Radiometry and reflectance
Course announcements

• Homework assignment 4 due November 7th.
  - Generally shorter to accommodate final project proposals.
  - Two bonus parts.

• Homework assignment 5 will be posted tonight.

• No reading group this week, we’ll do one next week.

• Go over mid-semester survey.
Overview of today’s lecture

• Radiometric quantities.
• A little bit about color.
• Reflectance equation.
• Standard reflectance functions.
Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).
- Todd Zickler (Harvard).
- Srinivasa Narasimhan (CMU).
Appearance
Appearance
“Physics-based” computer vision
(a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are efficient: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena.

I $\rightarrow$ shape, illumination, reflectance
Example application: Photometric Stereo
Quantifying Light
Assumptions

Light sources, reflectance spectra, sensor sensitivity modeled separately at each wavelength

Geometric/ray optics

No polarization

No fluorescence, phosphorescence, ...
Radiometry

Radiometry studies the measurement of electromagnetic radiation, including visible light.
Radiometry

Assume light consists of photons with:

- \( \mathbf{x} \): Position
- \( \vec{\omega} \): Direction of travel
- \( \lambda \): Wavelength

Each photon has an energy of:

\[
\frac{hc}{\lambda}
\]

- \( h \approx 6.63 \times 10^{-34} \text{ m}^2 \text{ kg/s} \): Planck’s constant
- \( c = 299,792,458 \text{ m/s} \): speed of light in vacuum
- Unit of energy, Joule: \( [J = \text{kg m}^2/\text{s}^2] \)
Radiometry

How do we measure the energy flow?

Measuring energy means “counting photons”
Radiometry

Basic quantities (depend on wavelength)

- flux $\Phi$
- irradiance $E$
- radiosity $B$
- intensity $I$
- radiance $L$

will be the most important quantity for us
Flux (Radiant Flux, Power)

total amount of radiant energy passing through surface or space per unit time

\[ \Phi(A) \quad \left[ \frac{J}{s} = W \right] \]

examples:
- number of photons hitting a wall per second
- number of photons leaving a lightbulb per second (how do we quantify this exactly?)
Irradiance

*area density* of flux

flux per unit area *arriving* at a surface

\[ E(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right] \]

equation:

- number of photons *hitting* a small patch of a wall per second, *divided* by size of patch
Radiosity (Radiant Exitance)

**area density of flux**

flux per unit area **leaving** a surface

\[ B(x) = \frac{d\Phi(A)}{dA(x)} \quad \left[ \frac{W}{m^2} \right] \]

**example:**

- number of photons **reflecting off** a small patch of a wall per second, *divided* by size of patch
Radiant Intensity

directional density of flux

power (flux) per solid angle

\[ I(\omega) = \frac{d\Phi}{d\omega} \quad \left[ \frac{W}{\text{sr}} \right] \]
Solid Angle

Angle
- circle: $2\pi$ radians

\[ \theta = \frac{l}{r} \]

Solid angle
- sphere: $4\pi$ steradians

\[ \Omega = \frac{A}{r^2} \]
Subtended (Solid) Angle

Length/area of object’s *projection* onto a unit circle/sphere
Solid angle

The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

- Depends on:
  - orientation of patch
  - distance of patch

One can show: “surface foreshortening”

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]

Units: steradians [sr]
Solid angle

To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral):

\[ \Omega = \int \int \frac{\mathbf{r} \cdot \mathbf{n}}{|\mathbf{r}|^3} \, dS \]

One can show:

- "surface foreshortening"

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

Units: steradians [sr]
Radiant Intensity

directional density of flux

power (flux) per solid angle

\[ I(\vec{\omega}) = \frac{d\Phi}{d\omega} \quad \left[ \frac{W}{sr} \right] \]

\[ \Phi = \int_{S^2} I(\vec{\omega}) \, d\omega \]

example: \( \Phi = 4\pi I \) (for an isotropic point source)

- power per unit solid angle emanating from a point source
A hypothetical measurement device
Radiance

flux density per unit solid angle, per *perpendicular* unit area

\[ L(x, \omega) = \frac{d^2 \Phi(A)}{d\omega dA^\perp(x, \omega)} \left[ \frac{W}{m^2 \text{sr}} \right] \]

\[ = \frac{d^2 \Phi(A)}{d\omega dA(x) \cos \theta} \]
Radiance

fundamental quantity for vision and graphics

remains constant along a ray (in vacuum only!)

incident radiance $L_i$ at one point can be expressed as outgoing radiance $L_o$ at another point

$$L_i(x, \omega) = L_o(y, -\omega)$$
Overview of Quantities

- **flux**: $\Phi(A)$
- **irradiance**: $E(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right]$
- **radiosity**: $B(x) = \frac{d\Phi(A)}{dA(x)} \left[ \frac{W}{m^2} \right]$
- **intensity**: $I(\vec{\omega}) = \frac{d\Phi}{d\omega} \left[ \frac{W}{sr} \right]$
- **radiance**: $L(x, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos \theta dA(x) d\omega} \left[ \frac{W}{m^2 \text{sr}} \right]$
Radiance expressing *irradiance* in terms of radiance:

\[
L(x, \omega) = \frac{d^2 \Phi(A)}{\cos \theta dA(x) d\omega} \quad E(x) = \frac{d\Phi(A)}{dA(x)}
\]

\[
L(x, \omega) = \frac{dE(x)}{\cos \theta d\omega}
\]

\[
L(x, \omega) \cos \theta d\omega = dE(x)
\]

\[
\int_{H^2} L(x, \omega) \cos \theta d\omega = E(x)
\]

Integrate cosine-weighted radiance over hemisphere
Radiance

expressing *irradiance* in terms of radiance:

\[
\int_{H^2} L(x, \omega) \cos \theta \, d\omega = E(x)
\]

expressing *flux* in terms of radiance:

\[
\int_A E(x) \, dA(x) = \Phi(A) \quad E(x) = \frac{d\Phi(A)}{dA(x)}
\]

\[
\int_A \int_{H^2} L(x, \omega) \cos \theta \, d\omega \, dA(x) = \Phi(A)
\]

Integrate cosine-weighted radiance
over hemisphere and area
Radiance

Allows computing the radiant flux measured by any sensor

\[ \Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA \]

Cameras measure integrals of radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to (integrals of) radiance.

- “Processed” images (like PNG and JPEG) are not linear radiance measurements!!
Computing spherical integrals

Express function using spherical coordinates:

\[ \int_0^{2\pi} \int_0^{\pi} f(\theta, \phi) \, d\theta \, d\phi \]

\[ ? \]

**Warning**: this is not correct!
Differential Solid Angle

Differential area on the unit sphere around direction \( \vec{\omega} \)

\[
dA = (rd\theta)(r \sin \theta d\phi)
\]

\[
d\vec{\omega} = \frac{dA}{r^2} = \sin \theta d\theta d\phi
\]

\[
\Omega = \int_{S^2} d\vec{\omega} = \int_{0}^{2\pi} \int_{0}^{\pi} \sin \theta d\theta d\phi = 4\pi
\]
Overview of Quantities

- flux: \( \Phi(\mathbf{A}) \)
- irradiance: \( E(\mathbf{x}) = \frac{d\Phi(\mathbf{A})}{dA(\mathbf{x})} \)
- radiosity: \( B(\mathbf{x}) = \frac{d\Phi(\mathbf{A})}{dA(\mathbf{x})} \)
- intensity: \( I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \)
- radiance: \( L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(\mathbf{A})}{\cos \theta dA(\mathbf{x})d\vec{\omega}} \)

\[ \frac{J}{s} = W \]
\[ \frac{W}{m^2} \]
\[ \frac{W}{m^2} \]
\[ \frac{W}{sr} \]
\[ \frac{W}{m^2 \text{sr}} \]

All of these quantities can be a function of wavelength!
Handling color

• *Any* light sensor (digital or not) has different sensitivity to different wavelengths.

• This is described by the sensor’s *spectral sensitivity function* (SSF).

• When measuring some incident *spectral* flux, the sensor produces a *scalar color* response:

\[
R = \int_{\lambda} \Phi(\lambda) f(\lambda) d\lambda
\]
Handling color – the human eye

• The human eye is a collection of light sensors called cone cells.
• There are three types of cells with different spectral sensitivity functions.
• Human color perception is three-dimensional (*tristimulus color*).

\[
\begin{align*}
\text{“short”} \quad S &= \int_\lambda \Phi(\lambda) S(\lambda) d\lambda \\
\text{“medium”} \quad M &= \int_\lambda \Phi(\lambda) M(\lambda) d\lambda \\
\text{“long”} \quad L &= \int_\lambda \Phi(\lambda) L(\lambda) d\lambda
\end{align*}
\]

cone distribution for normal vision (64% L, 32% M)
Handling color – photography

Two design choices:

• What spectral sensitivity functions $f(\lambda)$ to use for each color filter?

• How to spatially arrange (“mosaic”) different color filters

Why more green pixels?

Generally do not match human LMS.

Bayer mosaic

SSF for Canon 50D

$\lambda$
Radiometry versus photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system to electromagnetic radiation
- Luminance ($Y$) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye’s luminous efficacy curve, e.g.:

$$Y(p, \omega) = \int_0^{\infty} L(p, \omega, \lambda) V(\lambda) \, d\lambda$$
## Radiometry versus photometry

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<th>Photometry</th>
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<td>Luminous Energy</td>
</tr>
<tr>
<td><strong>Flux (Power)</strong></td>
<td>Radiant Power</td>
<td>Luminous Power</td>
</tr>
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<td><strong>Flux Density</strong></td>
<td>Irradiance (incoming)</td>
<td>Illuminance (incoming)</td>
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<td></td>
<td>Radiosity (outgoing)</td>
<td>Luminosity (outgoing)</td>
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<td><strong>Angular Flux Density</strong></td>
<td>Radiance</td>
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<tr>
<td>Photometry</td>
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<tr>
<td>Luminous Energy</td>
<td>Talbot</td>
<td>Talbot</td>
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<tr>
<td>Luminous Power</td>
<td>Lumen</td>
<td>Lumen</td>
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<td>Illuminance</td>
<td>Lux</td>
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<td>Luminosity</td>
<td>Nit, Apostlib, Blondel</td>
<td>Stilb Lambert</td>
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<tr>
<td>Luminance</td>
<td>Candela</td>
<td>Candela</td>
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<tr>
<td>Luminous Intensity</td>
<td>Candela</td>
<td>Candela</td>
</tr>
</tbody>
</table>
Modern LED light

Input power: 11 W
Output: 815 lumens
(~ 80 lumens / Watt)

Incandescent bulbs:
~15 lumens / Watt)
Reflection equation
Light-Material Interactions
The BRDF

**Bidirectional Reflectance Distribution Function**

- how much light gets scattered from one direction into each other direction

- formally: ratio of outgoing *radiance* to incident *irradiance*
The reflection equation

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

Where does the cosine come from?

This describes a local illumination model
Motivation
Motivation
BRDF Properties

Real/physically-plausible BRDFs obey:
- Energy conservation

\[ \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_i \, d\vec{\omega}_i \leq 1, \quad \forall \vec{\omega}_r \]

Where does the cosine come from?
Helmholtz Reciprocity
BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

\[
\int_{H^2} f_r(x, \omega_i, \omega_r) \cos \theta_i \, d\omega_i \leq 1, \quad \forall \omega_r
\]

- Helmholtz reciprocity

\[
f_r(x, \omega_i, \omega_r) = f_r(x, \omega_r, \omega_i)
\]

\[
f_r(x, \omega_i \leftrightarrow \omega_r)
\]
BRDFs Properties

If the BRDF is unchanged as the material is rotated around the normal, then it is **isotropic**, otherwise it is **anisotropic**.

Isotropic BRDFs are functions of just 3 variables $(\theta_i, \theta_r, \Delta \phi)$.
Isotropic vs Anisotropic Reflection
Idealized materials
Diffuse reflection
Diffuse reflection
Lambertian reflection

Also called ideal diffuse reflection
Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

Scatters light equal in all directions
BRDF is a constant
Ideal Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

\[
L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i
\]

Note: we can drop \( \omega_r \)

\[
L_r(x) = f_r \int_{H^2} L_i(x, \omega_i) \cos \theta_i \, d\omega_i
\]

If all incoming light is reflected:

\[
E(x) = B(x)
\]

Note: can also be derived from energy conservation

\[
E(x) = \int_{H^2} L_r(x) \cos \theta \, d\omega
\]

\[
E(x) = L_r(x) \int_{H^2} \cos \theta \, d\omega
\]

\[
E(x) = L_r(x) \pi
\]

\[
f_r = \frac{1}{\pi}
\]
Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

\[
L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i
\]

\[
L_r(x) = \frac{\rho}{\pi} \int_{H^2} L_i(x, \omega_i) \cos \theta_i \, d\omega_i
\]

\(\rho\): Diffuse reflectance (albedo) \([0...1]\)
Specular/Mirror reflection
Mirror reflection

Assume $\mathbf{n}$ is unit length

What are the two properties that define the reflection direction?

- Reflected direction: $\mathbf{r}$
- View direction: $\mathbf{d}$
Mirror reflection

Assume \( n \) is unit length

What two properties defined reflection direction?

- co-planar view direction, reflected direction, and normal direction
- equal angles between normal-view directions, and normal-reflected directions
Mirror reflection

Assume $\mathbf{n}$ is unit length

Diagram:
- $\mathbf{r}$: reflected direction
- $\mathbf{d}$: view direction
- $\mathbf{n}$: normal vector
- $\mathbf{p}$: point on the surface

Angles: $\theta$
Mirror reflection

- reflected direction
- view direction
- \( \theta \)
- \( \theta \)
- \( r \)
- \( n \)
- \( d \)
- \( p \)
Mirror reflection

\[ r = -n(n \cdot d) \]

reflected direction

view direction
Mirror reflection

\[ r = -2n(n \cdot d) \]

- reflected direction
- view direction
Mirror reflection

Assumes $n$ is unit length

\[ r = -2n(n \cdot d) + d \]
Specular BRDF?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

\[ L_r(\mathbf{x}, \mathbf{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \mathbf{\omega}_i, \mathbf{\omega}_r) L_i(\mathbf{x}, \mathbf{\omega}_i) \cos \theta_i \, d\mathbf{\omega}_i \]

Scatters all light into one (or two) directions
Contains a Dirac delta
Integral drops out

What is the BRDF for specular reflection/refraction?
Dirac delta functions

\[ \int_{-\infty}^{\infty} f(x) \delta(x - a) \, dx = f(a) \]

Note: careful when performing changes of variables in Dirac delta functions!
BRDF of Ideal Specular Reflection

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

What is the BRDF for specular reflection?

\[ f_r(x, \omega_i, \omega_r) = F_r(\omega_i) \frac{\delta(\omega_i - R(\omega_r, \hat{n}))}{\cos \theta_i} \]

to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)
Specular refraction
Reflection vs. Refraction
The BSDF

Bidirectional Scattering Distribution Function

- informally: how much the material scatters light coming from one direction $\mathbf{l}$ into some other direction $\mathbf{v}$, at each point $\mathbf{p}$
Refraction
Refraction
Index of Refraction

Speed of light in vacuum / speed of light in medium

These are actually wavelength dependent!

<table>
<thead>
<tr>
<th>Some values of</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
</tr>
<tr>
<td>Air at STP</td>
<td>1.00029</td>
</tr>
<tr>
<td>Ice</td>
<td>1.31</td>
</tr>
<tr>
<td>Water</td>
<td>1.33</td>
</tr>
<tr>
<td>Crown glass</td>
<td>1.52 - 1.65</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.417</td>
</tr>
</tbody>
</table>
Dispersion
Double rainbow all the way across the sky!
Dispersion: “Halos” and “Sun dogs”
Halos and Sundogs

Sundogs are produced by hexagonal plate shaped ice crystals drifting with their large faces nearly horizontal.

Sundog rays pass through crystal faces inclined 60° to each other.

Rays are deviated by 22° or more. Red is deviated least, giving the 'dog' a red inner edge.

All crystals refract the sun’s rays but we see only those that glint their light towards our eyes. They are the crystals that, to us, are 22° or more from the sun and at the same altitude. Their collective glints form the sundogs.
Specular transmission/refraction

Snell’s law

\[ \eta_1 \sin \theta_1 = \eta_2 \sin \theta_2 \]
Specular transmission/refraction

Snell’s law

\[ t = \eta_1 / \eta_2 \left( d - (d \cdot n) n \right) - n \sqrt{1 - \eta_1^2 / \eta_2^2 \left( 1 - (d \cdot n)^2 \right)} \]
What is this dark circle?
What is this dark circle?

Called “Snell's window”

Caused by total internal reflection

source: mrreid.org
Recall...

Snell’s law

\[ t = \frac{\eta_1}{\eta_2} (d - (d \cdot n) n) - n \sqrt{1 - \frac{\eta_1^2}{\eta_2^2} \left(1 - (d \cdot n)^2\right)} \]

Can only happen when the ray starts in the higher index medium.
Total Internal Reflection
Total Internal Reflection
Total Internal Reflection
BTDF of Ideal Specular Refraction

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

What is the BTDF for specular refraction?

\[ f_t(x, \vec{\omega}_i, \vec{\omega}_r) = \frac{\eta_1^2}{\eta_2^2} (1 - F_r(\vec{\omega}_i)) \frac{\delta(\vec{\omega}_i - T(\vec{\omega}_r, \vec{n}))}{\cos \theta_i} \]

Fresnel reflection

Dirac delta

Refractive function

to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)
Reflection vs. Refraction

How much light is reflected vs. refracted?

- in reality determined by “Fresnel equations”
Fresnel Equations

*Reflection and refraction* from smooth *dielectric* (e.g. glass) surfaces

*Reflection* from *conducting* (e.g. metal) surfaces

Derived from Maxwell equations

Involves polarization of the wave
Fresnel Equations for Dielectrics

Reflection of light polarized parallel and perpendicular to the plane of refraction

\[ \rho_{\parallel} = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} \]
\[ \rho_{\perp} = \frac{\eta_1 \cos \theta_1 - \eta_2 \cos \theta_2}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2} \]

reflected: \[ F_r = \frac{1}{2} \left( \rho_{\parallel}^2 + \rho_{\perp}^2 \right) \]

refracted: \[ F_t = 1 - F_r \]
What’s happening in this photo?

source: flickr user neofob
Polarizing Filter
Polarization

Without Polarizer

With Polarizing Filter

source: photography.ca
Polarization

Without Polarizer

With Polarizing Filter

source: wikipedia
So Far: Idealized BRDF Models

Diffuse

Specular Reflection and Refraction
Real-world materials

Metals

Dielectric
Real-world materials

Metals

Dielectric
Real materials are more complex
Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

\[ f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_r \cdot \vec{\omega}_o)^e \]

\[ \vec{\omega}_r = (2\vec{n}(\vec{n} \cdot \vec{\omega}_i) - \vec{\omega}_i) \]
Blinn-Phong BRDF

Distribution of normals instead of reflection directions

\[ f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_h \cdot \vec{n})^e \]

\[ \vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|} \]

\[ \vec{n} \quad : \text{half-way vector} \]

incident direction

fr

\[ \vec{\omega}_i \]

\[ \vec{\omega}_o \]

outgoing direction
Microfacet Theory

Key idea:

- transition from individual interactions to statistical averages
Microfacet Theory

Assume surface consists of tiny facets

Assume that the differential area being viewed/illuminated is relatively large compared to the size of microfacets

A facet can be perfectly specular or diffuse
General Microfacet Model

\[ f(\tilde{\omega}_i, \tilde{\omega}_o) = \frac{F(\tilde{\omega}_h, \tilde{\omega}_o) \cdot D(\tilde{\omega}_h) \cdot G(\tilde{\omega}_i, \tilde{\omega}_o)}{4 |(\tilde{\omega}_i \cdot \bar{n})(\tilde{\omega}_o \cdot \bar{n})|} \]

\[ \tilde{\omega}_h = \frac{\tilde{\omega}_i + \tilde{\omega}_o}{\|\tilde{\omega}_i + \tilde{\omega}_o\|} \]
GGX and Beckmann

anti-glare (Beckman, $\alpha_b = 0.023$)  
ground (GGX, $\alpha_g = 0.394$)  
etched (GGX, $\alpha_g = 0.553$)
Interesting grazing angle behavior
Extension: Anisotropic Reflection
The Oren-Nayar Model

Same concept as the microfacet models, but assumes that the facets are diffuse

Shadowing/masking + interreflections

No analytic solution; fitted approximation

\[ f_r(\vec{n}_o, \vec{n}_i) = \frac{\rho}{\pi} \left( A + B \max(0, \cos(\phi_i - \phi_o)) \sin \alpha \tan \beta \right) \]

\[ A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \]
\[ B = \frac{0.45\sigma^2}{\sigma^2 + 0.09} \]

\[ \alpha = \max(\theta_i, \theta_o) \]
\[ \beta = \min(\theta_i, \theta_o) \]

Ideal Lambertian is just a special case (\(\sigma = 0\))
Measuring BRDFs
Measuring BRDFs

source: Matusik et al. 2003
The MERL Database

"A Data-Driven Reflectance Model"
Wojciech Matusik, Hanspeter Pfister, Matt Brand and Leonard McMillan.

- http://www.merl.com/brdf/