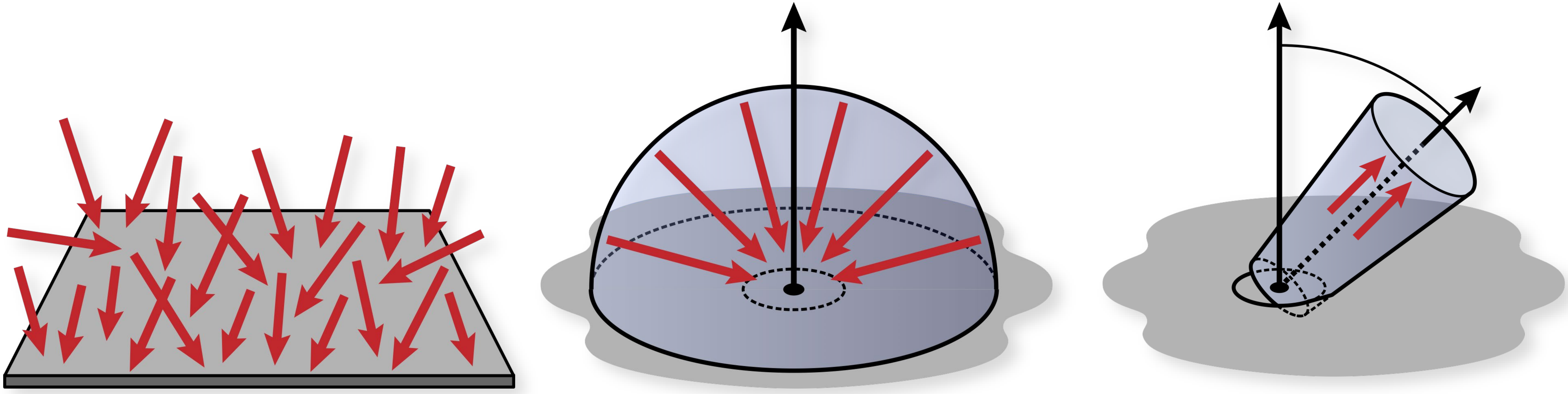
Radiometry



15-468, 15-668, 15-868
Physics-based Rendering
Spring 2022, Lecture 6

http://graphics.cs.cmu.edu/courses/15-468

Course announcements

- Take-home quiz 2 posted, due Tuesday 2/8 at 23:59.
  - How many of you have looked at/started/finished it?
  - Any questions?

- Programming assignment 1 posted, due Friday 2/11 at 23:59.
  - How many of you have looked at/started/finished it?
  - Any questions?

- First reading group on Friday 2/4, 4-6 pm.
  - Suggest topics on Piazza.

- First recitation took place yesterday, 4-5 pm.
  - Recording and photos of whiteboard on Canvas.
  - Any feedback?

Overview of today’s lecture

- Radiometric quantities.
- A little bit about color.
- Reflectance equation.
- Standard reflectance functions revisited.

Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

Quantifying Light

Assumptions

Light sources, reflectance spectra, sensor sensitivity modeled separately at each wavelength

Geometric/ray optics

No polarization

No fluorescence, phosphorescence, …

Radiometry

Radiometry studies the measurement of electromagnetic radiation, including visible light.

Radiometry

Assume light consists of photons with:

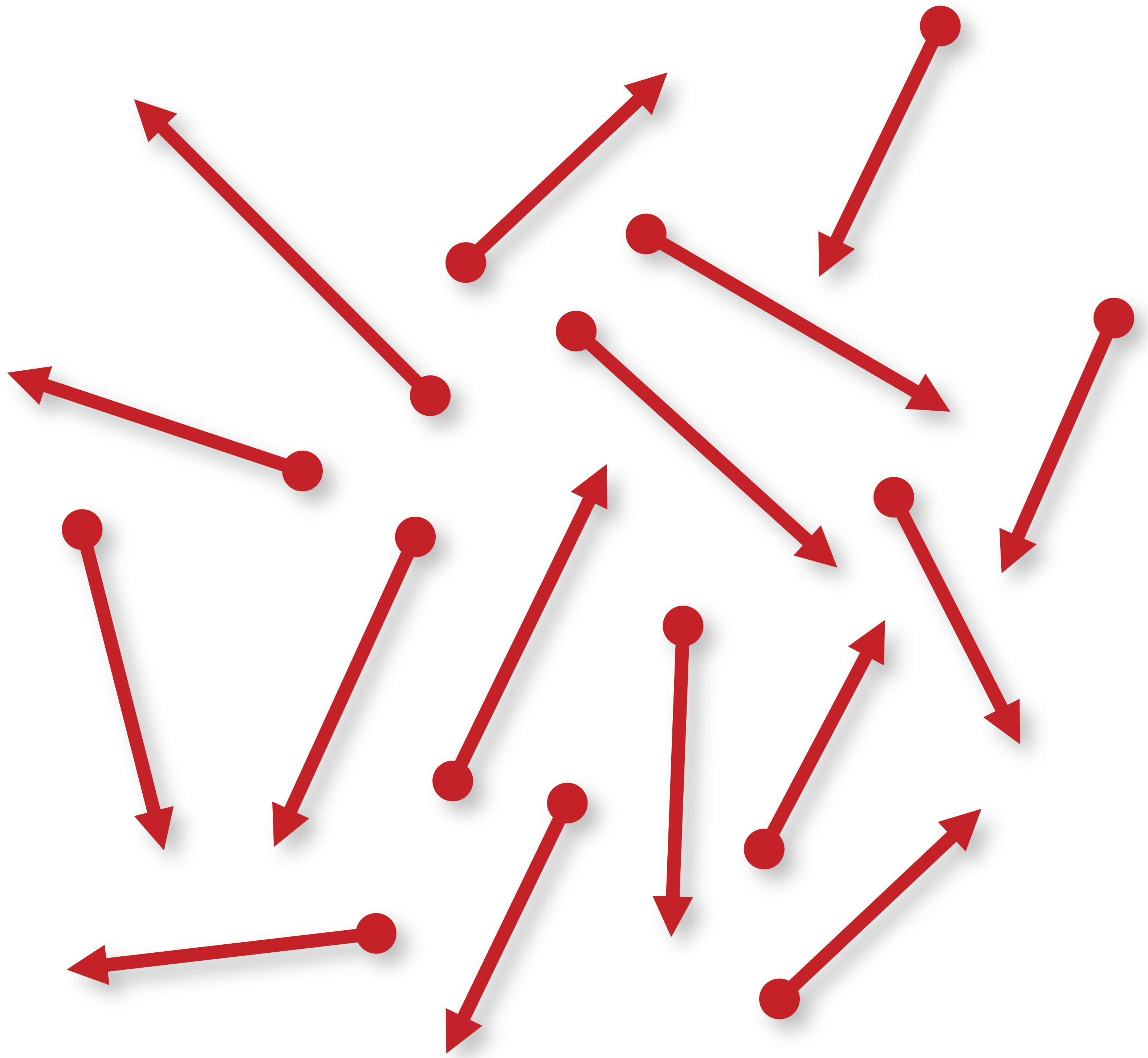
- $\mathbf{x}$: Position
- $\vec{\omega}$: Direction of travel
- λ: Wavelength

Each photon has an energy of: $\dfrac{h\,c}{\lambda}$

- $h \approx 6.63 \times 10^{-34}\,\mathrm{m^2\,kg/s}$: Planck's constant
- $c = 299{,}792{,}458\,\mathrm{m/s}$: speed of light in vacuum
- Unit of energy, Joule: $\left[\mathrm{J} = \mathrm{kg\,m^2/s^2}\right]$

Radiometry

How do we measure the energy flow?



Measuring energy means “counting photons”

Radiometry

Basic quantities (depend on wavelength)

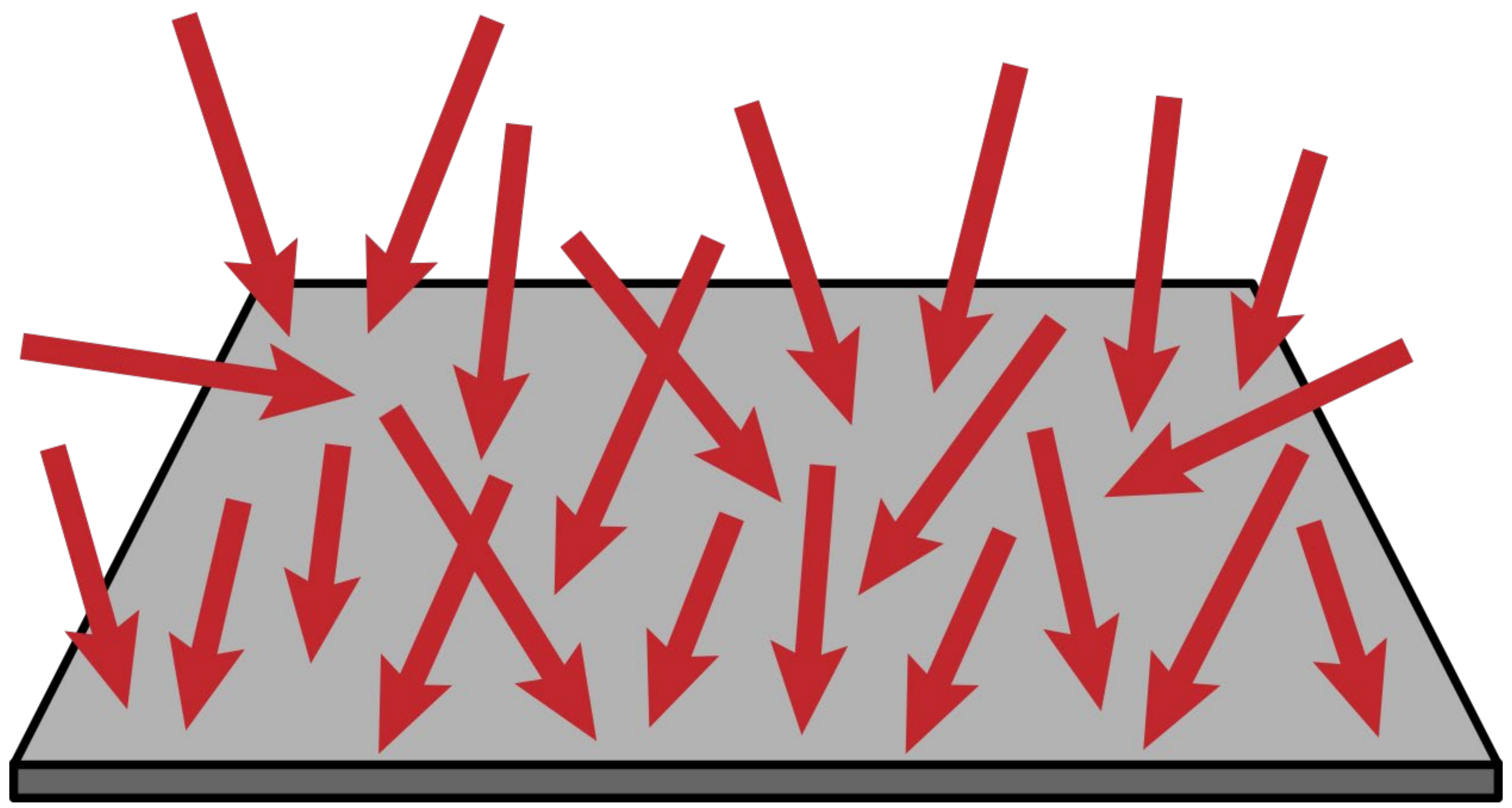
- flux Φ
- irradiance E
- radiosity B
- intensity I
- radiance L

will be the most important quantity for us

Flux (Radiant Flux, Power)

total amount of radiant energy passing through surface or space *per unit time*

$$\Phi(A) \qquad \left[\frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}\right]$$



examples:

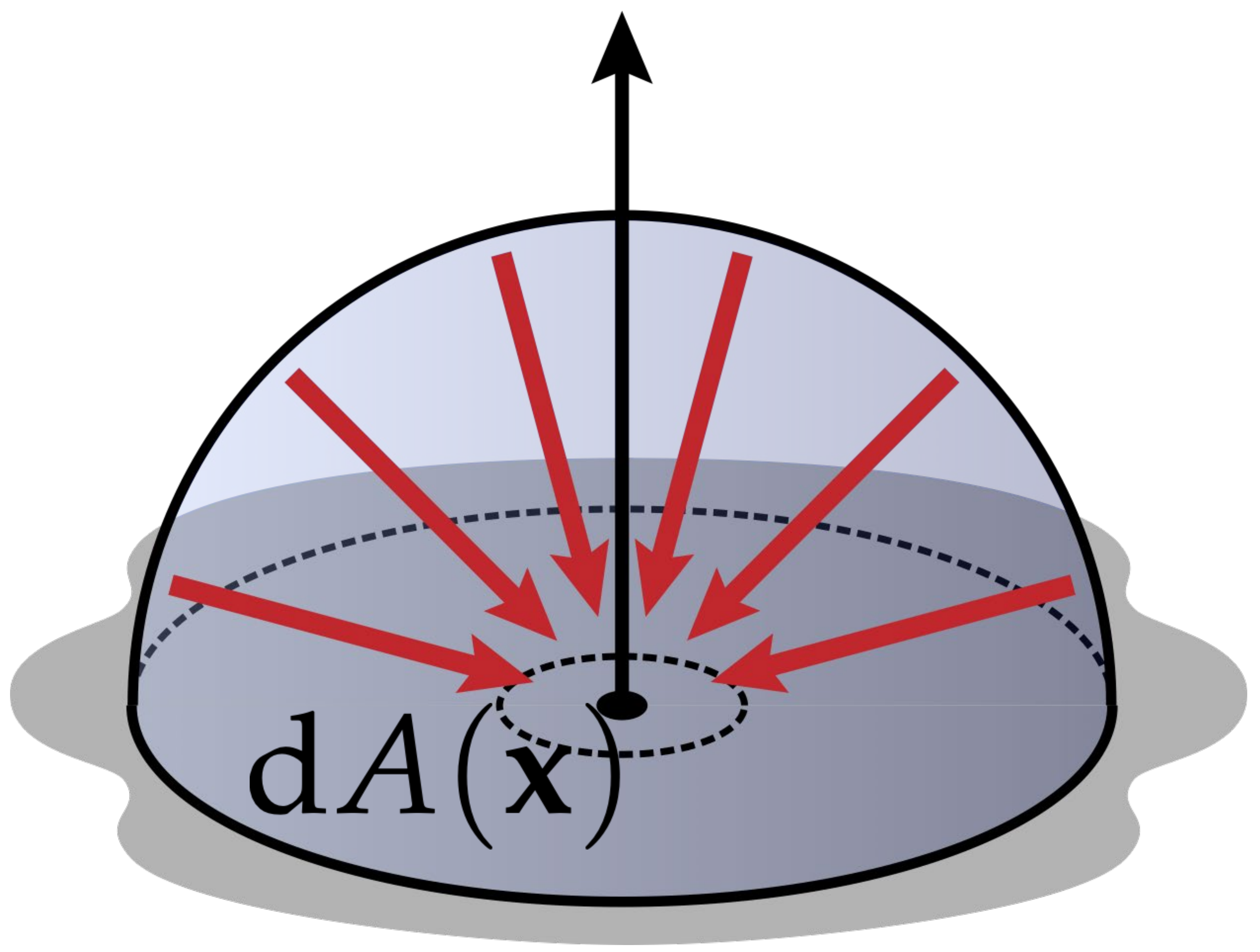
- number of photons hitting a wall per second
- number of photons leaving a lightbulb per second (how do we quantify this exactly?)

Irradiance

area density of flux

flux per unit area **arriving** at a surface

$$E(\mathbf{x}) = \frac{\mathrm{d}\Phi(A)}{\mathrm{d}A(\mathbf{x})} \qquad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$



example:

$\Phi(A)$

- number of photons **hitting** a small patch of a wall per second, *divided* by size of patch

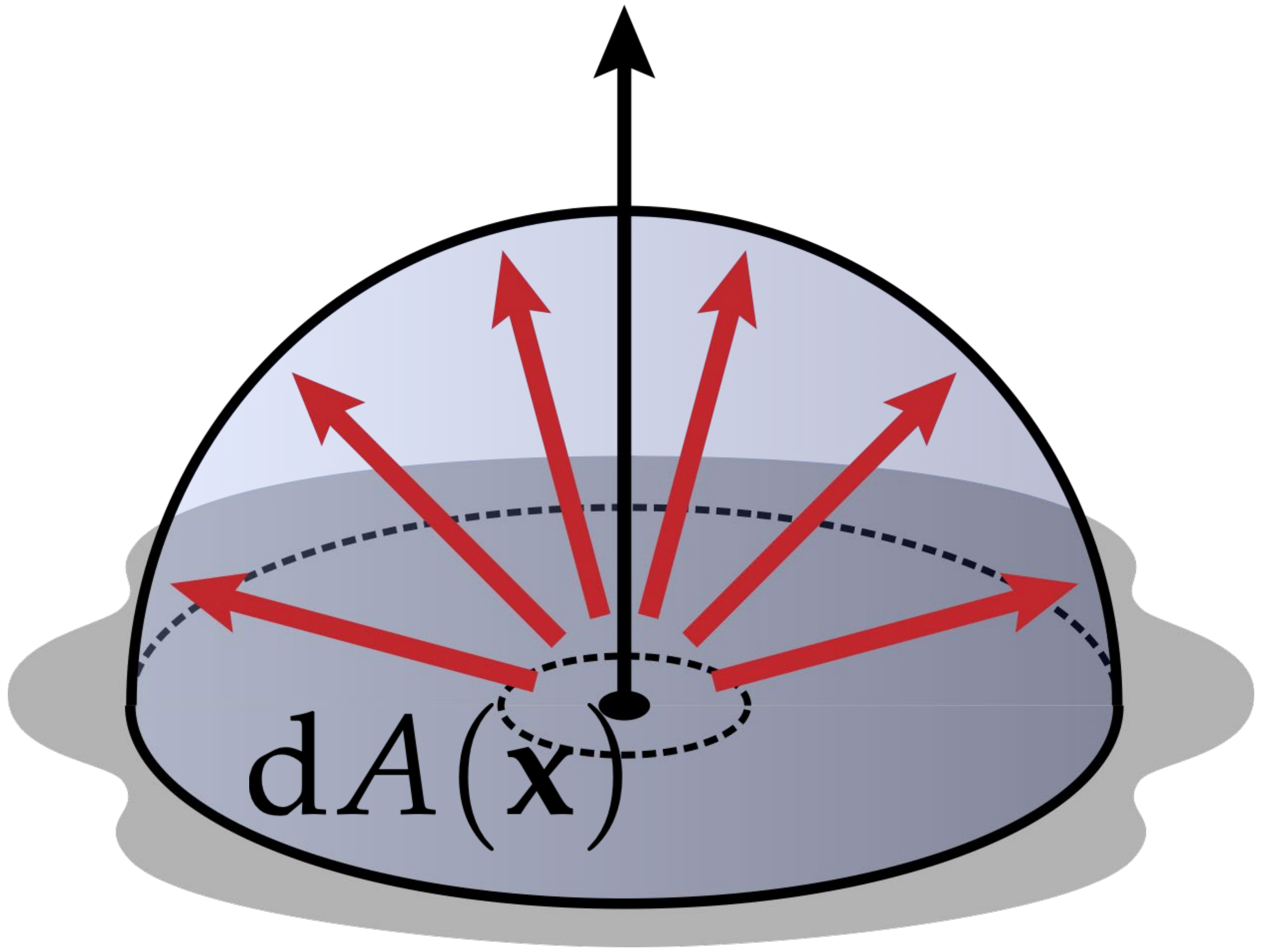
$A \to 0$ A

Radiosity (Radiant Exitance)

area density of flux

flux per unit area **leaving** a surface

$$B(\mathbf{x}) = \frac{\mathrm{d}\Phi(A)}{\mathrm{d}A(\mathbf{x})} \qquad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$



example:

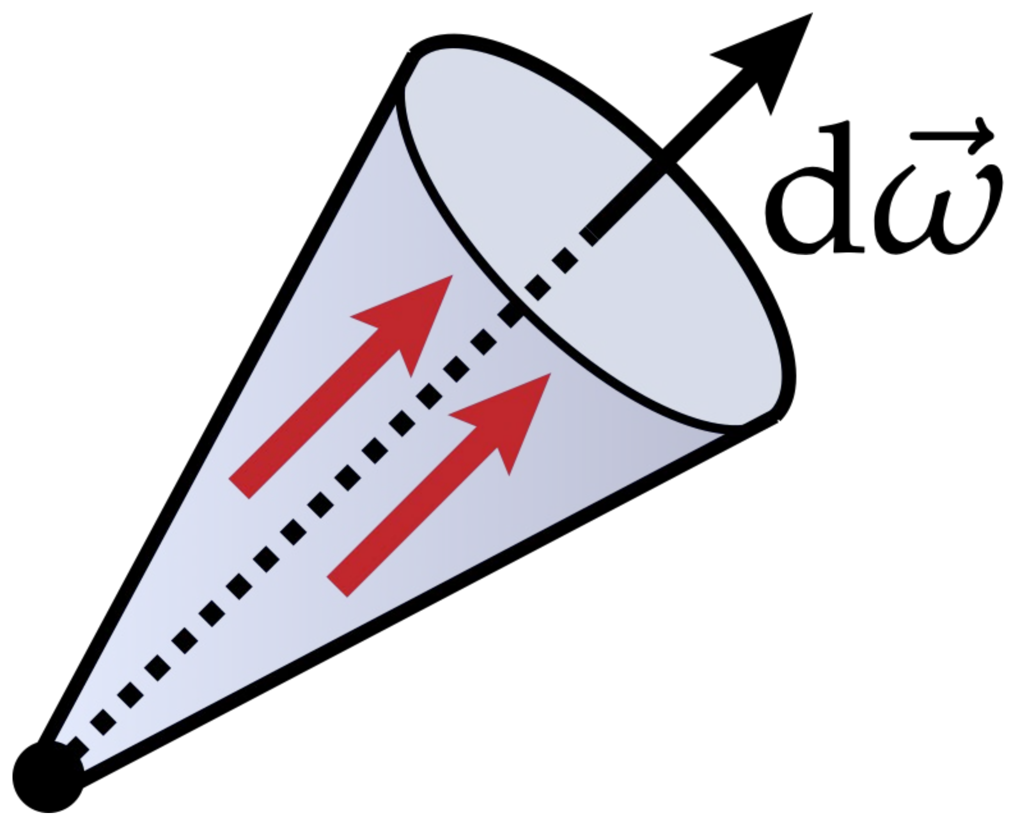
- number of photons **reflecting off** a small patch of a wall per second, *divided* by size of patch

Radiant Intensity

directional density of flux

power (flux) per solid angle

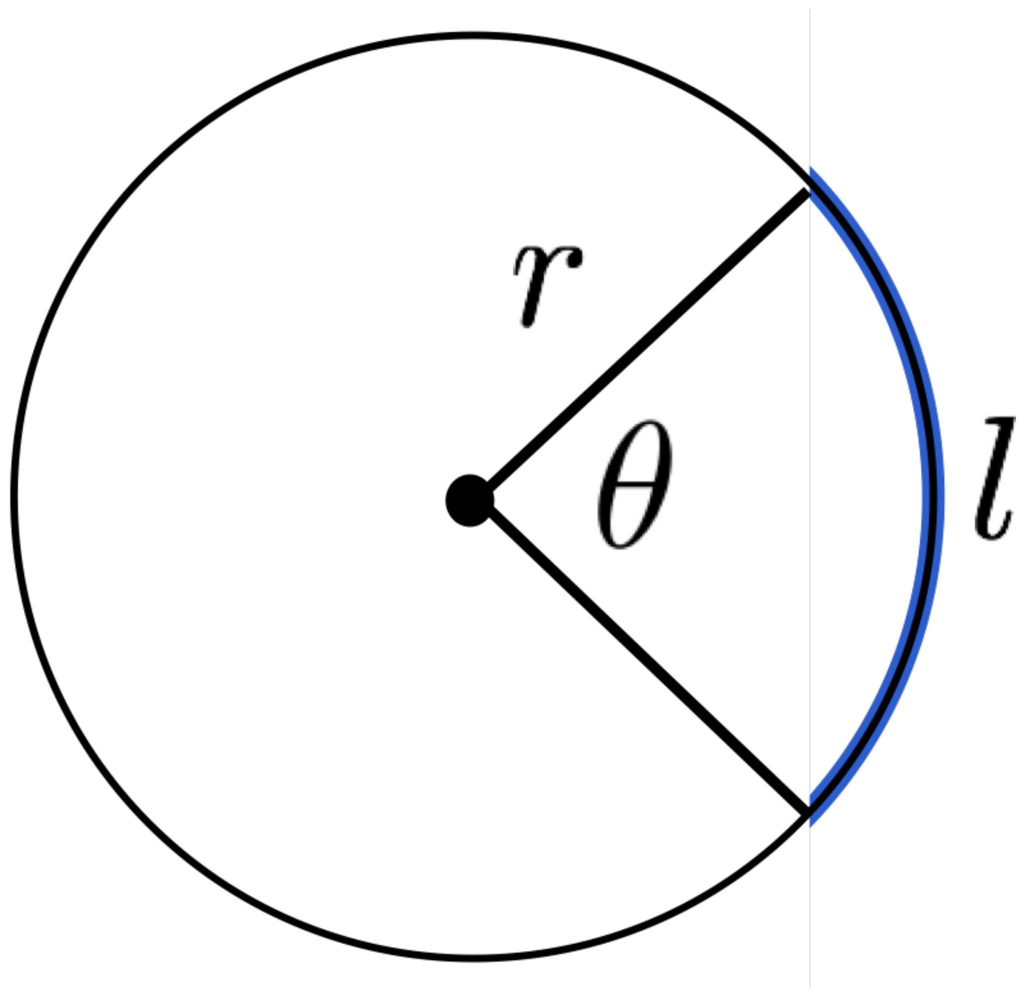
$$I(\vec{\omega}) = \frac{\mathrm{d}\Phi}{\mathrm{d}\vec{\omega}} \qquad \left[\frac{\mathrm{W}}{\mathrm{sr}}\right]$$



Solid Angle

Angle

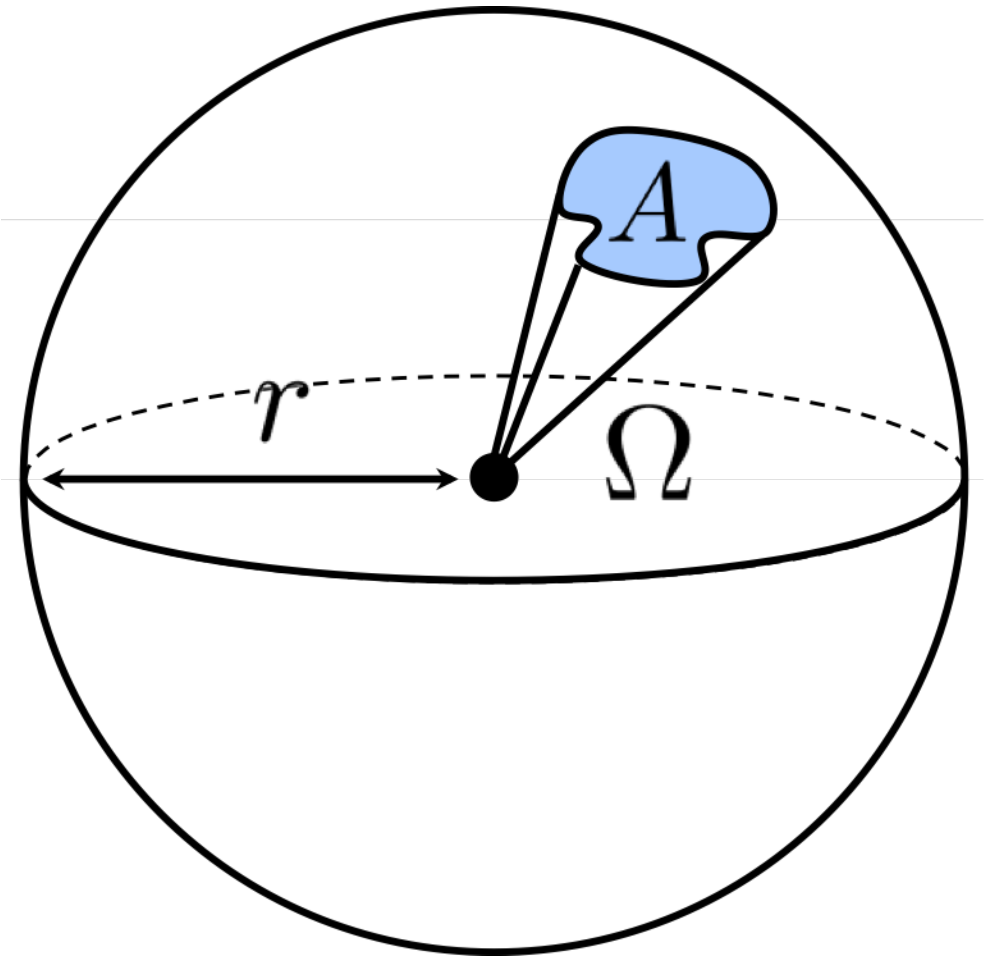
- circle: 2π radians



$$\theta = \frac{l}{r}$$

Solid angle

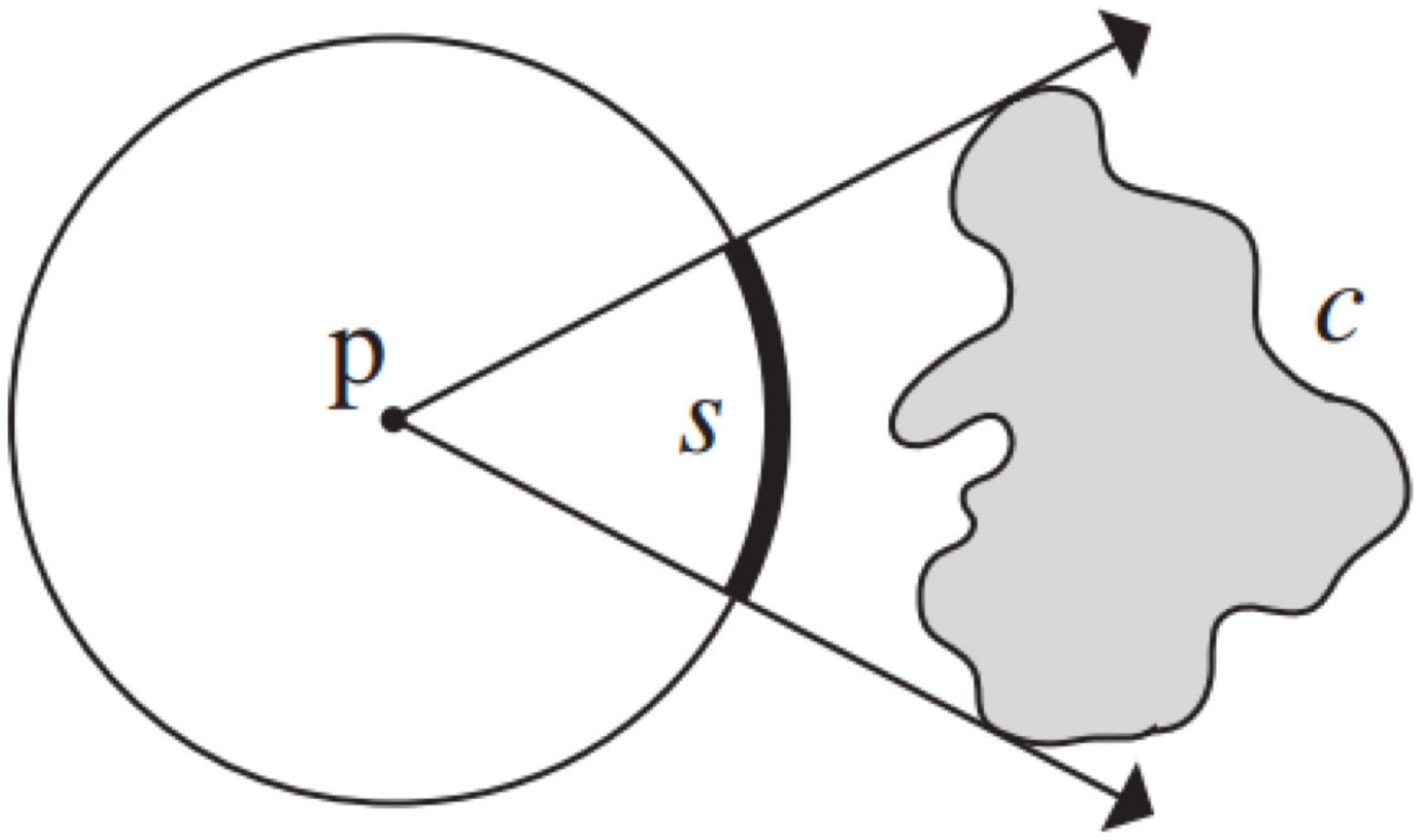
- sphere: 4π steradians

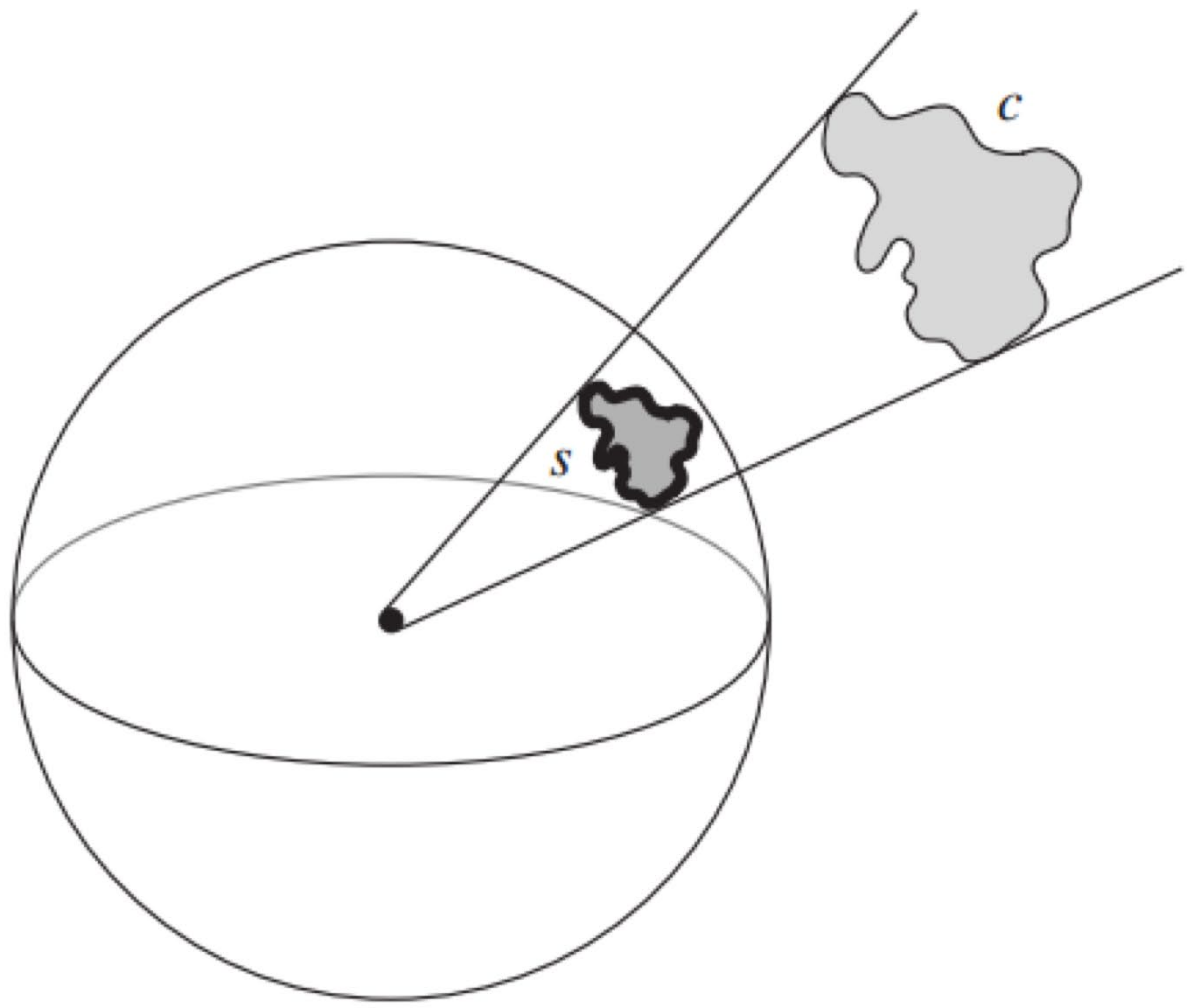


$$\Omega = \frac{A}{r^2}$$

Subtended (Solid) Angle

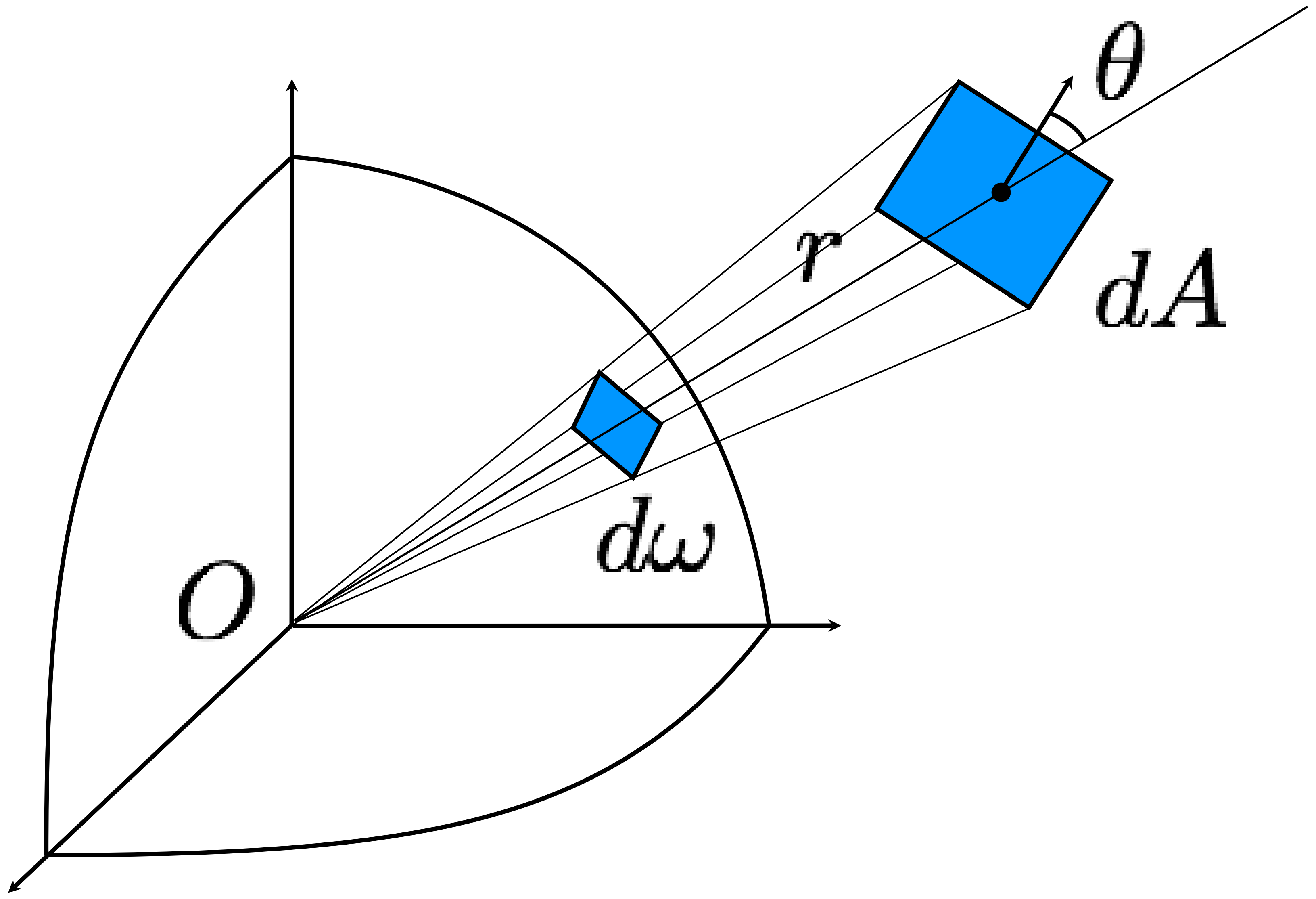
Length/area of object's *projection* onto a unit circle/sphere





Solid angle

The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

One can show:

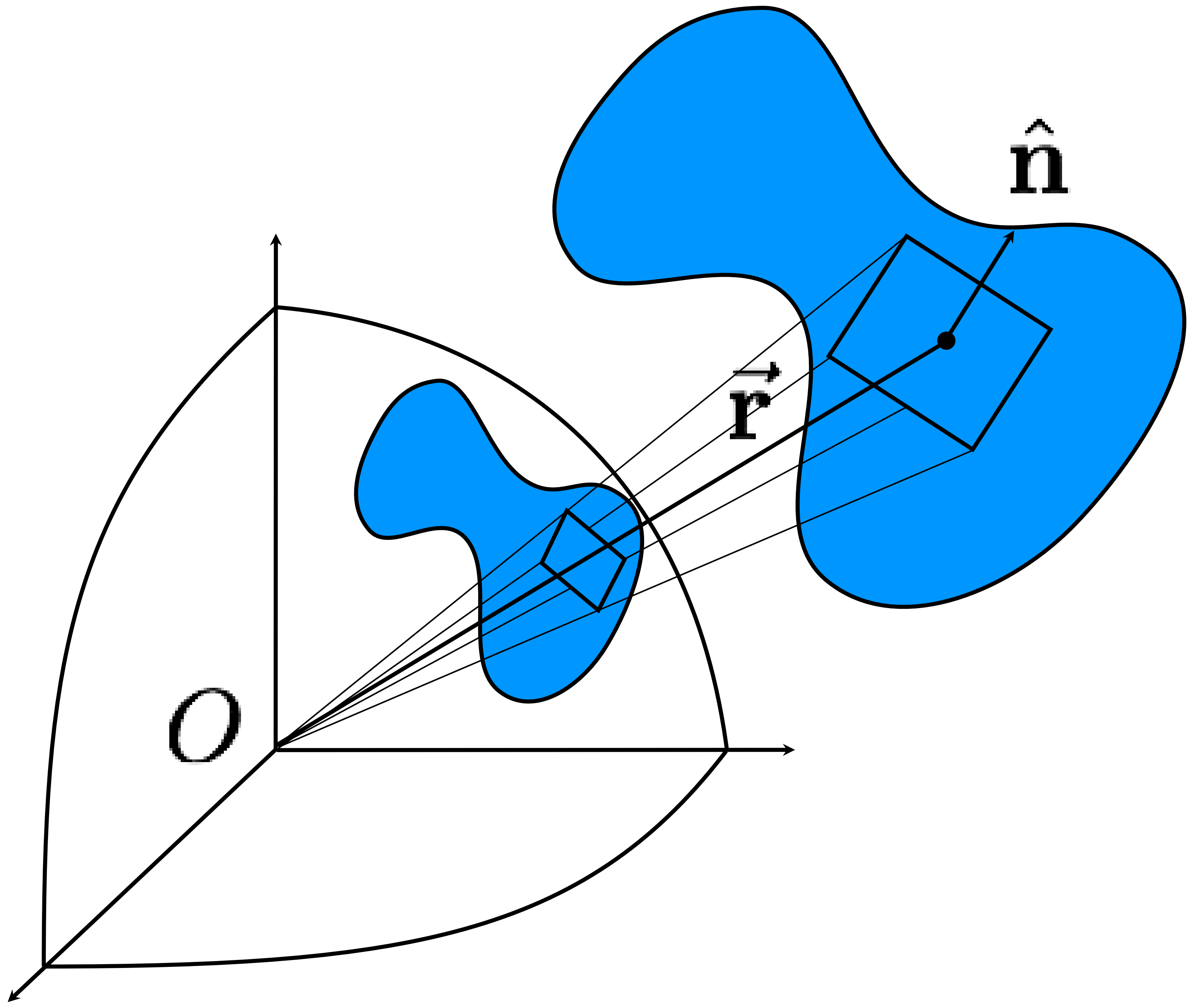
$$\boxed{d\omega = \frac{dA \cos\theta}{r^2}}$$

"surface foreshortening"

Units: steradians [sr]

Solid angle

To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{\mathbf{r}} \cdot \hat{\mathbf{n}} \; dS}{|\vec{\mathbf{r}}|^3}$$

One can show:

$$\boxed{d\omega = \frac{dA \cos\theta}{r^2}}$$

"surface foreshortening"

Units: steradians [sr]

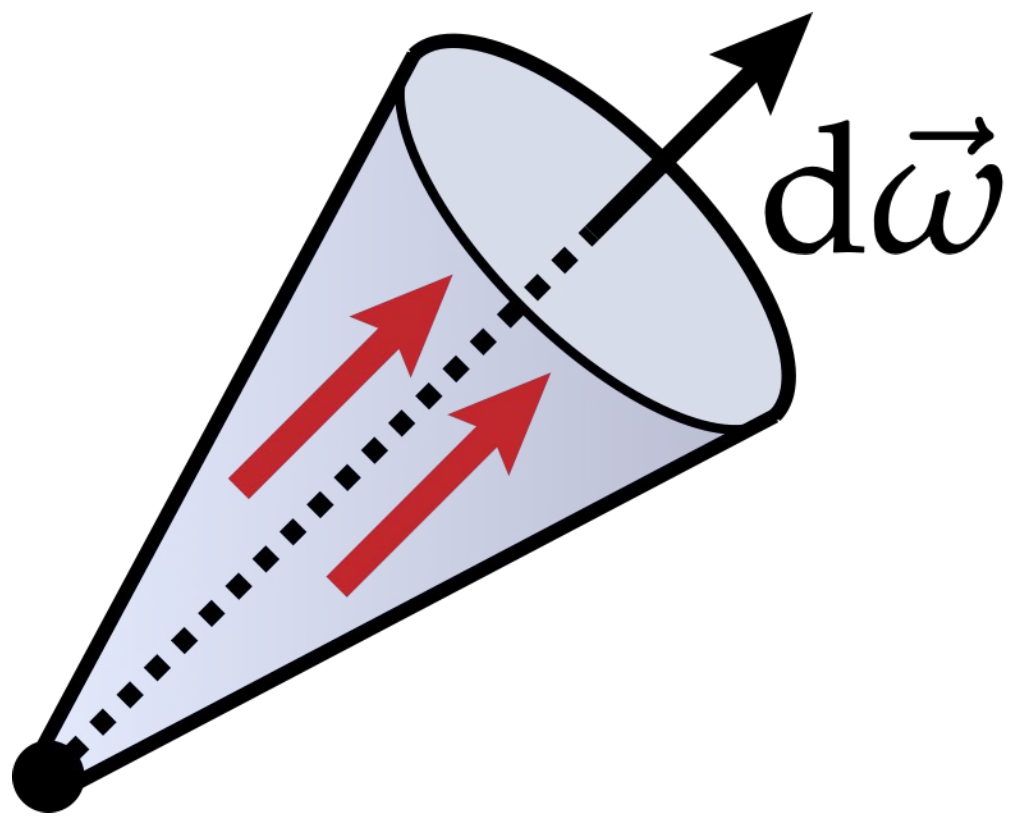
Radiant Intensity

directional density of flux

power (flux) per solid angle

$$I(\vec{\omega}) = \frac{\mathrm{d}\Phi}{\mathrm{d}\vec{\omega}} \qquad \left[\frac{\mathrm{W}}{\mathrm{sr}}\right]$$

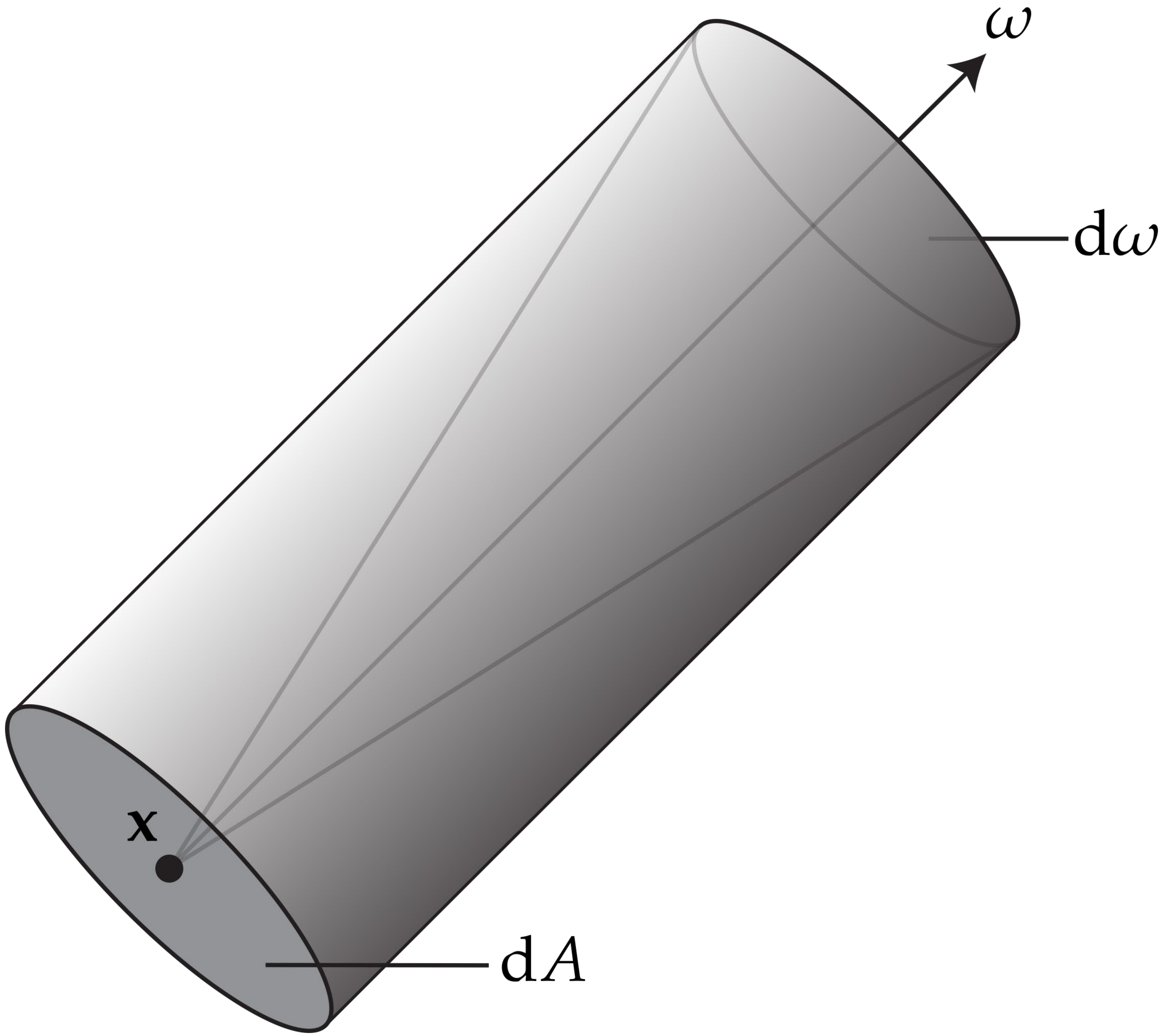
$$\Phi = \int_{S^2} I(\vec{\omega})\,\mathrm{d}\vec{\omega}$$



example: $\Phi = 4\pi I$ (for an isotropic point source)

- power per unit solid angle emanating from a point source

A hypothetical measurement device

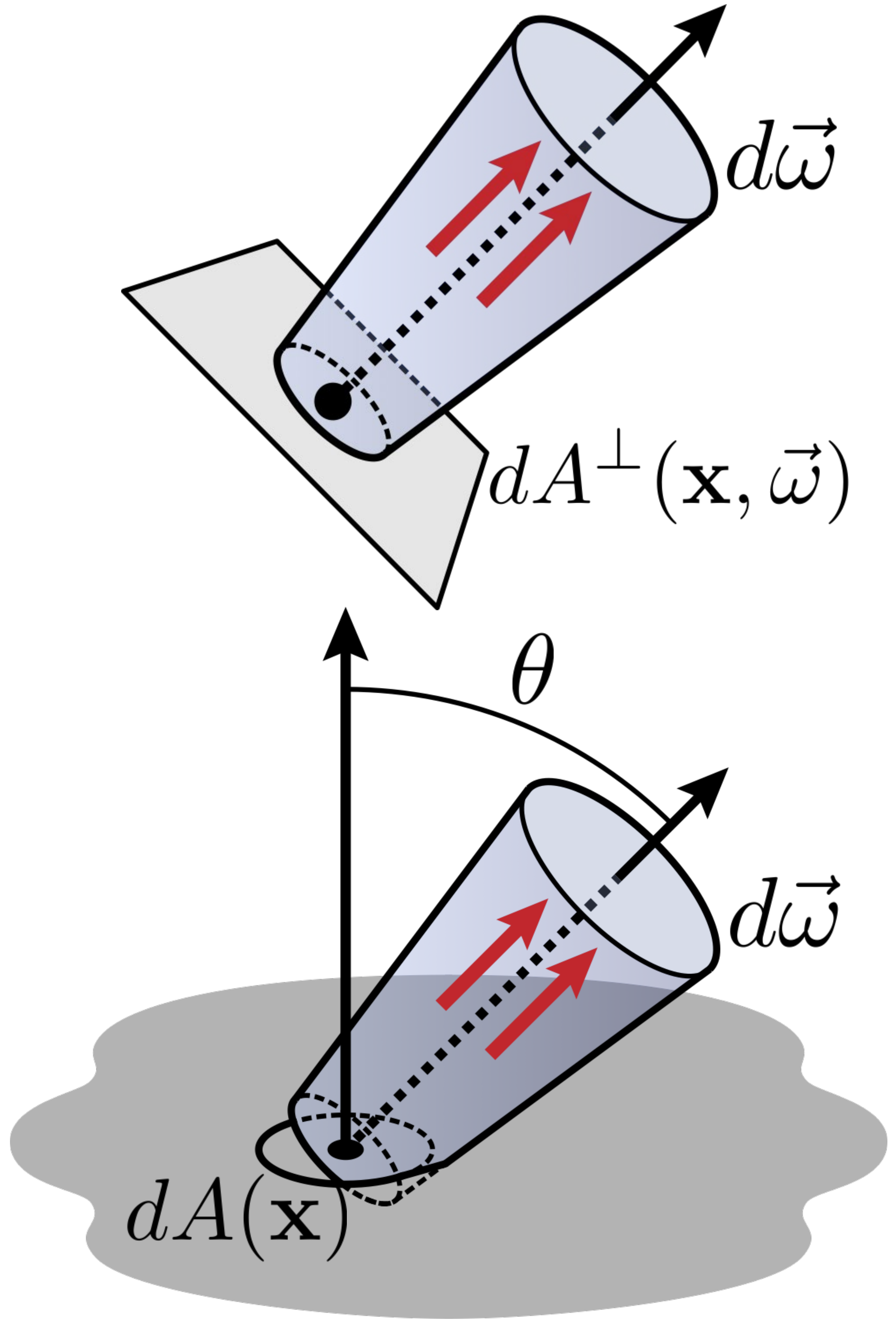


Radiance

flux density per unit solid angle, per *perpendicular* unit area

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{d\vec{\omega}\, dA^{\perp}(\mathbf{x}, \vec{\omega})} \left[\frac{W}{m^2 sr}\right]$$

$$= \frac{d^2\Phi(A)}{d\vec{\omega}\, dA(\mathbf{x}) \cos\theta}$$



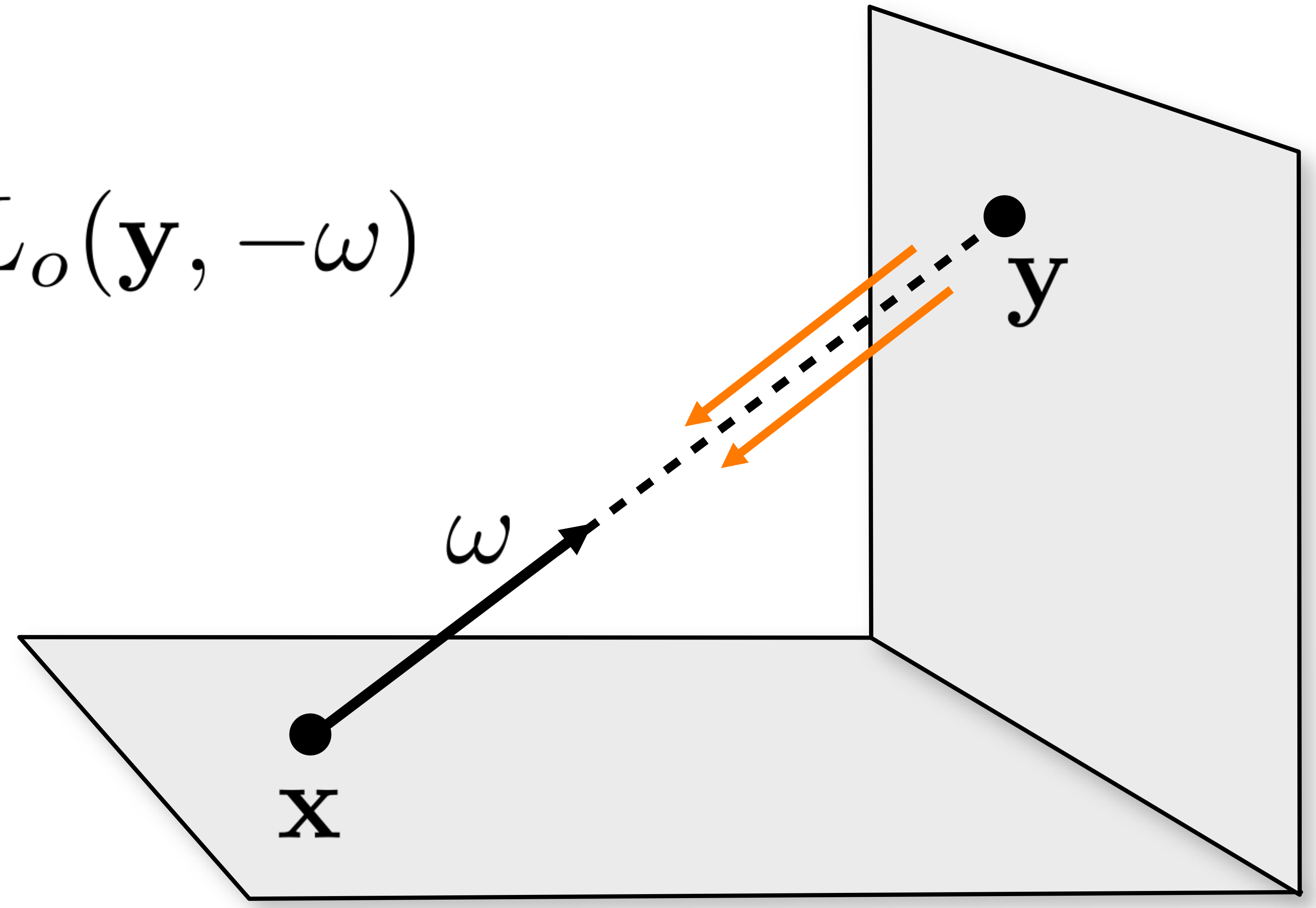
Radiance

fundamental quantity for ray tracing and physics-based rendering

remains constant along a ray (*in vacuum only*!)

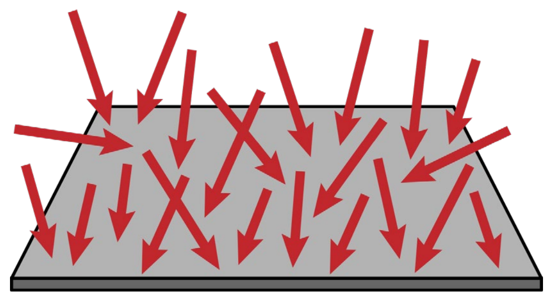
incident radiance L_i at one point can be expressed as outgoing radiance L_o at another point

$$L_i(\mathbf{x}, \omega) = L_o(\mathbf{y}, -\omega)$$

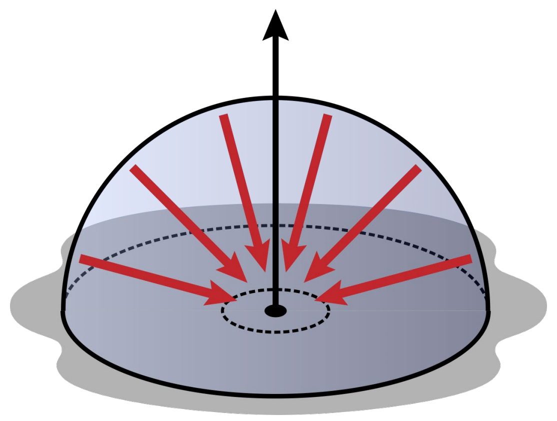


Overview of Quantities

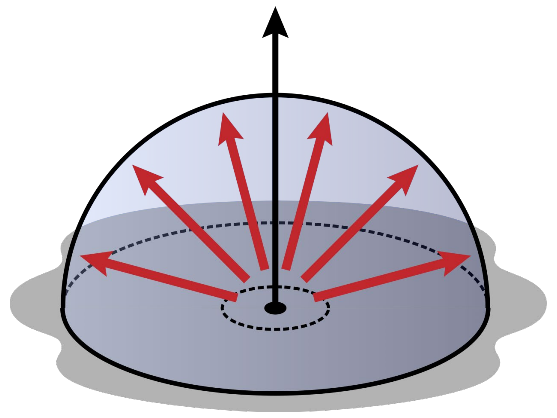
- flux: $\Phi(A)$ $\left[\frac{J}{s} = W\right]$



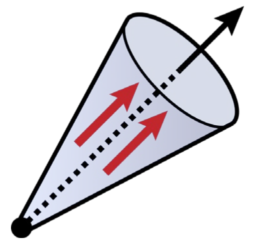
- irradiance: $E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



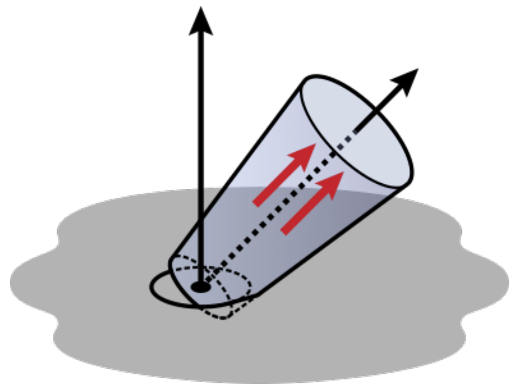
- radiosity: $B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



- intensity: $I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$ $\left[\frac{W}{sr}\right]$



- radiance: $L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}}$ $\left[\frac{W}{m^2 sr}\right]$



Radiance

expressing *irradiance* in terms of radiance:

$$L(\mathbf{x},\vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x})d\vec{\omega}} \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$L(\mathbf{x},\vec{\omega}) = \frac{dE(\mathbf{x})}{\cos\theta d\vec{\omega}}$$

$$L(\mathbf{x},\vec{\omega})\cos\theta\, d\vec{\omega} = dE(\mathbf{x})$$

$$\int_{H^2} L(\mathbf{x},\vec{\omega})\cos\theta\, d\vec{\omega} = E(\mathbf{x})$$

Integrate cosine-weighted radiance over hemisphere

Radiance

expressing *irradiance* in terms of radiance:

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = E(\mathbf{x})$$

expressing *flux* in terms of radiance:

$$\int_A E(\mathbf{x}) \, dA(\mathbf{x}) = \Phi(A) \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$\int_A \int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} dA(\mathbf{x}) = \Phi(A)$$

Integrate cosine-weighted radiance
over hemisphere and area

Radiance

Allows computing the radiant flux measured by *any* sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos\theta d\omega dA$$

Cameras measure integrals of radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to (integrals of) radiance.

- “Processed” images (like PNG and JPEG) are not linear radiance measurements!!

Computing spherical integrals

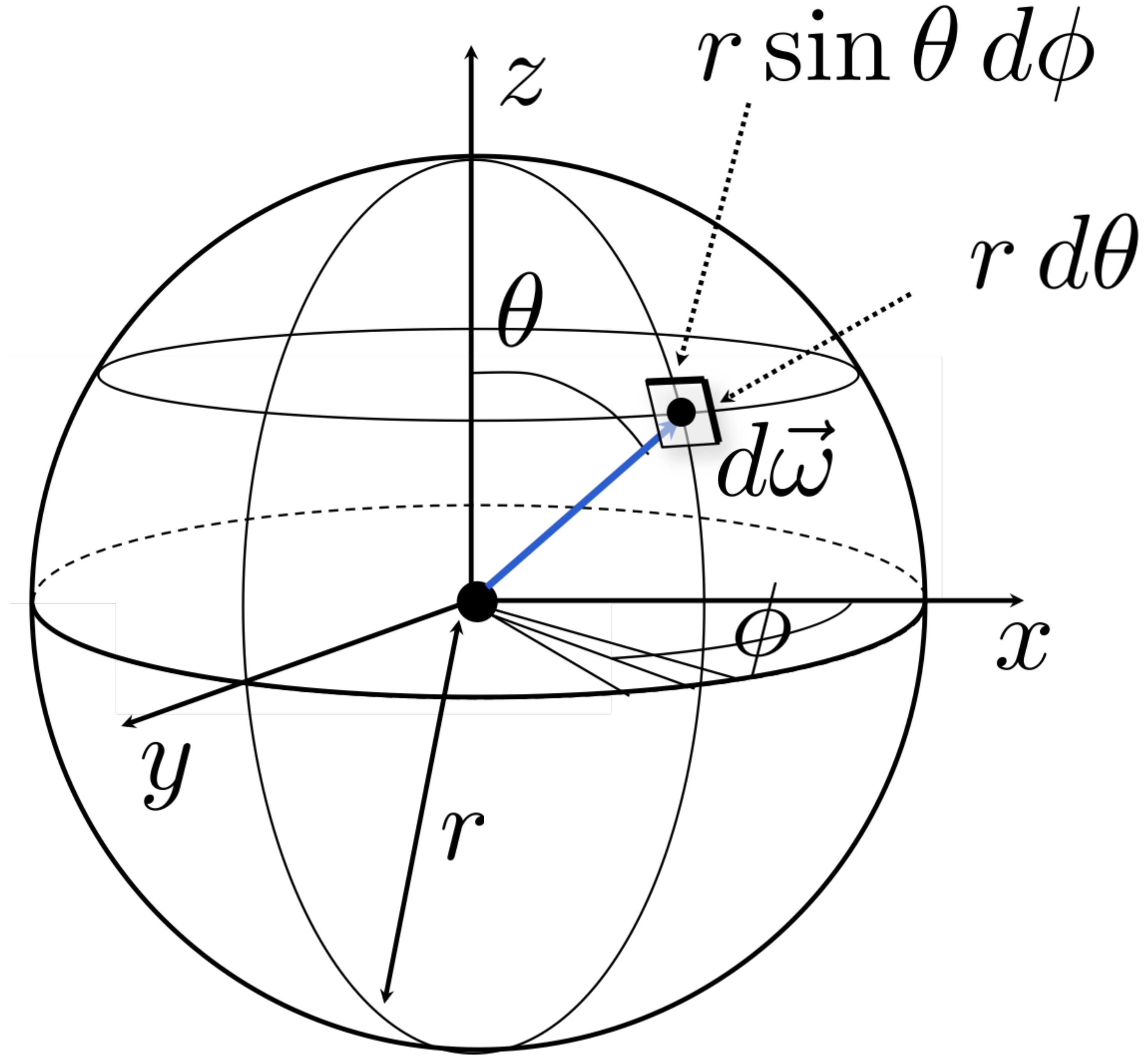
Express function using spherical coordinates:

$$\int_0^{2\pi}\int_0^{\pi} f(\theta,\phi)\ \mathrm{d}\theta\,\mathrm{d}\phi \quad ?$$

Warning: this is not correct!

Differential Solid Angle

Differential area on the unit sphere around direction



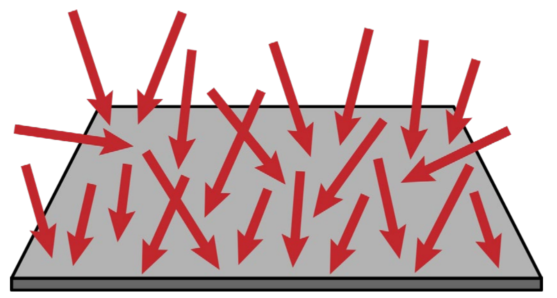
$$dA = (r d\theta)(r \sin\theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

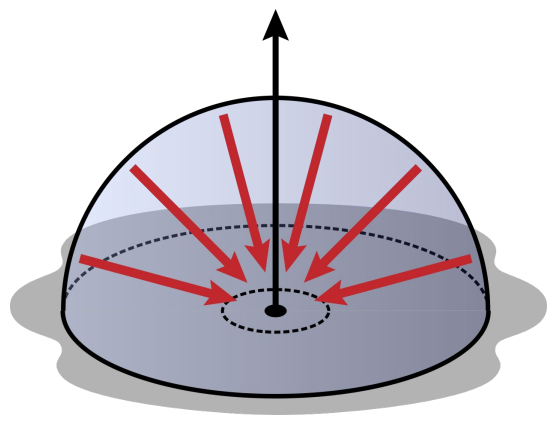
$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$

Overview of Quantities

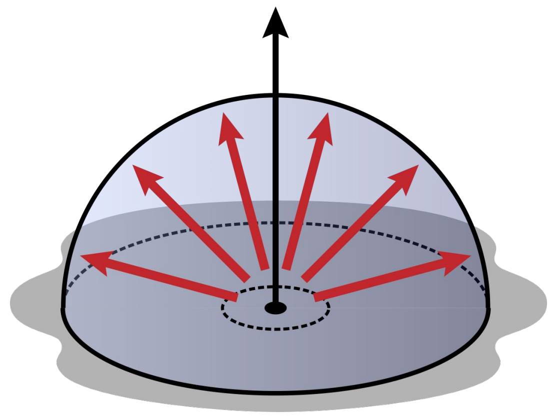
- flux: $\Phi(A)$ $\left[\frac{J}{s} = W\right]$



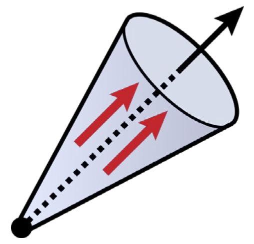
- irradiance: $E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



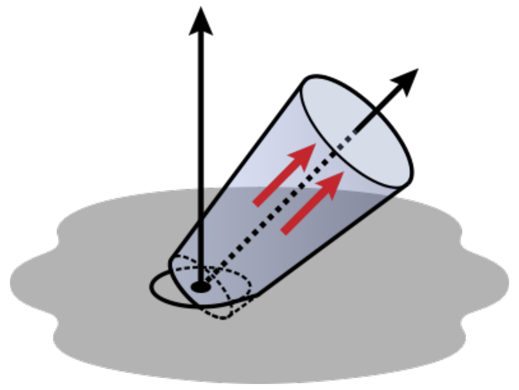
- radiosity: $B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



- intensity: $I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$ $\left[\frac{W}{sr}\right]$



- radiance: $L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}}$ $\left[\frac{W}{m^2 sr}\right]$



All of these quantities can be a function of wavelength!

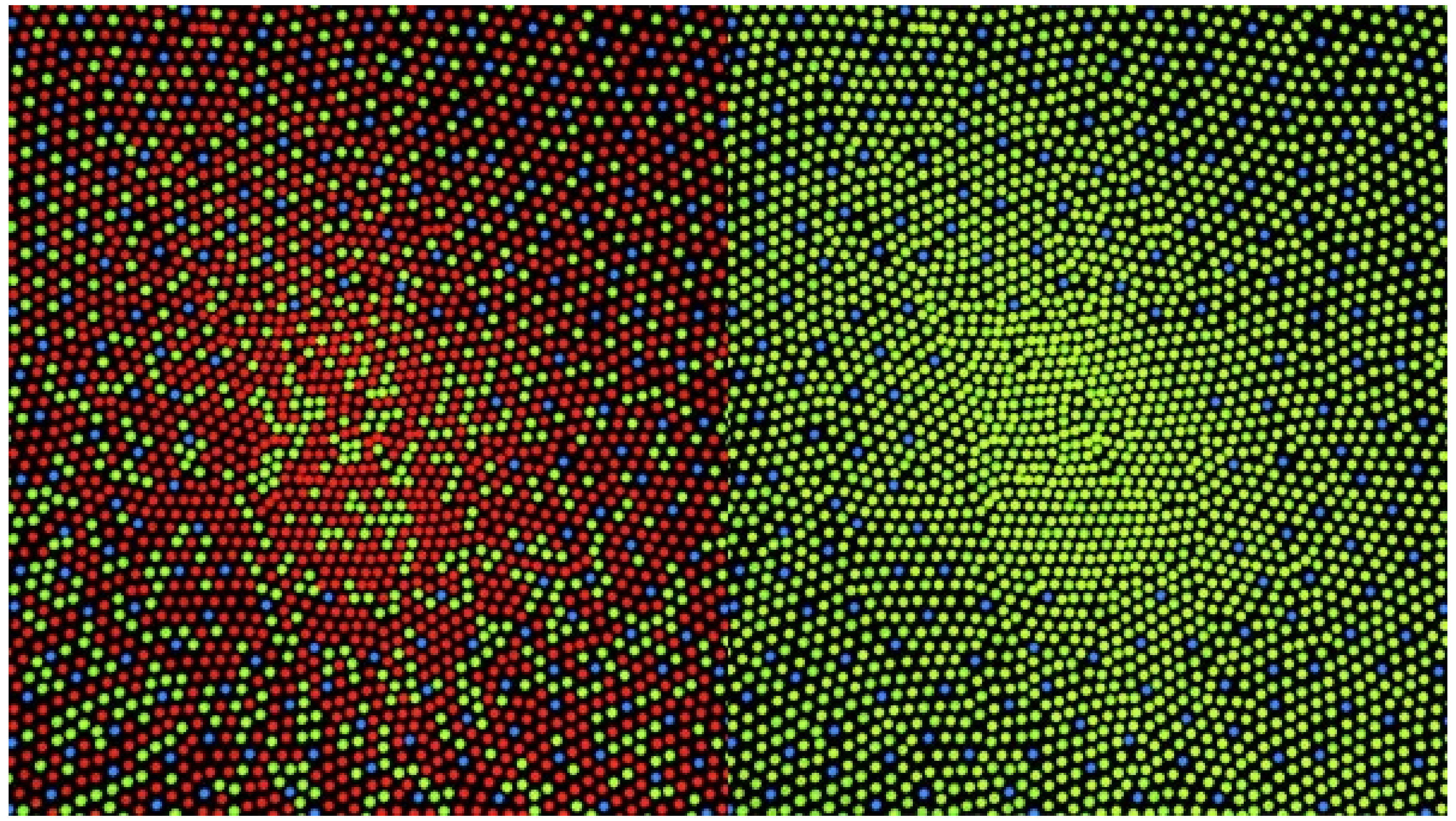
Handling color

- *Any* light sensor (digital or not) has different sensitivity to different wavelengths.
- This is described by the sensor's *spectral sensitivity function* (SSF).
- When measuring some incident *spectral* flux, the sensor produces a *scalar color* response:

$$\text{sensor response} \longrightarrow R = \int_\lambda \underbrace{\Phi(\lambda)}_{\text{spectral flux}} \underbrace{f(\lambda)}_{\text{sensor SSF}} d\lambda$$

Handling color – the human eye

- The human eye is a collection of light sensors called cone cells.
- There are three types of cells with different spectral sensitivity functions.
- Human color perception is three-dimensional (*tristimulus color*).

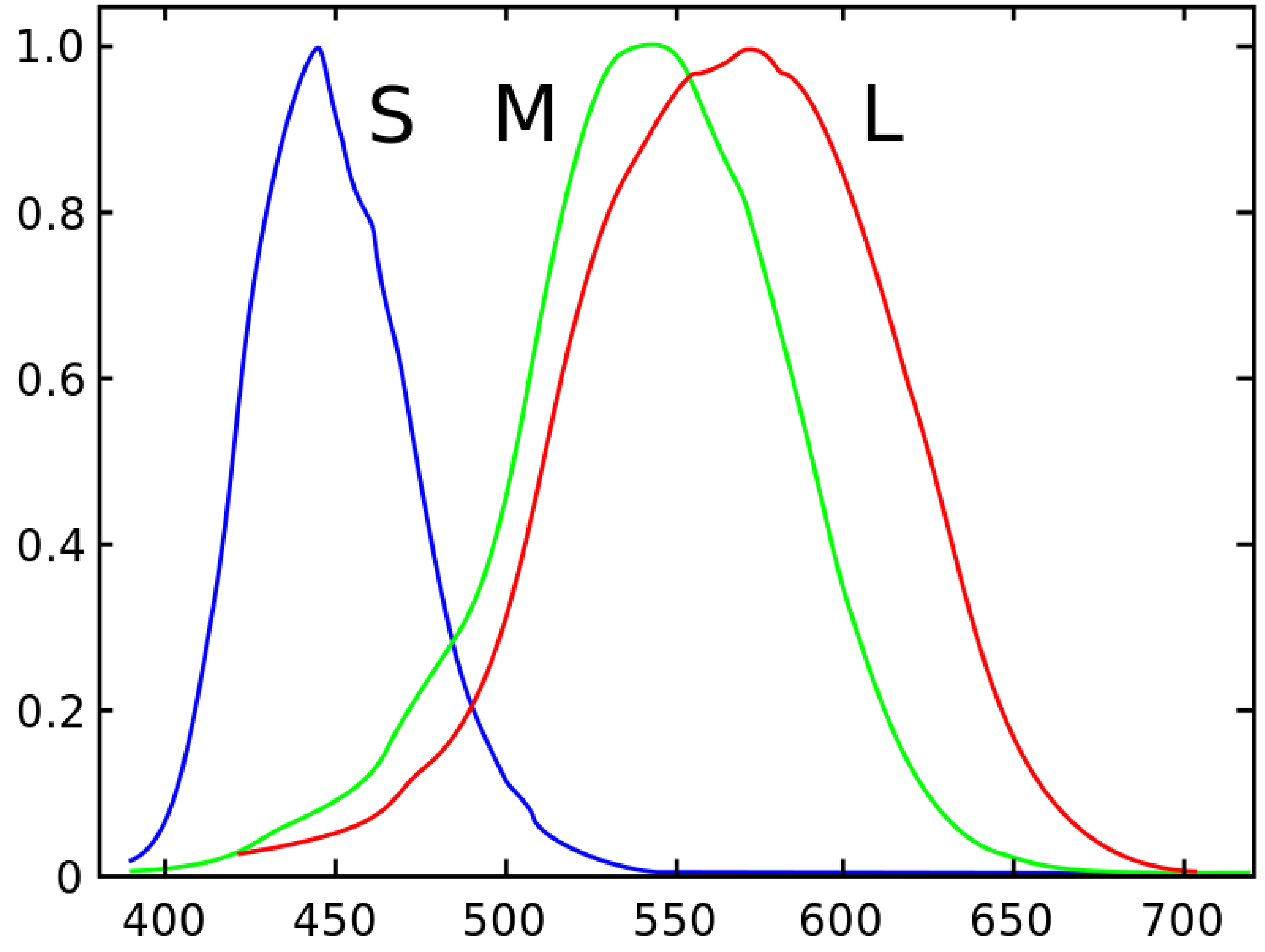


cone distribution
for normal vision
(64% L, 32% M)

"short" $S = \int_\lambda \Phi(\lambda) S(\lambda) d\lambda$

"medium" $M = \int_\lambda \Phi(\lambda) M(\lambda) d\lambda$

"long" $L = \int_\lambda \Phi(\lambda) L(\lambda) d\lambda$

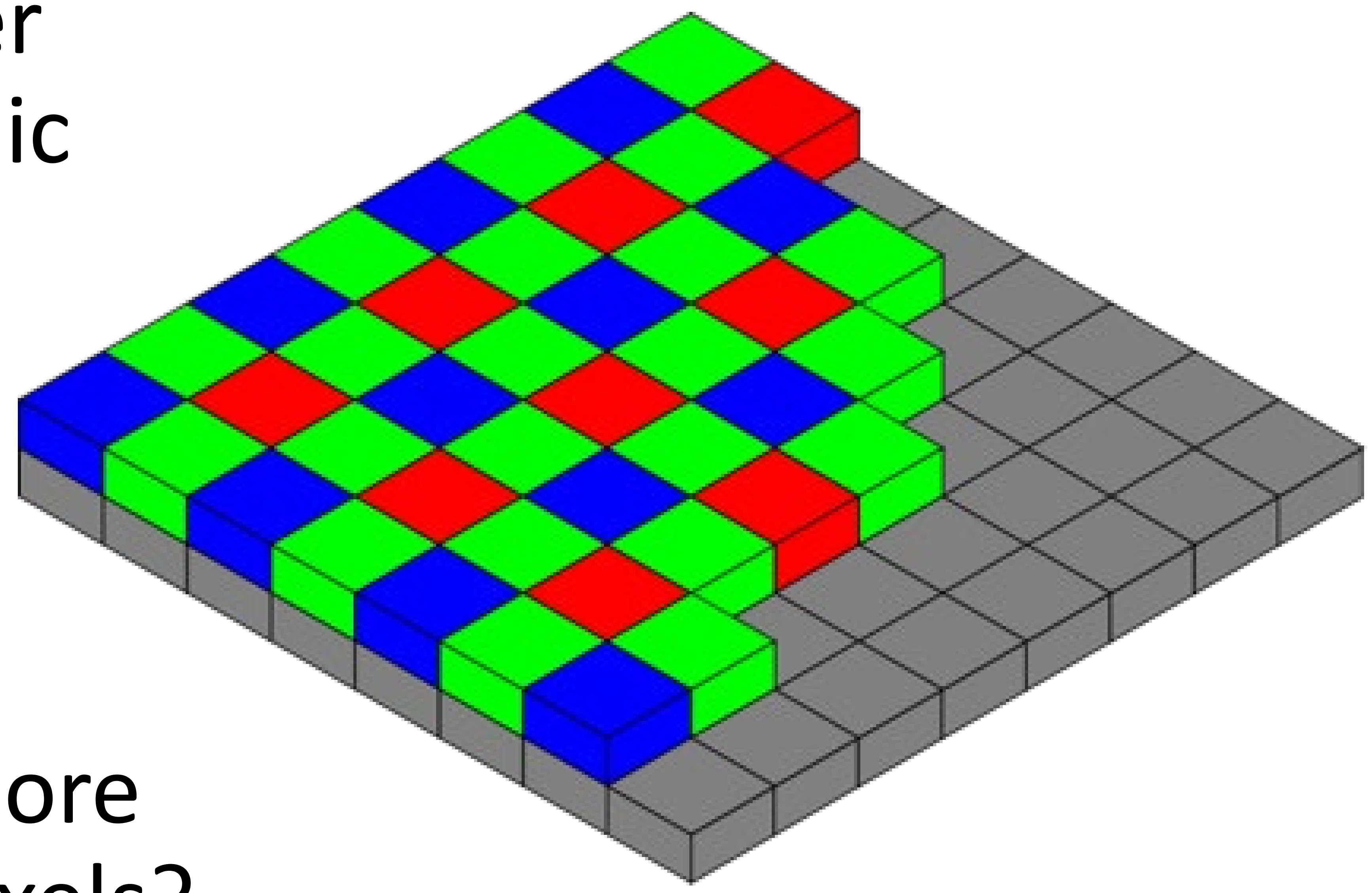


Handling color – photography

Two design choices:

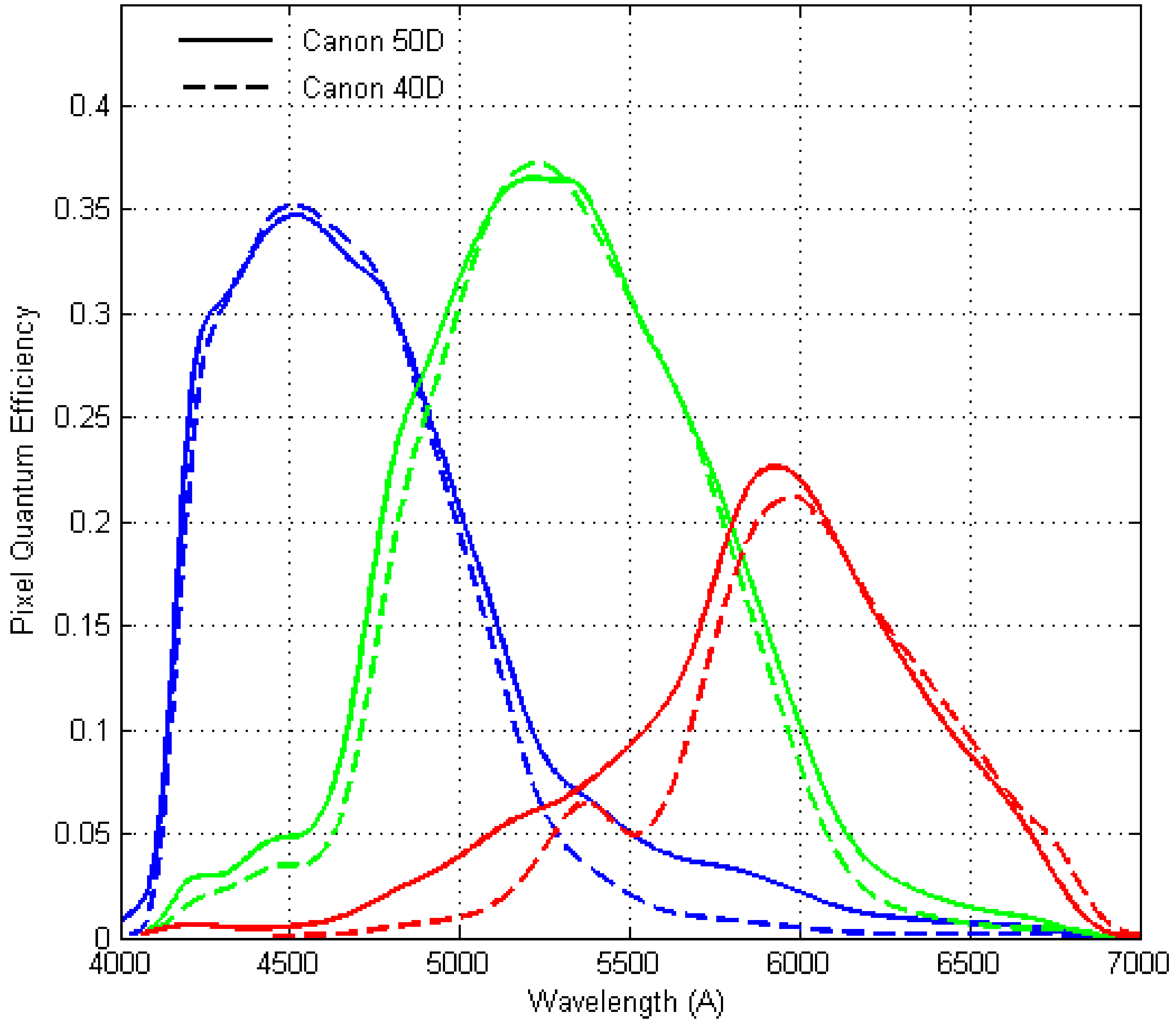
- What spectral sensitivity functions $f(\lambda)$ to use for each color filter?
- How to spatially arrange ("mosaic") different color filters?

Bayer mosaic



Why more green pixels?

SSF for Canon 50D

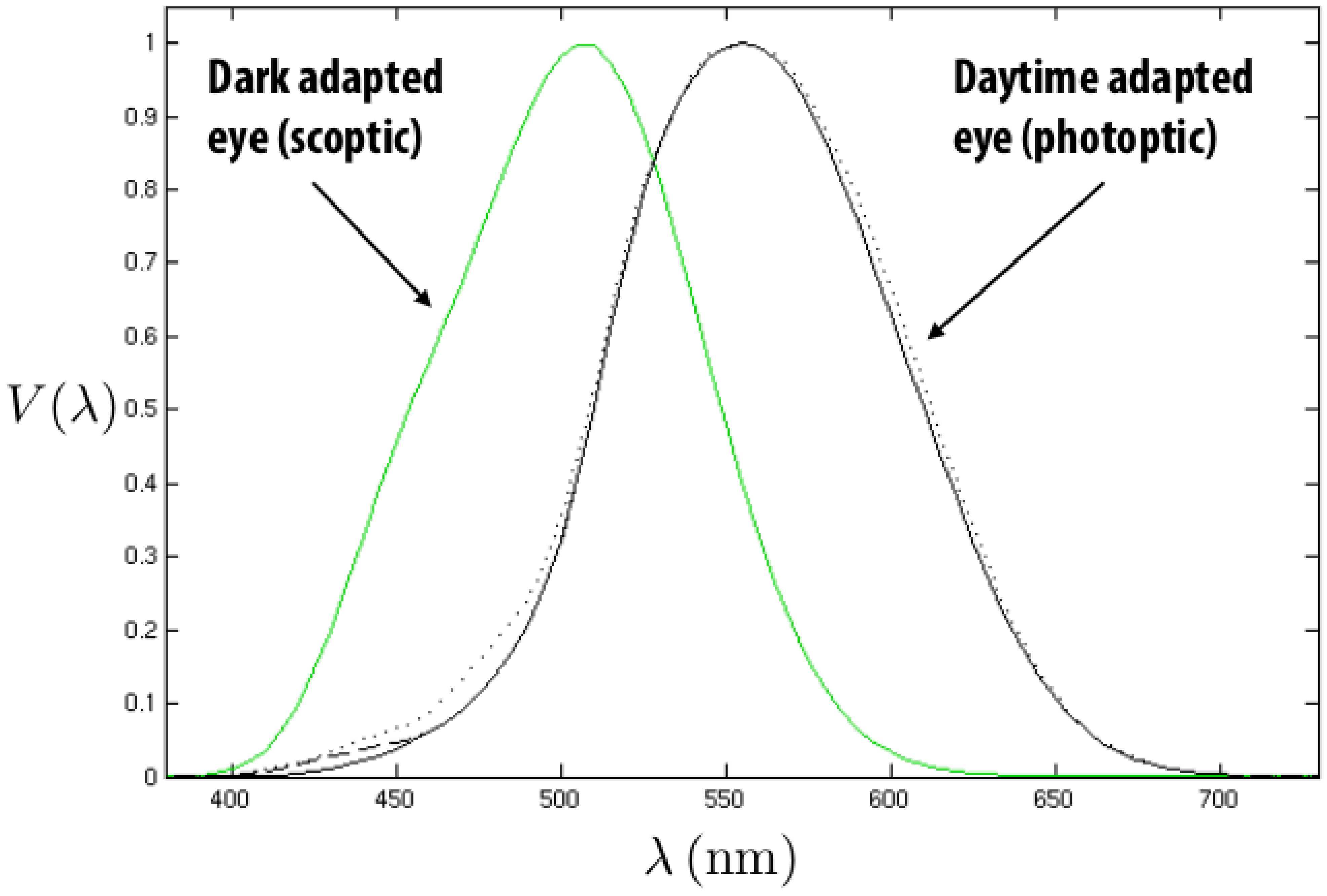


Generally do not match human LMS.

$f(\lambda)$

Radiometry versus photometry

- **All radiometric quantities have equivalents in photometry**
- **Photometry: accounts for response of human visual system to electromagnetic radiation**
- **Luminance (Y) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye's luminous efficacy curve, e.g.:**



$$Y(\mathrm{p},\omega) = \int_0^\infty L(\mathrm{p},\omega,\lambda)\,V(\lambda)\,\mathrm{d}\lambda$$

Radiometry versus photometry

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance (incoming) Radiosity (outgoing)	Illuminance (incoming) Luminosity (outgoing)
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Radiometry versus photometry

Photometry	MKS	CGS	British
Luminous Energy	Talbot	Talbot	Talbot
Luminous Power	Lumen	Lumen	Lumen
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit, Apostlib, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela	Candela	Candela

Modern LED light

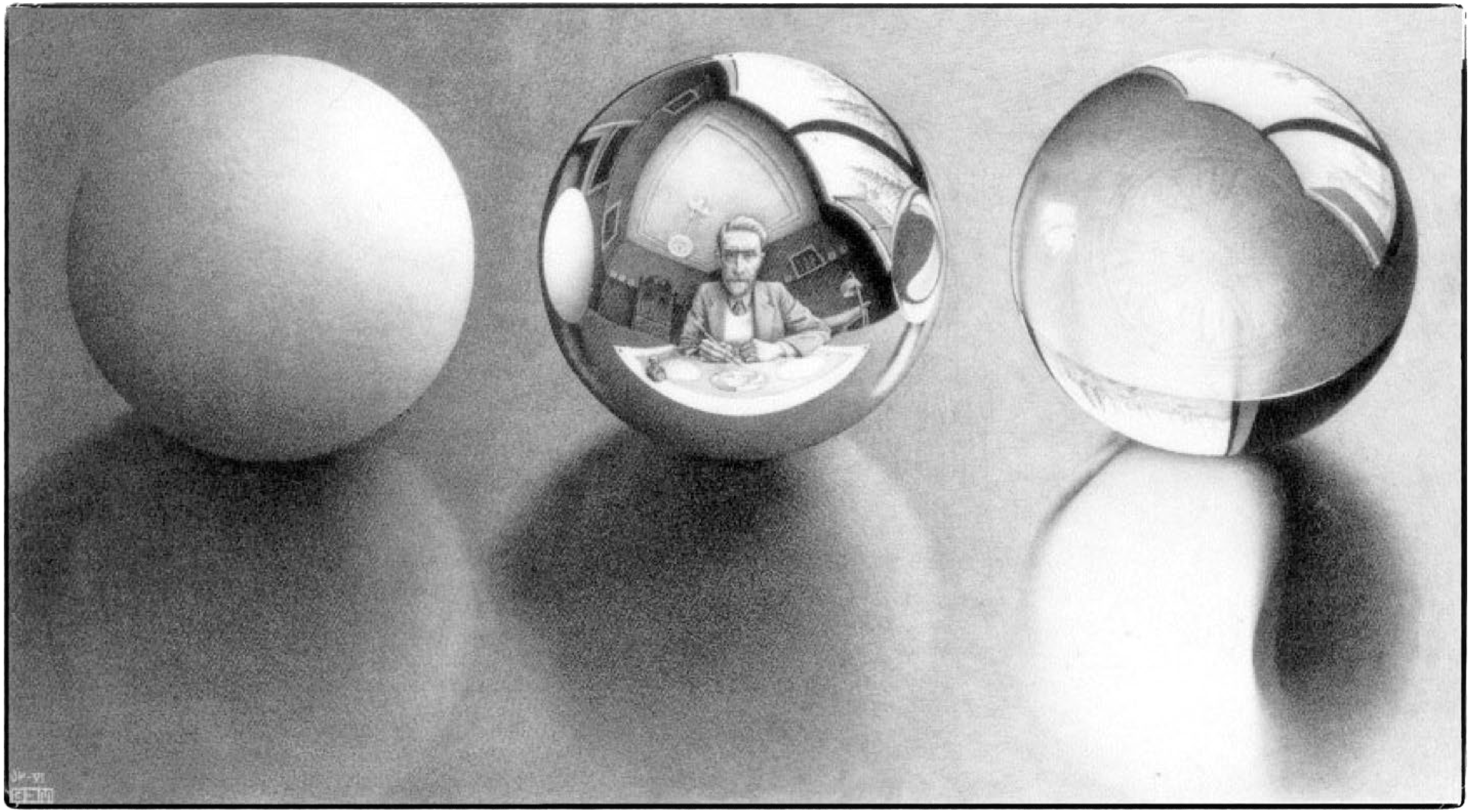
Input power: 11 W

Output: 815 lumens

(~ 80 lumens / Watt)

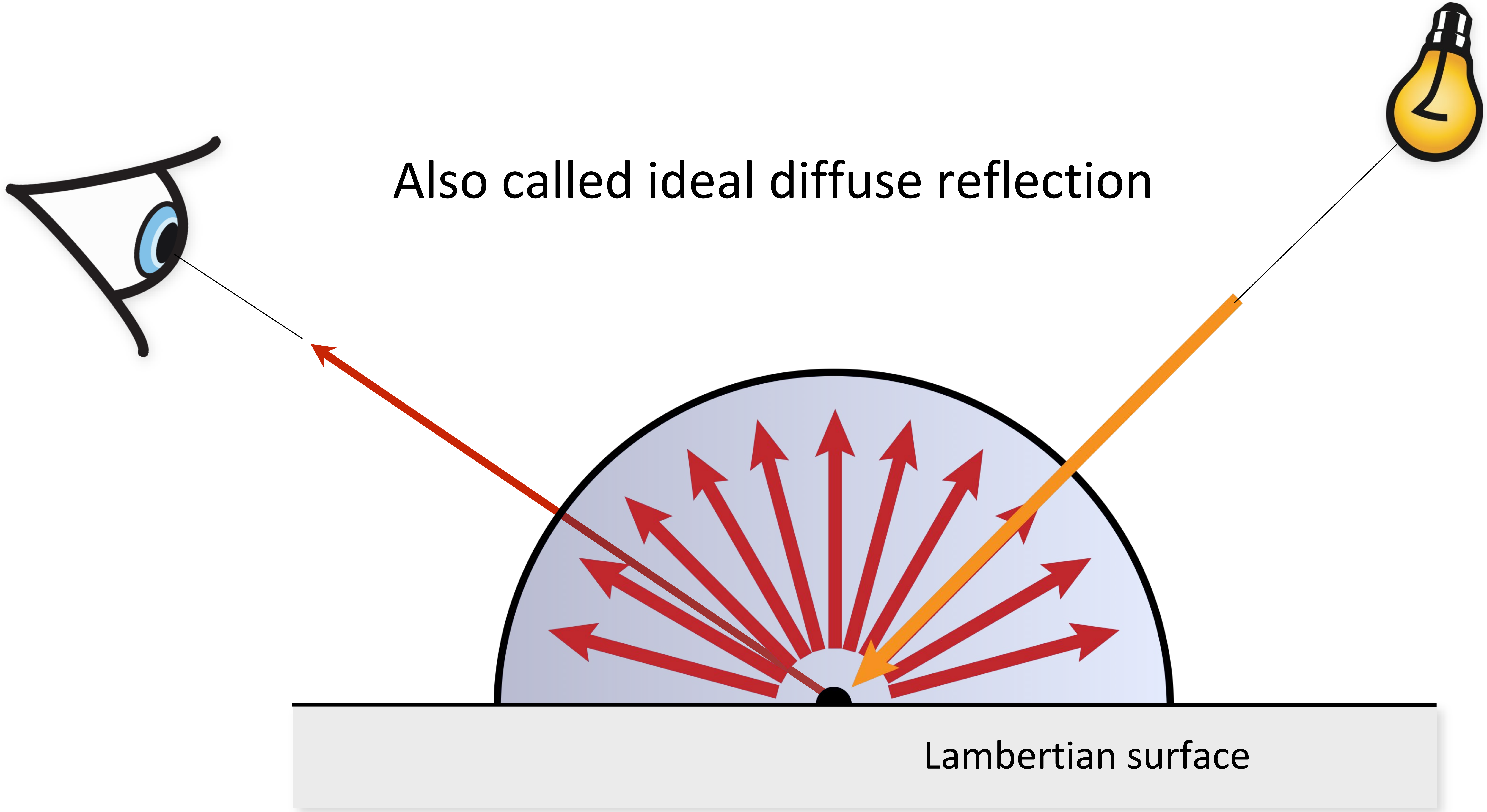
Incandescent bulbs:
~15 lumens / Watt)

Reflection equation

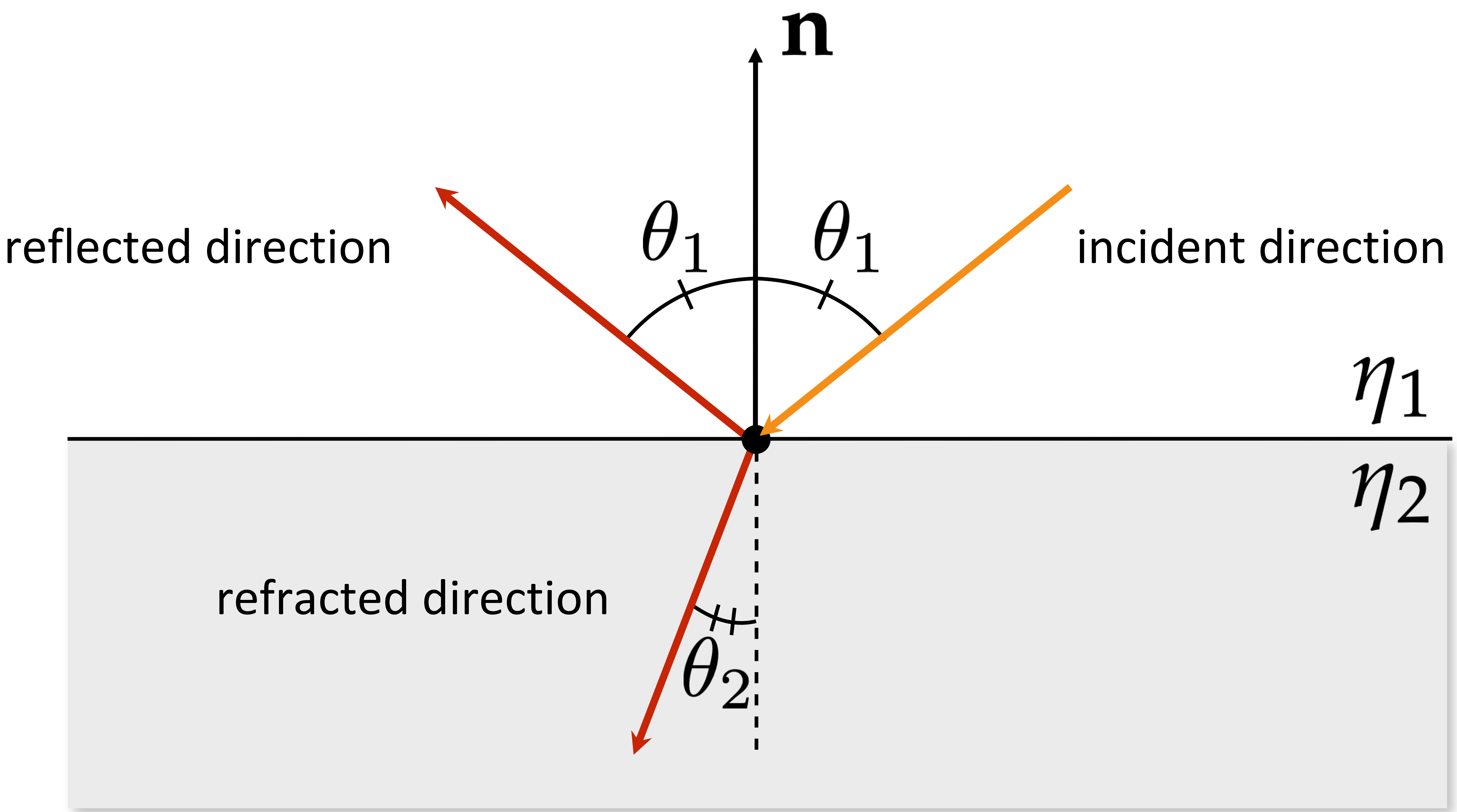


Lambertian reflection

Also called ideal diffuse reflection

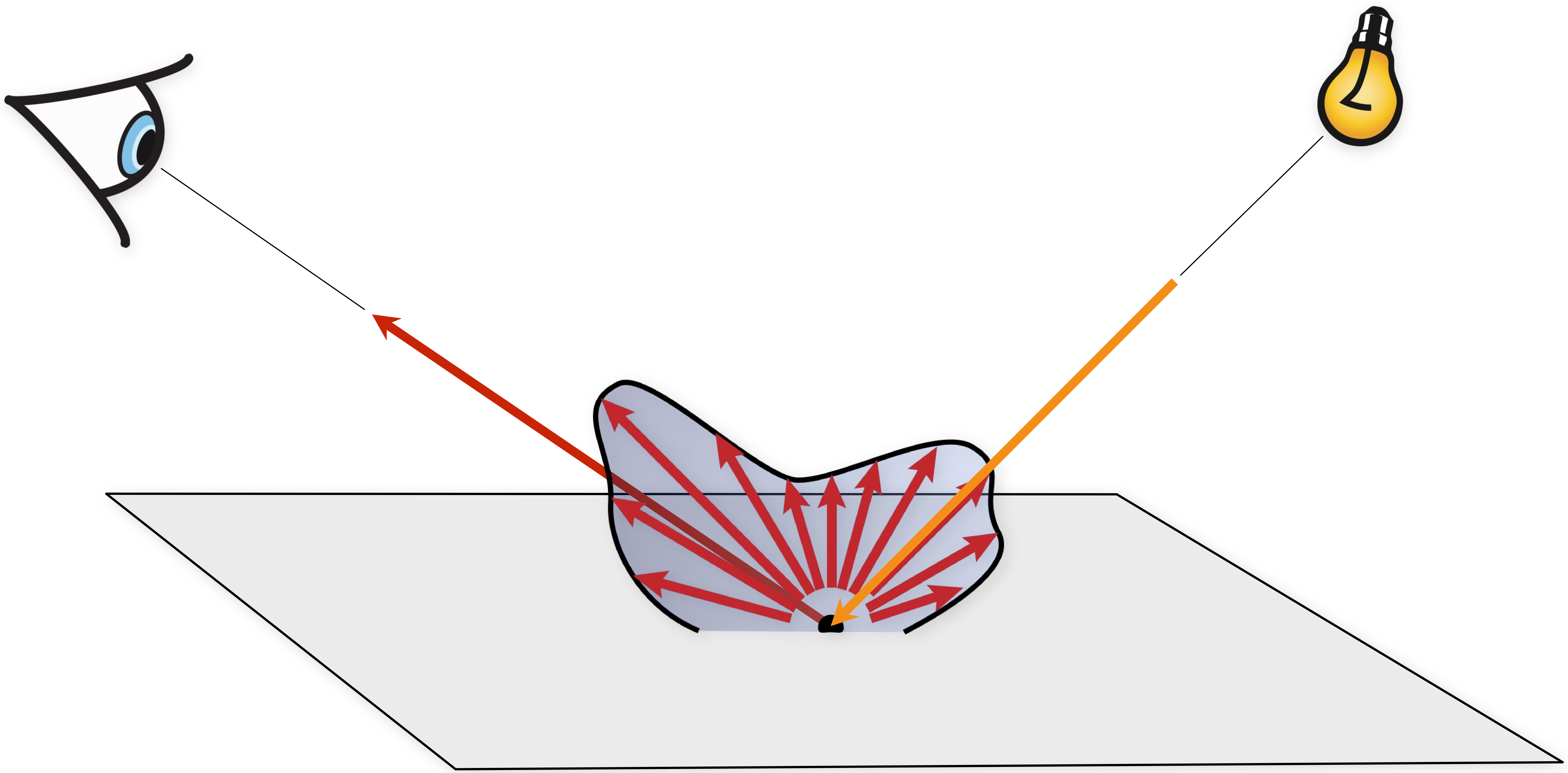


Ideal specular reflection/refraction



$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

Light-Material Interactions

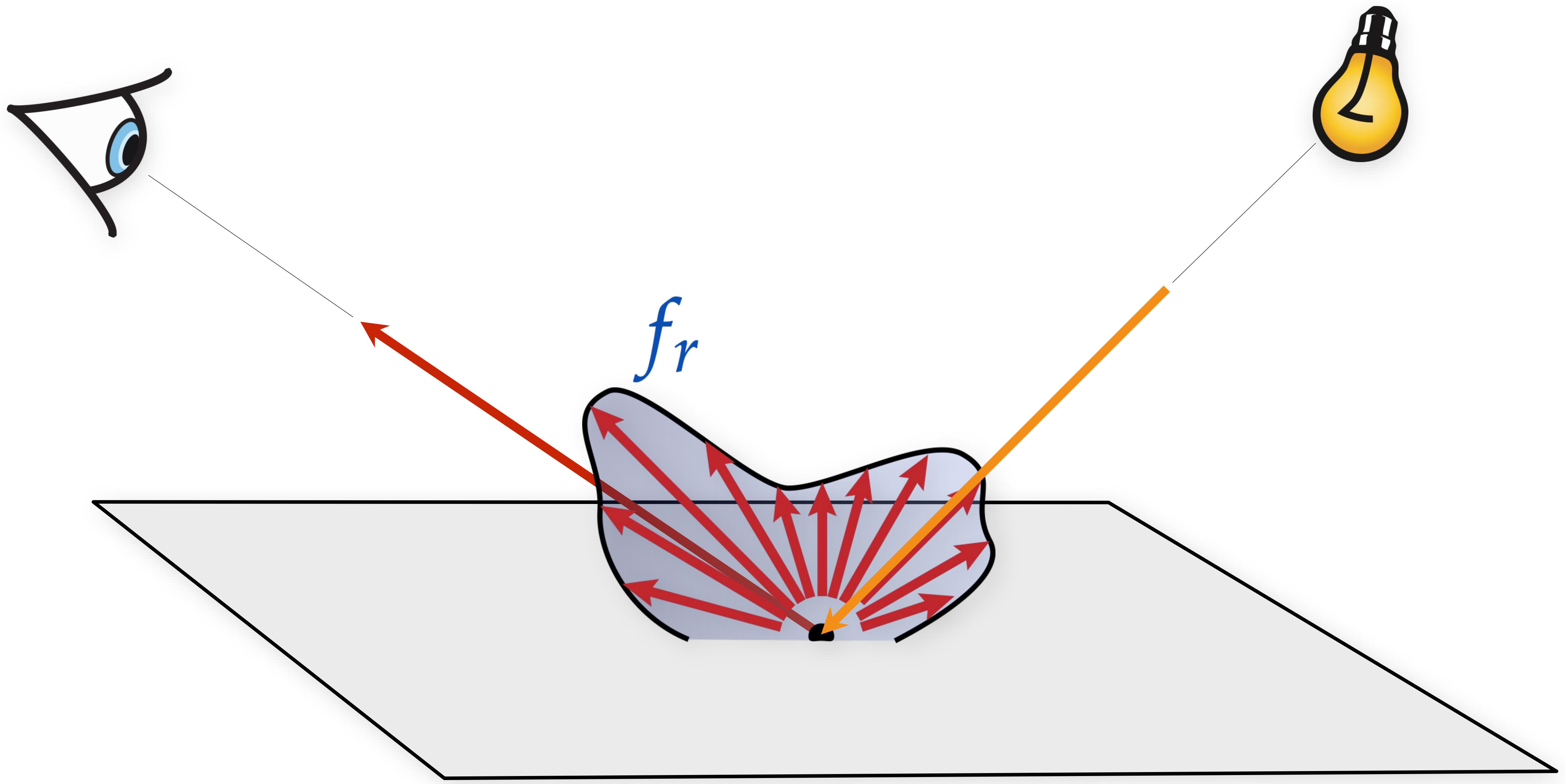


The BRDF

Bidirectional **R**eflectance **D**istribution **F**unction

- how much light gets scattered from one direction into each other direction

- formally: ratio of outgoing *radiance* to incident *irradiance*



The reflection equation

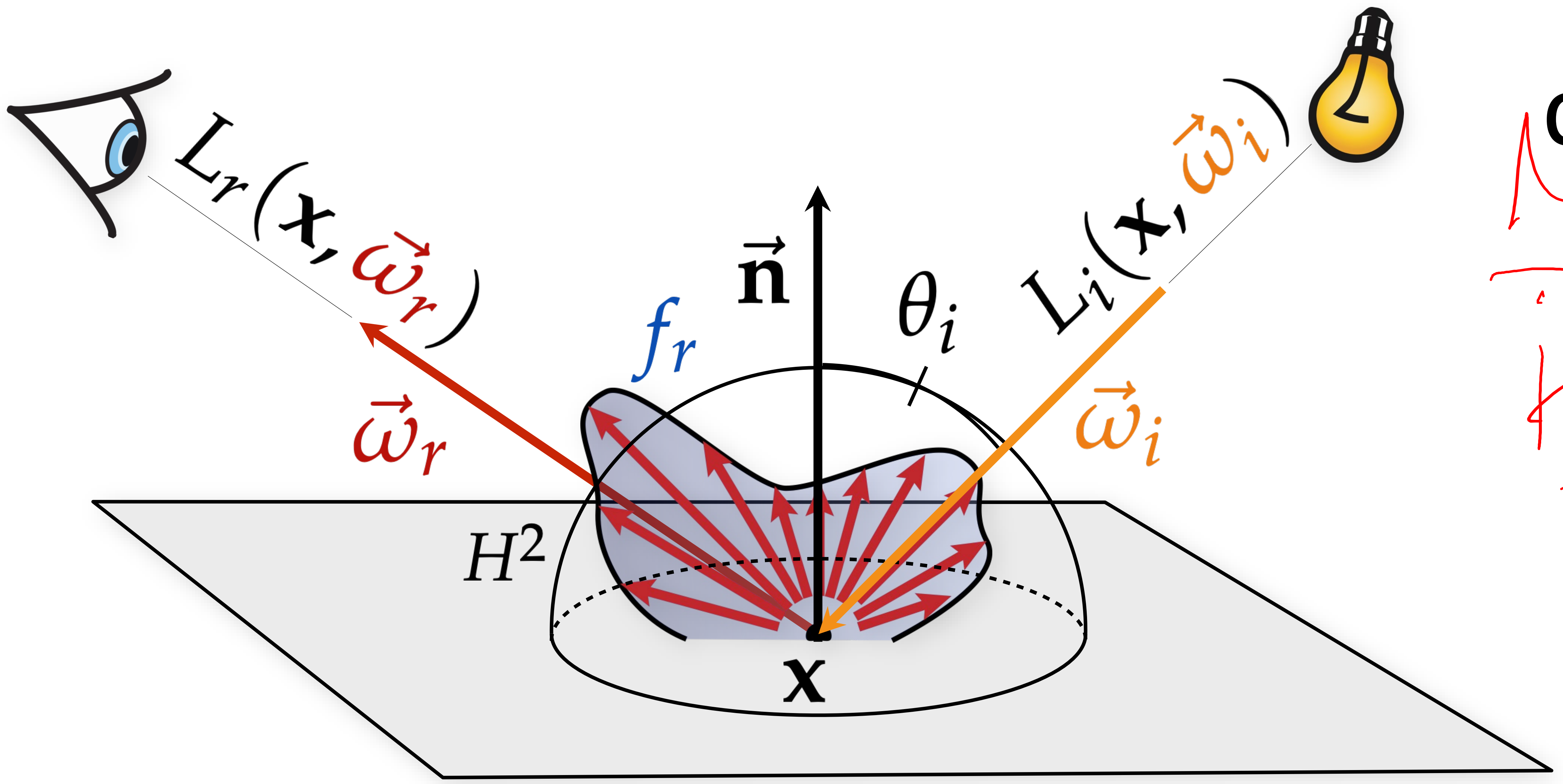
Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

Where does the cosine come from?

Irrad

rad cosθ



This describes a local illumination model

Motivation

Motivation

BRDF Properties

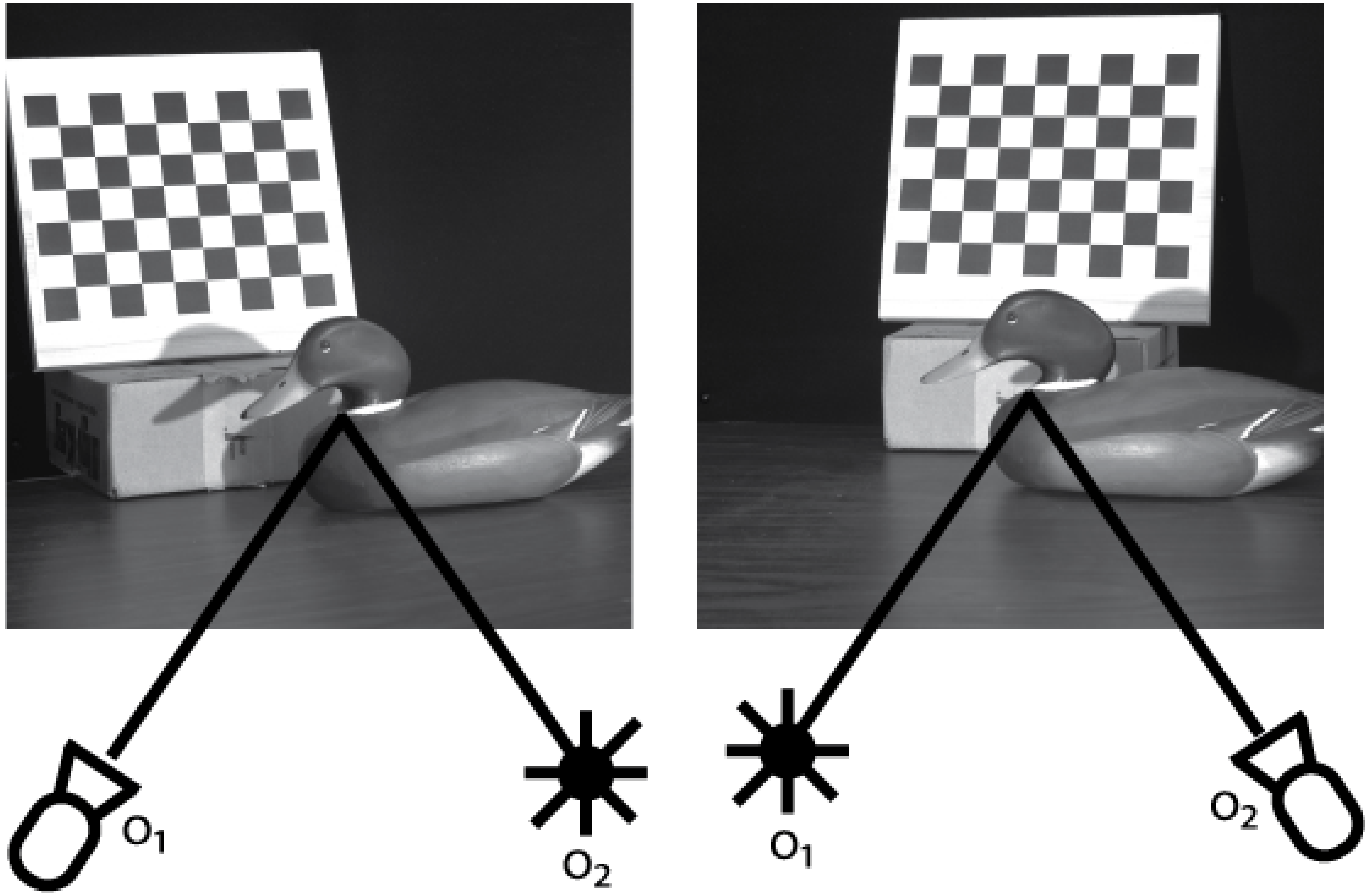
Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos\theta_r \, \mathrm{d}\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i$$

Where does the cosine come from?

Helmholtz Reciprocity



BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos\theta_r \, \mathrm{d}\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i$$

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$
$$f_r(\mathbf{x}, \vec{\omega}_i \leftrightarrow \vec{\omega}_r)$$

BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos\theta_r \,\mathrm{d}\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i$$

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$

- Together:

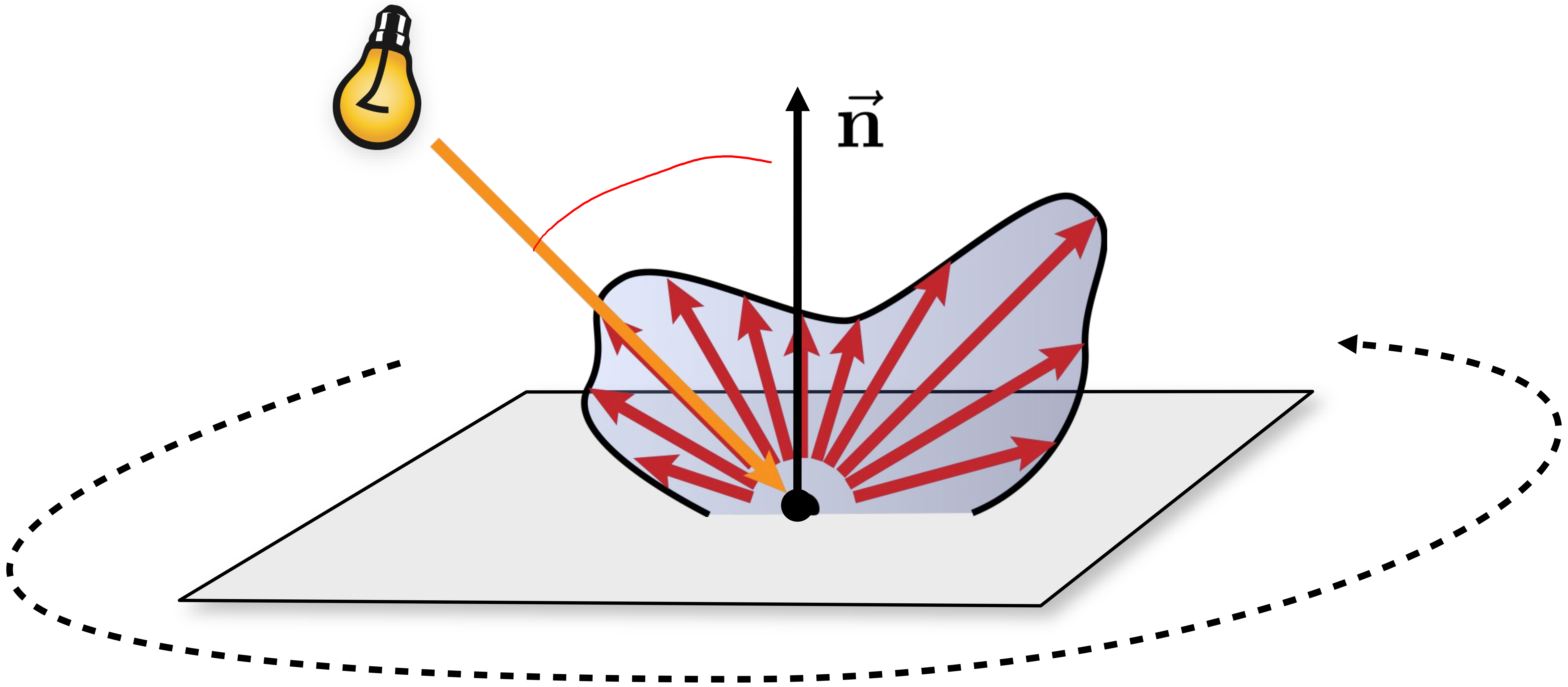
$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \,\mathrm{d}\vec{\omega}_i \leq 1, \quad \forall \vec{\omega}_r$$

BRDFs Properties

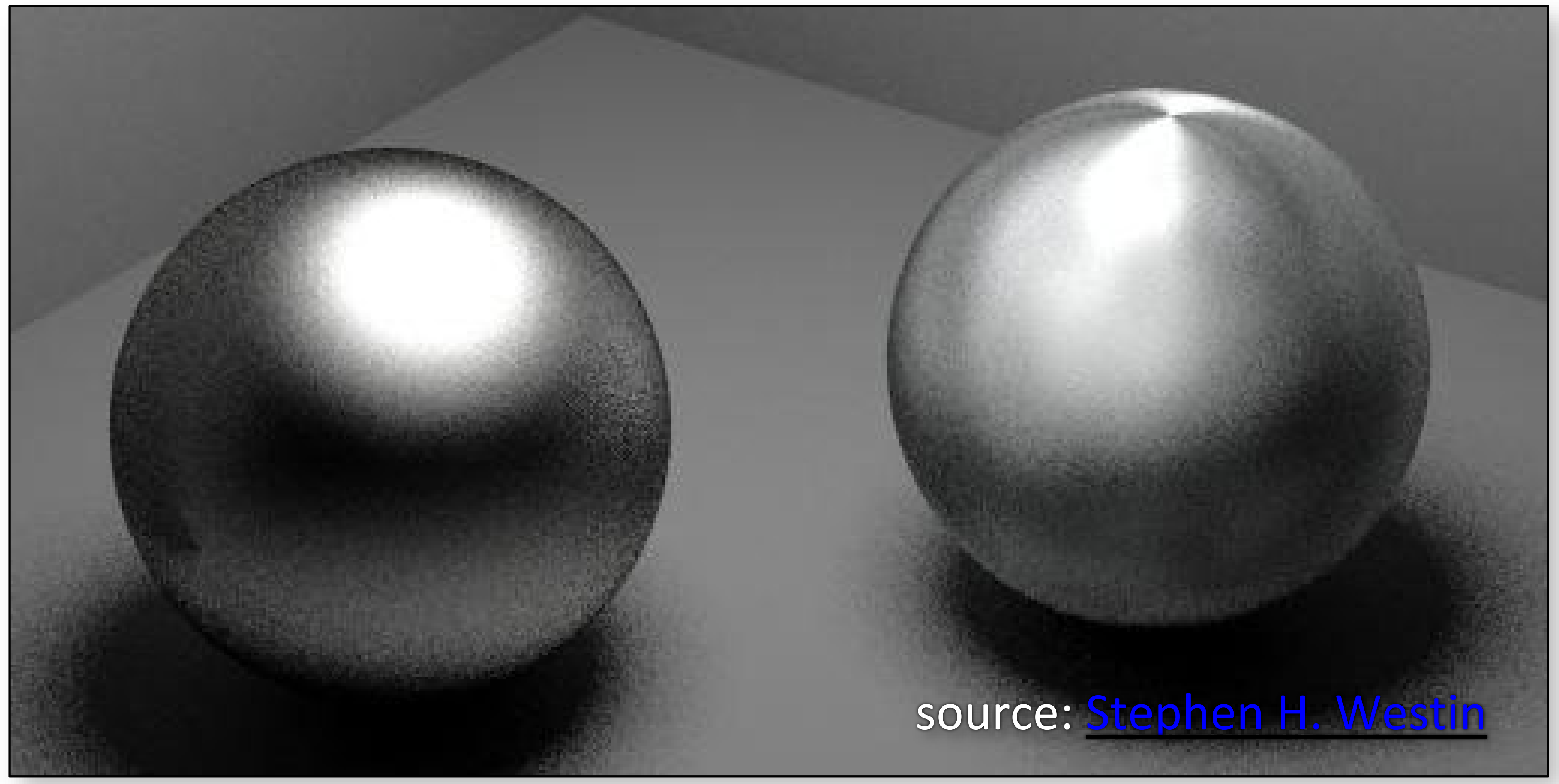
If the BRDF is unchanged as the material is rotated around the normal, then it is ***isotropic***, otherwise it is ***anisotropic***.

Isotropic BRDFs are functions of just 3 variables

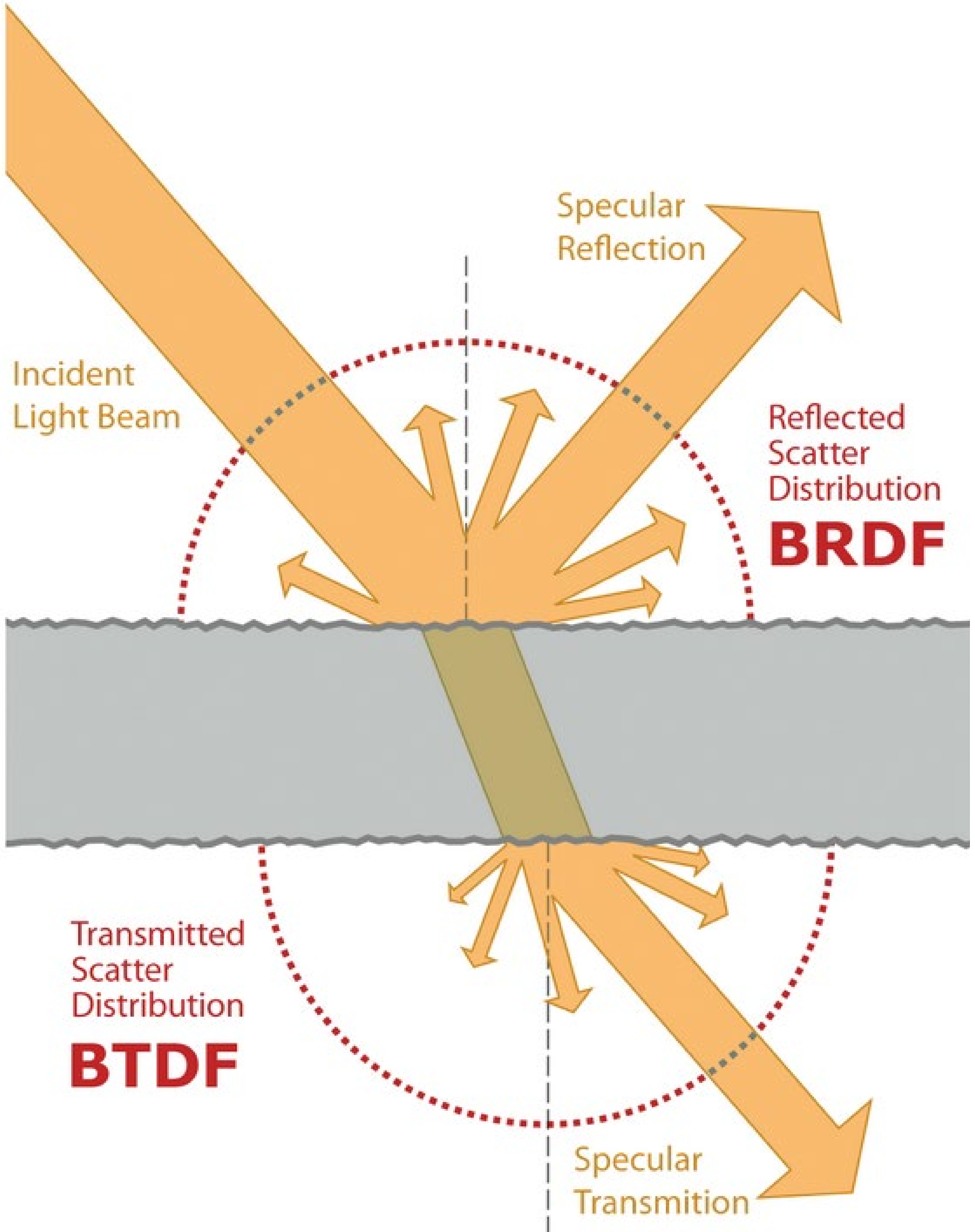
$$(\theta_i, \theta_r, \Delta\phi)$$



Isotropic vs Anisotropic Reflection

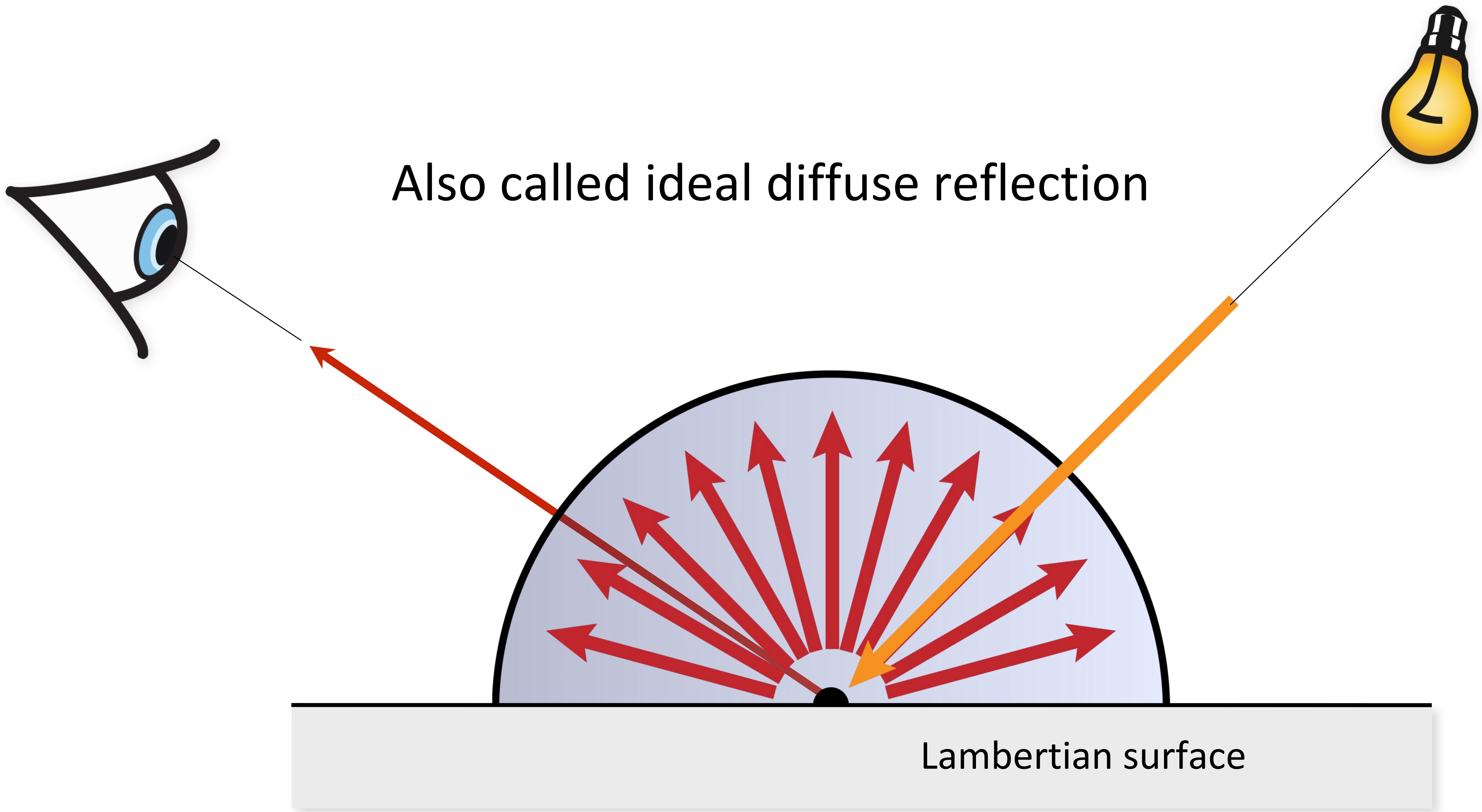


Reflection vs. Refraction



Lambertian reflection

Also called ideal diffuse reflection



BRDF for Lambertian reflection?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} \boxed{f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

Scatters light equal in all directions

BRDF is a constant

Lambertian BRDF

For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Note: we can drop ω_r

$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \, E(\mathbf{x})$$

If *all* incoming light is reflected:

$$E(\mathbf{x}) = B(\mathbf{x})$$

$$E(\mathbf{x}) = \int_{H^2} L_r(\mathbf{x}) \cos\theta \, d\vec{\omega}$$

Note: can also be derived from energy conservation

$$E(\mathbf{x}) = L_r(\mathbf{x}) \int_{H^2} \cos\theta \, d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \, \pi$$

$$f_r = \frac{1}{\pi}$$

Lambertian BRDF

For Lambertian reflection, the BRDF is a constant:

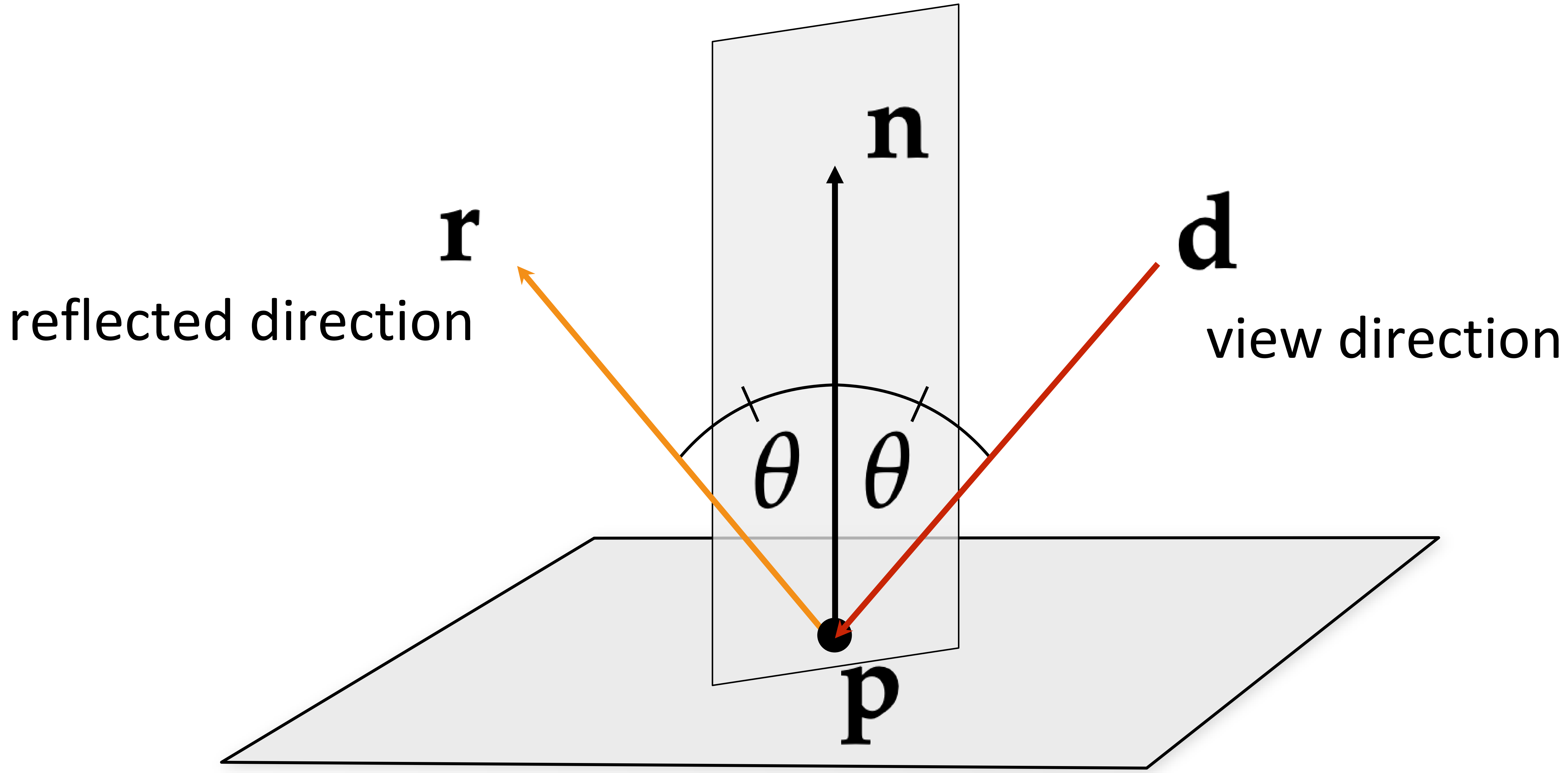
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

ρ: Diffuse reflectance (albedo) [0…1]

Specular BRDF

Assume $\mathbf{n}$ is unit length



$$\mathbf{r} = -2\mathbf{n}(\mathbf{n} \cdot \mathbf{d}) + \mathbf{d}$$

Specular BRDF?

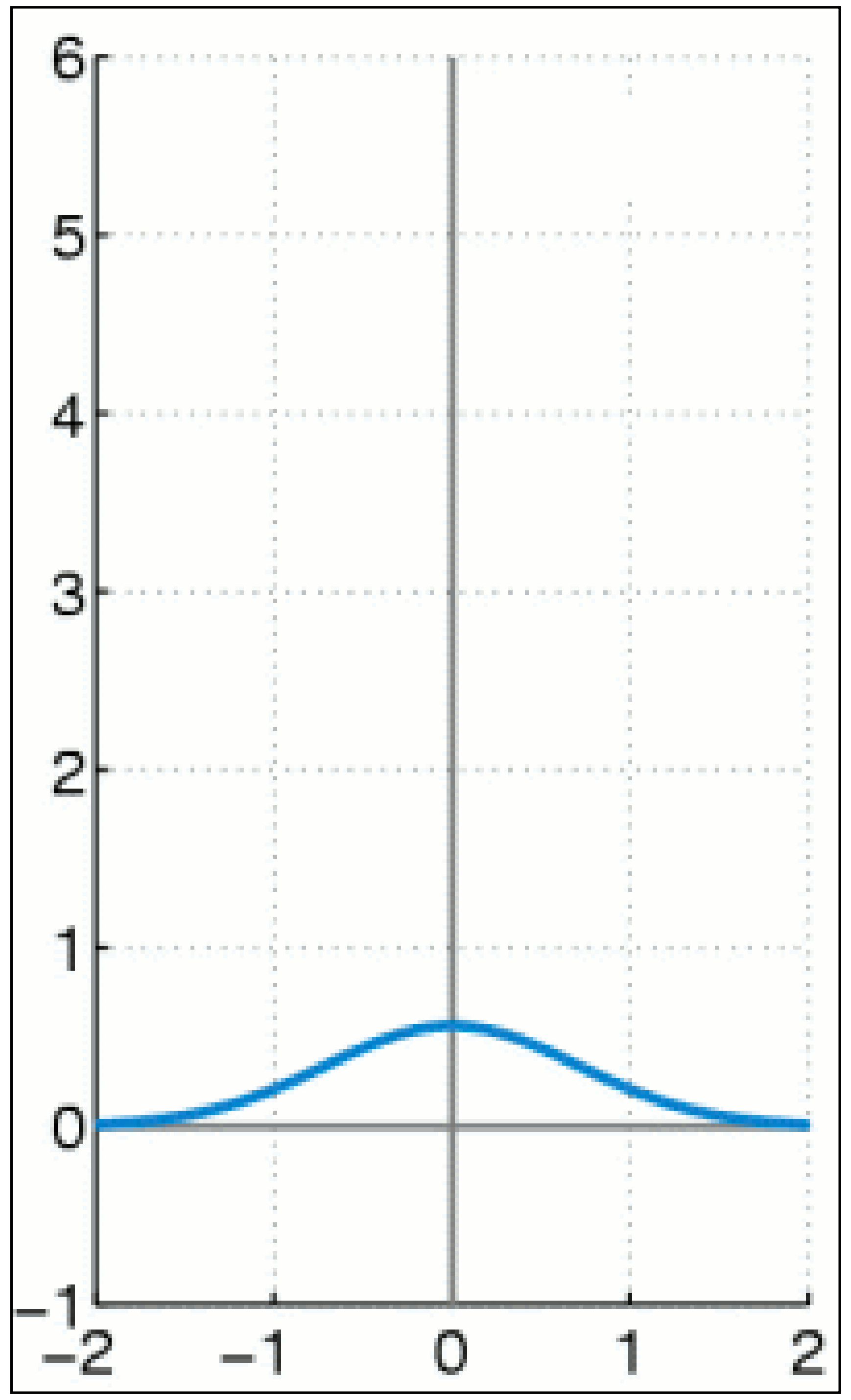
Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

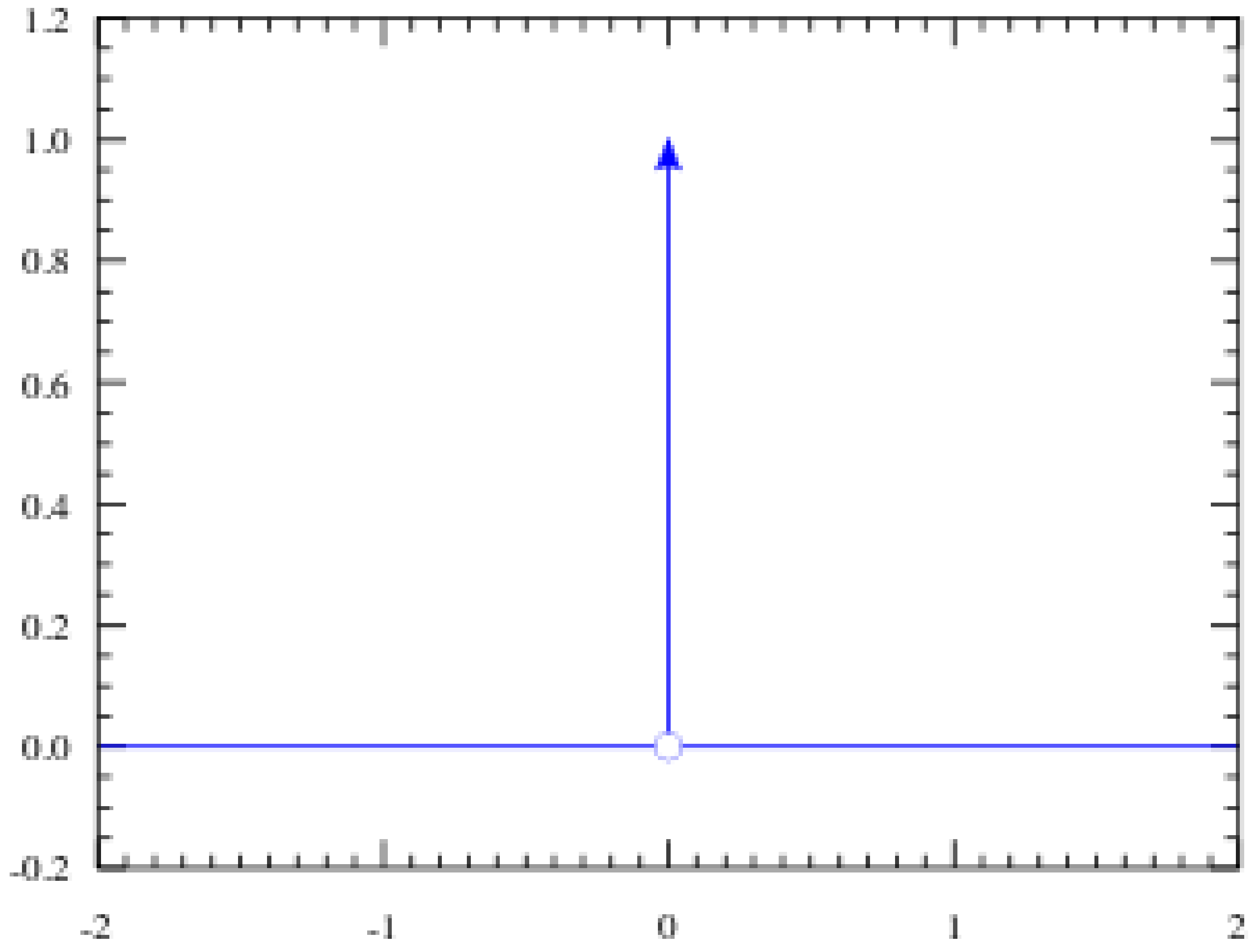
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} \boxed{f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

Scatters all light into one (or two) directions
Contains a Dirac delta
Integral drops out

What is the BRDF for specular reflection/refraction?

Dirac delta functions





$$\int_{-\infty}^{\infty} f(x)\delta(x-a)\,\mathrm{d}x = f(a)$$

Note: careful when performing changes of variables in Dirac delta functions!

BRDF of Ideal Specular Reflection

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

What is the BRDF for specular reflection?

Dirac delta

Fresnel reflection

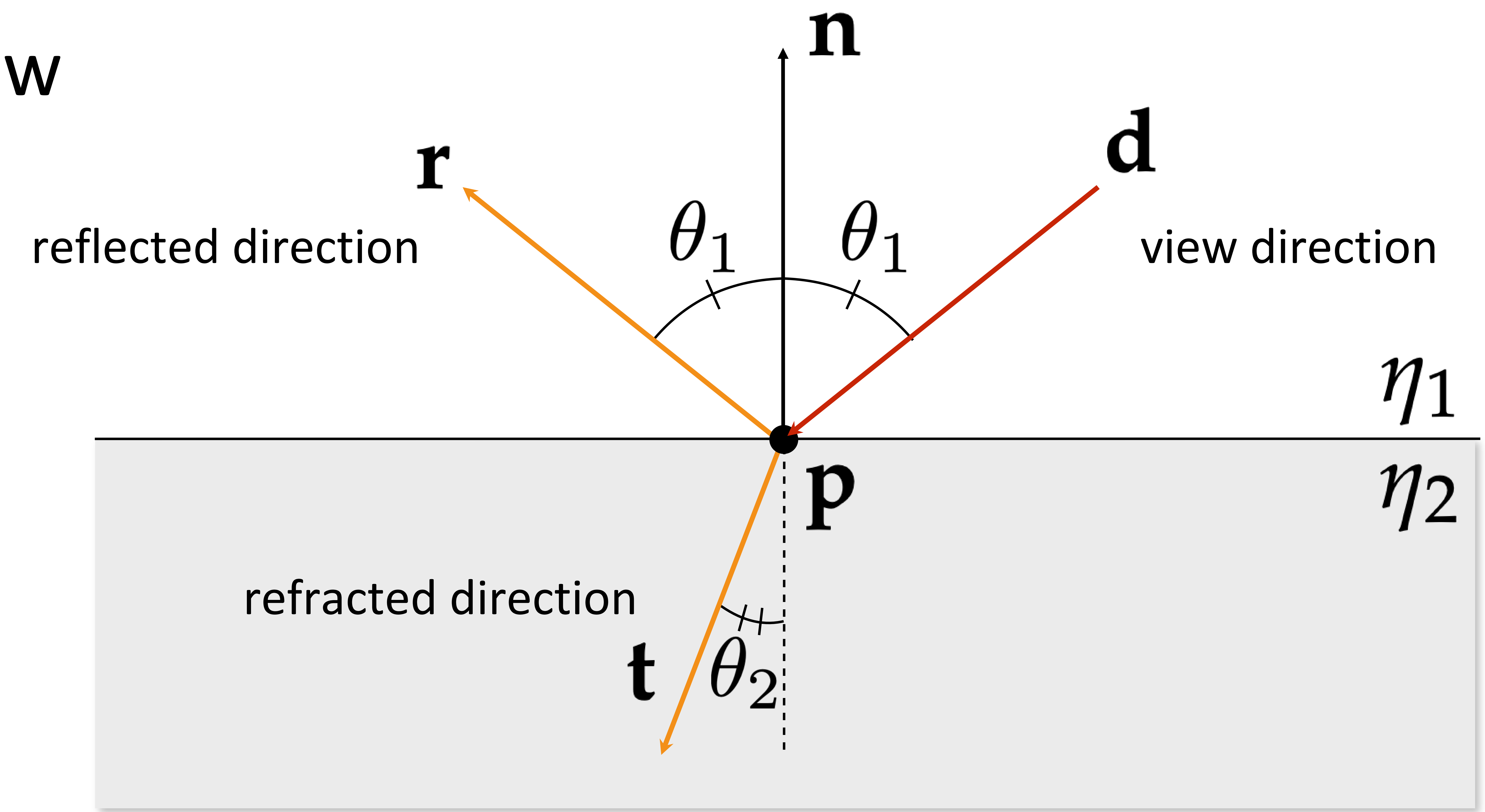
Reflection function
(flips about normal)

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_i - R(\vec{\omega}_r, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eqs. account for it)

Specular transmission/refraction

Snell's law



$$\mathbf{t} = \eta_1/\eta_2\left(\mathbf{d} - (\mathbf{d}\cdot\mathbf{n})\,\mathbf{n}\right) - \mathbf{n}\sqrt{1 - \eta_1^2/\eta_2^2\left(1 - (\mathbf{d}\cdot\mathbf{n})^2\right)}$$

BTDF of Ideal Specular Refraction

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

What is the BTDF for specular refraction?

Dirac delta

Refraction function

Fresnel reflection

$$f_t(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \boxed{\frac{\eta_1^2}{\eta_2^2}} (1 - F_r(\vec{\omega}_i)) \frac{\delta(\vec{\omega}_i - T(\vec{\omega}_r, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eqs. account for it)

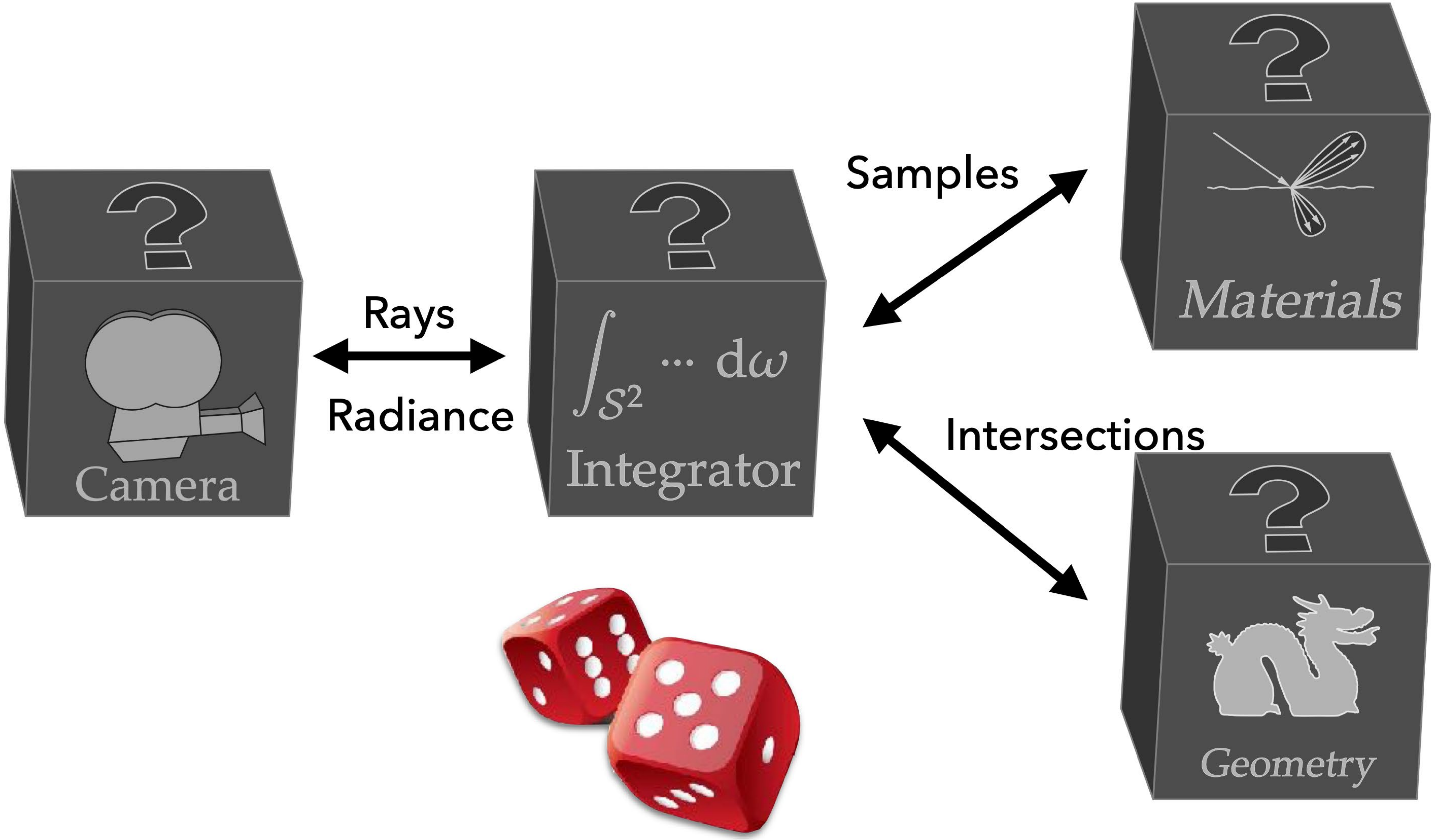
Approximating integrals with Monte Carlo

No need to be scared of math like this:

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = E(\mathbf{x})$$

- integrals will just turn into `for` loops in your code
- evaluating $L(\mathbf{x}, \omega)$ will correspond to tracing a ray

Architecture of a rendering system



Architecture of a rendering system