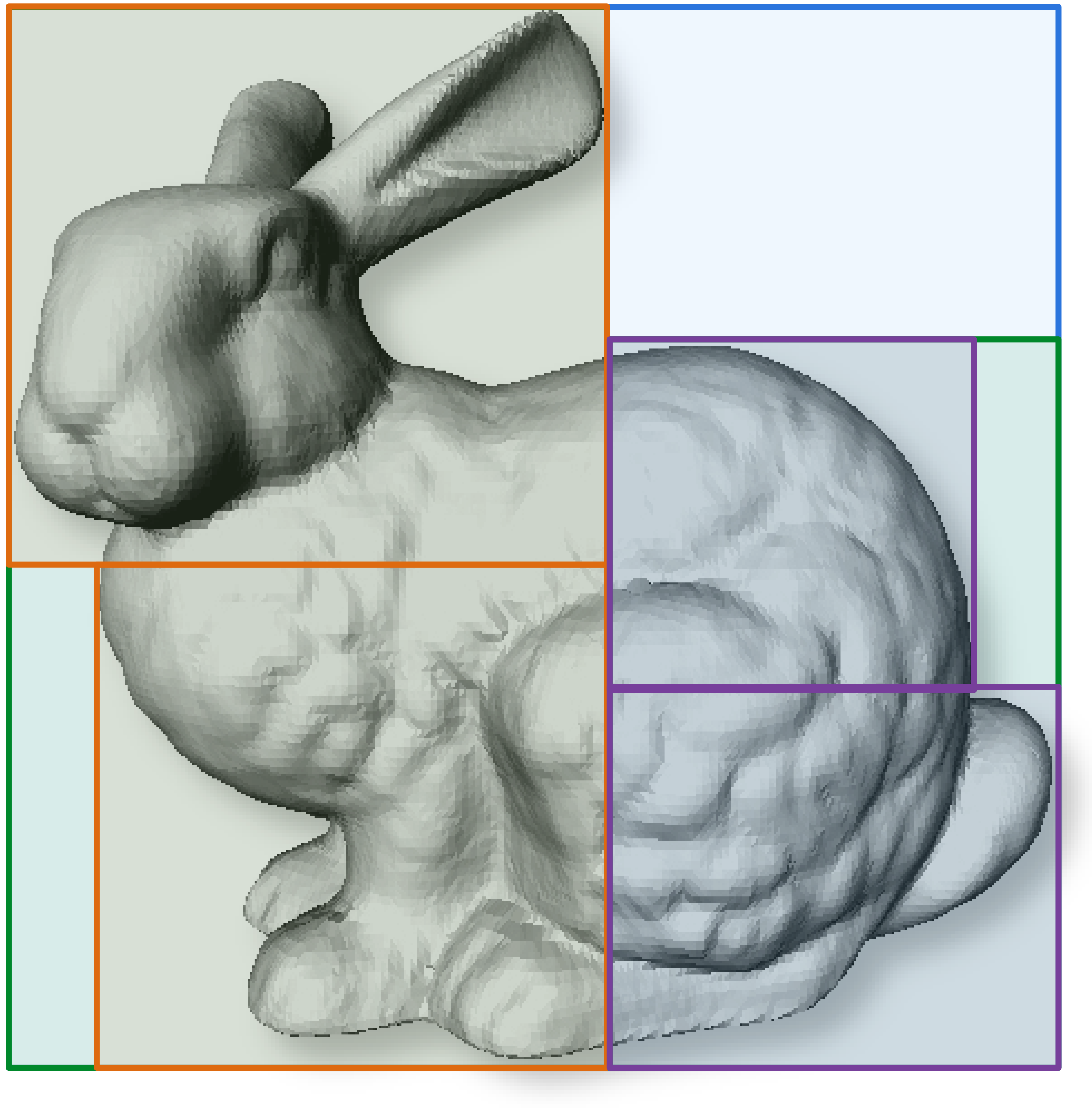
Ray tracing and geometric representations



15-468, 15-668, 15-868
Physics-based Rendering
Spring 2021, Lecture 2

http://graphics.cs.cmu.edu/courses/15-468

Course announcements

- Go over survey results.
- Take-home quiz 1 will be posted on Tuesday 2/9 and will be due a week later.
- Programming assignment 1 will be posted on Friday 2/12, will be due two weeks later.
- Office hours *for this week only* (will finalize starting next week based on survey results):
  - Yannis –Friday 4:30 – 6:30 pm.
  - Zoom details on Piazza and Canvas.

Course announcements

- Is there anyone not on Piazza?

https://piazza.com/class/kklw0l5me2or4

- Is there anyone not on Canvas?

https://canvas.cmu.edu/courses/22291

Survey results

Overview of today's lecture

- Introduction to ray tracing.
- Intersections with geometric primitives.
- Triangular meshes.

Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).

Two forms of 3D rendering

Rasterization: object point to image plane

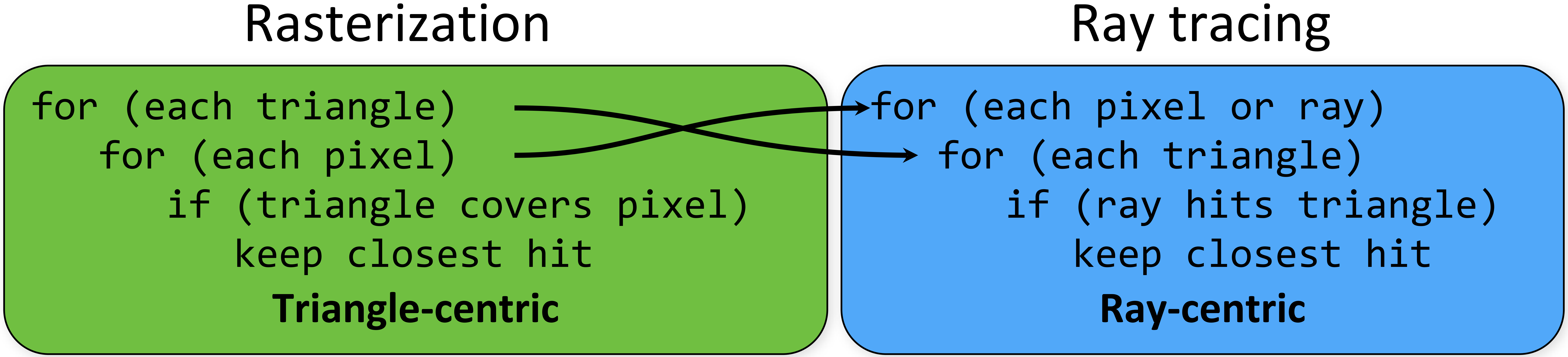
- start with a 3D object point
- apply transforms
- determine the 2D image plane point it projects to

Ray tracing: image plane to object point

- start with a 2D image point
- generate a ray
- determine the visible 3D object point

Inverse processes

Two forms of 3D rendering



Rasterization

```
for (each triangle)
    for (each pixel)
        if (triangle covers pixel)
            keep closest hit
```

Triangle-centric

Ray tracing

```
for (each pixel or ray)
    for (each triangle)
        if (ray hits triangle)
            keep closest hit
```

Ray-centric

After a slide by Frédo Durand

Rasterization advantages

Modern scenes are more complicated than images

- A 1920x1080 frame (1080p) at 64-bit color and 32-bit depth per pixel is 24 MB (not that much)
  - of course, if we have more than one sample per pixel this gets larger, but e.g. 4x supersampling is still a relatively comfortable ~100 MB
- Our scenes are routinely larger than this
  - This wasn’t always true

A rasterization-based renderer can *stream* over the triangles, no need to keep entire dataset around

- Allows parallelism and optimizations of memory systems

After a slide by Frédo Durand

Rasterization limitations

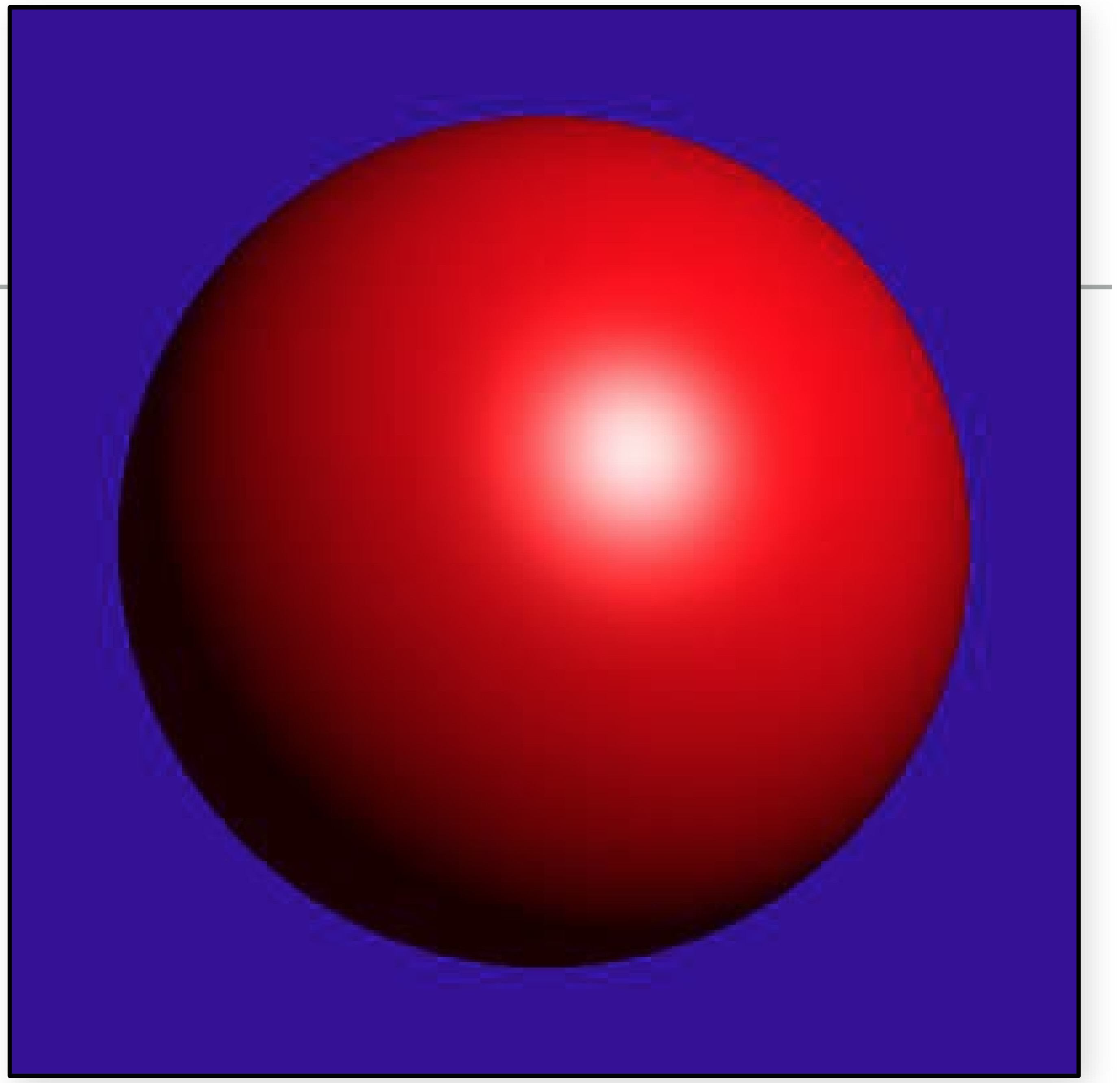
Restricted to scan-convertible primitives

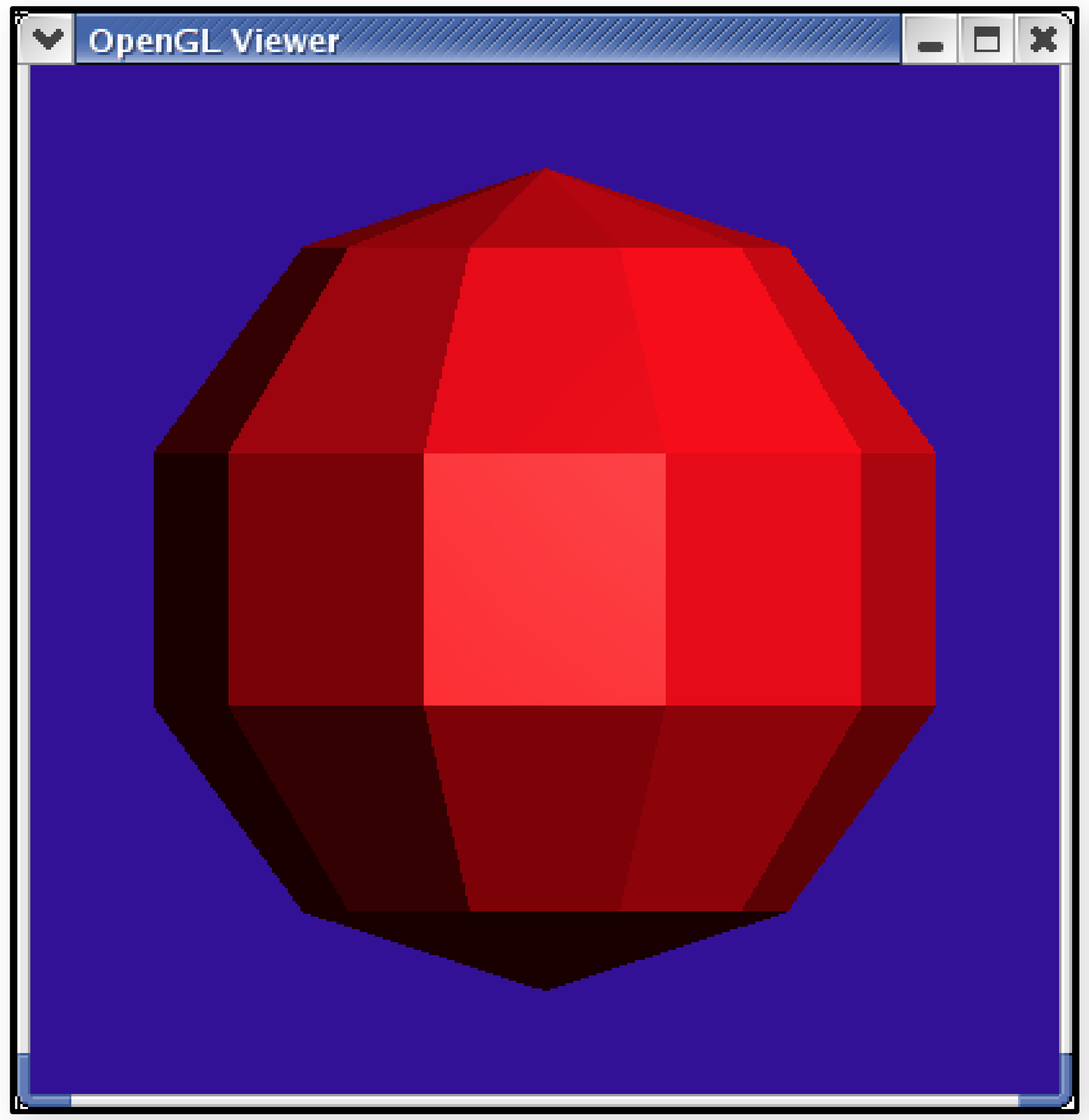
- Pretty much: triangles

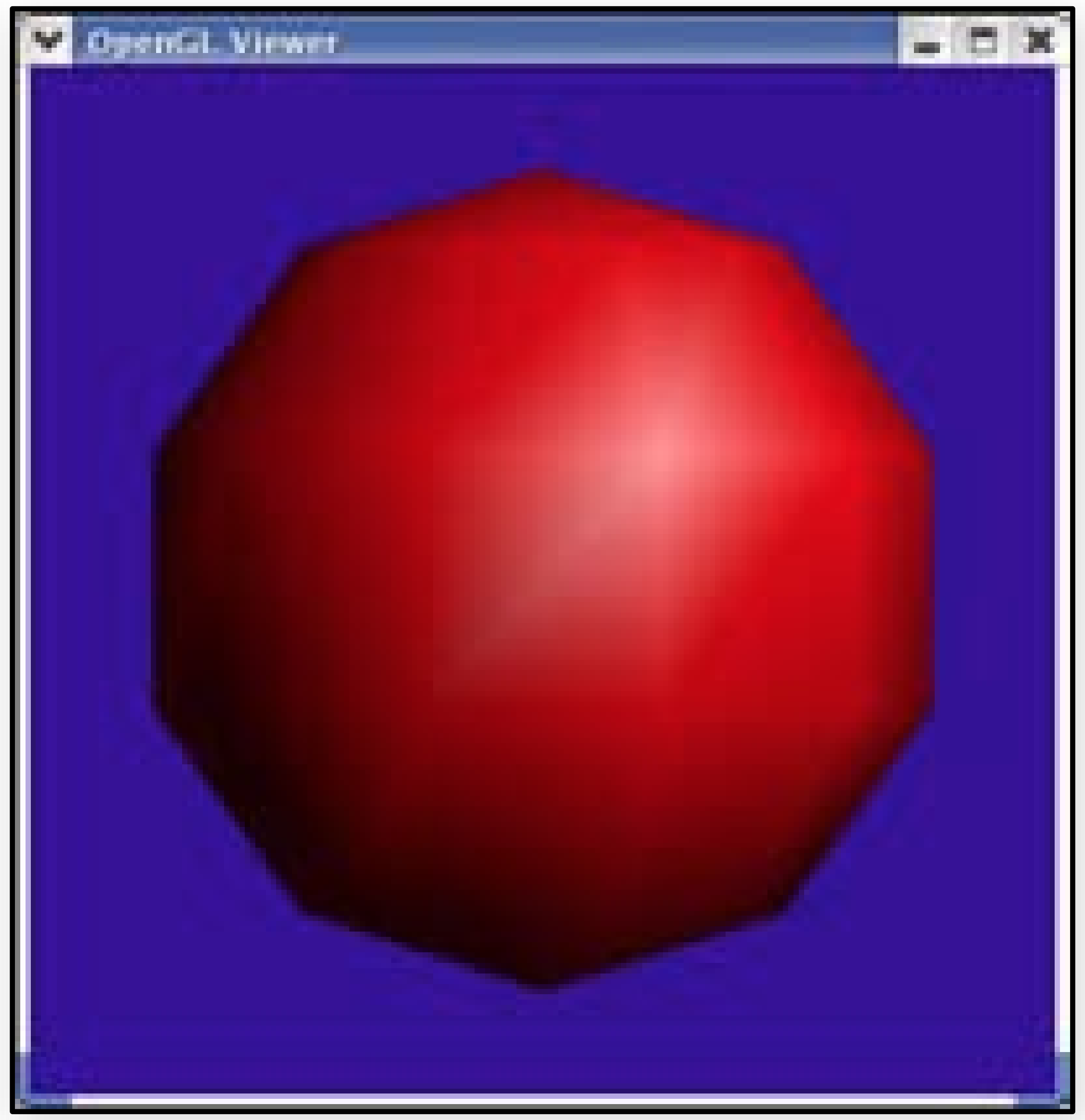
Faceting, shading artifacts

- This is largely going away with programmable per-pixel shading, though

No unified handling of shadows, reflection, transparency







After a slide by Frédo Durand

Ray/path tracing

Advantages

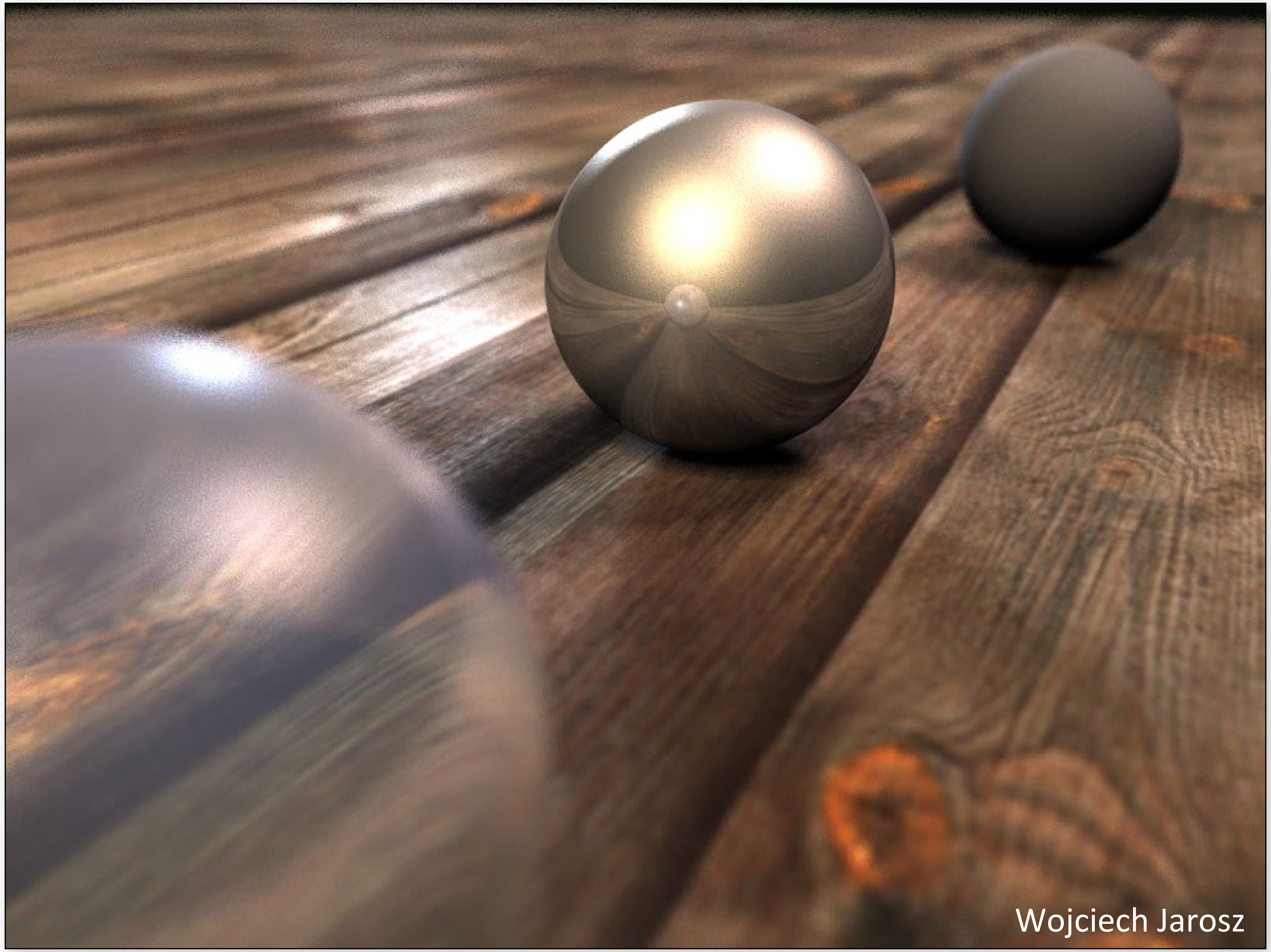
- Generality: can render anything that can be intersected with a ray
- Easily allows recursion (shadows, reflections, etc.)

Disadvantages

- Hard to implement in hardware (lacks computation coherence, must fit entire scene in memory, bad memory behavior)
  - Not such a big point anymore given general purpose GPUs
- Has traditionally been too slow for interactive applications
- Both of the above are changing rather rapidly right now!

After a slide by Frédo Durand

A ray-traced image



Wojciech Jarosz

Ray tracing today

Rapid change in film industry

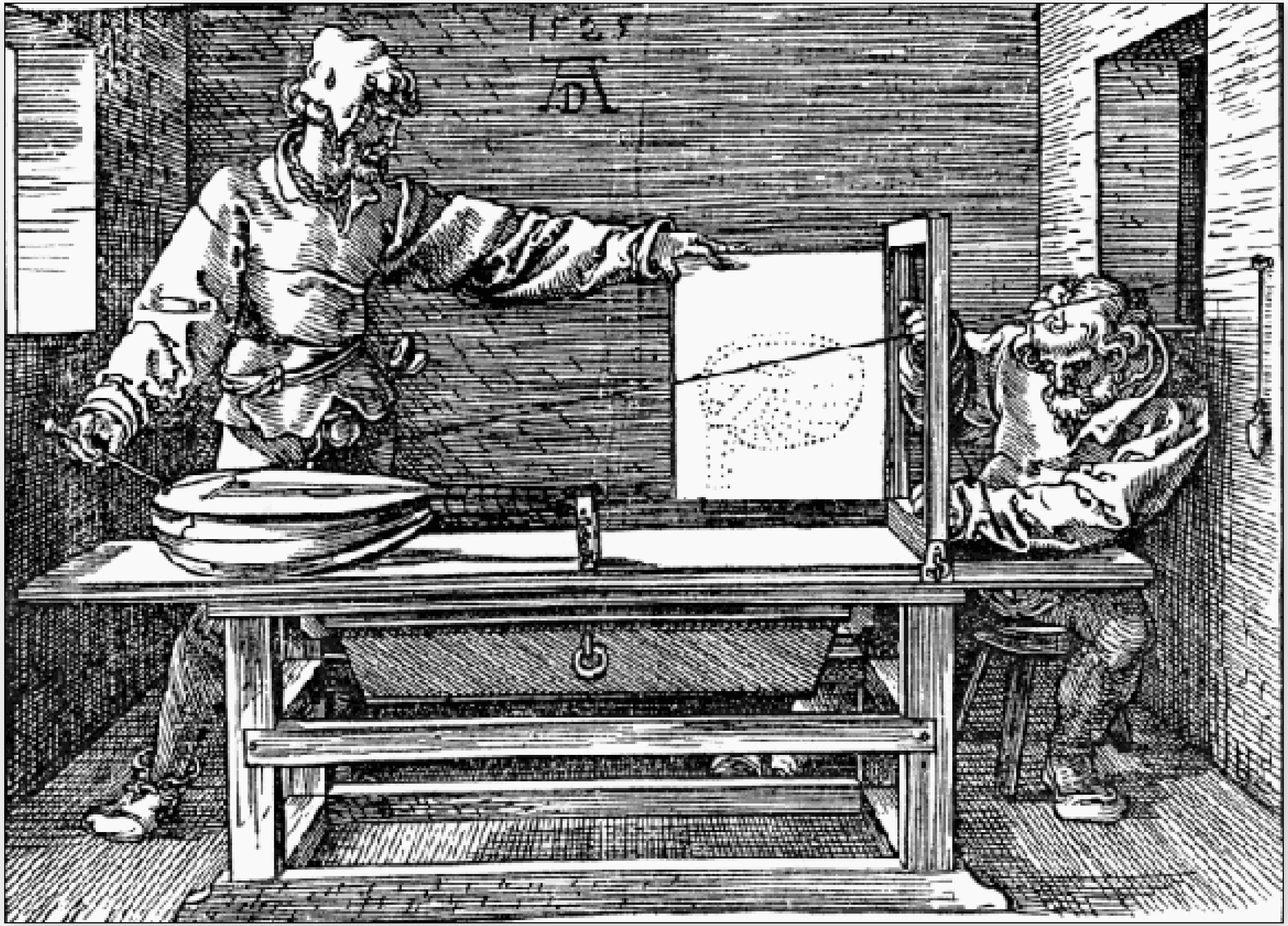
2008:

- Most CGI in films rendered using micro-polygon rasterization.
- “You’d be crazy to render a full-feature film with ray/path tracing.”
- Ray/path tracing mostly interesting to academics

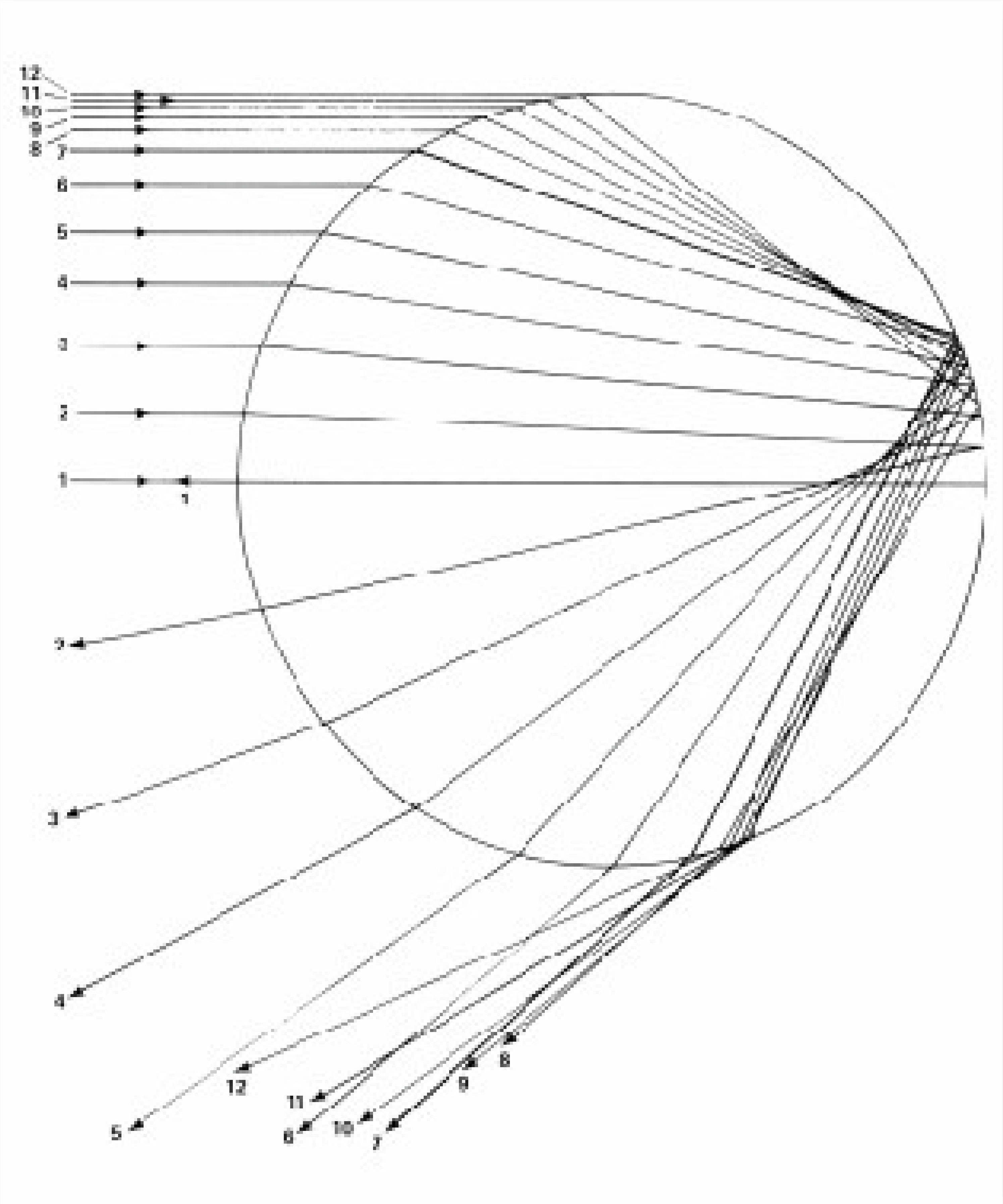
2018:

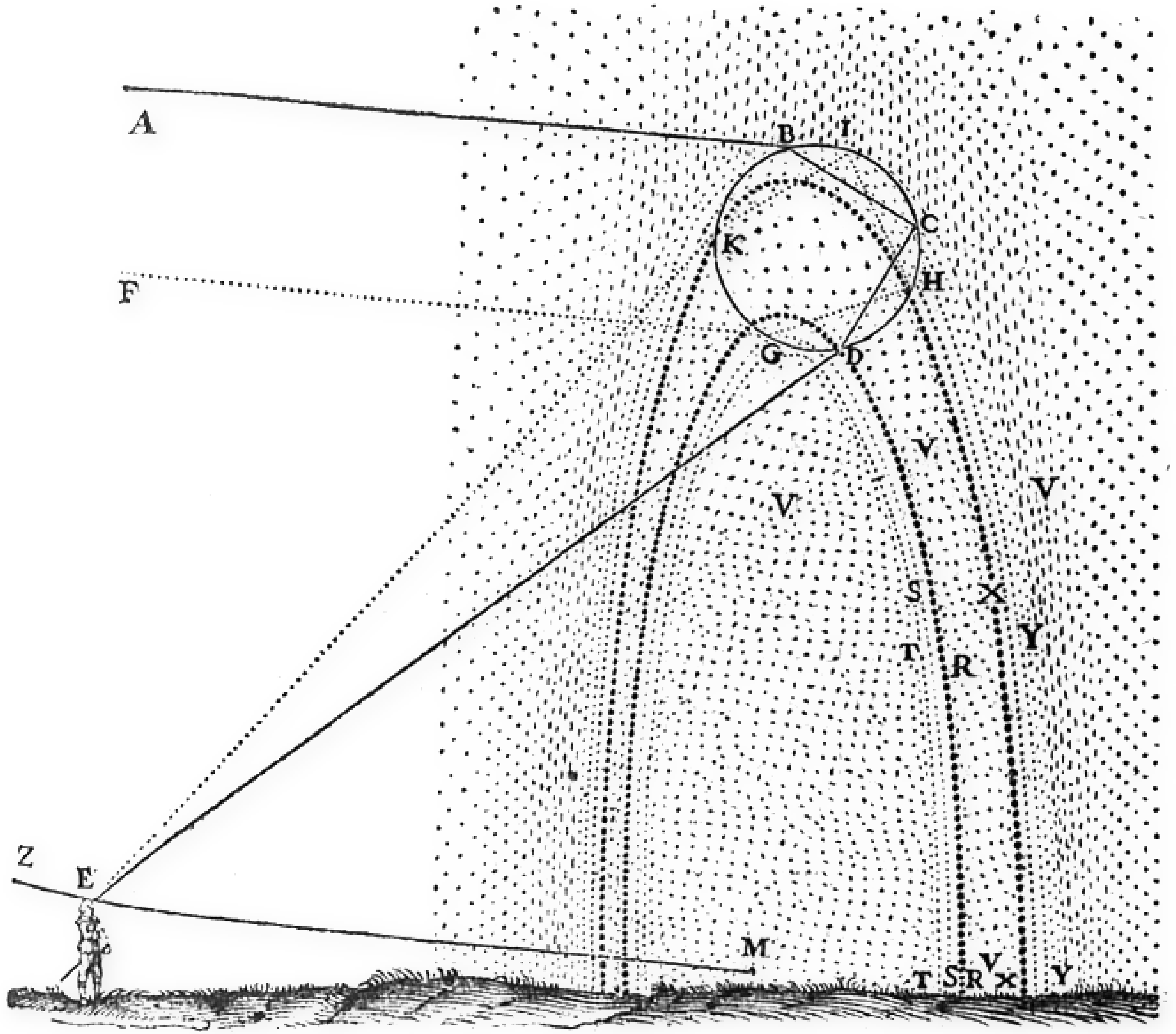
- Most major films now rendered using ray/path tracing.
- “You’d be crazy ***not*** to render a full-feature film using path tracing.”

Albrecht Dürer (1525)

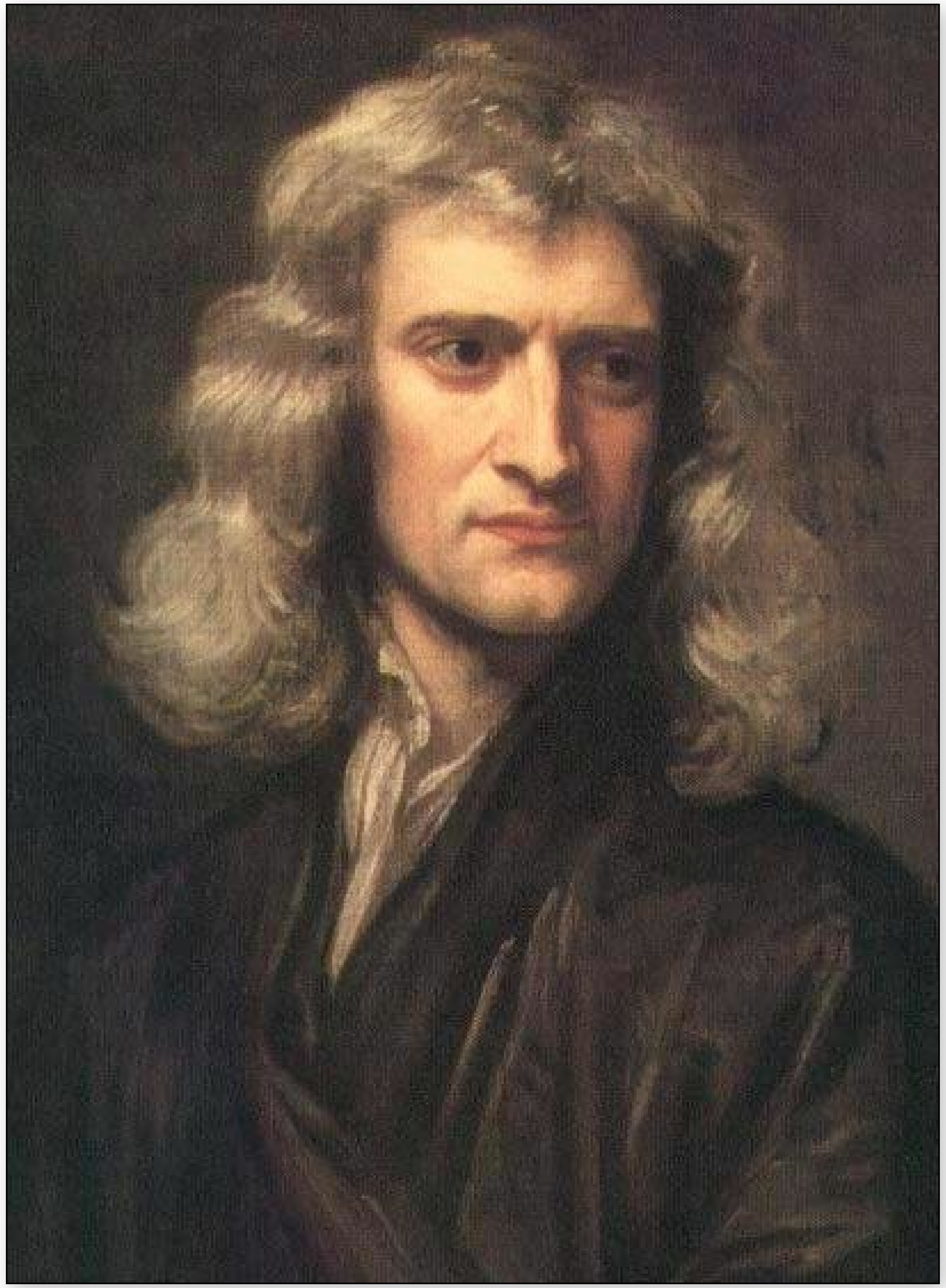


René Descartes (1650)

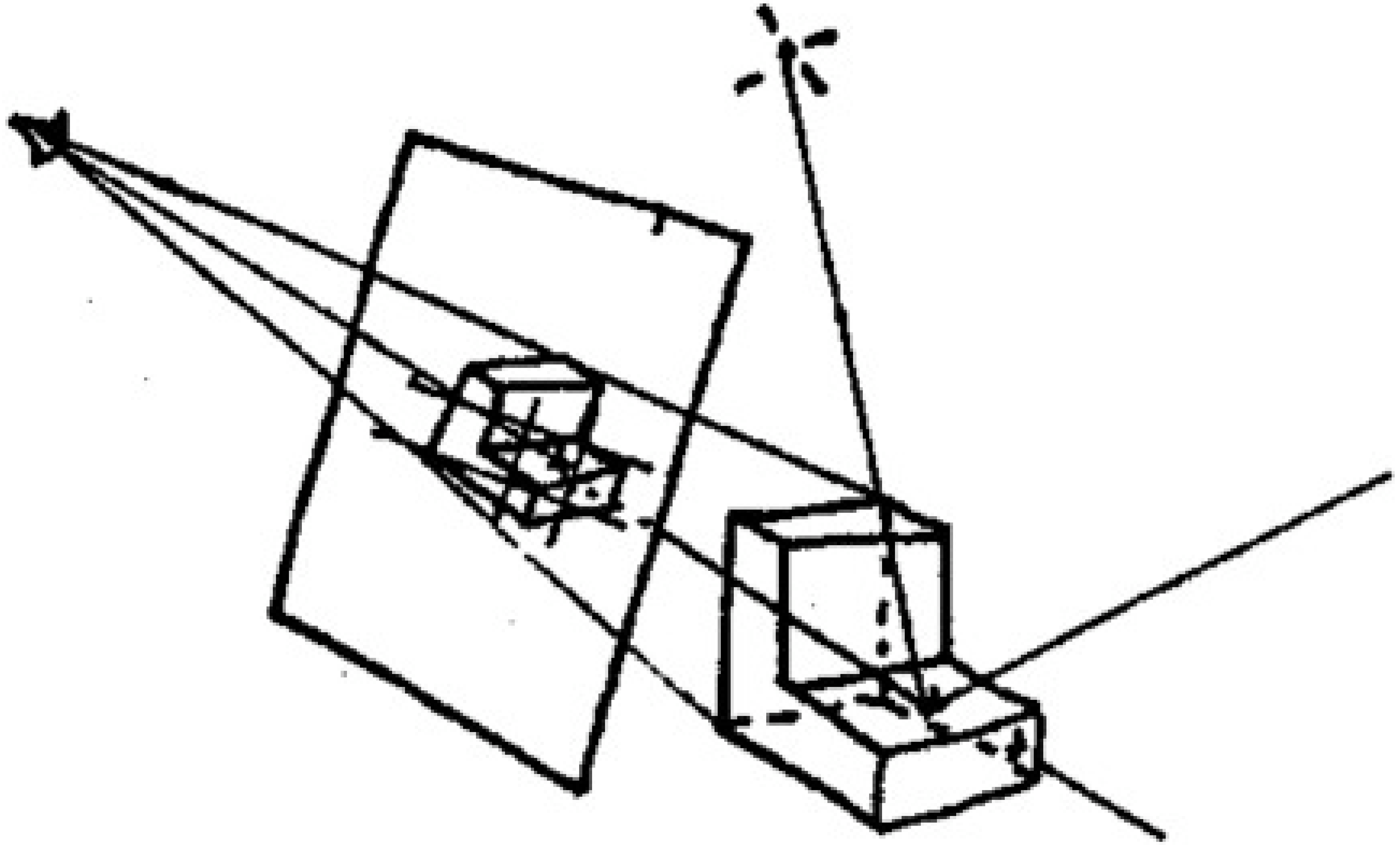




Isaac Newton (1670)



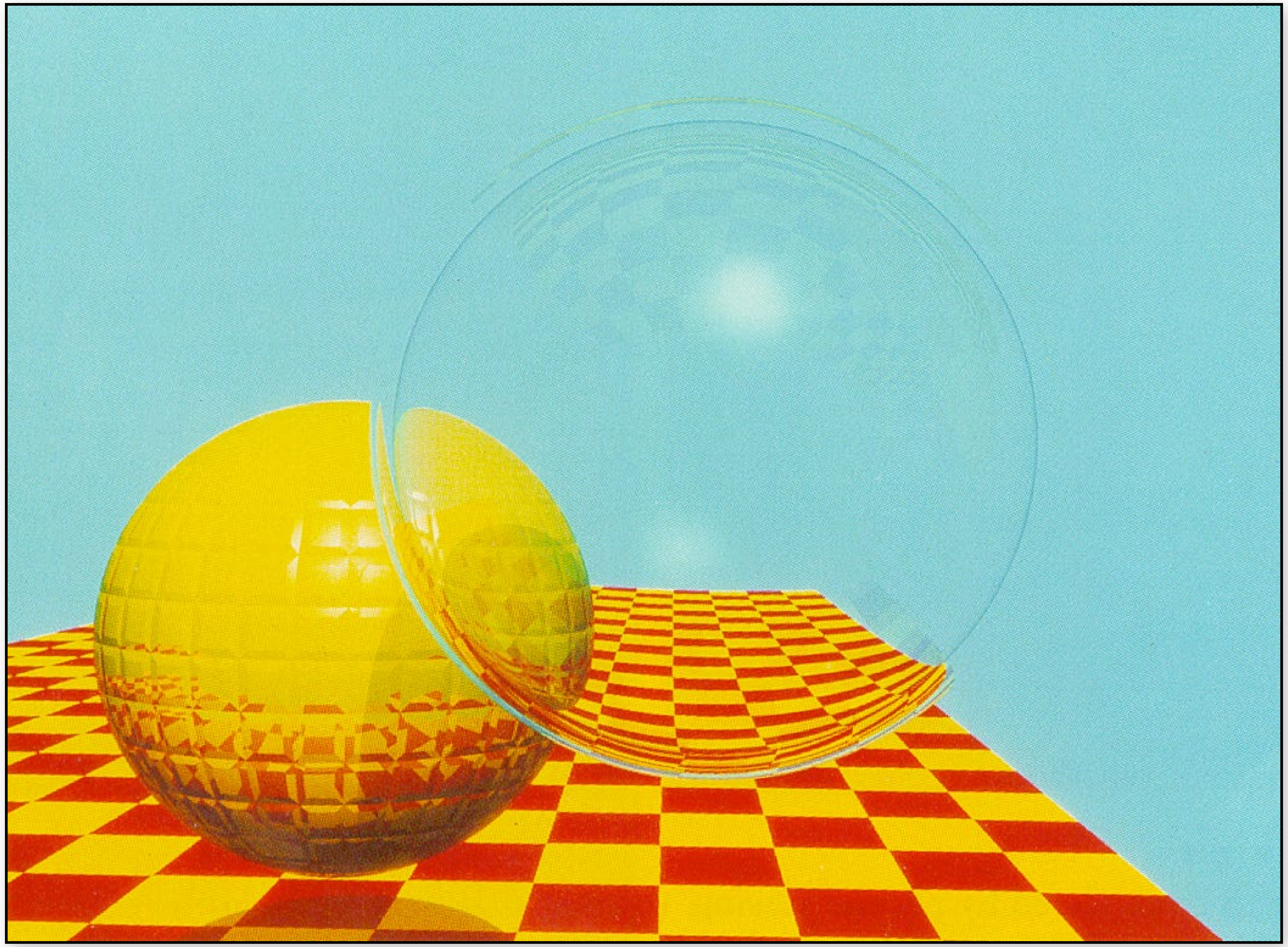
Appel (1968)



Ray casting

- Generate an image by sending one ray per pixel
- Check for shadows by sending a ray towards the light

Whitted (1979)



recursive ray tracing (reflection & refraction)

Light Transport - Assumptions

Geometric optics:

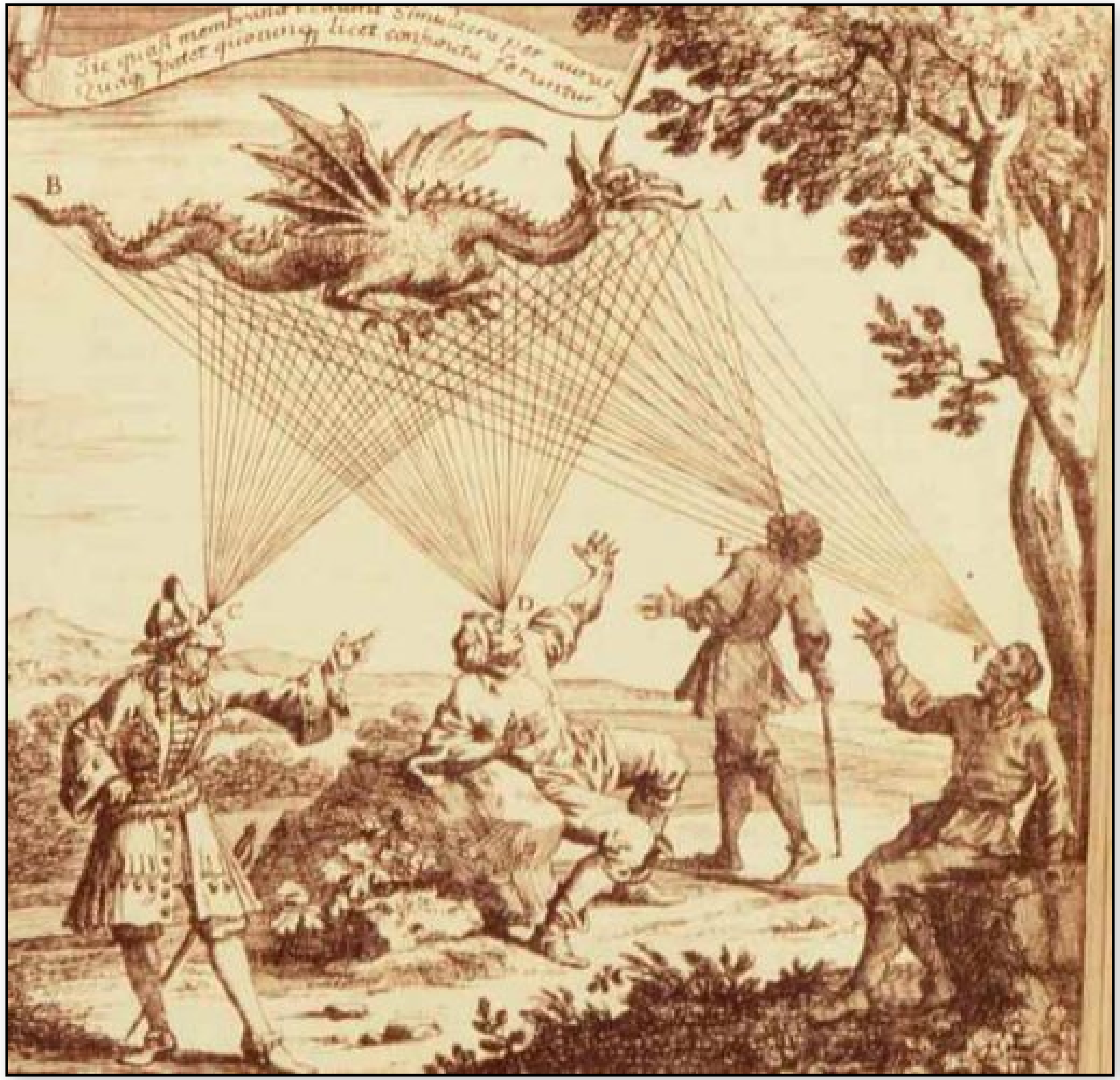
- no diffraction, no polarization, no interference

Light travels in a straight line in a vacuum

- no atmospheric scattering or refraction
- no gravity effects

Color can be represented as three numbers: (R,G,B)

Emission theory of vision



Eyes send out “feeling rays” into the world

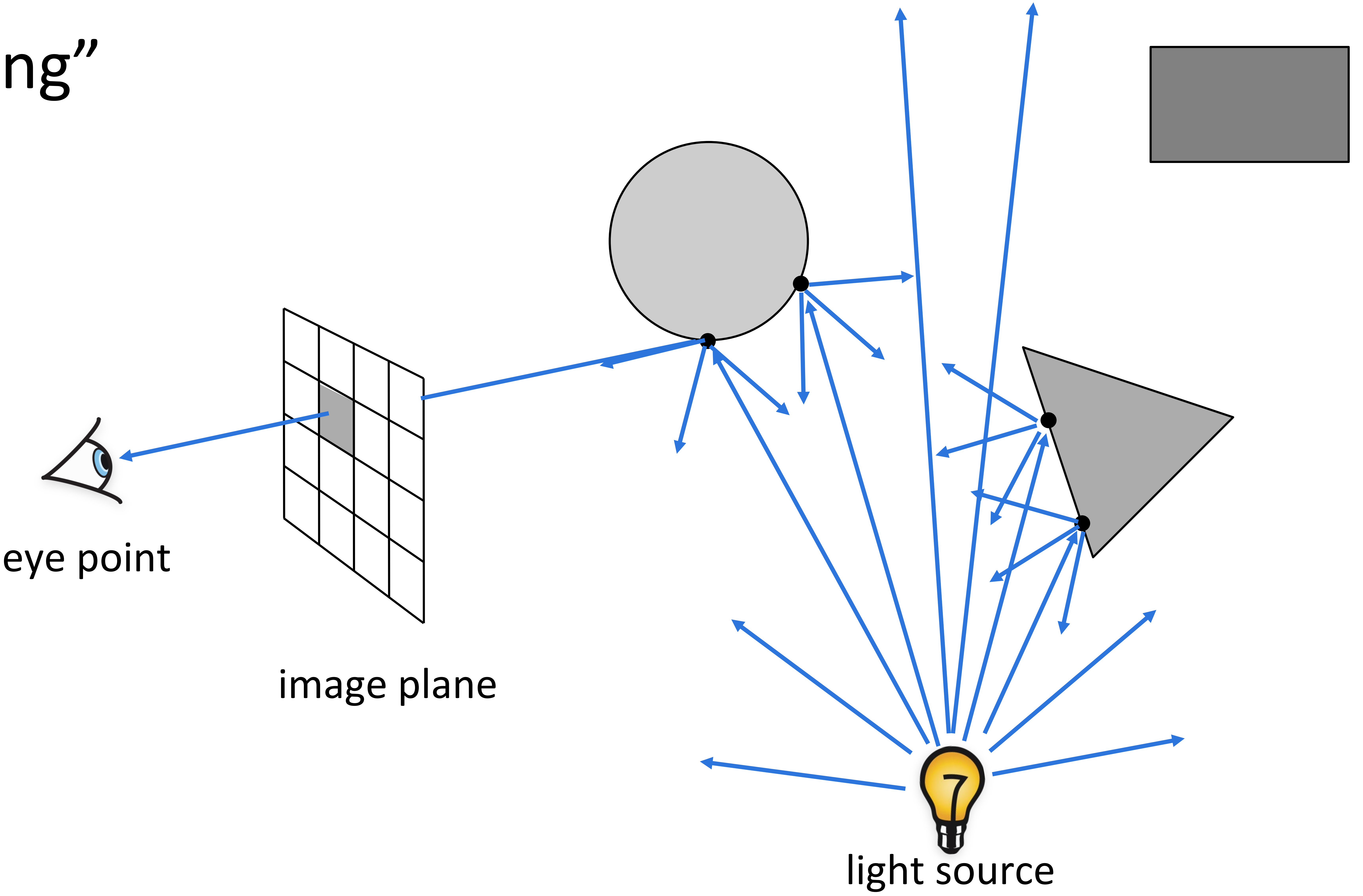
Supported by:

- Ancient greeks
- 50% of US college students*

*http://www.ncbi.nlm.nih.gov/pubmed/12094435?dopt=Abstract

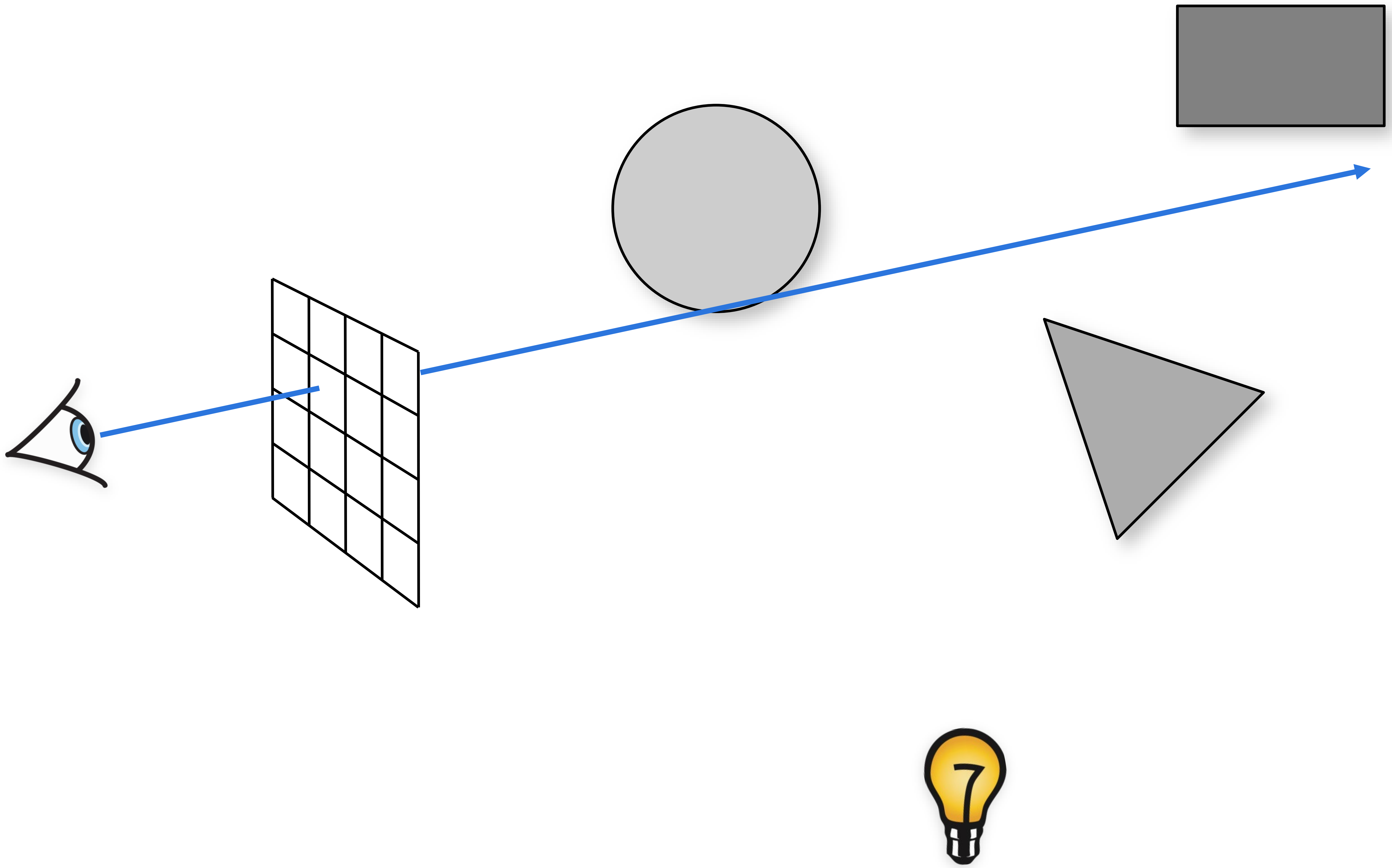
Ray Tracing - Overview

"light tracing"



Basic Ray Tracing Pipeline

Ray Generation

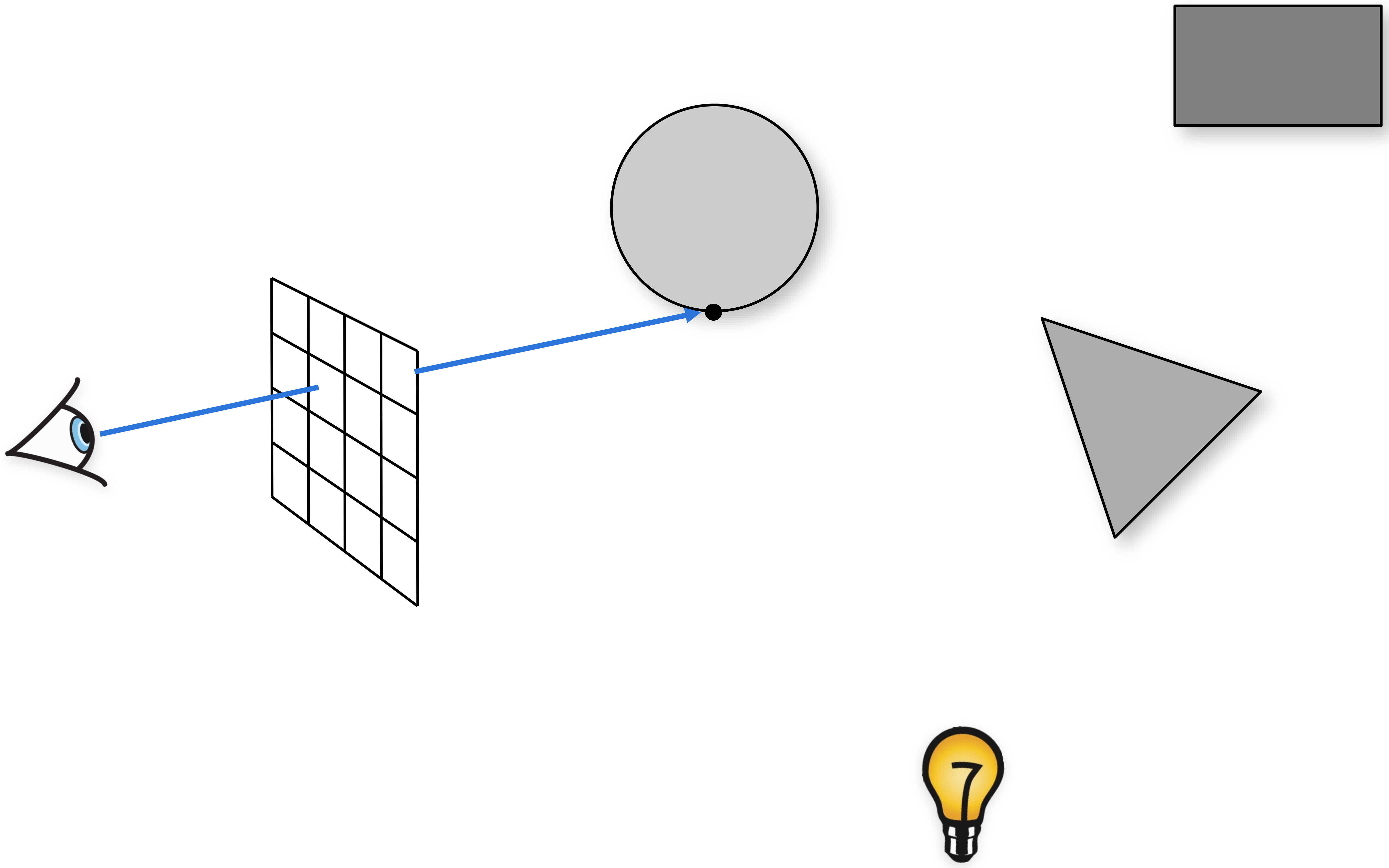


Basic Ray Tracing Pipeline

Ray Generation

↓

Intersection



Basic Ray Tracing Pipeline

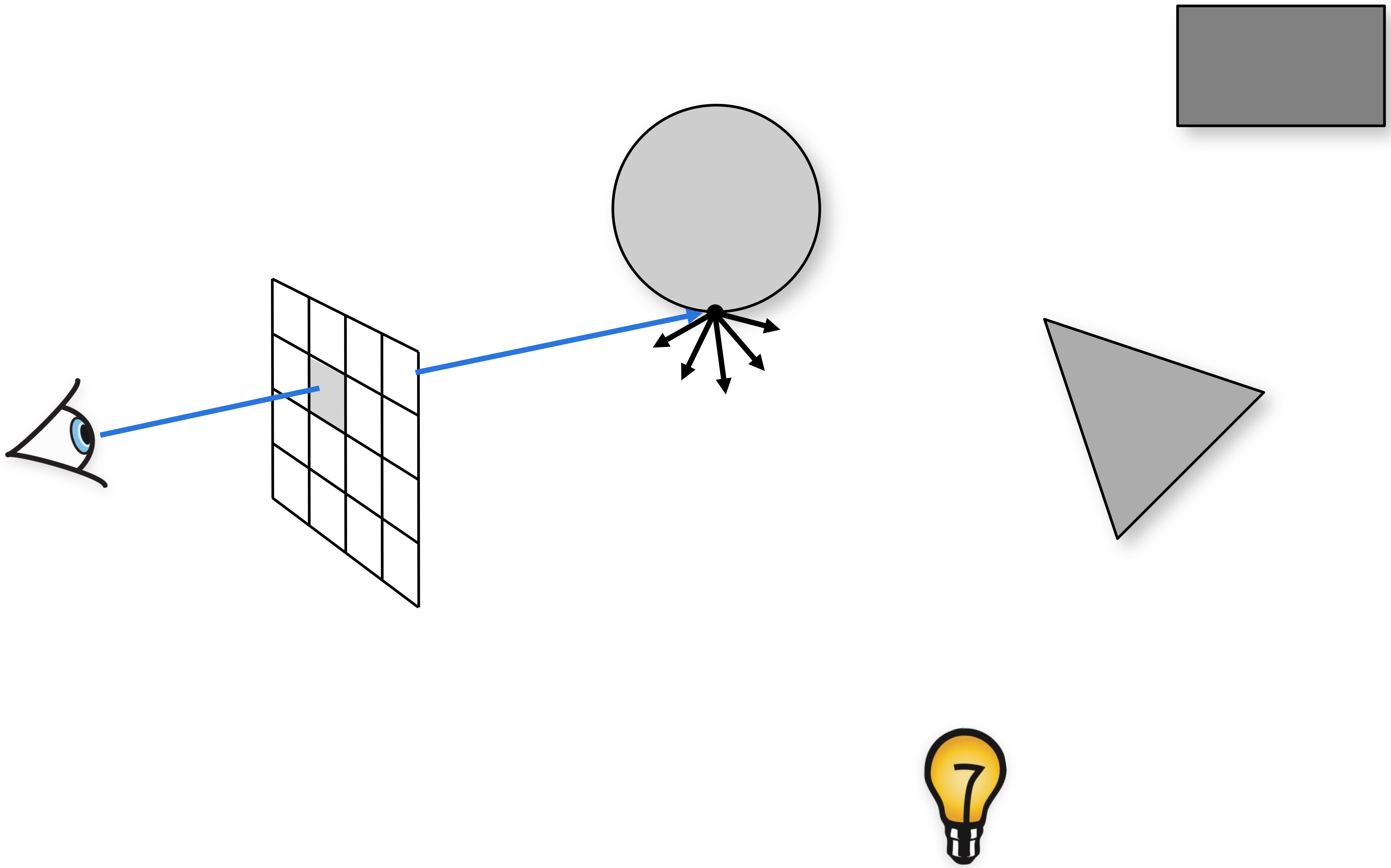
Ray Generation

↓

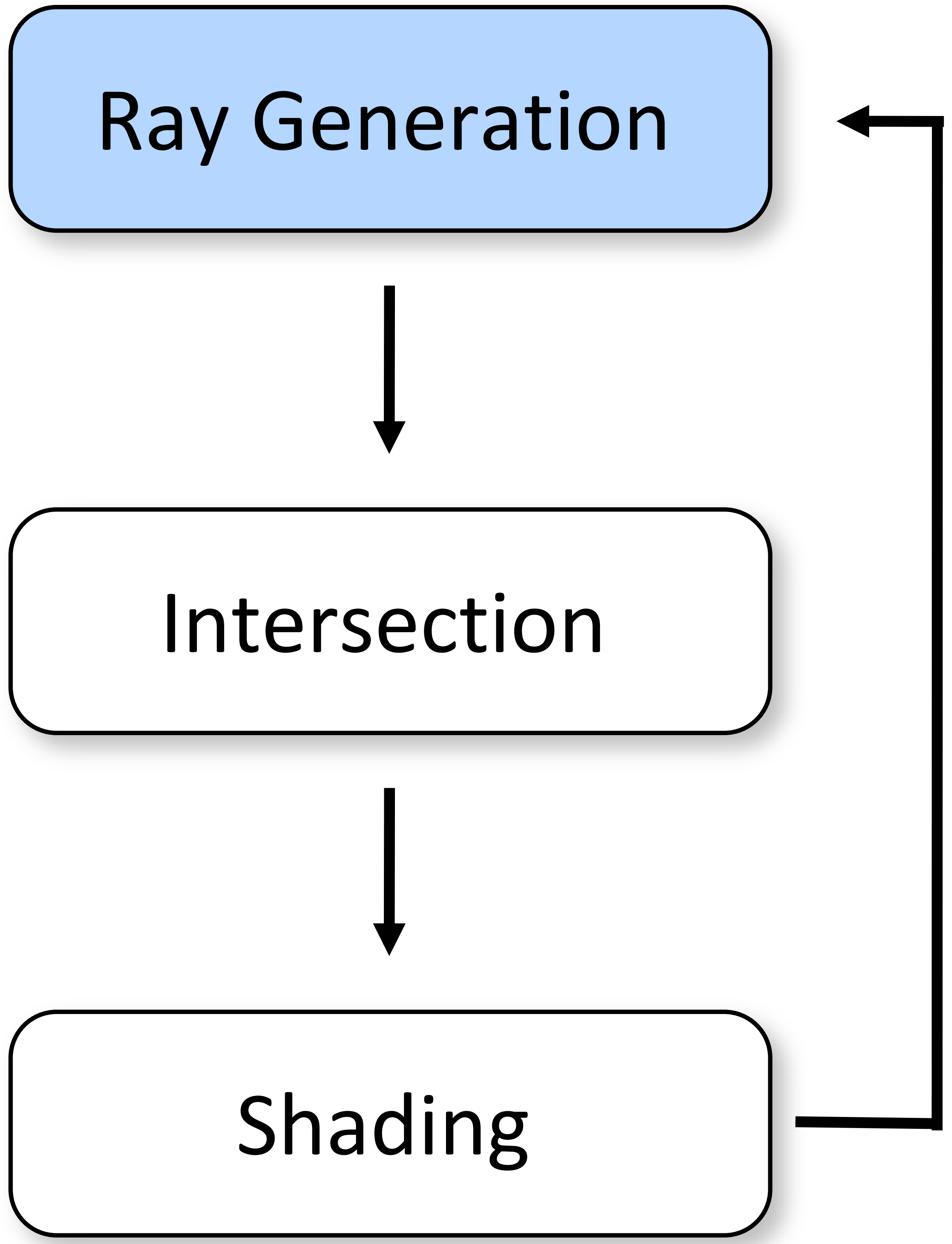
Intersection

↓

Shading



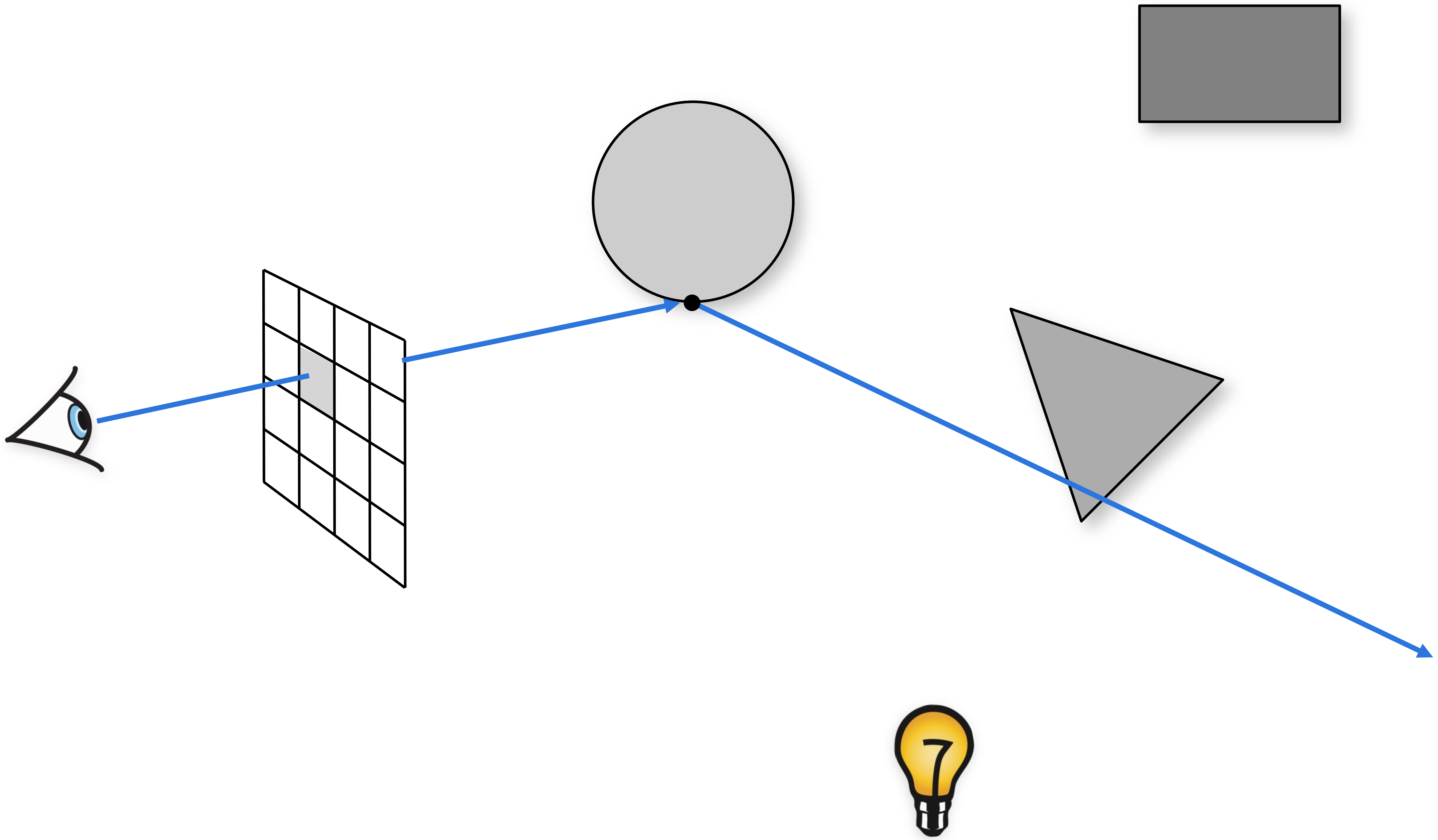
Basic Ray Tracing Pipeline



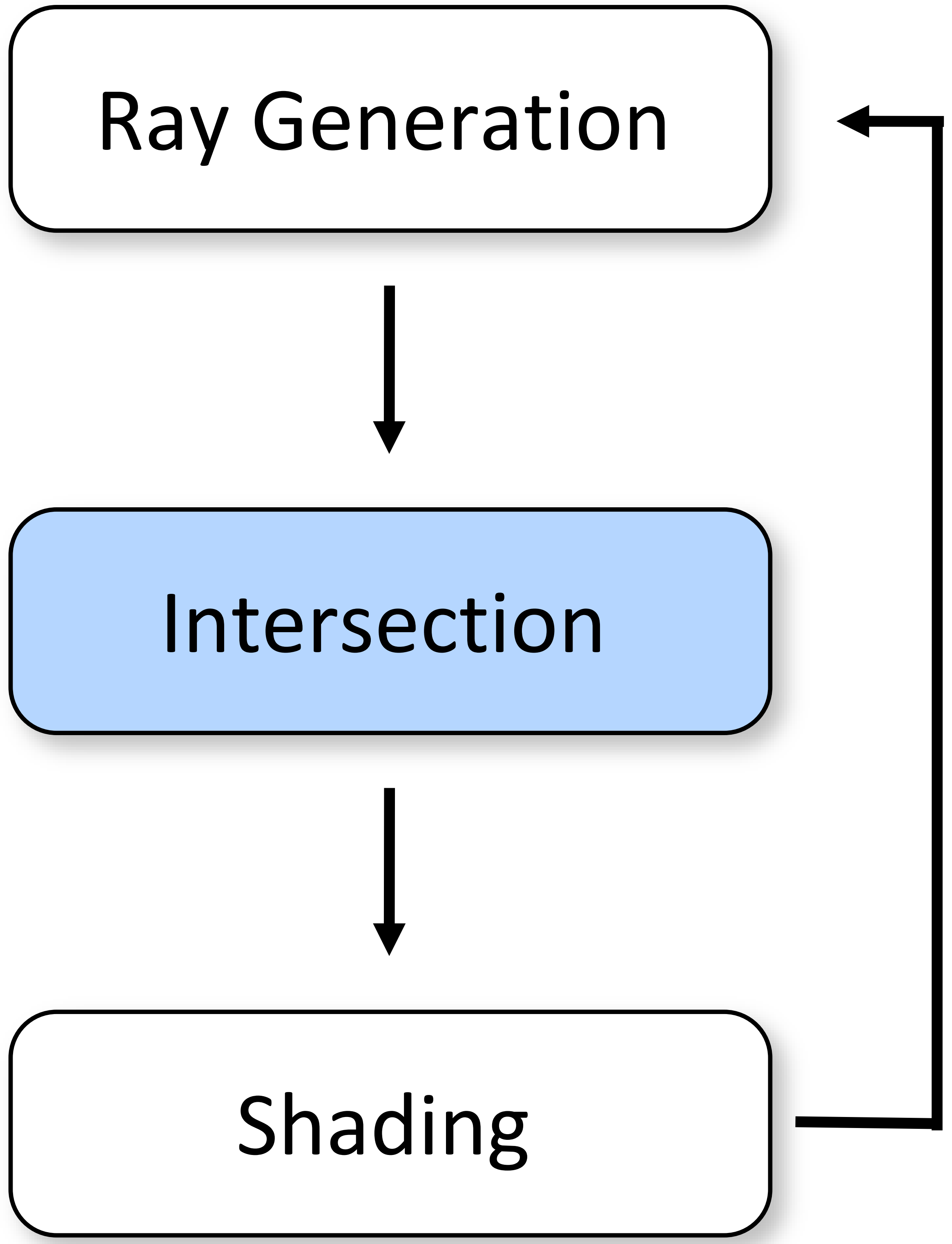
Ray Generation

Intersection

Shading



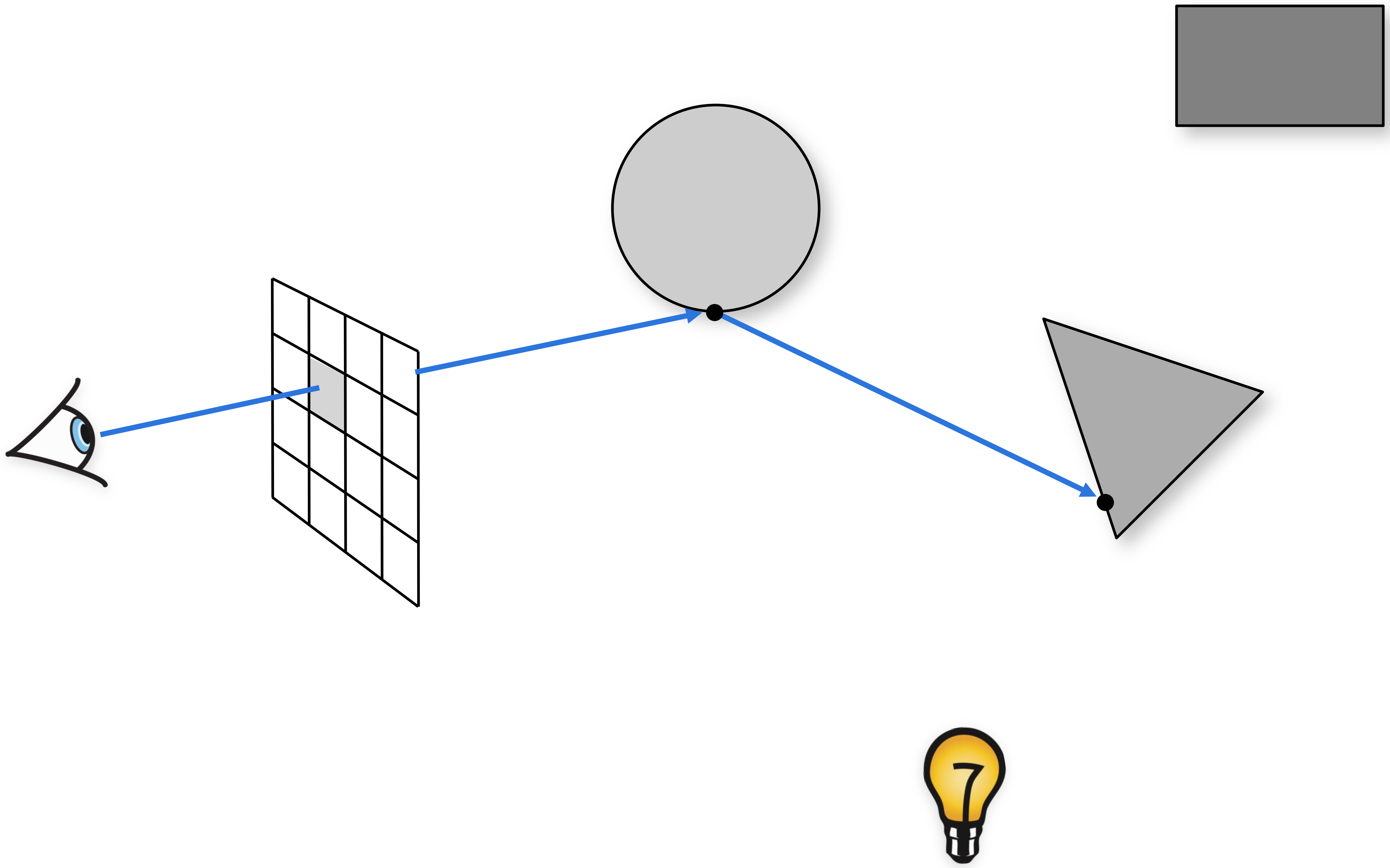
Basic Ray Tracing Pipeline



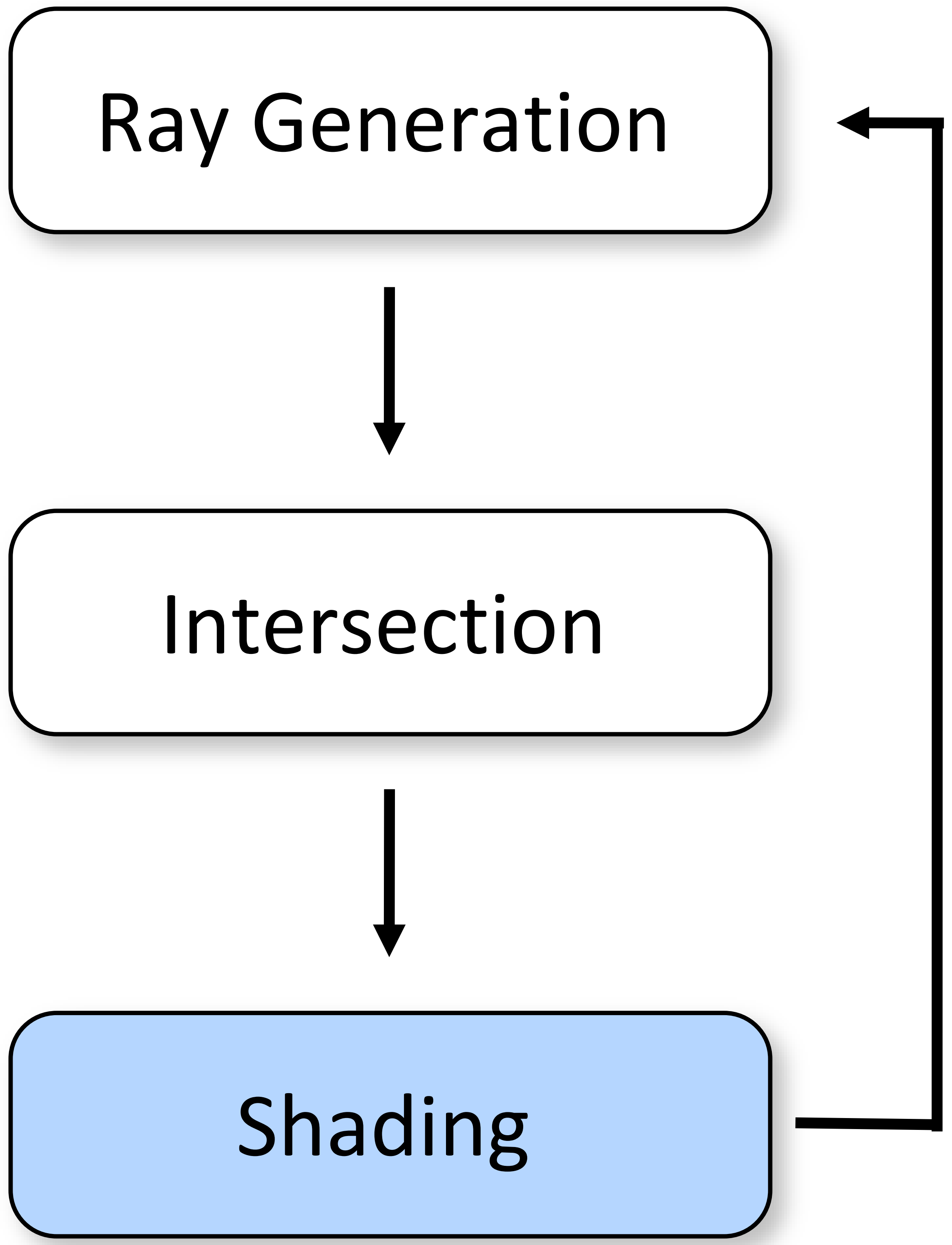
Ray Generation

Intersection

Shading



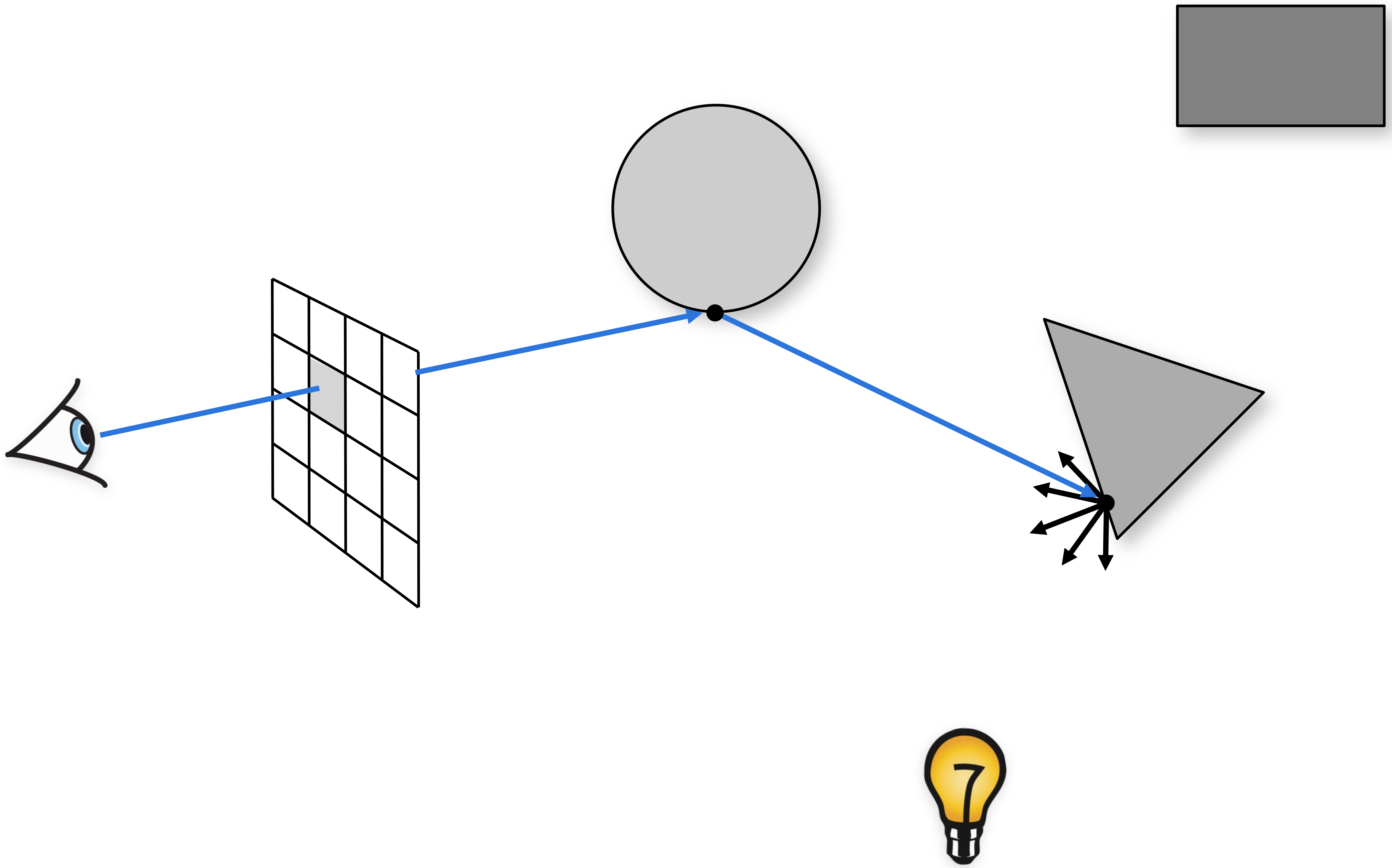
Basic Ray Tracing Pipeline



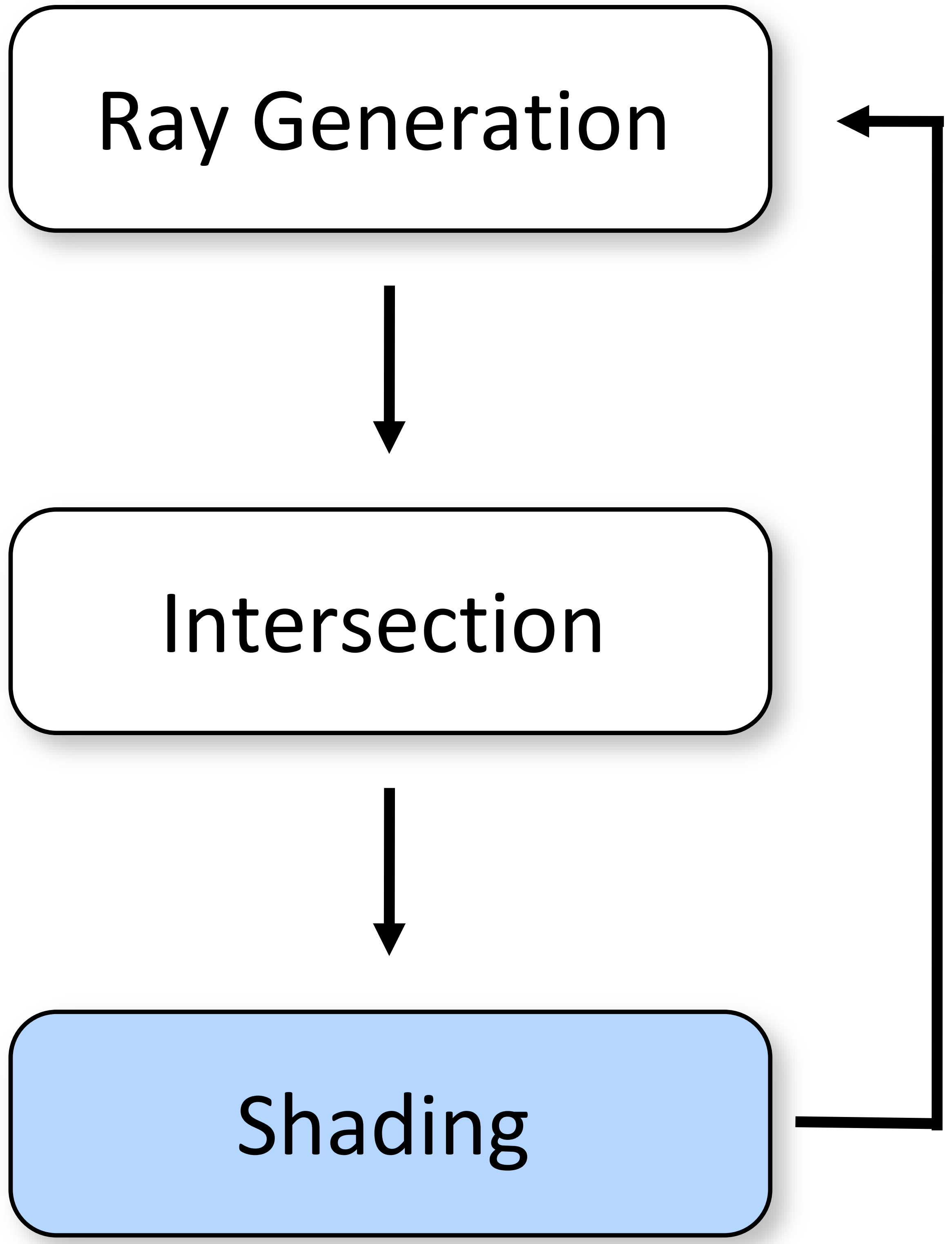
Ray Generation

Intersection

Shading



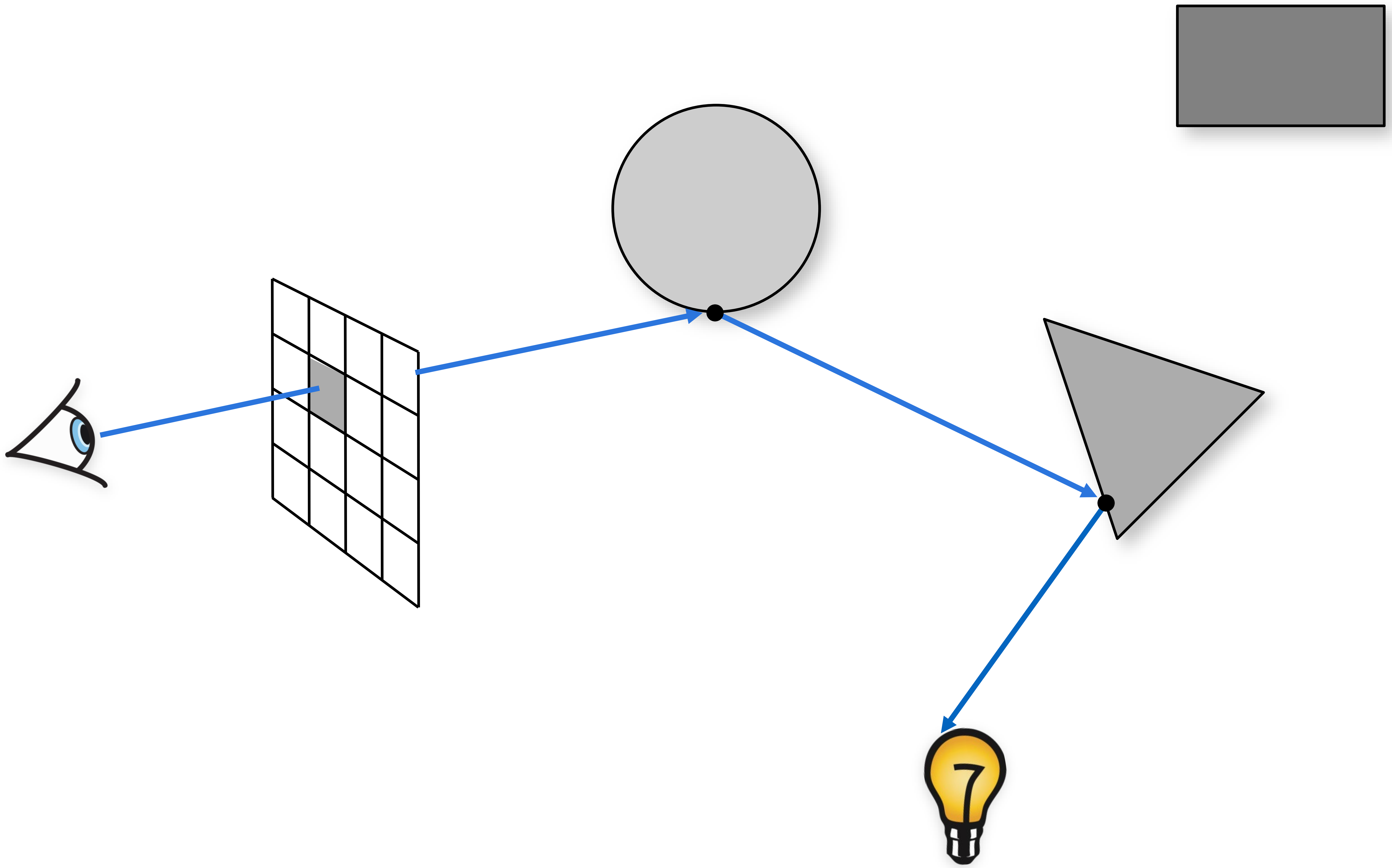
Basic Ray Tracing Pipeline



Ray Generation

Intersection

Shading



Ray Tracing Pseudocode

```
rayTraceImage()
{
   parse scene description

   for each pixel
      ray = generateCameraRay(pixel)
      pixelColor = trace(ray)
}
```

Ray Tracing Pseudocode

```
trace(ray)
{
   hit = find first intersection with scene objects

   color = shade(hit)
   return color
}
```

might **trace** more rays (recursive)

Ray Tracing Pseudocode

```
rayTraceImage()
{
   parse scene description

   for each pixel
      ray = generateCameraRay(pixel)
      pixelColor = trace(ray)
}
```

what is a ray?

how do we generate a camera ray?

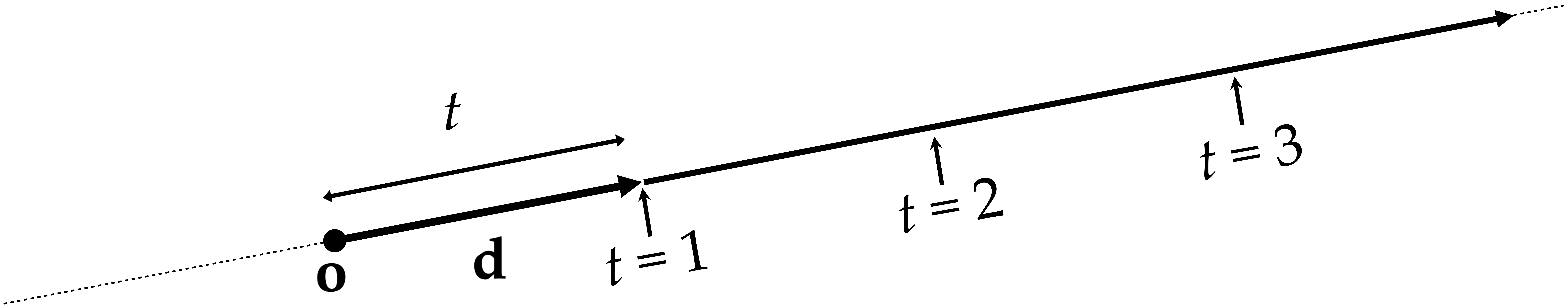
Ray: a half line

Standard representation: origin (point) $\mathbf{o}$ and direction $\mathbf{d}$

$$\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$$

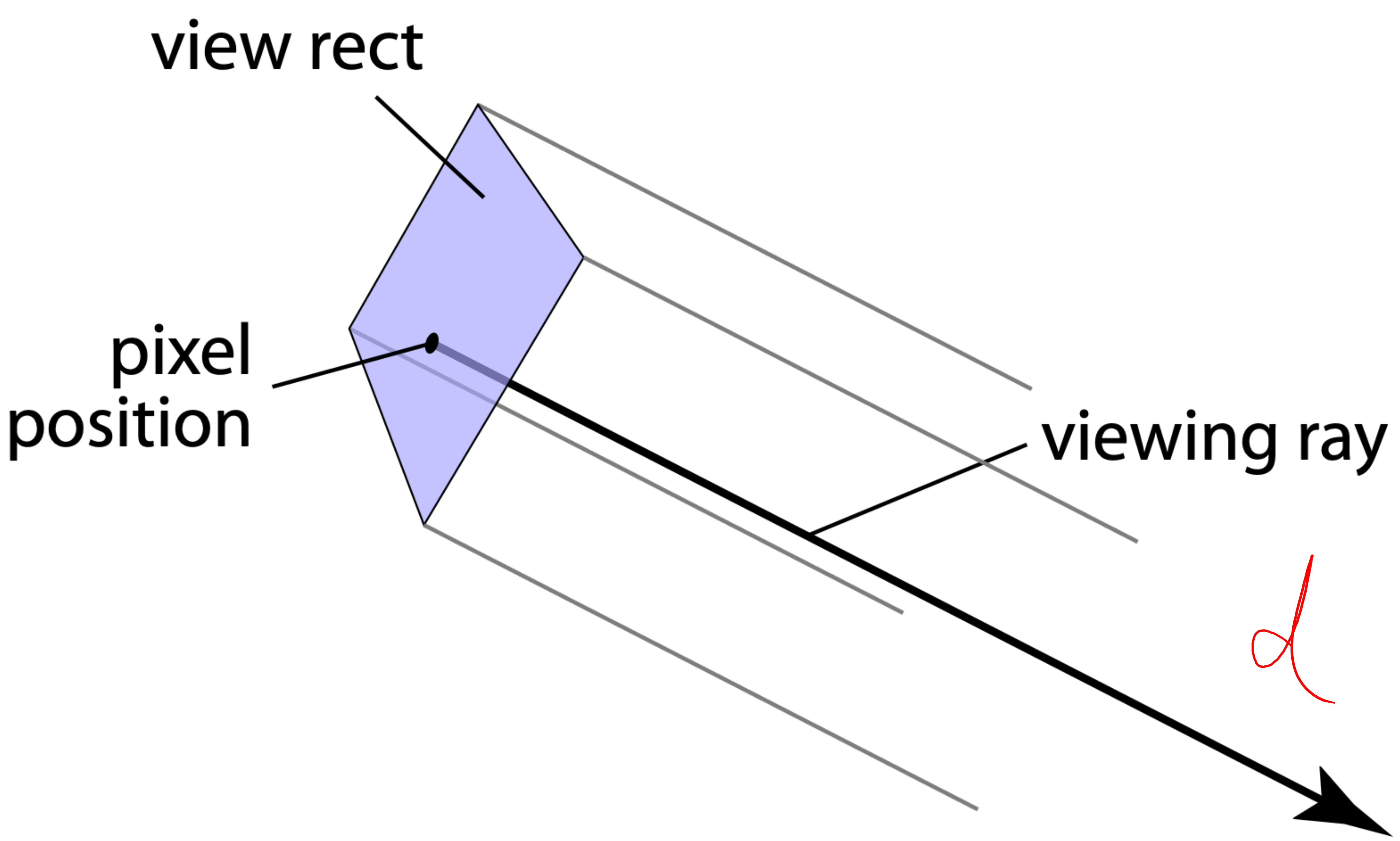
- this is a parametric equation for the line
- lets us directly generate the points on the line
- if we restrict to $t > 0$ then we have a ray
- note replacing $\mathbf{d}$ with $a\mathbf{d}$ doesn't change ray (for $a > 0$)

After a slide by Steve Marschner

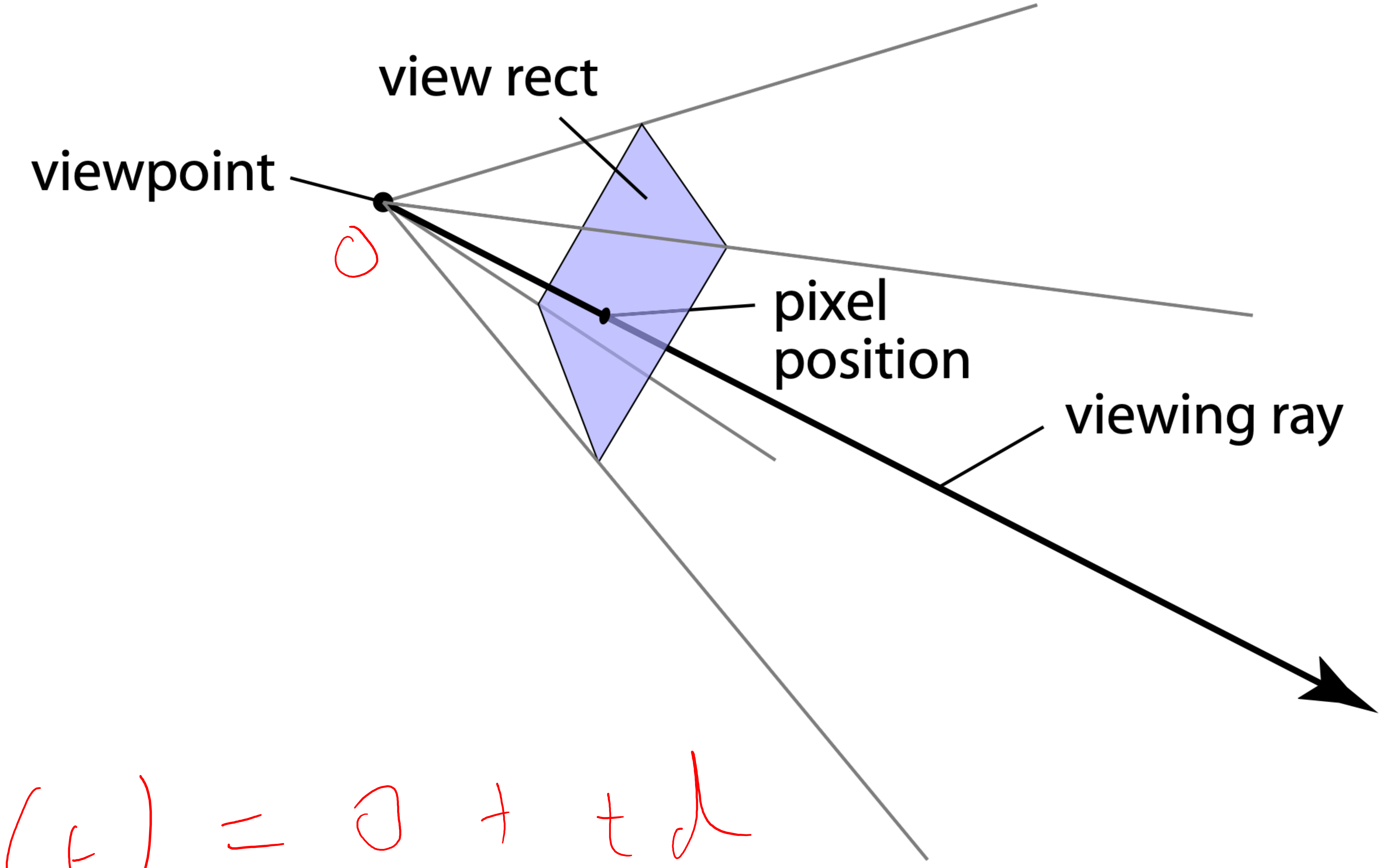


Generating eye rays

Orthographic



Perspective



$$r(t) = \underline{o} + t\underline{d}$$

After a slide by Steve Marschner

Pinhole Camera (Camera Obscura)

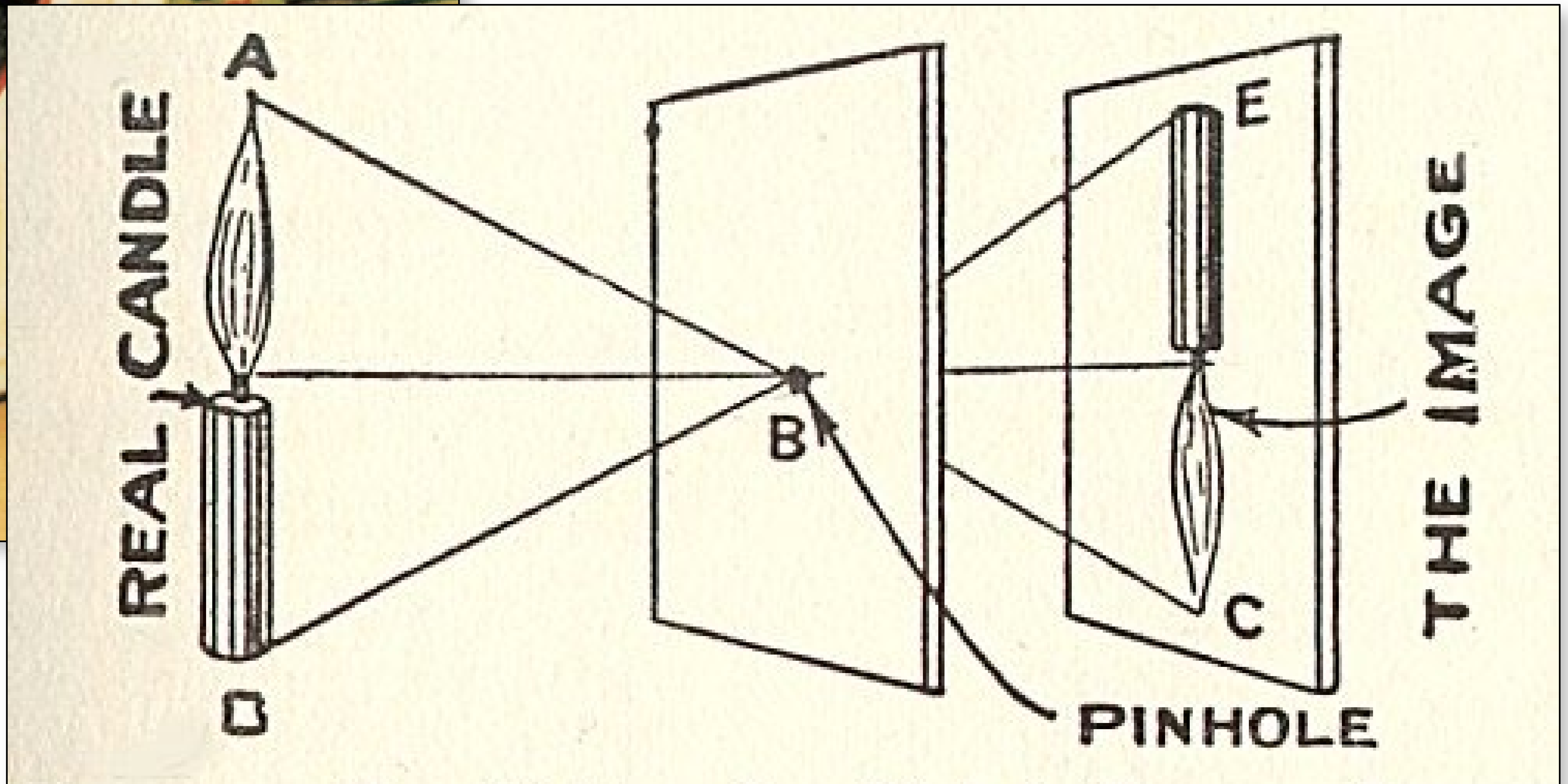
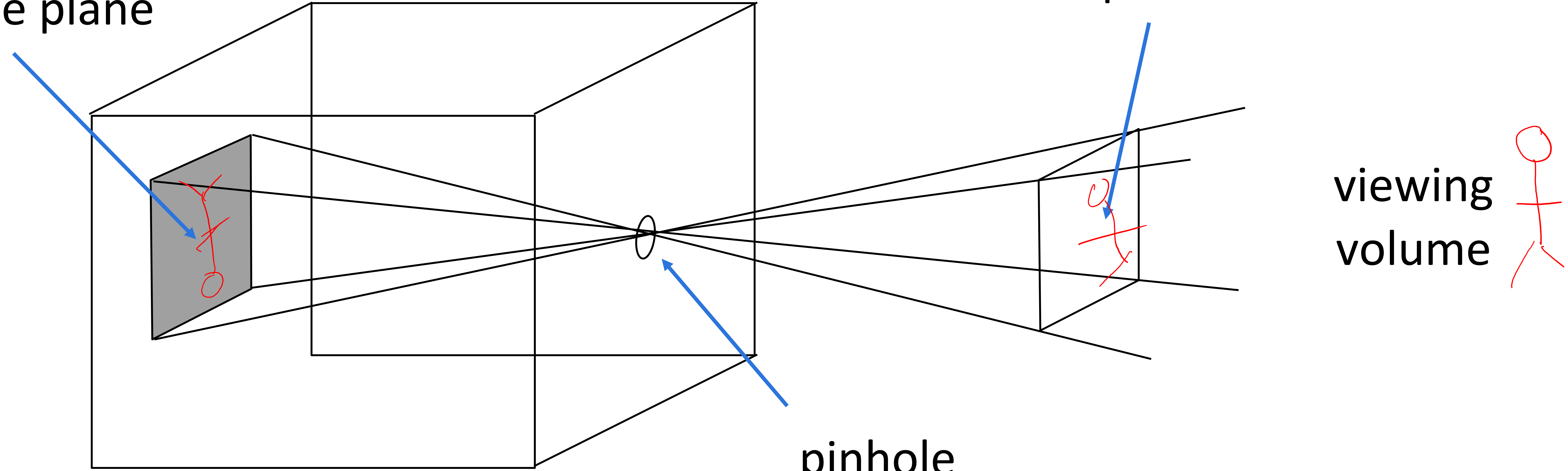


FIG. 131.—How Light and a Pinhole Form an Image.

Pinhole Camera

Pinhole Camera



film / physical image plane

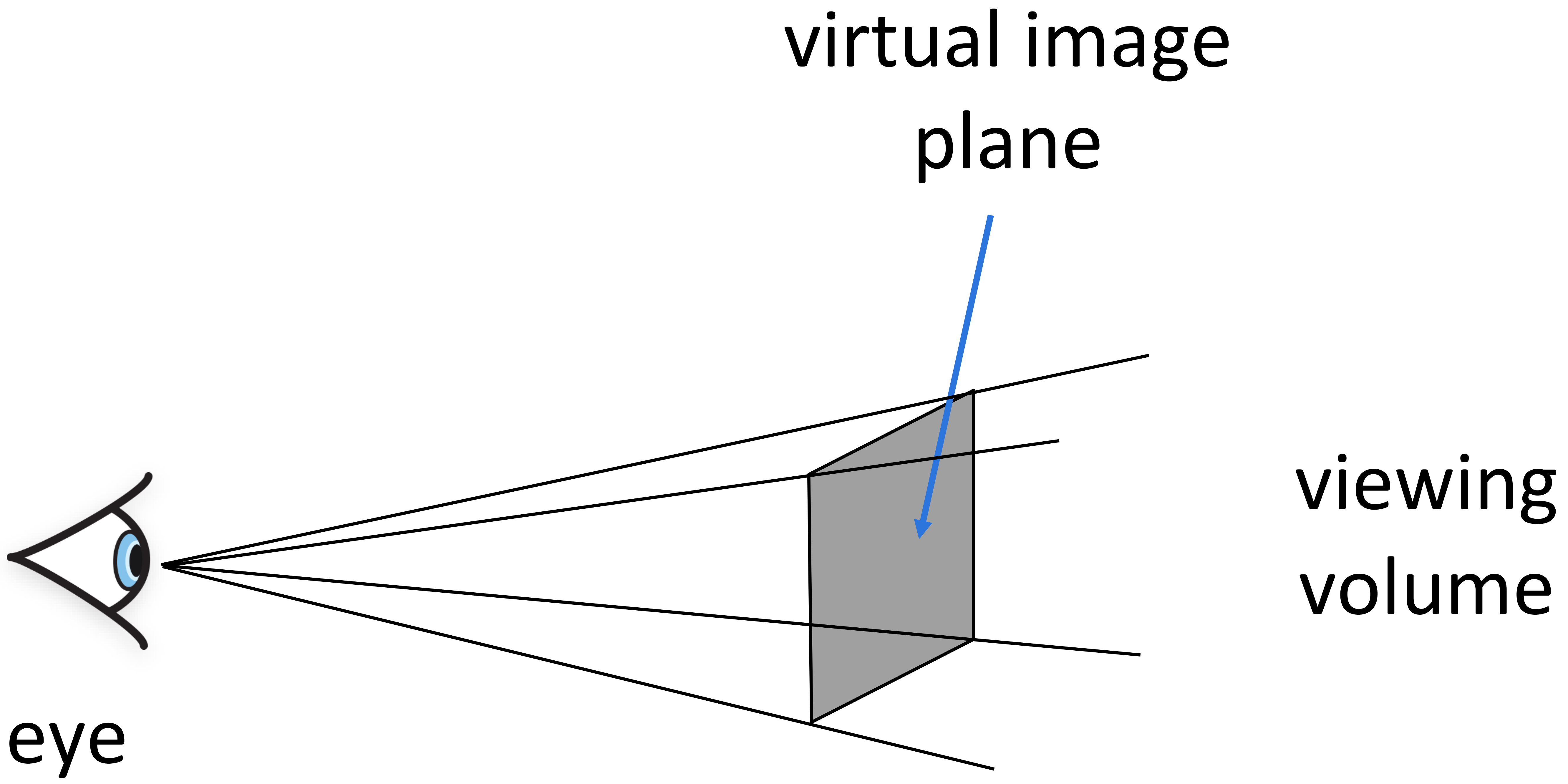
virtual image plane

viewing volume

pinhole

Pinhole Camera

Pinhole Camera



Generating eye rays—perspective

Establish view rectangle in X–Y plane, specified by, e.g.

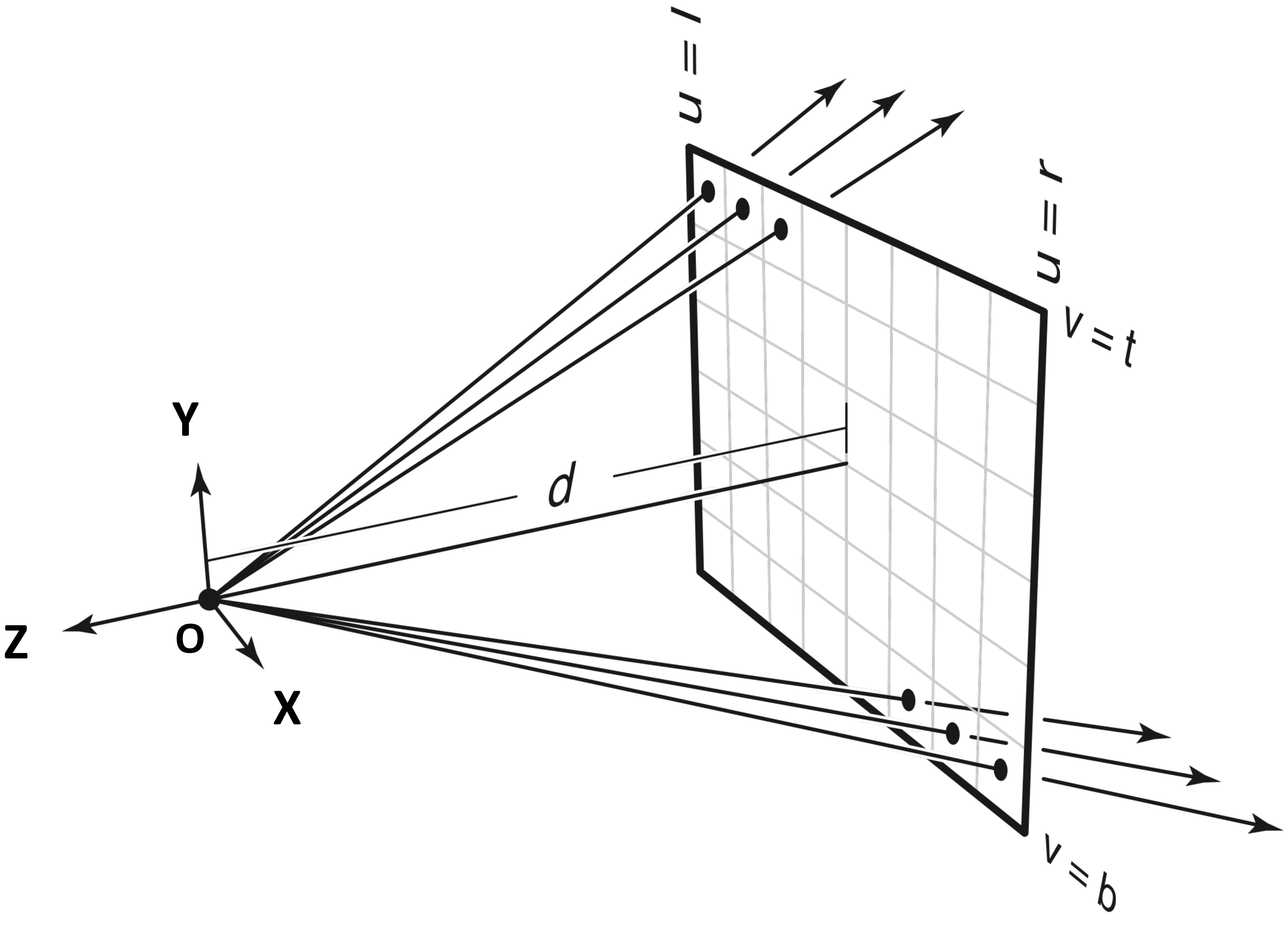
- l, r, t, b

Place rectangle at $z = -d$

$$\mathbf{s} = [u, v, -d]^T$$

$$\mathbf{d} = \mathbf{s}$$

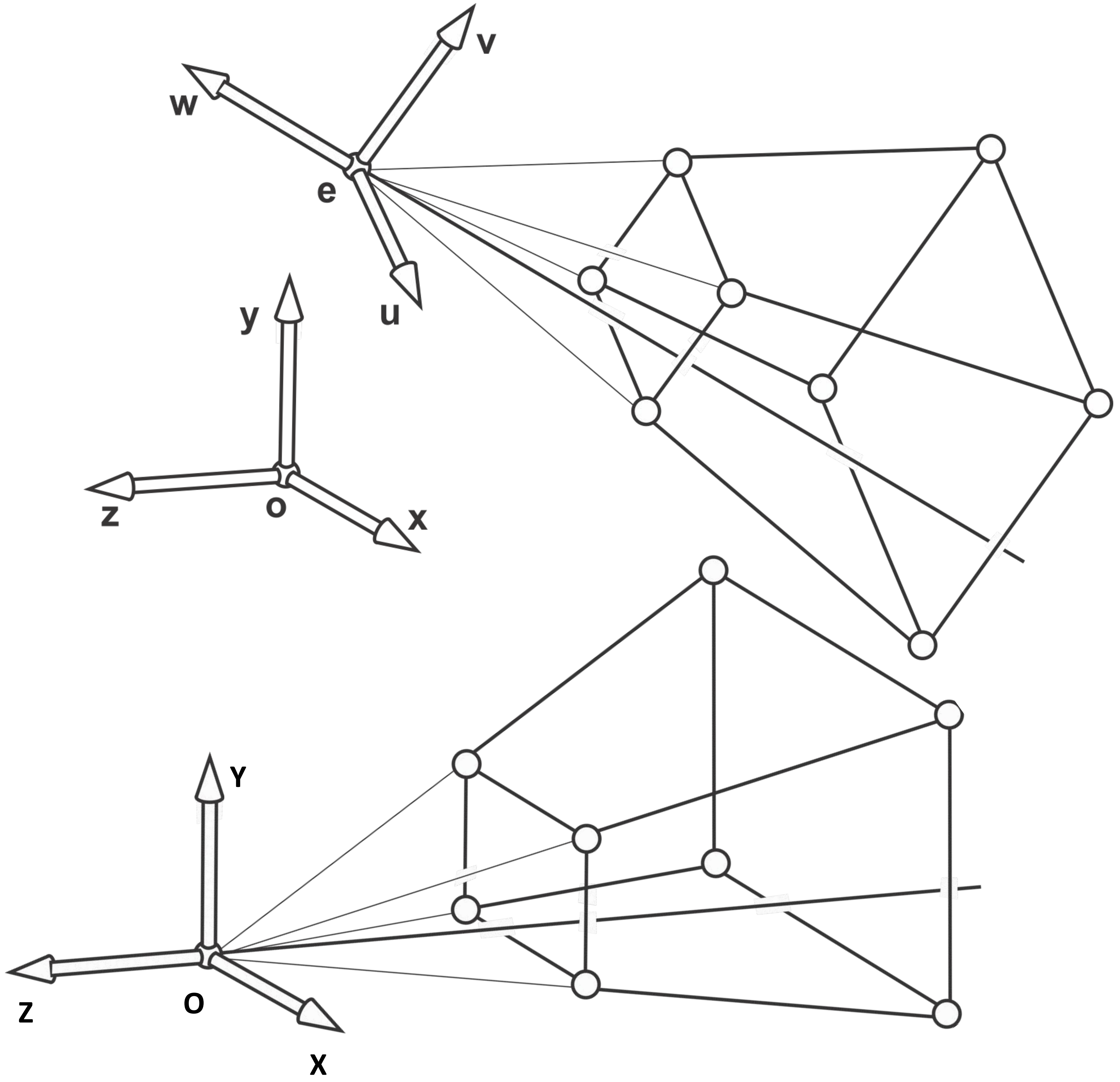
$$\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$$

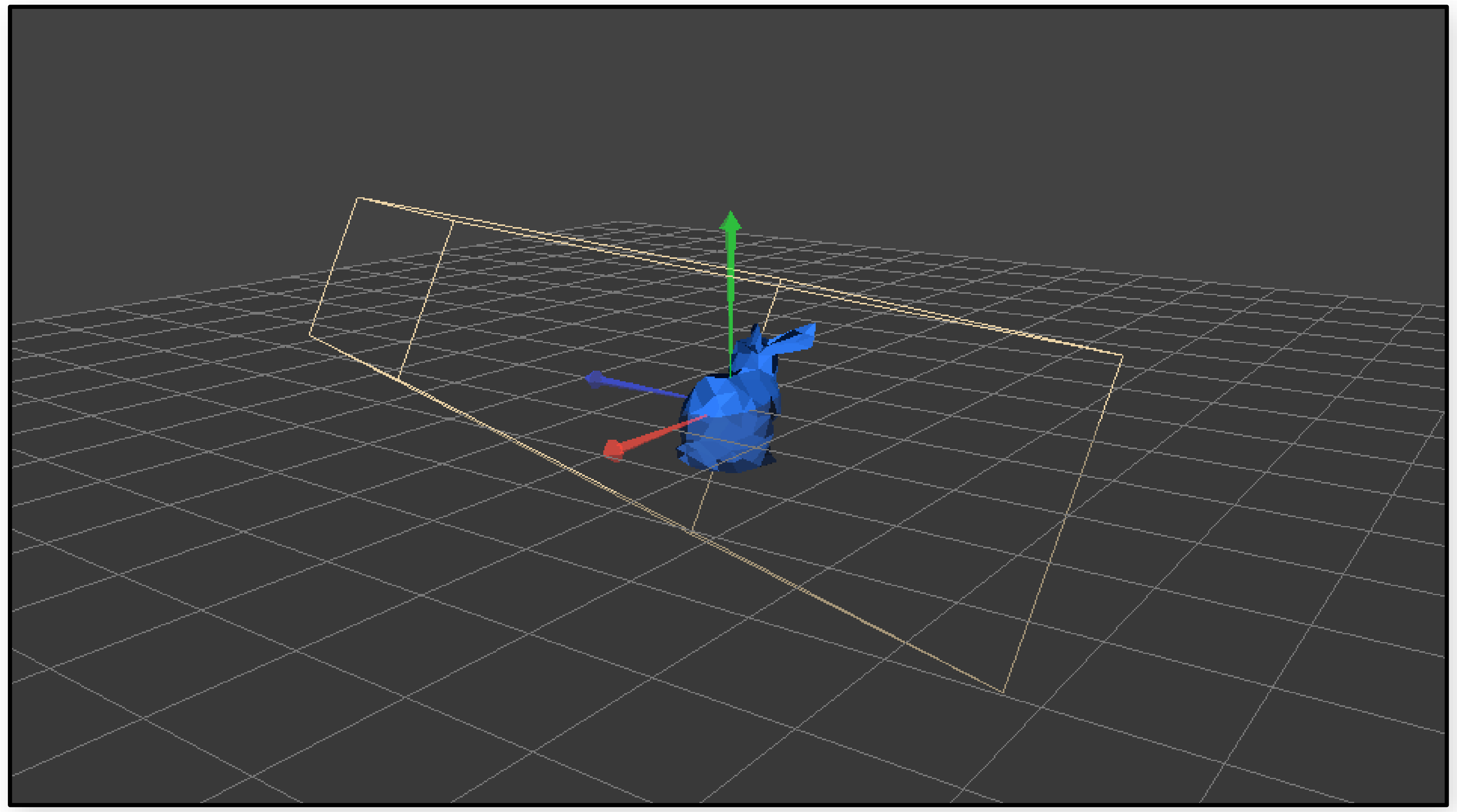


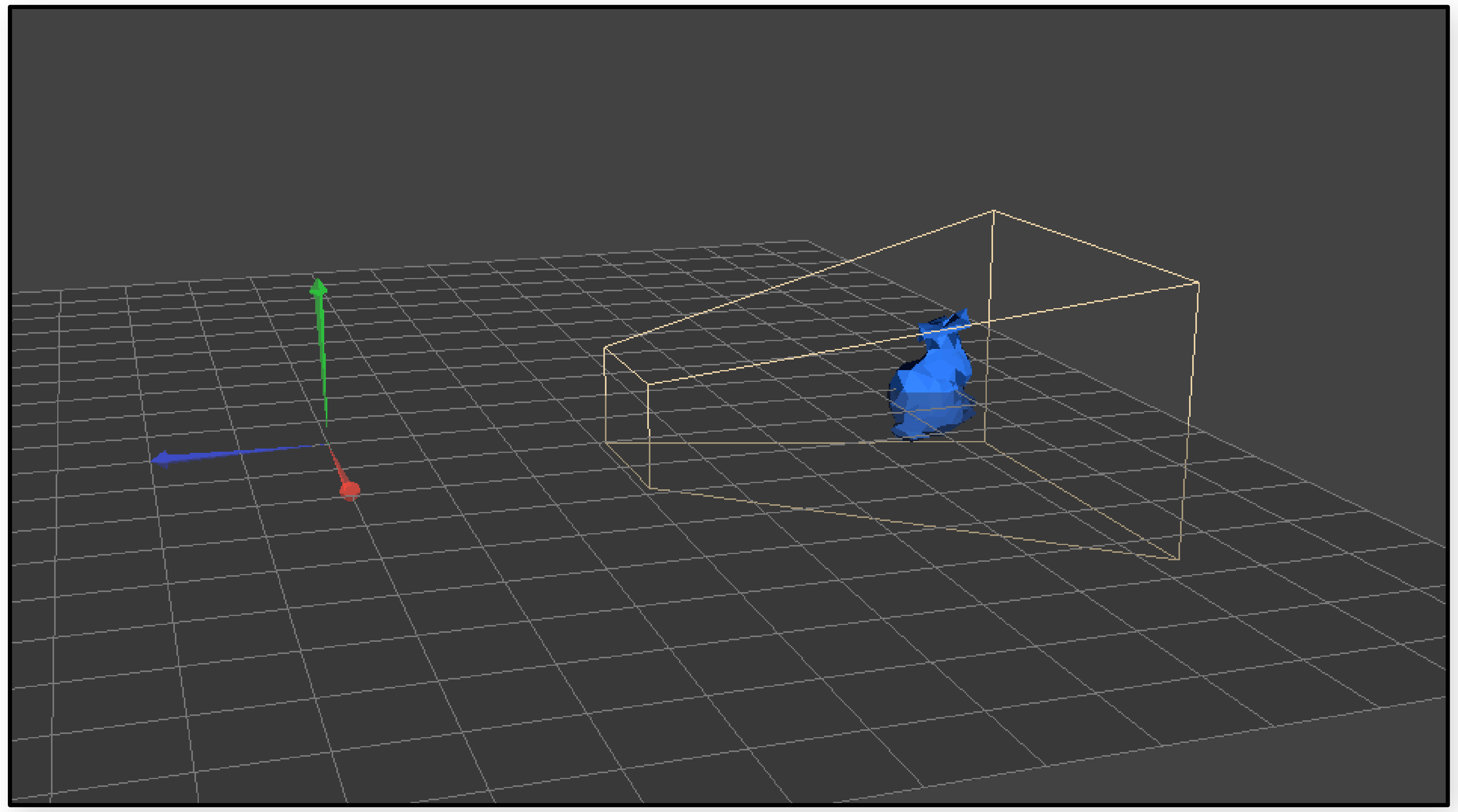
Does distance *d* matter?

After a slide by Steve Marschner

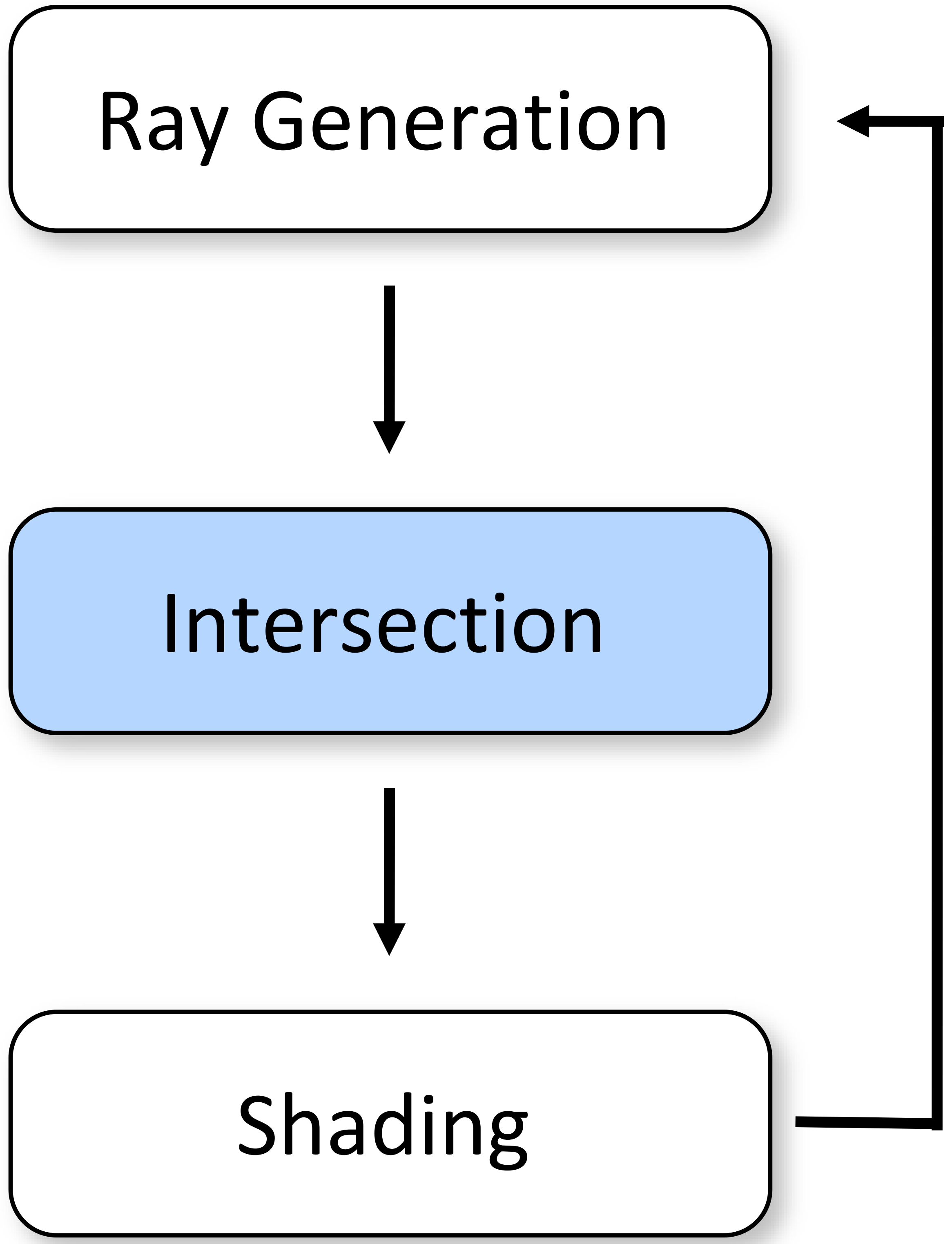
Placing the camera in the scene







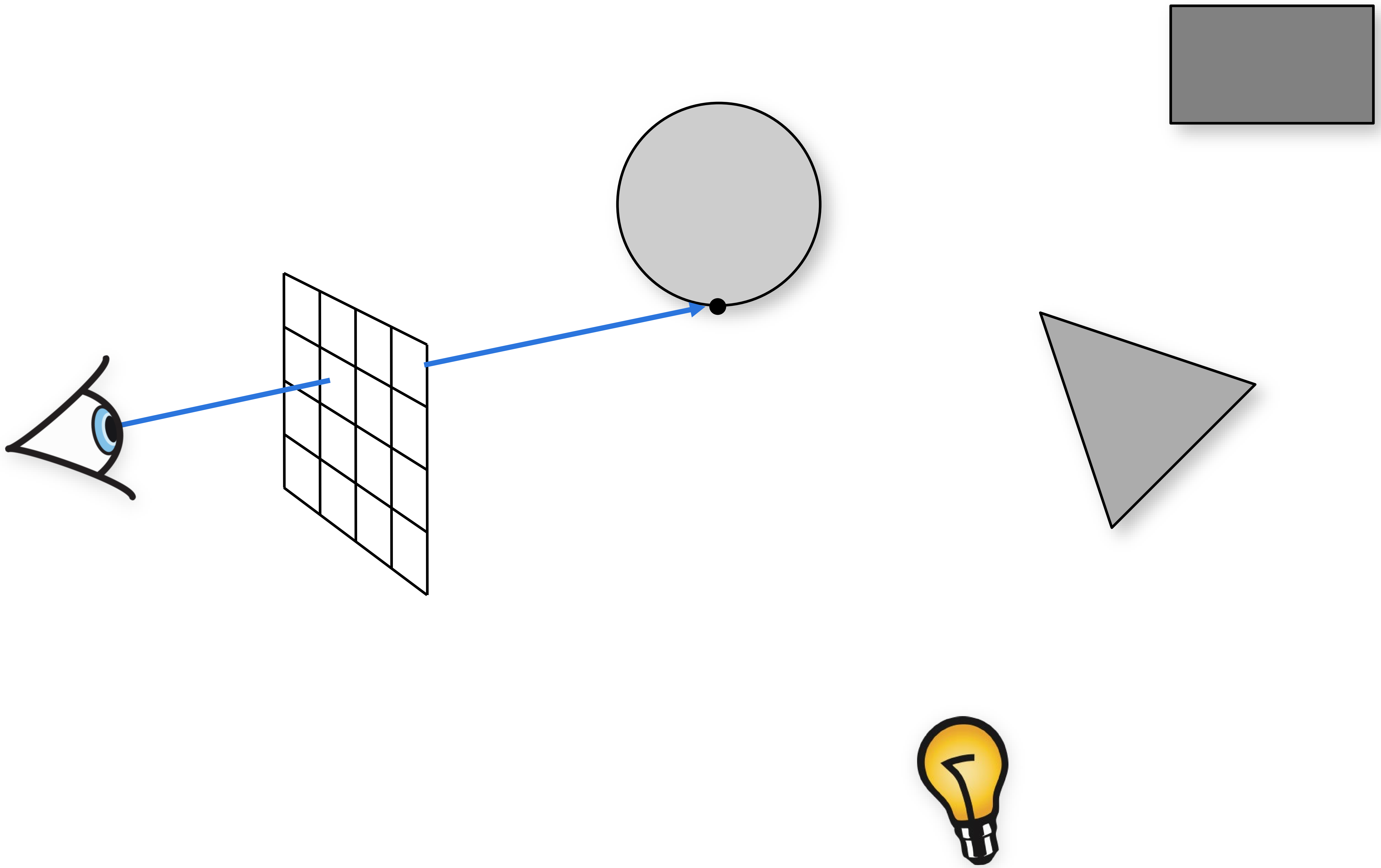
Ray-Surface Intersections



Ray Generation

Intersection

Shading



Ray-Surface Intersections

Surface primitives

- spheres
- planes
- triangles
- general implicits
- etc.

Ray-Sphere Intersection

Algebraic approach:

- Condition 1: point is on ray: $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$
- Condition 2: point is on sphere: $\|\mathbf{x} - \mathbf{c}\|^2 - r^2 = 0$

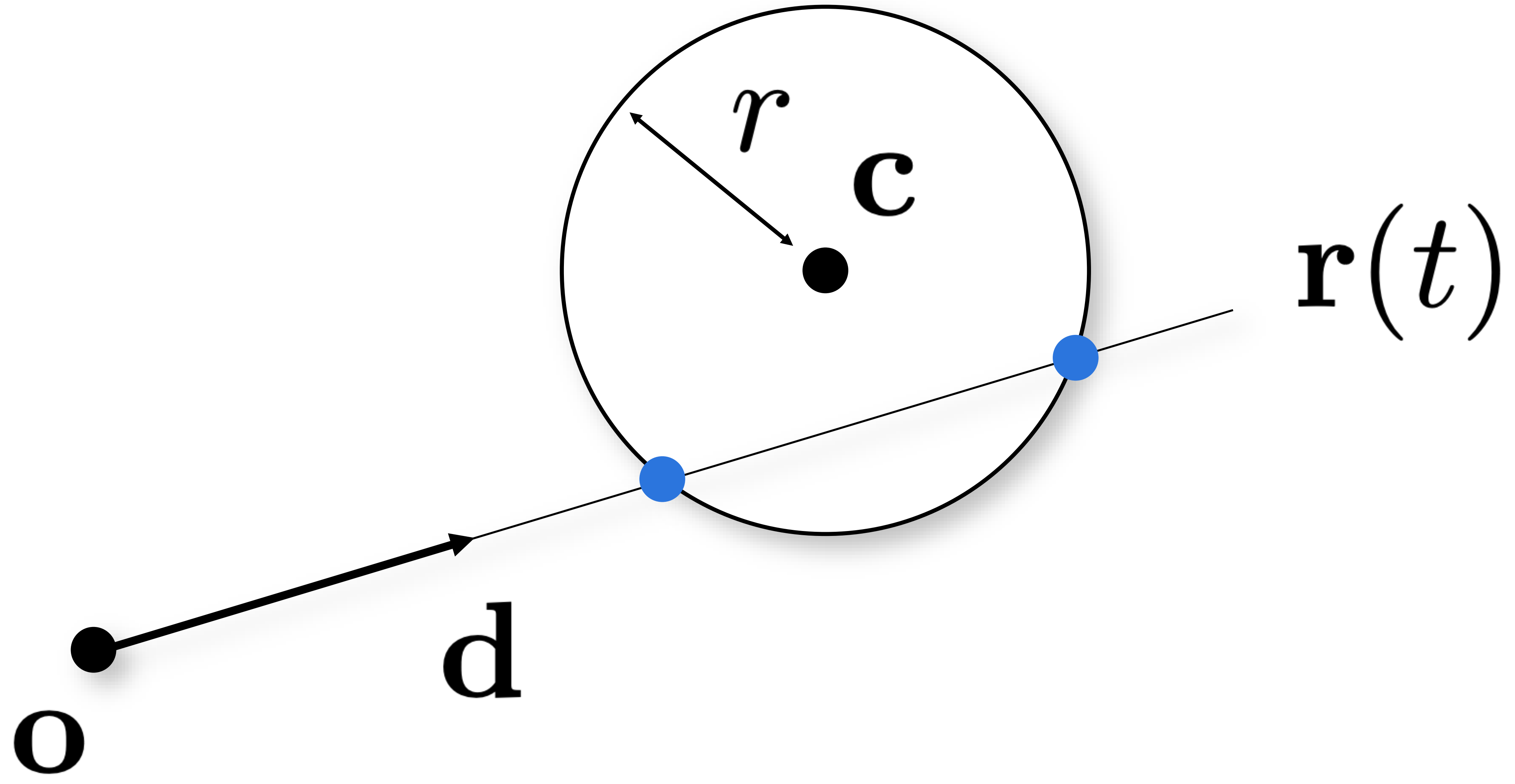
point of interest

center

radius

- substitute and solve for t:

$$\|\mathbf{o} + t\mathbf{d} - \mathbf{c}\|^2 - r^2 = 0$$



Ray-Sphere Intersection

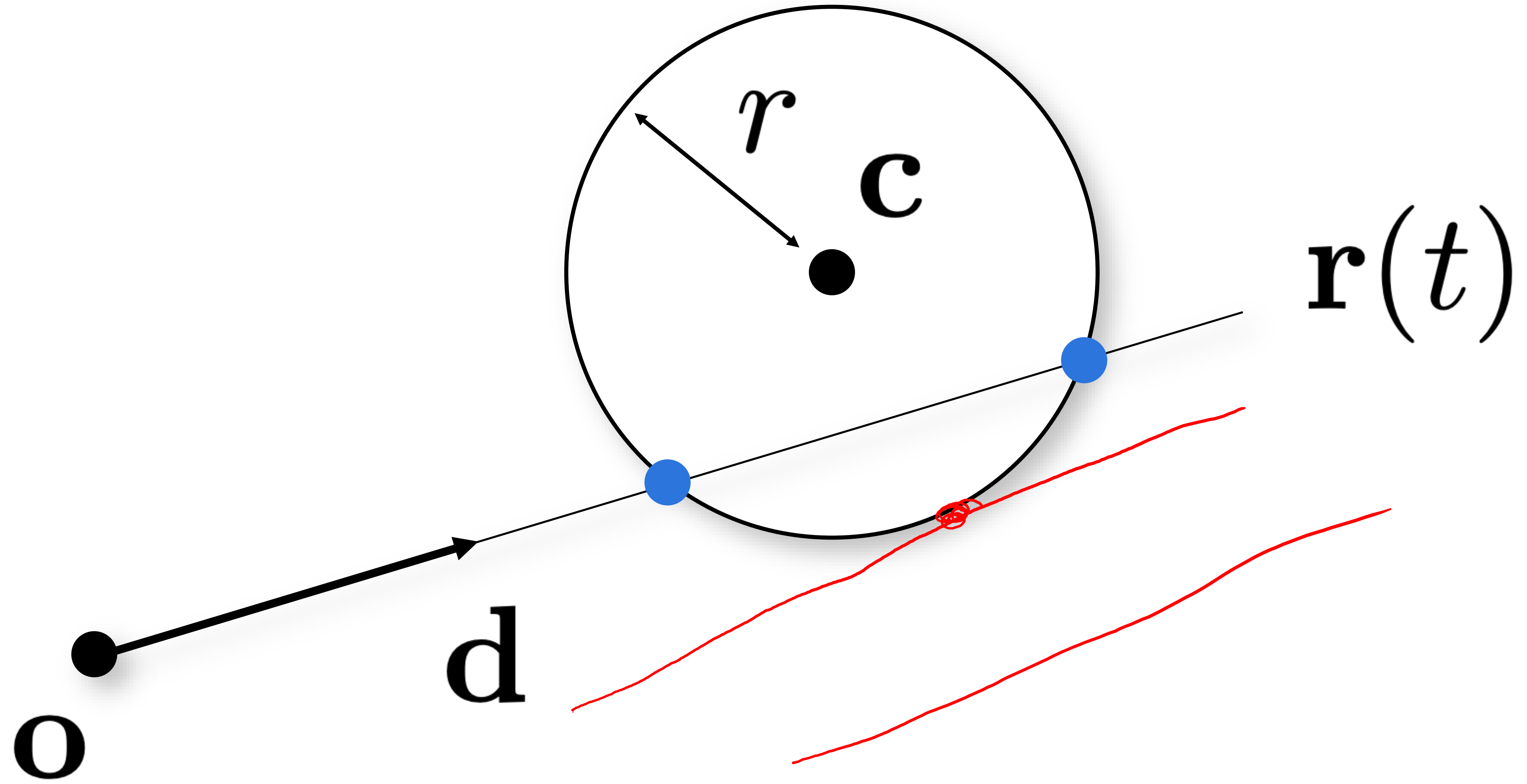
substitute and solve for t

$$\|\mathbf{o} + t\mathbf{d} - \mathbf{c}\|^2 - r^2 = 0 \longrightarrow \begin{aligned}&(\mathbf{o}_x + t\mathbf{d}_x - \mathbf{c}_x)^2 + \\ &(\mathbf{o}_y + t\mathbf{d}_y - \mathbf{c}_y)^2 + \\ &(\mathbf{o}_z + t\mathbf{d}_z - \mathbf{c}_z)^2 - r^2 = 0\end{aligned}$$

which reduces to: $At^2 + Bt + C = 0$

Solve for t using quadratic equation:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$



What happens when square root is zero or negative?

Ray-Surface Intersections

Surface primitives

- spheres
- planes
- triangles
- general implicits
- etc.

Ray-Plane Intersection

Plane equation (implicit)

Algebraic form:

$$ax + by + cz + d = 0$$

Ray-Plane Intersection

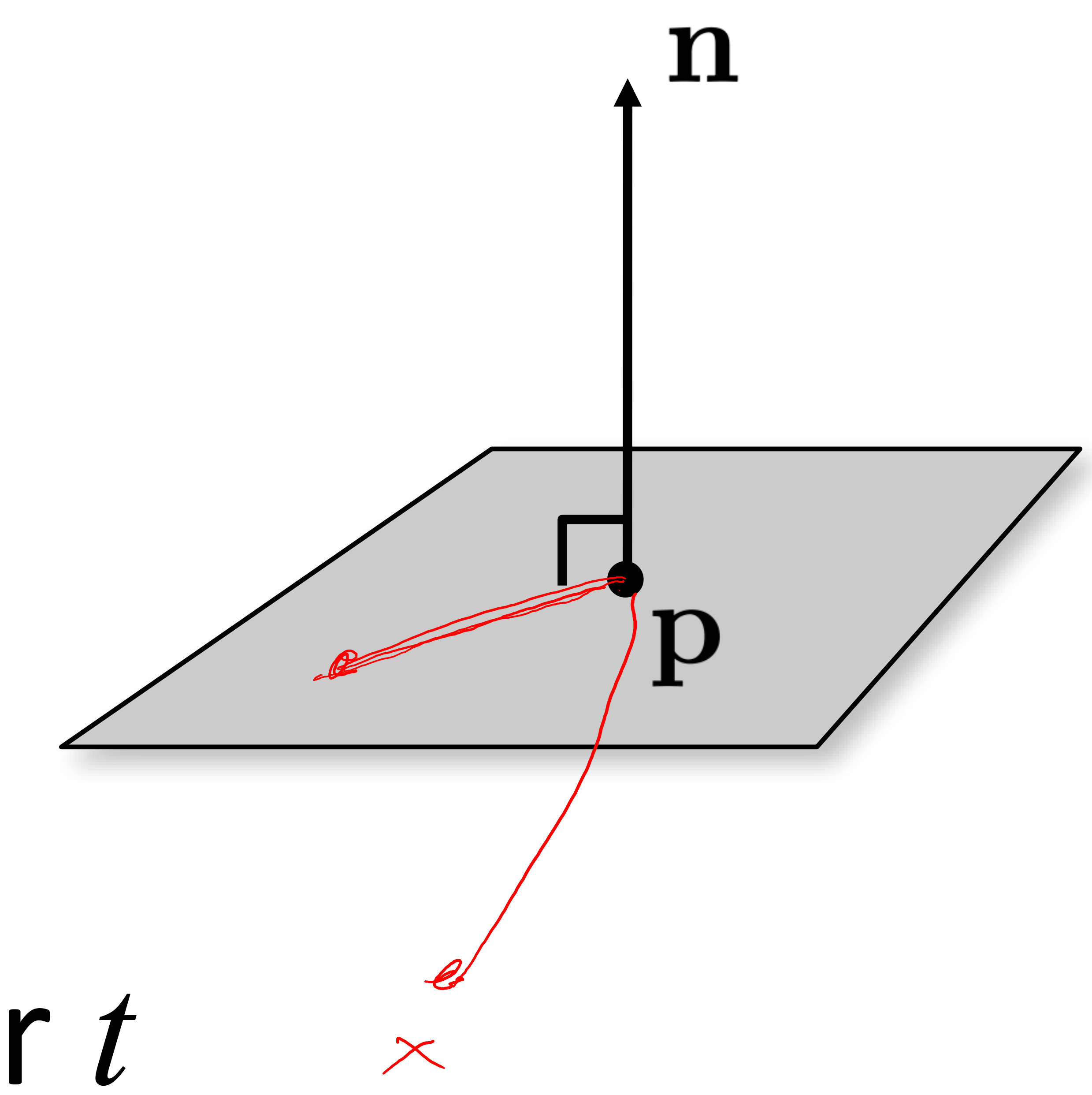
Plane equation (implicit)

$$(\mathbf{x} - \mathbf{p}) \cdot \mathbf{n} = 0$$

point of interest

point on plane

plane normal

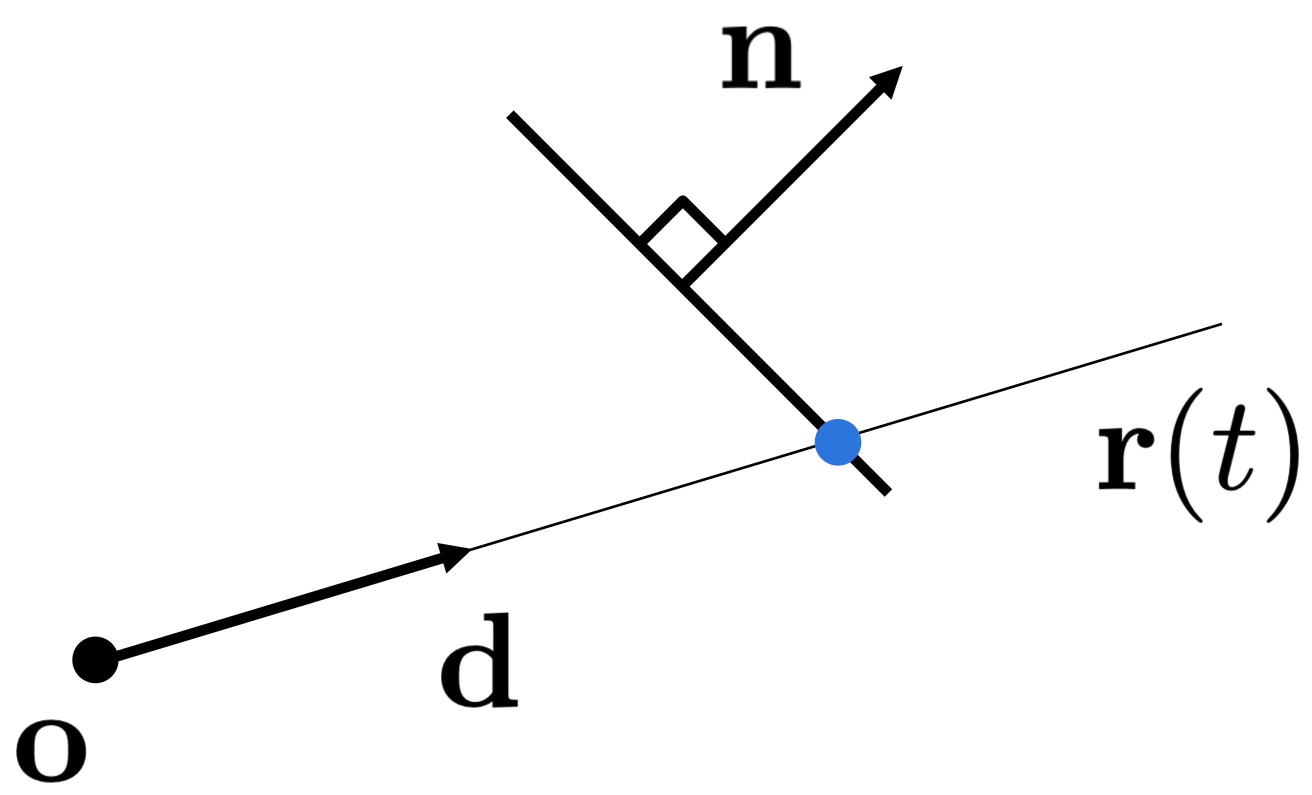


substitute ray equation for $\mathbf{x}$ and solve for t

$$(\mathbf{o} + t\mathbf{d} - \mathbf{p}) \cdot \mathbf{n} = 0$$

$$t\mathbf{d} \cdot \mathbf{n} + (\mathbf{o} - \mathbf{p}) \cdot \mathbf{n} = 0$$

$$t = -\frac{(\mathbf{o} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$



Ray-Surface Intersections

Surface primitives

- spheres
- planes
- triangles
- general implicits
- etc.

Ray-Triangle intersection

Condition 1: point is on ray: $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$

Condition 2: point is on plane: $(\mathbf{x} - \mathbf{p}) \cdot \mathbf{n} = 0$

Condition 3: point is on the inside of all three edges

First solve 1&2 (ray–plane intersection) for t:

$$(\mathbf{o} + t\mathbf{d} - \mathbf{p}) \cdot \mathbf{n} = 0$$

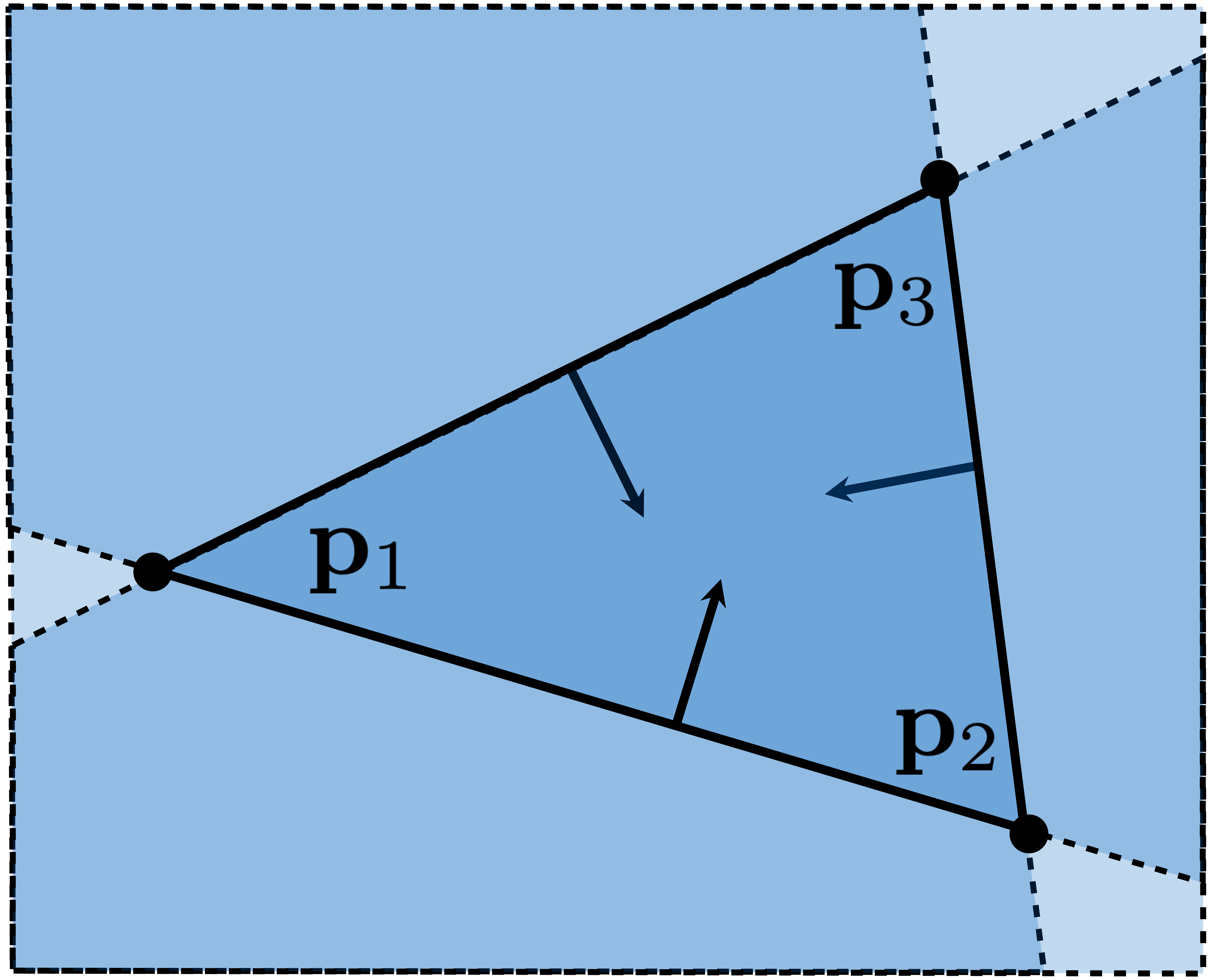
$$t = -\frac{(\mathbf{o} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$

Several options for 3

After a slide by Steve Marschner

Ray-Triangle intersection (Approach 1)

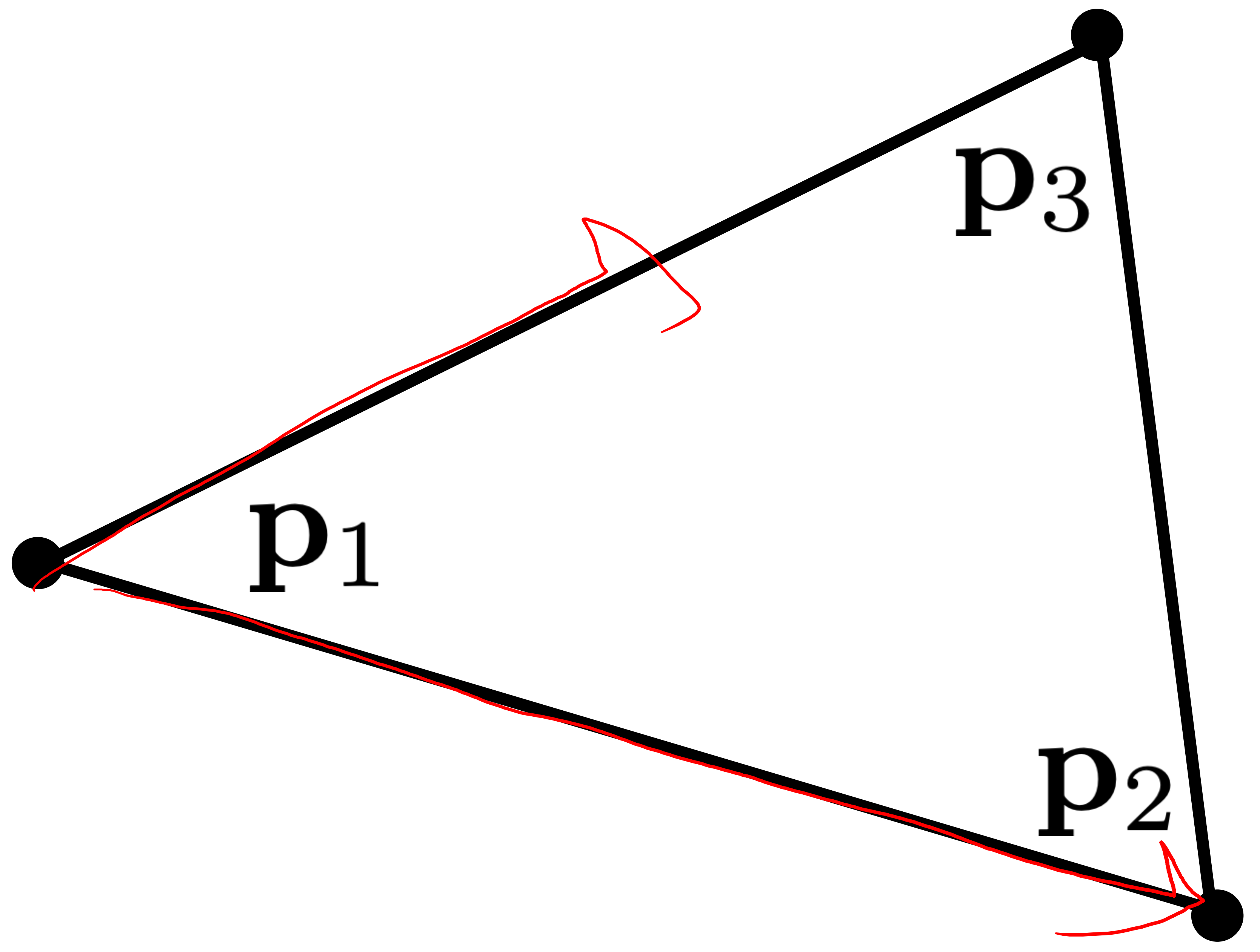
In plane, triangle is the intersection of 3 half spaces



Ray-Triangle intersection (Approach 1)

$$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

Which way does $\mathbf{n}$ point?



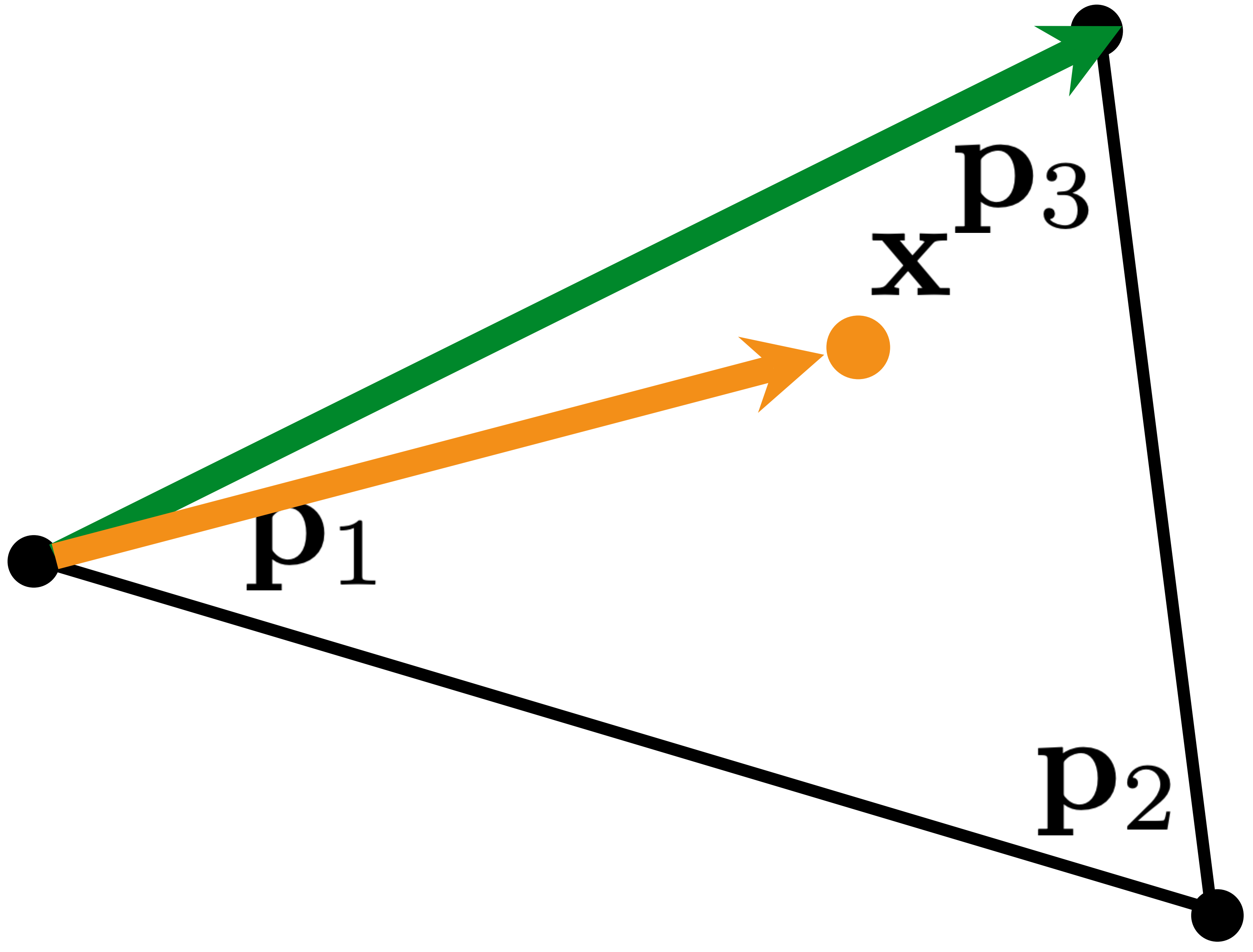
Ray-Triangle intersection (Approach 1)

$$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

$$\mathbf{n}_{\mathbf{x}13} = (\mathbf{x} - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

Which way does $\mathbf{n}$ point?

What about $\mathbf{n}_{\mathrm{x}13}$?



Ray-Triangle intersection (Approach 1)

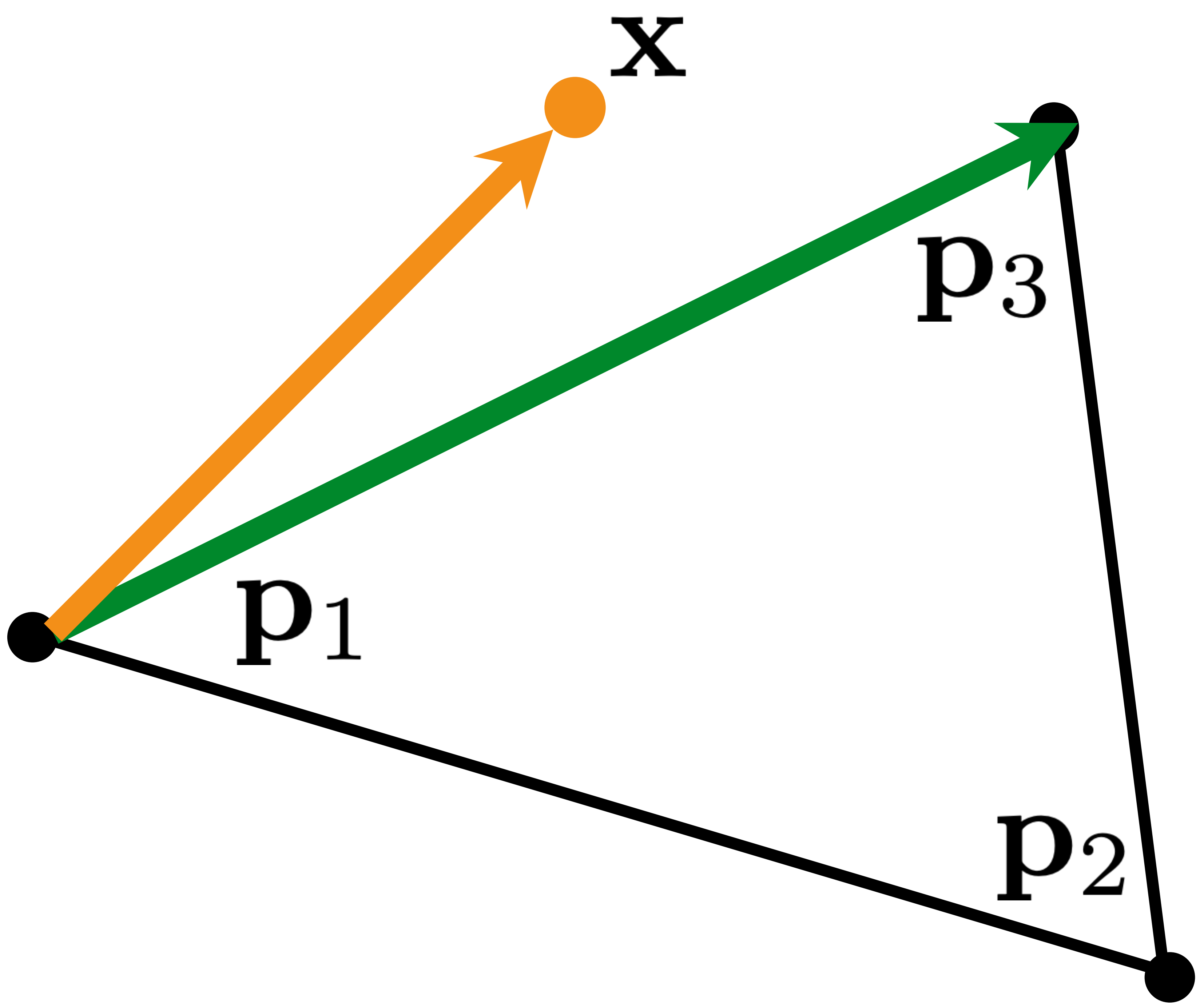
$$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

$$\mathbf{n}_{\mathbf{x}13} = (\mathbf{x} - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

Which way does $\mathbf{n}$ point?

What about $\mathbf{n}_{\mathrm{x}13}$?

- How about now?



Ray-Triangle intersection (Approach 1)

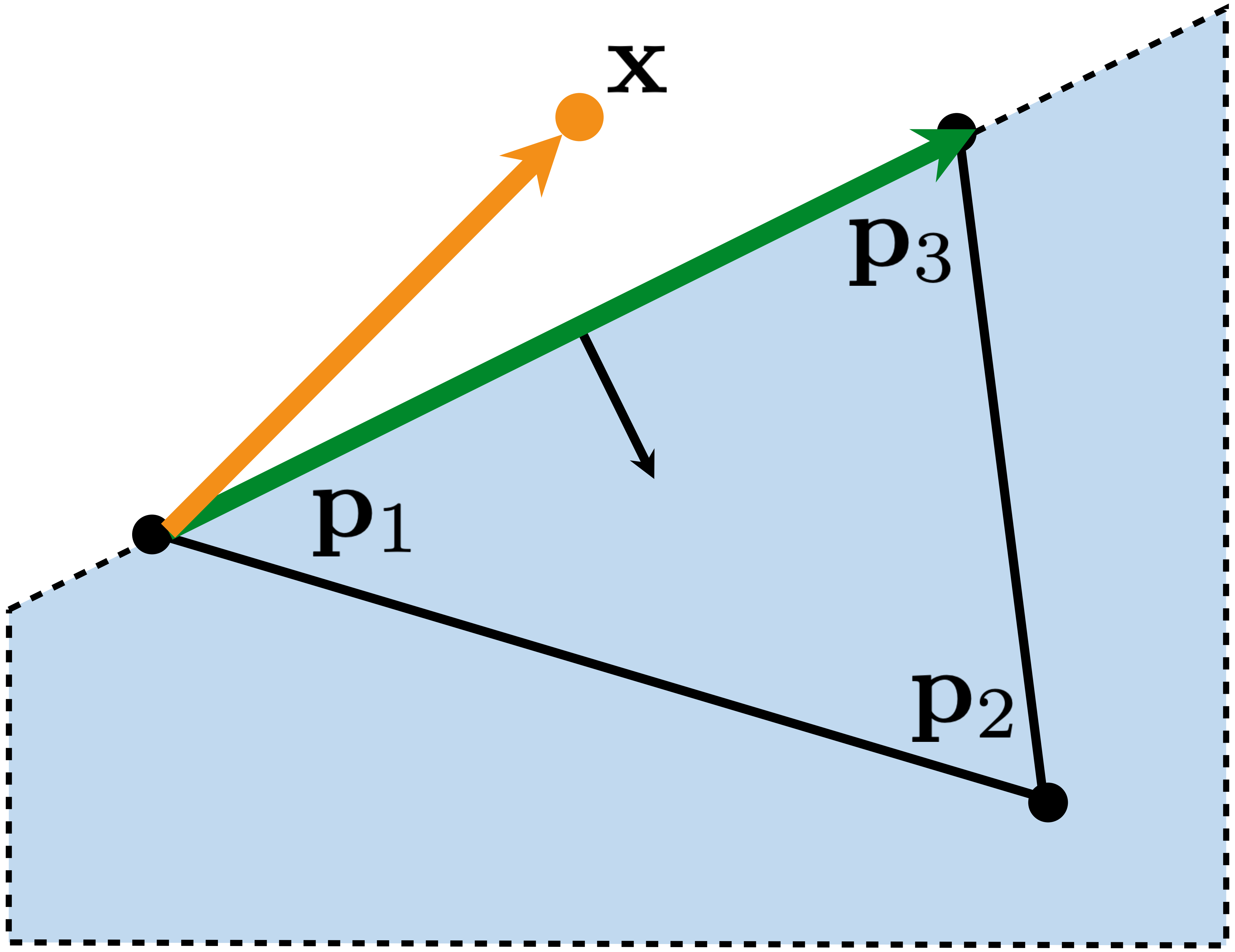
$$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

$$\mathbf{n}_{\mathrm{x}13} = (\mathbf{x} - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

Which way does $\mathbf{n}$ point?

What about $\mathbf{n}_{\mathrm{x}13}$?

- How about now?
- Edge test: $(\mathbf{n}_{\mathrm{x}13} \cdot \mathbf{n}) < 0$



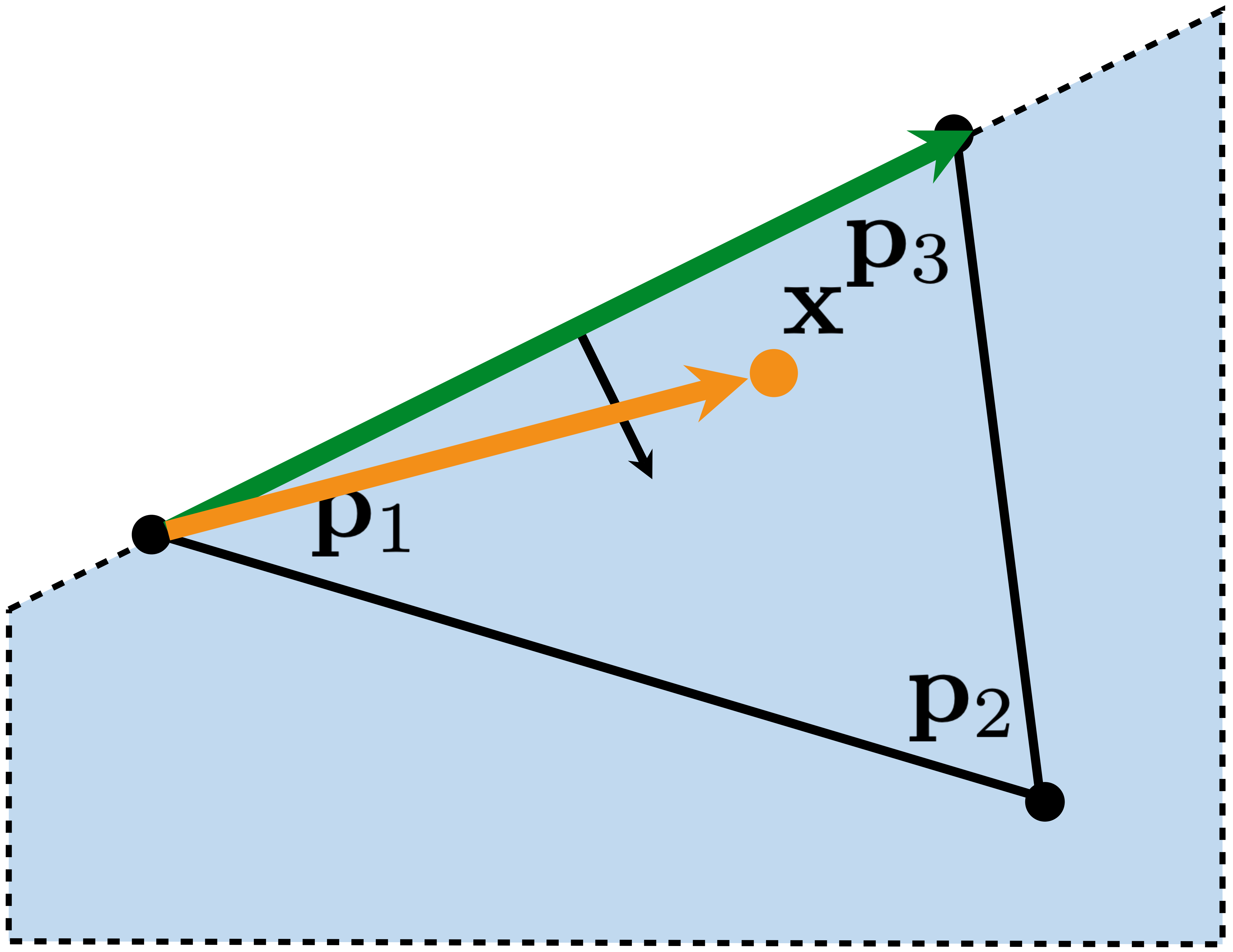
Ray-Triangle intersection (Approach 1)

$$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$
$$\mathbf{n}_{\mathbf{x}13} = (\mathbf{x} - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$$

Which way does $\mathbf{n}$ point?

What about $\mathbf{n}_{\mathbf{x}13}$?

- How about now?
- Edge test: $(\mathbf{n}_{\mathbf{x}13} \cdot \mathbf{n}) < 0$



Ray-Triangle Intersection (Approach 2)

Intersect ray with triangle’s plane

Test whether hit-point is within triangle

- compute sub-triangle areas α, β, γ
- test inside triangle conditions

Barycentric coordinates

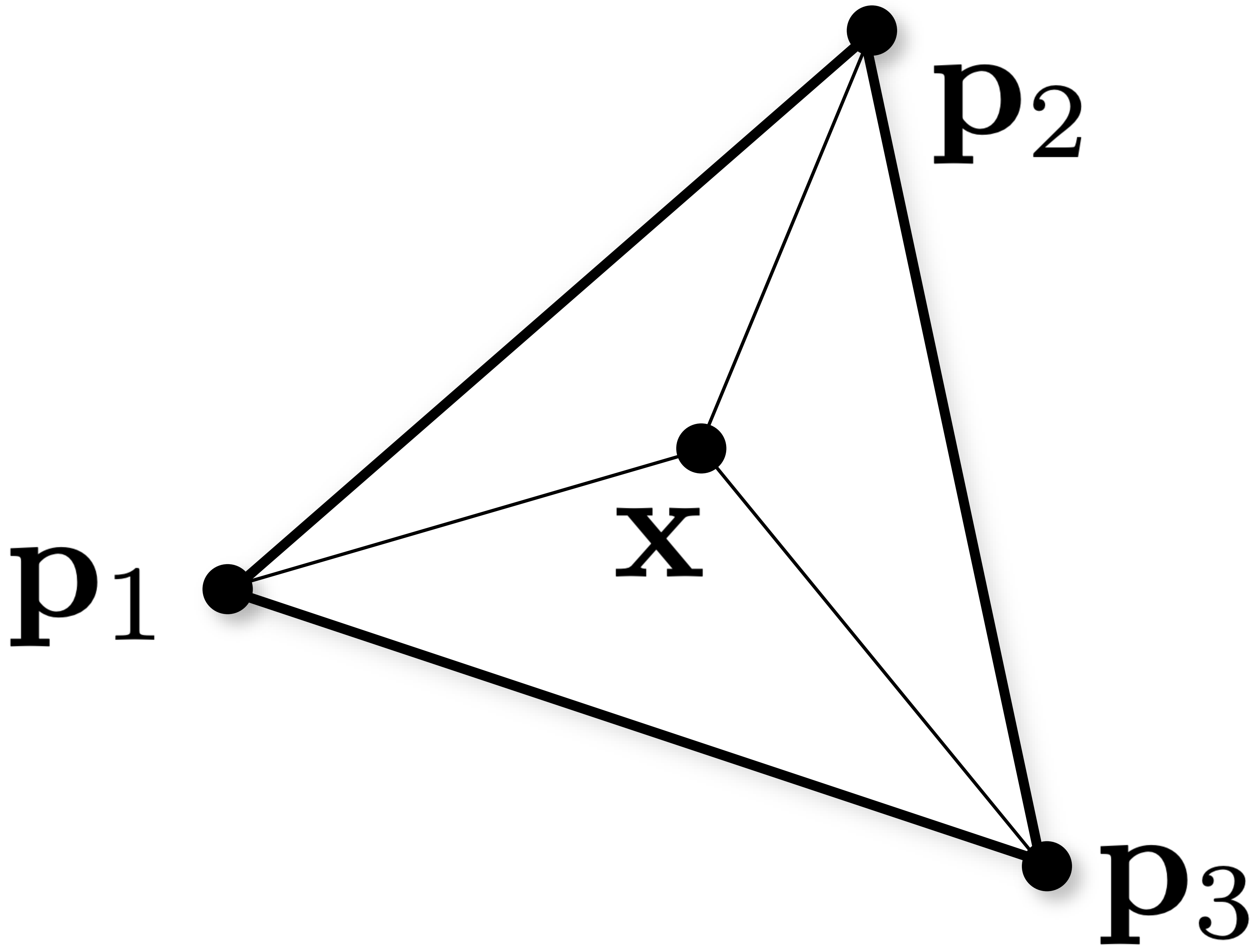
Barycentric coordinates: $\mathbf{x}(\alpha, \beta, \gamma) = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3$

Inside triangle conditions:

$$\alpha + \beta + \gamma = 1 \quad 0 \leq \alpha \leq 1$$

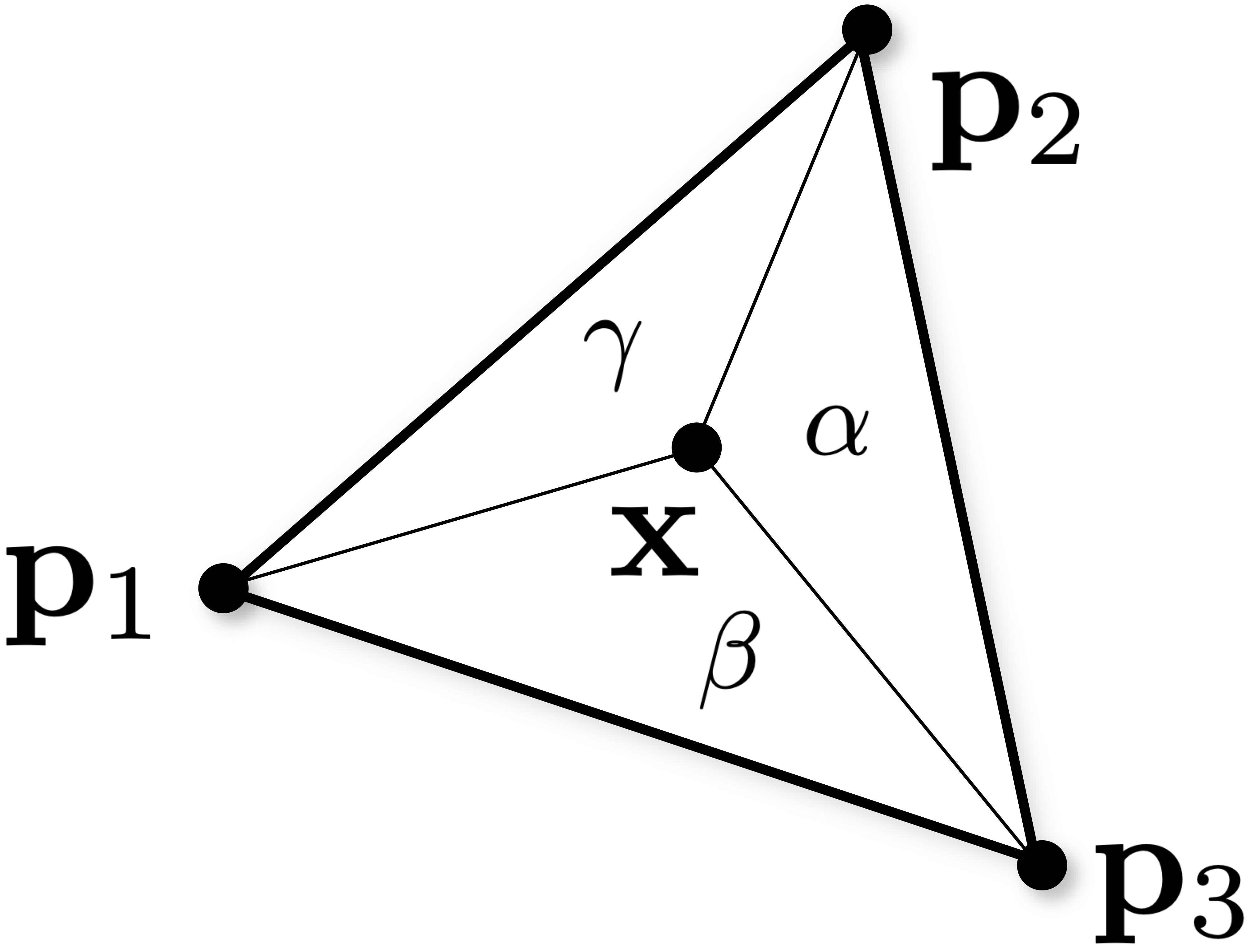
$$\gamma = 1 - \alpha - \beta \quad 0 \leq \beta \leq 1$$

$$0 \leq \gamma \leq 1$$



Interpretations of barycentric coords

Sub-triangle areas



$$\alpha = |\Delta \mathbf{p}_2\mathbf{p}_3\mathbf{x}|/|\Delta \mathbf{p}_1\mathbf{p}_2\mathbf{p}_3|$$

$$\beta = |\Delta \mathbf{p}_1\mathbf{p}_3\mathbf{x}|/|\Delta \mathbf{p}_1\mathbf{p}_2\mathbf{p}_3|$$

$$\gamma = |\Delta \underline{\mathbf{p}}_1\underline{\mathbf{p}_2}\mathbf{x}|/|\Delta \mathbf{p}_1\mathbf{p}_2\mathbf{p}_3|$$

$$\mathbf{x} = \alpha\mathbf{p}_1 + \beta\mathbf{p}_2 + \gamma\mathbf{p}_3$$

Ray-Triangle Intersection (Approach 3)

Insert ray equation:

$$\alpha\mathbf{p}_1 + \beta\mathbf{p}_2 + (1-\alpha-\beta)\mathbf{p}_3 = \mathbf{o} + t\mathbf{d}$$

$$\alpha(\mathbf{p}_1 - \mathbf{p}_3) + \beta(\mathbf{p}_2 - \mathbf{p}_3) + \mathbf{p}_3 = \mathbf{o} + t\mathbf{d}$$

$$\alpha\underbrace{(\mathbf{p}_1 - \mathbf{p}_3)} + \beta\underbrace{(\mathbf{p}_2 - \mathbf{p}_3)} - t\underline{\mathbf{d}} = \underbrace{\mathbf{o} - \mathbf{p}_3}$$

$$\alpha\underline{\mathbf{a}} + \beta\underline{\mathbf{b}} - t\underline{\mathbf{d}} = \underline{\mathbf{e}}$$

Solve directly

Can be much faster!

$$\underbrace{\begin{bmatrix} -\mathbf{d} & \mathbf{a} & \mathbf{b} \end{bmatrix}} \begin{bmatrix} t \\ \alpha \\ \beta \end{bmatrix} = \mathbf{e}$$

Ray-Surface Intersections

Other primitives

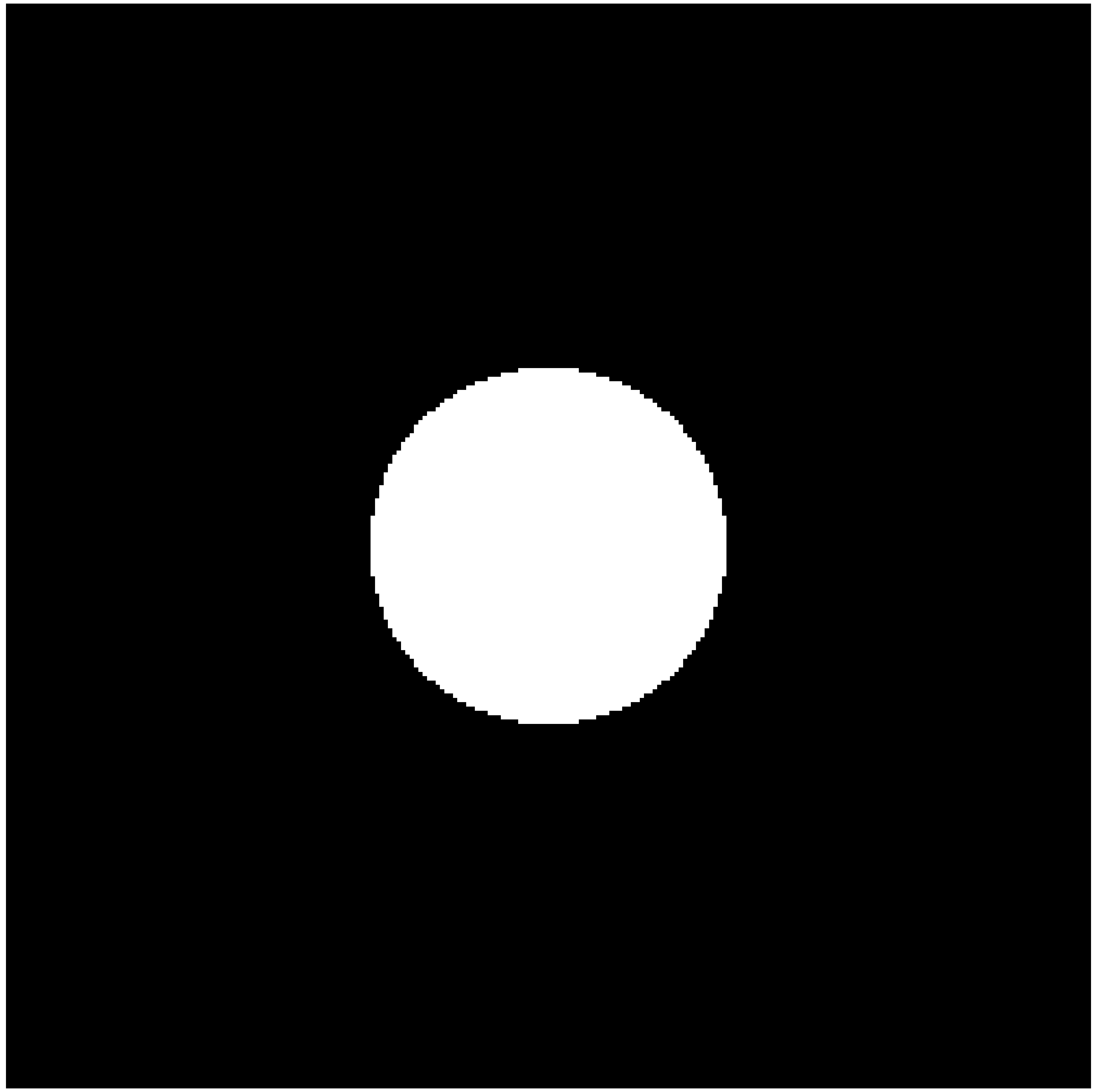
- cylinder
- cone, paraboloid, hyperboloid
- torus
- disk
- general polygons, meshes
- etc.

Image so far

With eye ray generation and sphere intersection

```
parse scene description

for each pixel:
   ray = camera.getRay(pixel);
   hit = s.intersect(ray, 0, +inf);
   if hit:
      image.set(pixel, white);
```



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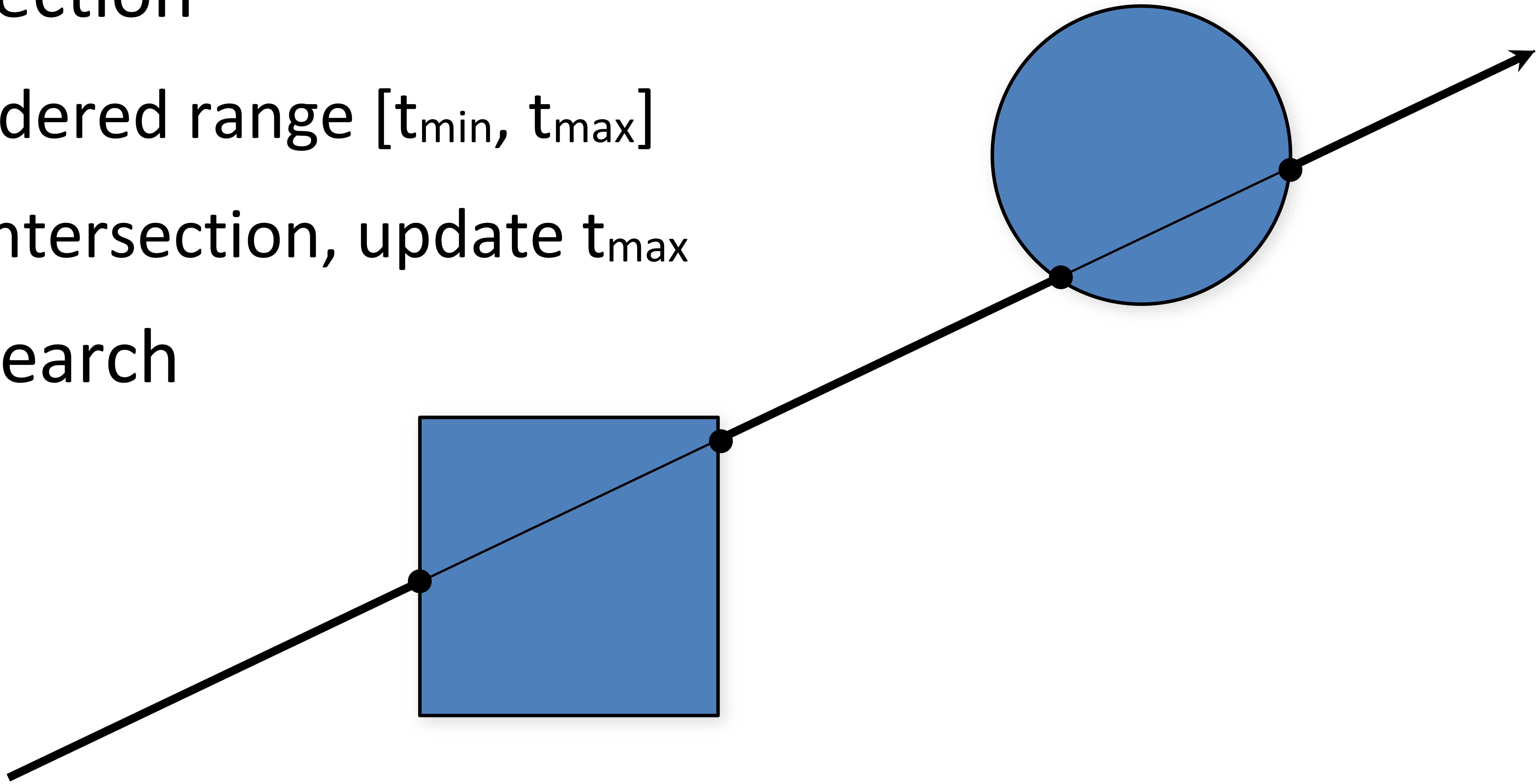
Intersecting many shapes

Intersect each primitive

Pick closest intersection

- Only within considered range [t_{min}, t_{max}]
- After each valid intersection, update t_{max}

Essentially a line search



Intersection against many shapes

The basic idea is:

```
Surfaces::intersect(ray, tMin, tMax):
   tBest = +inf; firstHit = null;
   for s in surfaces:
      hit = s.intersect(ray, tMin, tBest);
      if hit:
         tBest = hit.t;
         firstHit = hit;
   return firstHit;
```

- this is linear in number of surfaces but there are sublinear methods (acceleration structures)

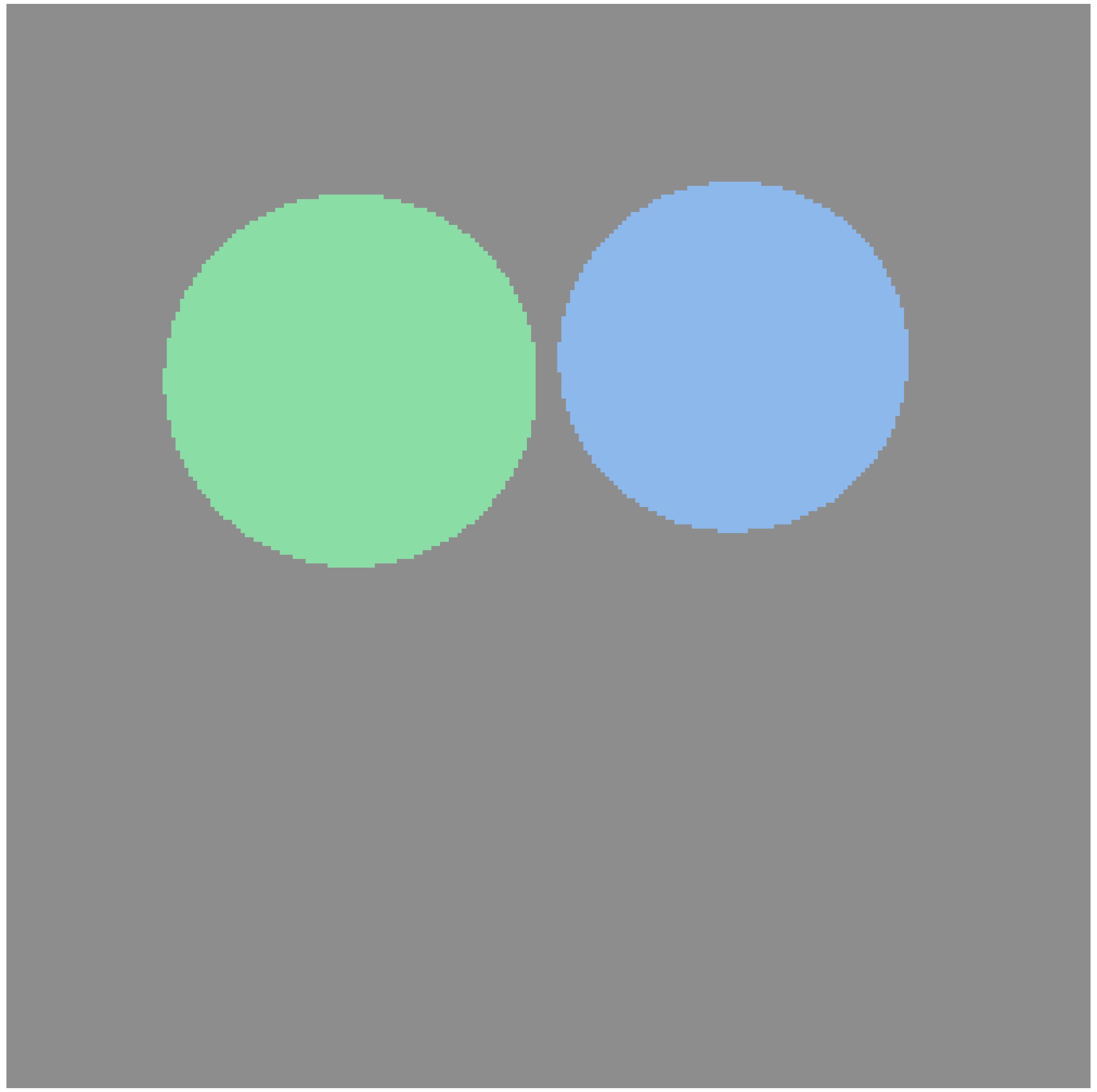
After a slide by Steve Marschner

Image so far

With eye ray generation and scene intersection

```
for each pixel:
   ray = camera.getRay(pixel);
   c = scene.trace(ray, 0, +inf);
   image.set(pixel, c);

Scene::trace(ray, tMin, tMax):
   hit = surfaces.intersect(ray, tMin, tMax);
   if (hit)
      return hit.color();
   else
      return backgroundColor;
```



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Intersecting transformed primitive?

Option 1: Transform the primitive

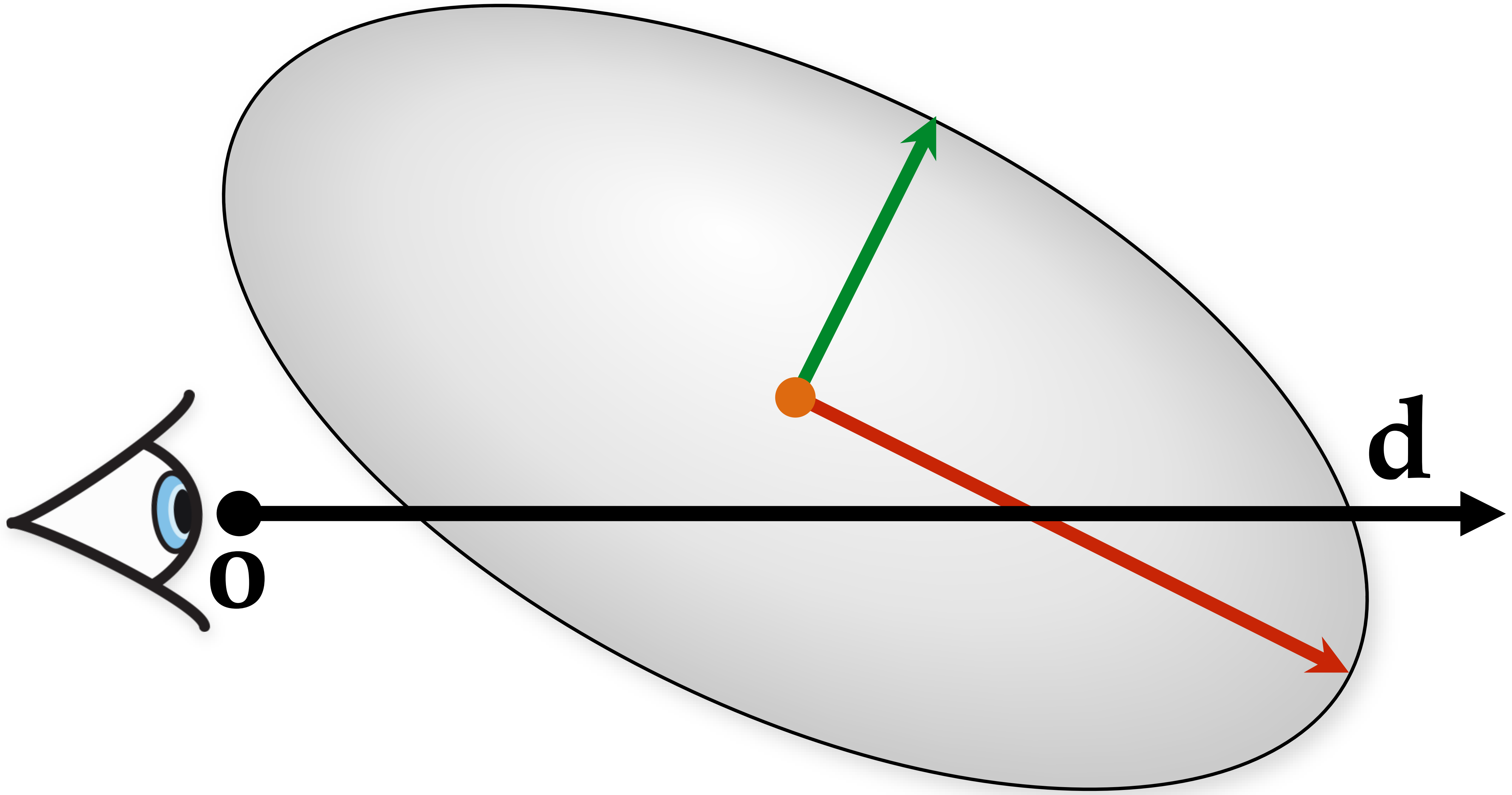
- simple for triangles, since they transform to triangles
- other primitives get more complicated (e.g. sphere ⟶ ellipsoid)

Option 2: Transform the ray (by the inverse transform)

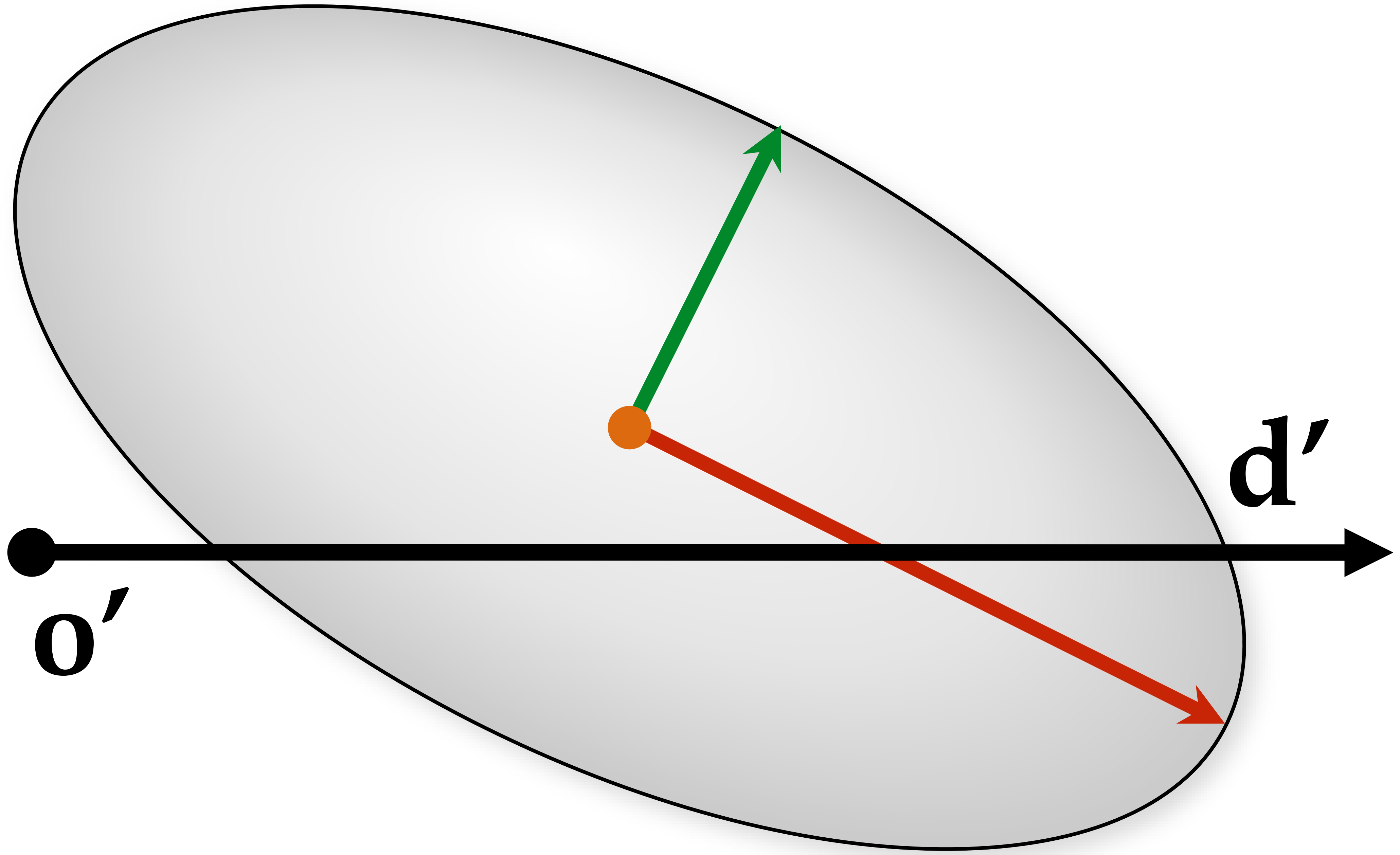
- more elegant; works on any primitive
- allows simpler intersection tests
  (e.g., just use existing sphere-intersection routine)

Intersection and coordinate systems

World space

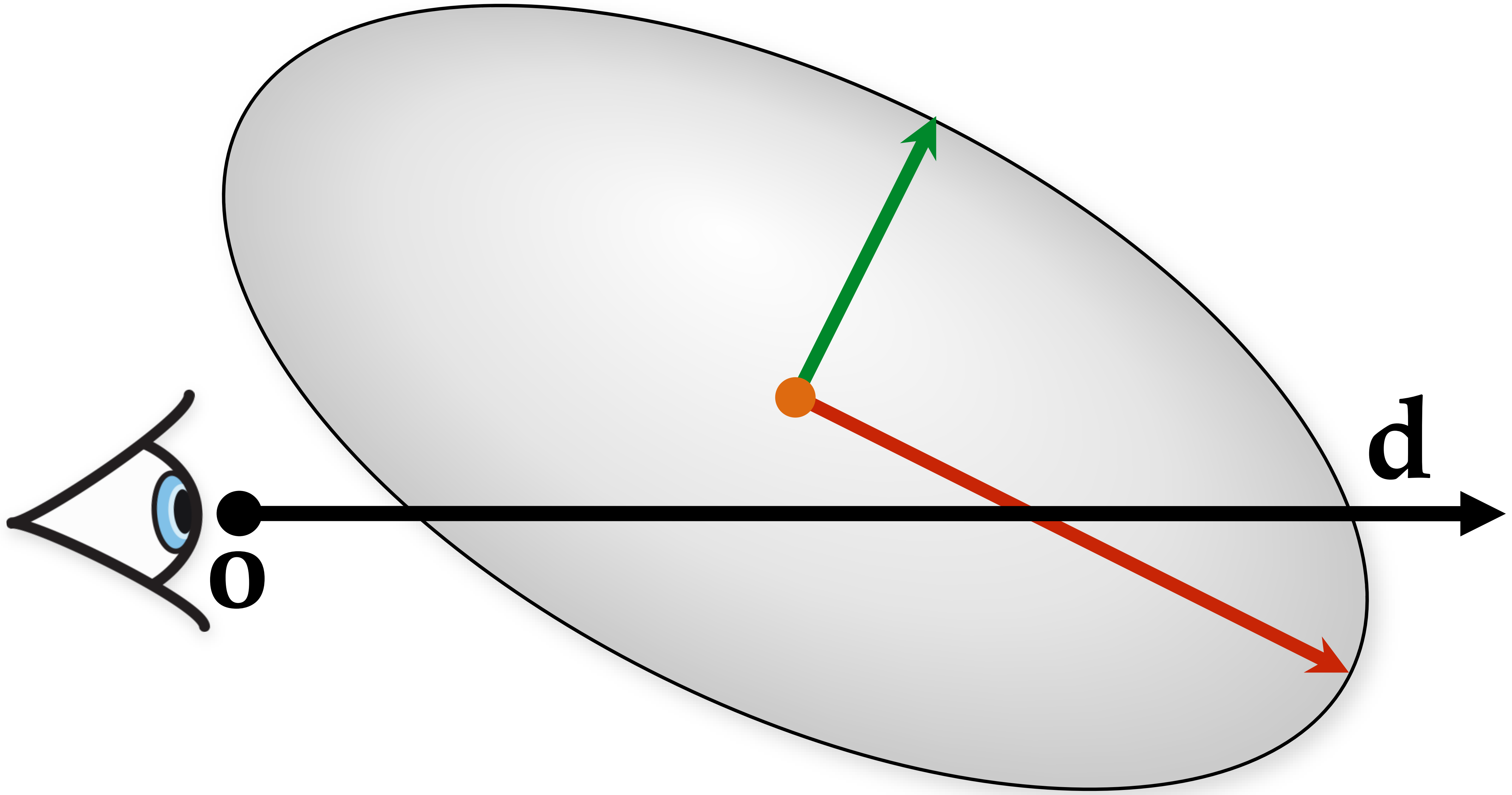


Local space

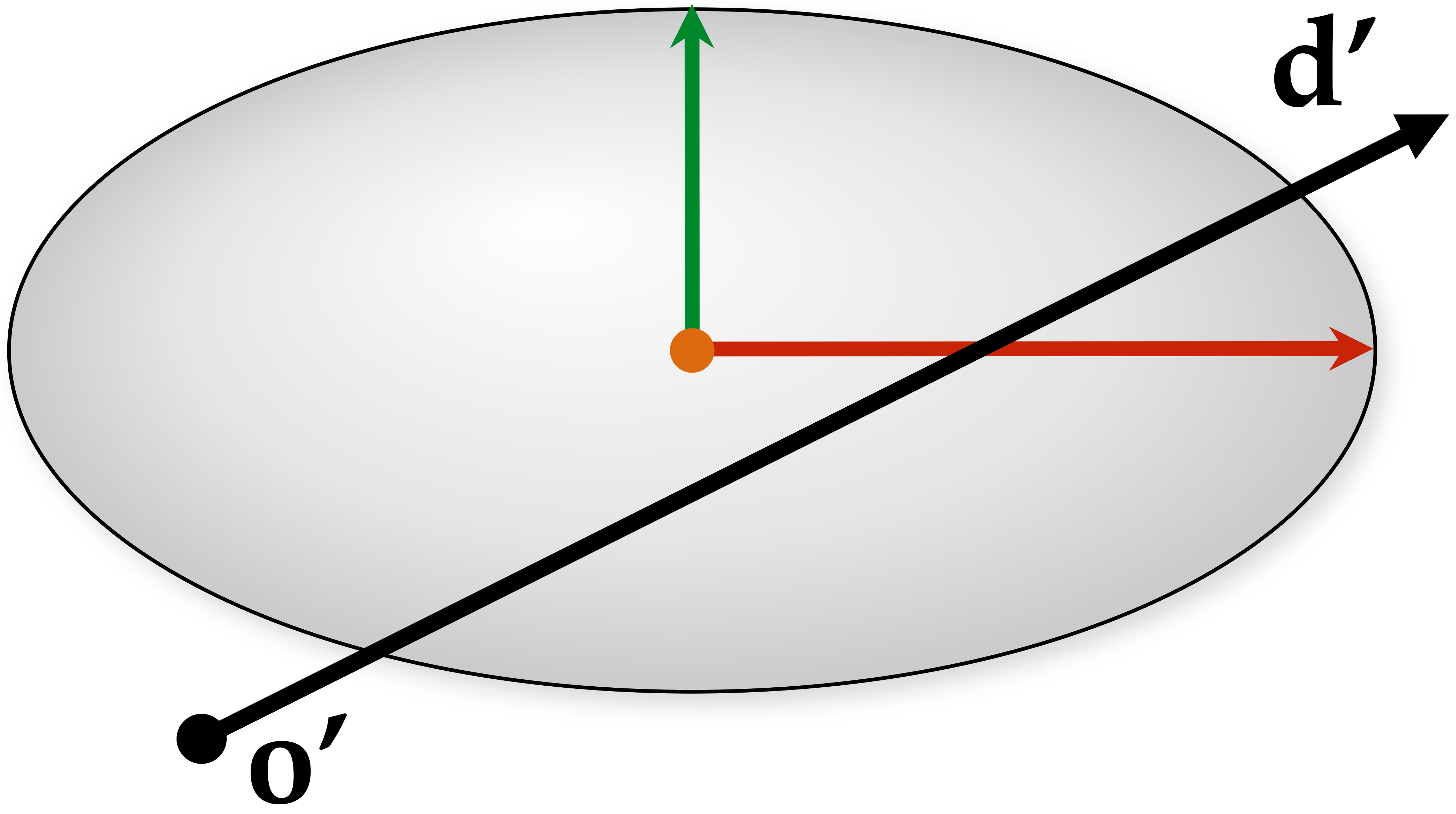


Intersection and coordinate systems

World space

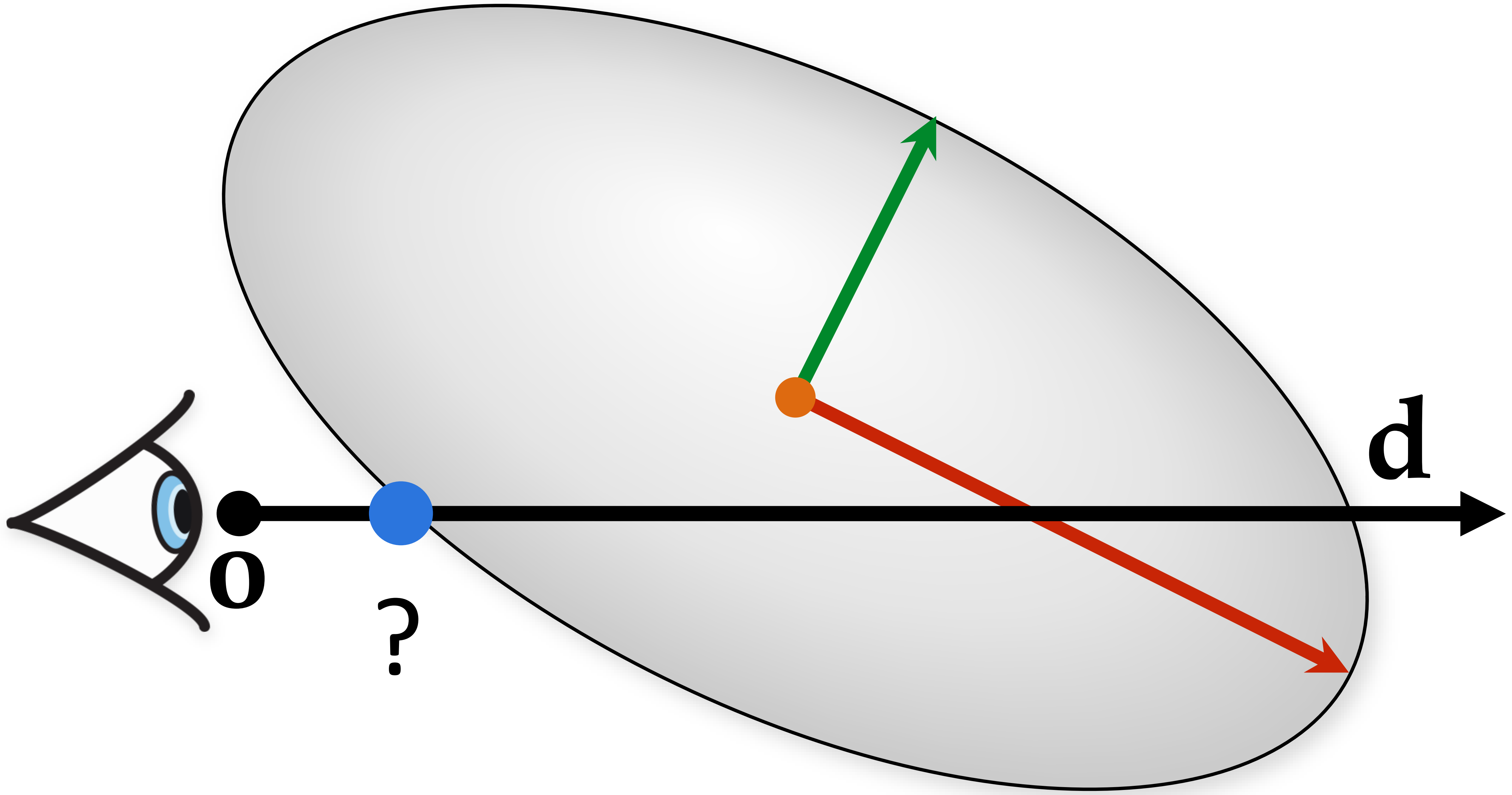


Local space

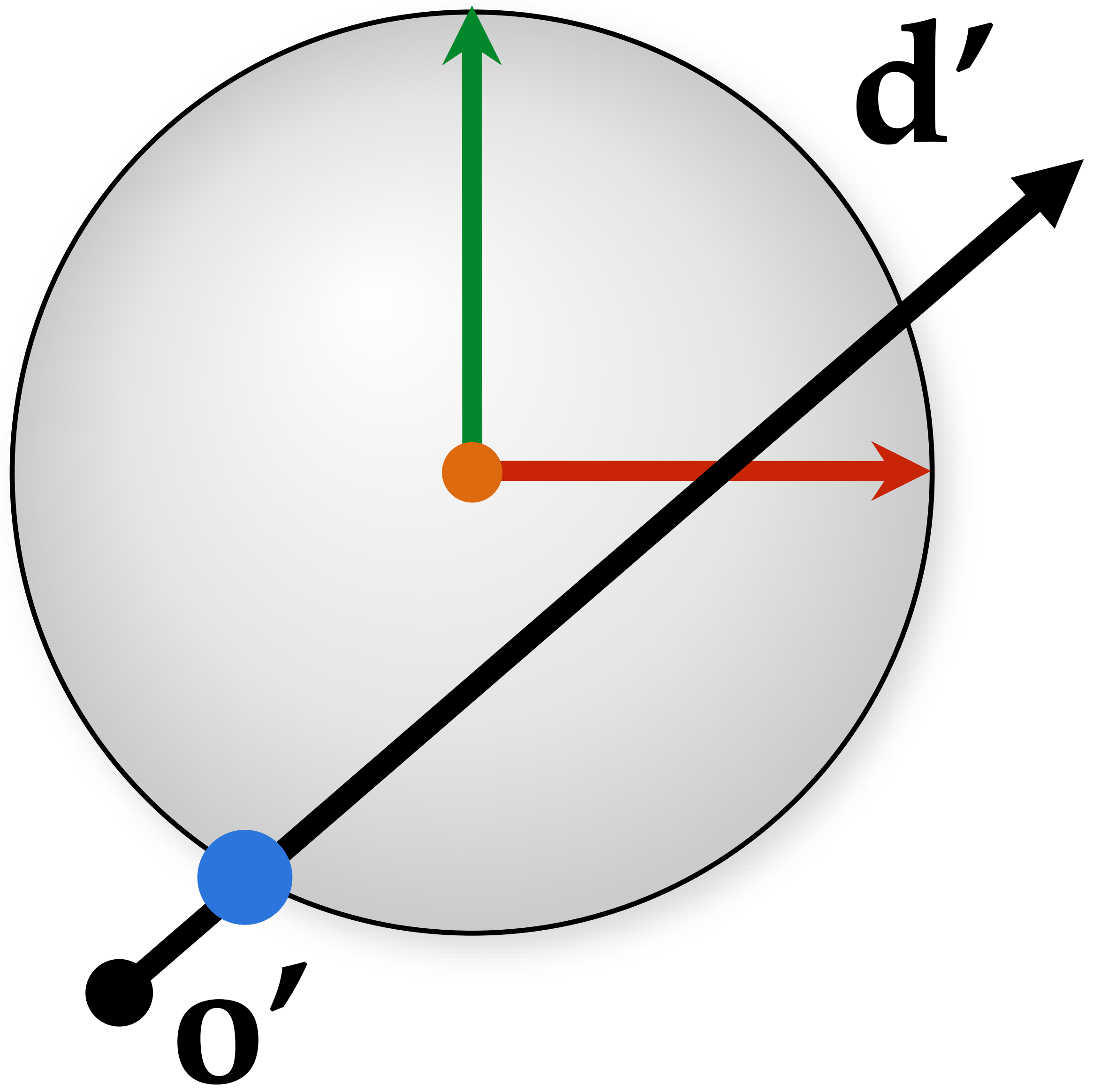


Intersection and coordinate systems

World space



Local space



We have a sphere now

But with a different ray

Transformations in homogeneous coords

A 3D transformation matrix:

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{24} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}$$

A 3D homogenous vector:

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

A position has $w \neq 0$, and a direction has $w = 0$

Transformations

Matrix-vector multiplication, $M\mathbf{v}$, transforms the vector

A translation matrix:

$$M_{\mathbf{t}} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A scaling matrix:

$$M_{\mathbf{s}} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Intersection and coordinate systems

Have a transform M, a ray $\mathbf{r}(t)$, and a surface S

To intersect:

1. Transform ray to local coords (by inverse of M)
2. Call surface intersection
3. Transform hit data back to global coords (by M)

How to transform a ray $\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$ by M^{-1}?

- $\mathbf{r}'(t) = M^{-1}\mathbf{p} + tM^{-1}\mathbf{d}$
- Remember: $\mathbf{p}$ forms as a point, $\mathbf{d}$ as a direction!

Ray-Surface Intersections

Other primitives

- cylinder
- cone, paraboloid, hyperboloid
- torus
- disk
- general polygons, meshes
- etc.

How should we represent complex geometry?

How are they obtained?

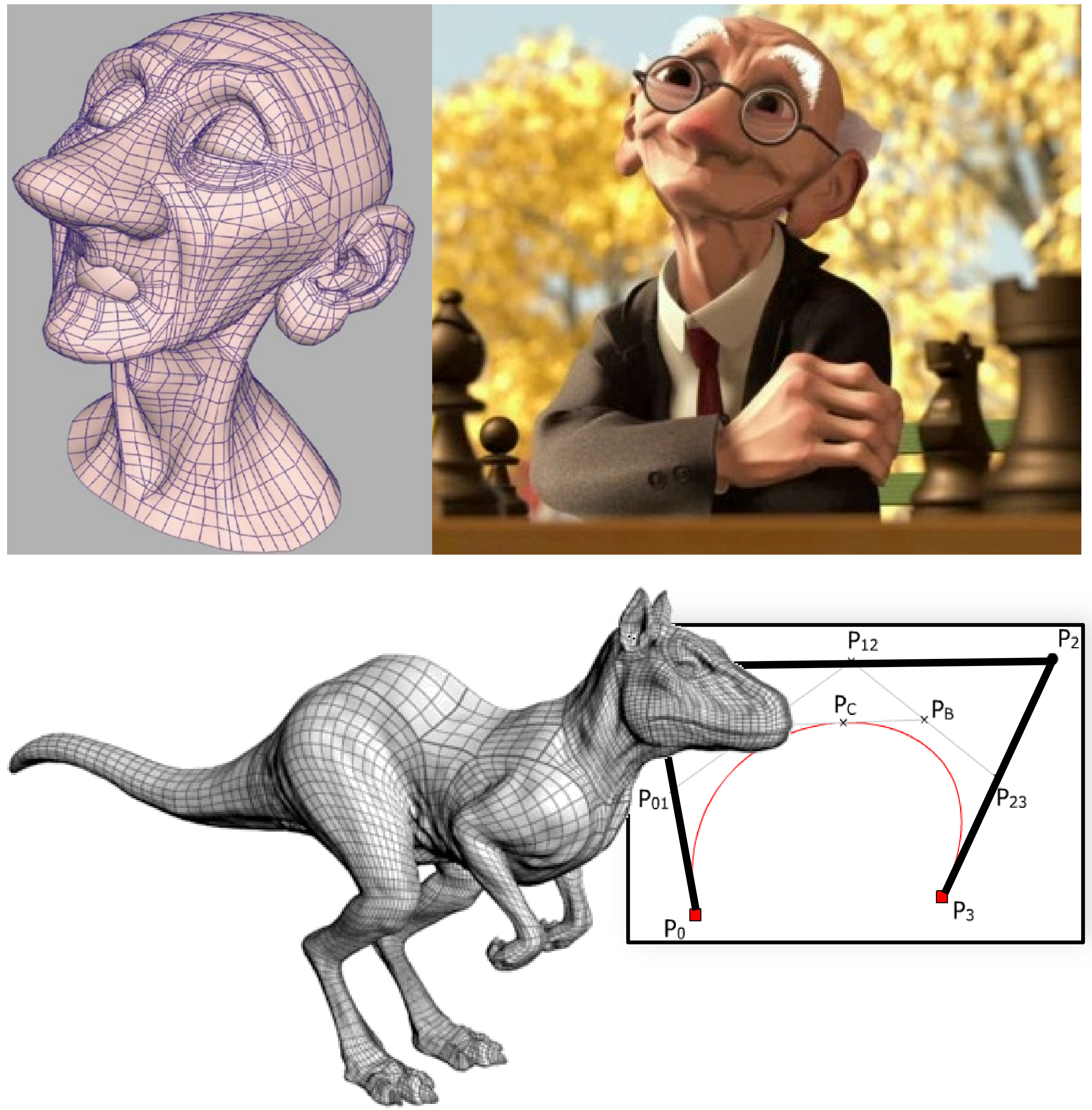
- modeled by hand
- scanned

What operations must we support?

- modeling/editing
- animating
- **texturing**
- **rendering**

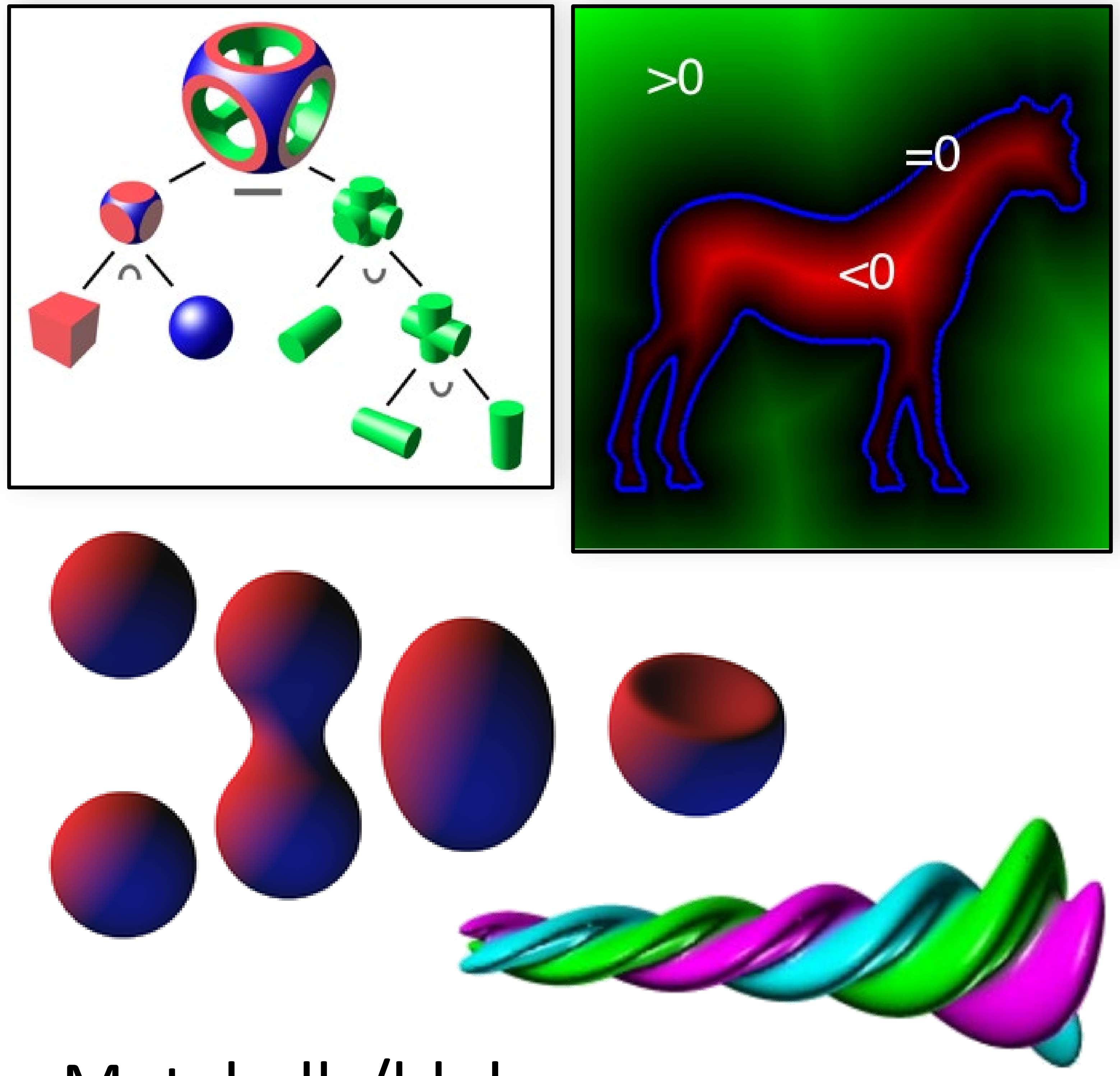
Surface representation zoo!

Parametric



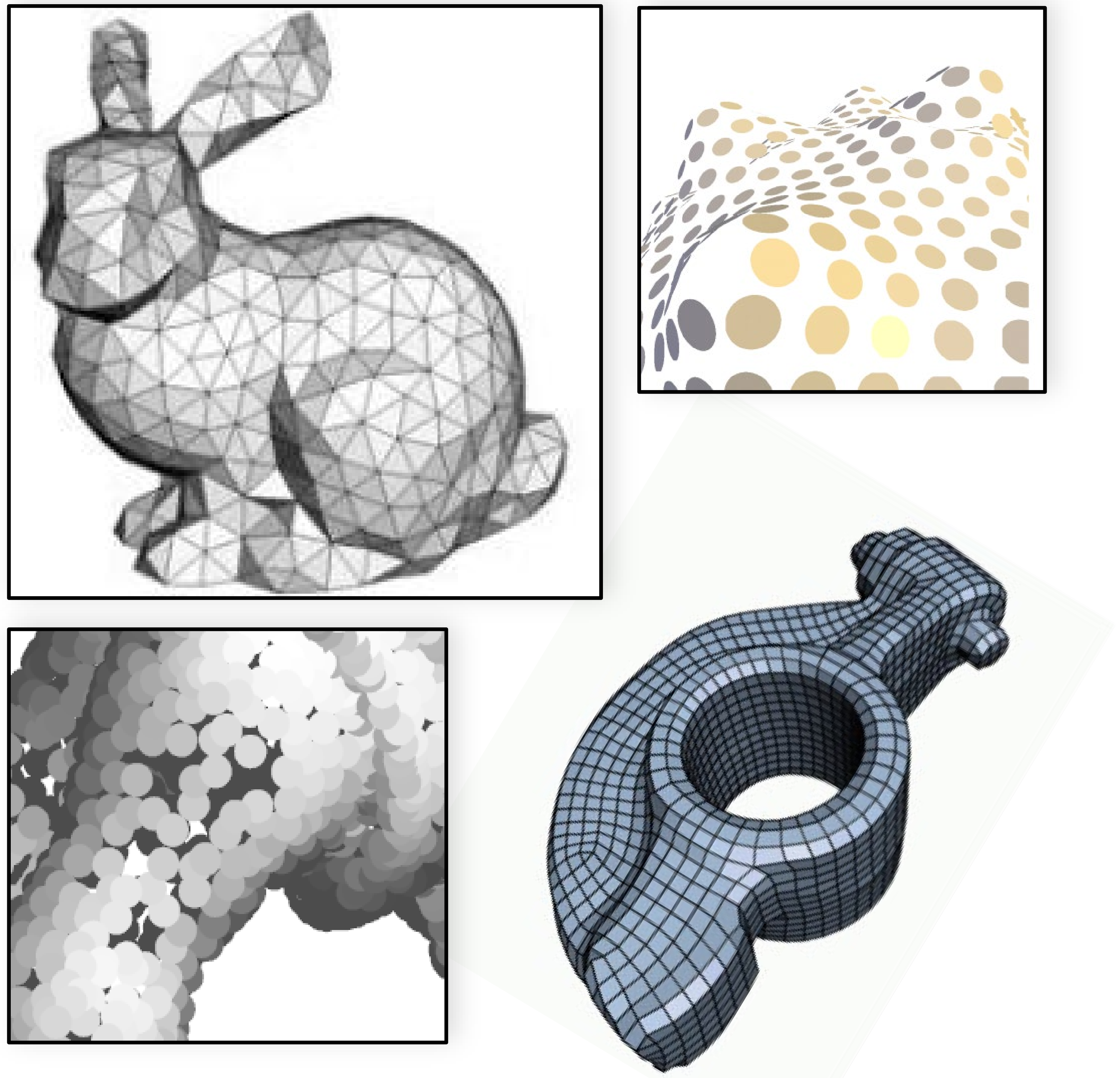
- Splines, tensor-product surfaces
- Subdivision surfaces

Implicit



- Metaballs/blobs
- Distance fields
- Procedural, CSG

Discrete/Sampled



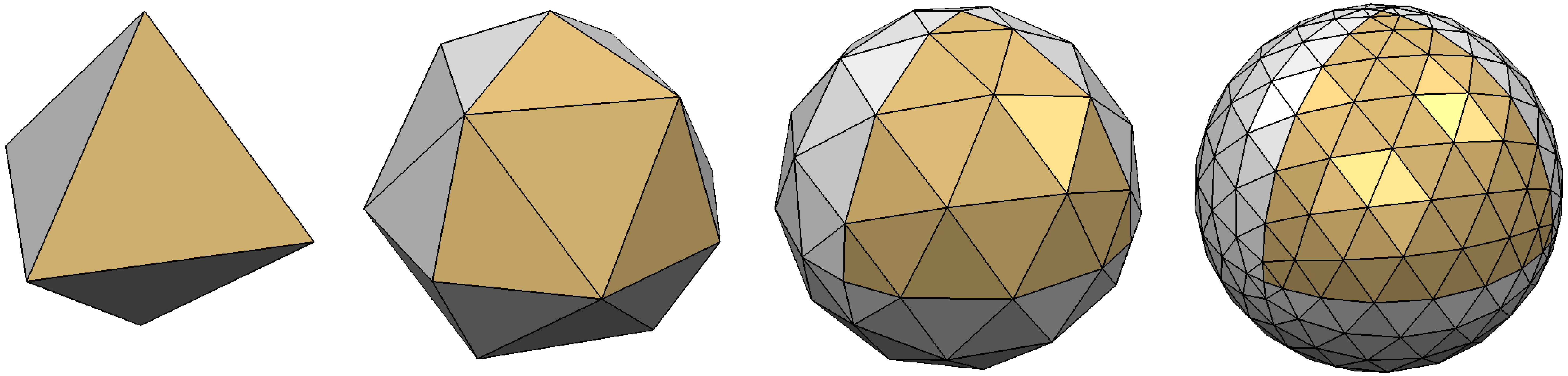
- Meshes
- Point set surfaces

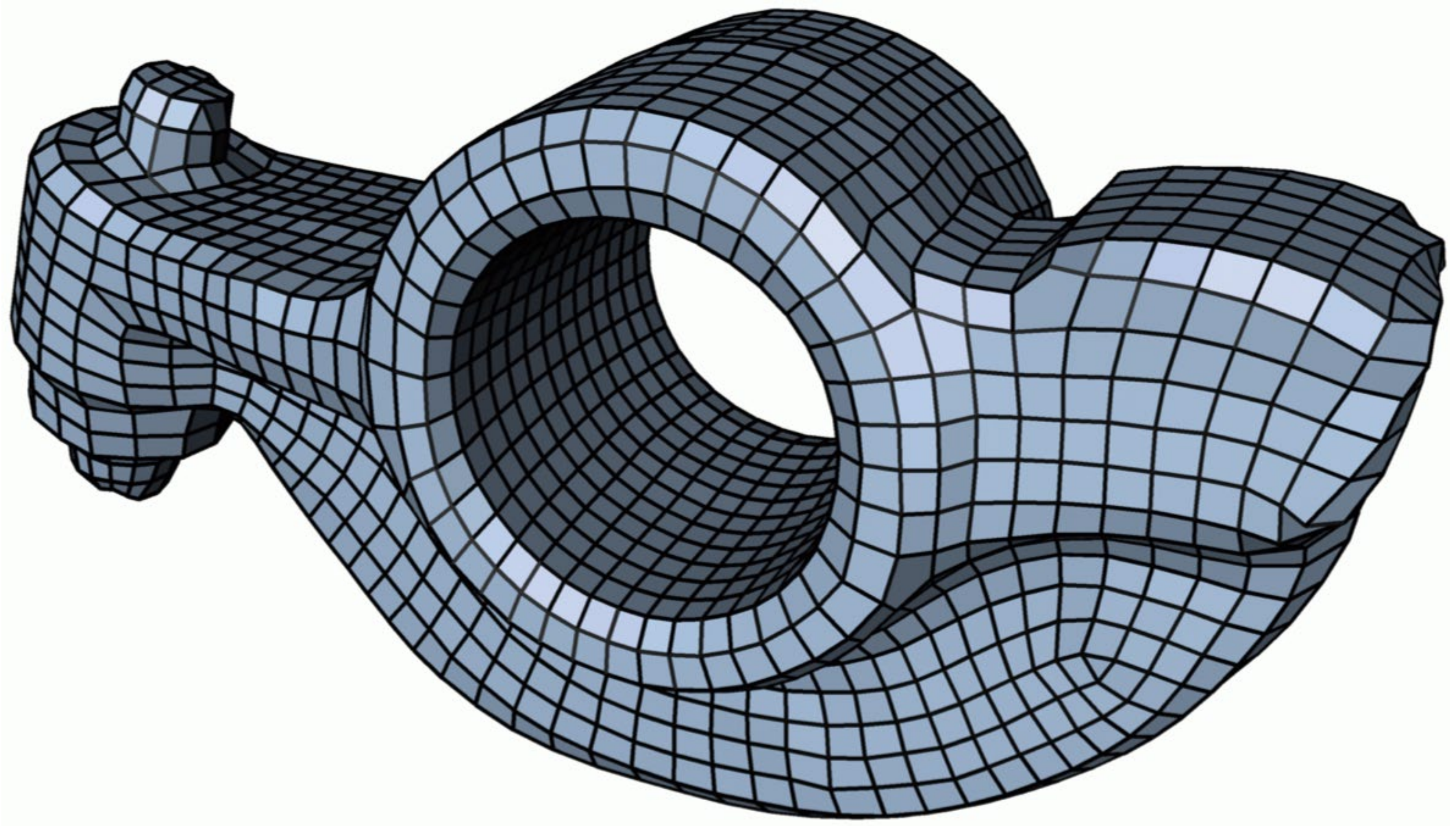
After a slide by Olga Sorkine-Hornung

Polygonal Meshes

Boundary representations of objects

- Piecewise linear





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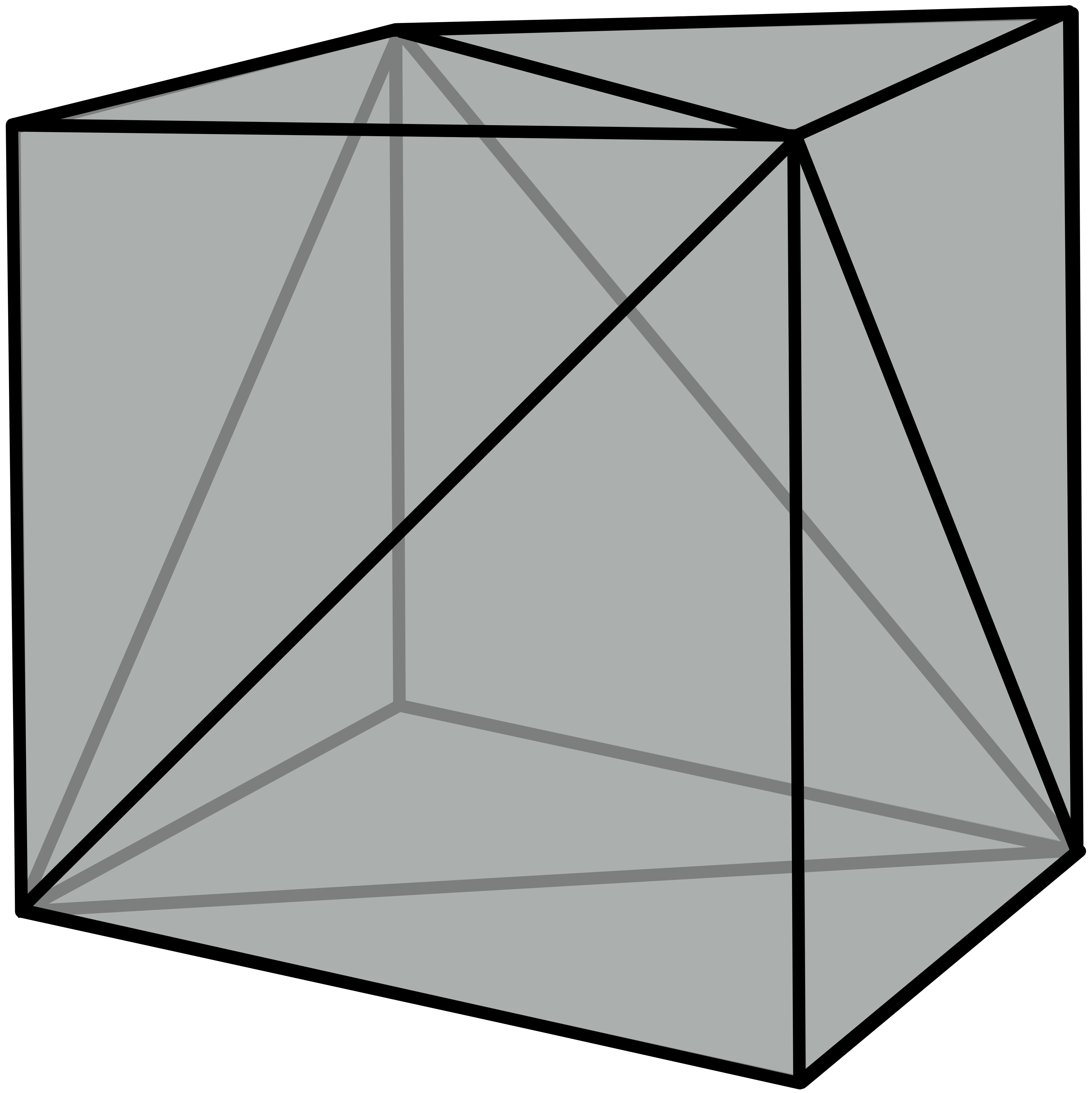
A small triangle mesh



12 triangles, 8 vertices

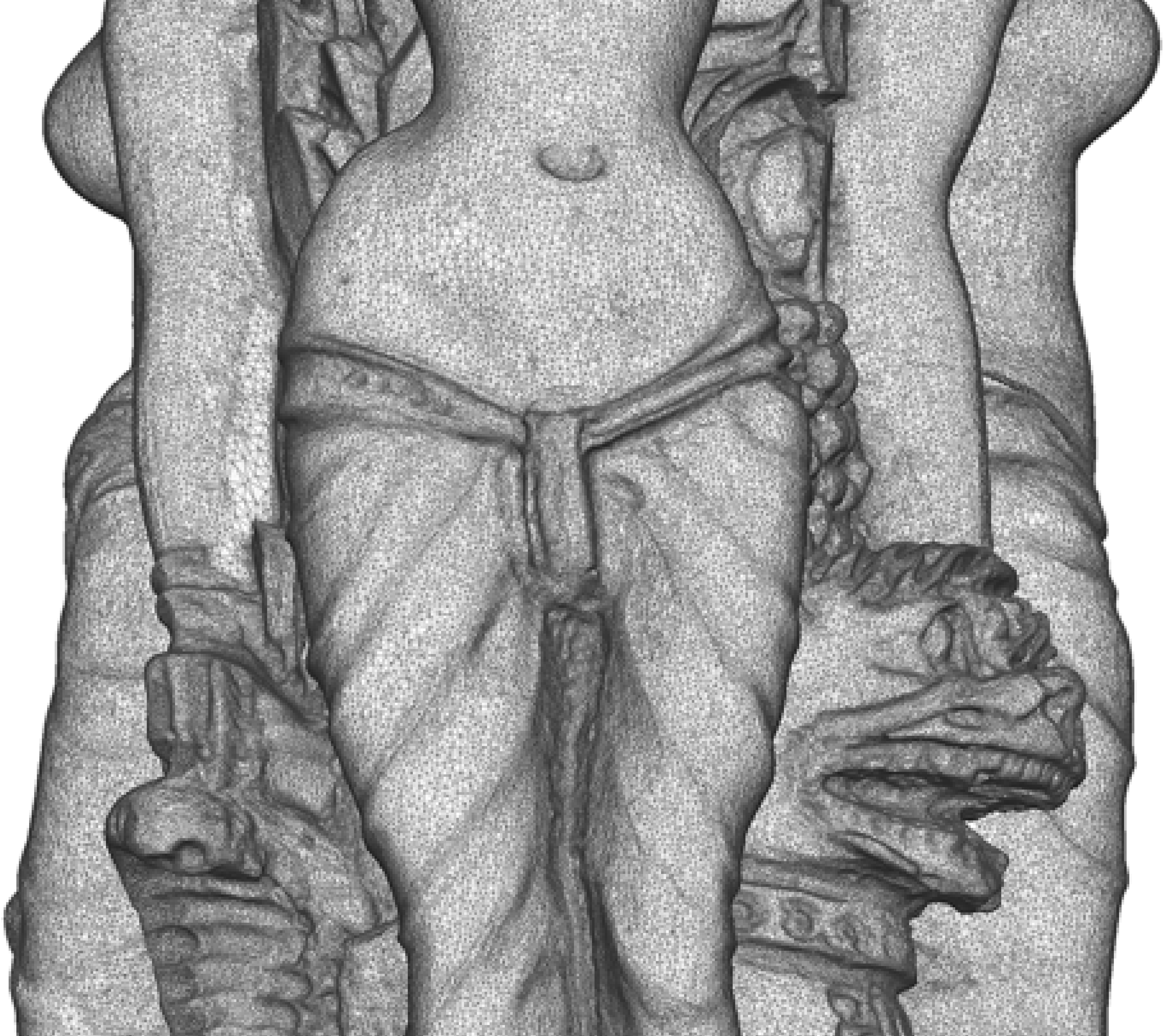
After a slide by Steve Marschner

A large mesh

10 million triangles from a high-resolution 3D scan

Traditional Thai sculpture—scan by XYZRGB, inc., image by MeshLab project

After a slide by Steve Marschner



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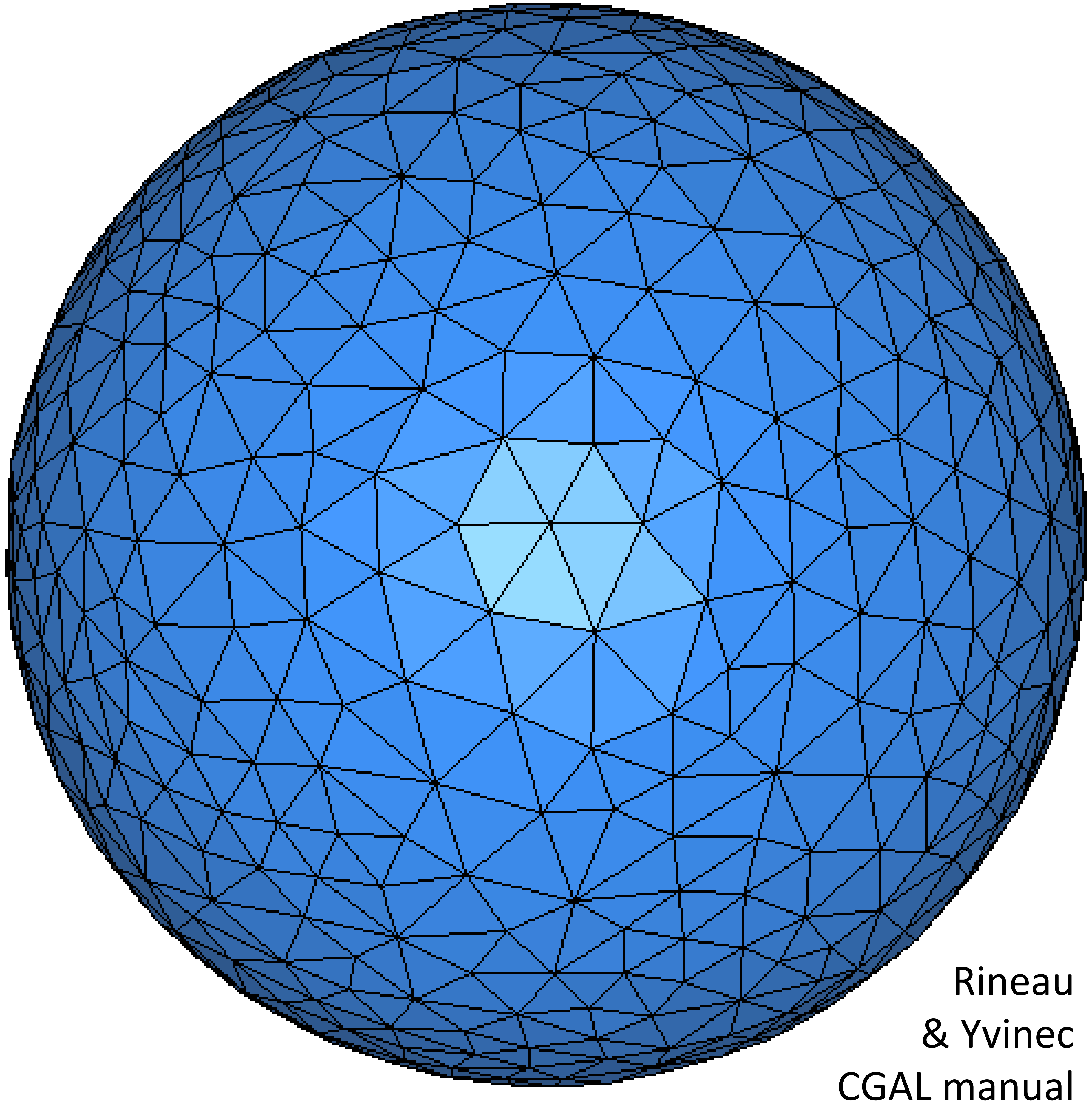
After a slide by Steve Marschner


Andrzej Barabasz

spheres


Rineau
& Yvinec
CGAL manual

approximate
sphere

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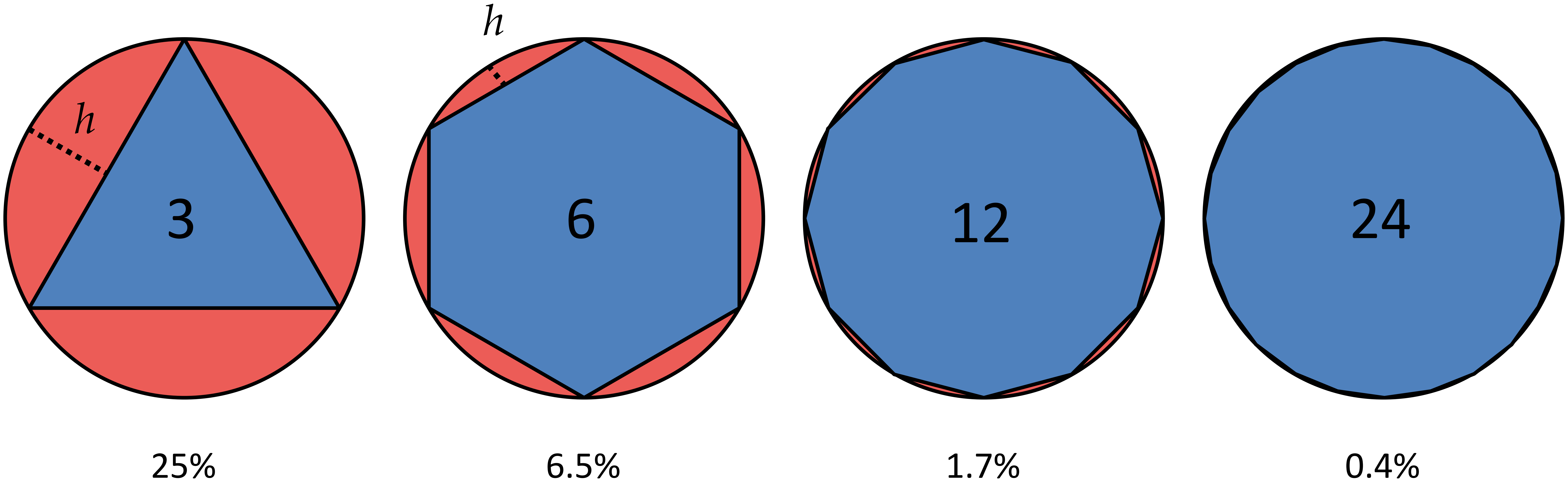
Meshes as Approx. of Smooth Surfaces

Piecewise linear approximation

- Error is $O(h^2)$



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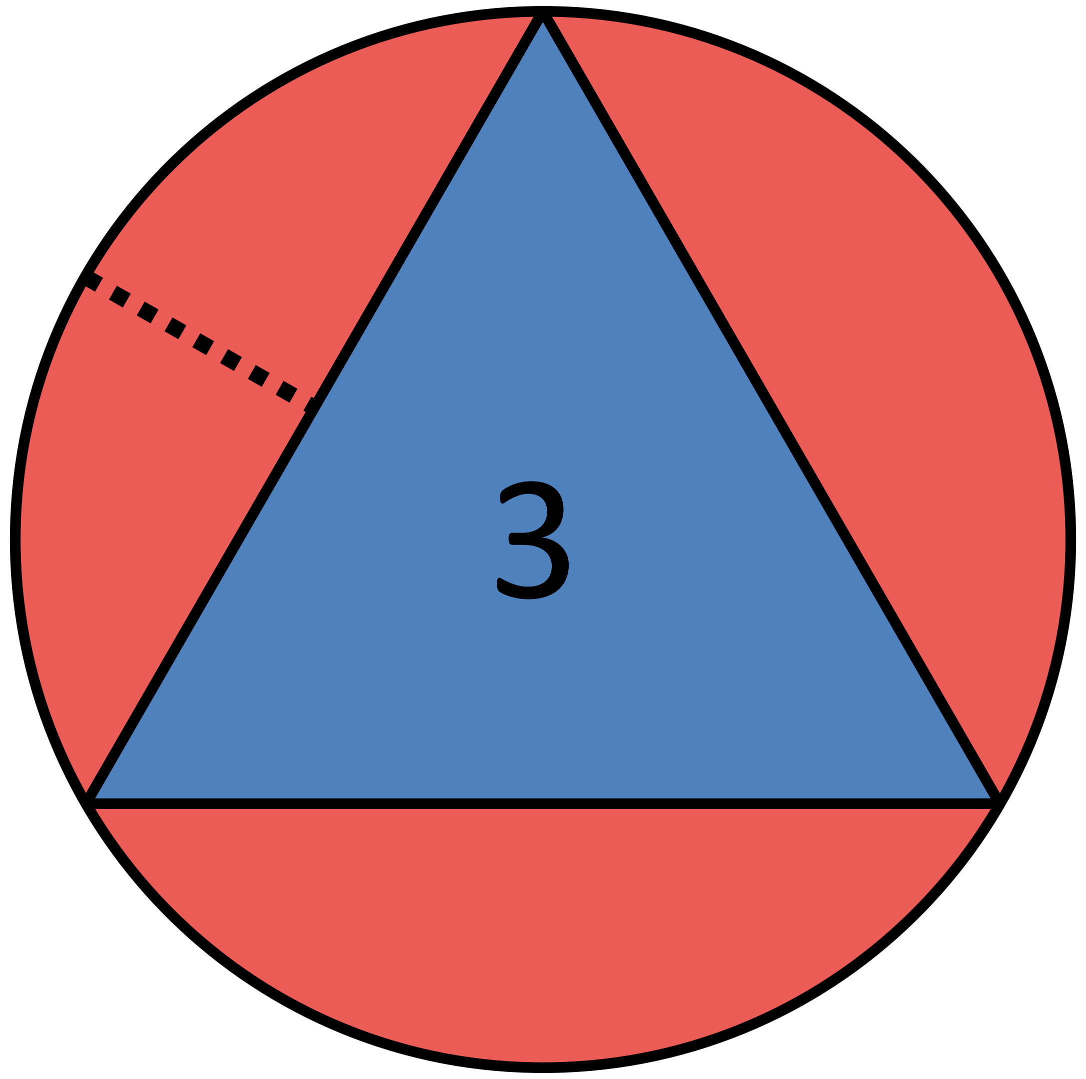
Meshes as Approx. of Smooth Surfaces

Piecewise linear approximation

- Error is $O(h^2)$



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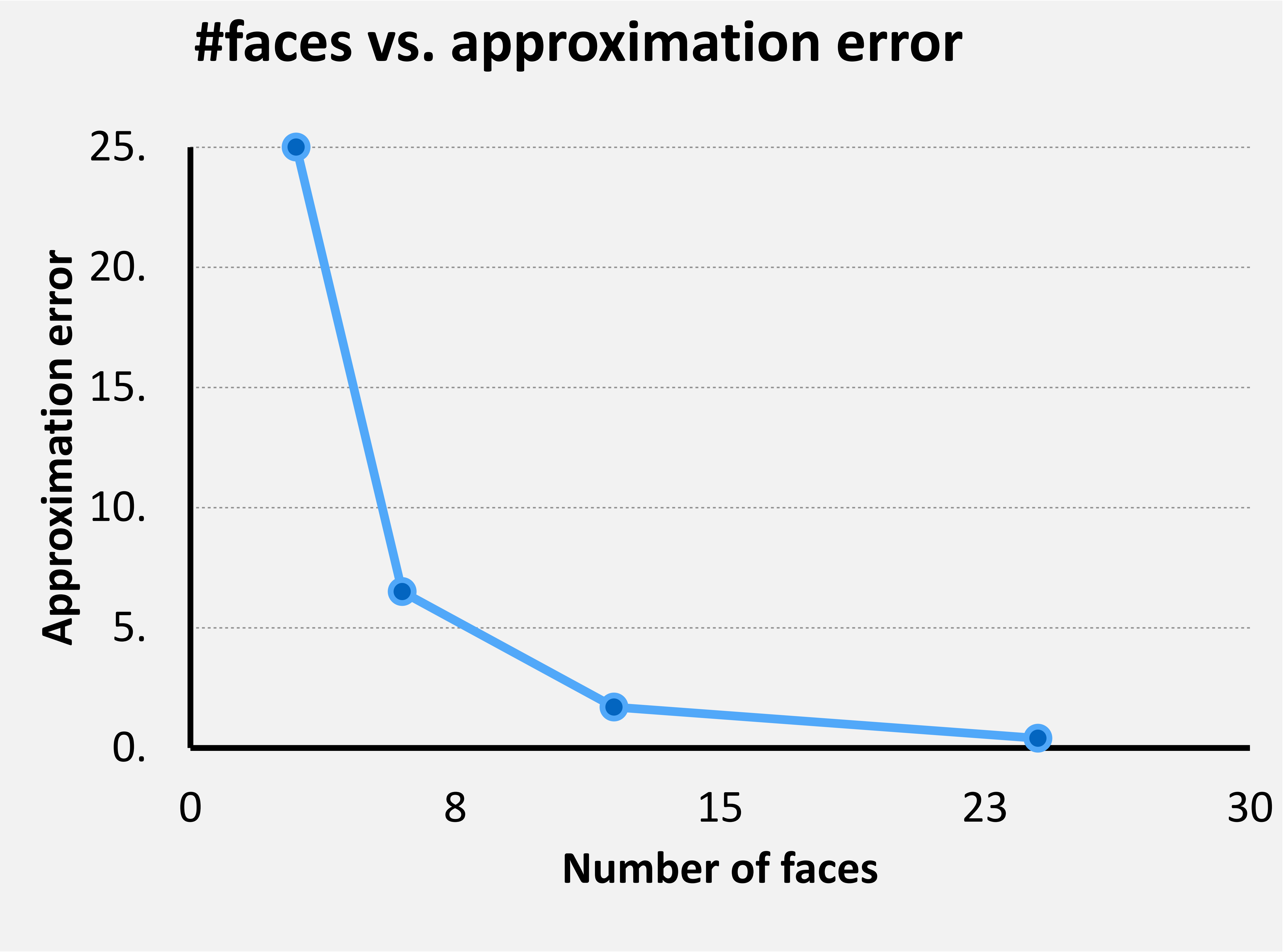

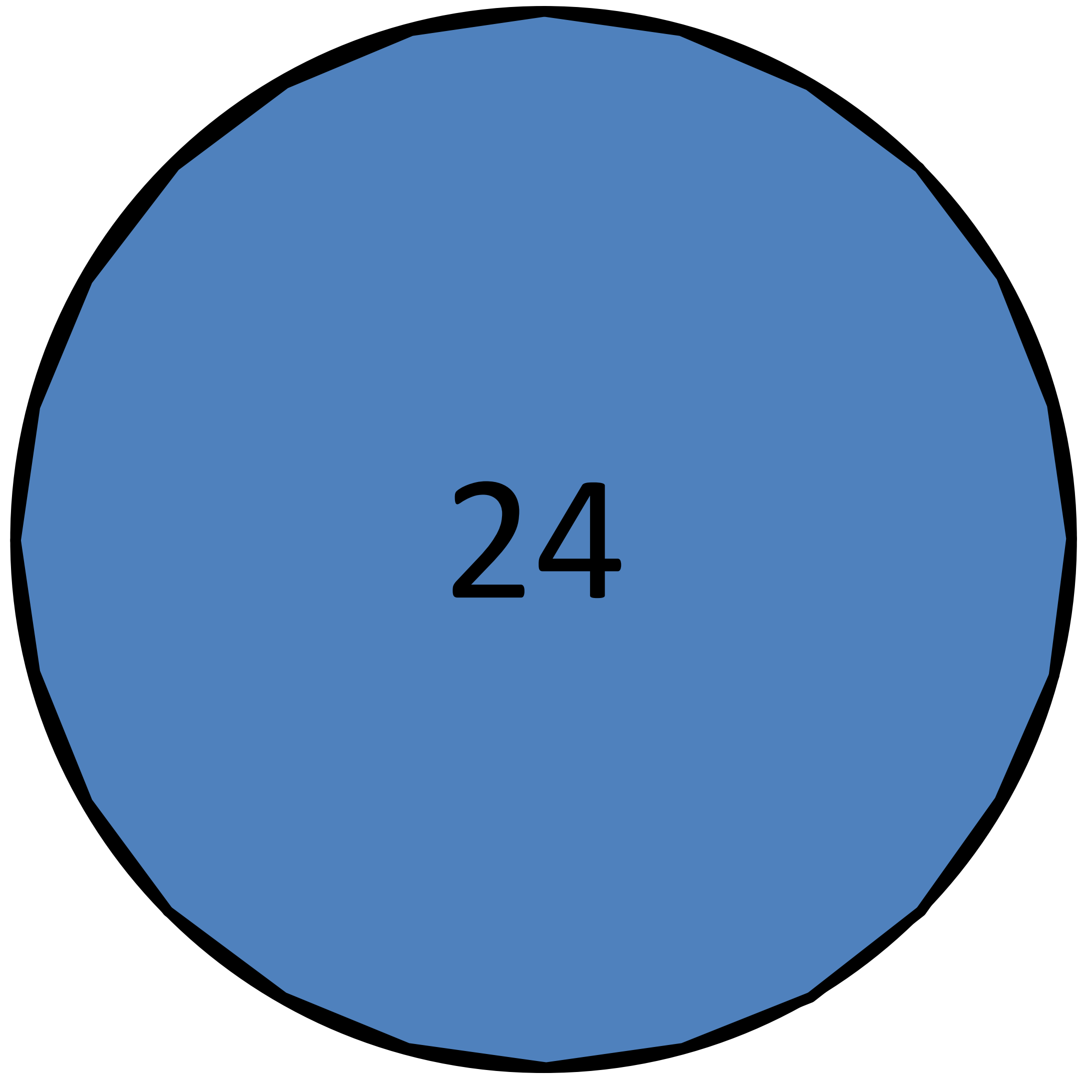

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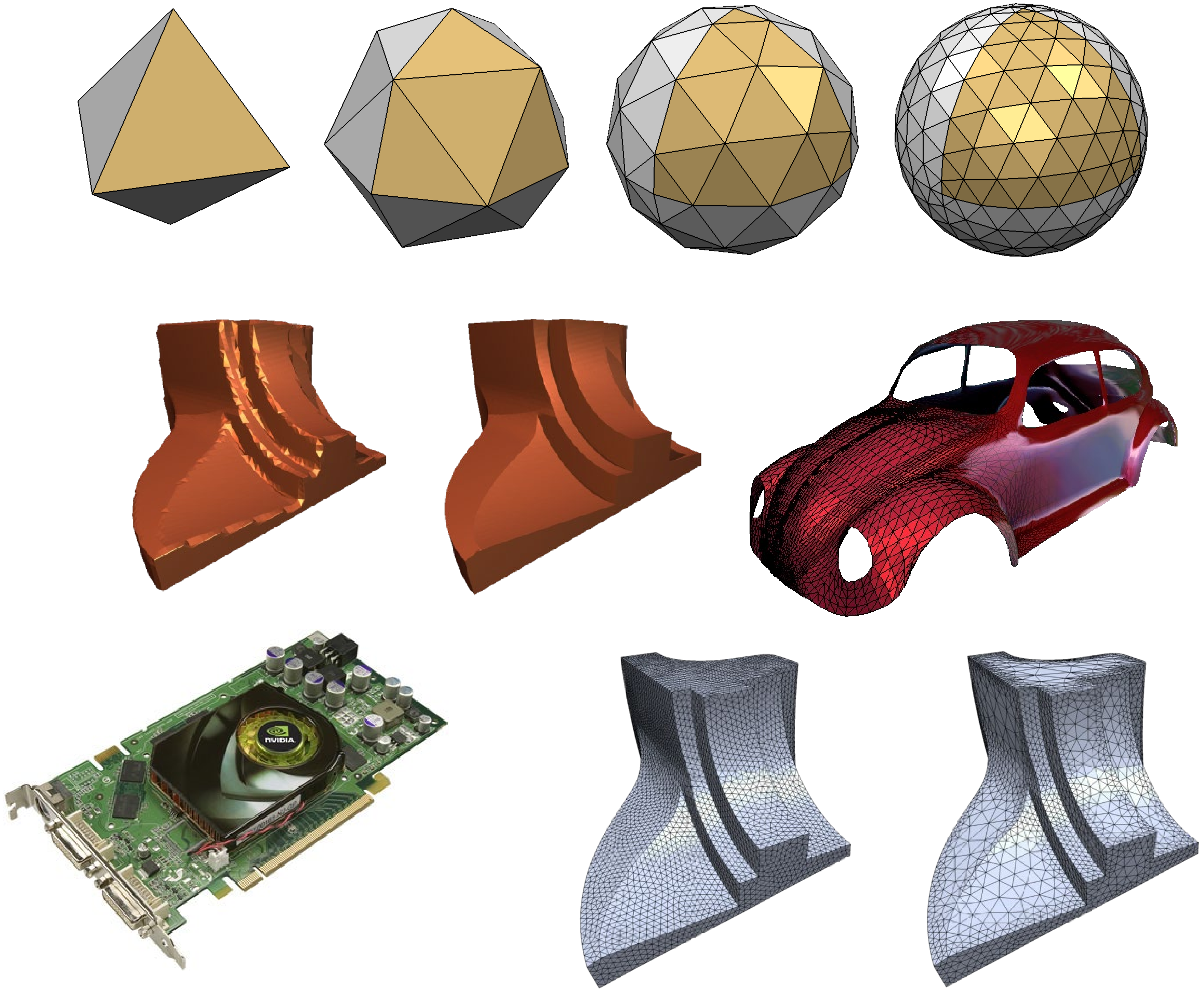
After a slide by Olga Sorkine-Hornung

Polygonal Meshes

Polygonal meshes are a good representation

- approximation $O(h^2)$
- arbitrary topology
- piecewise smooth surfaces
- adaptive refinement
- efficient rendering



After a slide by Olga Sorkine-Hornung

Data Structures: What should be stored?

After a slide by Olga Sorkine-Hornung

Geometry: 3D coordinates

Attributes

- Normal, color, texture coordinates
- Per vertex, face, edge

Connectivity

- Adjacency relationships

Separate Triangle List or Face Set (STL)

Face: 3 vertex positions

Storage:

- 4 Bytes/coordinate (using 32-bit floats)
- 36 Bytes/face

Wastes space

Triangles			
0	x0	y0	z0
1	x1	y1	z1
2	x2	y2	z2
3	x3	y3	z3
4	x4	y4	z4
5	x5	y5	z5
6	x6	y6	z6
...	...	...	...

Indexed Face Set (OBJ, OFF, WRL)

Vertex: position

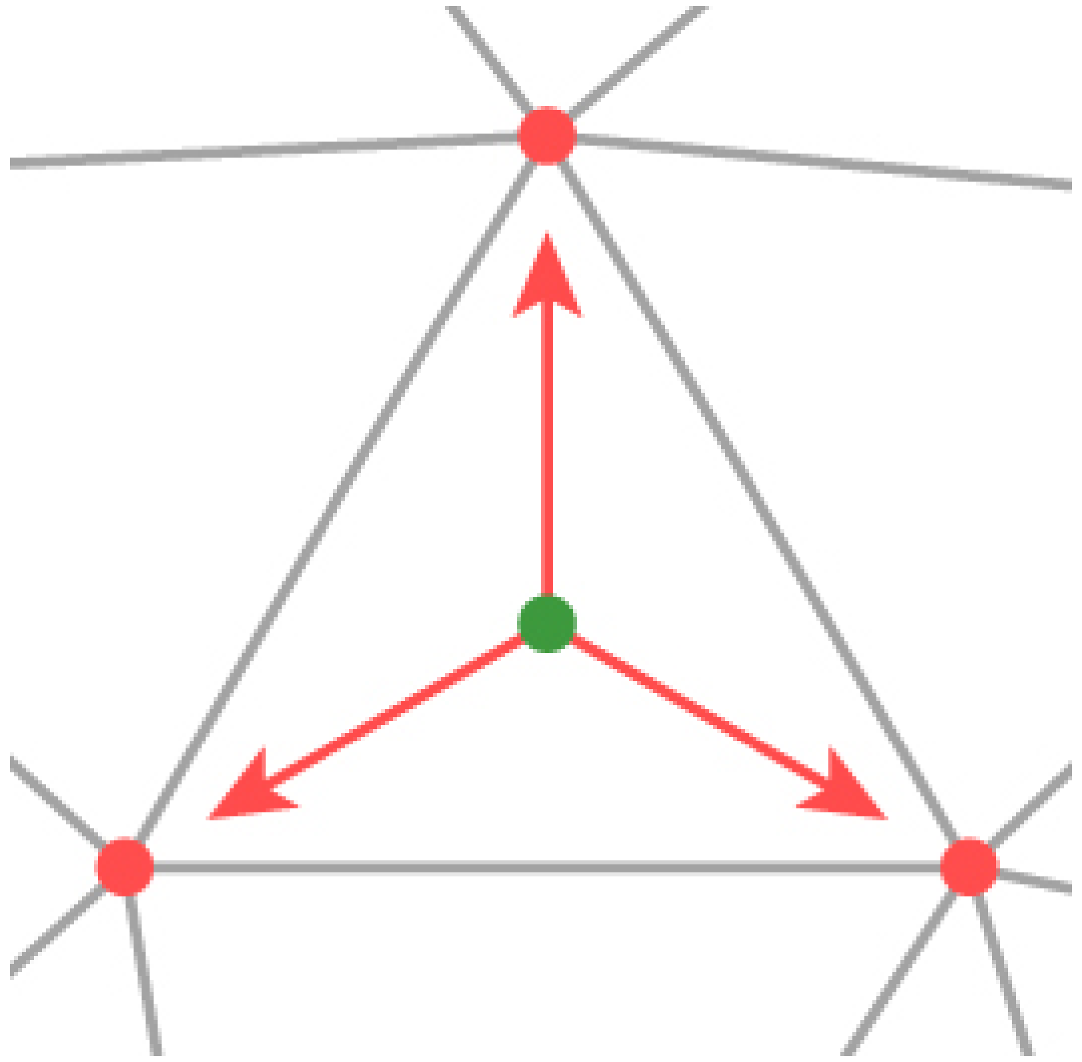
Face: vertex indices

Storage:

- 12 Bytes/vertex
- 12 Bytes/face

Reduces wasted space

Even better with per-vertex attributes



Triangles			
t0	v0	v1	v2
t1	v0	v1	v3
t2	v2	v4	v3
t3	v5	v2	v6
…	…	…	…

Vertices			
v0	x0	y0	z0
v1	x1	x1	z1
v2	x2	y2	z2
v3	x3	y3	z3
v4	x4	y4	z4
v5	x5	y5	z5
v6	x6	y6	z6
…	…	…	…

Data on meshes

Often need to store additional information besides just the geometry

Can store additional data at faces, vertices, or edges

Examples

- colors stored on faces, for faceted objects
- information about sharp creases stored at edges
- any quantity that varies *continuously* (without sudden changes, or *discontinuities*) gets stored at vertices

After a slide by Steve Marschner

Key types of vertex data

Surface normals

- when a mesh is approximating a curved surface, store normals at vertices

Texture coordinates

- 2D coordinates that tell you how to paste images on the surface

Positions

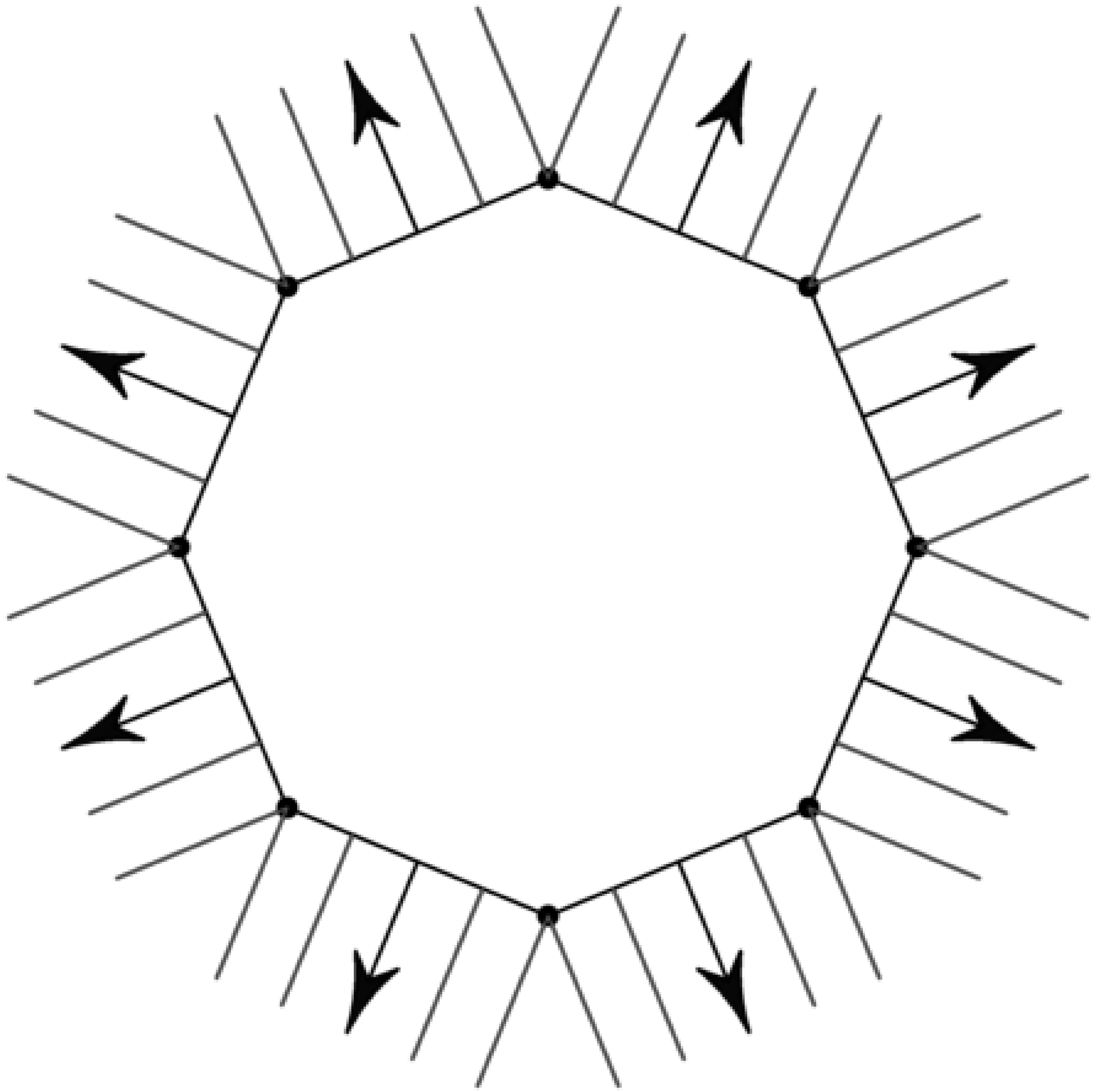
- at some level this is just another piece of data

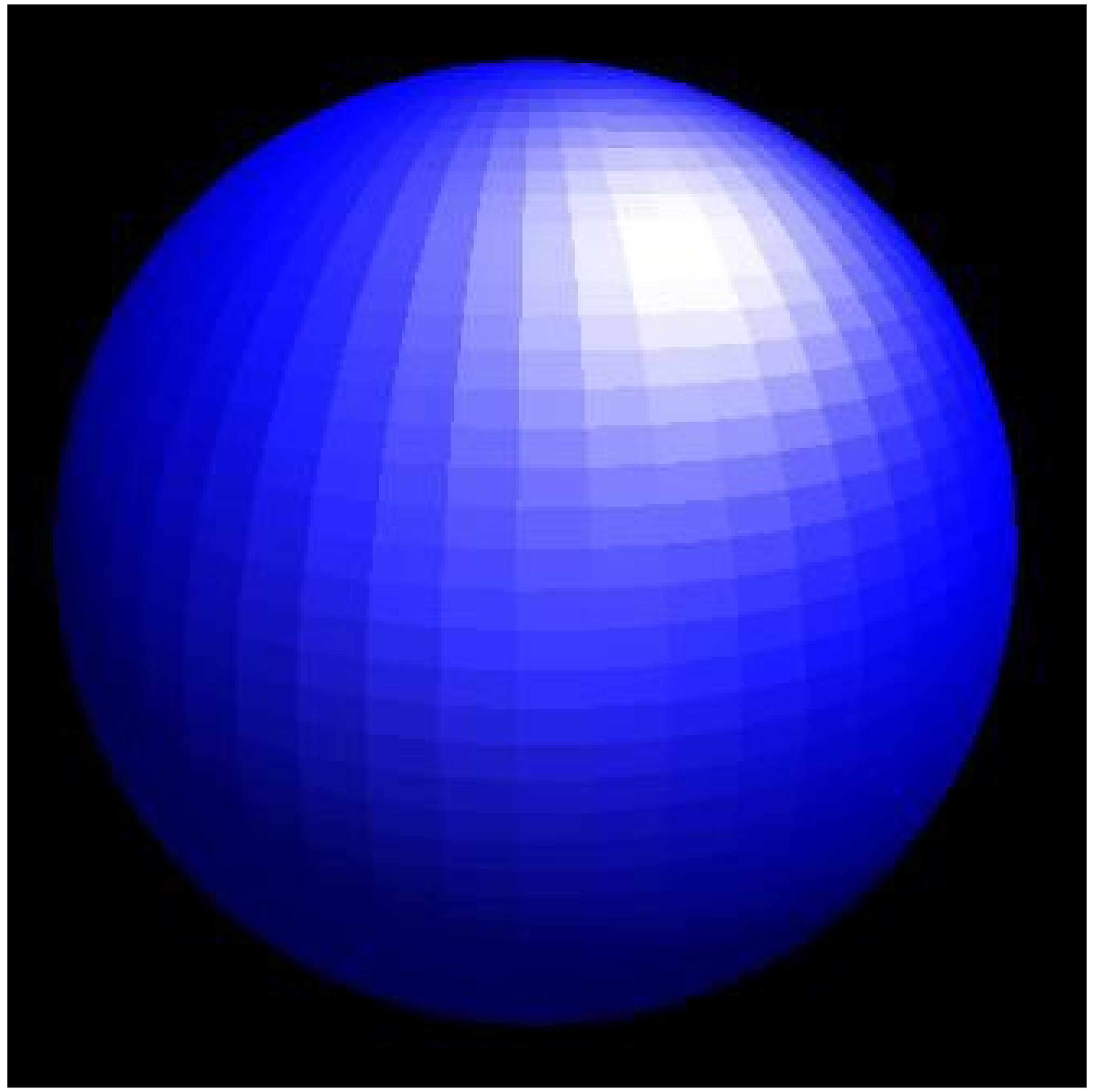
After a slide by Steve Marschner

Defining normals

Face normals: same normal for all points in face

- geometrically correct, but faceted look





After a slide by Jon Denning

Problems with face normals

Piecewise planar approximation converges pretty quickly to the smooth geometry as the number of triangles increases

- error is $O(h^2)$

But the surface normals don't converge so well

- normal is constant over each triangle, with discontinuous jumps across edges
- error is only $O(h)$

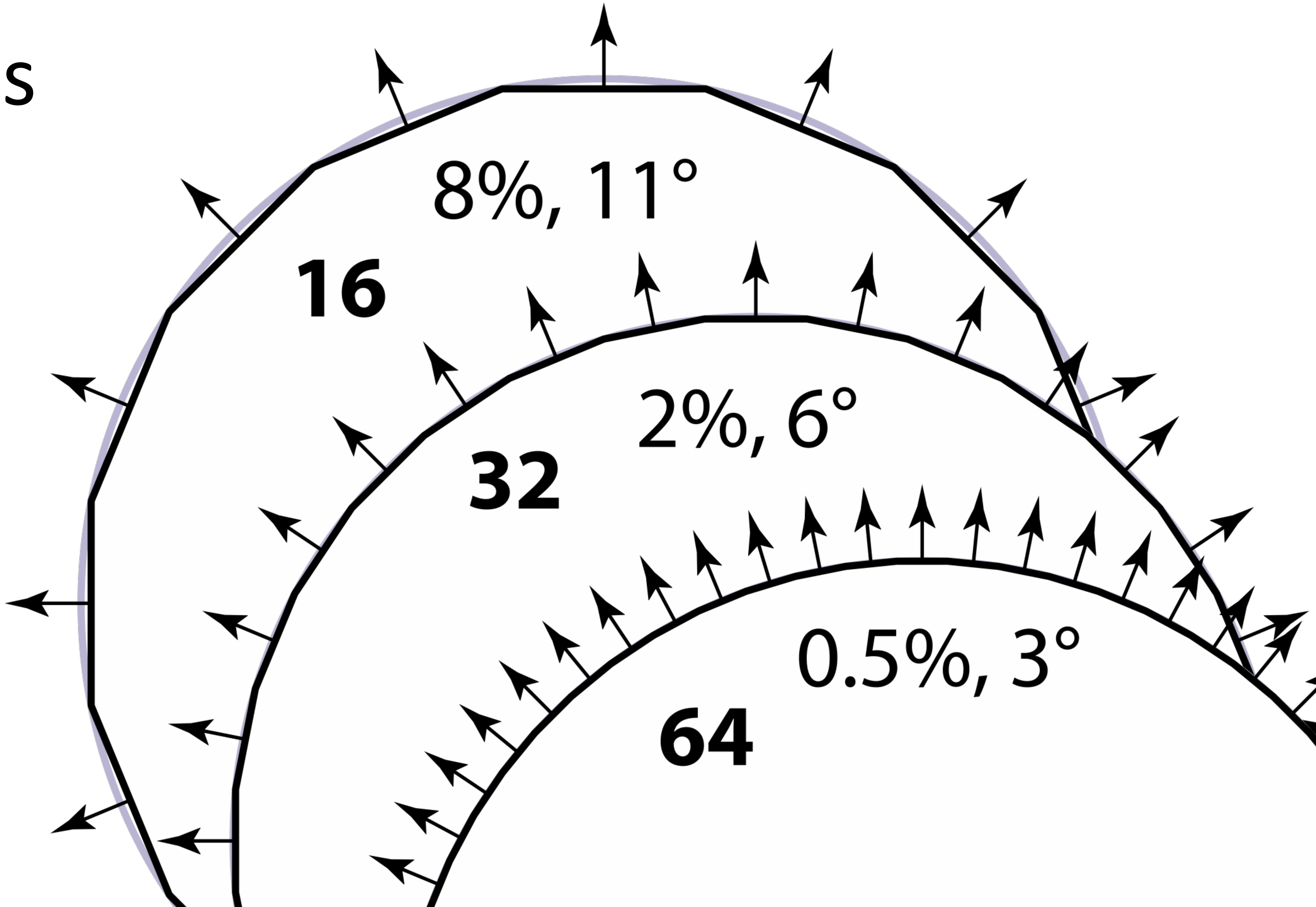
After a slide by Steve Marschner

Problems with face normals—2D example

Approximating circle with increasingly many segments

Max error in position error drops by factor of 4 each step

Max error in normal only drops by factor of 2



After a slide by Steve Marschner

Problems with face normals–solution

Piecewise planar approximation converges pretty quickly to the smooth geometry as the number of triangles increases

- for mathematicians: error is $O(h^2)$

But the surface normals don't converge so well

- normal is constant over each triangle, with discontinuous jumps across edges
- for mathematicians: error is only $O(h)$

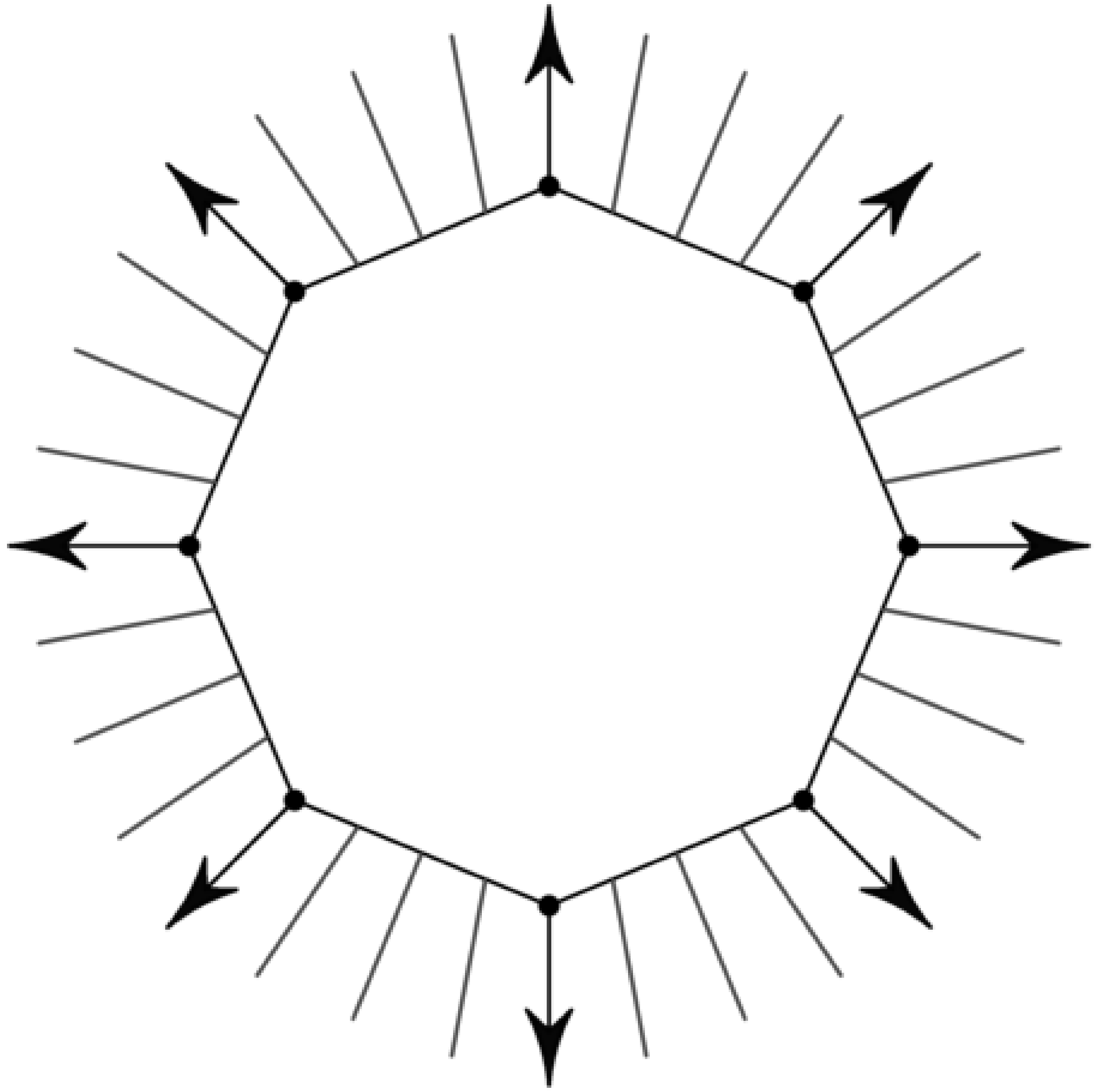
Better: store the "real" normal at each vertex, and *interpolate* to get normals that vary gradually across triangles

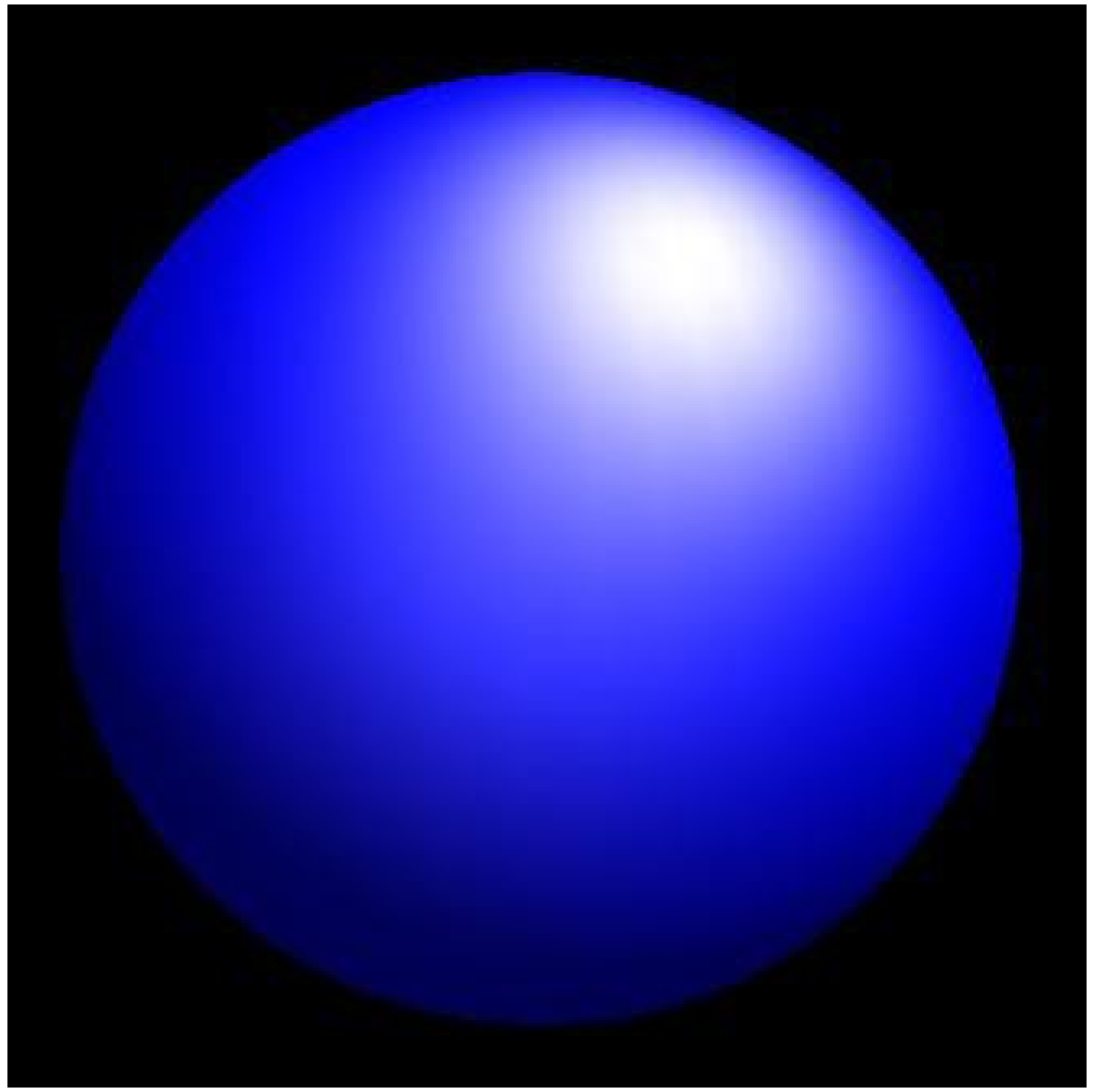
After a slide by Steve Marschner

Defining normals

Vertex normals: store normal at vertices, interpolate in face

- geometrically "inconsistent", but smooth look

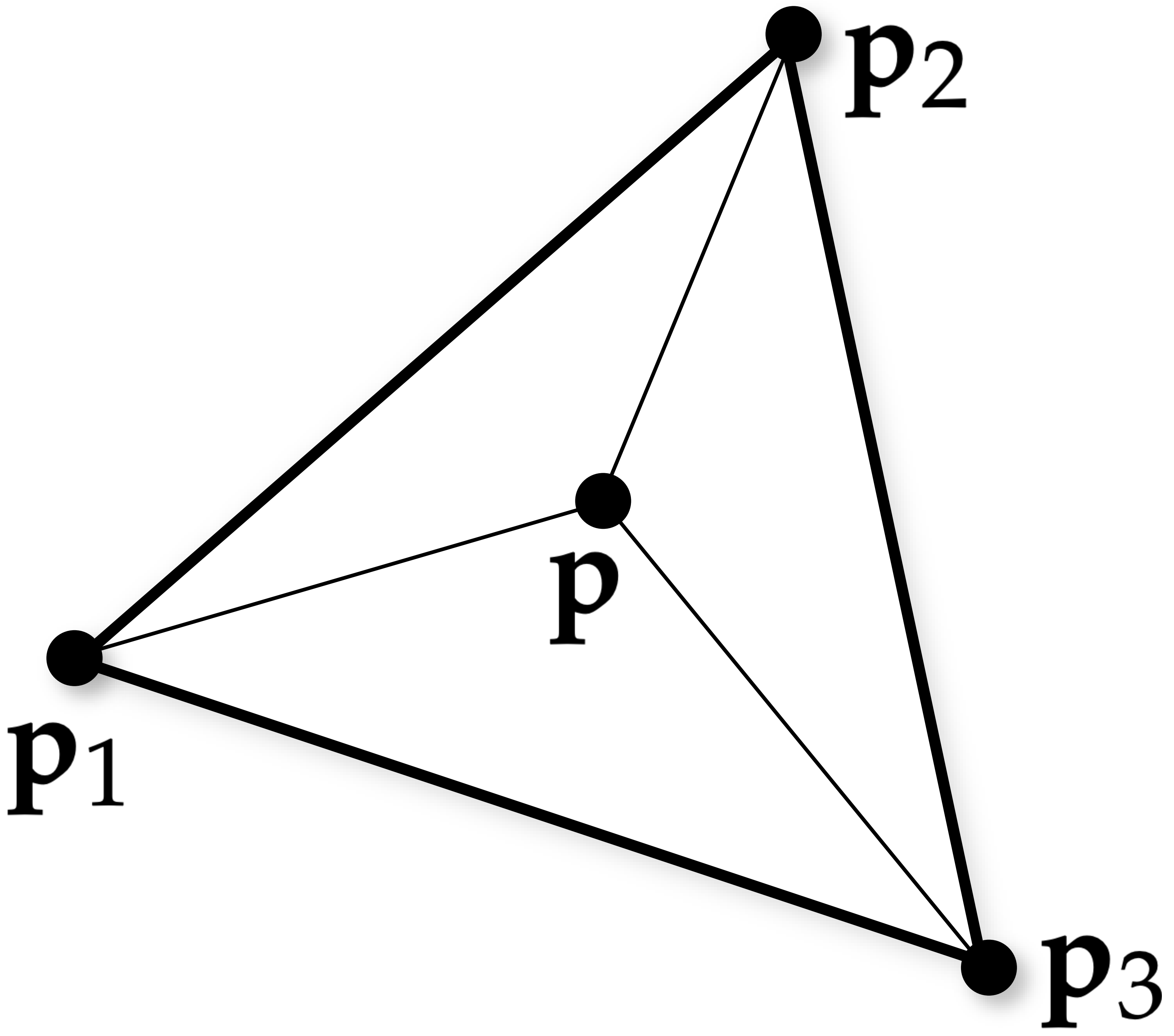




After a slide by Jon Denning

Barycentric coordinates

Barycentric interpolation: $\mathbf{p}(\alpha, \beta, \gamma) = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3$



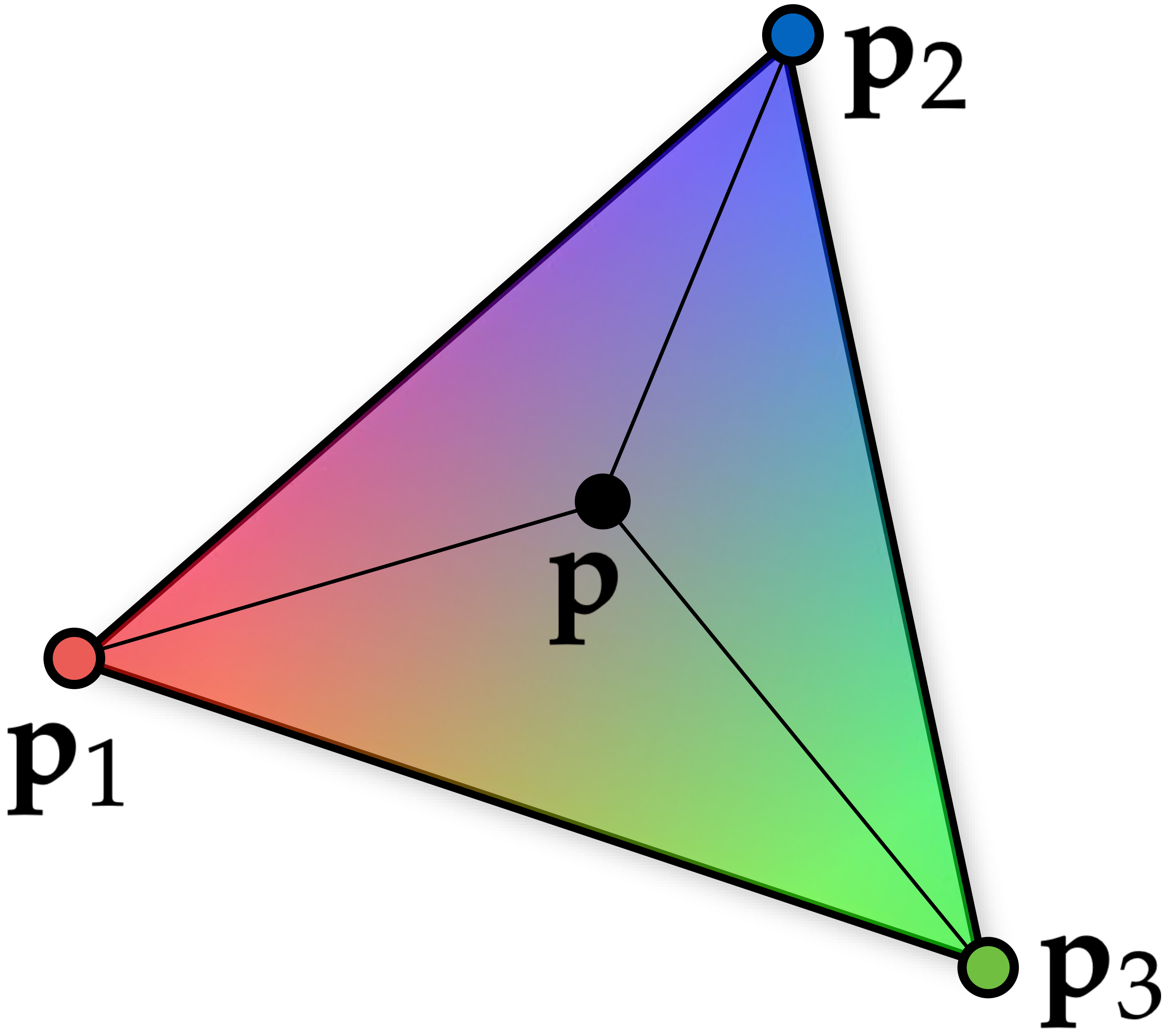
Can use this eqn. to interpolate any vertex quantity across triangle!

Barycentric coordinates

Barycentric interpolation:

$$\mathbf{p}(\alpha, \beta, \gamma) = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3$$

$$\mathbf{c}(\alpha, \beta, \gamma) = \alpha \mathbf{c}_1 + \beta \mathbf{c}_2 + \gamma \mathbf{c}_3$$



Can use this eqn. to interpolate any vertex quantity across triangle!

Barycentric coordinates

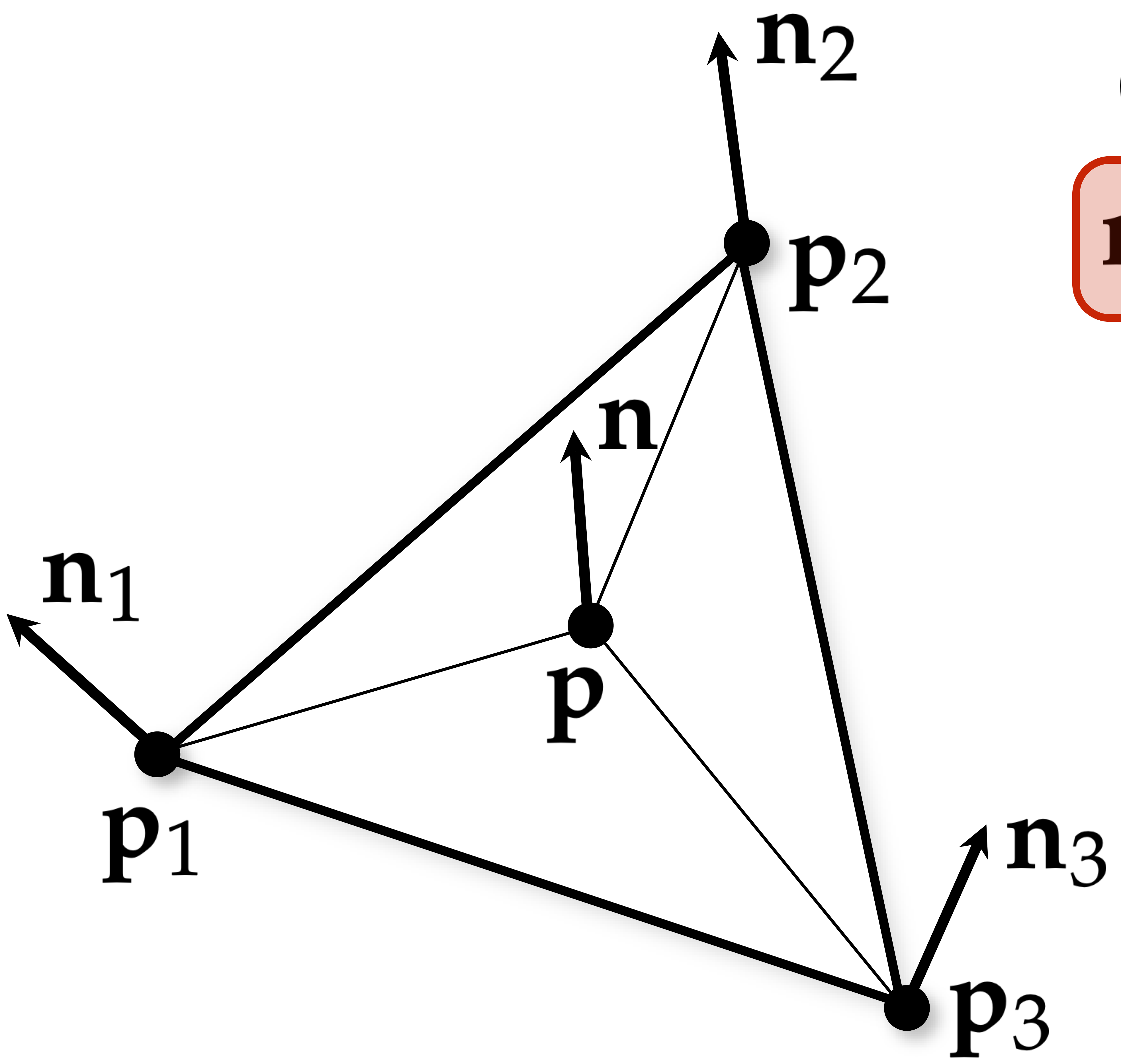
Barycentric interpolation:

$$\mathbf{p}(\alpha, \beta, \gamma) = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3$$

$$\mathbf{c}(\alpha, \beta, \gamma) = \alpha \mathbf{c}_1 + \beta \mathbf{c}_2 + \gamma \mathbf{c}_3$$

$$\boxed{\mathbf{n}(\alpha, \beta, \gamma)} = \alpha \mathbf{n}_1 + \beta \mathbf{n}_2 + \gamma \mathbf{n}_3$$

not guaranteed to be unit length



Can use this eqn. to interpolate any vertex quantity across triangle!

Realism through geometric complexity

Andreas Byström

Ray Tracing Acceleration

Ray-surface intersection is at the core of every ray tracing algorithm

Brute force approach:

- intersect every ray with every primitive
- many unnecessary ray-surface intersection tests

Andreas Byström

Ray Tracing Cost

“the time required to compute the intersections of rays and surfaces is over 95 percent” [Whitted 1980]

Cost = $O(n_x \cdot n_y \cdot n_o)$

- (number of pixels) · (number of objects)
- Assumes 1 ray per pixel

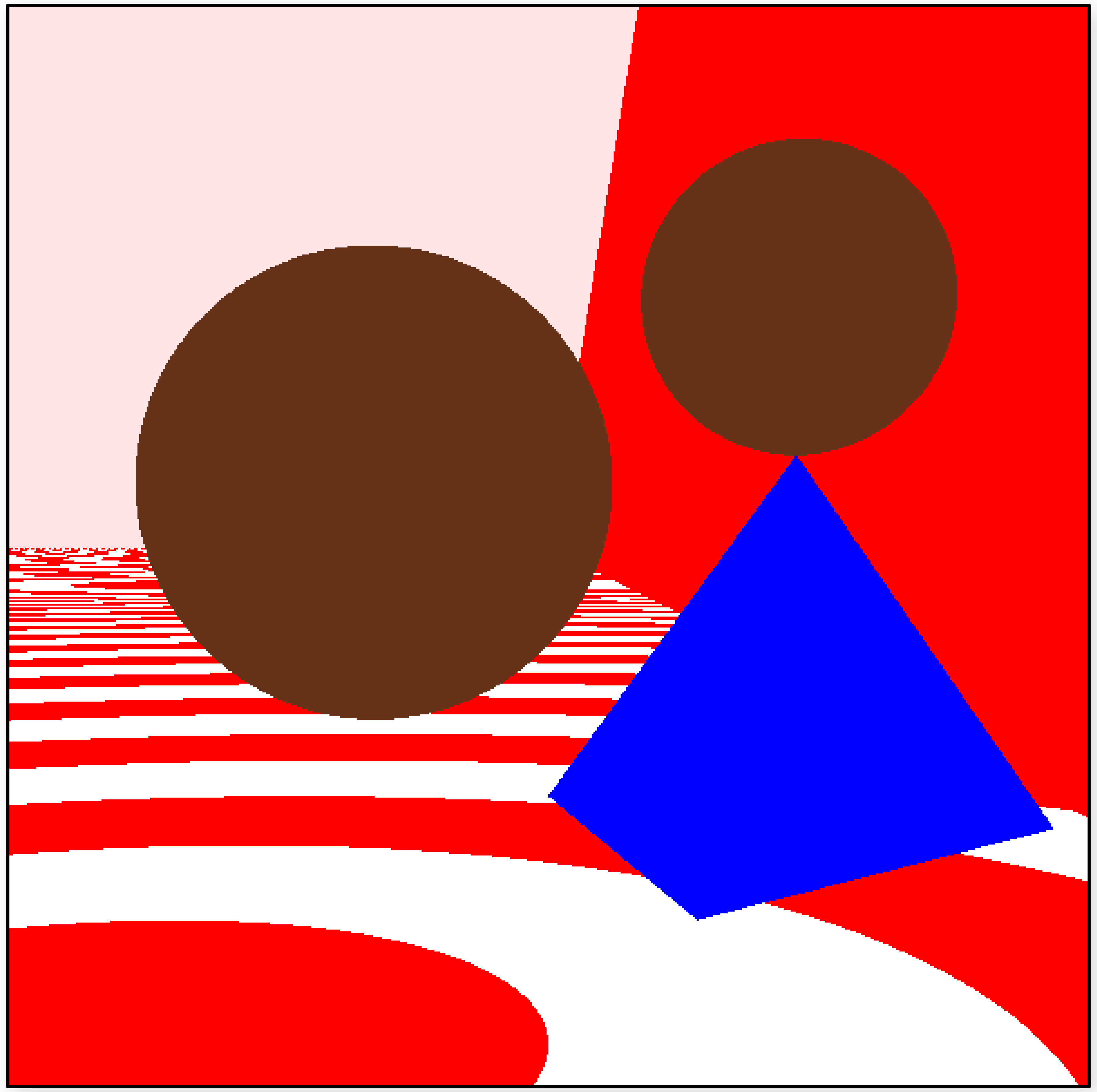
Example: 1024 x 1024 image of a scene with 1000 triangles

- Cost is (at least) 10^9 ray-triangle intersections

Typically measured per ray:

- Naive: $O(n_o)$ - linear with number of objects

O(n$_o$) Ray Tracing (The Problem)



8 primitives → 3 seconds

Andreas Byström

50K trees each with 1M polygons = 50B polygons

→ **594 years!**

Sub-linear Ray Tracing

50K trees each with 1M polygons = 50B polygons → **11 minutes**

300,000,000x speedup!

The solution

Improve efficiency of ray-surface intersections by constructing **acceleration structures**.

- A spatial organization of objects in a scene to minimize the necessary number of ray-object intersection tests.

Spatial sorting/subdivision (e.g. grid, kd-tree, ochre)

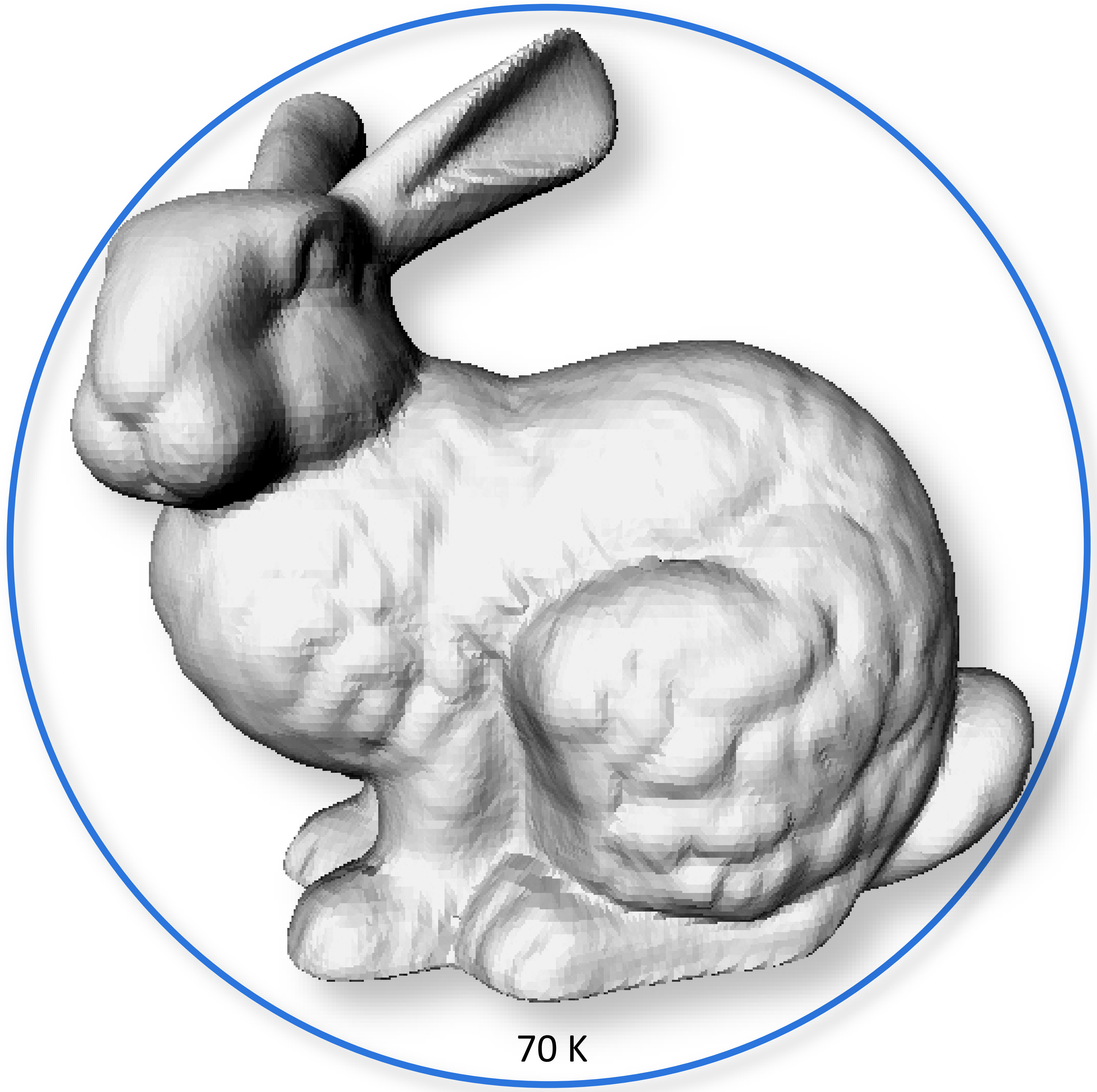
- Decompose **space** into disjoint **regions** & assign objects to regions

Object sorting/subdivision (bounding volume hierarchy)

- Decompose **objects** into disjoint **sets** & bound using simple volumes for fast rejection

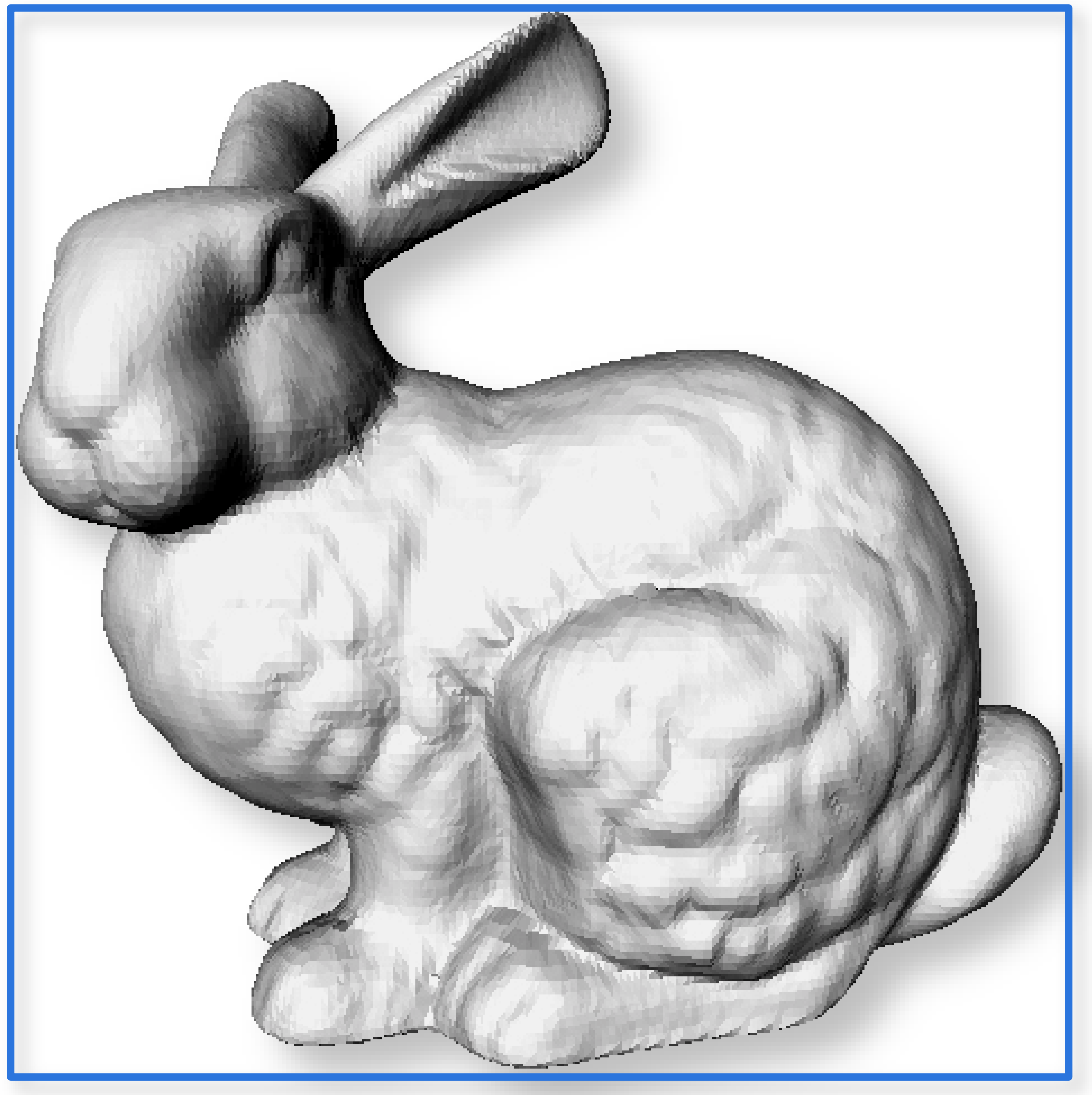
Bounding Volumes

Spheres



Bounding Volumes

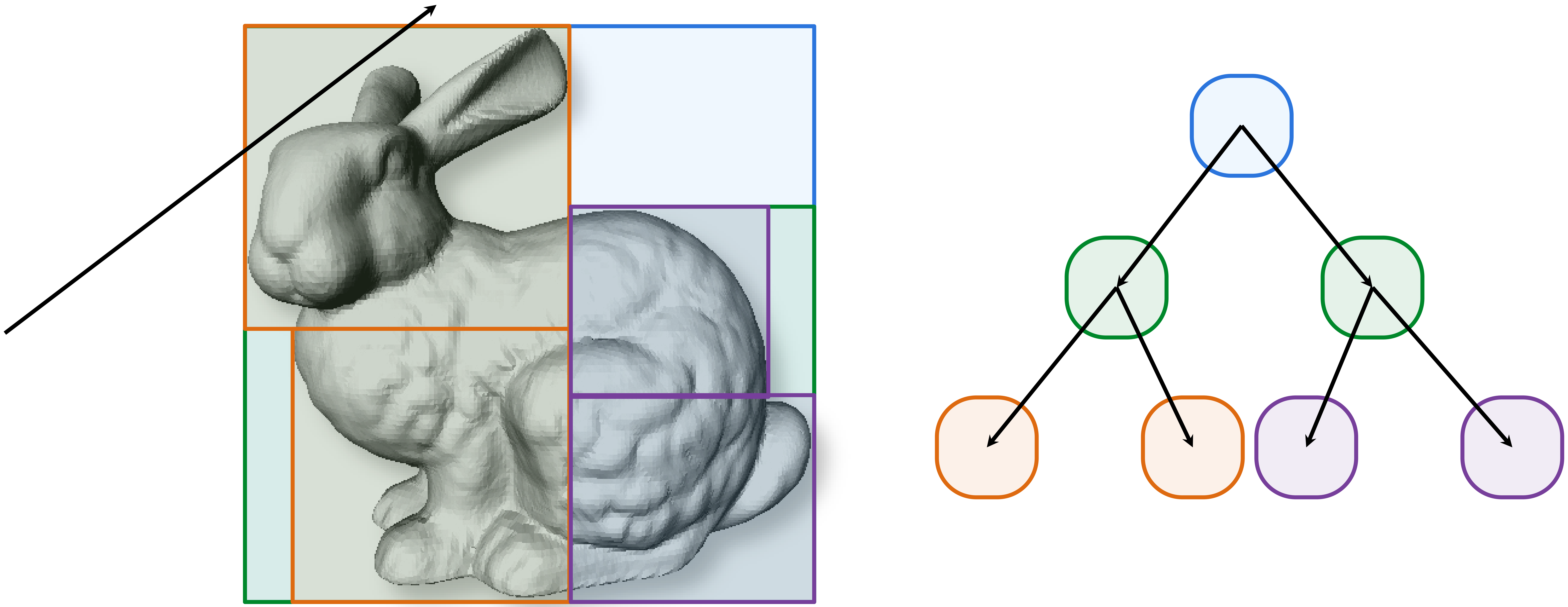
Axis-aligned bounding boxes (most common)



70 K

Bounding Volumes Hierarchies

Now do this hierarchically!



BVH Traversal

```
void BVHNode::intersectBVH(ray, &hit):
   if (bound.hit(ray)):
      if (leaf):
         leaf.intersect(ray, hit);
      else:
         leftChild.intersectBVH(ray, hit);
         rightChild.intersectBVH(ray, hit);
```

Constructing BVHs

Top-down:

- partition objects along an axis and create two sub-sets

Bottom-up:

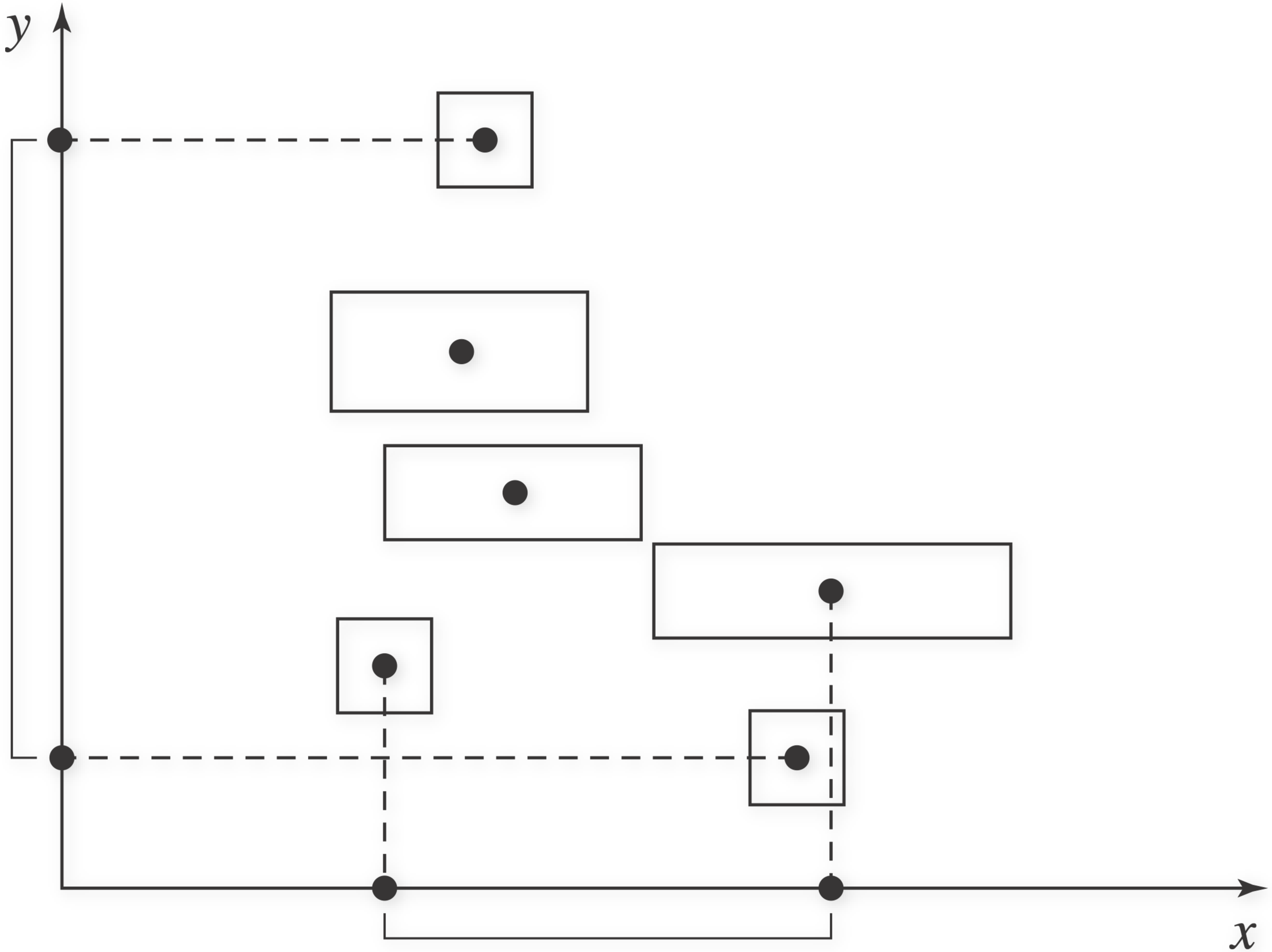
- recursively group nearby objects together

Divisive (top-down) BBH construction

1. Choose split axis
2. Choose split plane location
3. Choose whether to create leaf or split + repeat

Many strategies for each of these steps

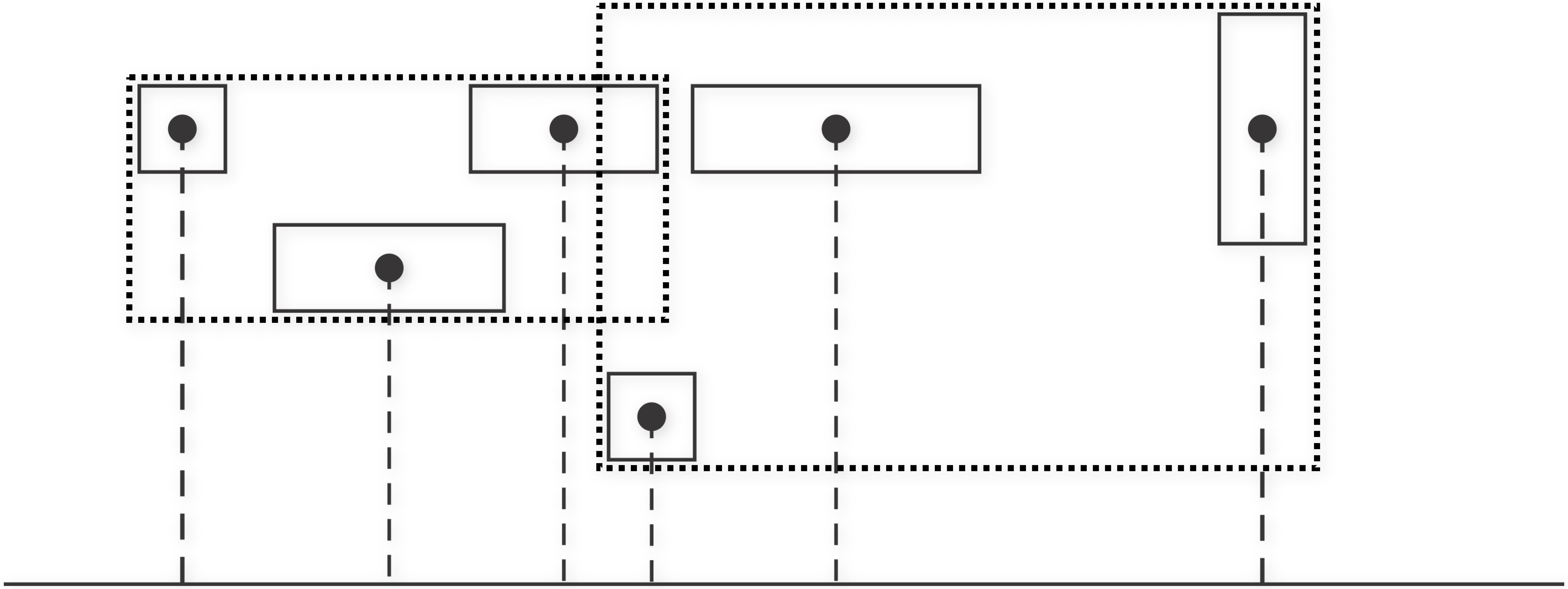
Choosing axis based on centroid extents



PBRe2 fig. 4.8

Object-median splitting

1. Sort bbox centroids along split axis
2. Take take first half as left child, second half as right



PBRe2 fig. 4.11