Direct illumination
Course announcements

• Take-home quiz 4 due Tuesday 2/22 at 23:59.

• Programming assignment 2 posted, due Friday 2/25 at 23:59.
  - How many of you have looked at/started/finished it?
  - Any questions?

• Tomorrow’s reading group.
  - Please post or vote for suggested topics on Piazza by Thursday early afternoon.

• Take-home quiz recitations.
  - Nobody showed up yesterday :-( .
  - I will try and do a recording of the solutions on my own.
Overview of today’s lecture

• Importance sampling the reflectance equation.

• BRDF importance sampling.

• Direct versus indirect illumination.

• Different forms of the reflectance equation.

• Environment lighting.

• Light sources.

• Mixture sampling.

• Multiple importance sampling.
Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).
Reflection equation

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

What terms can we importance sample?

- BRDF
- incident radiance
- cosine term
Reflection equation

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i d\bar{\omega}_i \]

What terms can we importance sample?

- BRDF
- incident radiance
- cosine term
This is what we did for ambient occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling
Reflection equation

\[
L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i
\]

What terms can we importance sample?

- BRDF
- incident radiance
- cosine term
Rough materials

In reality, most materials are neither perfectly diffuse nor specular, but somewhere in between

- Imagine a shiny surface scratched up at a microscopic level
- “Blurry” reflections of the light source
Importance Sampling the BRDF

Cosine-weighted importance sampling

BRDF importance sampling

\[ p(\bar{\omega}_i) \propto f(x, \bar{\omega}_i, \bar{\omega}_r) \]
Importance Sampling the BRDF

Uniform hemispherical sampling  BRDF importance sampling
Phong BRDF

Normalized exponentiated cosine lobe:

\[ f_r(\tilde{\omega}_o, \tilde{\omega}_i) = \frac{e + 2}{2\pi} (\tilde{\omega}_r \cdot \tilde{\omega}_o)^e \]

\[ \tilde{\omega}_r = (2\tilde{n}(\tilde{n} \cdot \tilde{\omega}_i) - \tilde{\omega}_i) \]
Phong BRDF

Normalized exponentiated cosine lobe:

\[ f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_r \cdot \vec{\omega}_o)^e \]

\[ \vec{\omega}_r = (2\vec{n}(\vec{n} \cdot \vec{\omega}_i) - \vec{\omega}_i) \]

Interpretation

- randomize reflection rays in a lobe about mirror direction
- perfect mirror reflection of a blurred light
Blinn-Phong BRDF

Randomize normals instead of reflection directions

\[ f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_h \cdot \vec{n})^e \]

\[ \vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|} \]

\( \vec{\omega}_i \) : incident direction

\( \vec{\omega}_h \) : half-way vector

\( \vec{\omega}_o \) : outgoing direction
Phong BRDF

\[ f_r(\mathbf{\omega}_o, \mathbf{\omega}_i) = \frac{e + 2}{2\pi} (\mathbf{\omega}_r \cdot \mathbf{\omega}_o)^e \]

\[ \mathbf{\omega}_r = (2\mathbf{n}(\mathbf{n} \cdot \mathbf{\omega}_i) - \mathbf{\omega}_i) \]
Halfway vector vs. mirror direction BRDFs

BRDFs based on mirror reflection direction have round highlights.

Highlights of BRDFs based on halfway vector get increasingly narrow at glancing angles.
Halfway vector vs. mirror direction BRDFs

Amount of difference depends on circumstance

- Significant for floors, walls, etc. at grazing angles
- Less for highly curvy surfaces and moderate angle

After a slide by Naty Hoffman, SIGGRAPH 2006
Importance Sampling the BRDF

Recipe:

1. Express the desired distribution in a convenient coordinate system
   - requires computing the Jacobian
2. Compute marginal and conditional 1D PDFs
3. Sample 1D PDFs using the inversion method
Sampling the Phong BRDF

Normalized Phong-like $\cos^e$ lobe:

\[ \vec{\omega} = (\theta, \phi) = \left( \arccos \left( \frac{e+1}{\sqrt{1 - \zeta_1}} \right), 2\pi \zeta_2 \right) \]

\[ p(\vec{\omega}) = \frac{e + 1}{2\pi} \vec{\omega}_z^e = \frac{e + 1}{2\pi} \cos^e \theta \]

Then rotate z axis to align with mirror reflection direction
Sampling the Blinn-Phong BRDF

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_h \cdot \vec{n})^e$$

Mirror reflection from random micro-normal

**General recipe:**
- randomly generate a $\omega_h$, with PDF proportional to $\cos^e$
- reflect incident direction $\omega_i$ about $\omega_h$ to obtain $\omega_o$
- convert PDF($\omega_h$) to PDF($\omega_o$) (change-of-variable)

Read PBRTv3 14.1
Half-direction transform

2D:

\[ \theta_h := \frac{\theta_i + \theta_o}{2} \]

\[ \frac{d\theta_h}{d\theta_o} = ? \]

3D:

\[ \omega_h := \frac{\omega_i + \omega_o}{||\omega_i + \omega_o||} \]

\[ \frac{d\omega_h}{d\omega_o} = ? \]
Reflection equation

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

What terms can we importance sample?

- BRDF
- incident radiance
- cosine term
Direct vs. Indirect illumination
Direct vs. Indirect Illumination

Where does $L_i$ “come from”? 

$$L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i$$
Direct vs. Indirect Illumination

Where does $L_i$ “come from”? $L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i$
Direct vs. Indirect Illumination

Direct illumination

Indirect illumination

Direct + indirect illumination
Direct vs. Indirect Illumination

Direct illumination only  

Direct + Indirect illumination

Images courtesy of PDI/DreamWorks
Direct Illumination

Where does $L_i$ “come from”? $L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i$

The incident radiance $L_i$ at $x$ from direction $\omega$ equals the emitted radiance $L_e$ at the end of the ray from $x$ towards $\omega$:

$$L_i(x, \bar{\omega}) = L_e(r(x, \bar{\omega}), -\bar{\omega})$$
Direct Illumination

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_e(r(x, \bar{\omega}_i), -\bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

The incident radiance \( L_i \) at \( x \) from direction \( \omega \) equals the emitted radiance \( L_e \) at the end of the ray from \( x \) towards \( \omega \):

\[ L_i(x, \bar{\omega}) = L_e(r(x, \bar{\omega}), -\bar{\omega}) \]
Direct Illumination

\[ L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

How can we estimate the integral?

\[ \langle L_r(\mathbf{x}, \vec{\omega}_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_{i,k}), -\vec{\omega}_{i,k}) \cos \theta_{i,k}}{p\Omega(\vec{\omega}_{i,k})} \]
Direct Illumination

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_e(r(x, \vec{\omega}_i), -\vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]
Direct Illumination

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_e(r(x, \omega_i), -\omega_i) \cos \theta_i \, d\omega_i \]
Direct Illumination

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_e(r(x, \omega_i), -\omega_i) \cos \theta_i \, d\omega_i \]

Any problems?
Direct Illumination

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_e(r(x, \bar{\omega}_i), -\bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]
Reflection equation

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \ d\bar{\omega}_i \]

What terms can we importance sample?

- BRDF
- incident radiance?
- cosine term
Importance Sampling Incident Radiance

Generally impossible, but...

for direct illumination we can explicitly sample emissive surfaces
Forms of Reflection Equation

Hemispherical integration

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r)L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

Surface Area integration

\[ L_r(x, z) = \int_{A} f_r(x, y, z)L_i(x, y)G(x, y) \, dA(y) \]
Forms of Reflection Equation

Change in notation:

\[ L_i(x, \bar{\omega}_i) = L_i(x, y) \]
\[ L_r(x, \bar{\omega}_r) = L_r(x, z) \]
\[ f_r(x, \bar{\omega}_i, \bar{\omega}_r) = f_r(x, y, z) \]

Transform integral over directions into integral over surface area.

Jacobian determinant of the trans.: 

\[ d\bar{\omega}_i = \frac{|\cos \theta_o|}{\|x - y\|^2} dA \]
Forms of Reflection Equation

\[ L_i(x, \bar{\omega}_i) = L_i(x, y) \]
\[ L_r(x, \bar{\omega}_r) = L_r(x, z) \]
\[ f_r(x, \bar{\omega}_i, \bar{\omega}_r) = f_r(x, y, z) \]
\[ d\bar{\omega}_i = \frac{|\cos \theta_o|}{||x - y||^2} dA \]

Hemispherical form:
\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i d\bar{\omega}_i \]

Surface area form:
\[ L_r(x, z) = \int_{A} f_r(x, y, z) L_i(x, y) G(x, y) dA(y) \]
Area Form of the Reflection Eq.

\[ L_r(x, z) = \int_A f_r(x, y, z) L_i(x, y) G(x, y) \, dA(y) \]

Geometry term:

\[ G'(x, y) = V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{\|x - y\|^2} \]

Visibility term:

\[ V(x, y) = \begin{cases} 
1 : & \text{visible} \\
0 : & \text{not visible} 
\end{cases} \]
Area Form of the Reflection Eq.

\[ L_r(x, z) = \int_A f_r(x, y, z) L_i(x, y) G(x, y) \, dA(y) \]

Geometry term:

\[ G'(x, y) = V(x, y) \frac{\begin{vmatrix} \cos \theta_i & \cos \theta_o \\ \|x - y\|^2 \end{vmatrix}}{\|x - y\|^2} \]

Visibility term:

\[ V(x, y) = \begin{cases} 1 : & \text{visible} \\ 0 : & \text{not visible} \end{cases} \]

Jacobian determinant of the transform

\[ d\tilde{\omega}_i = \frac{|\cos \theta_o|}{\|x - y\|^2} \, dA \]
Area Form of the Reflection Eq.

Interpreting \[
\frac{|\cos \theta_i| \cos \theta_o}{\|x - y\|^2}
\]

The chance that a photon emitted from a differential patch will hit another differential patch decreases as:

- the patches face away from each other (numerator)
- the patches move away from each other (denominator)
Area Form of the Reflection Eq.

Interpreting

\[
\frac{|\cos \theta_i|}{\|\mathbf{x} - \mathbf{y}\|^2} \quad \frac{|\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2}
\]

- numerator = 0
- 0 < numerator < 1
- numerator = 1
Direct Illumination

\[
L_r(\mathbf{x}, \tilde{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \tilde{\omega}_i, \tilde{\omega}_r) L_e(r(\mathbf{x}, \tilde{\omega}_i), -\tilde{\omega}_i) \cos \theta_i \, d\tilde{\omega}_i
\]
Direct Illumination

\[ L_r(x, z) = \int_A f_r(x, y, z)L_e(y, x)V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{||x - y||^2} dA(y) \]
Direct Illumination

\[ L_r(x, z) = \int_{A_s} f_r(x, y, z)L_e(y, x)V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{||x - y||^2} dA(y) \]
Direct Illumination

\[ L_r(x, z) = \int_{A_e} f_r(x, y, z) L_e(y, x) V(x, y) \frac{\cos \theta_i |\cos \theta_o|}{||x - y||^2} dA(y) \]
Direct Illumination

\[ L_r(x, z) = \int_{A_e} f_r(x, y, z) L_e(y, x) V(x, y) \frac{\cos \theta_i \cos \theta_o}{\|x - y\|^2} dA(y) \]
Sampling the hemisphere
Direct Illumination

Sampling the area of the light
Forms of Reflection Equation

Hemispherical integration

Surface Area integration

How do we decide which one to use?

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \ d\bar{\omega}_i \]

\[ L_r(x, z) = \int_{A} f_r(x, y, z) L_i(x, y) G(x, y) \ dA(y) \]
Environment Lighting
Environment Lighting

Environment map
(distant light source)

Scene
Environment Lighting

The image “wraps” around the virtual scene, serving as a *distant* source of illumination

\[ L_r(x, \omega_r) = \int_{\Omega} f_r(\omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

\[ = \int_{\Omega} f_r(\omega_i, \omega_r) L_{\text{env}}(\omega_i) V(x, \omega_i) \cos \theta_i \, d\omega_i \]
Environment Lighting
Environment Lighting

\[ L_r(x, \omega_r) = \int_{\Omega} f_r(\omega_i, \omega_r) L_{\text{env}}(\omega_i) V(x, \omega_i) \cos \theta_i \, d\omega_i \]
Importance Sampling $L_{\text{env}}$

$$p(\tilde{w}_i) \propto L_{\text{env}}(\tilde{w}_i)$$
Importance Sampling $L_{env}$

$$p(\tilde{\omega}_i) \propto L_{env}(\tilde{\omega}_i)$$

Several strategies exist
We’ll discuss:

- Marginal/Conditional CDF method
- Hierarchical warping method
Importance Sampling

Recipe:

1. Express the desired distribution in a convenient coordinate system
   - requires computing the Jacobian
2. Compute marginal and conditional 1D PDFs
3. Sample 1D PDFs using the inversion method
Marginal/Conditional CDF

Assume the lat/long parameterization

Draw samples from joint \( p(\theta, \phi) \propto L_{\text{env}}(\theta, \phi) \sin \theta \)
Why the Sine?

General case of integrating some $f(\vec{\omega})$ over $S^2$

If we set $d\vec{\omega} = \sin \theta d\theta d\phi$ we want to cancel out the sine.

Comes from the Jacobian

$$
\int_{S^2} f(\vec{\omega}) \, d\vec{\omega} = \int_0^{2\pi} \int_0^\pi f(\theta, \phi) \sin \theta \, d\theta d\phi
$$

$$
\approx \frac{1}{N} \sum_{i=1}^N \frac{f(\theta_i, \phi_i) \sin \theta_i}{p(\theta_i, \phi_i)}
$$

$$
p(\theta, \phi) \propto f(\theta, \phi) \sin \theta
$$
Marginal/Conditional CDF

Assume the lat/long parameterization

Draw samples from joint \( p(\theta, \phi) \propto L_{\text{env}}(\theta, \phi) \sin \theta \)

- Step 1: create scalar version \( L'(\theta, \phi) \) of \( L_{\text{env}}(\theta, \phi) \sin \theta \)

- Step 2: compute marginal PDF

\[
p(\theta) = \int_0^{2\pi} L'(\theta, \phi) \, d\phi
\]

- Step 3: compute conditional PDF

\[
p(\phi|\theta) = \frac{p(\theta, \phi)}{p(\theta)}
\]

- Step 4: draw samples \( \theta_i \sim p(\theta) \) and \( \phi_i = p(\phi|\theta) \)
Step 1: Scalar Importance Func.

Original environment map
Step 1: Scalar Importance Func.

Scalar version
(average, max, or luminance of RGB channels)
Step 1: Scalar Importance Func.

Multiplied by $\sin \theta$
Step 2: Marginalization

Marginalize to get $p(\theta)$
Step 3: Conditional PDFs

Once normalized, each row can serve as the conditional PDF

\[ \phi \]

\[ \theta \]
Step 4: Sampling
Step 4: Sampling
Sampling Discrete 1D PDFs
Sampling Discrete 1D PDFs

Given a uniform random value $\xi$

Find $x_i$ and $x_{i+1}$ using binary search

Linearly interpolate to find $x$
C++ details

Don’t need to implement binary search yourself!
- Given sorted list, use std::lower_bound(...)
- See implementation in PBRT
Resulting Sample Distribution
Light Sources
## Light Sources

<table>
<thead>
<tr>
<th>Point light</th>
<th>Spot light</th>
<th>Directional light</th>
<th>Quad light</th>
<th>Sphere light</th>
<th>Mesh light</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Point light icon" /></td>
<td><img src="image" alt="Spot light icon" /></td>
<td><img src="image" alt="Directional light icon" /></td>
<td><img src="image" alt="Quad light icon" /></td>
<td><img src="image" alt="Sphere light icon" /></td>
<td><img src="image" alt="Mesh light icon" /></td>
</tr>
</tbody>
</table>

- **Delta lights** (create hard shadows)
- **Area/Shape lights** (create soft shadows)
Point Light

Omnidirectional emission from a single point

Typically defined using a point $p$ and emitted power $\Phi$

- delta function with respect to which form of the reflection equation?

$$L_r(x, z) = \int_{A_e} f_r(x, y, z) L_e(y, x) V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{\|x - y\|^2} \, dA(y)$$

$$L_r(x, z) = \frac{\Phi}{4\pi} f_r(x, p, z) V(x, p) \frac{|\cos \theta_i|}{\|x - p\|^2}$$
Point Light

Omnidirectional emission from a single point

Typically defined using a point $p$ and emitted power $\Phi$

- delta function with respect to surface integral

$$L_r(x, z) = \frac{\Phi}{4\pi} f_r(x, p, z) V(x, p) \frac{|\cos \theta_i|}{\|x - p\|^2}$$
Spot Light?

**Directionally dependent** emission from a single point

Typically defined using a point \( \mathbf{p} \) and ...

\[
L_r(x, z) = \Phi \frac{f_r(x, \mathbf{p}, z)V(x, \mathbf{p})}{4\pi} \frac{|\cos \theta_i|}{||x - \mathbf{p}||^2}
\]
Spot Light

Directionally dependent emission from a single point

Typically defined using a point $p$ and a directionally dependent radiant intensity function $I$

$$L_r(x, z) = I(p, x)f_r(x, p, z)V(x, p)\frac{\cos \theta_i}{\|x - p\|^2}$$

The intensity can be defined using IES profiles:
Typically defined using direction $\omega$ and radiance $L_d(\omega)$ coming from direction $\omega$

$$L_r(x, \omega_r) = f_r(x, \omega, \omega_r)V(x, \omega)L_d(\omega) \cos \theta$$

- delta function with respect to which form of the reflection equation?
Quad Light

Has finite area... creates soft shadows
Quad Light

Point light

Quad light
Sphere Light

Typically defined using a center $\mathbf{p}$, radius $r$, and emitted power $\Phi$ (or emitted radian $L_e$)

Has finite surface area $4\pi r^2$
Sphere Light

How to sample points on the sphere light?

**Approach 1:** uniformly sample *sphere area*
How to sample points on the sphere light?

**Approach 1:** uniformly sample *sphere area*.

Many samples are not visible from the shading point!
Sphere Light

How to sample points on the sphere light?

**Approach 2** (better): uniformly sample *area* of the visible *spherical cap*

Can sample a spherical cap using Hat-Box theorem!
Sphere Light

How to sample points on the sphere light?

**Approach 2 (better):** uniformly sample *area* of the *visible spherical cap*

Uniform area-density is not ideal as emitted radiance is weighted by the cosine term (recall the form factor in the G term)
Sphere Light

How to sample points on the sphere light?

**Approach 3** (even better): uniformly sample *solid angle* subtended by the sphere
Sphere Light

How to sample points on the sphere light?

**Approach 3** (even better): uniformly sample *solid angle* subtended by the sphere
Sphere Light

How to sample points on the sphere light?

Caution!

- Approaches use PDFs defined wrt different measures
- Make sure to convert the PDF into the measure of the integral!

\[
p_A(x) = \frac{\cos \theta}{d^2} p_\Omega(\vec{\omega})
\]

\[
p_\Omega(\vec{\omega}) = \frac{d^2}{\cos \theta} p_A(x)
\]
Sphere Light

How to sample points on the sphere light?

Caution!

- Approaches use PDFs defined wrt different measures
- Make sure to convert the PDF into the measure of the integral!
- Example: using approach 1 for MC integration of the hemispherical formulation of the reflection eq.

\[
\langle L_r(x, \bar{\omega}_r) \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(x, \bar{\omega}_{i,k}, \bar{\omega}_r) L_i(x, \bar{\omega}_{i,k}) \cos \theta_{i,k}}{p_\Omega(\bar{\omega}_{i,k})}
\]

\[
p_A(y) = \frac{1}{4\pi r^2}
\]

\[
p_\Omega(\bar{\omega}_i) = \frac{||x - y||^2}{|\omega_i \cdot n_y| 4\pi r^2}
\]
Sphere Light

**Validation:** irradiance is independent of radius
(assuming it emits always the same power & no occluders)

A sphere light    A smaller sphere light    A point light

Identical irradiance profiles
Mesh Light

An emissive mesh where every surface point emits given radiance $L_e$

Total area: $\sum A(k)$
How to importance sample?

Preprocess:
- build a discrete PDF, $p_\Delta$, for choosing polygons (triangles) proportional to their area:

$$p_\Delta(i) = \frac{A(i)}{\sum_k A(k)}$$

Run-time:
- sample a polygon $i$ and a point $x$ on $i$
- compute the PDF of choosing the point:

$$p_A(x) = p_\Delta(i)p_A(x|i) = \frac{1}{\sum A(k)}$$
Light Sources

Point light
Spot light
Directional light
Quad light
Sphere light
Mesh light

Delta lights
(create hard shadows)

Area/Shape lights
(create soft shadows)
Reflection Equation

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

What terms can we importance sample?

- BRDF
- incident radiance
- cosine term

What terms \textbf{should} we importance sample?

- depends on the context, hard to make a general statement
Multiple Strategies

Cosine-weighted hemisphere

Uniform surface area

Few samples wasted, high concentration (good)

Most samples wasted (bad)

pdf \times contribution (bad)

No samples wasted pdf \times contribution (good)
Combining Multiple Strategies

Cosine-weighted hemisphere

Uniform surface area

\[ p_1(\vec{w}) = \frac{\cos \theta}{\pi} \]

\[ p_2(x) = \frac{1}{A} \]
Combining Multiple Strategies

Cosine-weighted hemisphere

\[ p_1(\vec{w}) = \frac{\cos \theta}{\pi} \]

Uniform surface area

\[ p_2(x) = \frac{1}{A} \quad p_2(\vec{w}) = \frac{1}{A} \frac{d^2}{\cos \theta} \]
Combining Multiple Strategies

Could just average two different estimators:

\[
\frac{0.5}{N_1} \sum_{i=1}^{N_1} \frac{f(x_i)}{p_1(x_i)} + \frac{0.5}{N_2} \sum_{i=1}^{N_2} \frac{f(x_i)}{p_2(x_i)}
\]

– doesn’t really help: \textit{variance is additive}

Instead, sample from the average PDF

\[
\frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{0.5(p_1(x_i) + p_2(x_i))}
\]
Sample from Average PDF (mixture sampling)

You are given two sampling functions and their corresponding pdfs:

```c
float sample1(float rnd); float pdf1(float x);
float sample2(float rnd); float pdf2(float x);
```

Create a new function:

```c
float sampleAvg(float rnd);
```

which has the corresponding pdf:

```c
float pdfAvg(float x)
{
    return 0.5 * (pdf1(x) + pdf2(x));
}
```
float sampleAvg(float rnd)
{
    float Prob1 = 0.5;
    if (rand.nextFloat() < Prob1)
        return sample1(rnd);
    else
        return sample2(rnd);
}
float sampleAvg(float \textit{rnd})
{
    float Prob1 = 0.5;
    if (rnd < Prob1)
        return sample1(rnd);
    else
        return sample2(rnd);
}

These need to be uniform random numbers in [0..1)
float sampleAvg(float \texttt{rnd})
{
    float Prob1 = 0.5;
    if (\texttt{rnd} < Prob1)
        return sample1(\texttt{rnd});
    else
        return sample2(\texttt{rnd});
}

These need to be uniform random numbers in [0..1)
float sampleAvg(float \texttt{rnd})
{
  float Prob1 = 0.5;
  if (rnd < Prob1)
    return sample1(rnd / Prob1);
  else
    return sample2(rnd);
}
Sample from Average PDF (mixture sampling)

```c
float sampleAvg(float rnd)
{
    float Prob1 = 0.5;
    if (rnd < Prob1)
        return sample1(rnd / Prob1);
    else
        return sample2((rnd-Prob1) / (1-Prob1));
}
```
Sample from Weighted Average

```c
float sampleWeightedAvg(float rnd) {
    float Prob1 = 0.25;
    if (rnd < Prob1) {
        return sample1(rnd / Prob1);
    } else {
        return sample2((rnd-Prob1)/(1-Prob1));
    }
}

float pdfWeightedAvg(float x) {
    return 0.25 * pdf1(x) + 0.75 * pdf2(x);
}
```

Still works, just change Prob1
Cosine-weighted sampling
Uniform surface area sampling
Mixture sampling (average PDF)
Cosine-weighted sampling (× 4)
Uniform surface area (× 4)
Mixture sampling (× 4)
Cosine-weighted sampling (/ 2)
Uniform surface area (/ 2)
Mixture sampling (\ 2)
Light sampling
Mixture sampling
Multiple Importance Sampling (MIS)
Motivation

In MC integration, variance is high when the PDF is not proportional to the integrand

Worst case: rare samples with huge contributions

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

“fireflies”
Motivation

In MC integration, variance is high when the PDF is not proportional to the integrand

Worst case: rare samples with huge contributions

\[ \langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)} \]

We often have multiple sampling strategies

If at least one covers each part of the integrand well, then combining them should reduce fireflies
Multiple Importance Sampling

Weighted combination of 2 strategies

\[
\langle F^{\text{MIS}} \rangle = \frac{w_1(x_1) f(x_1)}{p_1(x_1)} + \frac{w_2(x_2) f(x_2)}{p_2(x_2)}
\]

– where:

\[
w_1(x) + w_2(x) = 1
\]
Multiple Importance Sampling

Weighted combination of $M$ strategies

$$
\langle F\sum N_s \rangle = \sum_{s=1}^{M} \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(x_i) \frac{f(x_i)}{p_s(x_i)}
$$

– where:

$$
\sum_{s=1}^{M} w_s(x) = 1
$$

How to choose the weights?
Multiple Importance Sampling

Balance heuristic (provably good):

\[ w_s(x) = \frac{p_s(x)}{\sum_j p_j(x)} \]

Power heuristic (more aggressive, can be better):

\[ w_s(x) = \frac{p_s(x)^\beta}{\sum_j p_j(x)^\beta} \]

Other heuristics exist

- e.g. cutoff heuristic, maximum heuristic, ...
Multiple Importance Sampling

Multi-sample model

\[ \langle F^{\sum N_s} \rangle = \sum_{s=1}^{M} \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(x_i) \frac{f(x_i)}{p_s(x_i)} \]

What if we want to draw just one sample?

One-sample model:

\[ \langle F^1 \rangle = w_s(x) \frac{f(x)}{q_s p_s(x)} \]

where \( q_s \) is the probability of using strategy \( s \), and \( \sum_{s=1}^{N} q_s = 1 \)
Interpreting the Balance Heuristic

Balance heuristic for the one-sample model:

\[ w_s(x) = \frac{q_s p_s(x)}{\sum_j q_j p_j(x)} \]

Plugged into the one-sample model:

\[ \langle F^1 \rangle = w_s(x) \frac{f(x)}{q_s p_s(x)} = \frac{q_s p_s(x)}{\sum_j q_j p_j(x)} \frac{f(x)}{q_s p_s(x)} = \frac{f(x)}{\sum_j q_j p_j(x)} \]

Balance heuristic samples from average PDF
Why Does it Work?

Using a single strategy:

\[
\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}
\]

large value
small value

Combining multiple strategies using balance heuristic:

\[
\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{\sum_j q_j p_j(x_i)}
\]

large value
relatively large value
(as long as at least one PDF is large)
Sampling the Light

sphere lights

highly glossy surface

moderately glossy surface
Sampling the BRDF

sphere lights

highly glossy surface

moderately glossy surface
Multiple Importance Sampling

- sphere lights
- highly glossy surface
- moderately glossy surface

Eric Veach and Leonidas J. Guibas 1995.
Multiple Importance Sampling

See PBRe3 13.10.1 for more details