Radiometry
Course announcements

- Take-home quiz 3 will be posted today.

- Programming assignment 2 posted, due Friday 3/12 at 23:59.
  - How many of you have looked at/started/finished it?
  - Any questions?

- Second reading group on Friday 3/5, 3-5 pm.
  - Suggest topics on Piazza.

- Please participate in the discussion on take-home quiz solutions on Piazza:
  https://piazza.com/class/kklw0l5me2or4?cid=64
Overview of today’s lecture

• Radiometric quantities.
• A little bit about color.
• Reflectance equation.
• Standard reflectance functions revisited.
Slide credits

Most of these slides were directly adapted from:

• Wojciech Jarosz (Dartmouth).
Assumptions

Light sources, reflectance spectra, sensor sensitivity modeled separately at each wavelength

Geometric/ray optics

No polarization

No fluorescence, phosphorescence, ...
Quantifying Light
Radiometry

Radiometry studies the measurement of electromagnetic radiation, including visible light.
Radiometry

Assume light consists of photons with:

- \( \mathbf{x} \): Position
- \( \mathbf{\omega} \): Direction of travel
- \( \lambda \): Wavelength

Each photon has an energy of:

\[
\frac{h c}{\lambda}
\]

- \( h \approx 6.63 \times 10^{-34} \text{ m}^2 \text{ kg/s} \): Planck's constant
- \( c = 299,792,458 \text{ m/s} \): speed of light in vacuum
- Unit of energy, Joule: \( \left[ J = \text{kg m}^2 / \text{s}^2 \right] \)
Radiometry

How do we measure the energy flow?

Measuring energy means “counting photons”
Radiometry

Basic quantities (depend on wavelength)

- flux $\Phi$
- irradiance $E$
- radiosity $B$
- intensity $I$
- radiance $L$

will be the most important quantity for us
Flux (Radiant Flux, Power)

total amount of radiant energy passing through surface or space per unit time

\[ \Phi(A) \left[ \frac{J}{s} = W \right] \]

examples:

- number of photons hitting a wall per second

- number of photons leaving a lightbulb per second (how do we quantify this exactly?)
Irradiance

*area density of flux*

flux per unit area **arriving** at a surface

\[ E(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right] \]

dexample:

- number of photons **hitting** a small patch of a wall per second, *divided* by size of patch
Radiosity (Radiant Exitance)

(area density of flux)

flux per unit area **leaving** a surface

\[ B(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right] \]

eample:

- number of photons **reflecting off** a small patch of a wall per second, **divided** by size of patch
Radiant Intensity

directional density of flux

power (flux) per solid angle

\[ I(\bar{\omega}) = \frac{d\Phi}{d\bar{\omega}} \quad \text{[W/sr]} \]
Solid Angle

Angle
- circle: $2\pi$ radians

$$\theta = \frac{l}{r}$$

Solid angle
- sphere: $4\pi$ steradians

$$\Omega = \frac{A}{r^2}$$
Subtended (Solid) Angle

Length/area of object’s *projection* onto a unit circle/sphere
Radiant Intensity

directional density of flux

power (flux) per solid angle

\[ I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \quad \text{[W/sr]} \]

\[ \Phi = \int_{S^2} I(\vec{\omega}) \, d\vec{\omega} \]

example: \( \Phi = 4\pi I \) (for an isotropic point source)

- power per unit solid angle emanating from a point source
A hypothetical measurement device
Radiance

flux density per unit solid angle, per *perpendicular* unit area

\[
L(x, \omega) = \frac{d^2 \Phi(A)}{d\omega dA^\perp(x, \omega)} \left[ \frac{W}{m^2 \text{sr}} \right]
\]

\[
= \frac{d^2 \Phi(A)}{d\omega dA(x) \cos \theta}
\]
Radiance

Lambert’s Cosine Law

\[ E = \frac{\Phi}{A} \]

\[ E = \frac{\Phi}{A / \cos \theta} = \frac{\Phi}{A} \cos \theta \]
Radiance

Lambert’s Cosine Law

\[ E = \frac{\Phi}{A / \cos \theta} = \frac{\Phi}{A} \cos \theta \]
Radiance

fundamental quantity for ray tracing and physics-based rendering
remains constant along a ray (in vacuum only!)

incident radiance $L_i$ at one point can be expressed as outgoing radiance $L_o$ at another point

$$L_i(x, \omega) = L_o(y, -\omega)$$
Overview of Quantities

- **flux:** $\Phi(A)$

- **irradiance:**
  
  $E(x) = \frac{d\Phi(A)}{dA(x)}$ 

- **radiosity:**
  
  $B(x) = \frac{d\Phi(A)}{dA(x)}$ 

- **intensity:**
  
  $I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$ 

- **radiance:**
  
  $L(x, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos \theta dA(x)d\vec{\omega}}$ 

\[ \frac{J}{s} = W \]
Radiance

expressing *irradiance* in terms of radiance:

\[
L(x, \omega) = \frac{d^2\Phi(A)}{\cos \theta dA(x) d\omega}
\]

\[
E(x) = \frac{d\Phi(A)}{dA(x)}
\]

\[
L(x, \omega) = \frac{dE(x)}{\cos \theta d\omega}
\]

\[
L(x, \omega) \cos \theta d\omega = dE(x)
\]

\[
\int_{H^2} L(x, \omega) \cos \theta d\omega = E(x)
\]

Integrate cosine-weighted radiance over hemisphere
Radiance

expressing \textit{irradiance} in terms of radiance:

\[
\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos \theta d\vec{\omega} = E(\mathbf{x})
\]

expressing \textit{flux} in terms of radiance:

\[
\int_{A} E(\mathbf{x}) dA(\mathbf{x}) = \Phi(A) \quad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}
\]

\[
\int_{A} \int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos \theta d\vec{\omega} dA(\mathbf{x}) = \Phi(A)
\]

Integrate cosine-weighted radiance over hemisphere and area
Computing spherical integrals

Express function using spherical coordinates:

$$\int_0^{2\pi} \int_0^\pi f(\theta, \phi) \, d\theta \, d\phi$$

**Warning**: this is not correct!
Differential Solid Angle

Differential area on the unit sphere around direction $\vec{\omega}$

$$dA = (rd\theta)(r \sin \theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi = 4\pi$$
Overview of Quantities

- **flux:** $\Phi(A)$
  
  \[
  \frac{J}{s} = W
  \]

- **irradiance:** $E(x) = \frac{d\Phi(A)}{dA(x)}$
  
  \[
  \frac{W}{m^2}
  \]

- **radiosity:** $B(x) = \frac{d\Phi(A)}{dA(x)}$
  
  \[
  \frac{W}{m^2}
  \]

- **intensity:** $I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$
  
  \[
  \frac{W}{sr}
  \]

- **radiance:** $L(x, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos \theta dA(x)d\vec{\omega}}$
  
  \[
  \frac{W}{m^2 \, sr}
  \]

All of these quantities can be a function of wavelength!
Handling color

• *Any* light sensor (digital or not) has different sensitivity to different wavelengths.

• This is described by the sensor’s *spectral sensitivity function* (SSF).

• When measuring some incident *spectral flux*, the sensor produces a *scalar color* response:

\[ R = \int_\lambda \Phi(\lambda) f(\lambda) d\lambda \]
Handling color – the human eye

- The human eye is a collection of light sensors called cone cells.
- There are three types of cells with different spectral sensitivity functions.
- Human color perception is three-dimensional (tristimulus color).

“short” \[ S = \int_{\lambda} \Phi(\lambda)S(\lambda) d\lambda \]

“medium” \[ M = \int_{\lambda} \Phi(\lambda)M(\lambda) d\lambda \]

“long” \[ L = \int_{\lambda} \Phi(\lambda)L(\lambda) d\lambda \]

cone distribution for normal vision (64% L, 32% M)
Handling color – photography

Two design choices:

• What spectral sensitivity functions $f(\lambda)$ to use for each color filter?

• How to spatially arrange (“mosaic”) different color filters

Bayer mosaic

Why more green pixels?

Generally do not match human LMS.

SSF for Canon 50D
Radiometry versus photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system to electromagnetic radiation
- Luminance ($Y$) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye’s luminous efficacy curve, e.g.:

$$Y(p, \omega) = \int_{0}^{\infty} L(p, \omega, \lambda) \, V(\lambda) \, d\lambda$$
# Radiometry versus photometry

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# Radiometry versus photometry

<table>
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<tr>
<th>Photometry</th>
<th>MKS</th>
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<td>Stilb Lambert</td>
<td>Footlambert</td>
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<tr>
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Modern LED light

Input power: 11 W
Output: 815 lumens
(~ 80 lumens / Watt)

Incandescent bulbs:
~15 lumens / Watt)
Reflection equation
Lambertian reflection

Also called ideal diffuse reflection
Ideal specular reflection/refraction

\[ \eta_1 \sin \theta_1 = \eta_2 \sin \theta_2 \]
Light-Material Interactions
The BRDF

**Bidirectional Reflectance Distribution Function**

- how much light gets scattered from one direction into each other direction

- formally: ratio of outgoing *radiance* to incident *irradiance*
The reflection equation

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(x, \tilde{\omega}_r) = \int_{H^2} f_r(x, \tilde{\omega}_i, \tilde{\omega}_r) L_i(x, \tilde{\omega}_i) \cos \theta_i \, d\tilde{\omega}_i$$

Where does the cosine come from?

This describes a local illumination model
Motivation
BRDF Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

\[
\int_{H^2} f_r(x, \omega_i, \omega_r) \cos \theta_i \, d\omega_i \leq 1, \quad \forall \omega_r
\]

Where does the cosine come from?
Helmholtz Reciprocity
BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

\[
\int_{H^2} f_r(x, \omega_i, \omega_r) \cos \theta_i \, d\omega_i \leq 1, \quad \forall \omega_r
\]

- Helmholtz reciprocity

\[
f_r(x, \omega_i, \omega_r) = f_r(x, \omega_r, \omega_i)
\]

\[
f_r(x, \omega_i \leftrightarrow \omega_r)
\]
BRDFs Properties

If the BRDF is unchanged as the material is rotated around the normal, then it is \textit{isotropic}, otherwise it is \textit{anisotropic}.

Isotropic BRDFs are functions of just 3 variables

\[(\theta_i, \theta_r, \Delta \phi)\]
Isotropic vs Anisotropic Reflection

source: luxology.com

source: Stephen H. Westin

source: Stephen H. Westin
Reflection vs. Refraction
BRDF for ideal diffuse reflection?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i$$

Scatters light equal in all directions

BRDF is a constant
Ideal Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

Note: we can drop \( \omega_r \)

\[ L_r(x) = f_r \int_{H^2} L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

\[ L_r(x) = f_r \, E(x) \]

If all incoming light is reflected:

\[ E(x) = B(x) \]

\[ E(x) = \int_{H^2} L_r(x) \cos \theta \, d\bar{\omega} \]

\[ f_r = \frac{1}{\pi} \]

Note: can also be derived from energy conservation

\[ E(x) = L_r(x) \int_{H^2} \cos \theta \, d\bar{\omega} \]

\[ E(x) = L_r(x) \pi \]
Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

\[ L_r(x) = \frac{\rho}{\pi} \int_{H^2} L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

\( \rho \): Diffuse reflectance (albedo) [0...1)
Specular BRDF?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i$$

Scatters all light into one (or two) directions

Contains a Dirac delta

Integral drops out

What is the BRDF for specular reflection/refraction?
Dirac delta functions

\[ \int_{-\infty}^{\infty} f(x) \delta(x - a) \, dx = f(a) \]

Note: careful when performing changes of variables in Dirac delta functions!
BRDF of Ideal Specular Reflection

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

What is the BRDF for specular reflection?

\[ f_r(x, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_i - R(\vec{\omega}_r, \vec{n}))}{\cos \theta_i} \]

- Fresnel reflection
- Dirac delta
- Reflection function (flips about normal)
- to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)
BTDF of Ideal Specular Refraction

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

What is the BTDF for specular refraction?

\[ f_t(x, \bar{\omega}_i, \bar{\omega}_r) = \frac{\eta_1^2}{\eta_2^2} (1 - F_r(\bar{\omega}_i)) \frac{\delta(\bar{\omega}_i - T(\bar{\omega}_r, \bar{n}))}{\cos \theta_i} \]

to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)

Fresnel reflection
Dirac delta
Refraction function
Turning math into algorithms

No need to be scared of math like this:

\[ \int_{H^2} L(x, \omega) \cos \theta \, d\omega = E(x) \]

- integrals will just turn into for loops in your code
- evaluating \( L(x, \omega) \) will correspond to tracing a ray
Next time:

Approximating these integrals with Monte Carlo
Architecture of a rendering system
Architecture of a rendering system

Chapter 6

Chapter 14

Chapter 8

Materials

Geometry

Chapter 3