Noise
Course announcements

- Homework assignment 2 is out.
  - Due October 1st.
  - Requires camera \textit{and} tripod.
  - Start early! \textit{Substantially} larger programming and imaging components than in Homework assignment 1.
  - Generous bonus component, up to 50\% extra credit.
  - Homework assignment 2 (and many later homeworks) requires submitting large files, so be mindful of that.
  - Do not leave uploading your Homework assignment 2 solution for the last minute!
Overview of today’s lecture

• Leftover from lecture 6: other aspects of HDR imaging.
• Leftover from lecture 6: tonemapping.
• Leftover from lecture 6: Some notes about HDR imaging and tonemapping.
• A few motivating examples.
• Sensor noise.
• Noise calibration.
• Optimal weights for HDR.
Slide credits

Many of these slides were inspired or adapted from:

- James Hays (Georgia Tech).
- Fredo Durand (MIT).
- Gordon Wetzstein (Stanford).
- Marc Levoy (Stanford, Google).
- Sylvain Paris (Adobe).
- Sam Hasinoff (Google).
A few motivating questions from things we’ve seen
Side-effects of increasing ISO

Image becomes very grainy because noise is amplified.
- Why does increasing ISO increase noise?
Tonemapping for a single image

Modern DSLR sensors capture about 3 stops of dynamic range.
- Tonemap single RAW file instead of using camera’s default rendering.

Careful not to “tonemap” noise.
- Why is this not a problem with multi-exposure HDR?
Merging non-linear exposure stacks

1. Calibrate response curve

2. Linearize images

For each pixel:

3. Find “valid” images

4. Weight valid pixel values appropriately

5. Form a new pixel value as the weighted average of valid pixel values

Same steps as in the RAW case.

Note: many possible weighting schemes

(pixel value) / t_i

(noise) 0.05 < pixel < 0.95 (clipping)
Many possible weighting schemes

“Confidence” that pixel is noisy/clipped

\[ w_{ij} = \exp \left( -4 \frac{(I_{linj} - 0.5)^2}{0.5^2} \right) \]
Sensor noise
A quick note

• We will only consider per-pixel noise.
• We will not consider cross-pixel noise effects (blooming, smearing, cross-talk, and so on).
Noise in images

Results in “grainy” appearance.
The (in-camera) image processing pipeline

Which part introduces noise?

1. Analog front-end
2. RAW image (mosaiced, linear, 12-bit)
3. Final RGB image (non-linear, 8-bit)
4. Demosaicing
5. White balance
6. Compression
7. Tone reproduction
8. Color transforms
9. Denoising
The (in-camera) image processing pipeline

Which part introduces noise?
• Noise is introduced in the green part.

RAW image (mosaiced, linear, 12-bit)

final RGB image (non-linear, 8-bit)
The noisy image formation process

What are the various parts?
The noisy image formation process

- Scene radiant flux $\Phi$
- Dark current $D$
- Sensor (exposure $t$, quantum efficiency $\alpha$)
- Analog voltage $L$
- Analog amplifier (gain $g = k \cdot \text{ISO}$)
- Analog voltage $G$
- Analog-to-digital converter (ADC)
- Discrete signal $I$
The noisy image formation process

scene radiant flux $\Phi$

dark current D

sensor (exposure $t$, quantum efficiency $\alpha$)

introduces photon noise and dark noise

analog voltage $L$

analog amplifier (gain $g = k \cdot \text{ISO}$)

introduces read noise

analog voltage $G$

analog-to-digital converter (ADC)

introduces ADC noise

discrete signal $I$

- We will be ignoring saturation, but it can be modeled using a clipping operation.
Background: Normal distribution

Is it a continuous or discrete probability distribution?
Background: Normal distribution

Is it a continuous or discrete probability distribution?
• It is continuous.

How many parameters does it depend on?
Background: Normal distribution

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How many parameters does it depend on?
• Two parameters, the mean $\mu$ and the standard deviation $\sigma$.

What is its probability distribution function?
Background: Normal distribution

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How many parameters does it depend on?
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What is its probability distribution function?

$$ n \sim \text{Normal}(\mu, \sigma) \iff p(n = x; \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} $$

What are its mean and variance?
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What are its mean and variance?
• Mean: $\mu(n) = \mu$
• Variance: $\sigma(n)^2 = \sigma^2$

What is the distribution of the sum of two independent Normal random variables?
Background: Normal distribution

Is it a continuous or discrete probability distribution?
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How many parameters does it depend on?
• Two parameters, the *mean* $\mu$ and the standard deviation $\sigma$.

What is its probability distribution function?

$$n \sim \text{Normal}(\mu, \sigma) \iff p(n = x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

What are its mean and variance?
• Mean: $\mu(n) = \mu$
• Variance: $\sigma(n)^2 = \sigma^2$

What is the distribution of the sum of two independent Normal random variables?

$$n_1 \sim \text{Normal}(0, \sigma_1), n_2 \sim \text{Normal}(0, \sigma_2) \Rightarrow n_1 + n_2 \sim \text{Normal}\left(0, \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$
Background: Poisson distribution

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• One parameter, the rate $\lambda$.

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What is its probability mass function?

$$N \sim \text{Poisson}(\lambda) \iff P(N = k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

What are its mean and variance?
Background: Poisson distribution

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• Variance: $\sigma(N)^2 = \lambda$

The mean and variance of a Poisson random variable both equal the rate $\lambda$.

What is the distribution of the sum of two independent Poisson random variables?
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$$N_1 \sim \text{Poisson}(\lambda_1), N_2 \sim \text{Poisson}(\lambda_2) \Rightarrow N_1 + N_2 \sim \text{Poisson}(\lambda_1 + \lambda_2)$$
The noisy image formation process

- Scene radiant flux $\Phi$
- Dark current $D$
- Sensor (exposure $t$, quantum efficiency $\alpha$)
  - Introduces photon noise and dark noise
- Analog amplifier (gain $g = k \cdot$ ISO)
  - Introduces read noise
- Analog-to-digital converter (ADC)
  - Introduces ADC noise

- We will be ignoring saturation, but it can be modeled using a clipping operation.
Photon noise

A consequence of the discrete (quantum) nature of light.
- Photon detections are independent random events.
- Total number of detections is Poisson distributed.
- Also known as shot noise and Schott noise.

\[ N_{\text{detections}} \sim \text{Poisson}[t \cdot \alpha \cdot \Phi] \]
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\[ N_{\text{detections}} \sim \text{Poisson}(t \cdot \alpha \cdot \Phi) \]

Photon noise depends on scene flux and exposure.
Dark noise

A consequence of “phantom detections” by the sensor.

• Electrons are randomly released without any photons.
• Total number of detections is Poisson distributed.
• Increases *exponentially* with sensor temperature (+6°C ≈ doubling).

\[ N_{\text{detections}} \sim \text{Poisson}[t \cdot D] \]
Dark noise

A consequence of “phantom detections” by the sensor.
• Electrons are randomly released without any photons.
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Can you think of examples when dark noise is important?
Dark noise

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Can you think of examples when dark noise is important?
- Very long exposures (astrophotography, pinhole camera).

Can you think of ways to mitigate dark noise?

\[
N_{\text{detections}} \sim \text{Poisson}(t \cdot D)
\]
Dark noise

A consequence of “phantom detections” by the sensor.

• Electrons are randomly released without any photons.
• Total number of detections is Poisson distributed.
• Increases exponentially with sensor temperature (+6°C ≈ doubling).

Can you think of examples when dark noise is important?

• Very long exposures (astrophotography, pinhole camera).

Can you think of ways to mitigate dark noise?

• Cool the sensor.

$N_{\text{detections}} \sim \text{Poisson}[t \cdot D]$
Fundamental question

Why are photon noise and dark noise Poisson random variables?
The noisy image formation process

- Scene radiant flux $\Phi$
- Dark current $D$
- Sensor (exposure $t$, quantum efficiency $\alpha$)
  - Introduces photon noise and dark noise
- Analog voltage $L$
- Analog amplifier (gain $g = k \cdot ISO$)
  - Introduces read noise
- Analog voltage $G$
- Analog-to-digital converter (ADC)
  - Introduces ADC noise

- What is the distribution of the sensor readout $L$?
The distribution of the sensor readout

We know that the sensor readout is the sum of all released electrons:

\[ L = N_{\text{photon\_detections}} + N_{\text{phantom\_detections}} \]

What is the distribution of photon detections?
The distribution of the sensor readout

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What is the distribution of phantom detections?
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We know that the sensor readout is the sum of all released electrons:

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The distribution of the sensor readout

We know that the sensor readout is the sum of all released electrons:

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\[ N_{\text{photon\_detecteds}} \sim \text{Poisson}(t \cdot \alpha \cdot \Phi) \]

What is the distribution of phantom detections?

\[ N_{\text{phantom\_detecteds}} \sim \text{Poisson}(t \cdot D) \]

What is the distribution of the sensor readout?

\[ L \sim \text{Poisson}(t \cdot (\alpha \cdot \Phi + D)) \]
The noisy image formation process

scene radiant flux $\Phi$

dark current $D$

sensor (exposure $t$, quantum efficiency $\alpha$)

introduces photon noise and dark noise

analogue voltage $L$, $L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D))$

analog amplifier (gain $g = k \cdot \text{ISO}$)

introduces read noise

analog-to-digital converter (ADC)

introduces ADC noise

discrete signal $I$
Read and ADC noise

A consequence of random voltage fluctuations before and after amplifier.
• Both are independent of scene and exposure.
• Both are normally (zero-mean Guassian) distributed.
• ADC noise includes quantization errors.

\[ n_{\text{read}} \sim \text{Normal}(0, \sigma_{\text{read}}) \]
\[ n_{\text{ADC}} \sim \text{Normal}(0, \sigma_{\text{ADC}}) \]

Very important for dark pixels.
The noisy image formation process

scene radiant flux $\Phi$

analog voltage $L$

$L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D))$

dark current $D$

sensor (exposure $t$, quantum efficiency $\alpha$)

introduces photon noise and dark noise

analog amplifier (gain $g = k \cdot \text{ISO}$)

introduces read noise

analog-to-digital converter (ADC)

introduces ADC noise

discrete signal $I$

• How can we express the voltage $G$ and discrete intensity $I$?
Expressions for the amplifier and ADC outputs

Both read noise and ADC noise are additive and zero-mean.

• How can we express the output of the amplifier?
Expressions for the amplifier and ADC outputs

Both read noise and ADC noise are **additive** and **zero-mean**.

- How can we express the output of the amplifier?

\[ G = L \cdot g + n_{\text{read}} \cdot g \]

- How can we express the output of the ADC?

  don’t forget to account for the ISO-dependent gain
Expressions for the amplifier and ADC outputs

Both read noise and ADC noise are additive and zero-mean.

• How can we express the output of the amplifier?

\[ G = L \cdot g + n_{\text{read}} \cdot g \]

• How can we express the output of the ADC?

\[ I = G + n_{\text{ADC}} \]
The noisy image formation process

- Scene radiant flux $\Phi$
  - Analog voltage $L$, $L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D))$
  - Sensor (exposure $t$, quantum efficiency $\alpha$)
  - Introduces photon noise and dark noise

- Dark current $D$

- Analog amplifier (gain $g = k \cdot \text{ISO}$)
  - Analog voltage $G = L \cdot g + n_{\text{read}} \cdot g$
  - Introduces read noise

- Analog-to-digital converter (ADC)
  - Discrete signal $I = G + n_{\text{ADC}}$
  - Introduces ADC noise
Putting it all together

Without saturation, the digital intensity equals:

\[ I = L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}} \]

where

\[ L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D)) \]
\[ n_{\text{read}} \sim \text{Normal}(0, \sigma_{\text{read}}) \]
\[ n_{\text{ADC}} \sim \text{Normal}(0, \sigma_{\text{ADC}}) \]

What is the mean of the digital intensity (assuming no saturation)?

\[ E(I) = \]
Putting it all together

Without saturation, the digital intensity equals:

\[ I = L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}} \]

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What is the mean of the digital intensity (assuming no saturation)?

\[ E(I) = E(L \cdot g) + E(n_{\text{read}} \cdot g) + E(n_{\text{ADC}}) = \]
Putting it all together

Without saturation, the digital intensity equals:

\[ I = L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}} \]

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\[ L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D)) \]
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\[ E(I) = E(L \cdot g) + E(n_{\text{read}} \cdot g) + E(n_{\text{ADC}}) \]
\[ = t \cdot (a \cdot \Phi + D) \cdot g \]

What is the variance of the digital intensity (assuming no saturation)?

\[ \sigma(I)^2 = \]
Putting it all together

Without saturation, the digital intensity equals:

$$I = L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}}$$

where

$$L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D))$$

$$n_{\text{read}} \sim \text{Normal}(0, \sigma_{\text{read}})$$

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What is the mean of the digital intensity (assuming no saturation)?

$$E(I) = E(L \cdot g) + E(n_{\text{read}} \cdot g) + E(n_{\text{ADC}})$$

$$= t \cdot (a \cdot \Phi + D) \cdot g$$

What is the variance of the digital intensity (assuming no saturation)?

$$\sigma(I)^2 = \sigma(L \cdot g)^2 + \sigma(n_{\text{read}} \cdot g)^2 + \sigma(n_{\text{ADC}})^2$$
Putting it all together

Without saturation, the digital intensity equals:

\[ I = L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}} \]

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What is the variance of the digital intensity (assuming no saturation)?

\[ \sigma(I)^2 = \sigma(L \cdot g)^2 + \sigma(n_{\text{read}} \cdot g)^2 + \sigma(n_{\text{ADC}})^2 \]
\[ = t \cdot (a \cdot \Phi + D) \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2 \]
How do we compute mean and variance in practice?
How do we compute mean and variance in practice?

Mean: capture multiple linear images with identical settings and average.

\[
\bar{I} = \frac{1}{N} \sum_{n=1}^{N} I_n \xrightarrow{N \to \infty} E(I)
\]
How do we compute mean and variance in practice?

Mean: capture multiple linear images with identical settings and average.

\[
\bar{I} = \frac{1}{N} \sum_{n=1}^{N} I_n \xrightarrow{N \to \infty} E(I)
\]

Variance: capture multiple linear images with identical settings and form variance estimator.

\[
\bar{\Sigma} = \frac{1}{N - 1} \sum_{n=1}^{N} (I_n - \bar{I})^2 \xrightarrow{N \to \infty} \sigma(I)^2
\]
The noisy image formation process

scene radiant flux $\Phi$

dark current $D$

sensor (exposure $t$, quantum efficiency $\alpha$)

analog voltage $L$, $L \sim \text{Poisson}(t \cdot (\alpha \cdot \Phi + D))$

analog voltage $G = L \cdot g + n_{\text{read}} \cdot g$, $n_{\text{read}} \sim \text{Normal}(0, \sigma_{\text{read}})$

analog amplifier (gain $g = k \cdot \text{ISO}$)

analog-to-digital converter (ADC)

discrete signal $I = G + n_{\text{ADC}}$, $n_{\text{ADC}} \sim \text{Normal}(0, \sigma_{\text{ADC}})$

discrete image intensity (with saturation): $I = \min(L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}}, I_{\max})$

intensity mean and variance (without saturation):

\[
E(I) = t \cdot (\alpha \cdot \Phi + D) \cdot g
\]

\[
\sigma(I)^2 = t \cdot (\alpha \cdot \Phi + D) \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2
\]
Affine noise model

Combine read and ADC noise into a single additive noise term:

\[ I = L \cdot g + n_{\text{add}} \]
where

\[ n_{\text{add}} = n_{\text{read}} \cdot g + n_{\text{ADC}} \]

What is the distribution of the additive noise term?
Affine noise model

Combine read and ADC noise into a single *additive* noise term:

$$ I = L \cdot g + n_{\text{add}} $$

where

$$ n_{\text{add}} = n_{\text{read}} \cdot g + n_{\text{ADC}} $$

What is the distribution of the additive noise term?

- Sum of two independent, normal random variables.

$$ n_{\text{add}} \sim \text{Normal}(0, \sqrt{\sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2}) $$
Affine noise model

scene radiant flux $\Phi$

dark current $D$

analog voltage $L$, $L \sim \text{Poisson}(t \cdot (\alpha \cdot \Phi + D))$

discrete image intensity (with saturation):
$I = \min(L \cdot g + n_{\text{add}}, I_{\text{max}})$

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intensity mean and variance (without saturation):
$E(I) = t \cdot (\alpha \cdot \Phi + D) \cdot g$
$\sigma(I)^2 = t \cdot (\alpha \cdot \Phi + D) \cdot g^2 + \sigma_{\text{add}}^2$
Some observations

Is image intensity an unbiased estimator of (scaled) scene radiant flux?
Some observations

Is image intensity an *unbiased* estimator of (scaled) scene radiant flux?
• No, because of dark noise (term $t \cdot D \cdot g$ in the mean).
• Averaging multiple images cancels out read and ADC noise, but *not* dark noise.

When are photon noise and additive noise dominant?
Some observations

Is image intensity an *unbiased* estimator of (scaled) scene radiant flux?
• No, because of dark noise (term $t \cdot D \cdot g$ in the mean).
• Averaging multiple images cancels out read and ADC noise, but *not* dark noise.

When are photon noise and additive noise dominant?
• Photon noise is dominant in very bright scenes.
• Additive noise is dominant in very dark scenes.

Can we ever completely remove noise?
Some observations

Is image intensity an *unbiased* estimator of (scaled) scene radiant flux?

• No, because of dark noise (term $t \cdot D \cdot g$ in the mean).

• Averaging multiple images cancels out read and ADC noise, but *not* dark noise.

When are photon noise and additive noise dominant?

• Photon noise is dominant in very bright scenes.

• Additive noise is dominant in very dark scenes.

Can we ever completely remove noise?

• We cannot eliminate photon noise.

• Super-sensitive detectors have pure Poisson photon noise.

  single-photon avalanche photodiode (SPAD)
Summary: noise regimes

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discrete image intensity (with saturation):

\[ I = \min(L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}}, I_{\text{max}}) \]

intensity mean and variance (without saturation):

\[
E(I) = t \cdot (a \cdot \Phi + D) \cdot g
\]

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\sigma(I)^2 = t \cdot (a \cdot \Phi + D) \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2
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## Summary: noise regimes

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Discrete image intensity (with saturation):

\[ I = \min(L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}}, I_{\text{max}}) \]

Intensity mean and variance (without saturation):

\[ E(I) = t \cdot (a \cdot \Phi + D) \cdot g \]
\[ \sigma(I)^2 = t \cdot (a \cdot \Phi + D) \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2 \]

Does this mean that using high exposure makes images more “noisy”?

---

\[ \quad \]
Signal-to-noise ratio
Variance versus signal-to-noise ratio

Variance?
Variance versus signal-to-noise ratio

Variance is an *absolute* measure of the (squared) magnitude of noise:

\[
\sigma(I)^2 = E \left( (I - E(I))^2 \right) = E(I^2) - E(I)^2
\]

Signal-to-noise ratio (SNR)?
Variance versus signal-to-noise ratio

Variance is an *absolute* measure of the (squared) magnitude of noise:

\[ \sigma(I)^2 = E \left( (I - E(I))^2 \right) = E(I^2) - E(I)^2 \]

Signal-to-noise ratio (SNR) is a *relative* measure of the (inverse squared) magnitude of noise:

\[ \text{SNR} = \frac{E(I)^2}{\sigma(I)^2} \]

When noise *decreases*:

- The variance...
- The SNR...
Variance versus signal-to-noise ratio

Variance is an *absolute* measure of the (squared) magnitude of noise:

\[ \sigma(I)^2 = E \left( (I - E(I))^2 \right) = E(I^2) - E(I)^2 \]

Signal-to-noise ratio (SNR) is a *relative* measure of the (inverse squared) magnitude of noise:

\[ \text{SNR} = \frac{E(I)^2}{\sigma(I)^2} \]

When noise *decreases*:
- The variance decreases.
- The SNR increases.
The case of sensor noise

Assuming for simplicity that there is no dark current:

\[
\text{SNR} = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2} \\
\sigma(I)^2 = t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2
\]

What happens when the exposure time or flux are very large?
The case of sensor noise

Assuming for simplicity that there is no dark current:

\[
\text{SNR} = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2}
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\[
\sigma(I)^2 = t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2
\]

What happens when the exposure time or flux are very large?

- We can ignore additive (read and ADC) noise terms.

\[
\text{SNR} = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{t \cdot a \cdot \Phi \cdot g^2} = t \cdot a \cdot \Phi
\]

\[
\sigma(I)^2 = t \cdot a \cdot \Phi \cdot g^2
\]

What happens when the flux or exposure time are very small?
The case of sensor noise

Assuming for simplicity that there is no dark current:

\[
SNR = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2}
\]

\[
\sigma(I)^2 = t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2
\]

What happens when the exposure time or flux are very large?

• We can ignore additive (read and ADC) noise terms.

\[
SNR = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{t \cdot a \cdot \Phi \cdot g^2} = t \cdot a \cdot \Phi
\]

\[
\sigma(I)^2 = t \cdot a \cdot \Phi \cdot g^2
\]

What happens when the flux or exposure time are very small?

• We can ignore scene-dependent noise terms.

\[
SNR = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{\sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2}
\]

\[
\sigma(I)^2 = \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2
\]
The case of sensor noise

Assuming for simplicity that there is no dark current:

\[
SNR = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2} = \frac{\sigma(I)^2}{t \cdot a \cdot \Phi \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2}
\]

What happens when the exposure time or flux are very large?
• We can ignore additive (read and ADC) noise terms.

\[
SNR = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{t \cdot a \cdot \Phi} = t \cdot a \cdot \Phi
\]

What happens when the flux or exposure time are very small?
• We can ignore scene-dependent noise terms.

\[
SNR = \frac{(t \cdot a \cdot \Phi \cdot g)^2}{\sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2}
\]
The case of sensor noise

As flux or exposure time increase:

• The noise variance increases.
• The SNR also increases.

Even though the absolute magnitude of noise increases, its relative magnitude compared to the signal we are measuring decreases.

→ Our measurements become *less noisy* as flux or exposure time increase.

(For the case of exposure time, we need to be careful to also take into account dark noise.)
Pop quiz

Is it better to use one long exposure or multiple short exposures?
Pop quiz

Is it better to use one long exposure or multiple short exposures?

• Using one long exposure is better, because additive noise is only added once.
• Using multiple short exposures is worse, because the result (after summing all images) will have additive noise variance increased by number of exposures.
• This assumes no saturation, and using RAW images.
Pop quiz

Is it better to increase the exposure, increase the ISO, or brighten digitally?
Pop quiz

Is it better to increase the exposure, increase the ISO, or brighten digitally?

• Increasing the exposure is the best, as it increases Poisson noise but leaves read noise and ADC noise fixed.

• Increasing the ISO is the second best, as it increases Poisson noise and read noise, but leaves ADC noise fixed.

• Brightening digitally is the worst, as it increases all three types of noise.

• This assumes no motion blur, no saturation, and using RAW images.
Pop quiz

Is it better to downsample digitally, or use a sensor with fewer pixels?
Pop quiz

Is it better to downsample digitally, or use a sensor with fewer pixels?

• Decreasing the number of pixels is better, as it increases the Poisson, but leaves additive noise fixed.
• Downsampling digitally is worse, as it increases both the Poisson noise and additive noise.
• This assumes that the total photosensitive area remains the same, the per-pixel additive noise remains the same, and no saturation.
Noise calibration
How can we estimate the various parameters?

scene radiant flux $\Phi$
dark current $D$
sensor (exposure $t$, quantum efficiency $\alpha$)
discrete image intensity (with saturation):

$$I = \min(L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}}, I_{\text{max}})$$

saturation level

analog voltage $L$, $L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D))$

analog voltage $G = L \cdot g + n_{\text{read}} \cdot g,$

analog amplifier (gain $g = k \cdot \text{ISO}$)

$\text{sensor (exposure } t, \text{ quantum efficiency } \alpha)$

intensity mean and variance:

$$E(I) = t \cdot (a \cdot \Phi + D) \cdot g$$

$$\sigma(I)^2 = t \cdot (a \cdot \Phi + D) \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2$$

discrete signal $I = G + n_{\text{ADC}},$

$n_{\text{ADC}} \sim \text{Normal}(0, \sigma_{\text{ADC}})$

analog-to-digital converter (ADC)
Estimating the dark current

Can you think of a procedure for estimating the dark current $D$?
Estimating the dark current

Can you think of a procedure for estimating the dark current $D$?

- Capture multiple images with the sensor completely blocked and average to form the *dark frame*.

Why is the dark frame a valid estimator of the dark current $D$?
Estimating the dark current

Can you think of a procedure for estimating the dark current $D$?

- Capture multiple images with the sensor completely blocked and average to form the *dark frame*.

Why is the dark frame a valid estimator of the dark current $D$?

- By blocking the sensor, we effectively set $\Phi = 0$.
- Average intensity becomes:

$$E(I) = t \cdot (a \cdot 0 + D) \cdot g = t \cdot D \cdot g$$

- The dark frame needs to be computed separately for each ISO setting, unless we can also calibrate the gain $g$.

For the rest of these slides, we assume that we have calibrated $D$ and removed it from captured images (by subtracting from them the dark frame).
Noise model before dark frame subtraction

scene radiant flux $\Phi$

dark current $D$

sensor (exposure $t$, quantum efficiency $\alpha$)

analog voltage $L$, $L \sim \text{Poisson}(t \cdot (a \cdot \Phi + D))$

analog amplifier (gain $g = k \cdot \text{ISO}$)

analog-to-digital converter (ADC)

discrete signal $I = G + n_{\text{ADC}}$, $n_{\text{ADC}} \sim \text{Normal}(0, \sigma_{\text{ADC}})$

discrete image intensity (with saturation):

$E(I) = t \cdot (a \cdot \Phi + D) \cdot g$

$\sigma(I)^2 = t \cdot (a \cdot \Phi + D) \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2$

saturation level

$\min(L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}}, I_{\text{max}})$
Noise model after dark frame subtraction

scene radiant flux $\Phi$

sensor (exposure $t$, quantum efficiency $\alpha$)

analog voltage $L$, $L \sim \text{Poisson}(t \cdot (a \cdot \Phi))$

analog amplifier (gain $g = k \cdot \text{ISO}$)

analog voltage $G = L \cdot g + n_{\text{read}} \cdot g$

$n_{\text{read}} \sim \text{Normal}(0, \sigma_{\text{read}})$

discrete signal $I = G + n_{\text{ADC}}$

$n_{\text{ADC}} \sim \text{Normal}(0, \sigma_{\text{ADC}})$

discrete image intensity (with saturation): intensity mean and variance:

$I = \min(L \cdot g + n_{\text{read}} \cdot g + n_{\text{ADC}}, I_{\text{max}})$

saturation level

intensity mean:

$E(I) = t \cdot (a \cdot \Phi) \cdot g$

intensity variance:

$\sigma(I)^2 = t \cdot (a \cdot \Phi) \cdot g^2 + \sigma_{\text{read}}^2 \cdot g^2 + \sigma_{\text{ADC}}^2$
Affine noise model after dark frame subtraction

scene radiant flux $\Phi$

sensor (exposure $t$, quantum efficiency $\alpha$)

discrete image intensity (with saturation):
$$I = \min(L \cdot g + n_{\text{add}}, I_{\text{max}})$$

discrete signal $I = L \cdot g + n_{\text{add}}$
$$n_{\text{add}} \sim \text{Normal}(0, \sigma_{\text{add}})$$

analog voltage $L$,
$$L \sim \text{Poisson}(t \cdot (a \cdot \Phi))$$

analog amplifier (gain $g = k \cdot \text{ISO}$)

analog-to-digital converter (ADC)

intensity mean and variance:
$$E(I) = t \cdot (a \cdot \Phi) \cdot g$$
$$\sigma(I)^2 = t \cdot (a \cdot \Phi) \cdot g^2 + \sigma_{\text{add}}^2$$
Estimating the gain and additive noise variance

Can you think of a procedure for estimating these quantities?
Estimating the gain and additive noise variance

1. Capture a large number of images of a grayscale target.
Estimating the gain and additive noise variance

1. Capture a large number of images of a grayscale target.

2. Compute the empirical mean and variance for each pixel, then form a mean-variance plot.

What do you expect the measurements to look like?
Estimating the gain and additive noise variance

1. Capture a large number of images of a grayscale target.

2. Compute the empirical mean and variance for each pixel, then form a mean-variance plot.

\[
E(I) = t \cdot (a \cdot \Phi) \cdot g
\]

\[
\sigma(I)^2 = t \cdot (a \cdot \Phi) \cdot g^2 + \sigma_{\text{add}}^2
\]

\[
\Rightarrow \sigma(I)^2 = E(I) \cdot g + \sigma_{\text{add}}^2
\]
Estimating the gain and additive noise variance

1. Capture a large number of images of a grayscale target.
2. Compute the empirical mean and variance for each pixel, then form a mean-variance plot.
3. Fit a line and use slope and intercept to estimate the gain and variance.

\[ \sigma(I)^2 = E(I) \cdot g + \sigma_{\text{add}}^2 \]

How would you modify this procedure to separately estimate read and ADC noise?
Estimating the gain and additive noise variance

1. Capture a large number of images of a grayscale target.
2. Compute the empirical mean and variance for each pixel, then form a mean-variance plot.
3. Fit a line and use slope and intercept to estimate the gain and variance.

\[
\sigma(I)^2 = E(I) \cdot g + \sigma_{\text{add}}^2
\]

How would you modify this procedure to separately estimate read and ADC noise?

- Perform it for a few different ISO settings (i.e., gains g).
Important notes

Noise calibration should be performed with RAW images!

The above procedure assumes that all pixels have the same noise characteristics.
• If that is not the case, then you need to capture multiple images under multiple exposure times, and use those to form the mean-variance plot for each pixel.
Optimal weights for HDR merging
Merging non-linear exposure stacks

1. Calibrate response curve
2. Linearize images

For each pixel:

3. Find “valid” images

4. Weight valid pixel values appropriately

\[
\text{(pixel value)} / t_i
\]

5. Form a new pixel value as the weighted average of valid pixel values

Same steps as in the RAW case.

Note: many possible weighting schemes
Many possible weighting schemes

“Confidence” that pixel is noisy/clipped

\[ w_{ij} = \exp \left( -4 \frac{(I_{\text{lin},ij} - 0.5)^2}{0.5^2} \right) \]

• What are the optimal weights for merging an exposure stack?
RAW (linear) image formation model

(Weighted) radiant flux for image pixel \((x,y)\): \(\alpha \cdot \Phi(x, y)\)

Exposure time:

\[ t_5 \quad t_4 \quad t_3 \quad t_2 \quad t_1 \]

What weights should we use to merge these images, so that the resulting HDR image is an optimal estimator of the weighted radiant flux?

Different images in the exposure stack will have different noise characteristics.
Simple estimation example

We have two noisy *unbiased* estimators $x$ and $y$ of the same quantity (e.g., a pixel’s intensity).

• The two estimators have variance $\sigma[x]^2$ and $\sigma[y]^2$. 
Simple estimation example

We have two noisy unbiased estimators $x$ and $y$ of the same quantity (e.g., a pixel’s intensity).
• The two estimators have variance $\sigma^2[x]$ and $\sigma^2[y]$.

Assume we form a new estimator from the convex combination of the other two:

$$z = a \cdot x + (1 - a) \cdot y$$
Simple estimation example

We have two noisy *unbiased* estimators $x$ and $y$ of the same quantity (e.g., a pixel’s intensity).

- The two estimators have variance $\sigma[x]^2$ and $\sigma[y]^2$.

Assume we form a new estimator from the *convex* combination of the other two:

$$z = a \cdot x + (1 - a) \cdot y$$

What criterion should we use in selecting the mixing weight $a$?
Simple estimation example

We have two noisy \textit{unbiased} estimators \( x \) and \( y \) of the same quantity (e.g., a pixel’s intensity).

- The two estimators have variance \( \sigma[x]^2 \) and \( \sigma[y]^2 \).

Assume we form a new estimator from the \textit{convex} combination of the other two:

\[
z = a \cdot x + (1 - a) \cdot y
\]

What criterion should we use in selecting the mixing weight \( a \)?

- We should select \( a \) to minimize the variance of estimator \( z \). (Why?)

What is the variance of \( z \) as a function of \( a \)?
Simple estimation example

We have two noisy unbiased estimators $x$ and $y$ of the same quantity (e.g., a pixel’s intensity).

- The two estimators have variance $\sigma[x]^2$ and $\sigma[y]^2$.

Assume we form a new estimator from the convex combination of the other two:

$$z = a \cdot x + (1 - a) \cdot y$$

What criterion should we use in selecting the mixing weight $a$?

- We should select $a$ to minimize the variance of estimator $z$.

What is the variance of $z$ as a function of $a$?

$$\sigma[z]^2 = a^2 \cdot \sigma[x]^2 + (1 - a)^2 \cdot \sigma[y]^2$$

What value of $a$ minimizes $\sigma[z]^2$?
Simple estimation example

Simple optimization problem:

\[ \frac{\partial \sigma[z]^2}{\partial a} = 0 \]

\[ \Rightarrow \frac{\partial (a^2 \cdot \sigma[x]^2 + (1-a)^2 \cdot \sigma[y]^2)}{\partial a} = 0 \]

\[ \Rightarrow 2 \cdot a \cdot \sigma[x]^2 - 2 \cdot (1-a) \cdot \sigma[y]^2 = 0 \]

\[ \Rightarrow a = \frac{\sigma[y]^2}{\sigma[x]^2 + \sigma[y]^2} \quad \text{and} \quad 1 - a = \frac{\sigma[x]^2}{\sigma[x]^2 + \sigma[y]^2} \]
Simple estimation example

Putting it all together, the optimal linear combination of the two estimators is

\[ z = \frac{\sigma[x]^2 \sigma[y]^2}{\sigma[x]^2 + \sigma[y]^2} \cdot \left( \frac{1}{\sigma[x]^2} x + \frac{1}{\sigma[y]^2} y \right) \]

- normalization factor
- weights inversely proportional to variance
Simple estimation example

Putting it all together, the optimal linear combination of the two estimators is

\[
z = \frac{\sigma[x]^2 \sigma[y]^2}{\sigma[x]^2 + \sigma[y]^2} \cdot \left( \frac{1}{\sigma[x]^2} x + \frac{1}{\sigma[y]^2} y \right)
\]

where the normalization factor weights inversely proportional to variance.

More generally, for more than two estimators,

\[
z = \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma[x_i]^2}} \cdot \sum_{i=1}^{N} \frac{1}{\sigma[x_i]^2} x_i
\]

This is weighting scheme is called Fisher weighting and is a BLUE estimator.
Back to HDR

Given *unclipped* and *dark-frame-corrected* intensity measurements $I_i[x, y]$ at pixel $[x, y]$ and exposures $t_i$, we can merge them optimally into a single HDR intensity $I[x, y]$ as

$$I[x, y] = \frac{1}{\sum_{i=1}^{N} w_i[x, y]} \cdot \sum_{i=1}^{N} w_i[x, y] \frac{1}{t_i} I_i[x, y]$$

The per-pixel weights $w_i[x, y]$ should be selected to be inversely proportional to that pixel’s variance at each image in the exposure stack.

- How do we compute this variance?
Back to HDR

Given unclipped and dark-frame-corrected intensity measurements $I_i[x, y]$ at pixel $[x, y]$ and exposures $t_i$, we can merge them optimally into a single HDR intensity $I[x, y]$ as

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The per-pixel weights $w_i[x, y]$ should be selected to be inversely proportional to that pixel’s variance at each image in the exposure stack.

• How do we compute this variance?  →  Use affine noise model.

$$\sigma_{\frac{1}{t_i} I_i[x, y]}^2 = ?$$
Back to HDR

Given unclipped and dark-frame-corrected intensity measurements $I_i[x, y]$ at pixel $[x, y]$ and exposures $t_i$, we can merge them optimally into a single HDR intensity $I[x, y]$ as

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The per-pixel weights $w_i[x, y]$ should be selected to be inversely proportional to that pixel’s variance at each image in the exposure stack.

- How do we compute this variance? → Use affine noise model.

$$\sigma \left[ \frac{1}{t_i} I_i[x, y] \right]^2 = \frac{1}{t_i^2} \sigma [I_i[x, y]]^2$$

$$\Rightarrow \sigma \left[ \frac{1}{t_i} I_i[x, y] \right]^2 = ?$$
Back to HDR

Given unclipped and dark-frame-corrected intensity measurements $I_i[x, y]$ at pixel $[x, y]$ and exposures $t_i$, we can merge them optimally into a single HDR intensity $I[x, y]$ as

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The per-pixel weights $w_i[x, y]$ should be selected to be inversely proportional to that pixel’s variance at each image in the exposure stack.

• How do we compute this variance? → Use affine noise model.

$$\sigma\left[\frac{1}{t_i} I_i[x, y]\right]^2 = \frac{1}{t_i^2} \sigma[I_i[x, y]]^2$$

$$\Rightarrow \sigma\left[\frac{1}{t_i} I_i[x, y]\right]^2 = \frac{1}{t_i^2} \left(t_i \cdot \alpha \cdot \Phi[x, y] \cdot g^2 + \sigma_{\text{add}}^2\right)$$

Computing the optimal weights requires:
1. calibrated noise characteristics.
2. knowing the radiant flux $\alpha \cdot \Phi[x, y]$. 
Back to HDR

Given unclipped and dark-frame-corrected intensity measurements $I_i[x, y]$ at pixel $[x, y]$ and exposures $t_i$, we can merge them optimally into a single HDR intensity $I[x, y]$ as

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The per-pixel weights $w_i[x, y]$ should be selected to be inversely proportional to that pixel’s variance at each image in the exposure stack.

• How do we compute this variance? → Use affine noise model.

$$\sigma[\frac{1}{t_i} I_i[x, y]]^2 = \frac{1}{t_i^2} \sigma[I_i[x, y]]^2$$

$$\Rightarrow \sigma[\frac{1}{t_i} I_i[x, y]]^2 = \frac{1}{t_i^2} \left( t_i \cdot \alpha \cdot \Phi[x, y] \cdot g^2 + \sigma_{\text{add}}^2 \right)$$

Computing the optimal weights requires:

1. calibrated noise characteristics.
2. knowing the radiant flux $\alpha \cdot \Phi[x, y]$.

This is what we wanted to estimate!
Simplification: only photon noise

If we assume that our measurements are dominated by photon noise, the variance becomes:

\[
\sigma \left[ \frac{1}{t_i} I_i[x, y] \right]^2 = \frac{1}{t_i^2} \left( t_i \cdot \alpha \cdot \Phi[x, y] \cdot g^2 + \sigma_{\text{add}}^2 \right) \approx ?
\]
Simplification: only photon noise

If we assume that our measurements are dominated by photon noise, the variance becomes:

\[
\sigma^2 \left( \frac{1}{t_i} I_i[x,y] \right) = \frac{1}{t_i^2} \left( t_i \cdot \alpha \cdot \Phi[x,y] \cdot g^2 + \sigma_{\text{add}}^2 \right) \approx \frac{1}{t_i} \alpha \cdot \Phi[x,y] \cdot g^2
\]

By replacing in the HDR merging formula and \textit{assuming only valid pixels}, the HDR estimate simplifies to:

\[
I[x,y] = \frac{1}{\sum_{i=1}^{N} t_i} \cdot \sum_{i=1}^{N} I_i[x,y]
\]

Notice that we no longer weight each image in the exposure stack by its exposure time!
Some comparisons

original weights

optimal weights assuming only photon noise
Simplification: only photon noise

When is this a good assumption?
More general case

If we cannot assume that our measurements are dominated by photon noise, we can approximate the variance as:

$$\sigma\left[\frac{1}{t_i} I_i[x,y]\right]^2 = \frac{1}{t_i^2} \left(t_i \cdot \alpha \cdot \Phi[x,y] \cdot g^2 + \sigma_{\text{add}}^2\right) \approx \frac{1}{t_i^2} \left(I_i[x,y] \cdot g + \sigma_{\text{add}}^2\right)$$

Where does this approximation come from?
More general case

If we cannot assume that our measurements are dominated by photon noise, we can approximate the variance as:

$$\sigma\left[\frac{1}{t_i} I_i[x, y]\right]^2 = \frac{1}{t_i^2} \left(t_i \cdot \alpha \cdot \Phi[x, y] \cdot g^2 + \sigma_{\text{add}}^2\right) \approx \frac{1}{t_i^2} \left(I_i[x, y] \cdot g + \sigma_{\text{add}}^2\right)$$

Where does this approximation come from?

- We use the fact that each pixel intensity (after dark frame subtraction) is an unbiased estimate of the radiant flux, weighted by exposure and gain:

$$E[I_i[x, y]] = t_i \cdot \alpha \cdot \Phi[x, y] \cdot g$$
Some comparisons

standard weights  optimal weights  ground-truth

tone-mapped merged HDR

2.8 dB  14.6 dB

Some comparisons
What about ISO?

- We need to separately account for read and ADC noise, as read noise is gain-dependent.
- We can optimize our exposure bracket by varying both shutter speed and ISO.

Bonus part of Homework 2 (+ 50%)!
Real capture results

<table>
<thead>
<tr>
<th>Standard exposure bracketing</th>
<th>SNR-optimal sequence</th>
<th>“Ground truth”</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO 6400 1/125 s</td>
<td>ISO 6400 1/1600 s</td>
<td>ISO 100 1/200 s</td>
</tr>
<tr>
<td>ISO 800 1/800 s</td>
<td>ISO 100 1/400 s</td>
<td>ISO 100 1/400 s</td>
</tr>
<tr>
<td>124 dB</td>
<td>16.2 dB</td>
<td>+15 more</td>
</tr>
</tbody>
</table>
References

Basic reading:
• Szeliski textbook, Sections 10.1, 10.2.
  A paper on weighting different exposures based on a very detailed sensor noise model, additionally discussing combining shutter speed and ISO changes.
  A detailed paper on radiometric and noise calibration based on the noise model we discussed.
  A very detailed discussion of noise characteristics and other performance aspects of digital sensors.

Additional reading:
• Kirk and Andersen, “Noise characterization of weighting schemes for combination of multiple exposures,” BMVC 2006.
  A great paper on the variance characteristics of most common HDR weighting schemes.
  This paper extends the analysis of optimal HDR weights to consider spatially-varying noise effects.
  A detailed tutorial on sensors and noise.
  These two papers examine noise-optimal acquisition and merging schemes for focal and aperture stacks, rather than exposure stacks.
  These two papers discuss the effect of different types of noise when fusing multiple images in the context of illumination multiplexing.
  A paper on the noise characteristics of single-photon-sensitive cameras.