Radiometry
Course announcements

• Take-home quiz 2 posted, due Tuesday 2/8 at 23:59.
  - How many of you have looked at/started/finished it?
  - Any questions?

• Programming assignment 1 posted, due Friday 2/11 at 23:59.
  - How many of you have looked at/started/finished it?
  - Any questions?

• First reading group on Friday 2/4, 4-6 pm.
  - Suggest topics on Piazza.

• First recitation took place yesterday, 4-5 pm.
  - Recording and photos of whiteboard on Canvas.
  - Any feedback?
Overview of today’s lecture

• Radiometric quantities.
• A little bit about color.
• Reflectance equation.
• Standard reflectance functions revisited.
Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).
Quantifying Light
Assumptions

Light sources, reflectance spectra, sensor sensitivity modeled separately at each wavelength

Geometric/ray optics

No polarization

No fluorescence, phosphorescence, ...
Radiometry

Radiometry studies the measurement of electromagnetic radiation, including visible light.
Radiometry

Assume light consists of photons with:

- \( \mathbf{x} \): Position
- \( \mathbf{w} \): Direction of travel
- \( \lambda \): Wavelength

Each photon has an energy of:

\[
\frac{hc}{\lambda}
\]

- \( h \approx 6.63 \times 10^{-34} \text{ m}^2 \text{ kg/s} \): Planck’s constant
- \( c = 299,792,458 \text{ m/s} \): speed of light in vacuum
- Unit of energy, Joule: \( J = \text{kg m}^2/\text{s}^2 \)
Radiometry

How do we measure the energy flow?

Measuring energy means “counting photons”
Radiometry

Basic quantities (depend on wavelength)

- flux $\Phi$
- irradiance $E$
- radiosity $B$
- intensity $I$
- radiance $L$

will be the most important quantity for us
Flux (Radiant Flux, Power)

Total amount of radiant energy passing through surface or space per unit time.

\[ \Phi(A) \quad \left[ \frac{J}{s} = W \right] \]

Examples:
- Number of photons hitting a wall per second.
- Number of photons leaving a lightbulb per second (how do we quantify this exactly?)
Irradiance

*area density of flux*

flux per unit area **arriving** at a surface

\[ E(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right] \]

example:

- number of photons **hitting** a small patch of a wall per second, *divided* by size of patch

\[ \Phi(A) \]

\[ A \]
Radiosity (Radiant Exitance)

*area density of flux*

flux per unit area **leaving** a surface

\[
B(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right]
\]

example:

- number of photons **reflecting off** a small patch of a wall per second, *divided* by size of patch
Radiant Intensity

directional density of flux

power (flux) per solid angle

\[ I(\omega) = \frac{d\Phi}{d\omega} \quad \left[ \frac{W}{sr} \right] \]
Solid Angle

Angle
- circle: $2\pi$ radians

Solid angle
- sphere: $4\pi$ steradians

\[
\theta = \frac{l}{r}
\]

\[
\Omega = \frac{A}{r^2}
\]
Subtended (Solid) Angle

Length/area of object’s projection onto a unit circle/sphere
The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

Depends on:
- orientation of patch
- distance of patch

One can show: 

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]

“surface foreshortening”

Units: steradians [sr]
Solid angle

To calculate solid angle subtended by a surface $S$ relative to $O$ you must add up (integrate) contributions from all tiny patches (nasty integral)

$$\Omega = \iint_S \frac{\mathbf{r} \cdot \hat{n}}{|\mathbf{r}|^3} dS$$

One can show:

$$d\omega = \frac{dA \cos \theta}{r^2}$$

“surface foreshortening”

Units: steradians [sr]
Radiant Intensity

directional density of flux

power (flux) per solid angle

\[ I(\hat{\omega}) = \frac{d\Phi}{d\hat{\omega}} \quad \left[ \frac{W}{sr} \right] \]

\[ \Phi = \int_{S^2} I(\hat{\omega}) \, d\hat{\omega} \]

example: \( \Phi = 4\pi I \) (for an isotropic point source)

- power per unit solid angle emanating from a point source
A hypothetical measurement device
Radiance

flux density per unit solid angle, per *perpendicular* unit area

\[
L(x, \omega) = \frac{d^2 \Phi(A)}{d\omega dA^\perp(x, \omega)} \left[ \frac{W}{m^2 \text{sr}} \right]
\]

\[
= \frac{d^2 \Phi(A)}{d\omega dA(x) \cos \theta}
\]
Radiance

fundamental quantity for ray tracing and physics-based rendering

remains constant along a ray (*in vacuum only!*)

incident radiance \( L_i \) at one point can be expressed as outgoing radiance \( L_o \) at another point

\[
L_i(x, \omega) = L_o(y, -\omega)
\]
Overview of Quantities

- **flux:** $\Phi(A)$
- **irradiance:** $E(x) = \frac{d\Phi(A)}{dA(x)}$ [W/m²]
- **radiosity:** $B(x) = \frac{d\Phi(A)}{dA(x)}$ [W/m²]
- **intensity:** $I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$ [W/sr]
- **radiance:** $L(x, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos \theta dA(x)d\vec{\omega}}$ [W/m²sr]
Radiance

expressing *irradiance* in terms of radiance:

\[ L(x, \omega) = \frac{d^2 \Phi(A)}{\cos \theta dA(x) d\omega} \quad E(x) = \frac{d\Phi(A)}{dA(x)} \]

\[ L(x, \omega) = \frac{dE(x)}{\cos \theta d\omega} \]

\[ L(x, \omega) \cos \theta d\omega = dE(x) \]

\[ \int_{H^2} L(x, \omega) \cos \theta d\omega = E(x) \]

Integrate cosine-weighted radiance over hemisphere
Radiance

expressing *irradiance* in terms of radiance:

\[
\int_{H^2} L(x, \bar{\omega}) \cos \theta \, d\bar{\omega} = E(x)
\]

expressing *flux* in terms of radiance:

\[
\int_{A} E(x) \, dA(x) = \Phi(A) \quad \quad \quad E(x) = \frac{d\Phi(A)}{dA(x)}
\]

\[
\int_{A} \int_{H^2} L(x, \bar{\omega}) \cos \theta \, d\bar{\omega} \, dA(x) = \Phi(A)
\]

Integrate cosine-weighted radiance
over hemisphere and area
Radiance

Allows computing the radiant flux measured by any sensor

\[ \Phi(W, X) = \int_x \int_w L(\hat{\omega}, x) \cos \theta d\omega dA \]

Cameras measure integrals of radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to (integrals of) radiance.

- “Processed” images (like PNG and JPEG) are not linear radiances measurements!!
Computing spherical integrals

Express function using spherical coordinates:

\[\int_0^{2\pi} \int_0^{\pi} f(\theta, \phi) \, d\theta \, d\phi \, ?\]

**Warning:** this is not correct!
Differential Solid Angle

Differential area on the unit sphere around direction

\[ dA = (rd\theta)(r \sin \theta d\phi) \]

\[ d\vec{\omega} = \frac{dA}{r^2} = \sin \theta d\theta d\phi \]

\[ \Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi = 4\pi \]
Overview of Quantities

- **flux:** \( \Phi(A) \)

- **irradiance:** \( E(x) = \frac{d\Phi(A)}{dA(x)} \) \( \left[ \frac{W}{m^2} \right] \)

- **radiosity:** \( B(x) = \frac{d\Phi(A)}{dA(x)} \) \( \left[ \frac{W}{m^2} \right] \)

- **intensity:** \( I(\vec{\omega}) = \frac{d\Phi}{d\Omega} \) \( \left[ \frac{W}{sr} \right] \)

- **radiance:** \( L(x, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos \theta dA(x)d\vec{\omega}} \) \( \left[ \frac{W}{m^2 sr} \right] \)

All of these quantities can be a function of wavelength!
Handling color

- Any light sensor (digital or not) has different sensitivity to different wavelengths.
- This is described by the sensor’s *spectral sensitivity function* (SSF).
- When measuring some incident *spectral flux*, the sensor produces a *scalar color* response:

\[ R = \int_{\lambda} \Phi(\lambda) \cdot f(\lambda) \, d\lambda \]
Handling color – the human eye

• The human eye is a collection of light sensors called cone cells.

• There are three types of cells with different spectral sensitivity functions.

• Human color perception is three-dimensional (*tristimulus color*).

"short" \[ S = \int_\lambda \Phi(\lambda) S(\lambda) d\lambda \]

"medium" \[ M = \int_\lambda \Phi(\lambda) M(\lambda) d\lambda \]

"long" \[ L = \int_\lambda \Phi(\lambda) L(\lambda) d\lambda \]
Handling color – photography

Two design choices:

• What spectral sensitivity functions $f(\lambda)$ to use for each color filter?

• How to spatially arrange (“mosaic”) different color filters

Why more green pixels? Generally do not match human LMS.

Bayer mosaic

SSF for Canon 50D
Radiometry versus photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system to electromagnetic radiation
- Luminance \( (Y) \) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye’s luminous efficacy curve, e.g.:

\[
Y(p, \omega) = \int_0^\infty L(p, \omega, \lambda) V(\lambda) \, d\lambda
\]
# Radiometry versus photometry

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## Radiometry versus photometry

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<td>Luminous Intensity</td>
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Modern LED light

Input power: 11 W
Output: 815 lumens
(~ 80 lumens / Watt)

Incandescent bulbs:
~15 lumens / Watt)
Reflection equation
Lambertian reflection

Also called ideal diffuse reflection

Lambertian surface
Ideal specular reflection/refraction

\[ \eta_1 \sin \theta_1 = \eta_2 \sin \theta_2 \]
Light-Material Interactions
The BRDF

**Bidirectional Reflectance Distribution Function**

- how much light gets scattered from **one direction** into **each other direction**

- formally: ratio of outgoing *radiance* to incident *irradiance*
The reflection equation

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

This describes a local illumination model

Where does the cosine come from?
Motivation
Motivation
BRDF Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

\[
\int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_r d\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i
\]

Where does the cosine come from?
Helmholtz Reciprocity
BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

\[ \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_r \, d\vec{\omega}_r \leq 1, \quad \forall \vec{\omega}_i \]

- Helmholtz reciprocity

\[ f_r(x, \vec{\omega}_i, \vec{\omega}_r) = f_r(x, \vec{\omega}_r, \vec{\omega}_i) \]
\[ f_r(x, \vec{\omega}_i \leftrightarrow \vec{\omega}_r) \]
BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

\[ \int_{H^2} f_r(x, \tilde{\omega}_i, \tilde{\omega}_r) \cos \theta_r \, d\tilde{\omega}_r \leq 1, \quad \forall \tilde{\omega}_i \]

- Helmholtz reciprocity

\[ f_r(x, \tilde{\omega}_i, \tilde{\omega}_r) = f_r(x, \tilde{\omega}_r, \tilde{\omega}_i) \]

- Together:

\[ \int_{H^2} f_r(x, \tilde{\omega}_i, \tilde{\omega}_r) \cos \theta_i \, d\tilde{\omega}_i \leq 1, \quad \forall \tilde{\omega}_r \]
BRDFs Properties

If the BRDF is unchanged as the material is rotated around the normal, then it is *isotropic*, otherwise it is *anisotropic*.

Isotropic BRDFs are functions of just 3 variables

\[(\theta_i, \theta_r, \Delta \phi)\]
Isotropic vs Anisotropic Reflection

source: luxology.com

source: Stephen H. Westin

source: Stephen H. Westin
Reflection vs. Refraction

- Incident Light Beam
- Specular Reflection
- Reflected Scatter Distribution
- Specular Transmission
- Transmitted Scatter Distribution
- BRDF
- BTDF
Lambertian reflection

Also called ideal diffuse reflection
BRDF for Lambertian reflection?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i \cos \theta_i) \, d\vec{\omega}_i \]

Scatters light equal in all directions
BRDF is a constant
Lambertian BRDF

For Lambertian reflection, the BRDF is a constant:

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

Note: we can drop \( \omega_r \)

\[ L_r(x) = f_r \int_{H^2} L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

\[ L_r(x) = f_r \, E(x) \]

If all incoming light is reflected:

\[ E(x) = B(x) \]

\[ E(x) = \int_{H^2} L_r(x) \cos \theta \, d\bar{\omega} \]

\[ f_r = \frac{1}{\pi} \]

Note: can also be derived from energy conservation

\[ E(x) = L_r(x) \int_{H^2} \cos \theta \, d\bar{\omega} \]

\[ E(x) = L_r(x) \pi \]
Lambertian BRDF

For Lambertian reflection, the BRDF is a constant:

\[
L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i
\]

\[
L_r(x) = \frac{\rho}{\pi} \int_{H^2} L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i
\]

\[\rho: \text{Diffuse reflectance (albedo)} \ [0...1]\]
Specular BRDF

Assume $n$ is unit length

$r = -2n(n \cdot d) + d$
Specular BRDF?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \bar{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \bar{\omega}_i, \bar{\omega}_r) L_i(\mathbf{x}, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i$$

Scatters all light into one (or two) directions
Contains a Dirac delta
Integral drops out

What is the BRDF for specular reflection/refraction?
Dirac delta functions

\[ \int_{-\infty}^{\infty} f(x) \delta(x - a) \, dx = f(a) \]

Note: careful when performing changes of variables in Dirac delta functions!
BRDF of Ideal Specular Reflection

$$L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i$$

What is the BRDF for specular reflection?

$$f_r(x, \omega_i, \omega_r) = F_r(\omega_i) \frac{\delta(\omega_i - R(\omega_r, \hat{n}))}{\cos \theta_i}$$

to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)

Fresnel reflection

Dirac delta

Reflection function (flips about normal)
Specular transmission/refraction

Snell’s law

\[ t = \frac{\eta_1}{\eta_2} (d - (d \cdot n) n) - n \sqrt{1 - \frac{\eta_1^2}{\eta_2^2} \left(1 - (d \cdot n)^2\right)} \]
BTDF of Ideal Specular Refraction

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

What is the BTDF for specular refraction?

\[ f_t(x, \omega_i, \omega_r) = \frac{\eta_1^2}{\eta_2^2} (1 - F_r(\omega_i)) \frac{\delta(\omega_i - T(\omega_r, \hat{n}))}{\cos \theta_i} \]

to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)
Approximating integrals with Monte Carlo

No need to be scared of math like this:

\[ \int_{H^2} L(x, \bar{\omega}) \cos \theta \, d\bar{\omega} = E(x) \]

- integrals will just turn into for loops in your code
- evaluating \( L(x, \omega) \) will correspond to tracing a ray
Architecture of a rendering system
Architecture of a rendering system

Chapter 6

Chapter 14

Chapter 3

Chapter 8

Materials

Geometry