Participating media

http://graphics.cs.cmu.edu/courses/15-468

Course announcements

- Take-home quiz 6 posted, due Tuesday 3/14 at 23:59 (after spring break).

- Programming assignment 3 posted, due Friday 3/17 at 23:59 (after spring break).
    - How many of you have looked at/started/finished it?
    - Any questions?

- Suggest topics for Friday's reading group on Piazza.

Overview of today's lecture

- Participating media.

- Scattering material characterization.

- Volume rendering equation.

- Ray marching.

- Volumetric path tracing.

- Delta tracking.

Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

Fog

Steve Lacey

Clouds & Crepuscular rays

http://mev.fopf.mipt.ru

Fire

Underwater

Surface or Volume?

source: Flickr

Antelope Canyon, Az.

Wojciech Jarosz

Aerial (Atmospheric) Perspective

Wojciech Jarosz

Henrik Wann Jensen
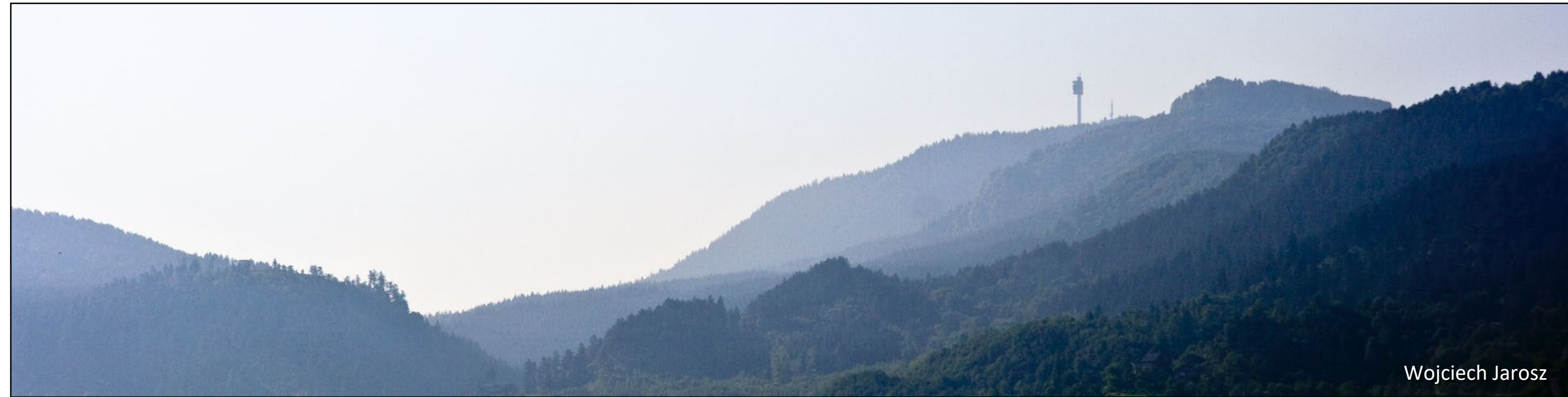
Leonardo da Vinci (1480)

Thus, if one is to be five times as distant, make it five times bluer.

—Treatise on Painting, Leonardo Da Vinci, pp 295, circa 1480.

Nebula

T.A.Rector (NOAO/AURA/NSF) and the Hubble Heritage Team (STScI/AURA/NASA)

Emission

http://wikipedia.org

Absorption

http://commons.wikimedia.org

Scattering

http://coclouds.com

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Defining Participating Media

Typically, we do not model particles of a medium explicitly (wouldn't fit in memory, completely impractical to ray trace)

The properties are described statistically using various coefficients and densities

- Conceptually similar idea as microfacet models

Defining Participating Media

Homogeneous:

- Infinite or bounded by a surface or simple shape
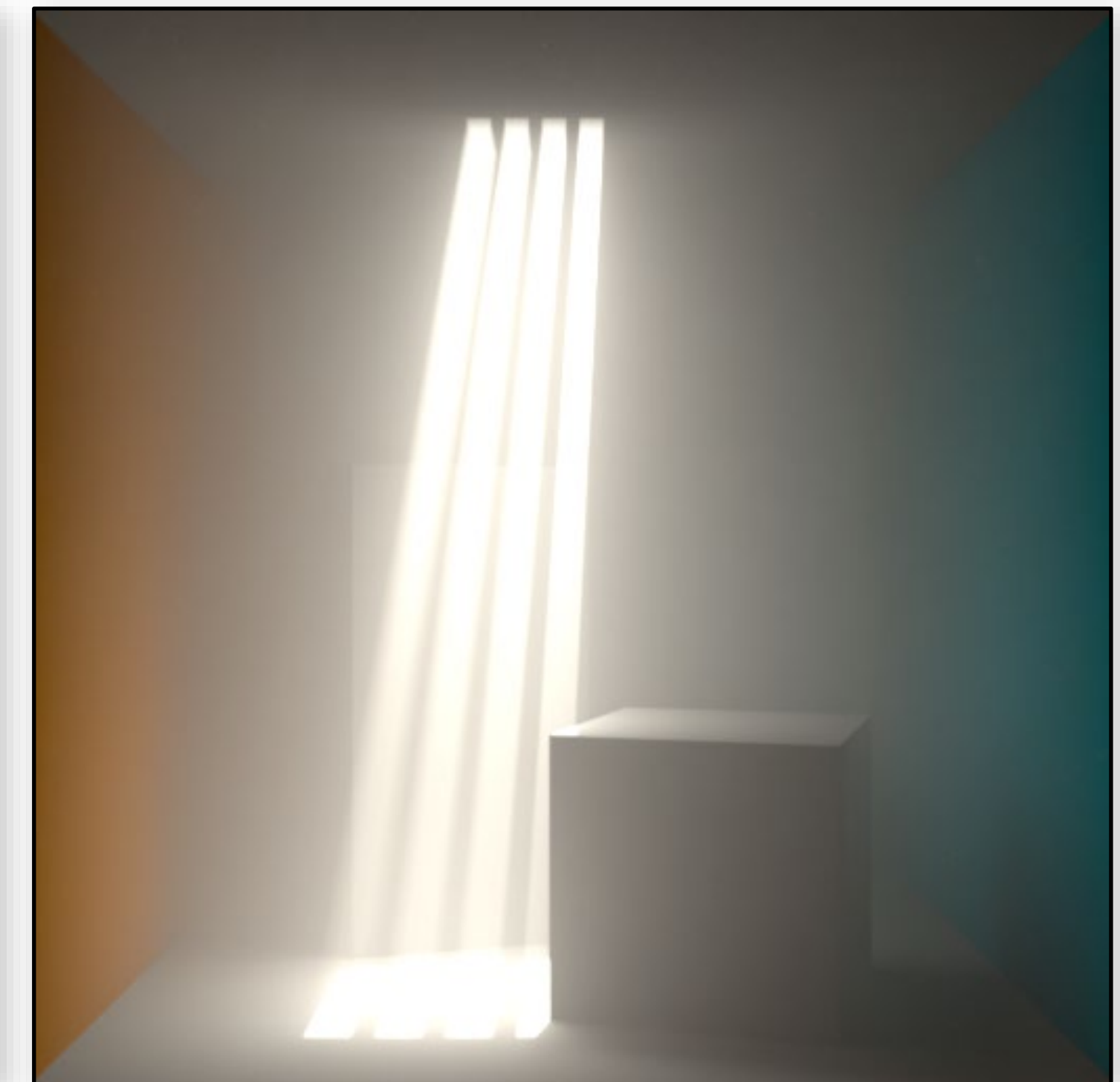
Defining Participating Media

Heterogeneous (spatially varying coefficients):

- Procedurally, e.g., using a noise function

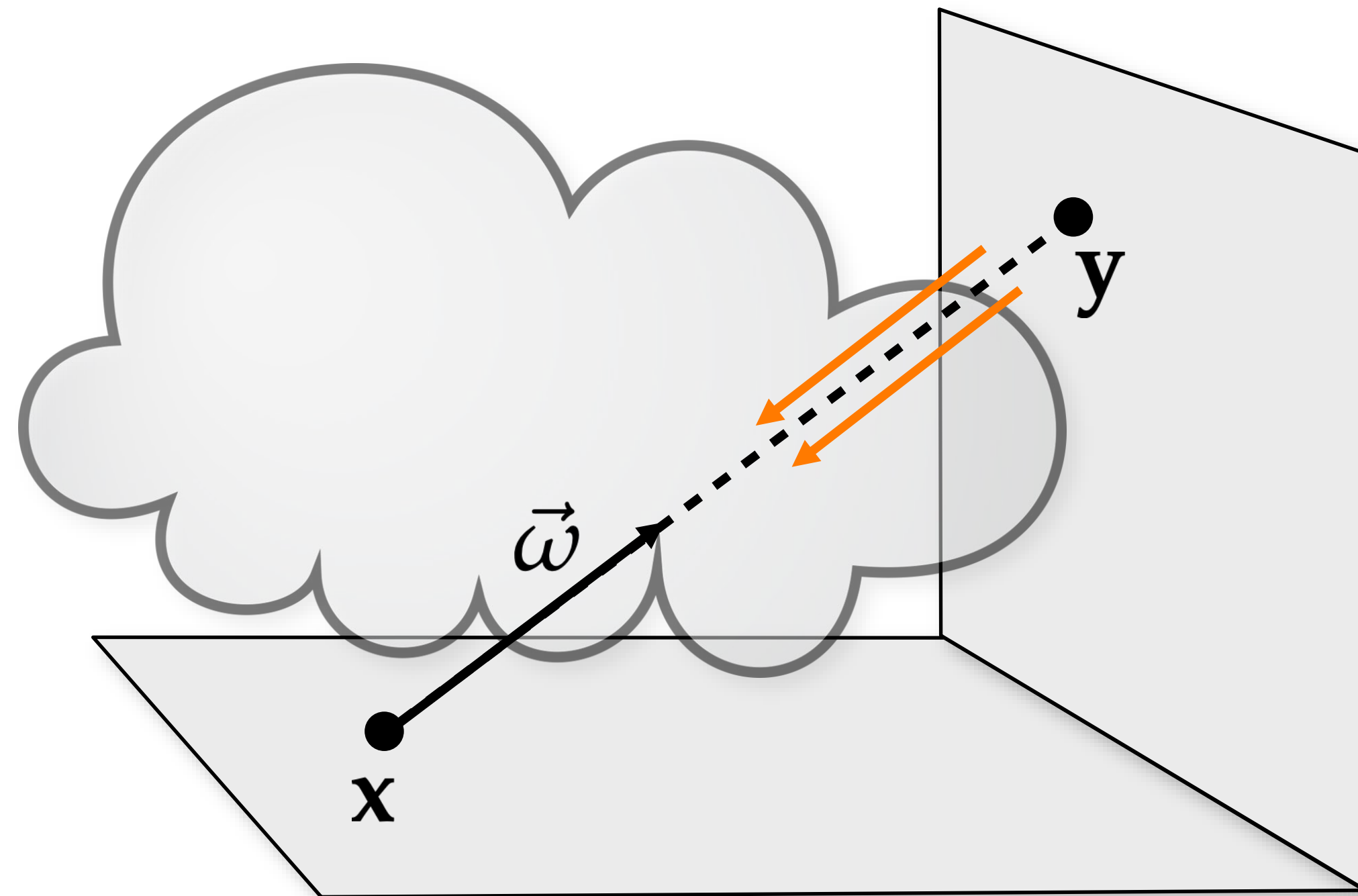- Simulation + volume discretization, e.g., a voxel grid
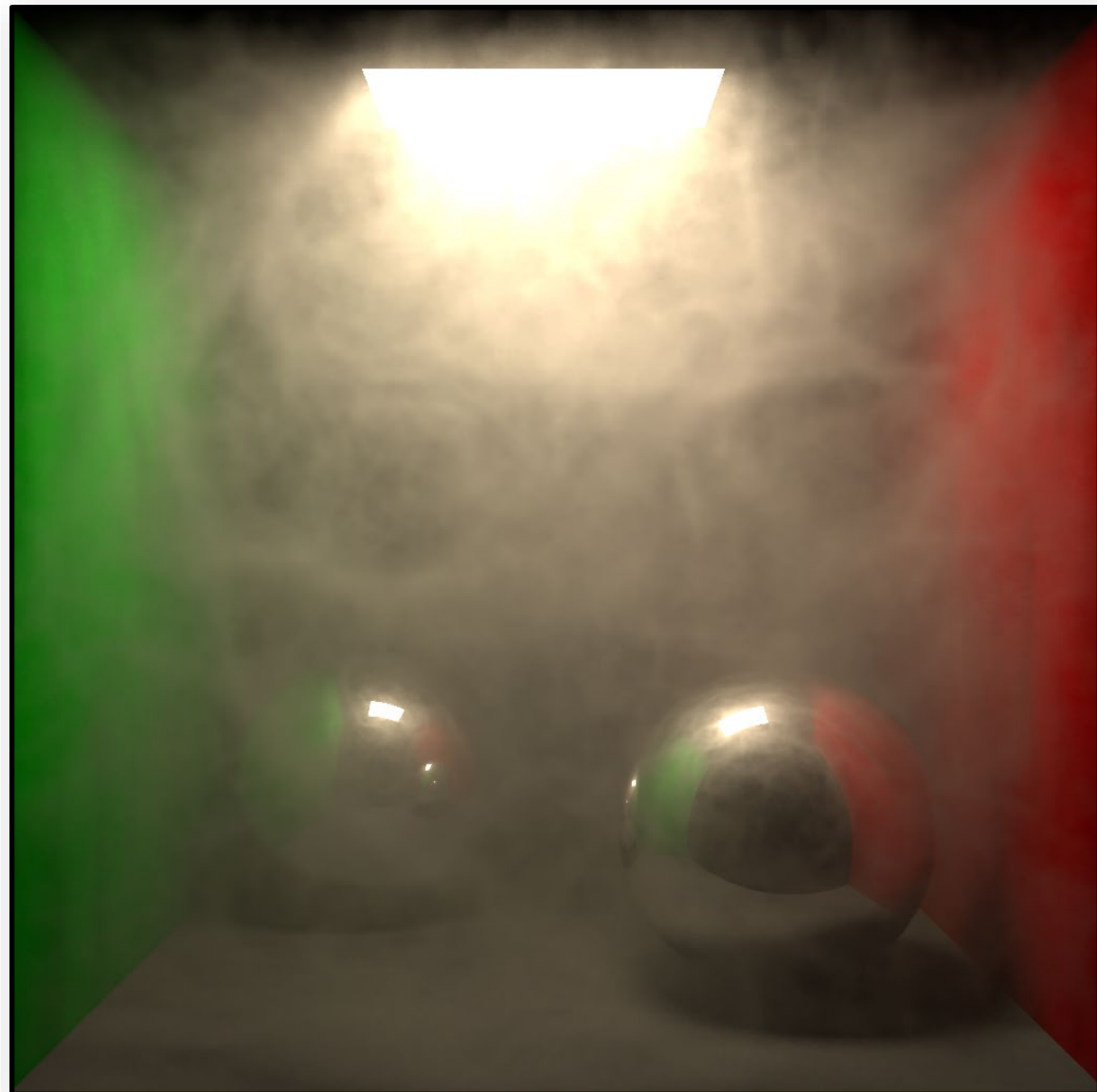
Radiance

The main quantity we are interested in for rendering is radiance
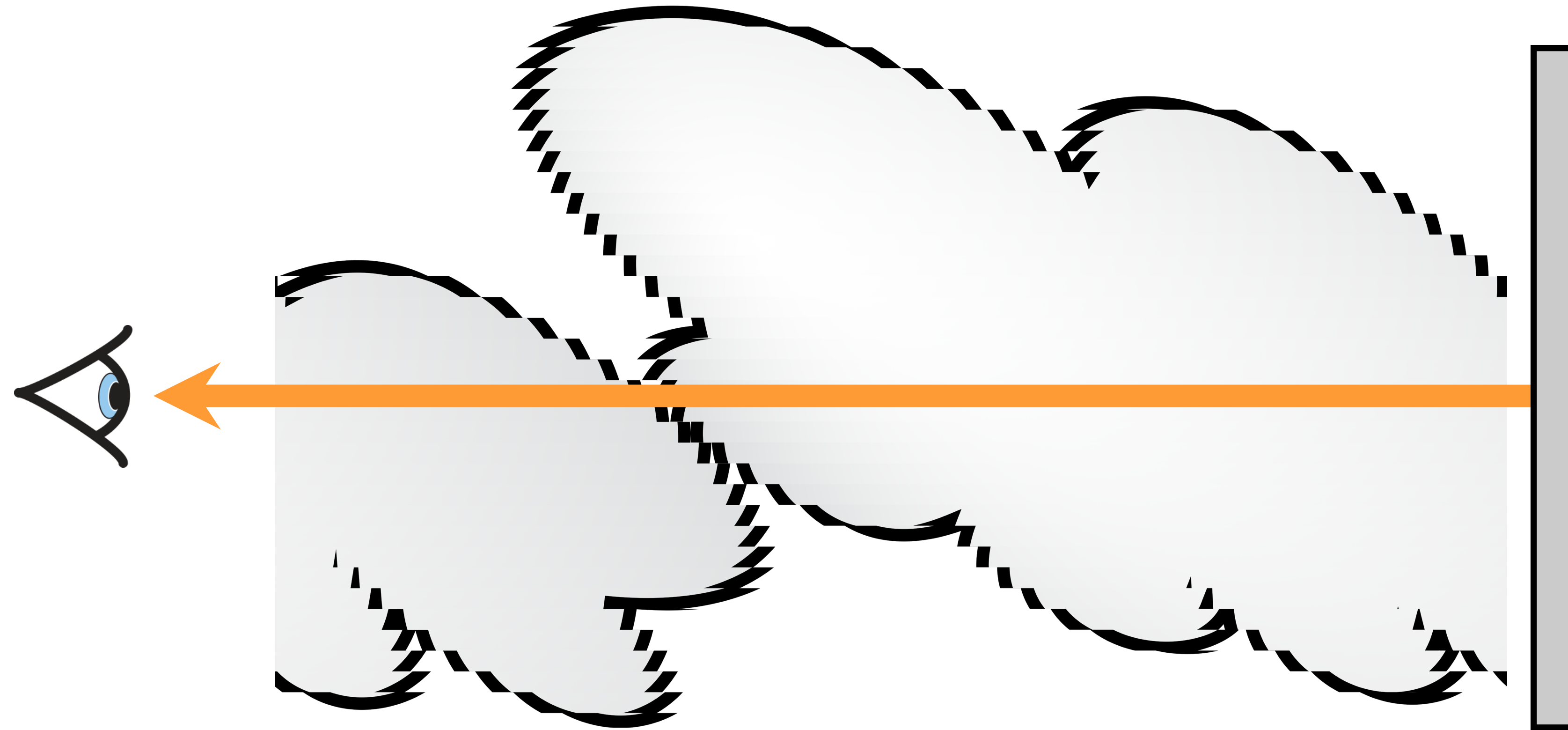
Previously: radiance *remains constant* along rays between surfaces
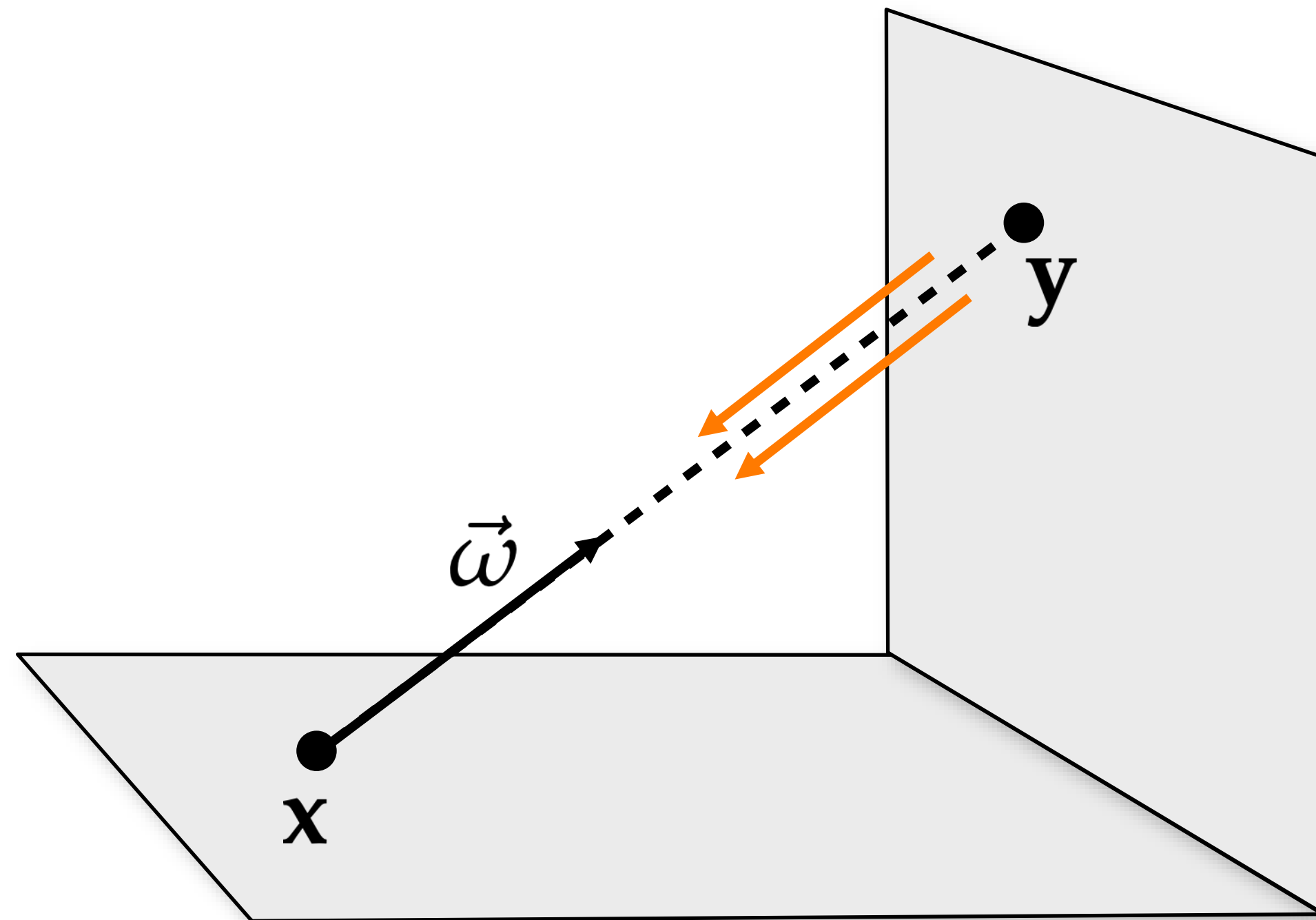
$$L_i(\mathbf{x}, \vec{\omega}) = L_o(\mathbf{y}, -\vec{\omega})$$
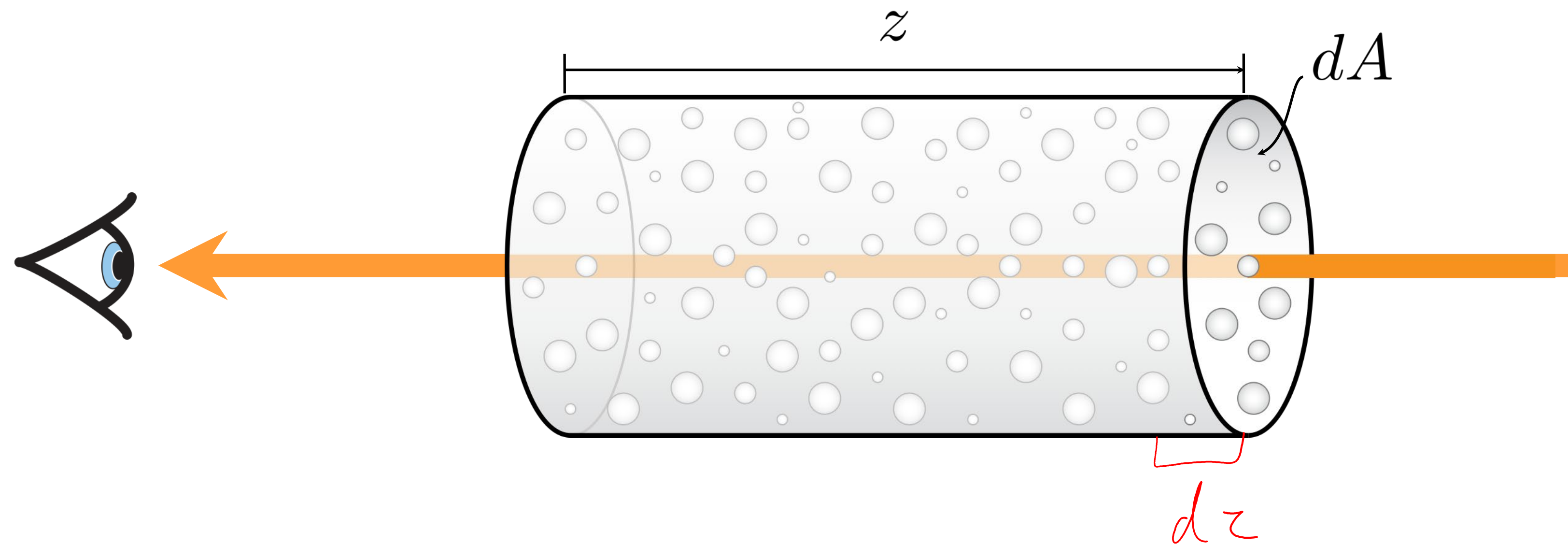$$\mathbf{y} = r(\mathbf{x}, \vec{\omega})$$

Radiance

The main quantity we are interested in for rendering is radiance

Now: radiance may *change* along rays between surfaces

$$L_i(\mathbf{x}, \vec{\omega}) \neq L_o(\mathbf{y}, -\vec{\omega})$$
$$\mathbf{y} = r(\mathbf{x}, \vec{\omega})$$

Participating Media

Differential Beam

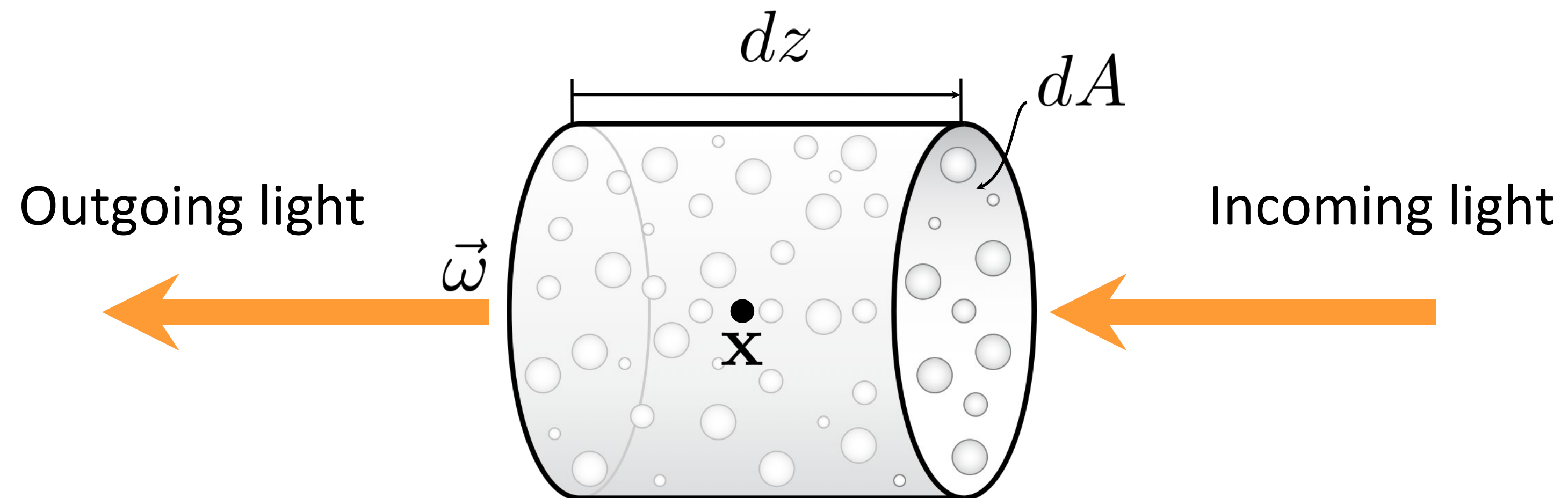How much light is *lost*/*gained* along the differential beam
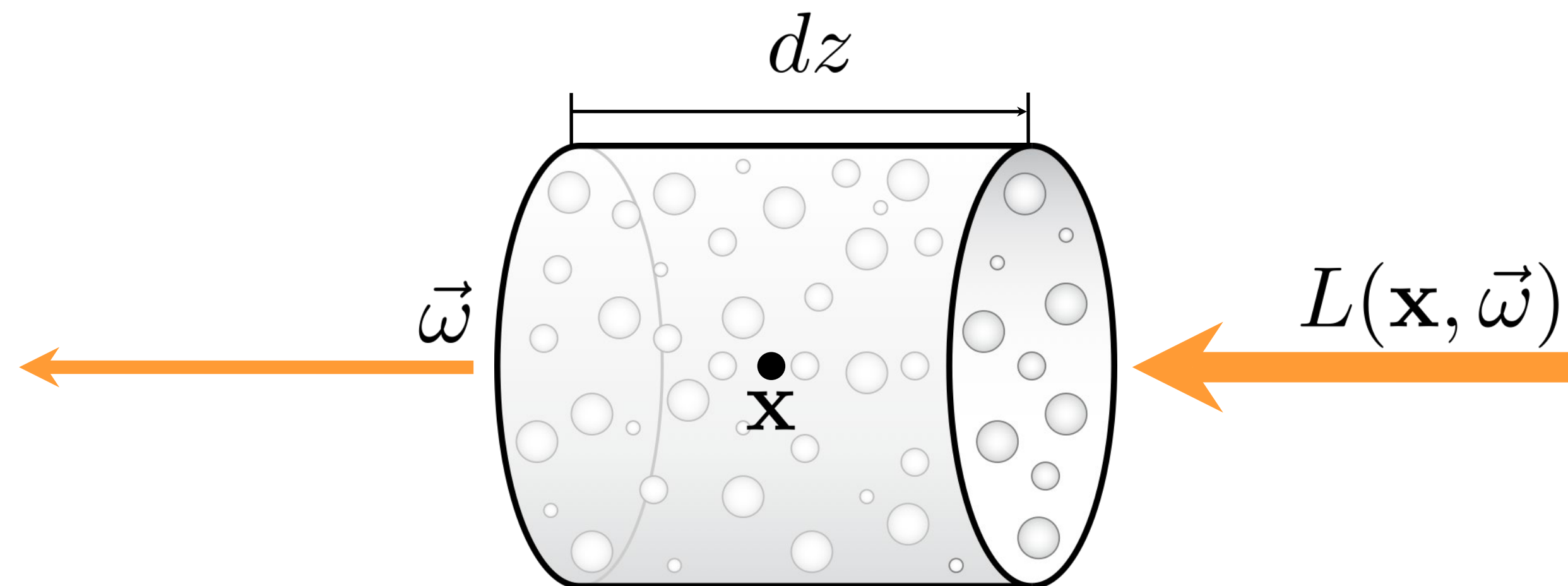due to interactions of light with the medium?

Differential Beam Segment

dz

dA

Outgoing light

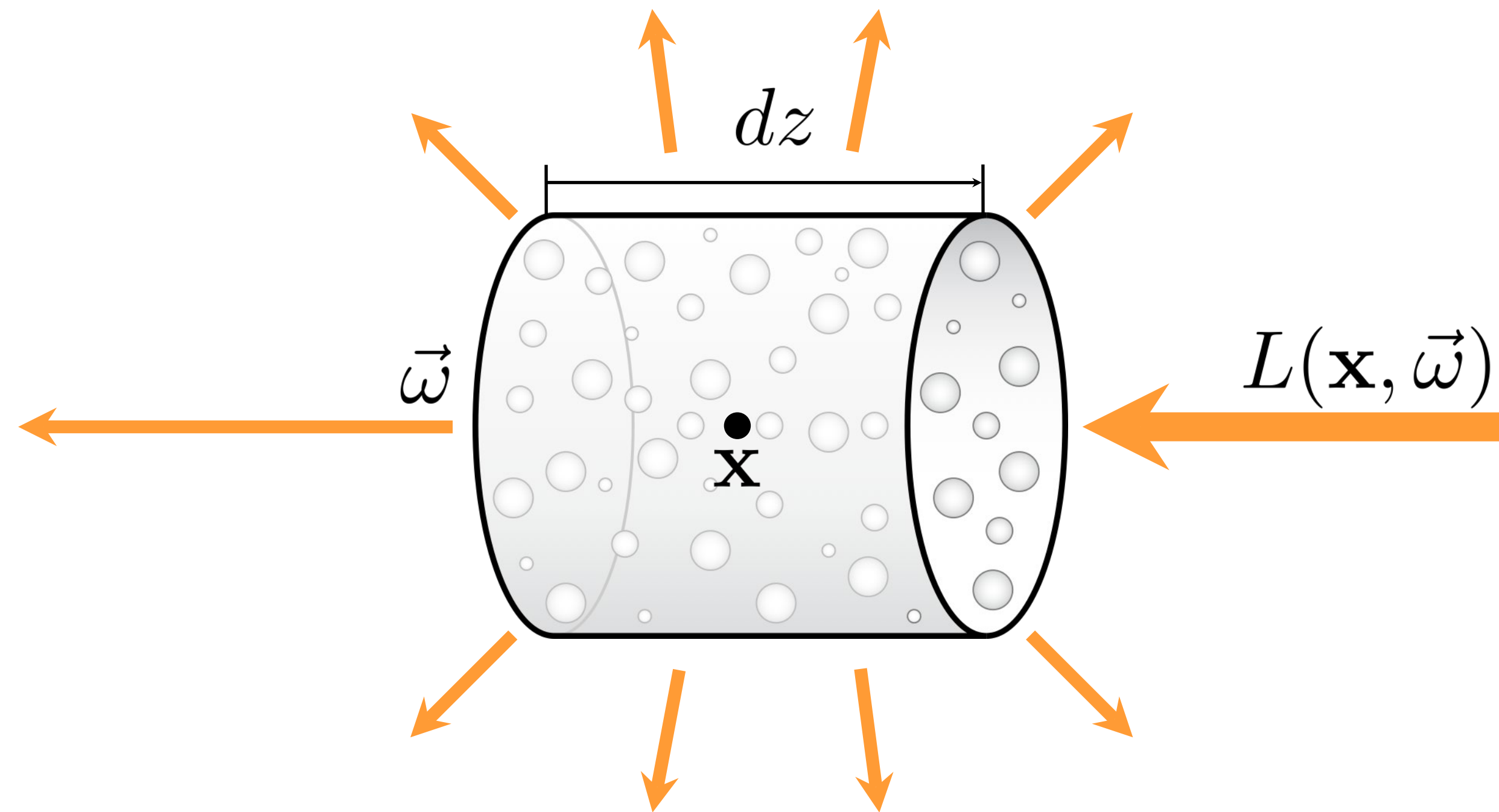Incoming light

$\vec{\omega}$

$\mathbf{x}$

Absorption

$$dL(\mathbf{x}, \vec{\omega}) = -\sigma_a(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz$$

$\sigma_a(\mathbf{x})$: absorption coefficient $\quad [m^{-1}]$
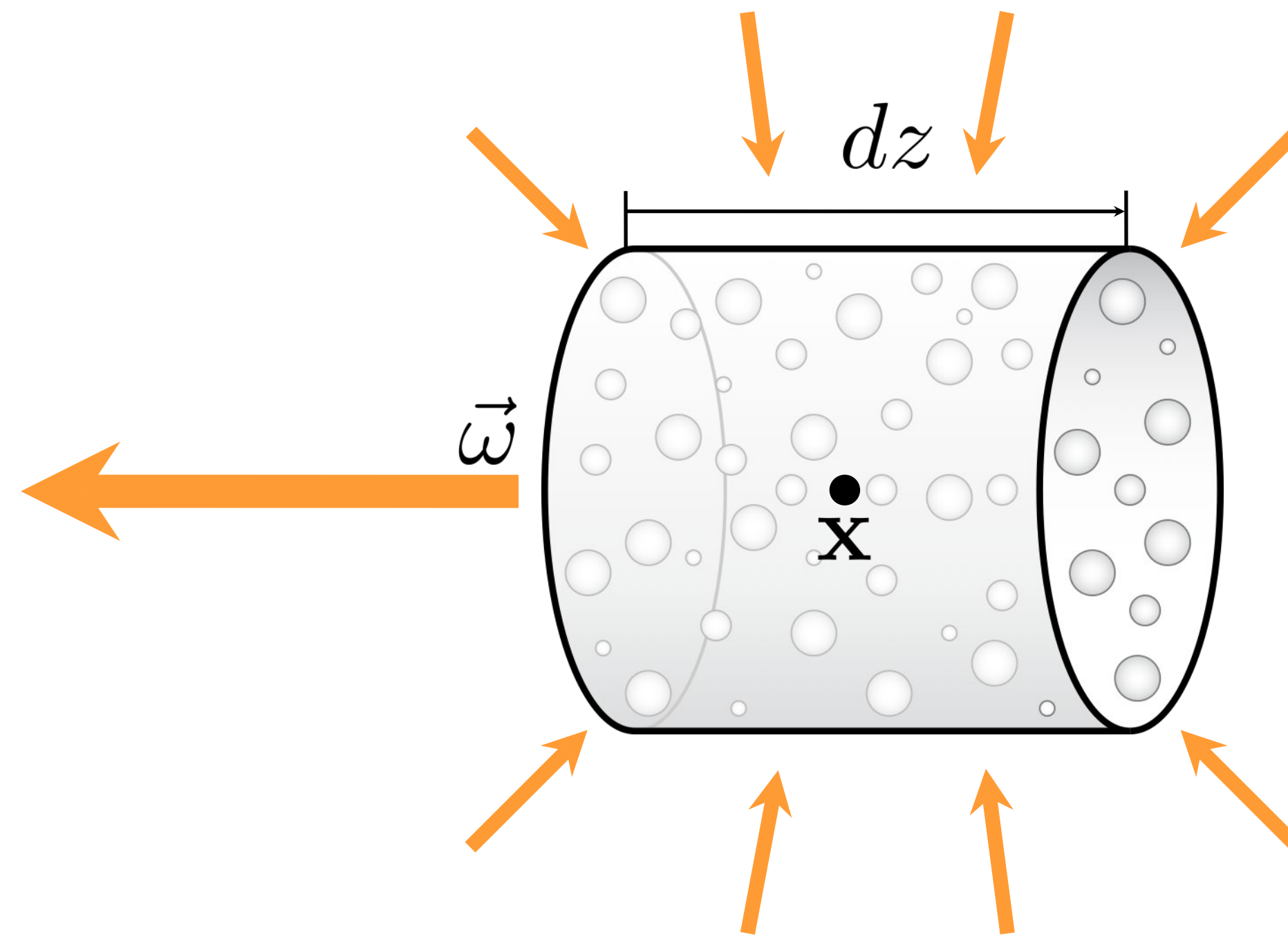
Out-scattering

$$dL(\mathbf{x}, \vec{\omega}) = -\sigma_s(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz$$

$\sigma_s(\mathbf{x})$: scattering coefficient $\quad [m^{-1}]$

In-scattering

$$dL(\mathbf{x}, \vec{\omega}) = \sigma_s(\mathbf{x}) L_s(\mathbf{x}, \vec{\omega}) dz$$
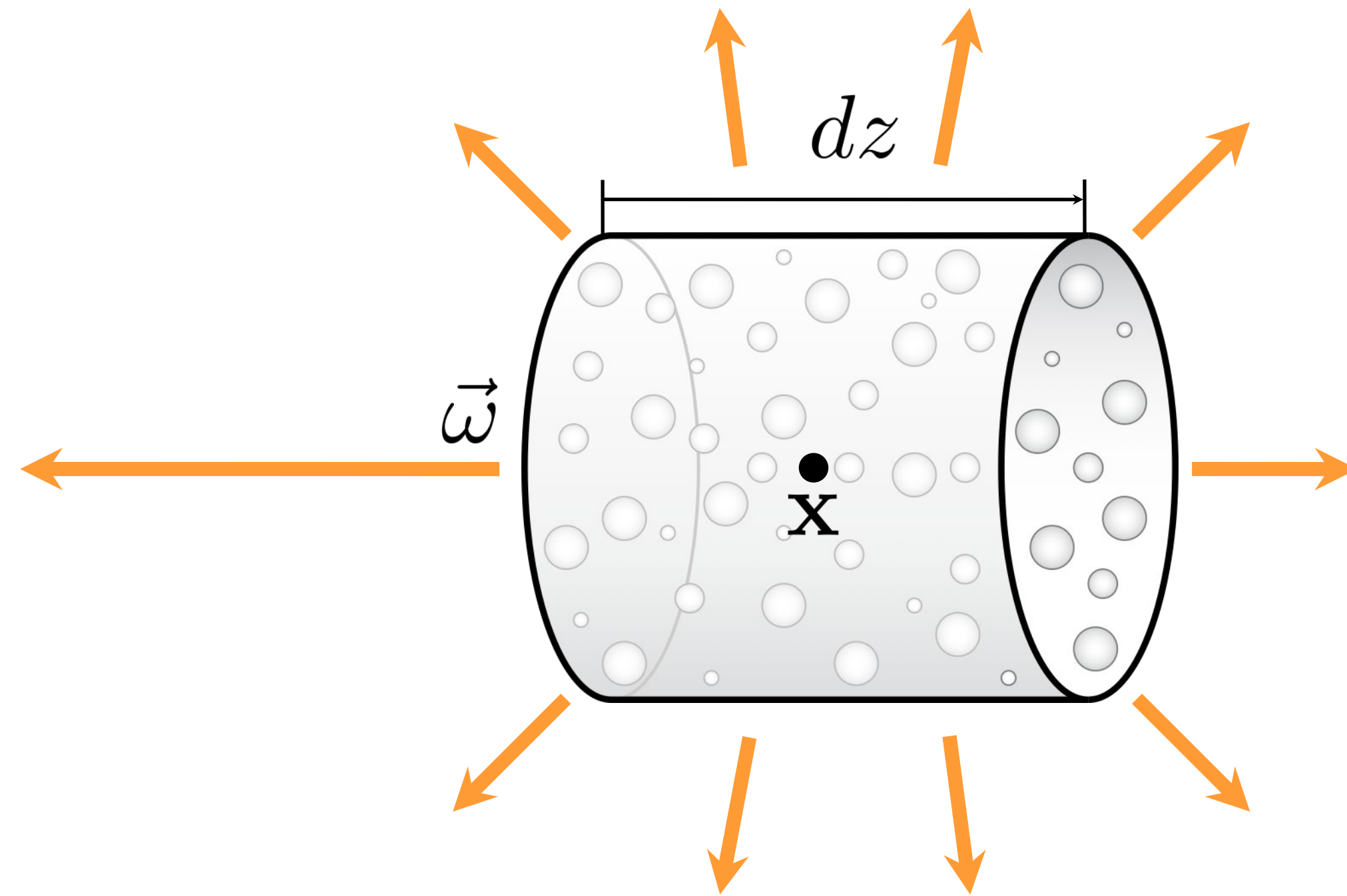
$\sigma_s(\mathbf{x})$: scattering coefficient $\quad [m^{-1}]$

$L_s(\mathbf{x}, \vec{\omega})$: in-scattered radiance

Emission

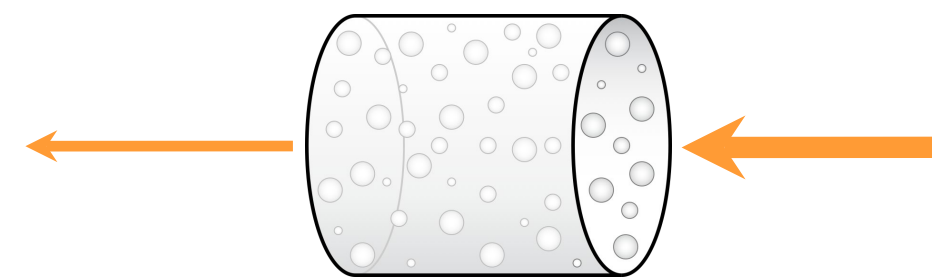$$dL(\mathbf{x}, \vec{\omega}) = \sigma_a(\mathbf{x}) L_e(\mathbf{x}, \vec{\omega}) dz$$

*Sometimes modeled without the absorption coefficient just by specifying a "source" term

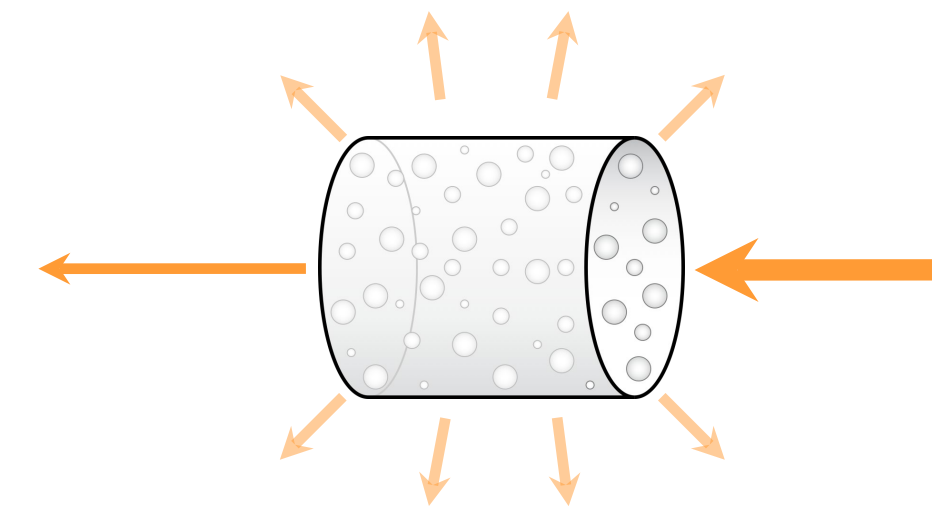$\sigma_a(\mathbf{x})$: absorption coefficient $\quad [m^{-1}]$

$L_e(\mathbf{x}, \vec{\omega})$: emitted radiance
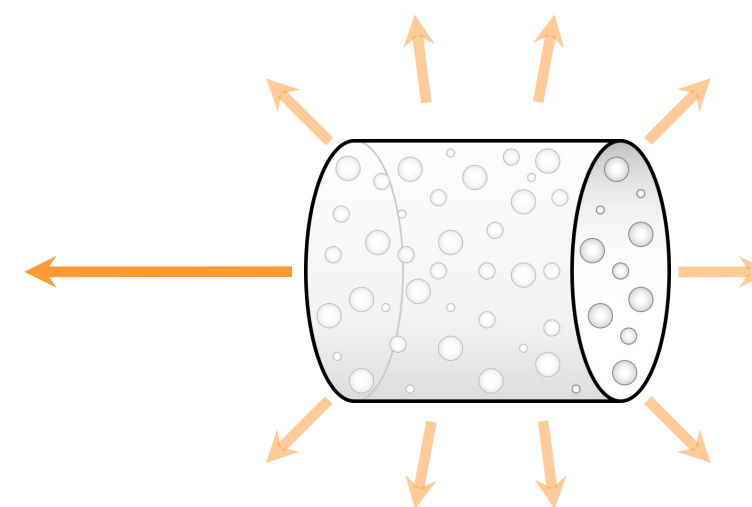
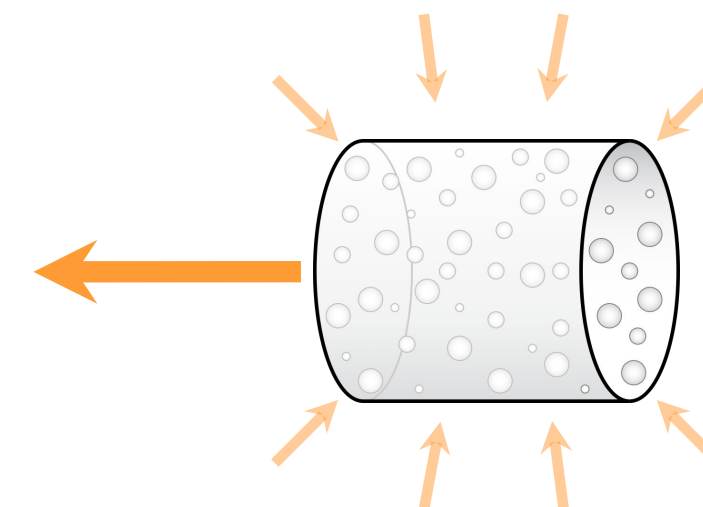# Radiative Transfer Equation (RTE)

Absorption   Out-scattering

$$dL(\mathbf{x}, \vec{\omega}) = \begin{array}{|cc|} -\sigma_a(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz & -\sigma_s(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz \\ \hline +\sigma_a(\mathbf{x})L_e(\mathbf{x}, \vec{\omega})dz & +\sigma_s(\mathbf{x})L_s(\mathbf{x}, \vec{\omega})dz \end{array}$$

Losses

Gains
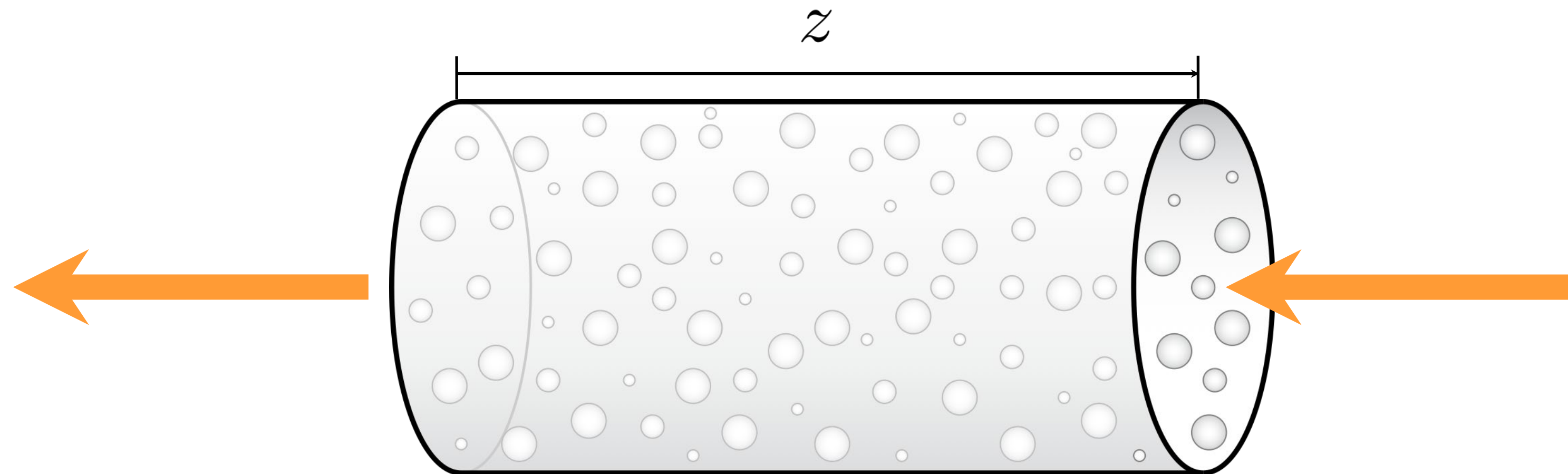
Emission   In-scattering

Losses (Extinction)

Absorption   Out-scattering

$$dL(\mathbf{x}, \vec{\omega}) = -\sigma_a(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz - \sigma_s(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz$$

$$= -\sigma_t(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz$$

$\sigma_t(\mathbf{x})$: extinction coefficient $[m^{-1}]$

: total loss of light per unit distance

What about a beam with a finite length?

Extinction Along a Finite Beam

$$dL(\mathbf{x}, \vec{\omega}) = -\sigma_t(\mathbf{x}) L(\mathbf{x}, \vec{\omega}) dz$$ // Assume constant $\sigma_t(\mathbf{x})$, reorganize

$$\frac{dL(\mathbf{x}, \vec{\omega})}{L(\mathbf{x}, \vec{\omega})} = -\sigma_t dz$$ // Integrate along beam from 0 to z

$$\ln(L_z) - \ln(L_0) = -\sigma_t z$$

$$\ln\left(\frac{L_z}{L_0}\right) = -\sigma_t z$$ // Exponentiate

$$\frac{L_z}{L_0} = e^{-\sigma_t z}$$

Beer-Lambert Law

Expresses the remaining radiance after traveling a finite distance through a medium with constant extinction coefficient
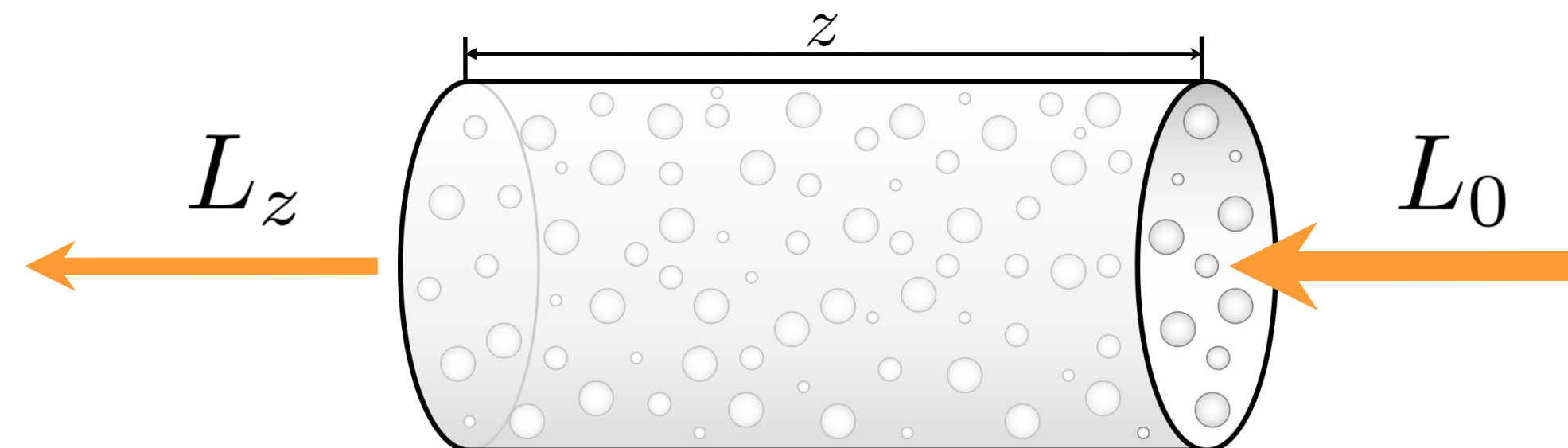
The fraction is referred to as the *transmittance*

Think of this as fractional visibility between points

Radiance at distance z

$$\frac{L_z}{L_0} = e^{-\sigma_t z}$$

Radiance at the beginning of the beam

L_z z L_0

$$\frac{L_z}{L_0} = V(0 \rightarrow z)$$

$$\frac{L_z}{L_0} = Exp(V(0 \rightarrow z))$$

Transmittance

Homogeneous volume:

$$T_r(\mathbf{x}, \mathbf{y}) = e^{-\boxed{\sigma_t \|\mathbf{x}-\mathbf{y}\|}}$$

Heterogeneous volume (spatially varying σ_t):

$$T_r(\mathbf{x}, \mathbf{y}) = e^{-\boxed{\int_0^{\|\mathbf{x}-\mathbf{y}\|} \sigma_t(t)dt}}$$

Optical thickness
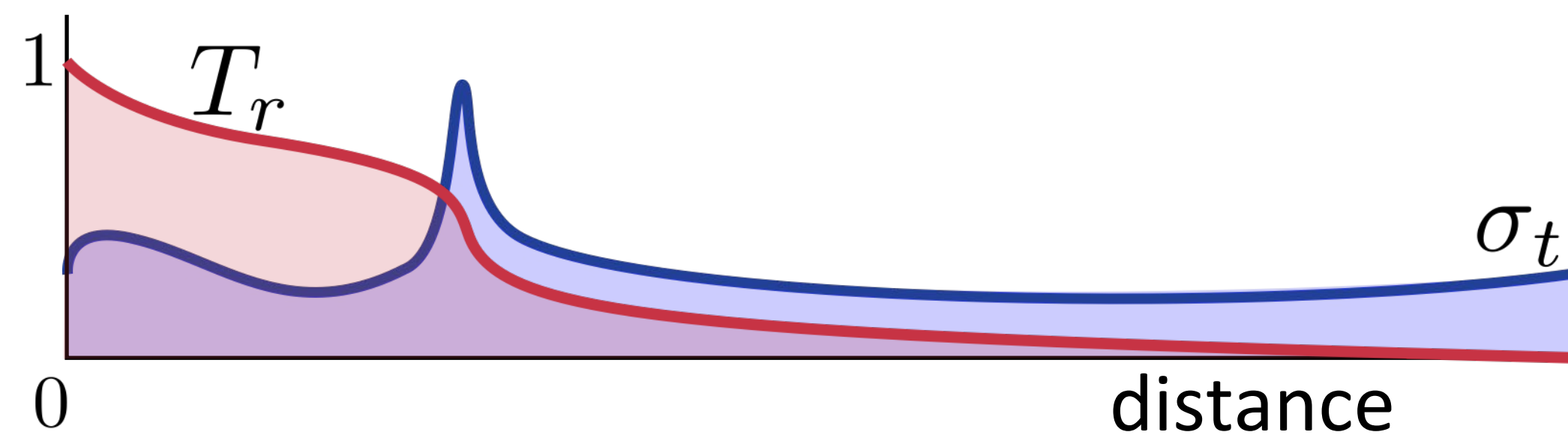
Transmittance

Homogeneous volume:

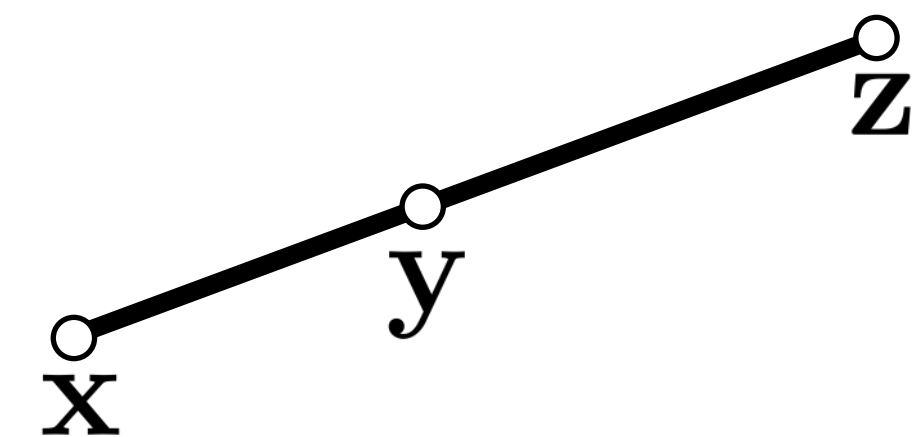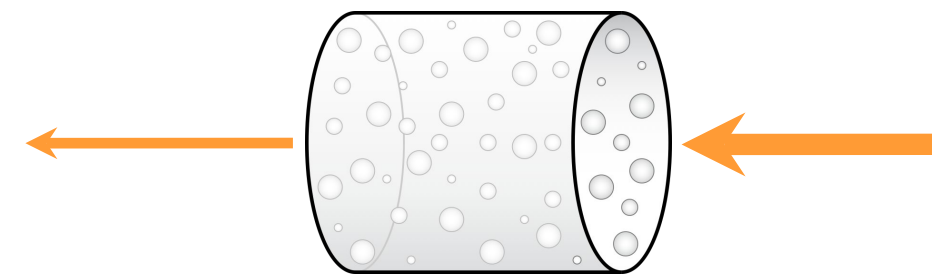$$T_r(\mathbf{x}, \mathbf{y}) = e^{-\sigma_t \|\mathbf{x} - \mathbf{y}\|}$$

Heterogeneous volume (spatially varying σ_t):

$$T_r(\mathbf{x}, \mathbf{y}) = e^{-\int_0^{\|\mathbf{x} - \mathbf{y}\|} \sigma_t(t) dt}$$

Optical thickness

Transmittance is multiplicative:

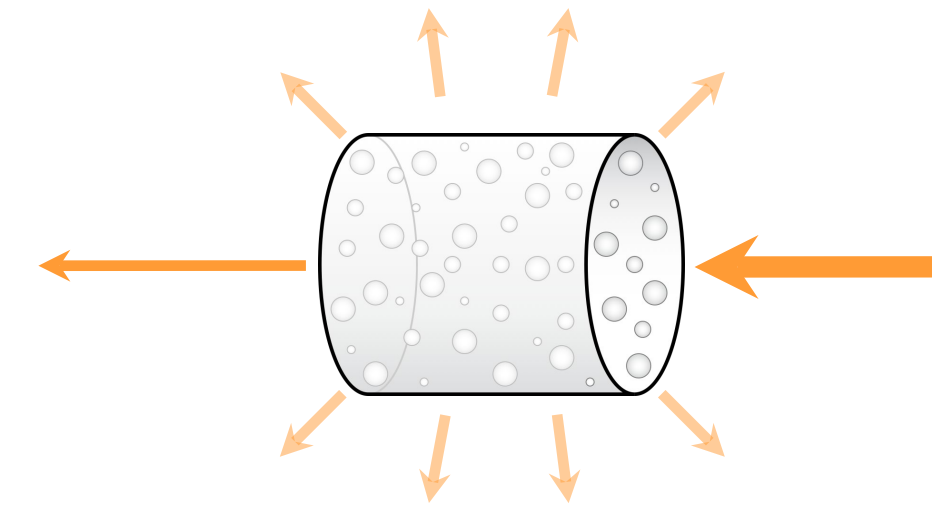$$T_r(\mathbf{x}, \mathbf{z}) = T_r(\mathbf{x}, \mathbf{y}) T_r(\mathbf{y}, \mathbf{z})$$
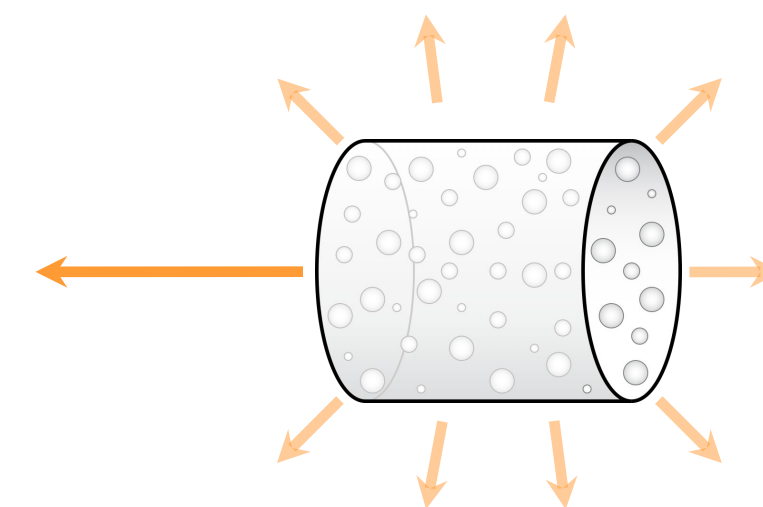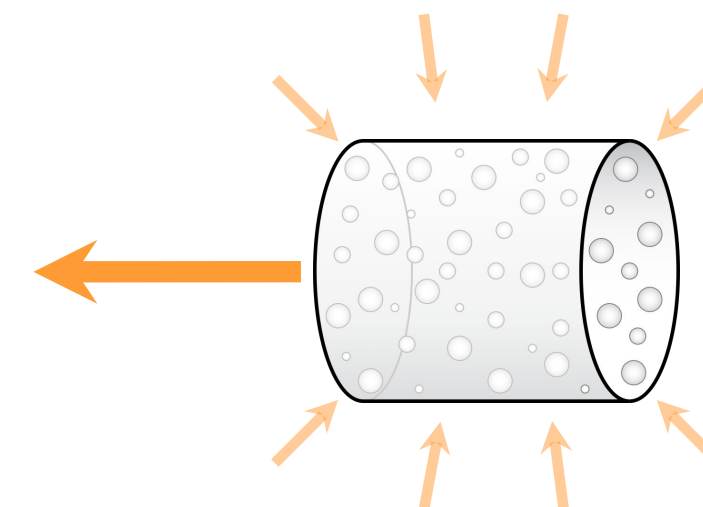
Radiative Transfer Equation (RTE)

Absorption

Out-scattering

$$dL(\mathbf{x}, \vec{\omega}) = -\sigma_a(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz \quad -\sigma_s(\mathbf{x})L(\mathbf{x}, \vec{\omega})dz$$
$$+\sigma_a(\mathbf{x})L_e(\mathbf{x}, \vec{\omega})dz \;+ \sigma_s(\mathbf{x})L_s(\mathbf{x}, \vec{\omega})dz$$
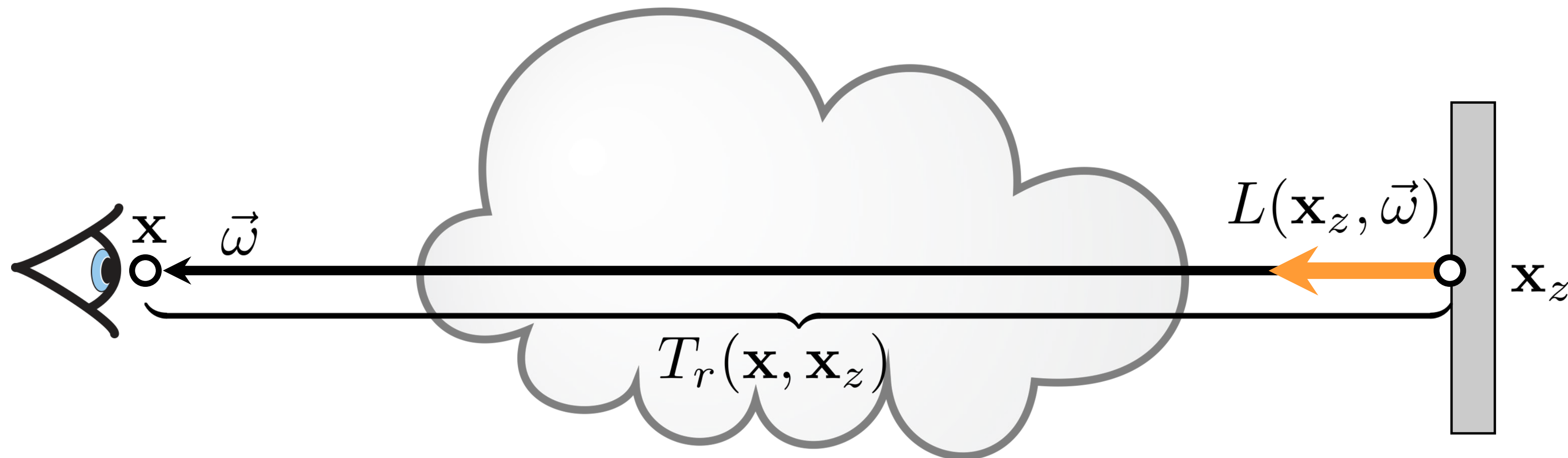
Emission

In-scattering

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = \boxed{T_r(\mathbf{x}, \mathbf{x}_z) L(\mathbf{x}_z, \vec{\omega})}$$

Reduced (background) surface radiance

$\mathbf{x}$ $\vec{\omega}$

$L(\mathbf{x}_z, \vec{\omega})$
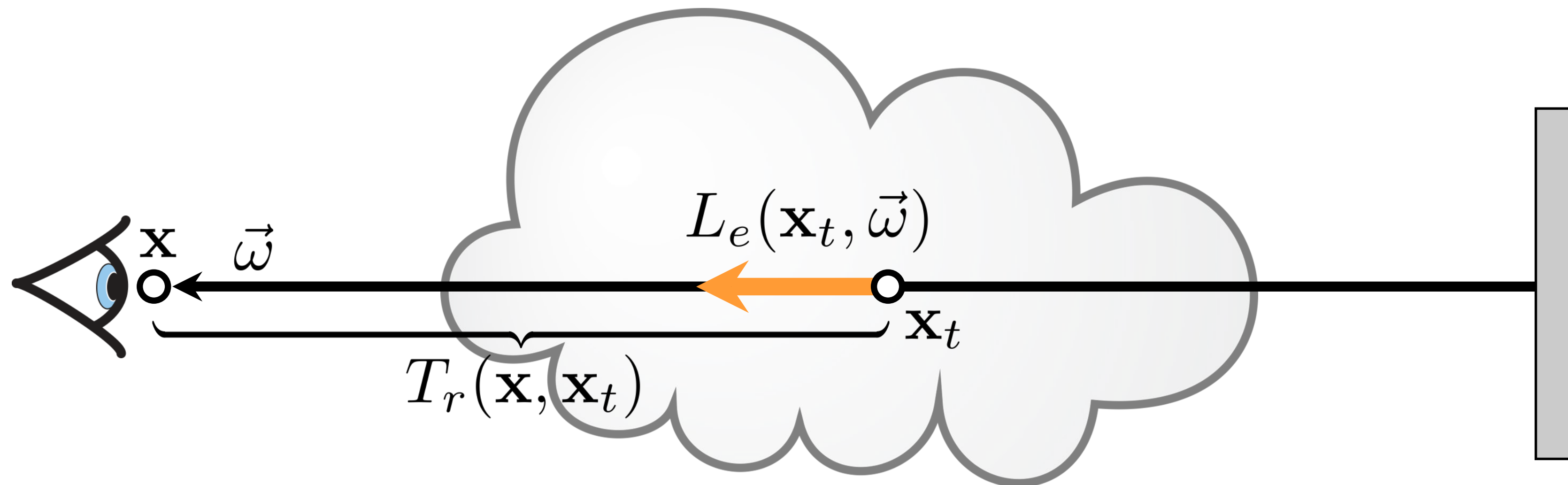
$\mathbf{x}_z$
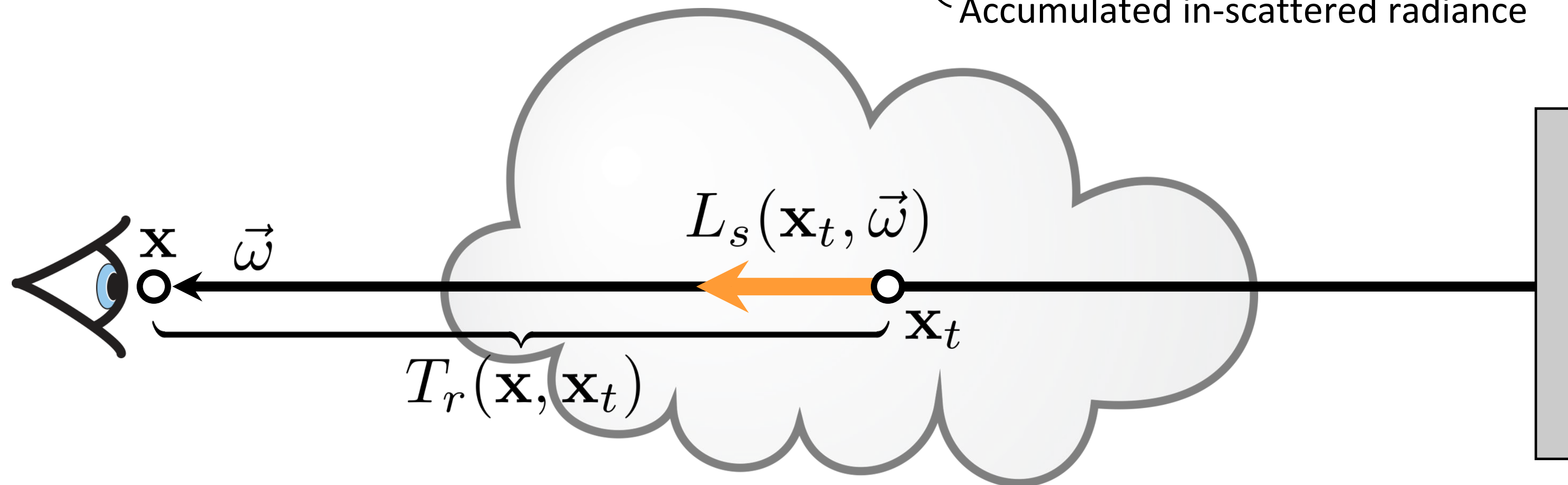
$T_r(\mathbf{x}, \mathbf{x}_z)$

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = T_r(\mathbf{x}, \mathbf{x}_z)L(\mathbf{x}_z, \vec{\omega})$$

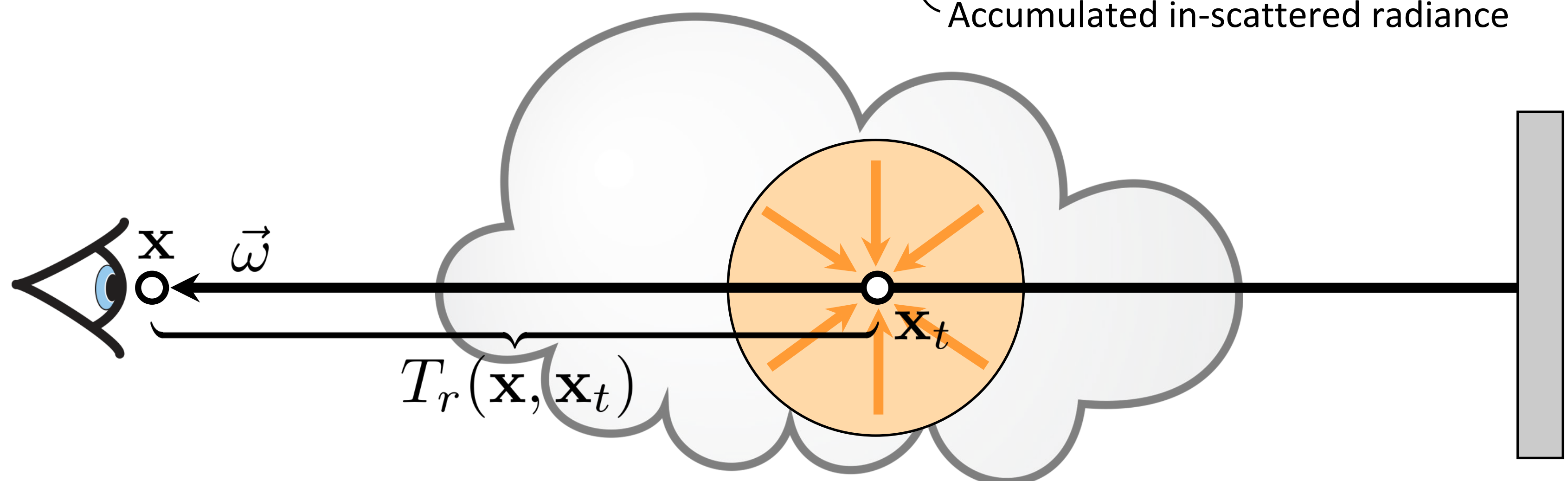$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_a(\mathbf{x}_t)L_e(\mathbf{x}_t, \vec{\omega})dt$$

Accumulated emitted radiance

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = T_r(\mathbf{x}, \mathbf{x}_z)L(\mathbf{x}_z, \vec{\omega})$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_a(\mathbf{x}_t)L_e(\mathbf{x}_t, \vec{\omega})dt$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_s(\mathbf{x}_t, \vec{\omega})dt$$

Accumulated in-scattered radiance

$L_s(\mathbf{x}_t, \vec{\omega})$

$\mathbf{x}$ $\vec{\omega}$

$\mathbf{x}_t$

$T_r(\mathbf{x}, \mathbf{x}_t)$

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = T_r(\mathbf{x}, \mathbf{x}_z) L(\mathbf{x}_z, \vec{\omega})$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) dt$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \sigma_s(\mathbf{x}_t) \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega}) L_i(\mathbf{x}_t, \vec{\omega}') d\vec{\omega}' dt$$

Accumulated in-scattered radiance

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = T_r(\mathbf{x}, \mathbf{x}_z)L(\mathbf{x}_z, \vec{\omega})$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_a(\mathbf{x}_t)L_e(\mathbf{x}_t, \vec{\omega})dt$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t)\int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega})L_i(\mathbf{x}_t, \vec{\omega}')d\vec{\omega}'dt$$

Scattering in Media

Phase Function f_p

Describes distribution of scattered light

Analog of BRDF but for scattering in media

Integrates to unity (unlike BRDF)

$$\int_{S^2} f_p(\mathbf{x}, \vec{\omega}', \vec{\omega}) d\vec{\omega}' = 1$$

Why do we have this property?

*We will use the same convention that phase function direction vectors always *point away* from the shading point **x**. Many publications, however, use a different convention for phase functions, in which direction vectors "follow" the light, i.e. one direction points *towards* **x** and the other *away* from **x**. When reading papers, be sure to clarify the meaning of the vectors to avoid misinterpretation.

Isotropic Scattering

$$f_p(\vec{\omega}',\vec{\omega}) = \langle \omega', \omega \rangle$$

isotropic

Uniform scattering, analogous to Lambertian BRDF

$$f_p(\vec{\omega}',\vec{\omega}) = \frac{1}{4\pi}$$

Where does this value come from?
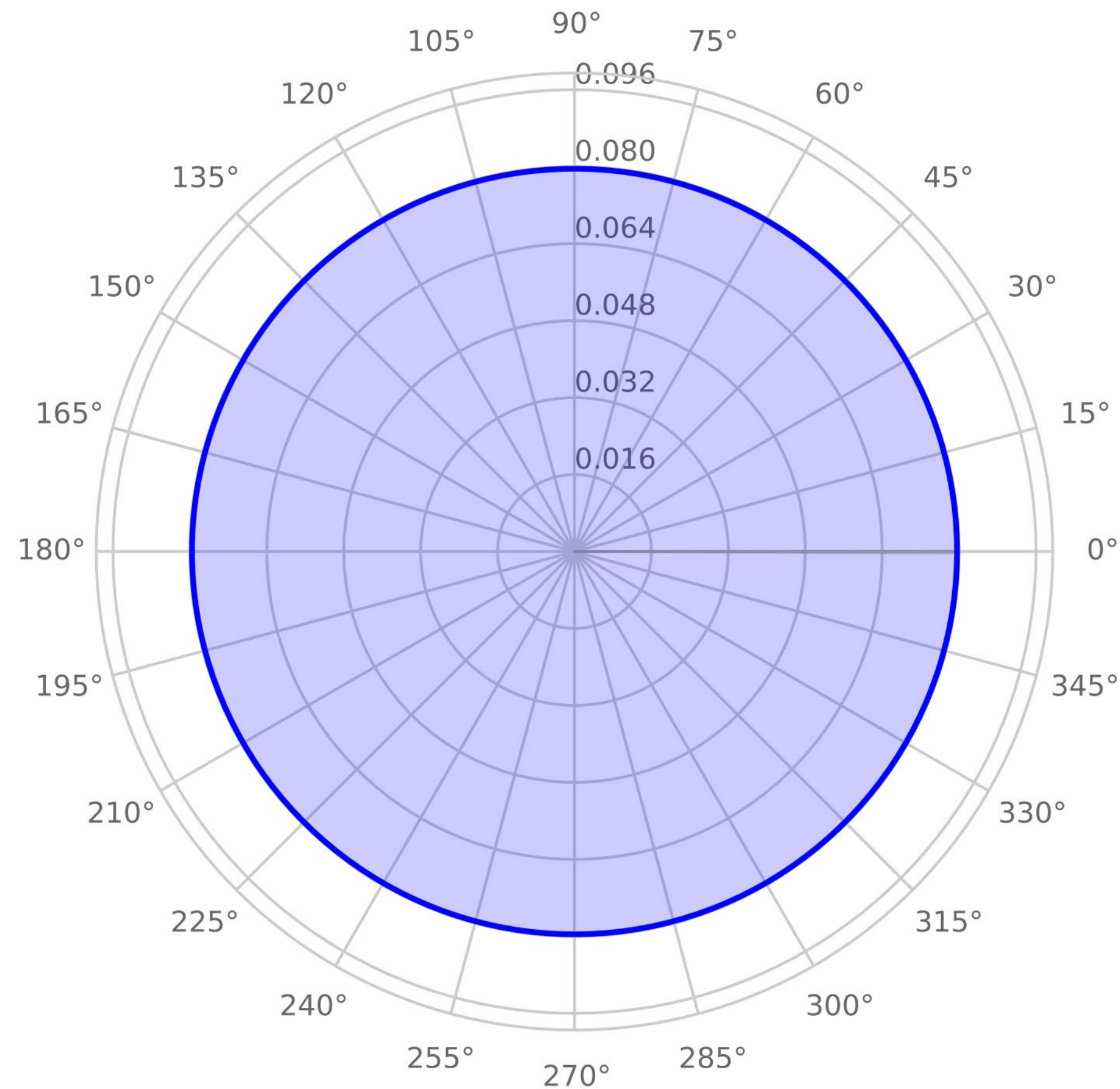
Anisotropic Scattering

Quantifying anisotropy (g, "average cosine"):

$$g = \int_{S^2} f_p(\mathbf{x}, \vec{\omega}', \vec{\omega}) \cos\theta \, d\vec{\omega}'$$

where:

$$\cos\theta = -\vec{\omega} \cdot \vec{\omega}'$$
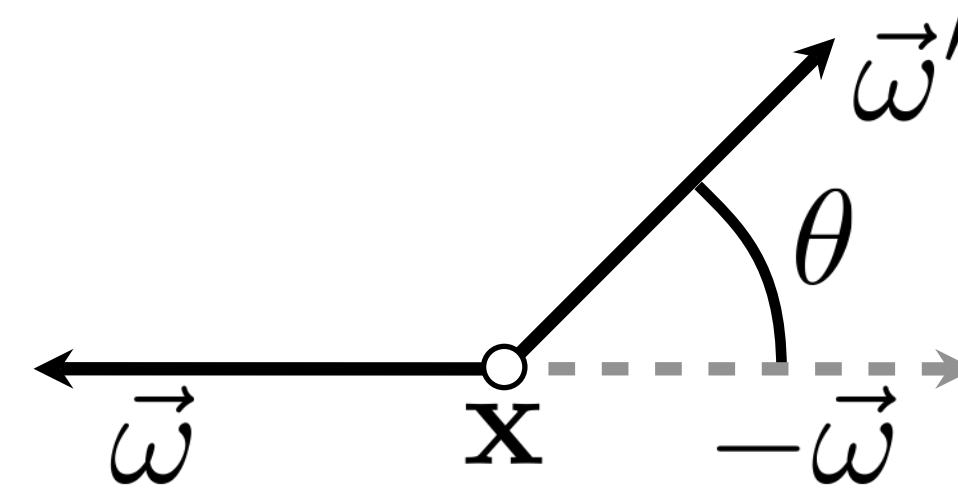
$g = 0$: isotropic scattering (on average)
$g > 0$: forward scattering
$g < 0$: backward scattering

Henyey-Greenstein Phase Function

Anisotropic scattering

$$f_{p\mathrm{HG}}(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{3/2}}$$
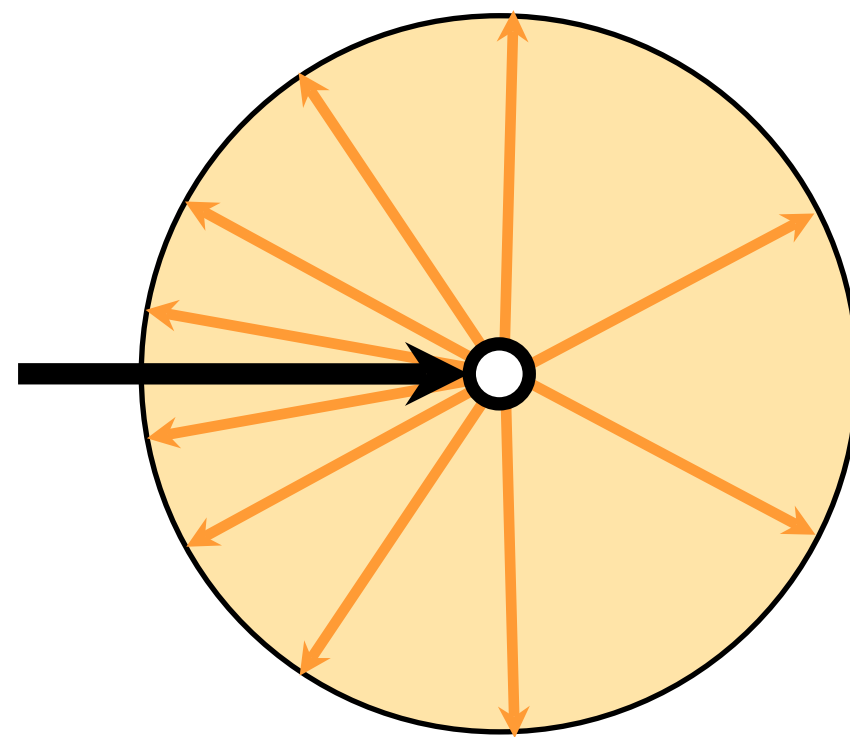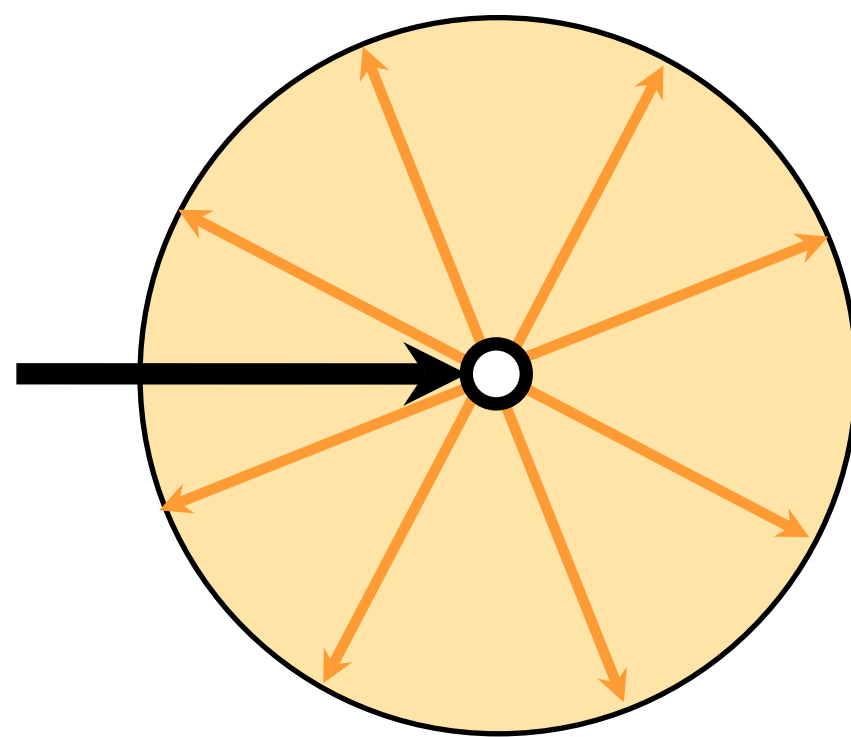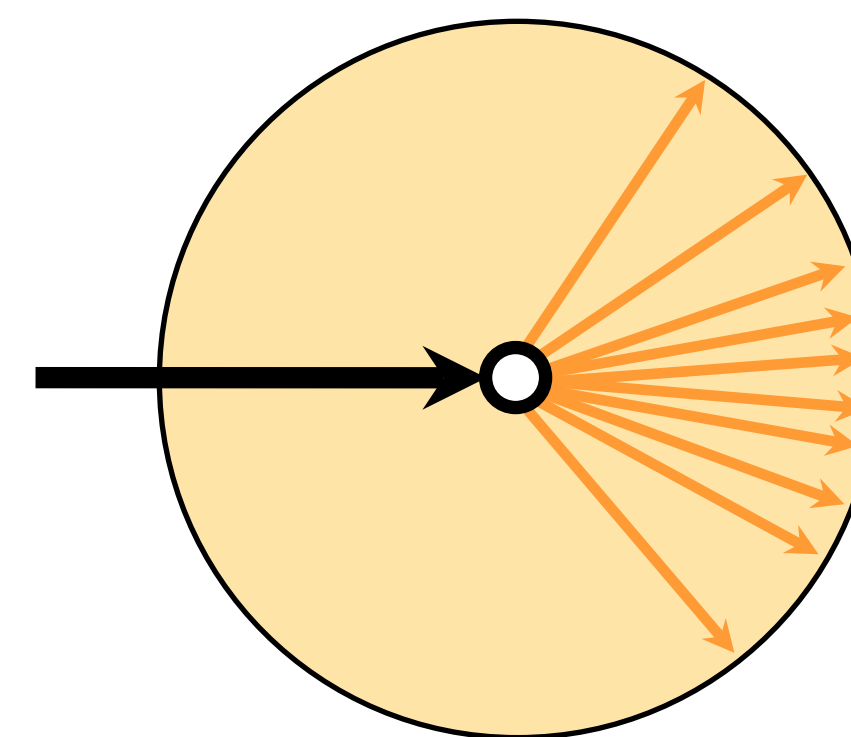
$g < 0$ $g = 0$ $g > 0$

Henyey-Greenstein Phase Function

Anisotropic scattering

$$f_{p\mathrm{HG}}(\theta) = \frac{1}{4\pi} \frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}}$$

$g = -0.5$

$g = 0$

$g = 0.8$
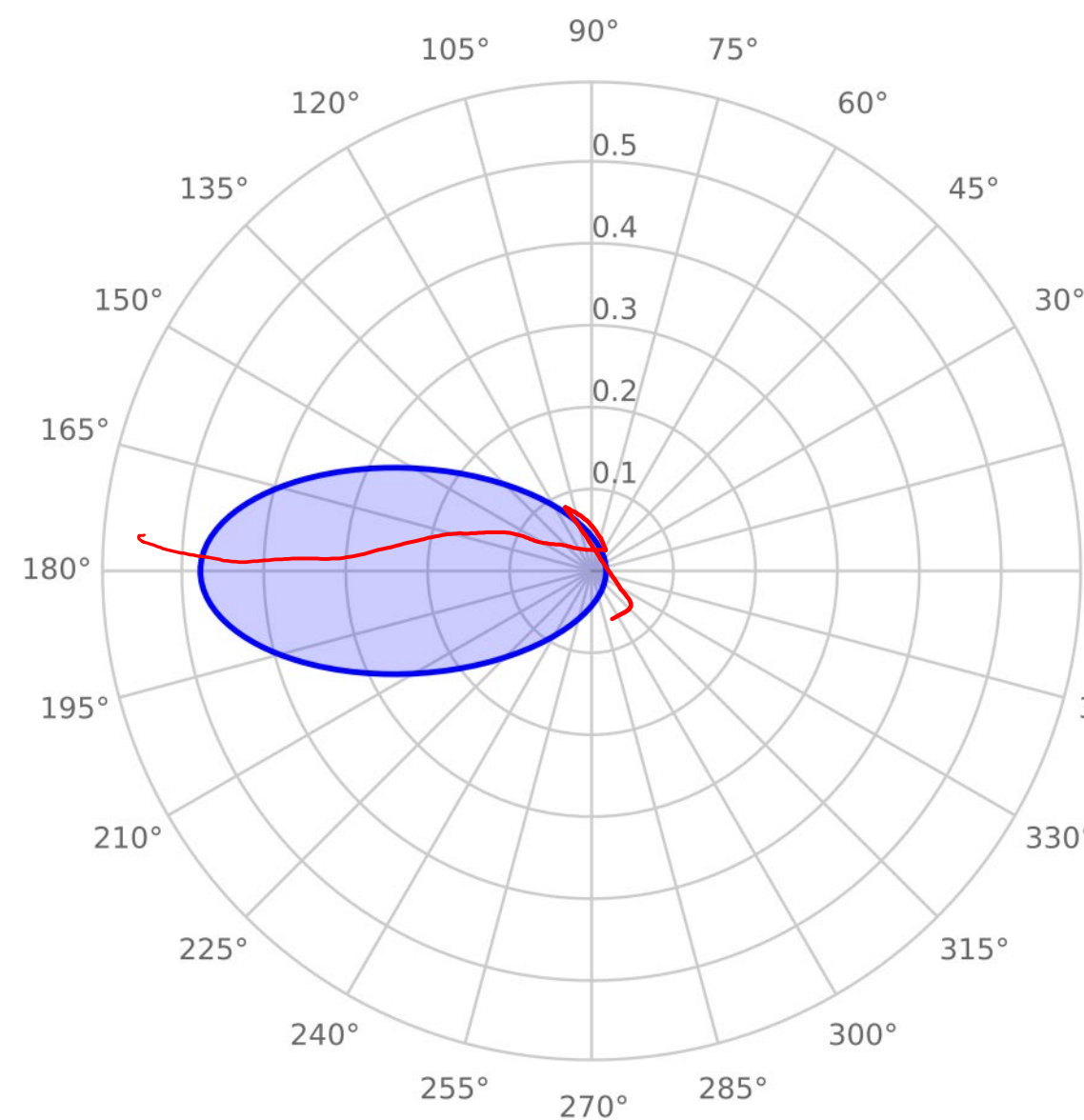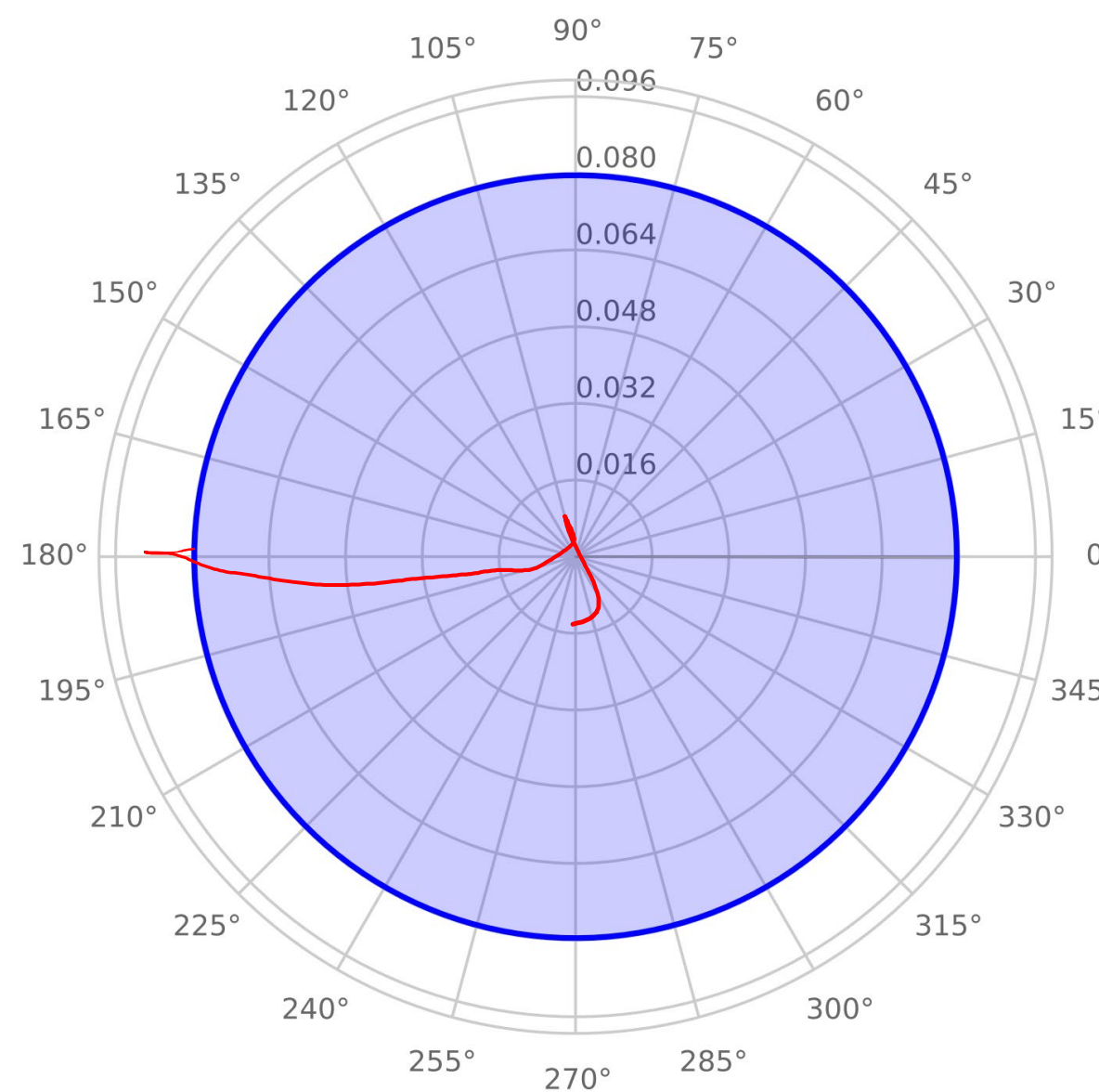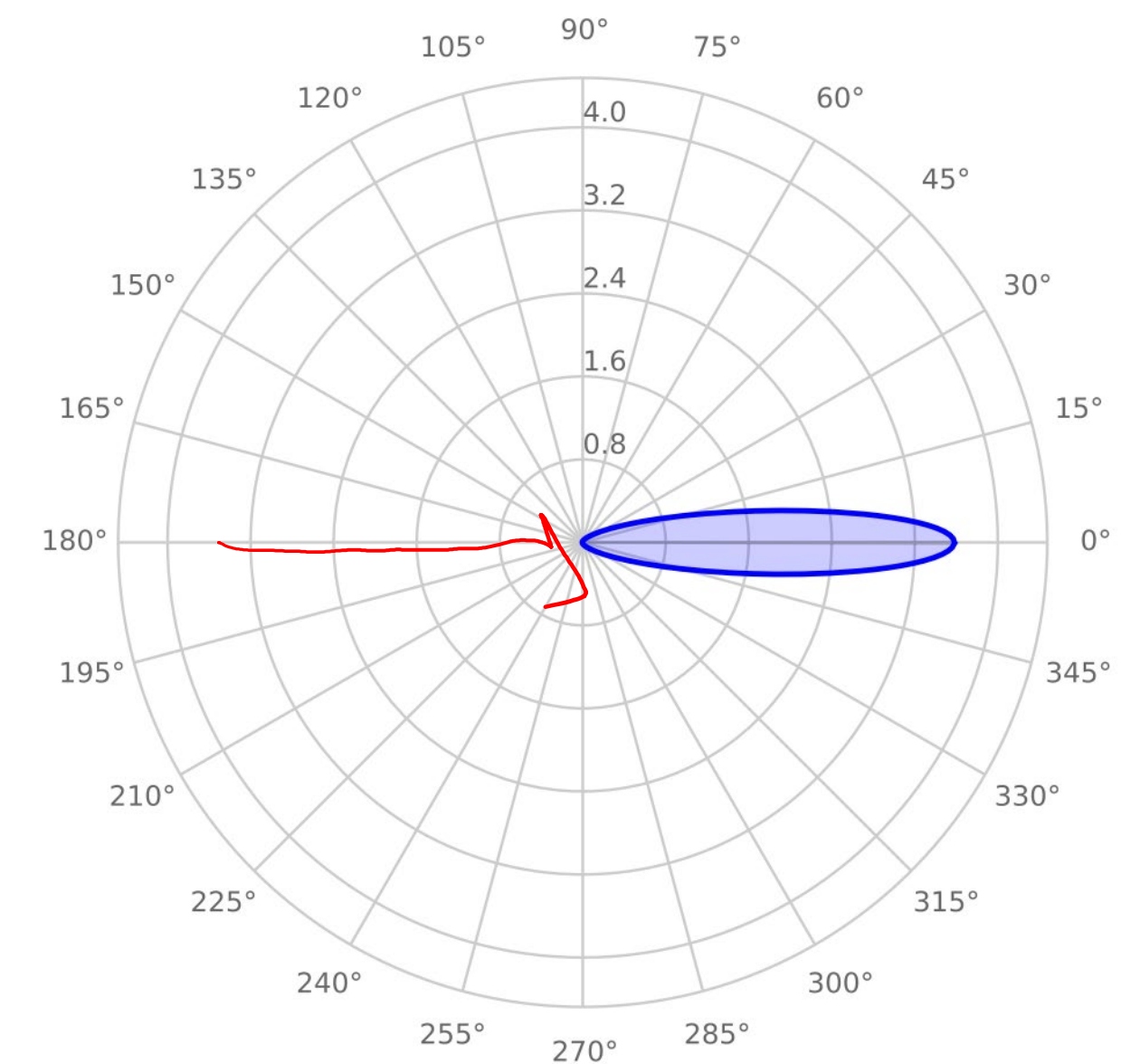
Henyey-Greenstein Phase Function
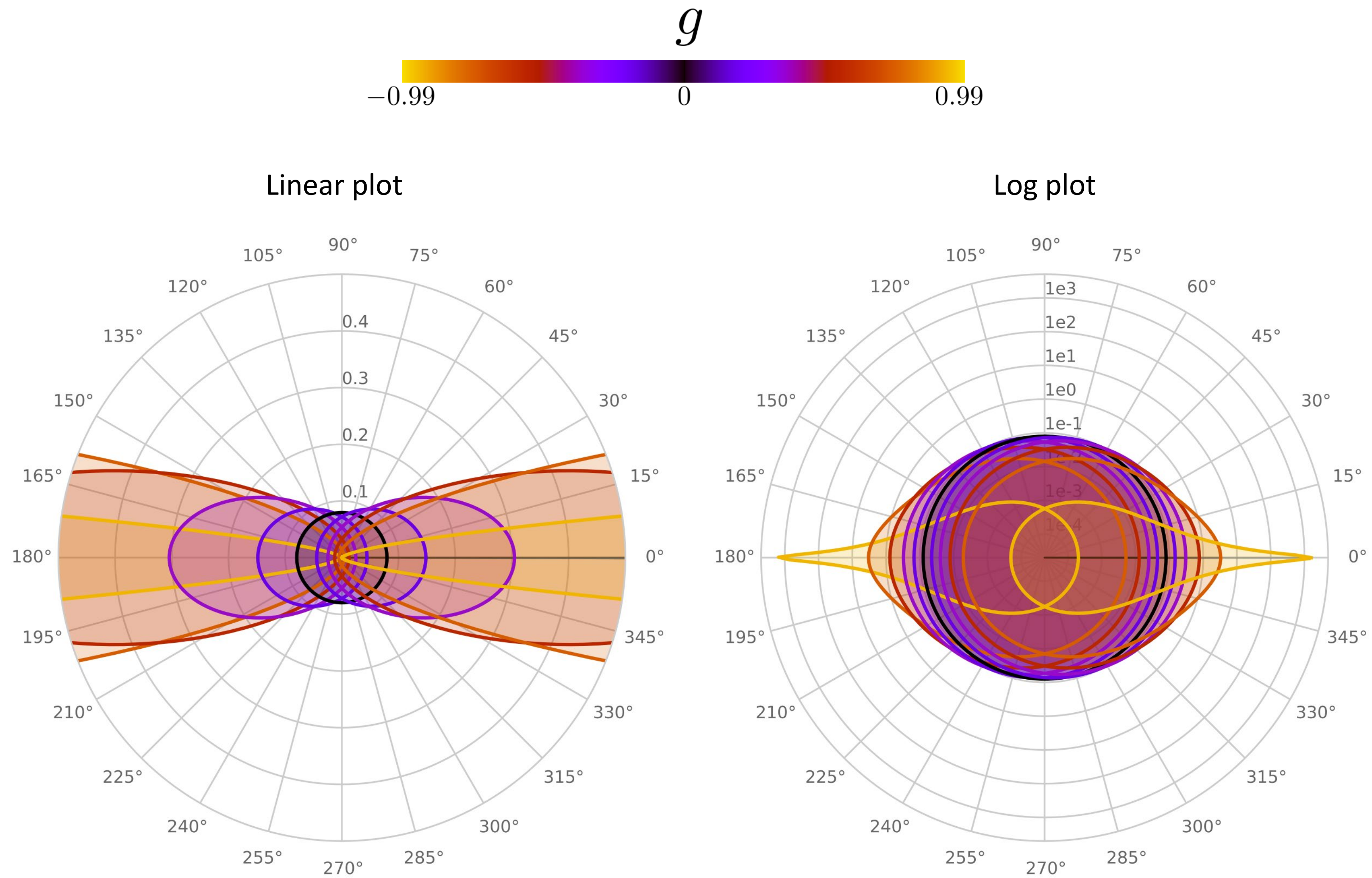
Henyey-Greenstein Phase Function

Empirical phase function

Introduced for intergalactic dust

Very popular in graphics and other fields

Schlick's Phase Function

Empirical phase function

Faster approximation of HG

$$f_{p\mathrm{Schlick}}(\theta) = \frac{1}{4\pi} \frac{1 - k^2}{(1 - k\cos\theta)^2}$$

$$k = 1.55g - 0.55g^3$$

Schlick's Phase Function

Empirical phase function

Faster approximation of HG

$g = -0.5 \quad k = -0.706$
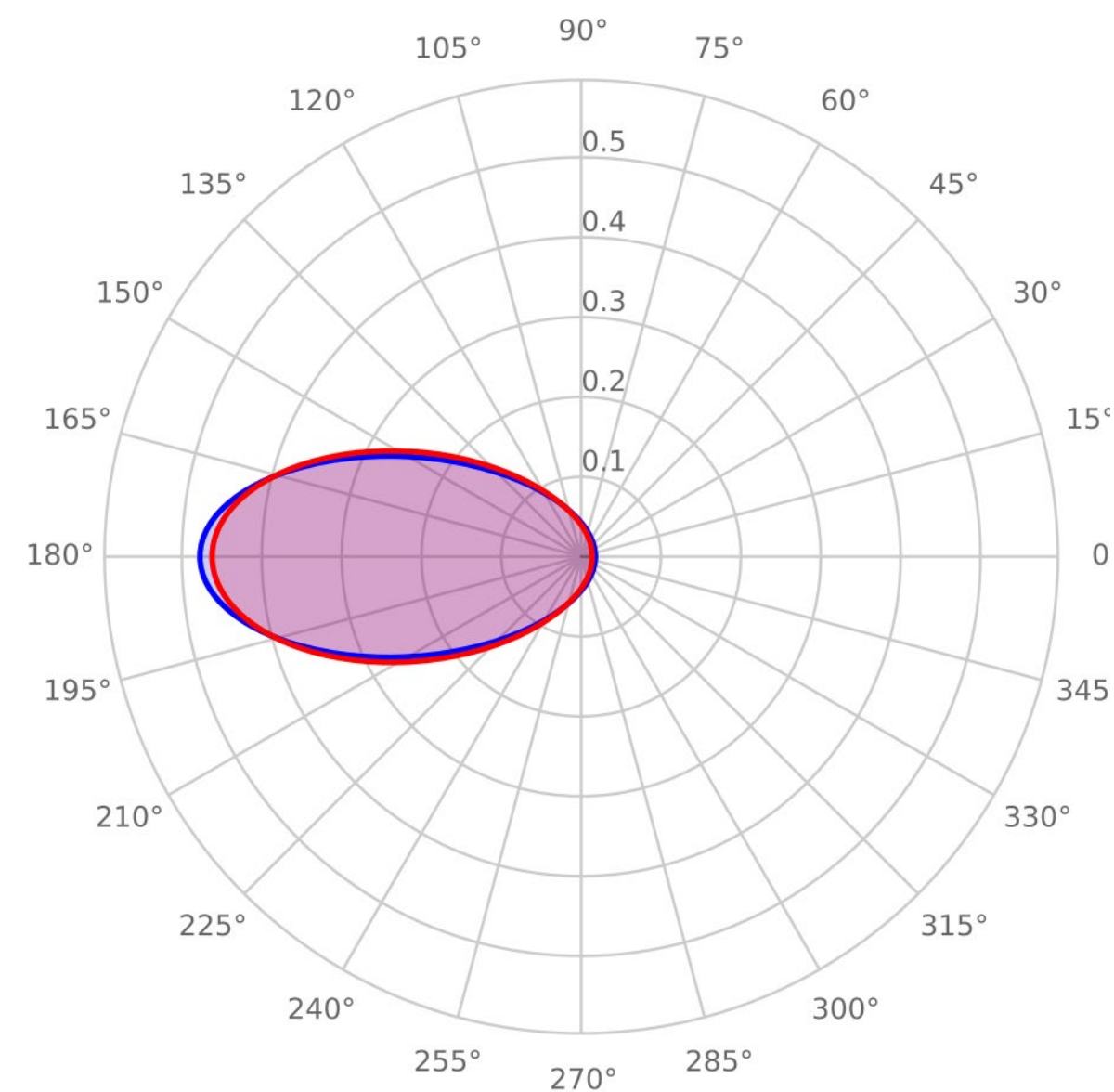
$g = 0 \quad k = 0$
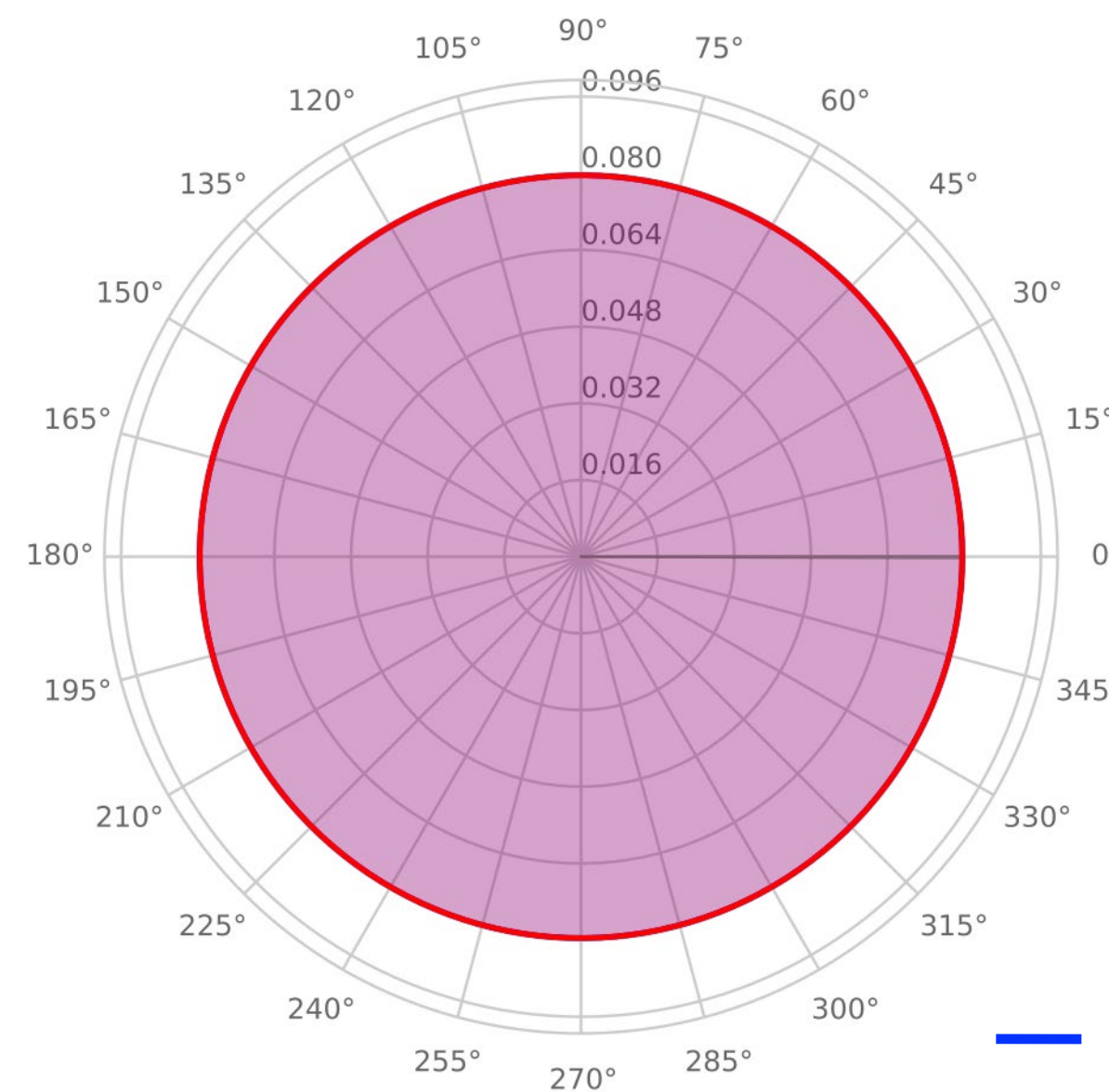
$g = 0.8 \quad k = 0.96$
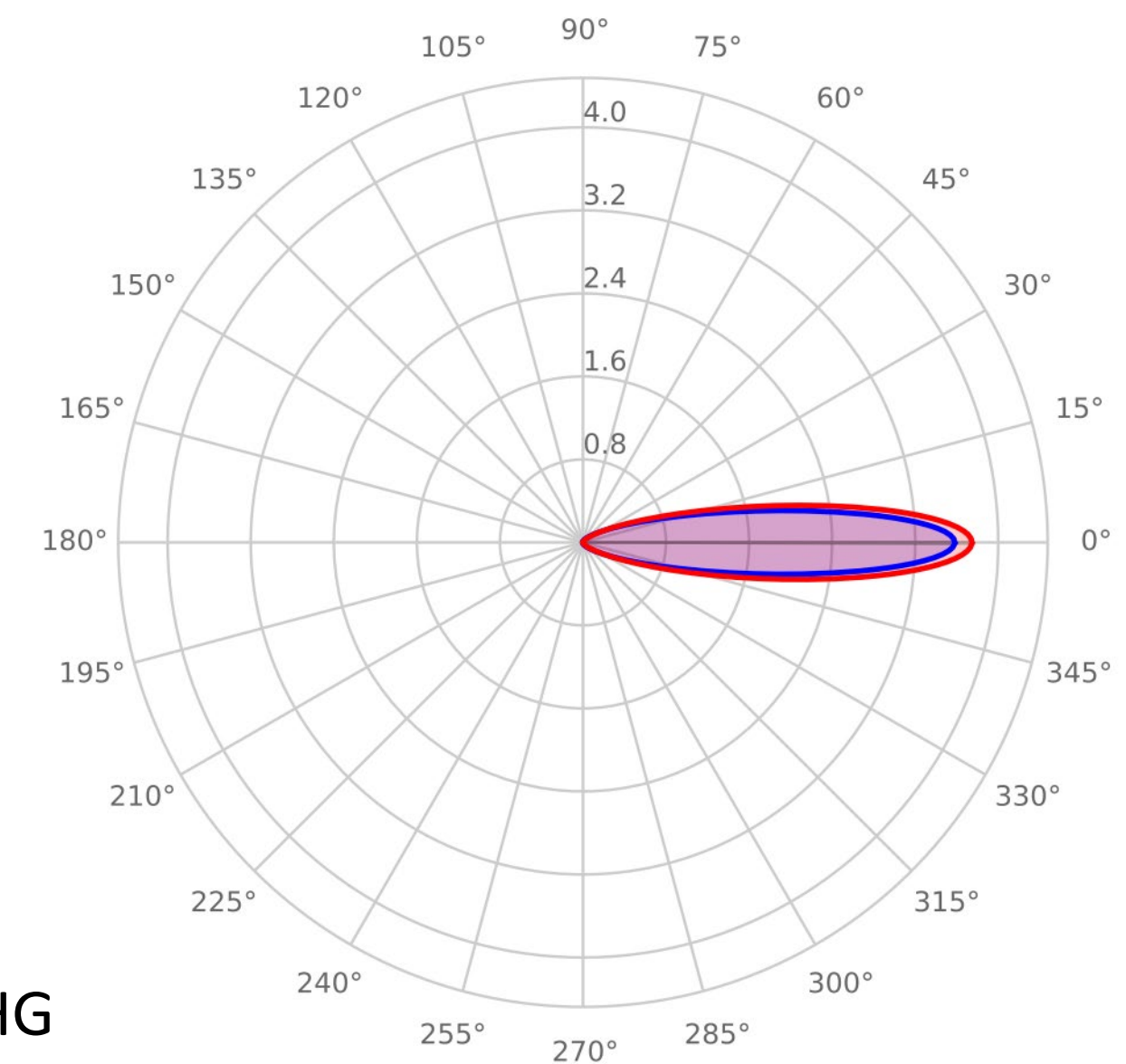
HG

Schlick

Lorenz-Mie Scattering

If the diameter of scatterers is on the order of light wavelength, we cannot neglect the wave nature of light

Solution to Maxwell's equations for scattering from any spherical dielectric particle

Explains many phenomena

Complicated:

- Solution is an infinite analytic series

Lorenz-Mie Phase Function
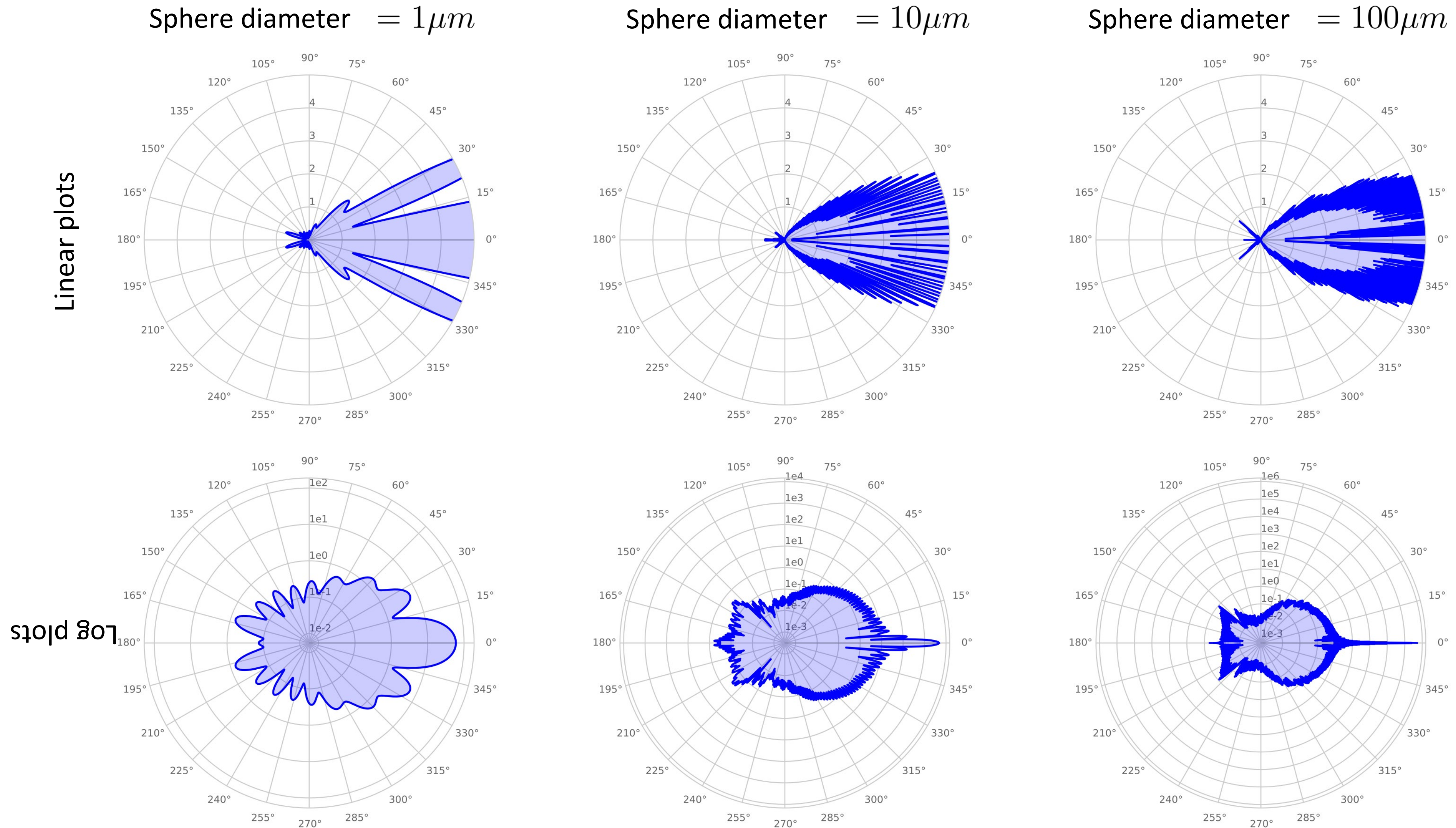
Data obtained from http://www.philiplaven.com/mieplot.htm

Rainbows

Lorenz-Mie Approximations

Hazy atmosphere
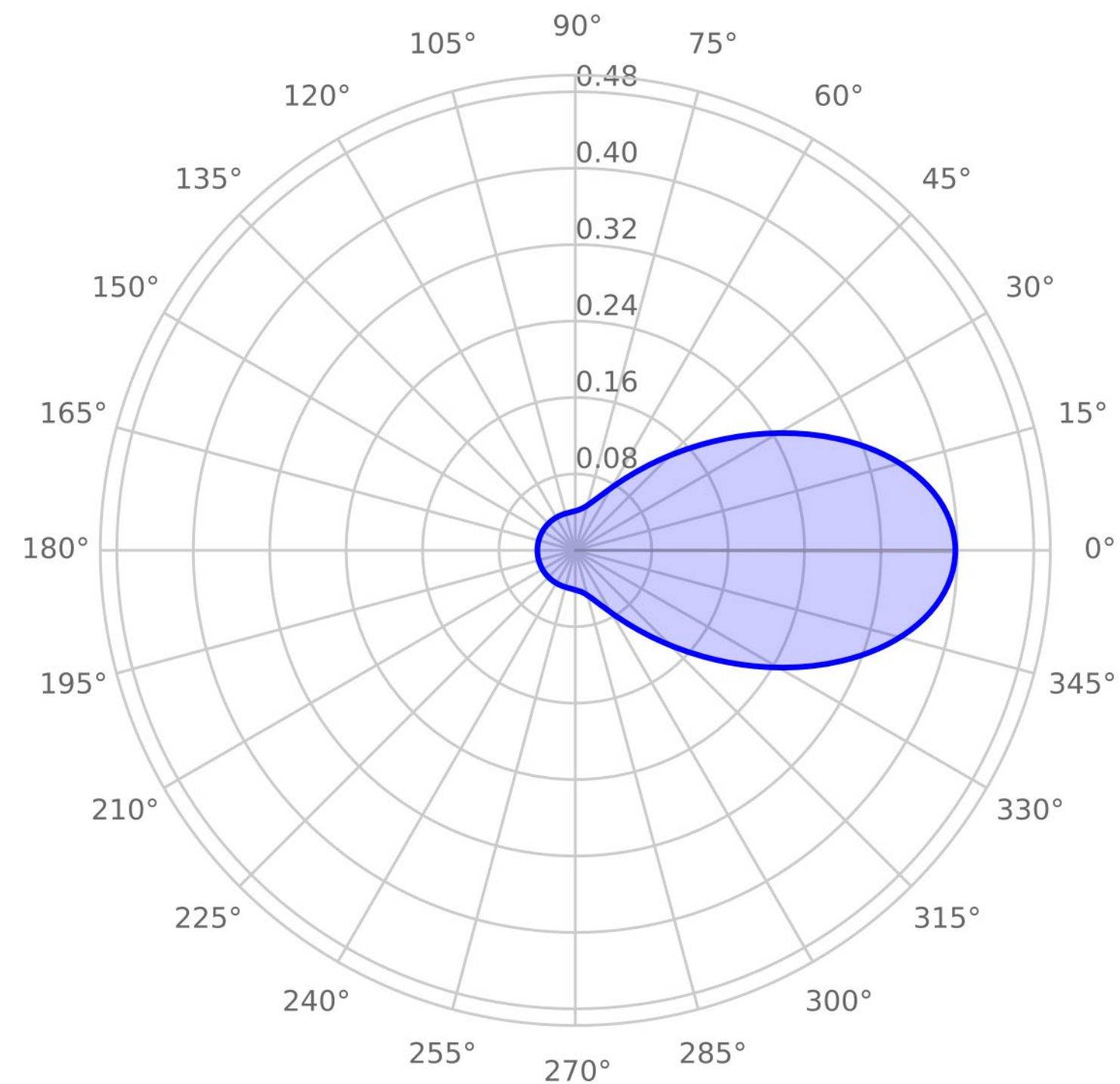
$$f_{p\,\text{hazy}}(\theta) = \frac{1}{4\pi}\left(5 + \left(\frac{1 + \cos\theta}{2}\right)^8\right)$$

Murky atmosphere

$$f_{p\,\text{murky}}(\theta) = \frac{1}{4\pi}\left(17 + \left(\frac{1 + \cos\theta}{2}\right)^{32}\right)$$

Lorenz-Mie Approximations

Hazy atmosphere

$$f_{p\,\mathrm{hazy}}(\theta) = \frac{1}{4\pi}\left(5 + \left(\frac{1+\cos\theta}{2}\right)^8\right)$$

Murky atmosphere

$$f_{p\,\mathrm{murky}}(\theta) = \frac{1}{4\pi}\left(17 + \left(\frac{1+\cos\theta}{2}\right)^{32}\right)$$

johnib.wordpress.com

srollinson.blogspot.com
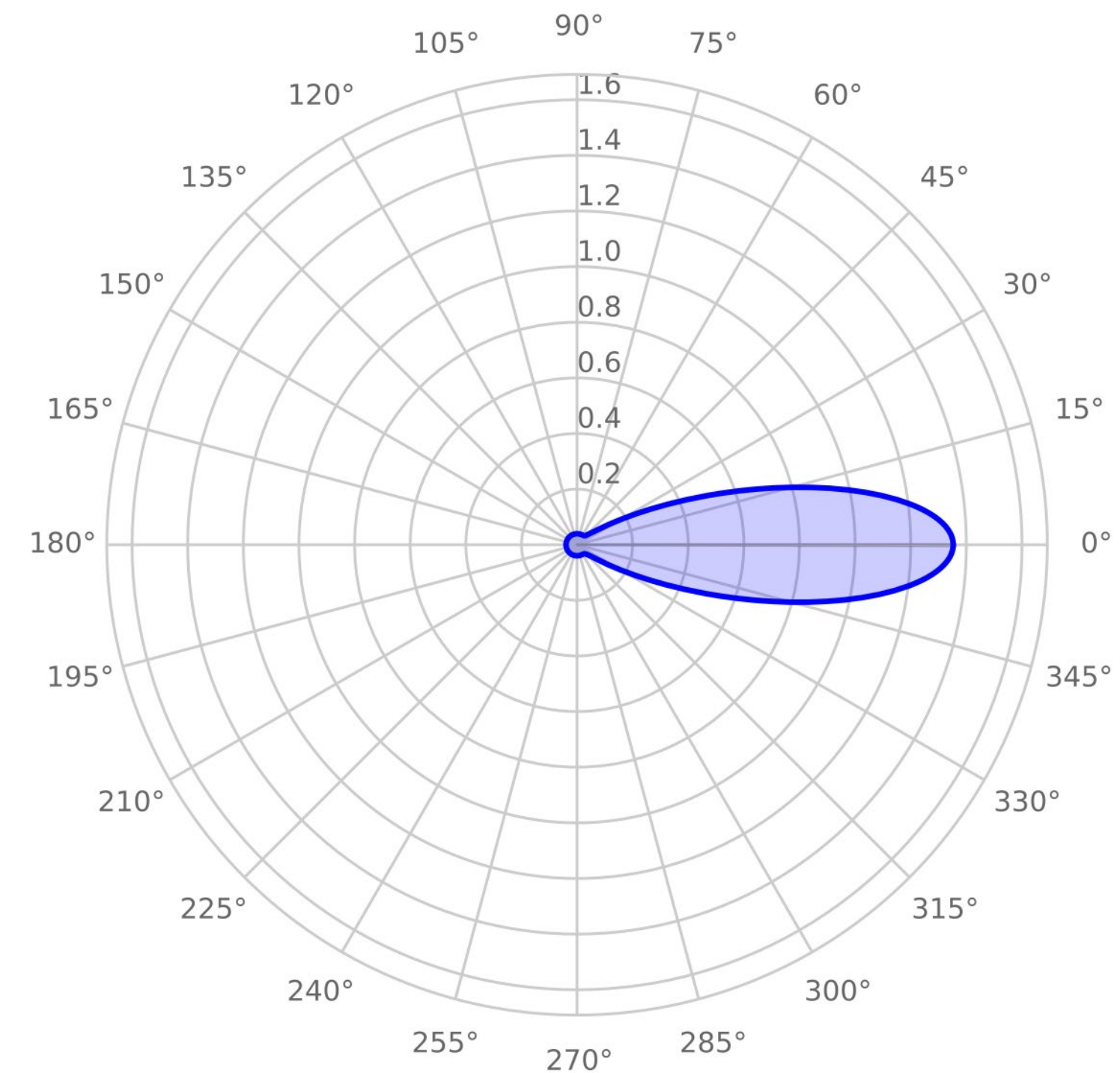
Rayleigh Scattering

Approximation of Lorenz-Mie for tiny scatterers that are typically smaller than 1/10th the wavelength of visible light

Used for atmospheric scattering, gasses, transparent solids

Highly wavelength dependent

Rayleigh Phase Function

$$f_{p\mathrm{Rayleigh}}(\theta) = \frac{3}{16\pi}(1 + \cos^2\theta)$$

Scattering at right angles is half as likely as scattering forward or backward
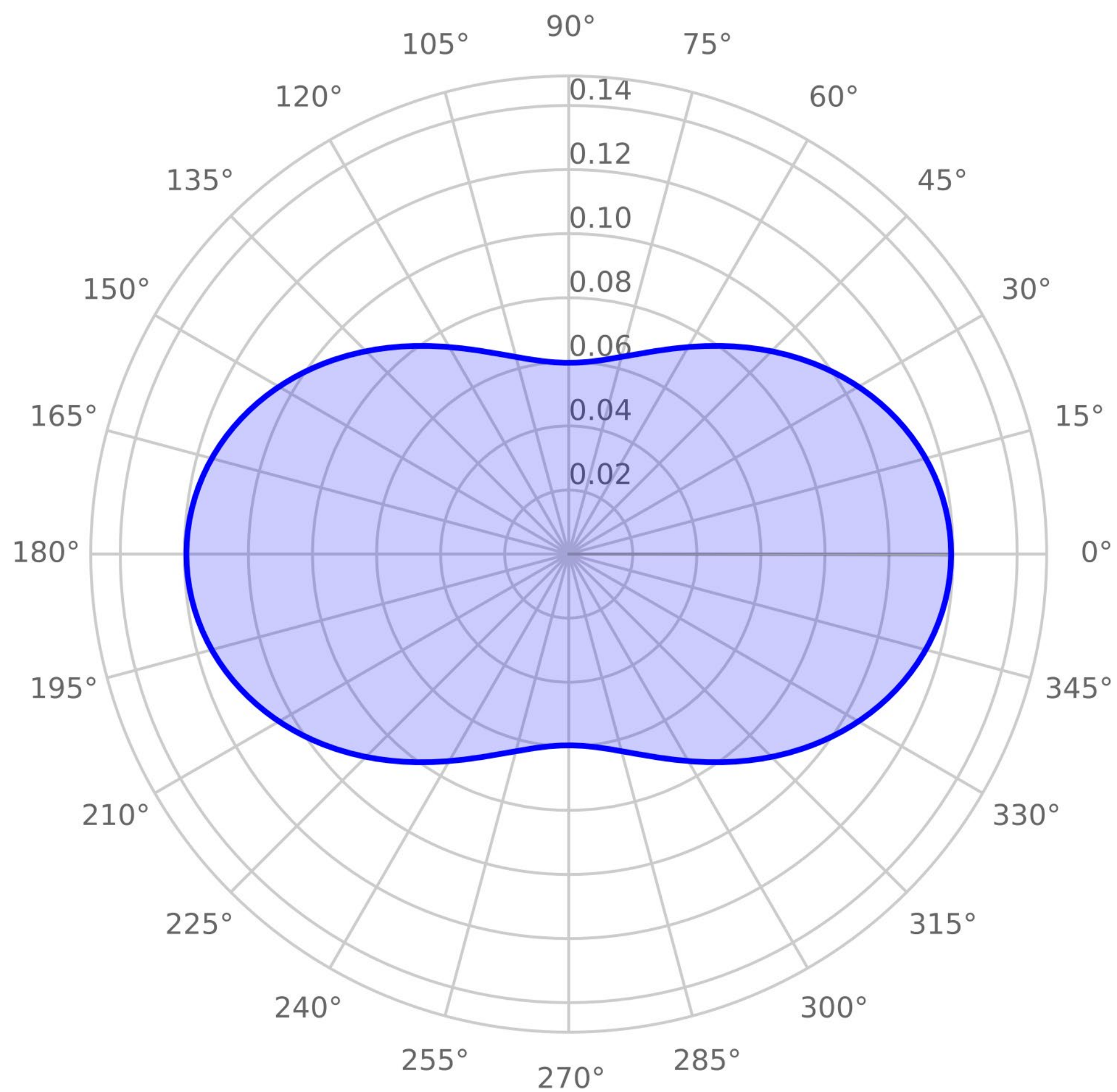
Rayleigh Scattering

Wavelength

Index of refraction

$$\sigma_{s\mathrm{Rayleigh}}(\lambda, d, \eta, \rho) = \rho \, \frac{2\pi^5 d^6}{3\lambda^4} \left(\frac{\eta^2 - 1}{\eta^2 + 2} \right)^2$$

Density of scatterers

Diameter of scatterers

Rayleigh Scattering

$$\sigma_{s\mathrm{Rayleigh}}(\lambda, d, \eta, \rho) = \rho\,\frac{2\pi^5 d^6}{3\lambda^4}\left(\frac{\eta^2 - 1}{\eta^2 + 2}\right)^2$$

Examples

Dana Stephenson/Getty Images
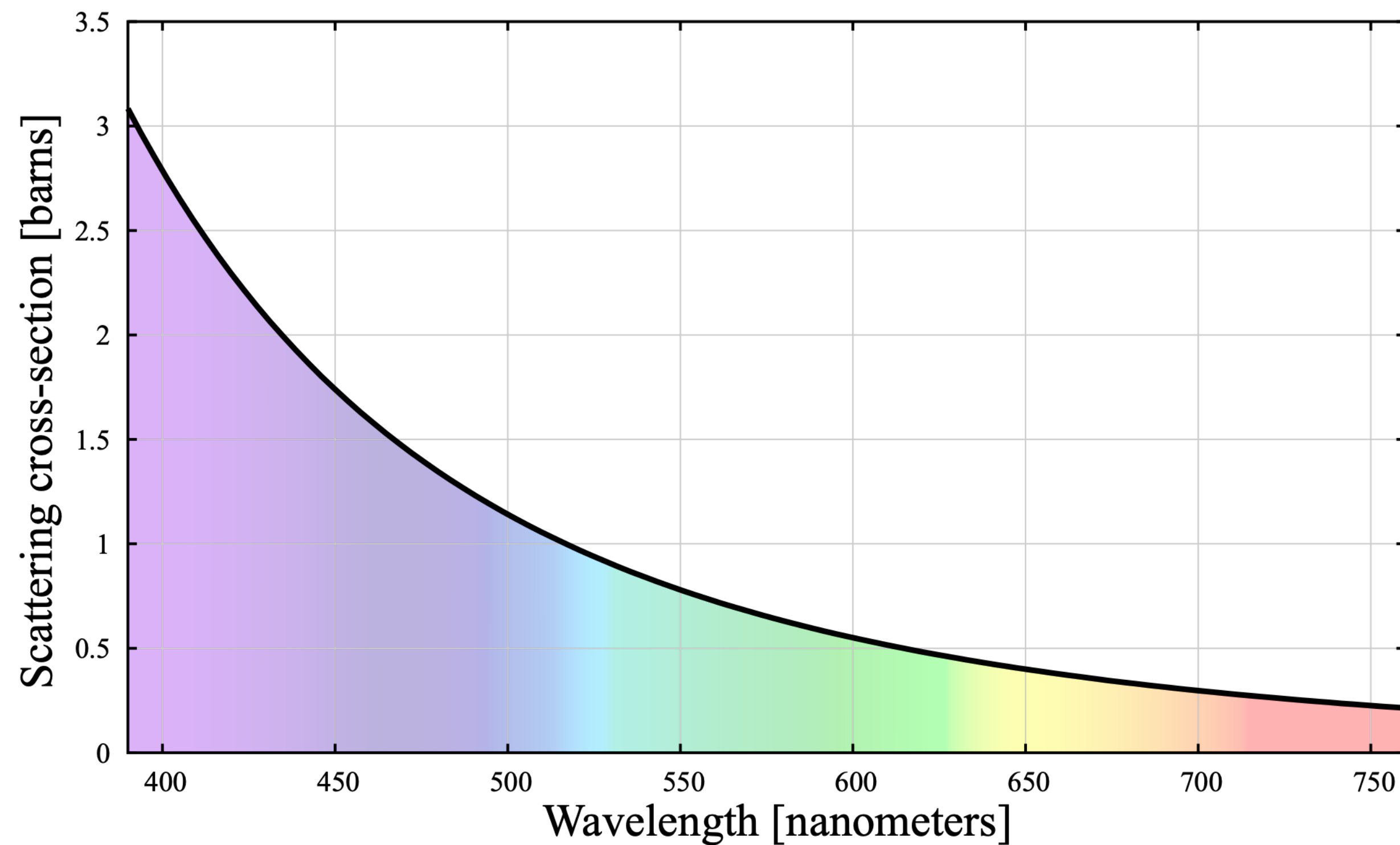
Examples

Steam

Smoke

Forward scattering

Backward scattering

Examples

Isotropic scattering

Examples

Forward scattering

Why is the Sky Blue?

Atmosphere

Earth
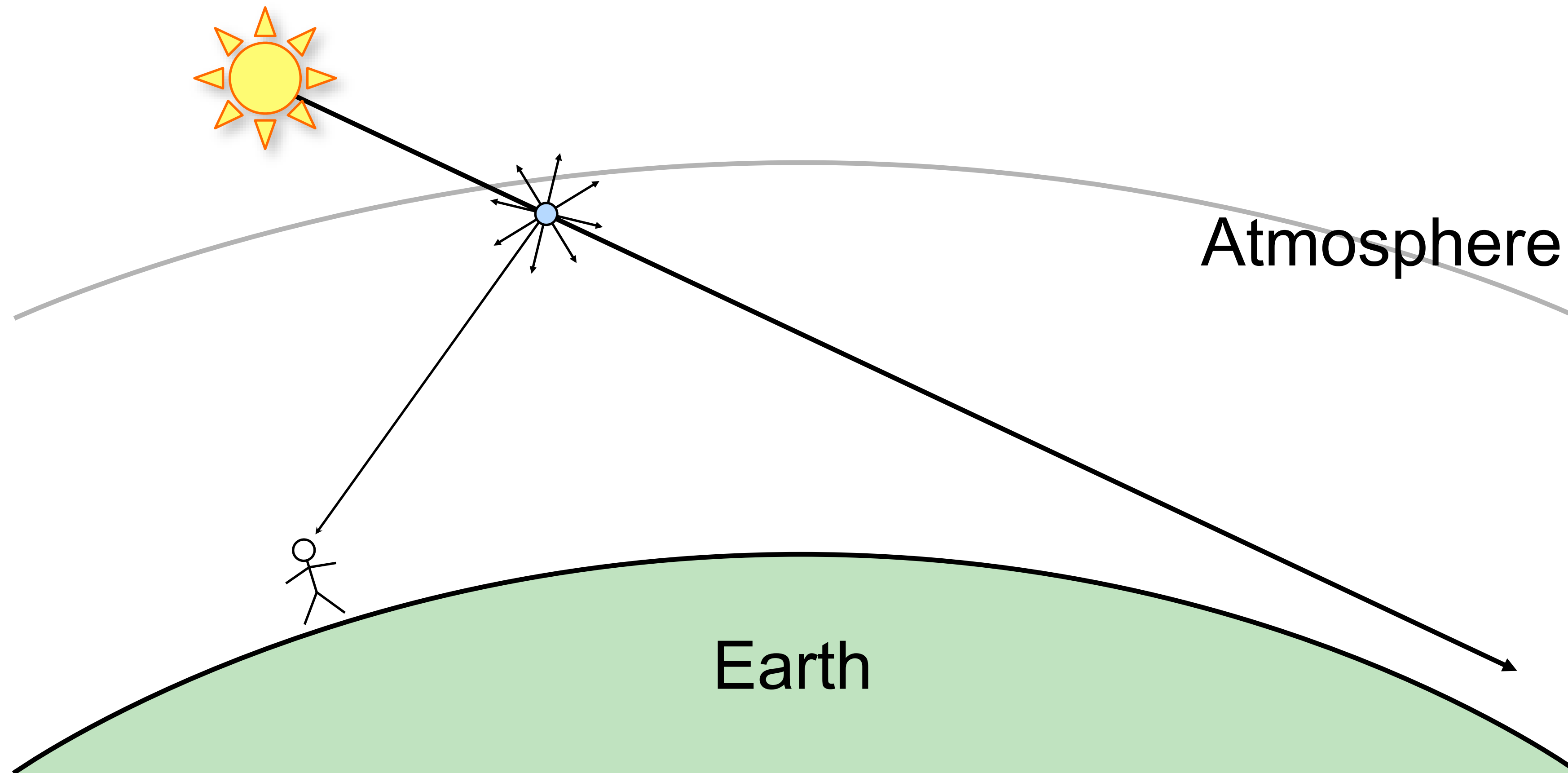
Why is the Sunset Red?

Atmosphere

Earth
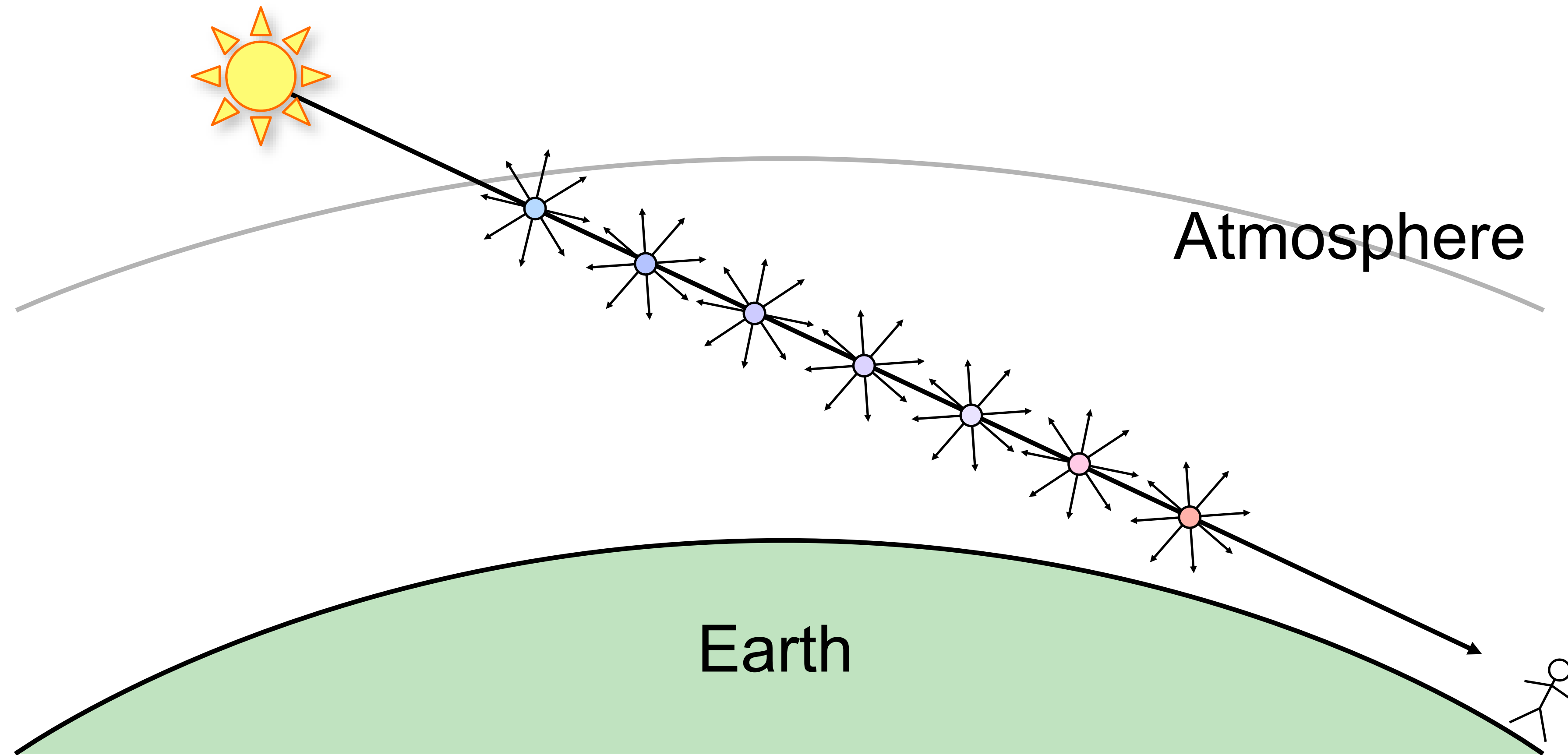
Rayleigh Scattering
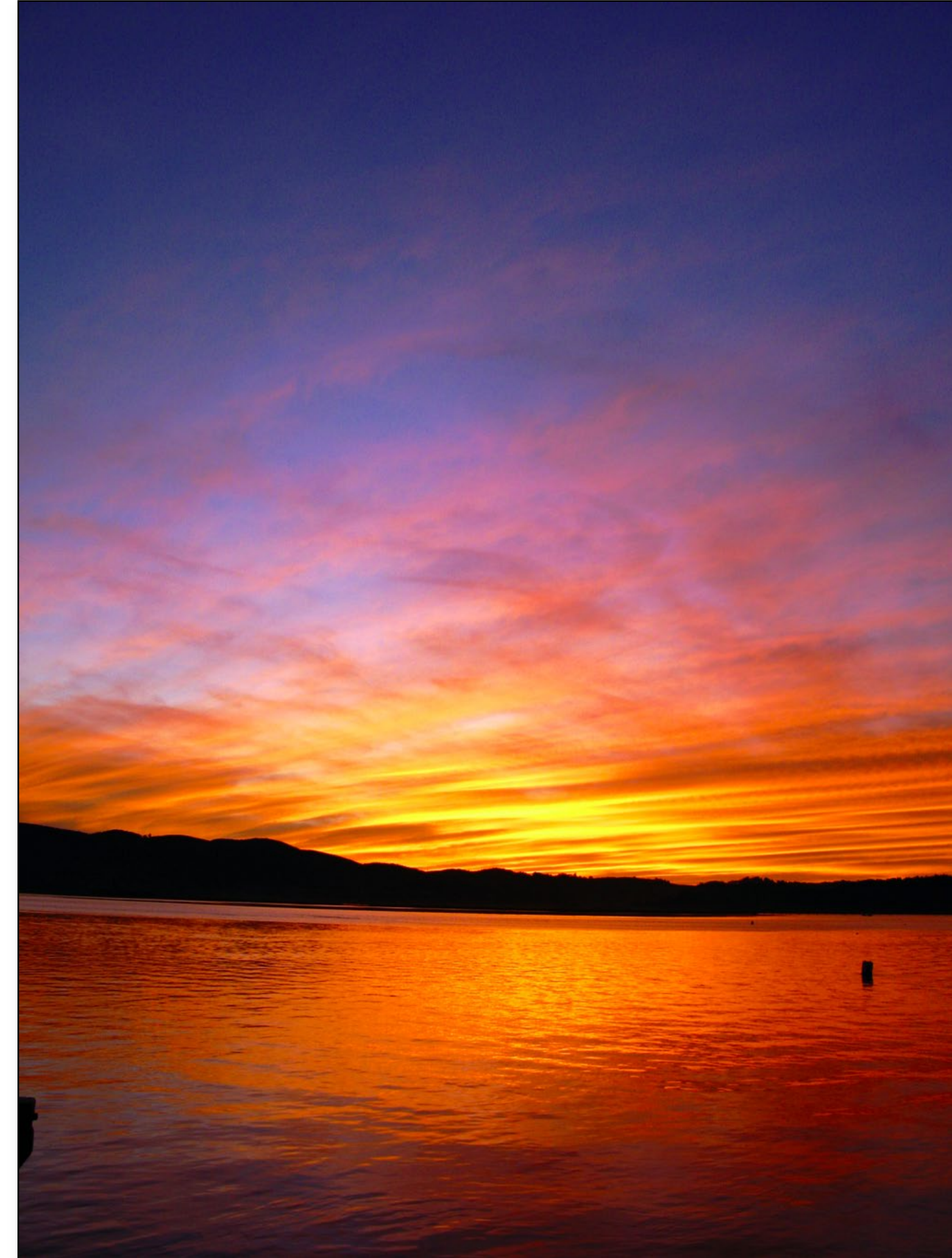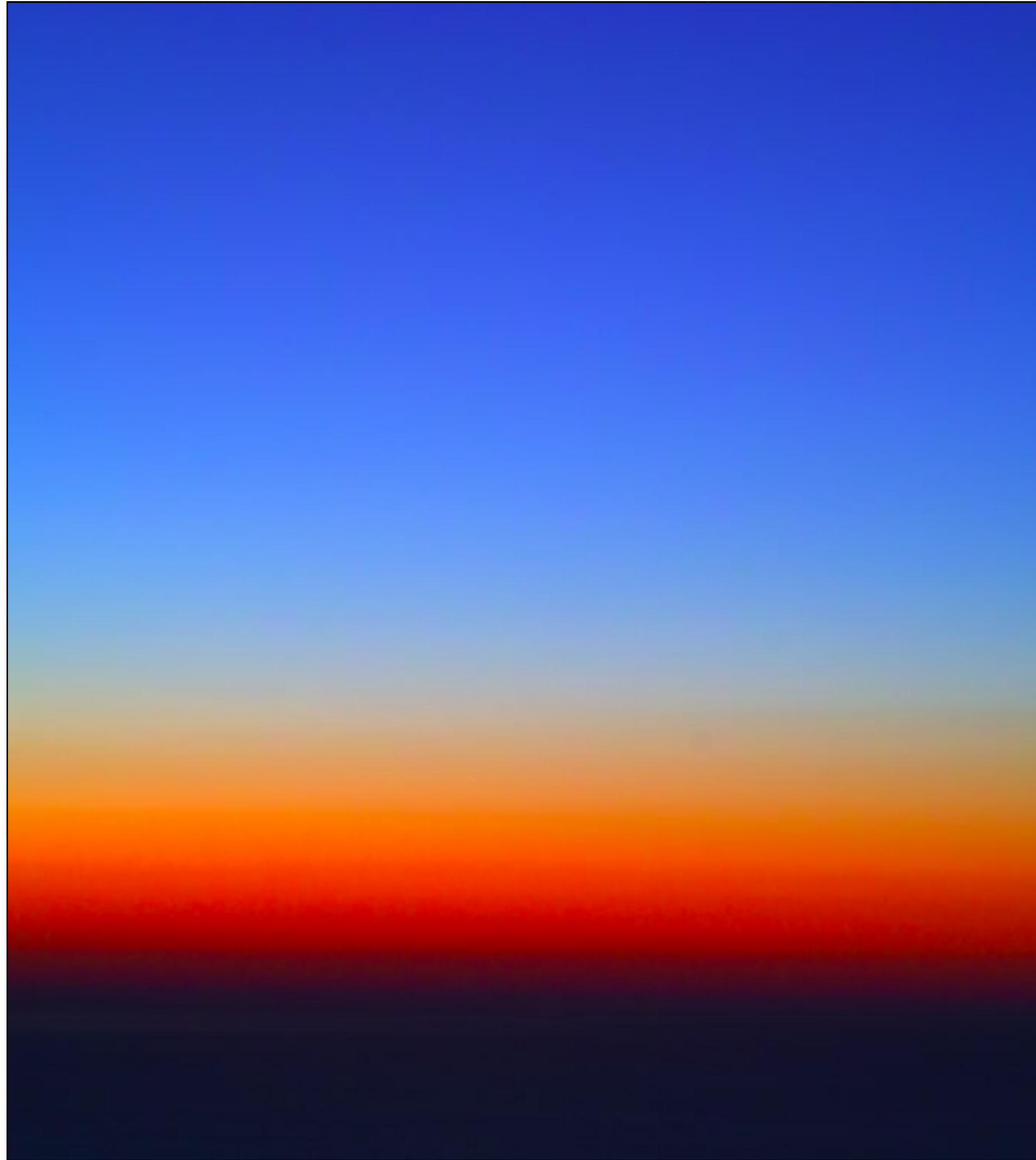
Media Properties (Recap)

Given:

- Absorption coefficient $\quad\quad \sigma_a(\mathbf{x})$ $\quad\quad\quad\quad\quad\quad\quad \left[\mathrm{m}^{-1}\right]$

- Scattering coefficient $\quad\quad \sigma_s(\mathbf{x})$ $\quad\quad\quad\quad\quad\quad\quad \left[\mathrm{m}^{-1}\right]$

- Phase function $\quad\quad\quad\quad f_p(\mathbf{x}, \vec{\omega}', \vec{\omega})$ $\quad\quad\quad\quad\quad \left[\mathrm{sr}^{-1}\right]$

Derived:

- Extinction coefficient $\quad \sigma_t(\mathbf{x}) = \sigma_a(\mathbf{x}) + \sigma_s(\mathbf{x}) \quad \left[\mathrm{m}^{-1}\right]$

- Albedo $\quad\quad\quad\quad\quad\quad \alpha(\mathbf{x}) = \sigma_s(\mathbf{x})/\sigma_t(\mathbf{x}) \quad\quad [\mathrm{none}]$

- Mean-free path $\quad\quad\quad\quad 1/\sigma_t(\mathbf{x})$ $\quad\quad\quad\quad\quad\quad [\mathrm{m}]$

- Transmittance $\quad\quad T_r(\mathbf{x}, \mathbf{y}) = e^{-\int_0^{\|\mathbf{x}-\mathbf{y}\|} \sigma_t(t)dt} \quad [\mathrm{none}]$

Homogeneous Isotropic Medium

Given:

- Absorption coefficient σ_a $\left[\mathrm{m}^{-1}\right]$

- Scattering coefficient σ_s $\left[\mathrm{m}^{-1}\right]$

- Phase function $\dfrac{1}{4\pi}$ $\left[\mathrm{sr}^{-1}\right]$

Derived:

- Extinction coefficient $\sigma_t = \sigma_a + \sigma_s$ $\left[\mathrm{m}^{-1}\right]$

- Albedo $\alpha = \sigma_s / \sigma_t$ $[\text{none}]$

- Mean-free path $1/\sigma_t$ $[\mathrm{m}]$

- Transmittance $T_r(\mathbf{x}, \mathbf{y}) = e^{-\sigma_t \|\mathbf{x} - \mathbf{y}\|}$ $[\text{none}]$

What is this?

source: wikipedia

Crepuscular Rays

source: wikipedia

Anti-Crepuscular Rays

source: wikipedia

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Crepuscular rays from space
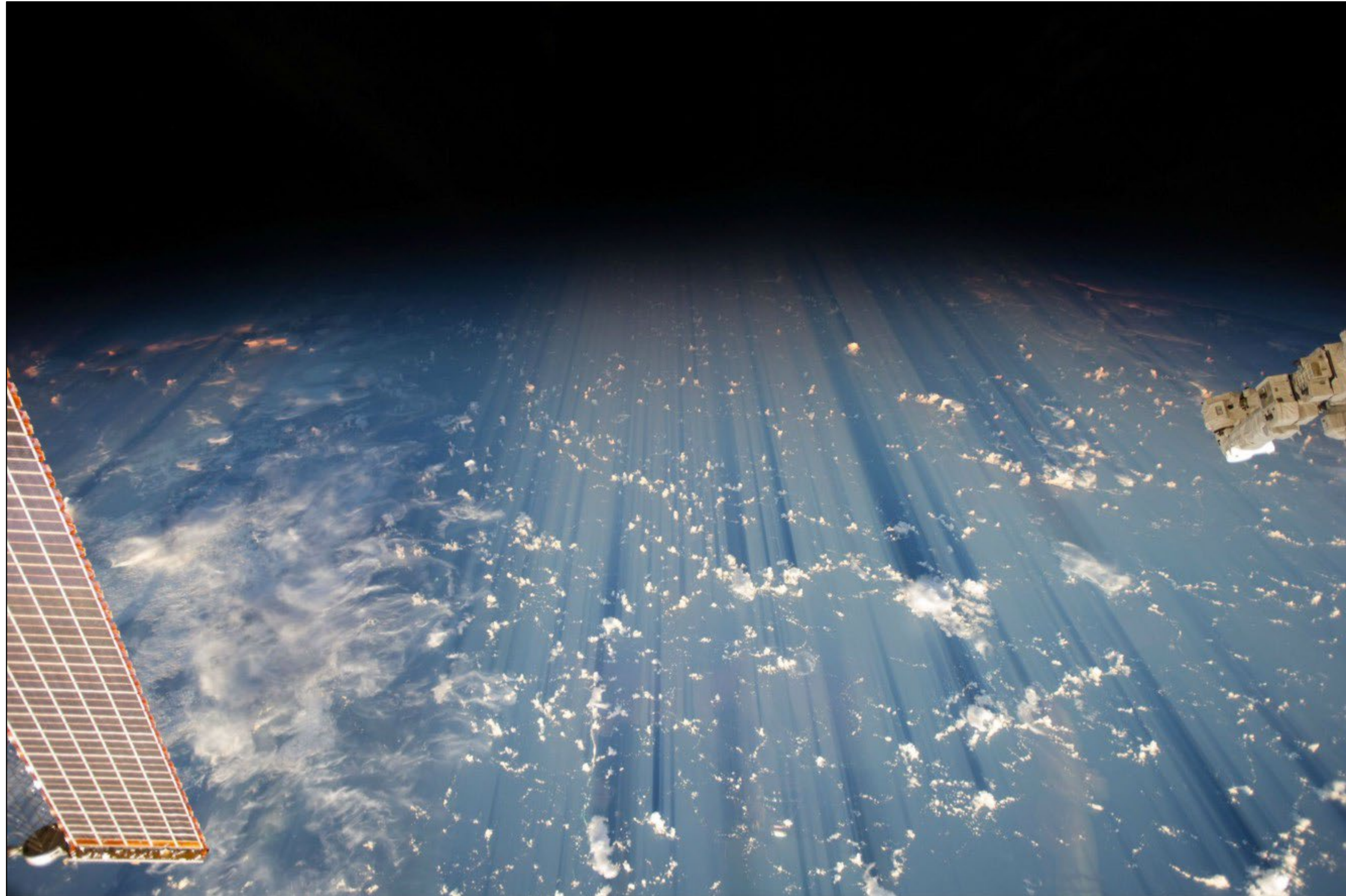
Solving the
Volume Rendering Equation

Complexity Progression

homogeneous vs. heterogeneous

scattering

- none

- fake ambient

- single

- multiple

Volume Rendering Equation

Attenuated background radiance

Accumulated emitted radiance

$$L(\mathbf{x}, \vec{\omega}) = \boxed{T_r(\mathbf{x}, \mathbf{x}_z) L(\mathbf{x}_z, \vec{\omega})}$$

$$+ \boxed{\int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) dt}$$

$$+ \boxed{\int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \sigma_s(\mathbf{x}_t) L_s(\mathbf{x}_t, \vec{\omega}) dt}$$

Accumulated in-scattered radiance

Purely absorbing media

Attenuated background radiance

$$L(\mathbf{x}, \vec{\omega}) = \boxed{T_r(\mathbf{x}, \mathbf{x}_z) L(\mathbf{x}_z, \vec{\omega})}$$

Fog

http://anordinarymom.wordpress.com

Participating Media

$$L(\mathbf{x}, \vec{\omega}) = \int_0^s T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t) \boxed{\sigma_s(\mathbf{x}_t)} L_i(\mathbf{x}_t, \vec{\omega}) \, dt + T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) L(\mathbf{x}_s, \vec{\omega})$$

$$L(\mathbf{x}, \vec{\omega}) = \boxed{\sigma_s} \int_0^s \boxed{T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t)} L_i(\mathbf{x}_t, \vec{\omega}) \, dt + \boxed{T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s)} L(\mathbf{x}_s, \vec{\omega})$$

$$L(\mathbf{x}, \vec{\omega}) = \sigma_s \int_0^s \boxed{e^{-t\sigma_t}} L_i(\mathbf{x}_t, \vec{\omega}) \, dt + \boxed{e^{-s\sigma_t}} L(\mathbf{x}_s, \vec{\omega})$$

Fog

http://anordinarymom.wordpress.com

$$L(\mathbf{x}, \vec{\omega}) = \sigma_s \int_0^s e^{-t\sigma_t} L_i(\mathbf{x}_t, \vec{\omega}) \, dt + e^{-s\sigma_t} L(\mathbf{x}_s, \vec{\omega})$$

Homogeneous Ambient Media

Assume in-scattered radiance is an ambient constant:

$$L(\mathbf{x}, \vec{\omega}) = \sigma_s \int_0^s e^{-t\sigma_t} \boxed{L_i(\mathbf{x}_t, \vec{\omega})} dt + e^{-s\sigma_t} L(\mathbf{x}_s, \vec{\omega})$$

Homogeneous Ambient Media

Assume in-scattered radiance is an ambient constant:

$$L(\mathbf{x}, \vec{\omega}) = \sigma_s \int_0^s e^{-t\sigma_t} L_i(\mathbf{x}_t, \vec{\omega}) \, dt + e^{-s\sigma_t} L(\mathbf{x}_s, \vec{\omega})$$

$$L(\mathbf{x}, \vec{\omega}) = \sigma_s \, L_i \int_0^s e^{-t\sigma_t} \, dt + e^{-s\sigma_t} L(\mathbf{x}_s, \vec{\omega})$$

$$L(\mathbf{x}, \vec{\omega}) = \sigma_s \, L_i \frac{1 - e^{-s\sigma_t}}{\sigma_t} + e^{-s\sigma_t} L(\mathbf{x}_s, \vec{\omega})$$

$$L(\mathbf{x}, \vec{\omega}) = \text{lerp}\left(\frac{\sigma_s}{\sigma_t} L_i, \; L(\mathbf{x}_s, \vec{\omega}), \; e^{-s\sigma_t} \right)$$

albedo

OpenGL Fog

Sun et al. 2005

OpenGL Clear Day

Sun et al. 2005
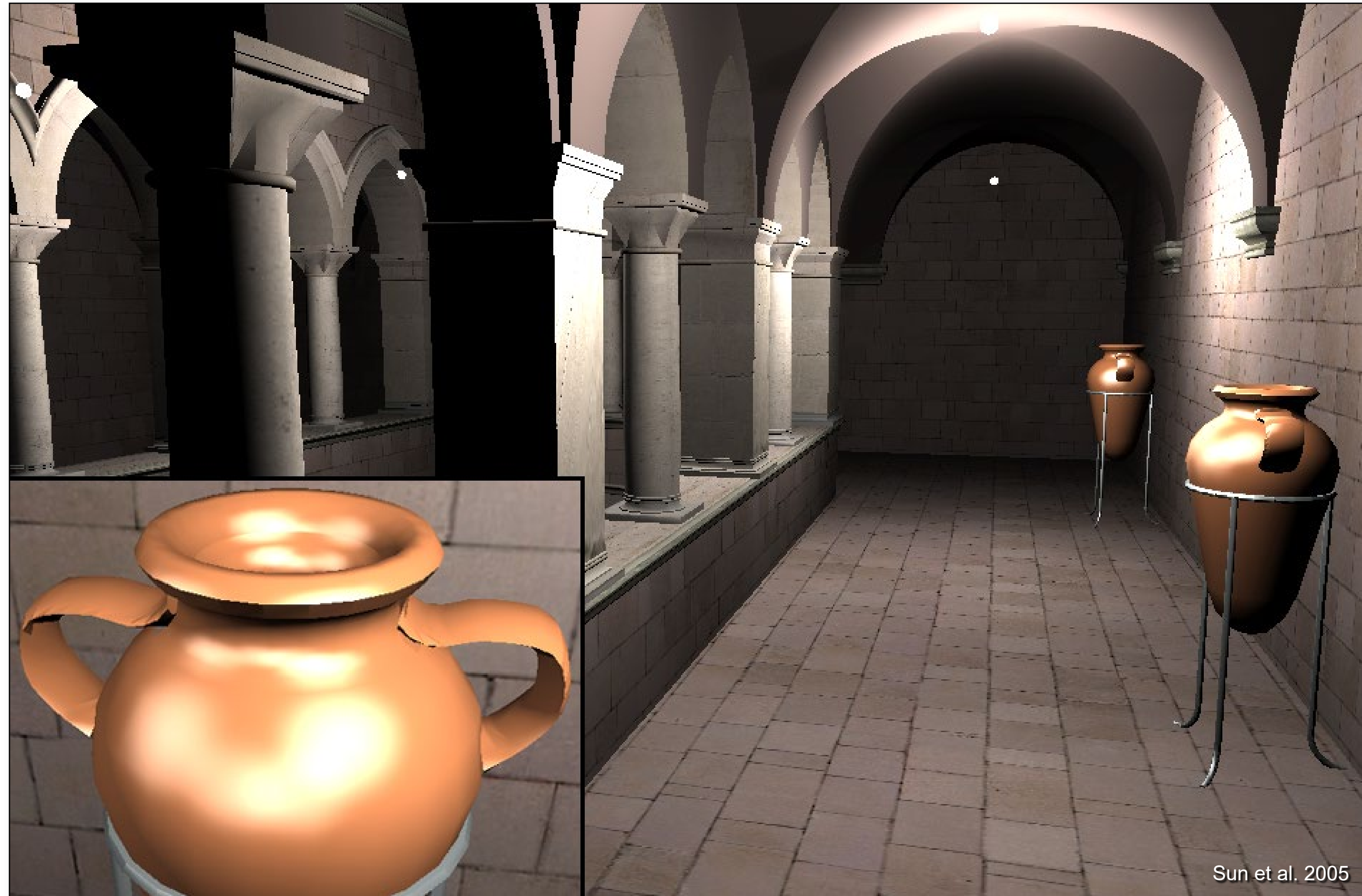
Fog

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = T_r(\mathbf{x}, \mathbf{x}_z)L(\mathbf{x}_z, \vec{\omega})$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_a(\mathbf{x}_t)L_e(\mathbf{x}_t, \vec{\omega})dt$$

$$+ \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_s(\mathbf{x}_t, \vec{\omega})dt$$

Accumulated in-scattered radiance

In-scattered Radiance

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \sigma_s(\mathbf{x}_t) \boxed{L_s(\mathbf{x}_t, \vec{\omega})} dt$$

$$\boxed{L_s(\mathbf{x}_t, \vec{\omega})} = \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega}) \boxed{L_i(\mathbf{x}_t, \vec{\omega}')} d\vec{\omega}'$$

Single scattering

- L_i arrives directly from a light source (direct illum.)
  i.e.:
  $$\boxed{L_i(\mathbf{x}, \vec{\omega})} = T_r(\mathbf{x}, r(\mathbf{x}, \vec{\omega})) \, L_e(r(\mathbf{x}, \vec{\omega}), -\vec{\omega})$$

Multiple scattering

- L_i arrives through multiple bounces (indirect illum.)

Single Scattering

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t) \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega}) T_r(\mathbf{x}_t, \mathbf{x}_e) L_e(\mathbf{x}_e, -\vec{\omega}') d\vec{\omega}' dt$$

Single Scattering

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t) \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega})T_r(\mathbf{x}_t, \mathbf{x}_e)L_e(\mathbf{x}_e, -\vec{\omega}')d\vec{\omega}'dt$$

Single Scattering

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \sigma_s(\mathbf{x}_t) \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega}) T_r(\mathbf{x}_t, \mathbf{x}_e) L_e(\mathbf{x}_e, -\vec{\omega}') d\vec{\omega}' dt$$

Single Scattering

$$L(\mathbf{x}, \vec{\omega}) = \boxed{\int_0^z} T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t) \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega}) T_r(\mathbf{x}_t, \mathbf{x}_e) L_e(\mathbf{x}_e, -\vec{\omega}') d\vec{\omega}' dt$$

$L_e(\mathbf{x}, -\vec{\omega}')$

$\mathbf{x}_e$

$\mathbf{x}$

Single Scattering

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t) \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega})T_r(\mathbf{x}_t, \mathbf{x}_e)L_e(\mathbf{x}_e, -\vec{\omega}')d\vec{\omega}'dt$$

(Semi-)analytic solutions:

- Sun et al. [2005]

- Pegoraro et al. [2009, 2010]

Numerical solutions:

- Ray-marching

- Equiangular sampling

Analytic Single Scattering

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t) \int_{S^2} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega})T_r(\mathbf{x}_t, \mathbf{x}_e)L_e(\mathbf{x}_e, -\vec{\omega}')d\vec{\omega}'dt$$

Assumptions:

- Homogeneous medium

- Point or spot light

- Relatively simple phase function

- No occlusion
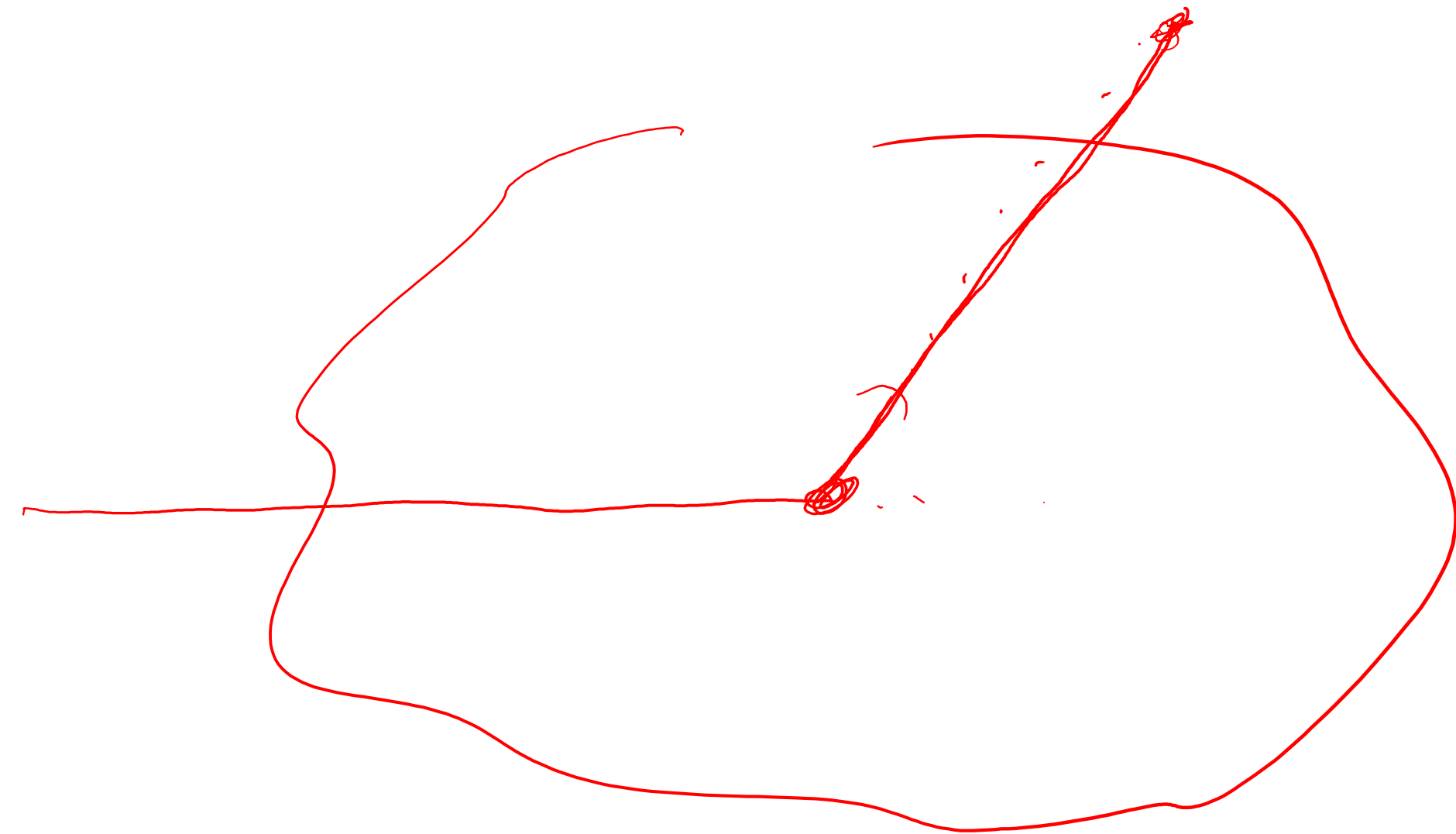
$$L(\mathbf{x}, \vec{\omega}) = \frac{\Phi}{4\pi}\frac{1}{4\pi}\sigma_s \int_0^z e^{-\sigma_t\|\mathbf{x},\mathbf{x}_t\|}\frac{e^{-\sigma_t\|\mathbf{x}_t,\mathbf{x}_p\|}}{\|\mathbf{x}_t,\mathbf{x}_p\|^2}\mathrm{d}t$$

OpenGL Fog

Sun et al. 2005

Analytic Single Scattering

Sun et al. 2005

Analytic Single Scattering

Pegoraro et al. 2009

Pegoraro et al. 2010
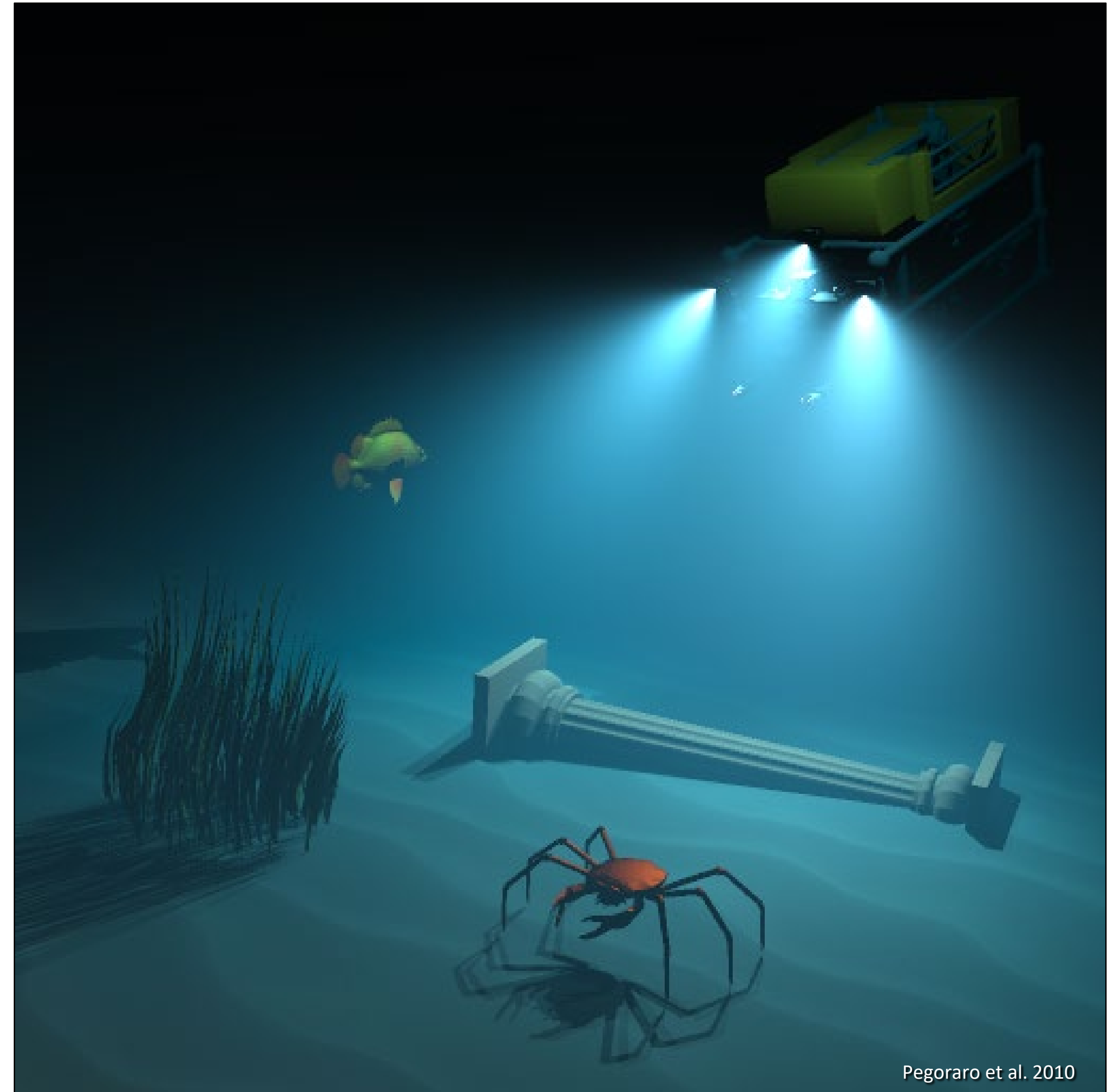
Andreas Levers

Andreas Levers

Analytic Single Scattering

$$L_m(x_a, x_b, \vec{\omega}) = \frac{\kappa_s}{h} e^{\kappa_t(x_a - x_h)} 2 \sum_{n=0}^{N-1} c(n) \sum_{k=0}^{2n} d(n,k) \int_{v_a}^{v_b} \frac{e^{-Hv}}{(v^2+1)^{n+1}} v^k dv$$

$$\int \frac{e^{av}}{(v^2+1)^m} v^n dv = \frac{1}{2^{m-1}} \sum_{l=0}^{m-1} \frac{1}{2^l} \binom{m-1+l}{m-1} \left(\sum_{k=0}^{\min\{m-1-l,n\}} \binom{n}{k} \left(\frac{a^{m-1-l-k}}{(m-1-l-k)!} E(a,v,m-n-l+k) \right. \right.$$
$$\left. - e^{av} \sum_{j=1}^{m-1-l-k} \frac{(j-1)!}{(m-1-l-k)!} \frac{a^{m-1-l-k-j}}{(v^2+1)^j} \sum_{\substack{i=(m-n-l+k-j) \bmod 2 \\ i+=2}}^{\leq j} (-1)^{\frac{m-n-l+k-j+i}{2}} \binom{j}{i} v^i \right)$$
$$\left. + \frac{e^{av}}{a} \sum_{k=0}^{\leq n-m+l} \binom{n}{k} \sum_{j=0}^{n-m+l-k} \frac{(n-m+l-k)!}{j!} \frac{1}{(-a)^{n-m+l-k-j}} \sum_{\substack{i=(-m+l+k-j) \bmod 2 \\ i+=2}}^{\leq j} (-1)^{\frac{-m+l+k-j+i}{2}} \binom{j}{i} v^i \right)$$

No shadows, implementation nightmare, computationally intensive...
Let's try brute force!

Ray-Marching

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_s(\mathbf{x}_t, \vec{\omega})dt$$

Approximate with Riemann sum
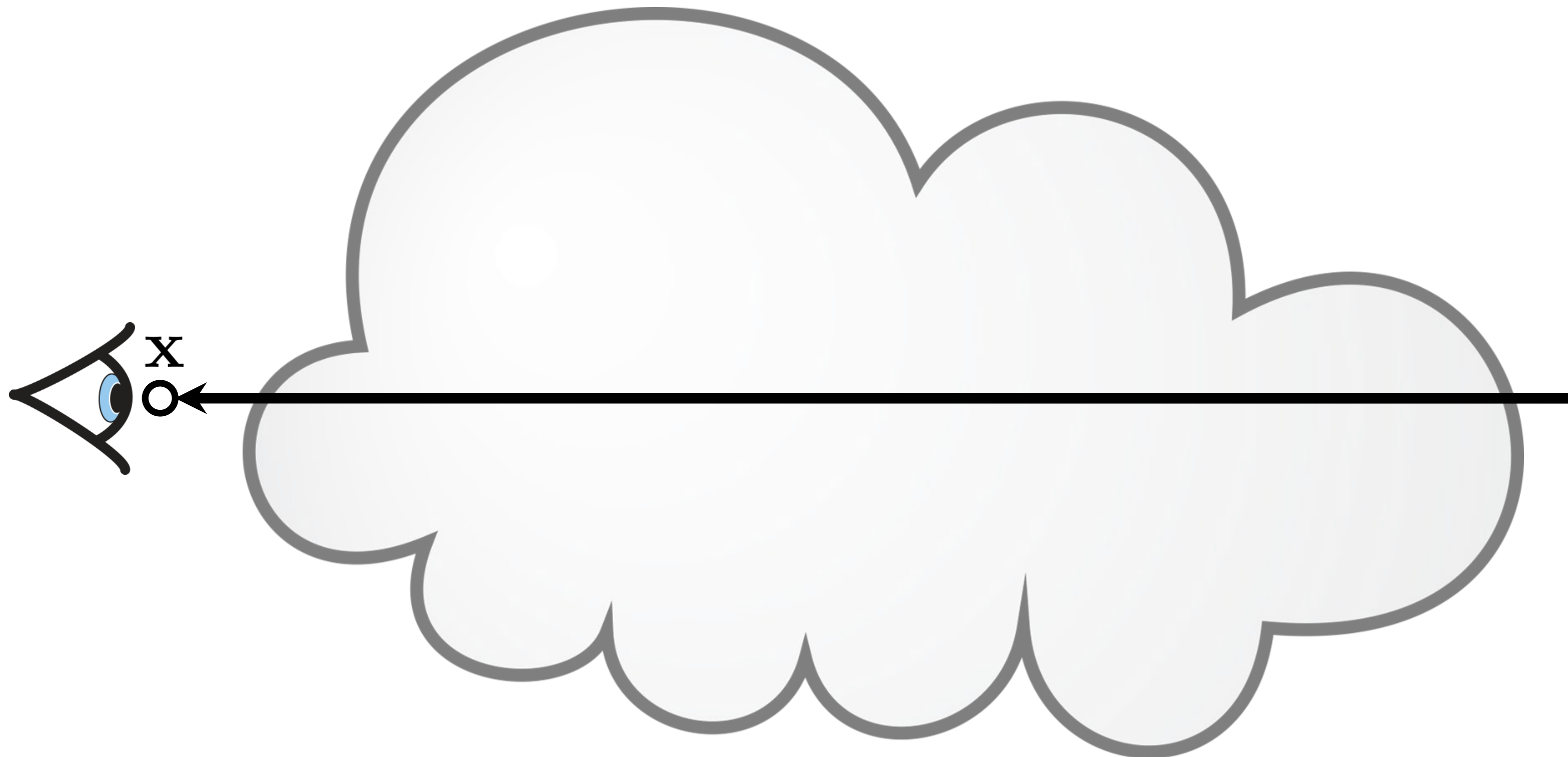
Ray-Marching

$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{i=1}^{N} T_r(\mathbf{x}, \mathbf{x}_{t,i}) \sigma_s(\mathbf{x}_{t,i}) L_s(\mathbf{x}_{t,i}, \vec{\omega}) \Delta t$$
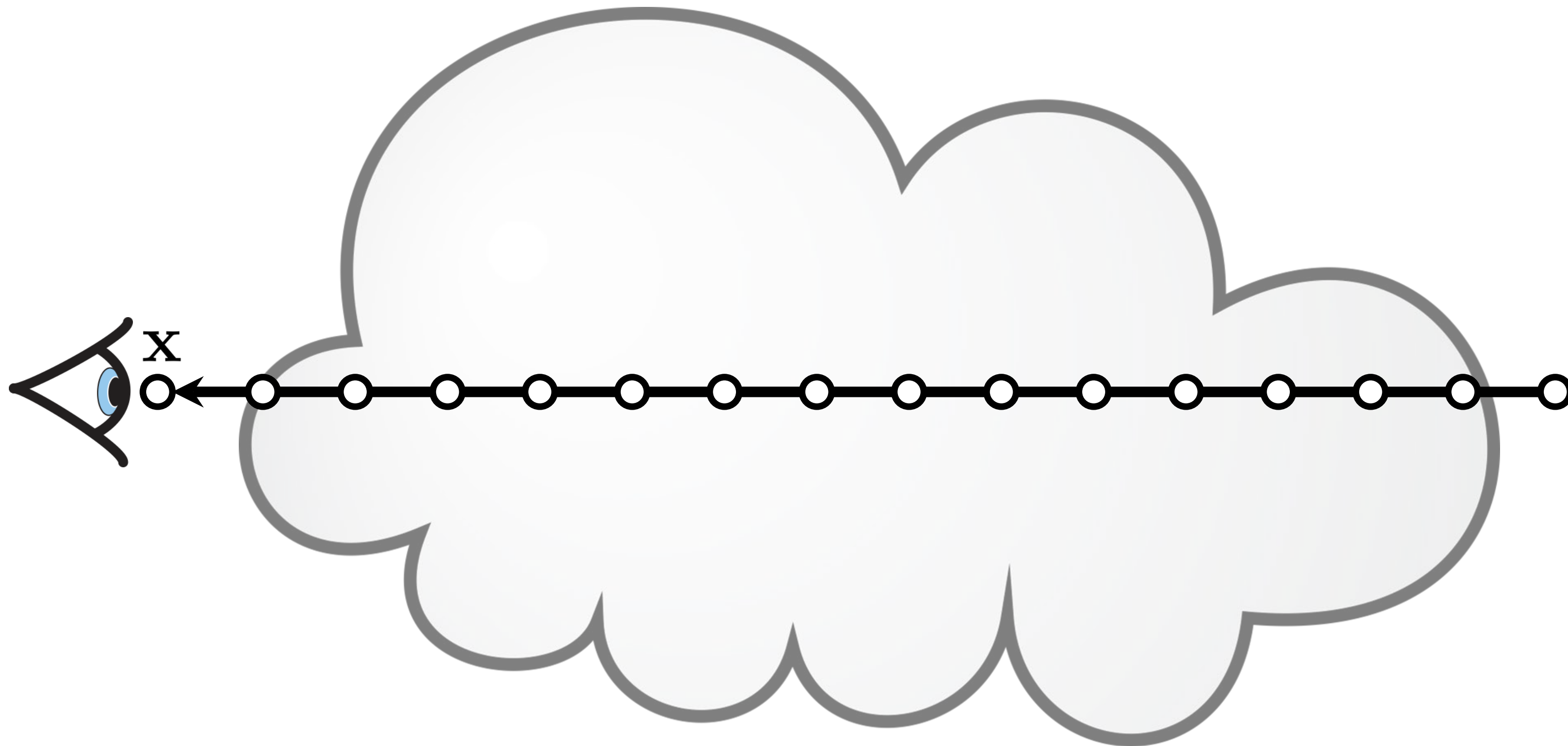
Ray-Marching

$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{i=1}^{N} T_r(\mathbf{x}, \mathbf{x}_{t,i}) \boxed{\sigma_s(\mathbf{x}_{t,i})} L_s(\mathbf{x}_{t,i}, \vec{\omega}) \Delta t$$
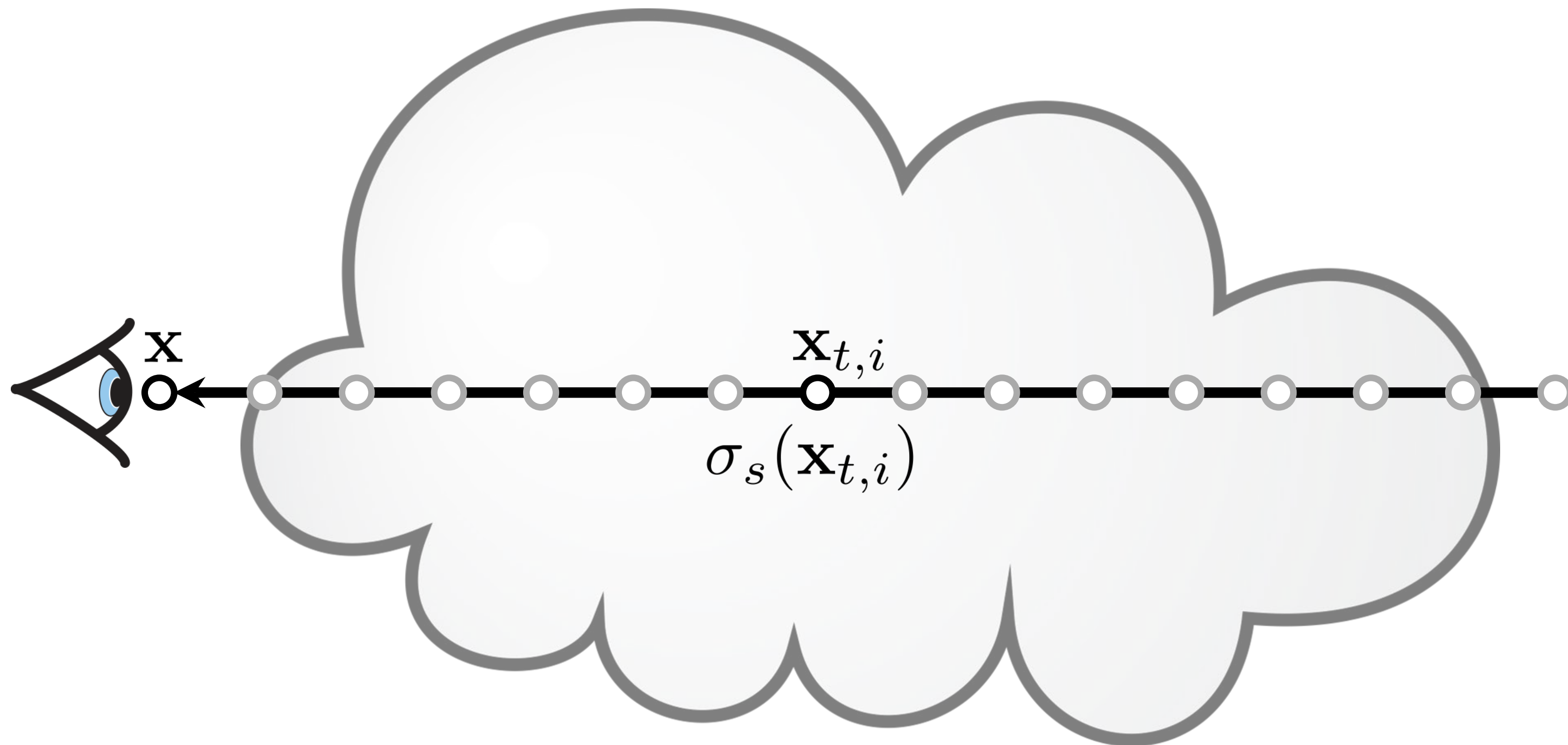
Ray-Marching

$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{i=1}^{N} \boxed{T_r(\mathbf{x}, \mathbf{x}_{t,i})} \sigma_s(\mathbf{x}_{t,i}) L_s(\mathbf{x}_{t,i}, \vec{\omega}) \Delta t$$

Homogeneous volume: $\quad T_r(\mathbf{x}, \mathbf{x}_{t,i}) = e^{-\sigma_t \|\mathbf{x}, \mathbf{x}_{t,i}\|}$
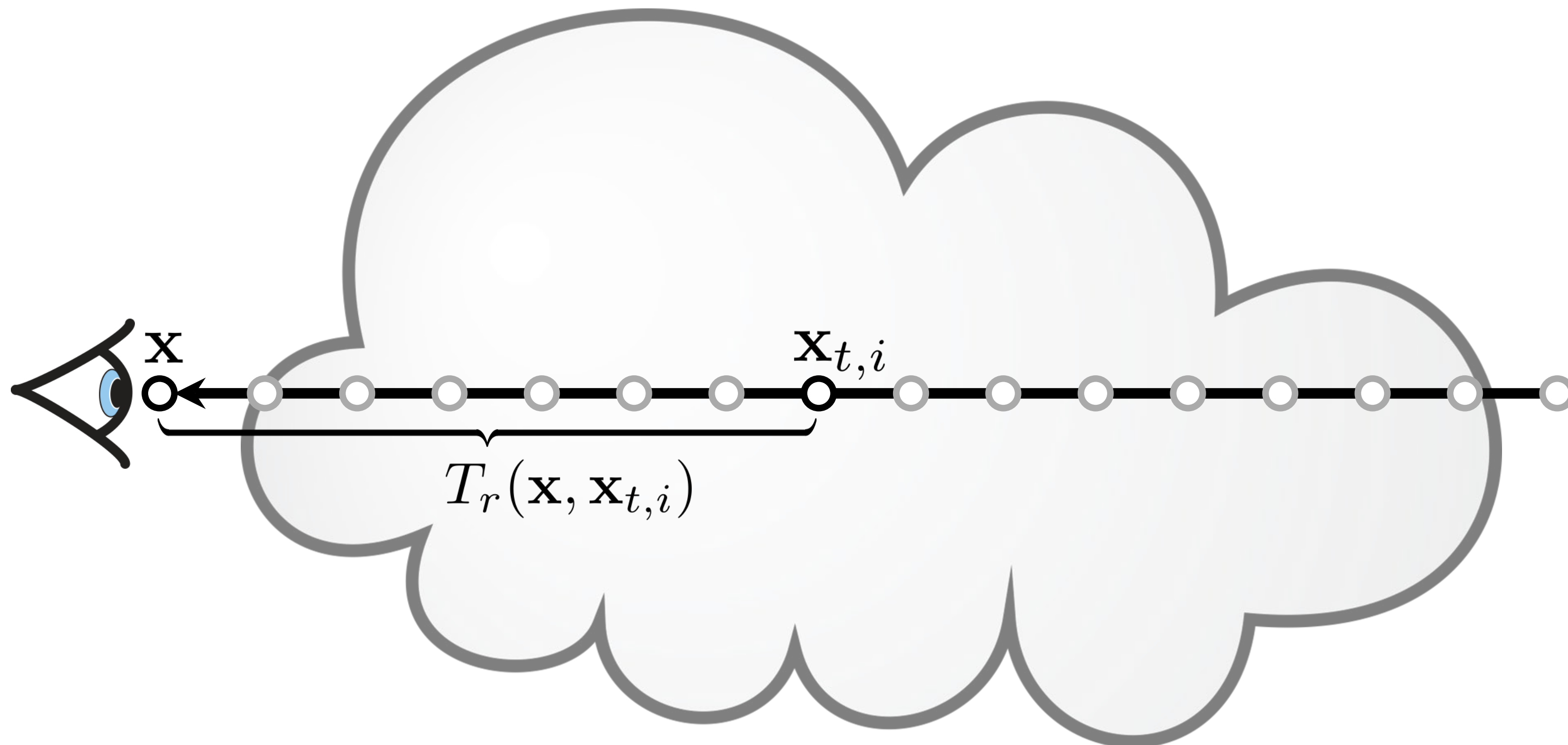
Ray-Marching

$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{i=1}^{N} \boxed{T_r(\mathbf{x}, \mathbf{x}_{t,i})} \sigma_s(\mathbf{x}_{t,i}) L_s(\mathbf{x}_{t,i}, \vec{\omega}) \Delta t$$

Heterogeneous volume: $\quad T_r(\mathbf{x}, \mathbf{x}_{t,i}) = T_r(\mathbf{x}, \mathbf{x}_{t,i-1}) e^{-\sigma_t(\mathbf{x}_{t,i}) \Delta t}$

Assume constant extinction along each segment

$\mathbf{x}$

$\mathbf{x}_{t,i}$

$T_r(\mathbf{x}, \mathbf{x}_{t,i-1})$

$e^{-\sigma_t(\mathbf{x}_{t,i}) \Delta t}$

$T_r(x, z) =$

$= T_r(x, y)$

$T_r(y, z)$
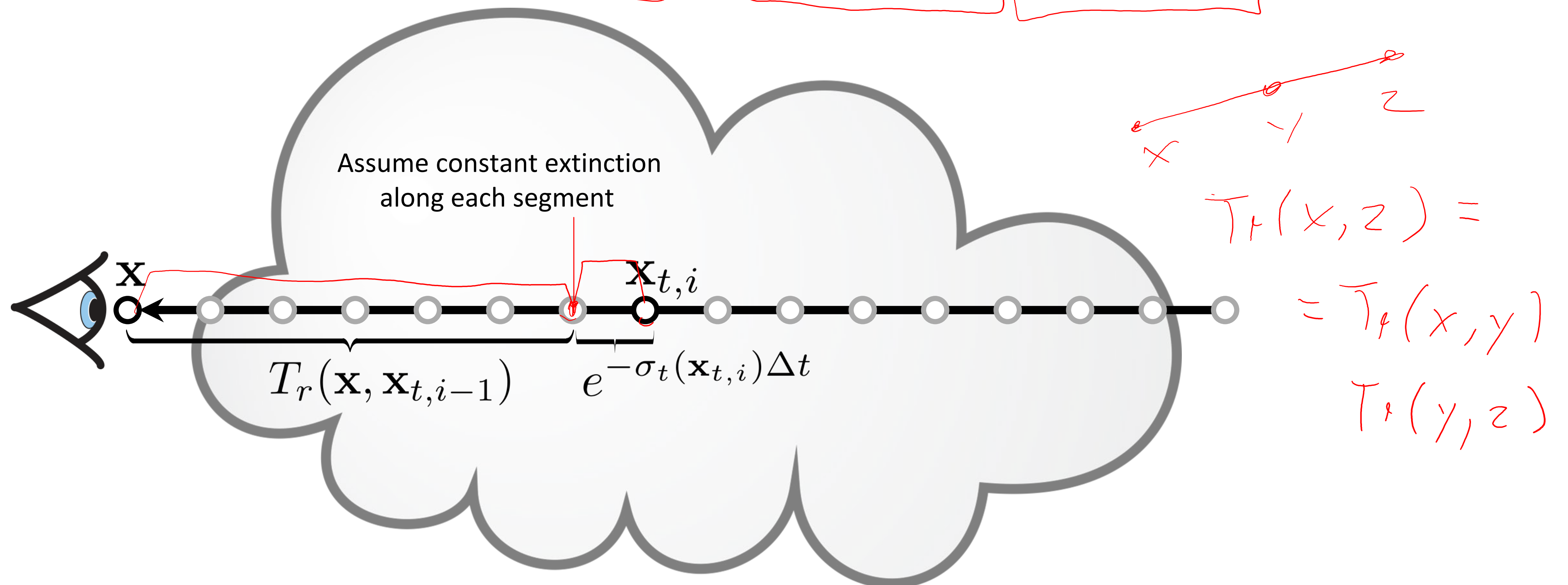
Ray-Marching

$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{i=1}^{N} T_r(\mathbf{x}, \mathbf{x}_{t,i}) \sigma_s(\mathbf{x}_{t,i}) \boxed{L_s(\mathbf{x}_{t,i}, \vec{\omega})} \Delta t$$
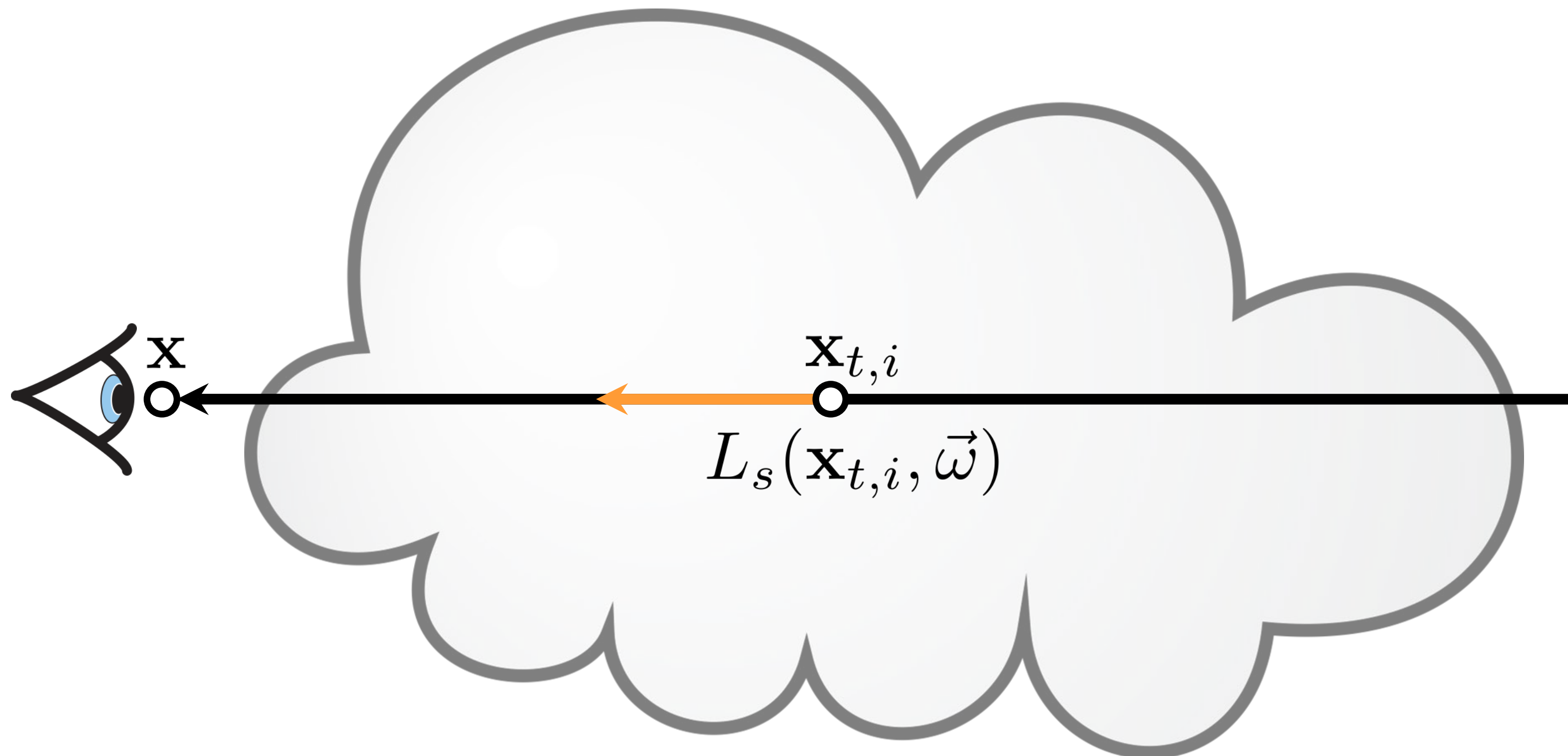
Ray-Marching

$$L_s(\mathbf{x}_t, \vec{\omega}) = \boxed{\int_{S^2}} f_p(\mathbf{x}_t, \vec{\omega}', \vec{\omega}) L_i(\mathbf{x}_t, \vec{\omega}') d\vec{\omega}'$$
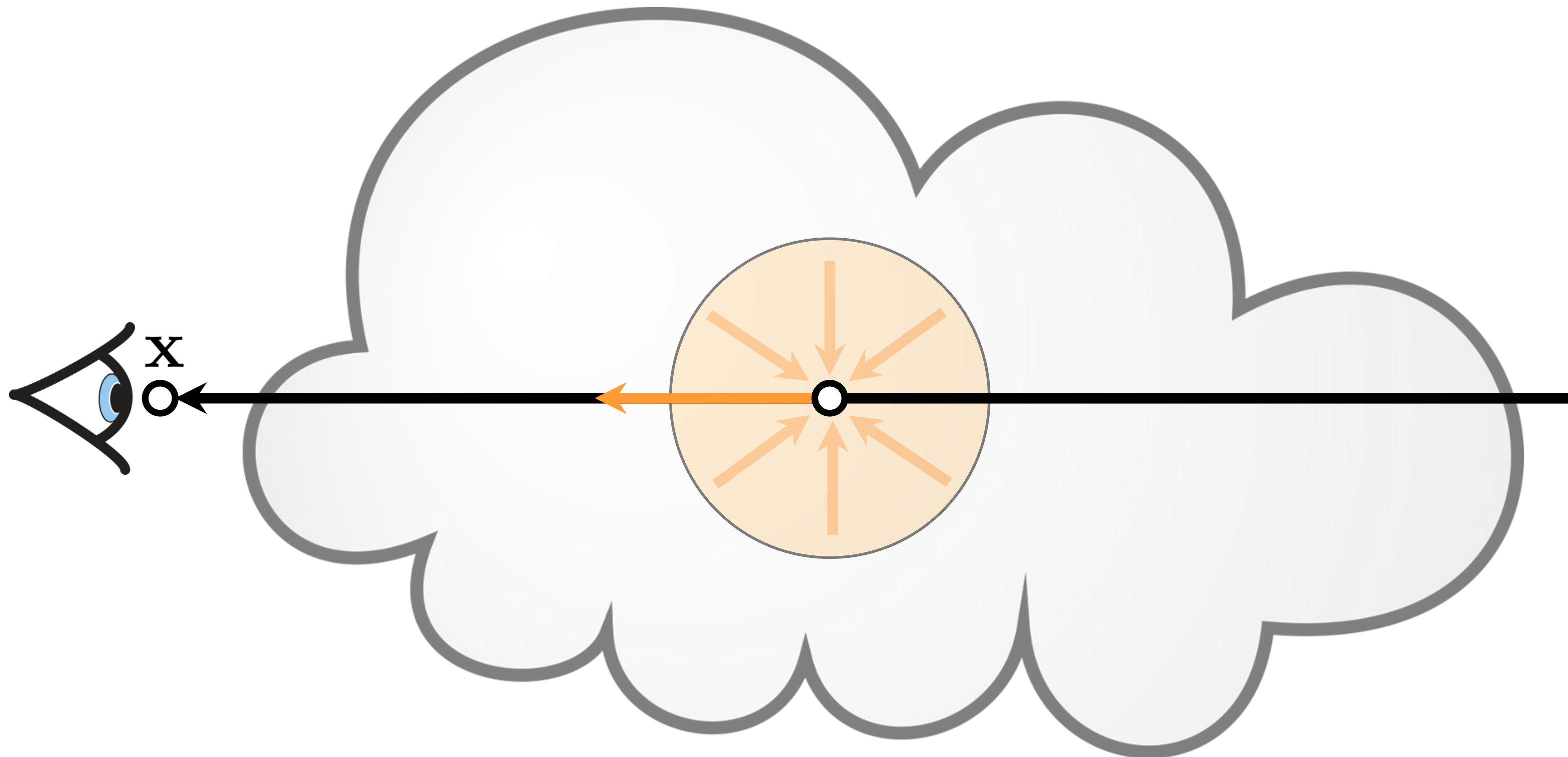
Ray-Marching

$$L_s(\mathbf{x}_t, \vec{\omega}) \approx \frac{1}{M} \sum_{j=0}^{M} \frac{f_p(\mathbf{x}_t, \vec{\omega}'_j, \vec{\omega}) L_i(\mathbf{x}_t, \vec{\omega}'_j)}{p(\vec{\omega}'_j)}$$
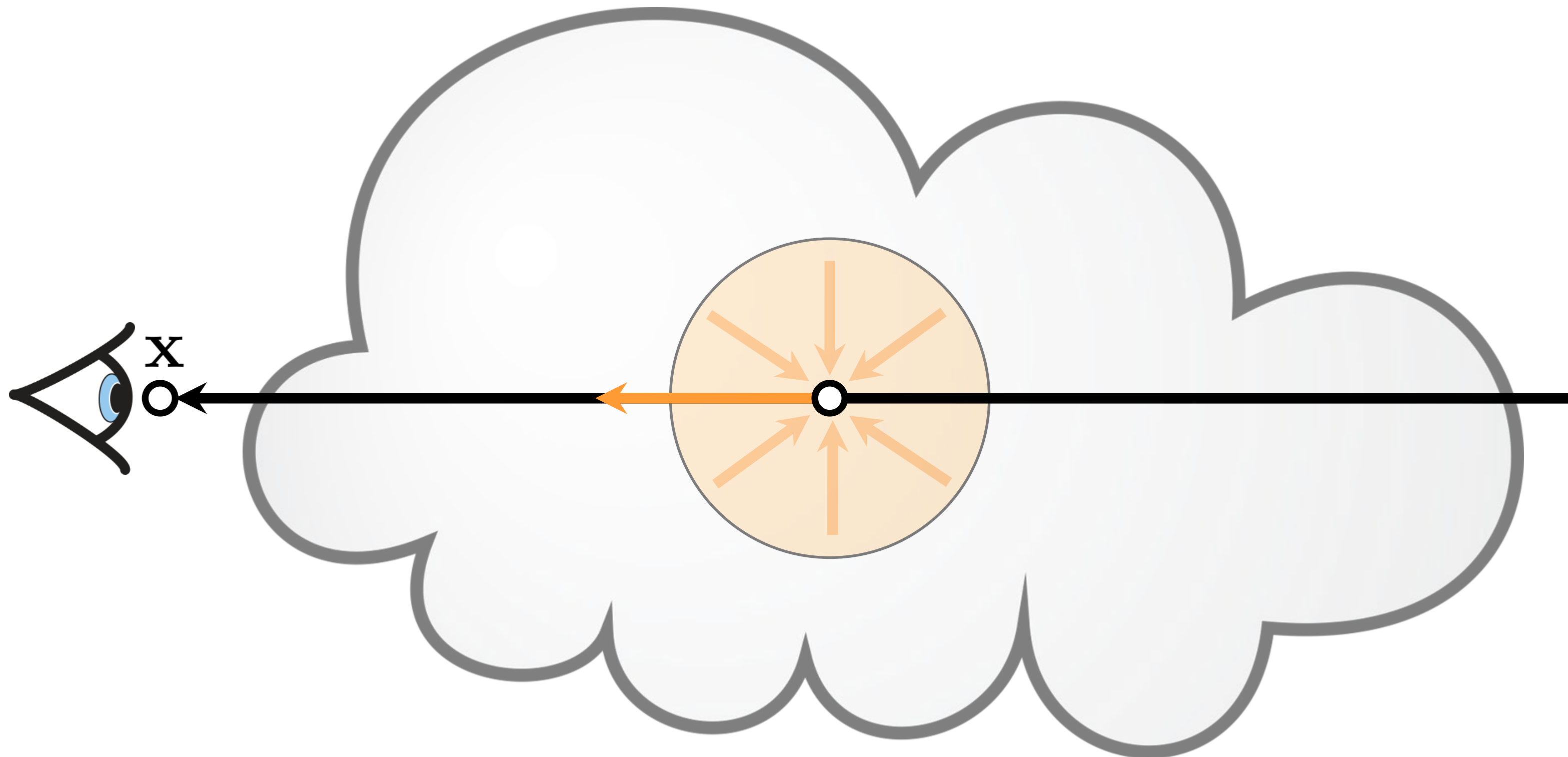
Ray-Marching

Single scattering:

$$L_i(\mathbf{x}_t, \vec{\omega}) = T_r(\mathbf{x}_t, \mathbf{x}_e)\, L_e(\mathbf{x}_e, -\vec{\omega})$$

Another ray-marching needed to estimate the transmittance along the connection ray (in heterogeneous media)
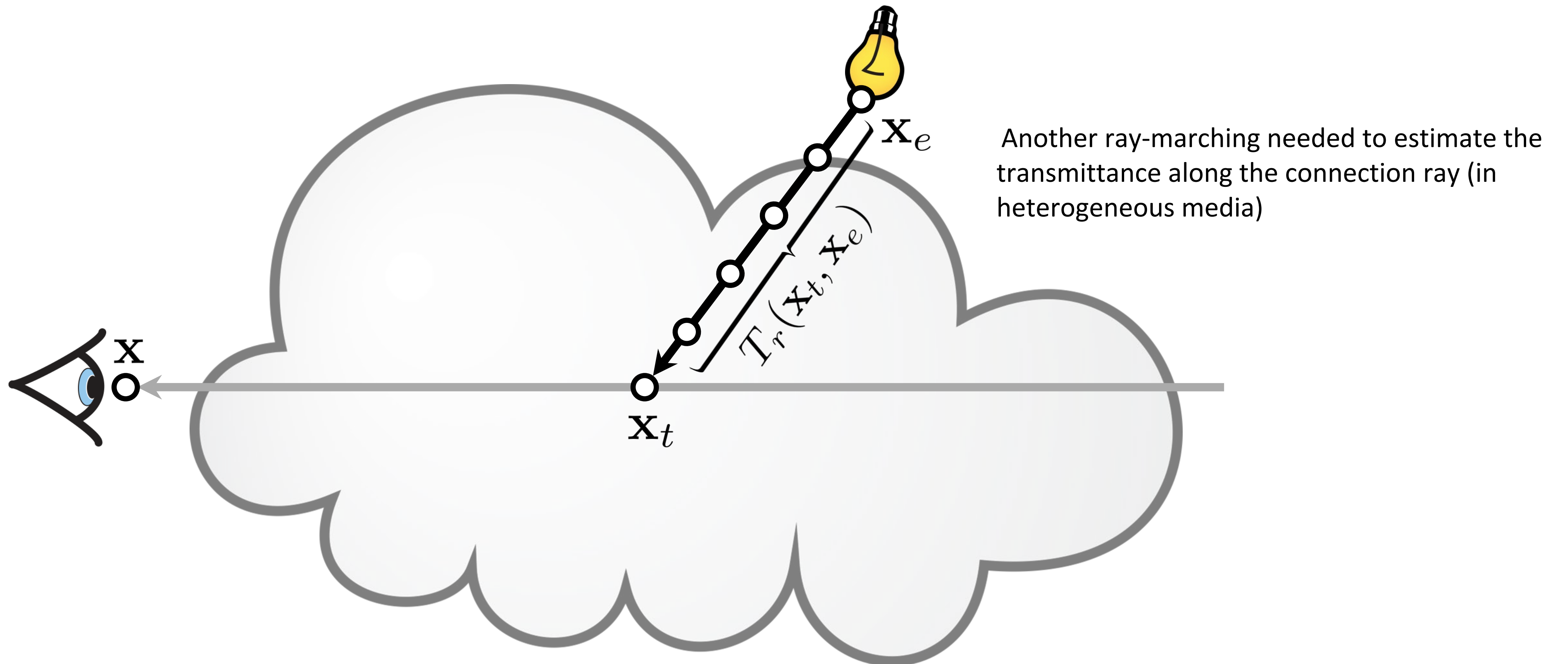
Ray-Marching in Heterogeneous Media

Marching towards the light source
- Connections are expensive, many, and uniformly distributed along the primary ray
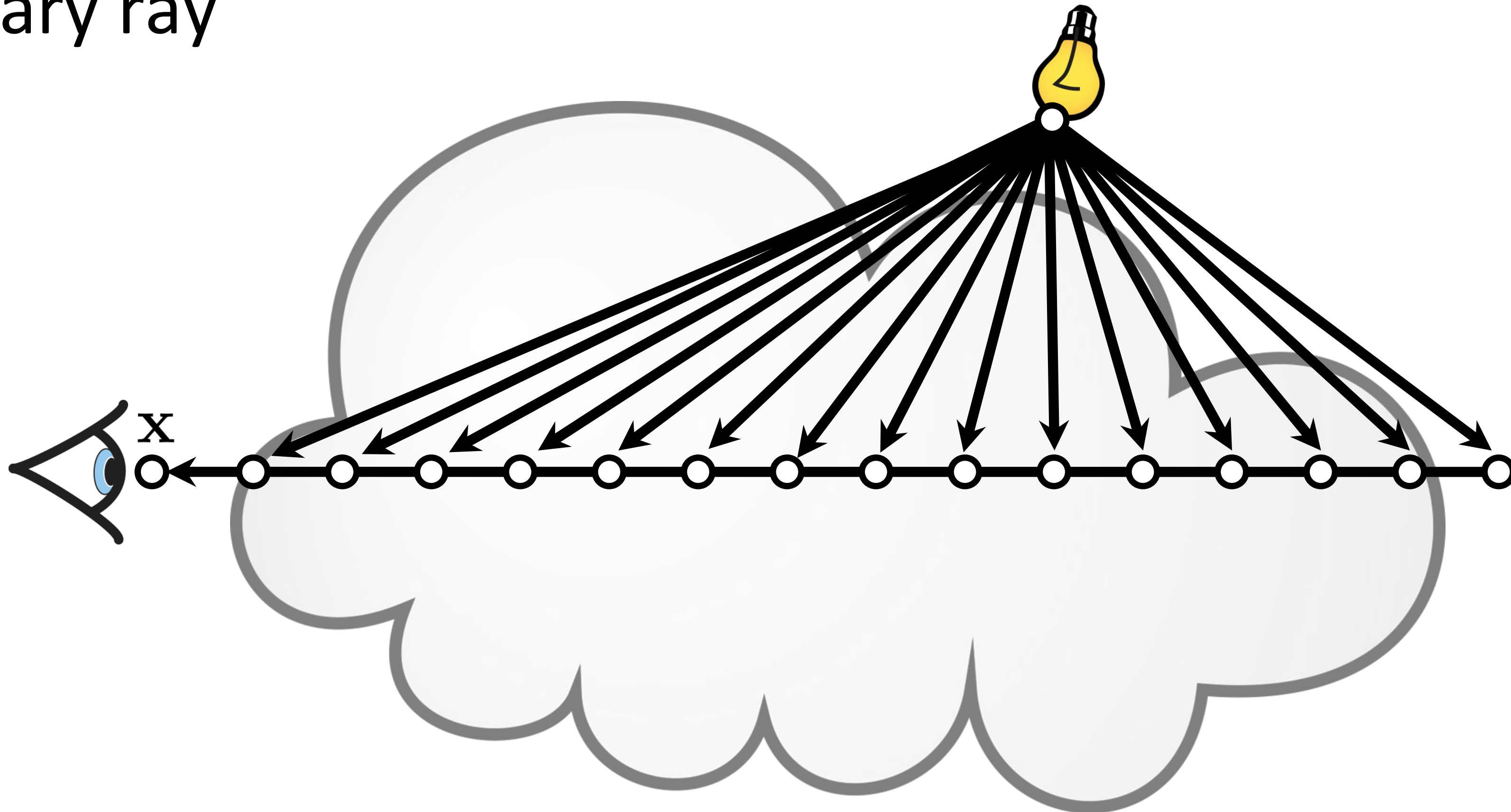
Decoupled Transmittance and In-scattering

1. Ray-march and cache transmittance

- Choose step-size w.r.t. frequency content to accurately capture variations
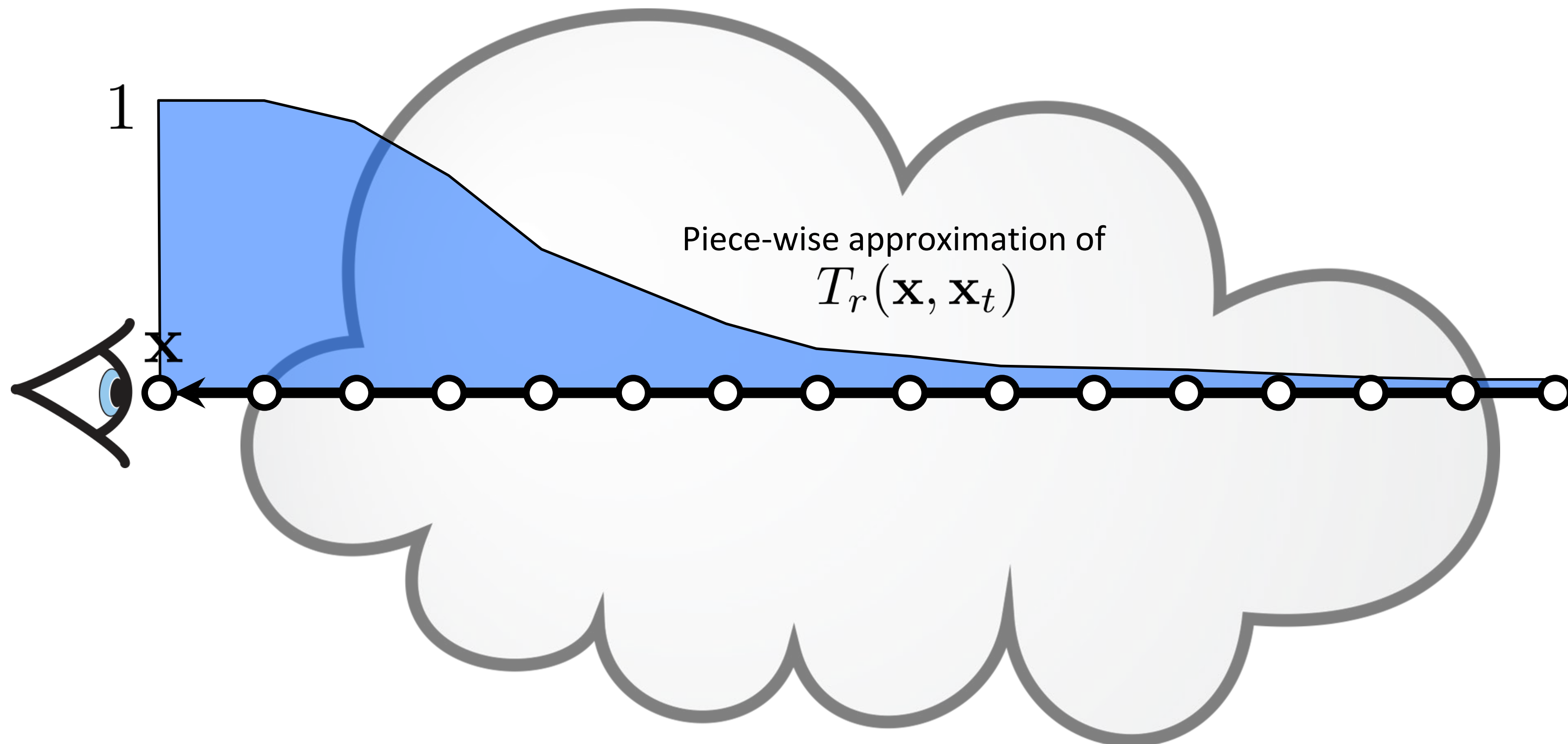
Piece-wise approximation of $T_r(\mathbf{x}, \mathbf{x}_t)$
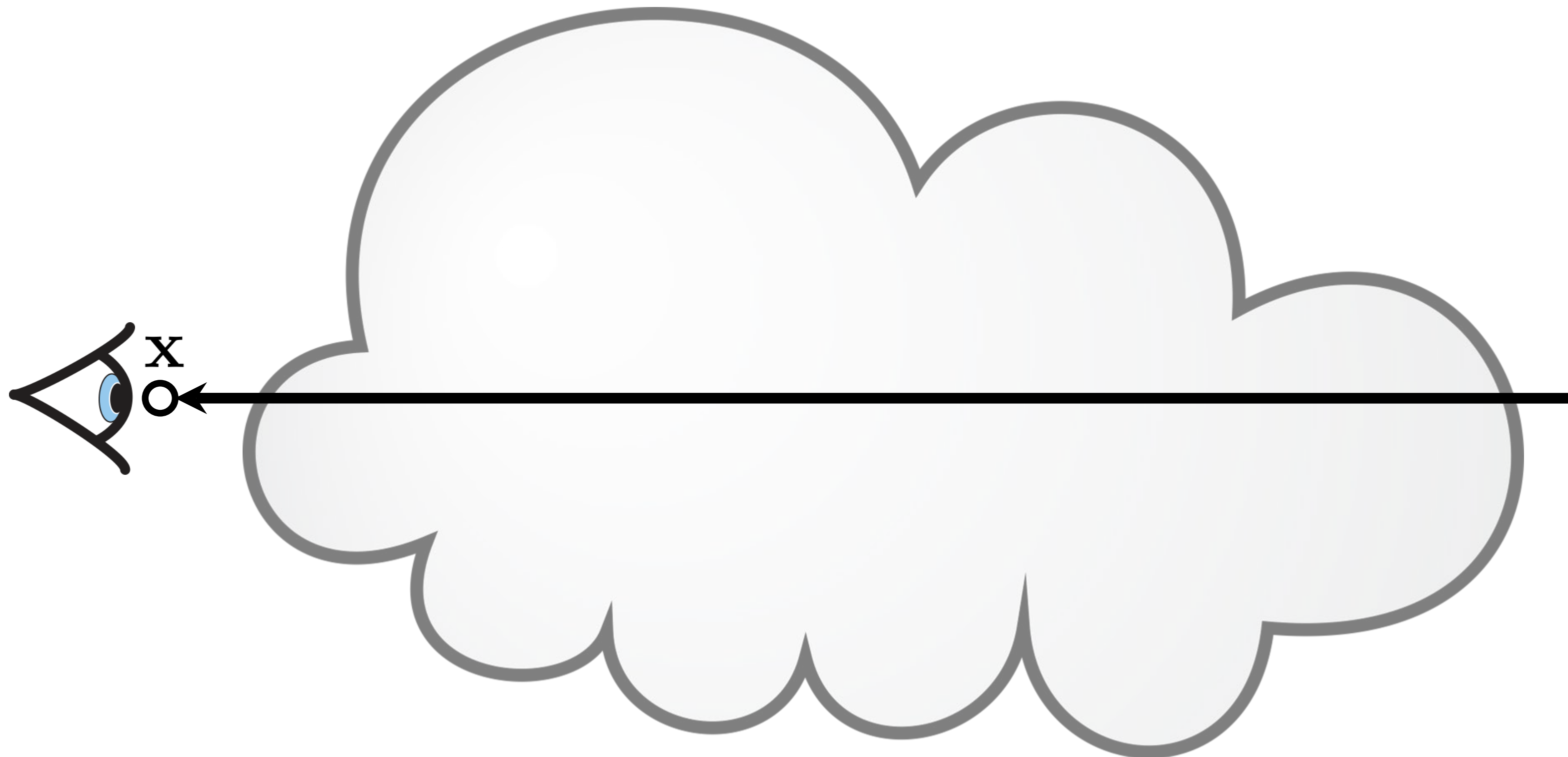
Decoupled Transmittance and In-scattering

2. Estimate in-scattering using MC integration

- Distribute samples $\propto$ (part of) the integrand

Decoupled Transmittance and In-scattering

2. Estimate in-scattering using MC integration

- Distribute samples ∝ (part of) the integrand

$$p(\mathbf{x}_t) \propto T_r(\mathbf{x}, \mathbf{x}_t)$$

Decoupled Transmittance and In-scattering

2. Estimate in-scattering using MC integration

- Distribute samples $\propto$ (part of) the integrand

$$p(\mathbf{x}_t) \propto \frac{1}{d^2}$$

d : distance to light

Decoupled Transmittance and In-scattering

2. Estimate in-scattering using MC integration

- Distribute samples ∝ (part of) the integrand

$$p(\mathbf{x}_t) \propto \frac{1}{d^2}$$

d : distance to light

Decoupled Transmittance and In-scattering

2. Estimate in-scattering using MC integration

- Distribute samples $\propto$ (part of) the integrand

$$p(\mathbf{x}_t) \propto \frac{1}{d^2}$$

d : distance to light

Decoupled Transmittance and In-scattering

2. Estimate in-scattering using MC integration

- Distribute samples $\propto$ (part of) the integrand

Equi-angular
sampling
[Kulla and Fajardo 2012]

$$p(\mathbf{x}_t) \propto \frac{1}{d^2}$$

d : distance to light

Decoupled Transmittance and In-scattering

Ray-marching
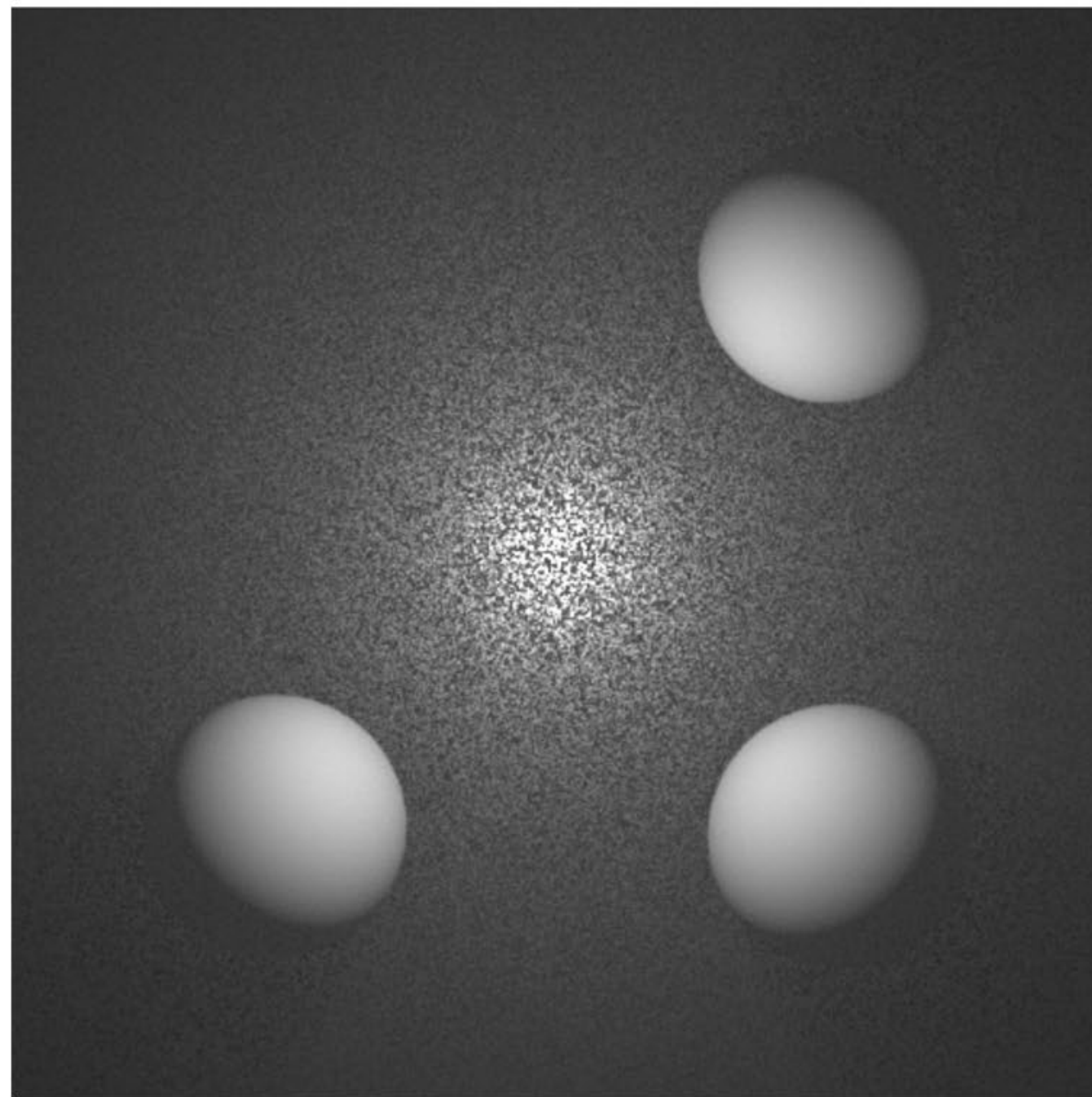
Equiangular sampling
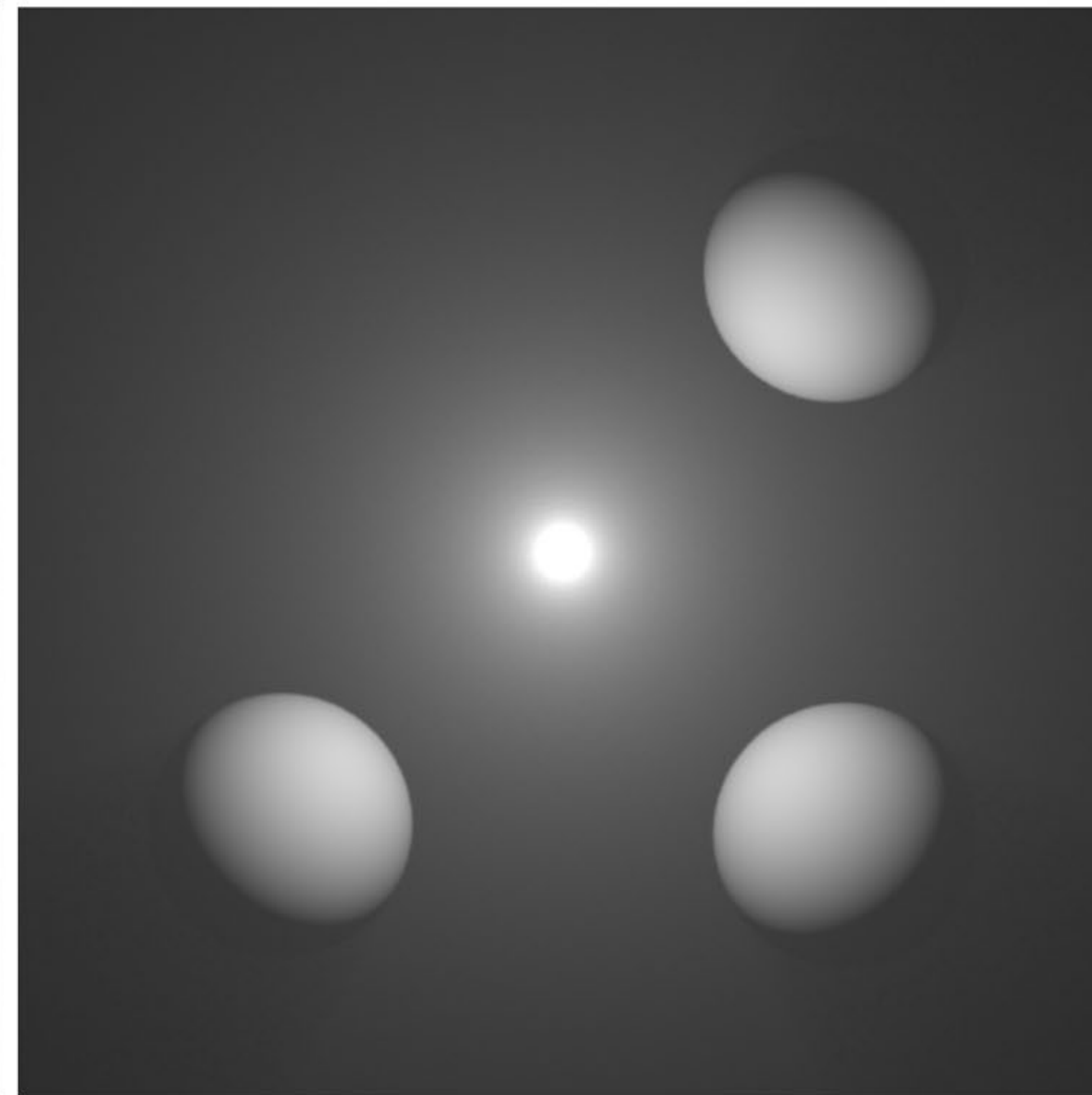
Multiple Bounces

Same concept as in recursive Monte Carlo ray tracing, but taking into account volumetric scattering

Exponential growth:

Visual Break

Single scattering

Multiple scattering
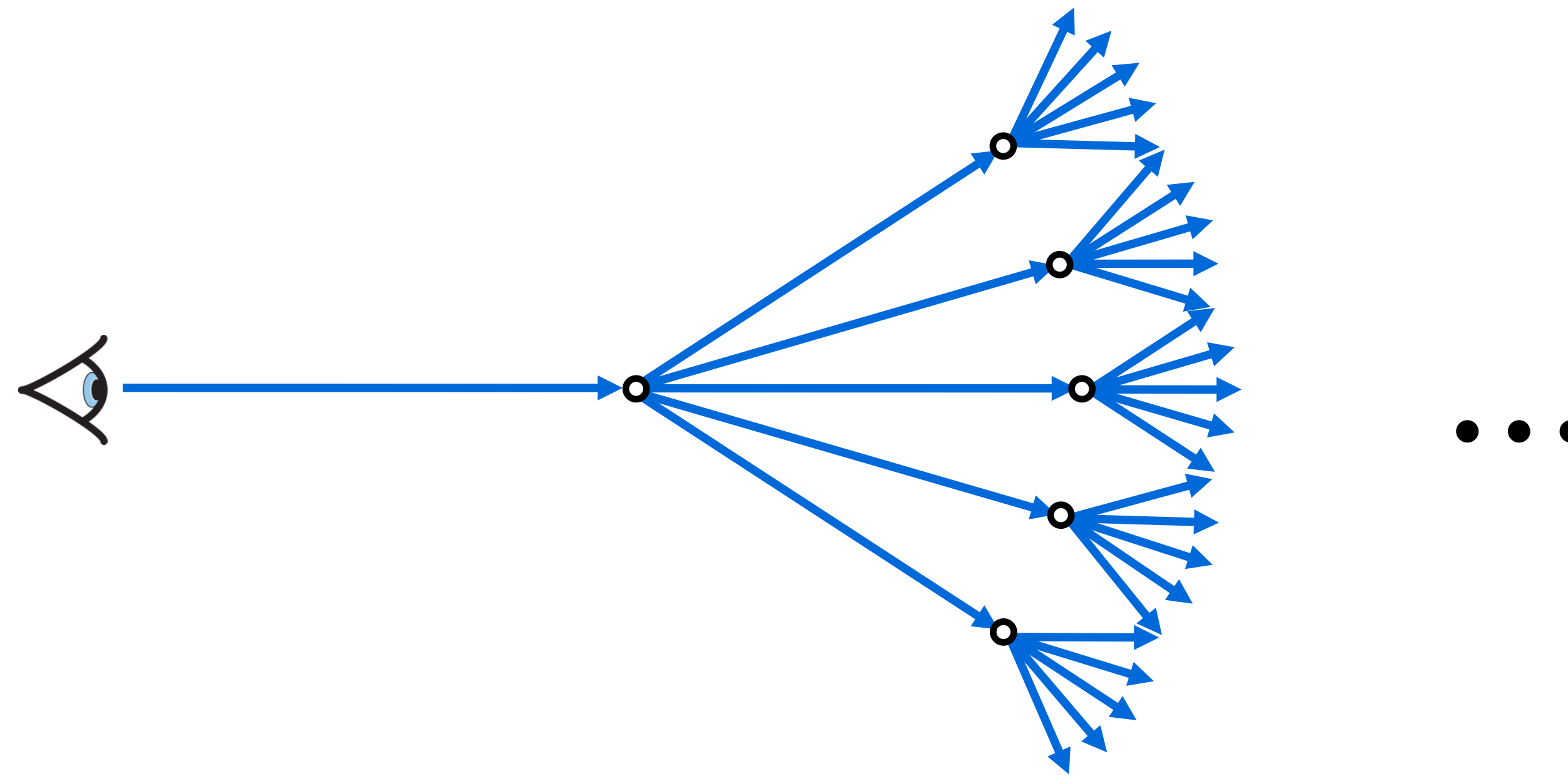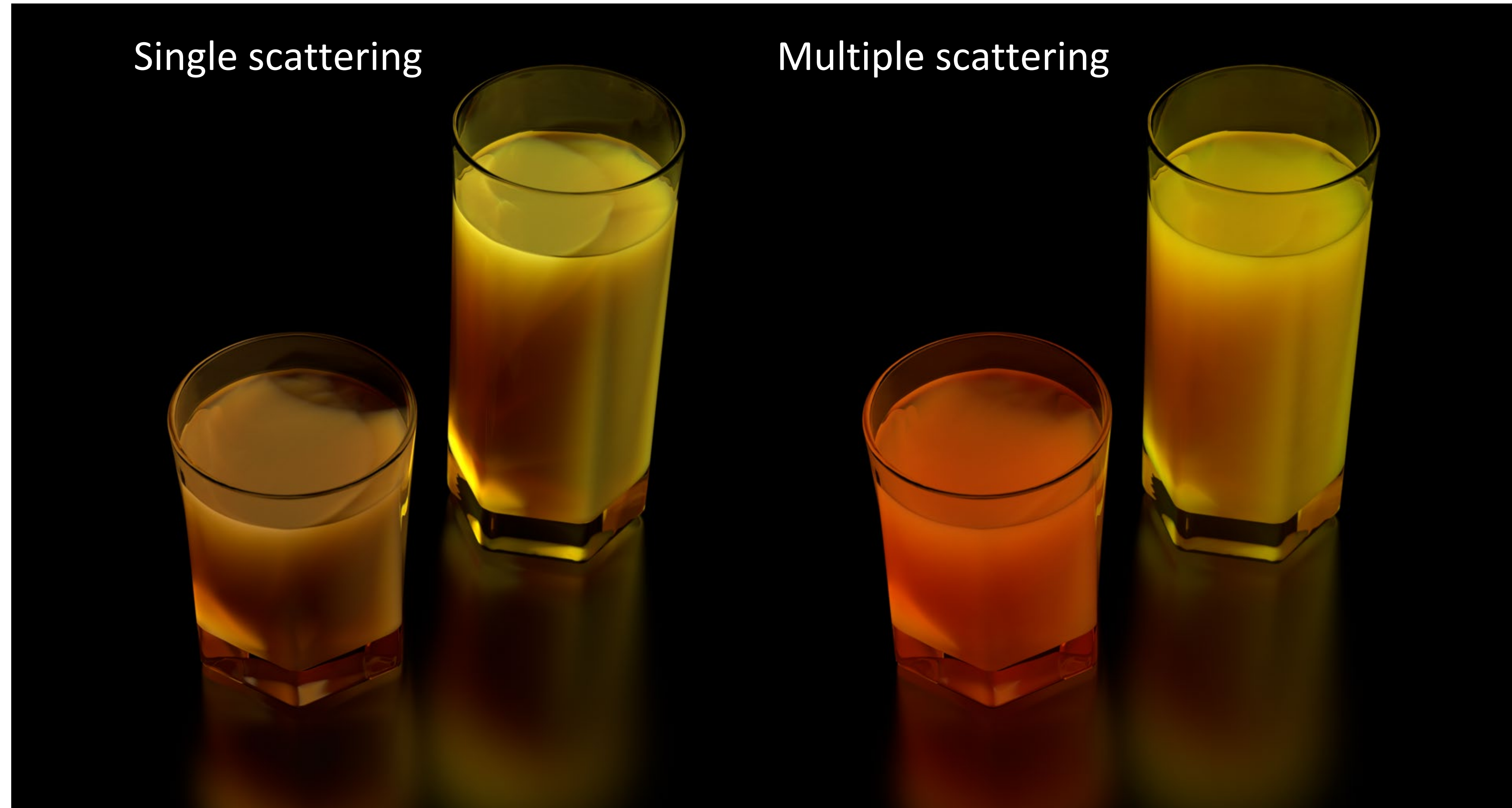
Volumetric Path Tracing

Volumetric Path Tracing

Motivation:

- Same as with standard path tracing: avoid the exponential growth

Paths can:

- Reflect/refract off surfaces

- Scatter inside a volume

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = \boxed{\int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_a(\mathbf{x}_t)L_e(\mathbf{x}_t, \vec{\omega})\,\mathrm{d}t}$$

Accumulated emitted radiance

$$+ \boxed{\int_0^z T_r(\mathbf{x}, \mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_s(\mathbf{x}_t, \vec{\omega})\,\mathrm{d}t}$$

Accumulated in-scattered radiance

$$+ \boxed{T_r(\mathbf{x}, \mathbf{x}_z)L(\mathbf{x}_z, \vec{\omega})}$$

Attenuated background radiance

Volume Rendering Equation

Accumulated emitted + in-scattered radiance

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \left[\sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) + \sigma_s(\mathbf{x}_t) L_s(\mathbf{x}_t, \vec{\omega}) \right] \, \mathrm{d}t$$
$$+ T_r(\mathbf{x}, \mathbf{x}_z) L(\mathbf{x}_z, \vec{\omega})$$

Attenuated background radiance

Volume Rendering Equation

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z T_r(\mathbf{x}, \mathbf{x}_t) \left[\sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) + \sigma_s(\mathbf{x}_t) L_s(\mathbf{x}_t, \vec{\omega}) \right] \, \mathrm{d}t$$

$$+ T_r(\mathbf{x}, \mathbf{x}_z) L(\mathbf{x}_z, \vec{\omega})$$

1-Sample Monte Carlo Estimator

$$\langle L(\mathbf{x}, \vec{\omega}) \rangle = \frac{T_r(\mathbf{x}, \mathbf{x}_t)}{p(t)} \left[\sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) + \sigma_s(\mathbf{x}_t) L_s(\mathbf{x}_t, \vec{\omega}) \right]$$

$$+ \frac{T_r(\mathbf{x}, \mathbf{x}_z)}{P(z)} L(\mathbf{x}_z, \vec{\omega})$$

$p(t)$ - probability *density* of distance t

$P(z)$ - *probability* of exceeding distance z

$$P(z) = \int_z^\infty p(t)\,dt$$
$$= 1 - \int_0^z p(t)\,dt$$

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1-Sample Monte Carlo Estimator

$$\langle L(\mathbf{x}, \vec{\omega}) \rangle = \frac{T_r(\mathbf{x}, \mathbf{x}_t)}{p(t)} \left[\sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) + \sigma_s(\mathbf{x}_t) \frac{f_p(\vec{\omega}, \vec{\omega}_i) L(\mathbf{x}_t, \vec{\omega}_i)}{p(\vec{\omega}_i)} \right]$$

$$+ \frac{T_r(\mathbf{x}, \mathbf{x}_z)}{P(z)} L(\mathbf{x}_z, \vec{\omega})$$

$p(t)$ - probability *density* of distance t

$P(z)$ - *probability* of exceeding distance z

$p(\vec{\omega}_i)$ - probability *density* of direction $\qquad \vec{\omega}_i$

Volumetric Path Tracing

1. Sample distance to next interaction

2. Scatter in the volume or bounce off a surface
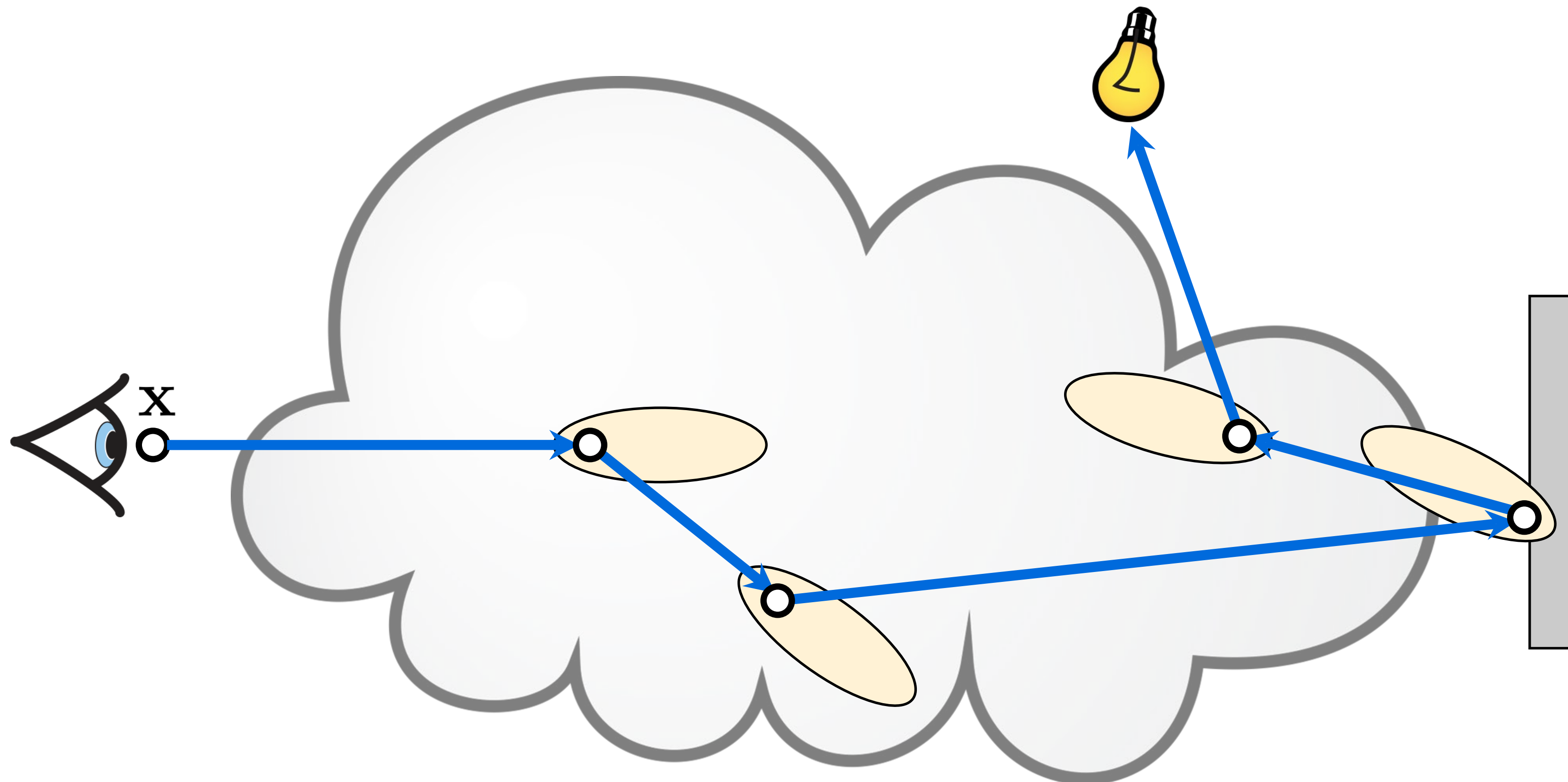
Volumetric Path Tracing with NEE
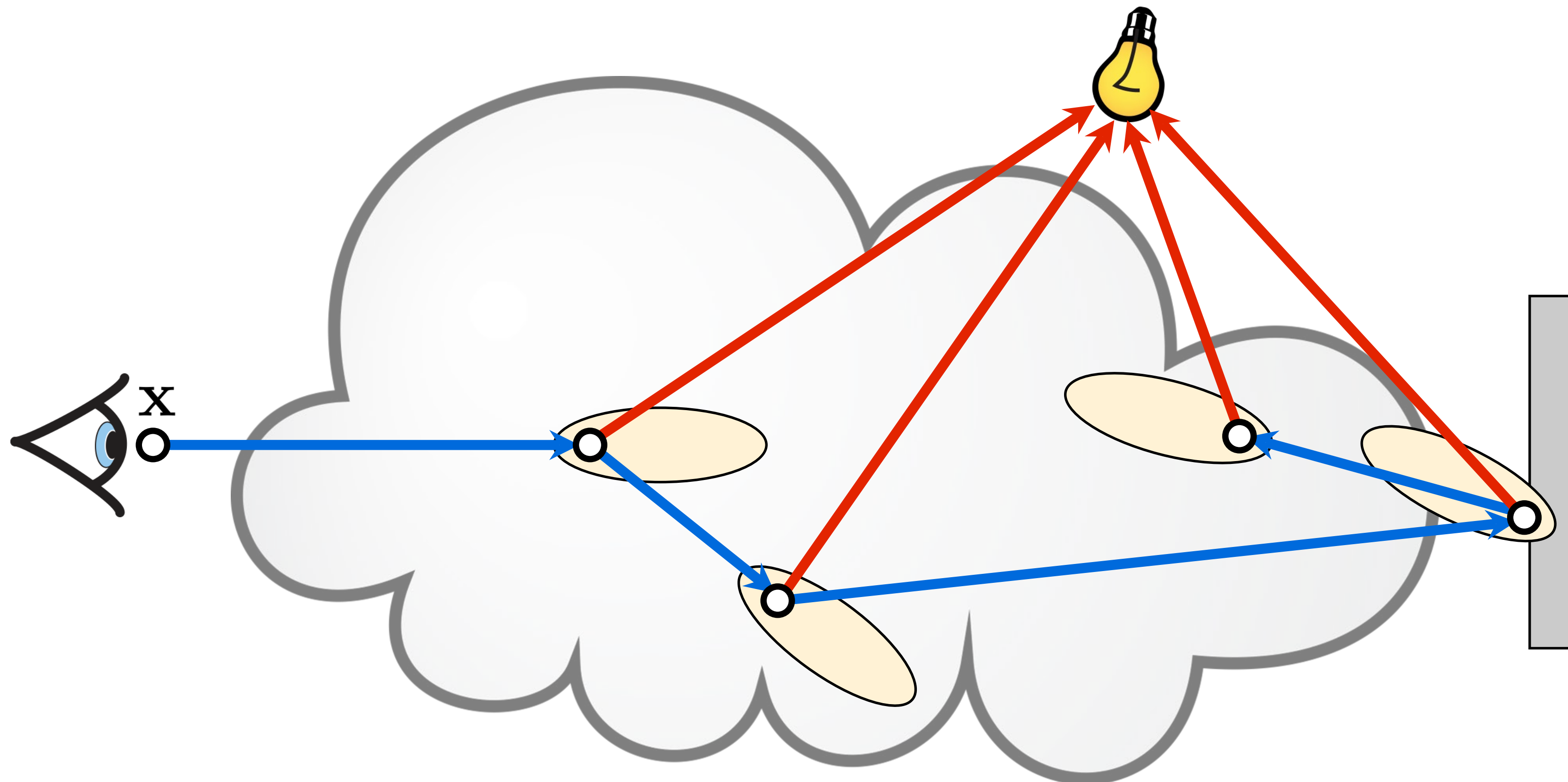
Sampling the Phase Function

Isotropic:

- Uniform sphere sampling

Henyey-Greenstein:

- Using the inversion method we can derive

$$\cos\theta = \frac{1}{2g}\left(1 + g^2 - \left(\frac{1 - g^2}{1 - g + 2g\xi_1)}\right)^2\right)$$

$$\phi = 2\pi\xi_2$$

- PDF is the value of the HG phase function

Free-path Sampling

Free-path (or free-flight distance):

- Distance to the next interaction within the medium

- Dense media (e.g. milk): short mean-free path

- Thin media (e.g. atmosphere): long mean-free path

Ideally, we want to sample proportional to (part of) integrand, e.g. transmittance:

$$p(\mathbf{x}_t | (\mathbf{x}, \vec{\omega})) \propto T_r(\mathbf{x}, \mathbf{x}_t)$$
$$p(t) \propto T_r(t)$$

$\Big)$ simplified notation for brevity

Free-path Sampling

Homogeneous media: $\qquad T_r(t) = e^{-\sigma_t t}$

- PDF: $\quad p(t) \propto e^{-\sigma_t t}$

$$p(t) = \frac{e^{-\sigma_t t}}{\int_0^\infty e^{-\sigma_t s} ds} = \sigma_t e^{-\sigma_t t}$$

- CDF: $\quad P(t) = \int_0^t \sigma_t e^{-\sigma_t s} ds = 1 - e^{-\sigma_t t}$

- Inverted CDF: $\quad P^{-1}(\xi) = -\dfrac{\ln(1 - \xi)}{\sigma_t}$

Free-path Sampling

Homogeneous media: $\quad T_r(t) = e^{-\sigma_t t}$

Recipe:

- Generate random number $\quad \xi$

- Sample distance $\quad t = -\dfrac{\ln(1-\xi)}{\sigma_t}$

- Compute PDF $\quad p(t) = \sigma_t e^{-\sigma_t t}$
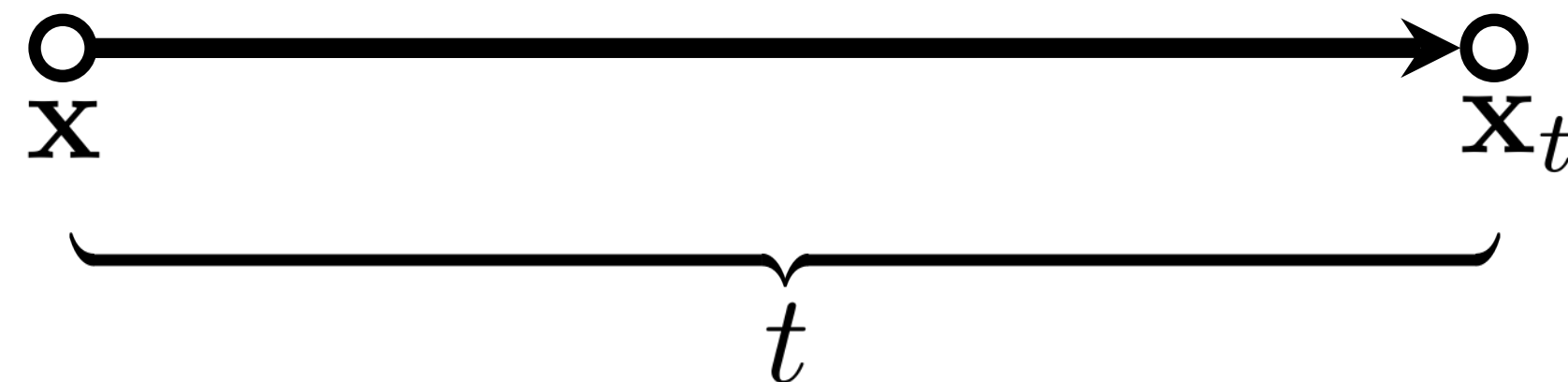
Free-path Sampling

Homogeneous media: $\quad T_r(t) = e^{-\sigma_t t}$

Recipe:

- Generate random number $\quad \xi$

- Sample distance $\quad t = -\dfrac{\cancel{\ln(1-\xi)}}{\cancel{\sigma_t}} = s$

- Compute PDF $\quad p(t) = \cancel{\sigma_t e^{-\sigma_t t}} \quad = e^{-\sigma_t s}$ Note: This is now a probability, not a probability density!

s

Surface hit before reaching $\quad t$

Volumetric PT for Homogeneous Volumes

```
Color vPT(x, ω)
tmax = nearestSurface(x, ω)
t = -log(1 - randf()) / σt   // Sample free path
if t < tmax: // Volume interaction
x += t * ω
pdf_t = σt * exp(-σt * t)
(ω', pdf_ω') = samplePF(ω)
return Tr(t) / pdf_t * (σa * Le(x, ω) + σs * PF(ω, ω') * vPT(x, ω') / pdf_ω')
else: // Surface interaction
x += tmax * ω
Pr_tmax = exp(-σt * tmax)
(ω', pdf_ω') = sampleBRDF(n, ω)
return Tr(tmax) / Pr_tmax * (Le(x, ω) + BRDF(ω, ω') * vPT(x, ω') / pdf_ω')
```

$$\langle L(\mathbf{x}, \vec{\omega}) \rangle = \frac{T_r(\mathbf{x}, \mathbf{x}_t)}{p(t)} \left[\sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) + \sigma_s(\mathbf{x}_t) \frac{f_p(\vec{\omega}, \vec{\omega}_i) L(\mathbf{x}_t, \vec{\omega}_i)}{p(\vec{\omega}_i)} \right] + \frac{T_r(\mathbf{x}, \mathbf{x}_z)}{P(z)} L(\mathbf{x}_z, \vec{\omega})$$

Volumetric PT for Homogeneous Volumes

```
Color vPT(x, ω)
tmax = nearestSurface(x, ω)
t = -log(1 - randf()) / σt    // Sample free path
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pdf_t = σt * exp(-σt * t)
(ω', pdf_ω') = samplePF(ω)
// Note: transmittance and PF cancel out with PDFs except for a constant factor 1/σt
return Tr(t) / pdf_t * (σa * Le(x, ω) + σs * PF(ω, ω') * vPT(x, ω') / pdf_ω')
else: // Surface interaction
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Pr_tmax = exp(-σt * tmax)
(ω', pdf_ω') = sampleBRDF(n, ω)
// Note: transmittance and prob of sampling the distance cancel out
return Tr(tmax) / Pr_tmax * (Le(x, ω) + BRDF(ω, ω') * vPT(x, ω') / pdf_ω')
```

$$\langle L(\mathbf{x}, \vec{\omega}) \rangle = \frac{T_r(\mathbf{x}, \mathbf{x}_t)}{p(t)} \left[\sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) + \sigma_s(\mathbf{x}_t) \frac{f_p(\vec{\omega}, \vec{\omega}_i) L(\mathbf{x}_t, \vec{\omega}_i)}{p(\vec{\omega}_i)} \right] + \frac{T_r(\mathbf{x}, \mathbf{x}_z)}{P(z)} L(\mathbf{x}_z, \vec{\omega})$$

Volumetric PT for Homogeneous Volumes

```
Color vPT(x, ω)
tmax = nearestSurface(x, ω)
t = -log(1 - randf()) / σₜ   // Sample free path
if t < tmax: // Volume interaction
x += t * ω
pdf_t = σₜ * exp(-σₜ * t)
(ω', pdf_ω') = samplePF(ω)
// Note: transmittance and PF cancel out with PDFs except for a constant factor 1/σₜ
return σₐ/σₜ * Lₑ(x, ω) + σₛ/σₜ * vPT(x, ω')
else: // Surface interaction
x += tmax * ω
Pr_tmax = exp(-σₜ * tmax)
(ω', pdf_ω') = sampleBRDF(n, ω)
// Note: transmittance and prob of sampling the distance cancel out
return Lₑ(x, ω) + BRDF(ω, ω') * vPT(x, ω') / pdf_ω'
```

$$\langle L(\mathbf{x}, \vec{\omega}) \rangle = \frac{T_r(\mathbf{x}, \mathbf{x}_t)}{p(t)} \left[\sigma_a(\mathbf{x}_t) L_e(\mathbf{x}_t, \vec{\omega}) + \sigma_s(\mathbf{x}_t) \frac{f_p(\vec{\omega}, \vec{\omega}_i) L(\mathbf{x}_t, \vec{\omega}_i)}{p(\vec{\omega}_i)} \right] + \frac{T_r(\mathbf{x}, \mathbf{x}_z)}{P(z)} L(\mathbf{x}_z, \vec{\omega})$$

What about heterogeneous media?

Free-path Sampling

Heterogeneous media: $\qquad T_r(t) = e^{\int_0^t -\sigma_t(s)ds}$

- Closed-form solutions exist only for simple media

  - e.g. linearly or exponentially varying extinction

- Other solutions:

  - Regular tracking (3D DDA)

  - Ray marching

  - Delta tracking

Free-path Sampling

How to sample the flight distance to the next interaction?

Random variable representing flight distance

$$T(t) = e^{-\int_0^t \sigma_t(s)\,ds} = \boxed{\begin{array}{c} P(X > t) \\ P(X \leq t) \end{array}} = F(t)$$

CDF

Partition of unity

$$F(t) = 1 - T(t)$$

Recipe for generating samples

Free-path Sampling

Cumulative distribution function (**CDF**)

$$F(t) = 1 - T(t) = 1 - e^{-\tau(t)}$$

Probability density function (**PDF**)

$$p(t) = \frac{dF(t)}{dt} = \frac{d}{dt}\left(1 - e^{-\tau(t)}\right) = \sigma_t(t)e^{-\tau(t)}$$

Inverted cumulative distr. function (**CDF^{-1}**)

$$\xi = 1 - e^{-\tau(t)}$$

↰**Solve for t**

$$\int_0^t \sigma_t(s)\, ds = -\ln(1 - \xi)$$

Approaches for finding t:
1) **ANALYTIC (closed-form CDF^{-1})**
2) **SEMI-ANALYTIC (regular tracking)**
3) **APPROXIMATE (ray marching)**

Regular Tracking (Semi-Analytic)

For piecewise-simple (e.g. piecewise-constant), summation replaces integration

$$\int_0^t \sigma_{\mathrm{t}}(s)\,\mathrm{d}s = -\ln(1-\xi)$$

$$\sum_{i=1}^k \sigma_{\mathrm{t},i}\,\Delta_i = -\ln(1-\xi)$$

(Hierarchical) voxel grid

LHS > RHS

Sampled collision
LHS=RHS

Start

Δ_1

Regular tracking:

1) Draw a random number ξ
2) While LHS < RHS
   move to the next intersection
3) Find the exact location
   in the last segment analytically

Ray Marching

Find the collision distance approximately

$$\int_0^t \sigma_\mathrm{t}(s)\,\mathrm{d}s = -\ln(1-\xi)$$

$$\sum_{i=1}^{k} \sigma_{\mathrm{t},i}\,\Delta = -\ln(1-\xi)$$

Constant step

(Hierarchical) voxel grid

LHS > RHS

Start

Sampled collision LHS=RHS

Ray marching:

1) Draw a random number ξ
2) While LHS < RHS make a (fixed-size) step
3) Find the exact location in the last segment analytically
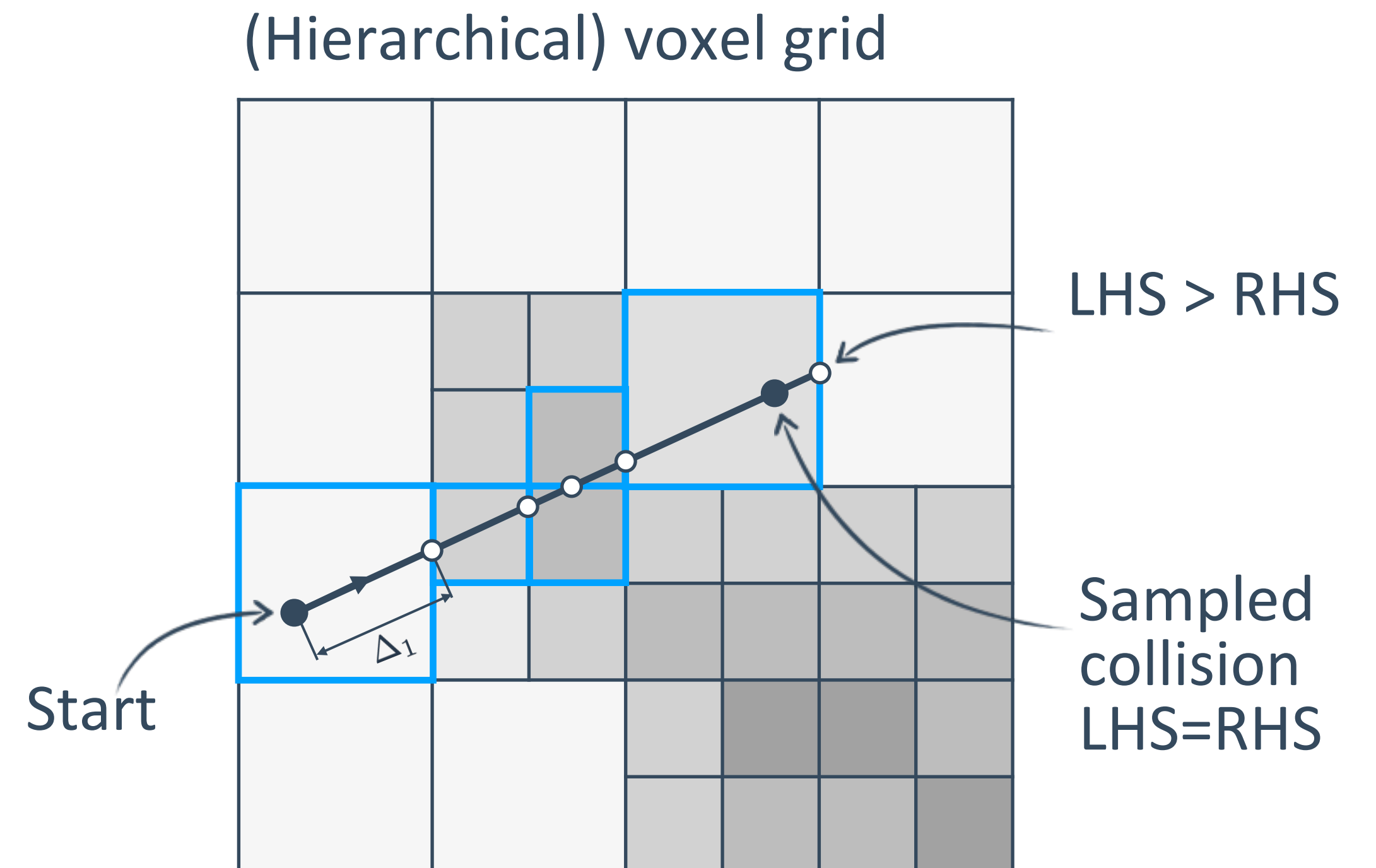
Ray Marching

Find the collision distance approximately

$$\int_0^t \sigma_{\mathrm{t}}(s)\,\mathrm{d}s = -\ln(1-\xi)$$

$$\sum_{i=1}^{k} \sigma_{\mathrm{t},i}\,\Delta = -\ln(1-\xi)$$

Constant step

Ray marching:

1) Draw a random number ξ

2) While LHS < RHS
   make a (fixed-size) step

3) Find the exact location
   in the last segment analytically

General volume

LHS > RHS

Start

Sampled
collision
LHS=RHS

Ray Marching

Find the collision distance approximately

$$\int_0^t \sigma_{\mathrm{t}}(s)\,\mathrm{d}s = -\ln(1-\xi)$$

$$\sum_{i=1}^{k} \sigma_{\mathrm{t},i}\,\Delta = -\ln(1-\xi)$$

Constant step

General volume

LHS > RHS

Ray marching:

1) Draw a random number ξ
2) While LHS < RHS
   make a (fixed-size) step
3) Find the exact location
   in the last segment analytically

Start

Ignored thin
features = bias

Sampled
collision
LHS=RHS
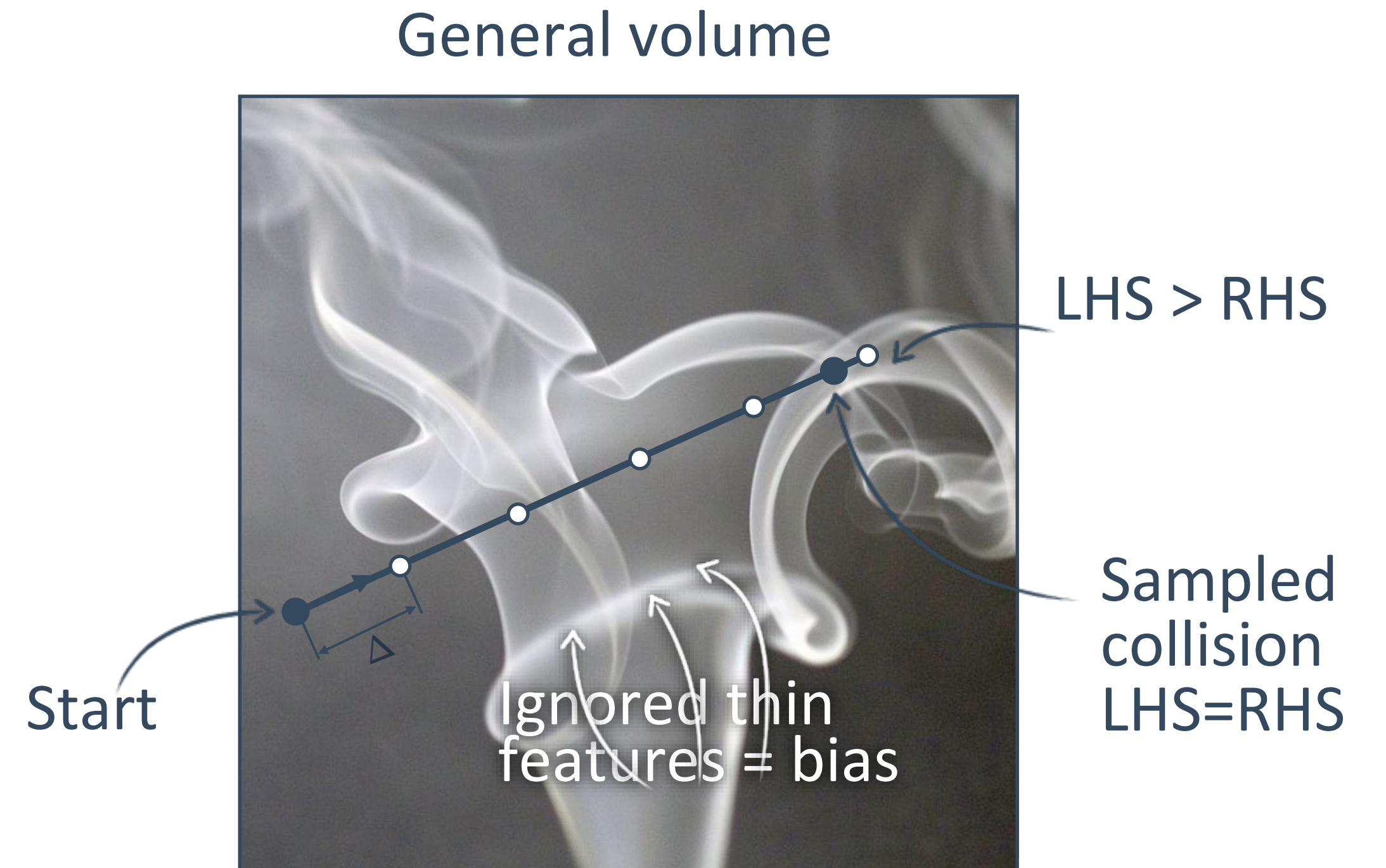
Free-path Sampling

ANALYTIC CDF^{-1}	REGULAR TRACKING	RAY MARCHING
▶ Efficient & simple, limited to few volumes	▶ Iterative, inefficient if free paths cross many boundaries	▶ Iterative, inaccurate (or inefficient) for media with high frequencies
▶ Simple volumes (e.g. homogeneous)	▶ Piecewise-simple volumes	▶ Any volume
▶ Unbiased	▶ Unbiased	▶ Biased

Common approach: sample optical thickness, find corresponding distance

Delta Tracking

(a.k.a. Woodcock tracking, pseudo scattering, hole tracking, null-collision method,...)

Delta tracking idea

Add **FICTITIOUS MATTER** to homogenize medium

- albedo: $\alpha(\mathbf{x}) = 1$

- phase function: $f_p(\vec{\omega}, \vec{\omega}') = \delta(\vec{\omega} - \vec{\omega}')$

Incident
light

Outgoing
light

Fictitious particle
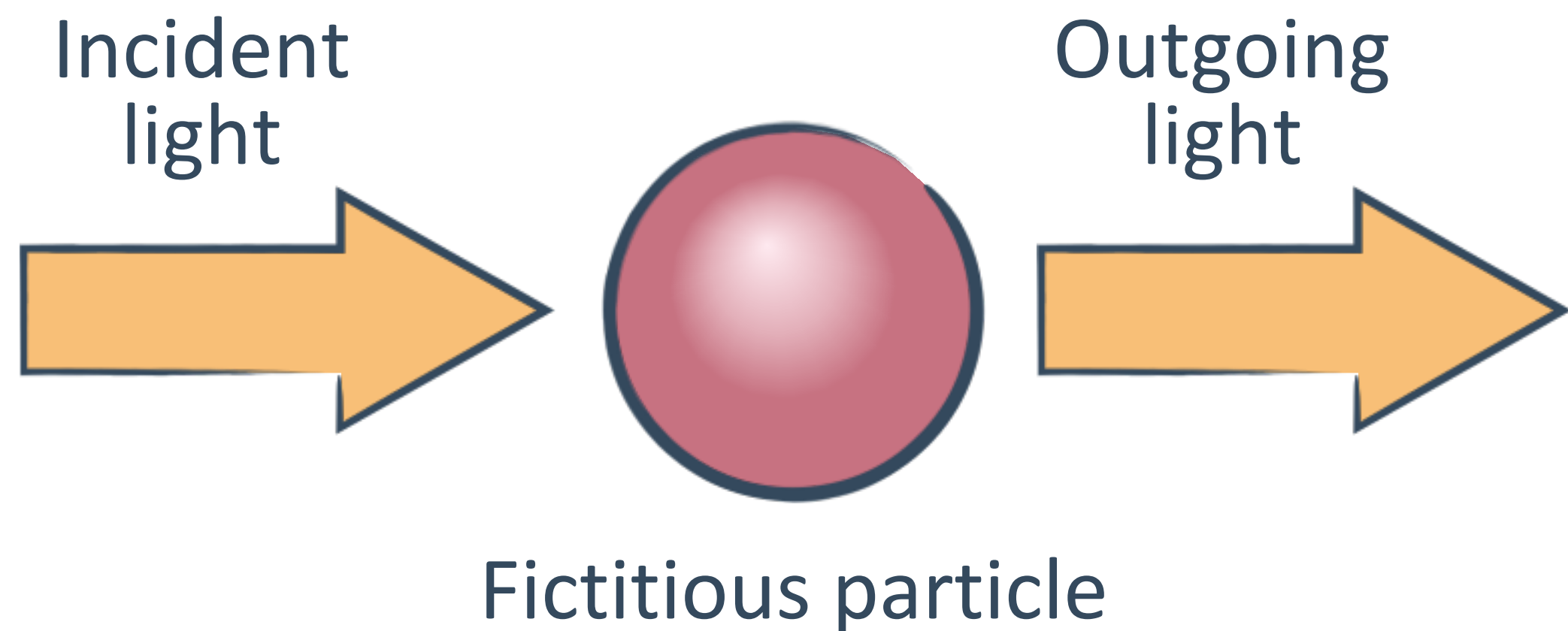
Presence of fictitious matter
does not impact light transport

Homogenization
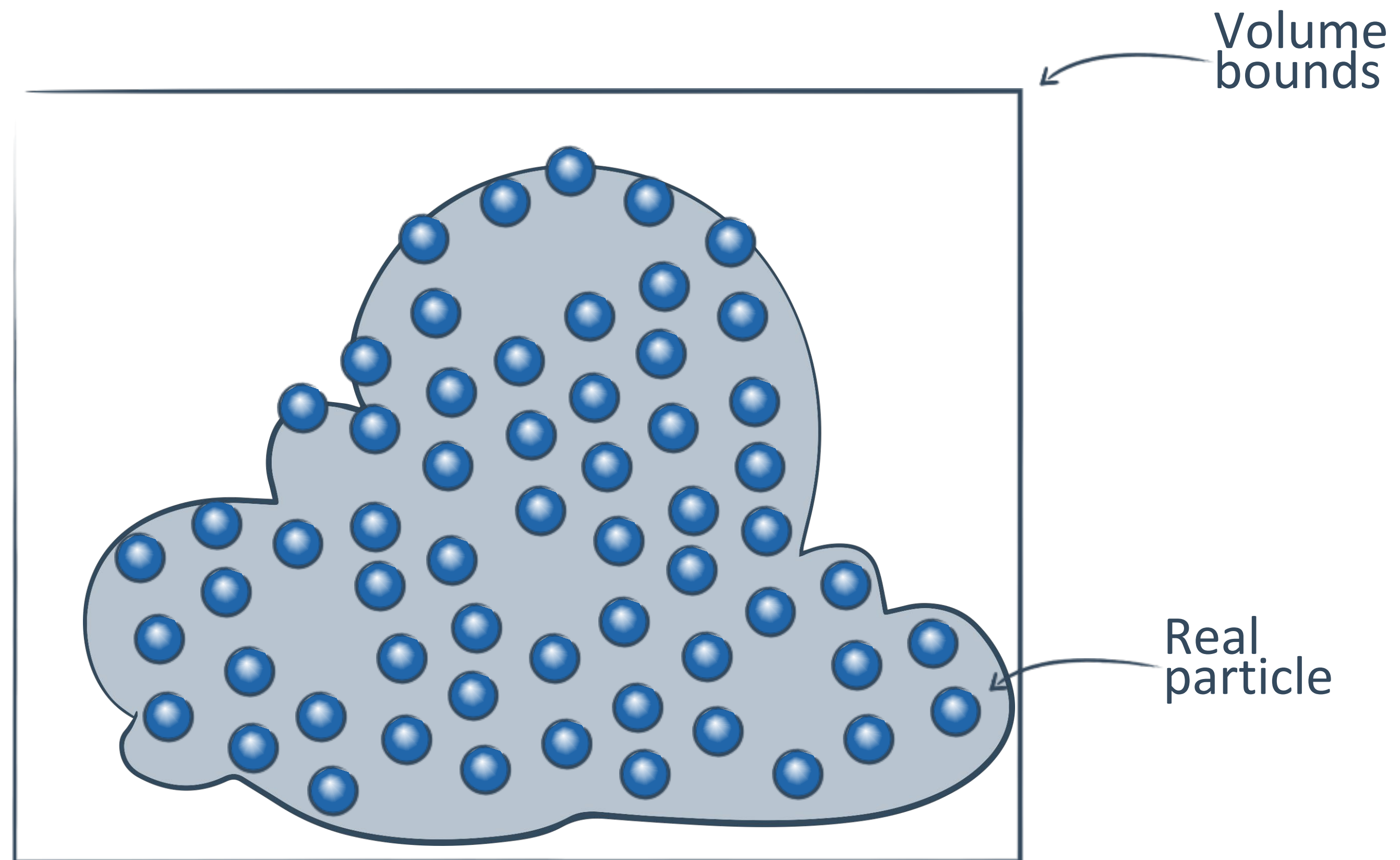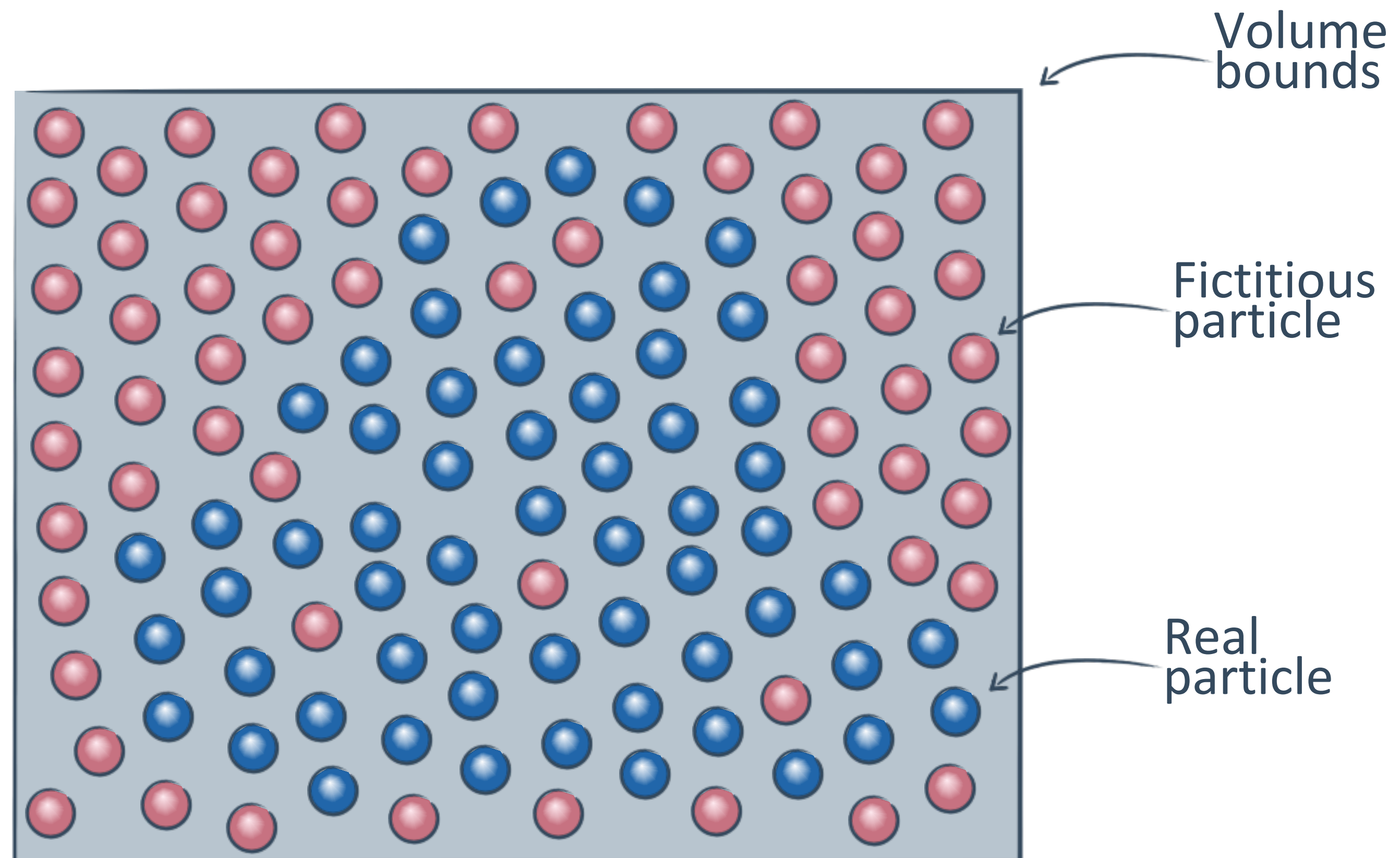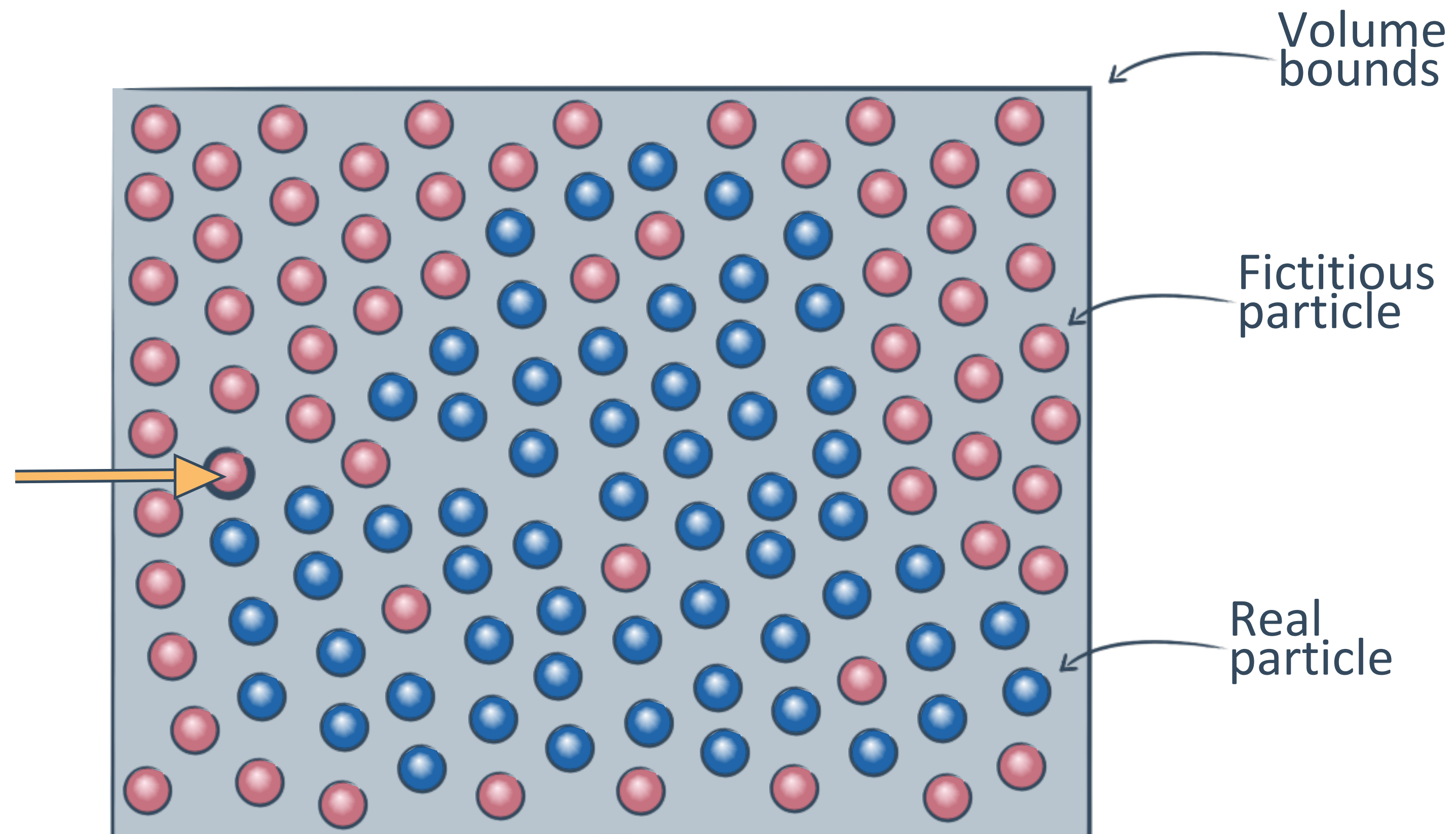
Volume bounds

Real particle

Homogenization

Homogenization

Homogenization
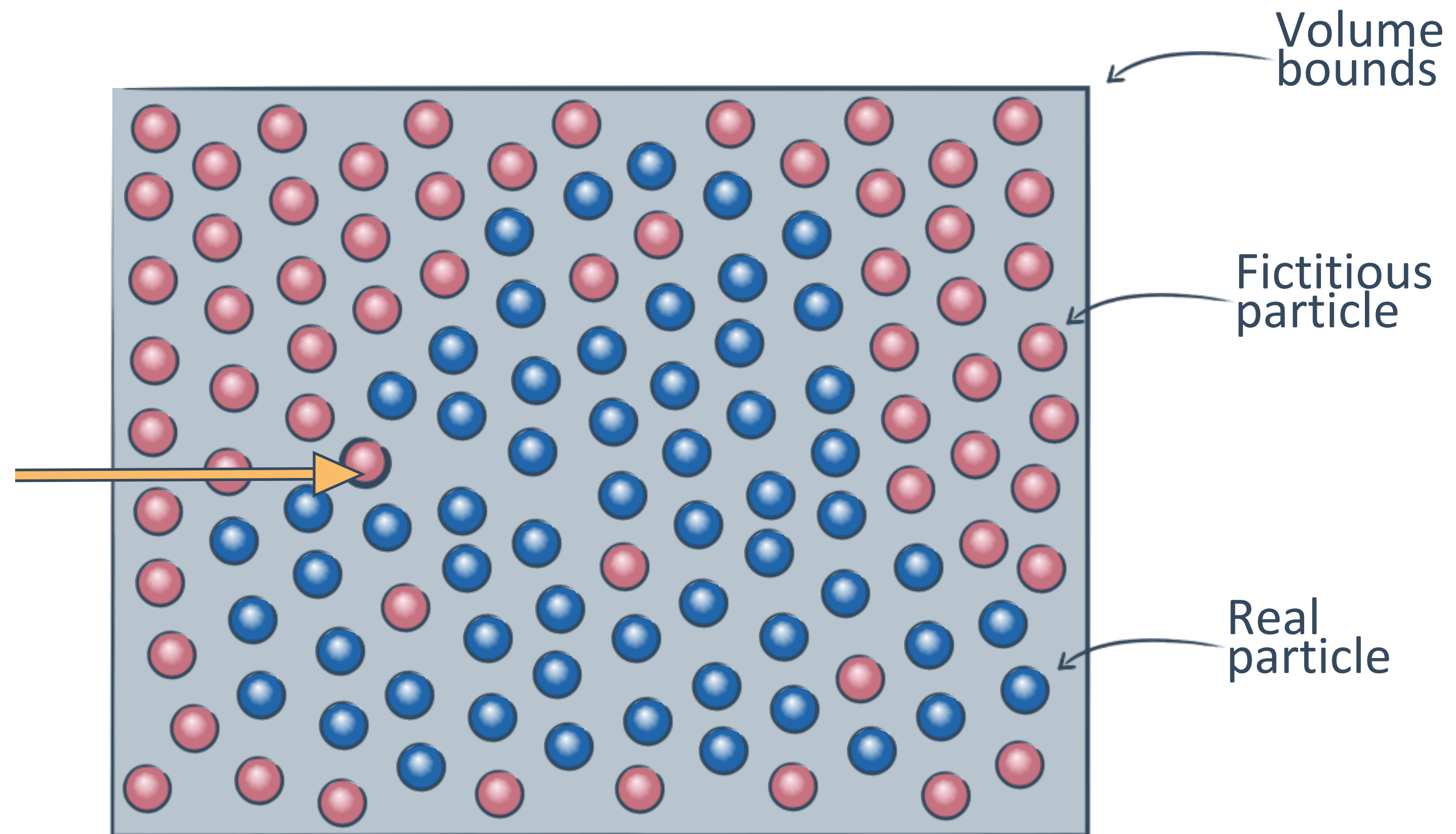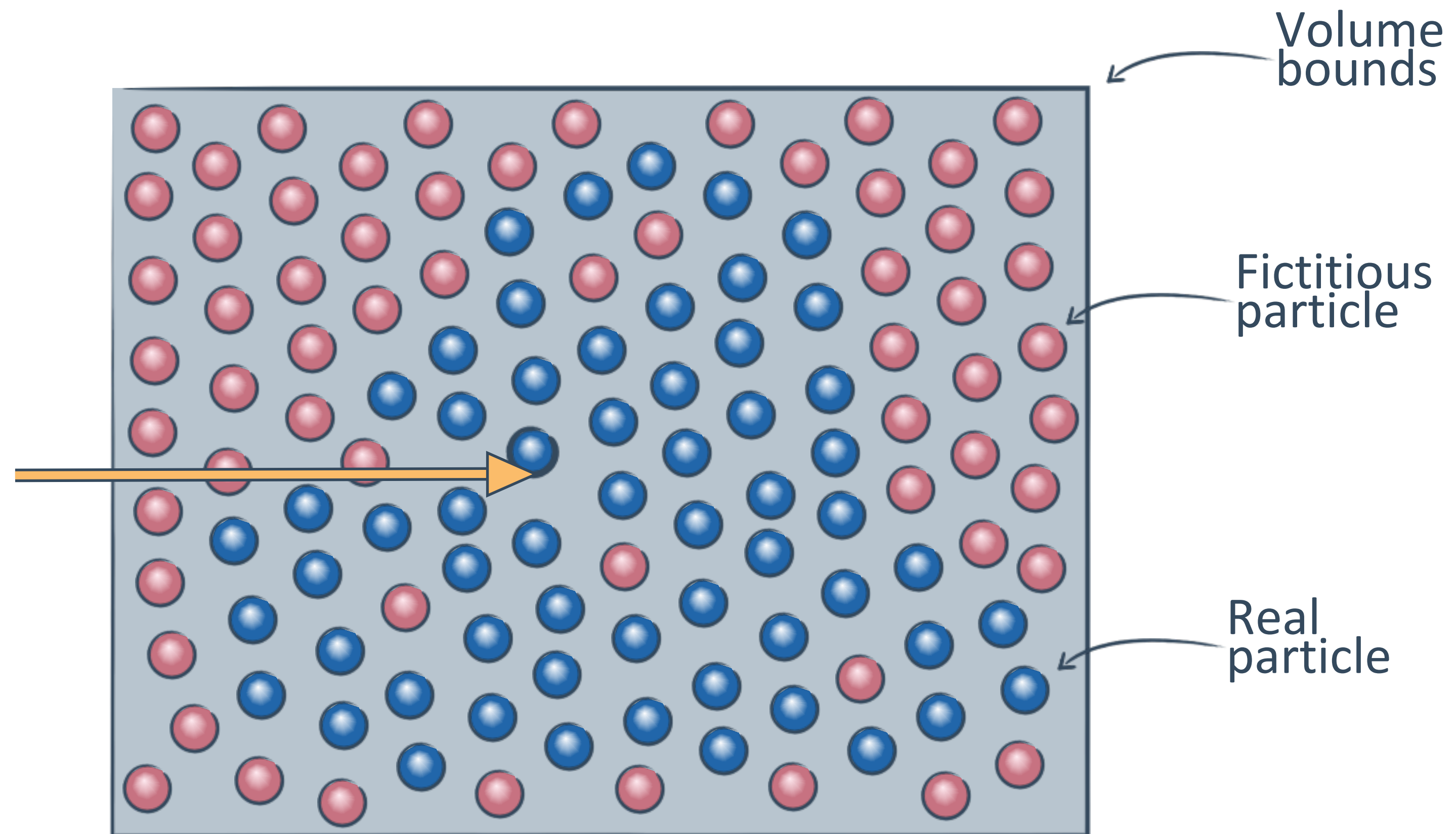
Volume bounds

Fictitious particle

Real particle

Homogenization

Volume bounds

Fictitious particle

Real particle

Homogenization

Homogenization

Volume bounds

Fictitious particle

Real particle

Homogenization

Volume bounds

Fictitious particle

Real particle

Homogenization

Homogenization

Volume bounds

Fictitious particle

Real particle

Homogenization

Volume bounds

Fictitious particle

Real particle
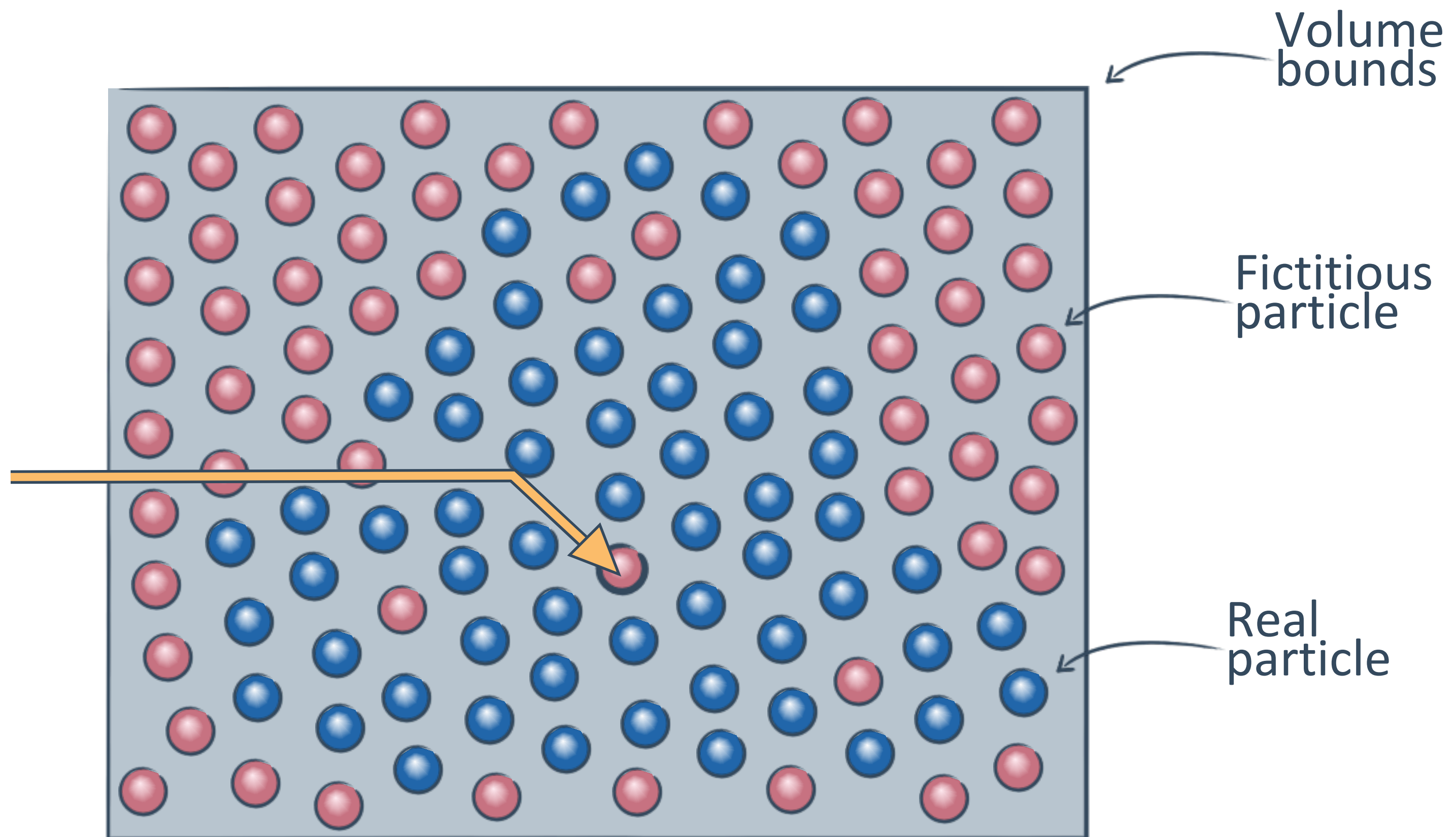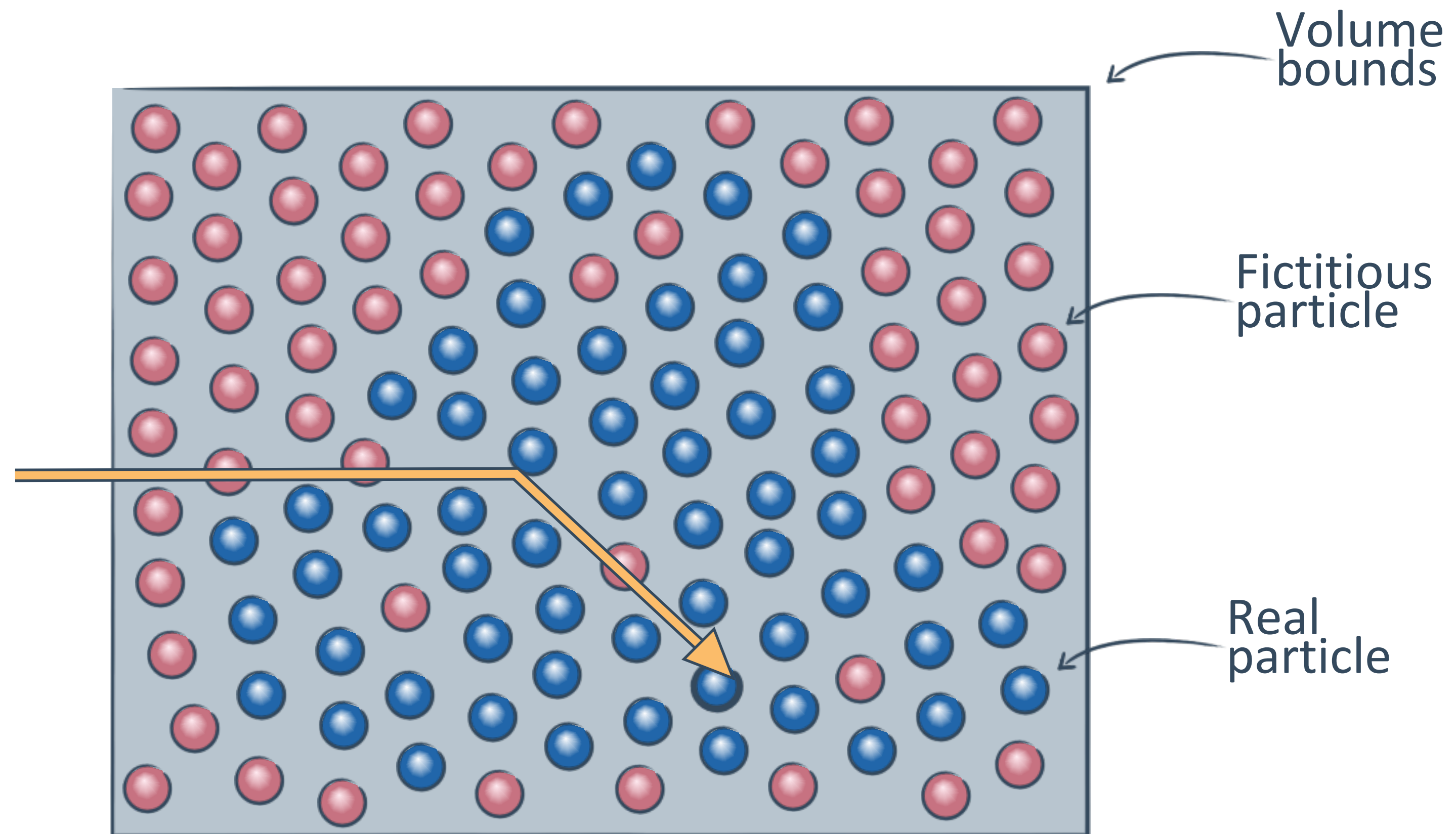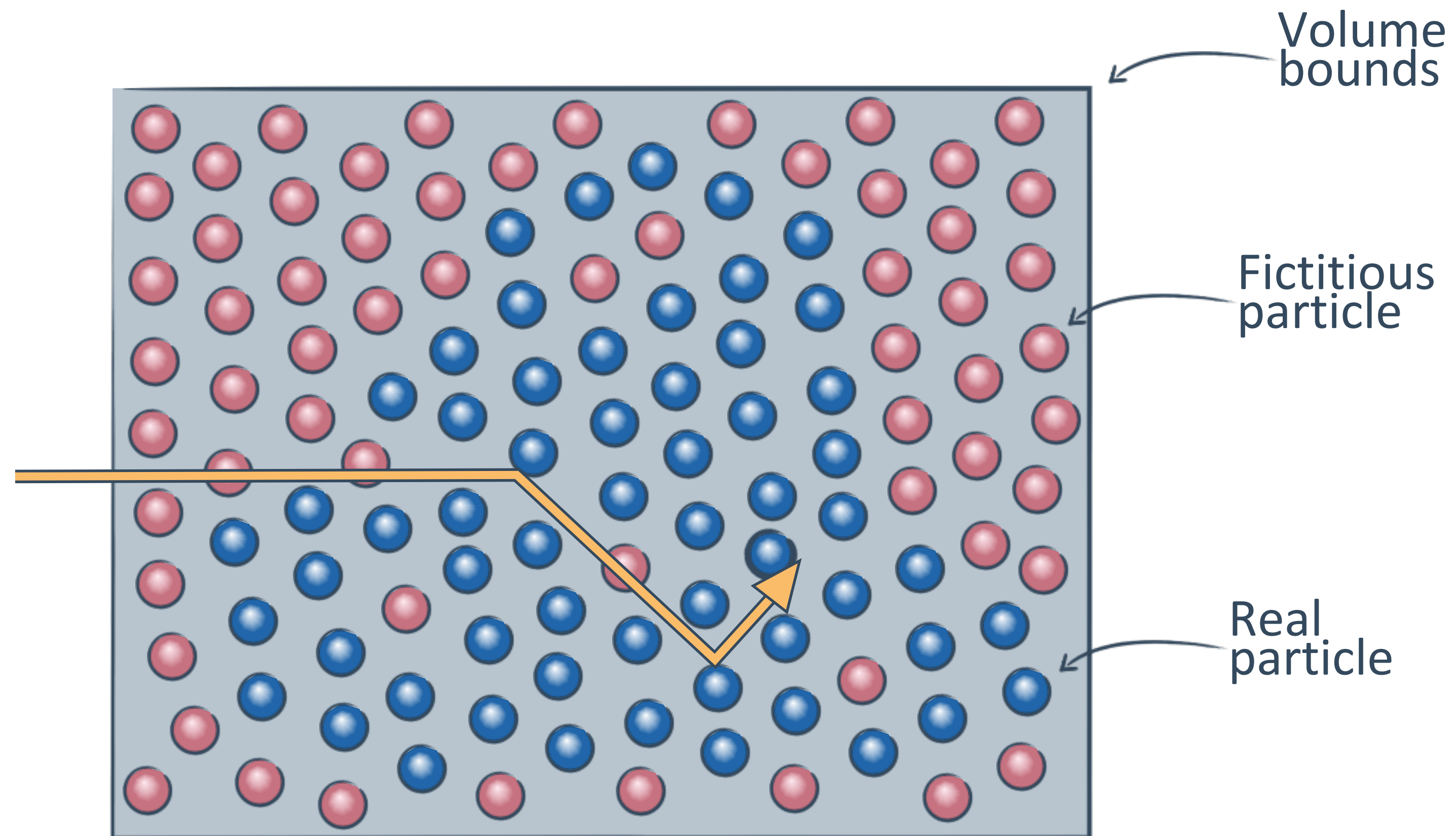
Homogenization

Stochastic Sampling
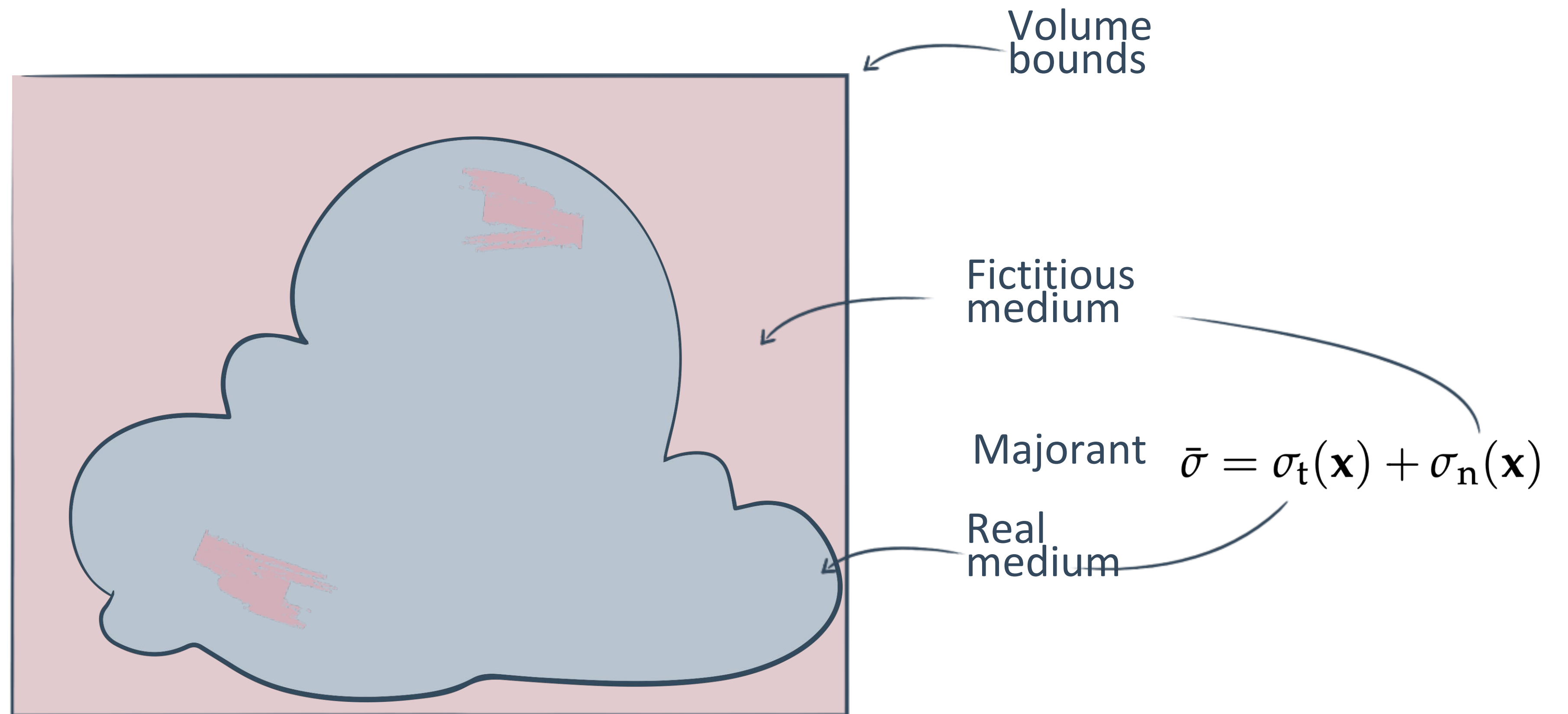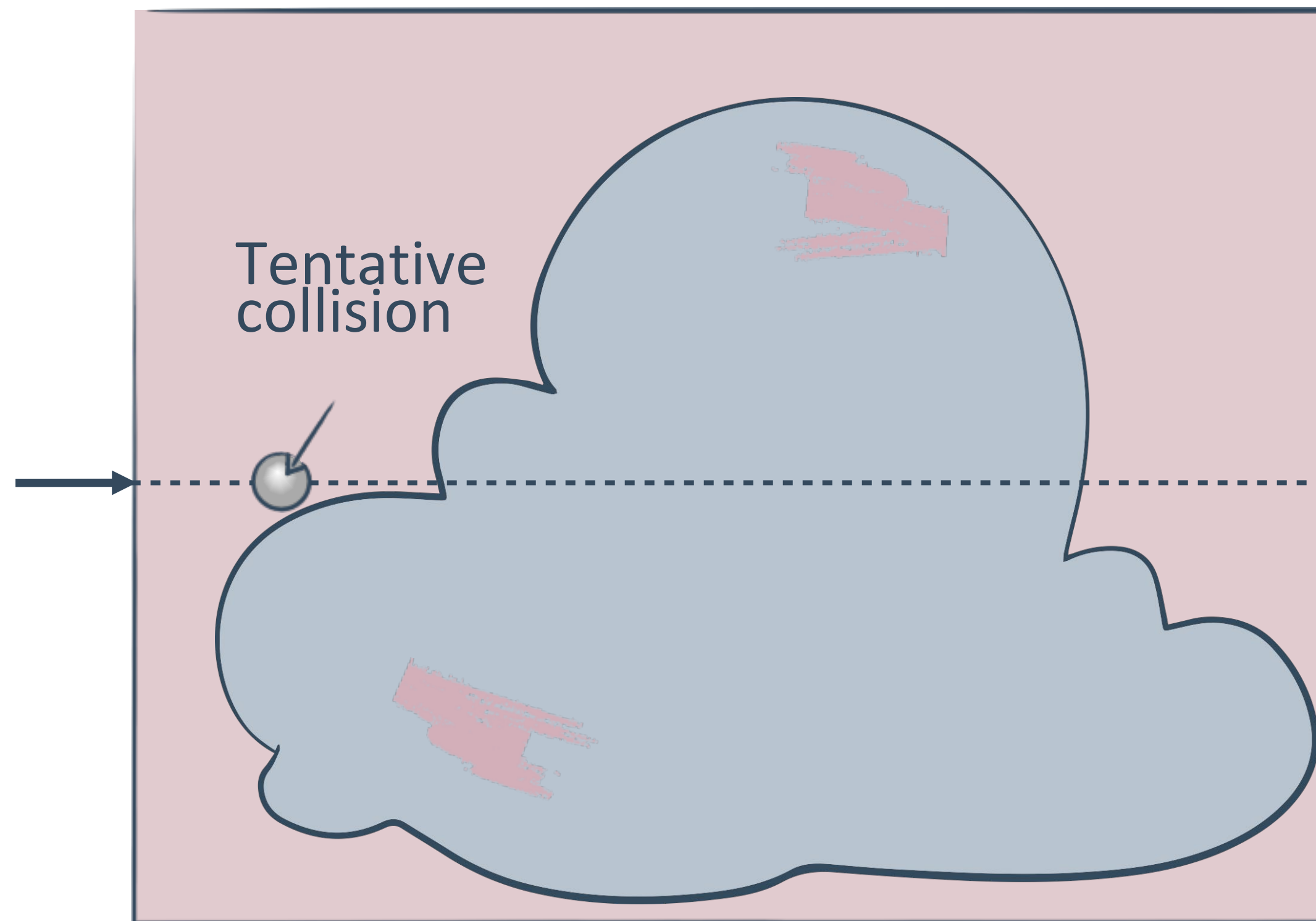
Volume bounds

Real medium

Stochastic Sampling

Volume
bounds

Fictitious
medium

Majorant $\quad \bar{\sigma} = \sigma_{\mathrm{t}}(\mathbf{x}) + \sigma_{\mathrm{n}}(\mathbf{x})$

Real

Stochastic Sampling

$$P_{\mathrm{r}}(\mathbf{x}) = \frac{\sigma_{\mathrm{t}}(\mathbf{x})}{\bar{\sigma}} \qquad P_{\mathrm{n}}(\mathbf{x}) = \frac{\sigma_{\mathrm{n}}(\mathbf{x})}{\bar{\sigma}}$$

Majorant $\bar{\sigma} = \sigma_{\mathrm{t}}(\mathbf{x}) + \sigma_{\mathrm{n}}(\mathbf{x})$

Tentative collision

σ_{t}

Extinction

Distance

$$-\frac{\ln(1-\xi)}{\bar{\sigma}}$$

Stochastic Sampling

$$P_{\mathrm{r}}(\mathbf{x}) = \frac{\sigma_{\mathrm{t}}(\mathbf{x})}{\bar{\sigma}} \qquad P_{\mathrm{n}}(\mathbf{x}) = \frac{\sigma_{\mathrm{n}}(\mathbf{x})}{\bar{\sigma}}$$

Majorant $\quad \bar{\sigma} = \sigma_{\mathrm{t}}(\mathbf{x}) + \sigma_{\mathrm{n}}(\mathbf{x})$

Tentative collision

Extinction

σ_{t}

Distance

Stochastic Sampling

$$P_{\mathrm{r}}(\mathbf{x}) = \frac{\sigma_{\mathrm{t}}(\mathbf{x})}{\bar{\sigma}} \qquad P_{\mathrm{n}}(\mathbf{x}) = \frac{\sigma_{\mathrm{n}}(\mathbf{x})}{\bar{\sigma}}$$

Tentative collision

Majorant $\quad \bar{\sigma} = \sigma_{\mathrm{t}}(\mathbf{x}) + \sigma_{\mathrm{n}}(\mathbf{x})$

Extinction

σ_{t}

Distance

Stochastic Sampling

$$P_{\mathrm{r}}(\mathbf{x}) = \frac{\sigma_{\mathrm{t}}(\mathbf{x})}{\bar{\sigma}} \qquad P_{\mathrm{n}}(\mathbf{x}) = \frac{\sigma_{\mathrm{n}}(\mathbf{x})}{\bar{\sigma}}$$

Real collision

Majorant $\quad \bar{\sigma} = \sigma_{\mathrm{t}}(\mathbf{x}) + \sigma_{\mathrm{n}}(\mathbf{x})$

Extinction

σ_{t}

Sampled free path

Distance

Impact of Majorant

Majorant $\bar{\sigma} = \sigma_{\mathrm{t}}(\mathbf{x}) + \sigma_{\mathrm{n}}(\mathbf{x})$

σ_{t}

Extinction

Distance

Sampled free path
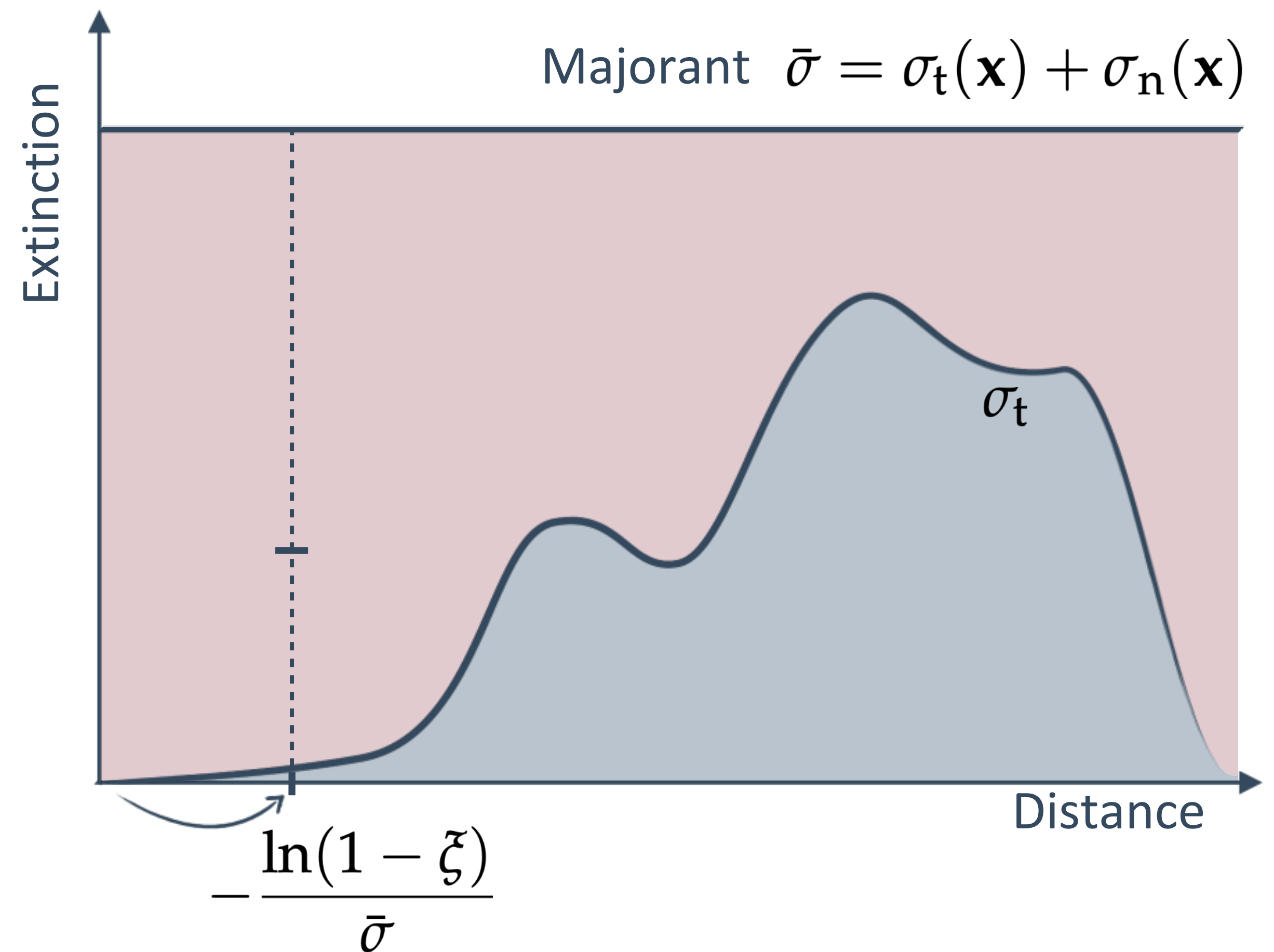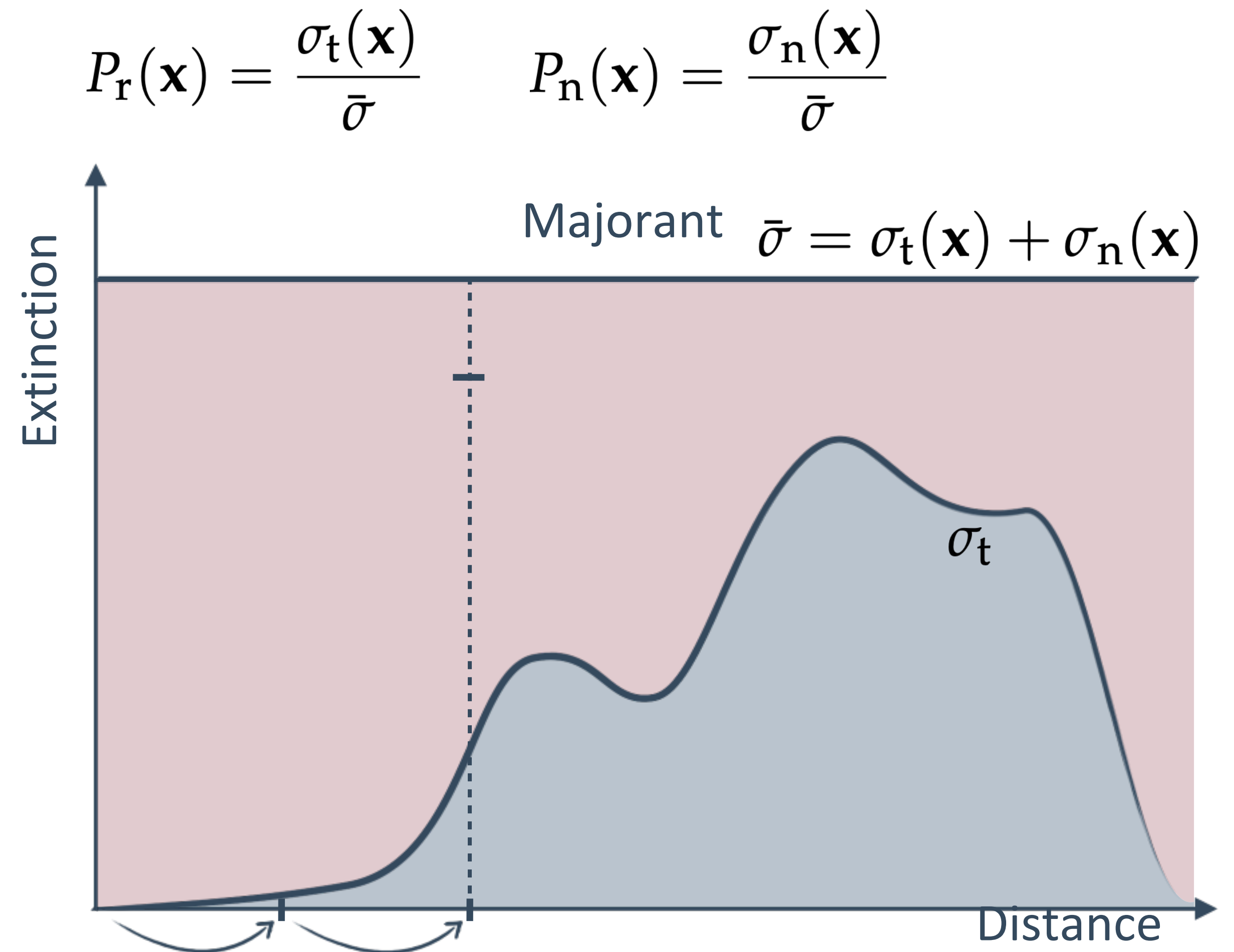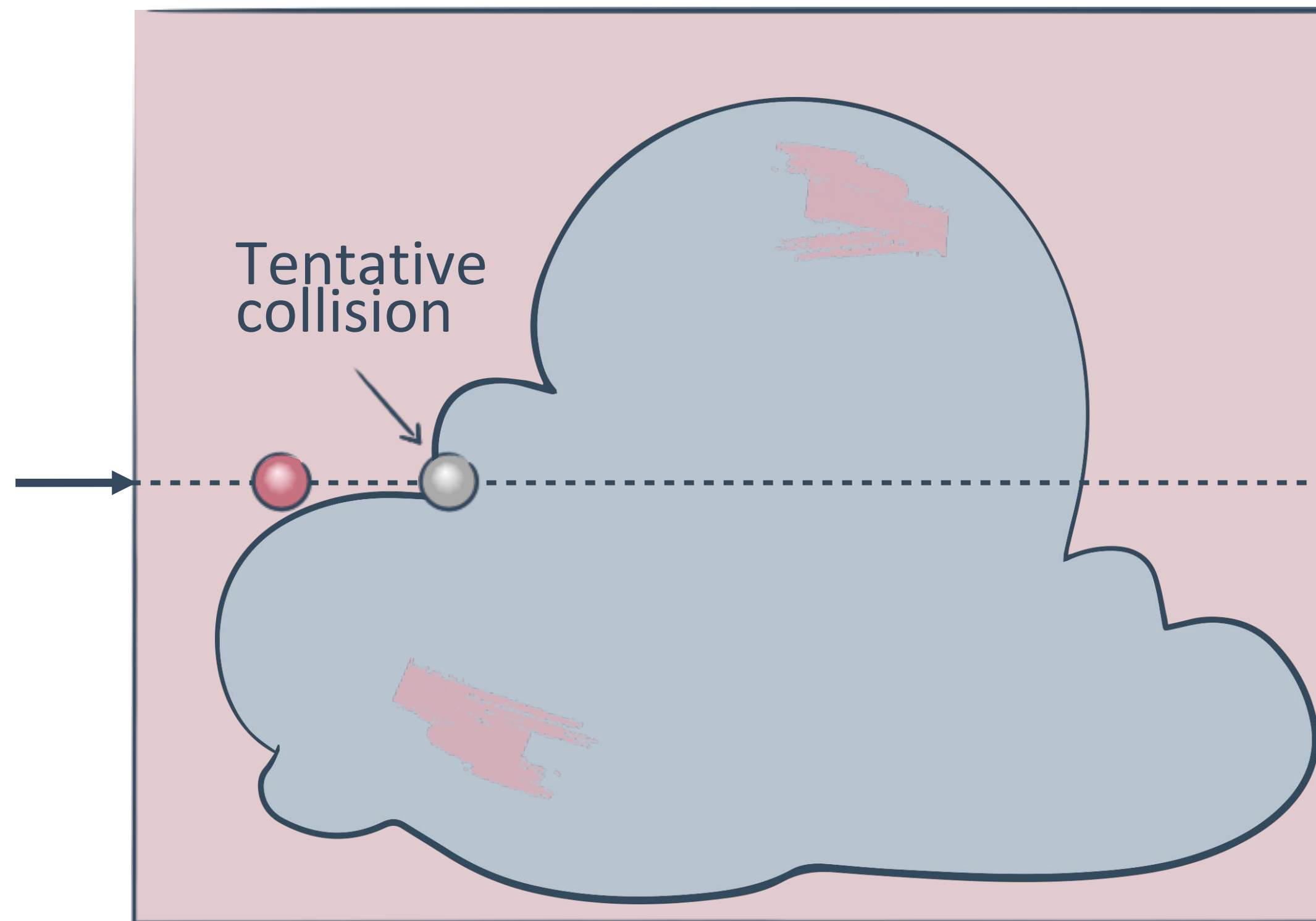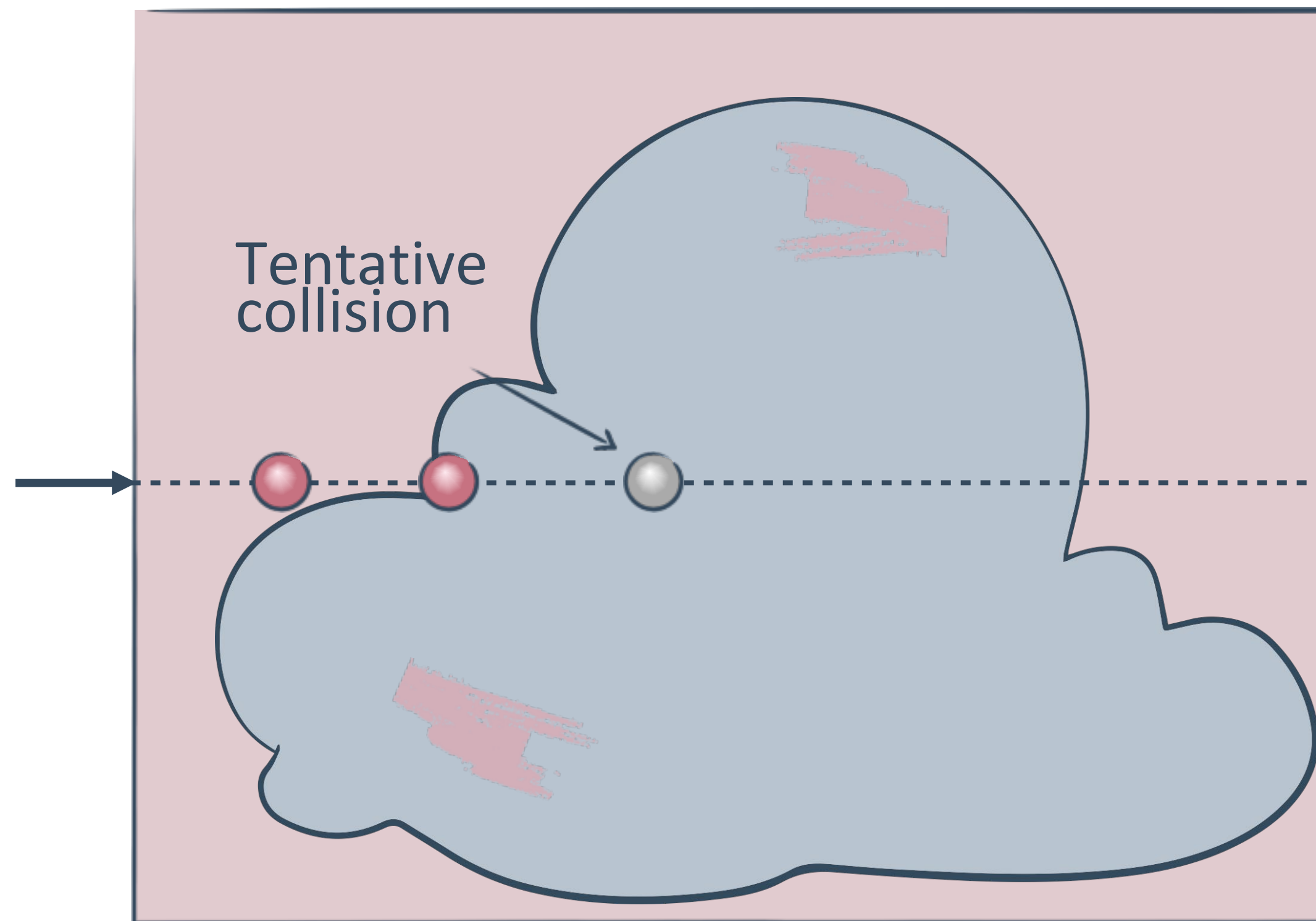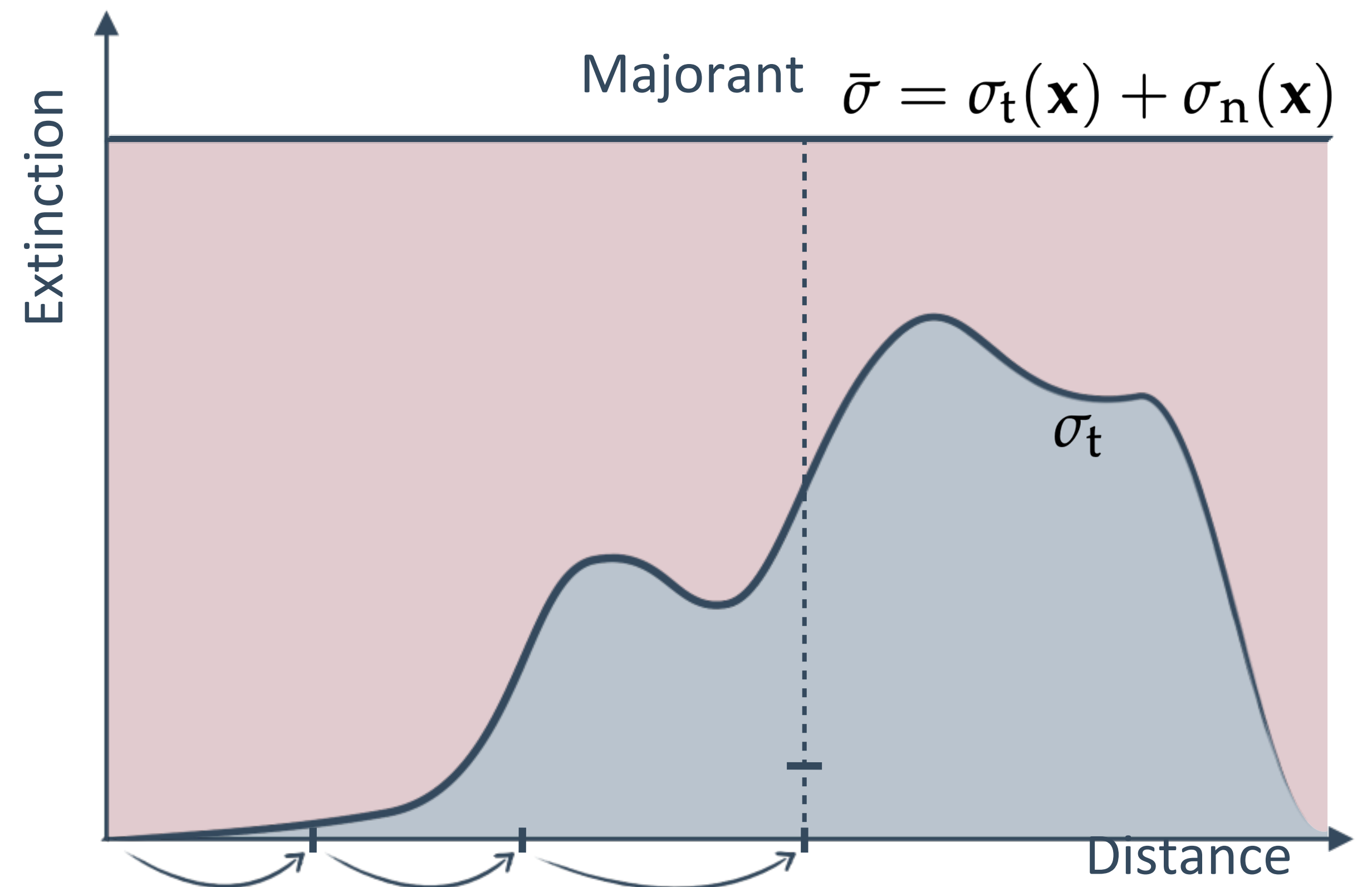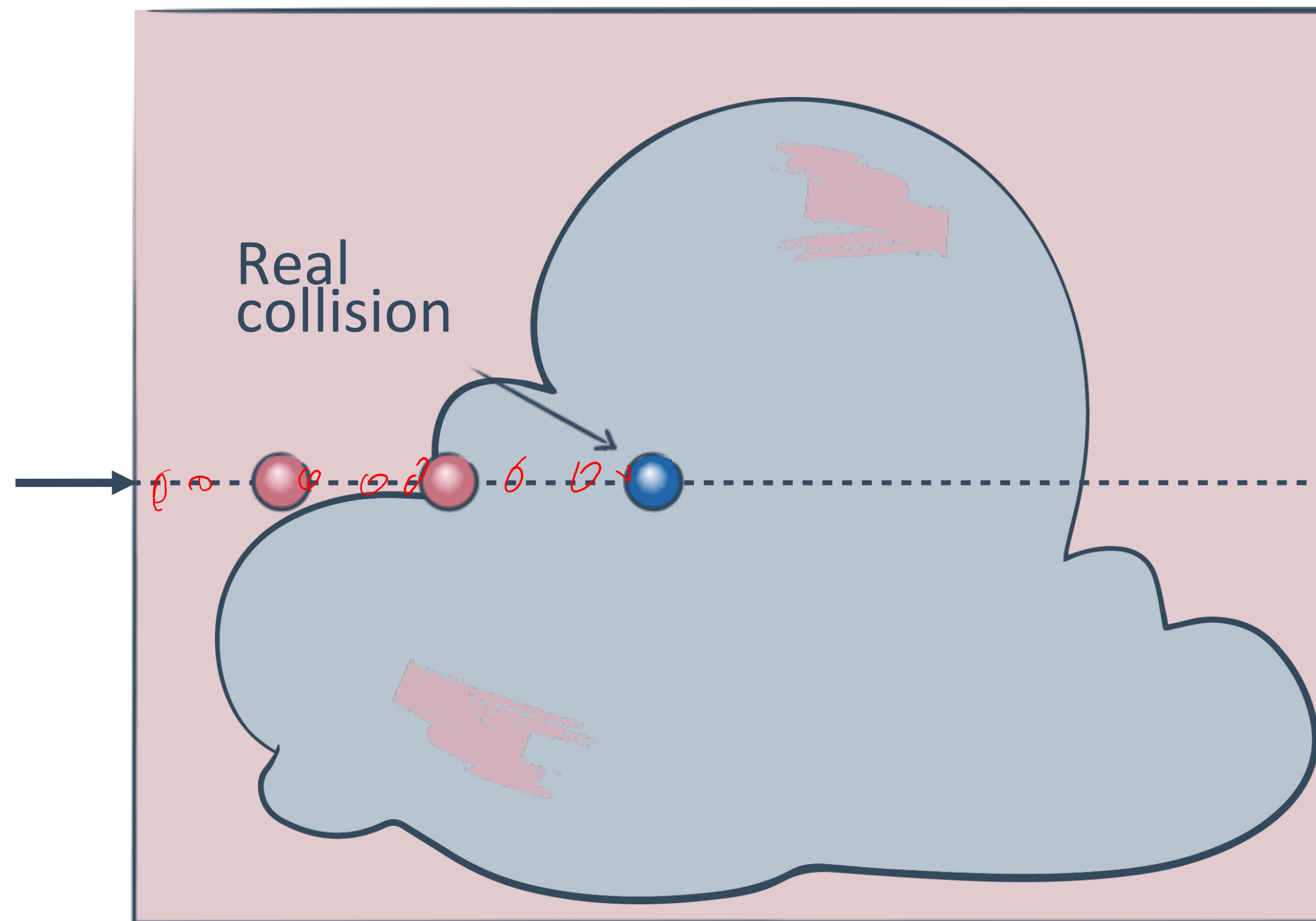
Impact of Majorant

Tight majorant = GOOD
(few rejected collisions)

Impact of Majorant

Majorant $\bar{\sigma} = \sigma_{\mathrm{t}}(\mathbf{x}) + \sigma_{\mathrm{n}}(\mathbf{x})$
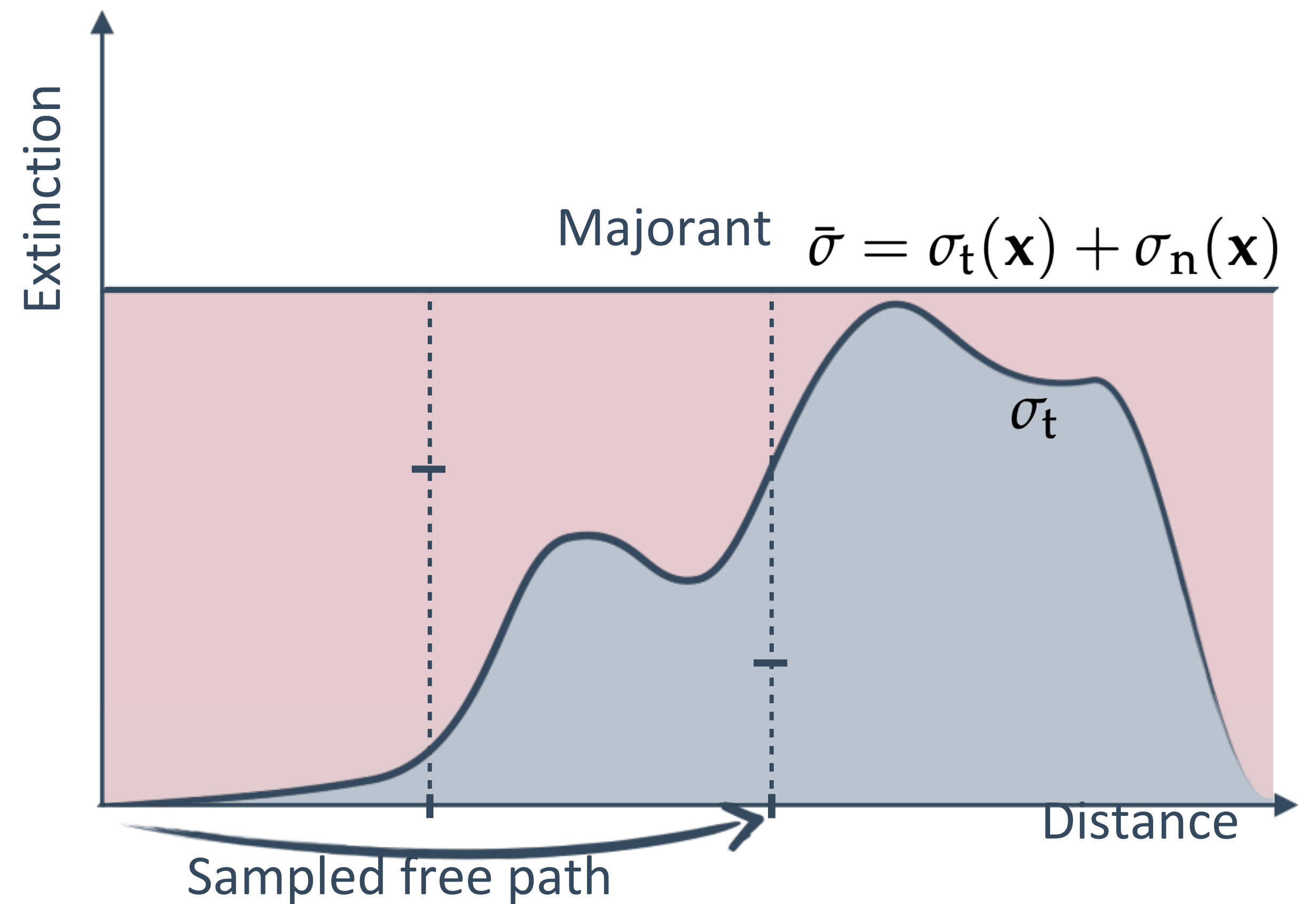
Loose majorant = BAD
(many expensive rejected collisions)

Extinction

σ_{t}

Distance
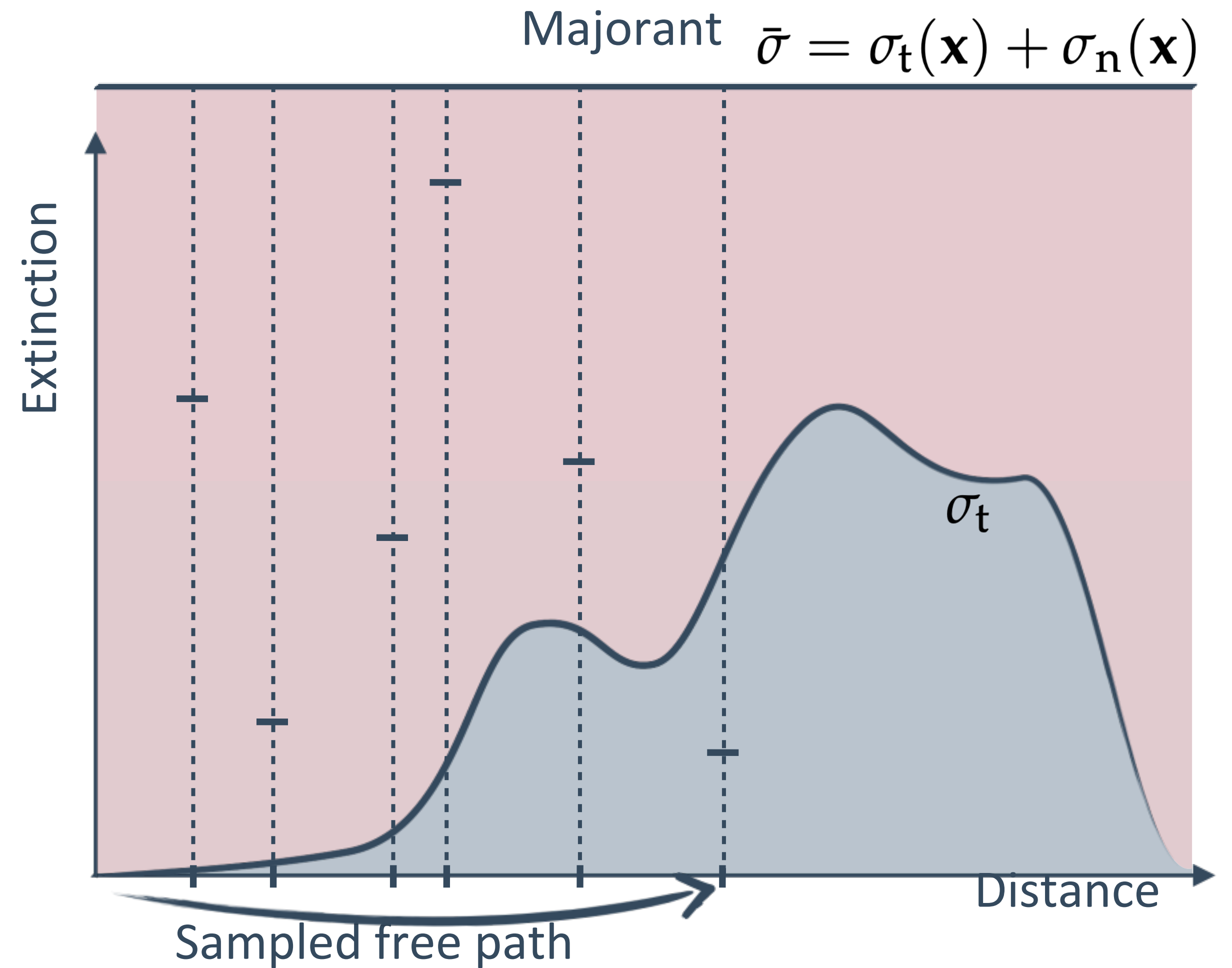
Sampled free path

Delta Tracking

```
void preprocess()

majorant = findMaximumExtinction()


void samplEFreePath(x, ω)

t = 0

do:

// Sample distance to next tentative collision

t += -ln(1 - randf()) / majorant

// Compute probability of a real collision

Pr = getExtinction(x + t*ω) / majorant

while Pr < randf()

return t
```

Delta Tracking Summary

Unbiased, see [Coleman 68] for a proof

Delta Tracking Summary

Unbiased, see [Coleman 68] for a proof

Majorant extinction

- defines the combined homogeneous volume

- must bound the real extinction

- loose majorants lead to many fictitious collisions