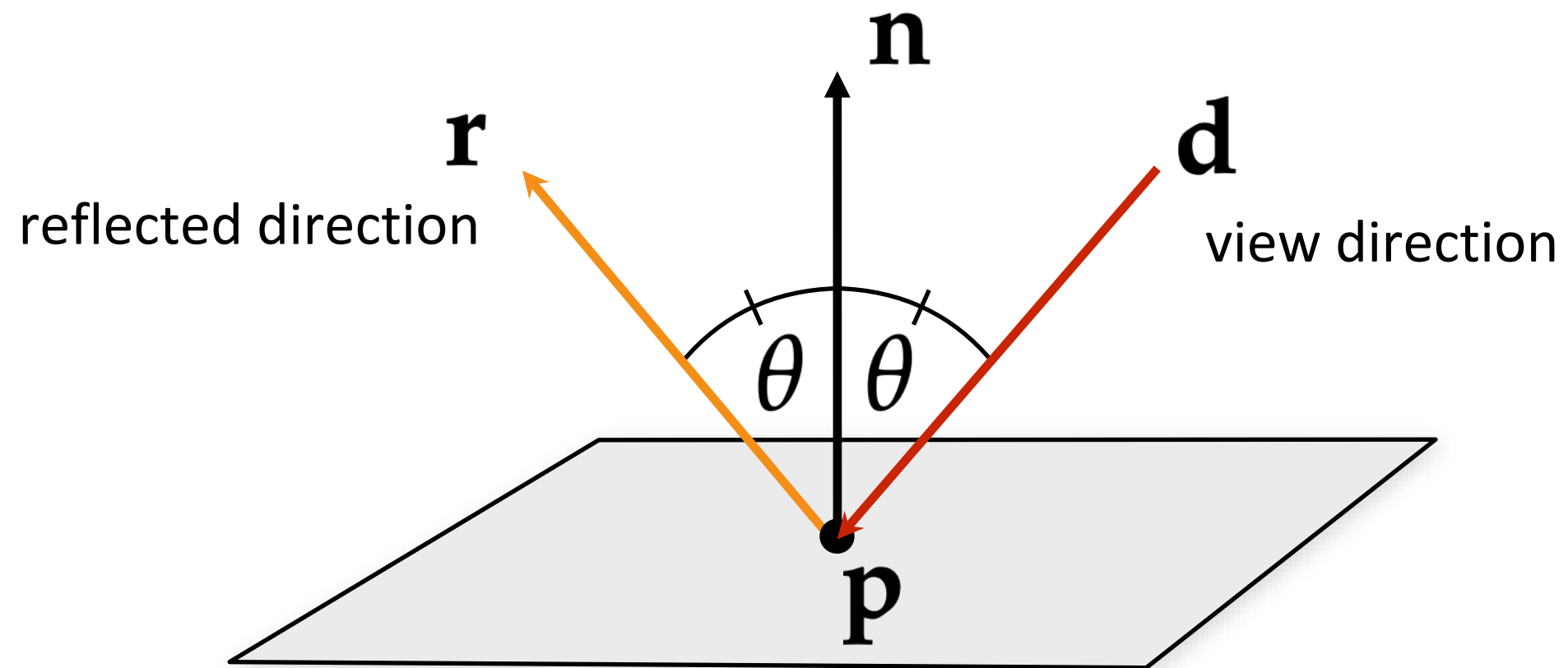
What two properties defined reflection direction?

- co-planar view direction, reflected direction, and normal direction
- equal angles between normal-view directions, and normal-reflected directions

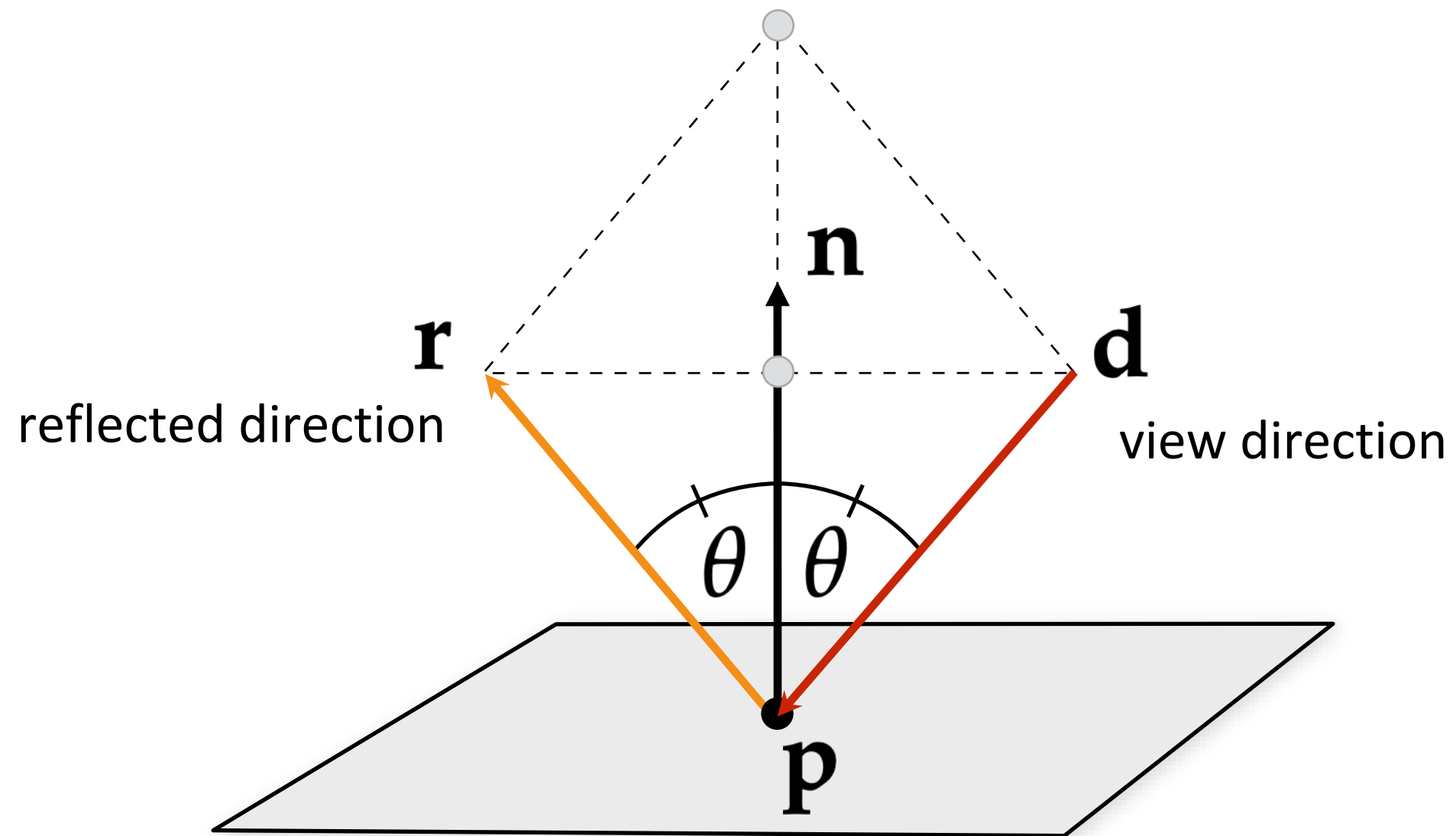


# Mirror reflection

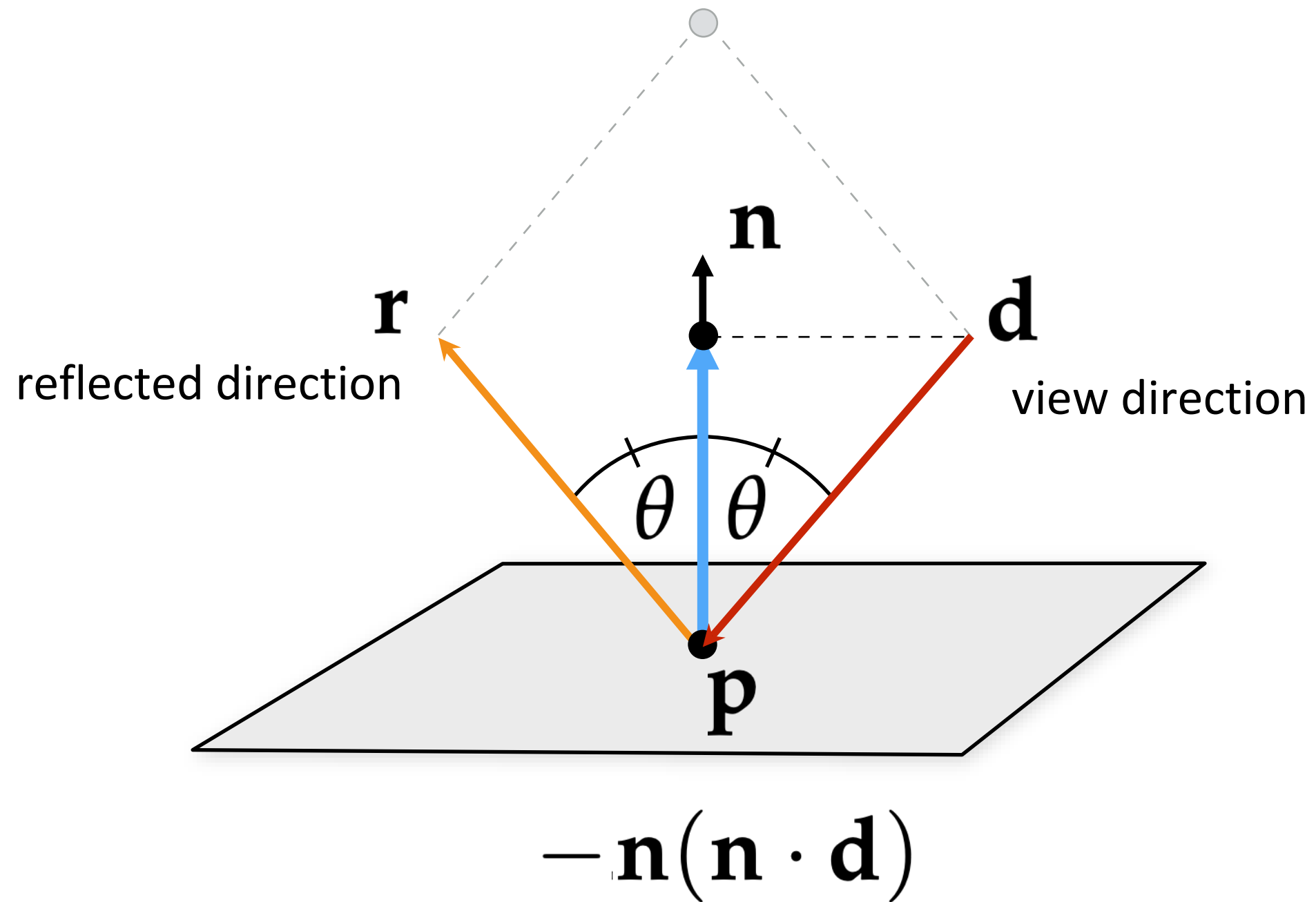
Assume  $\mathbf{n}$  is unit length



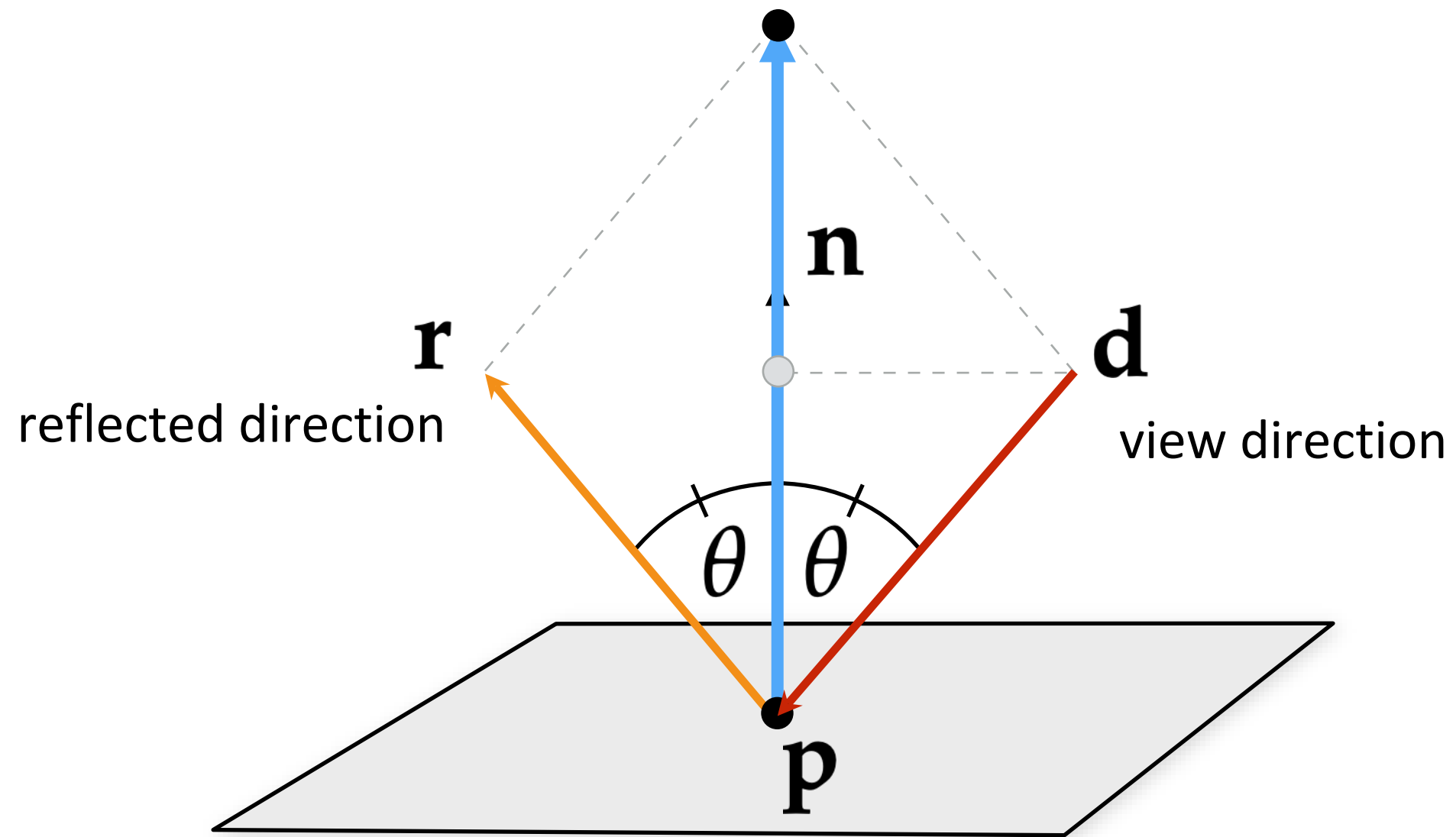
# Mirror reflection



# Mirror reflection



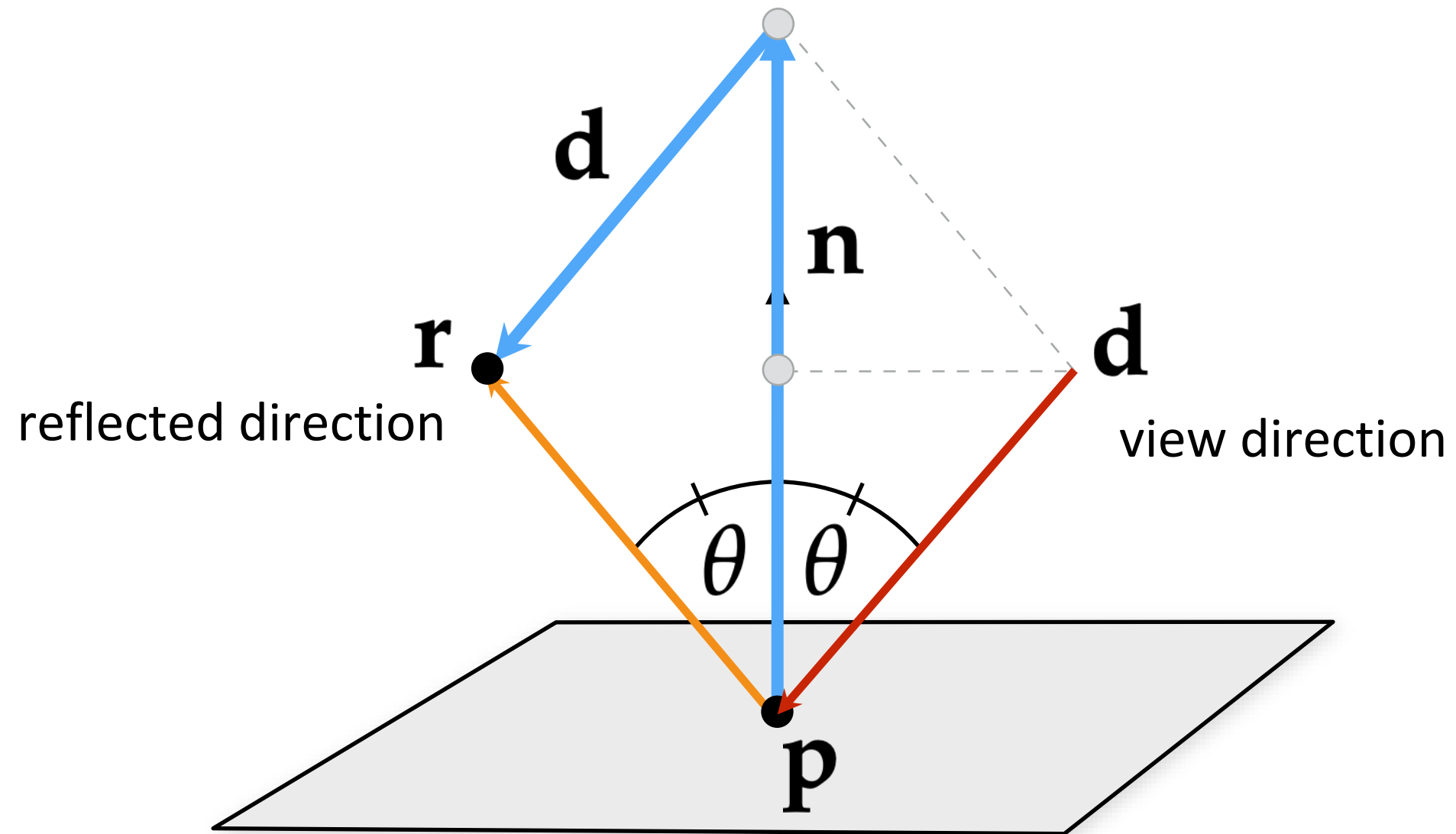
# Mirror reflection



$$-2\mathbf{n}(\mathbf{n} \cdot \mathbf{d})$$

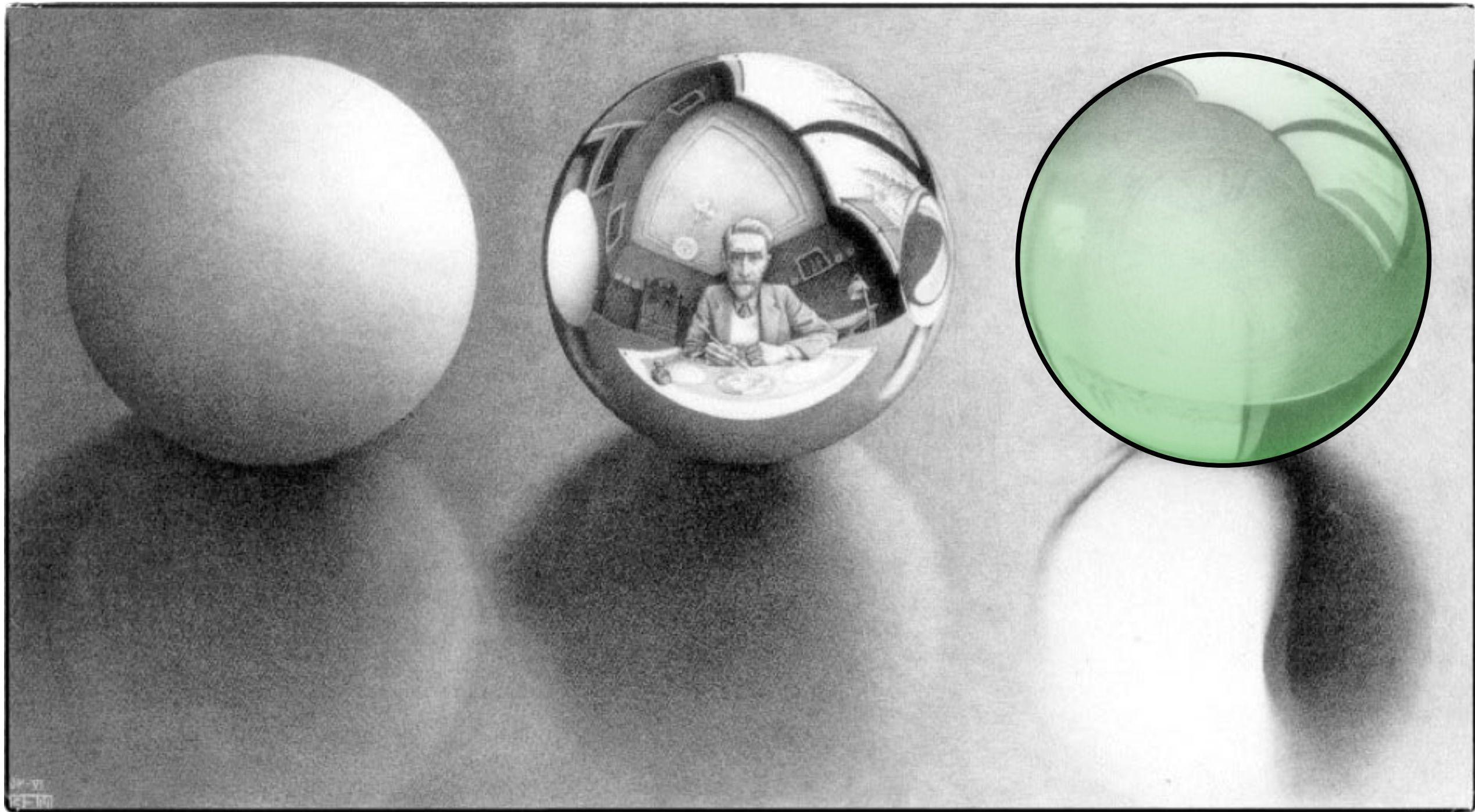
# Mirror reflection

Assumes  $\mathbf{n}$  is unit length



$$\mathbf{r} = -2\mathbf{n}(\mathbf{n} \cdot \mathbf{d}) + \mathbf{d}$$

# Specular refraction



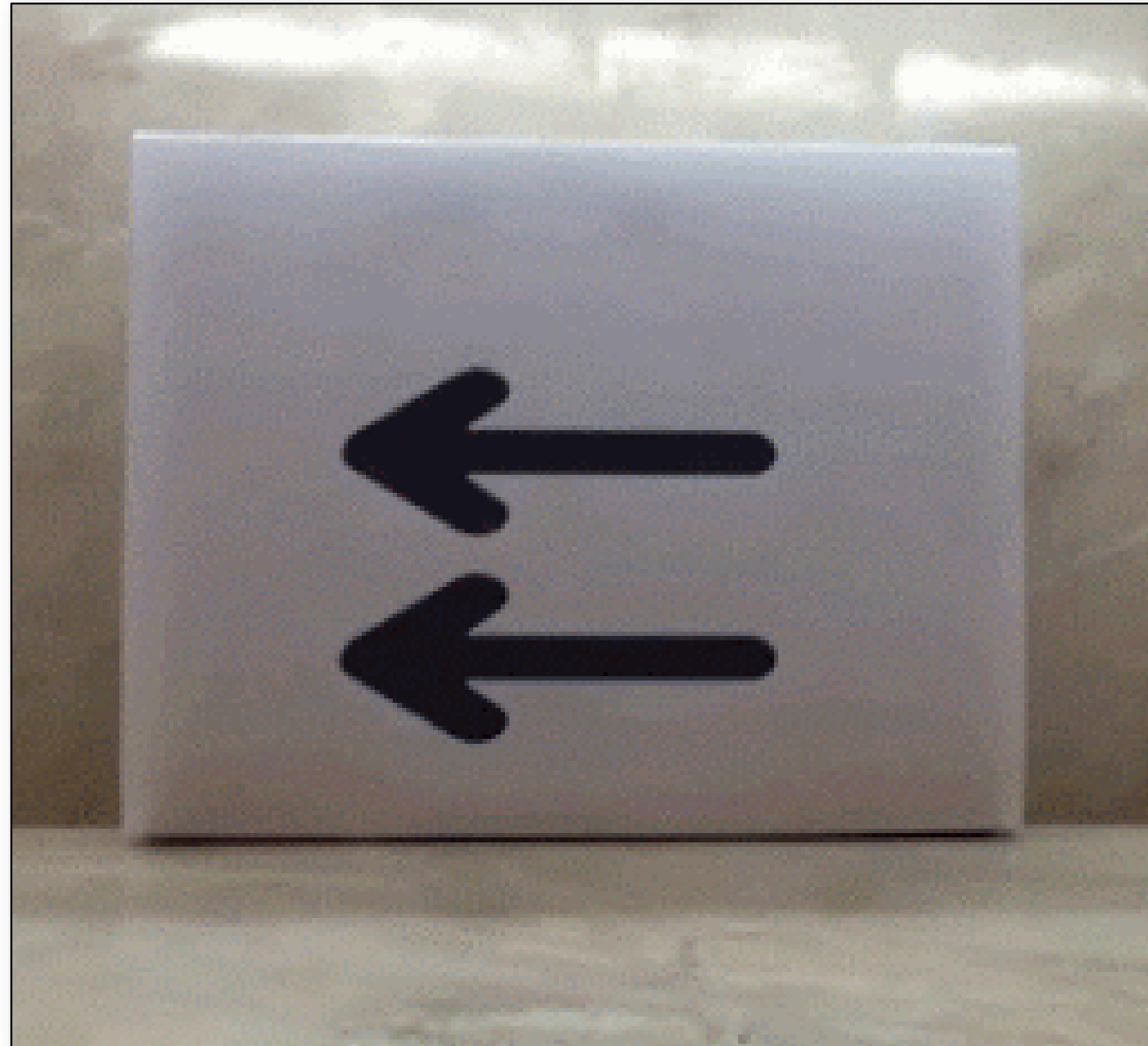
# Refraction

---



# Refraction

---





# Index of Refraction

---

Speed of light in vacuum / speed of light in medium

Some values of $\eta$	
Vacuum	1
Air at STP	1.00029
Ice	1.31
Water	1.33
Crown glass	1.52 - 1.65
Diamond	2.417

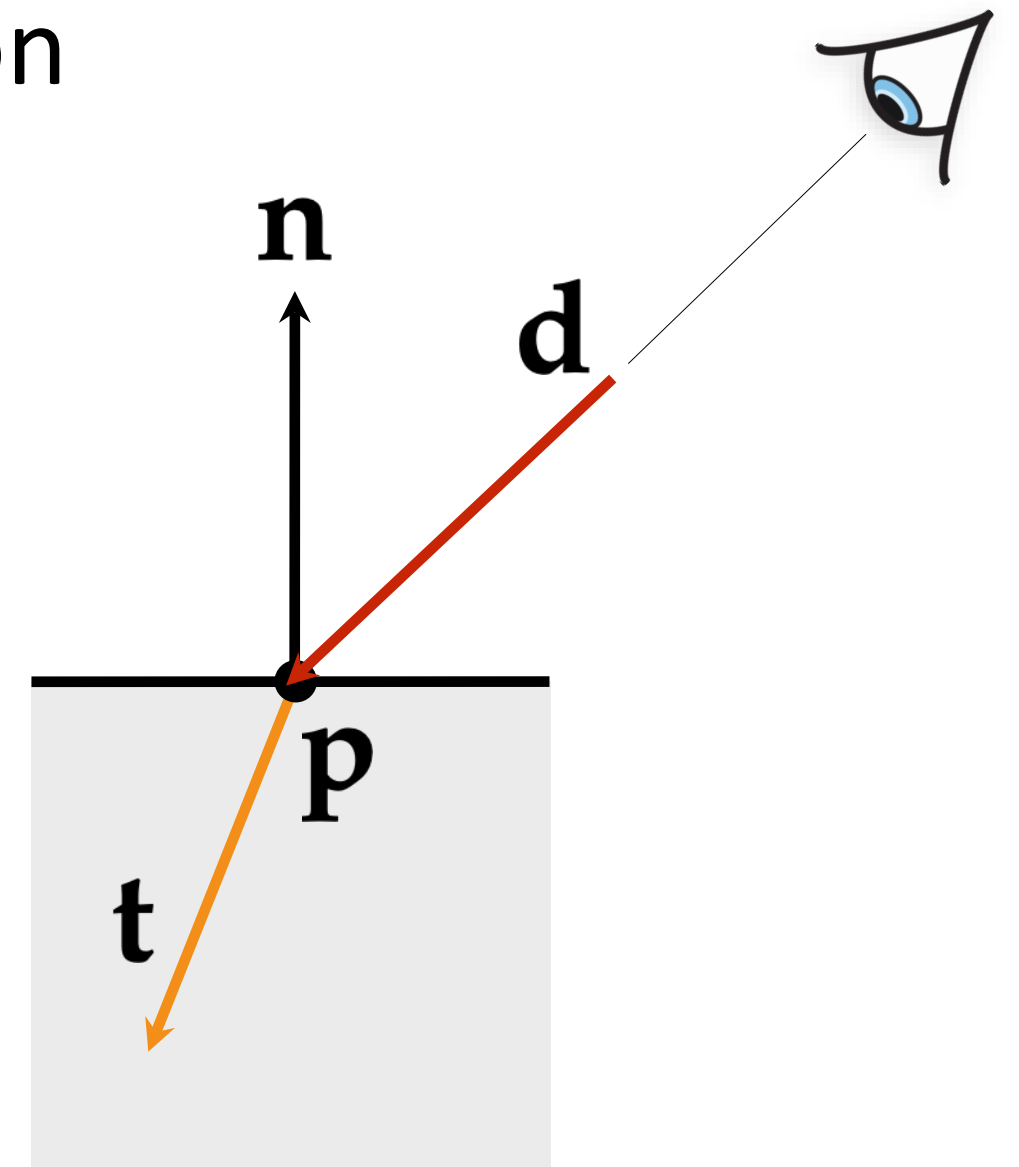
# Specular transmission/refraction

Materials like water, glass, etc., also refract/bend light

Trace a recursive ray in the refraction direction

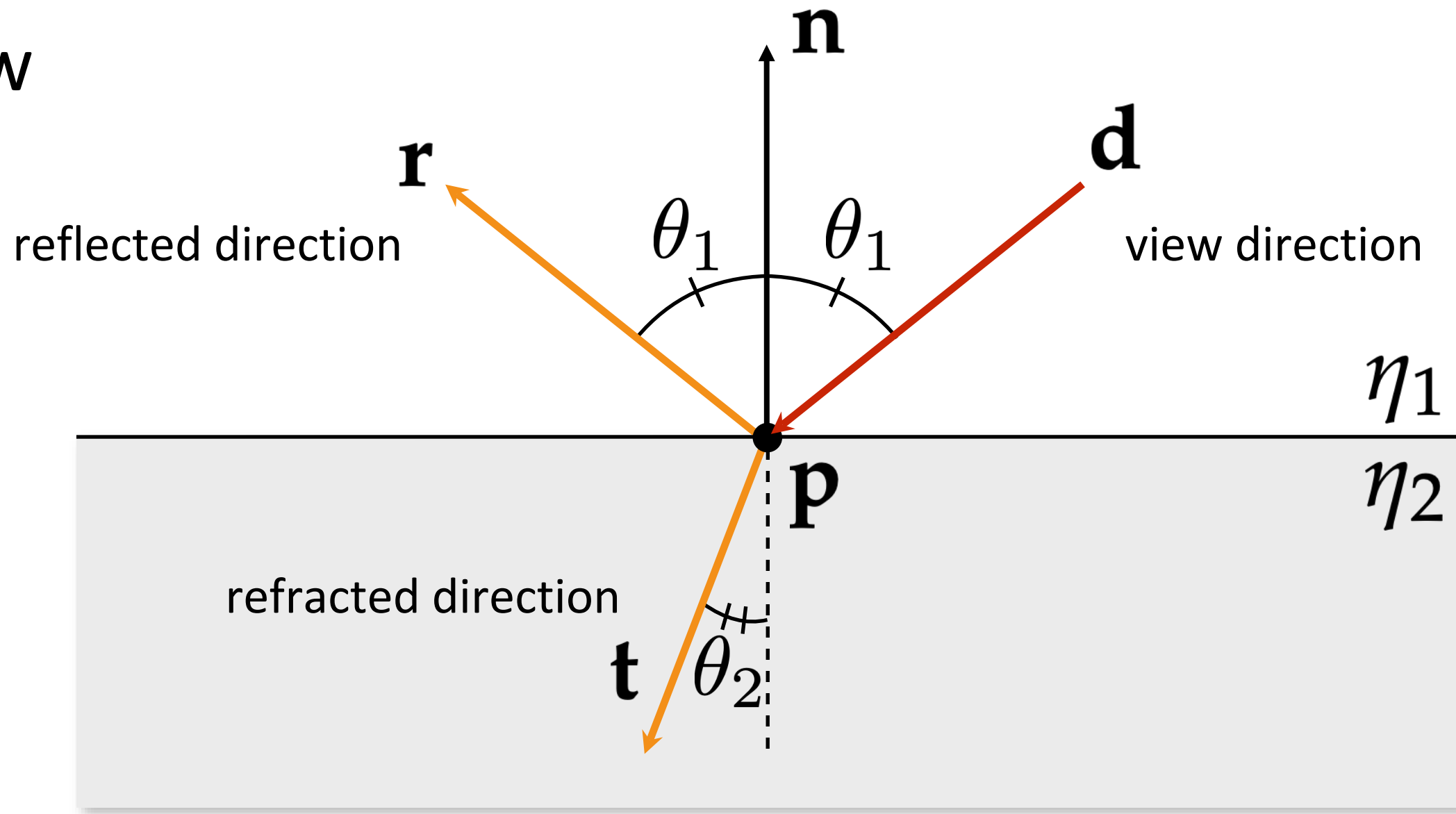
$$L_t = k_t \text{trace}(\mathbf{p}, \mathbf{t})$$

refracted light      refraction scaled down      recursive call



# Specular transmission/refraction

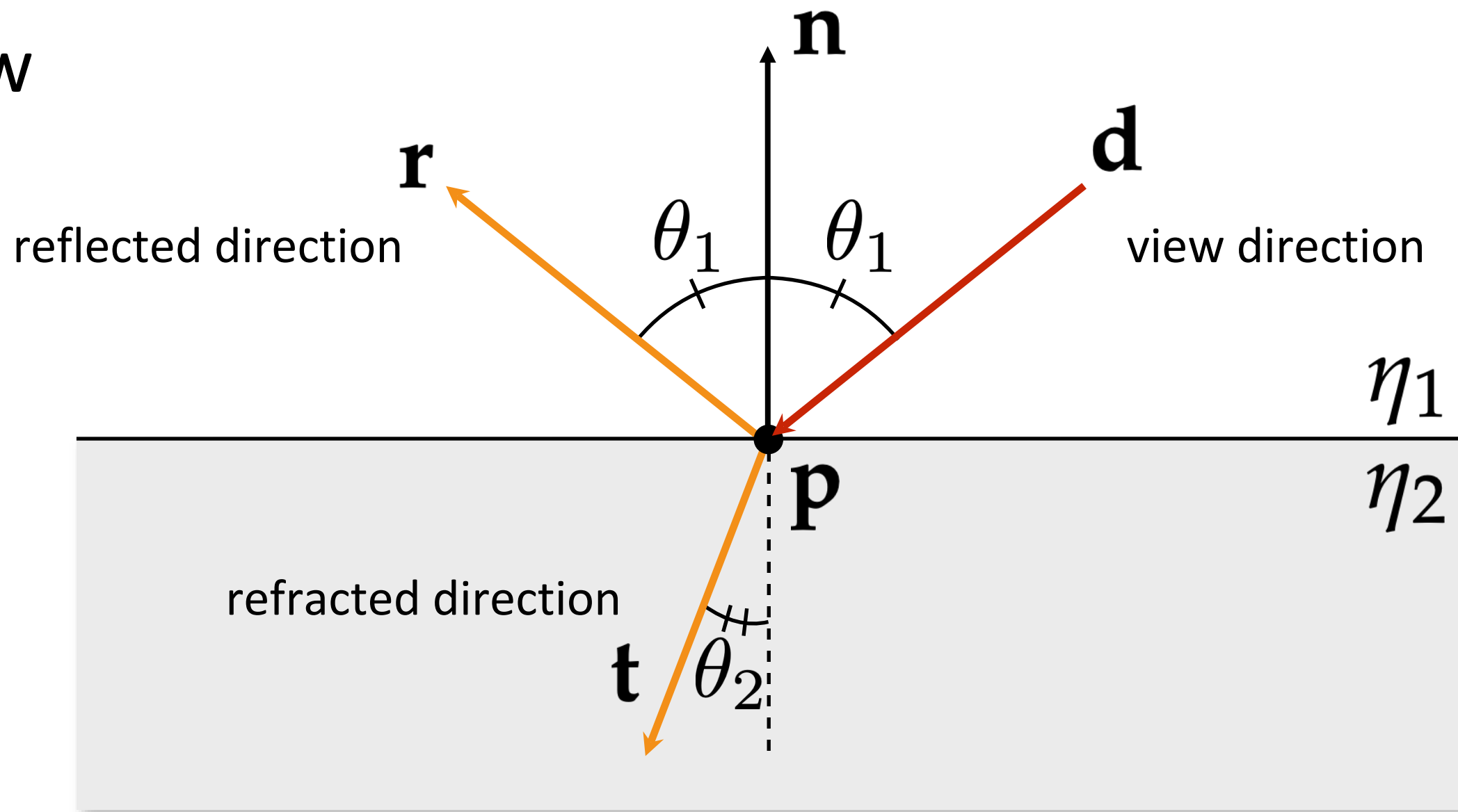
Snell's law



$$\eta_1 \sin \theta_1 = \eta_2 \sin \theta_2$$

# Specular transmission/refraction

Snell's law



$$\mathbf{t} = \eta_1/\eta_2 (\mathbf{d} - (\mathbf{d} \cdot \mathbf{n}) \mathbf{n}) - \mathbf{n} \sqrt{1 - \eta_1^2/\eta_2^2 (1 - (\mathbf{d} \cdot \mathbf{n})^2)}$$

# Index of Refraction

---

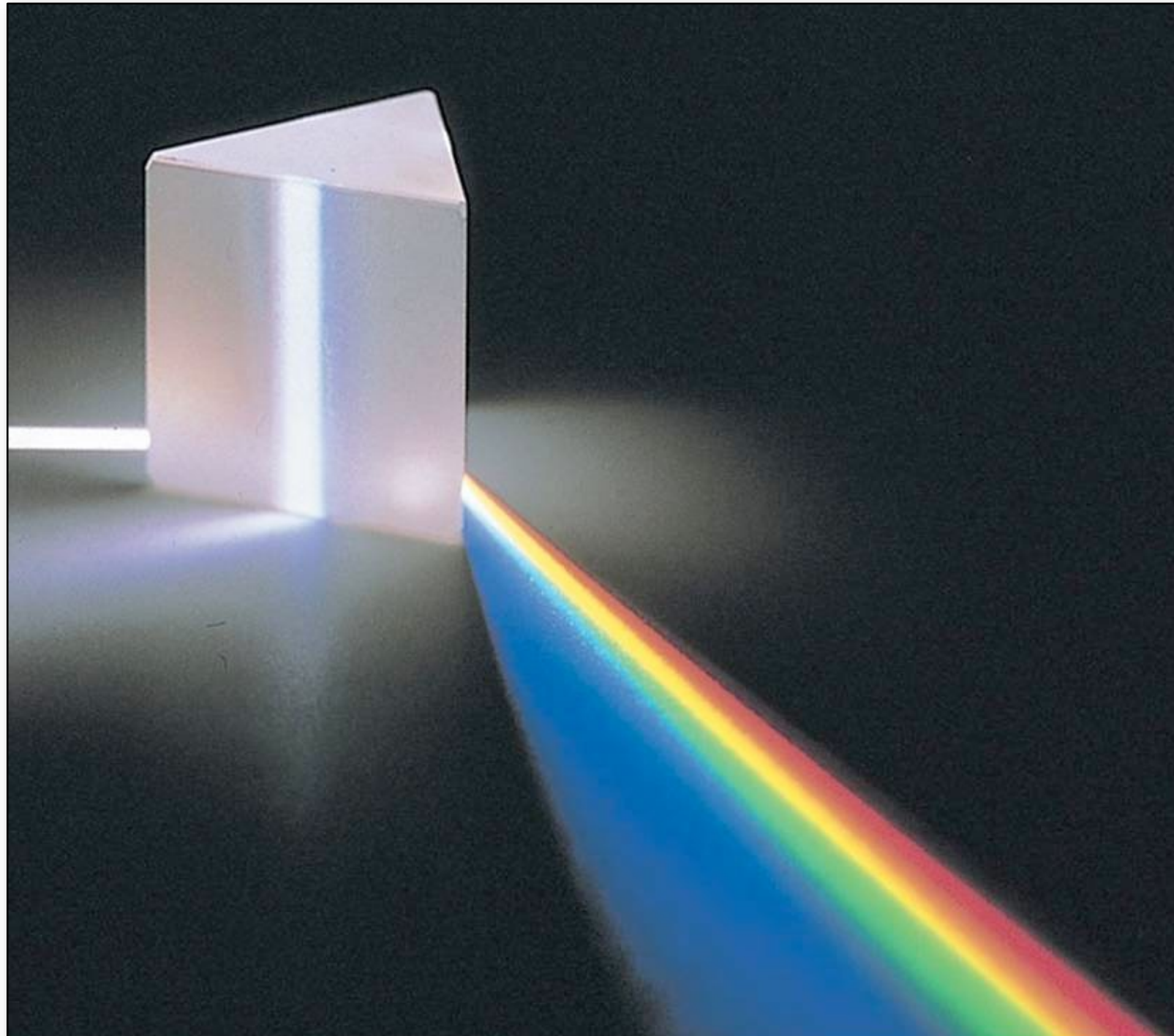
Speed of light in vacuum / speed of light in medium

Some values of $\eta$	
Vacuum	1
Air at STP	1.00029
Ice	1.31
Water	1.33
Crown glass	1.52 - 1.65
Diamond	2.417

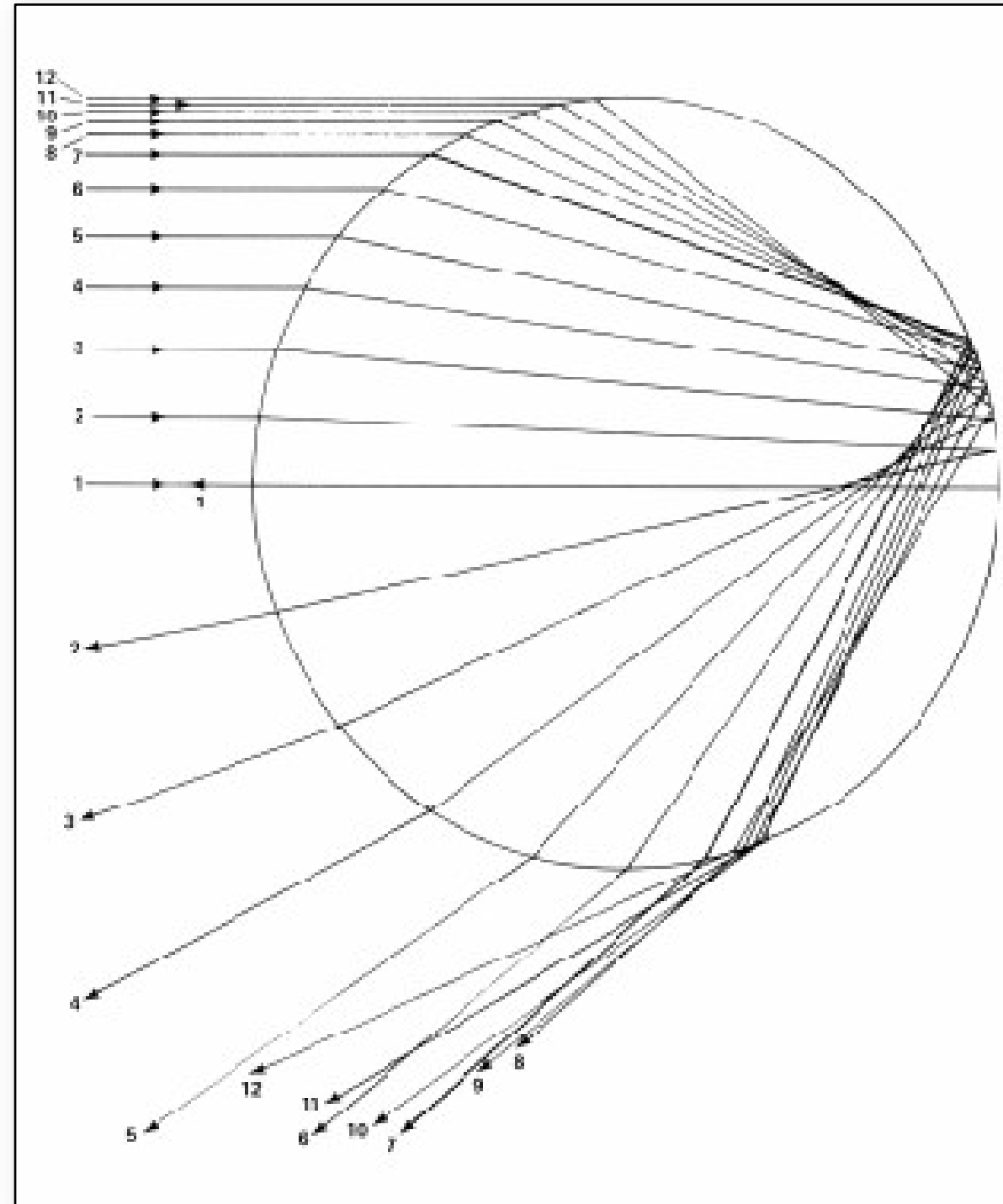
These are actually wavelength dependent!

# Dispersion

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# Refraction in a Waterdrop



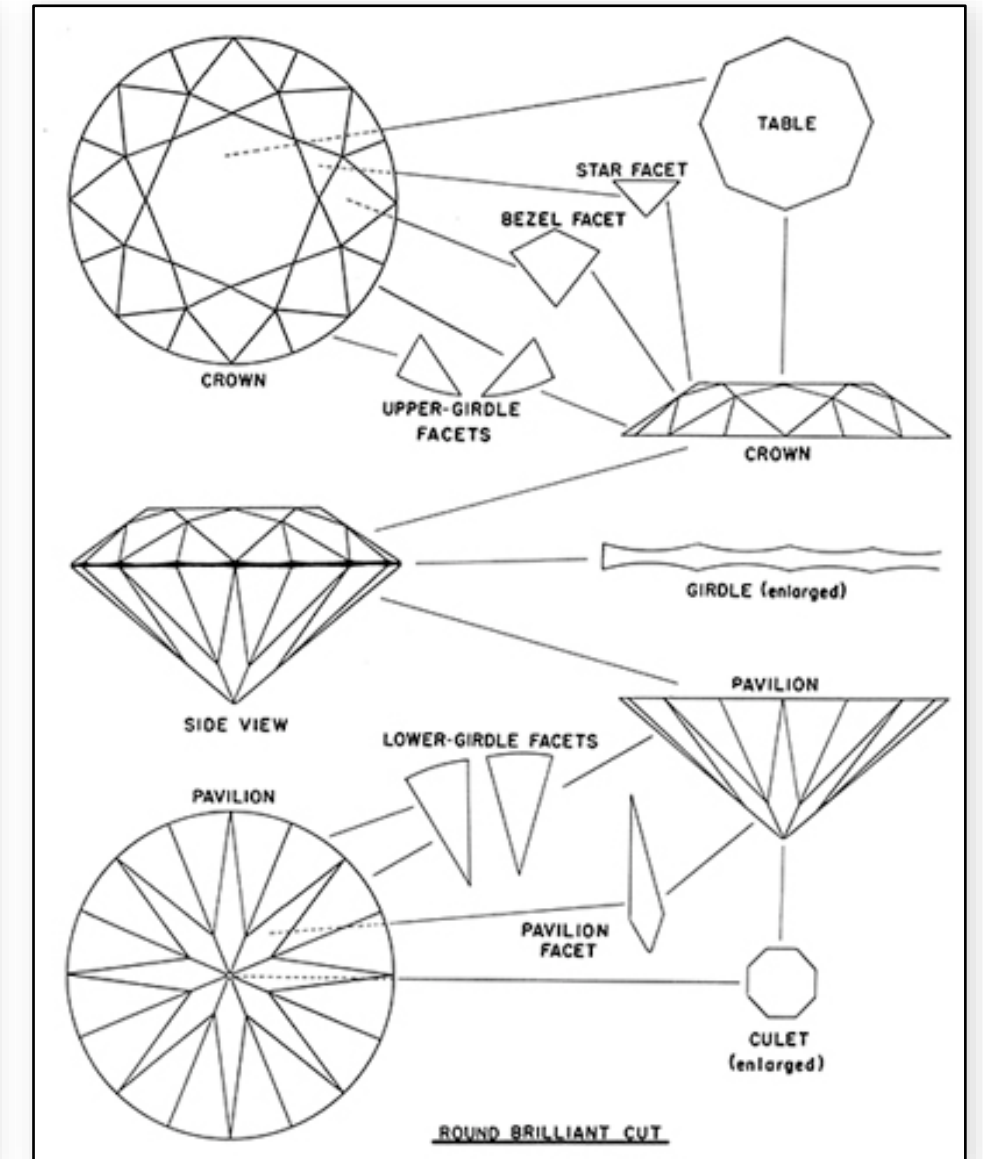
# Double rainbow all the way across the sky!

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# Dispersion



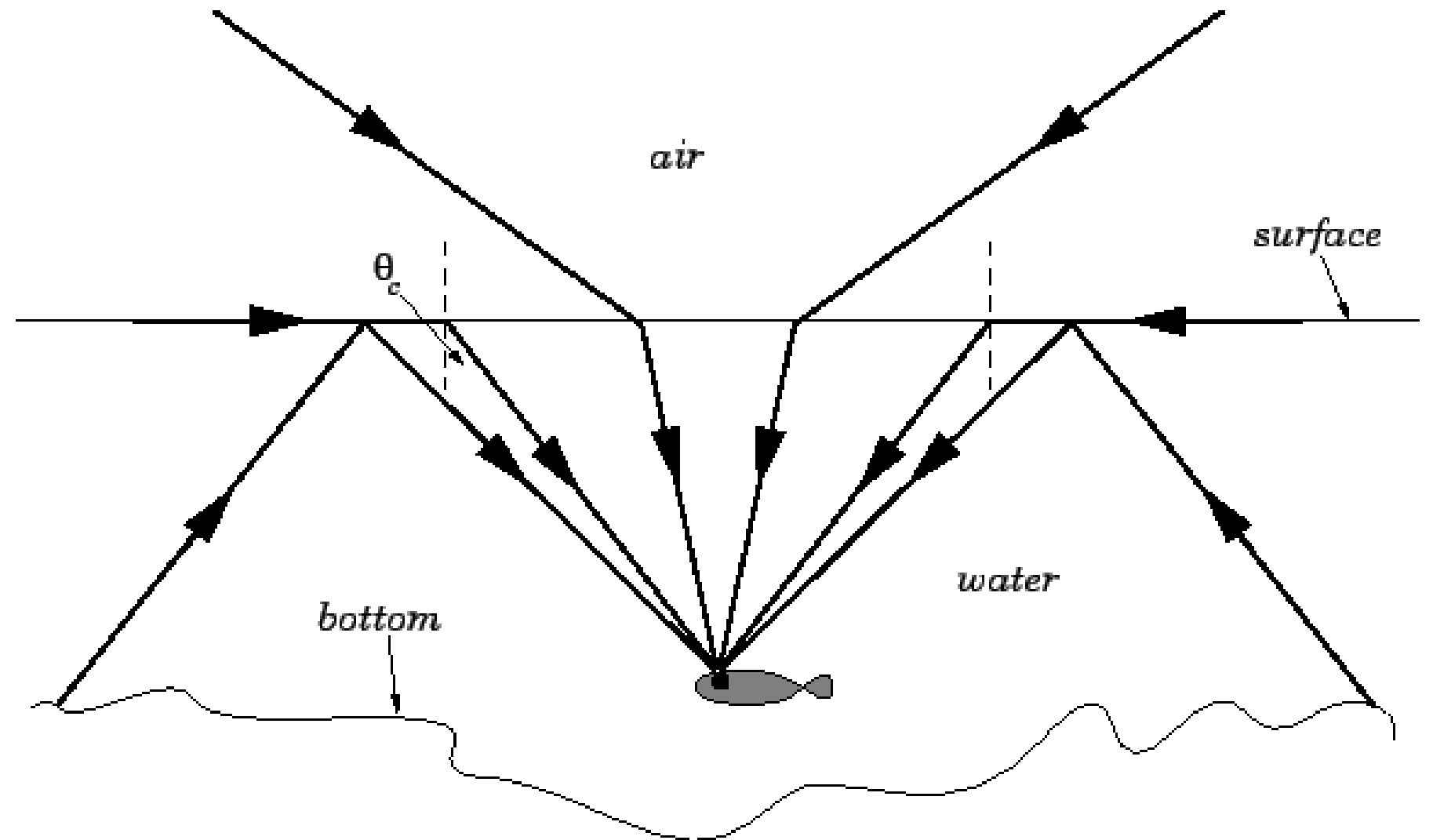
# What is this dark circle?



# What is this dark circle?

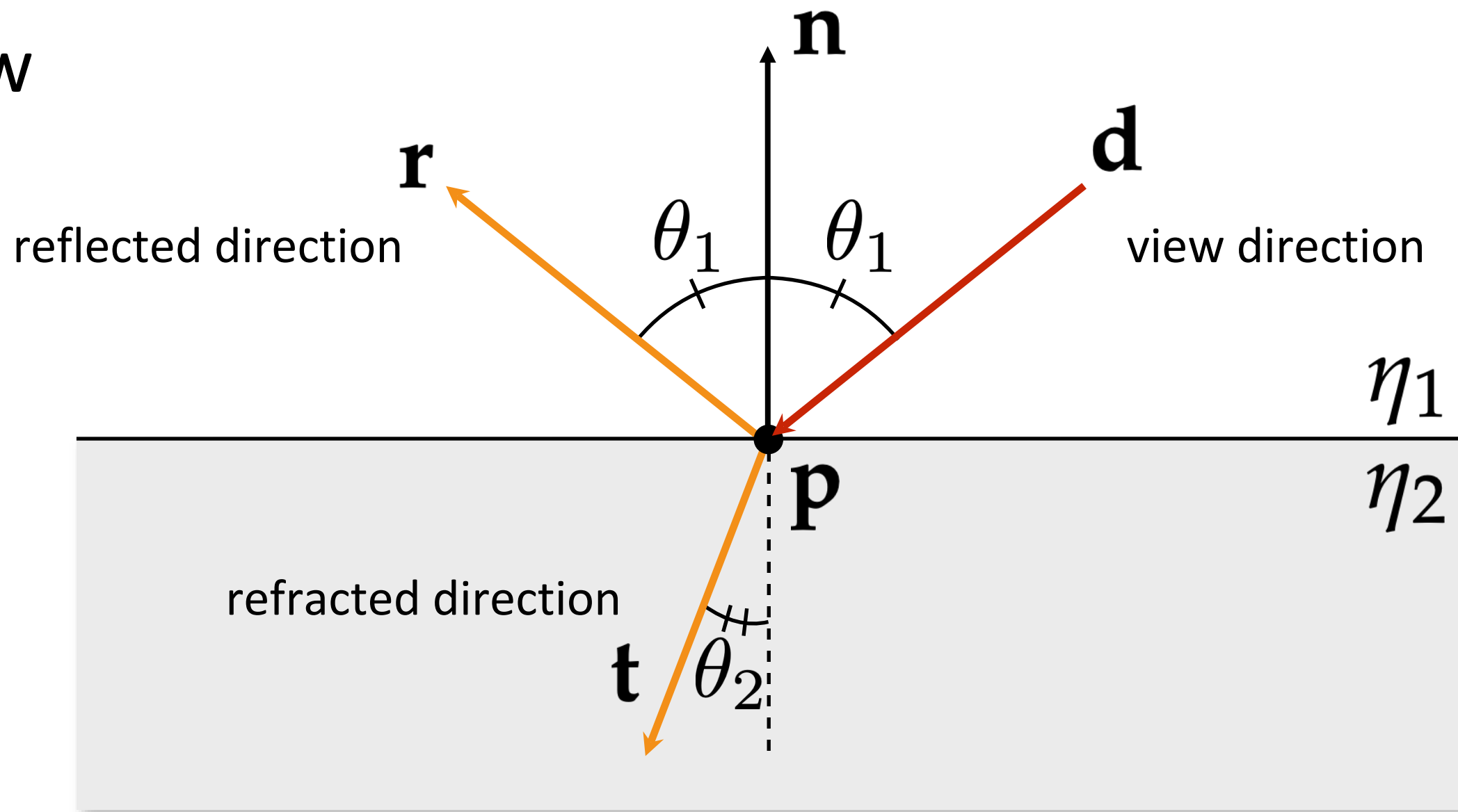


Called  
"Snell's window"  
Caused by total  
internal reflection



# Recall...

## Snell's law



$$\mathbf{t} = \eta_1/\eta_2 (\mathbf{d} - (\mathbf{d} \cdot \mathbf{n}) \mathbf{n}) - \mathbf{n} \sqrt{1 - \eta_1^2/\eta_2^2 (1 - (\mathbf{d} \cdot \mathbf{n})^2)}$$

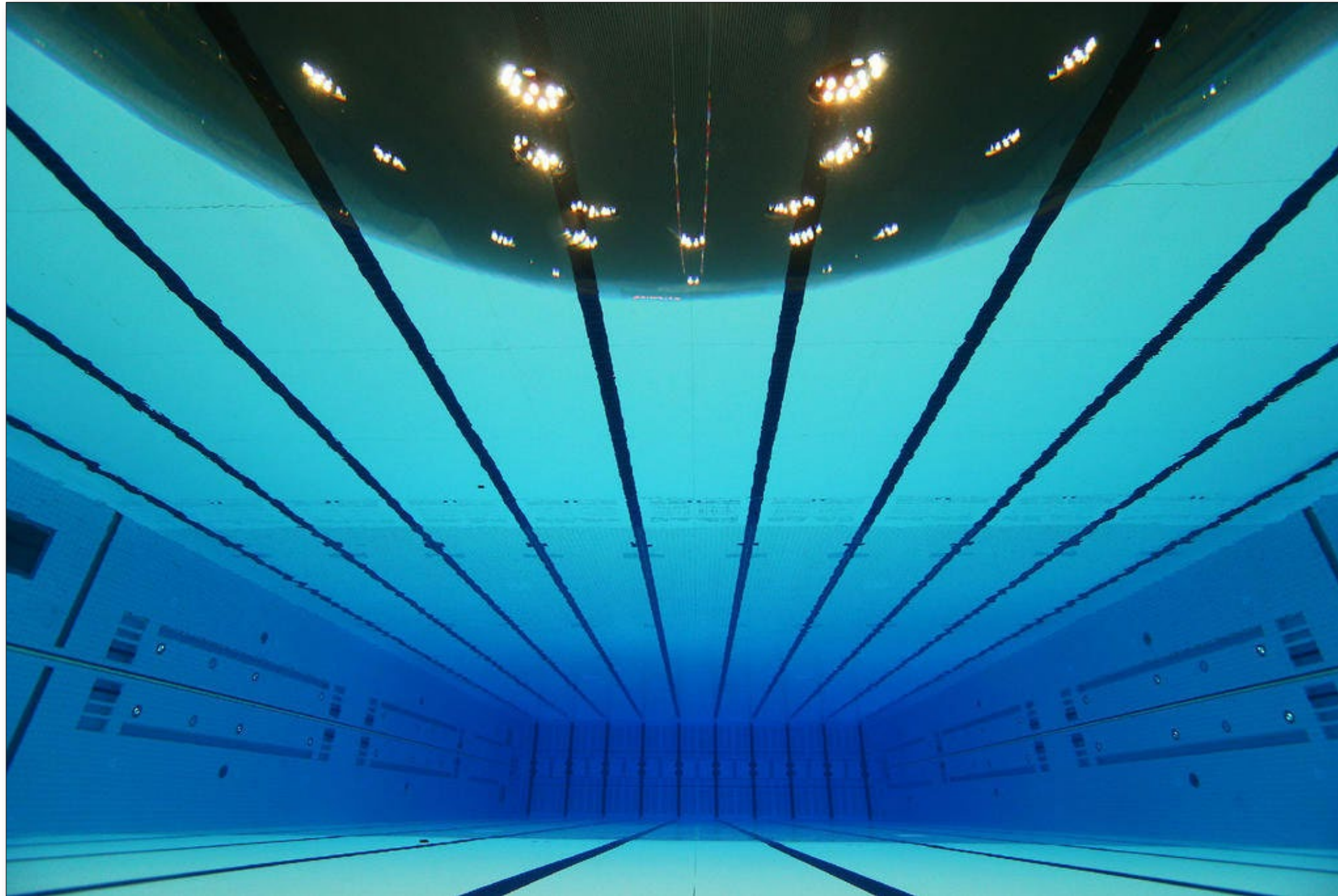
# When can total internal reflection happen?

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Can only happen when the ray starts in the higher index medium

# Total Internal Reflection

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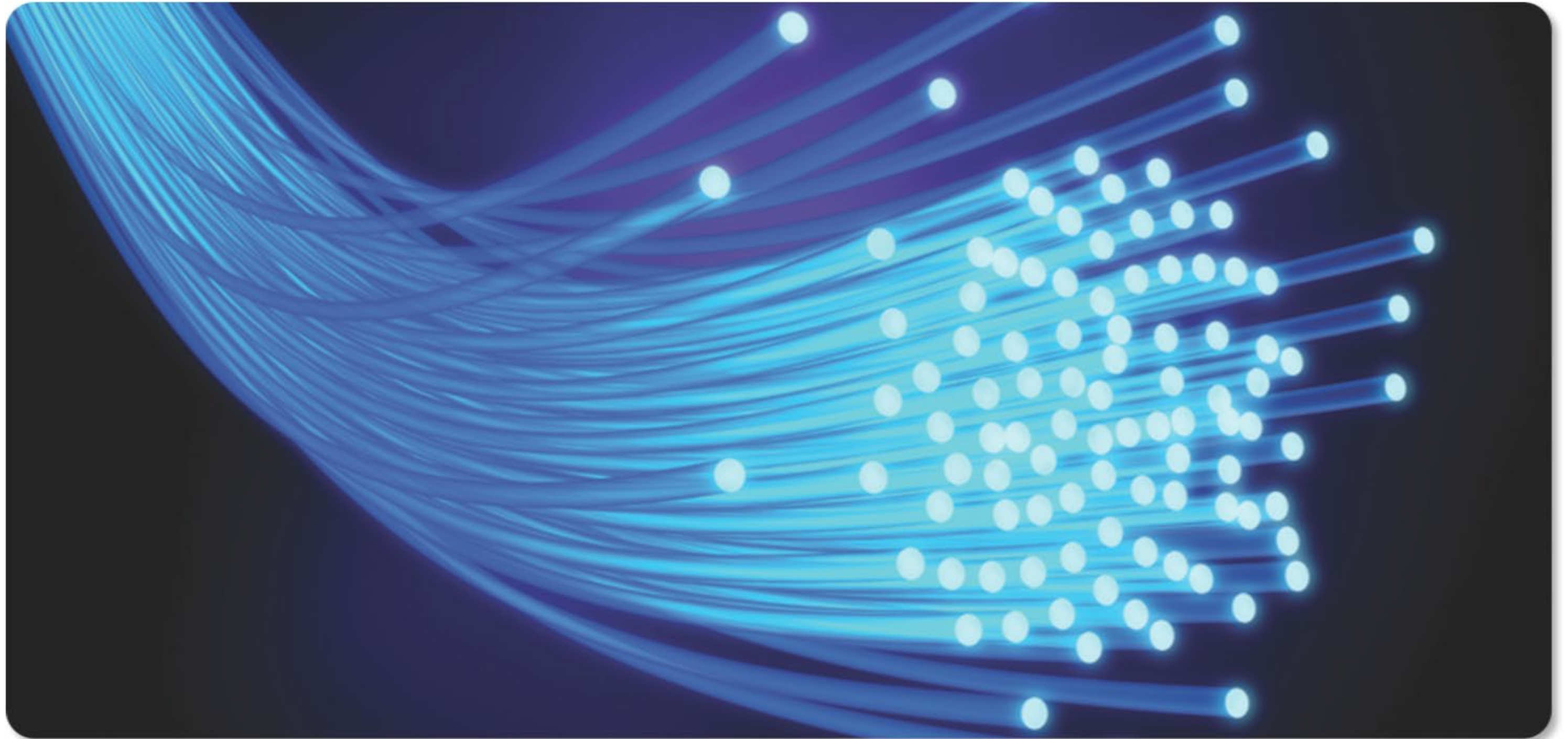
# Total Internal Reflection

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# Total Internal Reflection

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# Total Internal Reflection

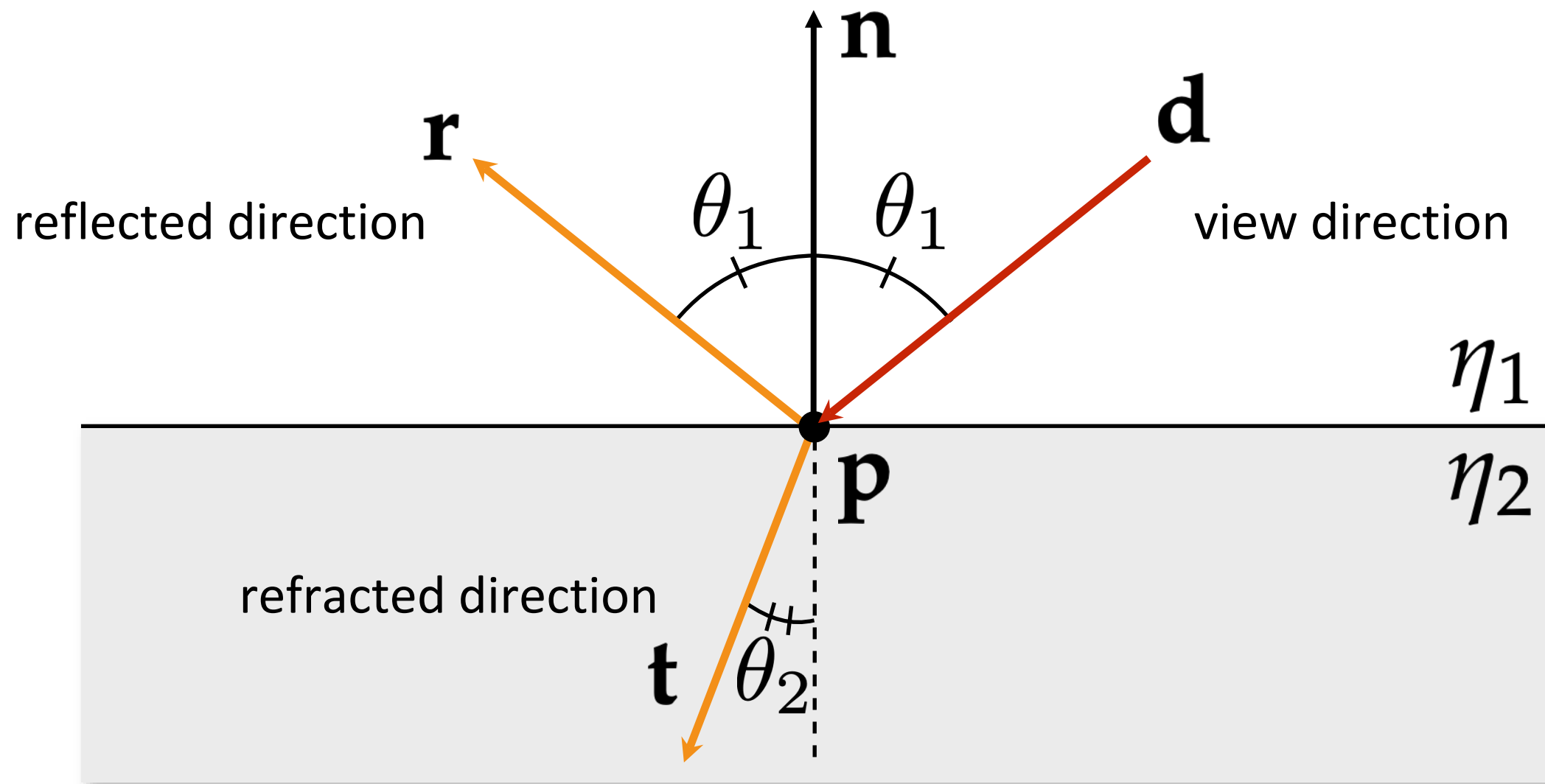
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# Reflection vs. Refraction

How much light is reflected vs. refracted?

- in reality determined by “Fresnel equations”



# Fresnel Equations

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*Reflection and refraction* from smooth *dielectric* (e.g. glass) surfaces

*Reflection* from *conducting* (e.g. metal) surfaces

Derived from Maxwell equations

Involves polarization of the wave

# Fresnel Equations for Dielectrics

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Reflection of light polarized parallel and perpendicular to the plane of refraction

$$\rho_{\parallel} = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2}$$

$$\rho_{\perp} = \frac{\eta_1 \cos \theta_1 - \eta_2 \cos \theta_2}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2}$$

reflected:  $F_r = \frac{1}{2} (\rho_{\parallel}^2 + \rho_{\perp}^2)$

refracted:  $F_t = 1 - F_r$

# What's happening in this photo?

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# Polarizing Filter

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# Polarization



Without Polarizer



With Polarizing Filter

# Polarization



Without Polarizer

With Polarizing Filter



# Effect of Polarization



# Effect of Polarization



# Fresnel Equations for Dielectrics

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Reflection of light polarized parallel and perpendicular to the plane of refraction

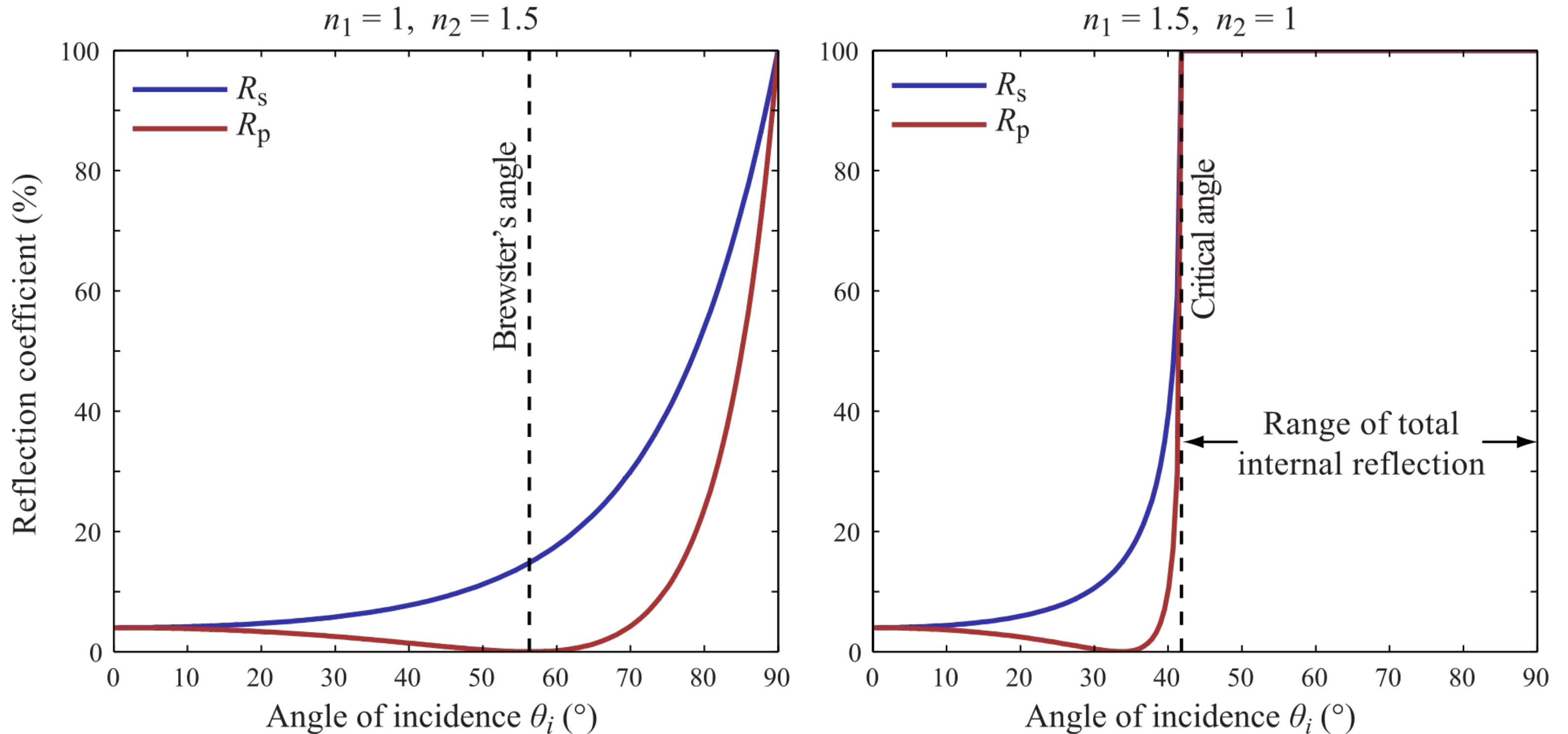
$$\rho_{\parallel} = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2}$$
$$\rho_{\perp} = \frac{\eta_1 \cos \theta_1 - \eta_2 \cos \theta_2}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2}$$

reflected:  $F_r = \frac{1}{2} (\rho_{\parallel}^2 + \rho_{\perp}^2)$

refracted:  $F_t = 1 - F_r$

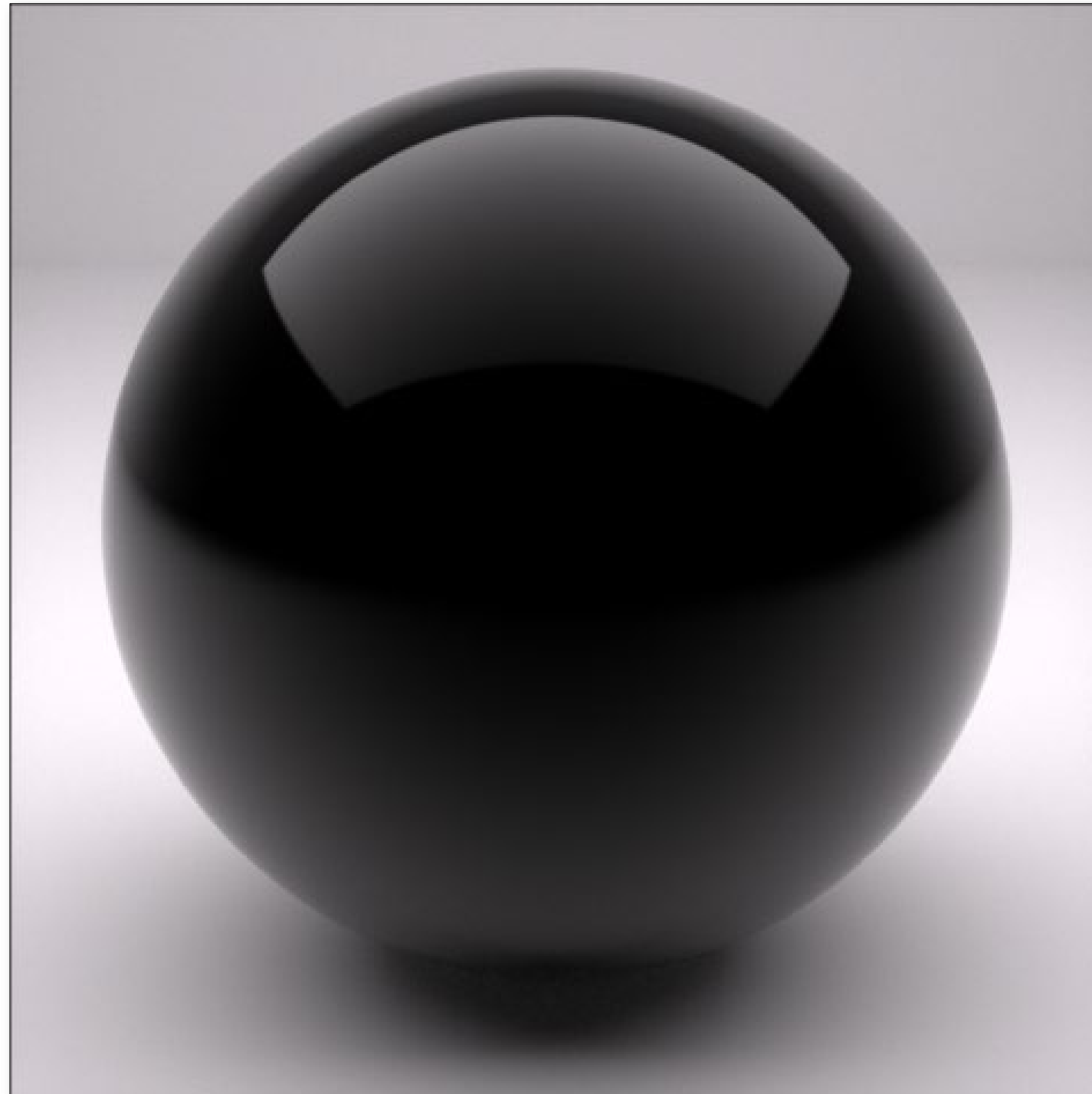
- The Shirley book uses a faster approximation (Schlick), but to get full accuracy you'd need to use these equations

# Fresnel equations for glass



# Fresnel reflection

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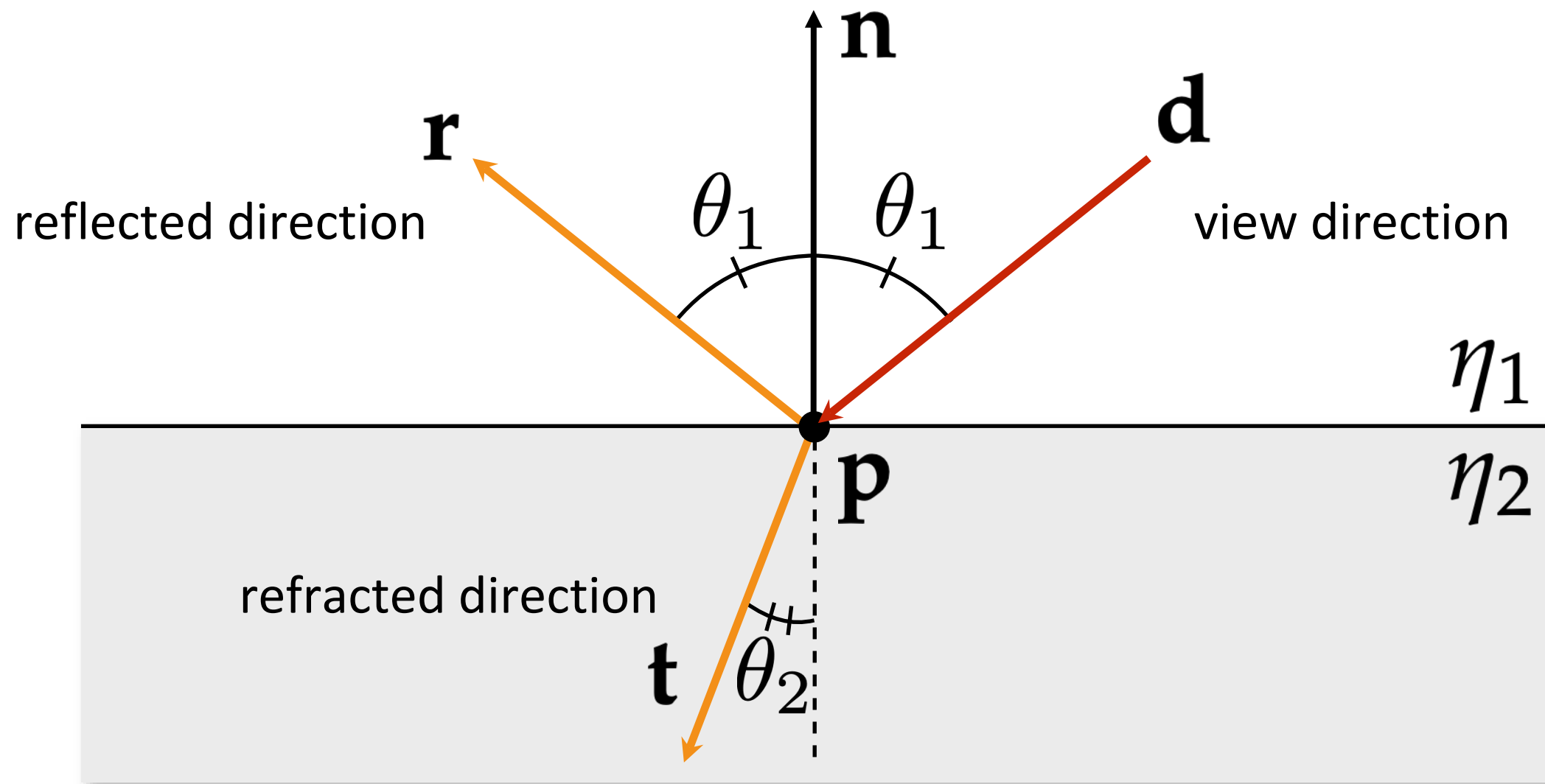
# Fresnel reflection/refraction

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# Reflection vs. Refraction

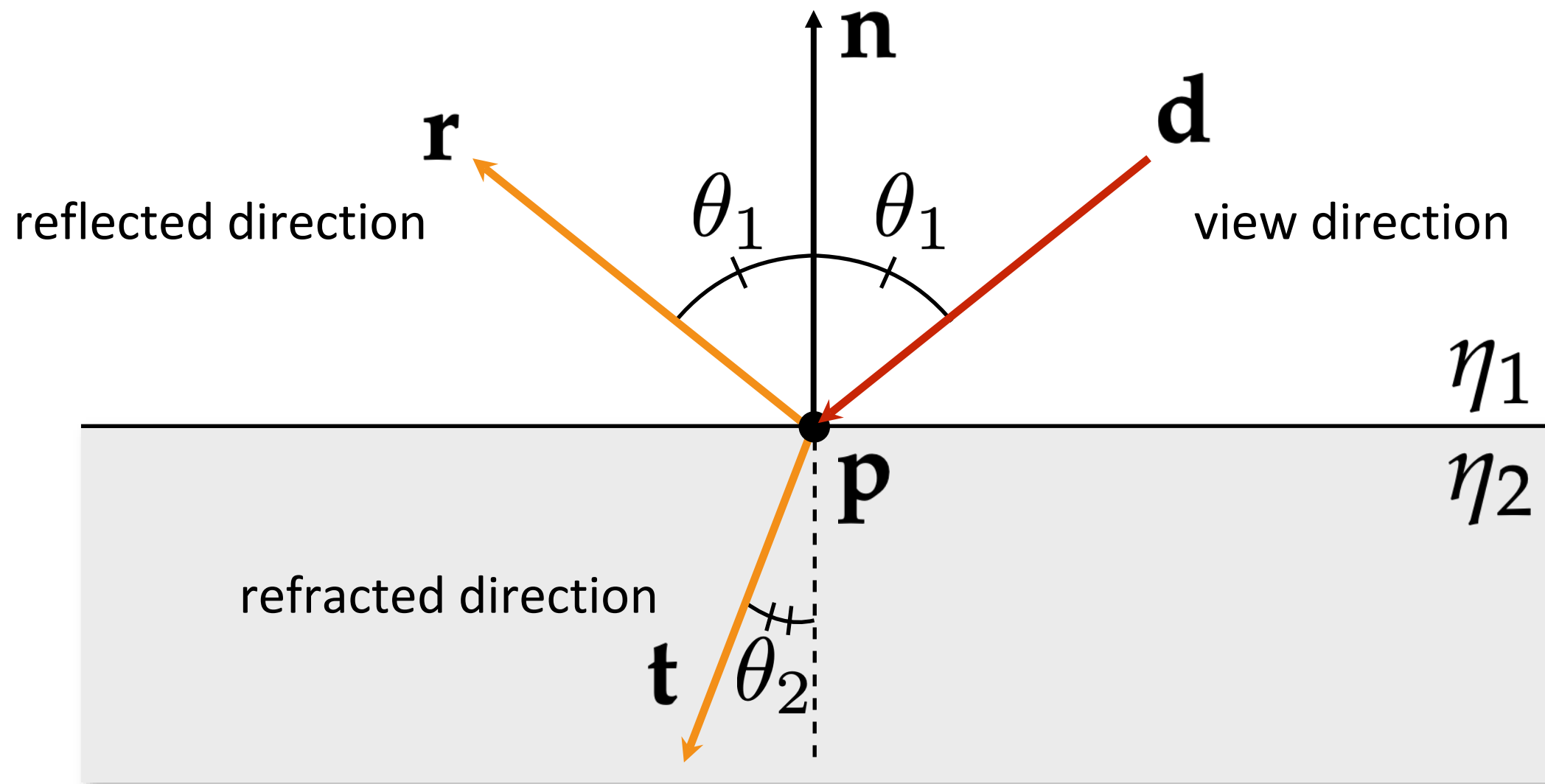
During ray tracing, how do we decide whether to reflect or refract?



# Reflection vs. Refraction

During ray tracing, how do we decide whether to reflect or refract?

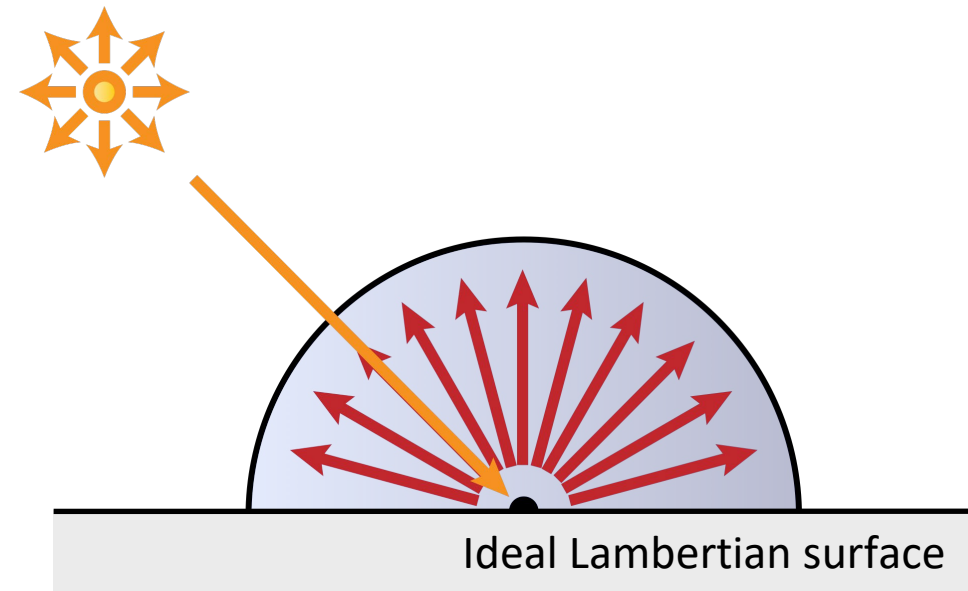
- Randomly! Using Fresnel coefficients as probabilities.



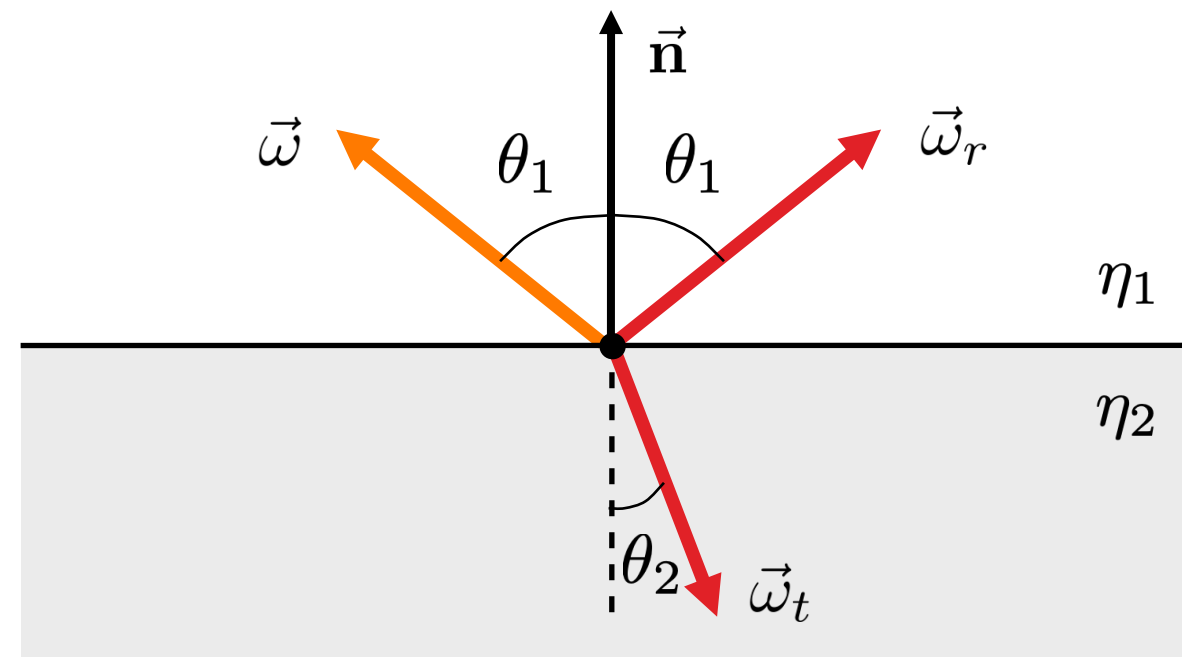


# So Far: Idealized BRDF Models

Diffuse



Specular Reflection and Refraction



Real materials are more complex



# Rough materials

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In reality, most materials are neither perfectly diffuse nor specular, but somewhere in between

- Imagine a shiny surface scratched up at a microscopic level

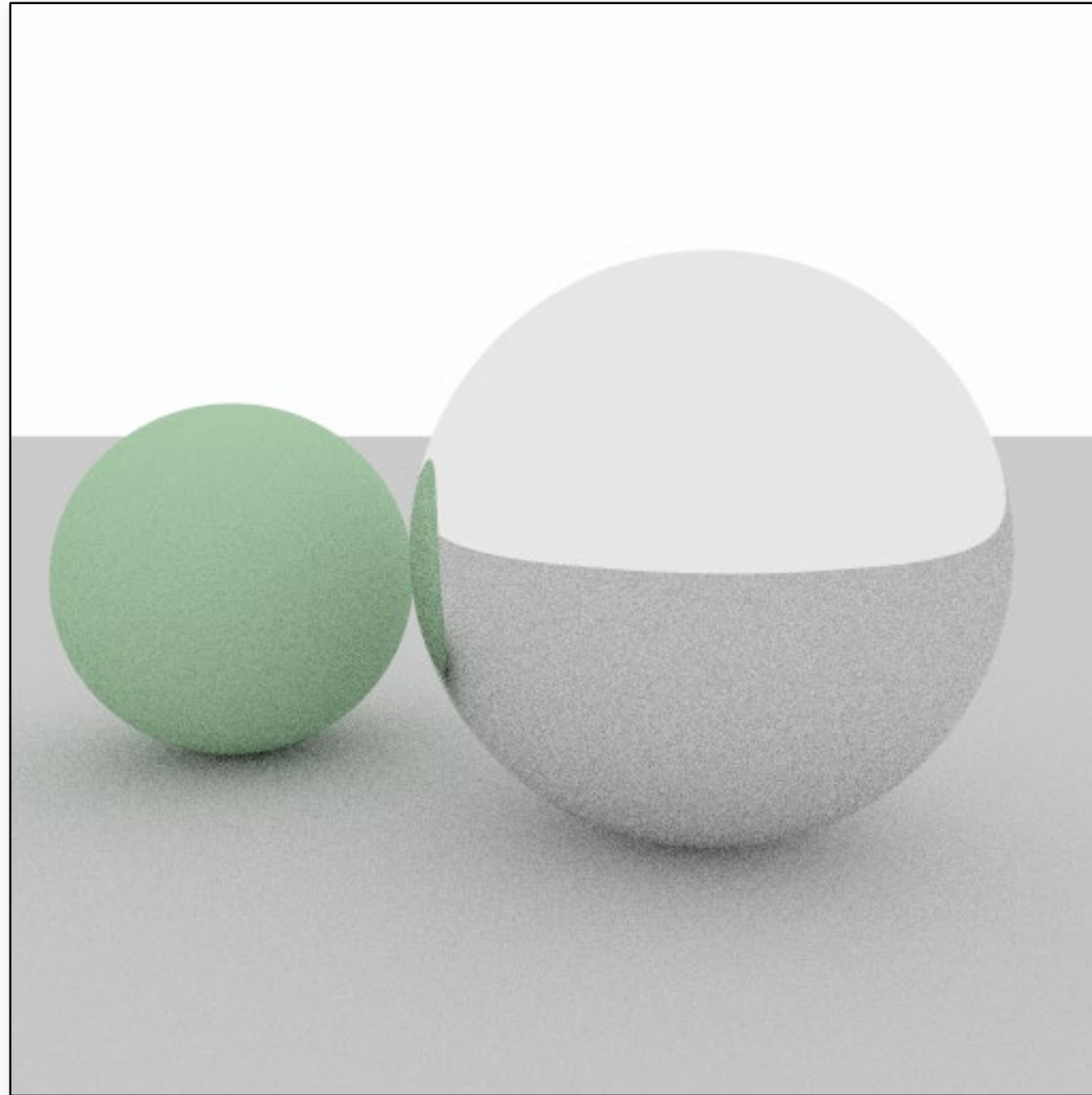
We will look at a more principled way to handle this later.

For now, we can easily approximate one important characteristic:  
blurred reflections

- Compute reflection direction, then add a random offset to it
- Sample random offset from sphere. Scale it to increase/decrease fuzziness

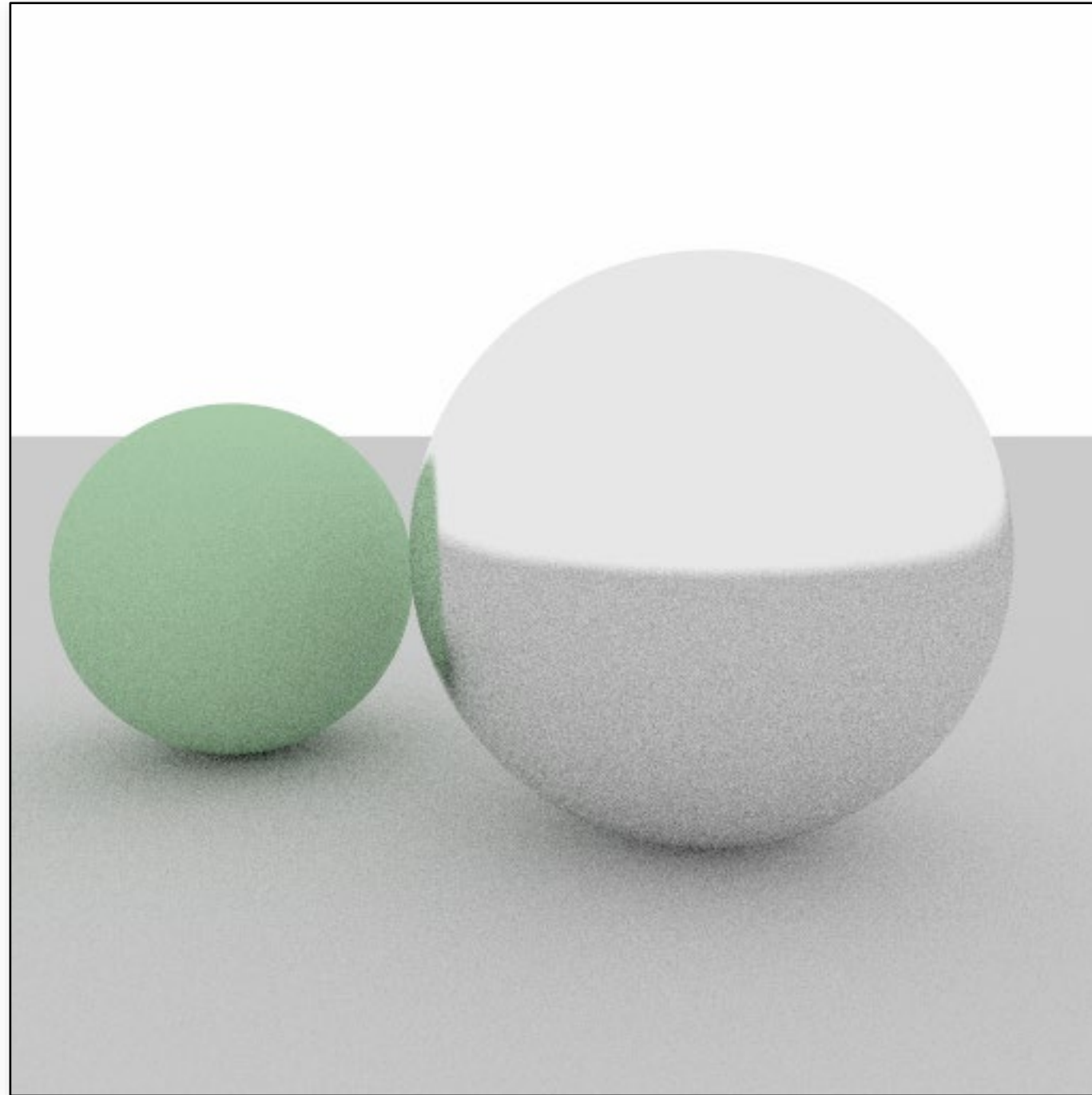
# Diffuse & mirror spheres

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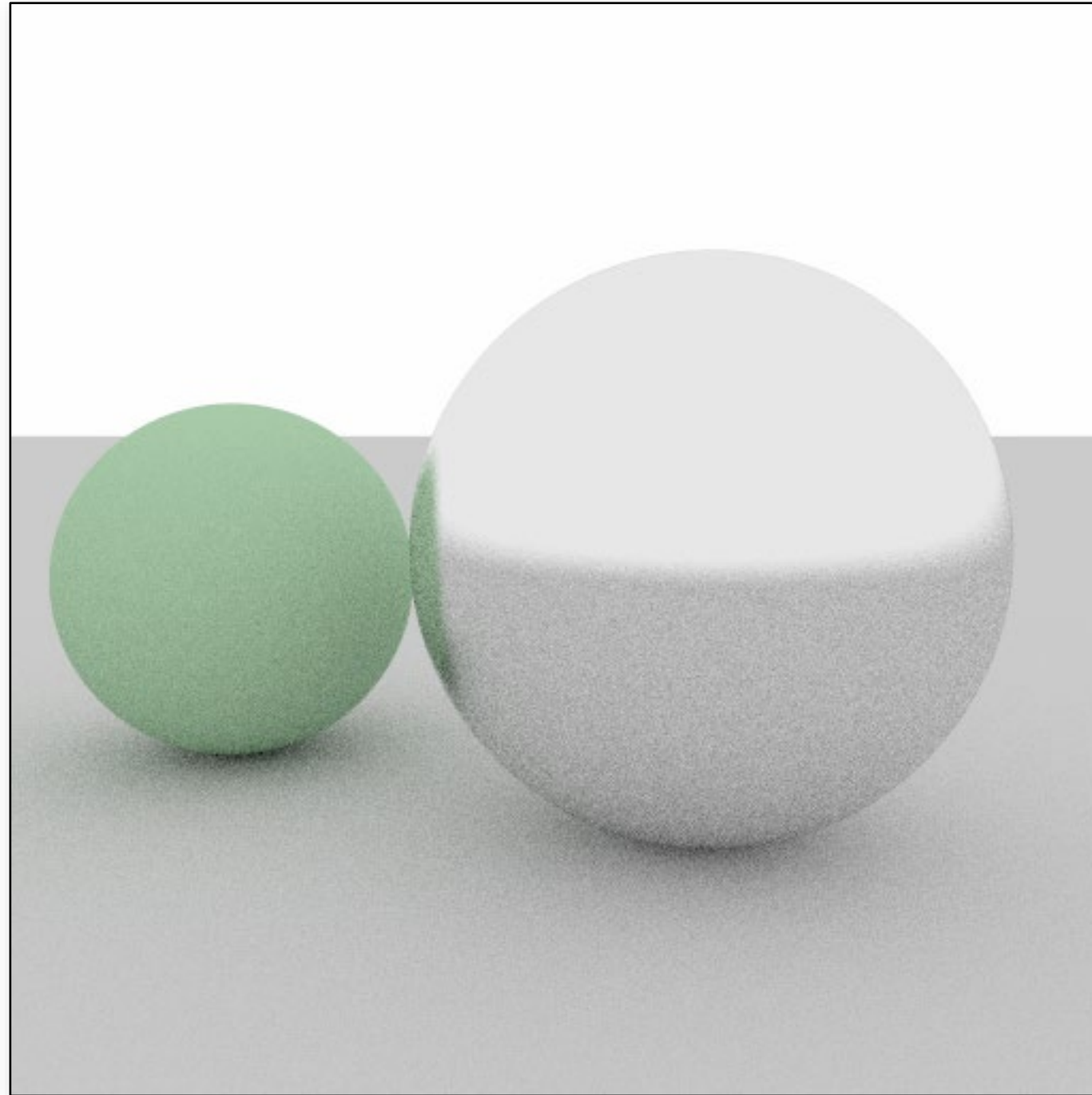
# Diffuse & rough mirror spheres

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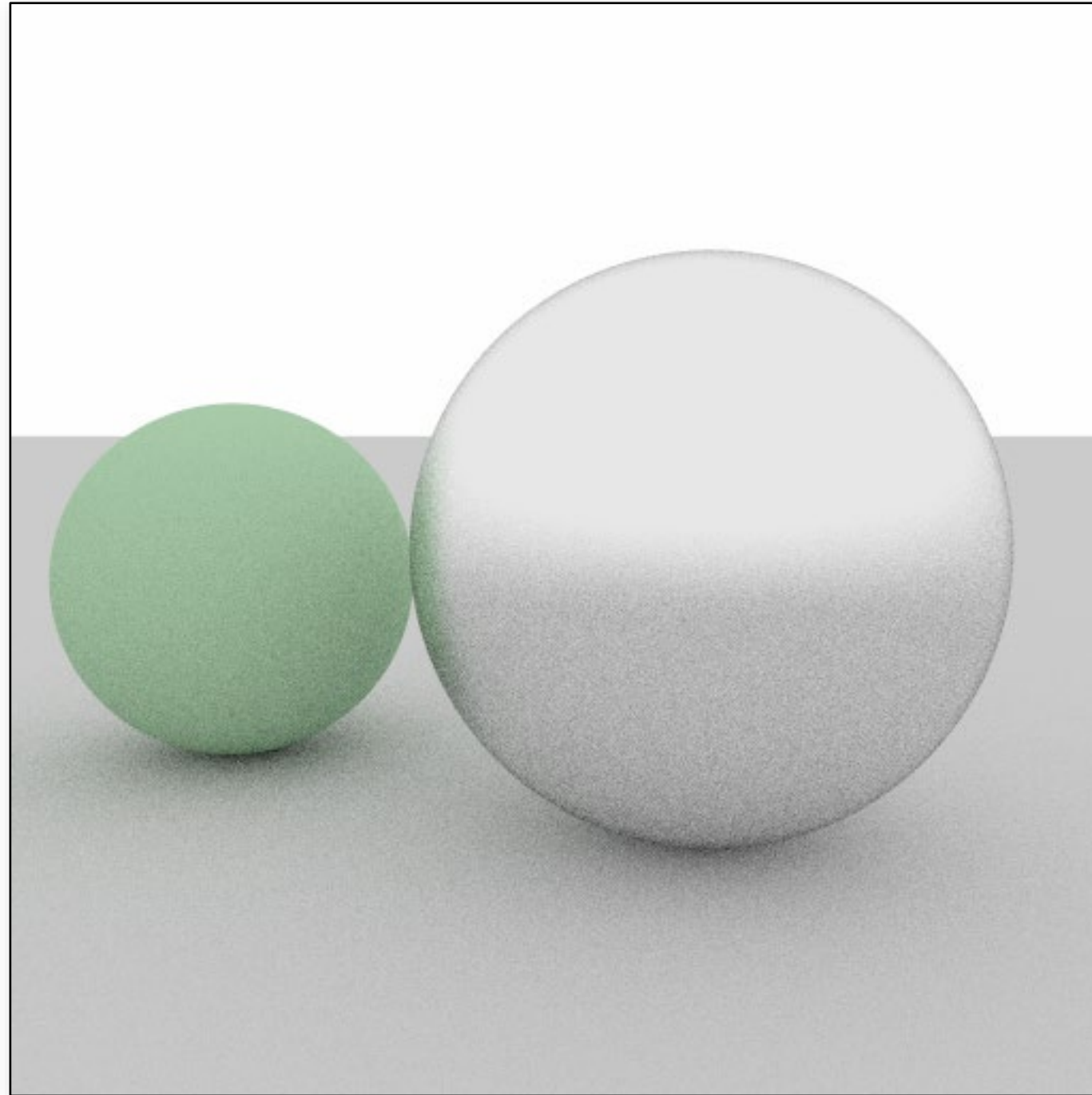
# Diffuse & rough mirror spheres

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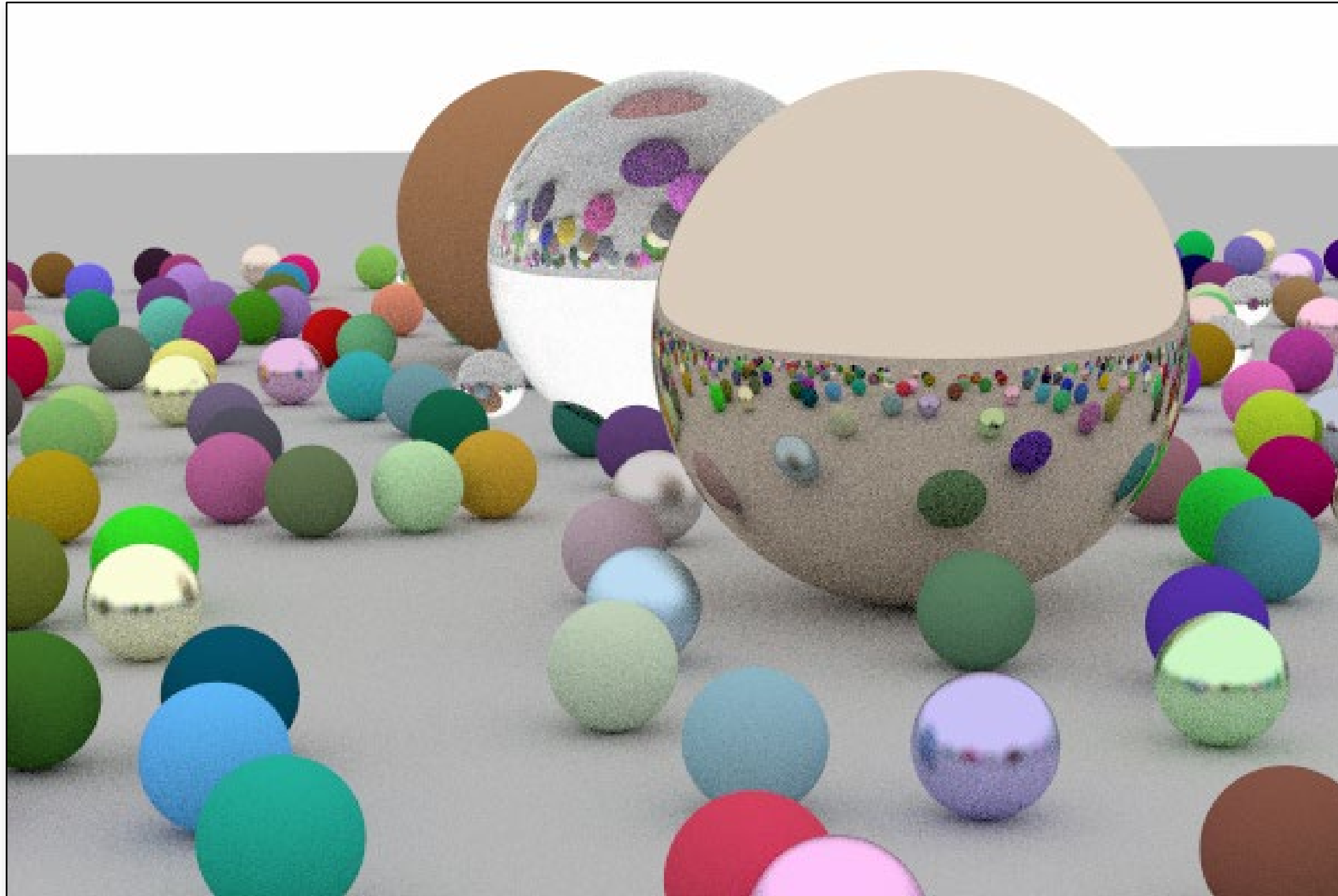


# Diffuse & rough mirror spheres

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# Putting it together





Lighting

# Lighting

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So far, the sky or background has been the only source of emitted light

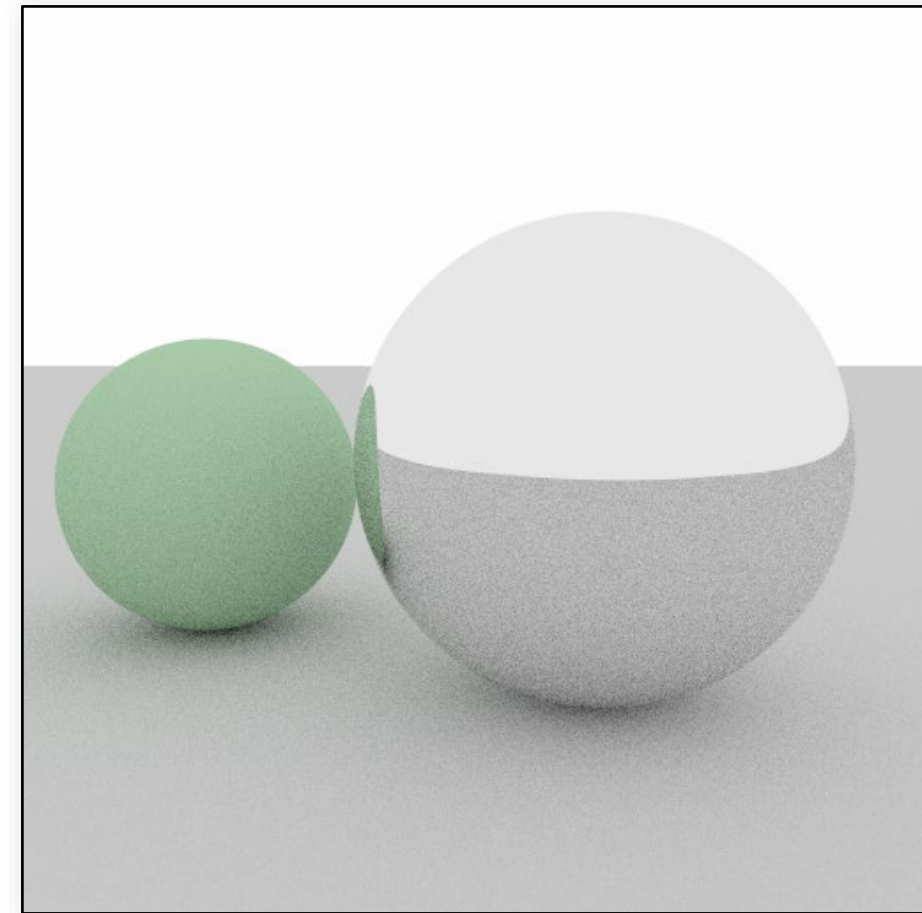
But we can easily make any surface a light source!

- Just return an emitted color when a ray hits that surface
- Add a function to material that returns emitted color
  - Returns black (all zeros) for regular (non-emissive) surfaces
  - Color will often be greater than (1,1,1)
- Also possible for surfaces to emit & scatter (but not common)

# Pseudo-code

---

```
Scene::trace(Ray ray)
  hit = surfaces.intersect(ray);
  if hit
    [col, sRay] = hit->mat->scatter(ray)
    return col * trace(sRay);
  else
    return backgroundColor;
```



# Pseudo-code

---

```
Scene::trace(Ray ray)
  hit = surfaces.intersect(ray);
  if hit
    emit = hit->mat->emit(ray)
    [col, sRay] = hit->mat->scatter(ray)
    return emit + col * trace(sRay);
  else
    return backgroundColor;
```

