

# Solid and procedural textures



15-468, 15-668, 15-868  
Physics-based Rendering  
Spring 2023, Lecture 5

# Course announcements

- Programming assignment 1 is due on Friday 2/10.
  - Any issues with the homework?
- Take-home quiz 2 due Tuesday 2/7.

# Overview of today's lecture

- 3D textures.
- Procedural textures.
- Generating “realistic noise”.

# Slide credits

Most of these slides were directly adapted from:

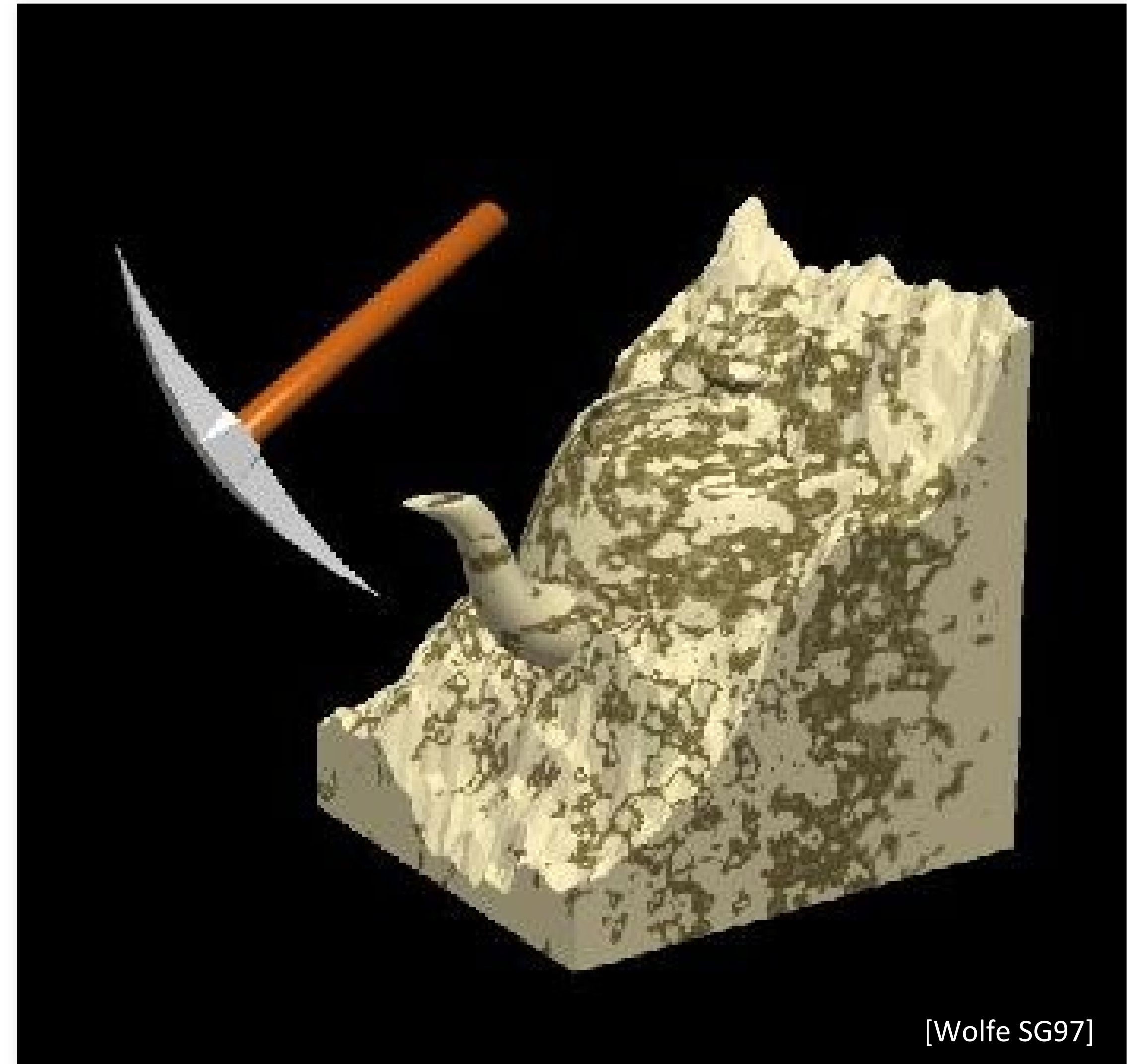
- Wojciech Jarosz (Dartmouth).



# 3D textures

Texture is a function of  $(u, v, w)$

- can evaluate texture at 3D point
- good for solid materials
- often defined procedurally



# Procedural texturing

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Instead of using rasterized image data, define texture procedurally

Simple example:

- $\text{color} = 0.5 * \sin(x) + 0.5$

Often called “solid texturing” because texture can easily vary in all 3 dimensions.

- but you can also do 2D or 1D procedural textures

# Raster vs. procedural textures

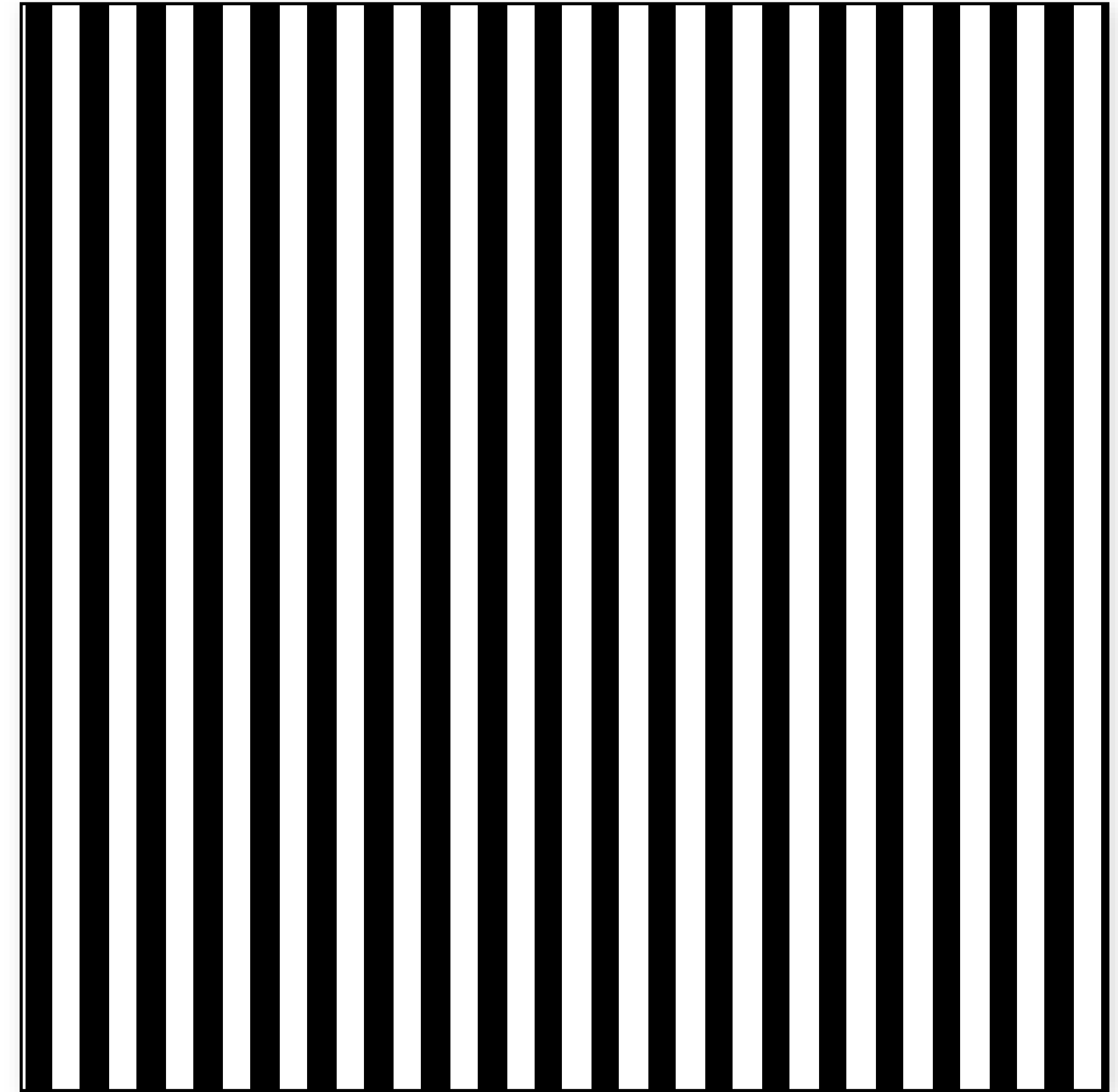
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Why use procedural textures?

- low memory usage
- infinite resolution
- solid texture: no need to parametrize surface

# 3D stripe texture

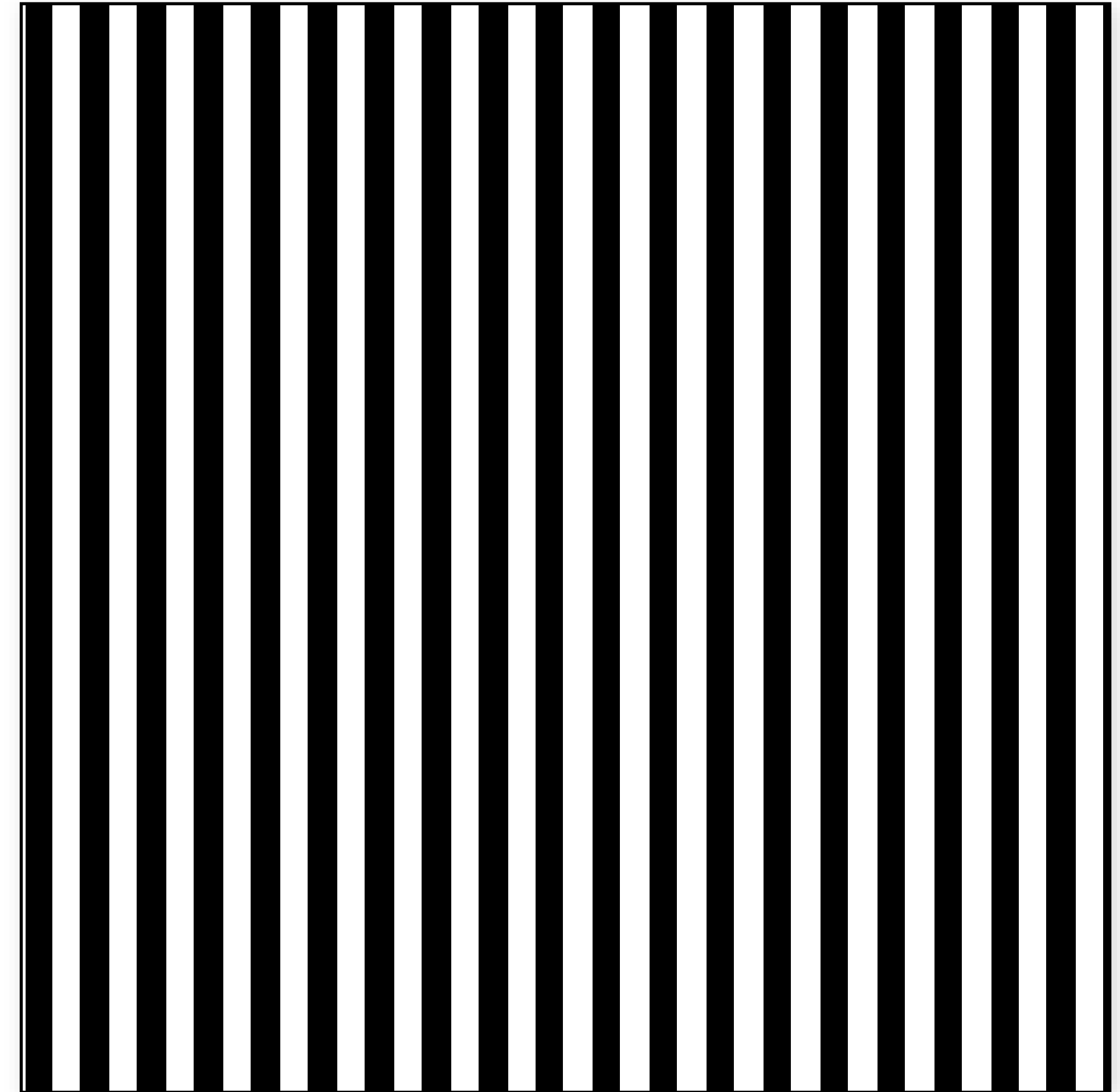
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# 3D stripe texture

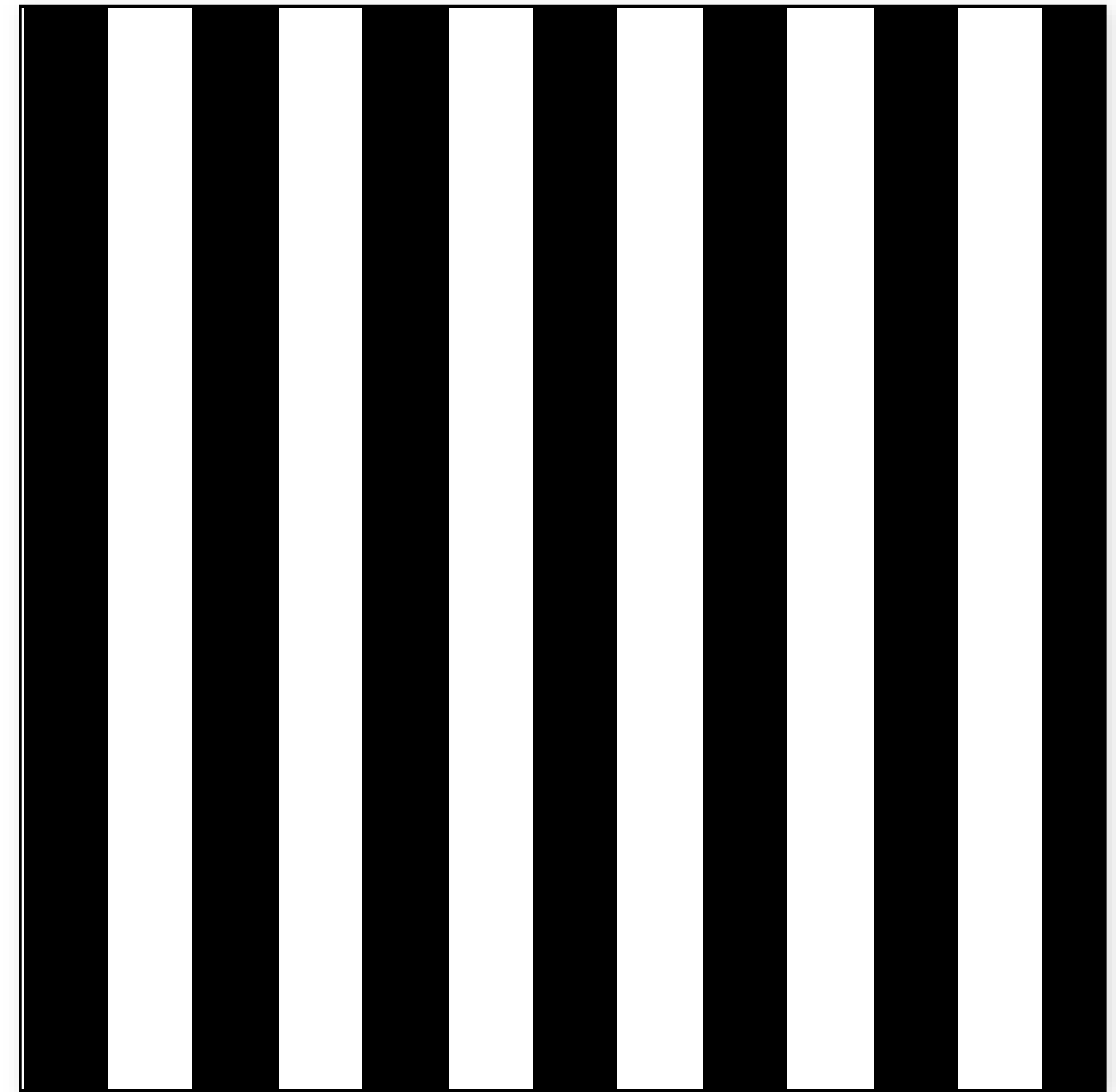
---

```
color stripe(point p):  
  if (sin(px) > 0)  
    return c0  
  else  
    return c1
```



# 3D stripe texture

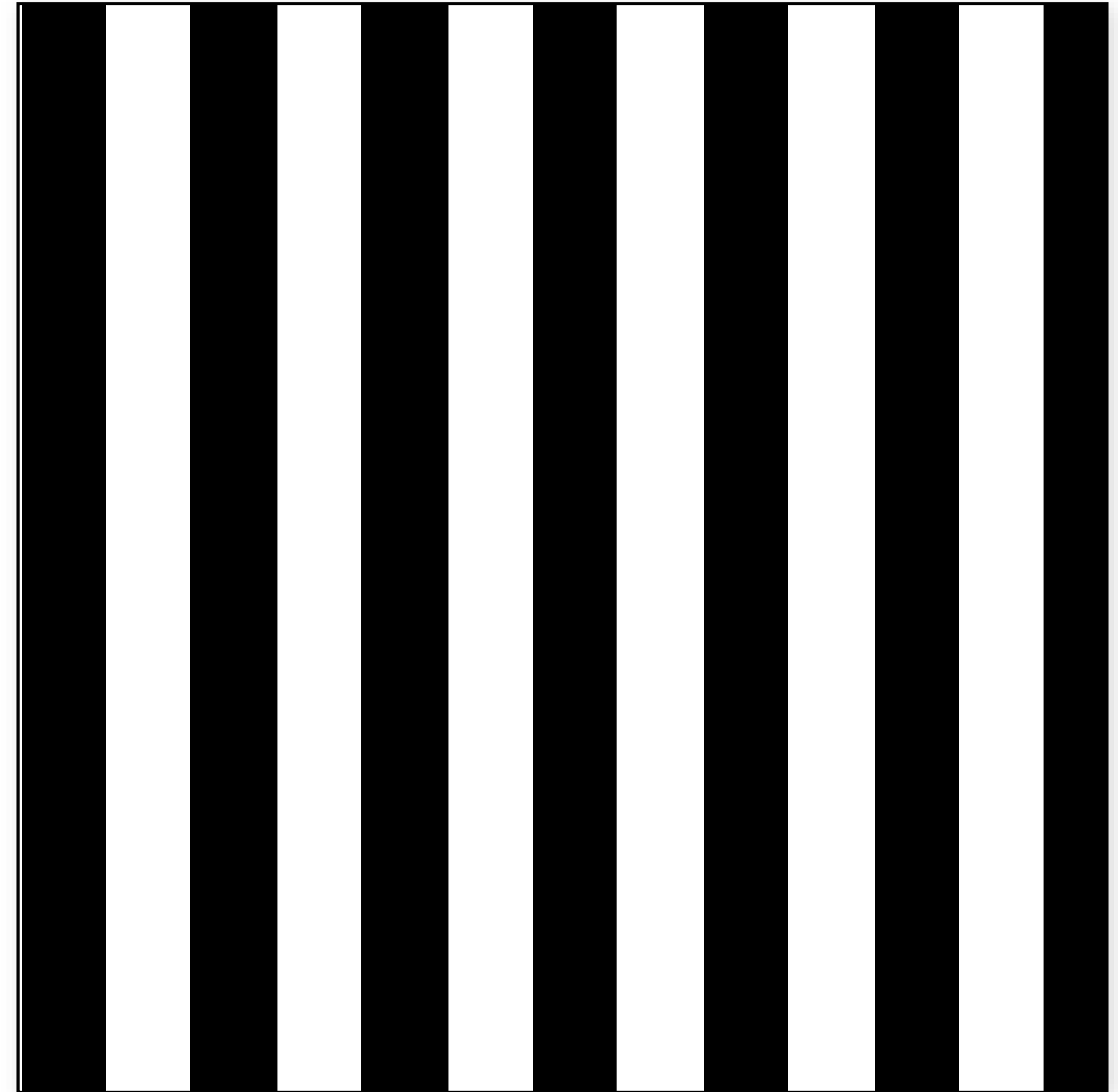
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# 3D stripe texture

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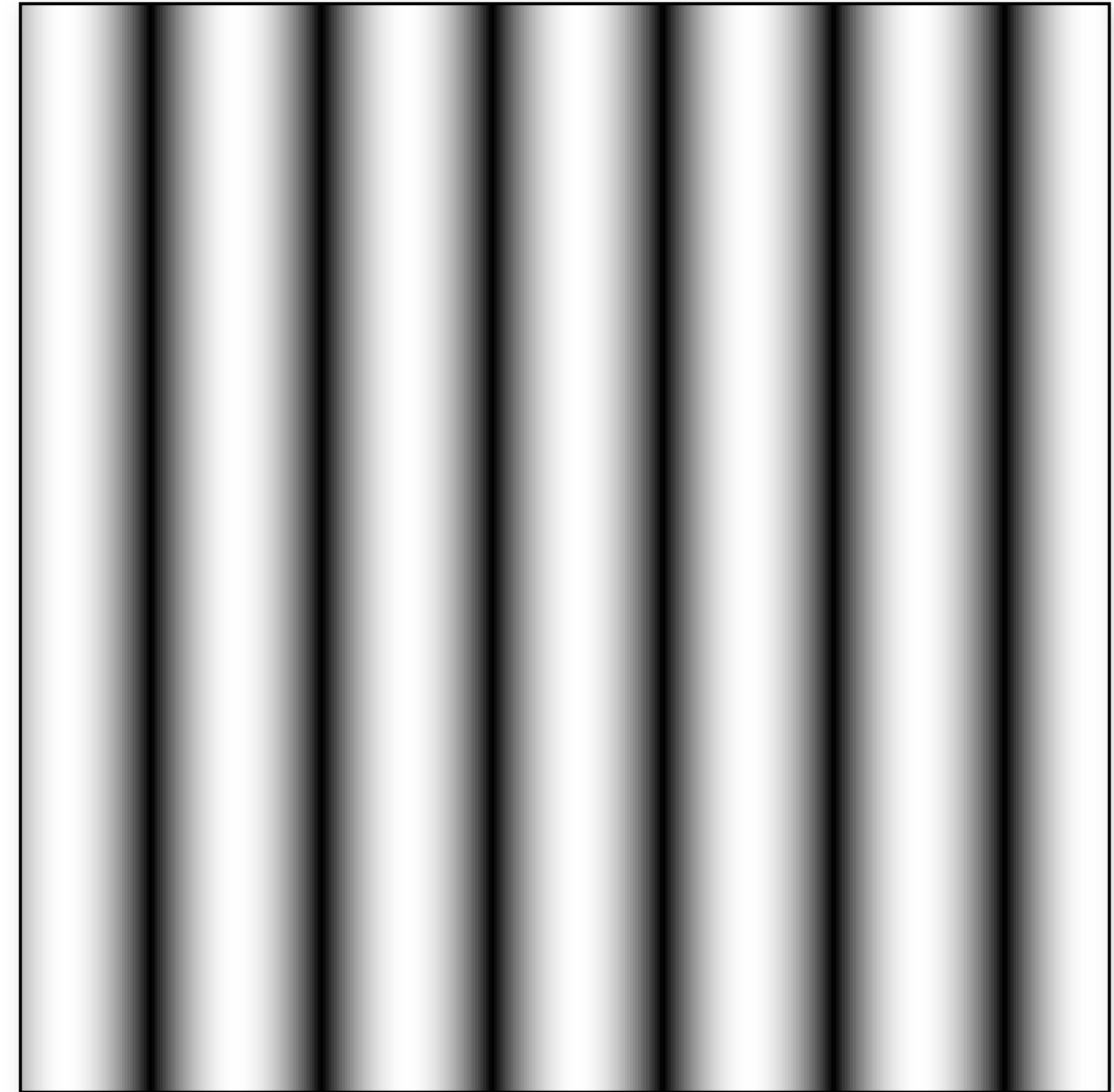
```
color stripe(point p, real w):  
  if ( $\sin(\pi \mathbf{p}_x / w) > 0$ )  
    return  $c_0$   
  else  
    return  $c_1$ 
```



# 3D stripe texture

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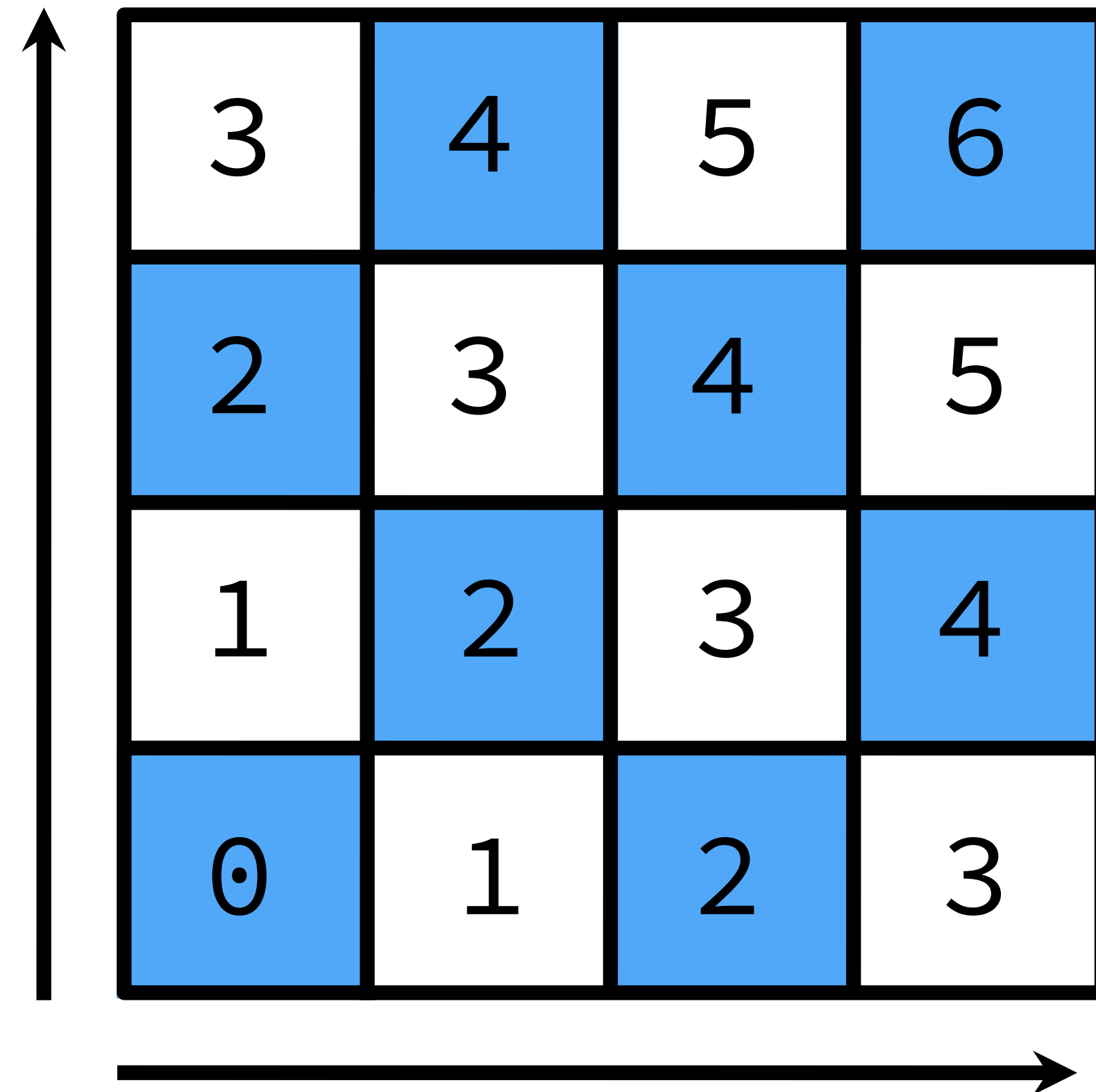
```
color stripe(point p, real w):  
   $t = (1 + \sin(\pi \mathbf{p}_x / w)) / 2$   
  return lerp(c0, c1, t)
```





# 2D checkerboard texture

```
color checkerboard(point p):  
  real a = floor(px)  
  real b = floor(py)  
  real val = a+b  
  if (isEven(val))  
    return c0  
  else  
    return c1
```



# 3D checkerboard texture

```
color checkerboard(point p):
```

```
  real a = floor(px)
```

```
  real b = floor(py)
```

```
  real c = floor(pz)
```

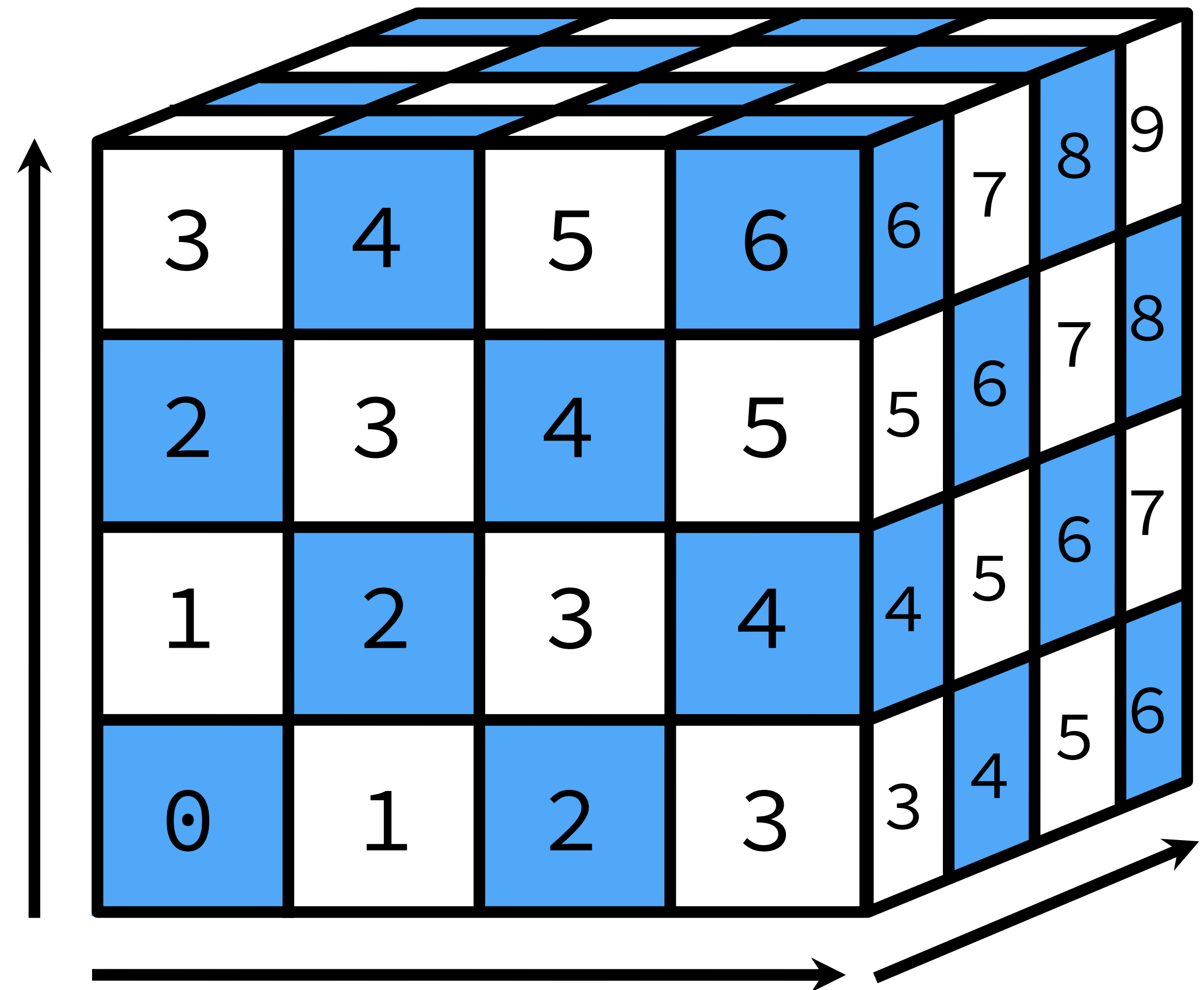
```
  real val = a+b+c
```

```
  if (isEven(val))
```

```
    return c0
```

```
  else
```

```
    return c1
```



# Procedural synthesis

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# Procedural synthesis

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© 2005  
Armands Auseklis  
terraescape.com

Created using MojoWorld by Pandromeda Inc. and Volumetric renderer by Dmytry Lavrov

# Procedural synthesis

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Industrial Light + Magic



# Procedural synthesis

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# Procedural textures

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Our procedurals are “too perfect”

Often want to add controlled variation to a texture

- Real textures have many imperfections

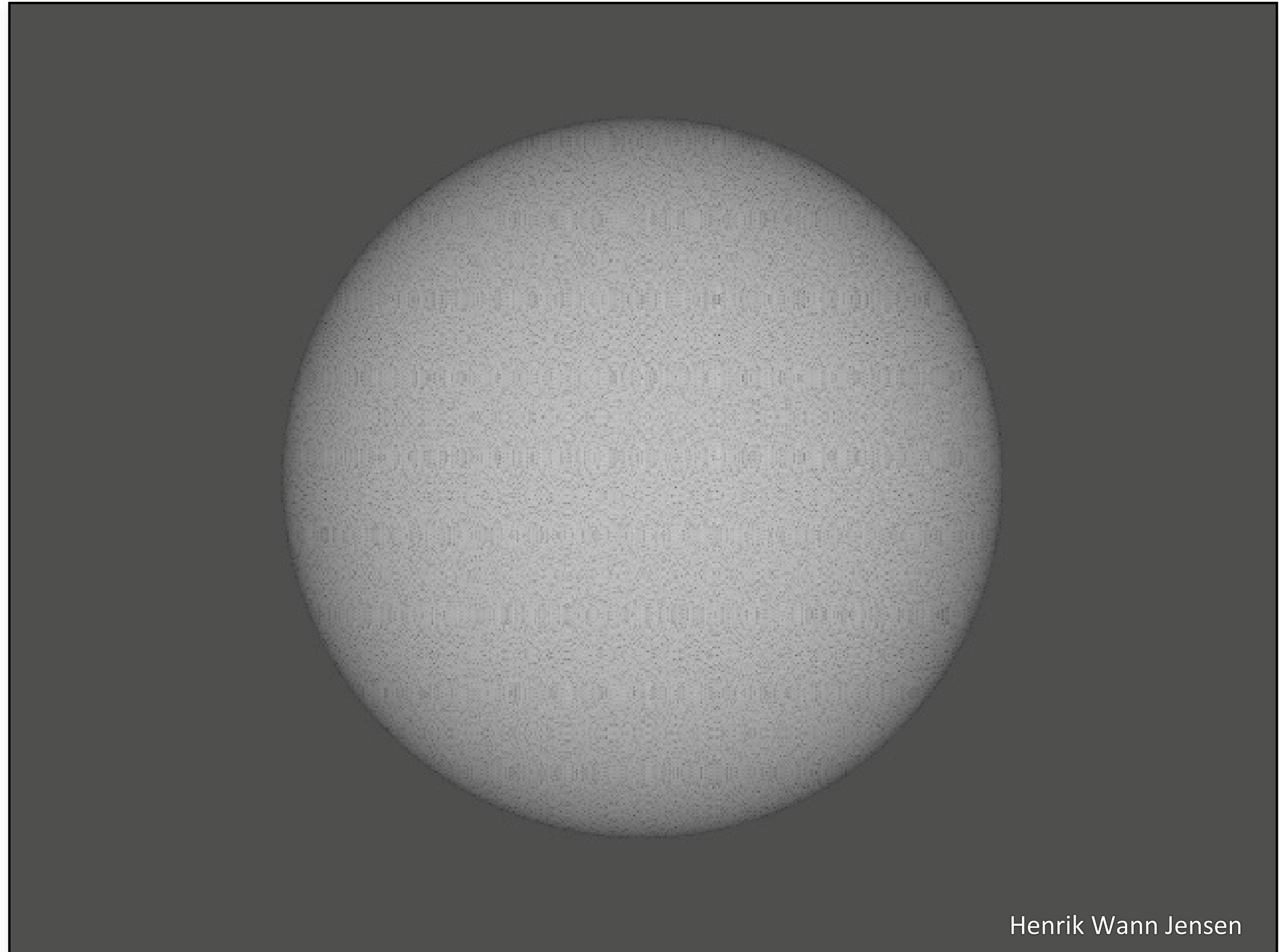
Just calling `rand()` is not that useful.

# Random noise

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*albedo* = randf();

Not band-limited,  
white noise.





# Noise functions

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Function:  $\mathbf{R}^n \rightarrow [-1, 1]$ , where  $n = 1, 2, 3, \dots$

Desirable properties:

- no obvious repetition
- rotation invariant
- band-limited (i.e., not scale-invariant)

Fundamental building block of most procedural textures

# Value noise

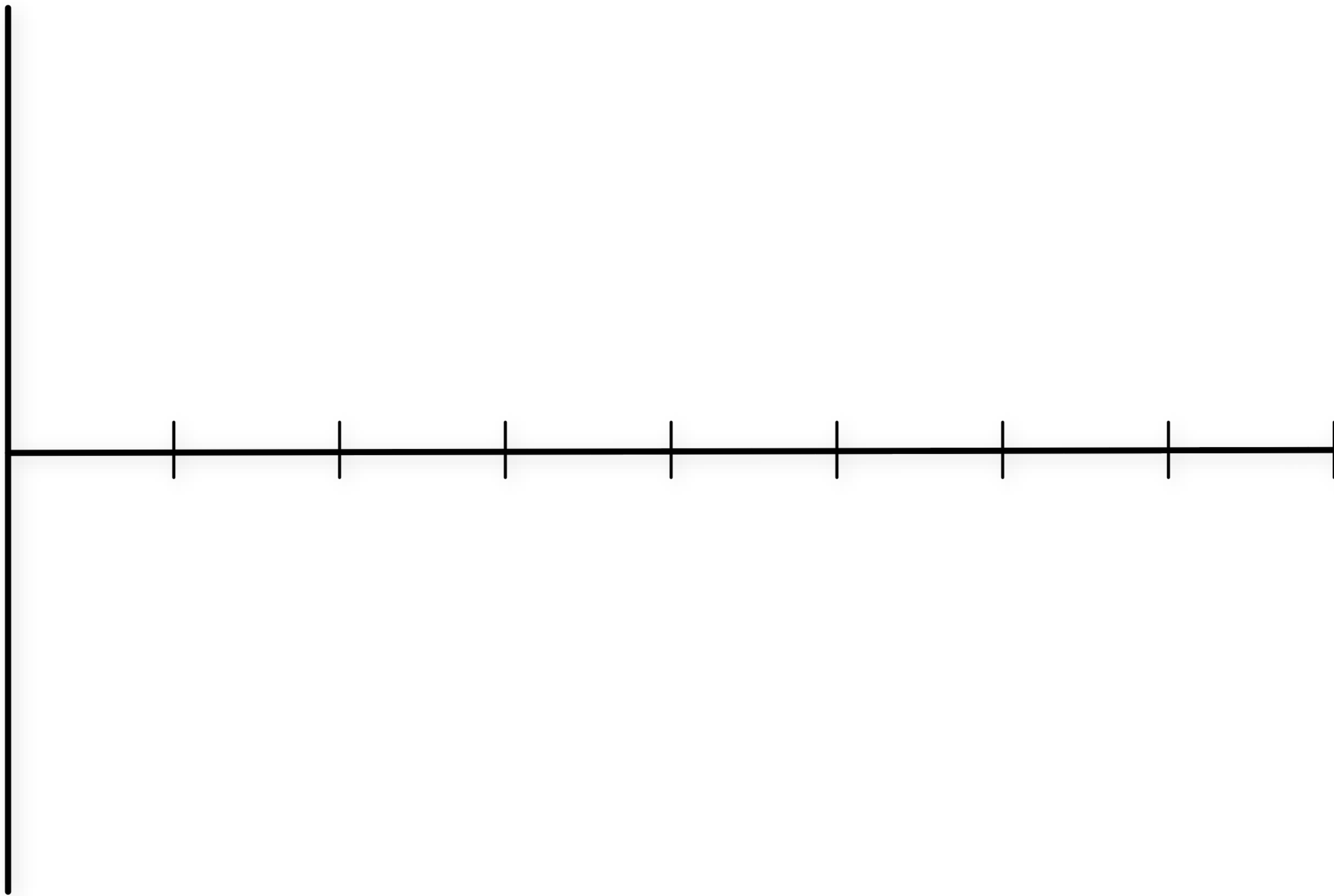
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Values associated with integer lattice locations

Given arbitrary position, interpolate value from neighboring lattice points

# Value noise example

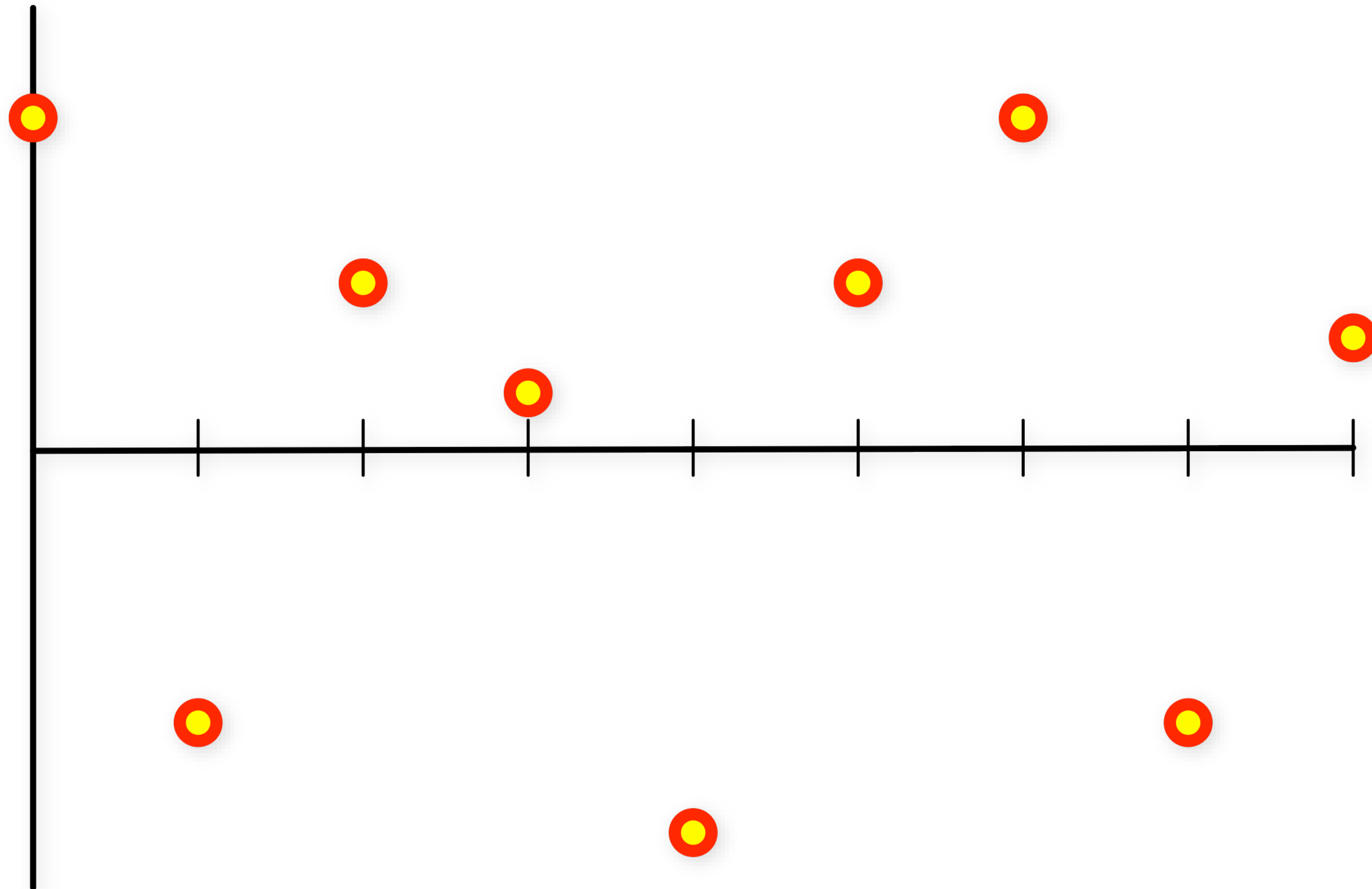
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# Value noise example

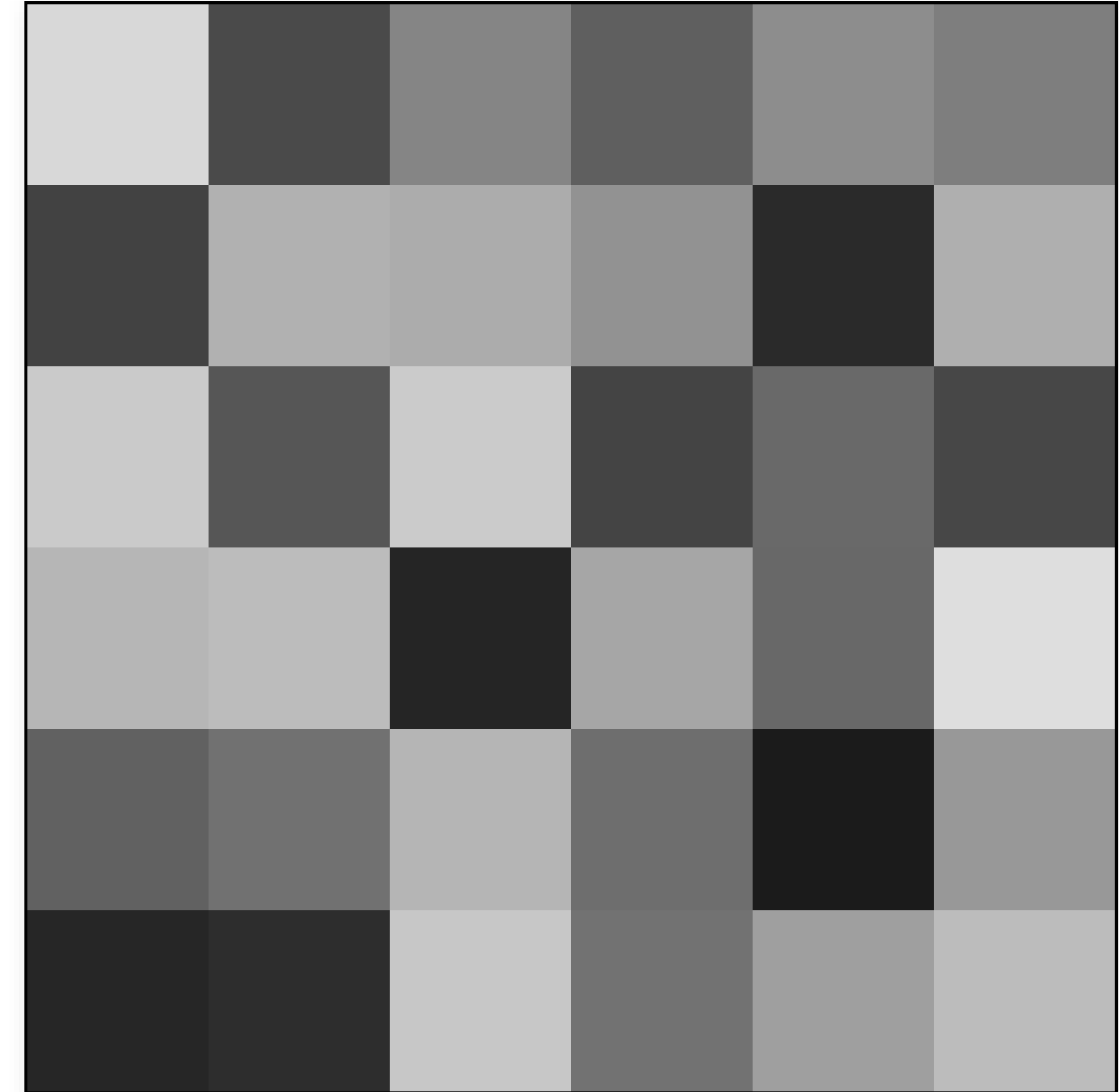
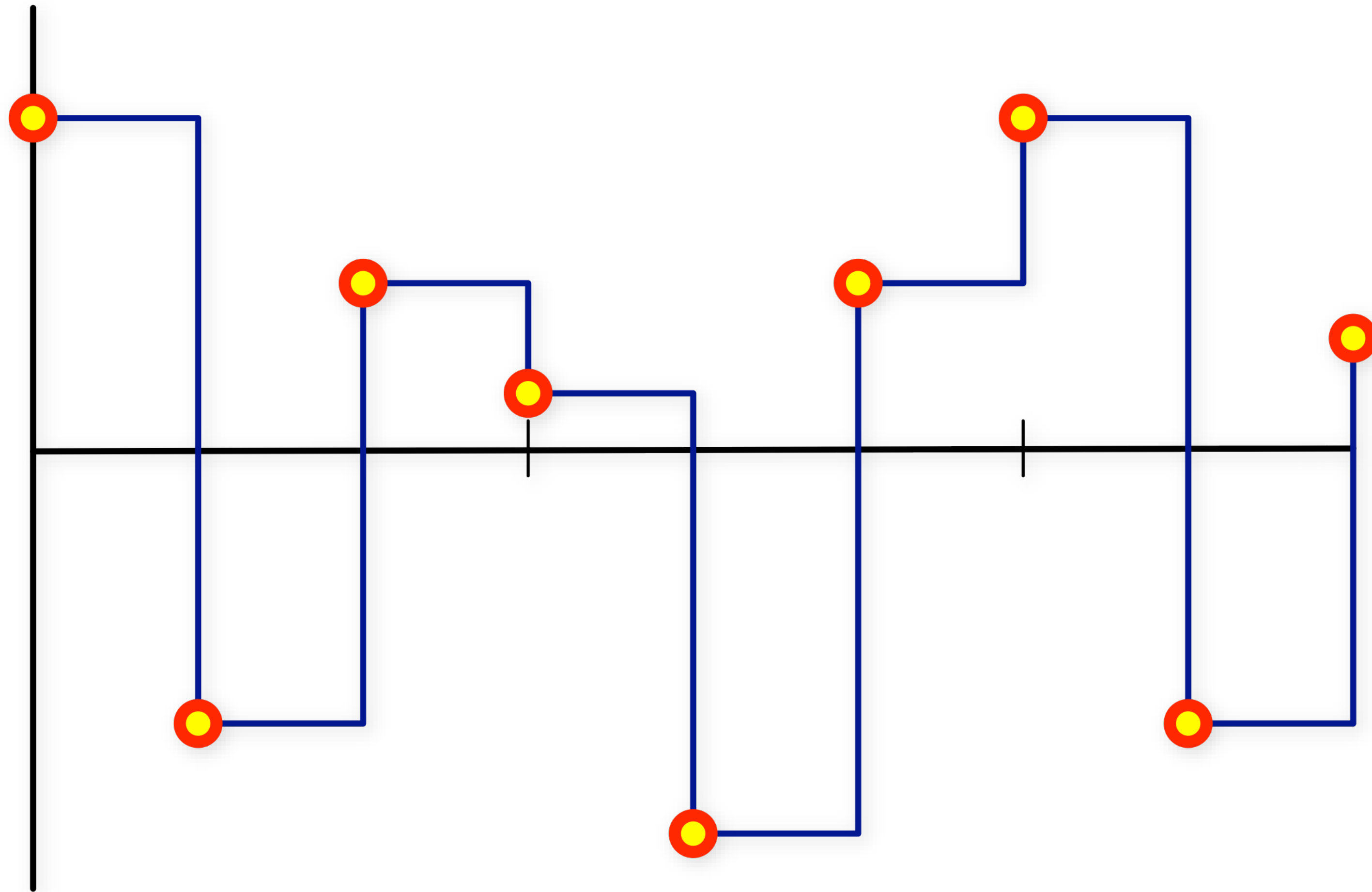
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Random values on grid



# Value noise example

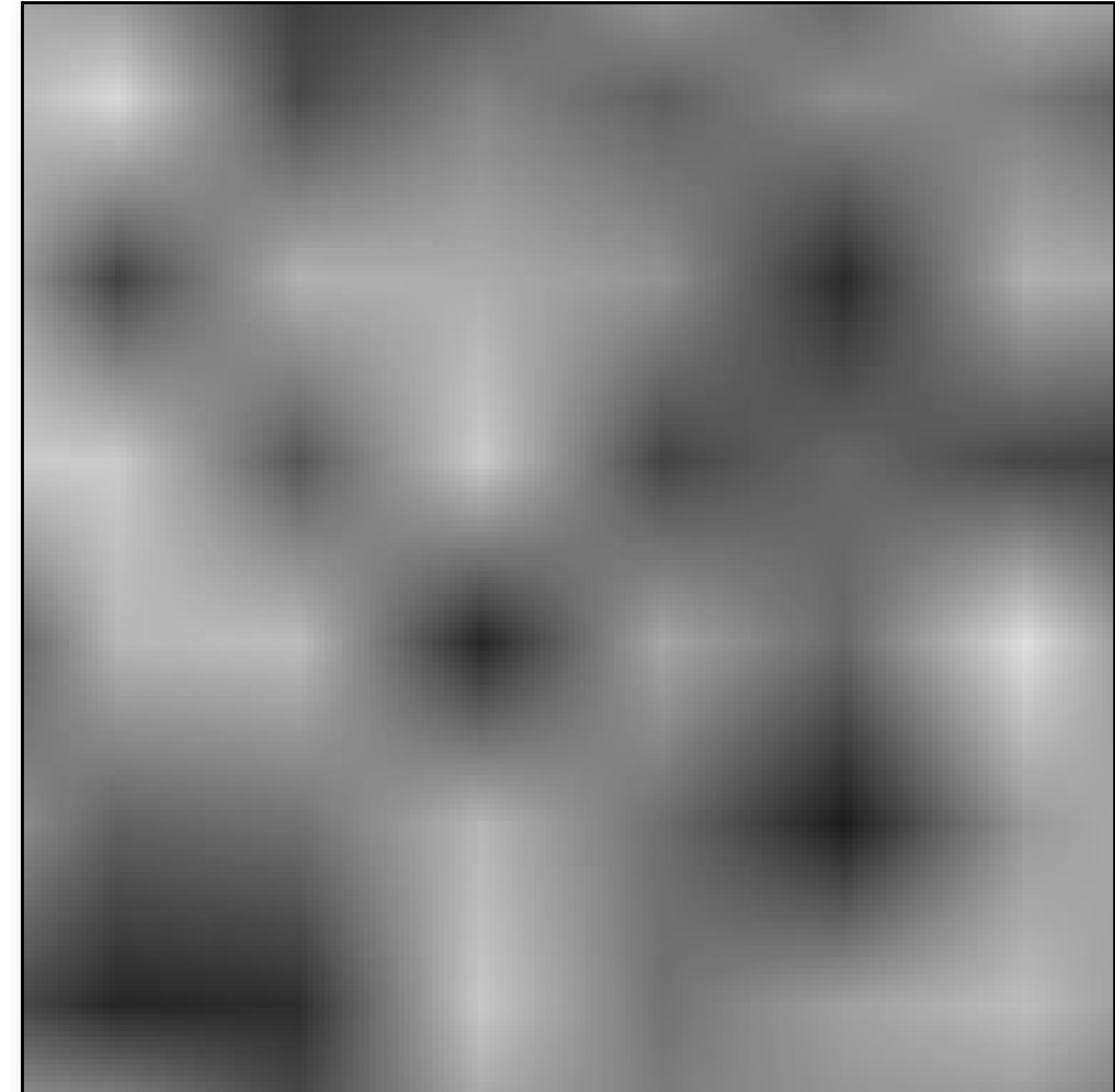
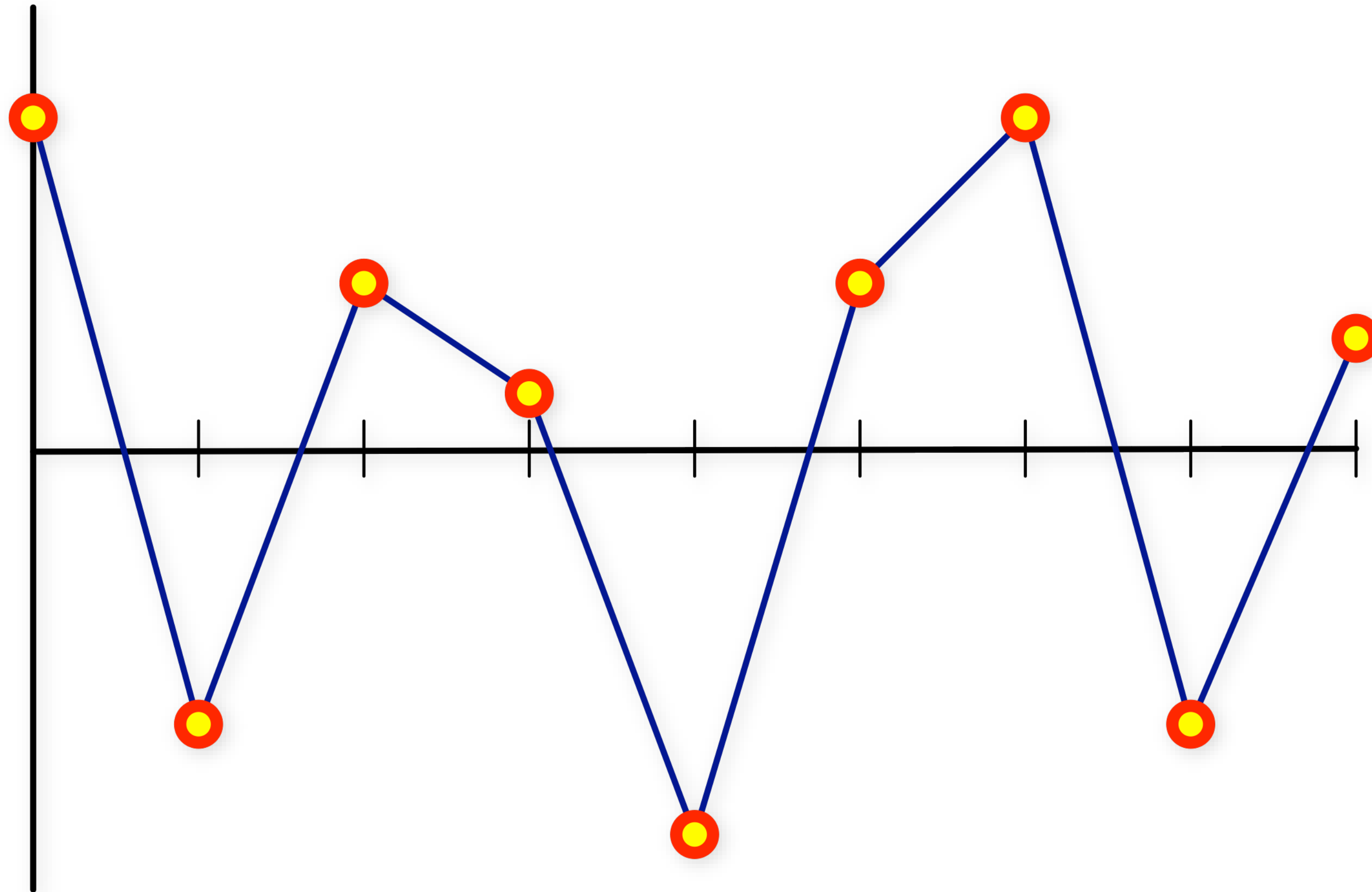
Random values on grid



Cell noise: use value of nearest point on grid

# Value noise example

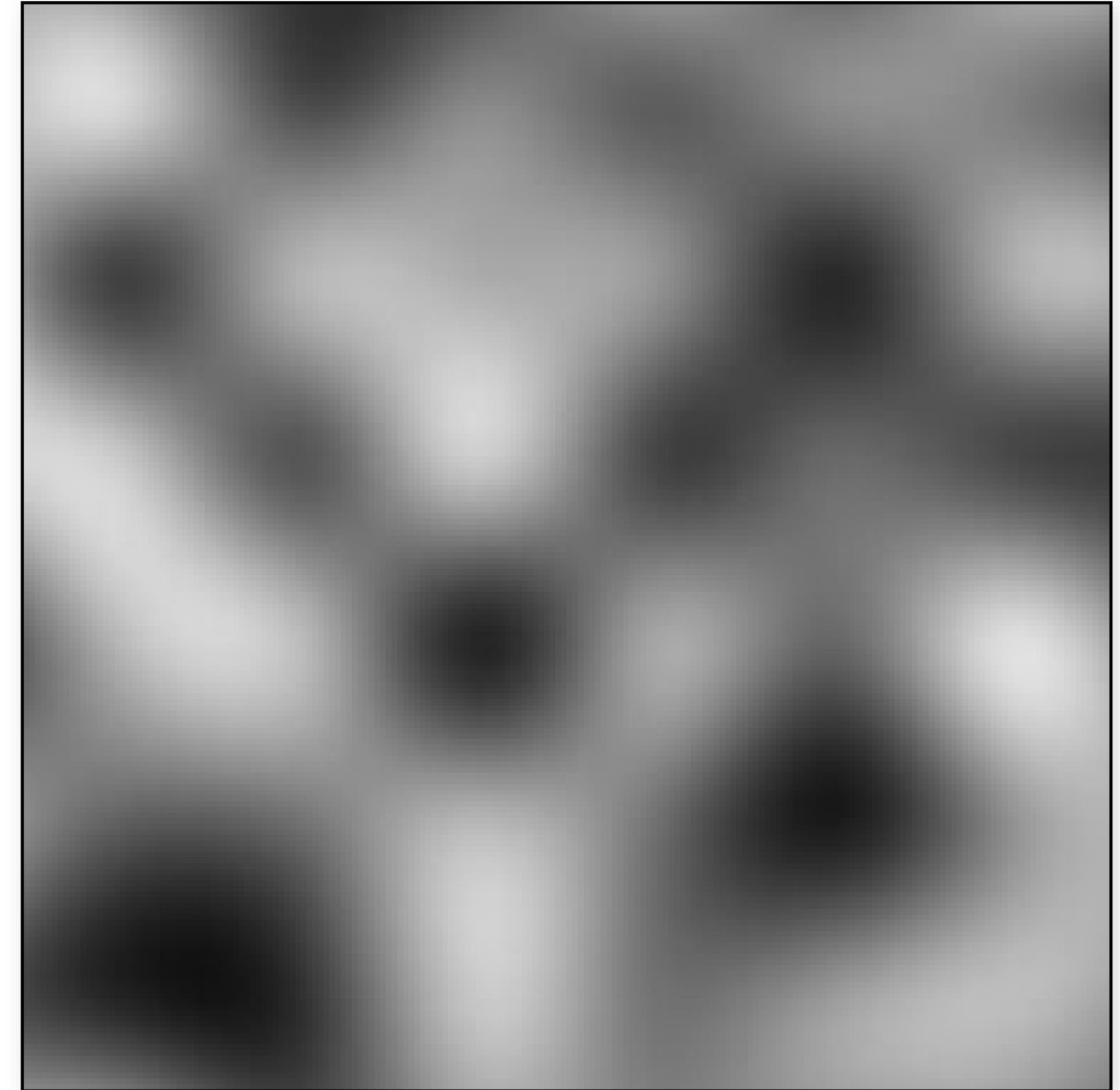
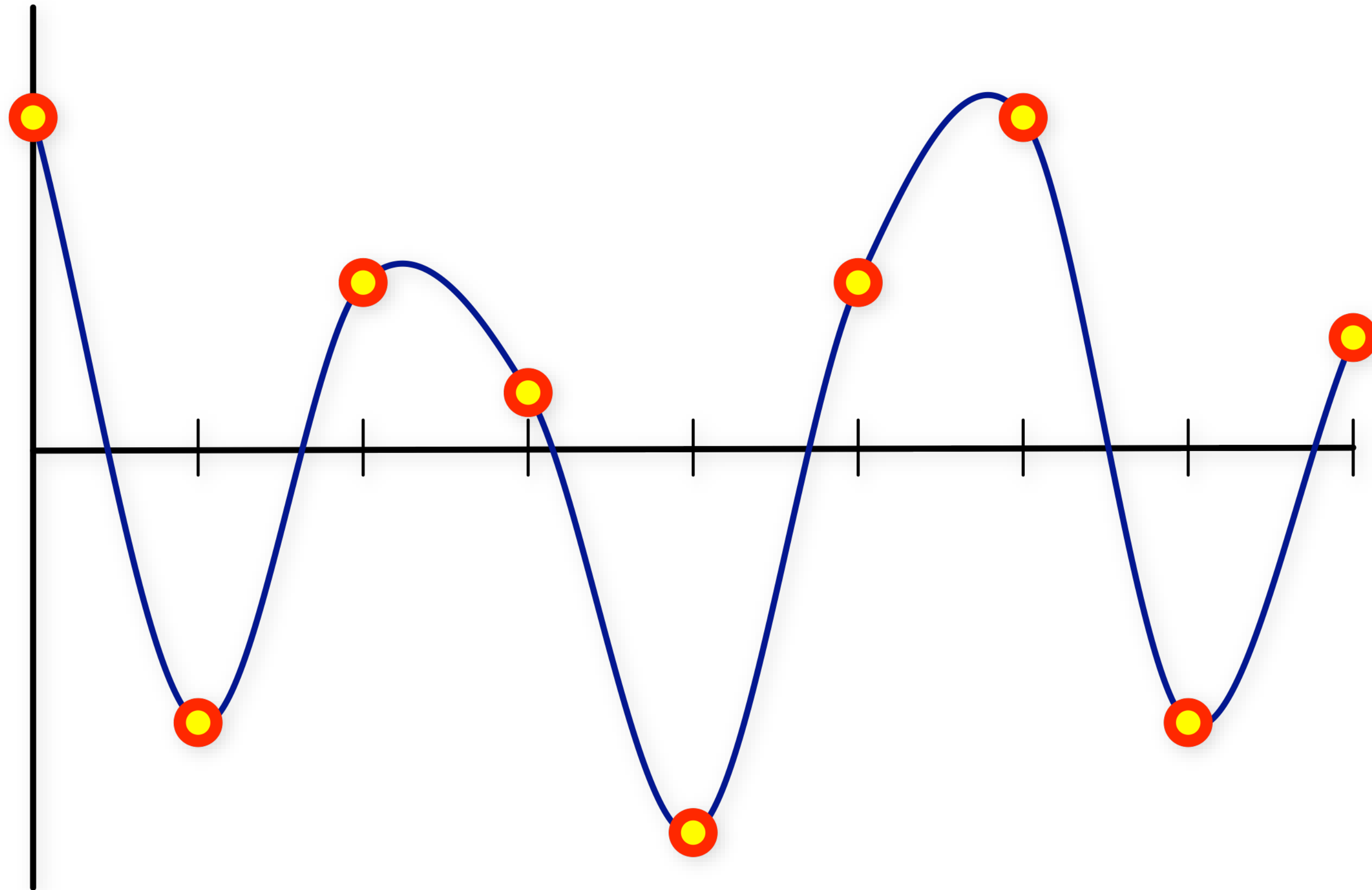
(Bi-) linearly interpolated values



Interpolate between  $2^n$  nearest grid points

# Value noise example

(Bi-) cubic interpolation



Interpolate between  $4^n$  nearest grid points

# Value noise - implementation issues

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Not feasible to store values at all integer locations

- pre-compute an array of pseudo-random values
- use a randomized hash function to map lattice locations to pseudo-random values



# Value noise - implementation details

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```
// randomly permuted array of 0...255, duplicated
const unsigned char values[256*2] = [1, 234, ...];

float noise1D(float x)
{
    int xi = int(floor(x)) & 255;
    return lerp(values[xi], values[xi+1], x-xi)/128.0-1;
}

// 2D hashing:
// values[xi + values[yi]];
// 3D hashing:
// values[xi + values[yi + values[zi]]];
// etc.
```

# Value noise - limitations

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# Value noise - limitations

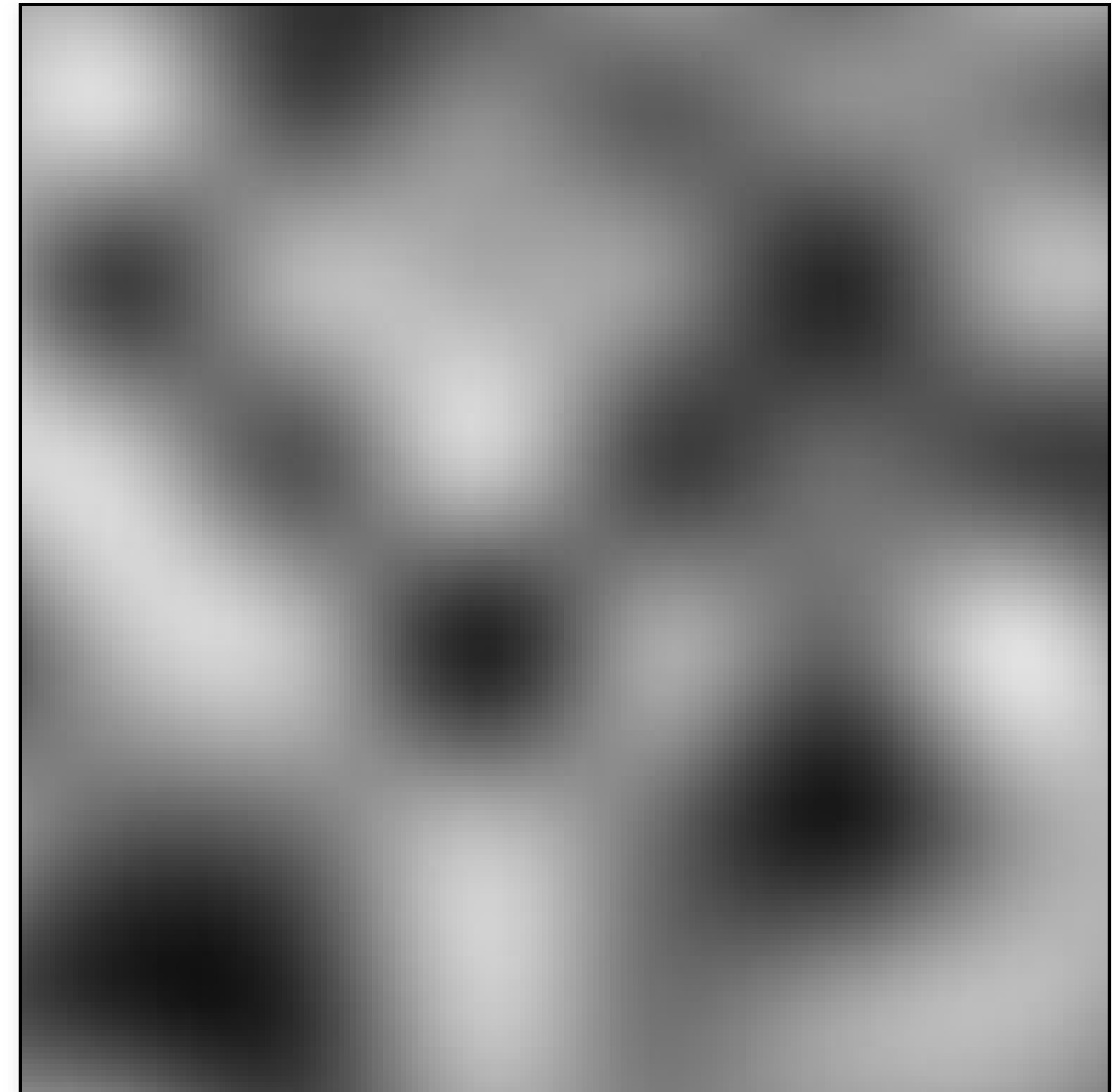
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Lattice structure apparent

- Minimal/maxima always on lattice

Slow/many lookups

- 8 values for trilinear
- 64 values for tricubic
  - $4^n$  for  $n$  dimensions



# Perlin noise

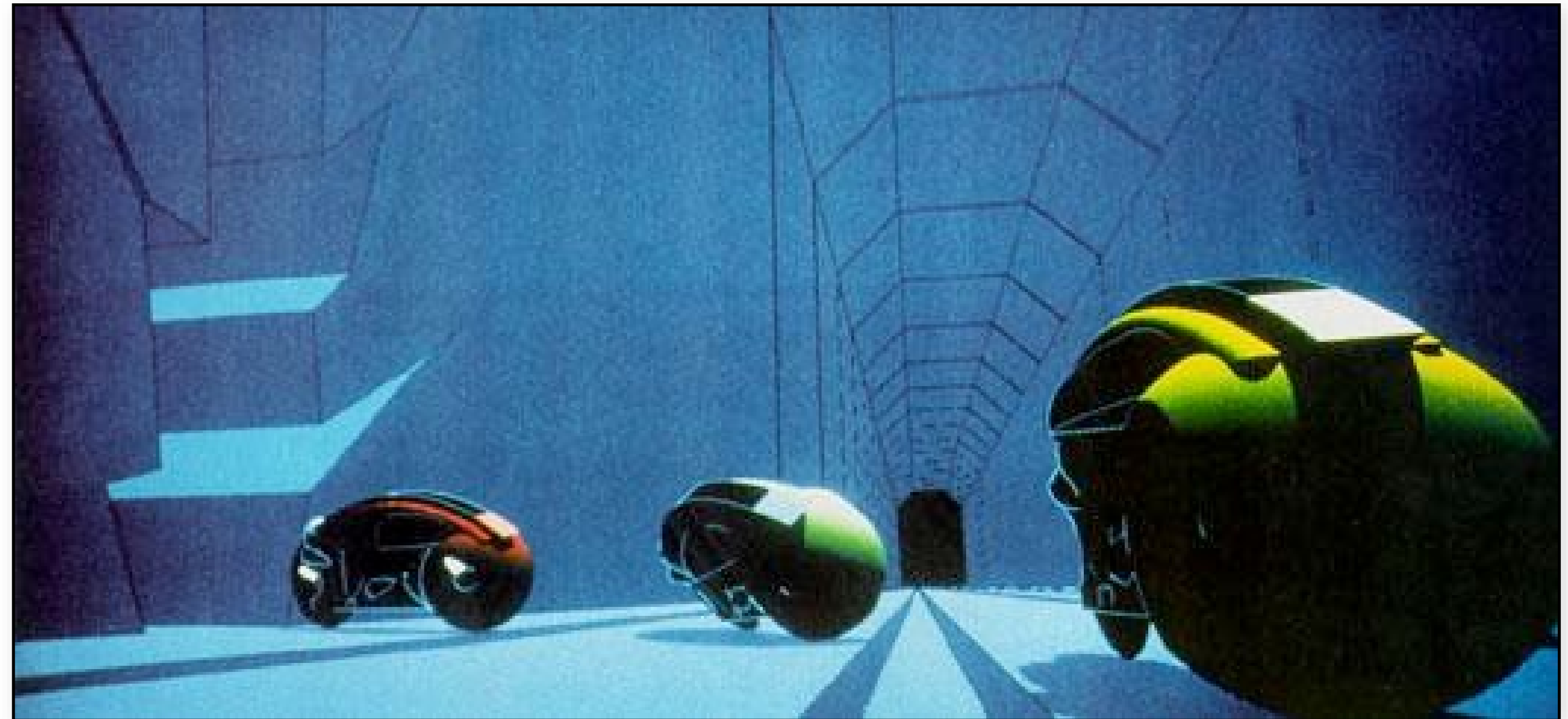
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Perlin noise, invented by Ken Perlin in 1982

- First used in the movie Tron!

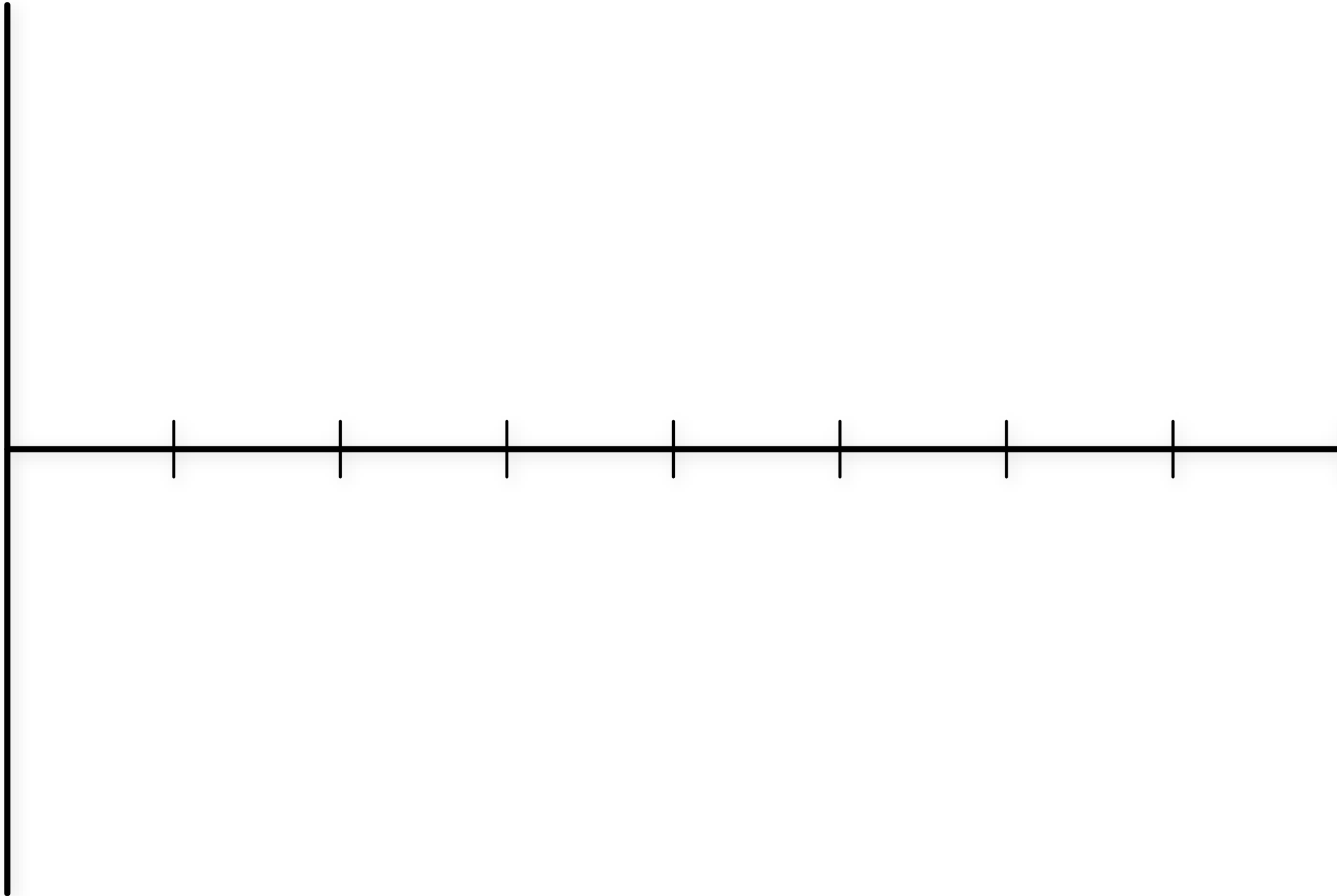
Store random vectors/gradients on lattice

- Use Hermite interp.
- a.k.a. “gradient noise”



# Classic Perlin noise

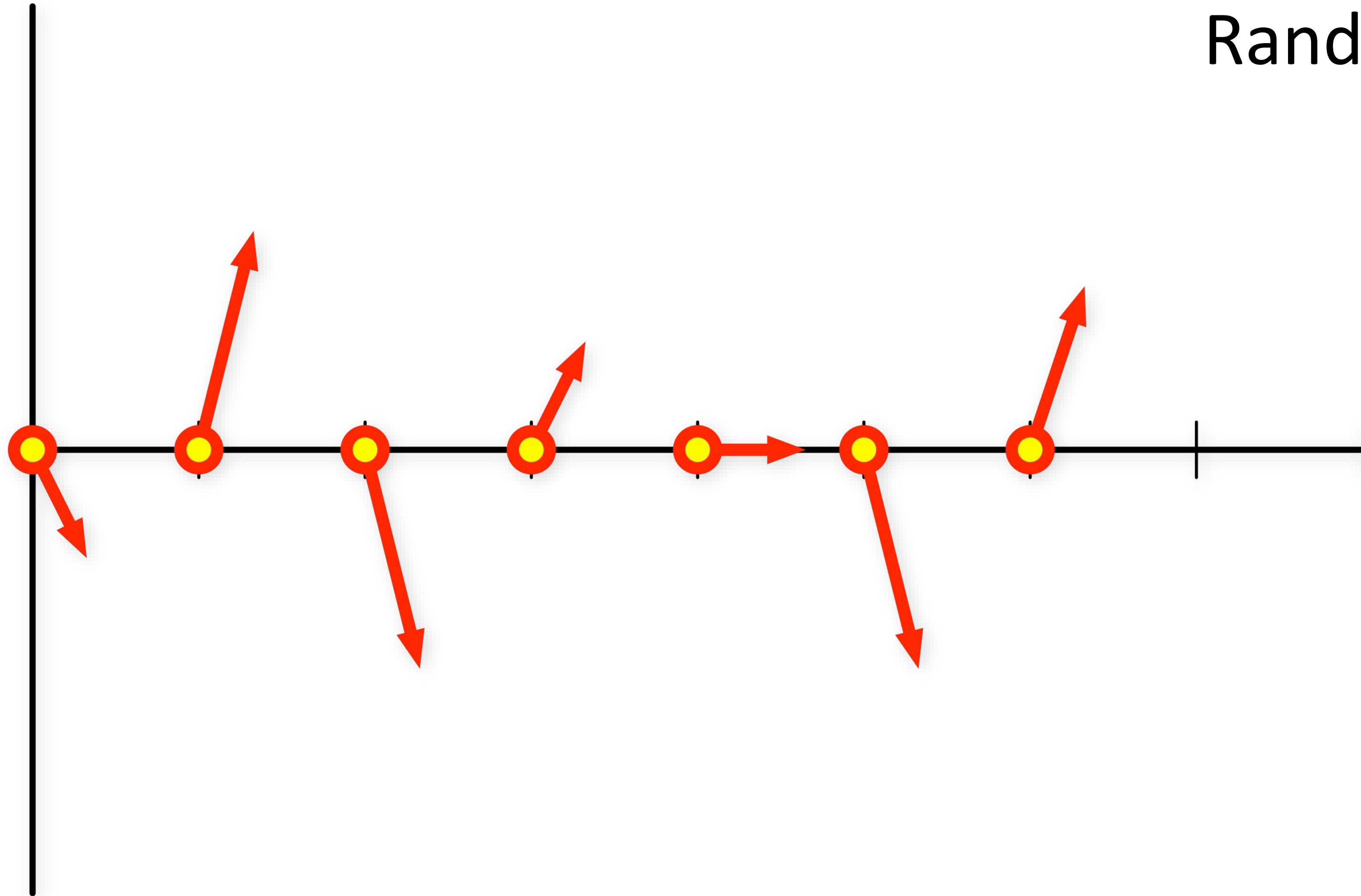
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# Classic Perlin noise

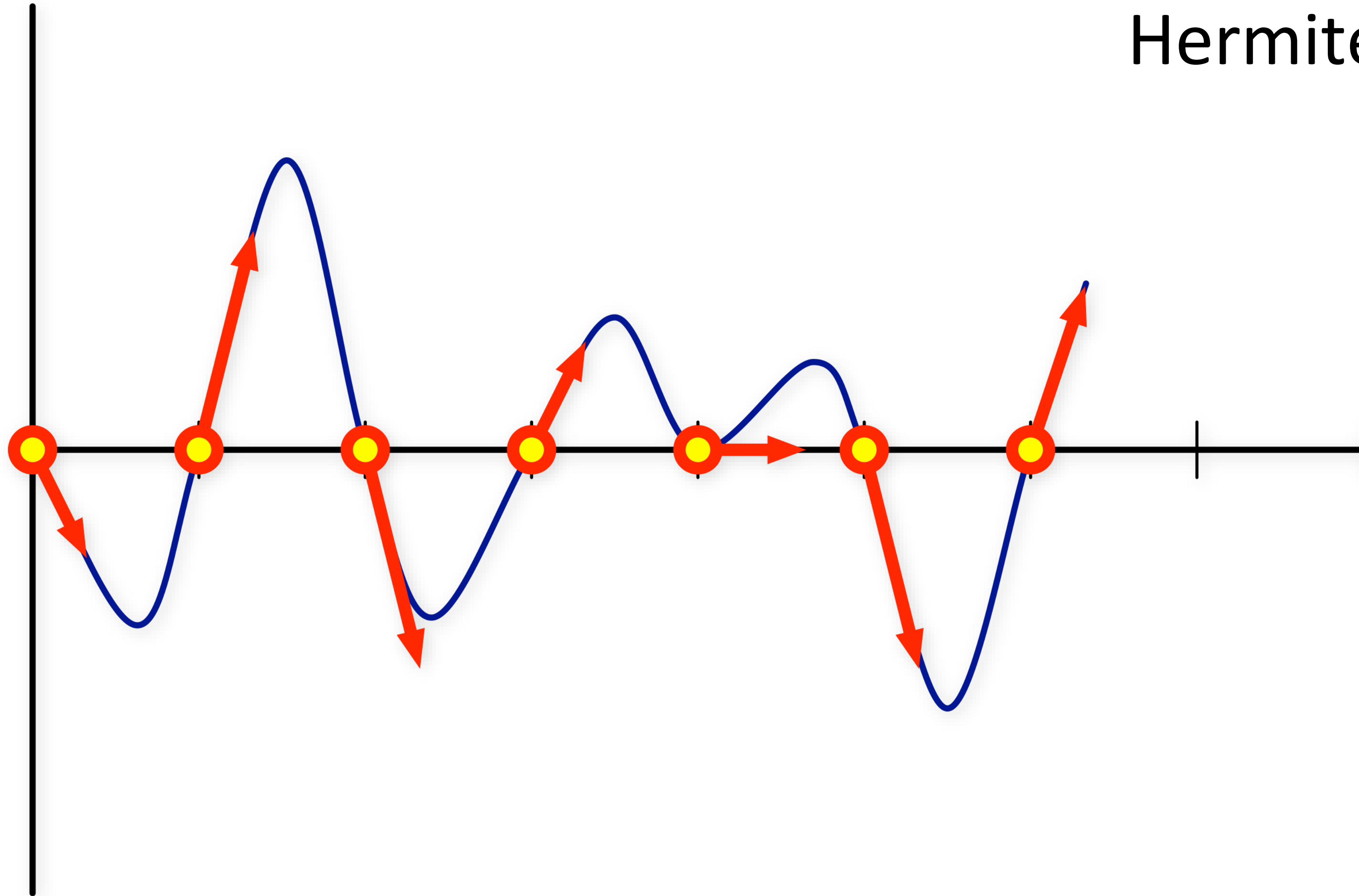
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Random gradients on grid



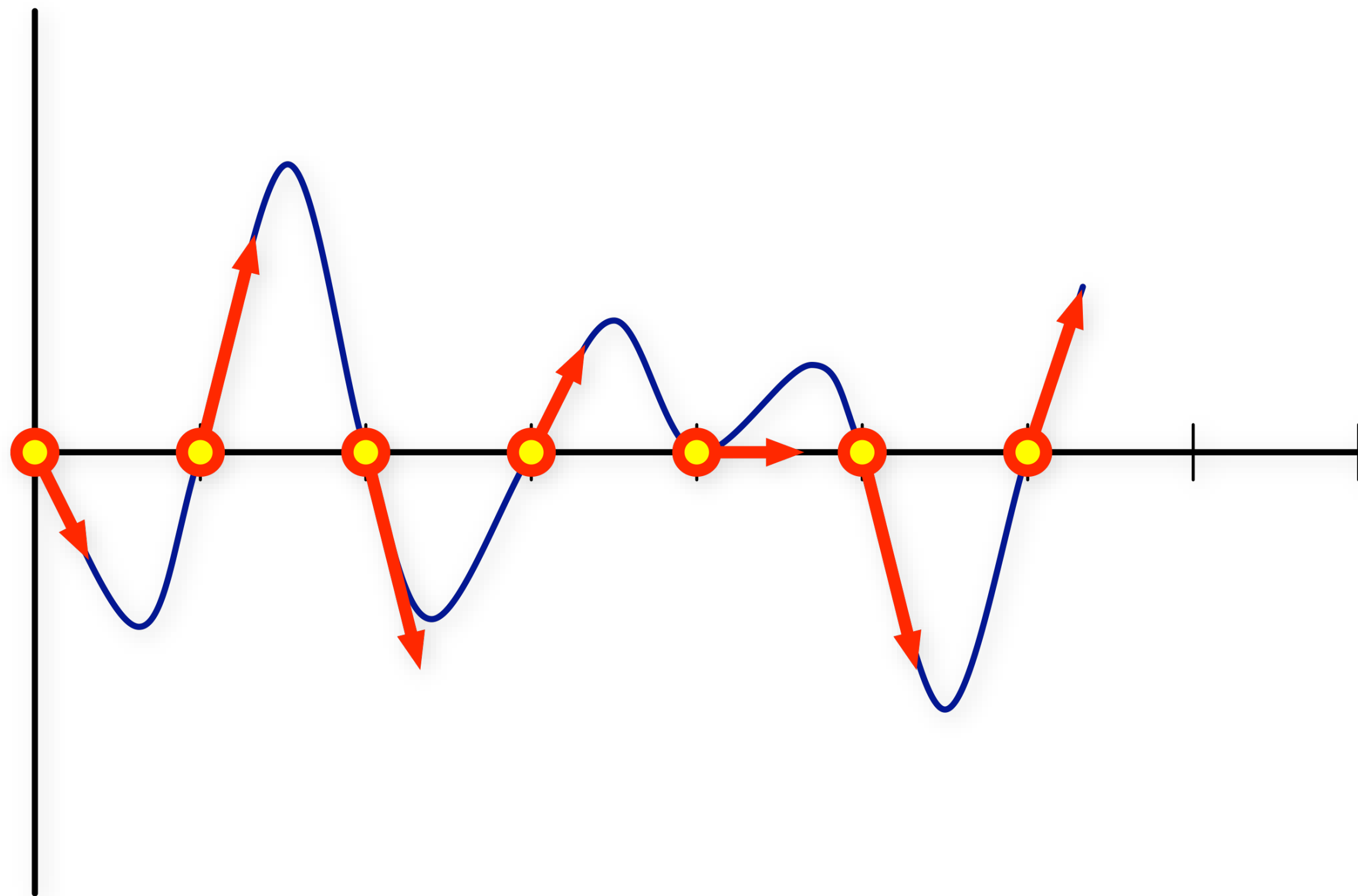
# Classic Perlin noise

Hermite-interpolated values

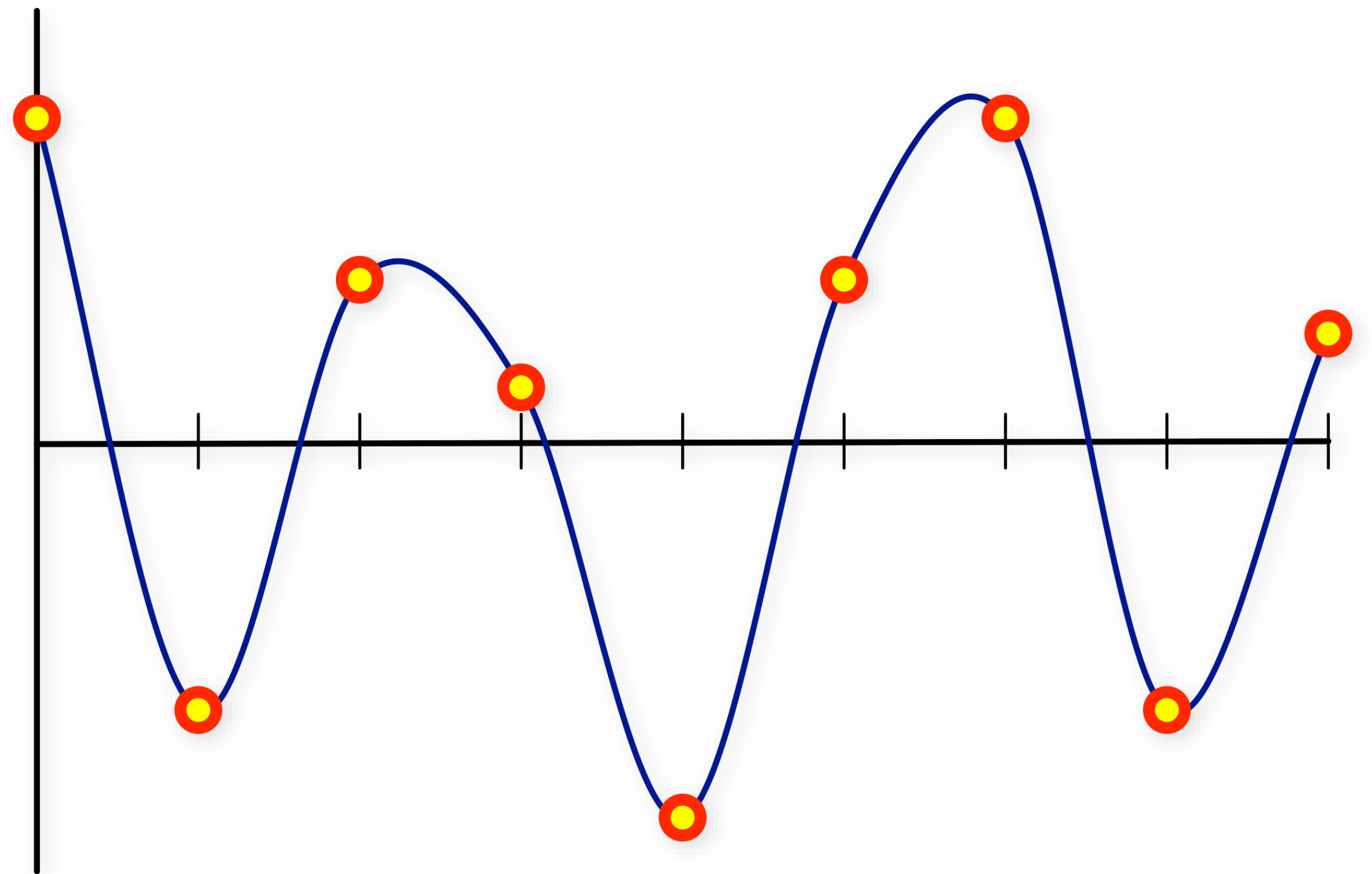


# Perlin noise vs. value noise

Perlin Noise  
(gradient noise)



Cubic Value Noise



Why is Perlin noise better?

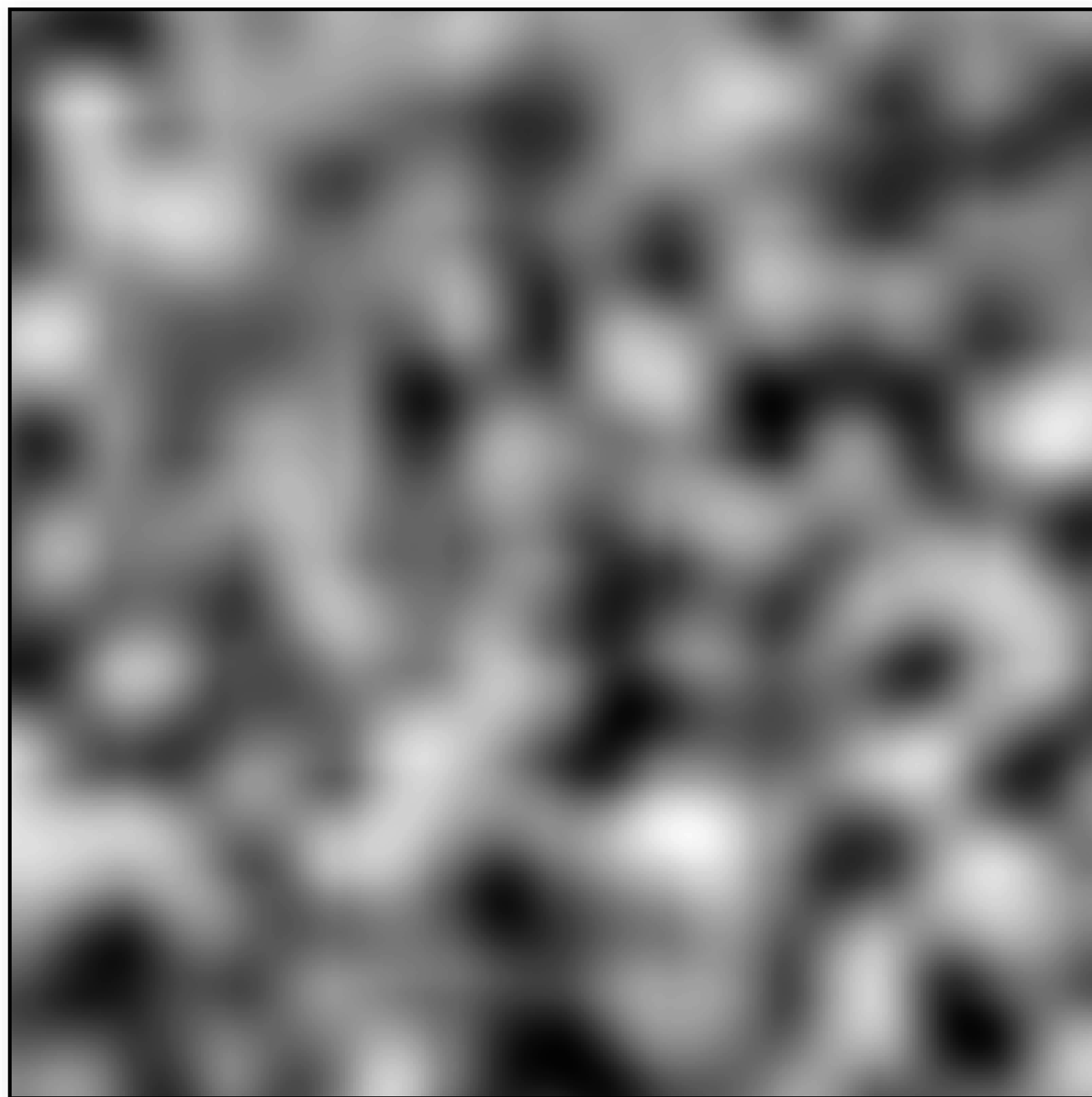


# Perlin noise

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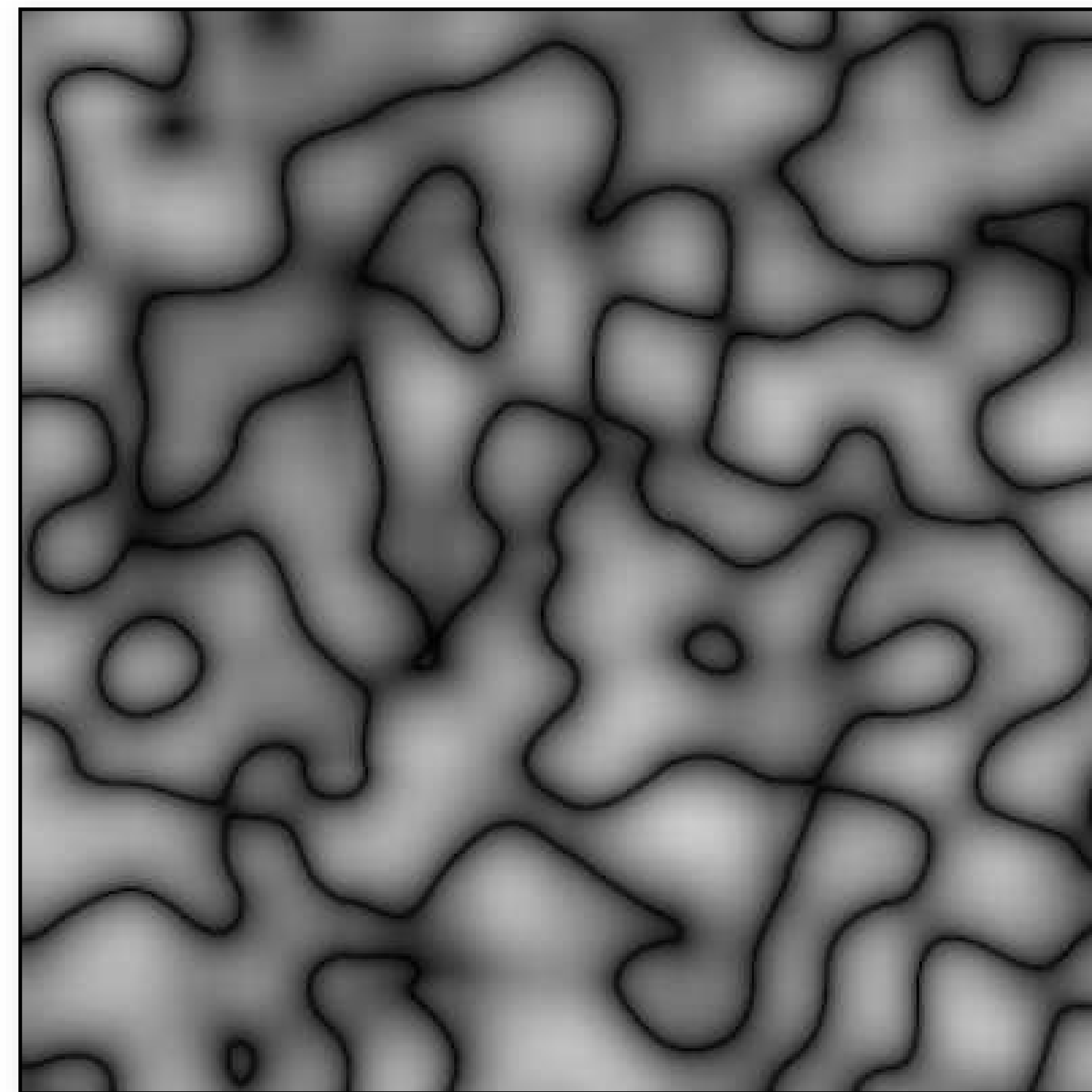
Typically signed by default,  $\sim$ in  $[-1,1]$  with a mean of 0

offset/scale to put into  $[0,1]$  range



$$(\text{noise}(\mathbf{p}) + 1) / 2$$

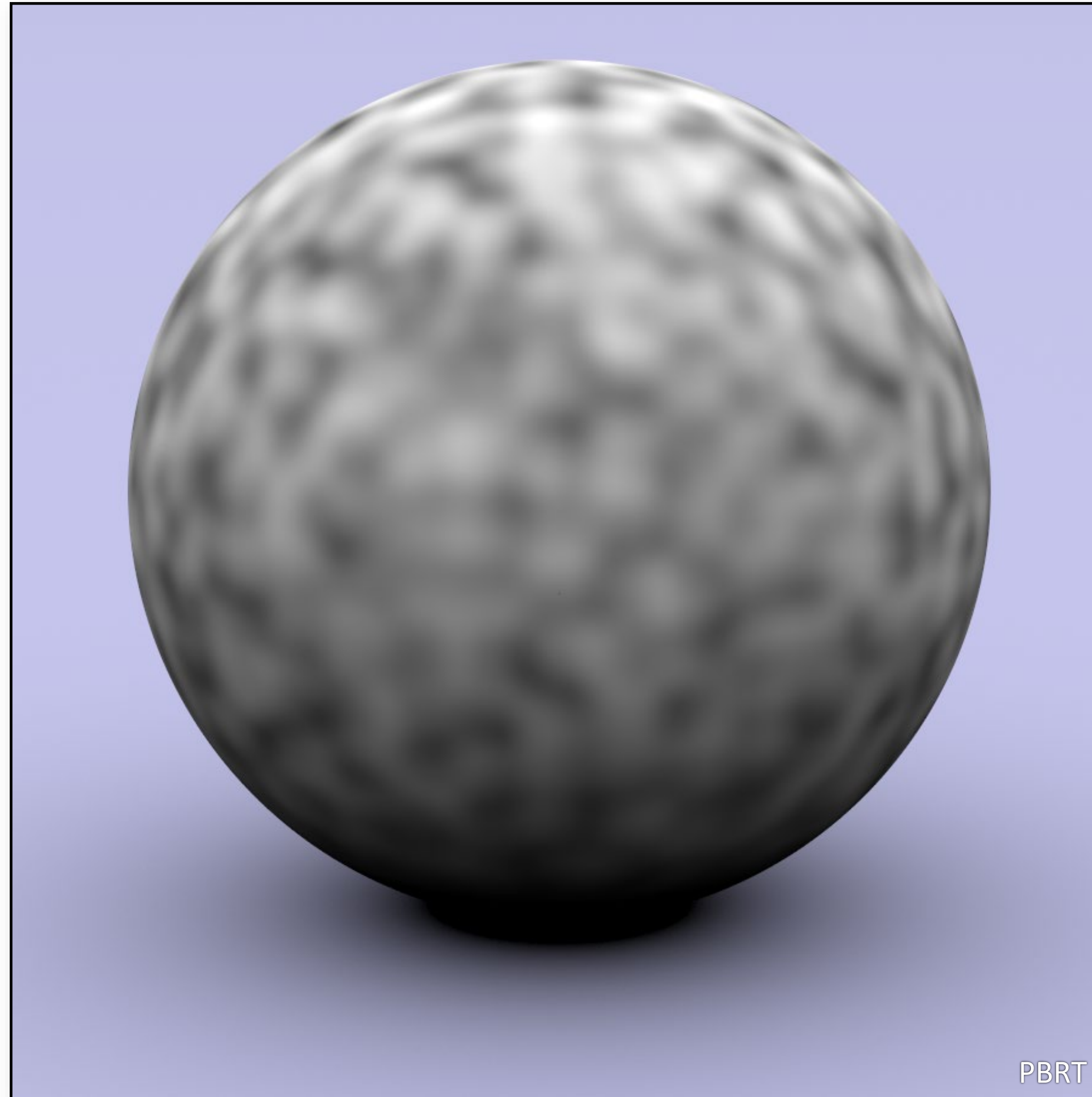
take absolute value



$$|\text{noise}(\mathbf{p})|$$

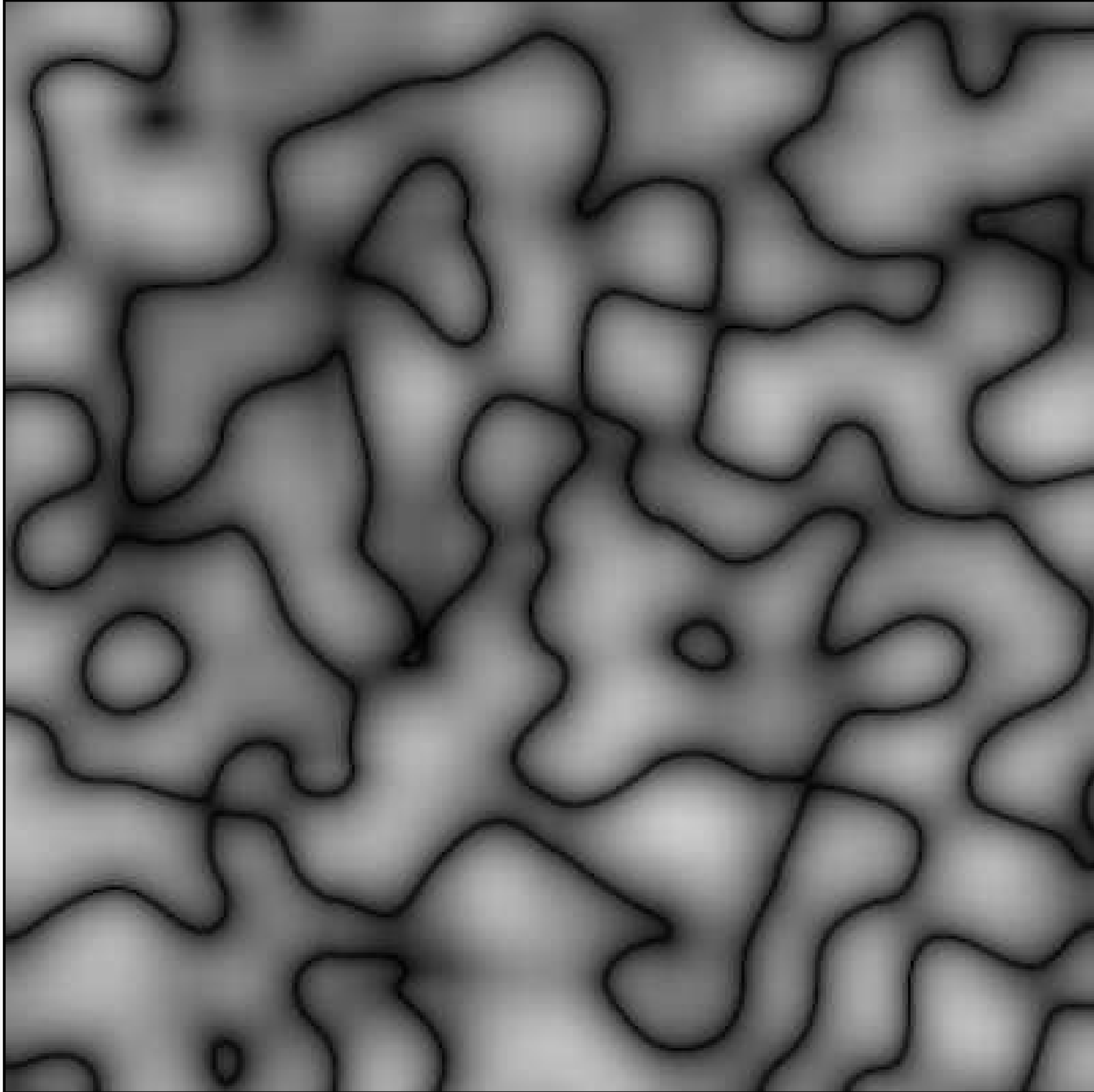
# 3D Perlin noise

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# Absolute value of noise

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# Perlin noise

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Change frequency: ?

Change amplitude: ?

# Perlin noise

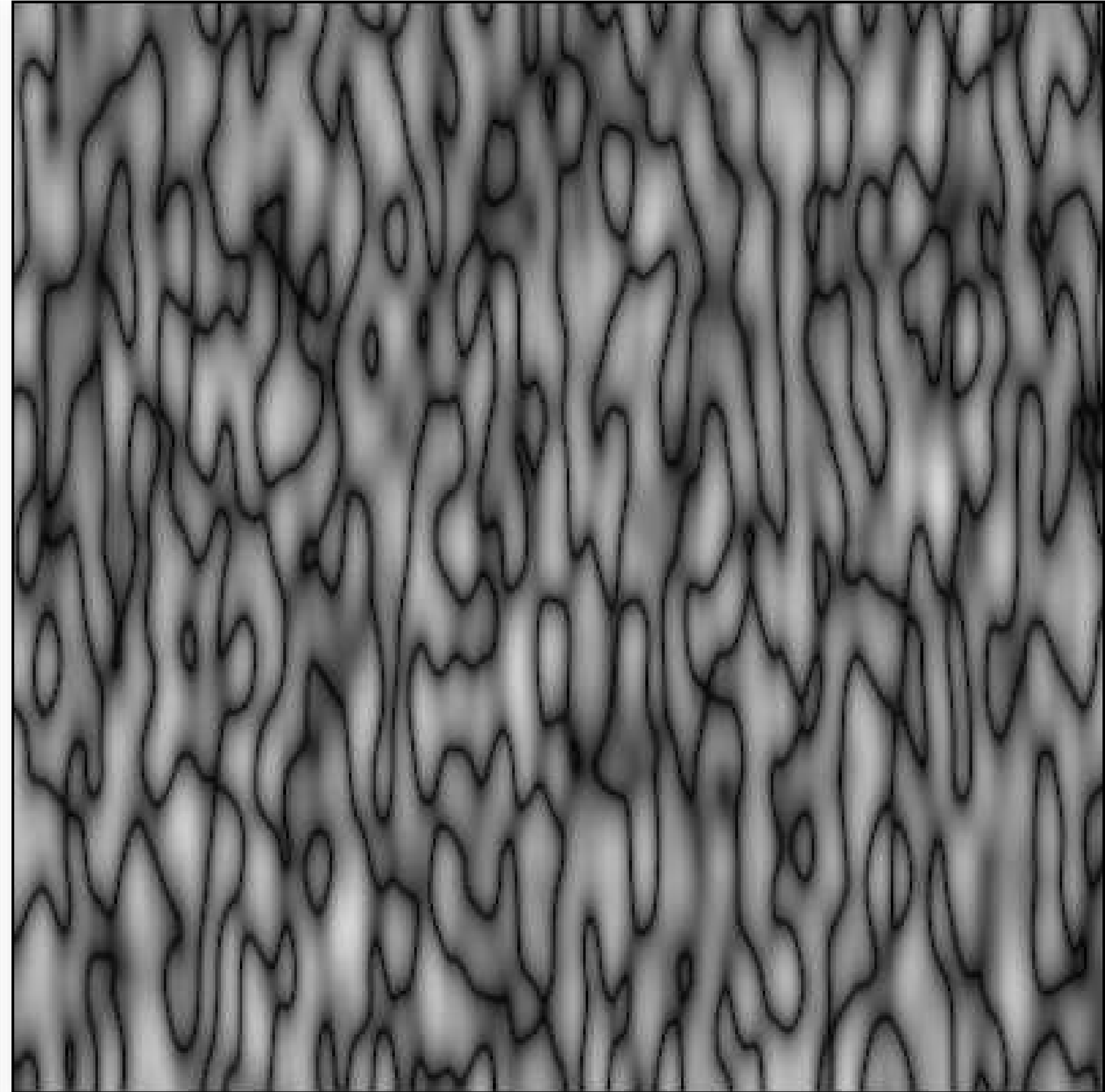
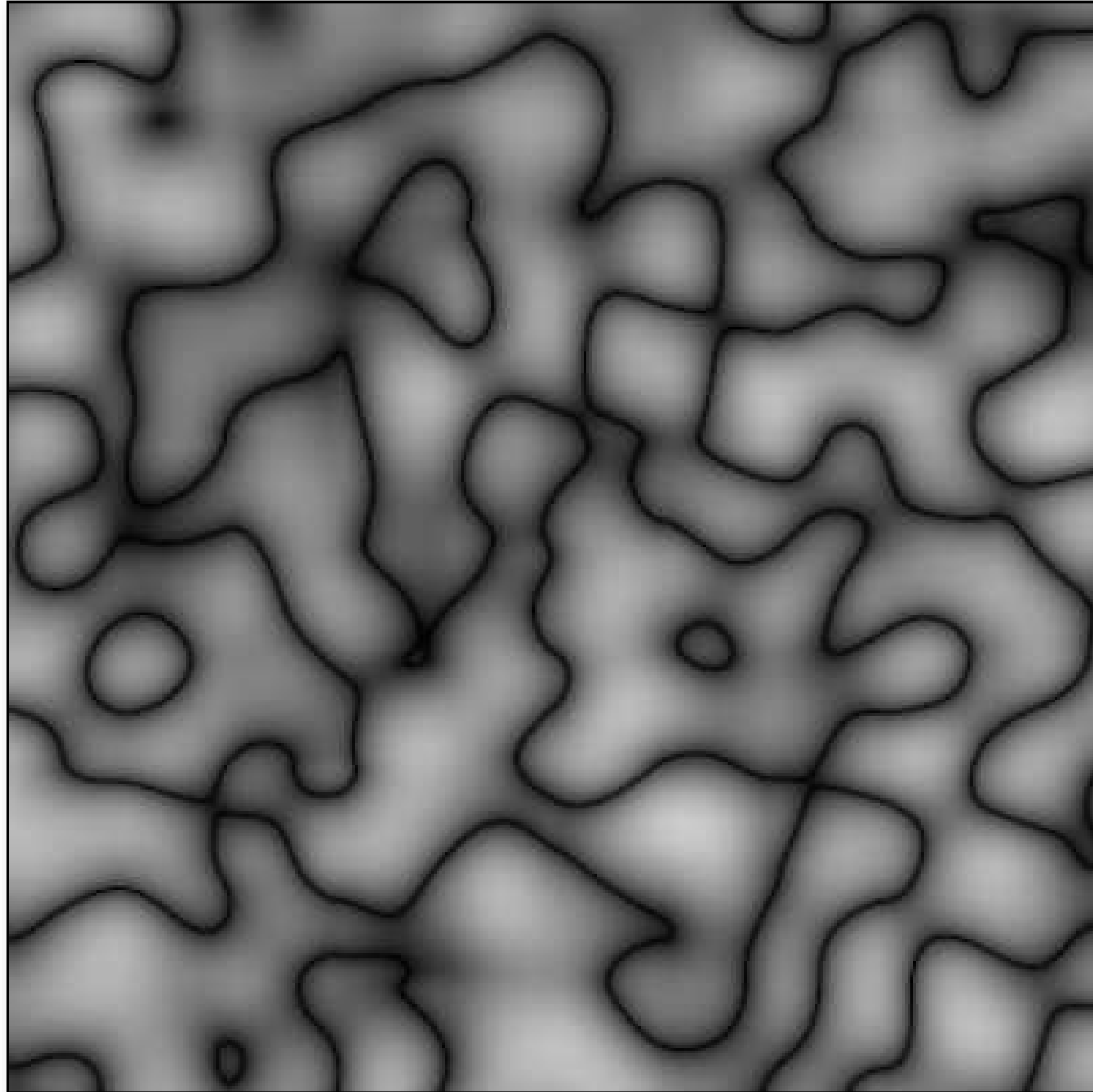
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Change frequency:  $\text{noise}(10 * x)$

Change amplitude:  $10 * \text{noise}(x)$

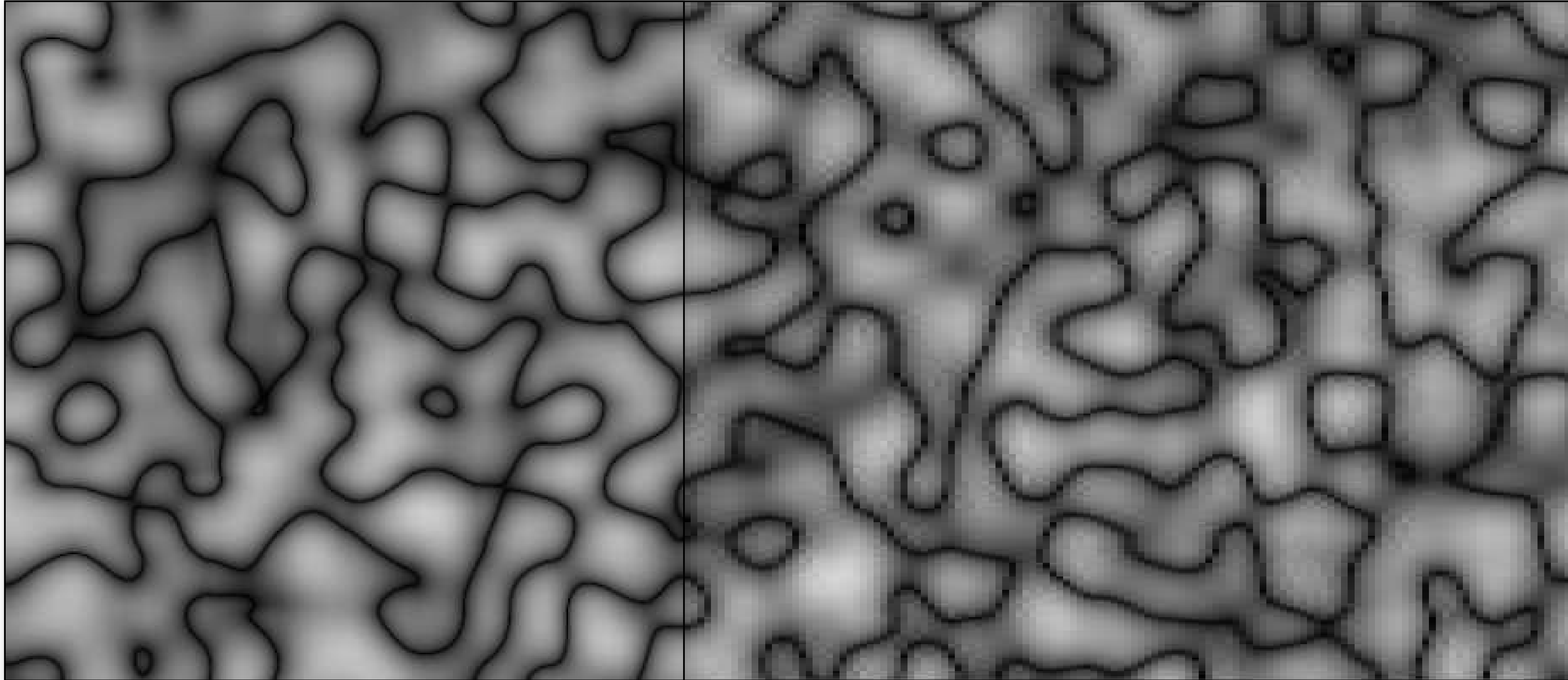
# Absolute value of noise

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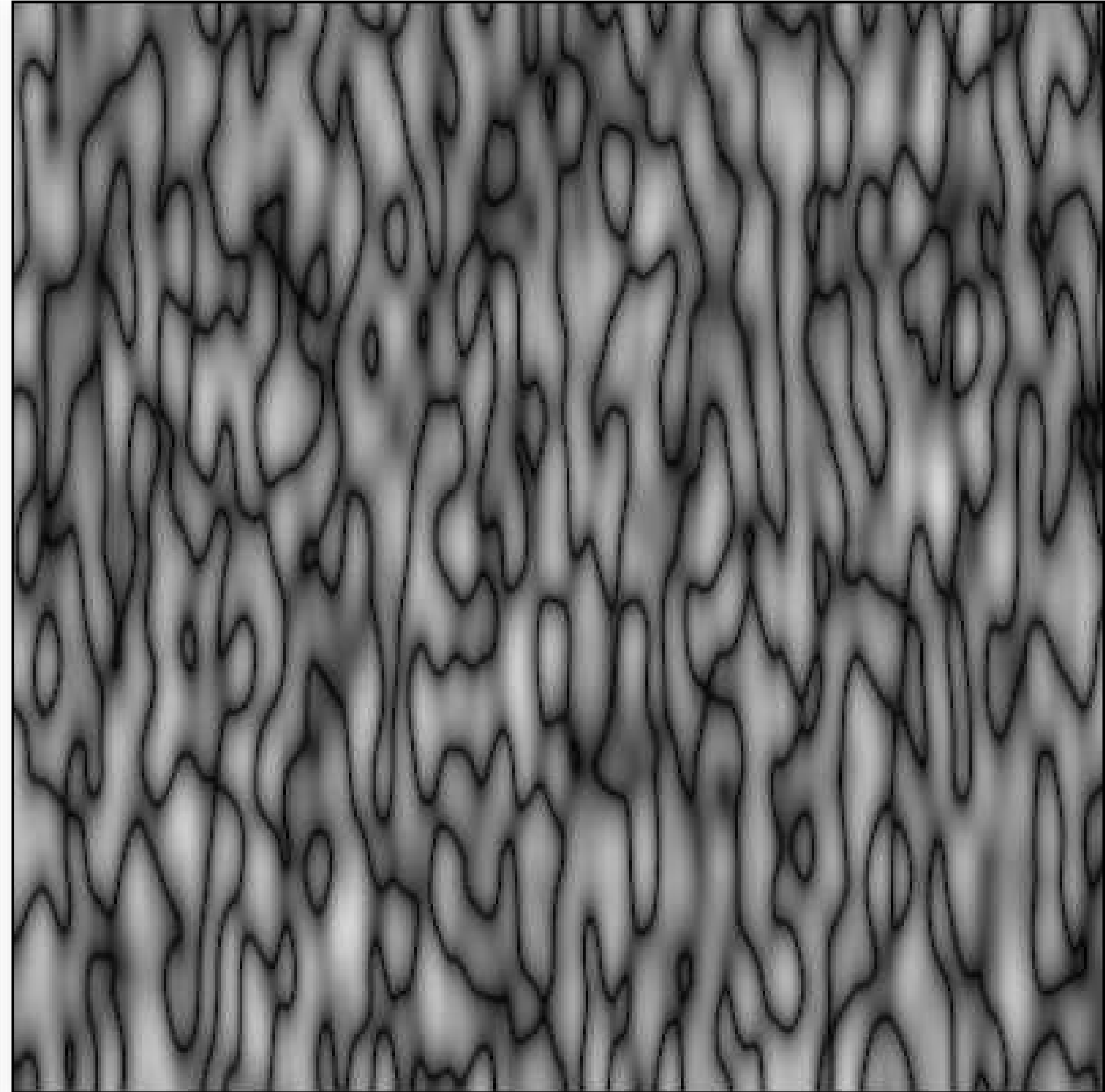
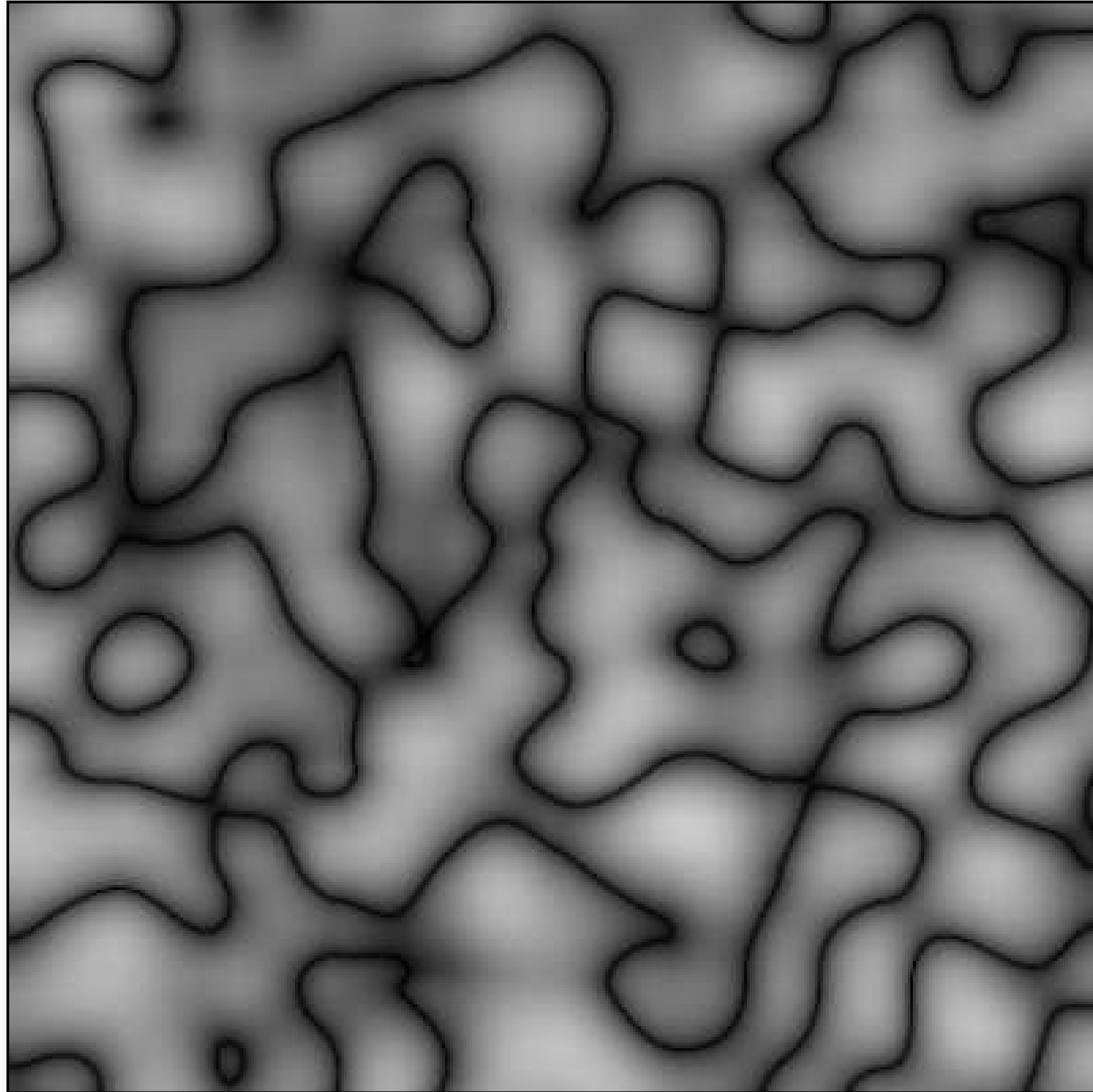
# Absolute value of noise

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# Absolute value of noise

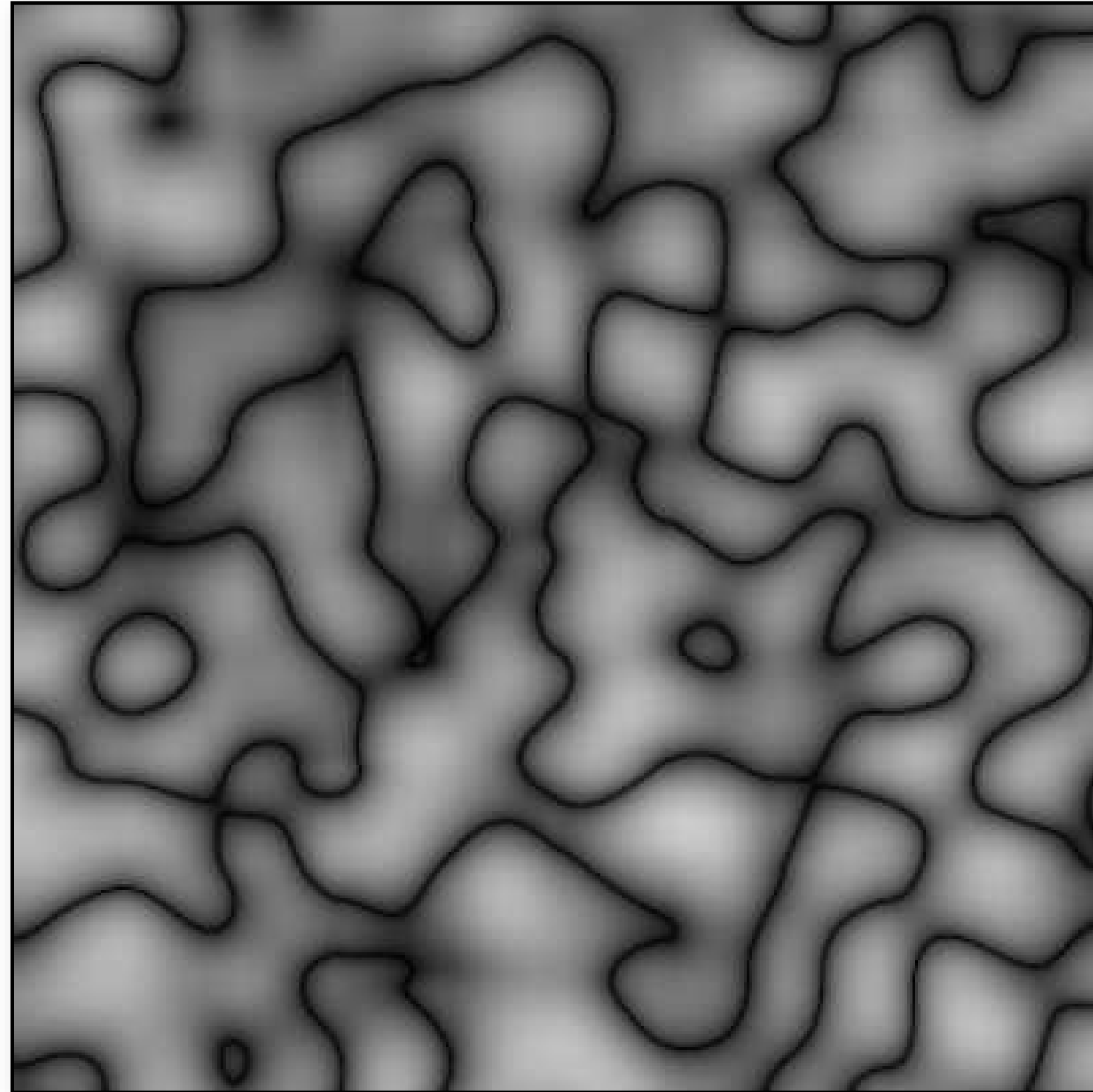
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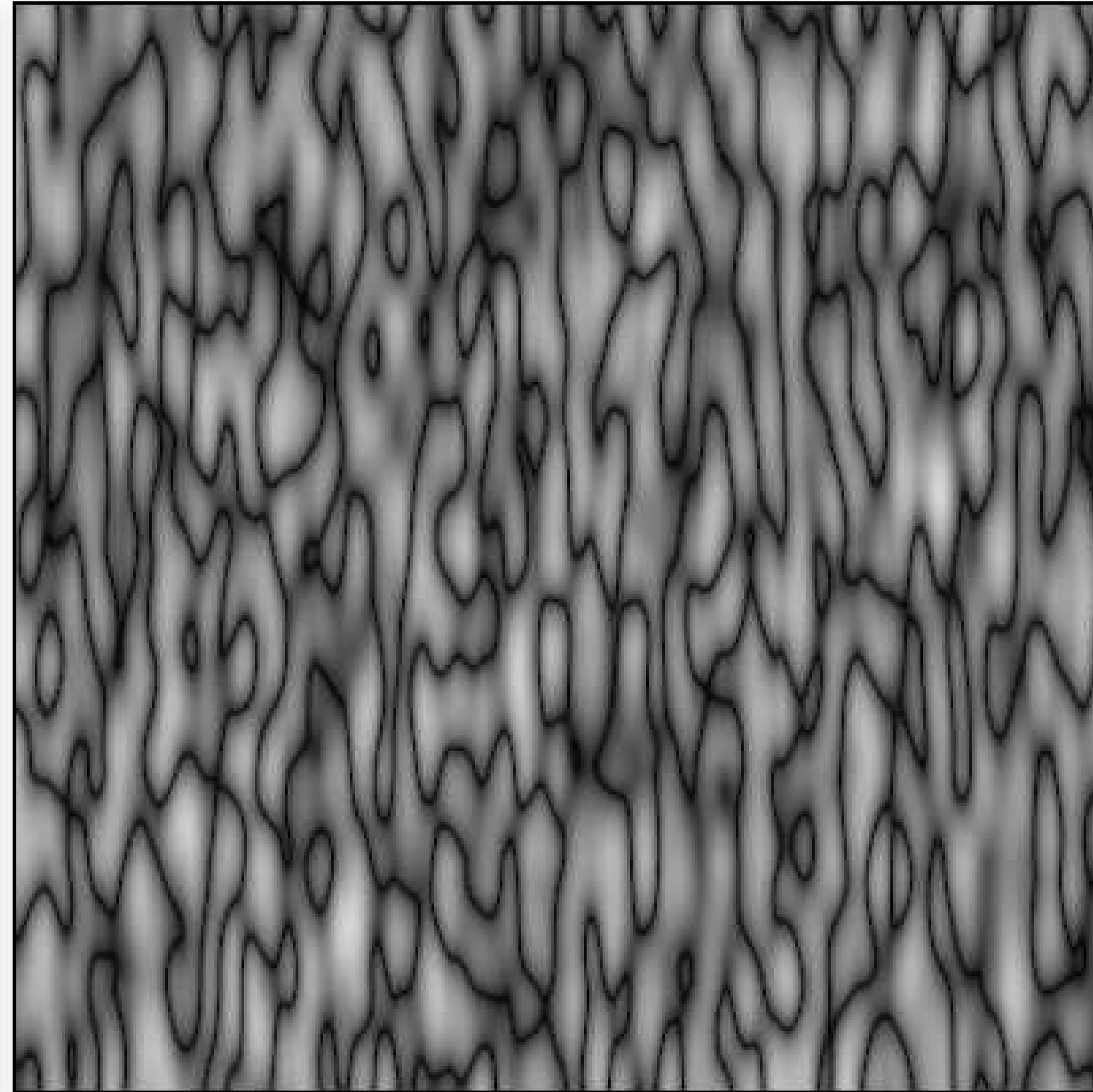


# Absolute value of noise

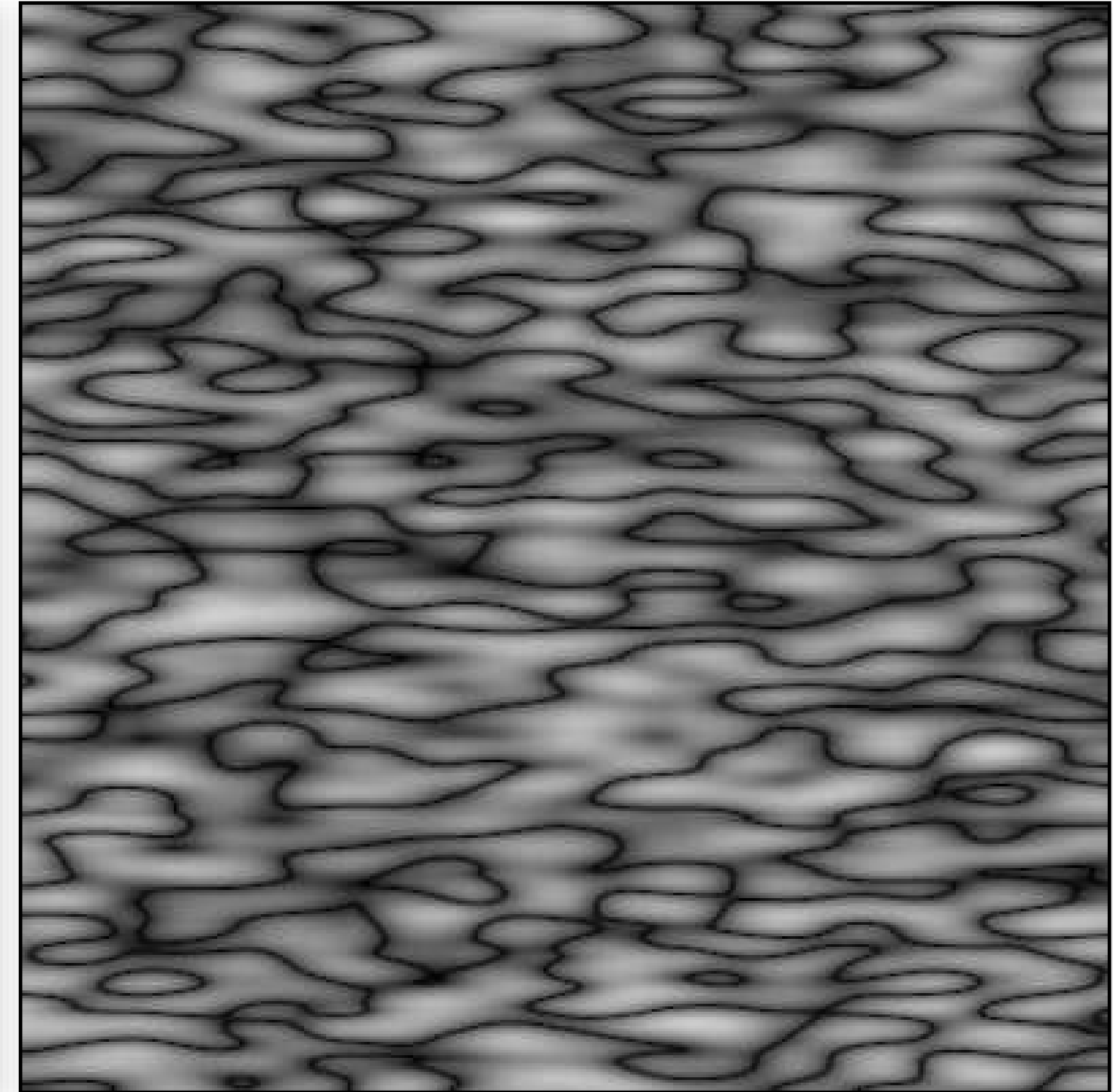
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$|\text{noise}(\mathbf{p})|$



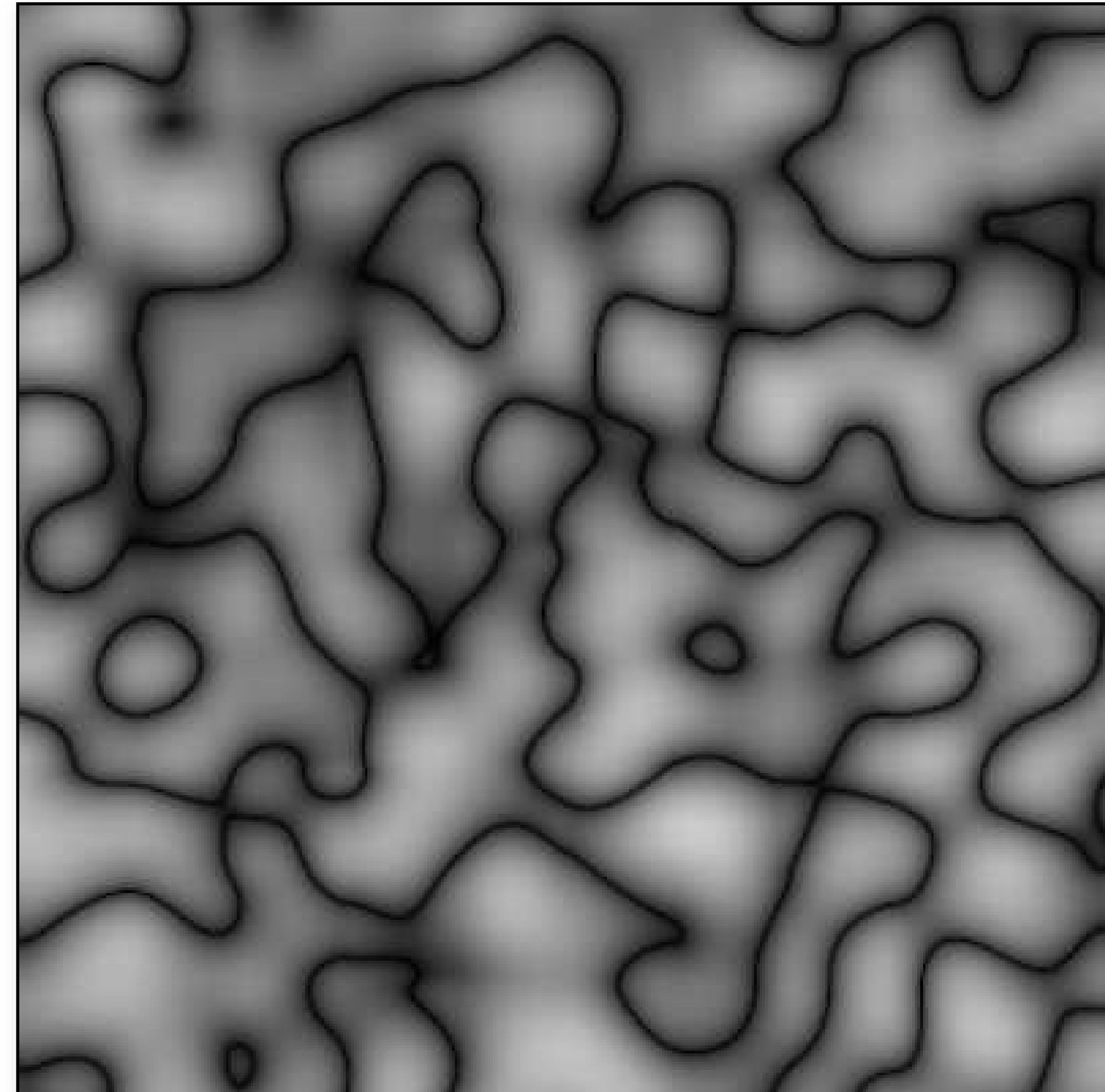
$|\text{noise}(4\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)|$



$|\text{noise}(\mathbf{p}_x, 4\mathbf{p}_y, \mathbf{p}_z)|$

# Perlin noise - limitations

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# Perlin noise - limitations

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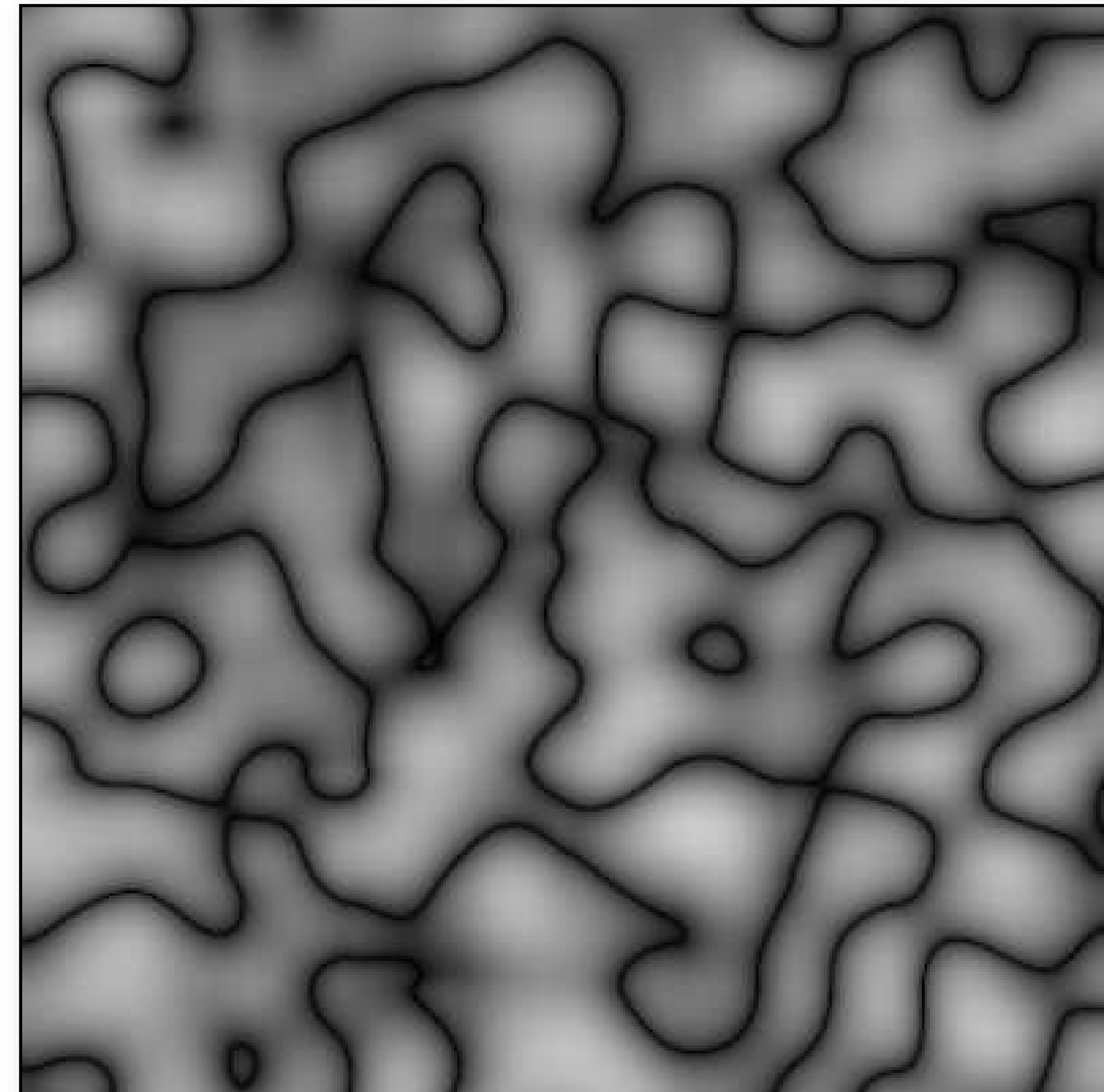
Lattice structure apparent for |noise|

- all lattice locations have value 0

Lookups faster, but still slow:

- Perlin is  $2^n$  for  $n$  dimensions instead of  $4^n$  for value noise
- other variations: simplex noise ( $O(n)$ )

Not quite rotation invariant



# More reading

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Fantastic explorable explanation by Andrew Kensler at Pixar

- [eastfarthing.com/blog/2015-04-21-noise](http://eastfarthing.com/blog/2015-04-21-noise)

# Spectral synthesis

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Representing a complex function  $f_s(\mathbf{p})$  by a sum of weighted contributions from a scaled function  $f(\mathbf{p})$ :

$$f_s(\mathbf{p}) = \sum_i w_i f(s_i \mathbf{p})$$

Called a “fractal sum” if  $w_i$  and  $s_i$  are set so:

- increasing frequencies have decreasing amplitude,  
e.g.:  $w_i = 2^{-i}$ ,  $s_i = 2^i$
- when  $s_i = 2^i$ , each term in summation is called an “octave”

What function  $f(\mathbf{p})$  should we use?

# fBm - fractional Brownian motion

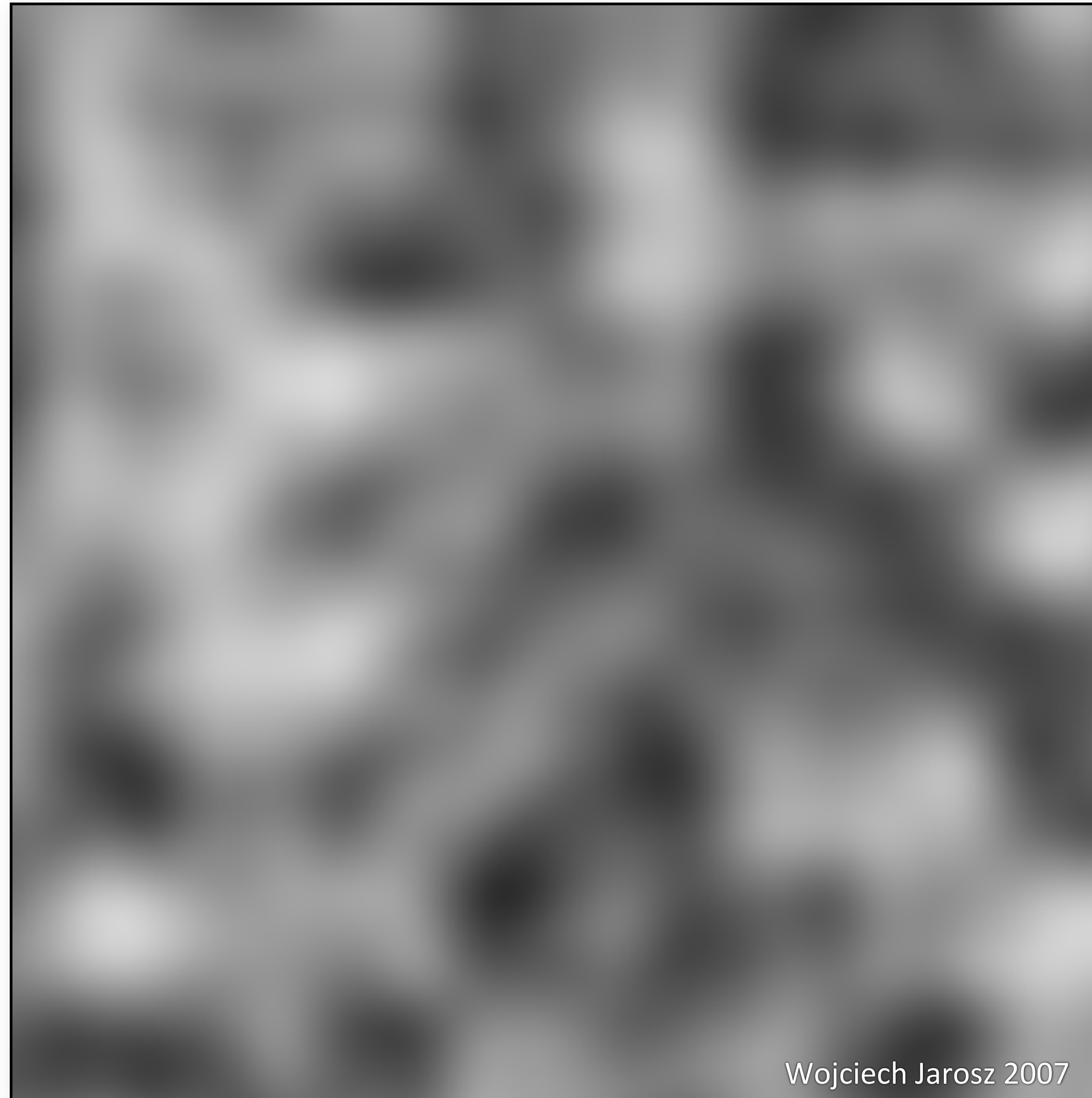
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In graphics:

- Fractal sum of Perlin noise functions
- “Fractal noise”

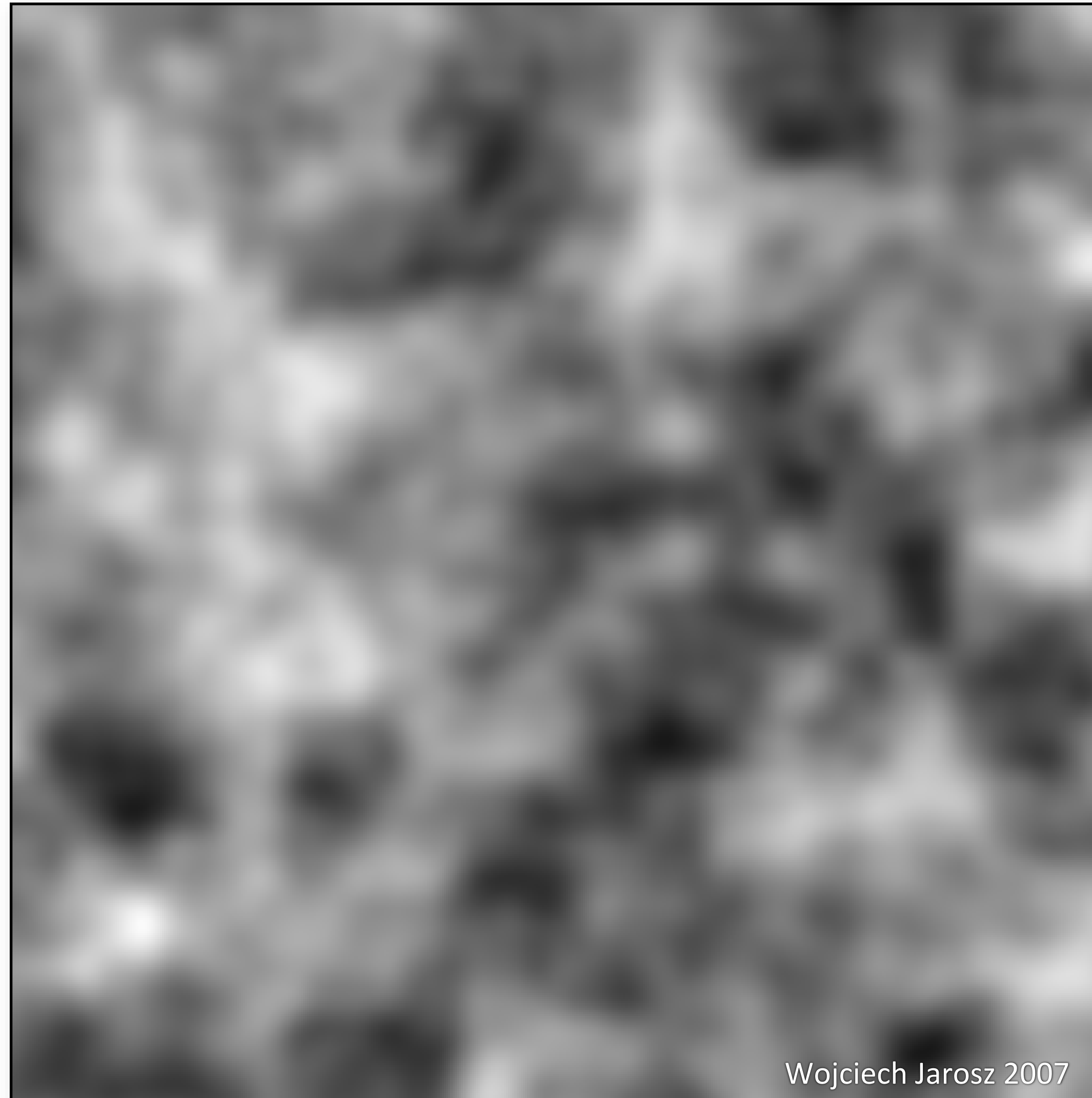
# fBm - 1 octave

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# fBm - 2 octaves

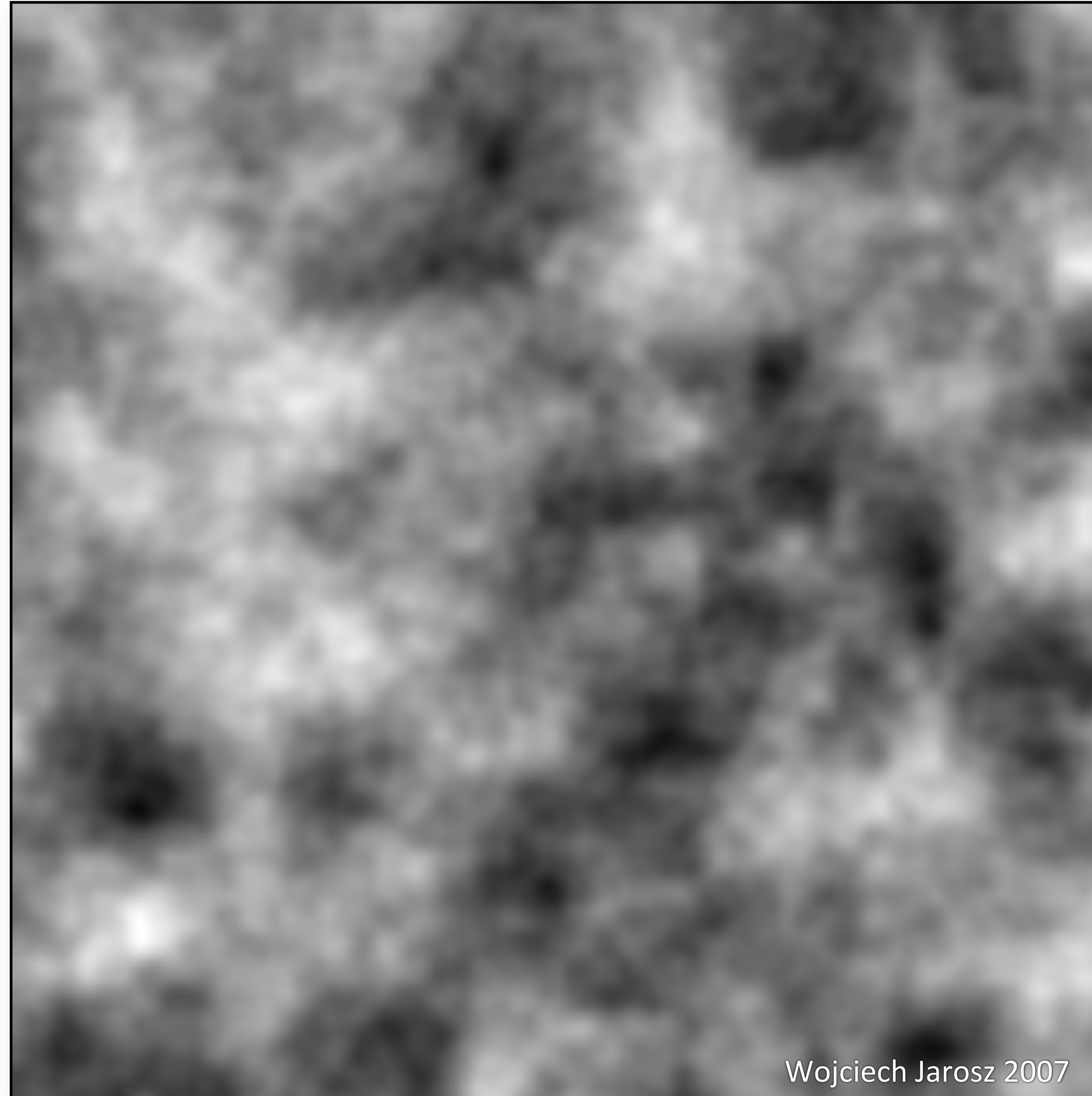
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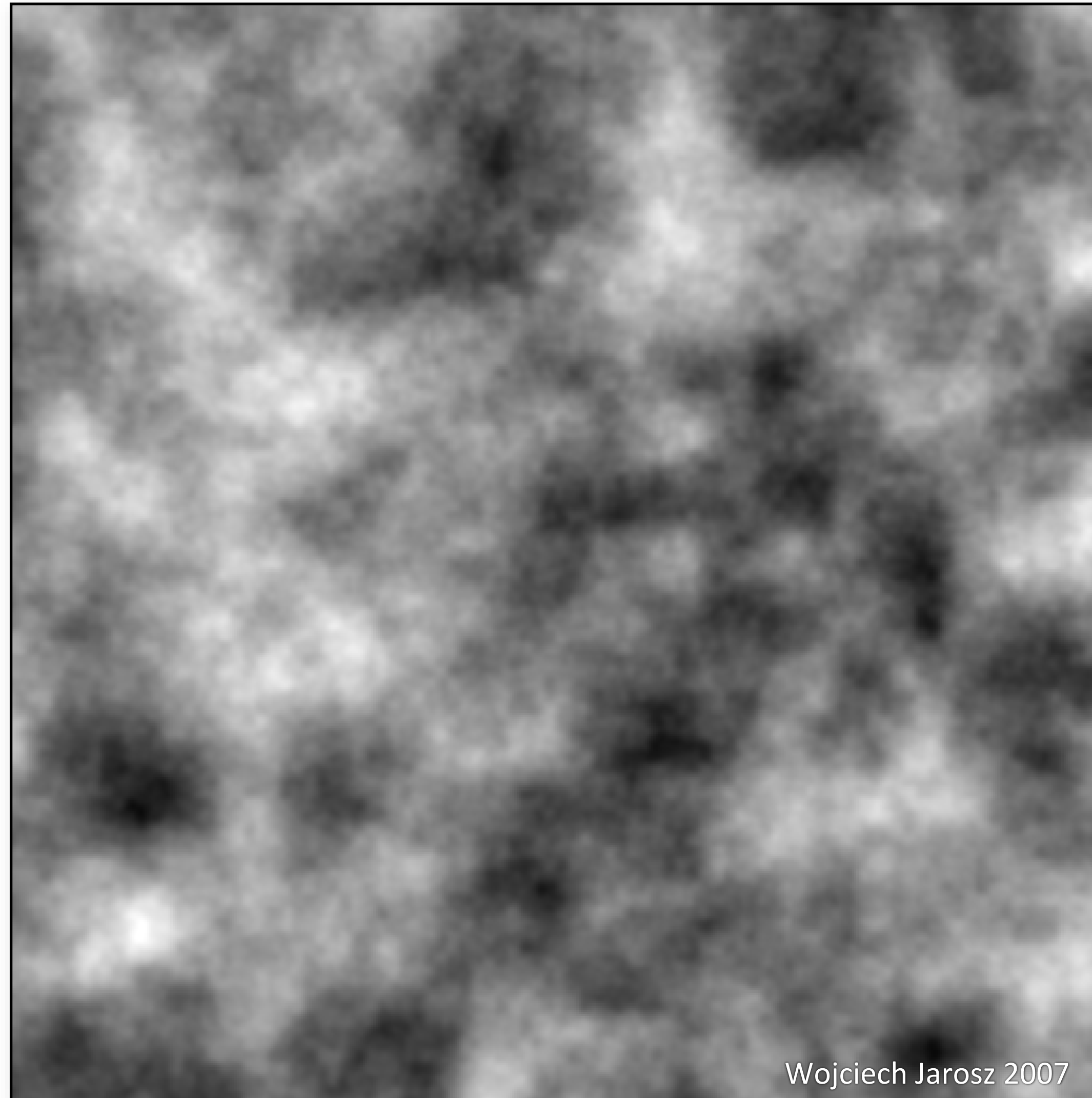
# fBm - 3 octaves

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# fBm - 4 octaves

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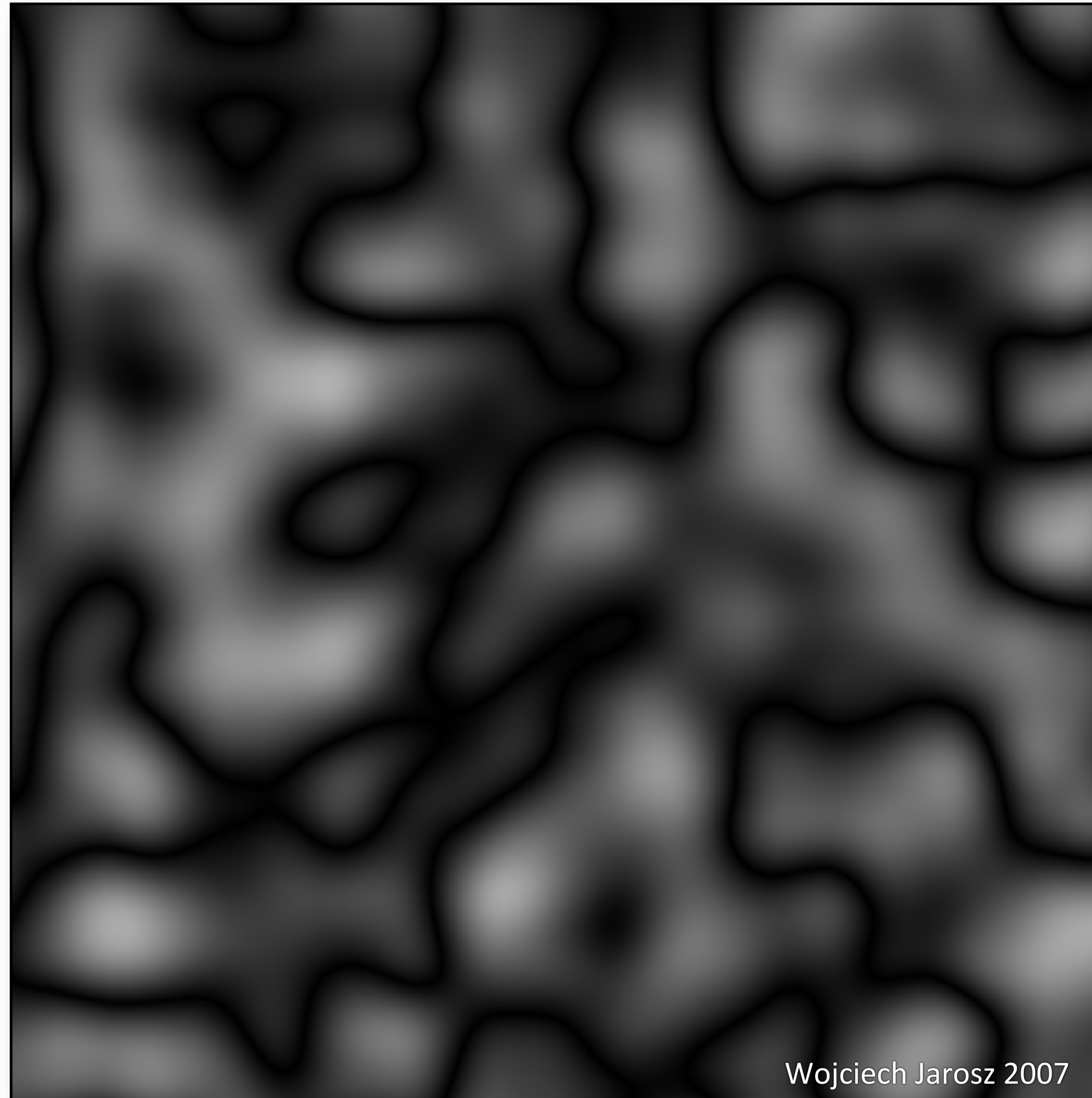
# Turbulence

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Same as fBm, but sum absolute value of noise function

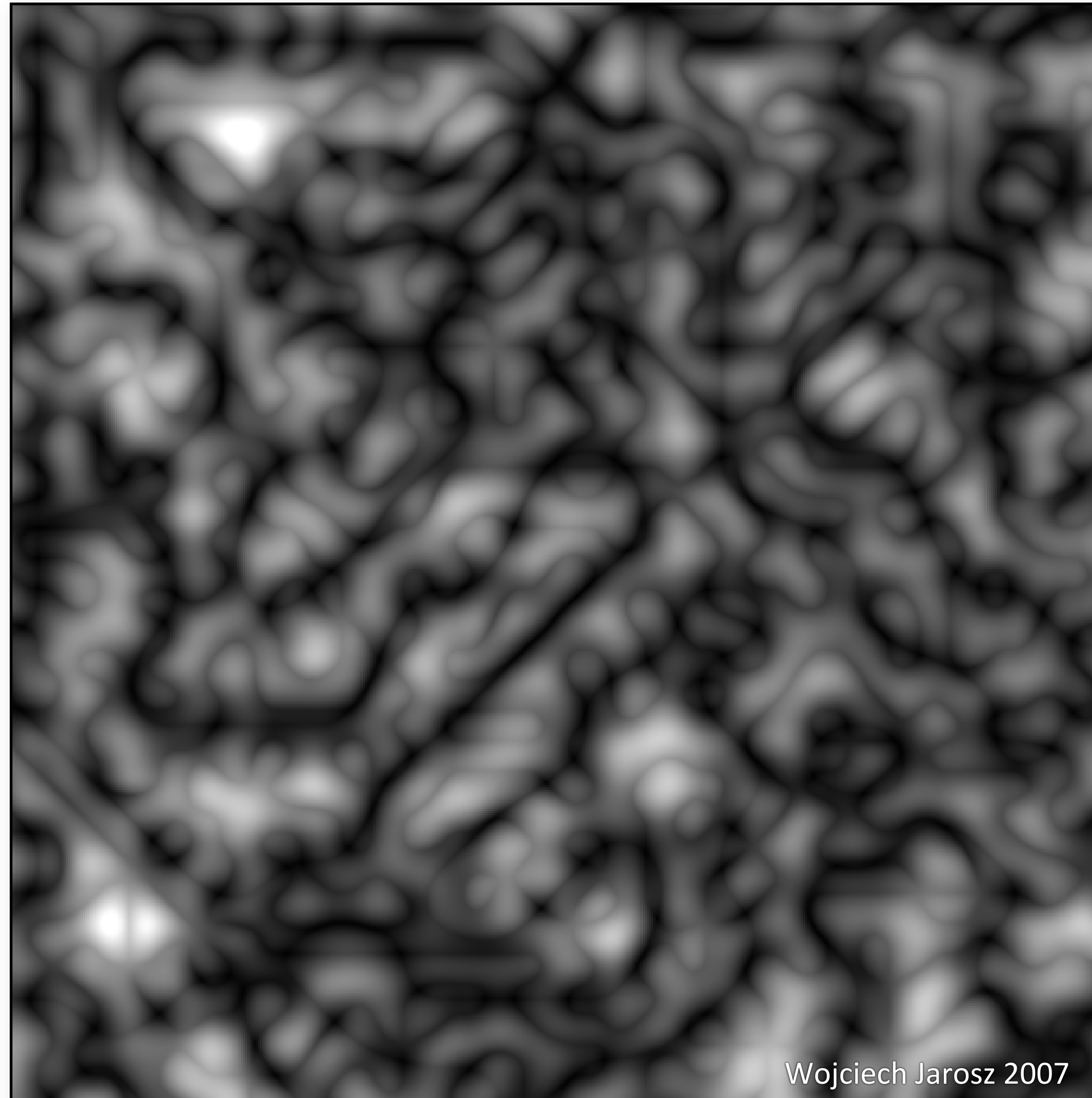
# Turbulence - 1 octave

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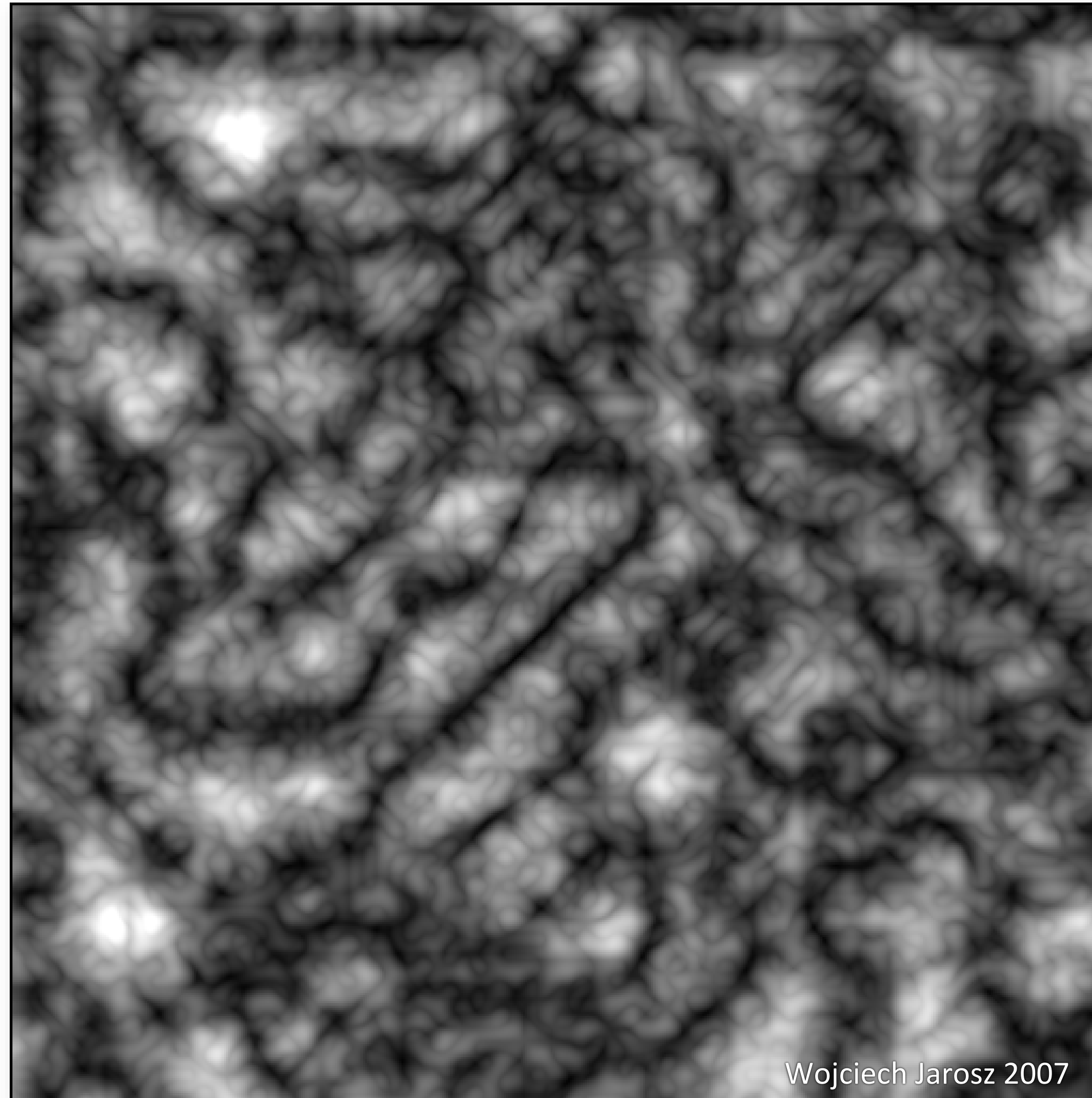
# Turbulence - 2 octaves

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# Turbulence - 3 octaves

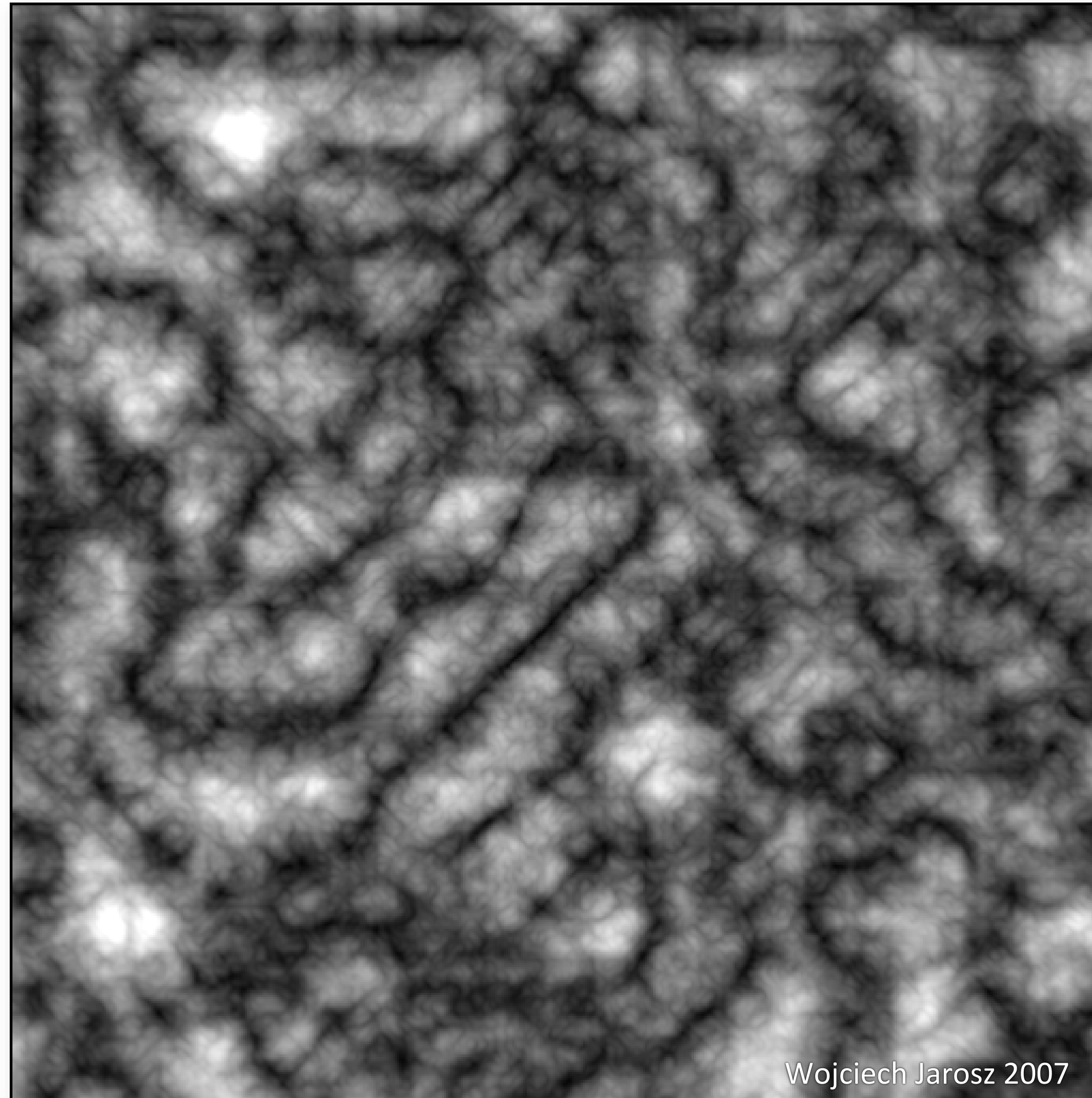
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Wojciech Jarosz 2007

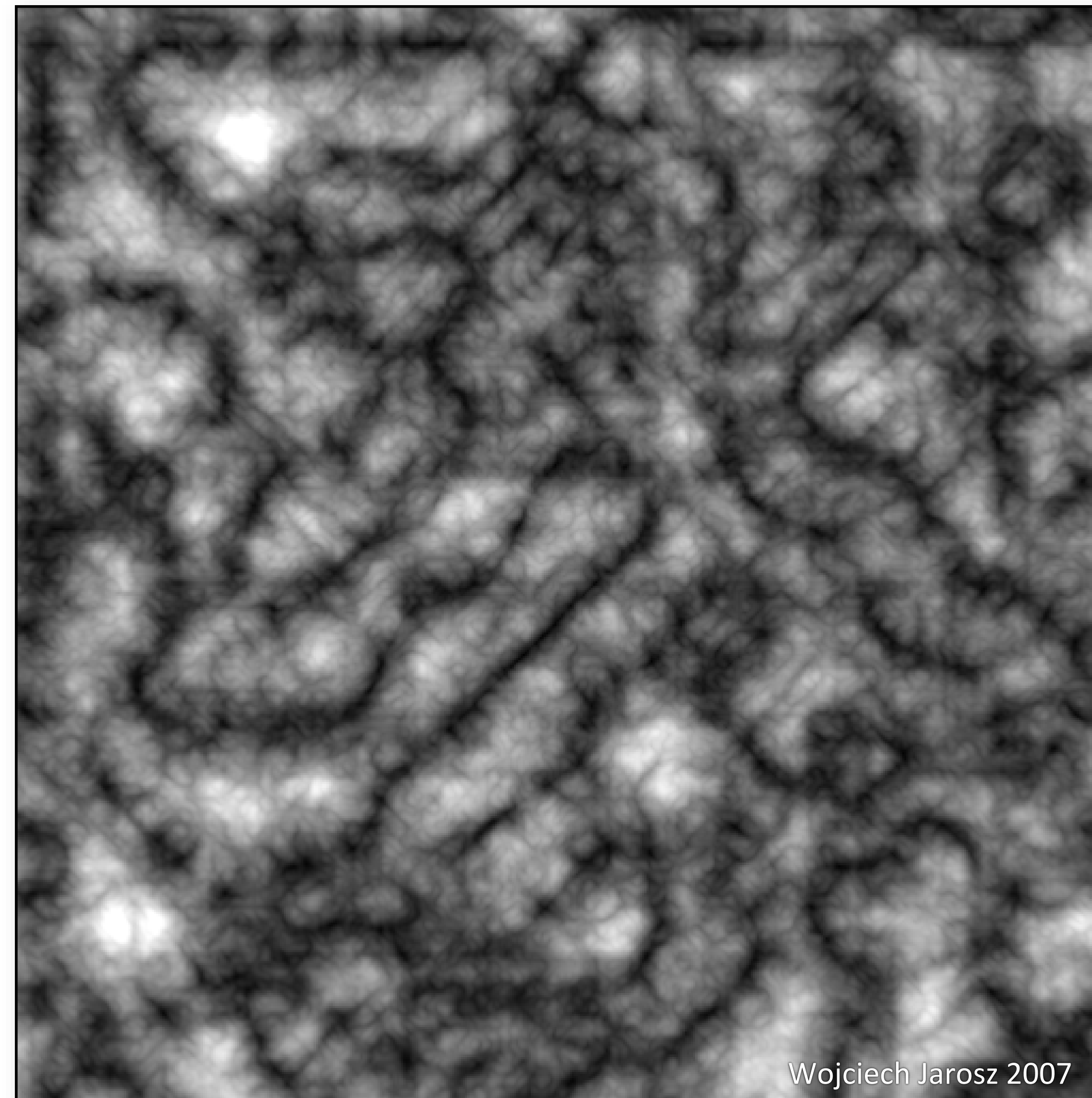
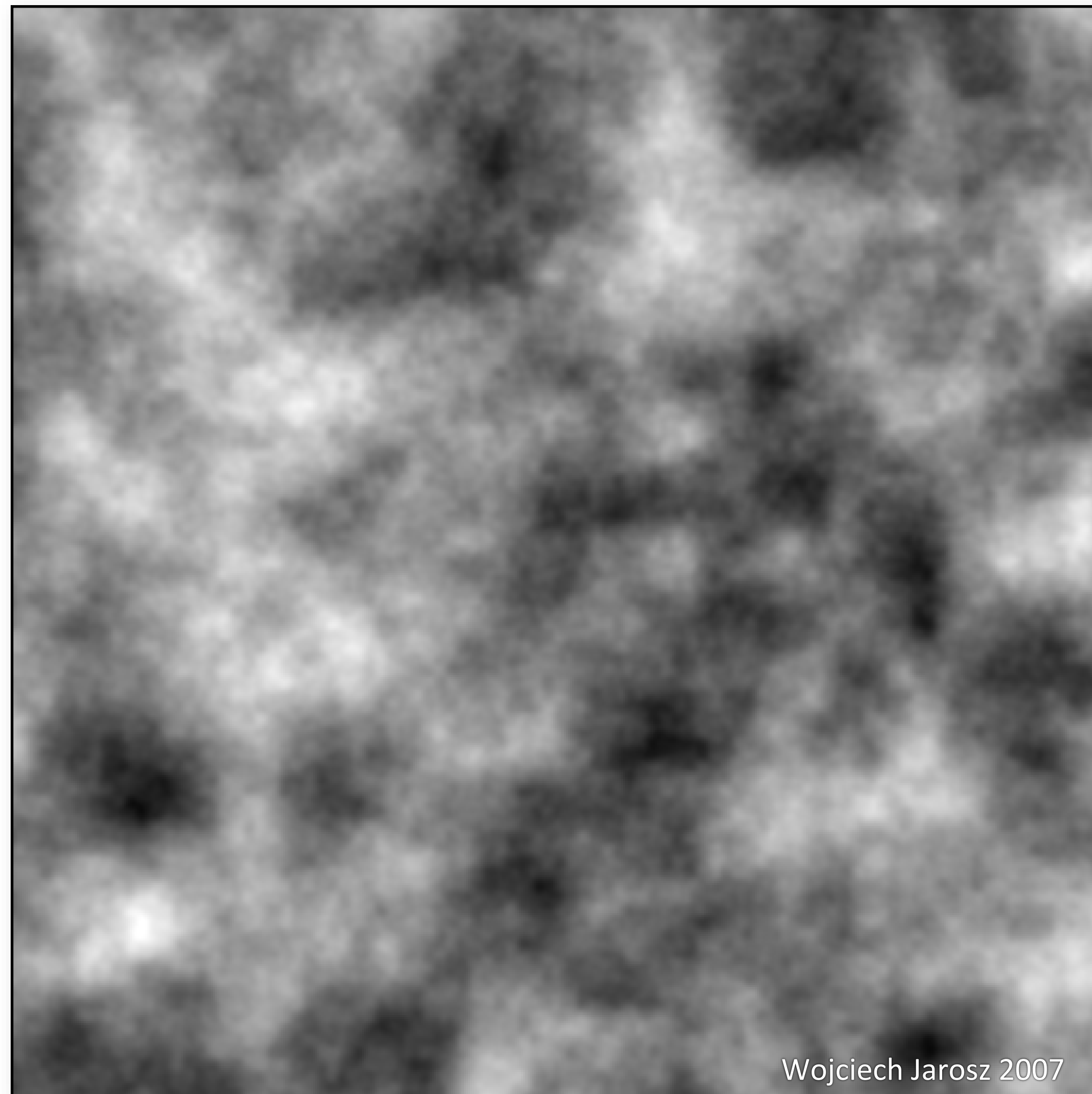
# Turbulence - 4 octaves

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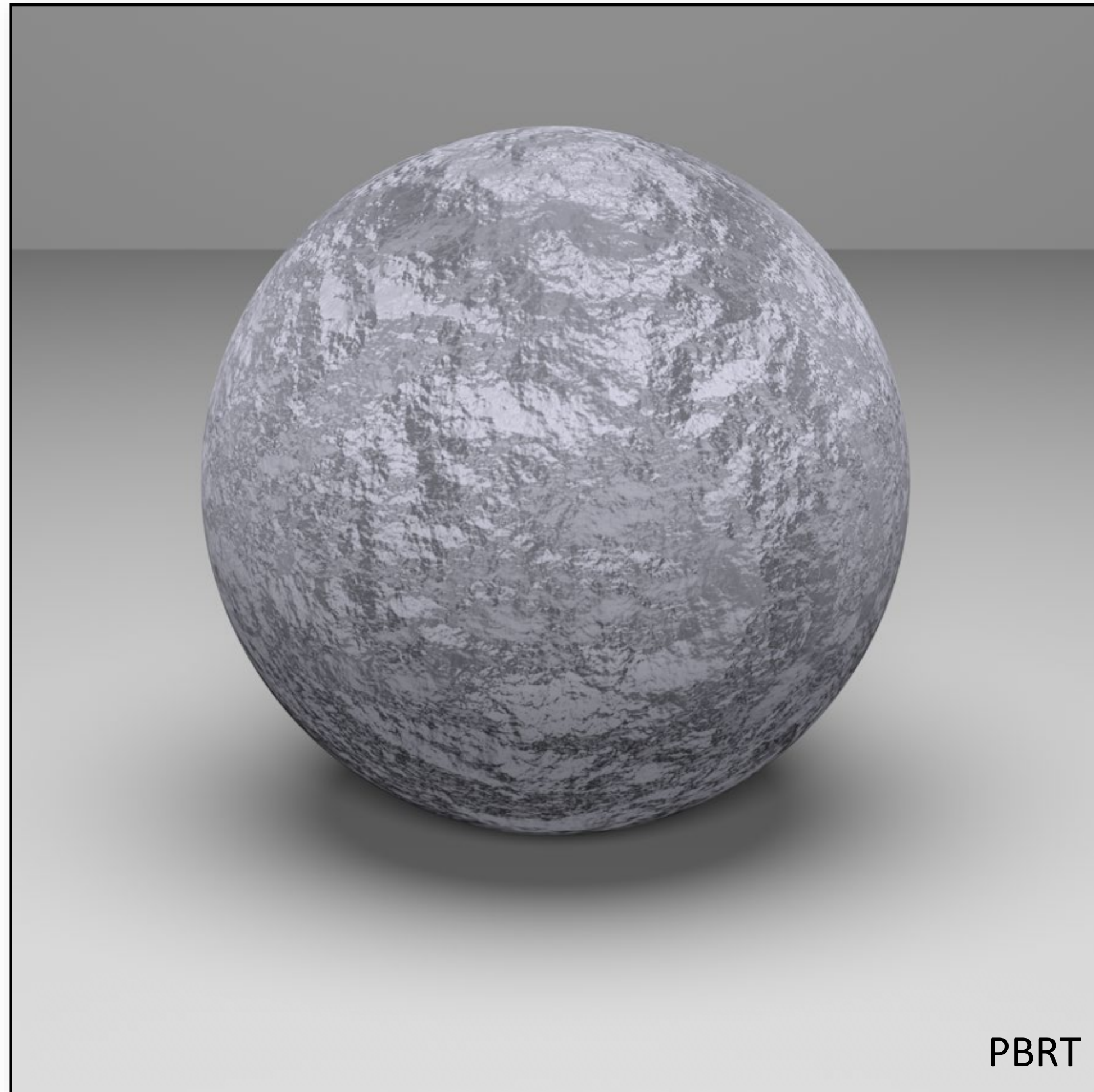
# fBm vs Turbulence

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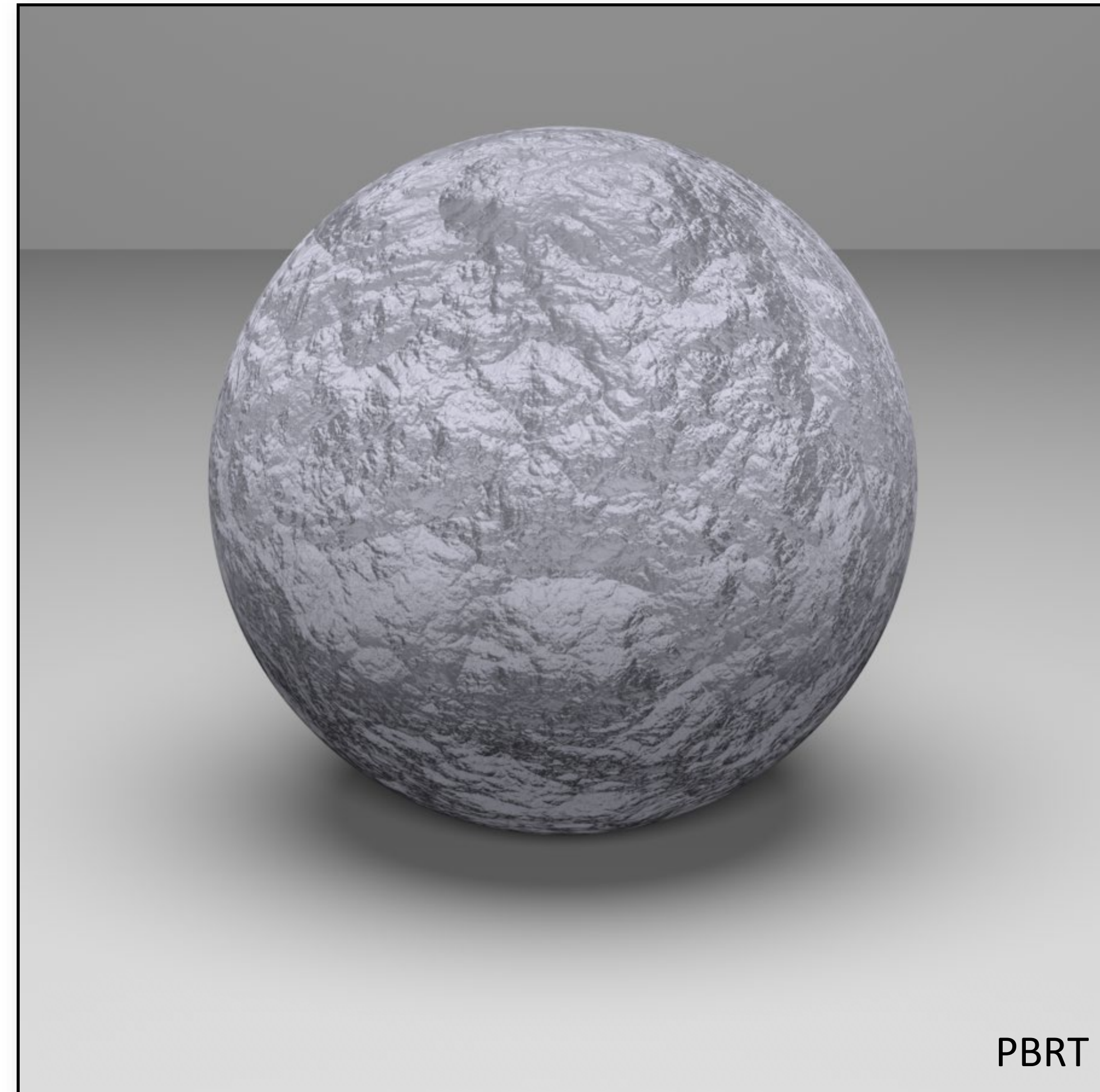




# Bump mapping



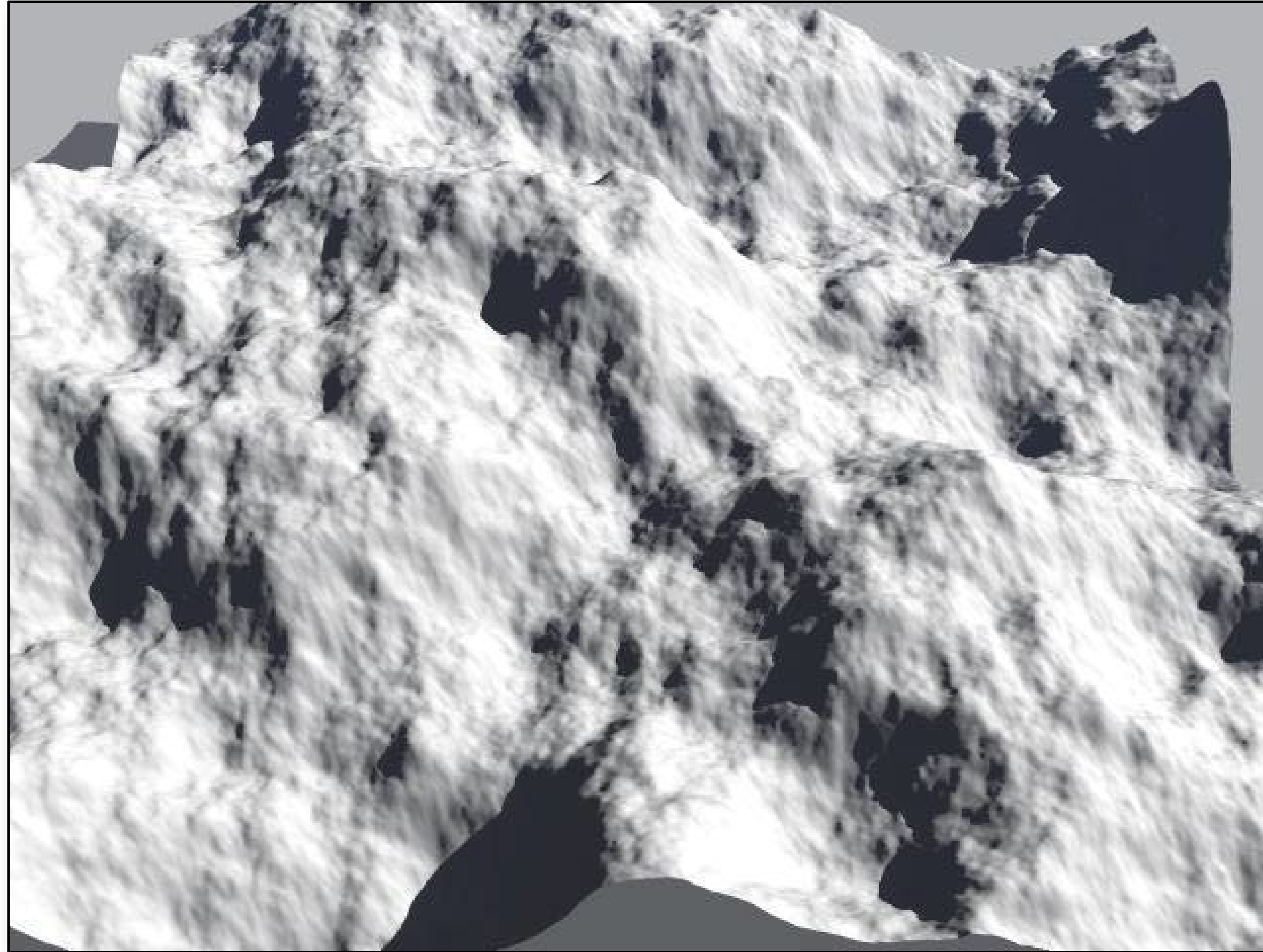
fBm



Turbulence

# 2D fBm

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A fractional Brownian motion (fBm) terrain patch of fractal dimension  $\sim 2.1$ .

# Fractal dimension

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Fractals have *fractional* dimension, e.g.  $D = 1.2$ .

- under some appropriate definition of dimension...

Integer component indicates the underlying Euclidean dimension of the fractal, in this case a line ("1" in 1.2).

Fractional part is called the fractal increment (".2" in 1.2).

Fractal increment varies from .0 to .999...

- fractal goes from (locally) occupying only its underlying Euclidean dimension (the line), to filling some part of the next higher dimension (the plane).

Continuous "slider" for the visual complexity of a fractal

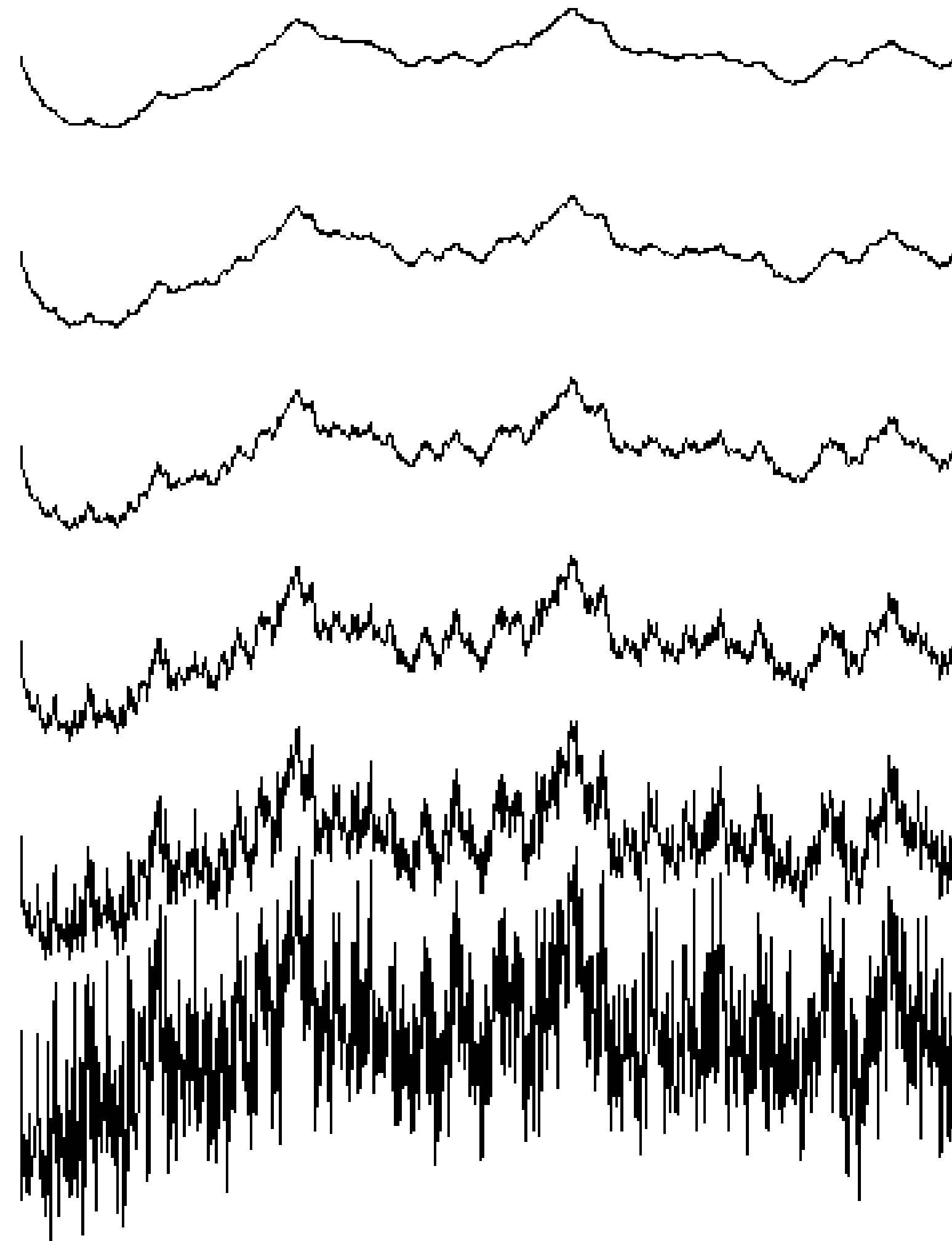
- "smoother"  $\Leftrightarrow$  "rougher"

What determines the dimension of fBm?



# Fractal dimension of fBm

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Traces of fBm for H varying from 1.0 to 0.0 in increments of 0.2

# fBm

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fBm is statistically homogeneous and isotropic.

- Homogeneous means "the same everywhere"
- Isotropic means "the same in all directions"

Fractal phenomena in nature are rarely so simple and well-behaved.

# Multifractals

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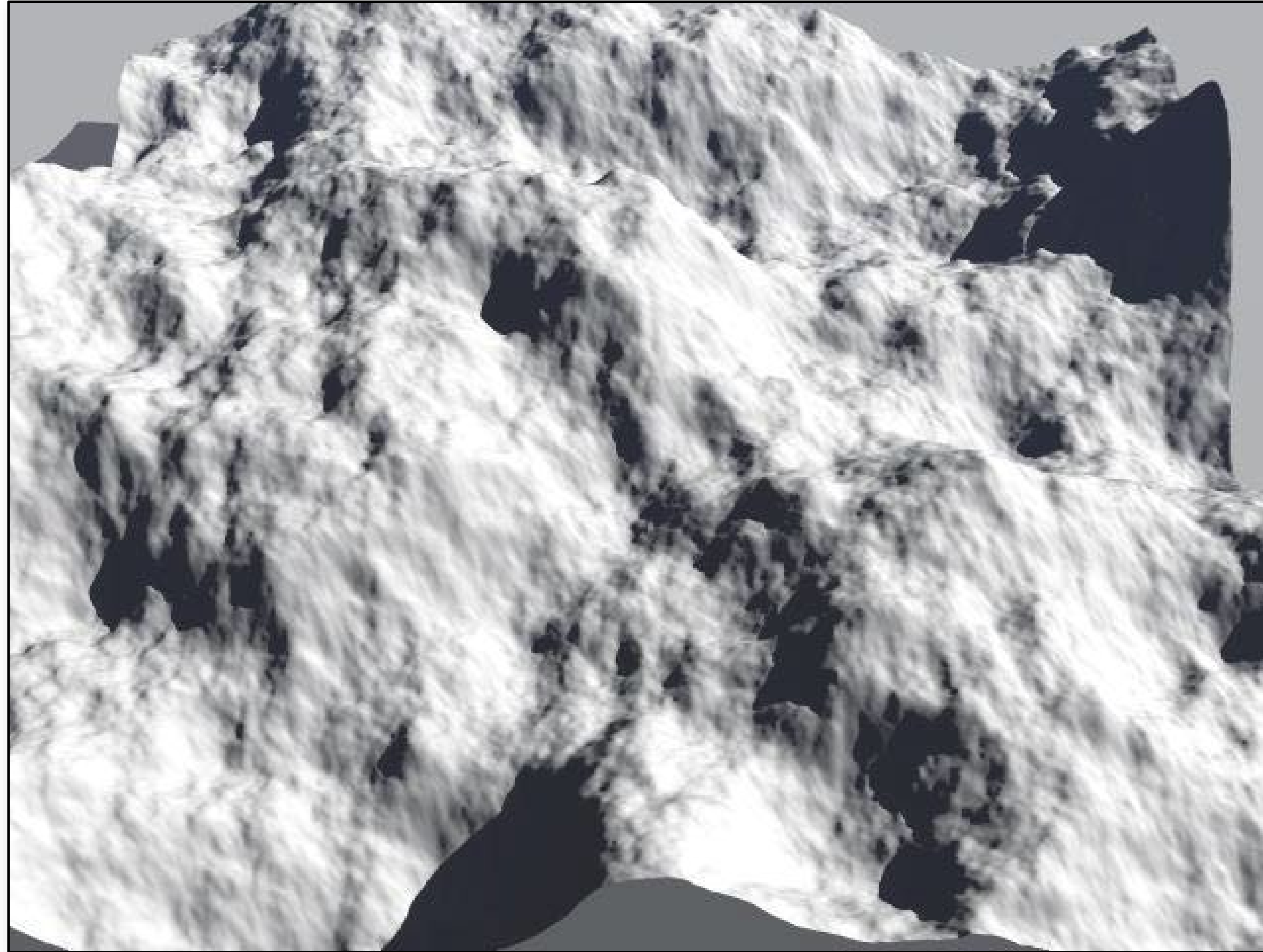
Fractal system which has a different fractal dimension in different regions

Heterogeneous fBm

- Scale higher frequencies in the summation by the value of the previous frequency.
- Many possibilities: hetero terrain, hybrid multifractal, ridged multifractal

# 2D fBm

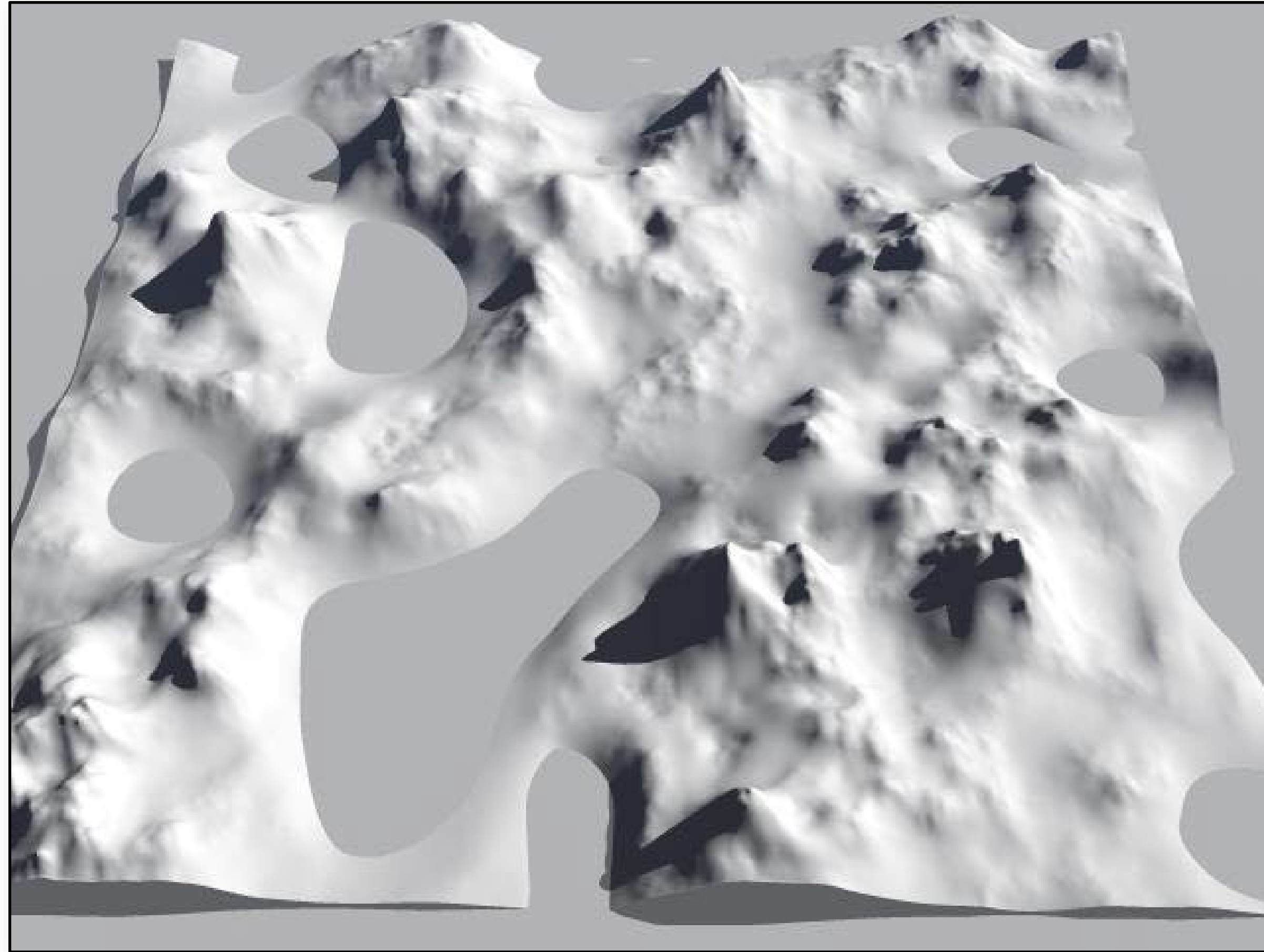
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A fractional Brownian motion (fBm) terrain patch of fractal dimension  $\sim 2.1$ .

# Heterogeneous fBm

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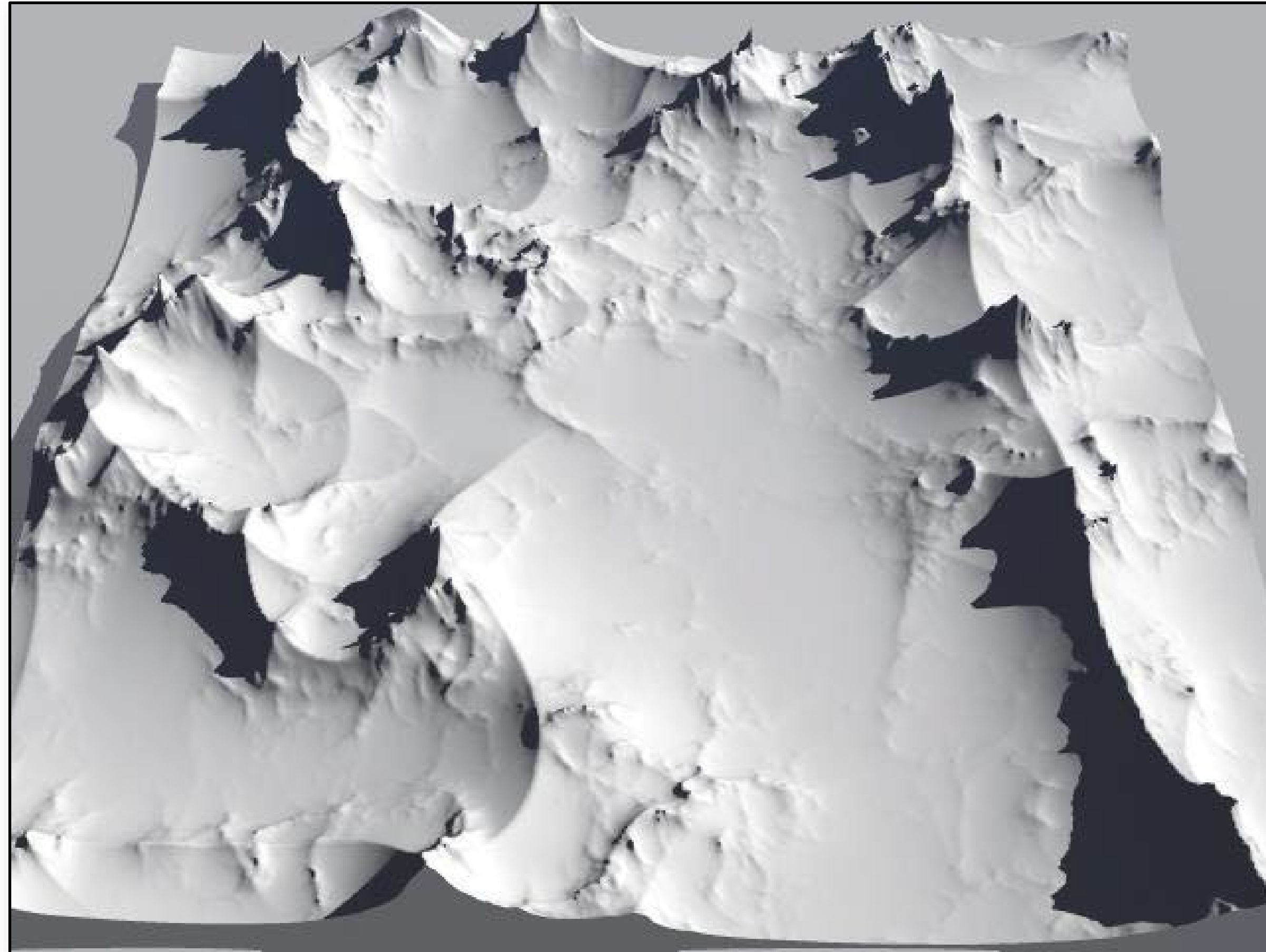


A hybrid multifractal terrain patch made with a Perlin noise basis: the “alpine hills” Bryce 4 terrain model.



# Heterogeneous fBm

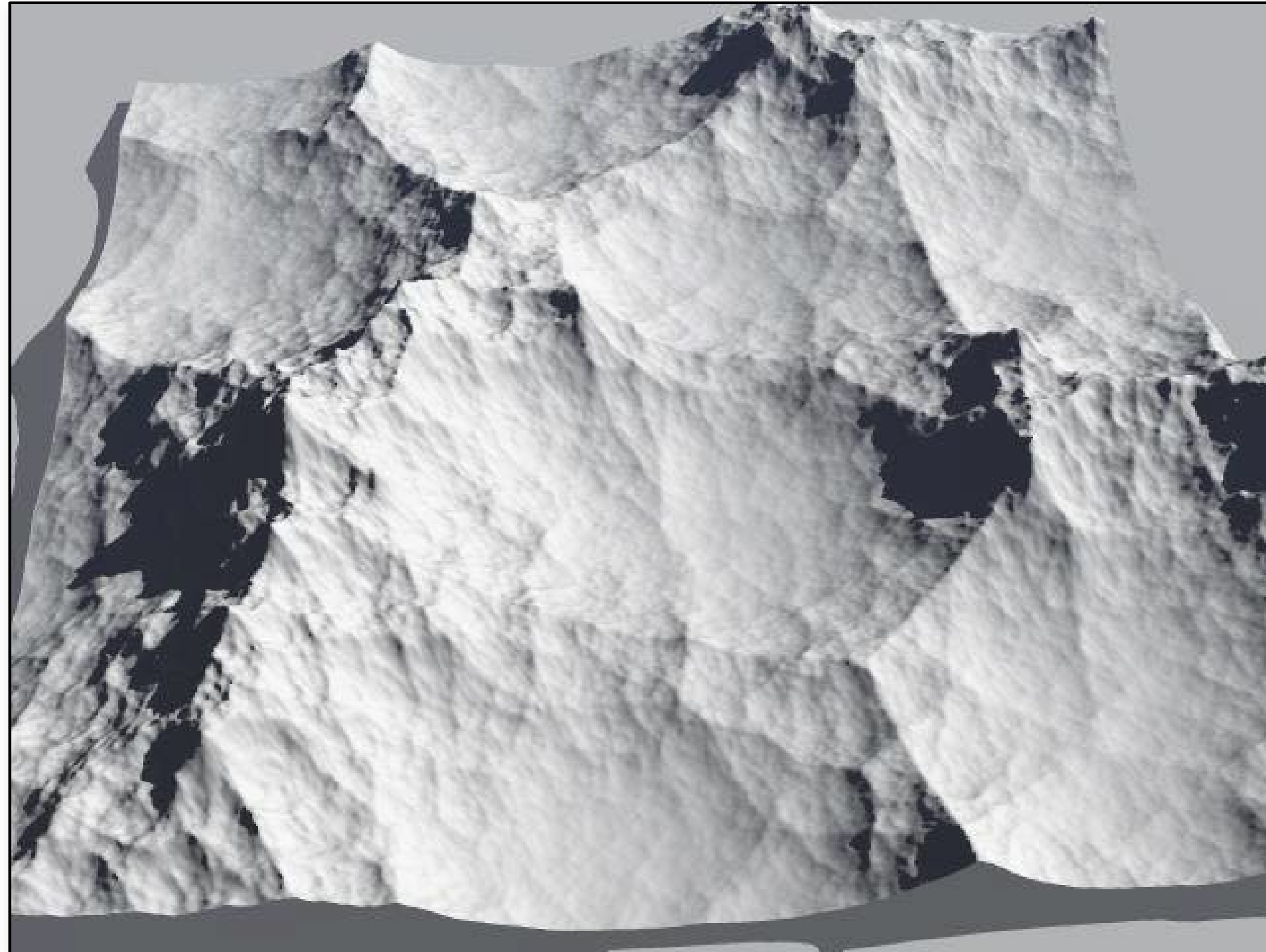
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The “ridges” terrain model from Bryce 4: a hybrid multifractal made from one minus the absolute value of Perlin noise.

# Heterogeneous fBm

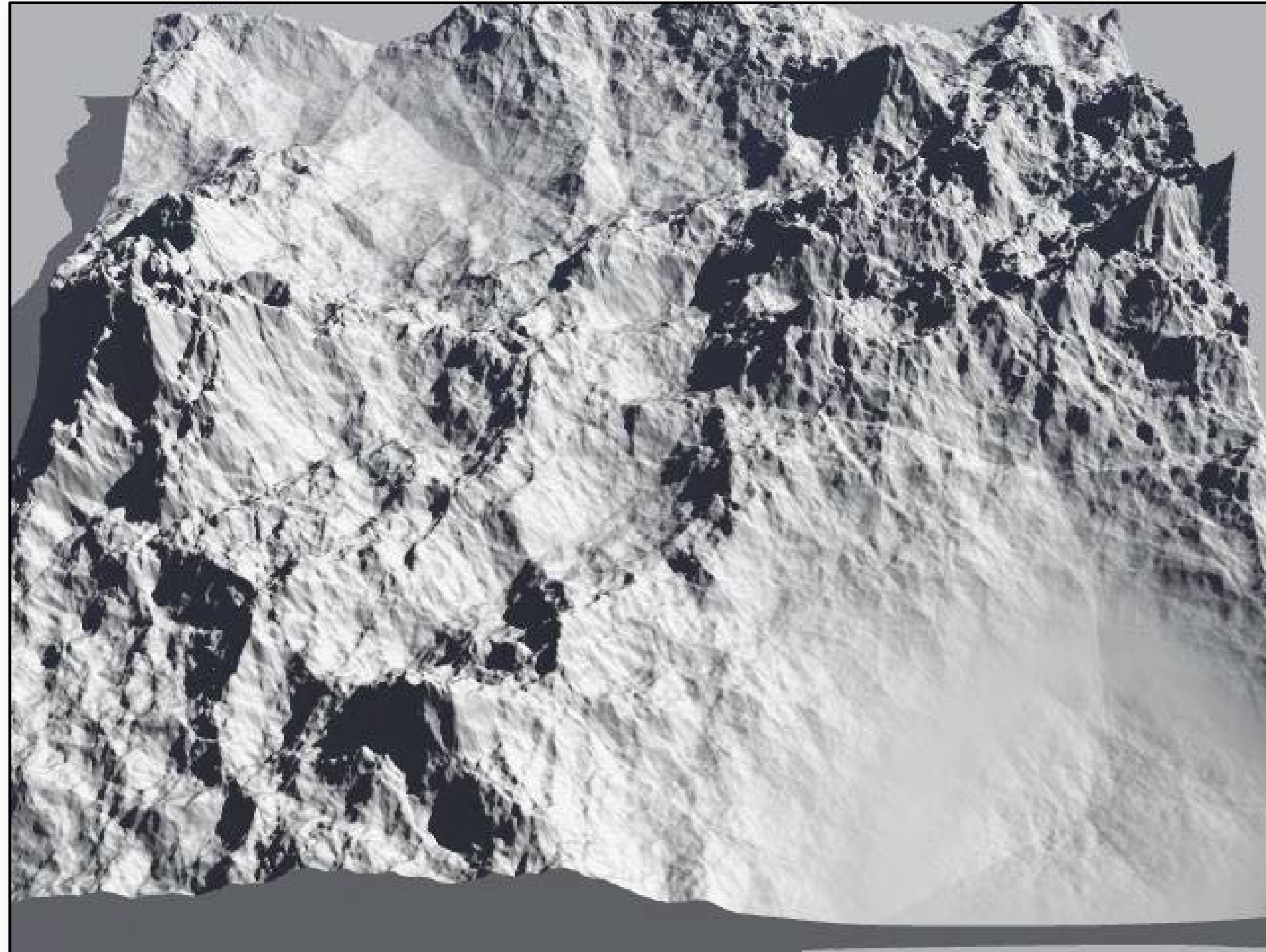
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A hybrid multifractal made from Worley's Voronoi distance-squared basis

# Heterogeneous fBm

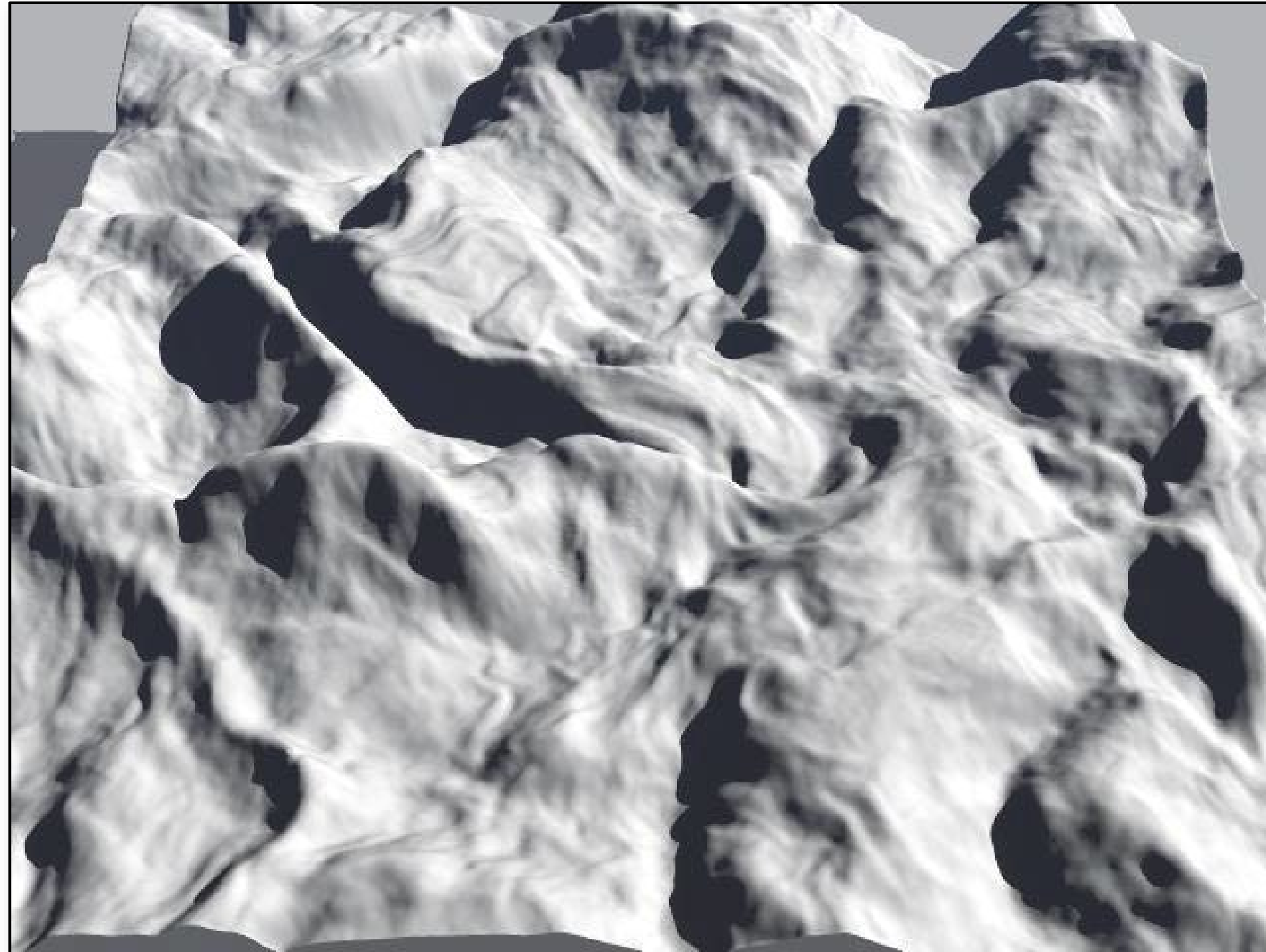
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A hybrid multifractal made from Worley's Voronoi distance basis

# Domain Distortion

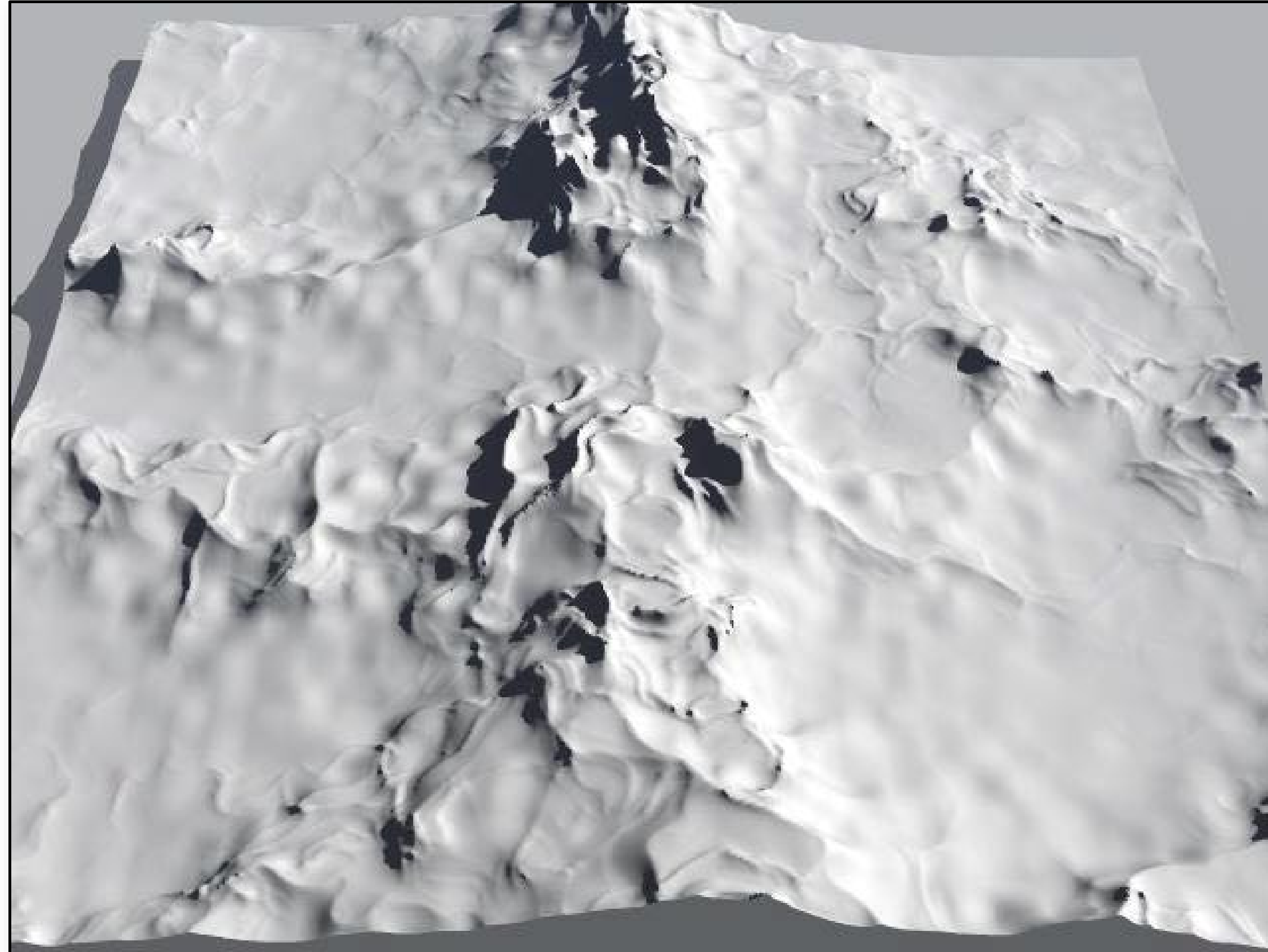
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fBm distorted with fBm

# Domain Distortion

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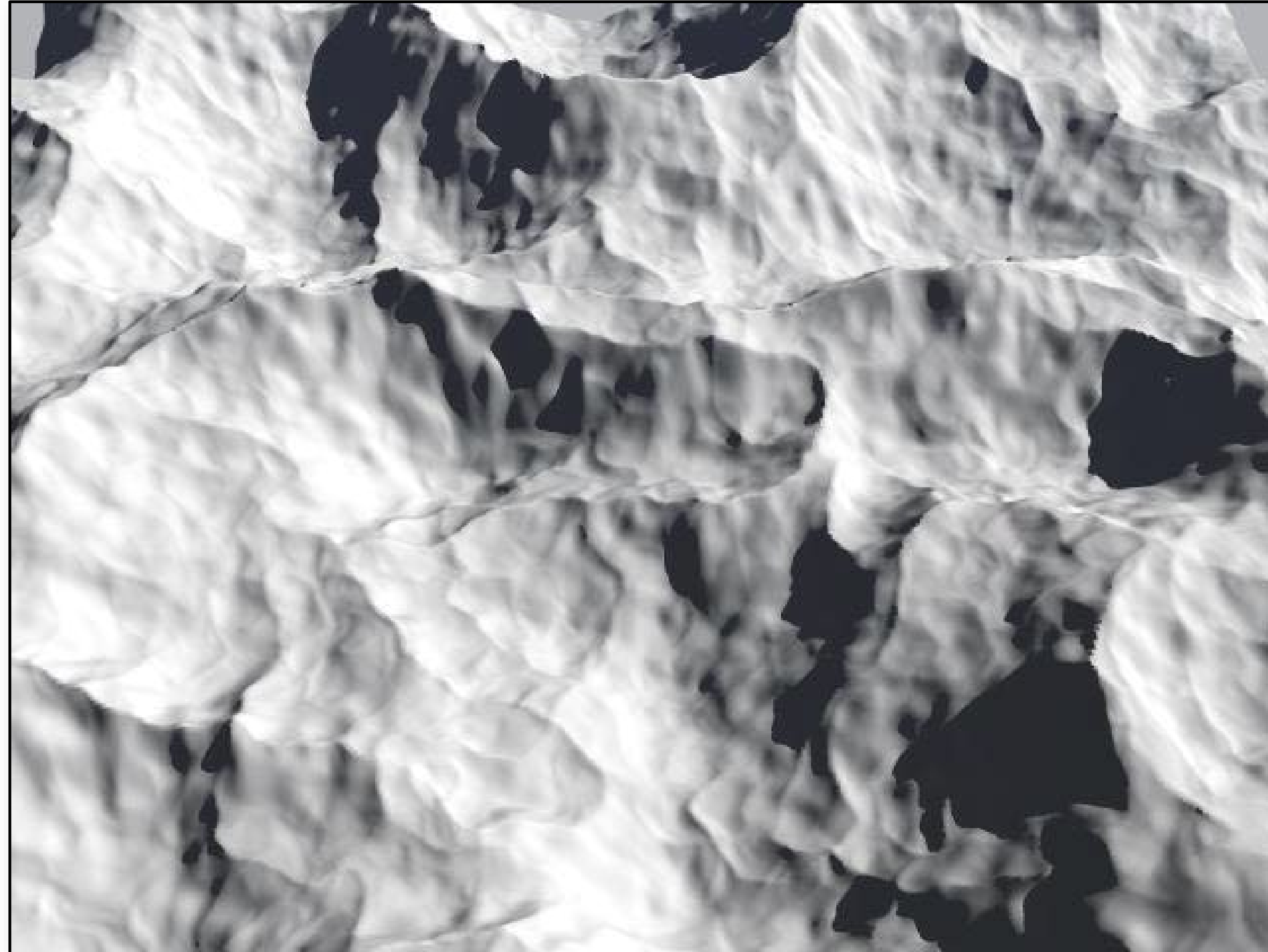


A sample of the “warped ridges” terrain model in Bryce 4: the “ridges” model distorted with fBm.



# Domain Distortion

---

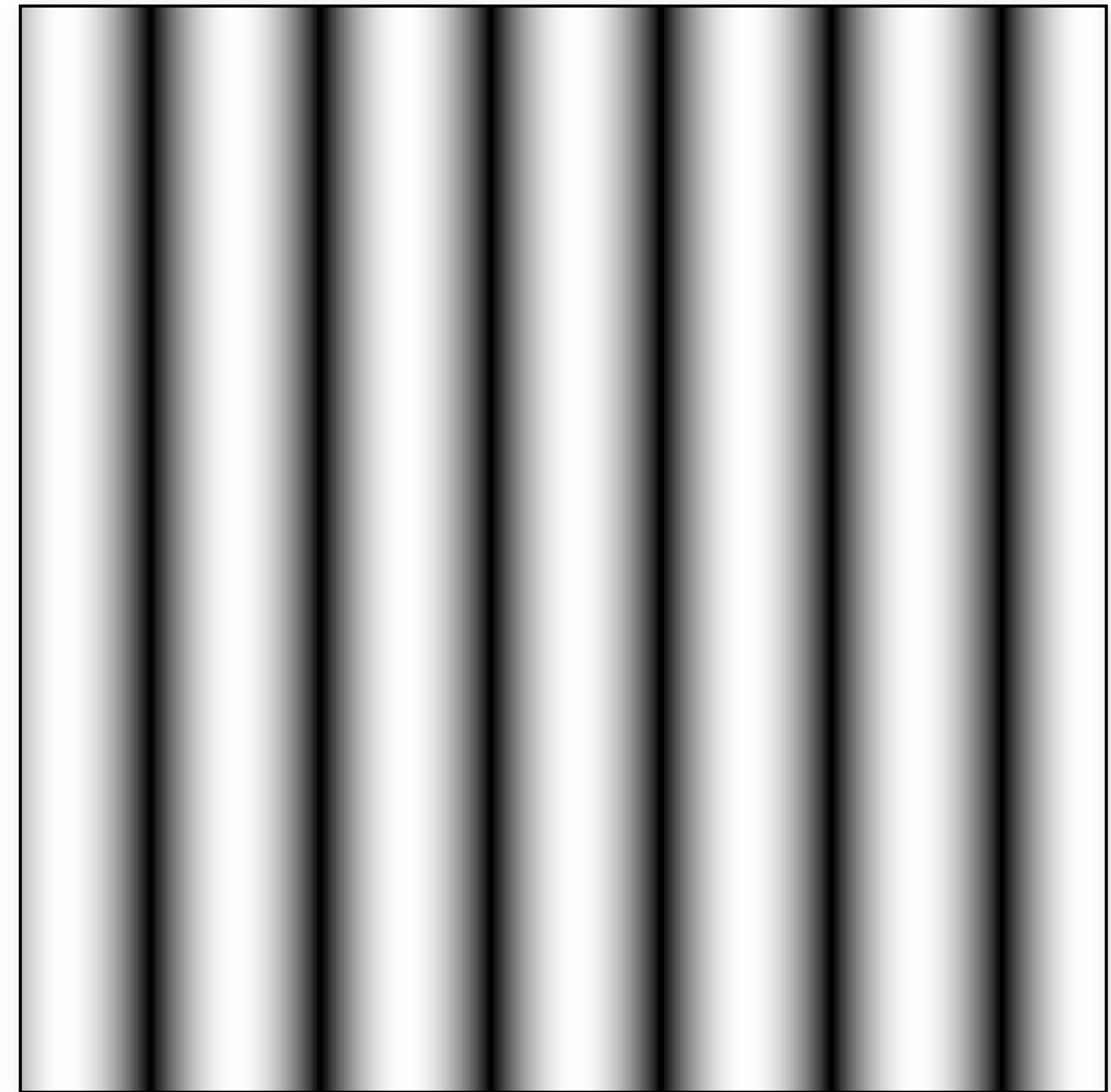


A sample of the “warped slickrock” terrain model in Bryce 4: fBm constructed from one minus the absolute value of Perlin noise, distorted with fBm.

# Recall: 3D stripe texture

---

```
color stripe(point p, real w):  
    t = (1 + sin( $\pi p_x/w$ ))/2  
    return lerp(c0, c1, t)
```

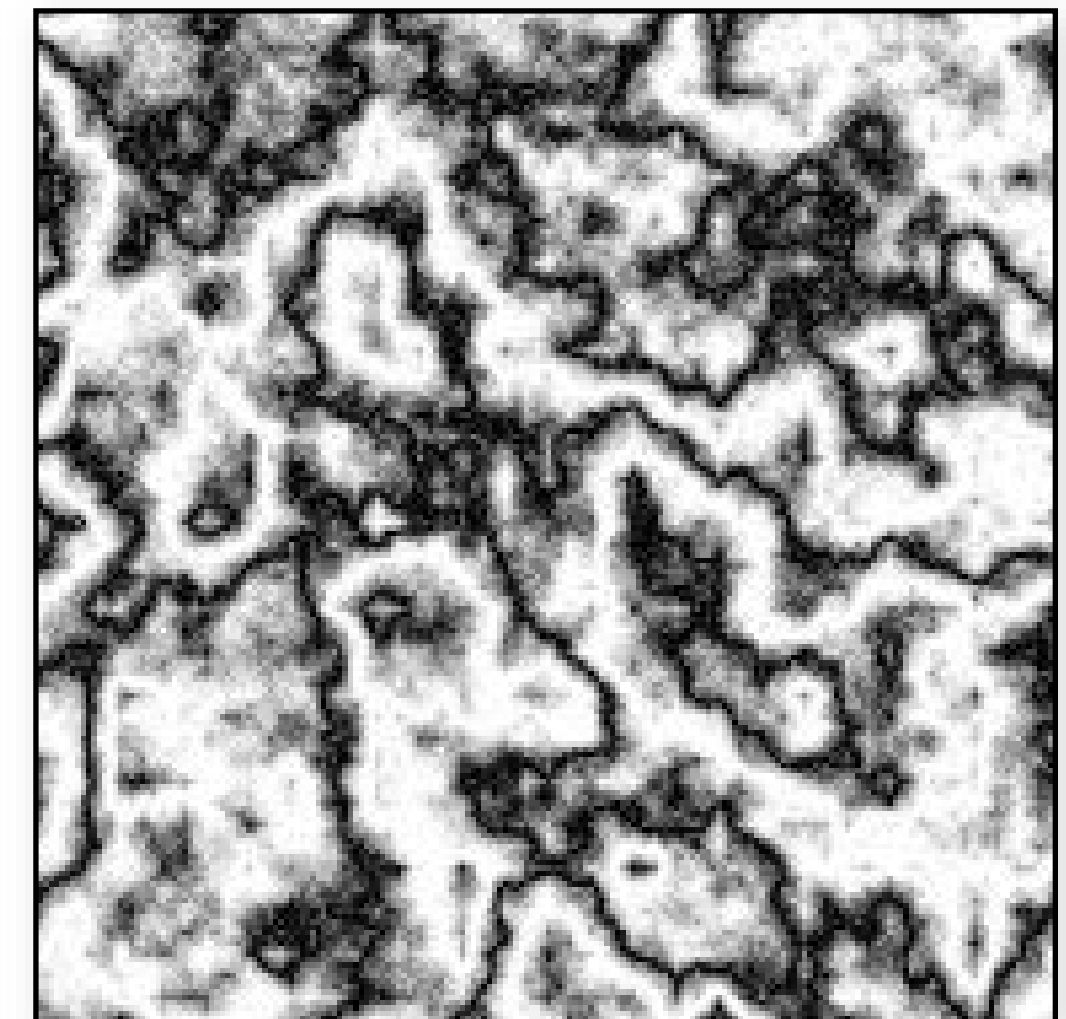
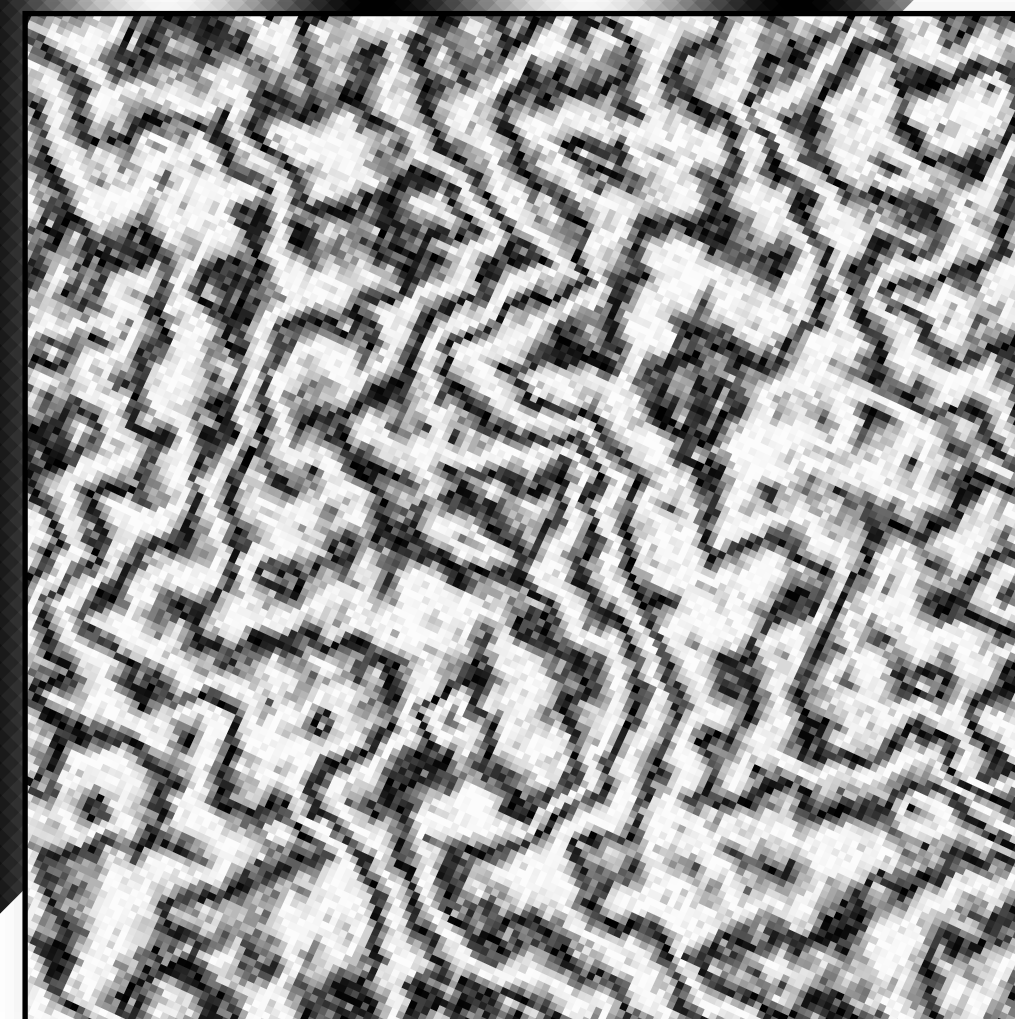
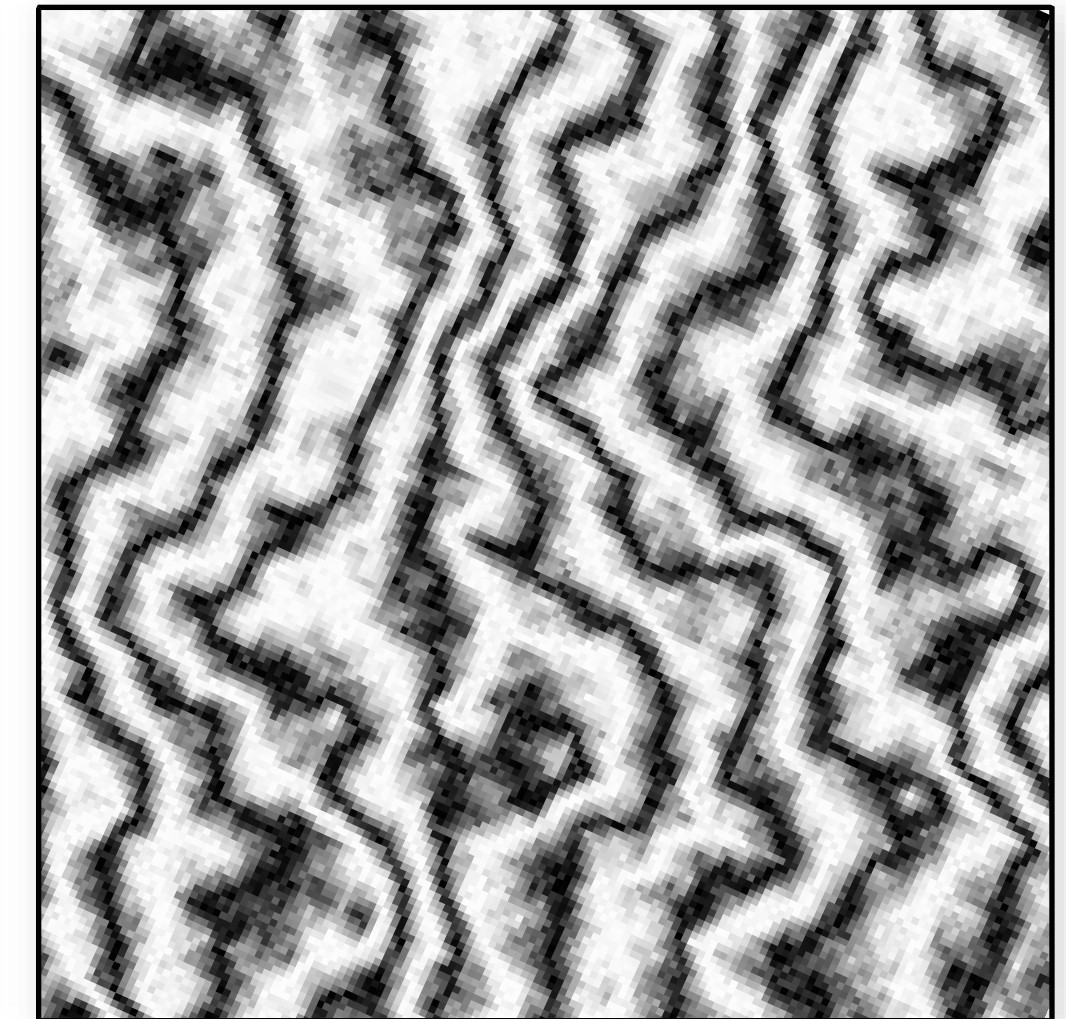
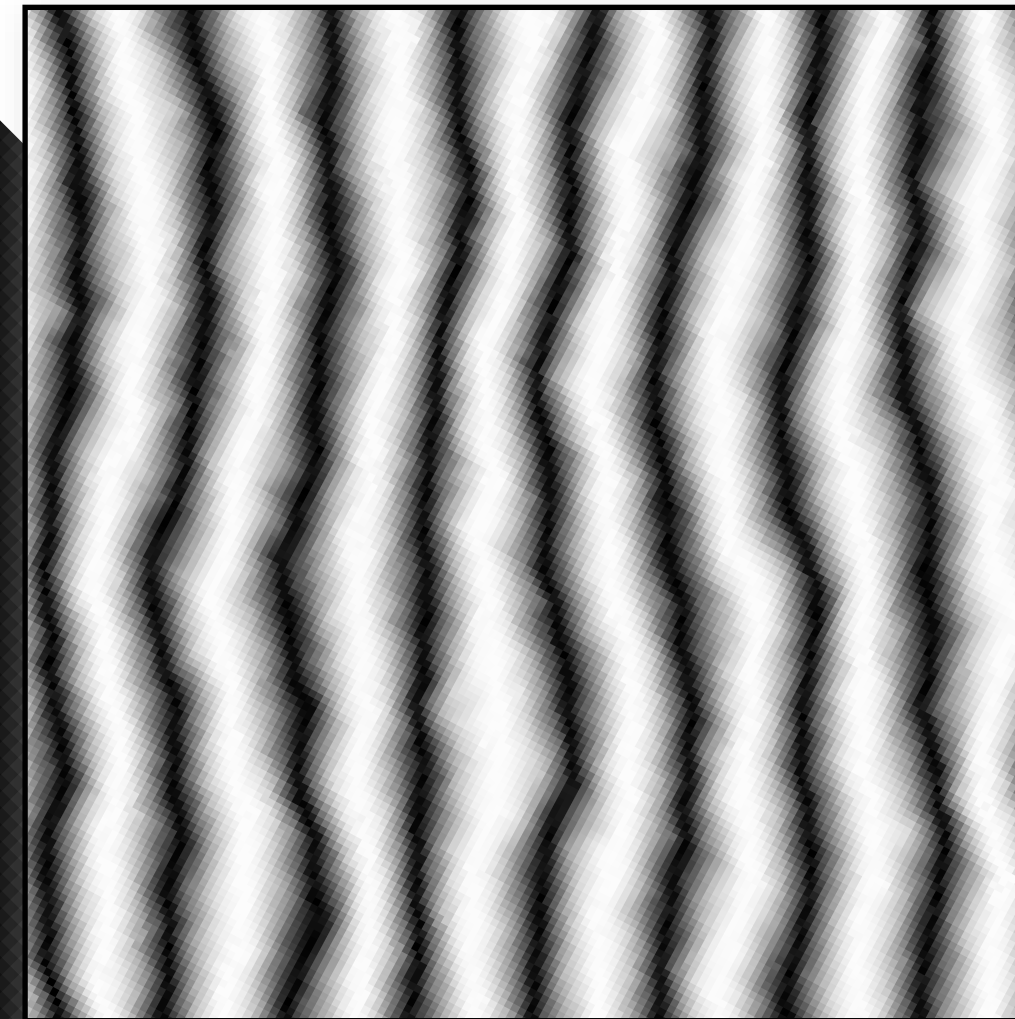


How can we make this less structured  
(less “boring”)?



# Mar

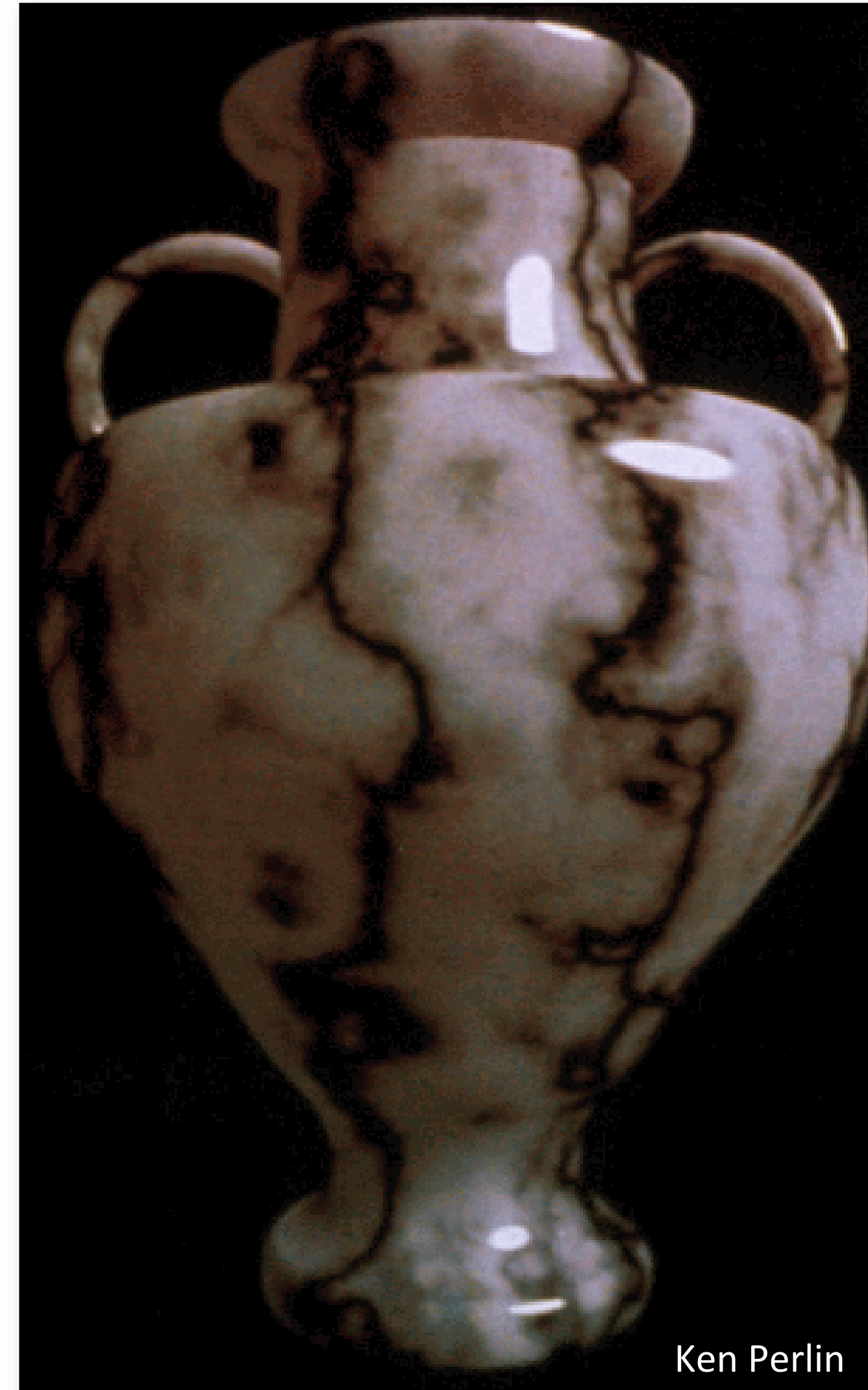
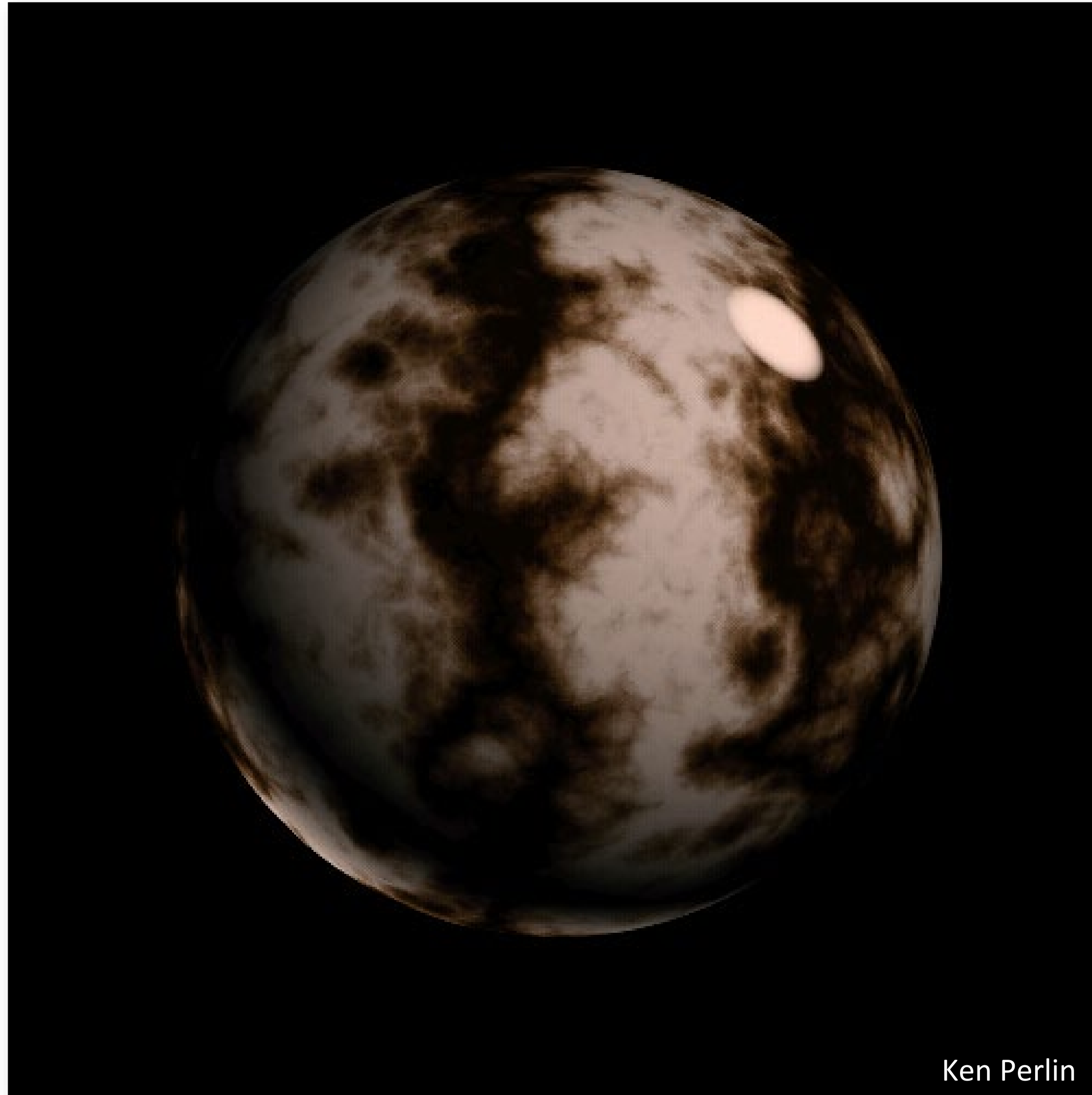
$$(1 + \sin(k_1 \mathbf{p}_x + \text{turbulence}(k_2 \mathbf{p})) / w) / 2$$





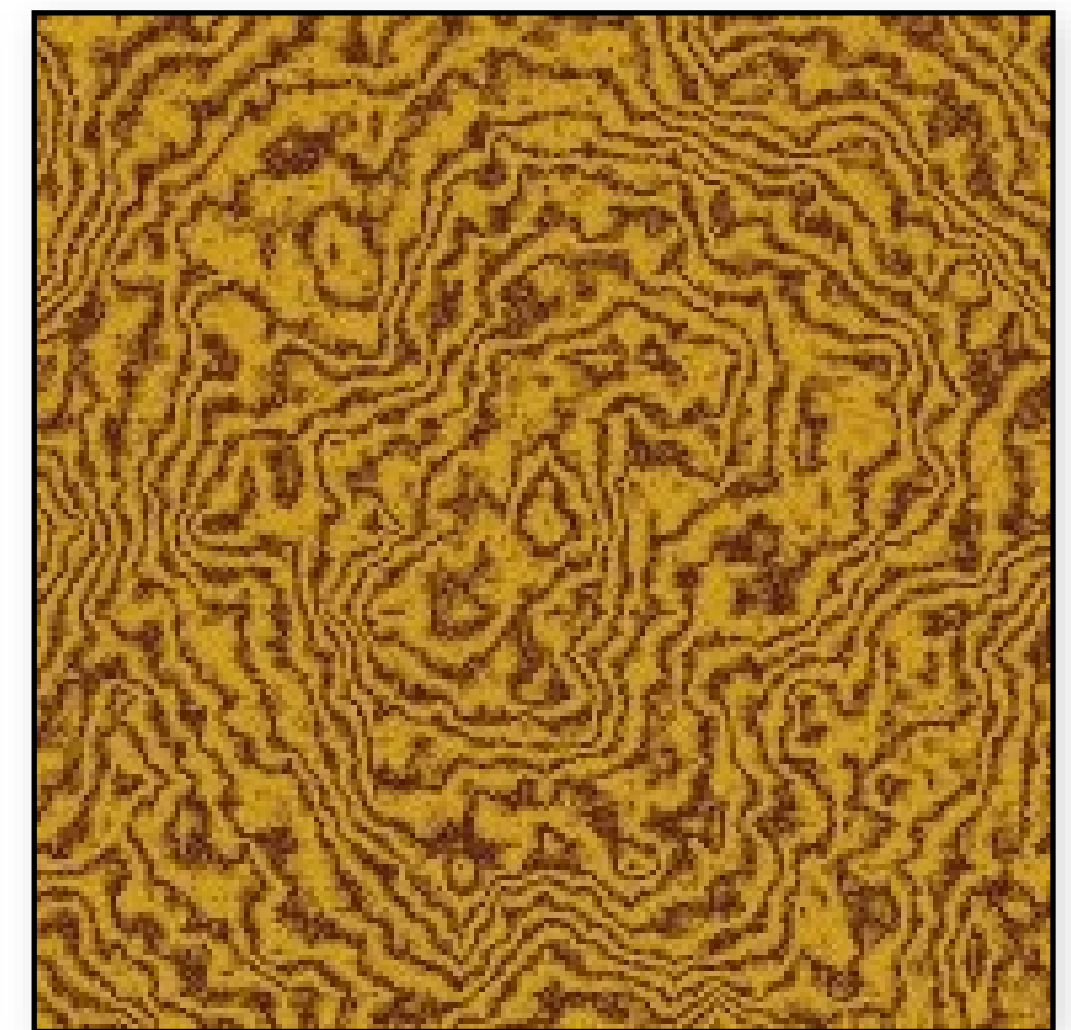
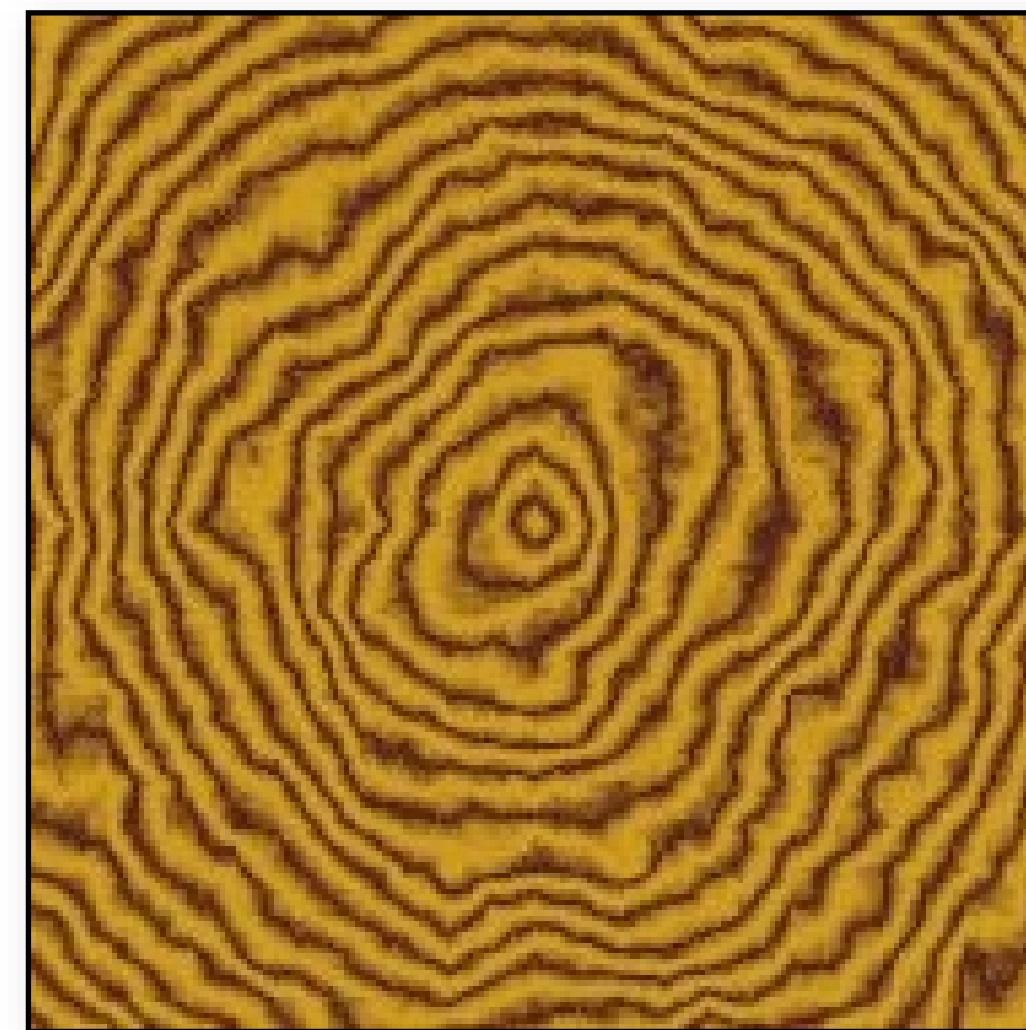
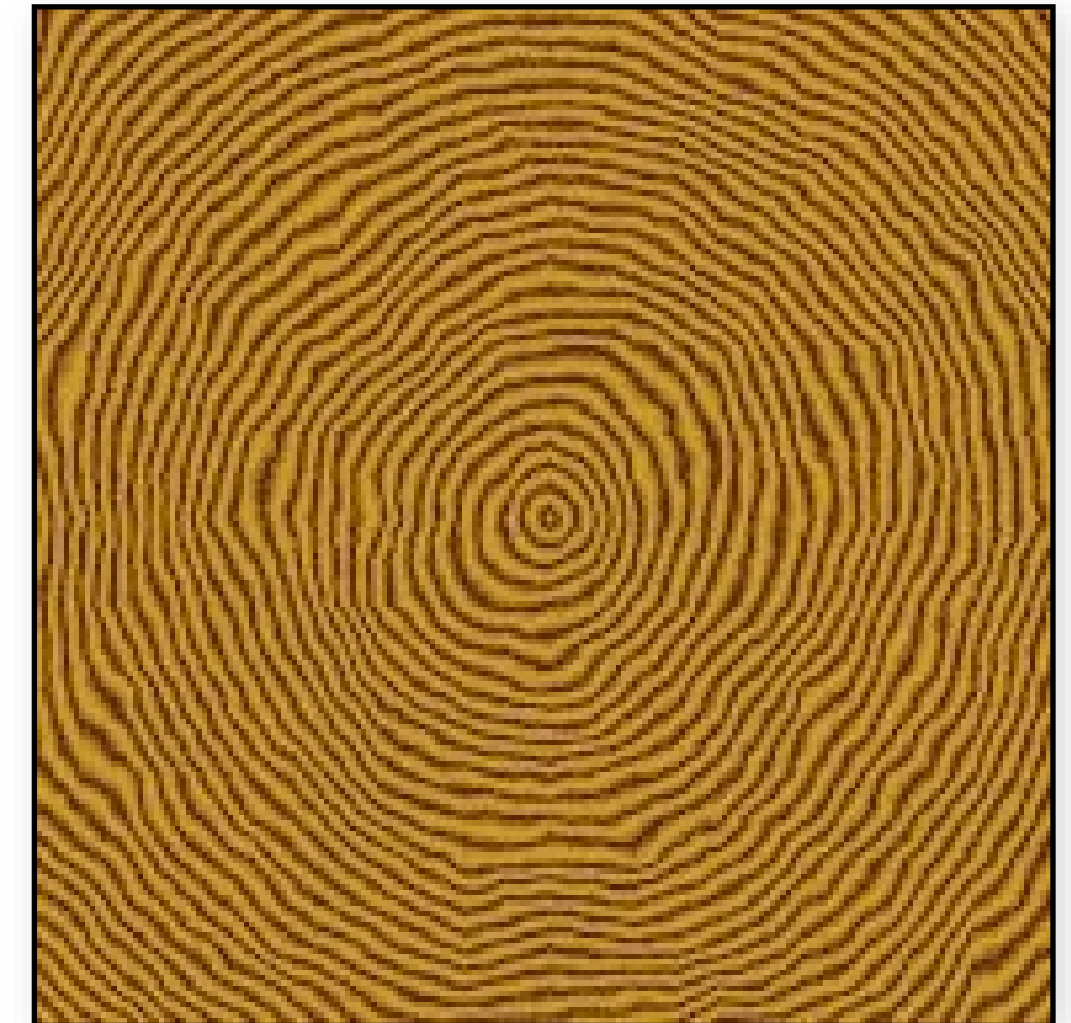
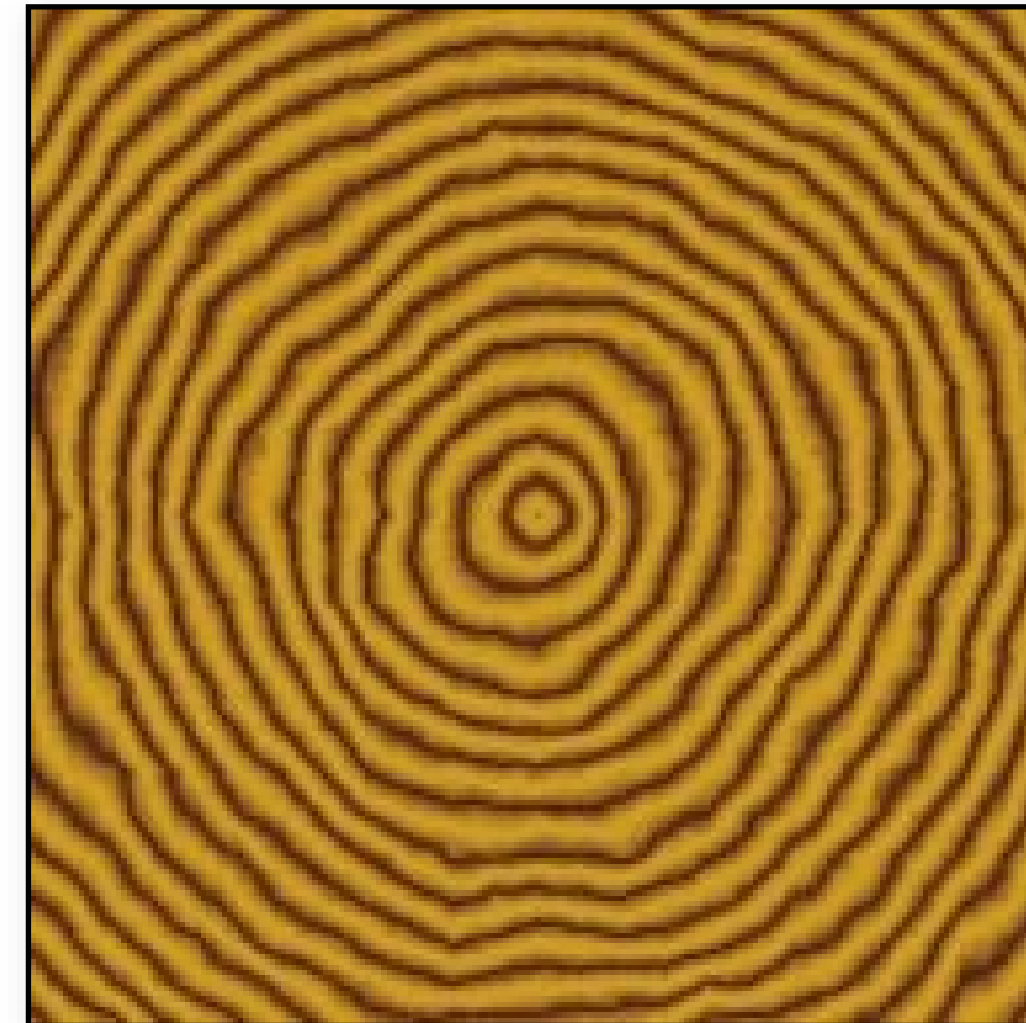
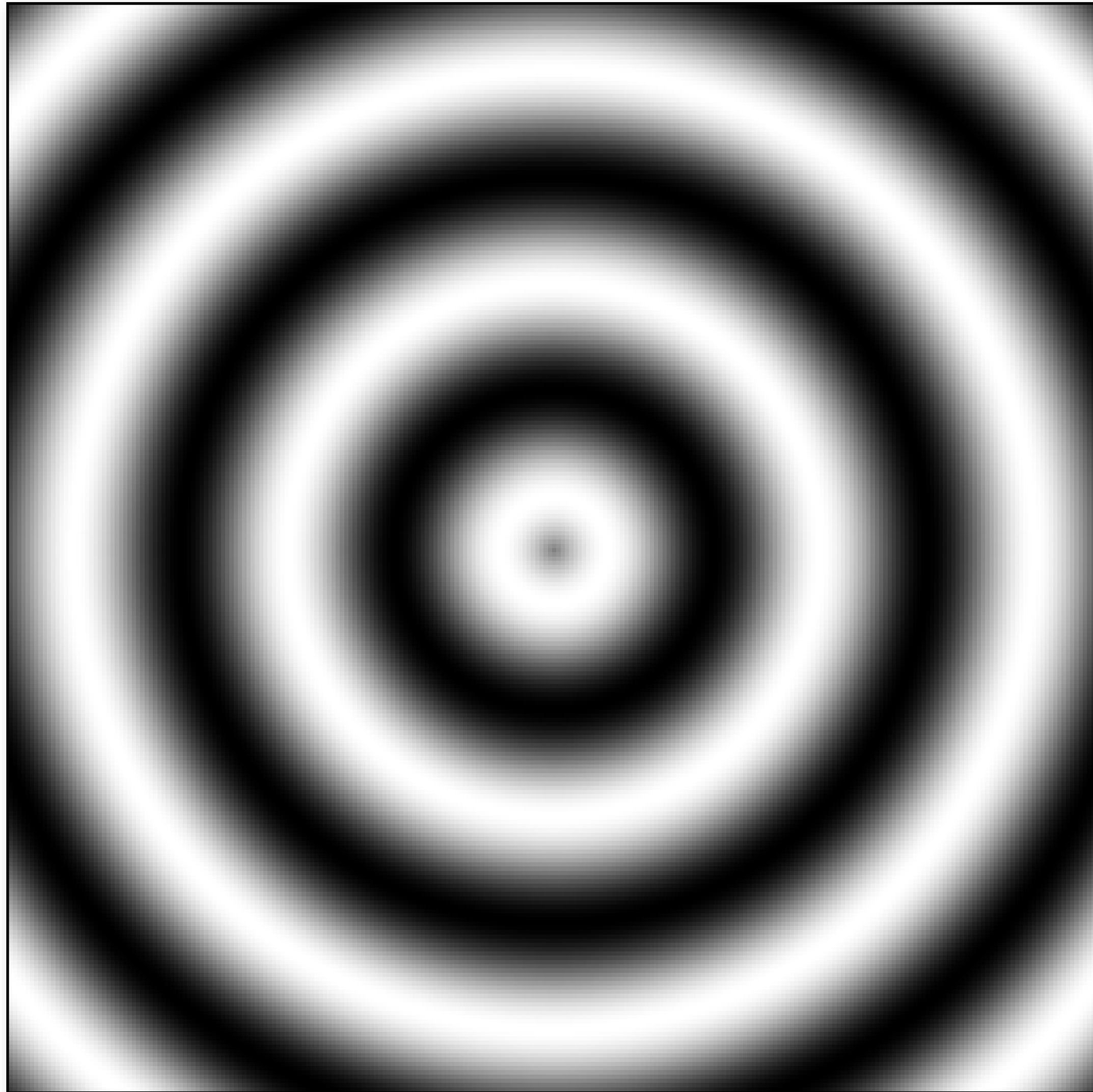
# Marble

$$(1 + \sin(k_1 \mathbf{p}_x + \text{turbulence}(k_2 \mathbf{p})) / w) / 2$$



# Wood

$$(1 + \sin(\sqrt{p_x^2 + p_y^2}) + \text{fBm}(\mathbf{p})) / 2$$



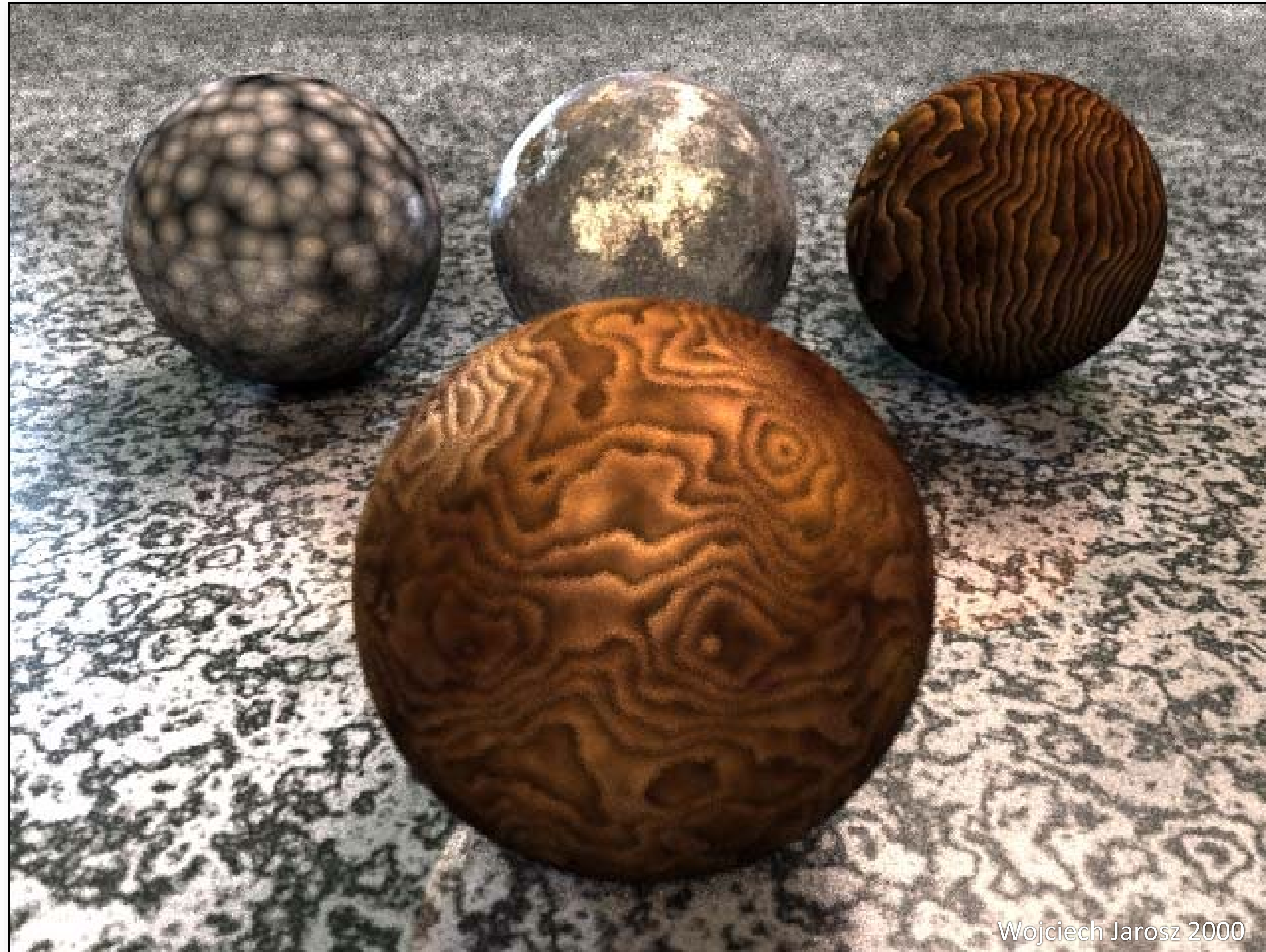
# Wood

$$(1 + \sin(\sqrt{p_x^2 + p_y^2}) + \text{fBm}(\mathbf{p})) / 2$$



# and more...

---



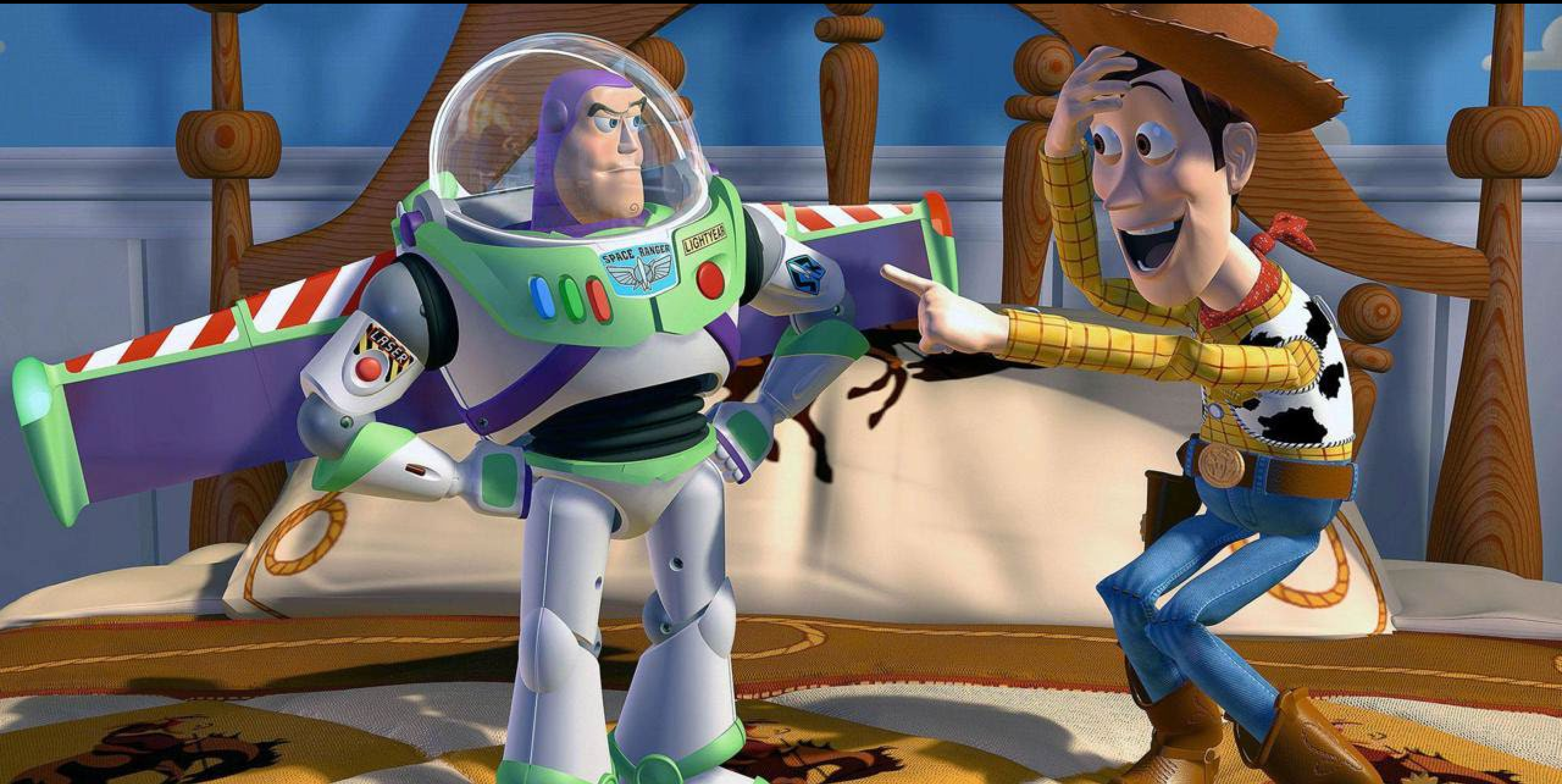
Wojciech Jarosz 2000



and more...

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# Worley noise

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“Cellular texture” function

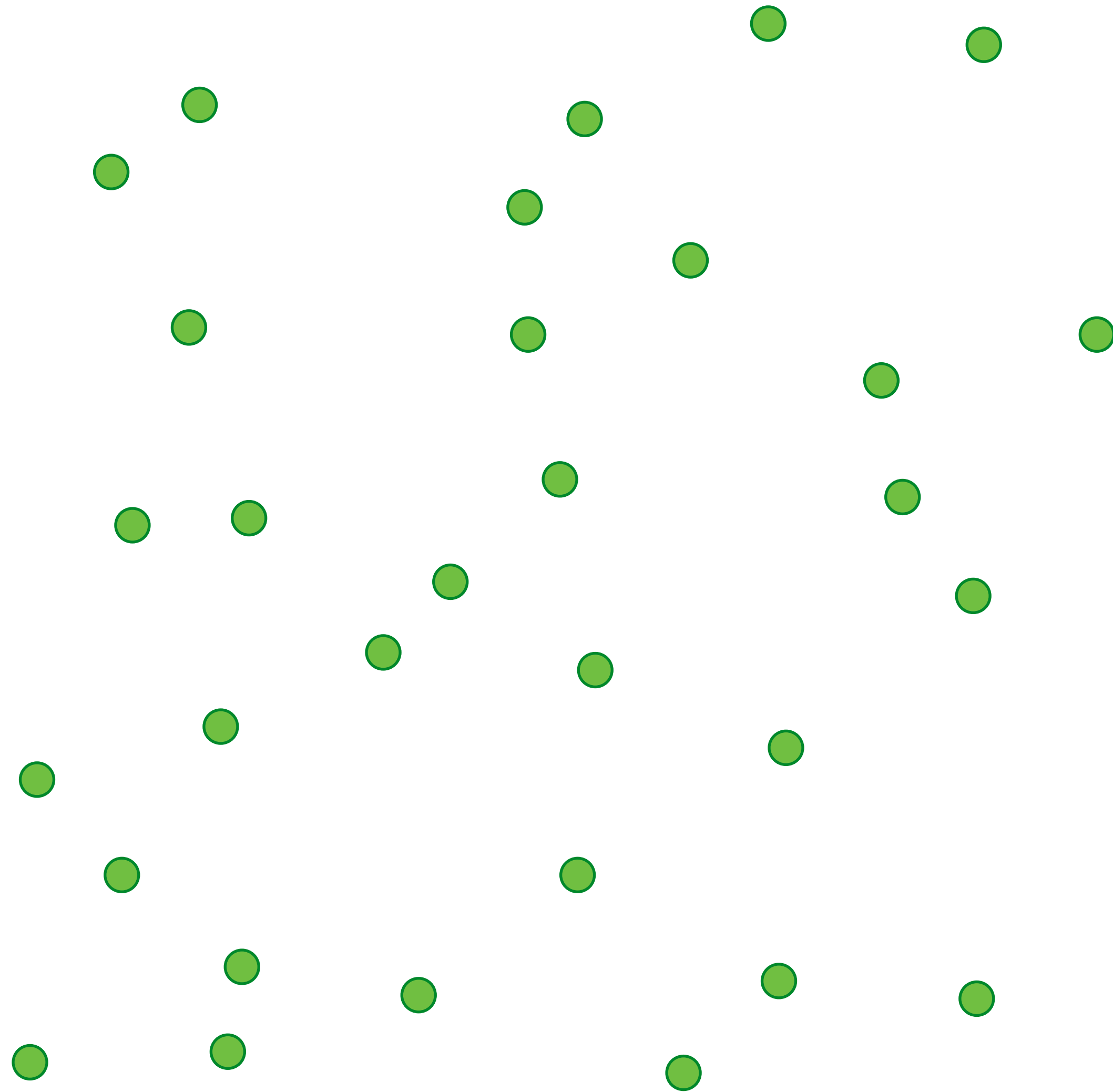
- Introduced in 1996 by Steve Worley
- Different from *cell* texture!

Randomly distribute “feature points” in space

- $f_n(x)$  = distance to  $n^{\text{th}}$  closest point to  $x$

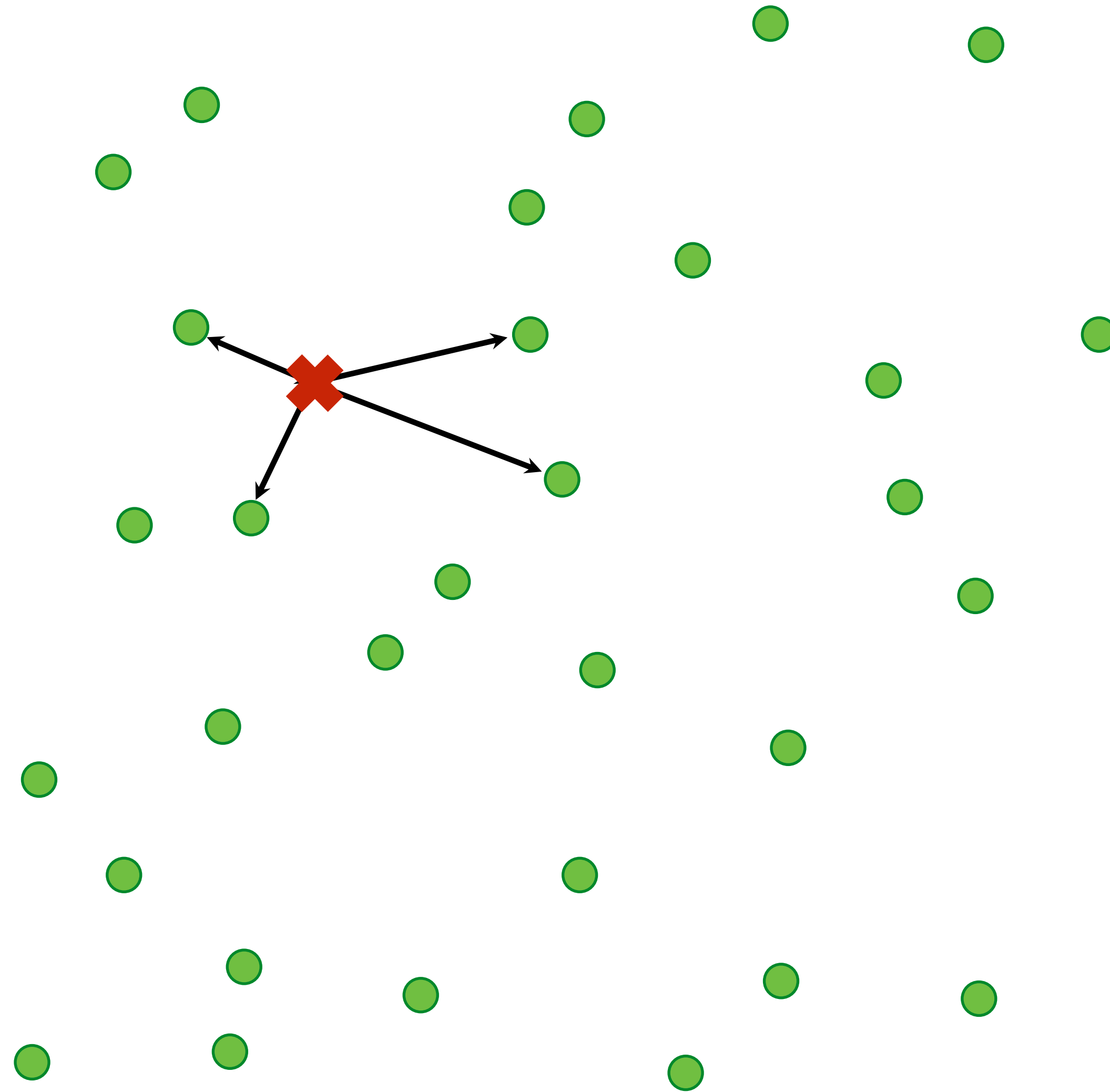
# 2D Worley noise: f1

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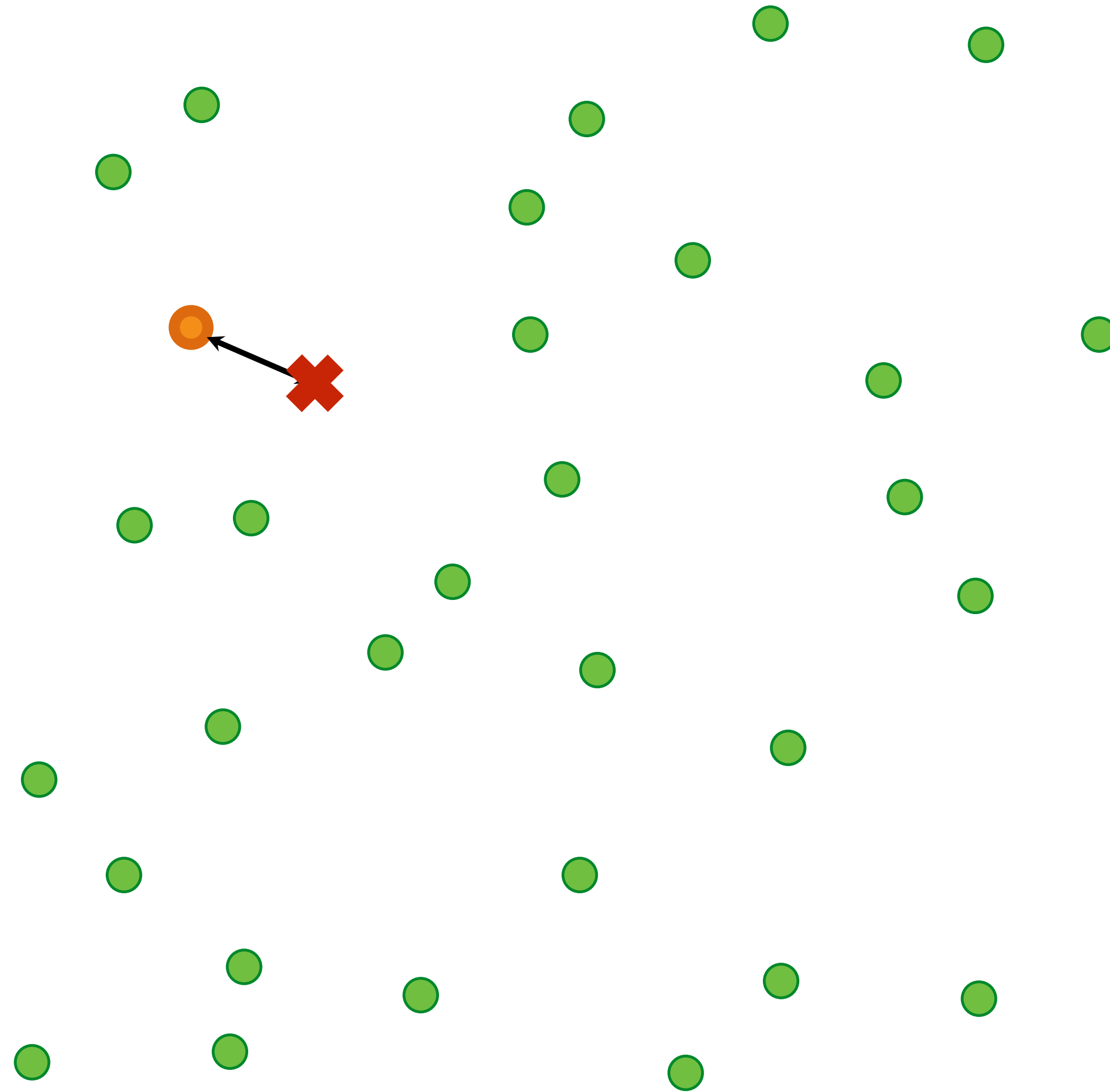
# 2D Worley noise: f1

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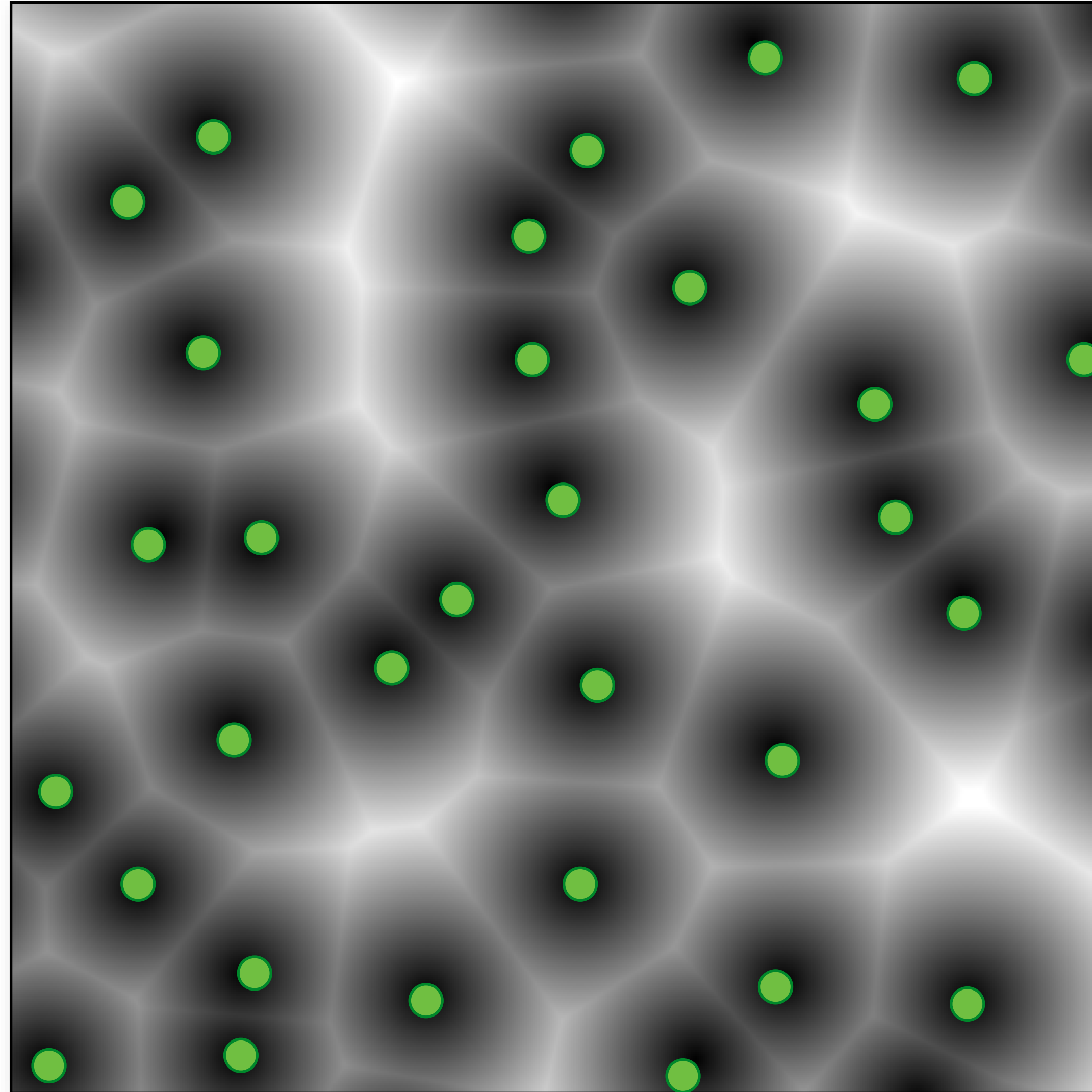
# 2D Worley noise: f1

---



# 2D Worley noise: $f_1$

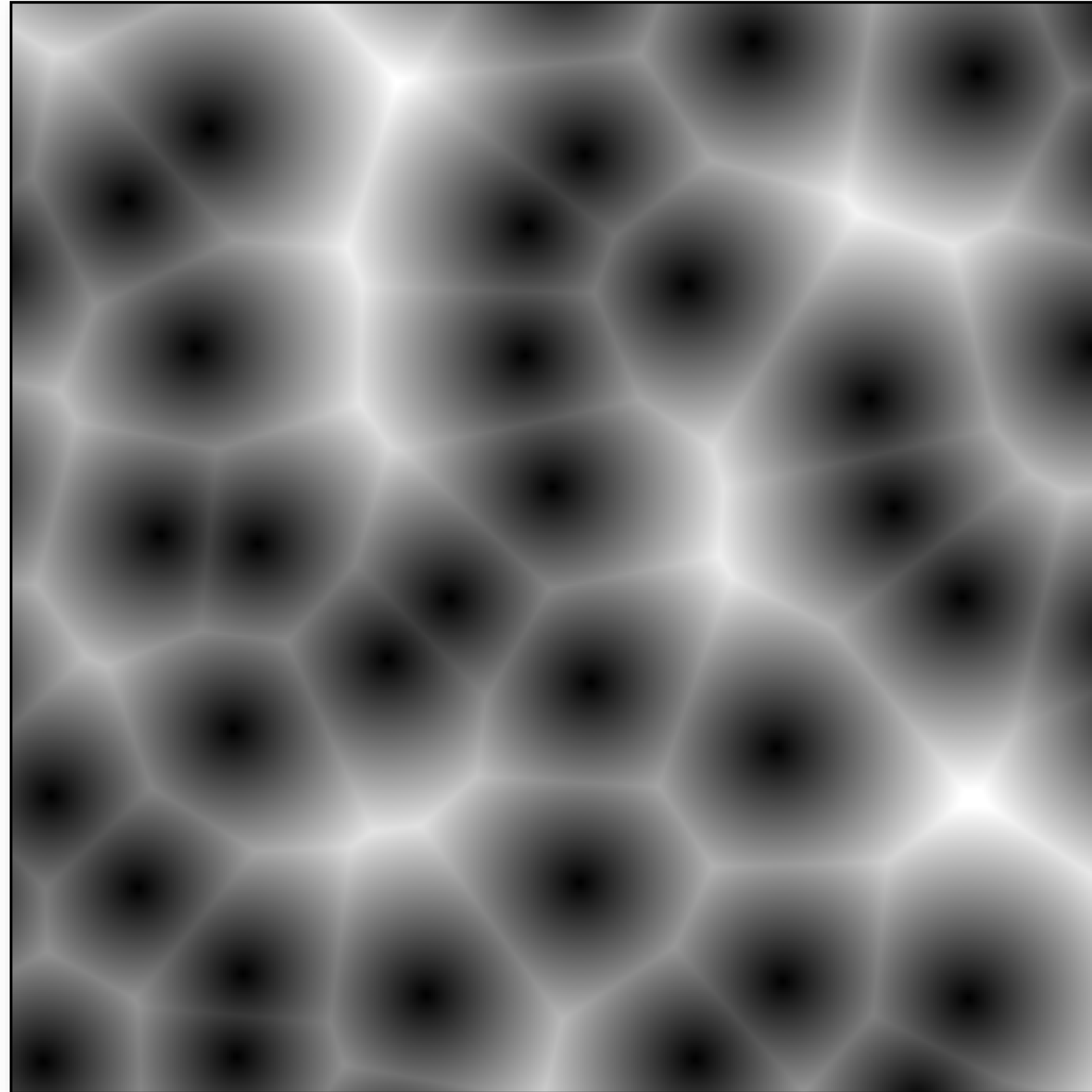
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What do we call this image in geometry?

# 2D Worley noise: $f_1$

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# Worley Noise

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fractal F1, bump map



# Worley Noise

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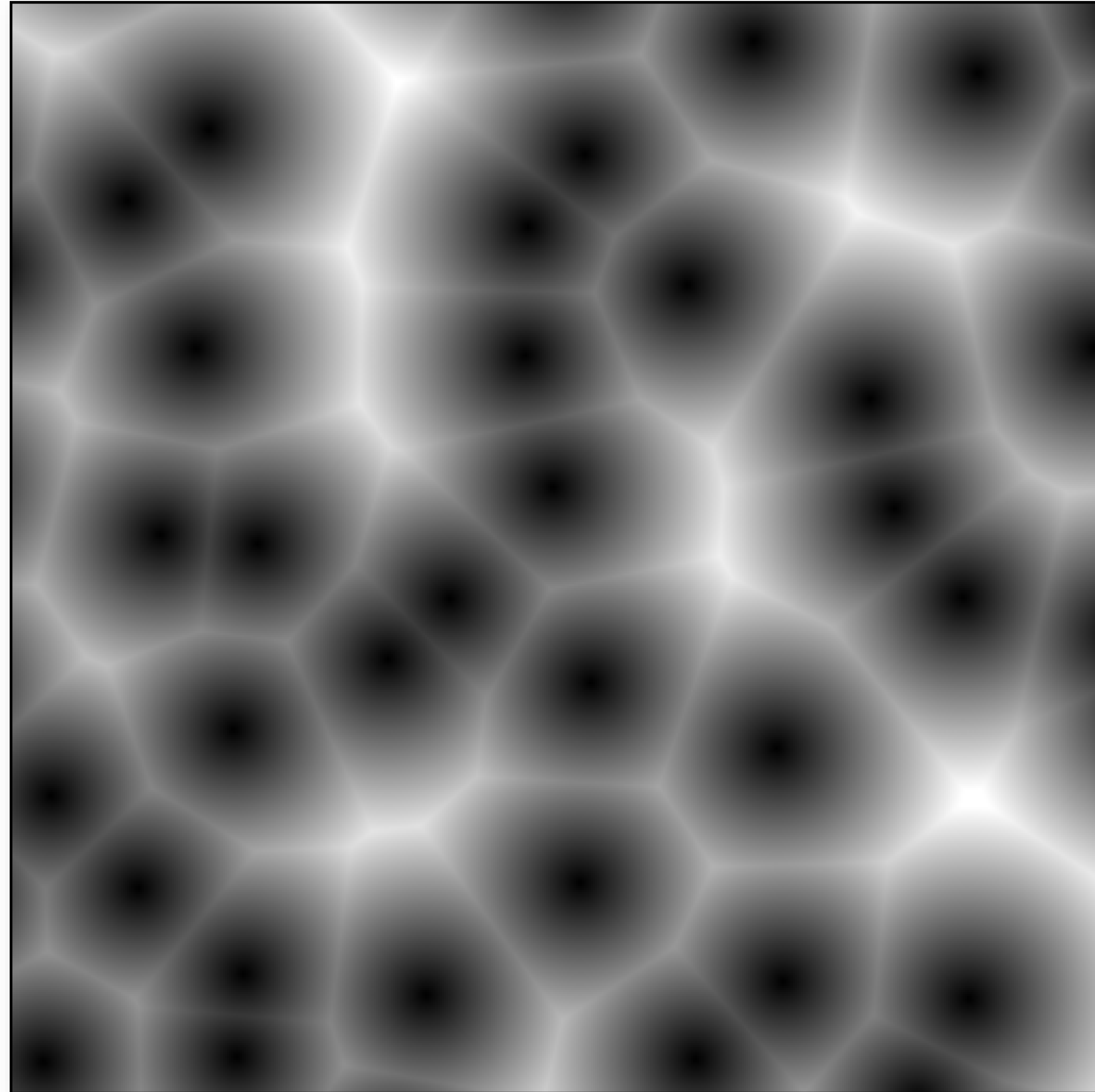


fractal F1, bump map



# 2D Worley noise: $f_1$

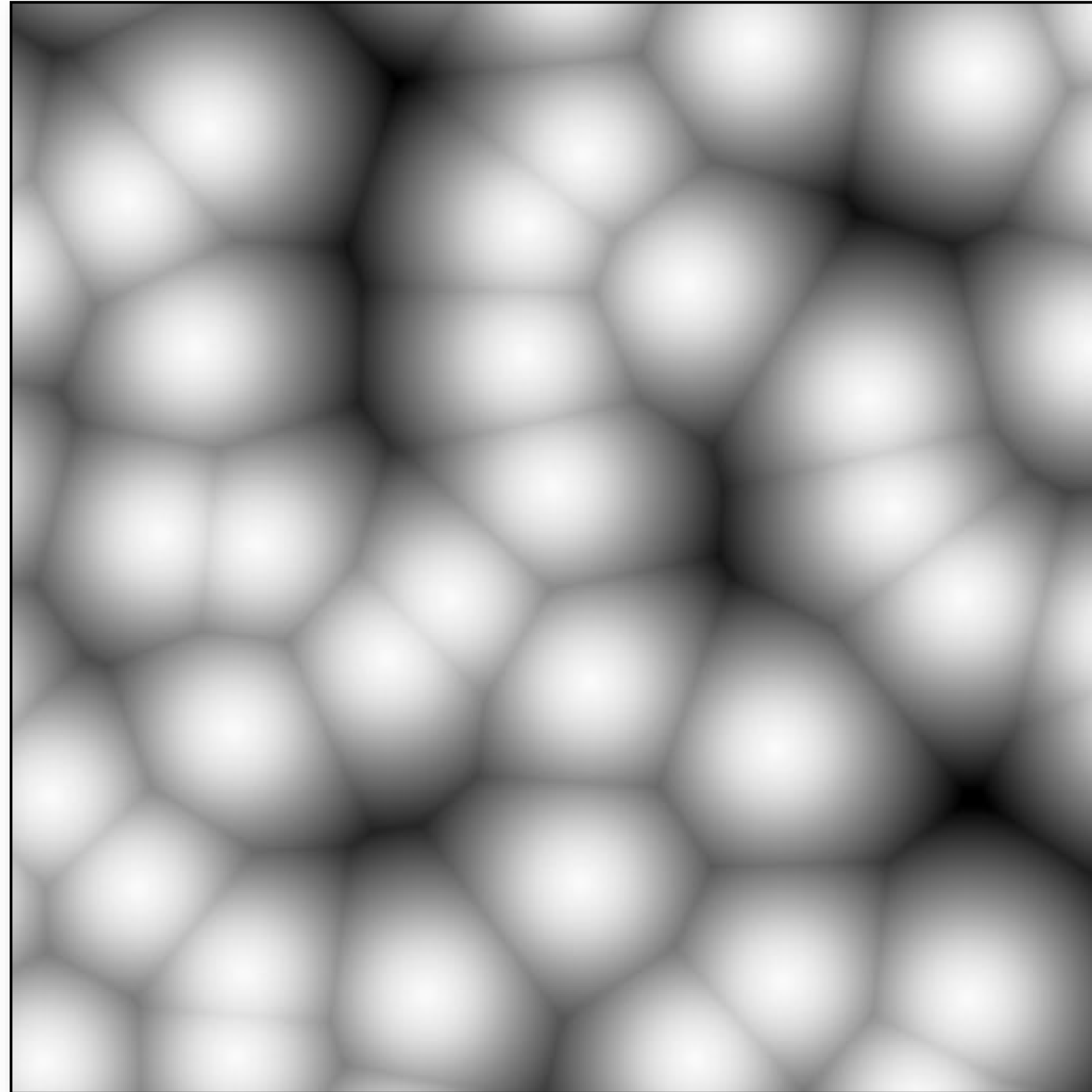
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Wojciech Jarosz 2007

# 2D Worley noise: $1-f_1$

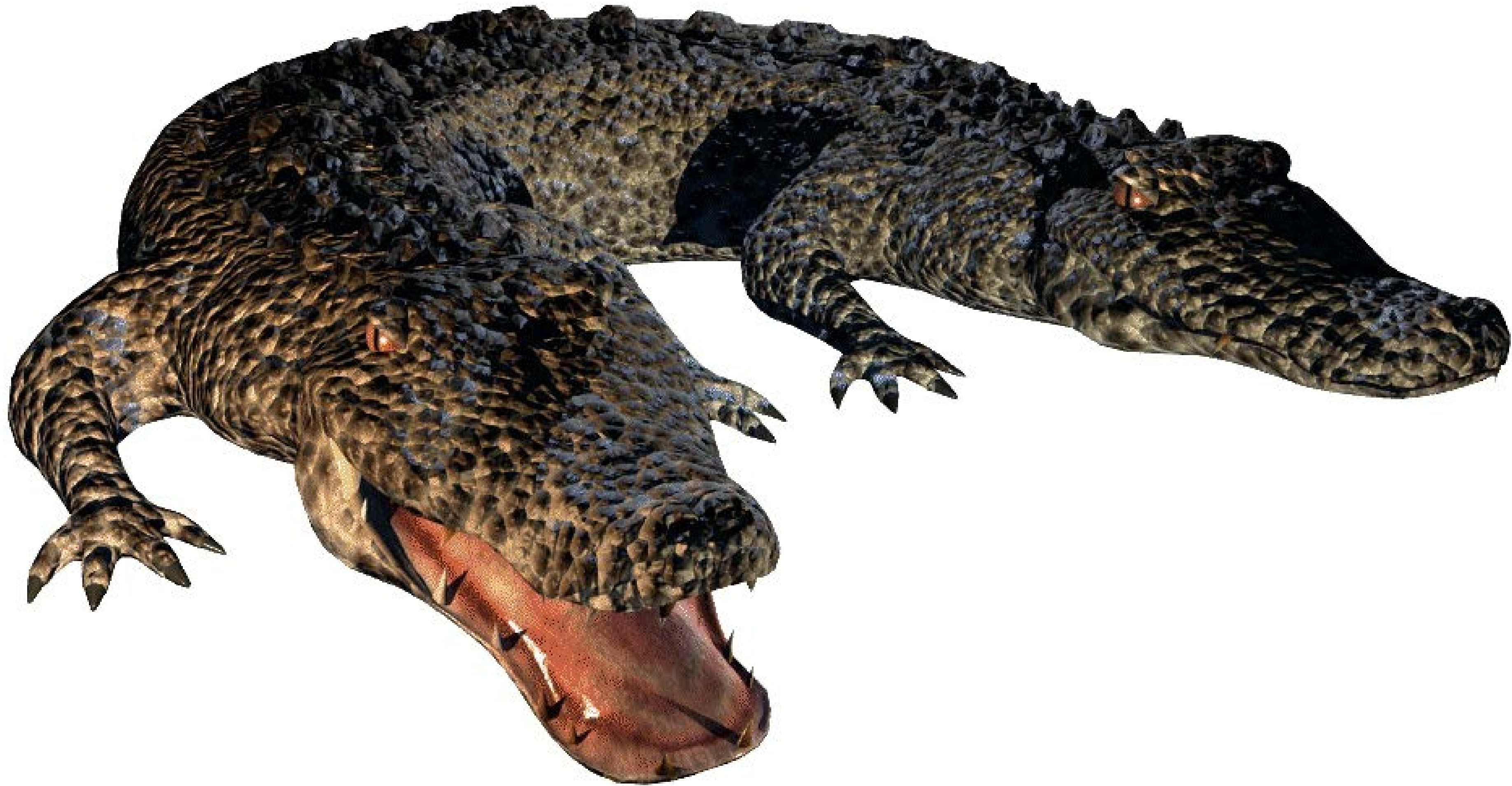
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Wojciech Jarosz 2007

# Worley Noise

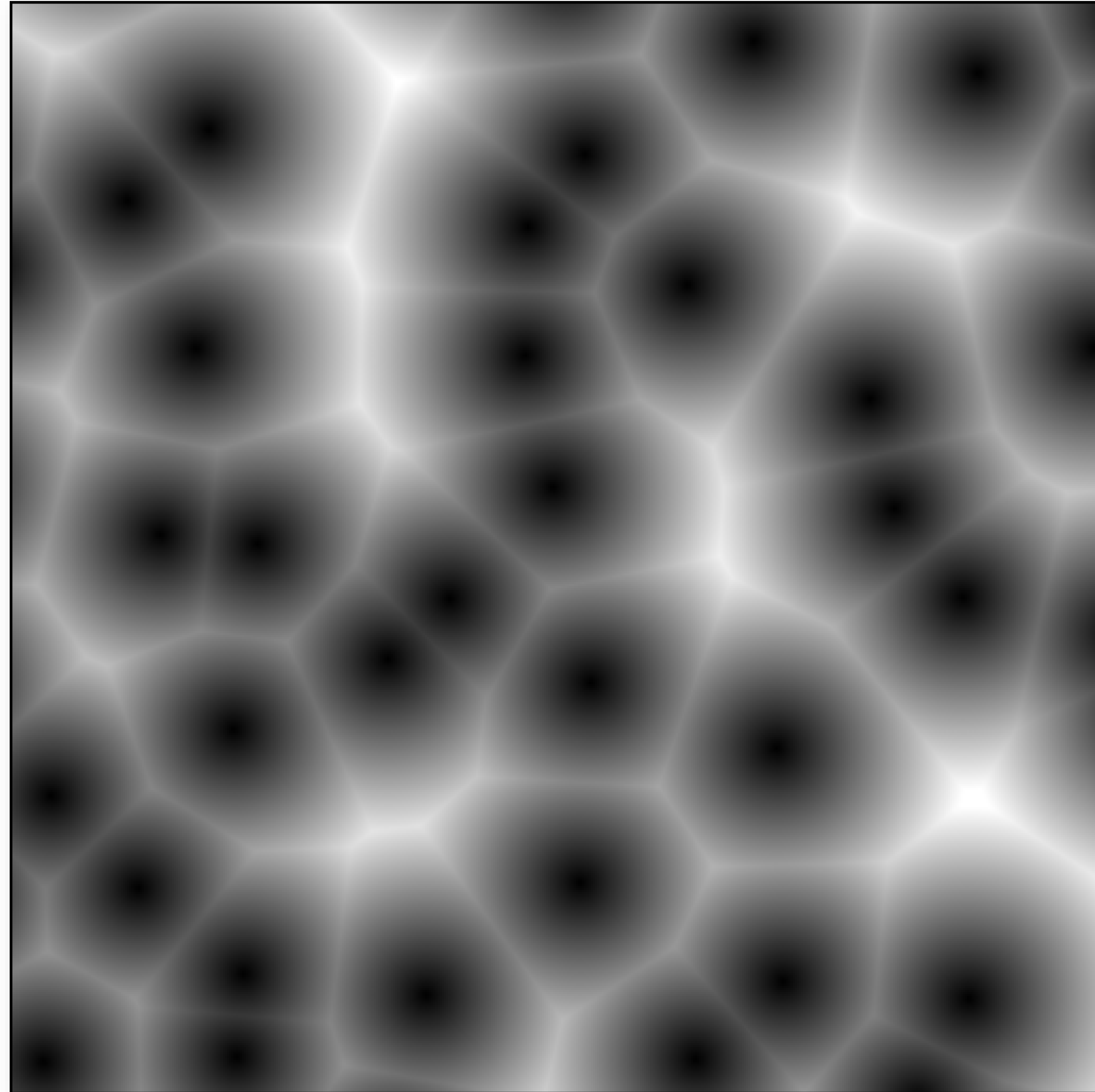
---



fractal  $1-f_1$ , color and bump map

# 2D Worley noise: $f_1$

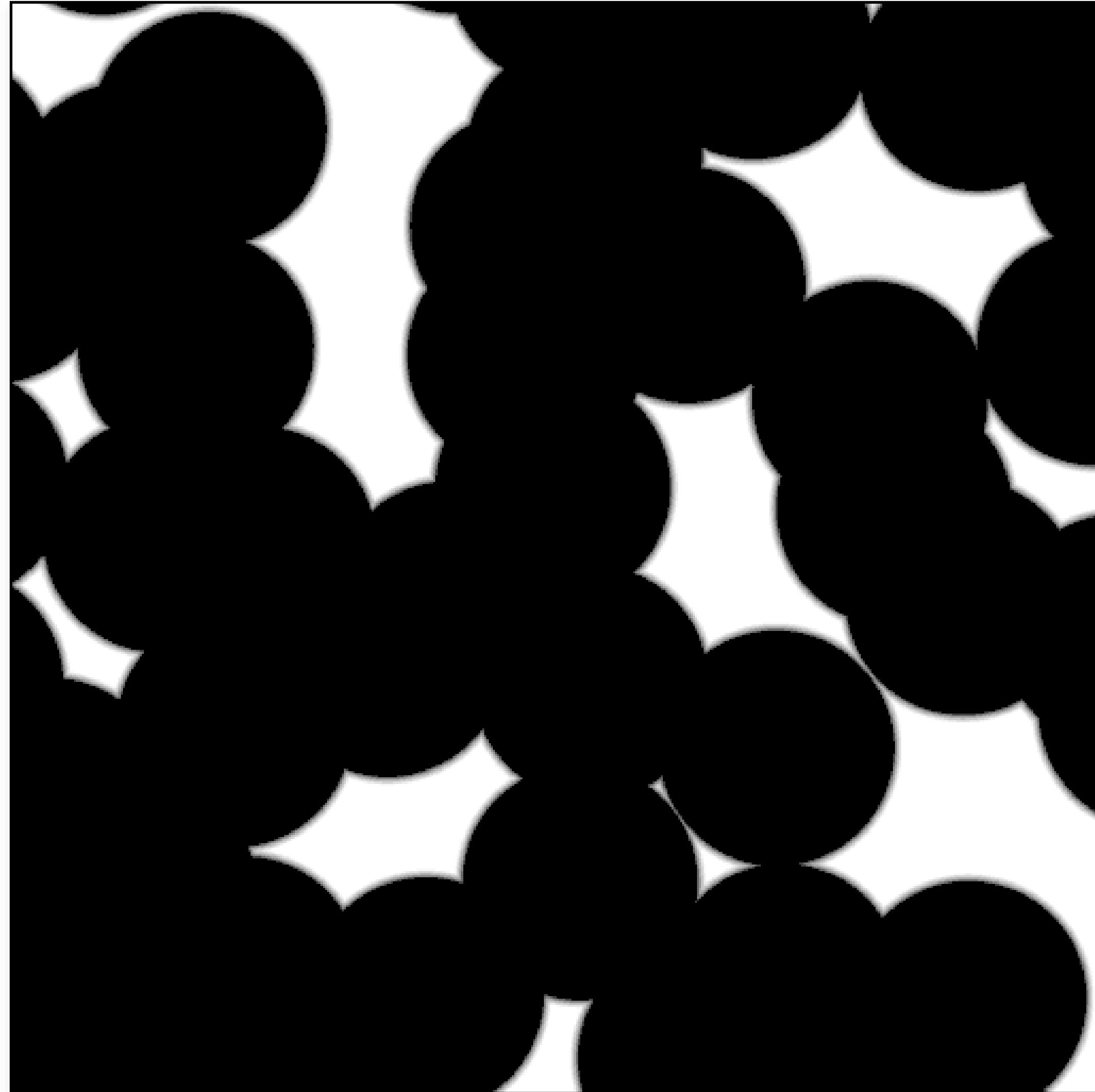
---



Wojciech Jarosz 2007

# 2D Worley noise: $f_1$ , thresholded

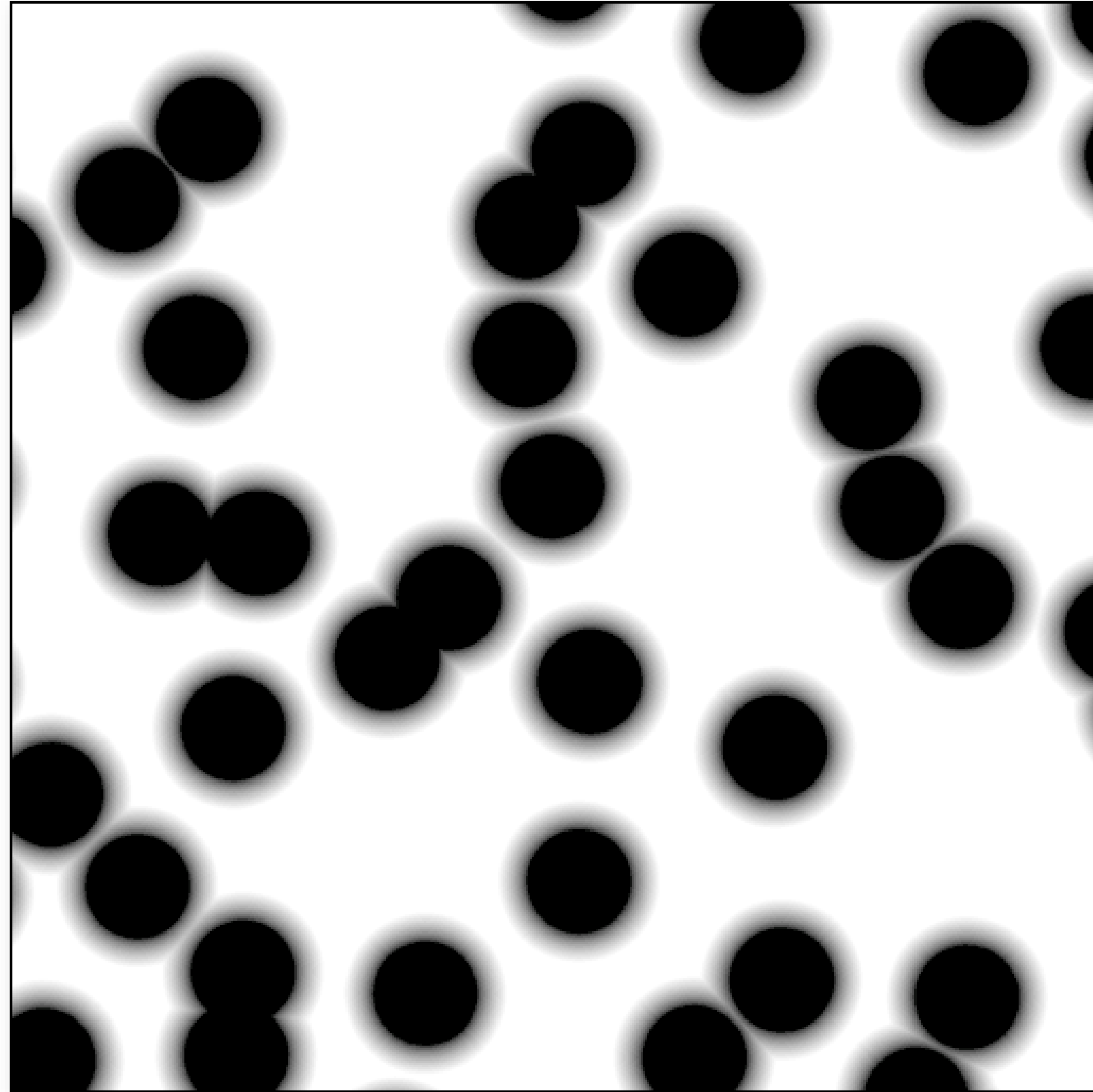
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Wojciech Jarosz 2007

# 2D Worley noise: $f_1$ , thresholded

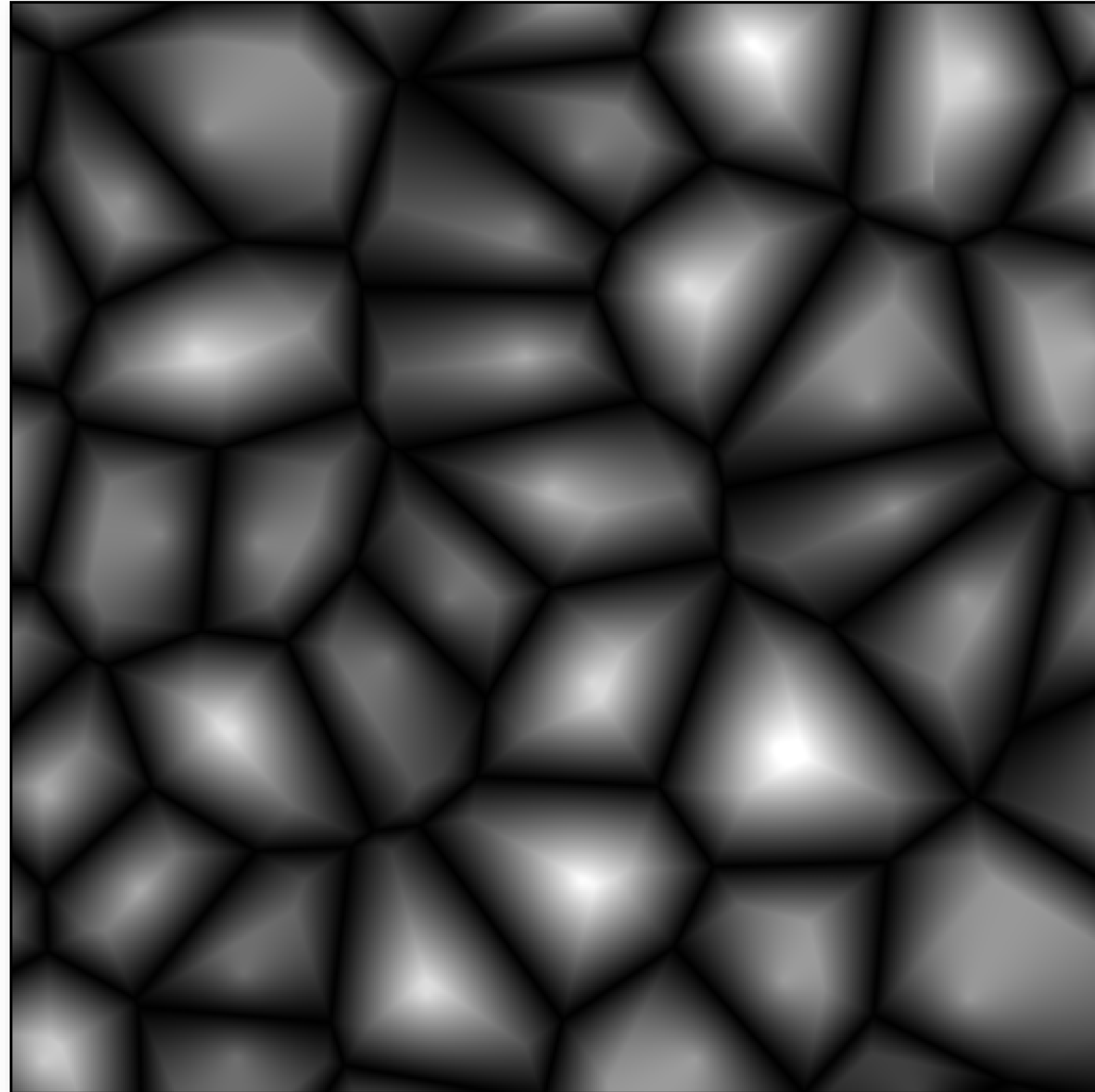
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Wojciech Jarosz 2007

# 2D Worley noise: $f_2-f_1$

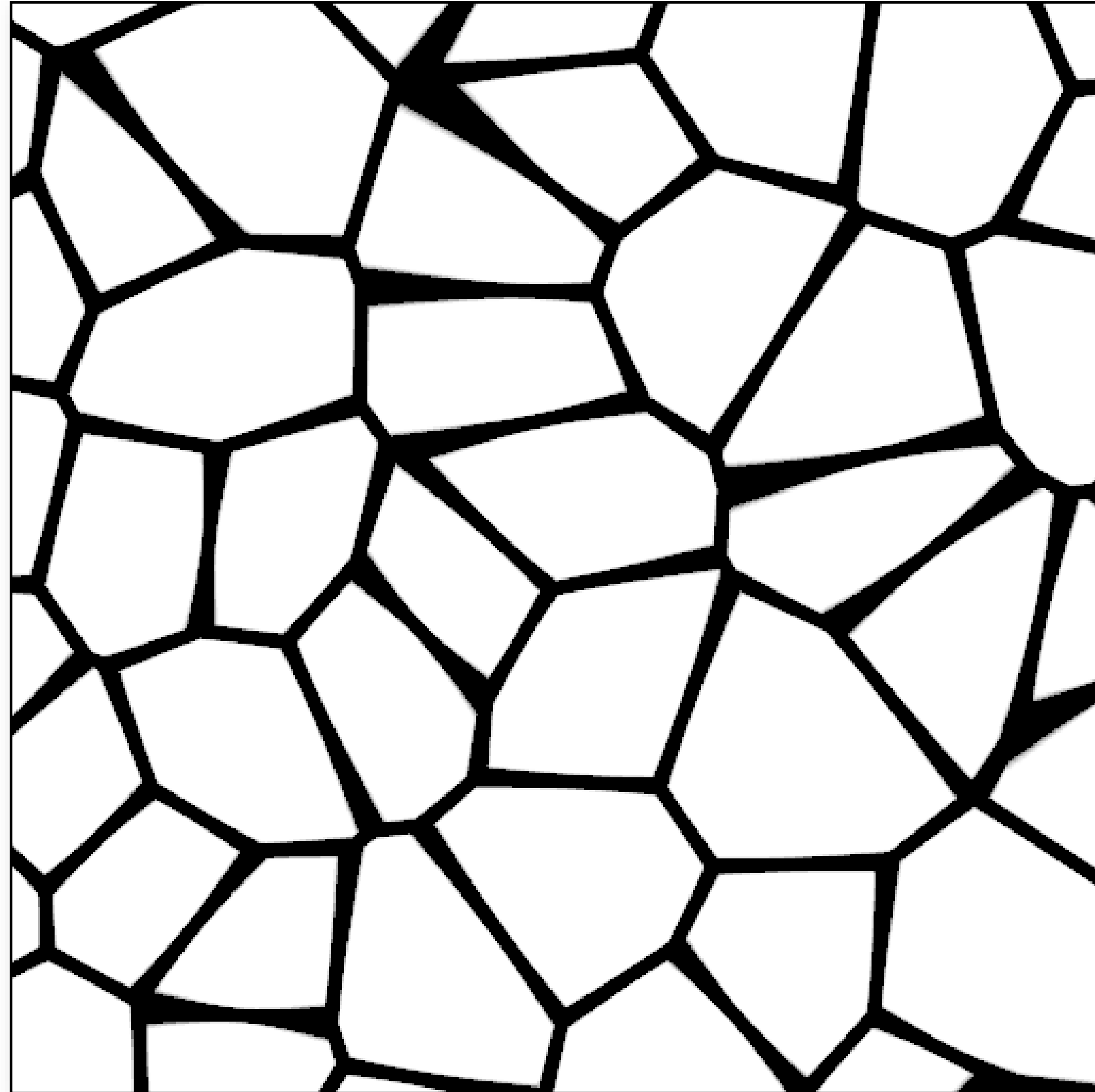
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Wojciech Jarosz 2007

# 2D Worley noise: $f_2-f_1$ , thresholded

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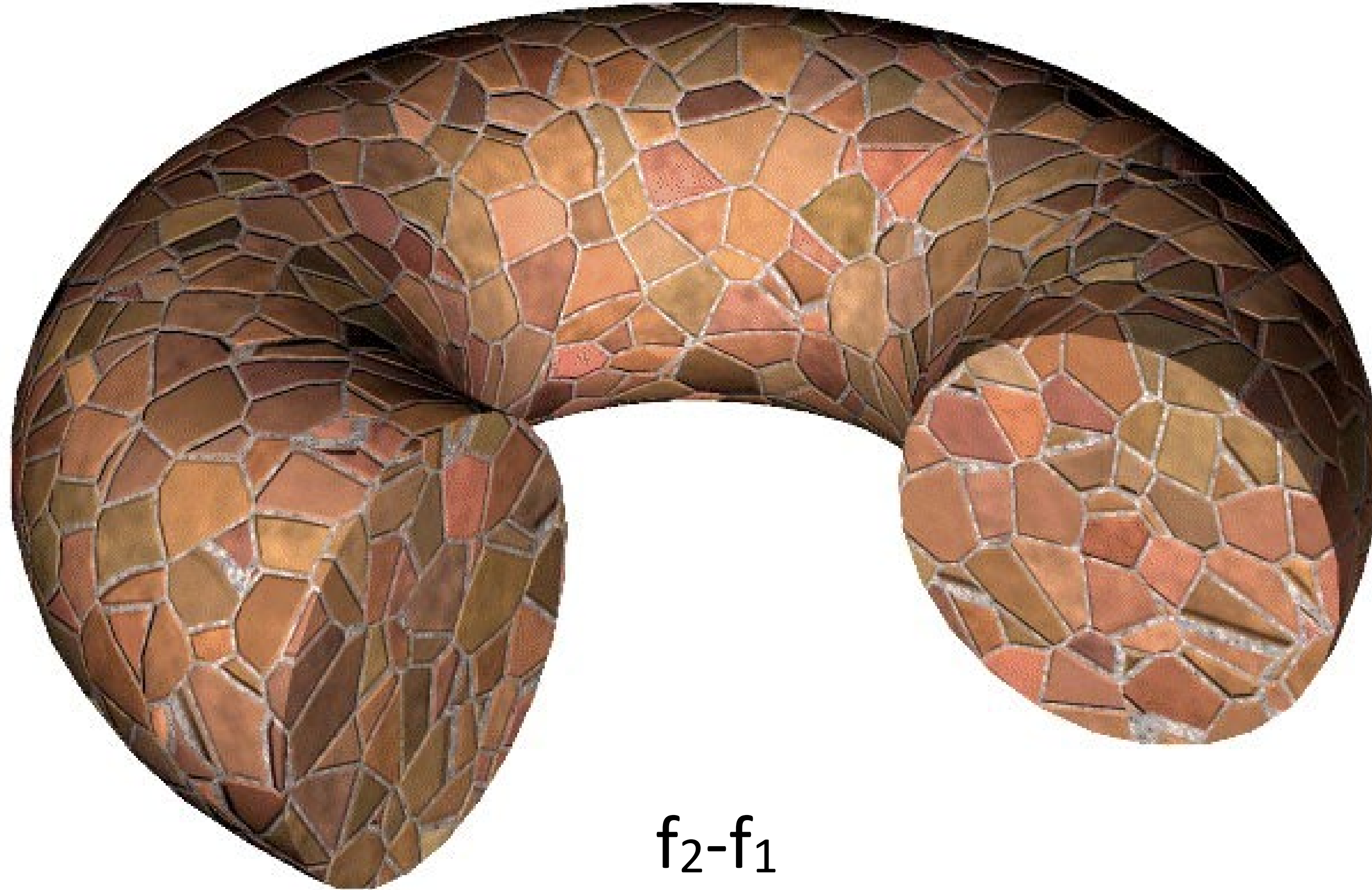


Wojciech Jarosz 2007



# 3D Worley noise

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$f_2 - f_1$

# Worley Noise

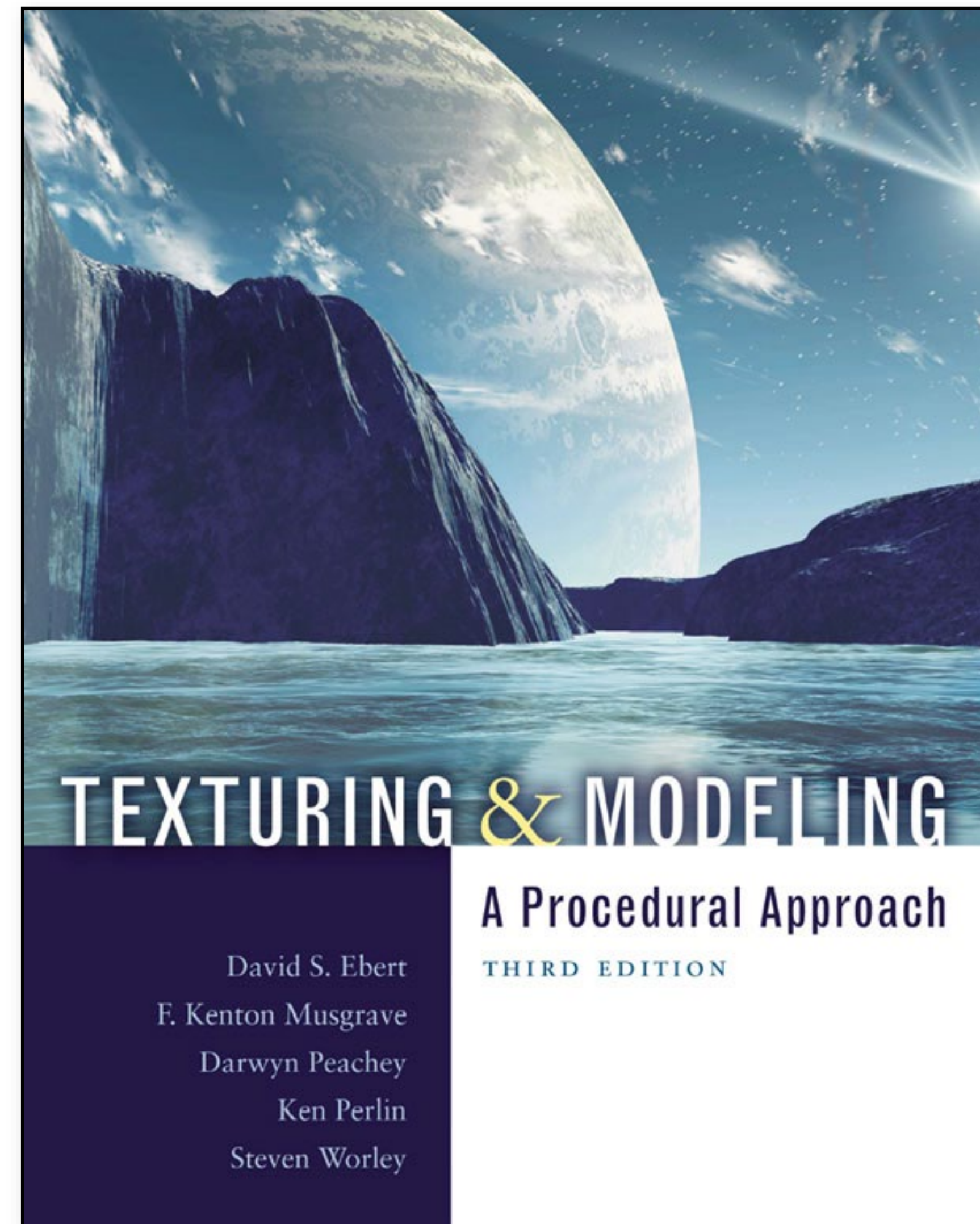
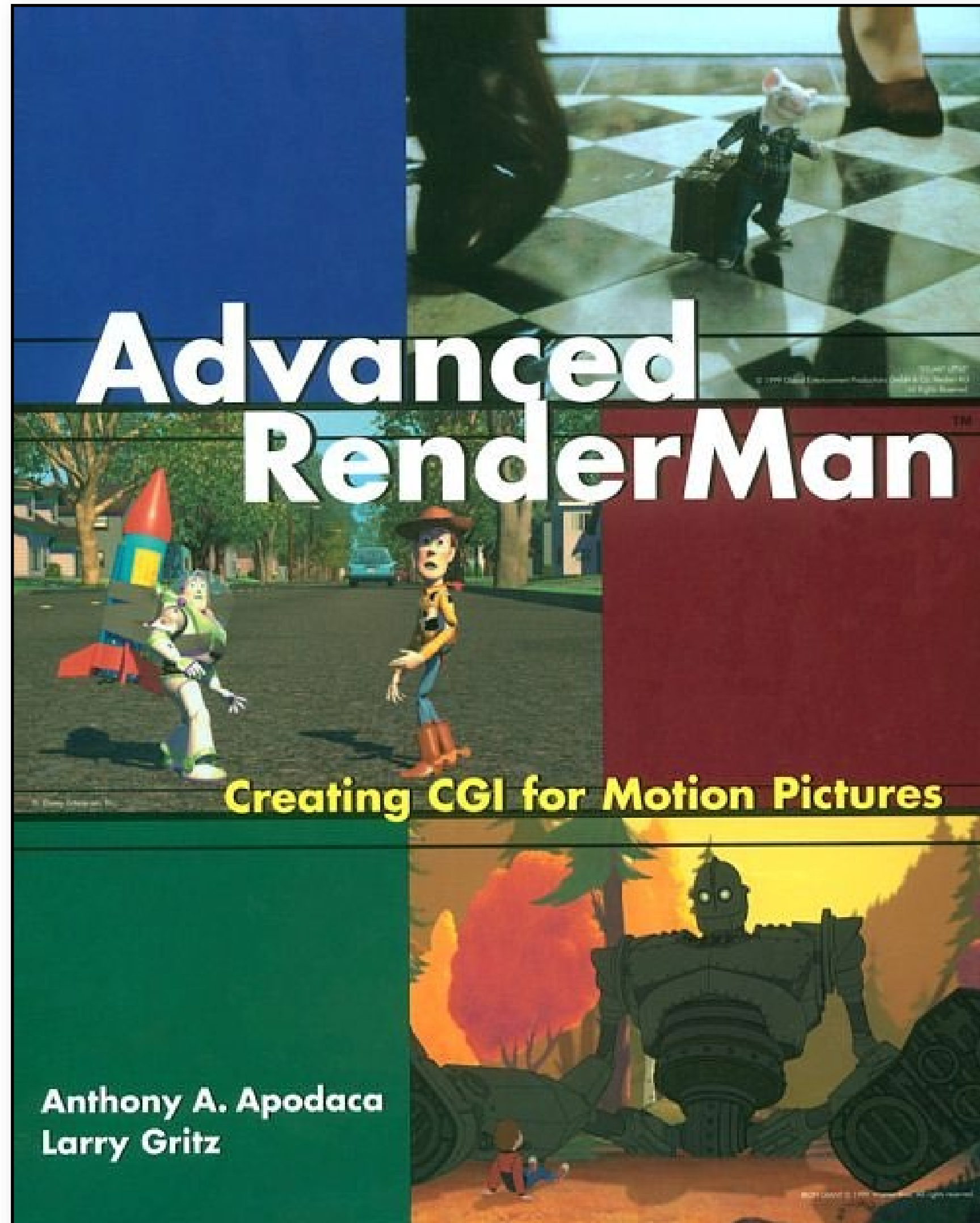
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fractal f1-f4 combinations



# Other Resources



# Demos

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Amazing realtime demos using fractal noise:

- <http://www.iquilezles.org/www/articles/morenoise/morenoise.htm>
- <https://www.shadertoy.com/view/4ttSWf>
- <https://www.shadertoy.com/view/XttSz2>