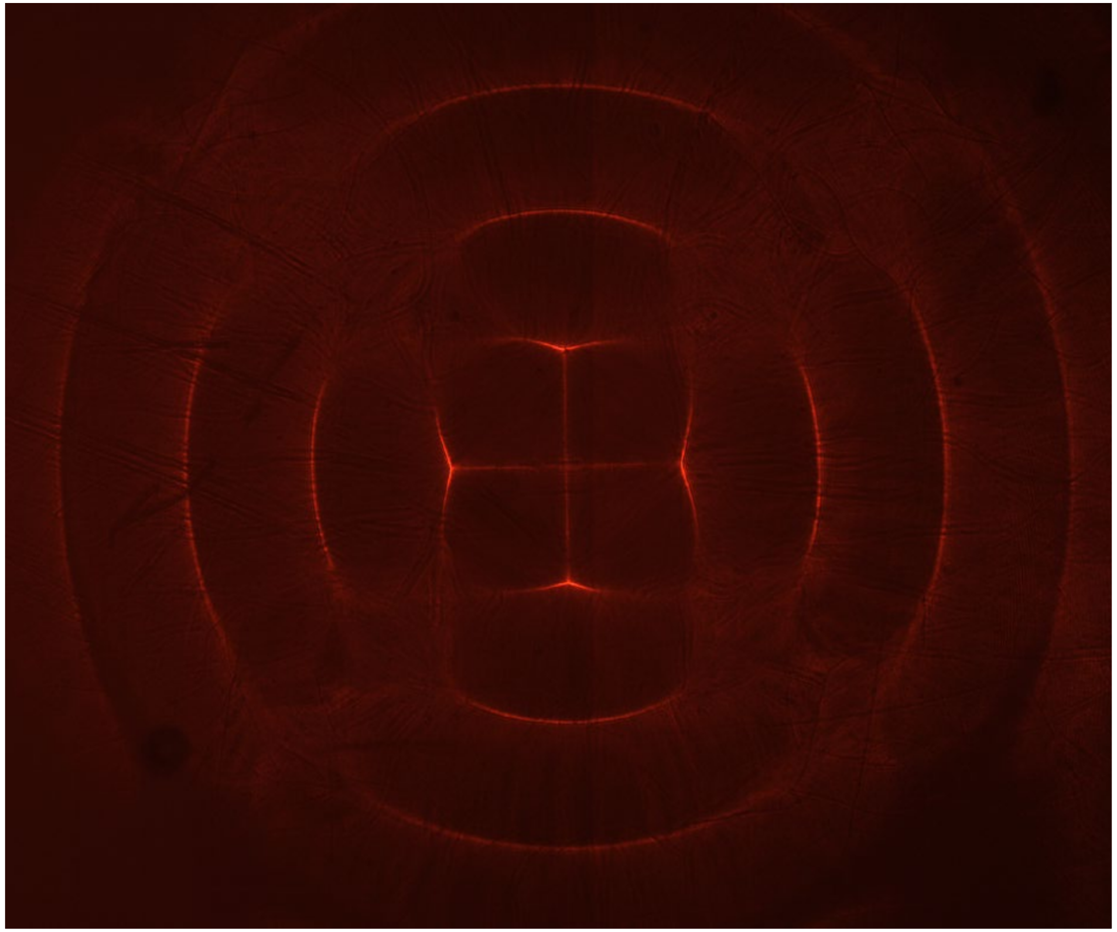
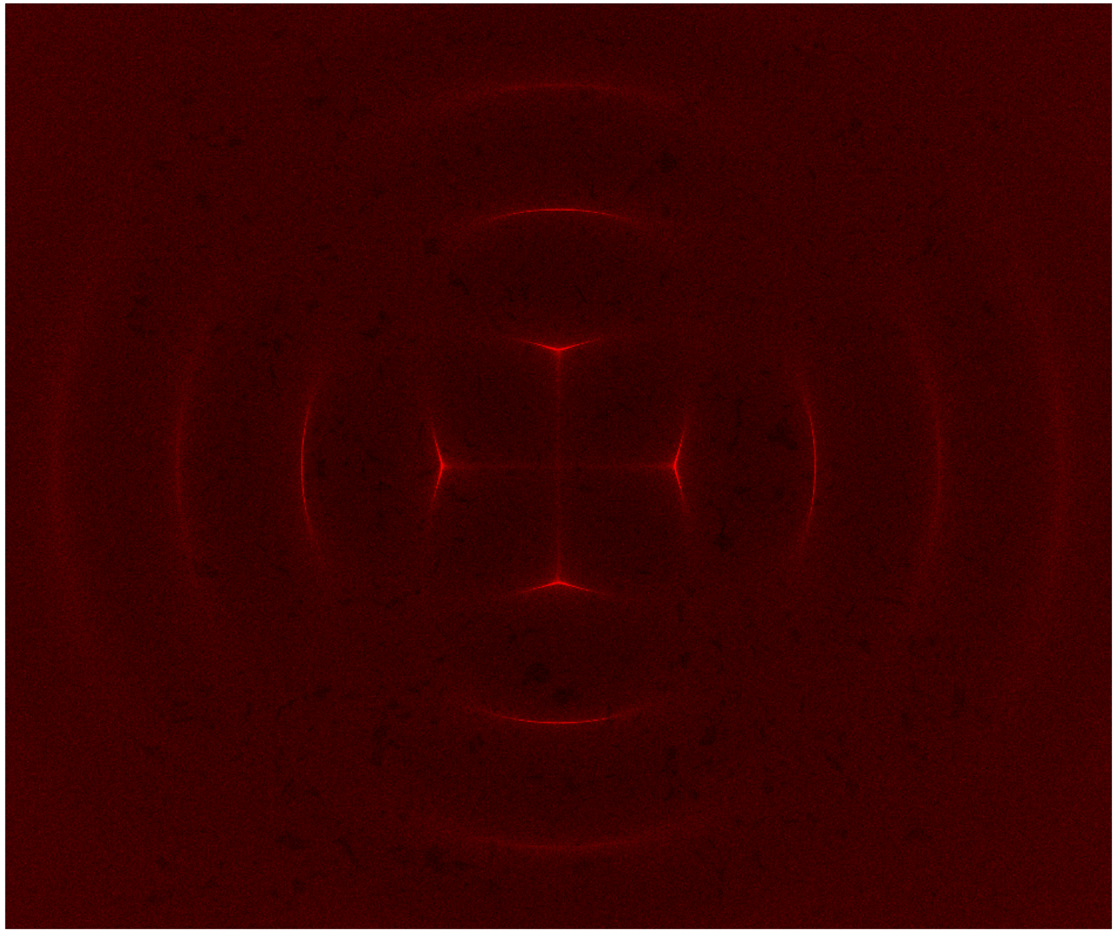
Rendering for scientific imaging applications





15-468, 15-668, 15-868
Physics-based Rendering
Spring 2023, Lecture 17

http://graphics.cs.cmu.edu/courses/15-468

Course announcements

- We’re all done with homework!
- *Please* vote for the topic of tomorrow’s reading group.

Overview of today's lecture

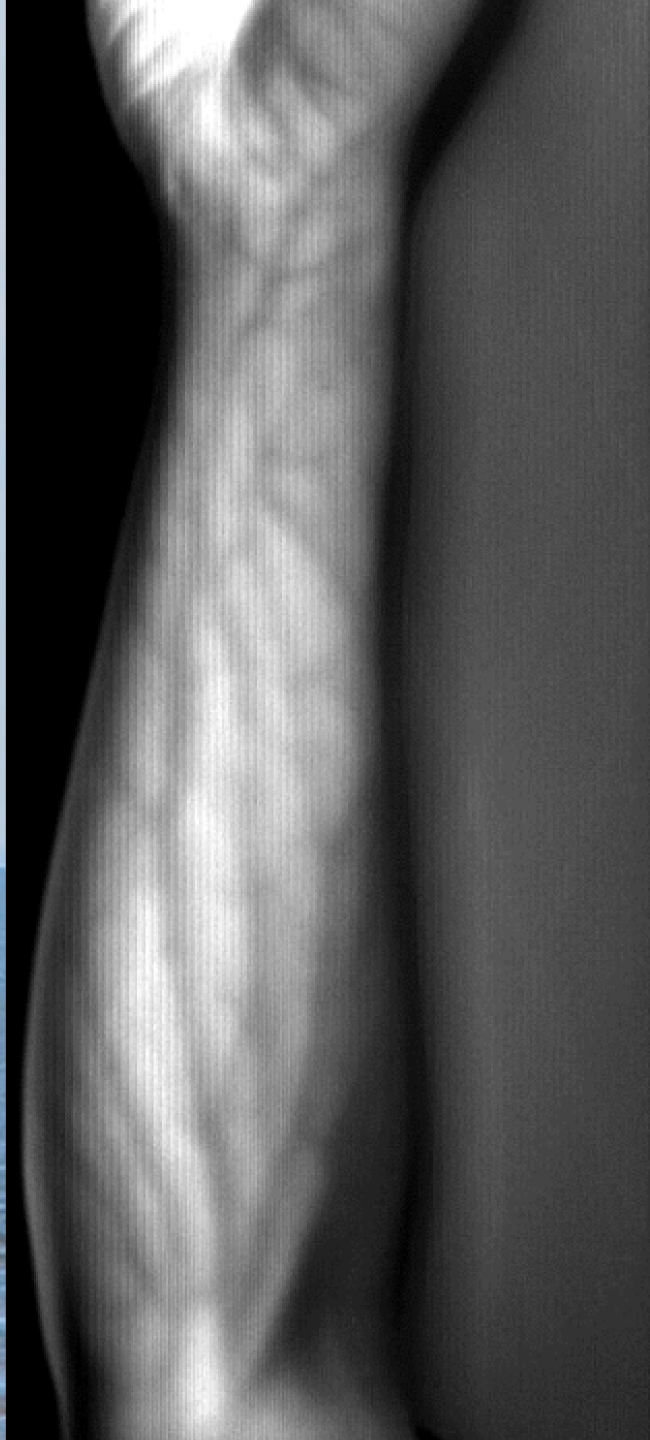
- Rendering continuous refraction.
- GRIN optics.
- Rendering the refractive radiative transfer equation.
- Acousto-optics.
- Rendering speckle.
- Fluorescence microscopy.

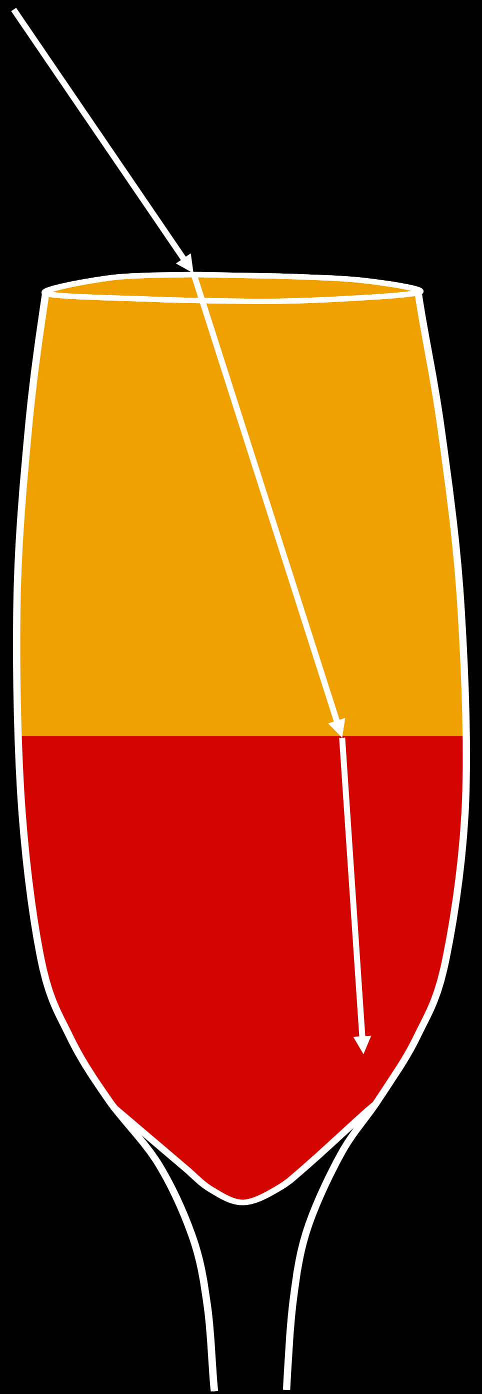
Slide credits

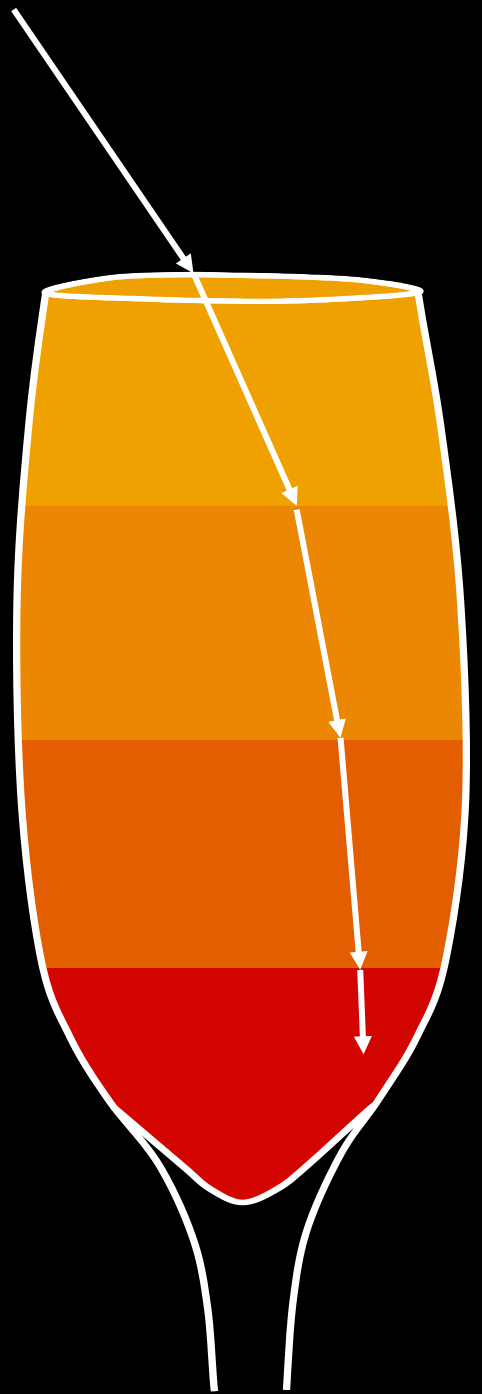
Many of these slides were directly adapted from:

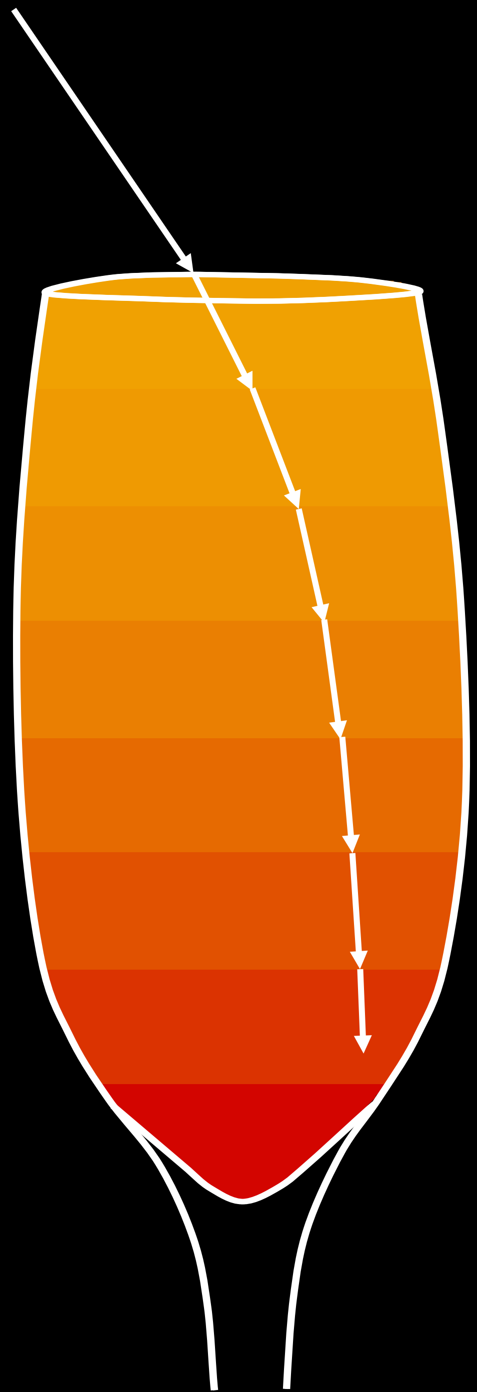
- Adithya Pediredla (CMU).
- Arjun Teh (CMU).
- Chen Bar (Technion).

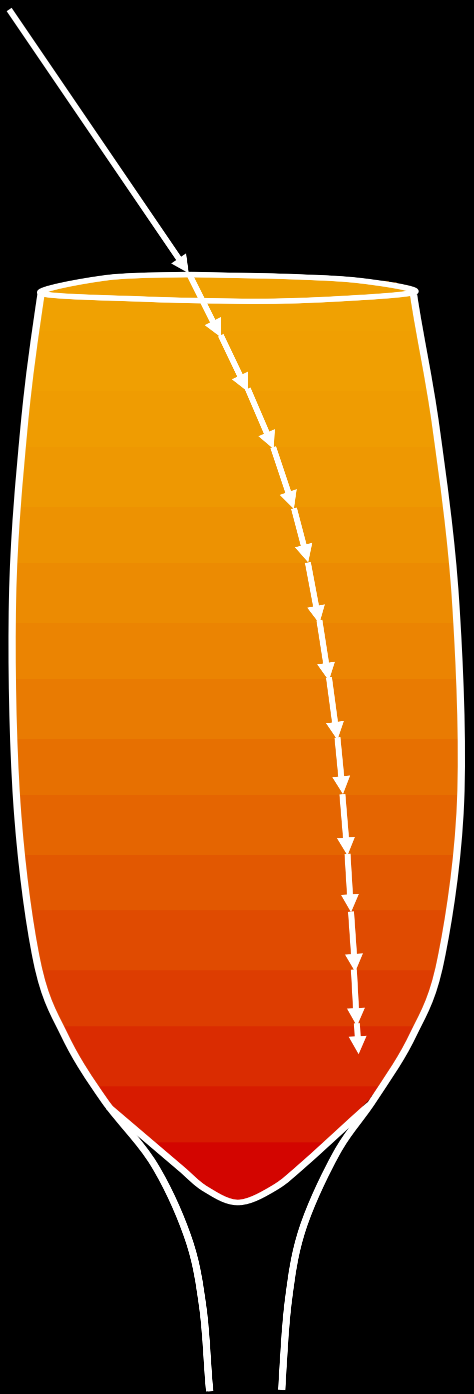
Media with continuously varying refractive index and scattering

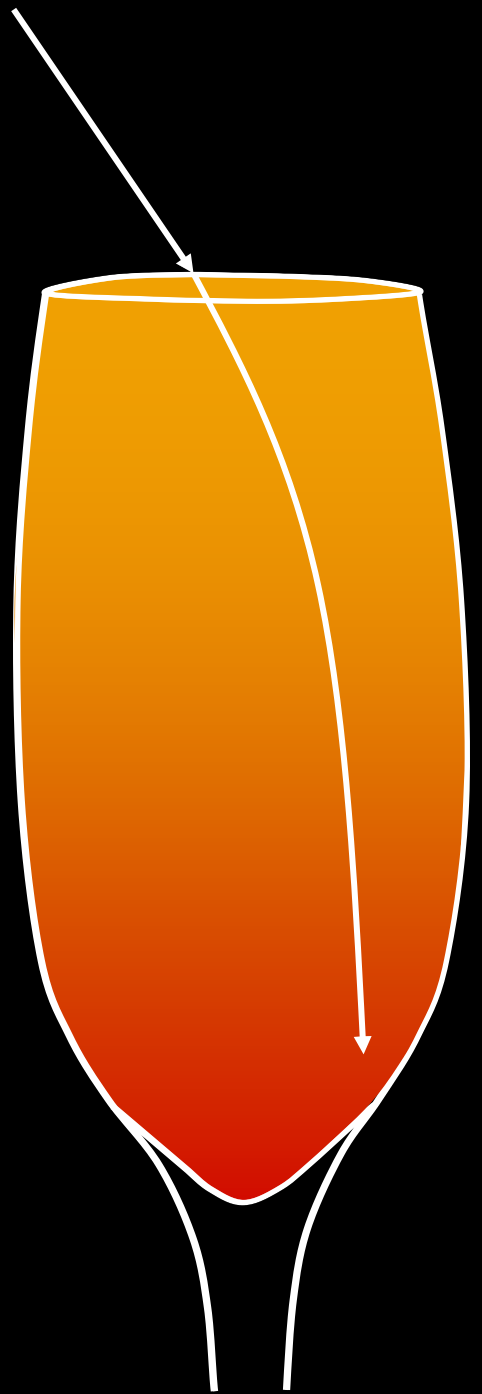








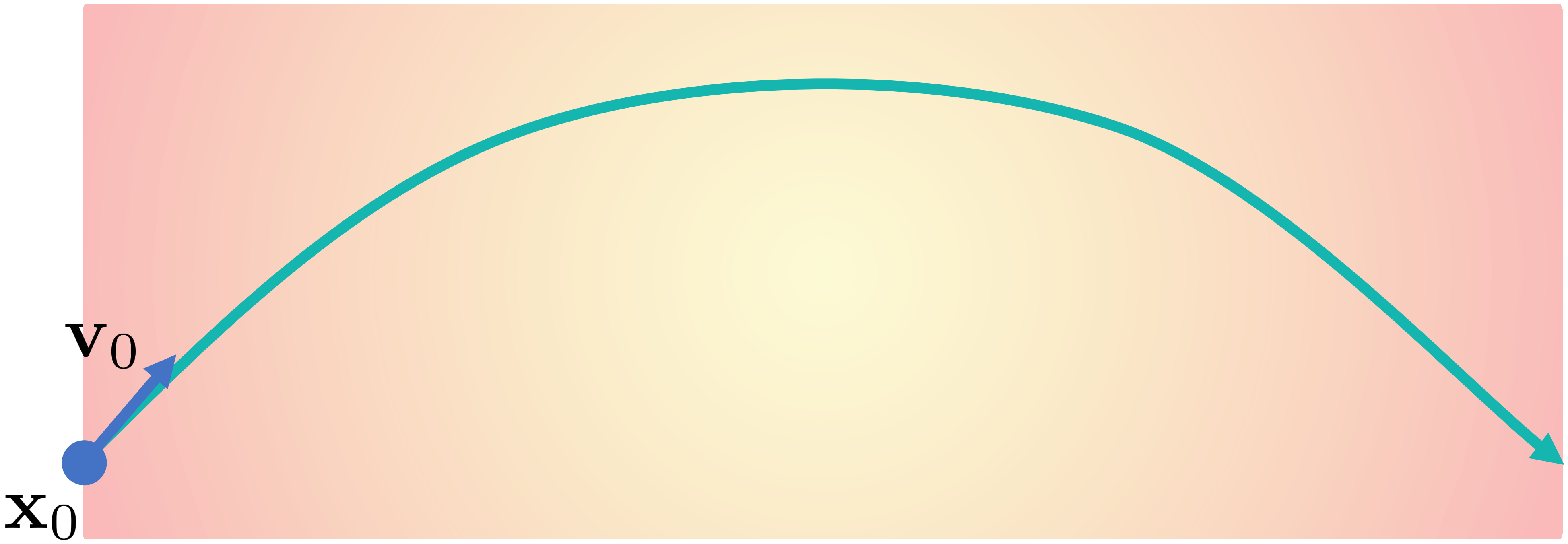




Nonlinear Ray Tracing

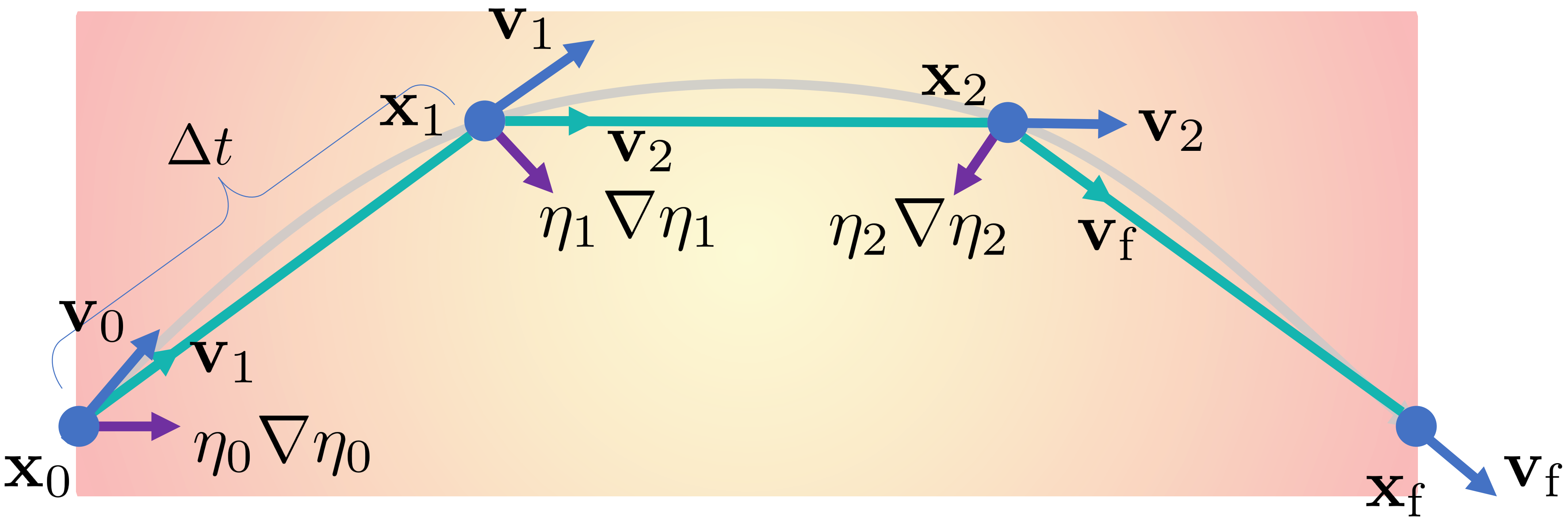
$\eta(\mathbf{x})$: refractive index of the volume at location, $\mathbf{x}$

Nonlinear Ray Tracing



$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v} \qquad \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \eta \nabla \eta$$

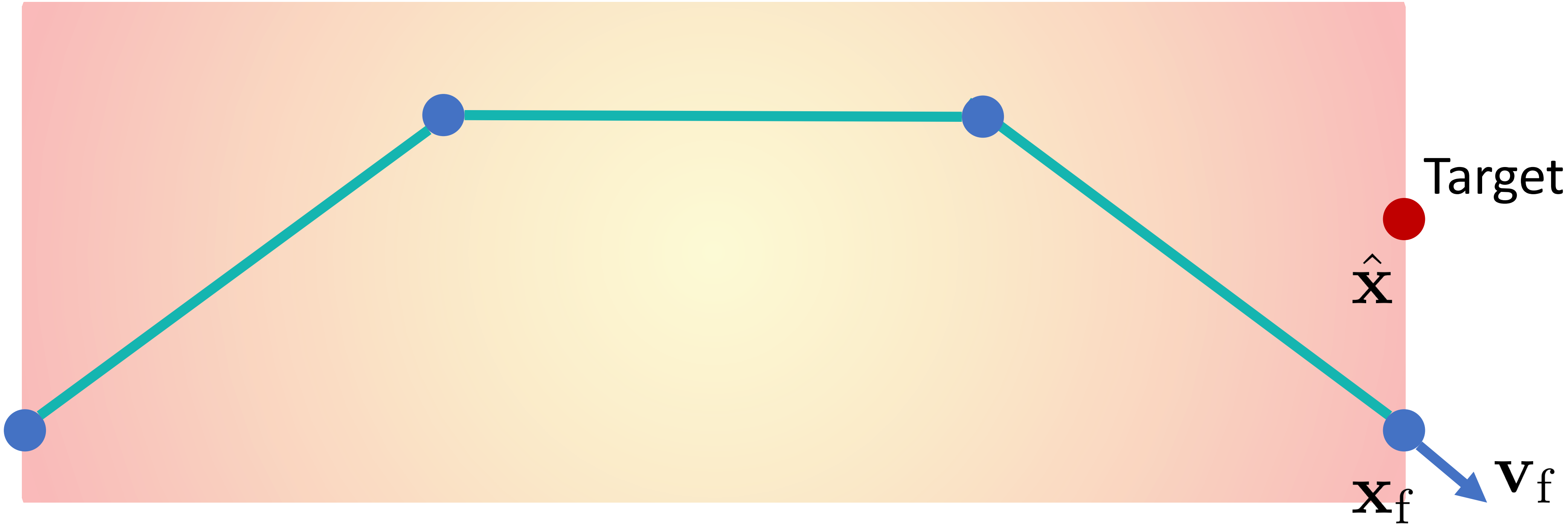
Nonlinear Ray Tracing



$$\boxed{\mathbf{x}_i = \mathbf{x}_{i-1} + \mathbf{v}_i \Delta t}$$

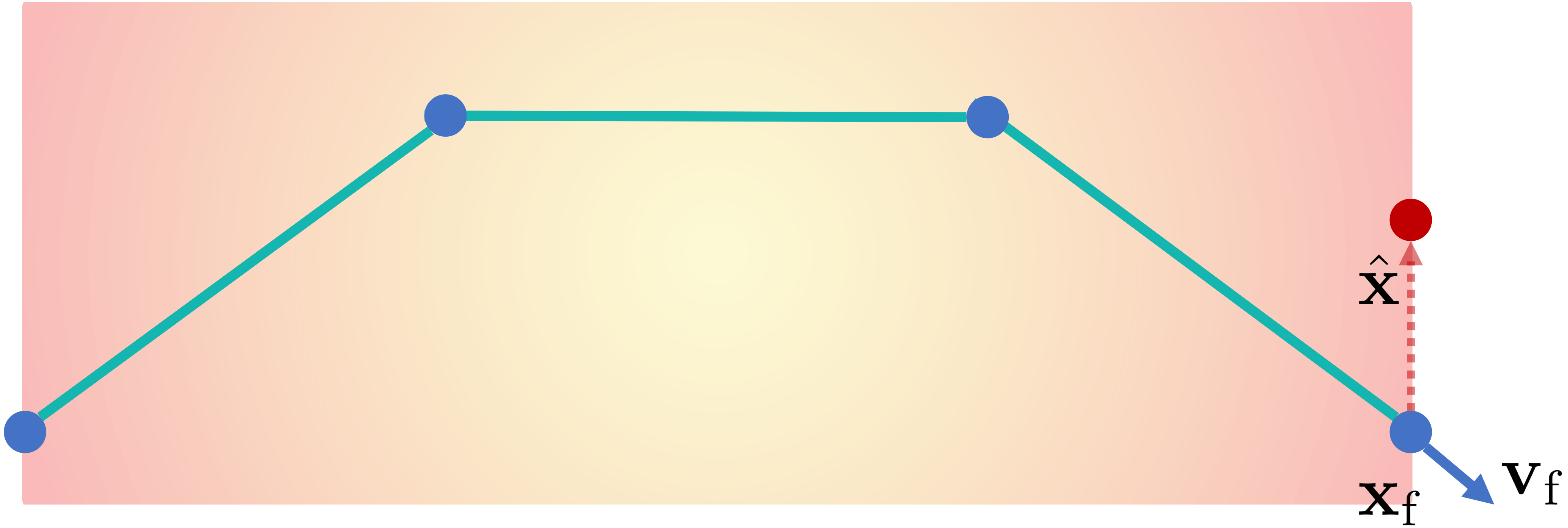
$$\boxed{\mathbf{v}_i = \mathbf{v}_{i-1} + \eta_{i-1} \nabla \eta_{i-1} \Delta t}$$

Nonlinear Ray Tracing



$$\min_{\eta} \left\|\hat{\mathbf{x}} - \mathbf{x}_\mathrm{f}\right\|^2$$

Nonlinear Ray Tracing



$$\frac{\mathrm{d}}{\mathrm{d}\eta}\|\hat{\mathbf{x}} - \mathbf{x}_{\mathrm{f}}\|^2$$

Nonlinear Ray Tracing in reverse

$$
\begin{aligned}
\min_{\eta} \sum_{i=1}^{N} \mathcal{F}_i \Bigg[\iint_{(\mathrm{x}_0,\mathrm{v}_0)\in\Omega} & C_i \left(\mathrm{x}\left(\sigma_f;\eta,\mathrm{x}_0,\mathrm{v}_0\right),\mathrm{v}\left(\sigma_f;\eta,\mathrm{x}_0,\mathrm{v}_0\right)\right) \,\mathrm{d}\mathrm{x}_0 \,\mathrm{d}\mathrm{v}_0 \Bigg] \\
\text{s.t. } \dot{\mathrm{x}}\left(\sigma;\eta,\mathrm{x}_0,\mathrm{v}_0\right) &= \mathrm{v}, \quad \forall\sigma\in\left[0,\sigma_f\right], \\
\dot{\mathrm{v}}\left(\sigma;\eta,\mathrm{x}_0,\mathrm{v}_0\right) &= \eta\nabla\eta, \quad \forall\sigma\in\left[0,\sigma_f\right], \\
\mathrm{x}\left(0;\eta,\mathrm{x}_0,\mathrm{v}_0\right) &= \mathrm{x}_0, \\
\mathrm{v}\left(0;\eta,\mathrm{x}_0,\mathrm{v}_0\right) &= \mathrm{v}_0,
\end{aligned}
\tag{15}
$$

$$\dot{\boldsymbol{\lambda}} = -\left(\nabla\eta\left(\nabla\eta\right)^{\top} + \eta\,\mathrm{Hess}\left(\eta\right)\right)\boldsymbol{\mu}, \qquad \forall\sigma\in\left[0,\sigma_f\right] \tag{19}$$

$$\dot{\boldsymbol{\mu}} = -\boldsymbol{\lambda}, \qquad \forall\sigma\in\left[0,\sigma_f\right] \tag{20}$$

$$\boldsymbol{\lambda}\left(\sigma_f\right) = \frac{\partial C}{\partial \mathrm{x}}, \tag{21}$$

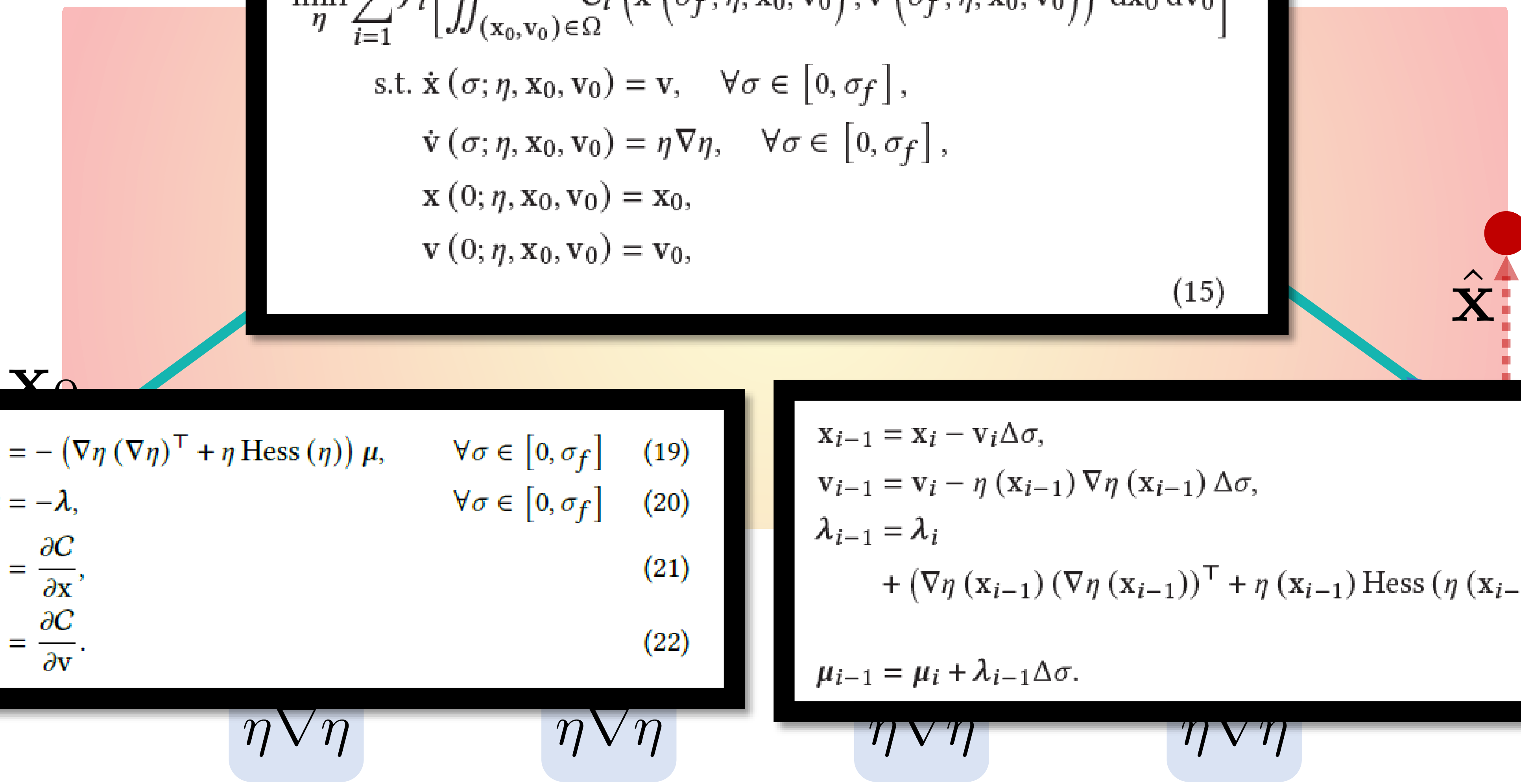
$$\boldsymbol{\mu}\left(\sigma_f\right) = \frac{\partial C}{\partial \mathrm{v}}. \tag{22}$$

$$\mathrm{x}_{i-1} = \mathrm{x}_i - \mathrm{v}_i\Delta\sigma, \tag{28}$$

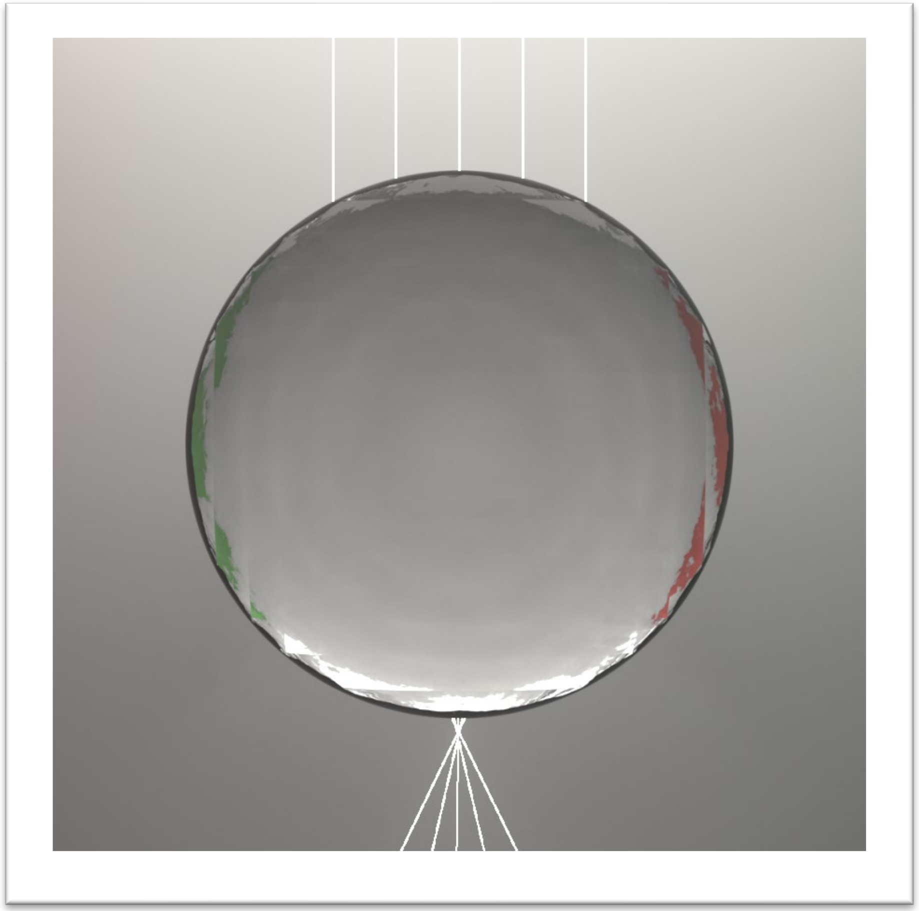
$$\mathrm{v}_{i-1} = \mathrm{v}_i - \eta\left(\mathrm{x}_{i-1}\right)\nabla\eta\left(\mathrm{x}_{i-1}\right)\Delta\sigma, \tag{29}$$

$$
\begin{aligned}
\boldsymbol{\lambda}_{i-1} = \boldsymbol{\lambda}_i & \\
&+ \left(\nabla\eta\left(\mathrm{x}_{i-1}\right)\left(\nabla\eta\left(\mathrm{x}_{i-1}\right)\right)^{\top} + \eta\left(\mathrm{x}_{i-1}\right)\mathrm{Hess}\left(\eta\left(\mathrm{x}_{i-1}\right)\right)\right)\boldsymbol{\mu}_i\Delta\sigma,
\end{aligned}
\tag{30}
$$

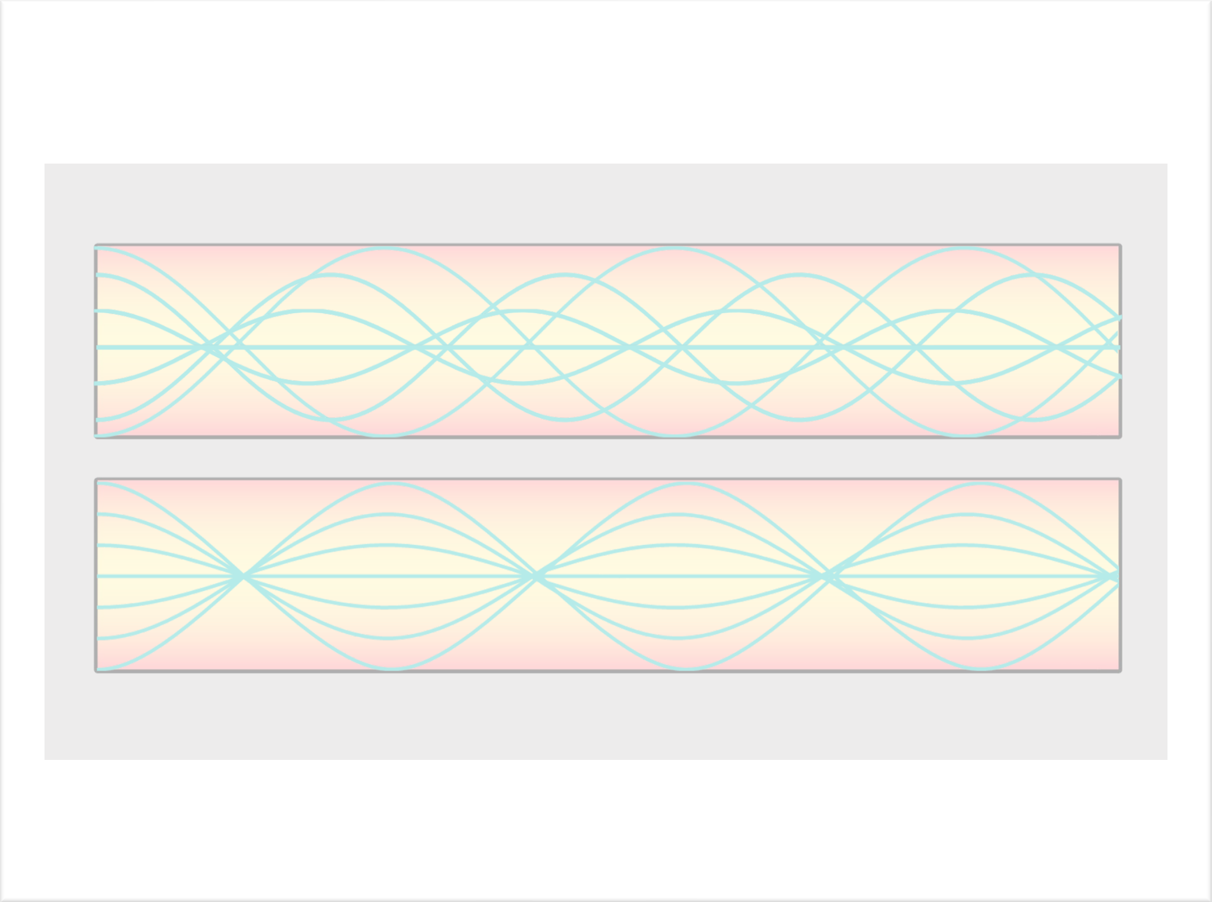
$$\boldsymbol{\mu}_{i-1} = \boldsymbol{\mu}_i + \boldsymbol{\lambda}_{i-1}\Delta\sigma. \tag{31}$$



Optimizing Gradient-Index (GRIN) Optics

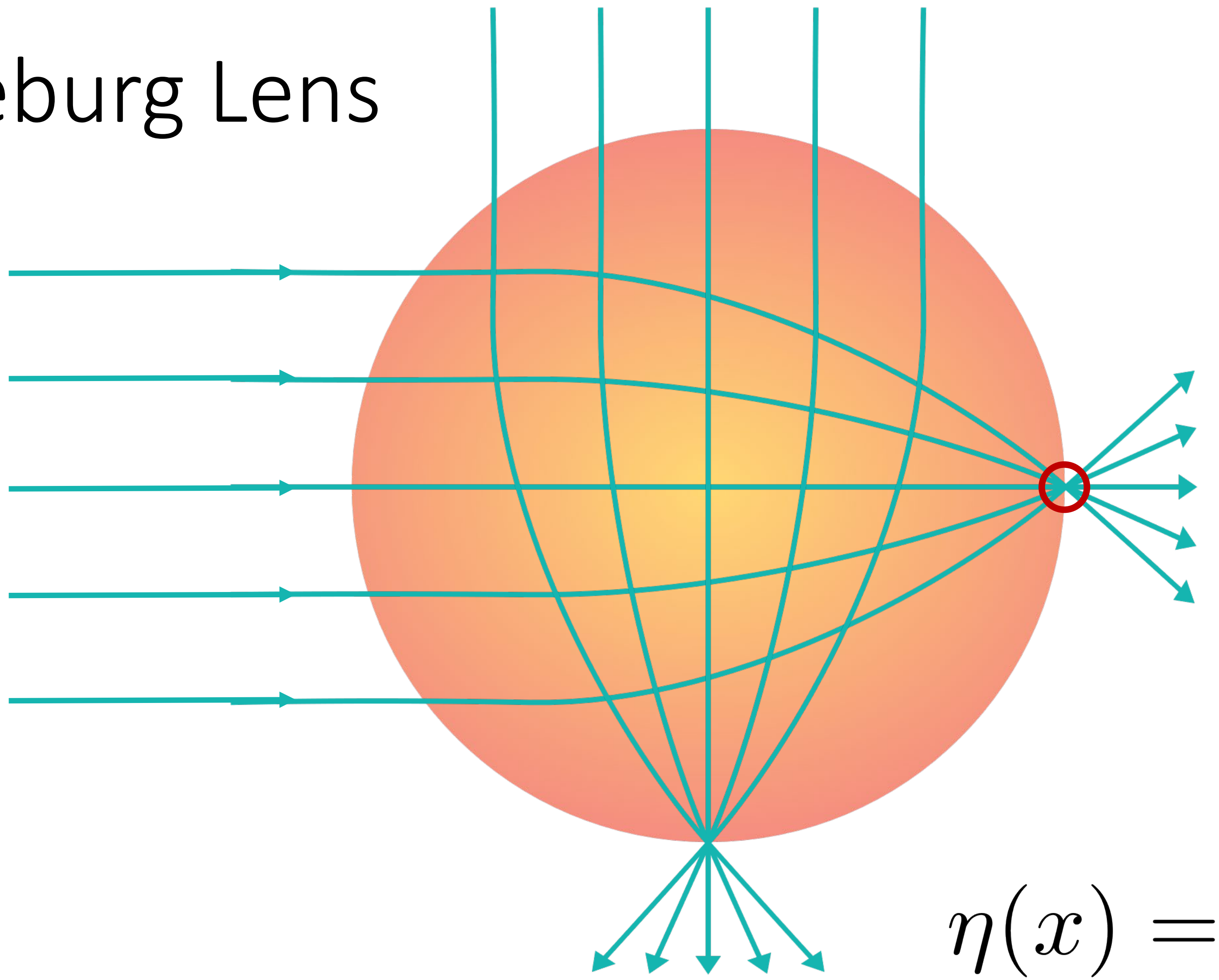


Luneburg Lens



GRIN Fiber

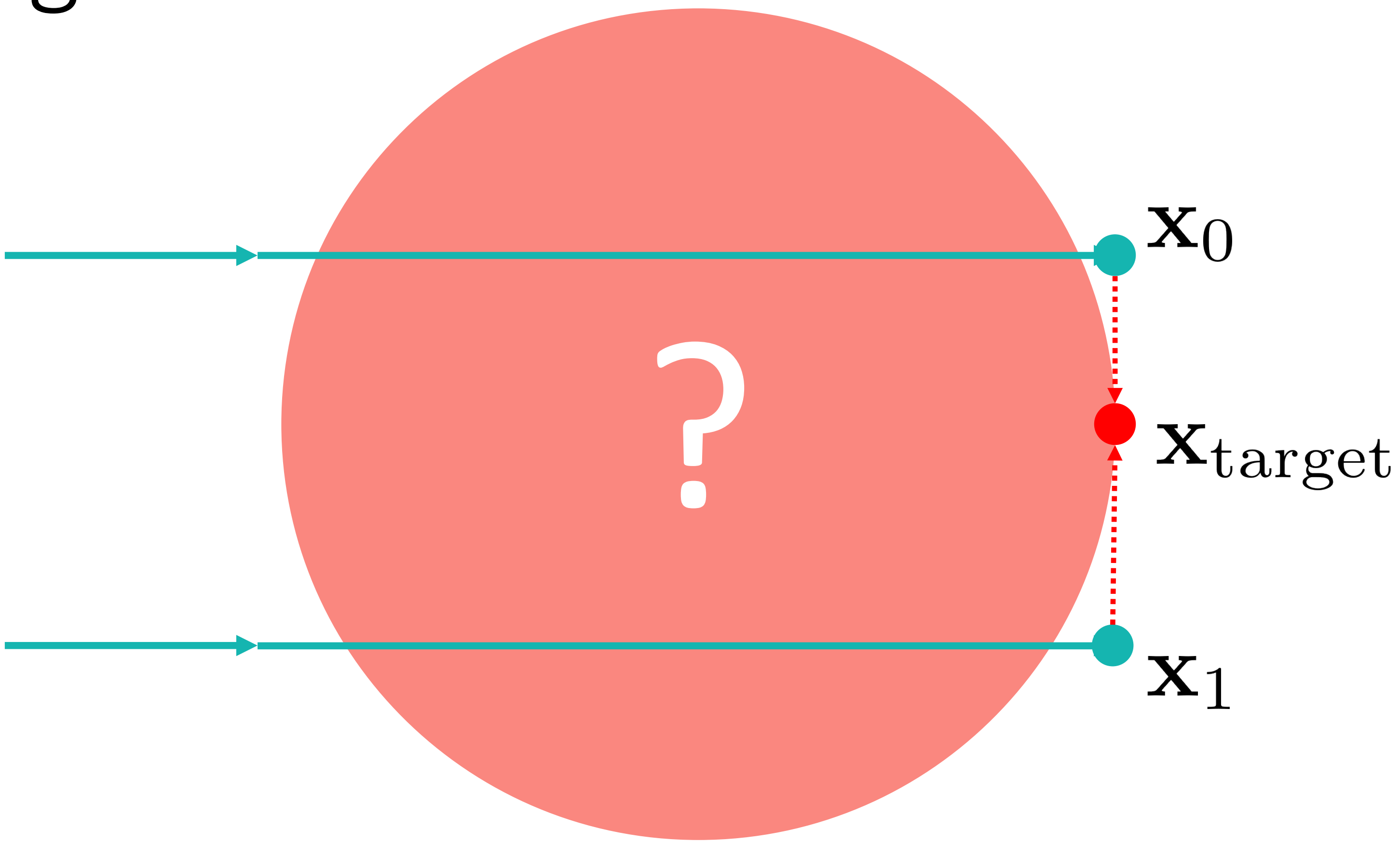
Luneburg Lens



$$\eta(x) = \sqrt{2 - \|x\|^2}$$

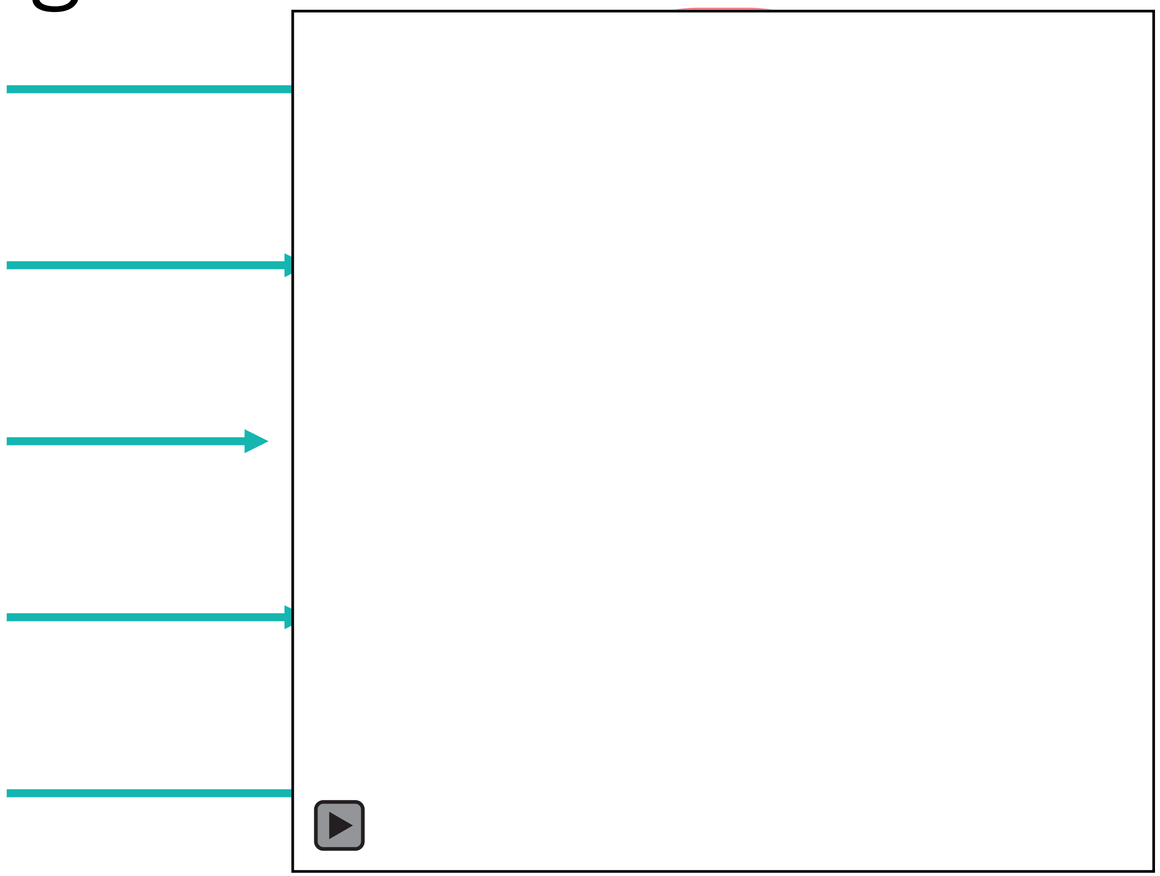
[Luneburg, R. K. 1944]

Luneburg Lens

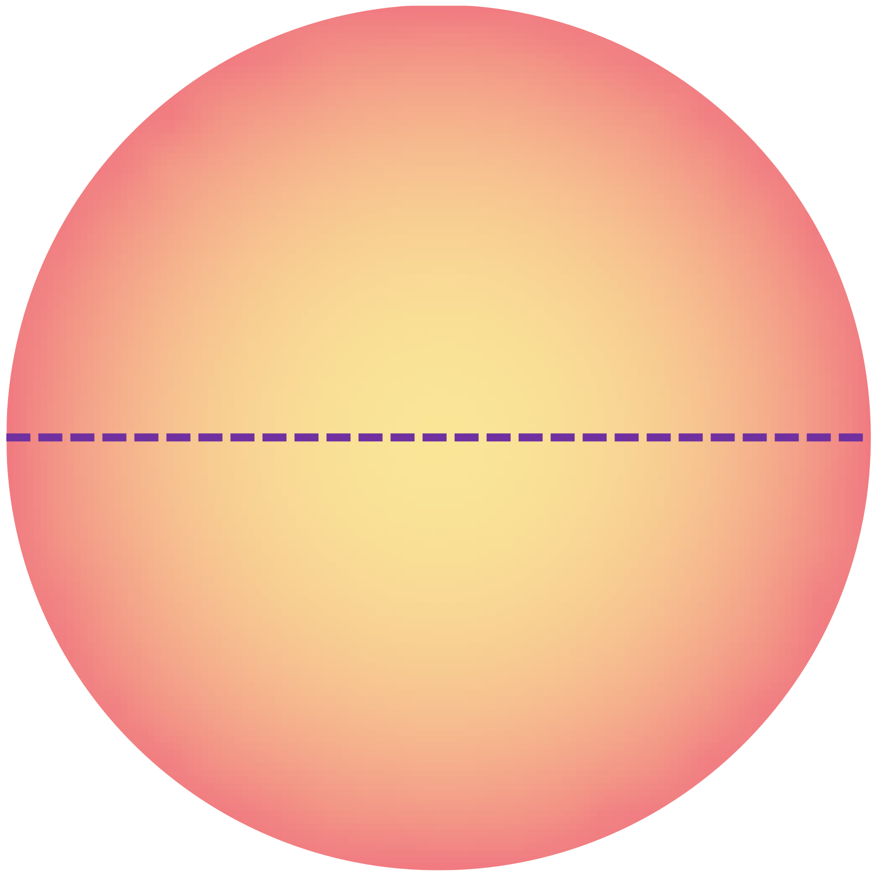


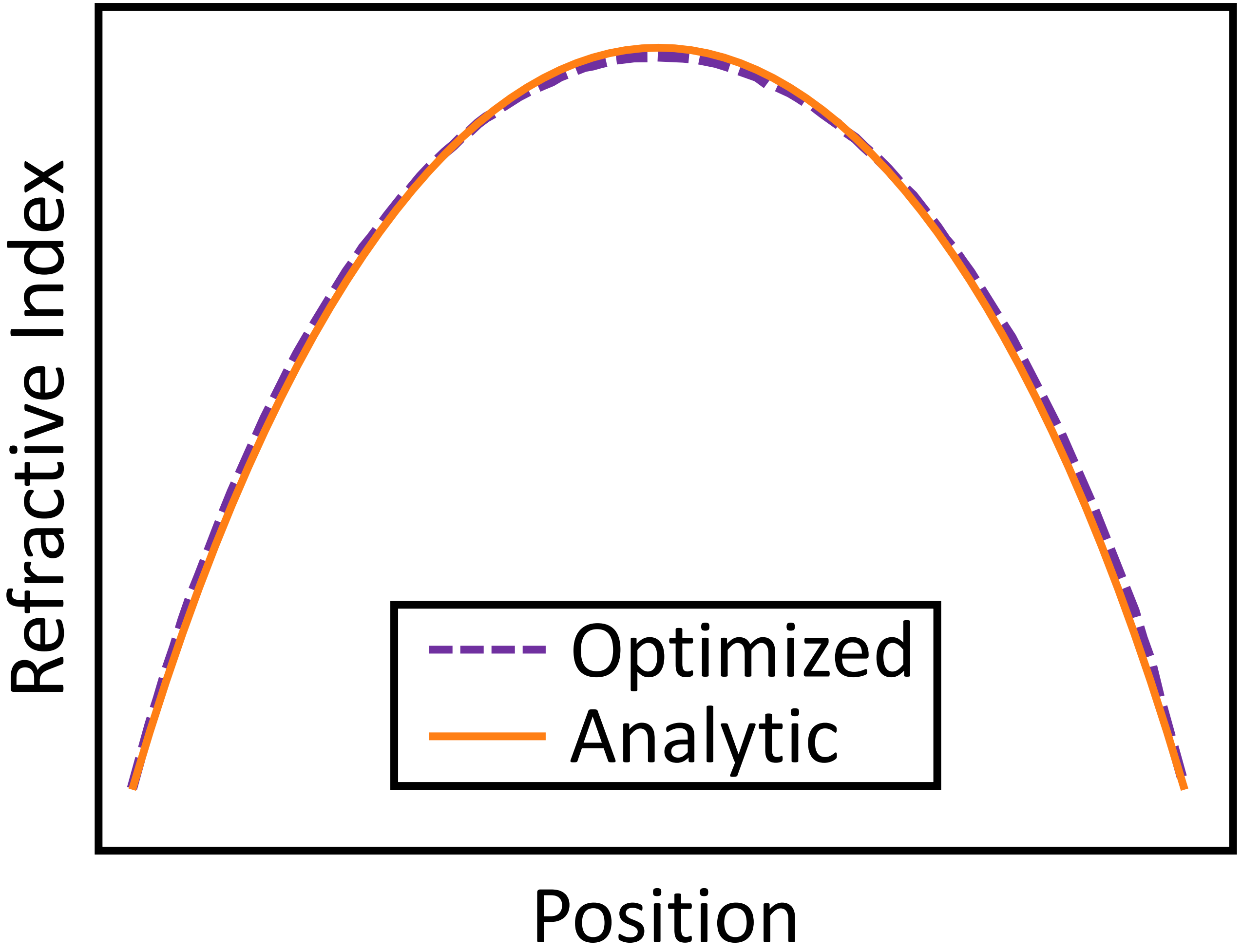
$$\min_{\eta} \sum_{i \in \text{rays}} \left\| \mathbf{x}_{\text{target}} - \mathbf{x}_{\text{i}} \right\|^2$$

Luneburg Lens



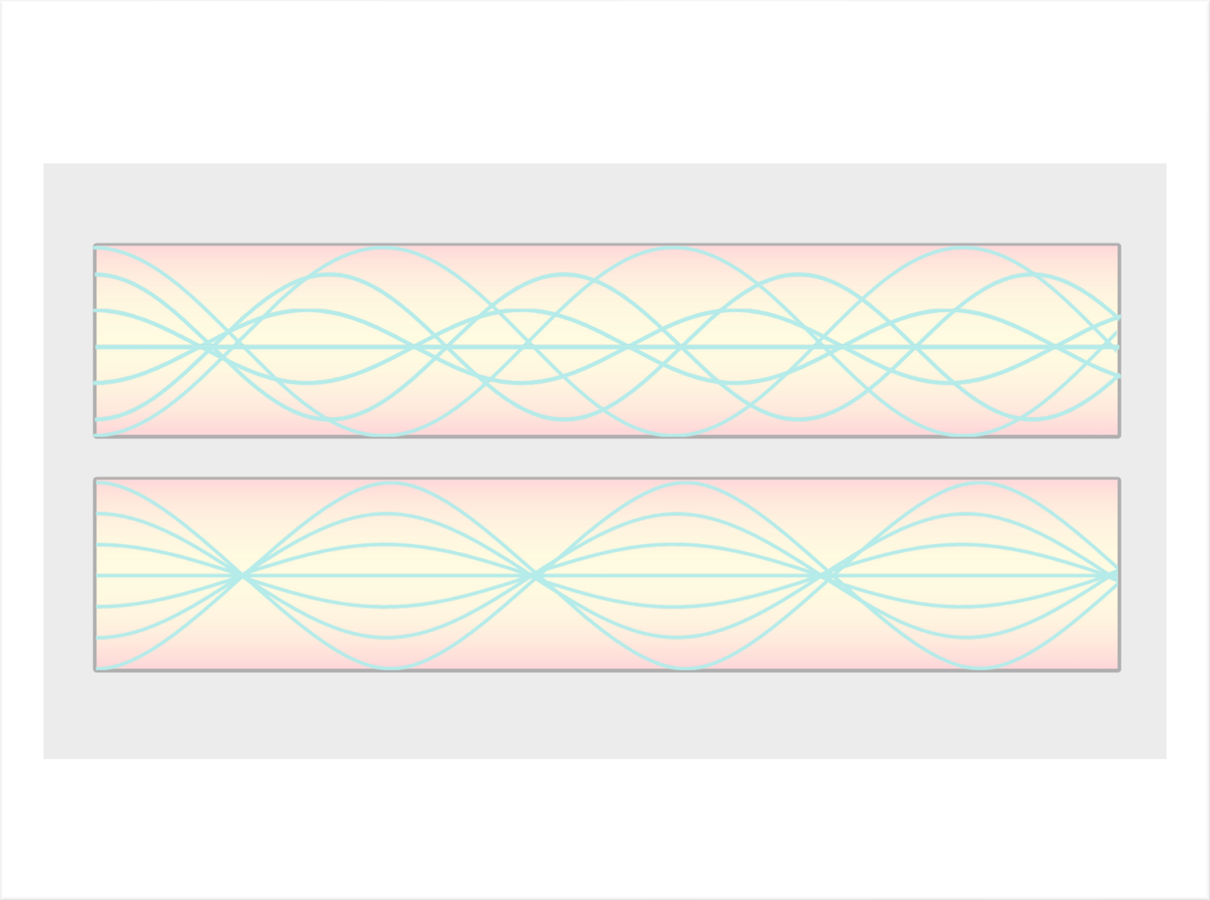
Luneburg Lens





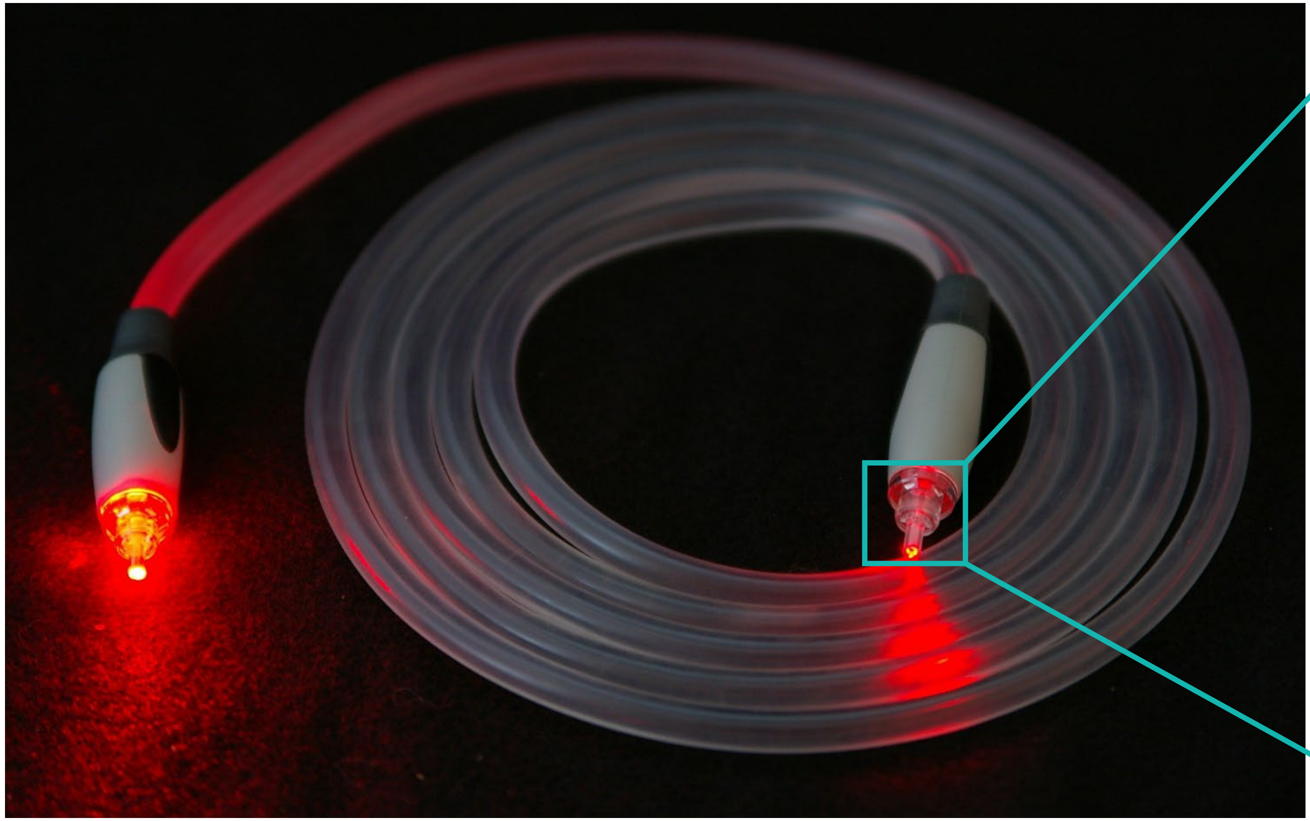
Optimizing Gradient-Index (GRIN) Optics

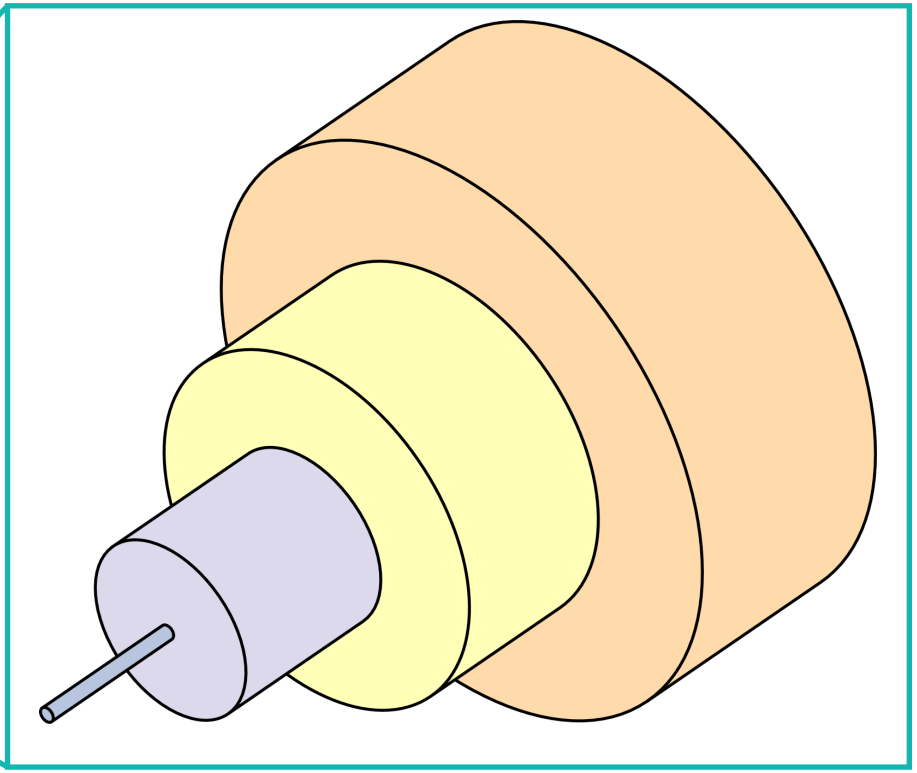
Luneburg Lens



GRIN Fiber

GRIN Fiber

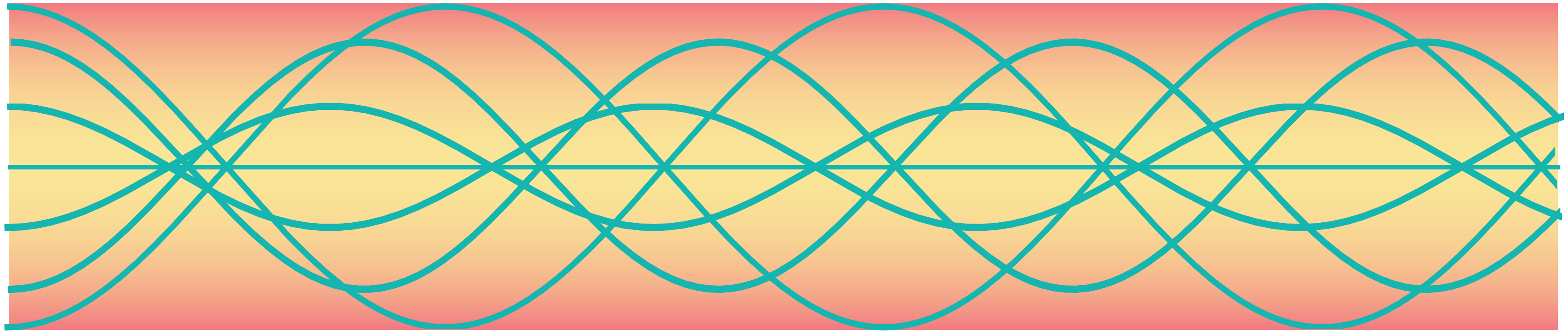


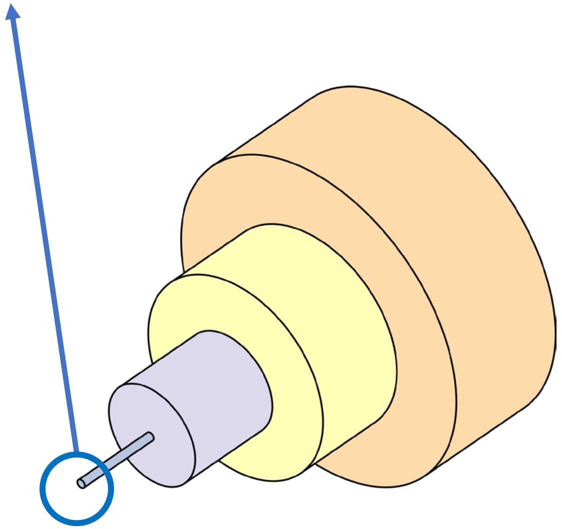


https://en.wikipedia.org/wiki/Optical_fiber

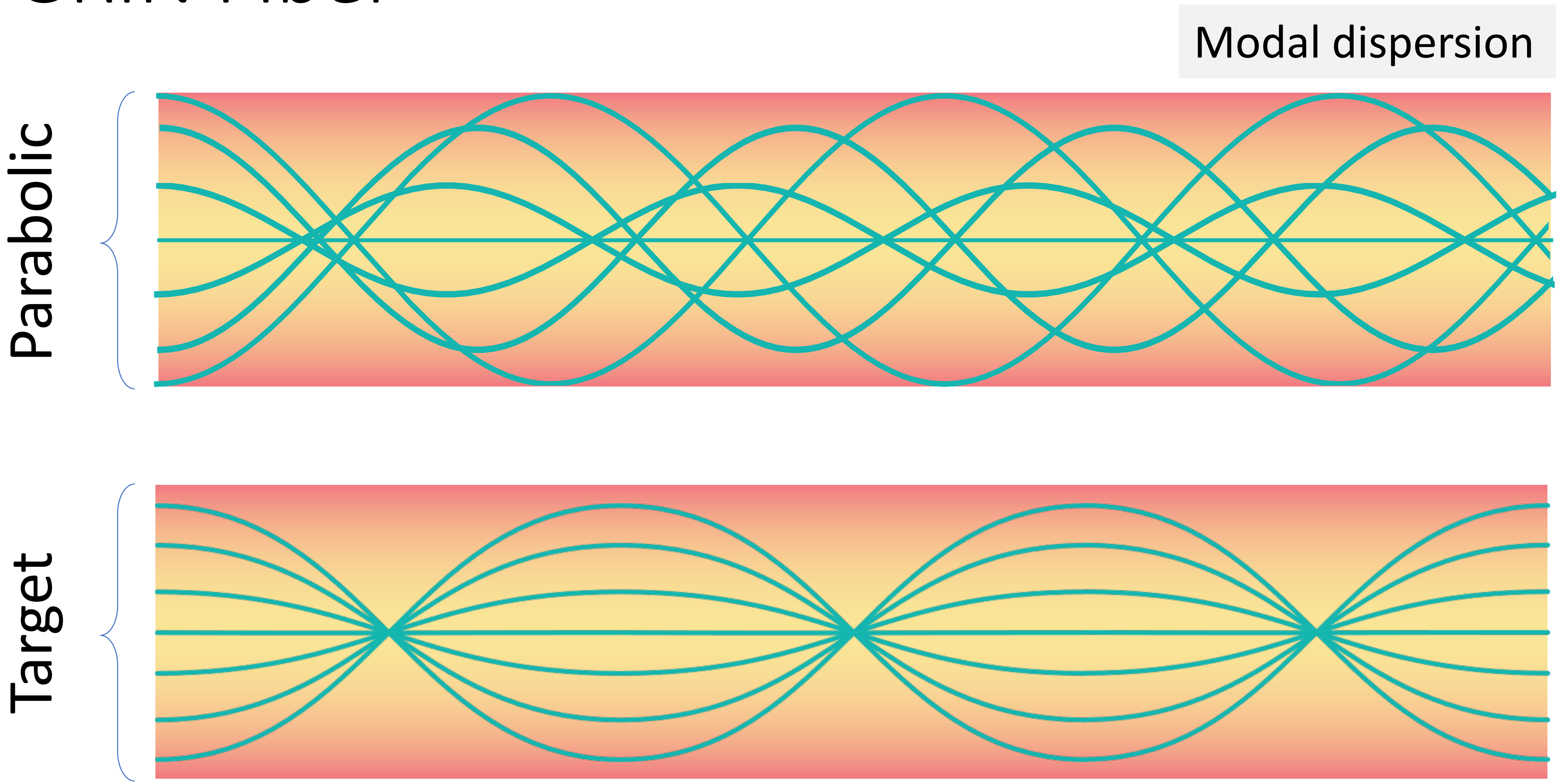
GRIN Fiber

Modal dispersion

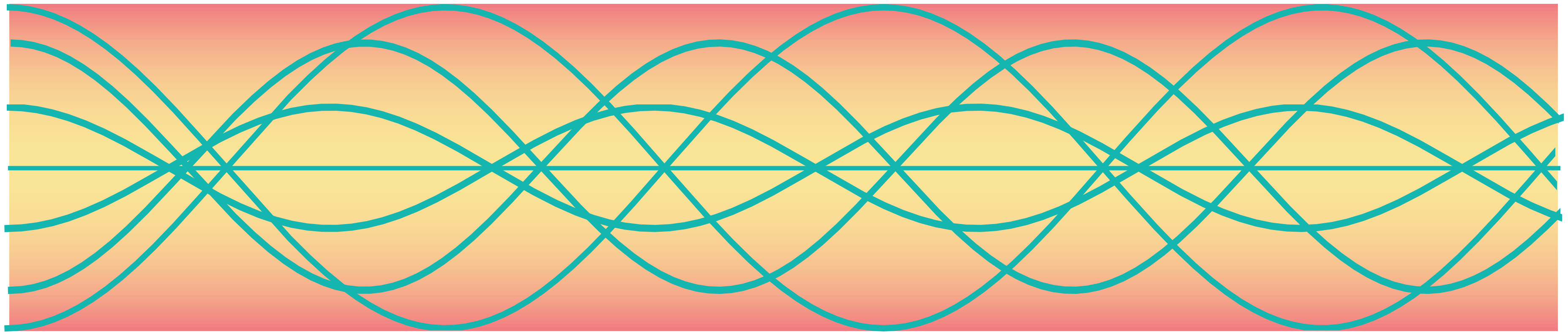




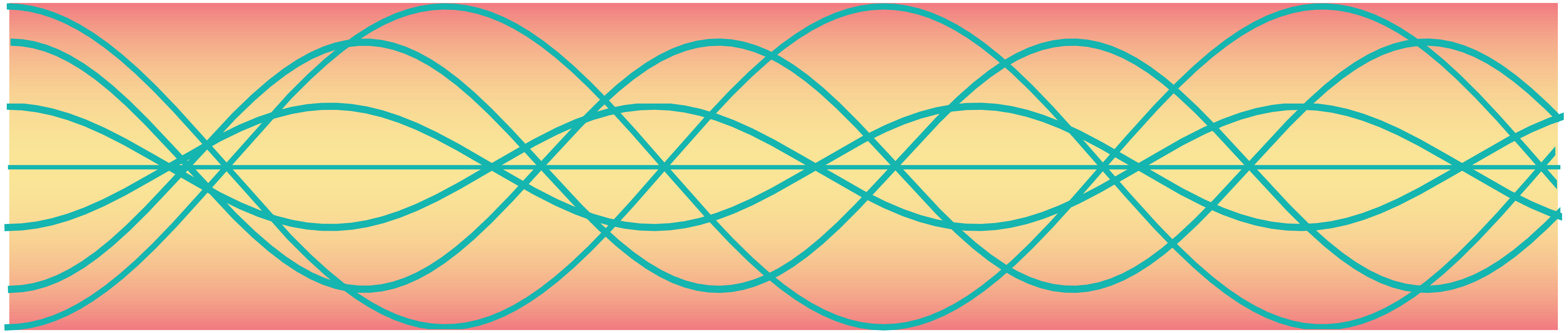
GRIN Fiber

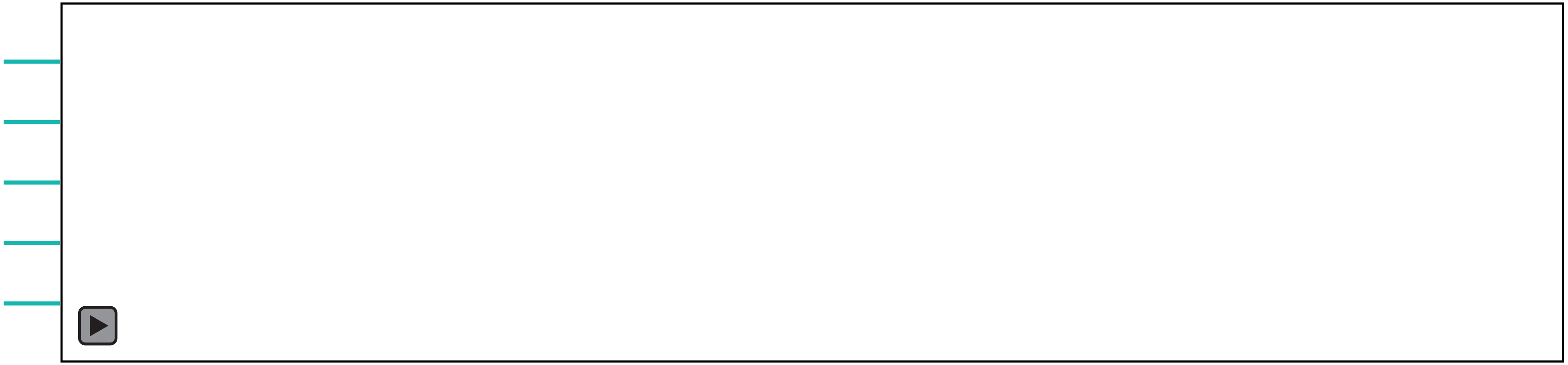


GRIN Fiber

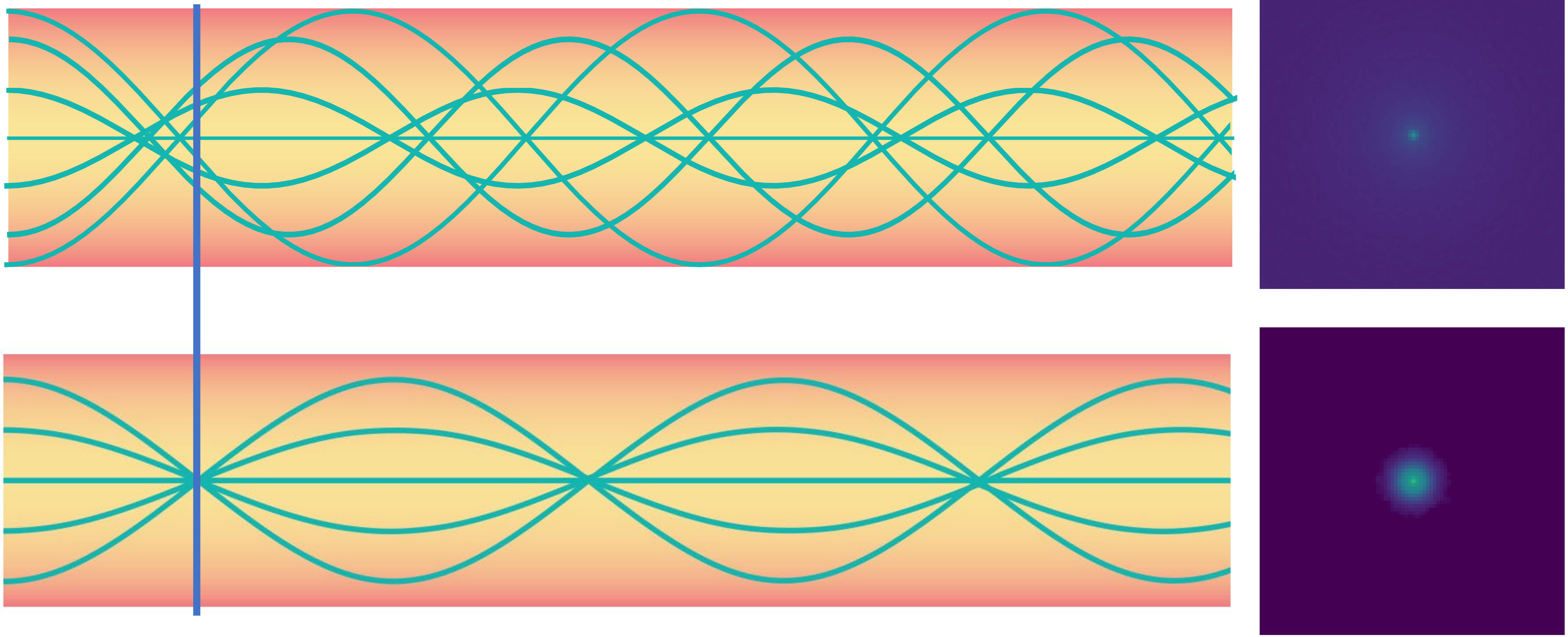


GRIN Fiber

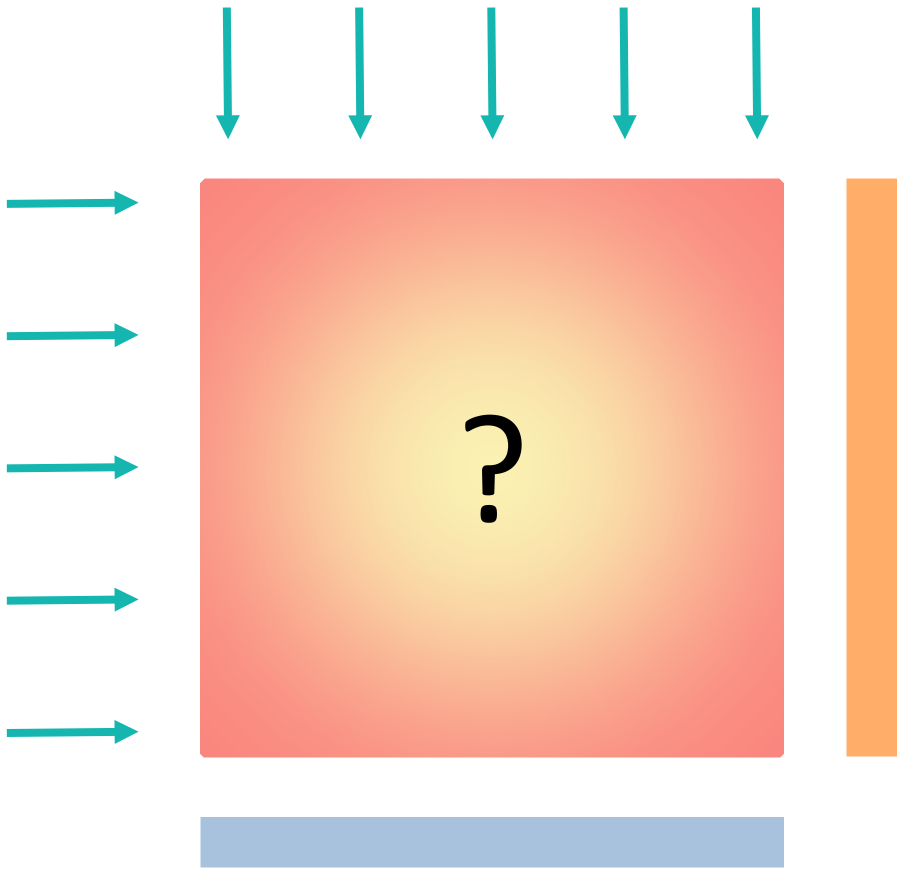


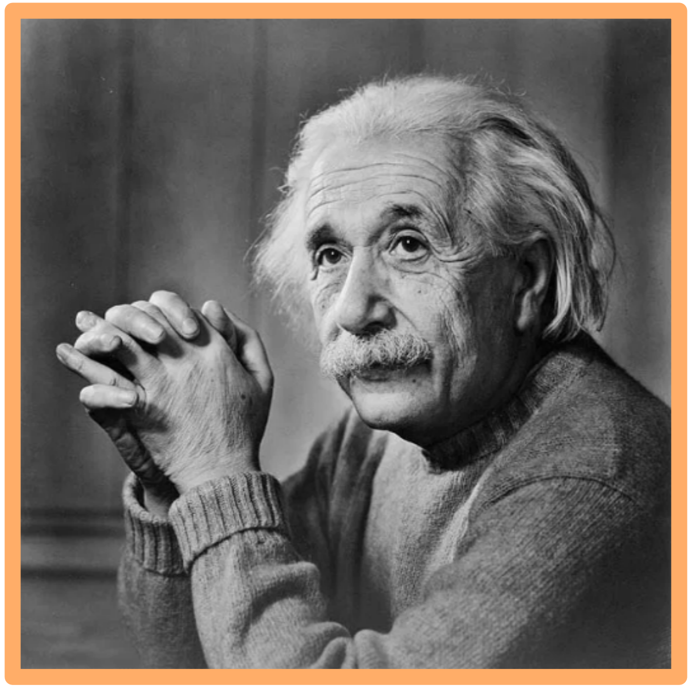


GRIN Fiber

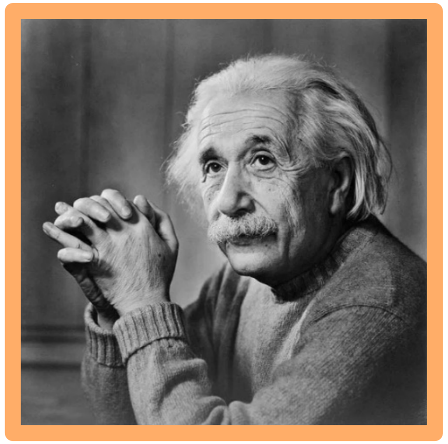


Multiview Display

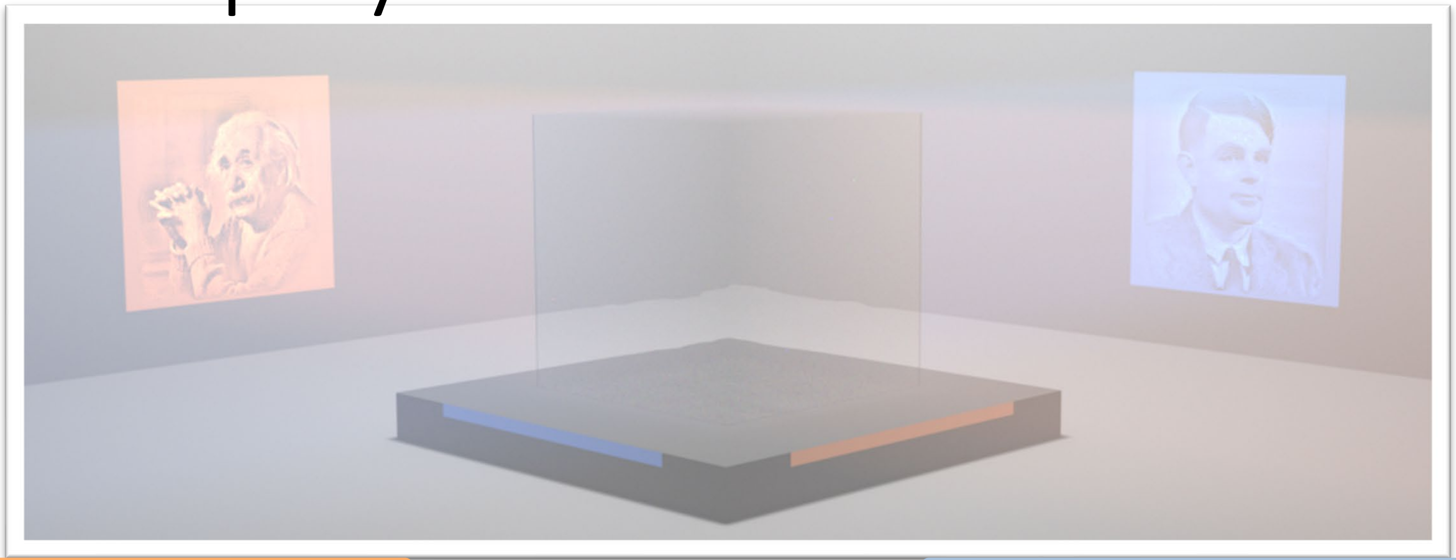


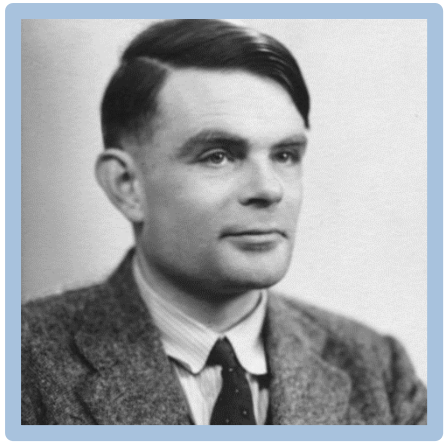


Multiview Display

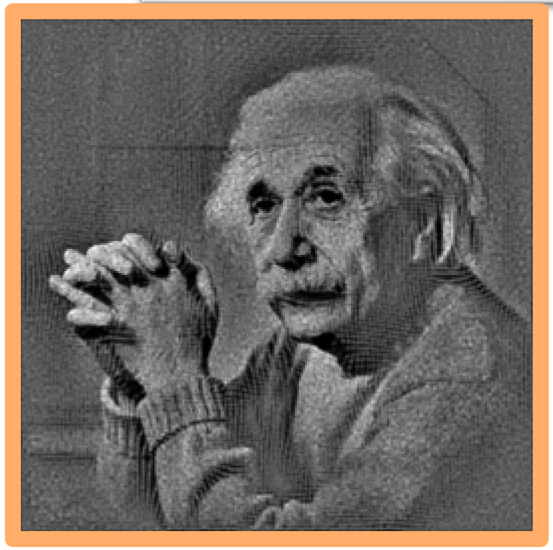


Target





Target

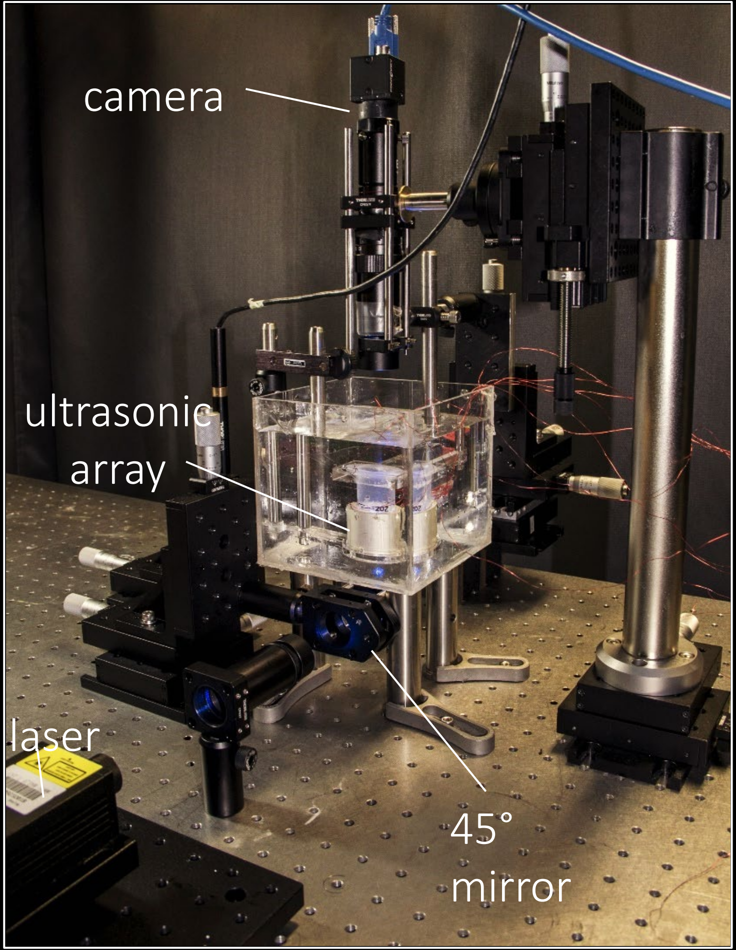


optimization results

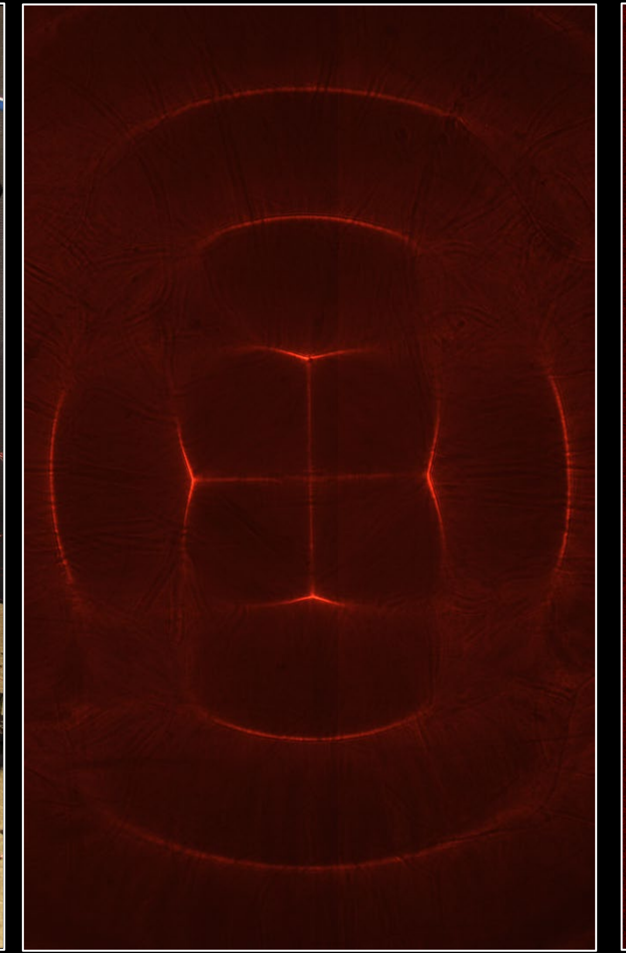
Media with continuously varying refractive index and scattering

unbiased techniques for scientific imaging

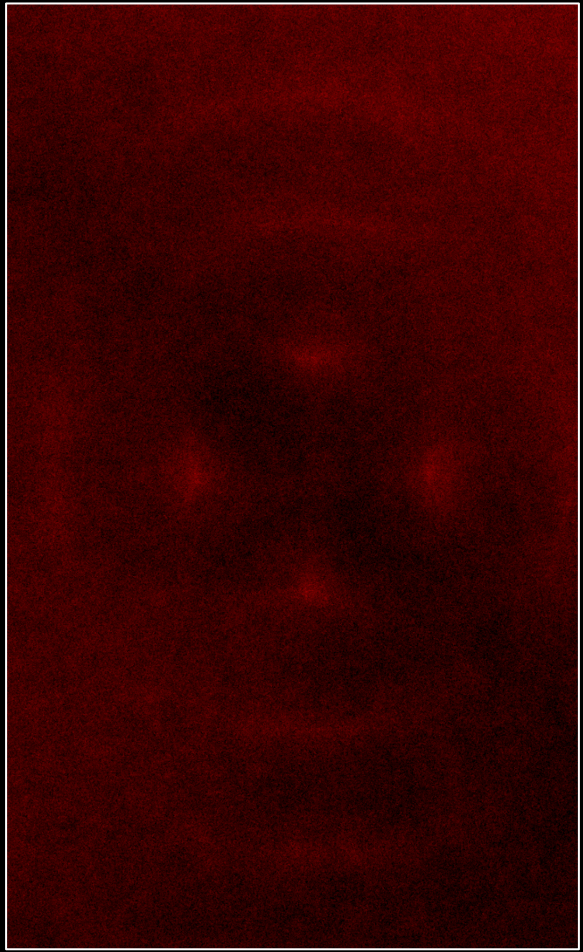
experimental hardware



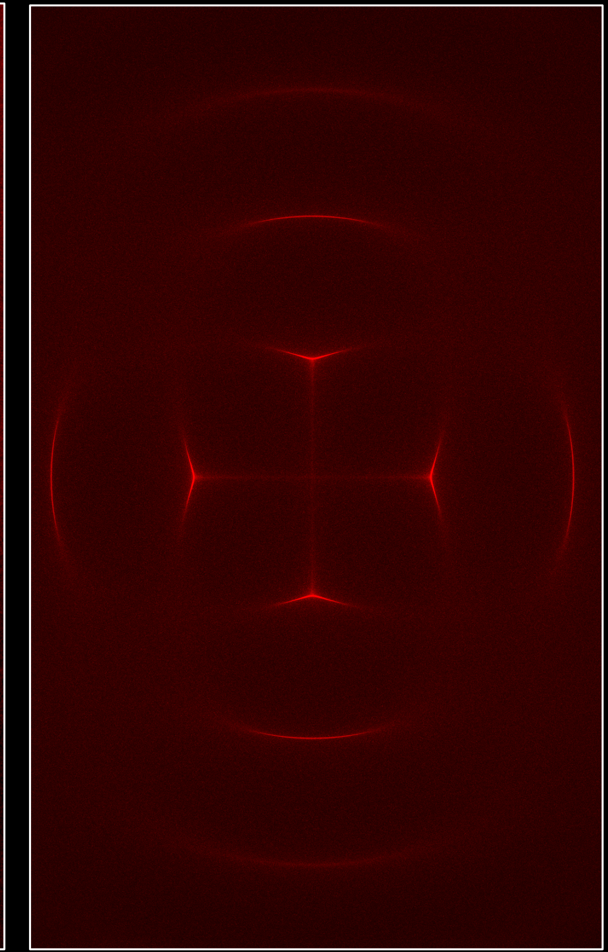
experimental capture

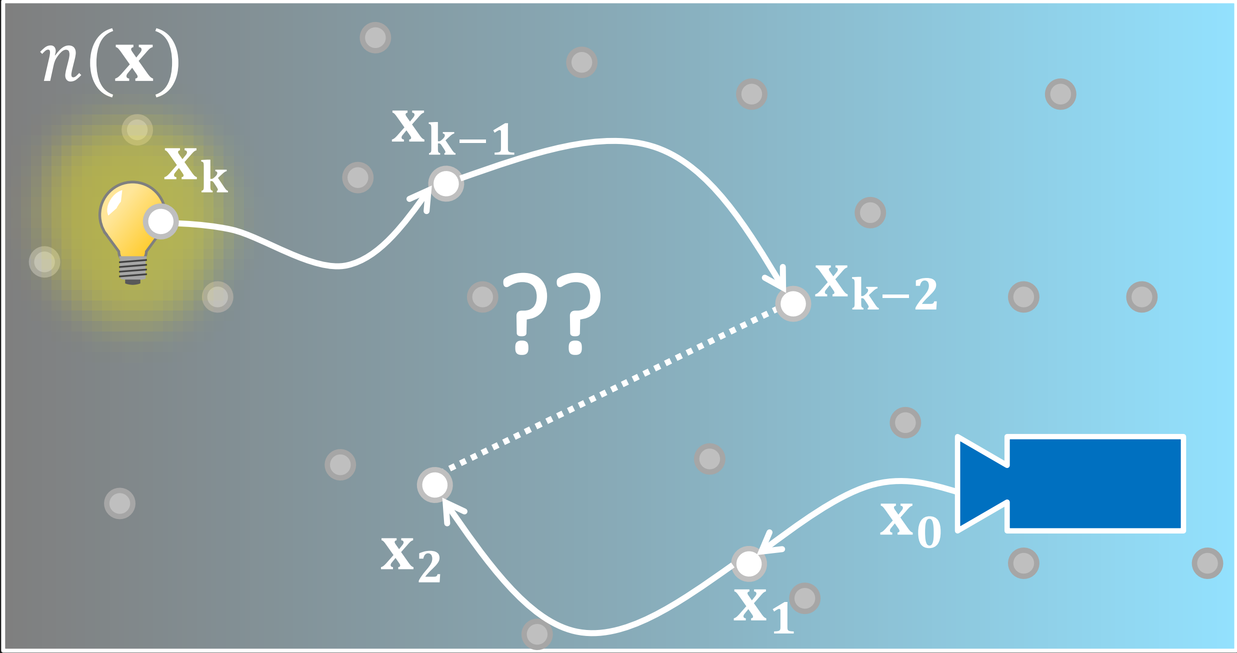


photon mapping

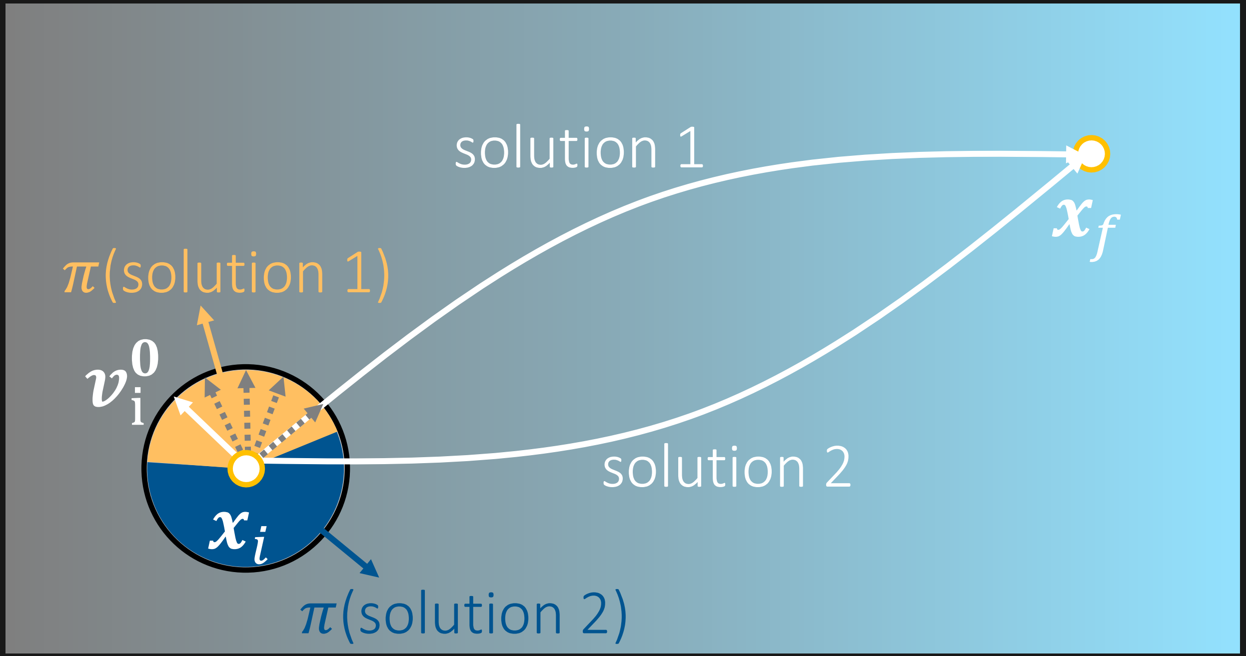


unbiased (ours)

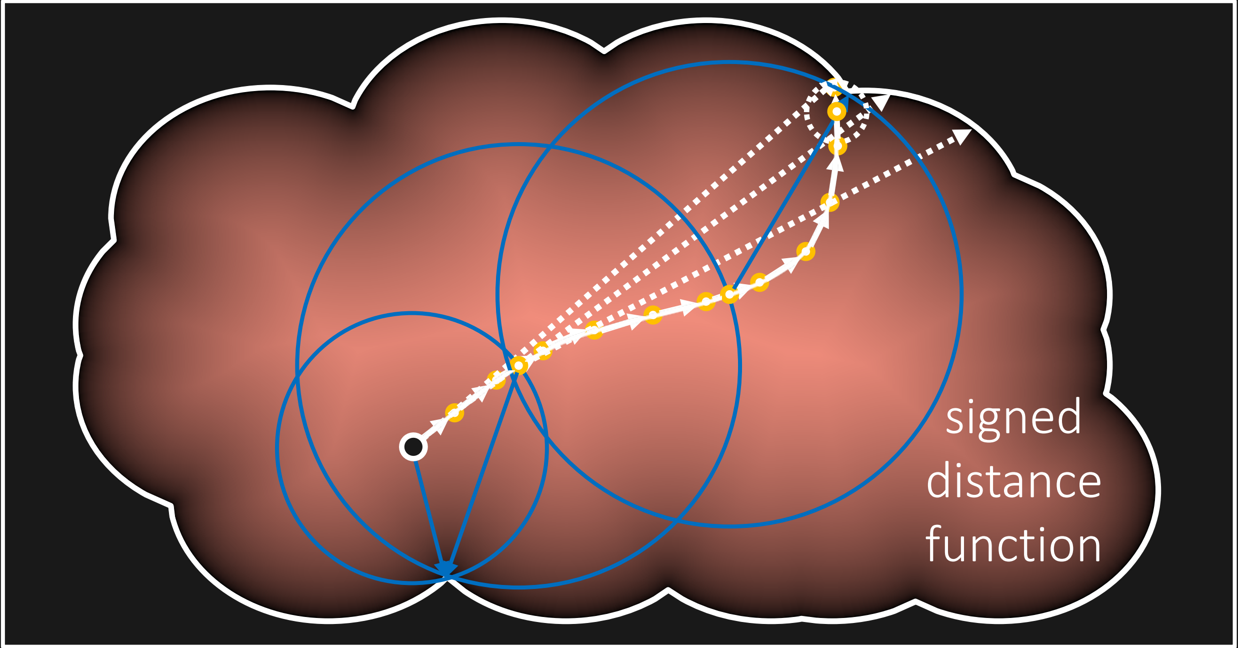




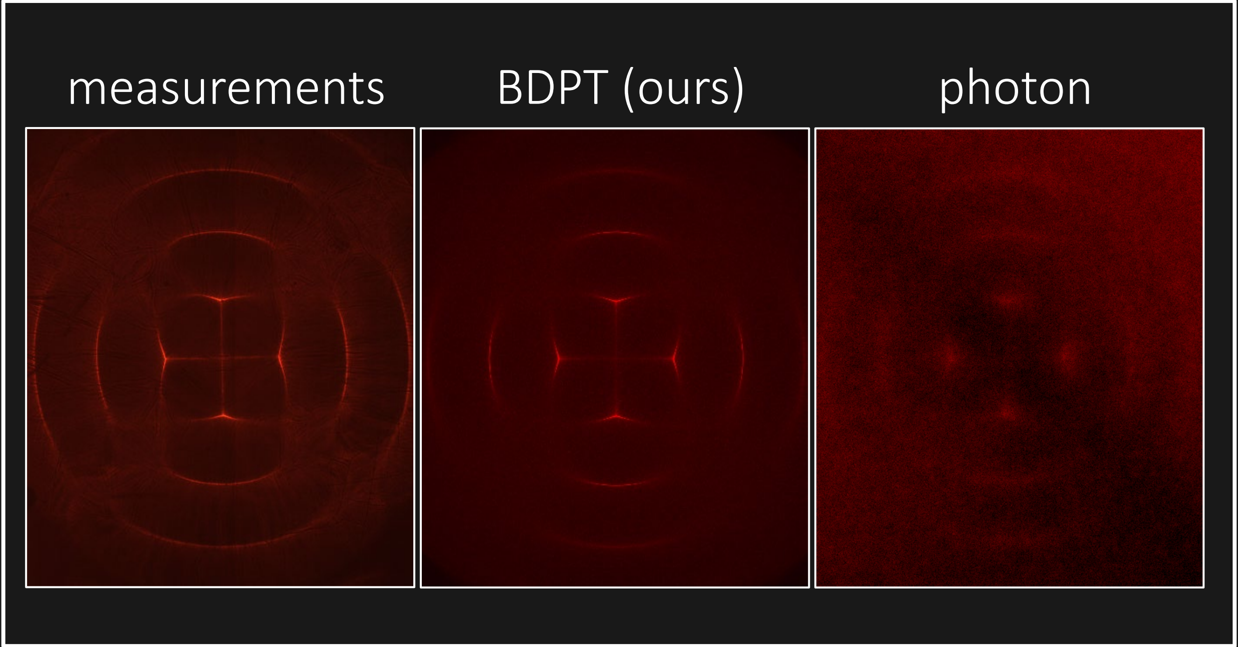
1. background on refractive radiative transfer equation



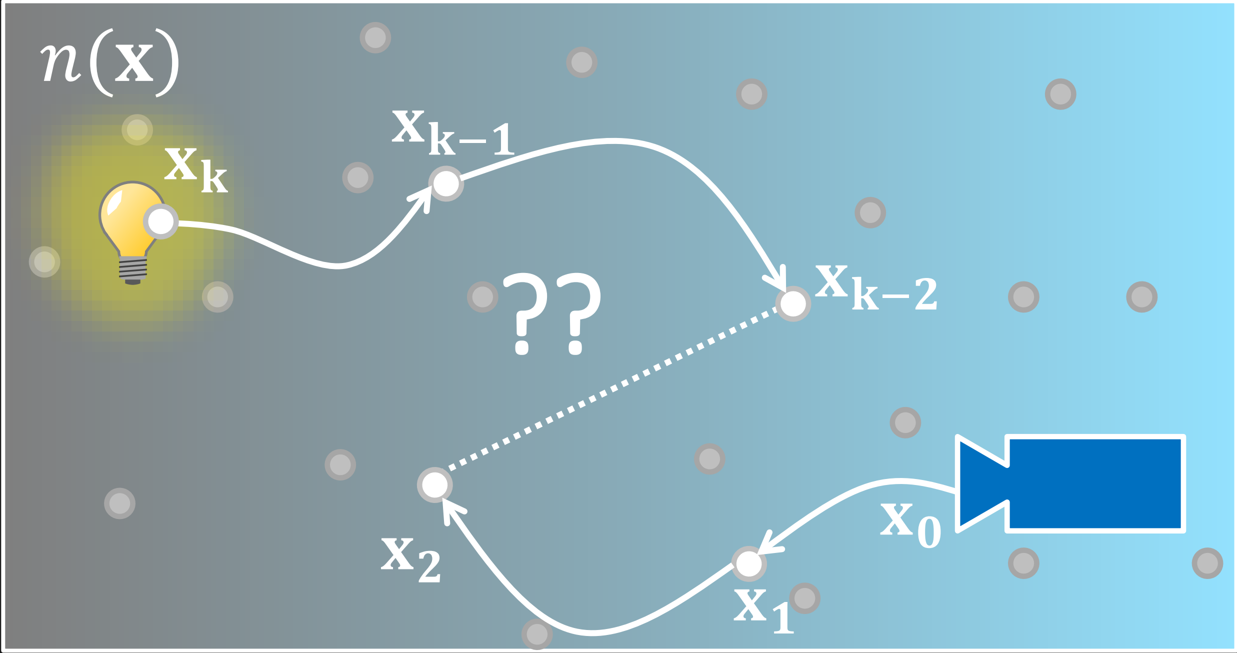
2. direct connections: our solution to unbiased rendering



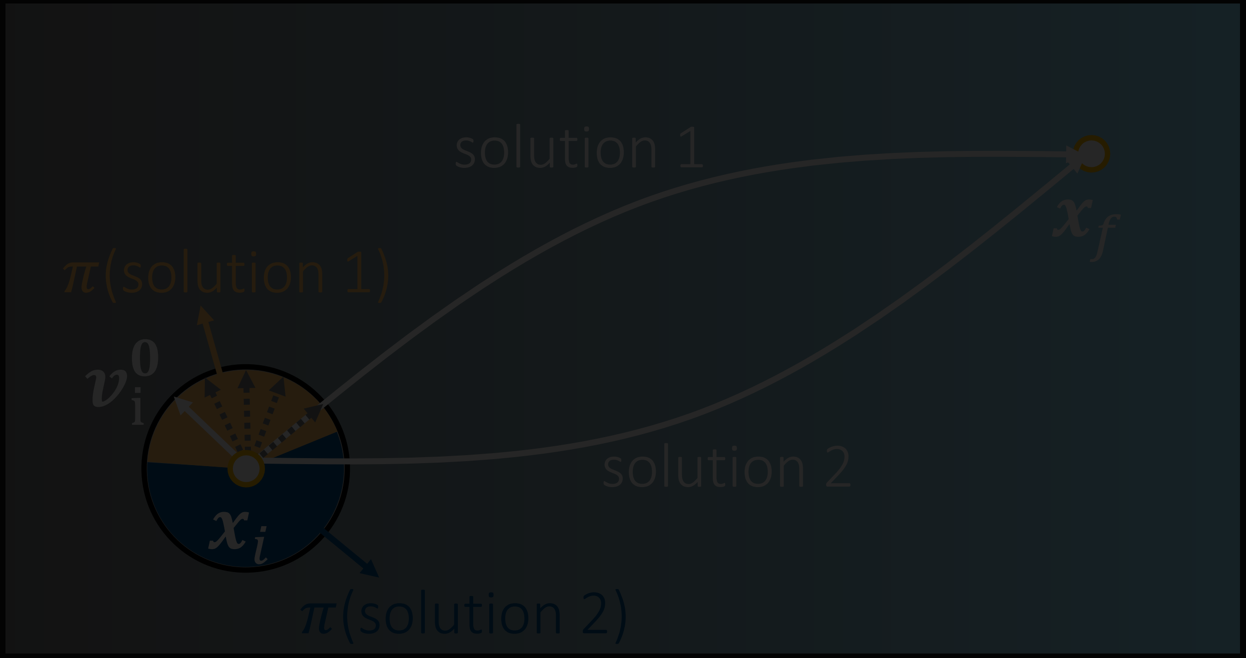
3. acceleration techniques



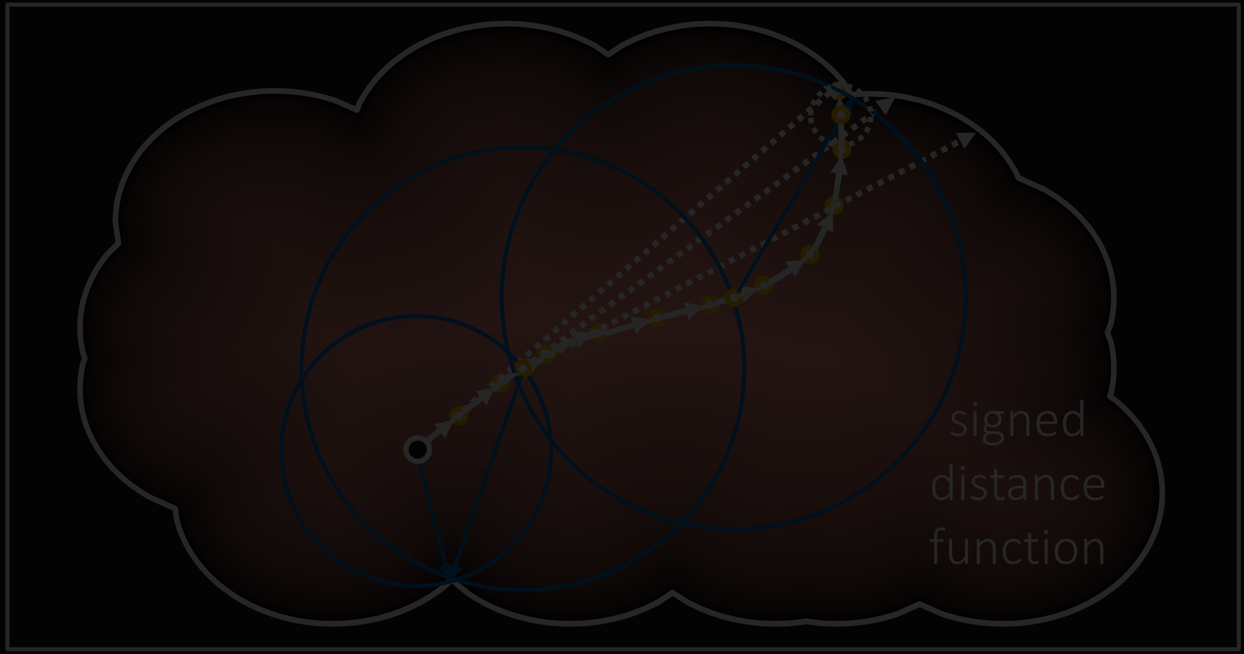
4. experiments



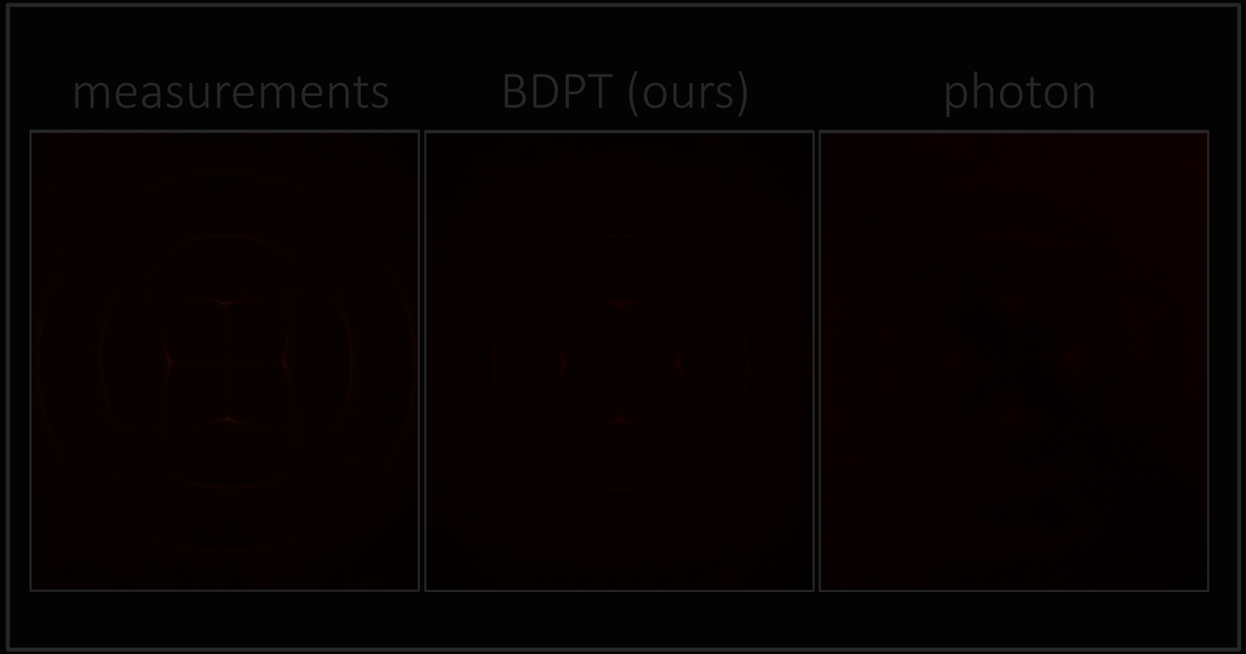
1. background on refractive radiative transfer equation



2. direct connections: our solution to unbiased rendering

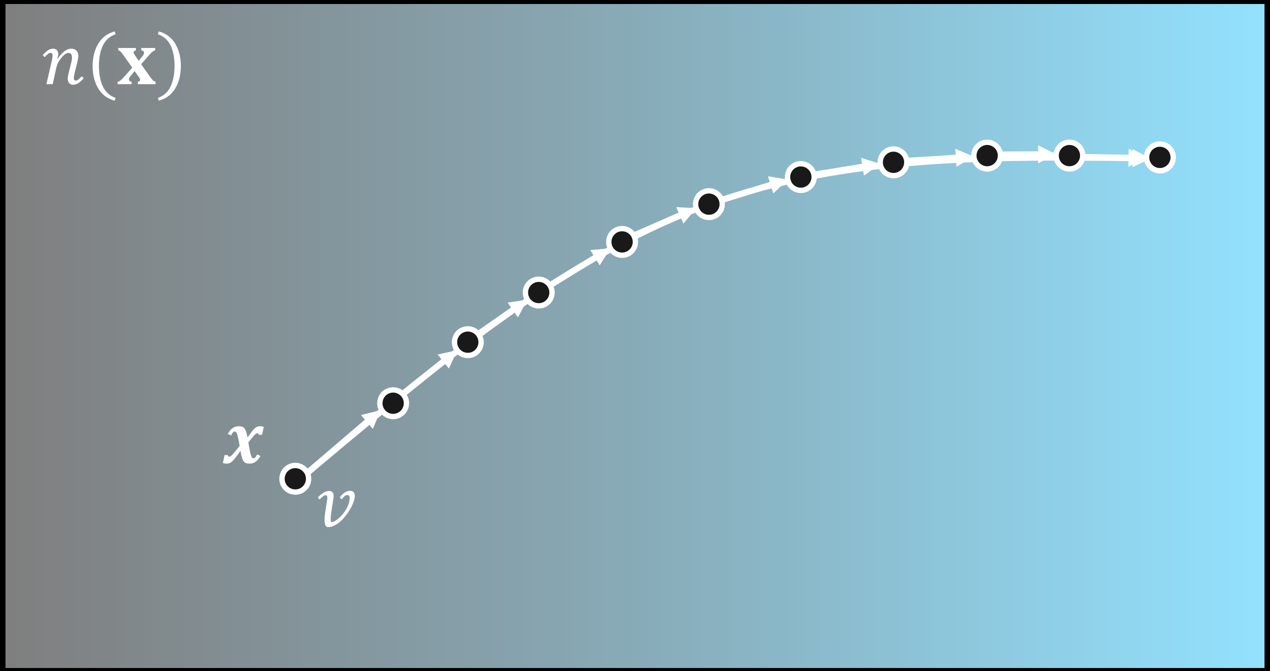


3. acceleration techniques



4. experiments

continuous refraction and no scattering

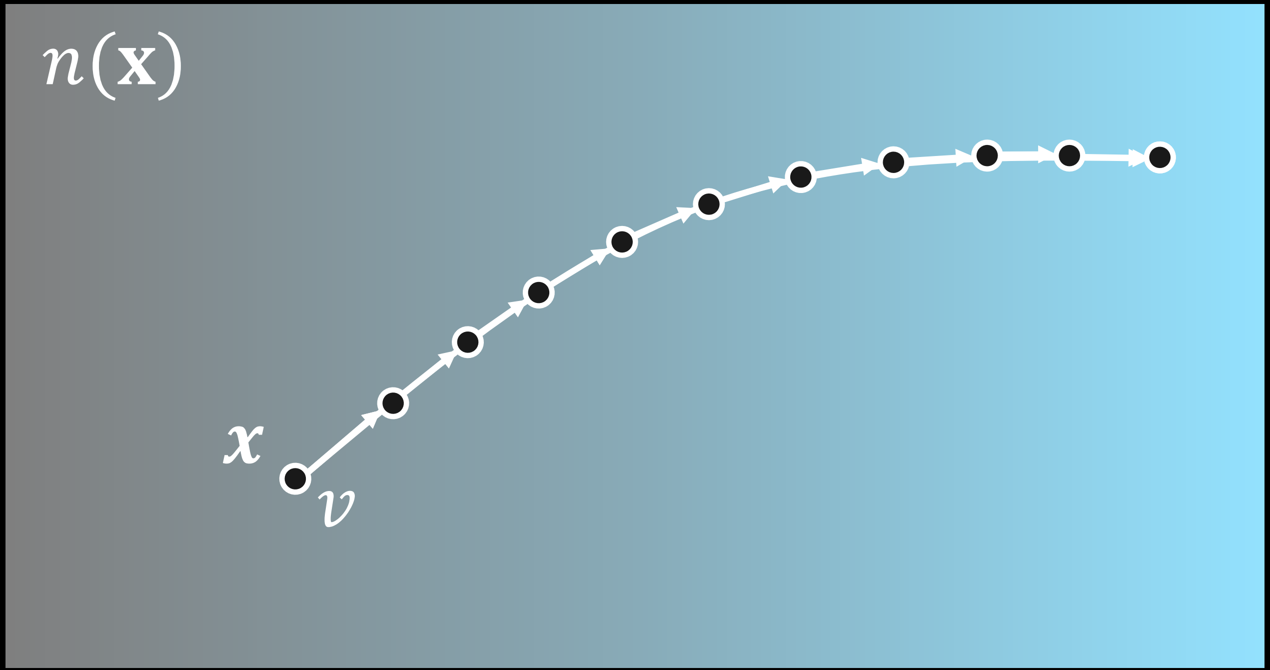


Hamilton's equations for refractive ray tracing

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x})$$

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})}$$

continuous refraction and no scattering



Hamilton's equations for refractive ray tracing

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x})$$

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})}$$

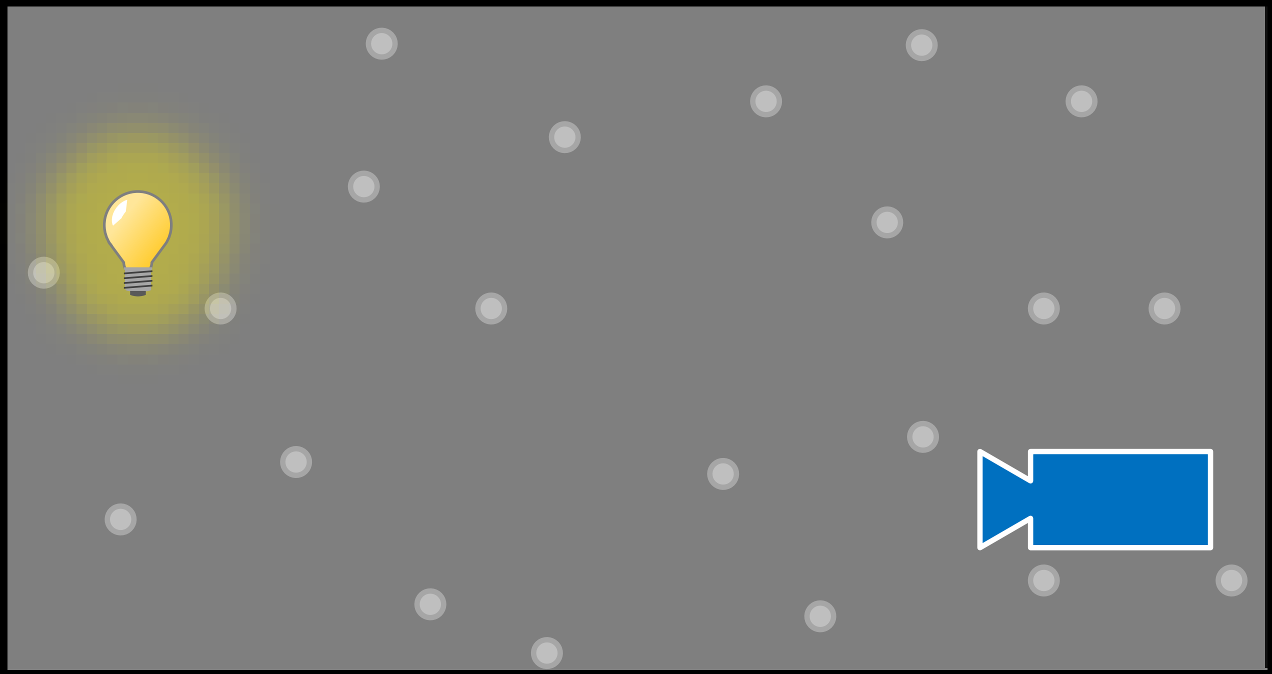
solved using symplectic integration

scattering and no continuous refraction

radiative transfer equation (RTE)

$$\frac{\mathrm{d}L}{\mathrm{d}s} = \sigma_a L_e - (\sigma_a + \sigma_s) L + \frac{\sigma_s}{4\pi} \int f_s(\omega', \omega) L \mathrm{d}\omega'$$

scattering and no continuous refraction

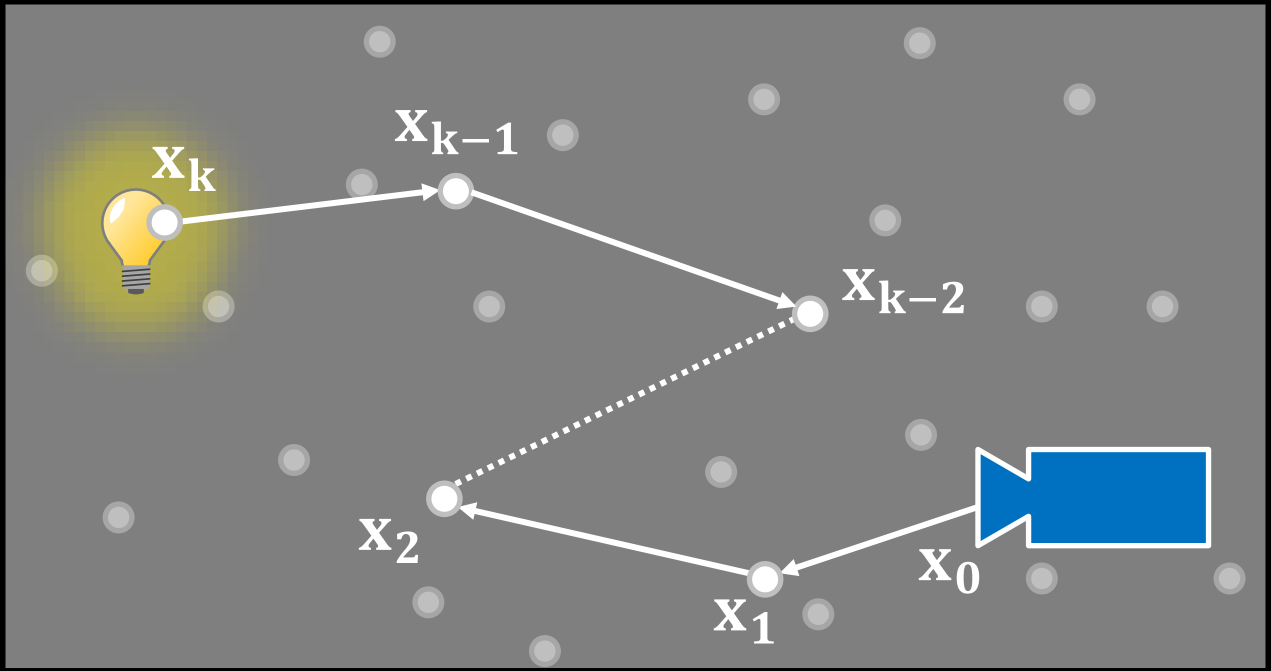


radiative transfer equation (RTE)

$$\frac{\mathrm{d}L}{\mathrm{d}s} = \sigma_a L_e - (\sigma_a + \sigma_s) L + \frac{\sigma_s}{4\pi}\int f_s(\omega', \omega) L \mathrm{d}\omega'$$

solved using Monte Carlo integration

scattering and no continuous refraction



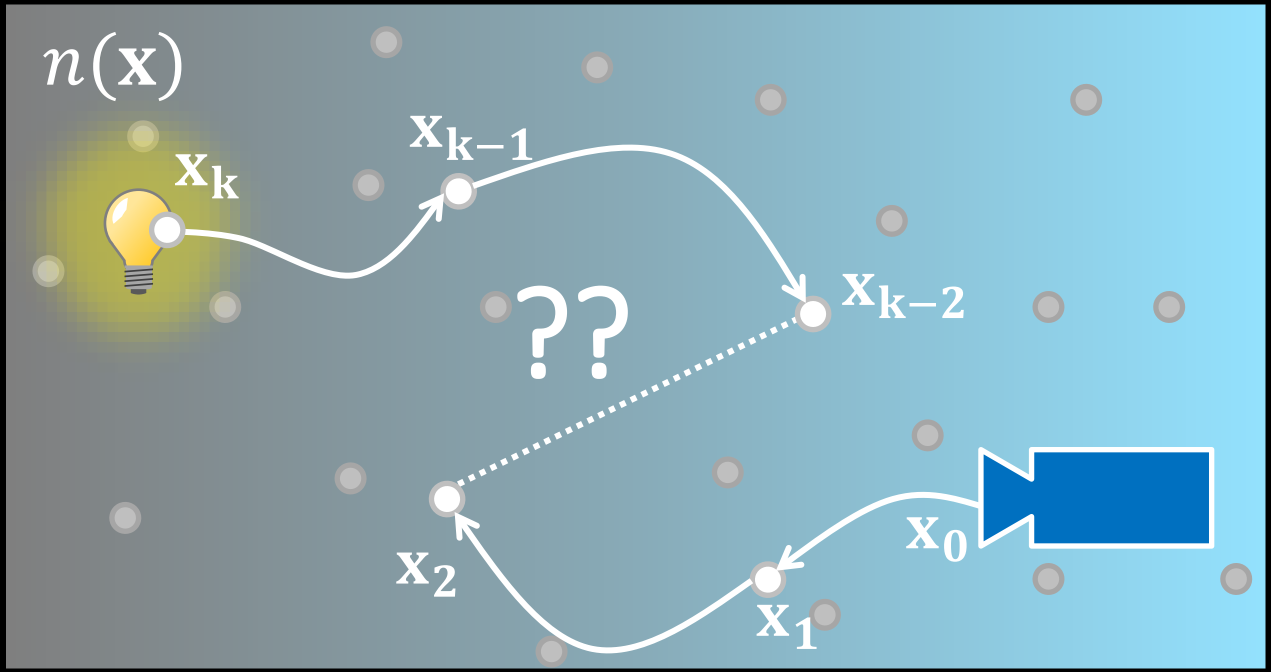
bidirectional path tracing (BDPT):

1. trace a random sensor subpath

2. trace a random emitter subpath

3. join vertices with a straight line

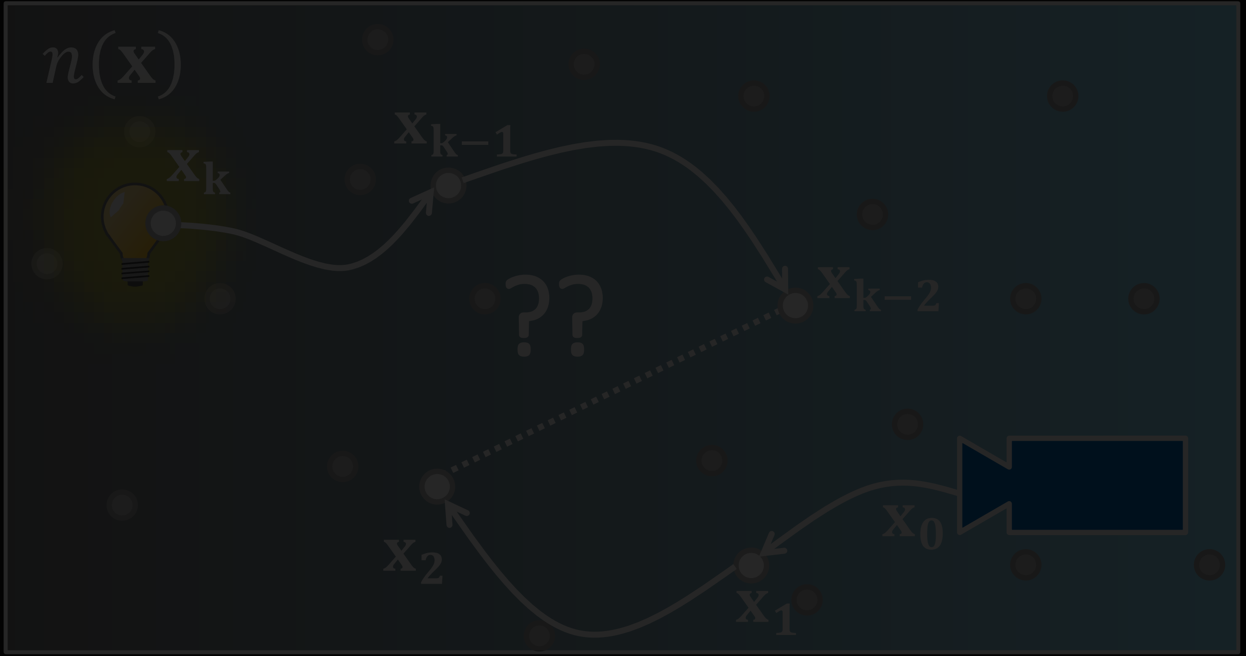
continuous refraction and scattering



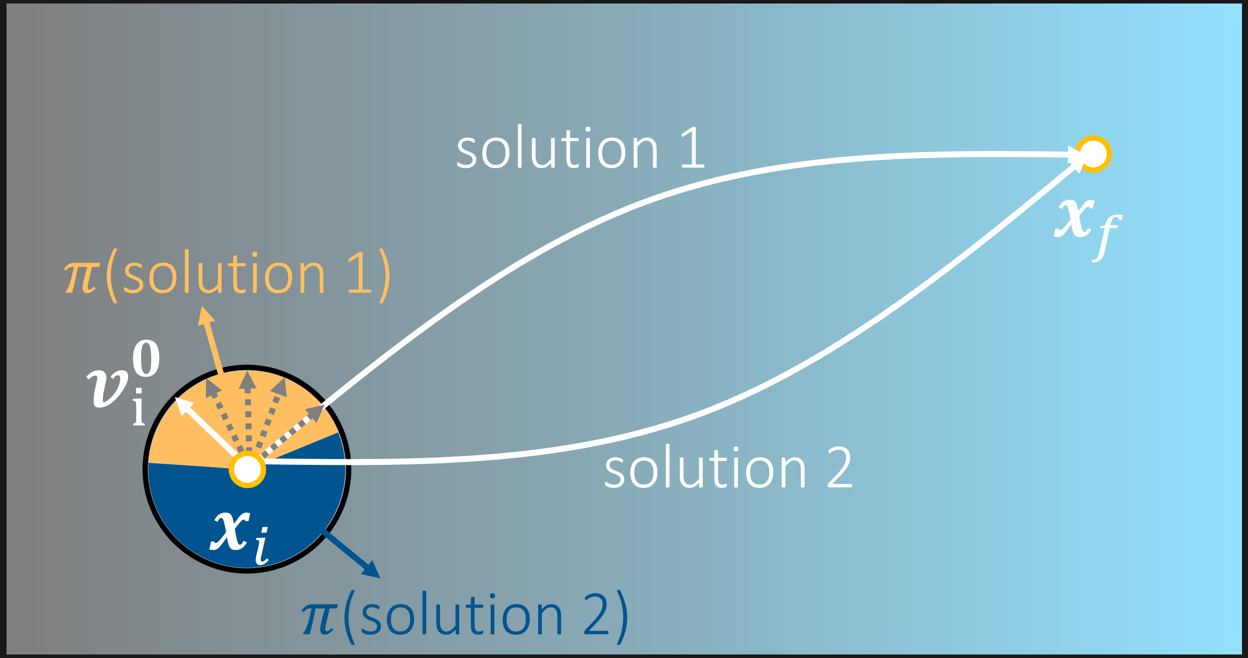
bidirectional path tracing (BDPT):

1. trace a random sensor subpath
   use refractive ray tracing
2. trace a random emitter subpath

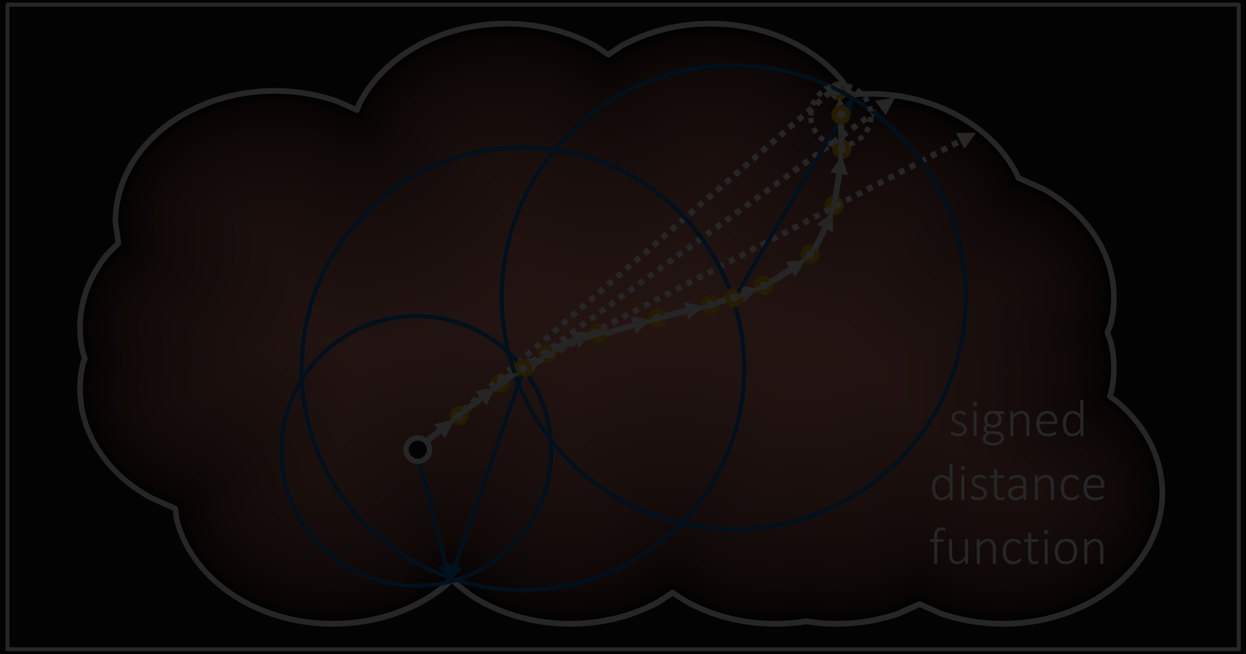
3. join vertices with a ~~straight line~~ curve



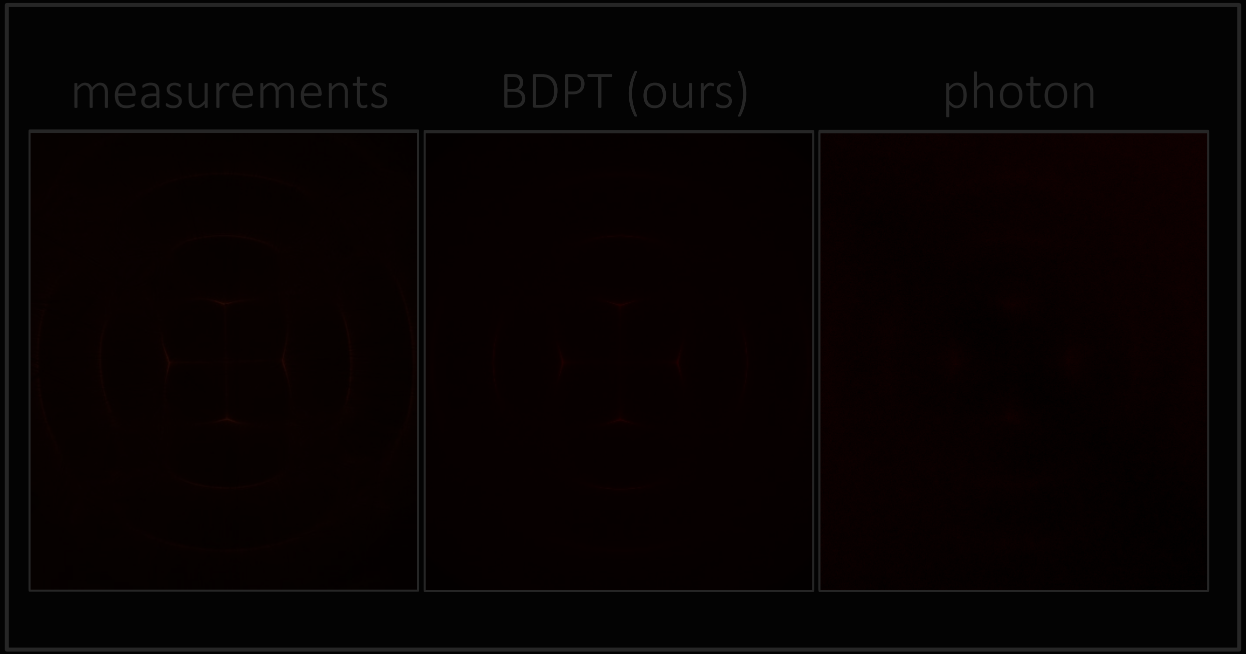
1. background on refractive radiative transfer equation



2. direct connections: our solution to unbiased rendering



3. acceleration techniques



4. experiments

direct connections

we have to solve this:

$$\boxed{\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x}), \quad \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})}}$$

boundary conditions: $\boldsymbol{x}_i, \boldsymbol{x}_f$

boundary value problem (BVP)

we know how to solve this:

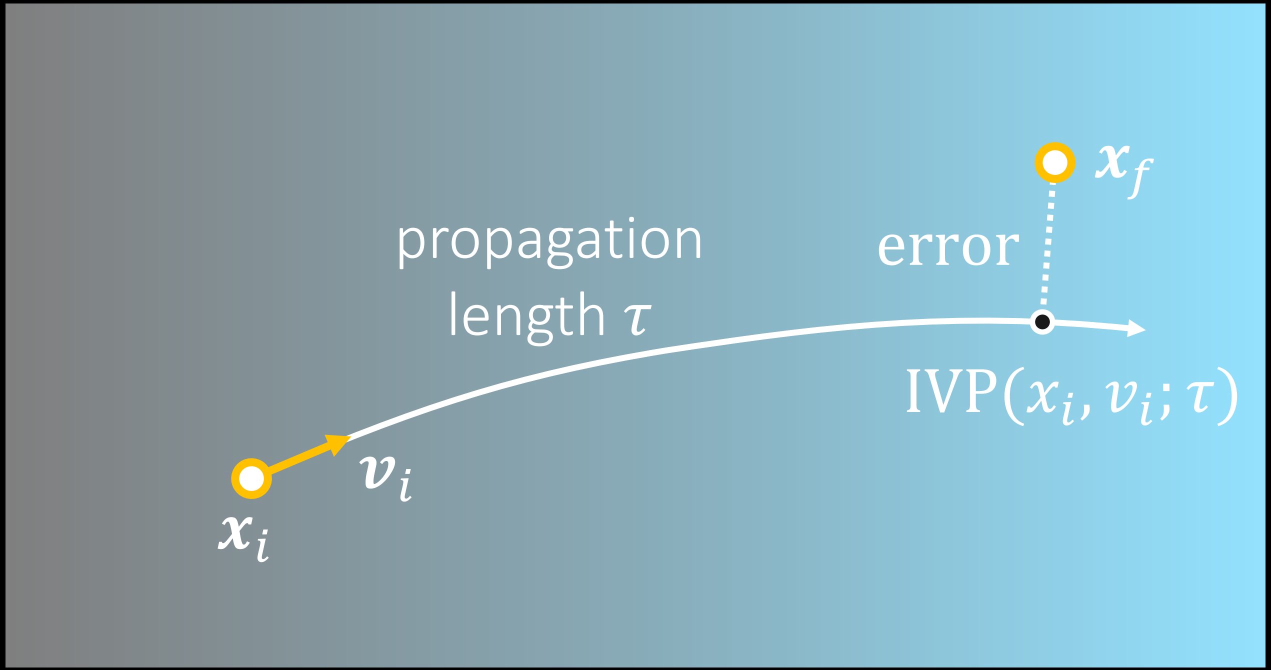
$$\boxed{\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x}), \quad \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})}}$$

boundary conditions: $\boldsymbol{x}_i, \boldsymbol{v}_i$

initial value problem (IVP),

a.k.a. refractive ray tracing

direct connections



$$\mathrm{error}\left(x_f, x_i, v_i\right) \equiv \min_{\tau} \left\| x_f - \mathrm{IVP}(x_i, v_i; \tau) \right\|^2$$

we have to solve this:

$$\boxed{\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x}), \quad \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})} \quad \text{boundary conditions: } \boldsymbol{x}_i, \boldsymbol{x}_f}$$

boundary value problem (BVP)

we know how to solve this:

$$\boxed{\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x}), \quad \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})} \quad \text{boundary conditions: } \boldsymbol{x}_i, \boldsymbol{v}_i}$$

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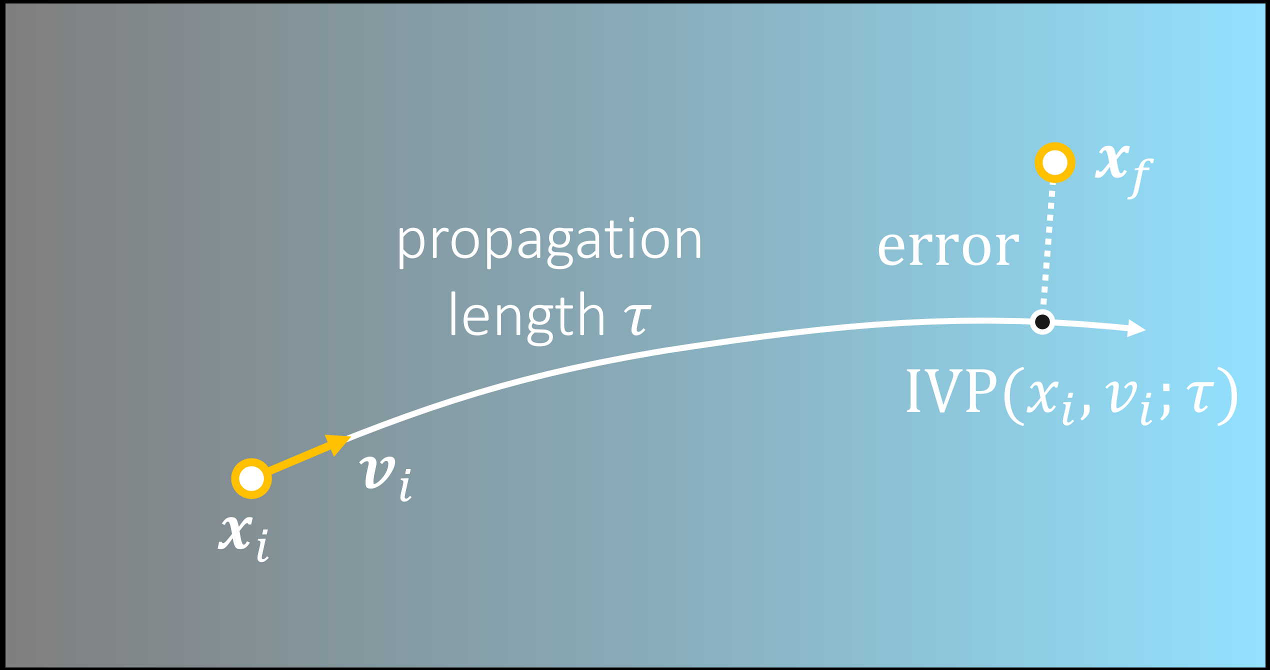

$$\mathrm{error}\left(x_f, x_i, v_i\right) \equiv \min_{\tau} \left\| x_f - \mathrm{IVP}(x_i, v_i; \tau) \right\|^2$$

we have to solve this:

$$\min_{v_i} \mathrm{error}\left(x_f, x_i, v_i\right)$$

boundary conditions: $\boldsymbol{x}_i, \boldsymbol{x}_f$

boundary value problem (BVP)

we know how to solve this:

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x}), \quad \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})}$$

boundary conditions: $\boldsymbol{x}_i, \boldsymbol{v}_i$

initial value problem (IVP),
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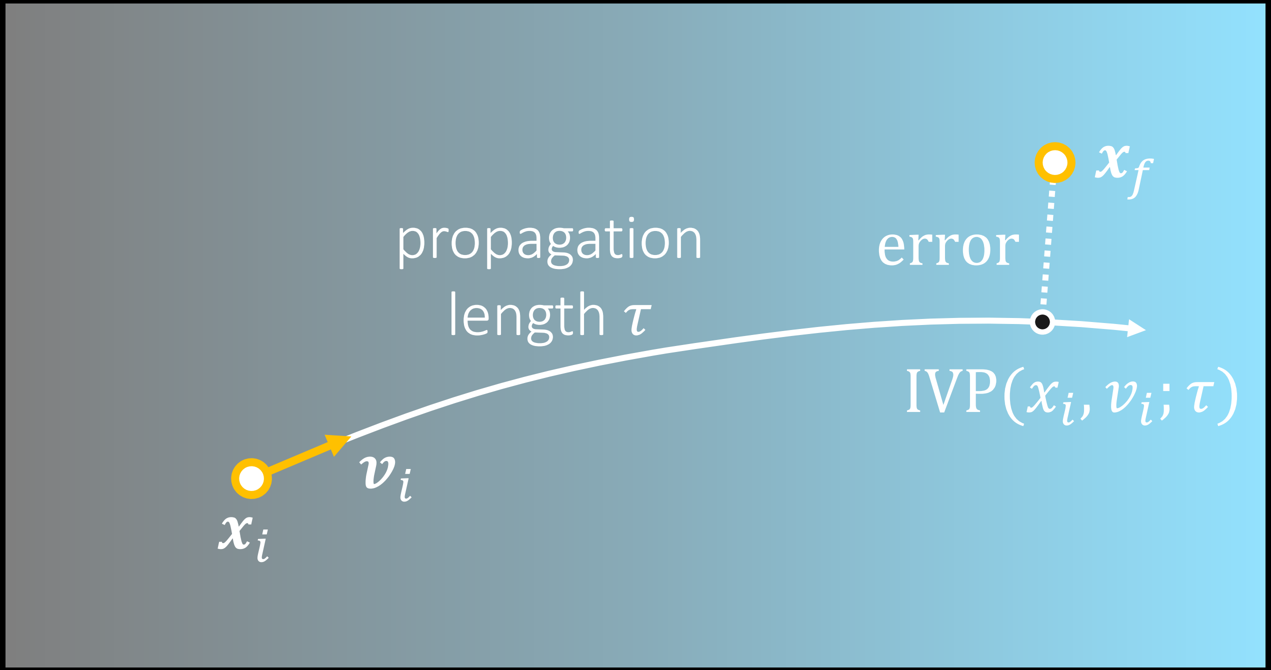
direct connections



differentiable

$$\boxed{\mathrm{error}\left(x_f, x_i, v_i\right) \equiv \min_{\tau} \left\| x_f - \mathrm{IVP}(x_i, v_i; \tau) \right\|^2}$$

we have to solve this:

$$\min_{v_i} \mathrm{error}\left(x_f, x_i, v_i\right)$$

boundary conditions: $\boldsymbol{x}_i, \boldsymbol{x}_f$

boundary value problem (BVP)

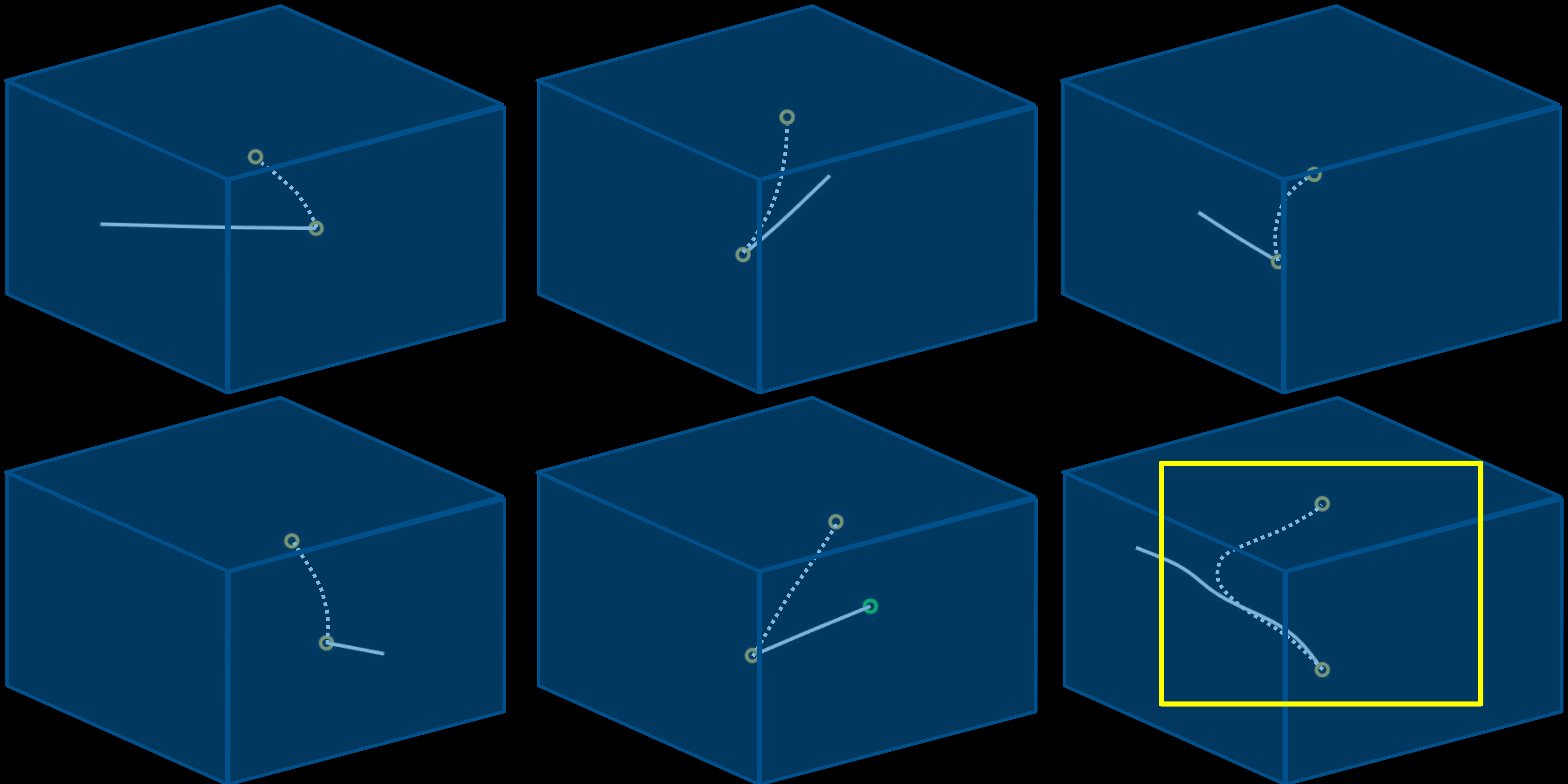
differentiable

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}s} = \nabla_x n(\boldsymbol{x}), \quad \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \frac{\boldsymbol{v}}{n(\boldsymbol{x})}$$

boundary conditions: $\boldsymbol{x}_i, \boldsymbol{v}_i$

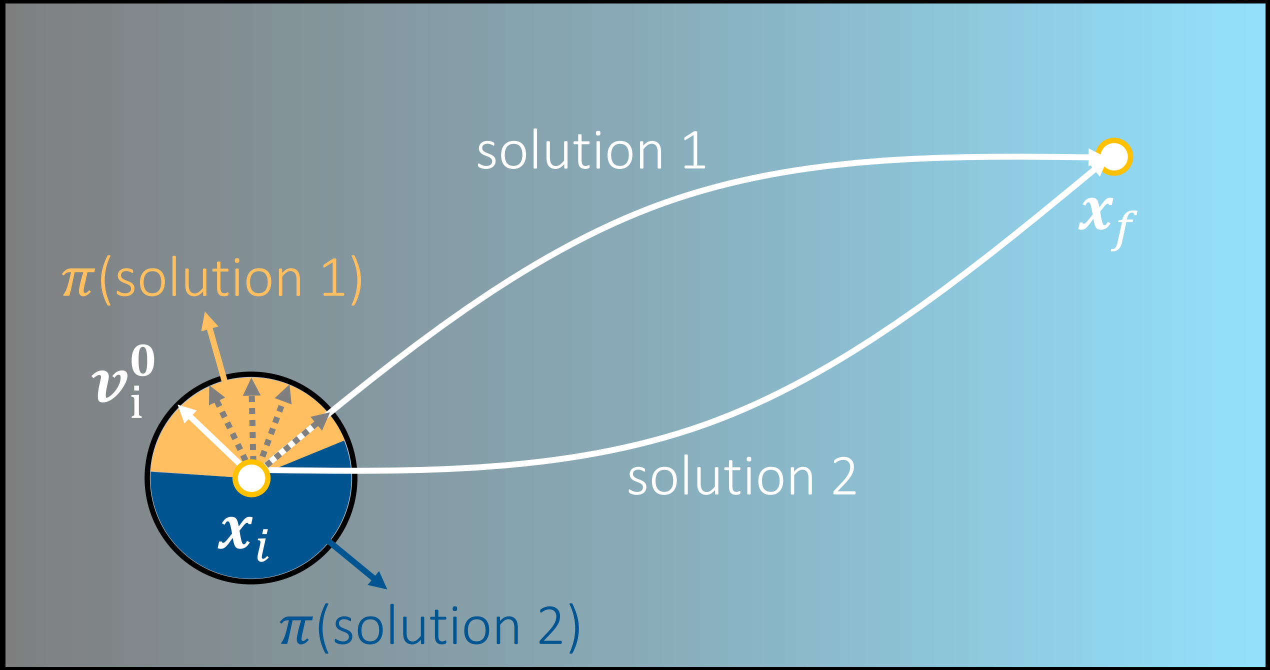
initial value problem (IVP),
a.k.a. refractive ray tracing

direct connections



multiple direct connections

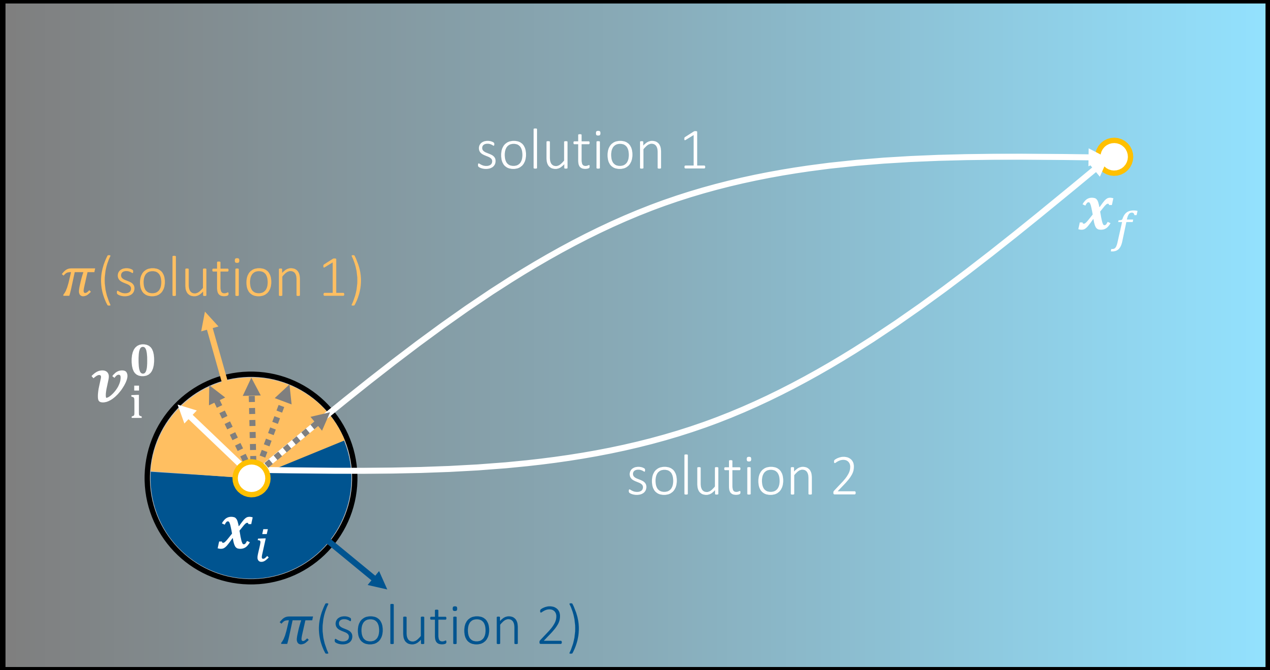
$$\text{total throughput} = \sum_{\text{all solutions}} \text{throughput(solution)}$$



approach 1:
exhaustively enumerate all solutions

multiple direct connections

$$\text{total throughput} = \sum_{\text{all solutions}} \text{throughput(solution)}$$



approach 1:
exhaustively enumerate all solutions

impractical

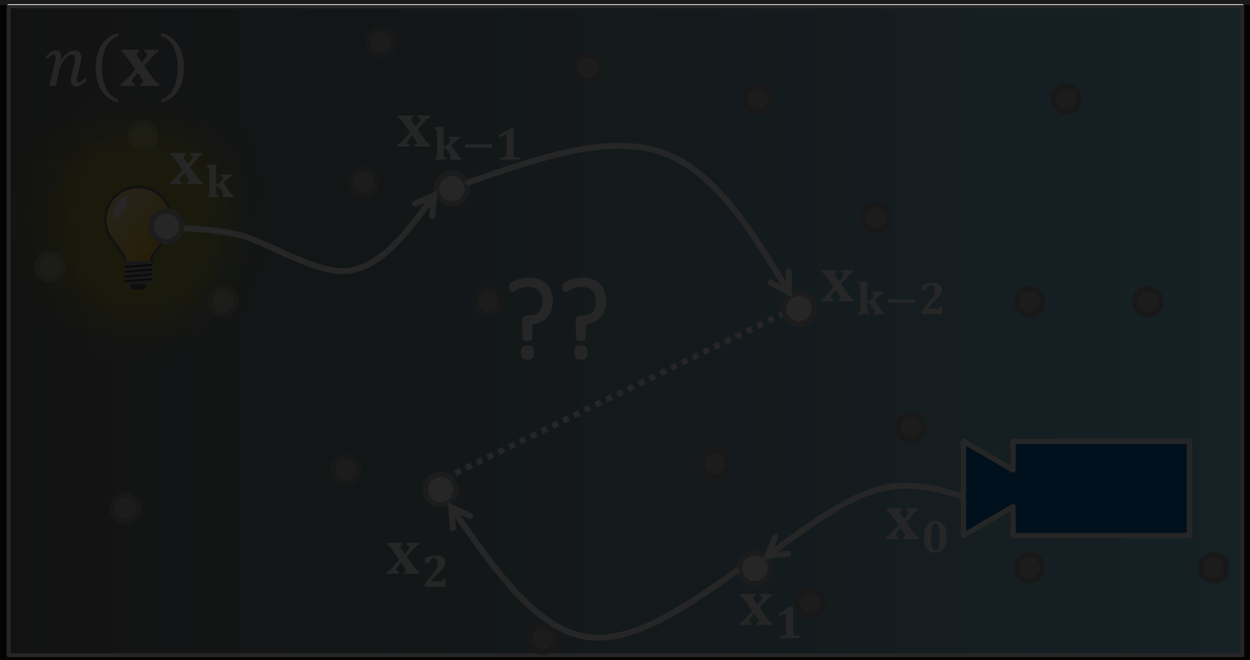
approach 2:
unbiased single-sample Monte Carlo

1. randomly sample initial direction
2. solve BVP
3. form estimate

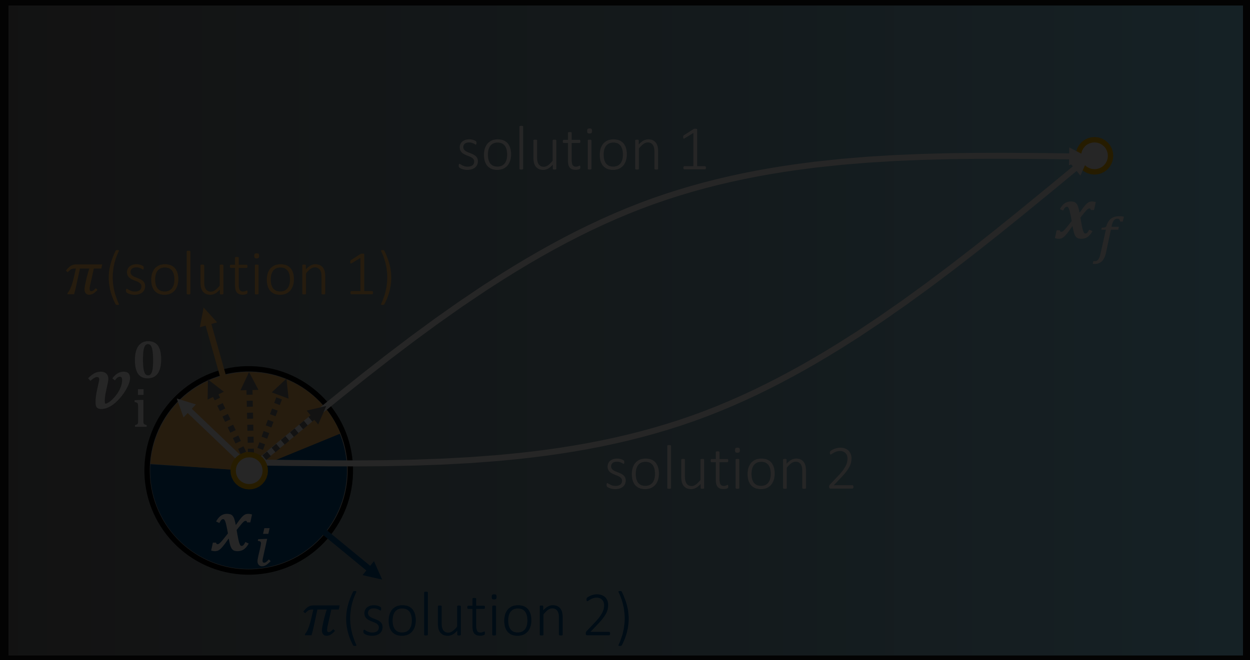
$$\text{total throughput} \approx \frac{\text{throughput(solution)}}{\boxed{\text{probability(solution)}}}$$

set of initial directions that converge to the solution

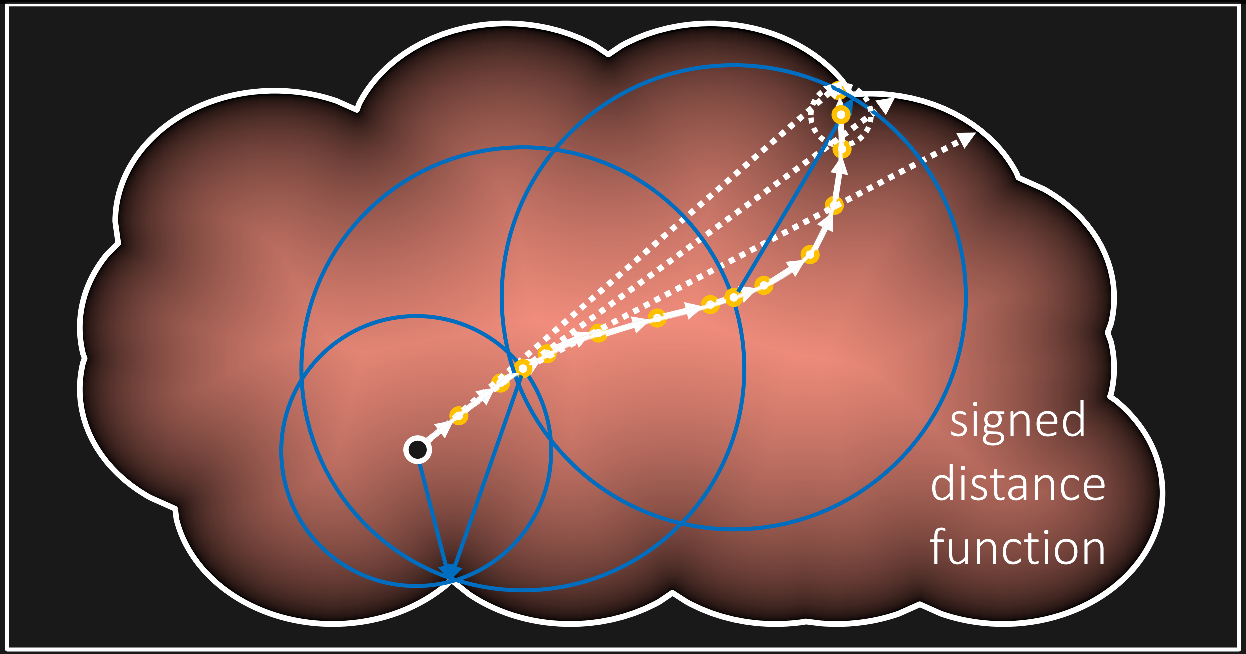
Zeltner et al. "Specular manifold sampling for rendering high-frequency caustics and glints", TOG 2020



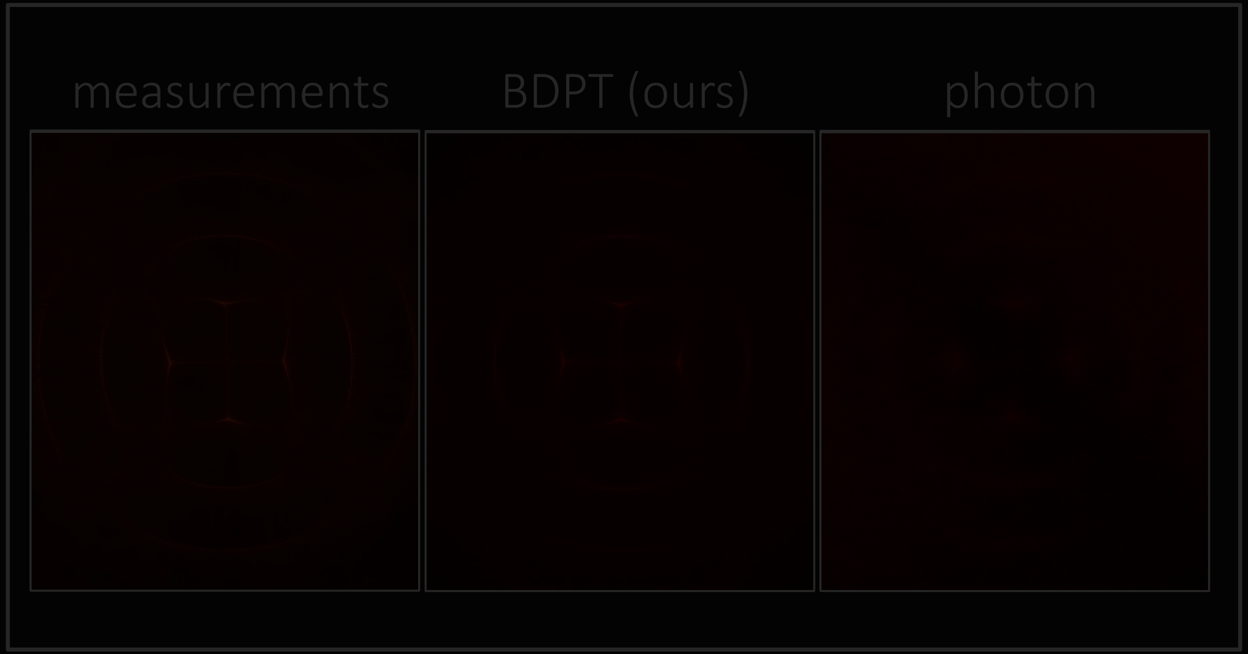
1. background on refractive radiative transfer equation



2. direct connections: our solution to unbiased rendering

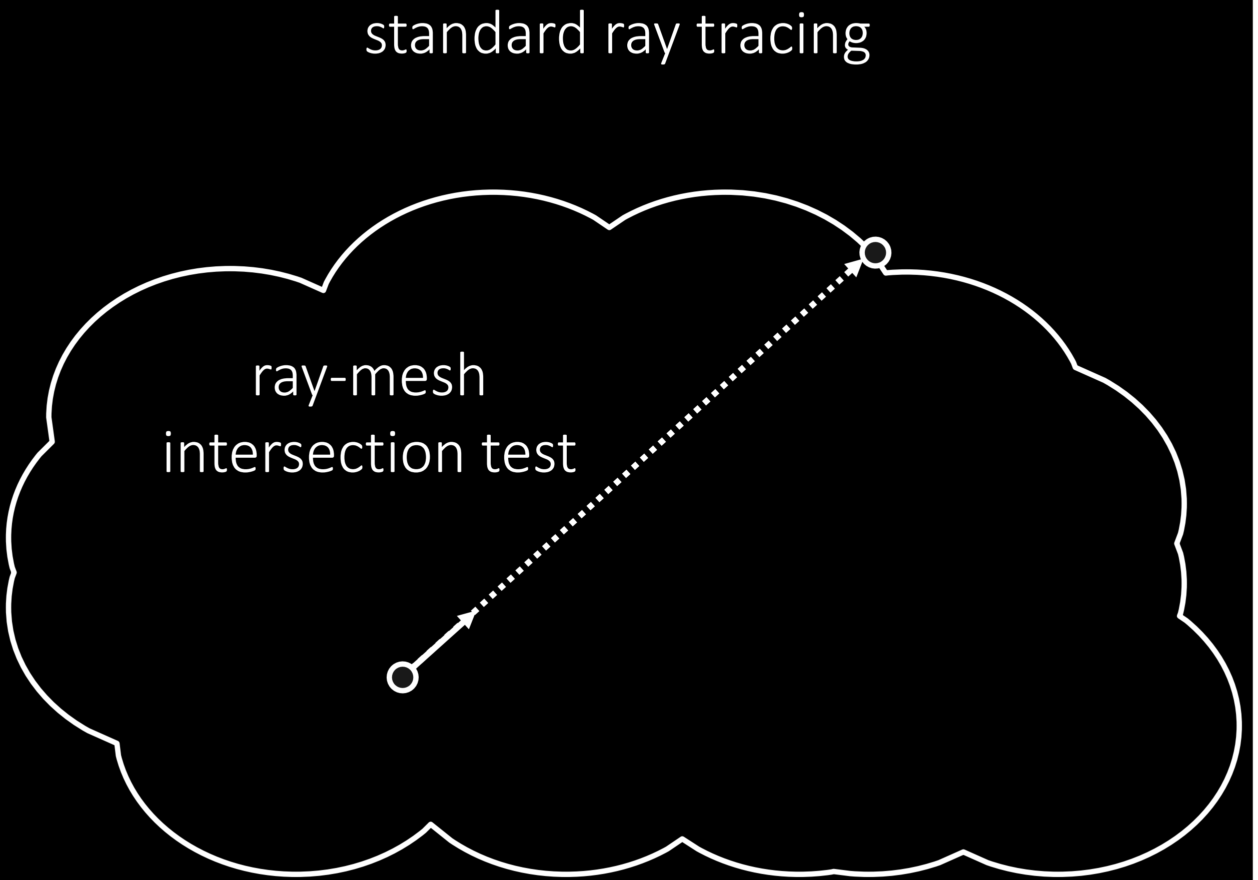


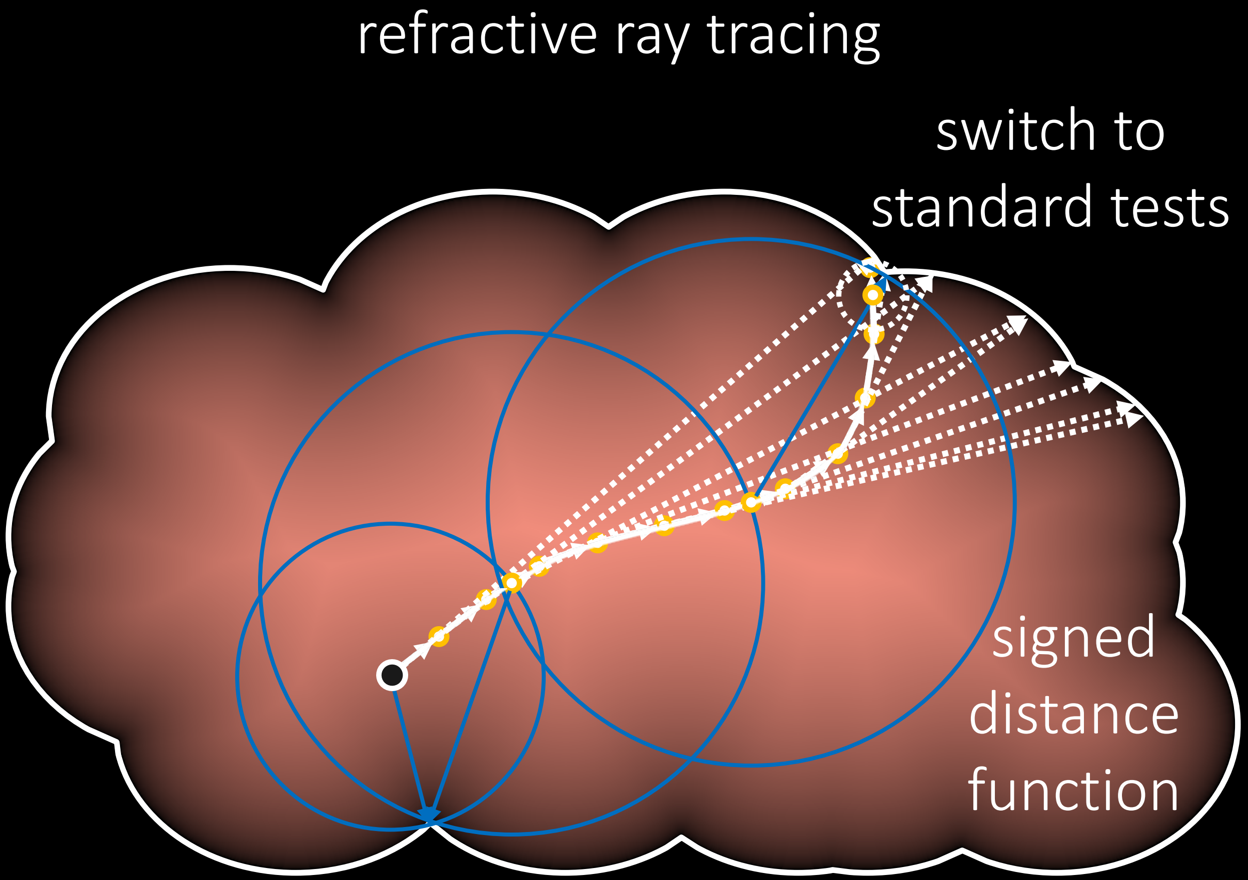
3. acceleration techniques



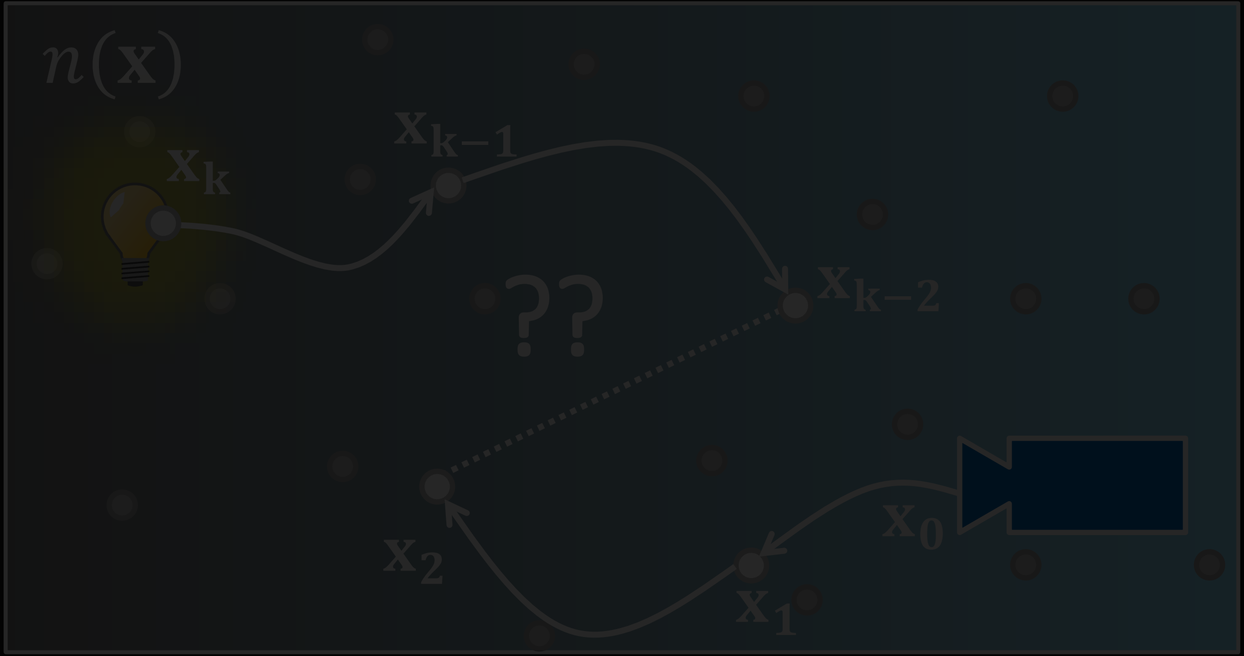
4. experiments

acceleration: sphere tracing

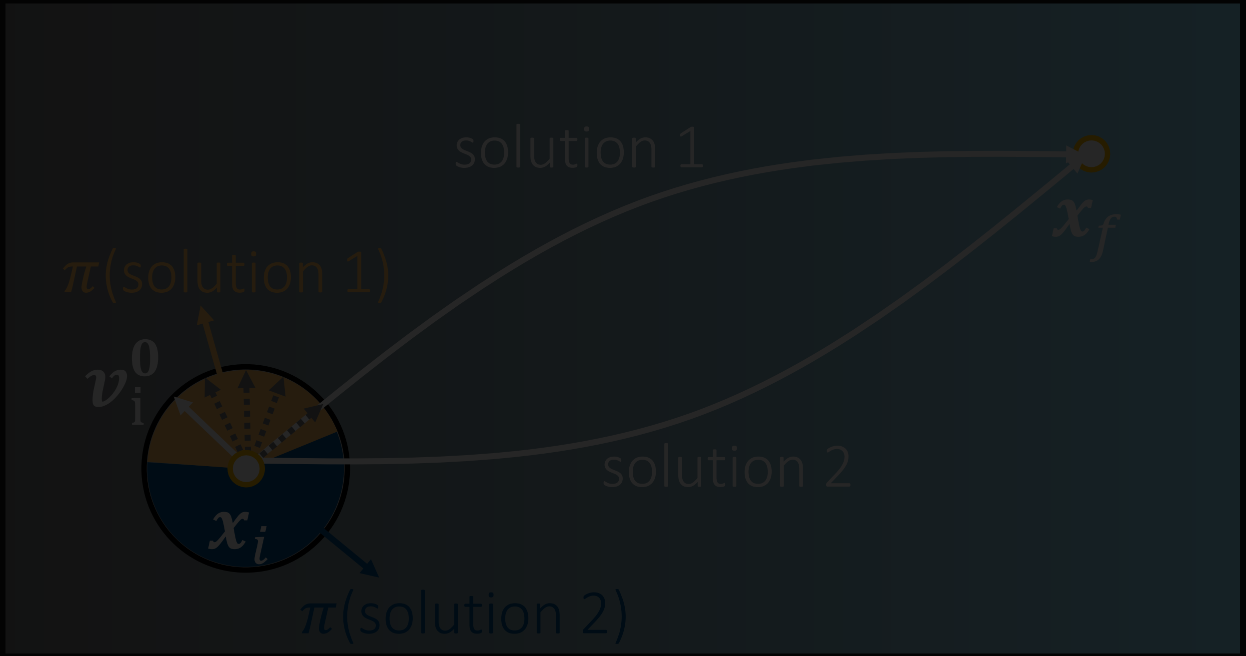




does not introduce bias



1. background on refractive radiative transfer equation



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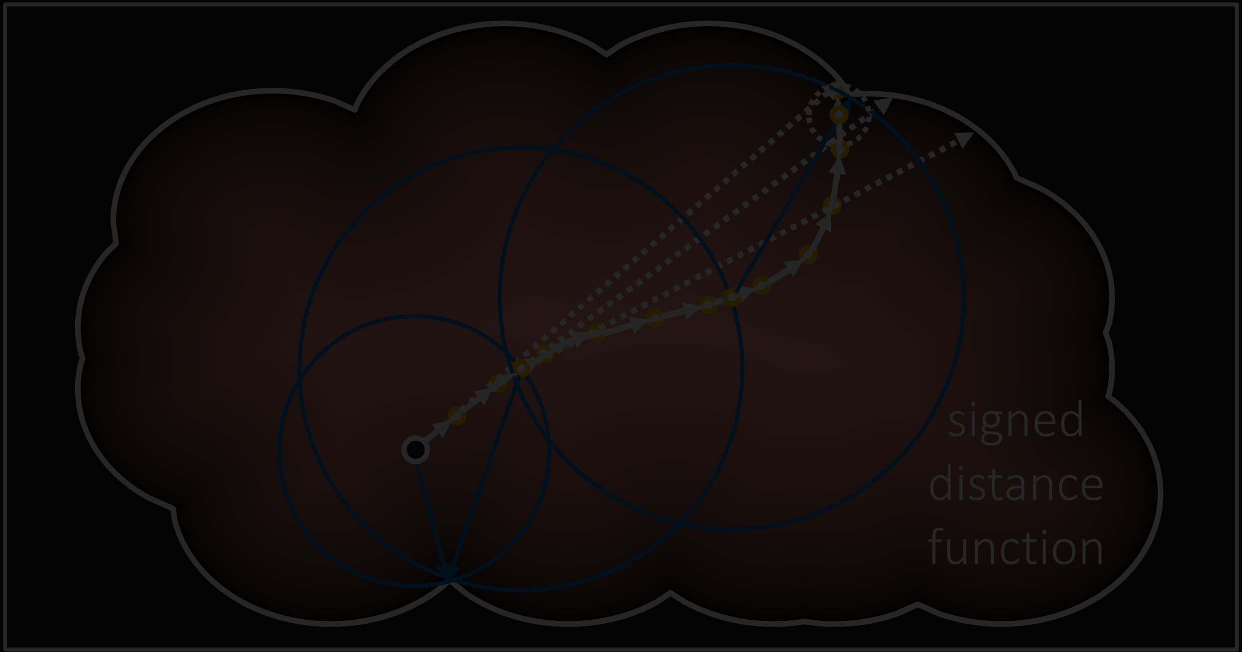
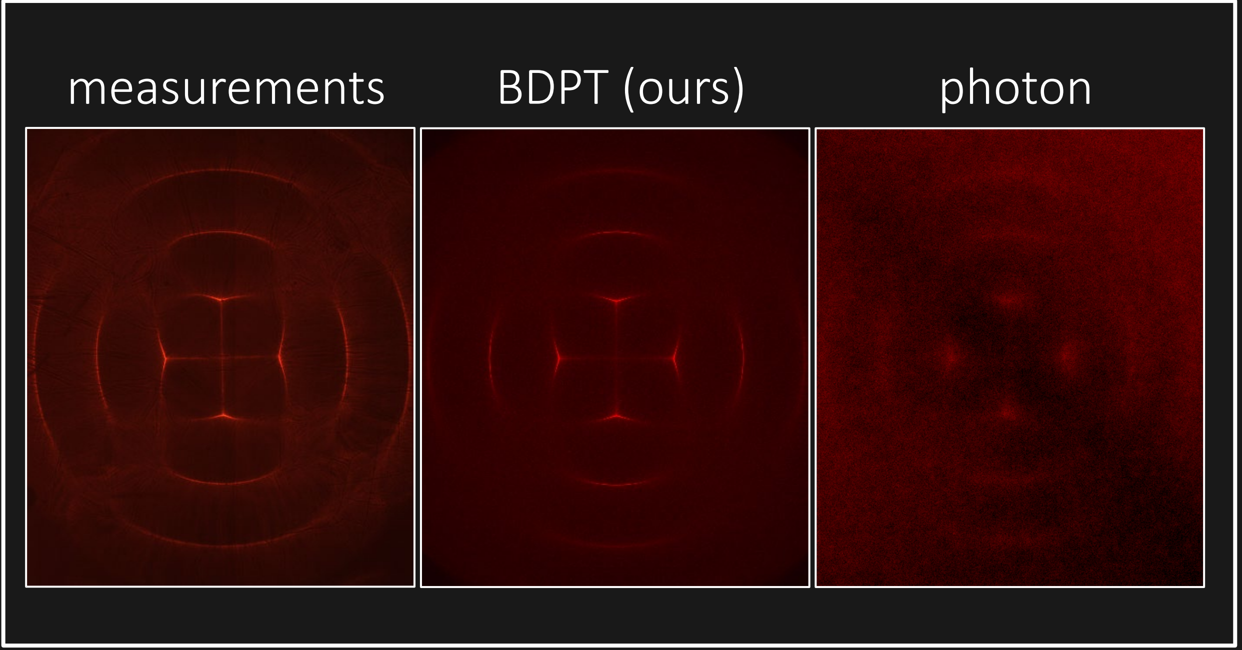
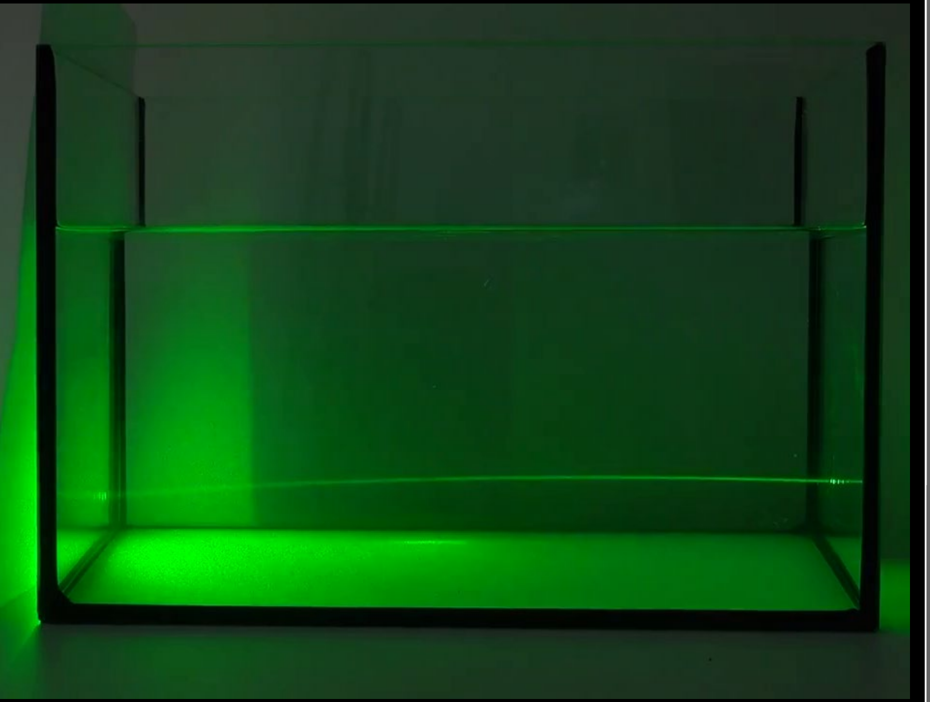


3. acceleration techniques

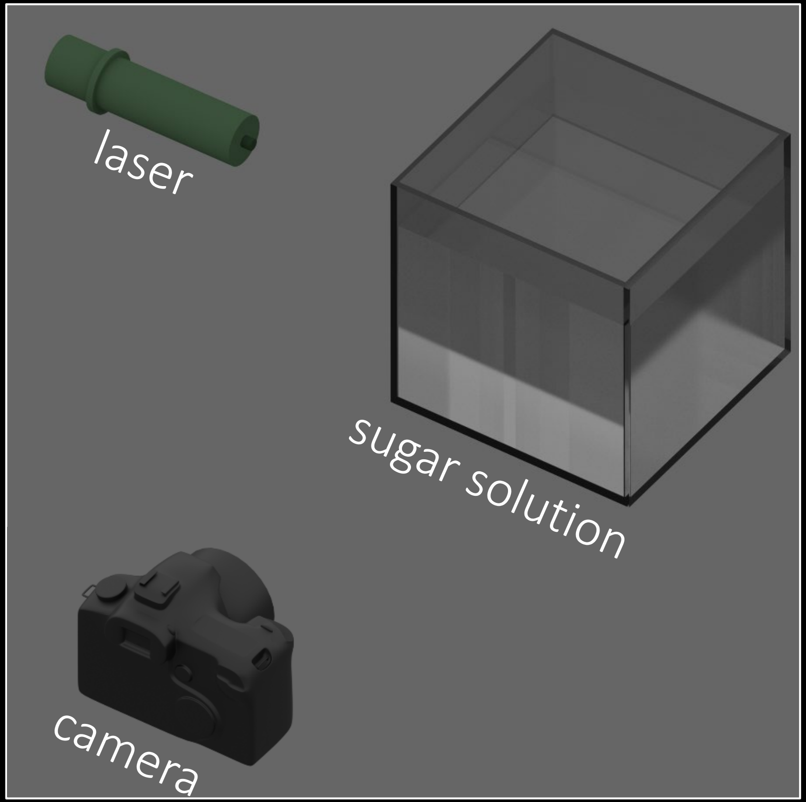


4. experiments

continuously refractive media and scattering



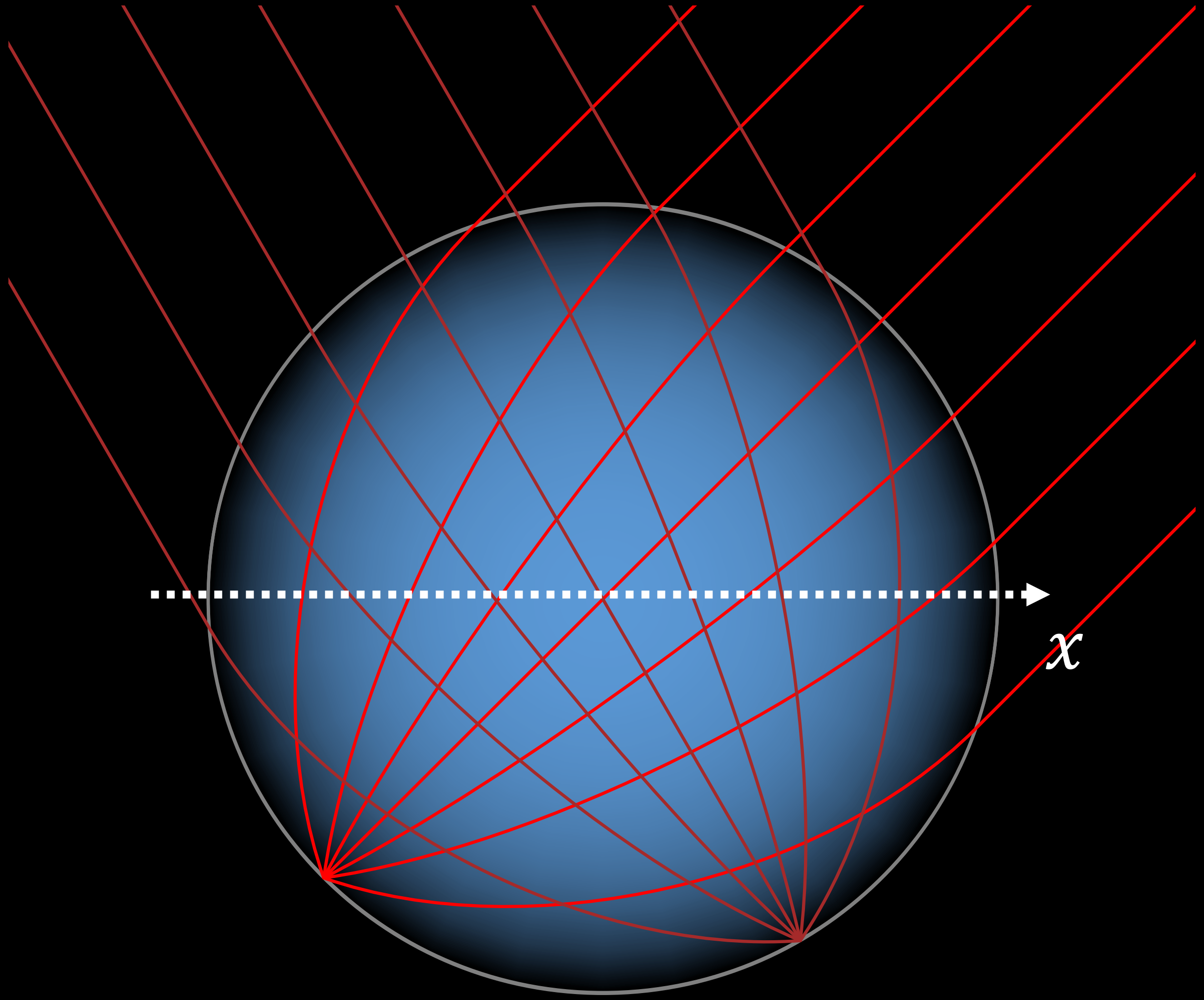
real scene

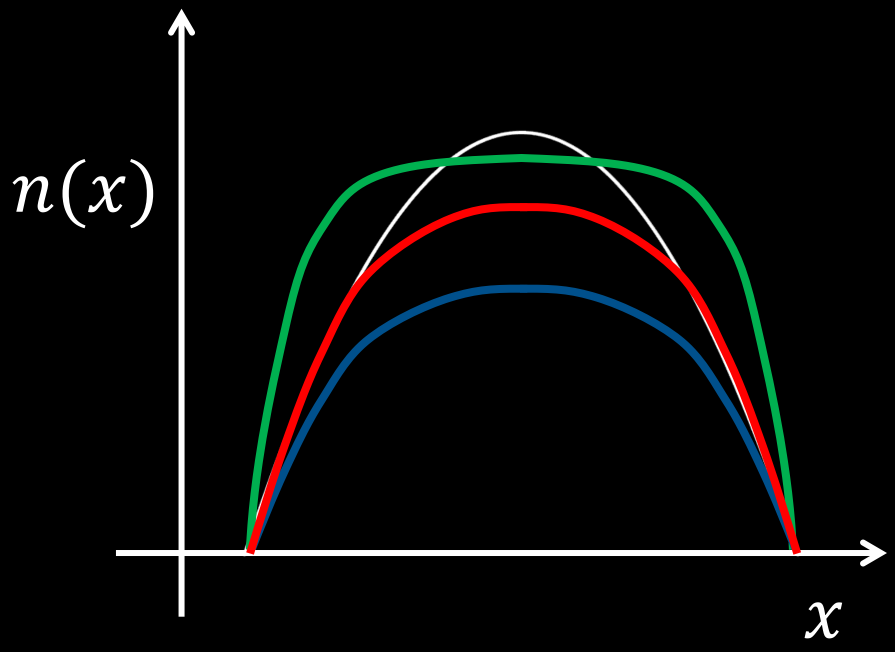


set up

light propagation

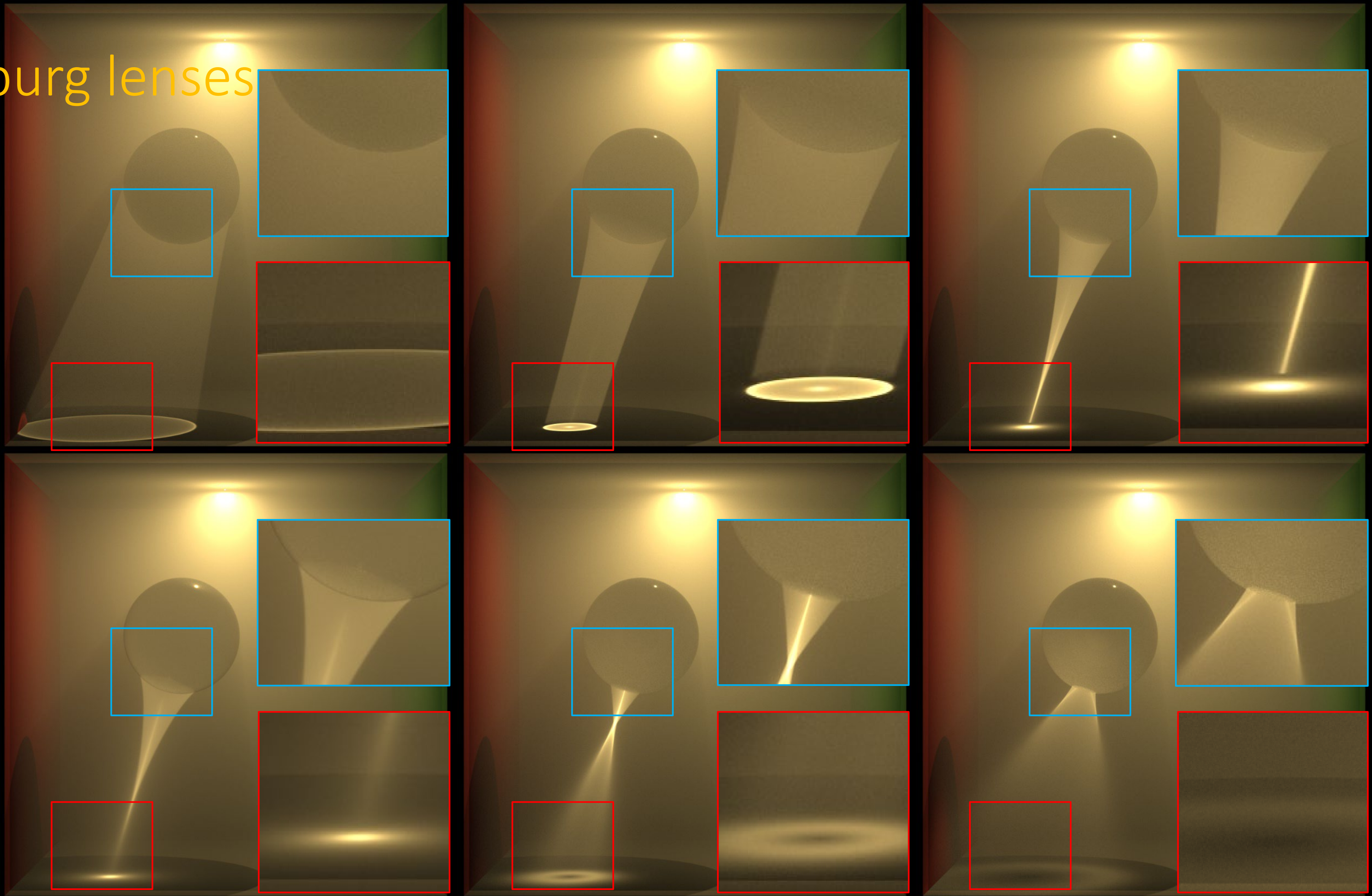
Luneburg lenses





https://en.wikipedia.org/wiki/Luneburg_lens

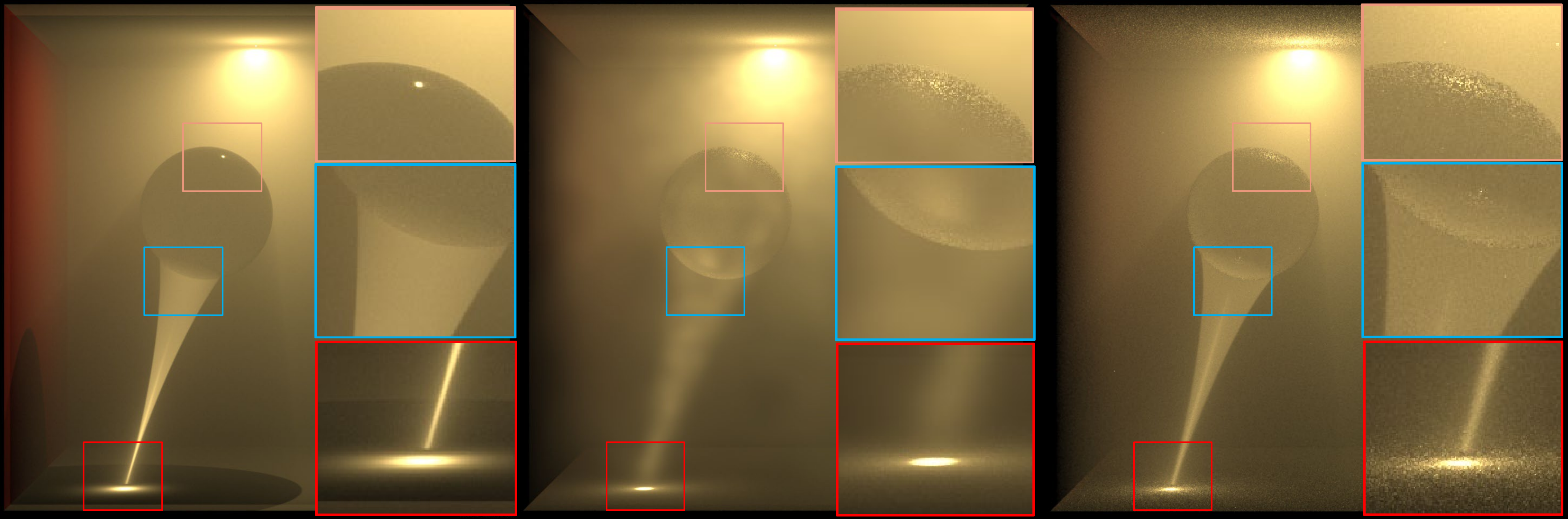
Luneburg lenses



power equal to standard lens

rendering time: 10 mins

comparison with photon mapping



BDPT (ours)

photon mapping
(default parameters)

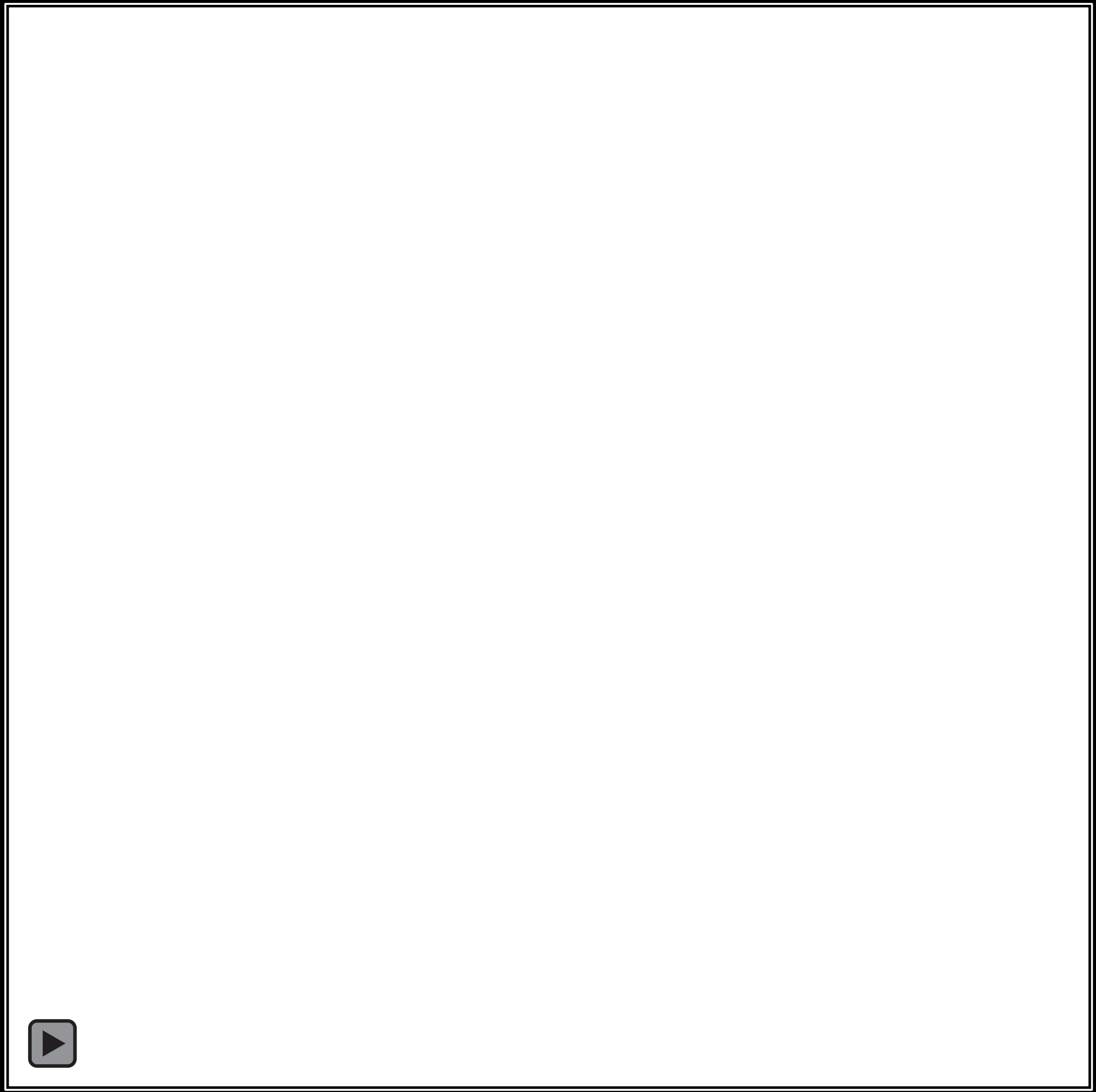
photon mapping
(optimized parameters)

BDPT is 5x faster than photon mapping

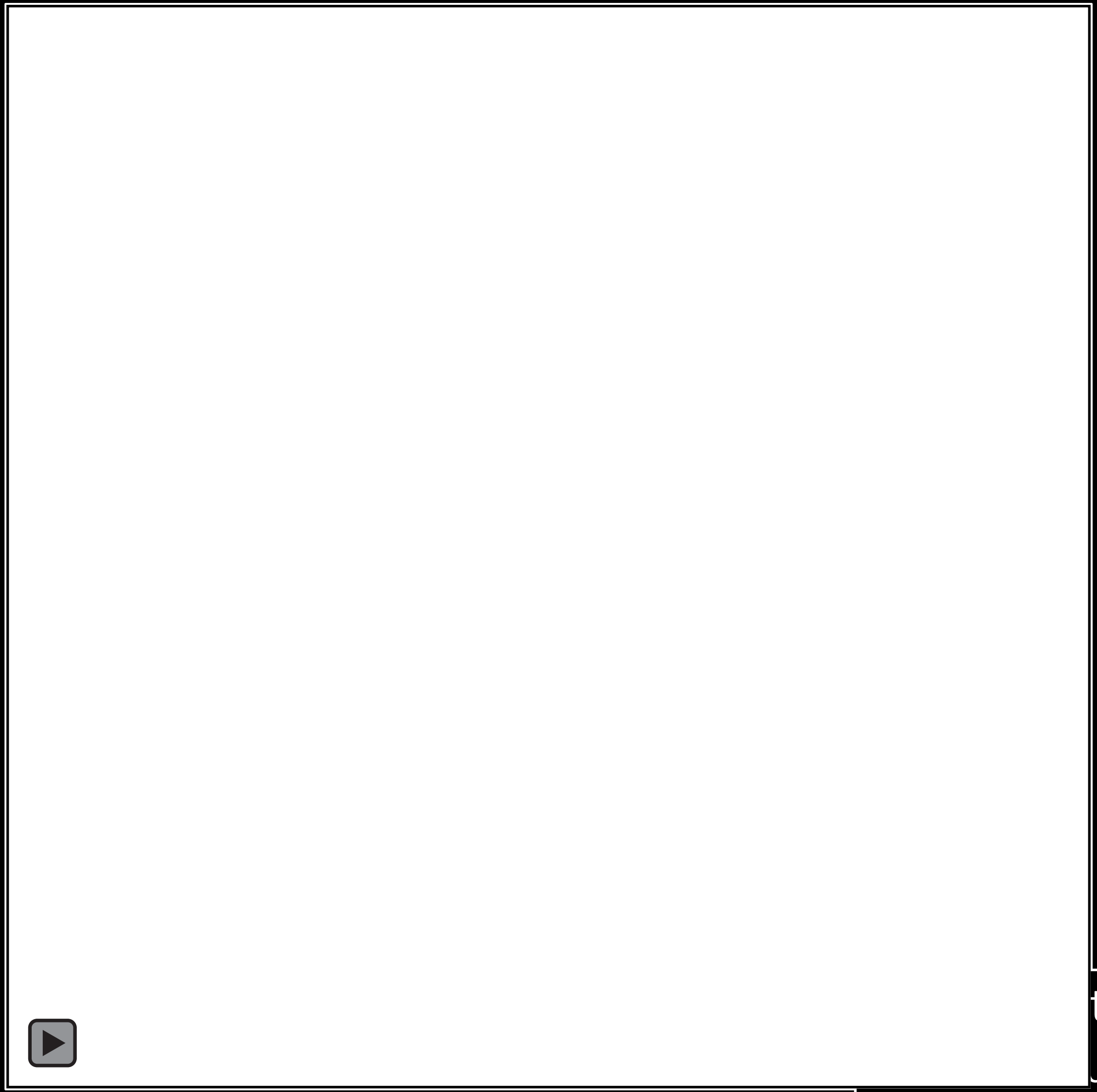
rendering time: 10 min

transient rendering (videos)

constant refractive index



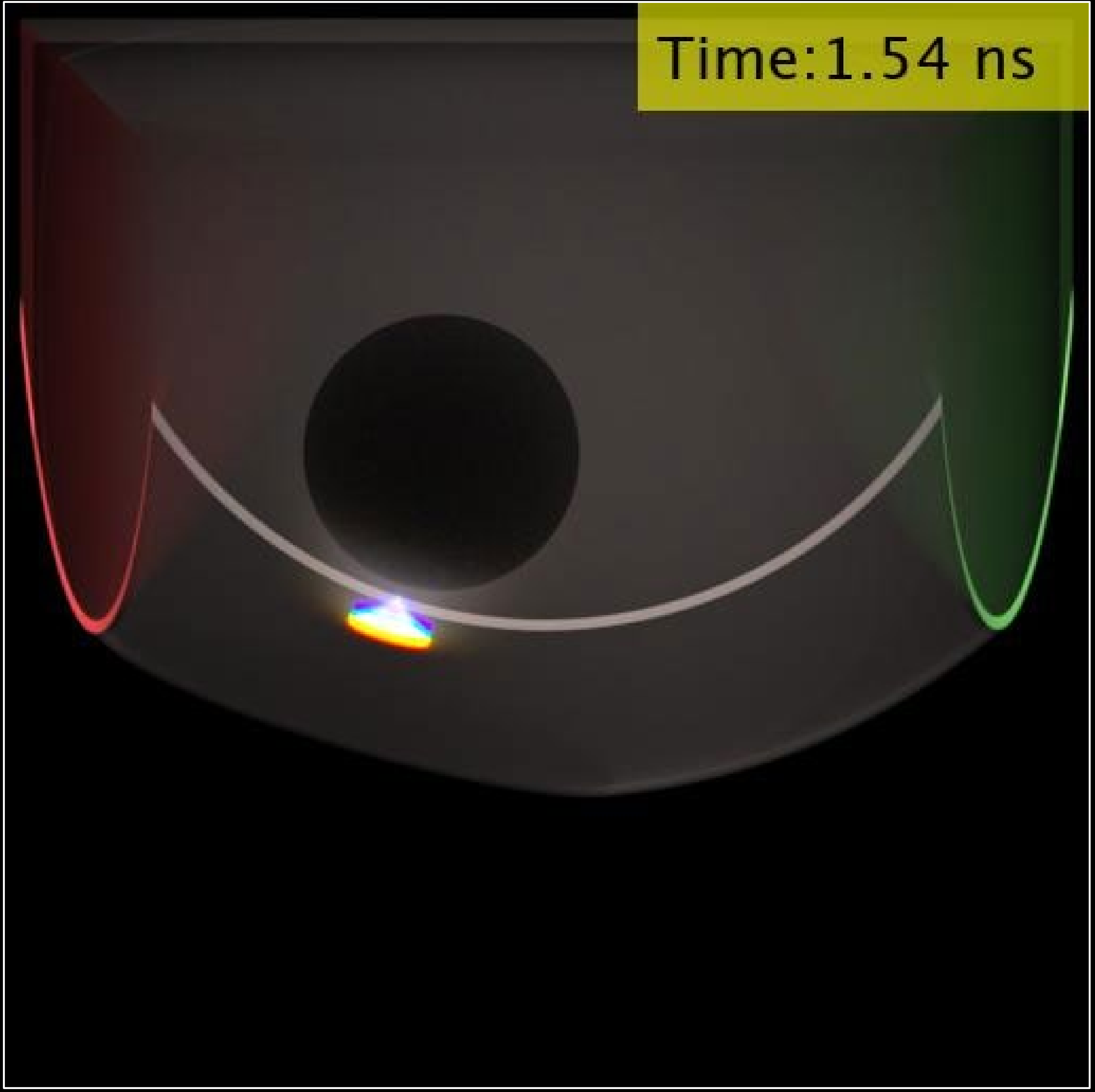
continuous refractive index



time:
urs

transient rendering

constant refractive index



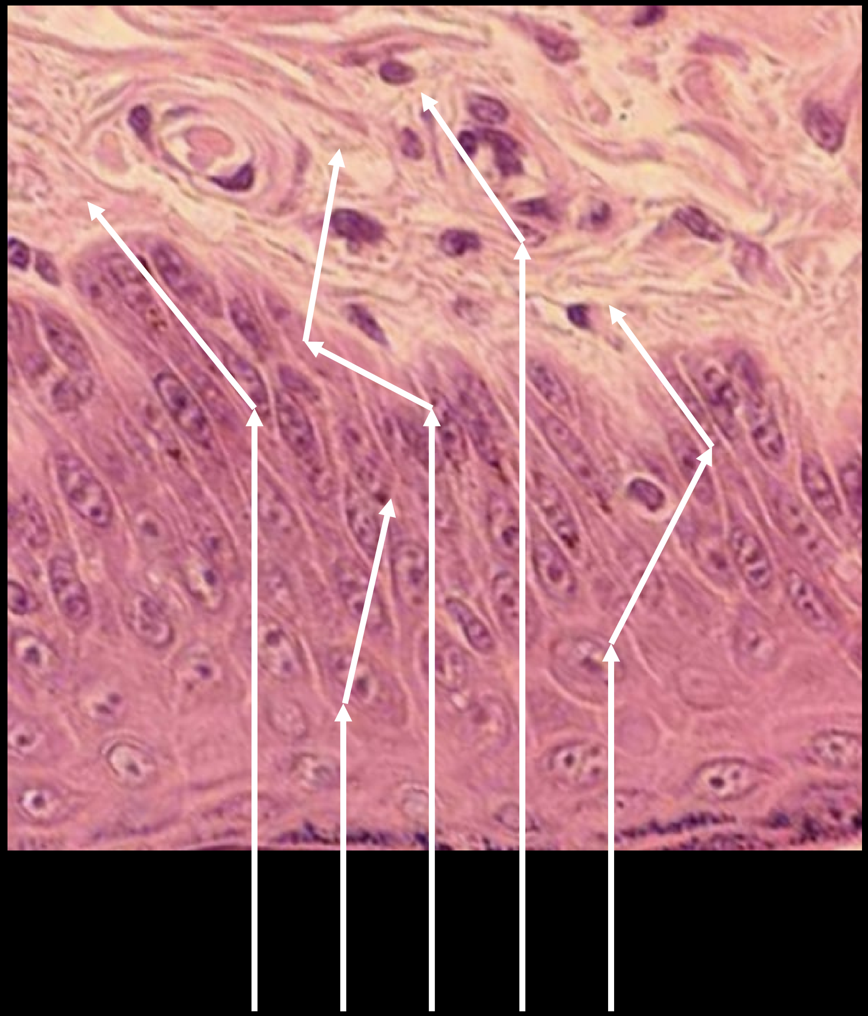
continuous refractive index

transient rendering

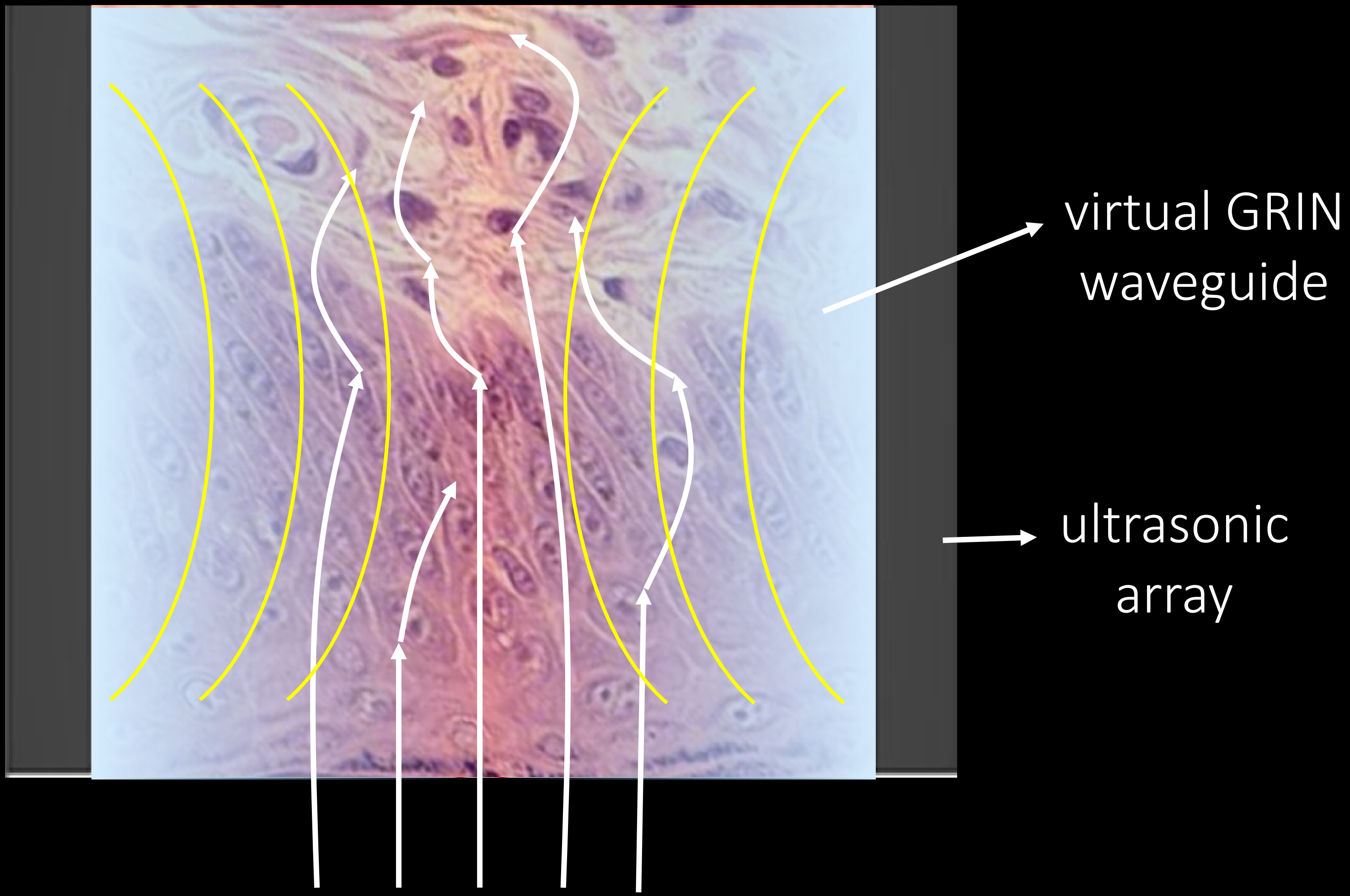
constant refractive index

continuous refractive index

virtual ultrasonic waveguides

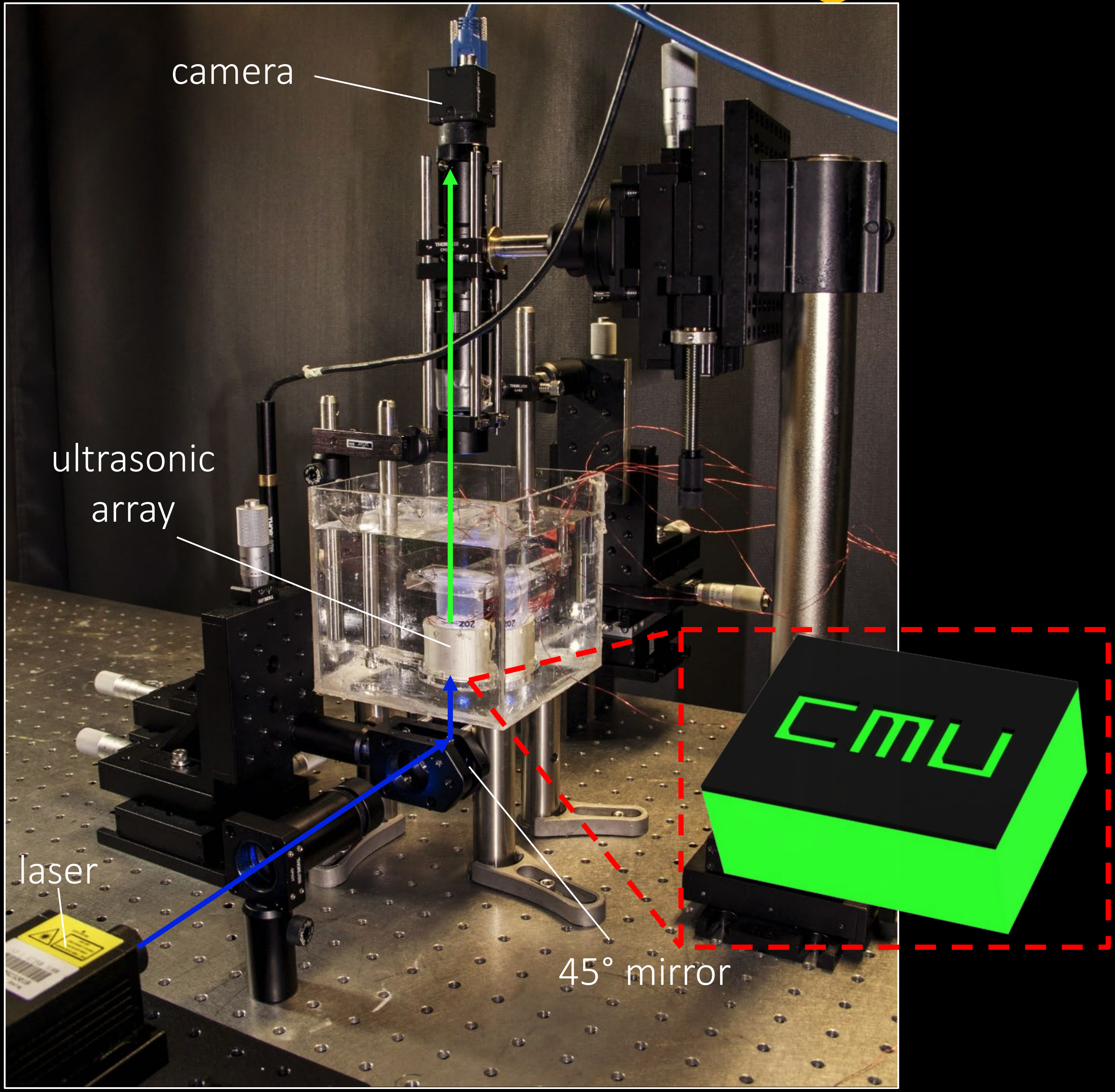


virtual ultrasonic waveguides

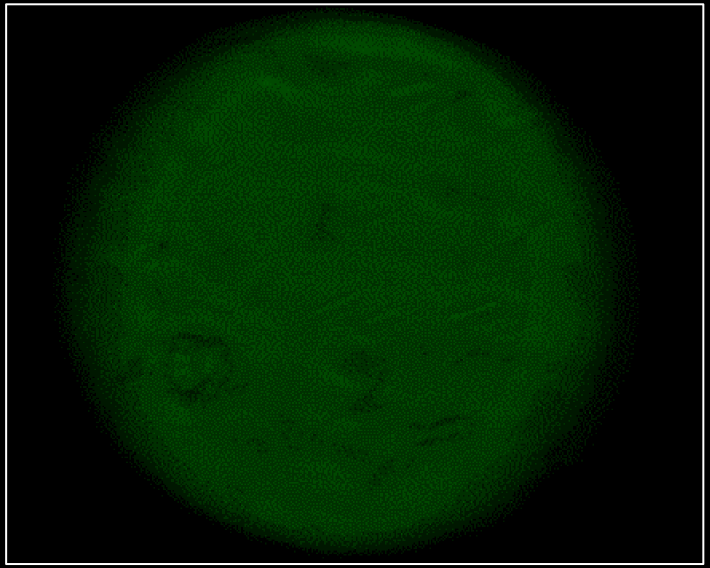


Chamanzar et al. “Ultrasonic sculpting of virtual optical waveguides in tissue”. Nature communications, 2019
Scopelliti et al. “Ultrasonically sculpted virtual relay lens for in situ microimaging”. Light: Science and Applications, 2019
Karimi et al. “In situ 3D reconfigurable ultrasonically sculpted optical beam paths”. Optics express, 2019

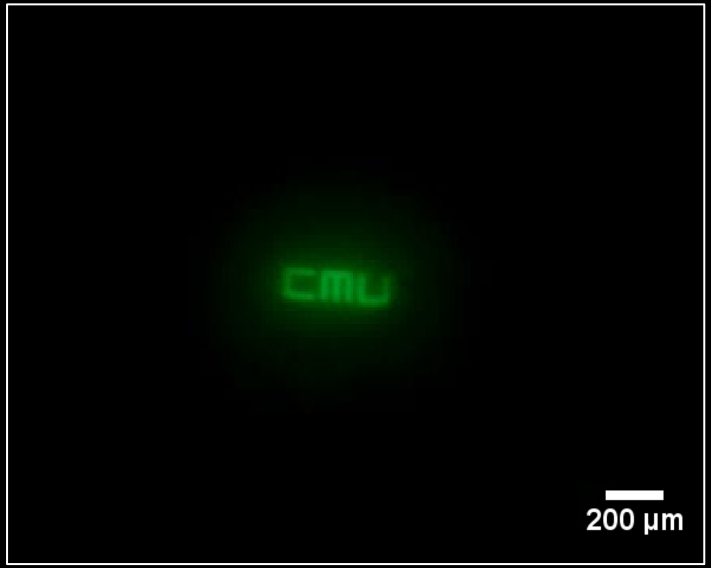
virtual ultrasonic waveguides



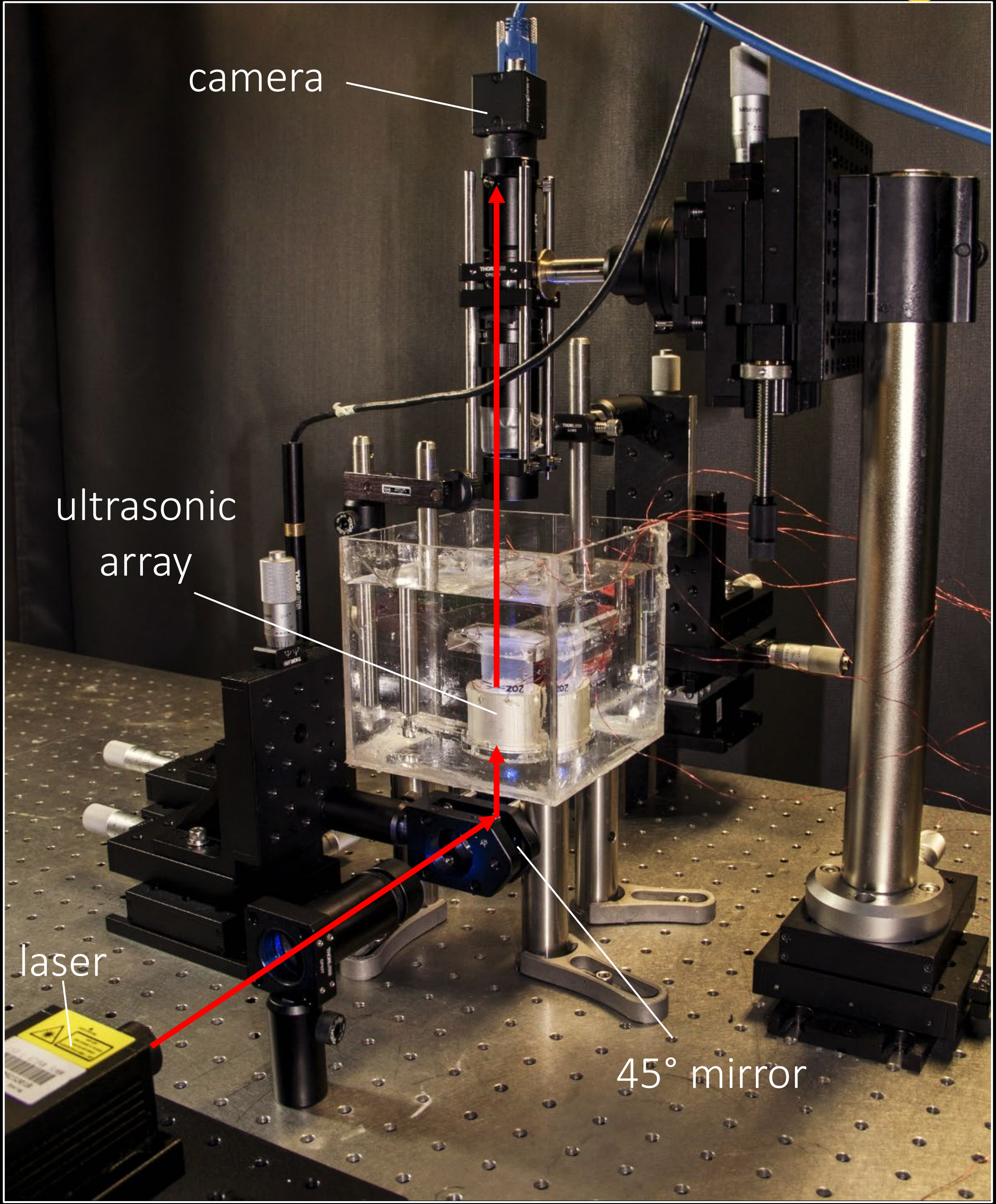
no waveguide

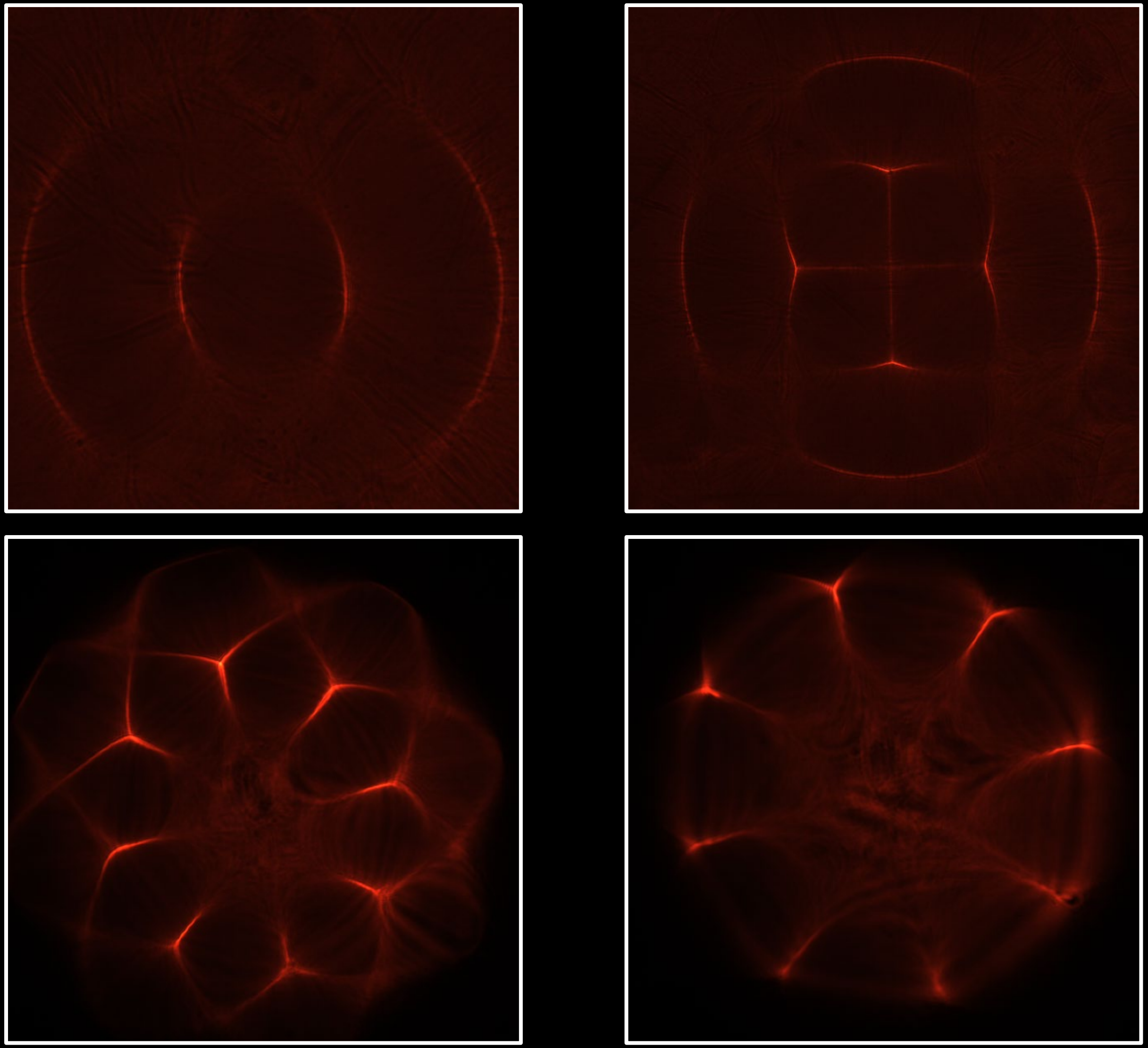


virtual waveguide



virtual ultrasonic waveguides

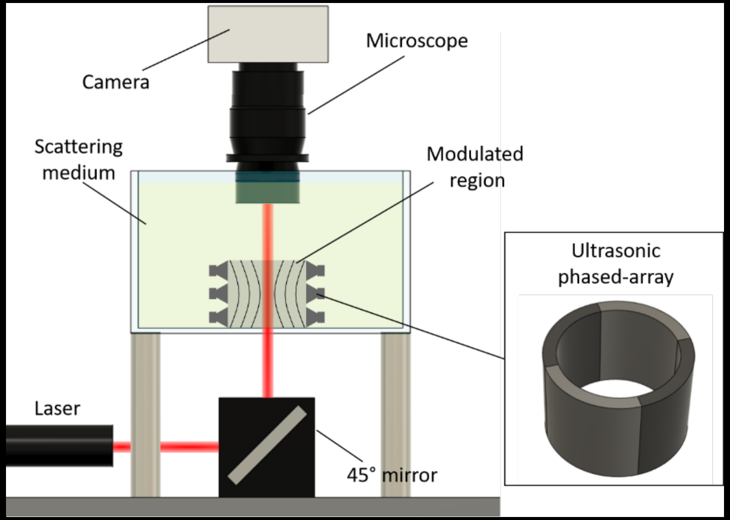




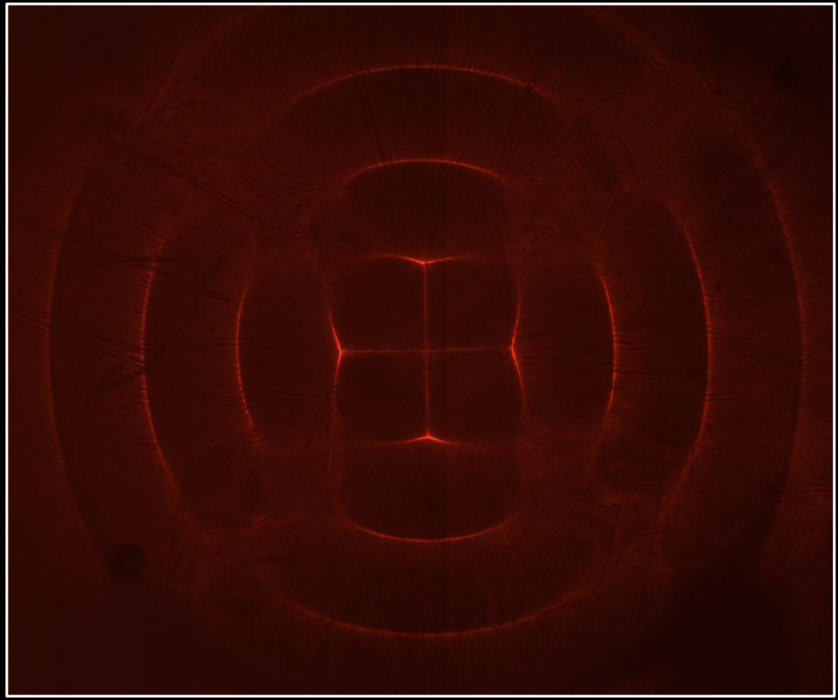
Karimi et al. "In situ 3D reconfigurable ultrasonically sculpted optical beam paths". Optics express, 2019

Rendering acousto-optics

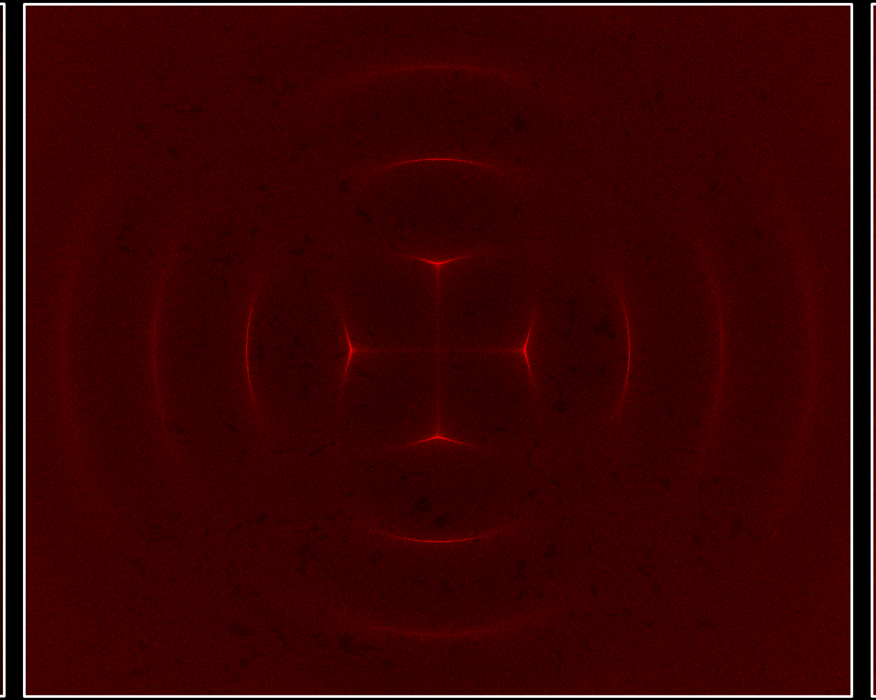
setup for ultrasonic lensing in scattering



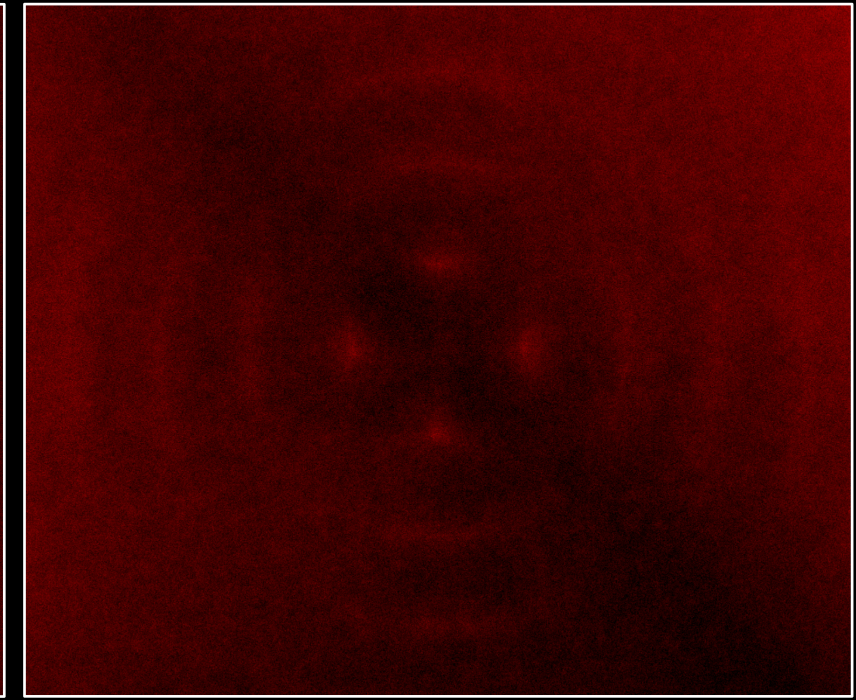
real capture



our algorithm

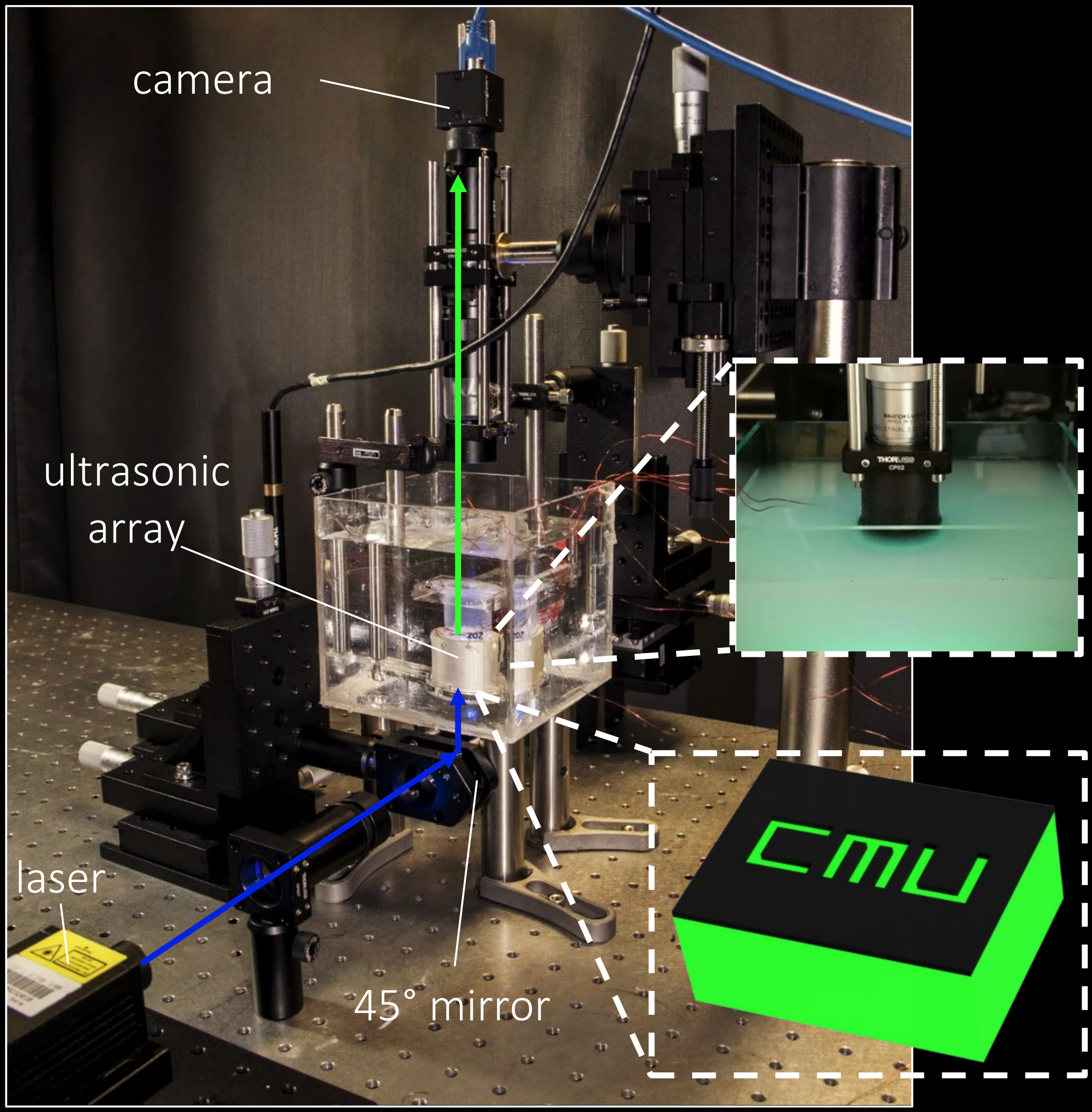


previous algorithm



[Pediredla et al. Transactions on Graphics 2020]

Ultrasonic light guiding inside tissue



High-dimensional, highly-non-linear design problem:

- ultrasound frequency
- ultrasound voltage
- shape of waveguides
- placement of transducers
- sensor size
- and more…

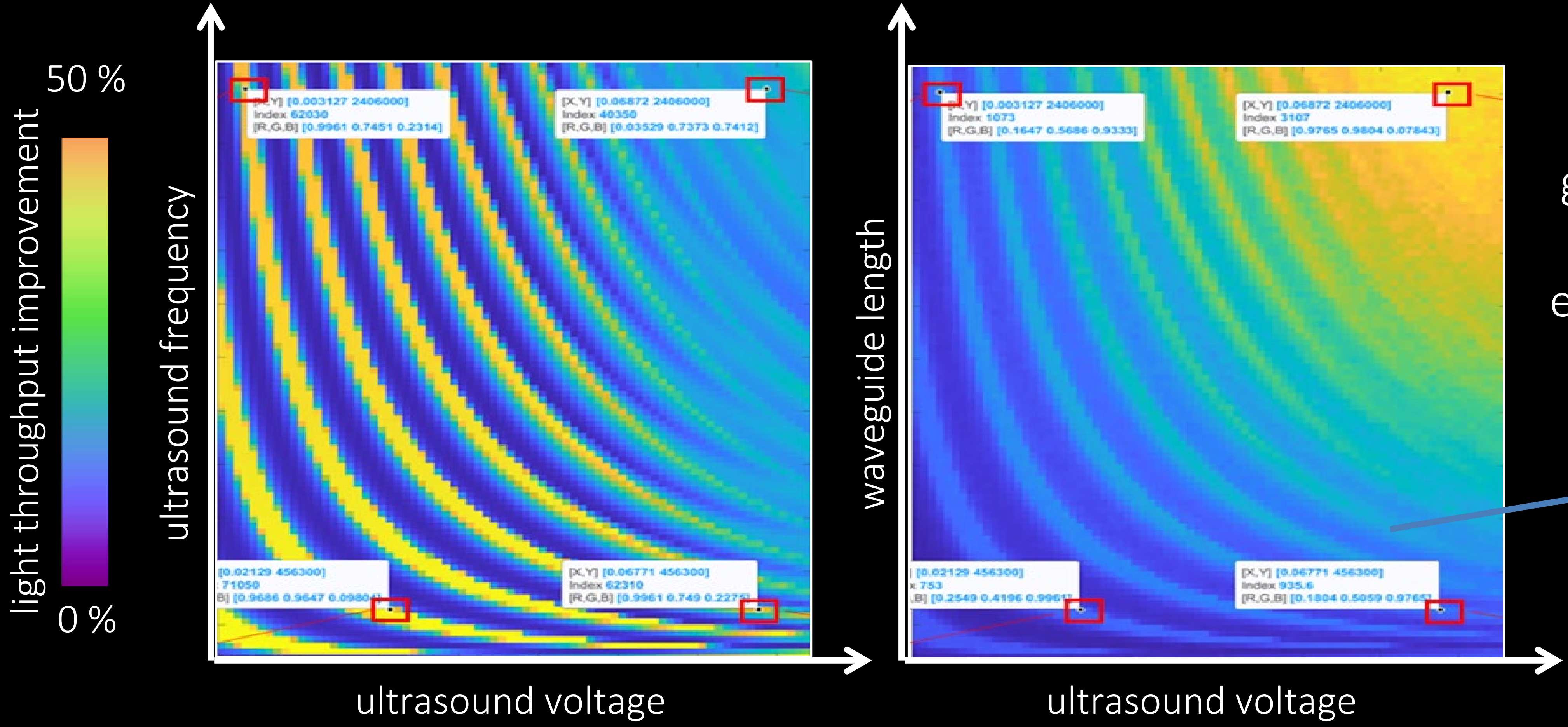
Guiding performance strongly affected by different parameter values

Painstaking experiments:

- several hours of work to test one set of parameter values

Optimizing ultrasonic GRIN waveguides

- Hundreds of thousands of virtual experiments.

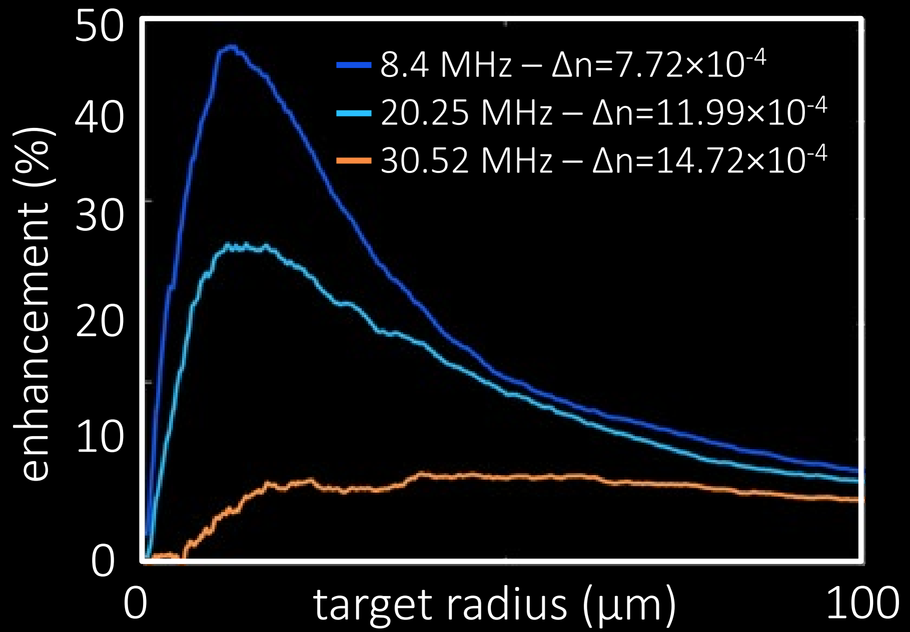


Each dot on these graphs would have been a real experiment taking a PhD student a full day's work

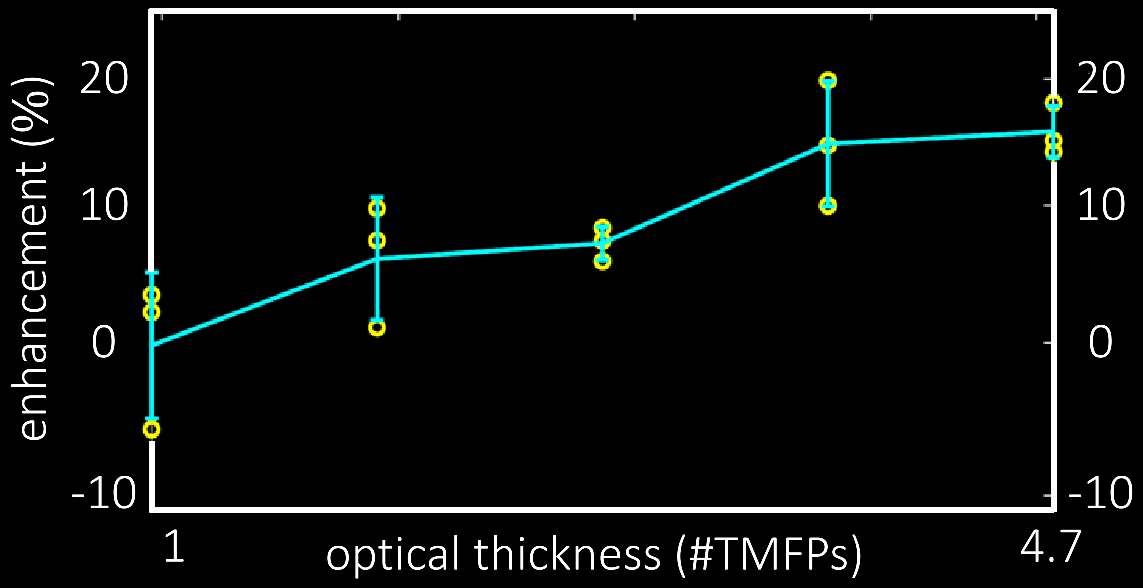
[Pediredla et al., submitted to Nature Communications 2021]

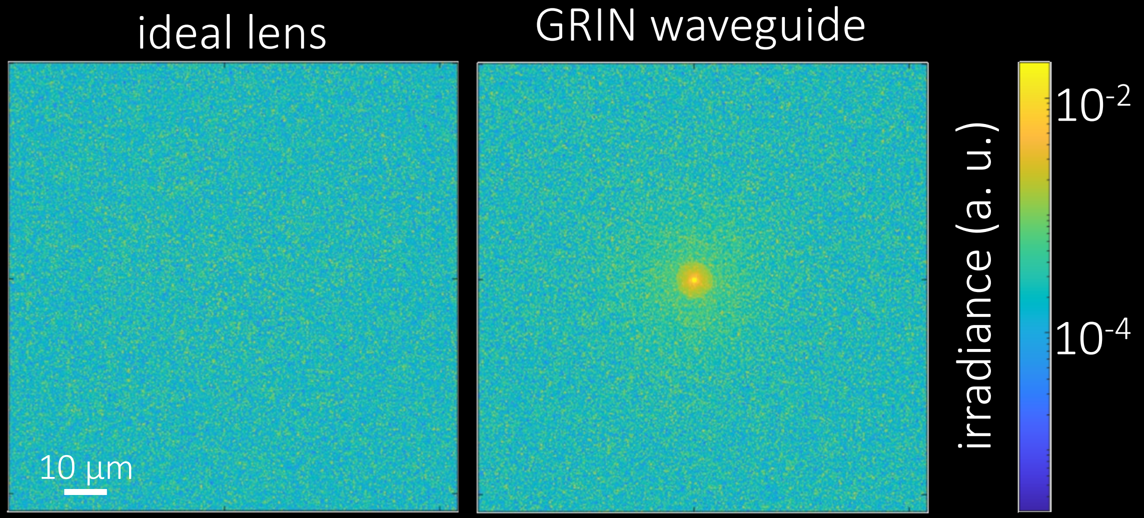
Improved light guiding in human bladder

simulations



real data





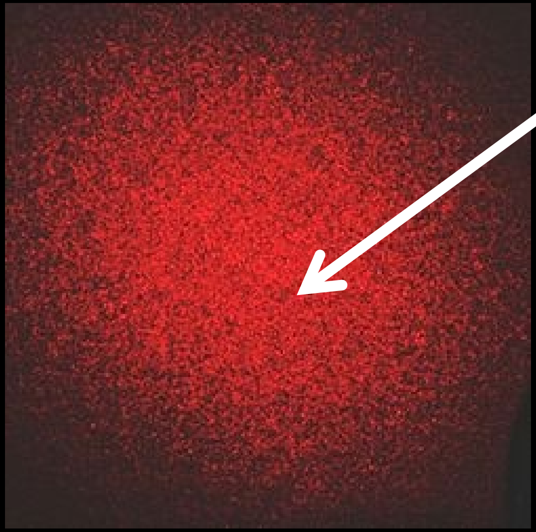
Improved light guiding performance by

- 200% compared to unoptimized waveguides
- 50% compared to external optics

Simulation predictions verified experimentally

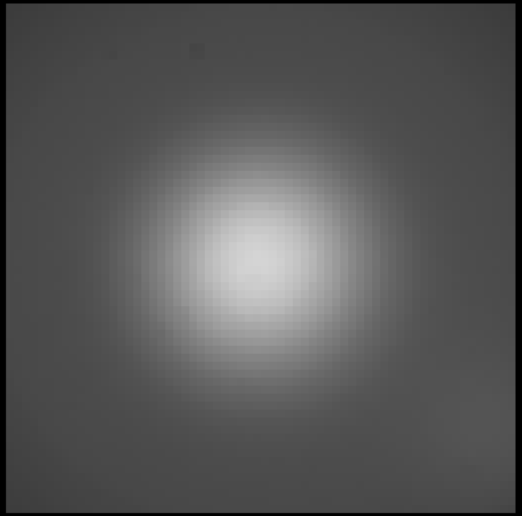
[Pediredla et al., submitted to Nature Communications 2021]

Speckle and memory effect



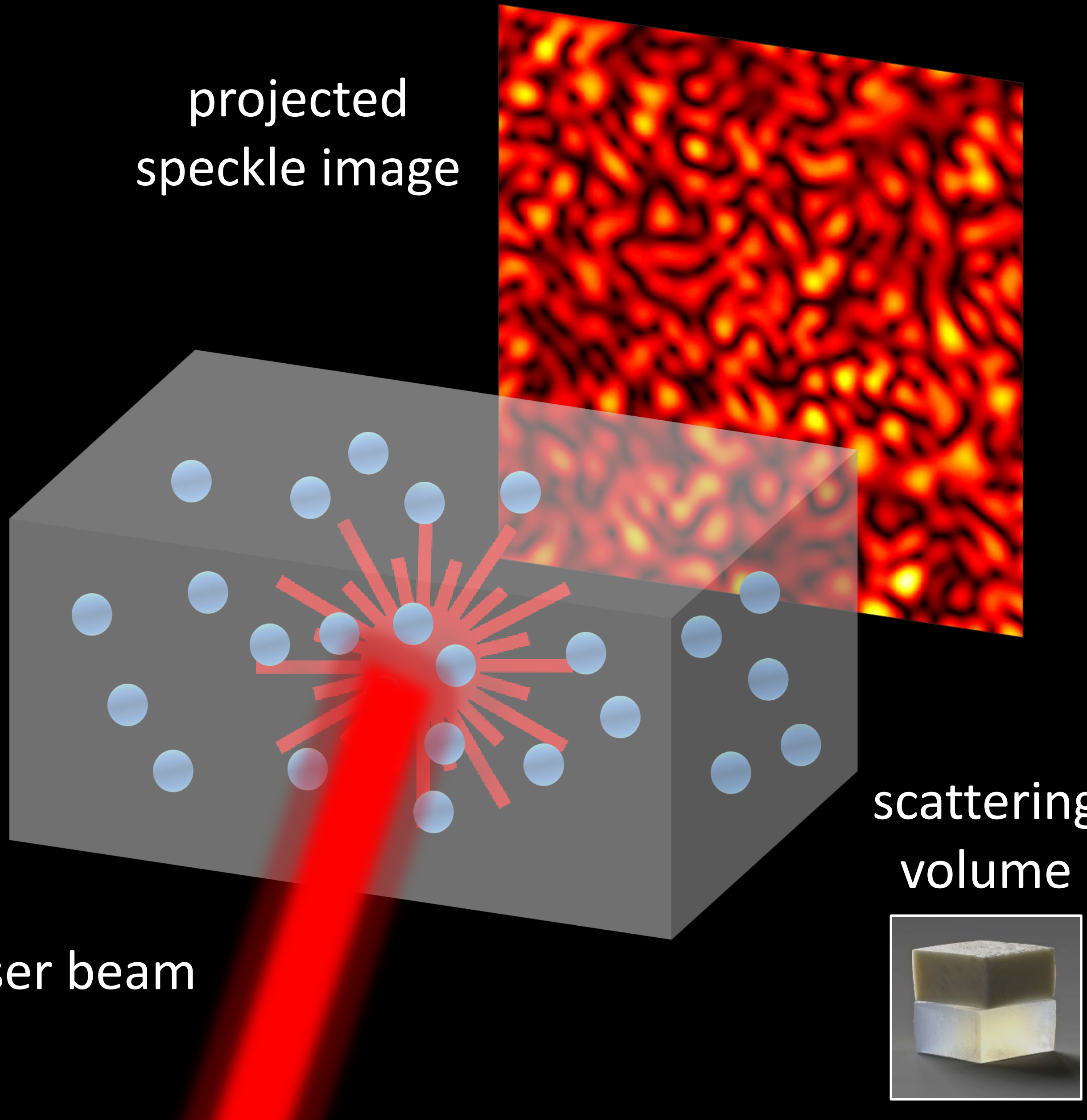
speckle: noise-like pattern

what real laser images look like



what standard rendered images look like

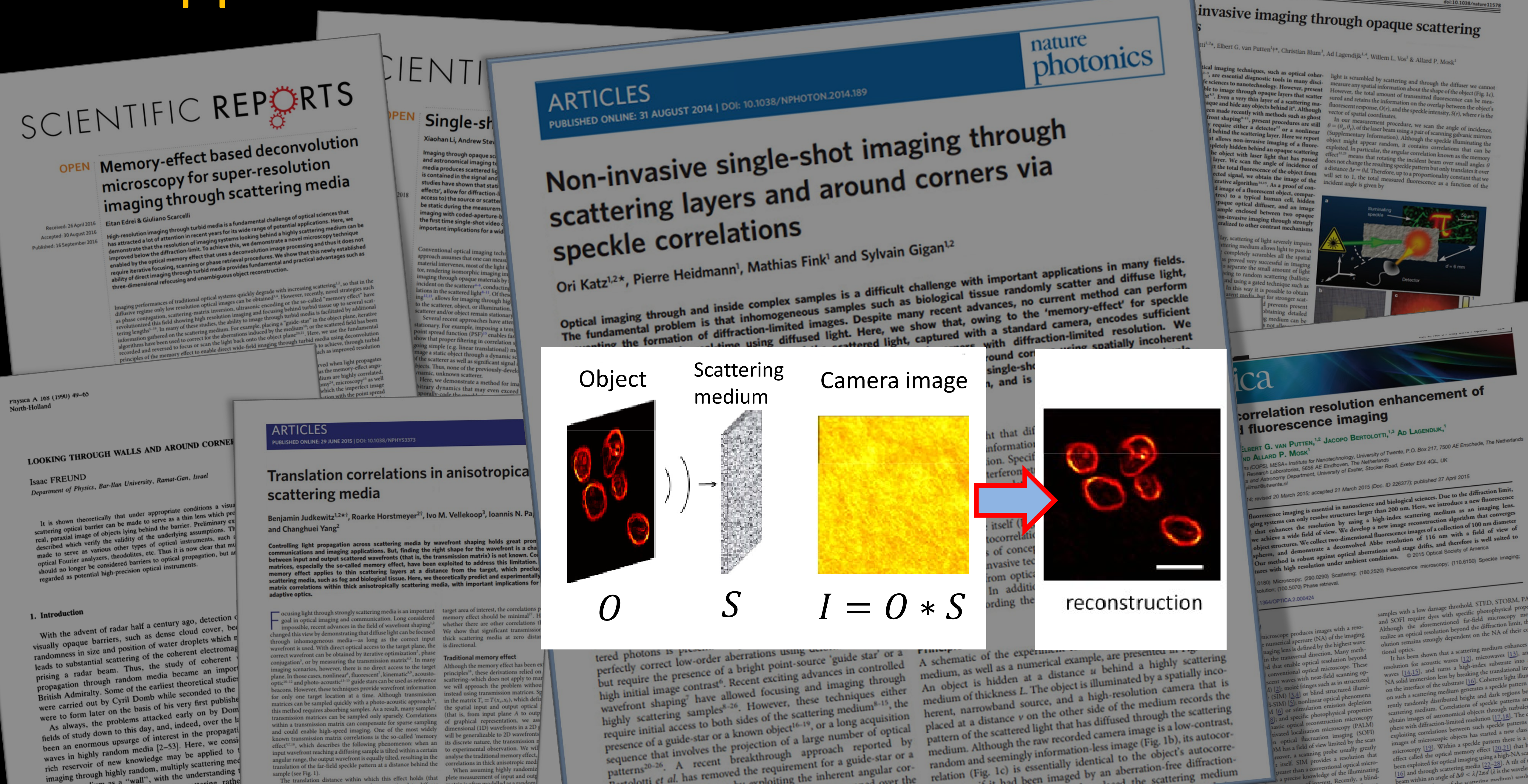
projected speckle image



laser beam

scattering volume

Applications and Related Work

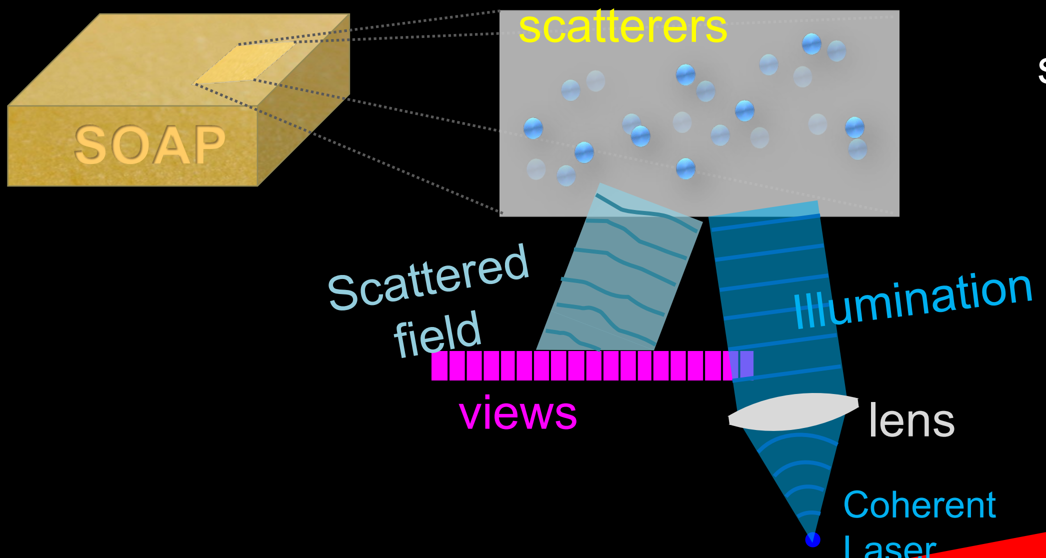


Simulating speckles

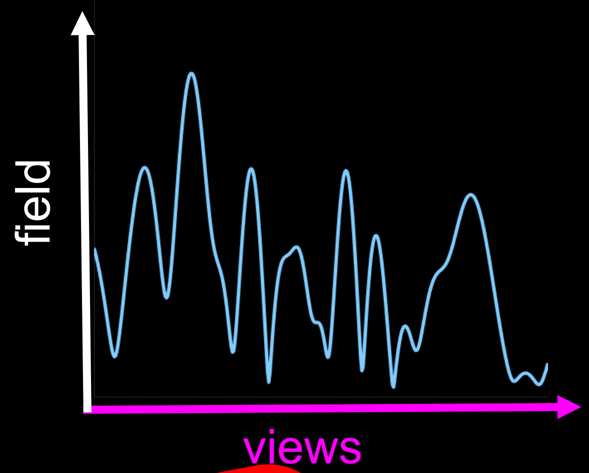
inefficient

Specify exact (sub-wavelength) position of scatterers

In graphics we describe materials by ***statistical*** bulk parameters, as the ***density*** of scatterers



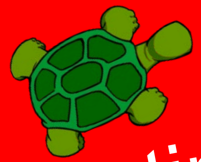
solve wave equations



Wave equation solvers

- Differential equation F…
- Integral equation (e.g.,…

Slow

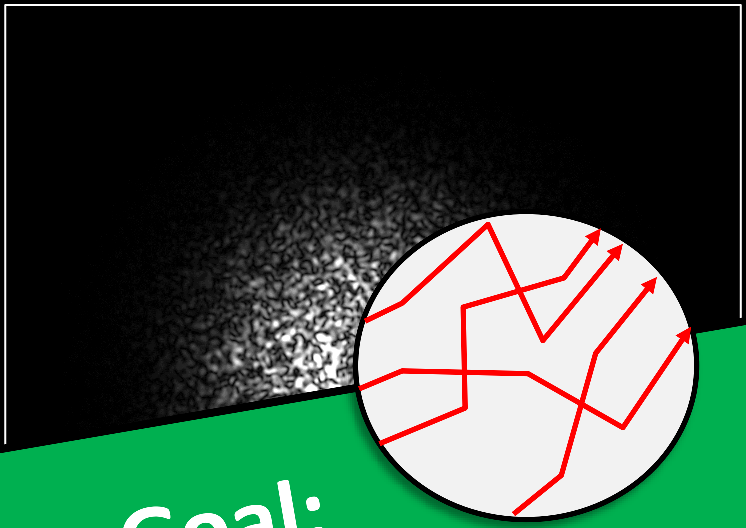


Practical only for tiny or optically thin media

For simplicity: **Flatland**
Scattering medium is **2D**
Sensor is **1D**
Speckle pattern is **1D**

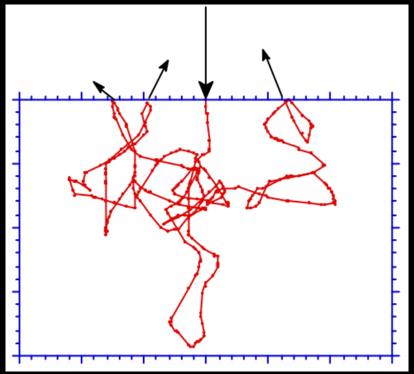
Monte Carlo (MC) Simulation of Speckles

MC Advantage:
1. **Fast**
2. input is scatterer **density** rather than exact scatterer locations



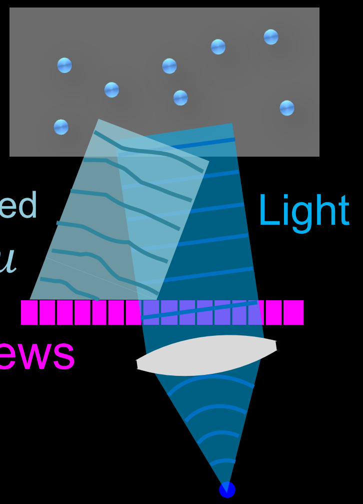
Our Goal:
Extend efficient MC tools to evaluate speckles and their coherent statistics

Standard intensity MC

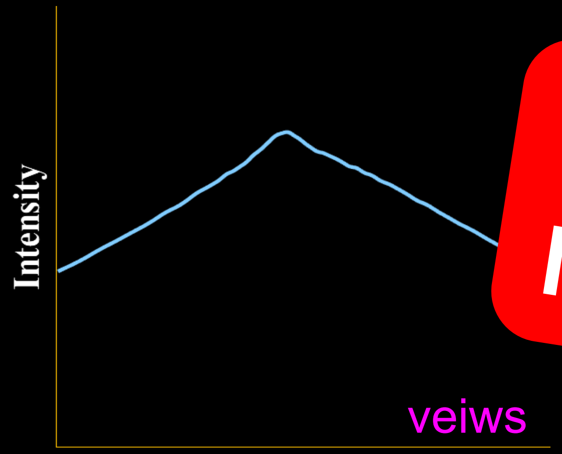


Monte Carlo Modeling of Light Transport in Multi-layered Tissues, **Wang & Jacques, 1992**

Wave Solution v.s. Monte Carlo

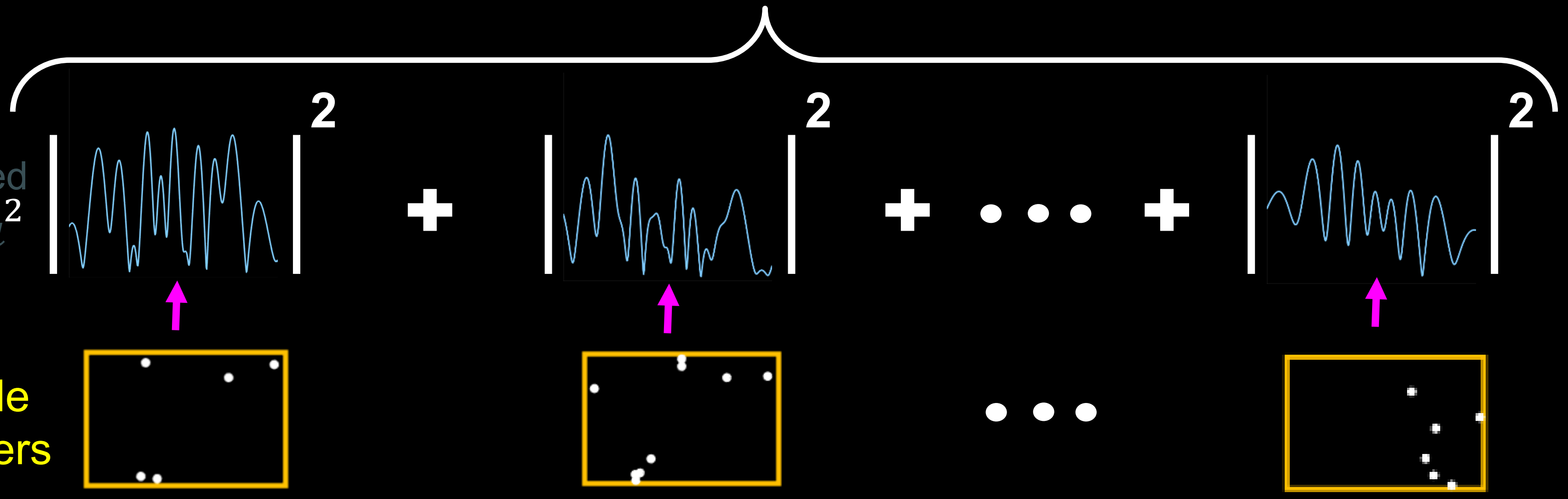


Intensity $= E_{\text{scatterers}}\left[\left|u^{\text{scatterers}}\right|^2\right]$



No Speckles
No Memory Effect

Scattered
Intensity $|u|^2$

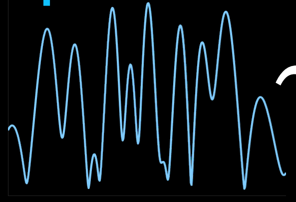


Sample
scatterers

MC requires the scatterers density – no need for exact positions

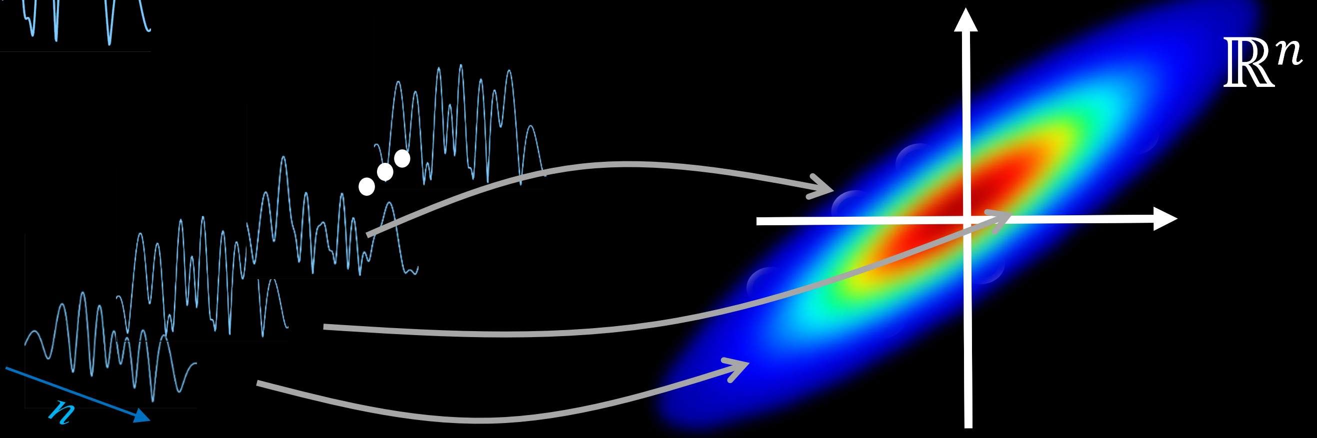
Speckle Statistics

Speckles



$$\sim \mathcal{N}\,(\text{Mean}, \text{Covariance})$$

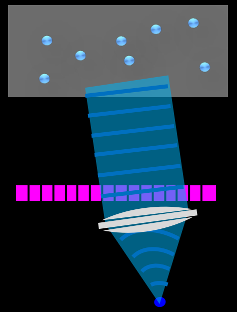
Sufficient Statistics



1st moment

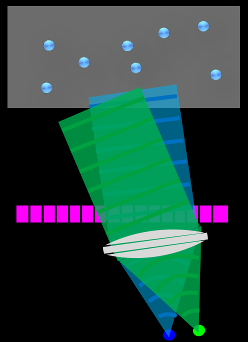
$$\cancel{\text{Intensity Mean} = \mathrm{E}_{\text{scatterers}}\left[\left|u^{\text{scatterers}}\right|^2\right]}$$

⇨ Incoherent Summation



Cross-Illumination

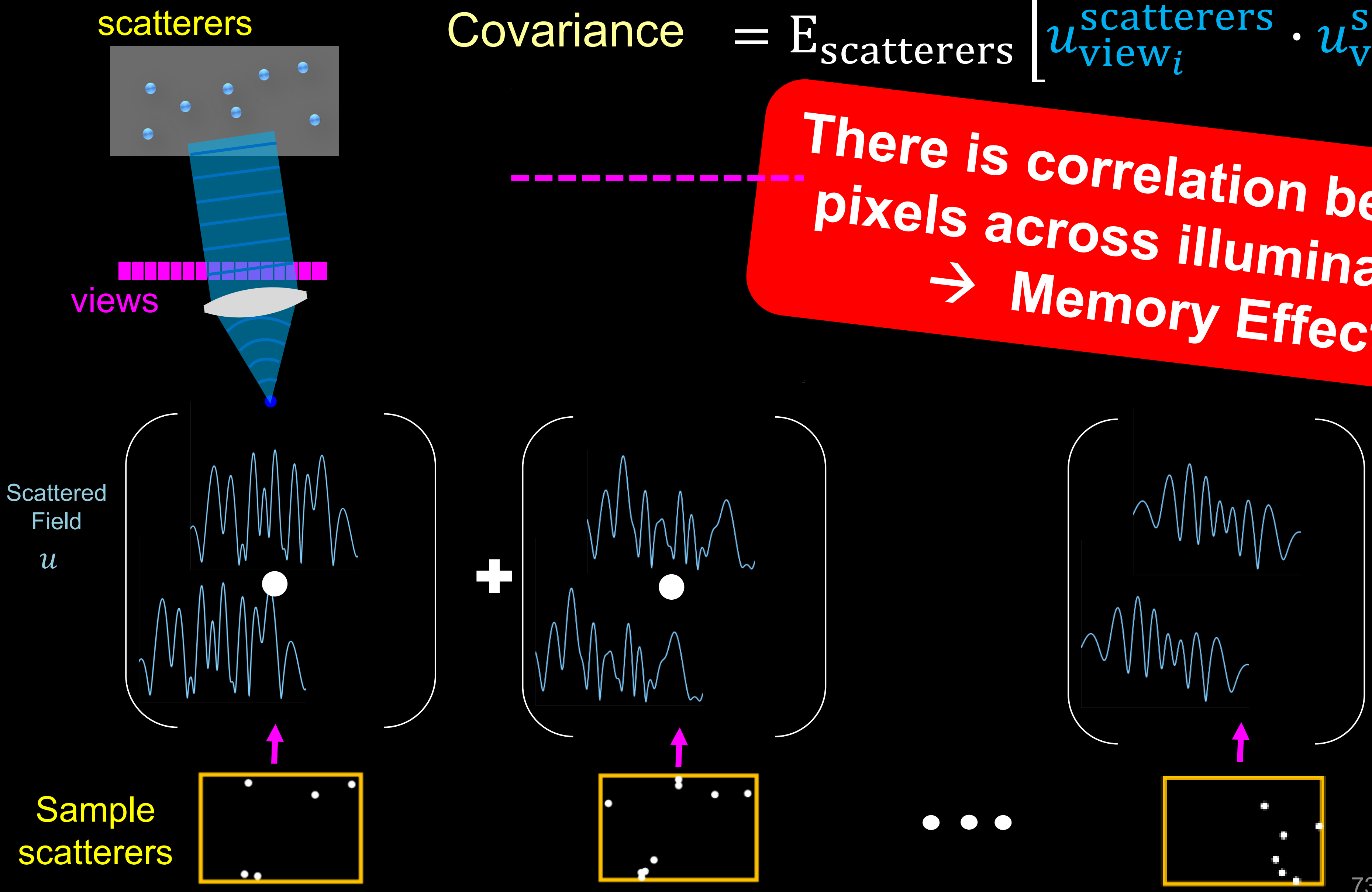
$$\text{Field Covariance} = \mathrm{E}_{\text{scatterers}}\left[u_{\text{view}_i}^{\text{light}_1,\text{scatterers}} \cdot u_{\text{view}_j}^{\text{light}_2,\text{scatterers}^*}\right]$$



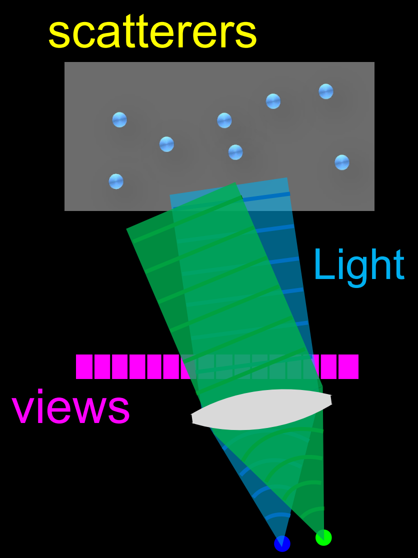
2nd Moment - Covariance

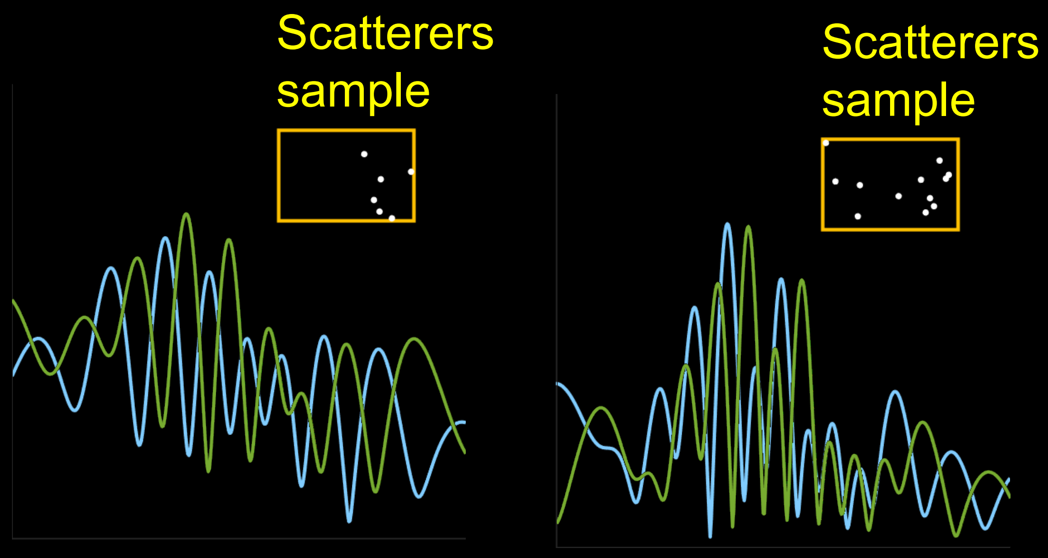
$$\text{Covariance} = \mathrm{E}_{\text{scatterers}}\left[u_{\text{view}_i}^{\text{scatterers}} \cdot {u_{\text{view}_j}^{\text{scatterers}}}^{*}\right]$$

There is correlation between pixels across illuminations → Memory Effect



Cross –illumination statistics





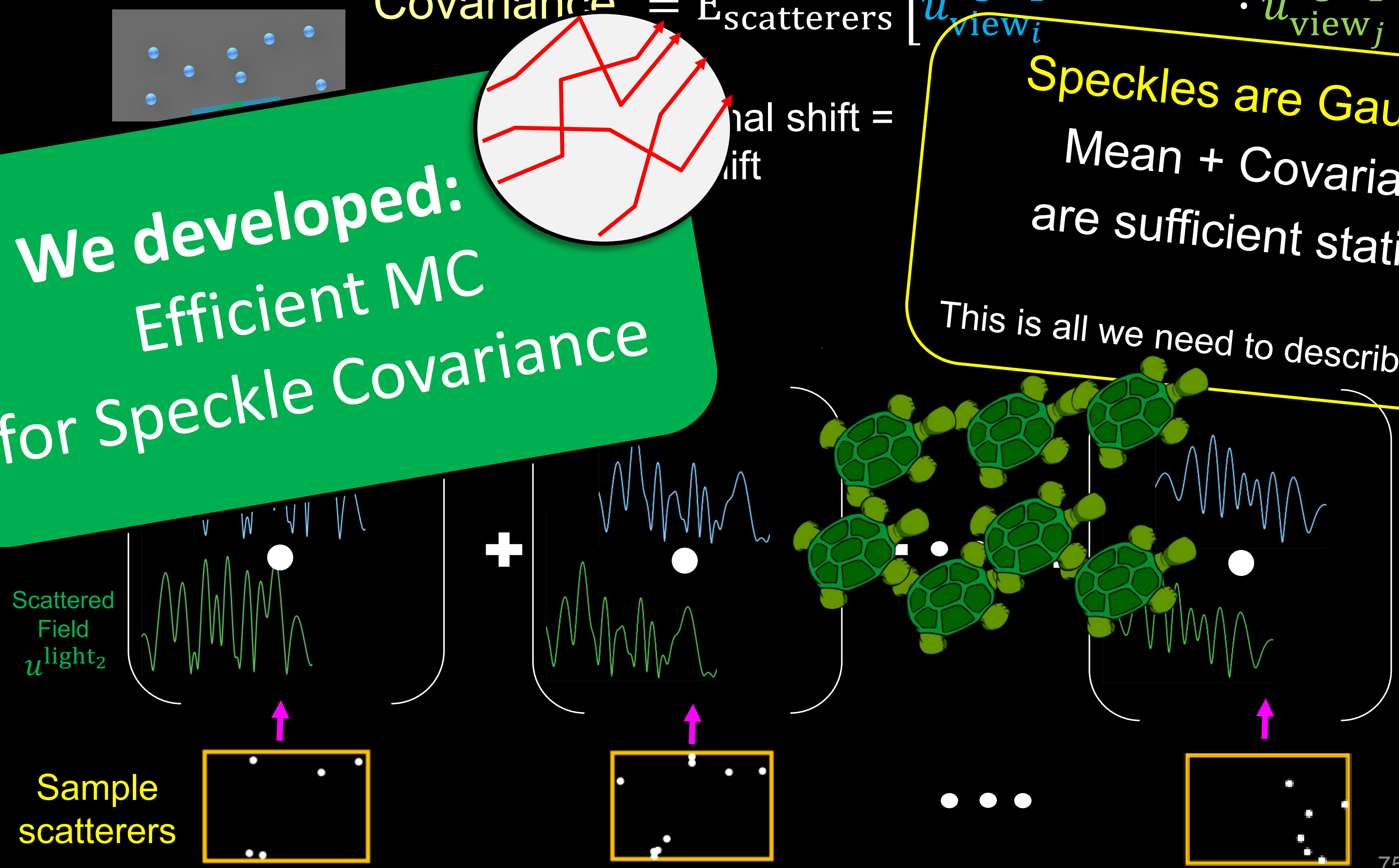
Memory Effect:

tilting illumination results in highly correlated shifted speckles

Next: Cross Illumination Covariance

Cross –illumination statistics

Covariance $= \mathrm{E}_{\text{scatterers}}\left[u_{\text{view}_i}^{\text{light}_1,\text{scatterers}} \cdot u_{\text{view}_j}^{\text{light}_2,\text{scatterers}^*}\right]$



We developed:
Efficient MC
for Speckle Covariance

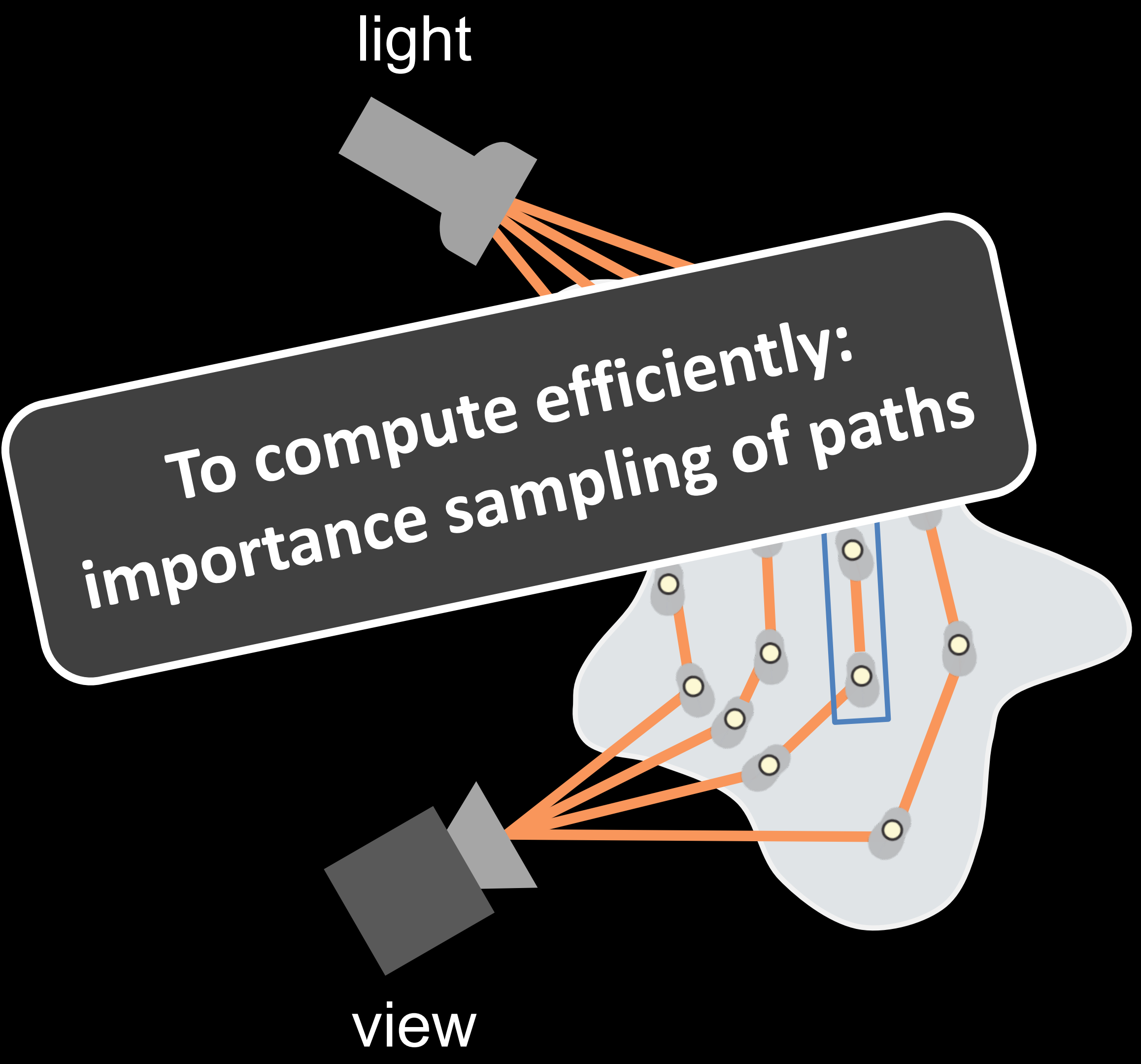
Speckles are Gaussian:
Mean + Covariance
are sufficient statistics

This is all we need to describe speckles

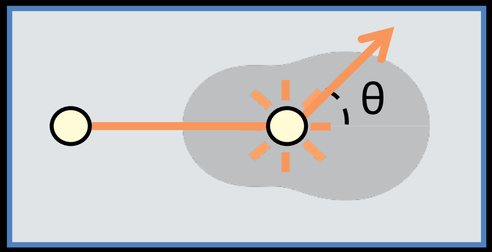
Monte Carlo Rendering 101

$$\text{Image} = \int_{\text{paths}} f(\text{path})$$

Throughput that acts on each path, depends on the scattering material



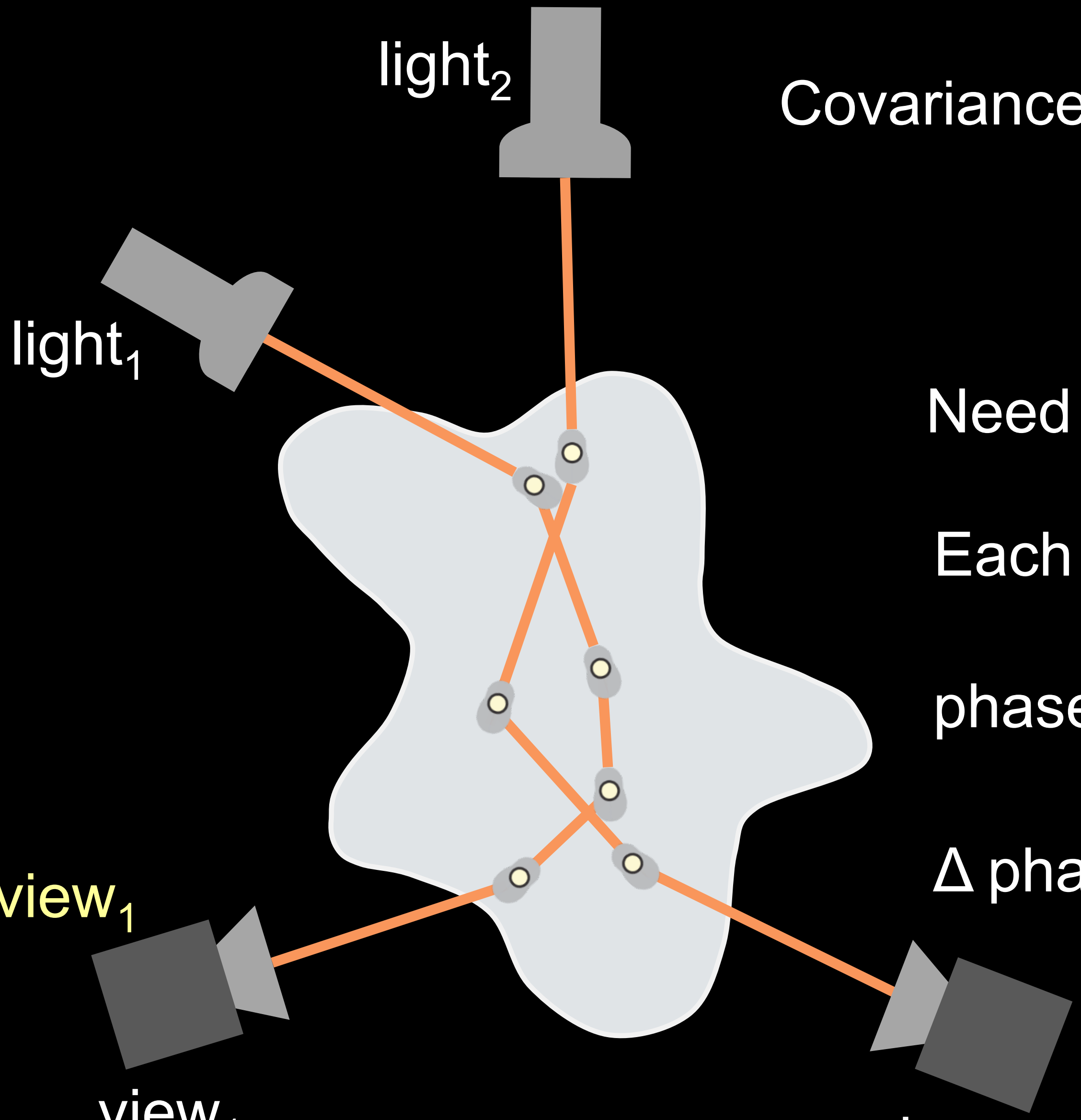
To compute efficiently: importance sampling of paths



material =

volumetric density (extinction coefficient)	σ
scattering albedo	a
phase function	p_θ

Covariance Rendering



$$\text{Covariance} = \int_{path_1, path_2} u(path_1) \cdot u^*(path_2)$$

$u = |u|\, e^{i \cdot phase}$

Need to consider products of ***pairs*** of paths

Each path contributes a complex number u

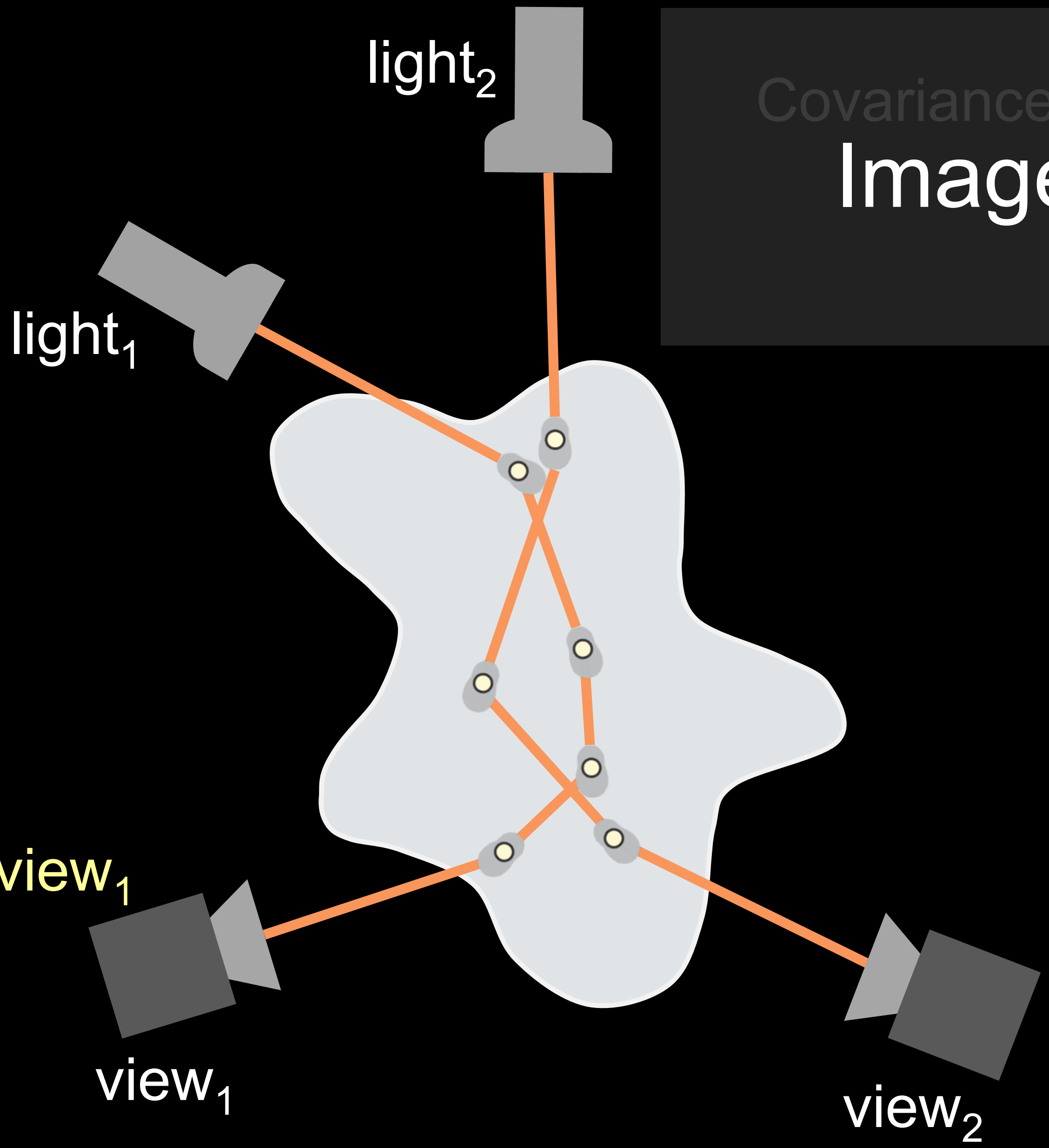
phase ∝ Length (path)

Δ phase ∝ Length ($path_1$) - Length ($path_2$)

$path_1$:
$light_1$ → $view_1$

$path_2$:
$light_2$ → $view_2$

Covariance Rendering



$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} \underbrace{u(\text{path}_1)\cdot u^*(\text{path}_2)}$$

Real throughput $i\cdot$phase

$u = |u|\, e^{i\cdot\text{phase}}$

$$\text{Image} = \int_{\text{paths}} f(\text{path})$$

$\text{path}_1 = \text{path}_2$

⇩

Same complex contribution

$u(\text{path}_1) = u(\text{path}_2)$

⇩

$\Delta\,\text{phase} = 0$

⇩

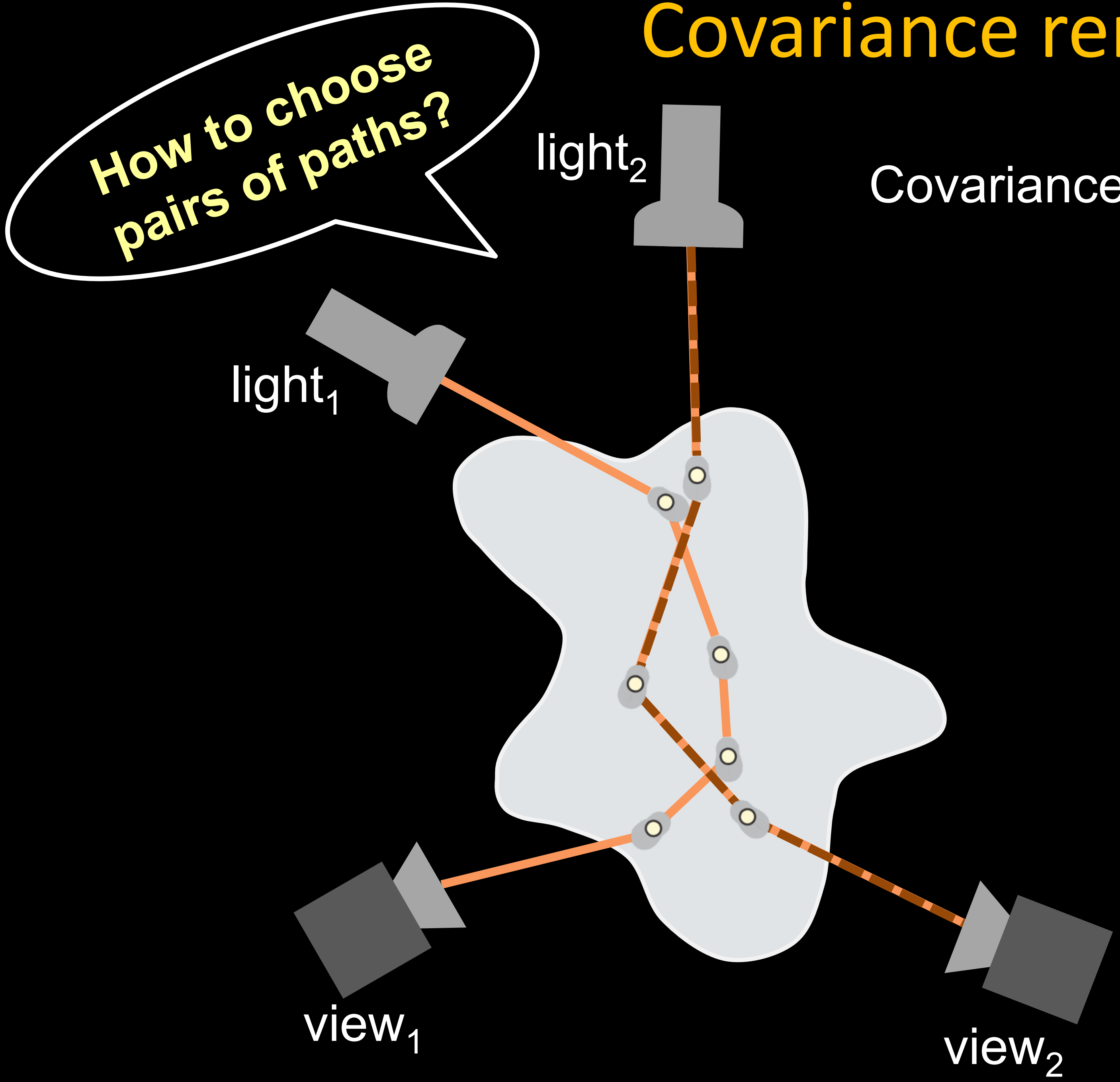
Real

$u(\text{path}) \cdot u^*(\text{path}) = f\,(\text{path})$

path$_1$:
light$_1$ → view$_1$

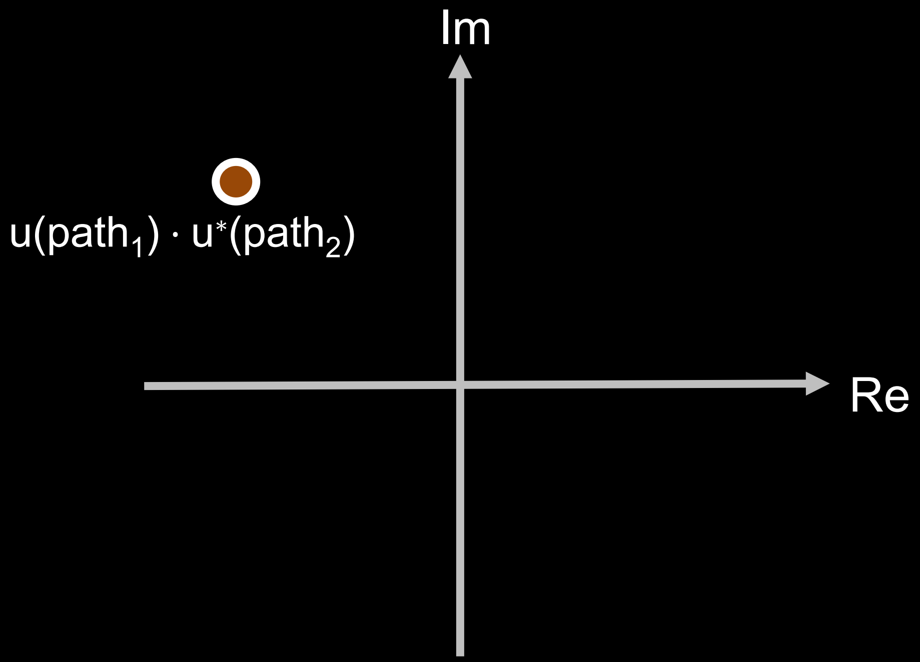
path$_2$:
light$_2$ → view$_2$

Covariance rendering



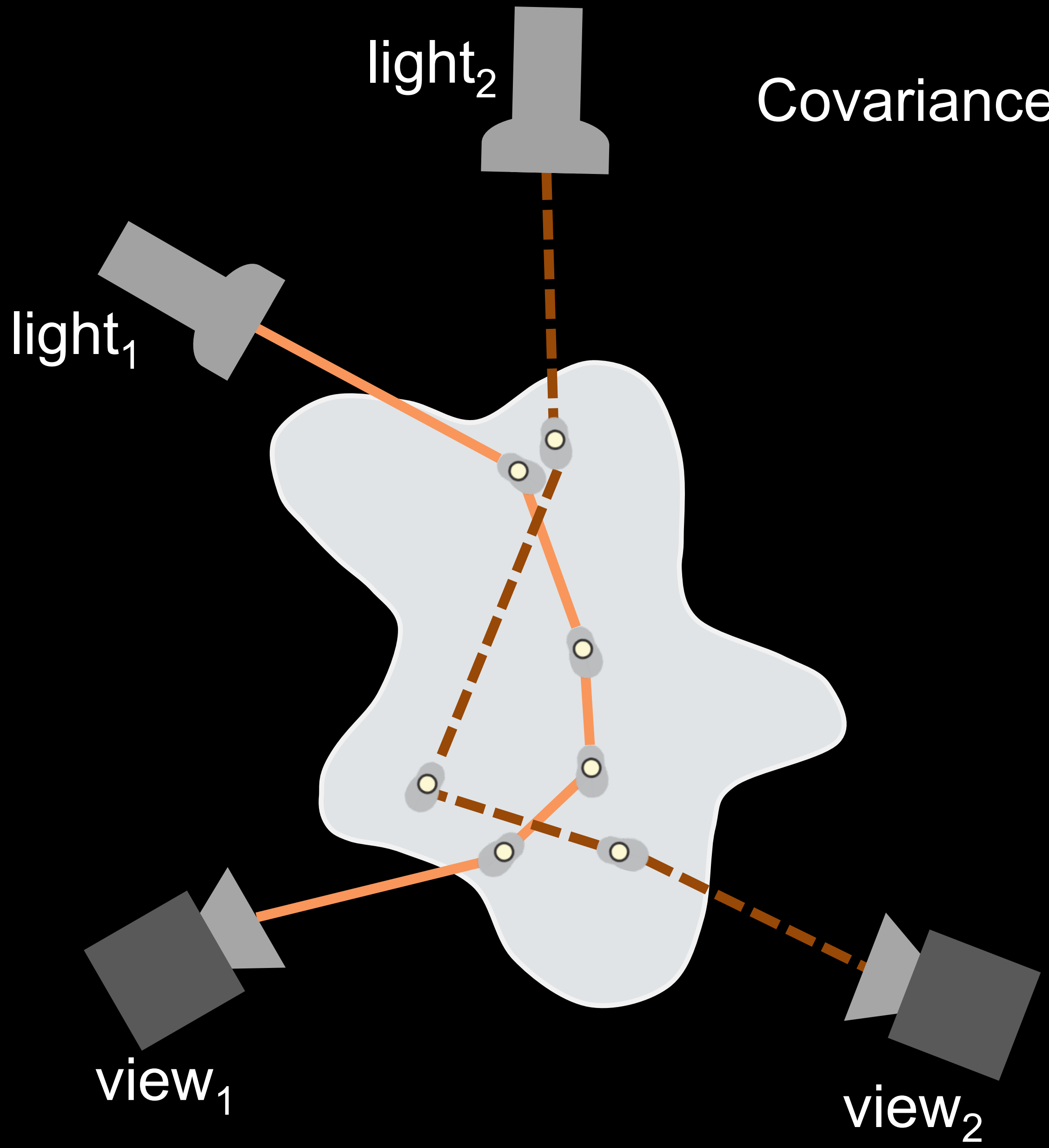
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

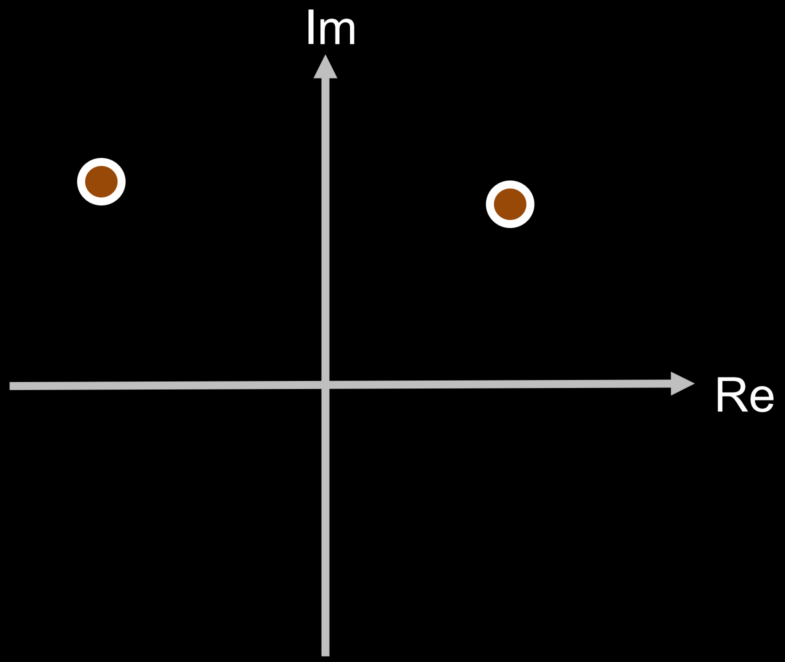
$u = |u|\, e^{i \cdot \text{phase}}$



Covariance rendering

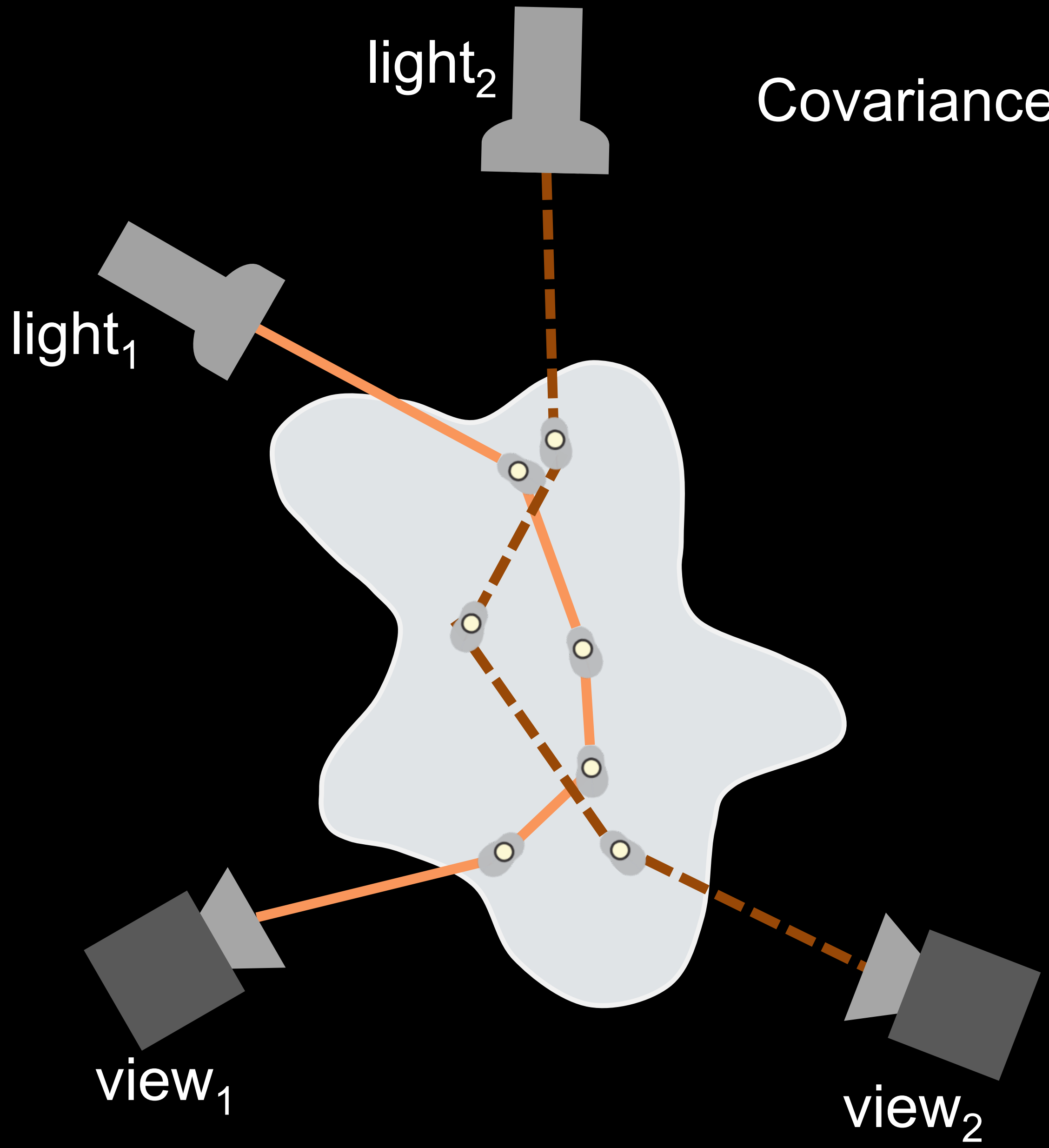
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

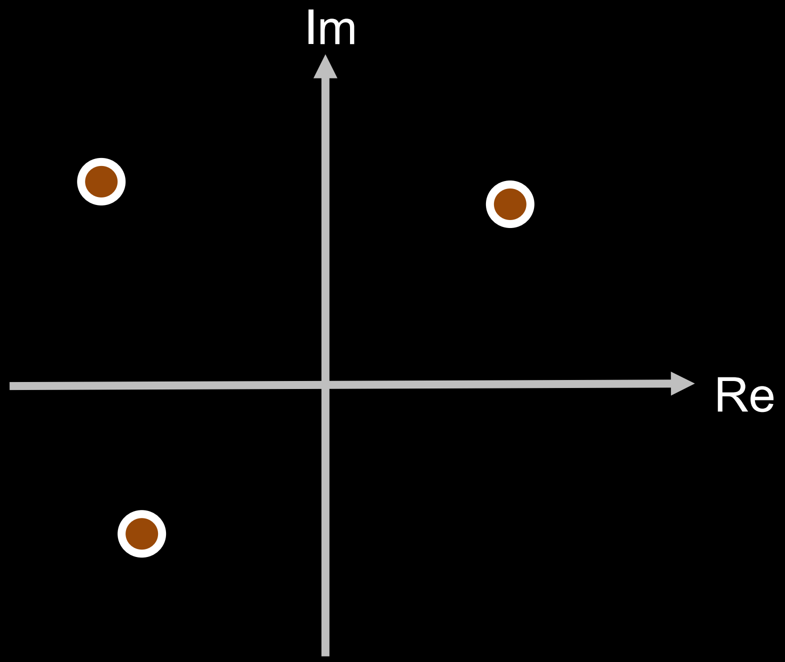




Covariance rendering

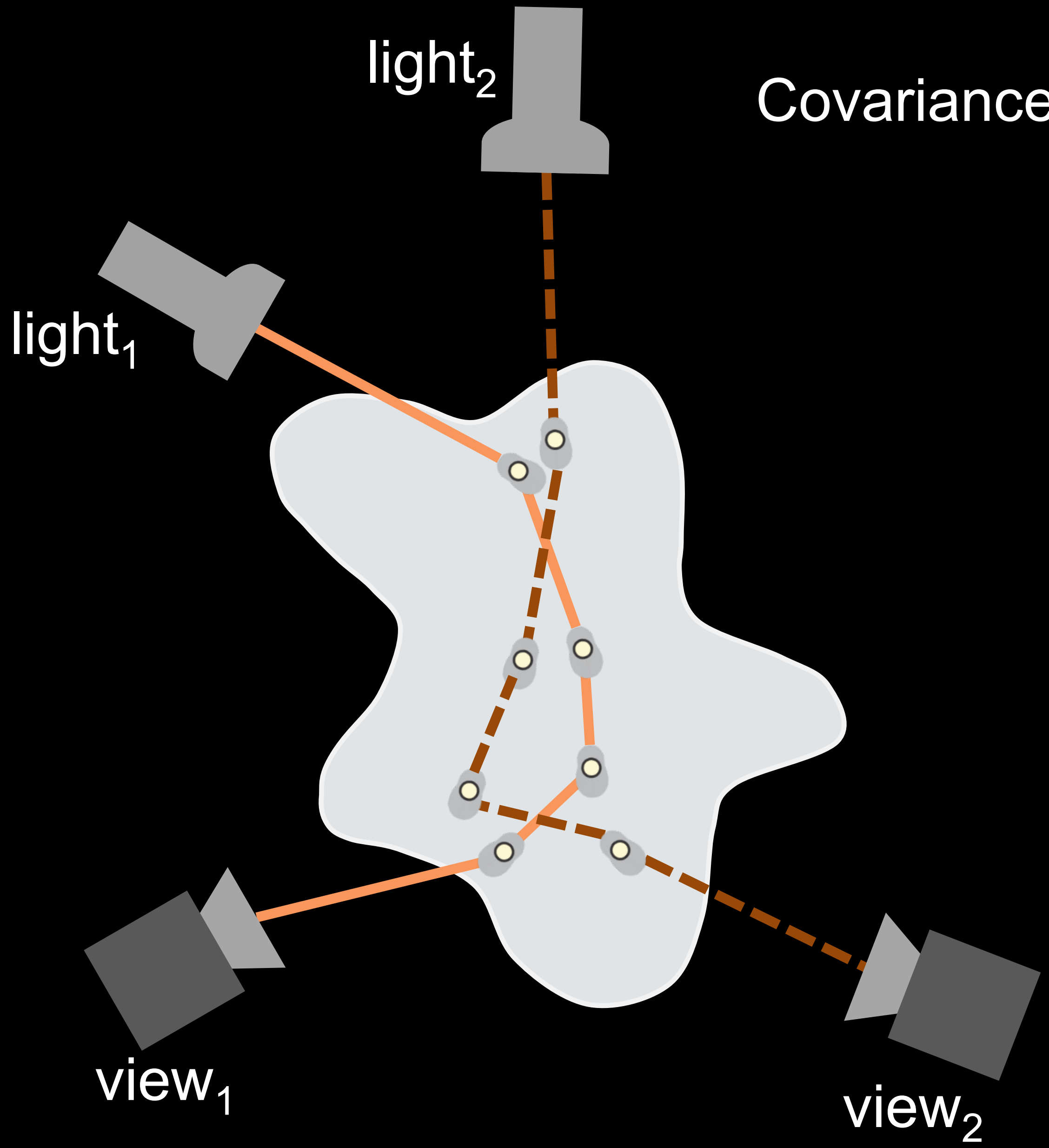
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

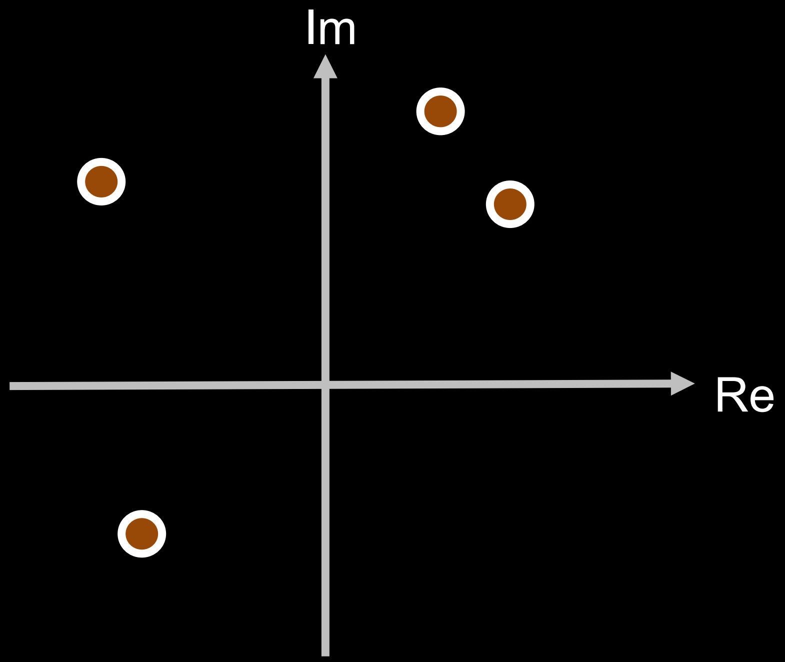




Covariance rendering

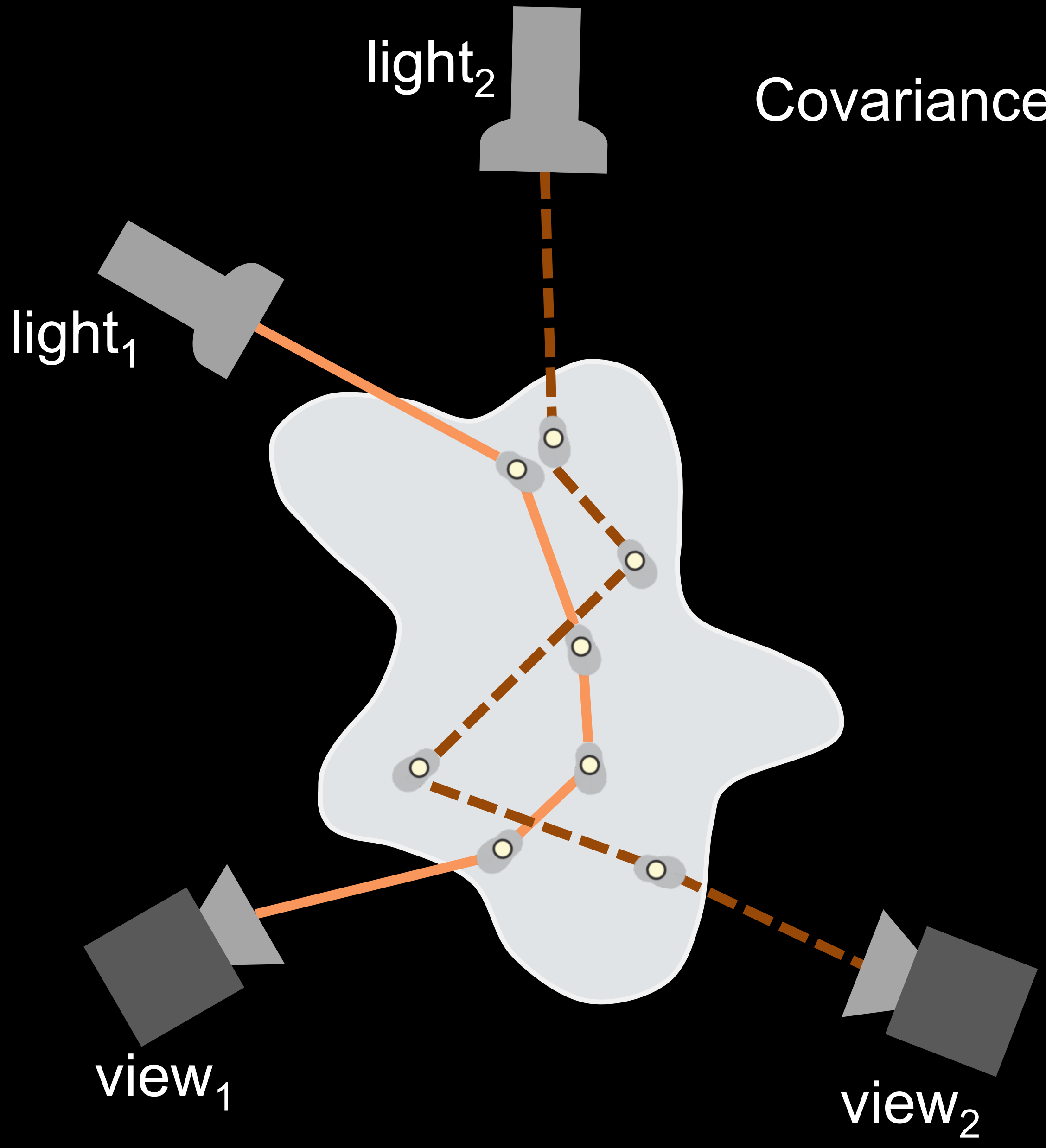
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

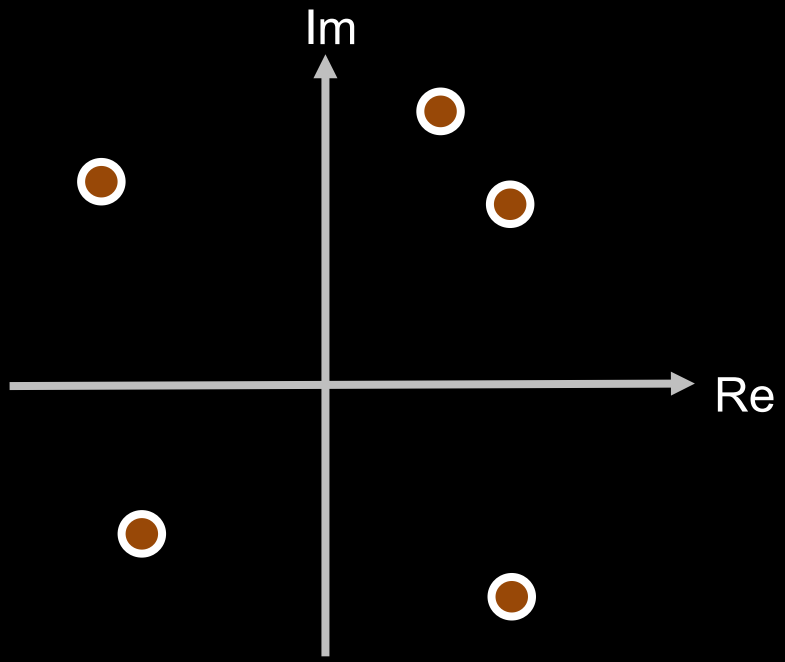




Covariance rendering

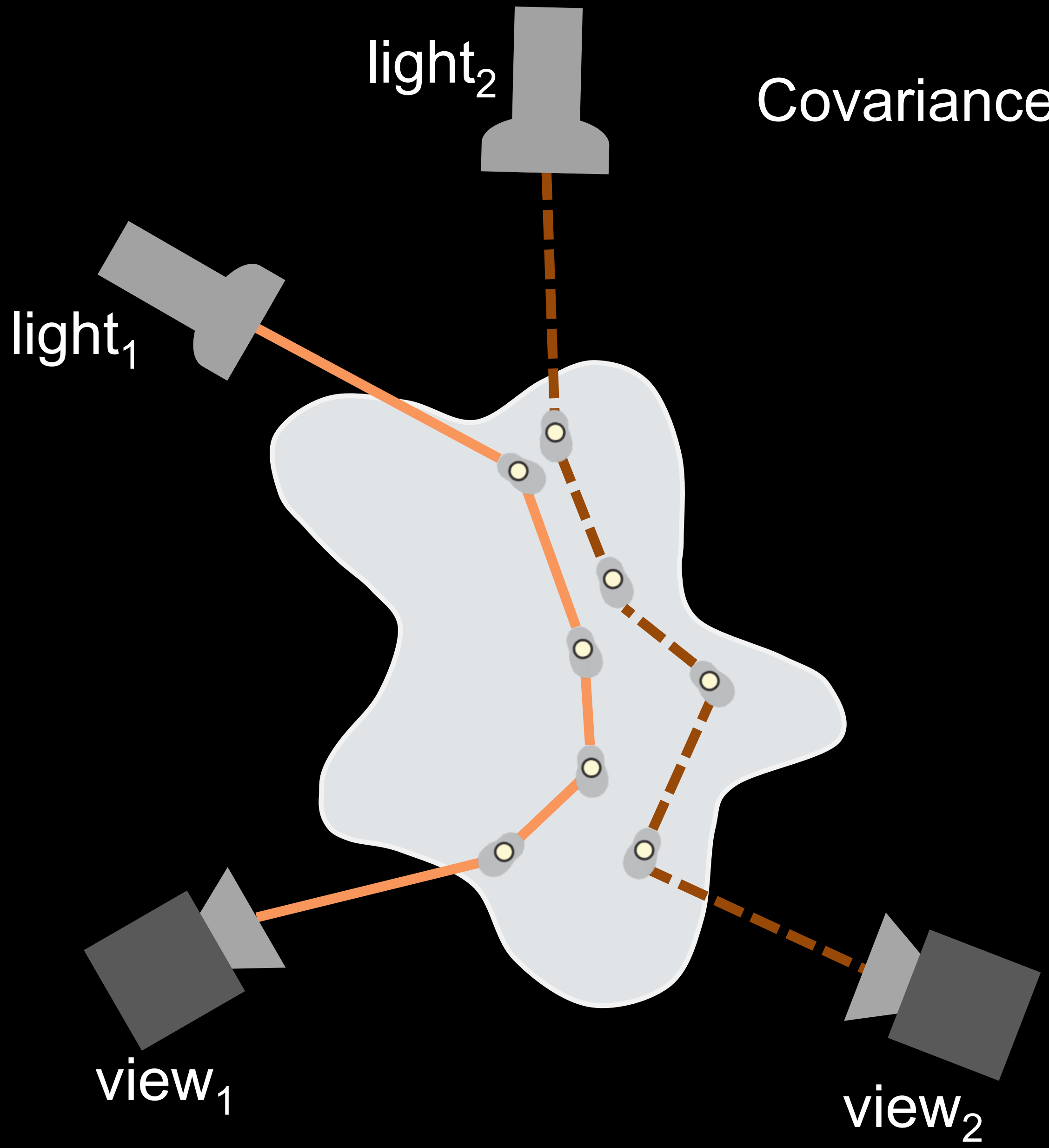
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

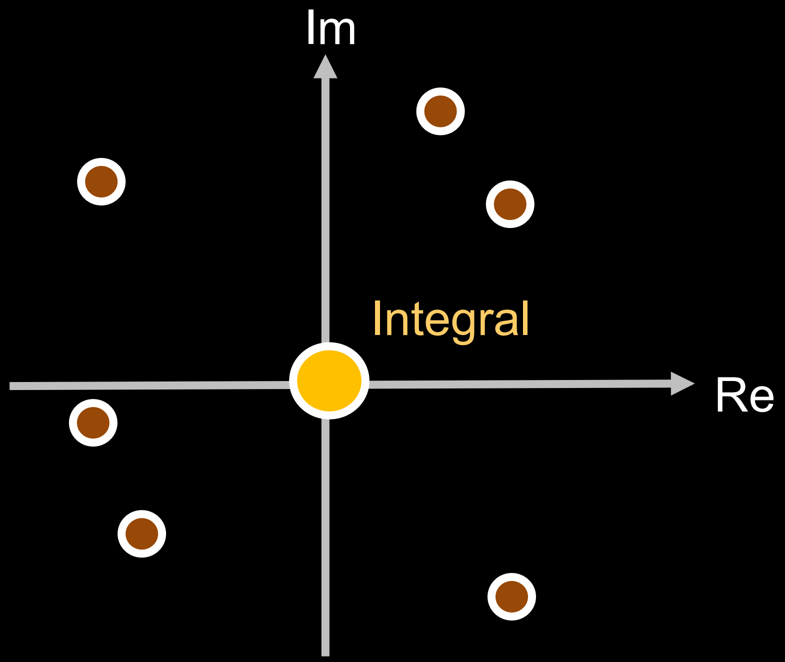




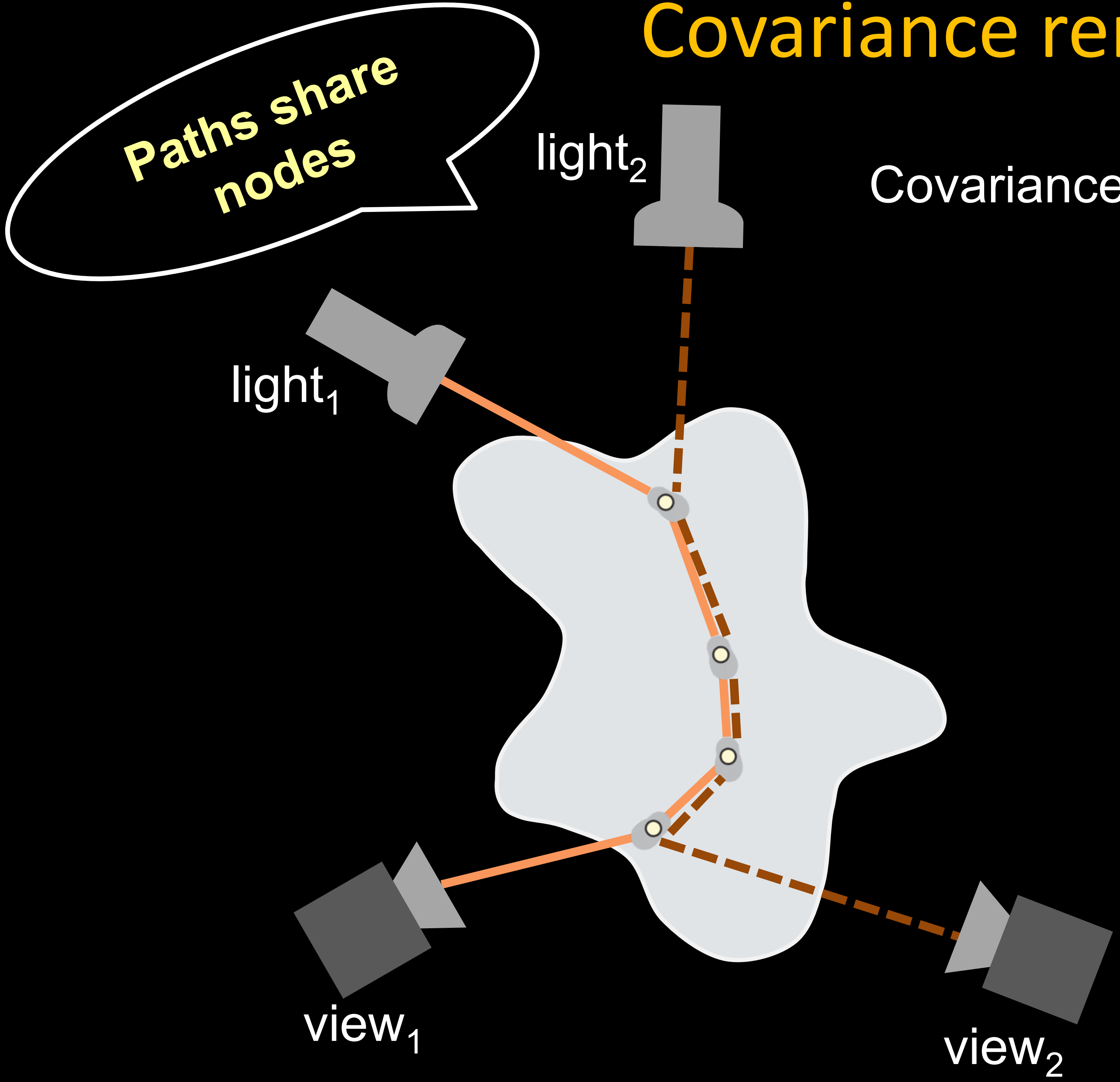
Covariance rendering

$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

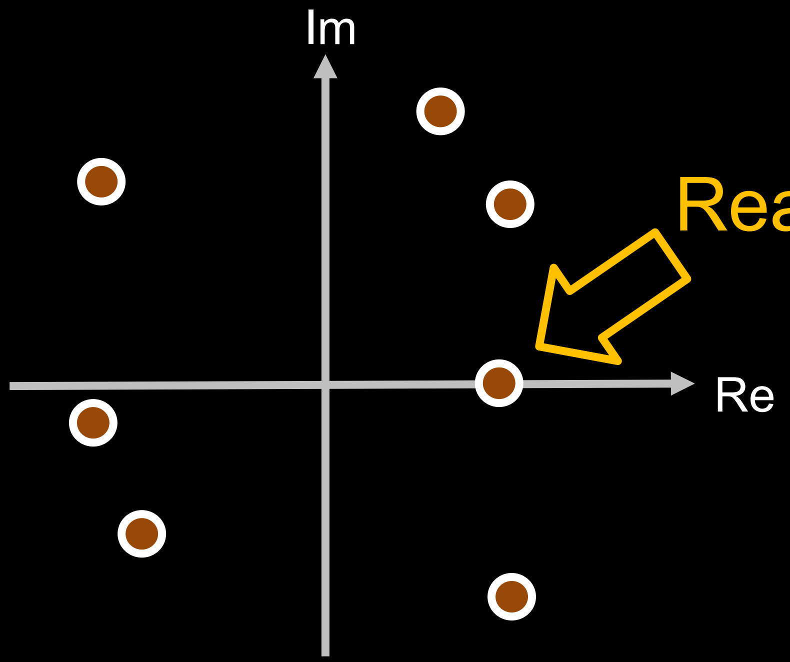




Covariance rendering

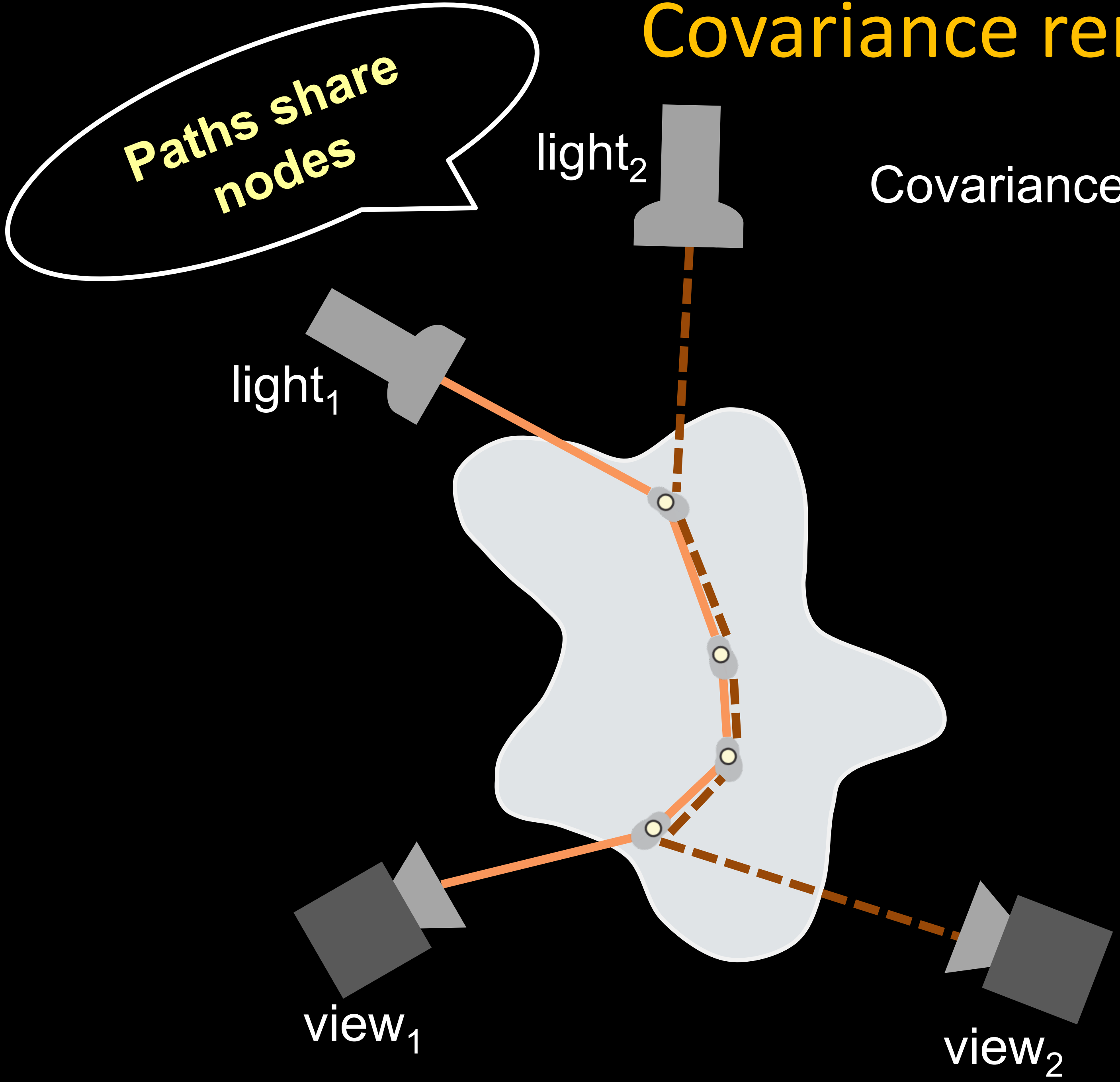


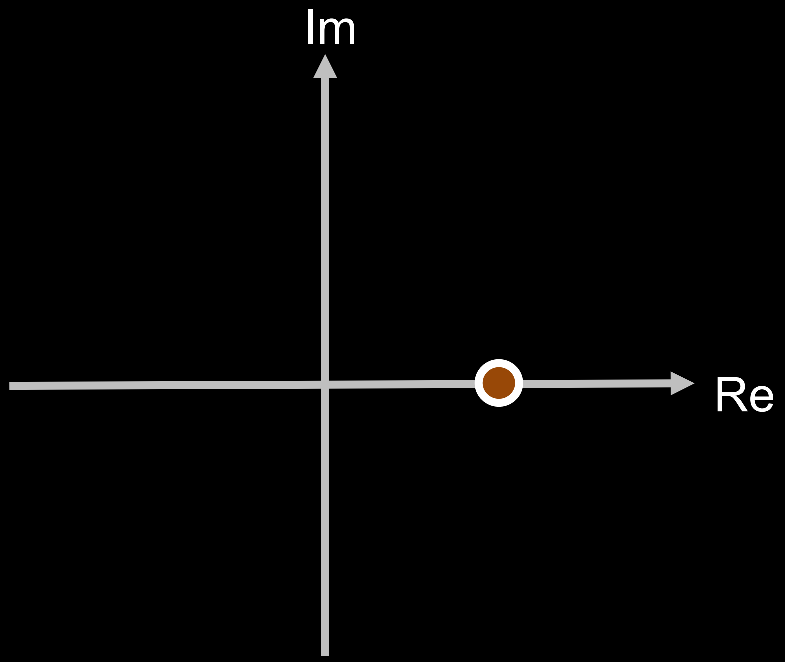
$$\text{Covariance} = \int_{path_1, path_2} u(path_1) \cdot u^*(path_2)$$



Covariance rendering

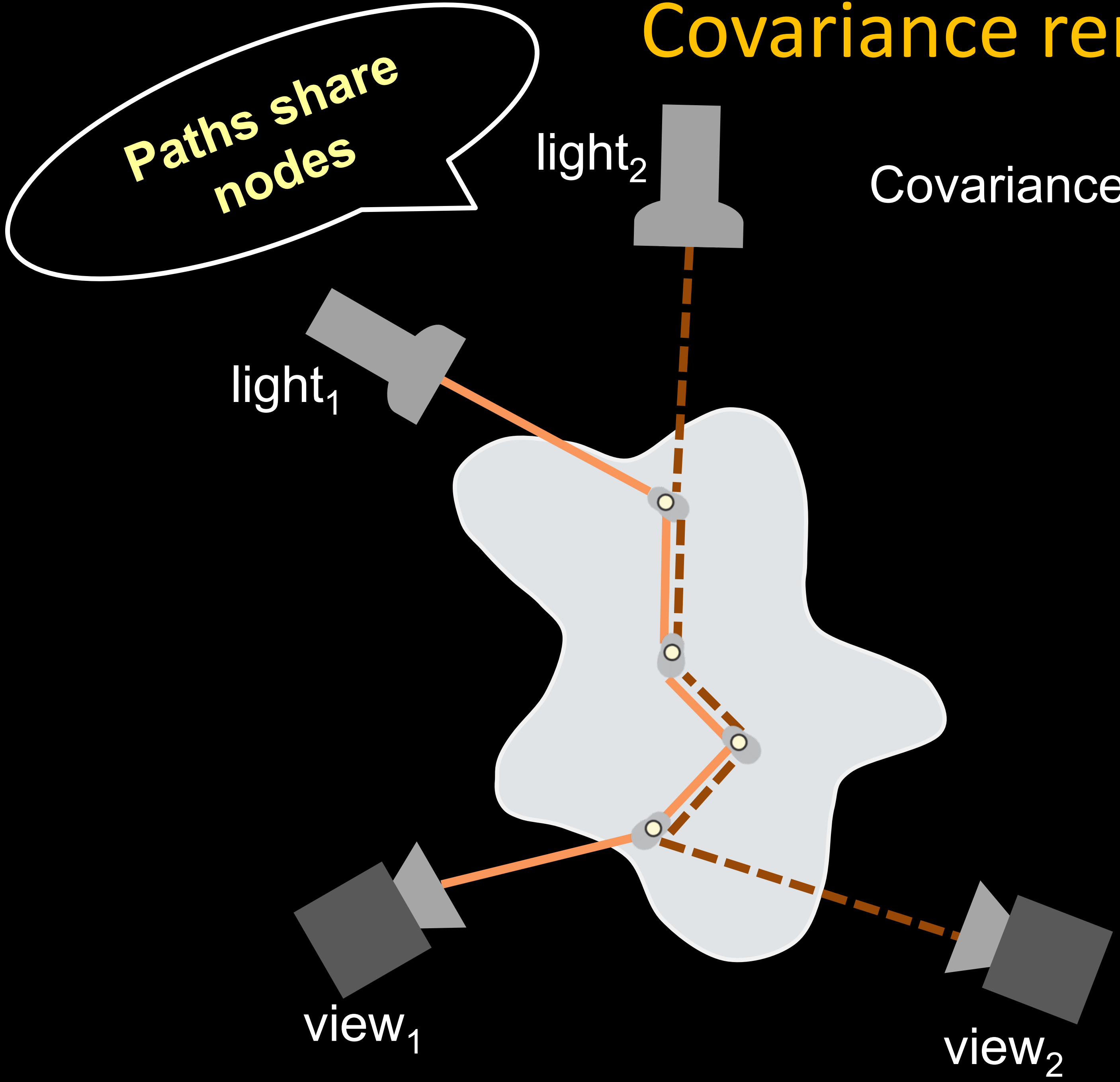
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

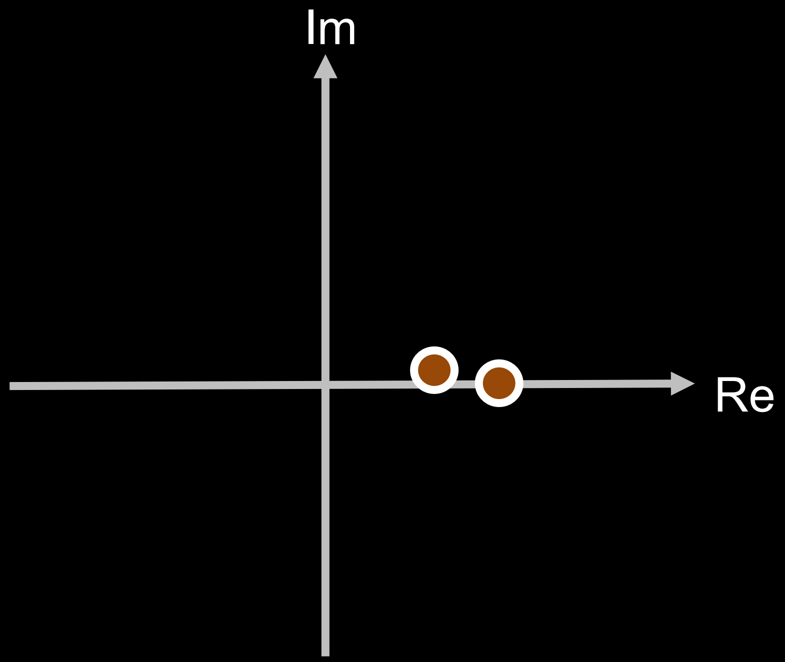




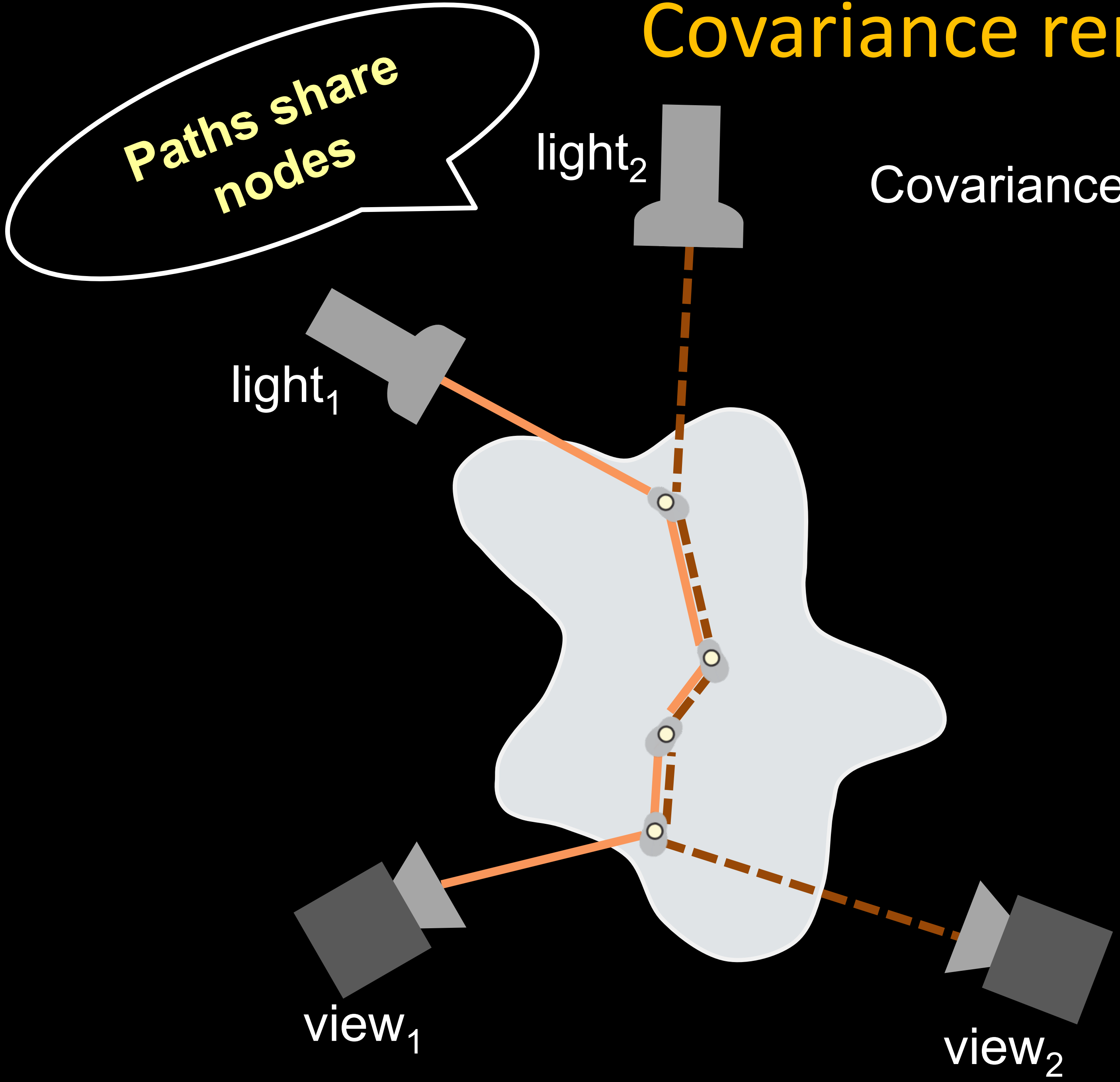
Covariance rendering

$$\text{Covariance} = \int_{path_1, path_2} u(path_1) \cdot u^*(path_2)$$

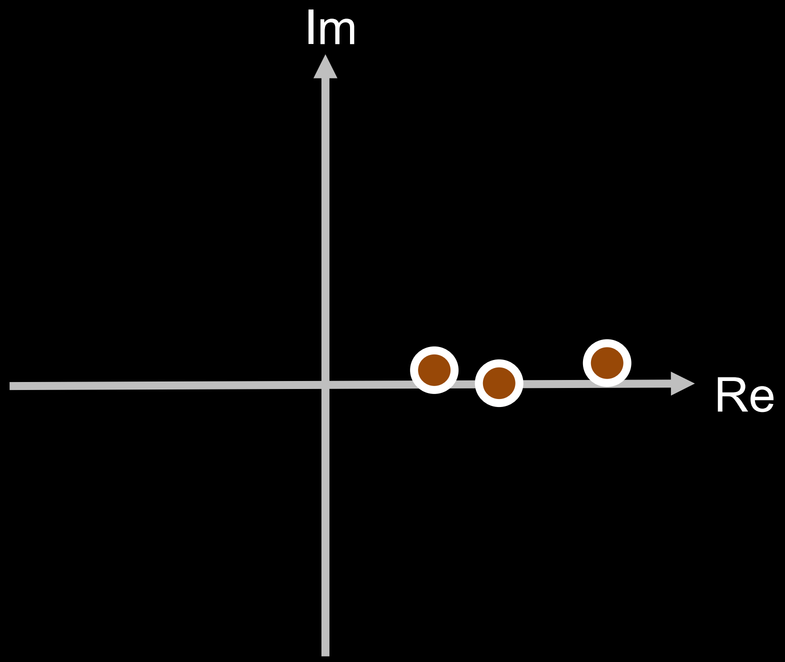




Covariance rendering

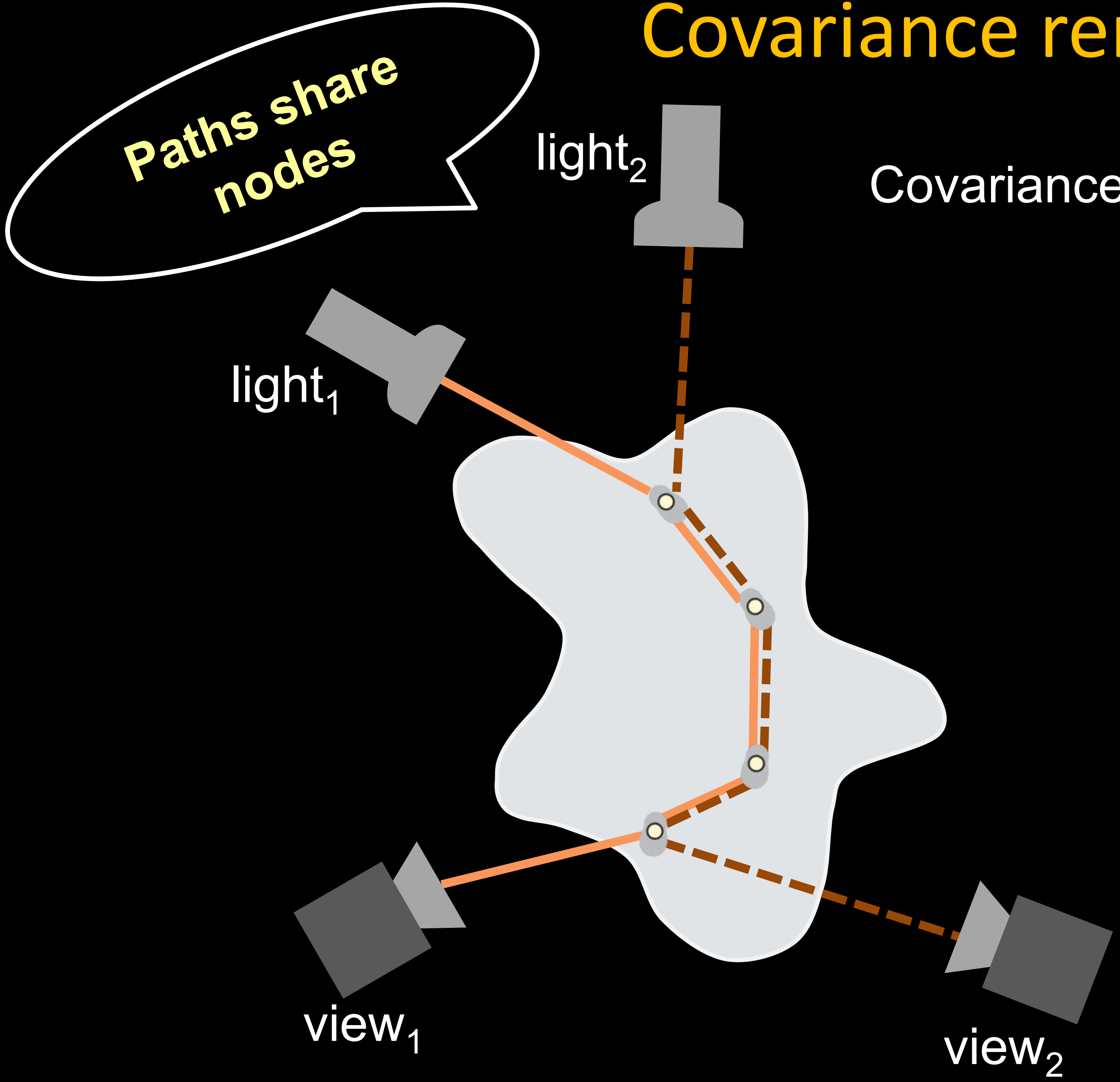


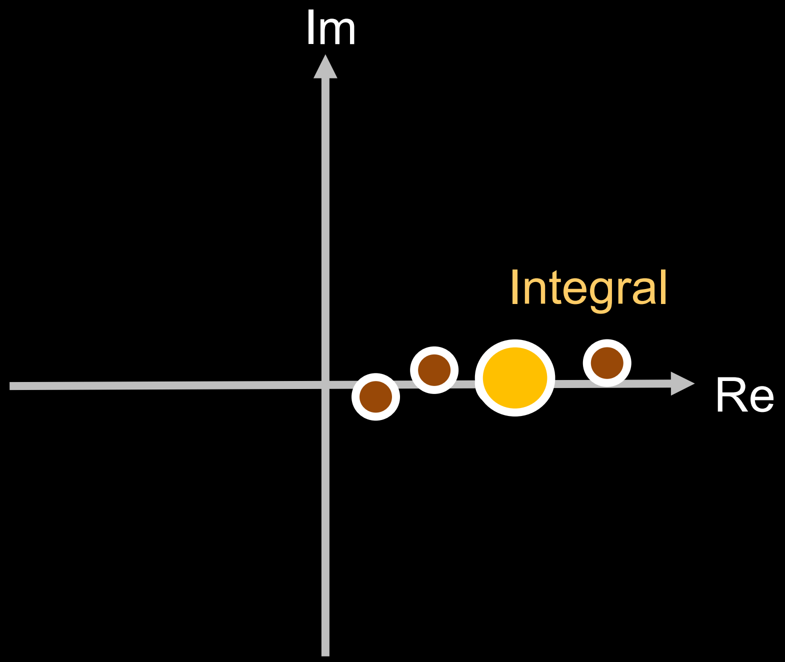
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1)\cdot u^*(\text{path}_2)$$



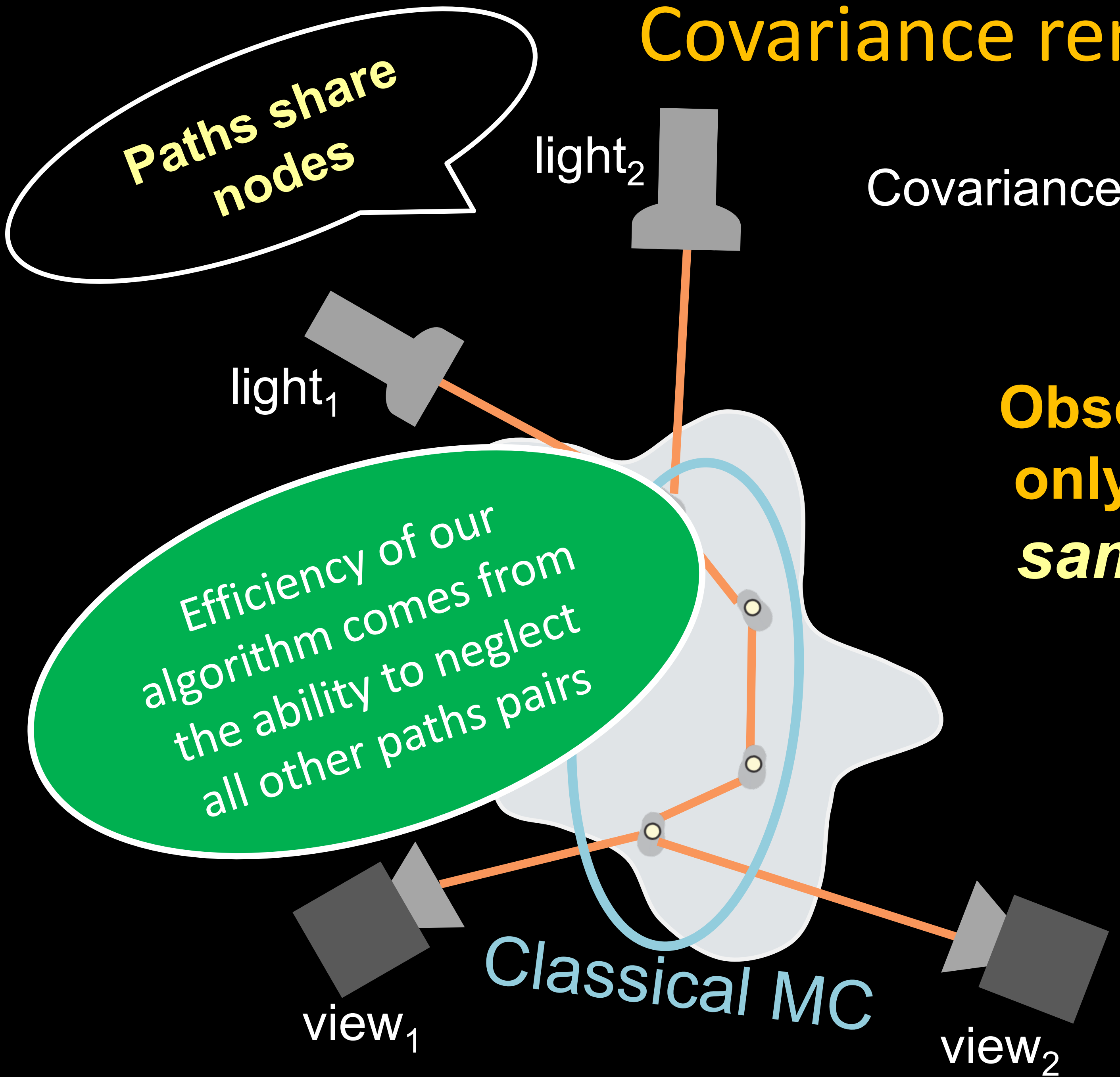
Covariance rendering

$$\text{Covariance} = \int_{path_1, path_2} u(path_1) \cdot u^*(path_2)$$



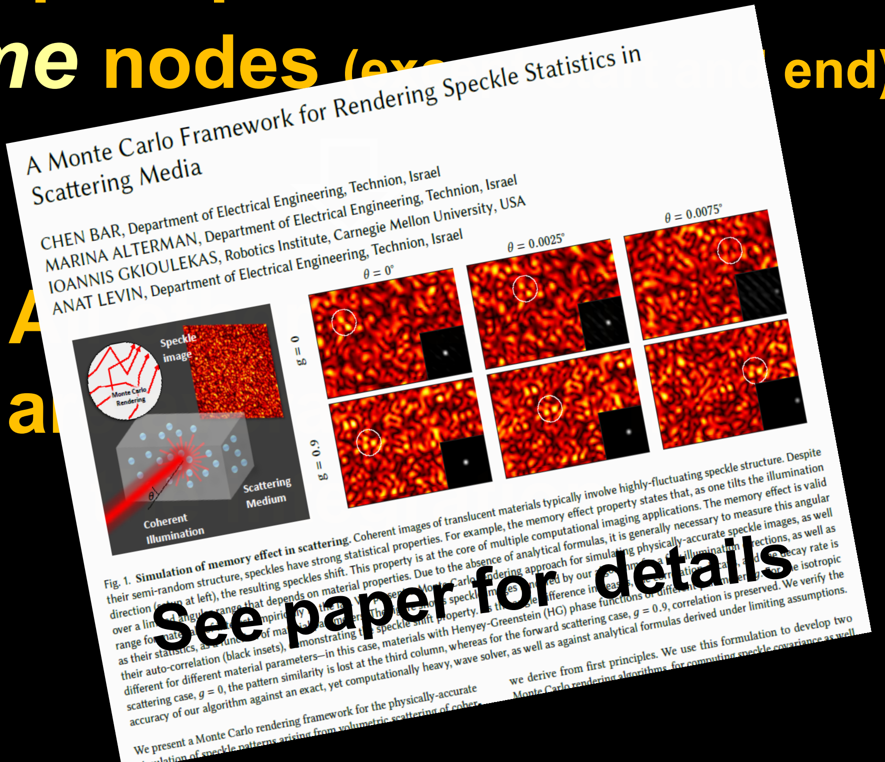


Covariance rendering



$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

Observation: need to consider only path pairs that share the *same* nodes (ex… end)



A Monte Carlo Framework for Rendering Speckle Statistics in Scattering Media

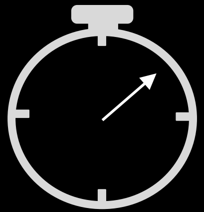
CHEN BAR, Department of Electrical Engineering, Technion, Israel
MARINA ALTERMAN, Department of Electrical Engineering, Technion, Israel
IOANNIS GKIOULEKAS, Robotics Institute, Carnegie Mellon University, USA
ANAT LEVIN, Department of Electrical Engineering, Technion, Israel

Fig. 1. **Simulation of memory effect in scattering.** Coherent images of translucent materials typically involve highly-fluctuating speckle structure. Despite their semi-random structure, speckles have strong statistical properties. For example, the memory effect property states that, as one tilts the illumination direction …

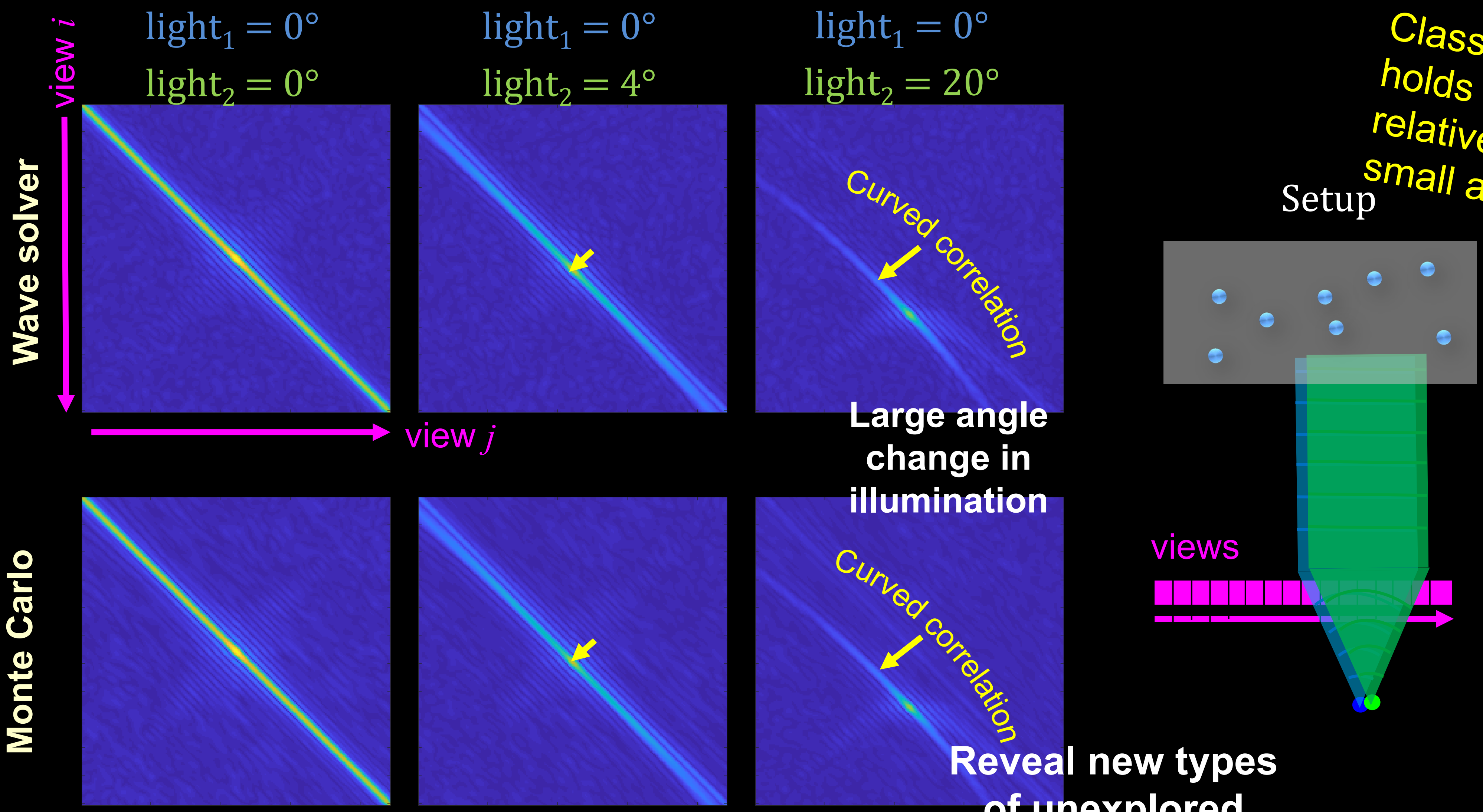
See paper for details

Validation: Wave Equation Covariances v.s. MC

Computation takes days



Several minutes

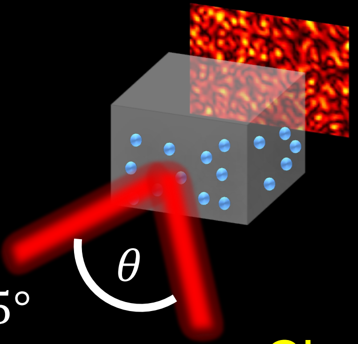


Classical ME holds for relatively small angles

Large angle change in illumination

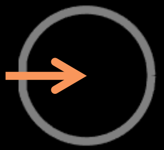
Reveal new types of unexplored correlations

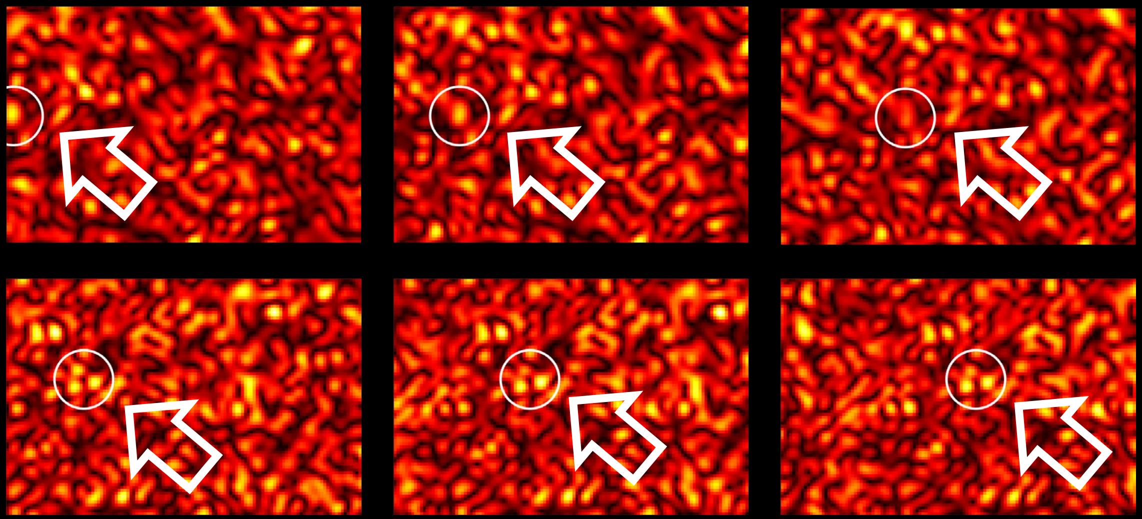
Rendering Speckles



Phase Function

$\theta = 0°$ $\theta = 0.0025°$ $\theta = 0.005°$

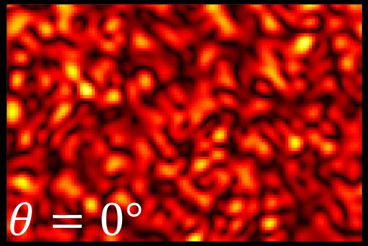


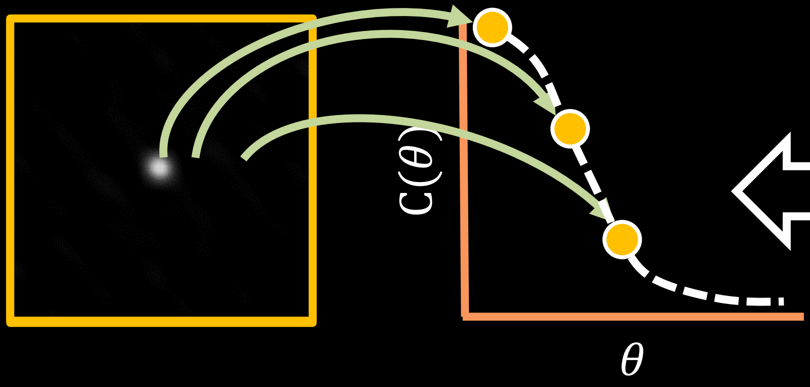


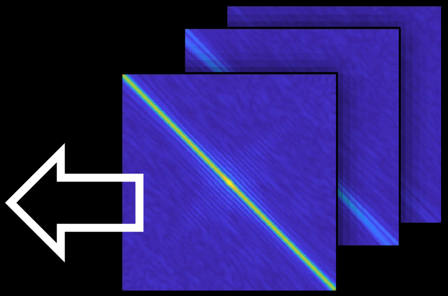
Classical ME holds for relatively small angles

Exact ME extent is different for different materials.

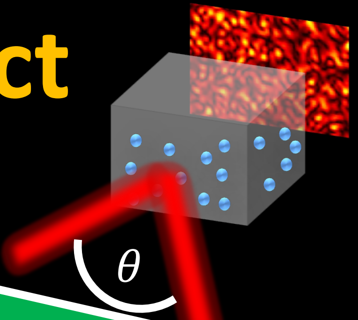
Computing ME extent as a function of θ :

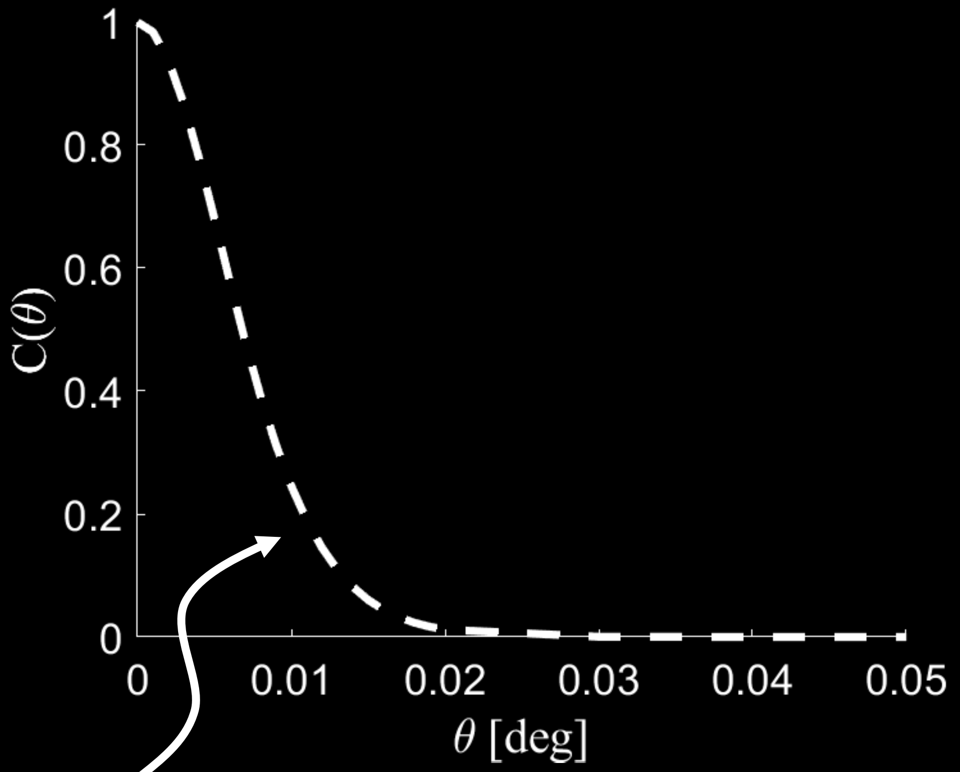






Evaluating the Memory Effect





Analytical solution based on diffusion approximation

This a
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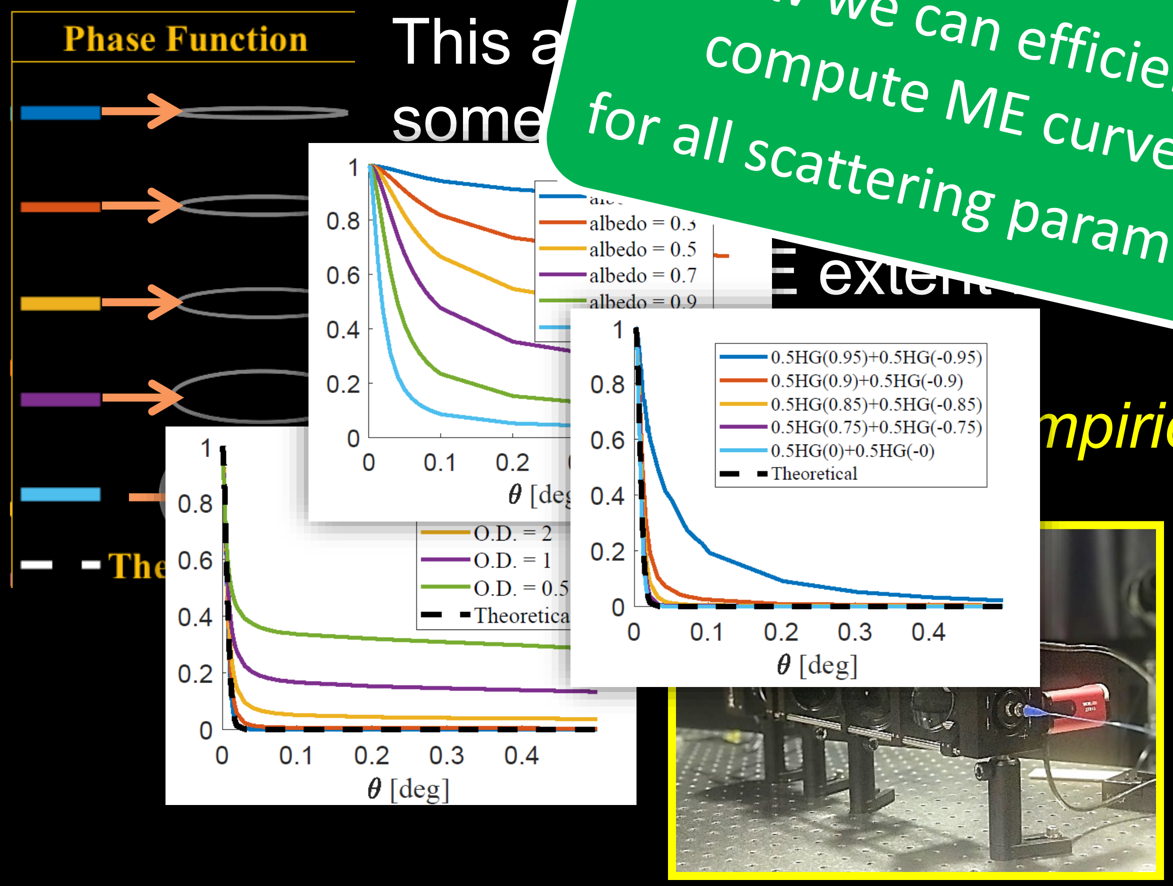
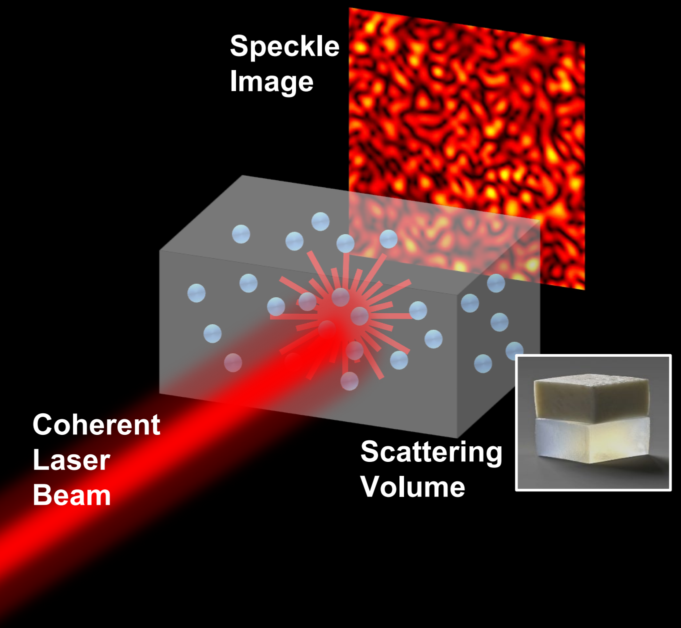


Now we can efficiently compute ME curves for all scattering parameters

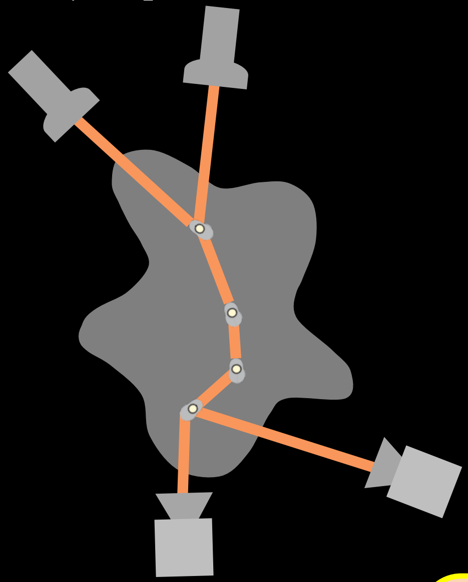
Summary

Problem:
Coherent Scattering

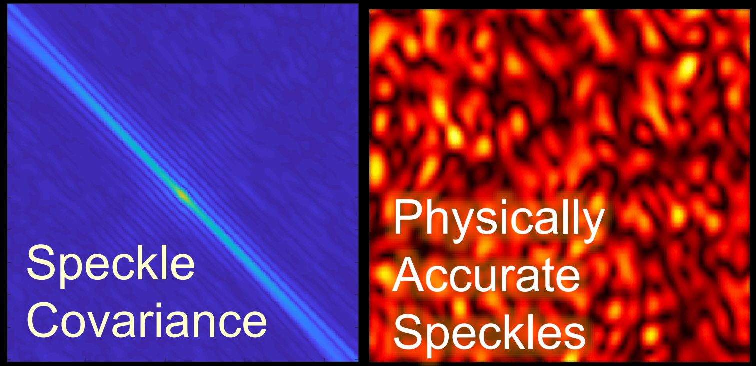


Path-integral formulation for speckle covariance

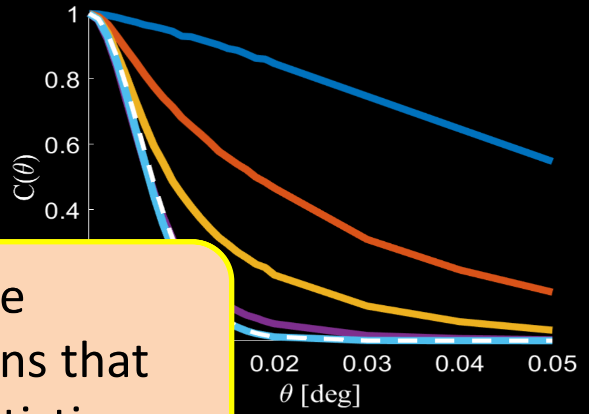
$$\text{Covariance} = \int_{\text{path}_1,\text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$



Efficient MC Rendering

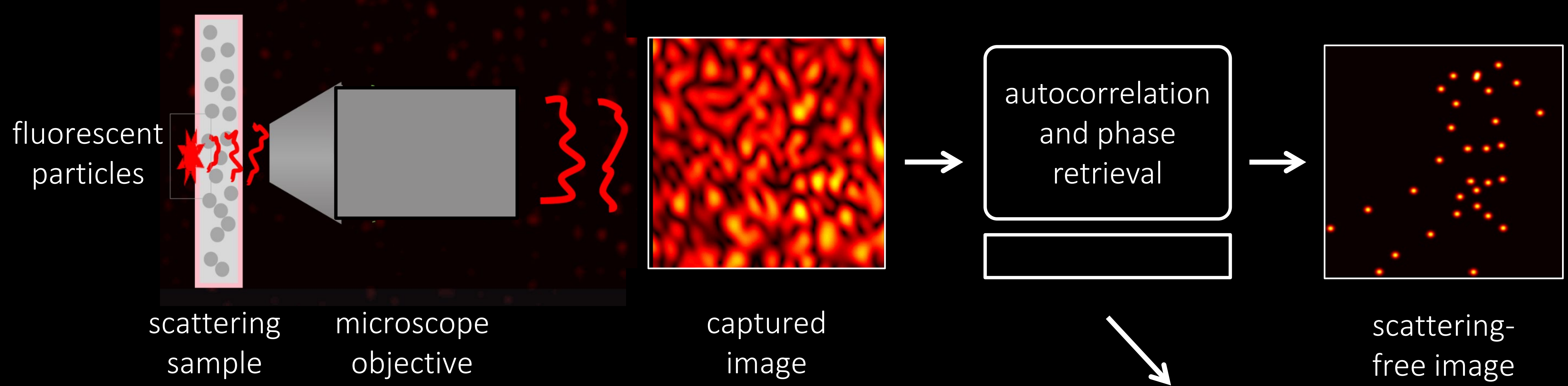


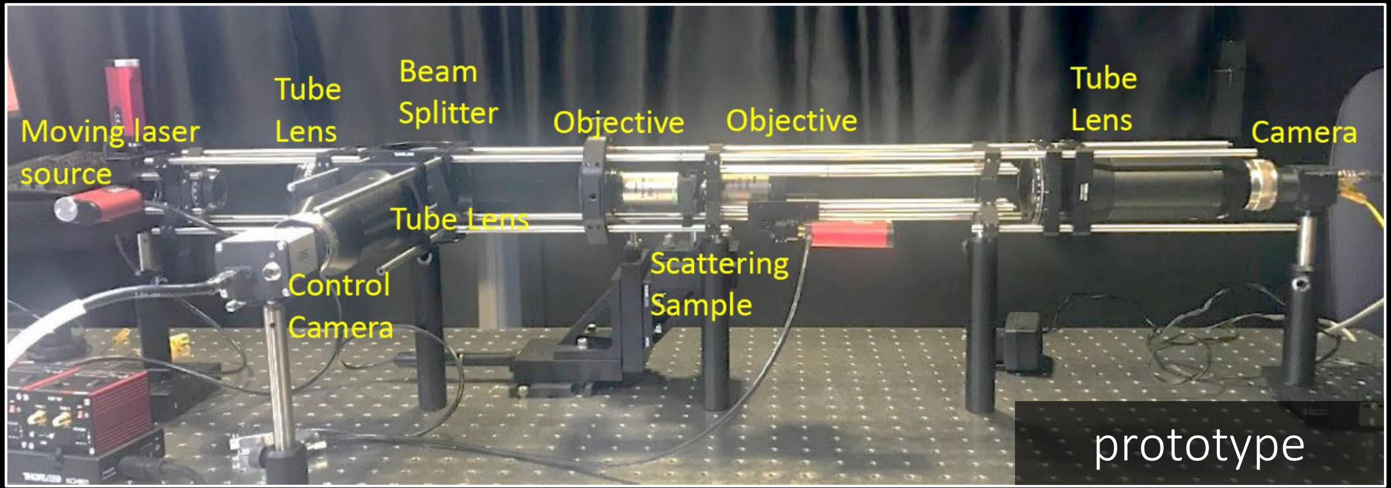
Memory Effect Evaluation



Potentially improve imaging applications that rely on speckle statistics

Speckle-based fluorescence microscopy



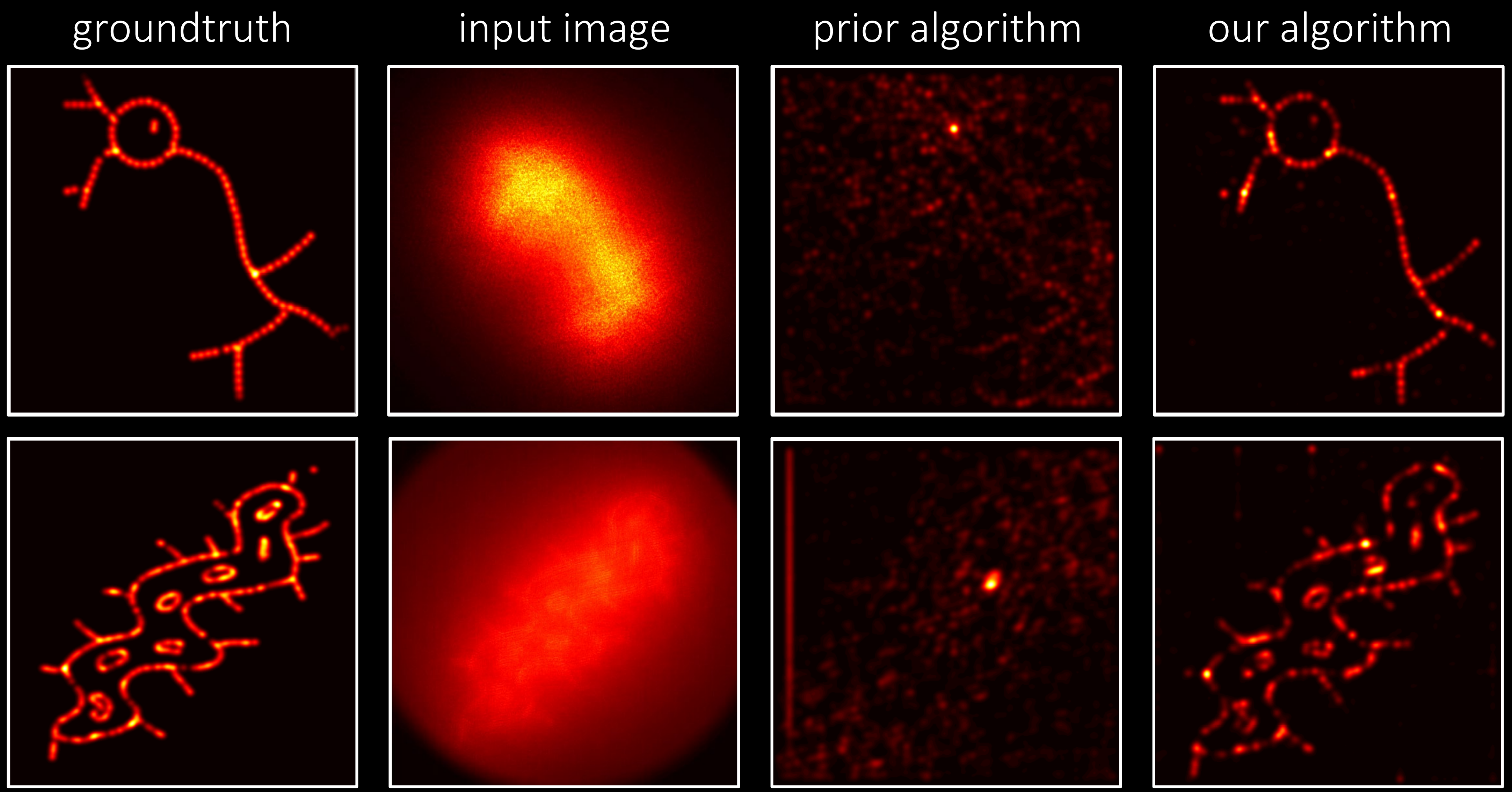


Performance strongly depends on:

- speckle statistics
- image priors
- tissue parameters

[PIs: Gkioulekas, Levin]

Better algorithms for fluorescence microscopy



[Alterman et al. Transactions on Graphics 2021]

Acquisition of scattering materials

Use differentiable speckle rendering to recover material parameters from speckle images