Pinholes and lenses
Course announcements

• Homework assignment 1 is out.
  - Due September 16th.
  - Start early! Second part takes a lot of time and requires considerable handiwork.
  - You can get boxes from my office.
  - Any issues with homework assignment 1?

• Camera distribution this week: drop by my office whenever is convenient.

• Apologies for having to cancel the first reading group, I’ll make up for it this Friday.
Overview of today’s lecture

- Some motivational imaging experiments.
- Pinhole camera.
- Accidental pinholes.
- The thin lens model.
- Lens camera and pinhole camera.
- Perspective.
- Field of view.
- Orthographic camera and telecentric lenses.
Slide credits

Many of these slides were adapted from:

• Kris Kitani (15-463, Fall 2016).
• Fredo Durand (MIT).
• Gordon Wetzstein (Stanford).
The modern photography pipeline

- Post-capture processing (lectures 5-10)
- Optics and optical controls (lectures 2-3, 11-20)
- Sensor, analog front-end, and color filter array (lectures 2, 23)
- In-camera image processing pipeline (lecture 2)
Some motivational imaging experiments
Let’s say we have a sensor...

digital sensor (CCD or CMOS)
... and an object we like to photograph

What would an image taken like this look like?

digital sensor (CCD or CMOS)

real-world object
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

All scene points contribute to all sensor pixels

real-world object

digital sensor (CCD or CMOS)

What does the image on the sensor look like?
Bare-sensor imaging

All scene points contribute to all sensor pixels
What can we do to make our image look better?

real-world object

digital sensor (CCD or CMOS)
Let’s add something to this scene

What would an image taken like this look like?
Pinhole imaging

real-world object

most rays are blocked

digital sensor (CCD or CMOS)

one makes it through
Pinhole imaging

real-world object

most rays are blocked

digital sensor (CCD or CMOS)

one makes it through
Pinhole imaging

Each scene point contributes to only one sensor pixel

What does the image on the sensor look like?
Pinhole imaging

real-world object

copy of real-world object (inverted and scaled)
Pinhole camera
Pinhole camera a.k.a. camera obscura
Pinhole camera a.k.a. camera obscura

First mention ...

Chinese philosopher Mozi
(470 to 390 BC)

First camera ...

Greek philosopher Aristotle
(384 to 322 BC)
Pinhole camera terms

- real-world object
- barrier (diaphragm)
- pinhole (aperture)
- digital sensor (CCD or CMOS)
Pinhole camera terms

- real-world object
- barrier (diaphragm)
- pinhole (aperture)
- camera center (center of projection)
- image plane
- digital sensor (CCD or CMOS)
Focal length

real-world object

focal length \( f \)
Focal length

What happens as we change the focal length?

real-world object

focal length 0.5 f
Focal length

What happens as we change the focal length?

real-world object

focal length 0.5 f
Focal length

What happens as we change the focal length?

object projection is half the size

real-world object

focal length 0.5 f
Pinhole size

Ideal pinhole has infinitesimally small size
• In practice that is impossible.
Pinhole size

What happens as we change the pinhole diameter?

real-world object

pinhole diameter
Pinhole size

What happens as we change the pinhole diameter?

real-world object
Pinhole size

What happens as we change the pinhole diameter?

real-world object
What happens as we change the pinhole diameter?

object projection becomes blurrier

real-world object
Pinhole size

What happens as we change the pinhole diameter?

real-world object

Will the image keep getting sharper the smaller we make the pinhole?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?  
What do wave optics predict will happen?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
Diffraction limit

Diffraction pattern = Fourier transform of the pinhole.
• Smaller pinhole means bigger Fourier spectrum.
• Smaller pinhole means more diffraction.
What about light efficiency?

- What is the effect of doubling the pinhole diameter?
- What is the effect of doubling the focal length?
What about light efficiency?

• 2x pinhole diameter → 4x light
• 2x focal length → ¼x light
Some terminology notes

A “stop” is a change in camera settings that changes amount of light by a factor of 2

The “f-number” is the ratio: focal length / pinhole diameter
Accidental pinholes
What does this image say about the world outside?
Accidental pinhole camera

Antonio Torralba, William T. Freeman
Computer Science and Artificial Intelligence Laboratory (CSAIL)
MIT
torralba@mit.edu, billf@mit.edu
Accidental pinhole camera

window is an aperture

projected pattern on the wall

upside down

window with smaller gap

view outside window
Accidental pinspeck camera

a) Difference image
b) Difference upside down
c) True outdoor view
Pinhole camera trade-off

Small (ideal) pinhole:
1. Image is sharp.
2. Signal-to-noise ratio is low.
Pinhole camera trade-off

Large pinhole:
1. Image is blurry.
2. Signal-to-noise ratio is high.

Can we get best of both worlds?
Almost, by using lenses

Lenses map “bundles” of rays from points on the scene to the sensor.

How does this mapping work exactly?
Lens (very) basics
What is a lens?

A piece of glass manufactured to have a specific shape
What is a lens?

A piece of glass manufactured to have a specific shape

- Shape of surfaces (usually spherical)
- Type of glass
- Aperture limiting the extent of the lens

Focal length is determined by the lens’ shape and material.
The lens on your camera
How does a lens work?
Refraction

Refraction is the bending of rays of light when they move from one material to another.
How does a lens work?

Lenses are designed so that their refraction makes light rays bend in a very specific way.
The thin lens model
Thin lens model

Simplification of geometric optics for well-designed lenses.
Thin lens model

Simplification of geometric optics for well-designed lenses.

Two assumptions:
1. Rays passing through lens center are unaffected.
Thin lens model

Simplification of geometric optics for well-designed lenses.

Two assumptions:
1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.
Thin lens model

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Simplification of geometric optics for well-designed lenses.

Two assumptions:
1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.

object distance $S$  focal length $f$
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.

object distance $S$  focal length $f$
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.

- object distance $S$
- focal length $f$
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\[ \text{object distance } S \quad \text{focal length } f \]
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object distance $S$  focal length $f$
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2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.

Focusing property:
1. Rays emitted from a point on one side converge to a point on the other side.
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.

Focusing property:
1. Rays emitted from a point on one side converge to a point on the other side.
2. Bundles emitted from a plane parallel to the lens converge on a common plane.
Gaussian lens formula

How can we relate scene-space \((S, y)\) and image space \((S', y')\) quantities?
Gaussian lens formula

How can we relate scene-space \((S, y)\) and image space \((S', y')\) quantities?

Use similar triangles.

object distance \(S\)

object height \(y\)

focal length \(f\)

image height \(y'\)

sensor distance \(S'\)
How can we relate scene-space \((S, y)\) and image space \((S', y')\) quantities?

\[
\frac{y'}{y} = ?
\]

Use similar triangles.
Gaussian lens formula

How can we relate scene-space \((S, y)\) and image space \((S', y')\) quantities?

\[
\frac{y'}{y} = \frac{S'}{S}
\]

Use similar triangles.
Gaussian lens formula

How can we relate scene-space \((S, y)\) and image space \((S', y')\) quantities?

\[
\frac{y'}{y} = \frac{S'}{S}
\]

Use similar triangles.
How can we relate scene-space \((S, y)\) and image space \((S', y')\) quantities?

\[
\frac{y'}{y} = \frac{S'}{S} \\
\frac{y'}{y} = \frac{S' - f}{f}
\]

Use similar triangles.
Gaussian lens formula

How can we relate scene-space \((S, y)\) and image space \((S', y')\) quantities?

\[
\frac{1}{S'} + \frac{1}{S} = \frac{1}{f} \\
\frac{S' - f}{f}
\]

Use similar triangles.
• We call \(m = y'/y\) the magnification.
Special focus distances

$S' = f, S = ?, m = ?$

\[
\frac{1}{S'} + \frac{1}{S} = \frac{1}{f} \\
m = \frac{S' - f}{f}
\]
Special focus distances

\[ S' = f, \ S = \infty, \ m = 0 \rightarrow \text{infinity focus (parallel rays)} \]

\[
\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}
\]

\[
m = \frac{S' - f}{f}
\]

\[ S' = S = \?, \ m = \? \]
Special focus distances

\[ S' = f, \ S = \infty, \ m = 0 \rightarrow \text{infinity focus (parallel rays)} \]

\[ m = \frac{S' - f}{f} \]

\[ \frac{1}{S'} + \frac{1}{S} = \frac{1}{f} \]

\[ S' = 2f, \ m = 1 \rightarrow \text{object is reproduced in real-life size} \]

\[ S = 2f \]

\[ S' = 2f \]
Free lunch?

By using a lens, we simultaneously achieve:
1. Image is sharp.
2. Signal-to-noise ratio is high.

Do we lose anything by using a lens?
Defocus

What happens if we don’t place the sensor at the focus distance?
Defocus

What happens if we don’t place the sensor at the focus distance?

We get a blurry image. This is called defocus.

- Defocus never happens with an ideal pinhole camera.
Defocus

Can’t we just move the sensor to the correct distance?
Defocus

Can’t we just move the sensor to the correct distance?

Unless our scene is just one plane, part of it will always be out of focus.
How do we control what is in focus?
How do we control what is in focus?

We change the distance between the sensor and the lens.
How do we control what is in focus?

We change the distance between the sensor and the lens

- What happens to plane in focus?

move lens further away from sensor
How do we control what is in focus?

We change the distance between the sensor and the lens

- move lens further away from sensor

• What happens to plane in focus? → It moves closer.
The lens on your camera

Focus ring: controls distance of lens from sensor
Demonstration
Defocus

Does the mean that lenses are only good for planar scenes?
Circle of confusion

How do we find where the point will focus?

in-focus object distance $S$  focal length $f$

actual object distance $O$  sensor distance $S'$
Circle of confusion

Will the point focus at a distance smaller or larger than $S'$?
Circle of confusion

How can we compute the diameter of the circle of confusion?
Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.
Circle of confusion

How can we compute the diameter of the circle of confusion? \( \rightarrow \) Use similar triangles.

\[
\frac{y}{D/2} = \frac{|O - S|}{O}
\]

\[
\frac{y}{c/2} = ?
\]
Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.

\[
\frac{y}{D/2} = \frac{|O - S|}{O}
\]

\[
\frac{y}{c/2} = \frac{1}{m}
\]
Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.

\[
\frac{y}{D/2} = \frac{|O - S|}{O}
\]

\[
\frac{y}{c/2} = \frac{1}{m}
\]

\[
c = mD \frac{|O - S|}{O}
\]
**Depth of field**

Distance from the in-focus object plane where the circle of confusion is *acceptably small*.

![Diagram of depth of field](image)

- $c < \varepsilon \Rightarrow$ DOF = \( \frac{2\varepsilon O}{mD} \)
- **Note:** in reality, DOF is slightly asymmetrical.

**Equation:**

\[
c = mD \frac{|O - S|}{O}
\]
Depth of field

depth of field

scene
Circle of Confusion

\[ c = mD \frac{|O - S|}{O} \]

**Canon 5D Mark III:** f=50mm, f/2.8 (N=2.8), focused at 5m, pixel size=7.5μm
Defocus depends on aperture

What happens to the circle of confusion as the aperture diameter is reduced?

\[ c = mD \frac{|O - S|}{O} \]
Defocus depends on aperture diameter

What happens to the circle of confusion as the aperture diameter is reduced? → It shrinks.

\[ c = mD \frac{|O - S|}{O} \]
Defocus depends on aperture diameter

What happens to the depth of field as the aperture diameter is reduced?

$$\text{DOF} = \frac{2\varepsilon O}{mD}$$

$$c = mD \frac{|O - S|}{O}$$
Defocus depends on aperture diameter

What happens to the depth of field as the aperture diameter is reduced? → It expands.

$\text{DOF} = \frac{2\varepsilon O}{mD} \left| \frac{O - S}{O} \right|$
Aperture size

Most lenses have apertures of variable size.
• The size of the aperture is expressed as the “f-number”: The bigger this number, the smaller the aperture.

You can see the aperture by removing the lens and looking inside it.
Demonstration
Depth of field

Form of defocus blur is determined by shape of aperture.
Defocus depends on aperture diameter

If small aperture sizes reduce defocus blur, should we always use the smallest aperture?

\[ c = mD \frac{|O - S|}{O} \]
Bokeh

Sharp depth of field ("bokeh") is often desirable.
What happens as the aperture keeps getting smaller?

- Point maps to area (blurry image)
- Point maps to point (sharp image)
What happens as the aperture keeps getting smaller?

Lens becomes equivalent to a pinhole.

- No defocus, everything is sharp regardless of depth.
- Very little light, signal-to-noise ratio is just as bad as pinhole.

point maps to point
(sharp image)

point maps to point
(sharp image)
Lens camera and pinhole camera
The lens camera
The pinhole camera
The pinhole camera

Central rays propagate in the same way for both models!
Describing both lens and pinhole cameras

We can derive properties and descriptions that hold for both camera models if:

- We consider only central rays.
- We assume that everything of interest in the scene is within the depth of field.
Important difference: focal length

In a pinhole camera, focal length is distance between aperture and sensor.
Important difference: focal length

In a lens camera, focal length is distance where parallel rays intersect.

- Object distance $S$
- Focal length $f$
- Sensor distance $S'$
Describing both lens and pinhole cameras

We can derive properties and descriptions that hold for both camera models if:

• We consider only central rays.
• We assume everything of interest in the scene is within the depth of field.
• We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.
Effect of aperture size on lens and pinhole cameras

Doubling the aperture diameter:
• Increases light throughput by four times.
• Increases circle of confusion for out-of-focus plane by two times.
• Decreases depth of field by two times.

Doubling the aperture diameter:
• Increases light throughput by four times.
• Increases circle of confusion for all planes by two times.
Field of view
Field of view

also described using angle of view

Note: here I drew a lens, but I could have just as well drawn a pinhole
Field of view

The part of the in-focus plane that gets mapped on the sensor.

• What happens to field of view as we focus closer?
Field of view

The part of the in-focus plane that gets mapped on the sensor.

- What happens to field of view as we focus closer? → It becomes smaller.

- Move in-focus plane closer to lens
- Move lens further away from sensor
- Angle of view also decreases
Comparison with pinhole camera

- What happens to field of view as we move closer?
Comparison with pinhole camera

No need to refocus: we can move object closer without changing aperture-sensor distance.

- What happens to field of view as we move closer? → It becomes smaller, but amount differs.
Comparison with pinhole camera

No need to refocus: we can move object closer without changing aperture-sensor distance.

This can be done with a lens only if depth of field is large enough. Then the two behave the same.
Field of view also depends on sensor size

- What happens to field of view when we reduce sensor size?
Field of view also depends on sensor size

- What happens to field of view when we reduce sensor size? → It decreases.

Lens and pinhole cameras behave the same in this case.
Field of view also depends on sensor size

- “Full frame” corresponds to standard film size.
- Digital sensors come in smaller formats due to manufacturing limitations (now mostly overcome).
- Lenses are often described in terms of field of view on film instead of focal length.
- These descriptions are invalid when not using full-frame sensor.
Crop factor

How much field of view is cropped when using a sensor smaller than full frame.
Magnification and perspective
Magnification depends on depth

- What happens to magnification as we focus further away?
Magnification depends on depth

- What happens to magnification as we focus further? → It becomes smaller.

- move object further away from lens
- move lens closer to sensor
Magnification depends on depth

\[ \frac{1}{S'} + \frac{1}{S} = \frac{1}{f} \]

\[ m = \frac{S' - f}{f} \]

We call \( m = \frac{y'}{y} \) the magnification.
Comparison with pinhole camera

No need to refocus: we can move object further without changing aperture-sensor distance.

This can be done with a lens only if depth of field is large enough. Then the two behave the same.
Forced perspective
The Ames room illusion
The Ames room illusion

- Actual position of Person A
- Apparent position of person A
- View of person A
- Apparent shape of room
- Actual and apparent position of person B
- Viewing peephole
The arrow illusion

Prof. Kokichi Sugihara has many other amazing illusions involving perspective distortion, check them out on YouTube or on his website:

http://www.isc.meiji.ac.jp/~kokichis/
Zooming
Zooming means changing the focal length

Very different process from refocusing
• What happens to field of view when we focus closer? → It decreases.
• What happens to field of view when we increase [redacted] focal length?
• When we increase lens focal length, field of view decreases (we “zoom in”).
Field of view

- 1000 mm: 2.5°
- 500 mm: 5°
- 350 mm: 7.5°
- 250 mm: 10°
- 135 mm: 18°
- 85 mm: 29°
- 50 mm: 43°
- 35 mm: 63°
- 28 mm: 75°
- 8 mm: 180°

Andrew McWilliams
Field of view

Increasing the lens focal length is similar to cropping

Is this effect identical to cropping?

\[f = 25 \text{ mm}\]
\[f = 50 \text{ mm}\]
\[f = 135 \text{ mm}\]
The lens on your camera

Focus ring: controls distance of lens from sensor

Zoom ring: controls focal length of lens
Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:
1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

When you turn the zoom ring to decrease the focal length of the lens:
1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).
Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:
1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

When you turn the zoom ring to decrease the focal length of the lens:
1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).

We can use both focus and zoom to cancel out their effects.
What if...

1. Set focal length to half depth 2 $Z$.

real-world object

focal length $f$

focal length 2 $f$

depth 2 $Z$
What if...

1. Set focal length to half
2. Set depth to half

Is this the same image as the one we had at focal length $2f$ and distance $2Z$?

Similar construction can be done with lenses, after taking care of refocusing.
Perspective distortion

long focal length  mid focal length  short focal length
Perspective distortion
What is the best focal length for portraits?

That’s like asking which is better, vi or emacs...

long focal length  mid focal length  short focal length
Vertigo effect

Named after Alfred Hitchcock’s movie
• also known as “dolly zoom”
Vertigo effect

How would you create this effect?
Orthographic camera and telecentric lenses
Can we make magnification depth-independent?
Orthographic camera

Depth-independent magnification $m = 1$ (real-life size).
Weak-perspective camera

Depth-independent magnification $m = \frac{f}{Z_0}$. 

Intermediate reference plane

Real-world object

Parallel projection

Fixed depth $Z_0$

Focal length $f$
How can we implement such a camera with lenses?
Telecentric lens

Place a pinhole at focal length, so that only rays parallel to primary ray pass through.
Telecentric lens

Place a pinhole at focal length, so that only rays parallel to primary ray pass through.

Magnification independent of object depth.

When is this lens equivalent to an orthographic camera?

Magnification depends only on sensor-lens distance $S'$. 
Regular vs telecentric lens

regular lens

telecentric lens
References

Basic reading:
• Szeliski textbook, Section 2.1.5, 2.2.3.
• Pedrotti, Pedrotti, and Pedrotti, Introduction to Optics. Chapters 2 and 3 have a detailed overview of basic geometric optics and lenses.

Additional reading:
• Torralba and Freeman, “Accidental Pinhole and Pinspeck Cameras,” CVPR 2012. The eponymous paper discussed in the slides.
• Watanabe and Nayar, “Telecentric Optics for Focus Analysis,” PAMI 1997. An early computational photography paper analyzing the ray optics and explaining the advantages and disadvantages of telecentric lenses relative to conventional lenses.