Course announcements

- Homework 2 is out.
  - Due September 27th.
  - Requires camera and tripod.
  - Start early! Substantially larger programming and imaging components than in Homework 1.
  - Generous bonus component, up to 50% extra credit.
  - No really: start early!
Overview of today’s lecture

• Leftover from lecture 6: optimal weights for HDR.
• Recap: color and human color perception.
• Retinal color space.
• Color matching.
• Linear color spaces.
• Chromaticity.
• Color calibration.
• Non-linear color spaces.
• Some notes about color reproduction.
Slide credits

Many of these slides were inspired or adapted from:

- Todd Zickler (Harvard).
- Fredo Durand (MIT).
Recap: color and human color perception
Color is an artifact of human perception

- “Color” is not an *objective* physical property of light (electromagnetic radiation).
- Instead, light is characterized by its wavelength.

What we call “color” is how we *subjectively* perceive a very small range of these wavelengths.
Light-material interaction

\[ \ell(\lambda) = r(\lambda)e(\lambda) \]

spectral radiance

\[ r(\lambda) \]

spectral reflectance

\[ e(\lambda) \]

illuminant spectrum
Light-material interaction

\[ \ell(\lambda) = r(\lambda)e(\lambda) \]

spectral radiance  
e(\lambda)  
illuminant spectrum  
r(\lambda)  
spectral reflectance
Illuminant Spectral Power Distribution (SPD)

- Most types of light “contain” more than one wavelengths.
- We can describe light based on the distribution of power over different wavelengths.

We call our sensation of all of these distributions “white”.
Light-material interaction

\[ \ell(\lambda) = r(\lambda)e(\lambda) \]

spectral radiance

illuminant spectrum

spectral reflectance
Spectral reflectance

- Most materials absorb and reflect light differently at different wavelengths.
- We can describe this as a ratio of reflected vs incident light over different wavelengths.
Light-material interaction

\[ \ell(\lambda) = r(\lambda)e(\lambda) \]

spectral radiance

\[ r(\lambda) \]

spectral reflectance

\[ e(\lambda) \]

illuminant spectrum
Human color vision

retinal color
\[ \mathbf{c}(\ell(\lambda)) = (c_s, c_m, c_l) \]

perceived color
object color
color names

\[ c_s = \int k_s(\lambda) \ell(\lambda) d\lambda \]

spectral radiance

LMS sensitivity functions
Retinal vs perceived color
Retinal vs perceived color

Retinal vs perceived color.
Retinal vs perceived color

- Our visual system tries to “adapt” to illuminant.
- We may interpret the same retinal color very differently.
Human color vision

We will exclusively discuss retinal color in this course.

Retinal color

\[ \mathbf{c}(\ell(\lambda)) = (c_s, c_m, c_l) \]

\[ c_s = \int k_s(\lambda)\ell(\lambda)d\lambda \]

Spectral radiance

Perceived color

Object color

Color names
Retinal color space
Spectral Sensitivity Function (SSF)

- Any light sensor (digital or not) has different sensitivity to different wavelengths.
- This is described by the sensor’s *spectral sensitivity function* \( f(\lambda) \).
- When measuring light of a some SPD \( \Phi(\lambda) \), the sensor produces a *scalar* response:

\[
R = \int_{\lambda} \Phi(\lambda) f(\lambda) \, d\lambda
\]

Weighted combination of light’s SPD: light contributes more at wavelengths where the sensor has higher sensitivity.
Spectral Sensitivity Function of Human Eye

• The human eye is a collection of light sensors called cone cells.
• There are three types of cells with different spectral sensitivity functions.
• Human color perception is three-dimensional (*tristimulus color*).

```
“short”  \[ S = \int_{\lambda} \Phi(\lambda) S(\lambda) d\lambda \]
“medium” \[ M = \int_{\lambda} \Phi(\lambda) M(\lambda) d\lambda \]
“long”  \[ L = \int_{\lambda} \Phi(\lambda) L(\lambda) d\lambda \]
```

cone distribution for normal vision (64% L, 32% M)
The retinal color space

\[ c(\ell_{\lambda_i}) = (c_s, c_m, c_l) \]

LMS sensitivity functions

“pure beam” (laser)
The retinal color space

\[ \mathbf{c}(\ell_{\lambda_i}) = (c_s, c_m, c_l) \]

- "lasso curve"
- contained in positive octant
- parameterized by wavelength
- starts and ends at origin
- never comes close to M axis

"pure beam" (laser)
The retinal color space

\[ c(\ell_{\lambda_i}) = (c_s, c_m, c_l) \]

if we also consider variations in the strength of the laser this “lasso” turns into (convex!) radial cone with a “horse-shoe shaped” radial cross-section
The retinal color space

$$c(\ell_{\lambda_i}) = (c_s, c_m, c_l)$$

The colors of mixed beams are at the interior of the convex cone with boundary the surface produced by monochromatic lights.

"mixed beam" = convex combination of pure colors
The retinal color space

\[ c(\ell_{\lambda_i}) = (c_s, c_m, c_l) \]

- distinct mixed beams can produce the same retinal color
- these beams are called metamers

= convex combination of pure colors

LMS sensitivity functions
There is an infinity of metamers

Ensemble of spectral reflectance curves corresponding to three chromatic-pigment recipes all matching a tan material when viewed by an average observer under daylight illumination. [Based on Berns (1988b).]
Example: illuminant metamerism

day light

scanned copy

halogen light
Color matching
Adjust the strengths of the primaries until they re-produce the test color. Then:

\[ c(\ell(\lambda)) = \alpha c(\ell_{435}) + \beta c(\ell_{535}) + \gamma c(\ell_{625}) \]

The equality symbol means “has the same retinal color as” or “is metameric to”.
To match some test colors, you need to add some primary beam on the left (same as “subtracting light” from the right)

\[ c(\ell(\lambda)) + \gamma c(\ell_{625}) = \alpha c(\ell_{435}) + \beta c(\ell_{535}) \]

\[ \longrightarrow c(\ell(\lambda)) = \alpha c(\ell_{435}) + \beta c(\ell_{535}) - \gamma c(\ell_{625}) \]
Repeat this matching experiments for pure test beams at wavelengths $\lambda_i$ and keep track of the coefficients (negative or positive) required to reproduce each pure test beam.

$$c(\lambda_i) = k_{435}(\lambda)c(\ell_{435}) + k_{535}(\lambda)c(\ell_{535}) + k_{625}(\lambda)c(\ell_{625})$$
Repeat this matching experiments for pure test beams at wavelengths $\lambda_i$ and keep track of the coefficients (negative or positive) required to reproduce each pure test beam.

$$
c(\lambda_i) = k_{435}(\lambda)c(\ell_{435}) + k_{535}(\lambda)c(\ell_{535}) + k_{625}(\lambda)c(\ell_{625}) $$
CIE color matching

What about “mixed beams”?
Two views of retinal color

**Analytic:** Retinal color is produced by analyzing spectral power distributions using the color sensitivity functions.

**Synthetic:** Retinal color is produced by synthesizing color primaries using the color matching functions.

What is each view of retinal color best suited for?
Two views of retinal color

**Analytic:** Retinal color is produced by analyzing spectral power distributions using the color sensitivity functions.

**Synthetic:** Retinal color is produced by synthesizing color primaries using the color matching functions.

How do they relate to each other?
Two views of retinal color

Analytic: Retinal color is produced by analyzing spectral power distributions using the color sensitivity functions.

Synthetic: Retinal color is produced by synthesizing color primaries using the color matching functions.

The two views are equivalent: Color matching functions are also color sensitivity functions. For each set of color sensitivity functions, there are corresponding color primaries.
Linear color spaces
Linear color spaces

1) Color matching experimental outcome:

\[ c(\lambda_i) = k_{435}(\lambda)c(\ell_{435}) + k_{535}(\lambda)c(\ell_{535}) + k_{625}(\lambda)c(\ell_{625}) \]

same in matrix form:

\[
\begin{bmatrix}
  c(\lambda_i) \\
  c(\ell_{435}) \\
  c(\ell_{545}) \\
  c(\ell_{625})
\end{bmatrix}
= \begin{bmatrix}
  k_{435} \\
  k_{535} \\
  k_{625}
\end{bmatrix}
\]

how is this matrix formed?
1) Color matching experimental outcome:

\[ c(\lambda_i) = k_{435}(\lambda)c(\ell_{435}) + k_{535}(\lambda)c(\ell_{535}) + k_{625}(\lambda)c(\ell_{625}) \]

same in matrix form:

\[
\begin{bmatrix}
   c(\lambda_i) \\
   c(\ell_{435}) \\
   c(\ell_{545}) \\
   c(\ell_{625})
\end{bmatrix} =
\begin{bmatrix}
   k_{435} \\
   k_{535} \\
   k_{625}
\end{bmatrix}
\]

2) Implication for arbitrary mixed beams:

\[
\begin{bmatrix}
   c(\ell(\lambda)) \\
   c(\ell_{435}) \\
   c(\ell_{545}) \\
   c(\ell_{625})
\end{bmatrix} =
\begin{bmatrix}
   \int k_{435}(\lambda)\ell(\lambda)d\lambda \\
   \int k_{535}(\lambda)\ell(\lambda)d\lambda \\
   \int k_{625}(\lambda)\ell(\lambda)d\lambda
\end{bmatrix}
\]

where do these terms come from?
Linear color spaces

1) Color matching experimental outcome:

\[ c(\lambda_i) = k_{435}(\lambda)c(\ell_{435}) + k_{535}(\lambda)c(\ell_{535}) + k_{625}(\lambda)c(\ell_{625}) \]

same in matrix form:

\[
\begin{bmatrix}
  c(\lambda_i) \\
  c(\lambda) \\
\end{bmatrix} =
\begin{bmatrix}
  c(\ell_{435}) & c(\ell_{545}) & c(\ell_{625}) \\
\end{bmatrix}
\begin{bmatrix}
  k_{435} \\
  k_{535} \\
  k_{625} \\
\end{bmatrix}
\]

2) Implication for arbitrary mixed beams:

\[
\begin{bmatrix}
  c(\ell(\lambda)) \\
  c(\ell(\lambda)) \\
\end{bmatrix} =
\begin{bmatrix}
  c(\ell_{435}) & c(\ell_{545}) & c(\ell_{625}) \\
\end{bmatrix}
\begin{bmatrix}
  \int k_{435}(\lambda)\ell(\lambda)d\lambda \\
  \int k_{535}(\lambda)\ell(\lambda)d\lambda \\
  \int k_{625}(\lambda)\ell(\lambda)d\lambda \\
\end{bmatrix}
\]

what is this similar to?
Linear color spaces

1) Color matching experimental outcome:
\[
c(\lambda_i) = k_{435}(\lambda)c(\ell_{435}) + k_{535}(\lambda)c(\ell_{535}) + k_{625}(\lambda)c(\ell_{625})
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same in matrix form:
\[
\begin{bmatrix}
c(\lambda_i) \\
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k_{435} \\
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k_{625}
\end{bmatrix}
\]

2) Implication for arbitrary mixed beams:
\[
\begin{bmatrix}
c(\ell(\lambda)) \\
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k_{435}(\lambda)\ell(\lambda)d\lambda \\
k_{535}(\lambda)\ell(\lambda)d\lambda \\
k_{625}(\lambda)\ell(\lambda)d\lambda
\end{bmatrix}
\]

representation of retinal color in LMS space
change of basis matrix
representation of retinal color in space of primaries
Linear color spaces

basis for retinal color $\Leftrightarrow$ color matching functions $\Leftrightarrow$ primary colors $\Leftrightarrow$ color space

$\begin{bmatrix} c(\ell(\lambda)) \end{bmatrix} = \begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} \int k_1(\lambda)\ell(\lambda)d\lambda \\ \int k_2(\lambda)\ell(\lambda)d\lambda \\ \int k_3(\lambda)\ell(\lambda)d\lambda \end{bmatrix}$

$\begin{bmatrix} c(\ell_{435}) & c(\ell_{545}) & c(\ell_{625}) \end{bmatrix} \text{M}^{-1}$

$\begin{bmatrix} k_1(\lambda) \\ k_2(\lambda) \\ k_3(\lambda) \end{bmatrix} = \text{M} \begin{bmatrix} k_{435}(\lambda) \\ k_{545}(\lambda) \\ k_{625}(\lambda) \end{bmatrix}$

$\text{M}^{-1}\text{M}$ can insert any invertible M

representation of retinal color in LMS space

change of basis matrix

representation of retinal color in space of primaries
A few important color spaces

LMS color space

CIE RGB color space

not the “usual” RGB color space encountered in practice
Two views of retinal color

**Analytic:** Retinal color is three numbers formed by taking the dot product of a power spectral distribution with three color matching/sensitivity functions.

**Synthetic:** Retinal color is three numbers formed by assigning weights to three color primaries to match the perception of a power spectral distribution.

How would you make a color measurement device?
How would you make a color measurement device?

Do what the eye does:
• Select three spectral filters (i.e., three color matching functions).
• Capture three measurements.

Can we use the CIE RGB color matching functions?
How would you make a color measurement device?

Do what the eye does:
• Select three spectral filters (i.e., three color matching functions.).
• Capture three measurements.

Can we use the CIE RGB color matching functions?

Negative values are an issue (we can’t “subtract” light at a sensor)

CIE RGB color space
How would you make a color measurement device?

Do what the eye does:
• Select three spectral filters (i.e., three color matching functions).
• Capture three measurements.

Can we use the LMS color matching functions?

LMS color space
How would you make a color measurement device?

Do what the eye does:
• Select three spectral filters (i.e., three color matching functions).
• Capture three measurements.

Can we use the LMS color matching functions?
• They weren’t known when CIE was doing their color matching experiments.
The CIE XYZ color space

- Derived from CIE RGB by adding enough blue and green to make the red positive.
- Probably the most important *reference* (i.e., device independent) color space.

Remarkable and/or scary: 80+ years of CIE XYZ is all down to color matching experiments done with 12 “standard observers”.

CIE XYZ color space
The CIE XYZ color space

- Derived from CIE RGB by adding enough blue and green to make the red positive.
- Probably the most important *reference* (i.e., device independent) color space.

How would you convert a color image to grayscale?

CIE XYZ color space

- Y corresponds to luminance (“brightness”)
- X and Z correspond to chromaticity
A few important color spaces

LMS color space

CIE RGB color space

CIE XYZ color space
Two views of retinal color

**Analytic:** Retinal color is three numbers formed by taking the dot product of a power spectral distribution with three color matching/sensitivity functions.

**Synthetic:** Retinal color is three numbers formed by assigning weights to three color primaries to match the perception of a power spectral distribution.

How would you make a color reproduction device?
How would you make a color reproduction device?

Do what color matching does:
• Select three color primaries.
• Represent all colors as mixtures of these three primaries.

Can we use the XYZ color primaries?

CIE XYZ color space
How would you make a color reproduction device?

Do what color matching does:
- Select three color primaries.
- Represent all colors as mixtures of these three primaries.

Can we use the XYZ color primaries?
- No, because they are not “real” colors (they require an SPD with negative values).
- Same goes for LMS color primaries.
The Standard RGB (sRGB) color space

- Derived by Microsoft and HP in 1996, based on CRT displays used at the time.
- Similar but not equivalent to CIE RGB.

Note the negative values

While it is called “standard”, when you grab an “RGB” image, it is highly likely it is in a different RGB color space...
The Standard RGB (sRGB) color space

- Derived by Microsoft and HP in 1996, based on CRT displays used at the time.
- Similar but not equivalent to CIE RGB.

There are really two kinds of sRGB color spaces: linear and non-linear.
- Non-linear sRGB images have the following tone reproduction curve applied to them.

\[
C_{\text{non-linear}} = \begin{cases} 
12.92 \cdot C_{\text{linear}}, & C_{\text{linear}} \leq 0.0031308 \\
(1 + 0.055) \cdot C_{\text{linear}}^{1.4} - 0.055, & C_{\text{linear}} > 0.0031308 
\end{cases}
\]
A few important color spaces

- LMS color space
- CIE RGB color space
- CIE XYZ color space
- sRGB color space
A few important color spaces

- LMS color space
- CIE RGB color space
- CIE XYZ color space
- sRGB color space

Is there a way to “compare” all these color spaces?
Chromaticity
CIE $xy$ (chromaticity)

\[ x = \frac{X}{X + Y + Z} \]
\[ y = \frac{Y}{X + Y + Z} \]

$$(X, Y, Z) \leftrightarrow (x, y, Y)$$

chromaticity

luminance/brightness

Perspective projection of 3D retinal color space to two dimensions.

Hue changes as one moves around the spectral locus

Saturation increases as one moves out radially from white

Green

Yellow

Orange

White

Red

Blue

Indigo

Violet
Note: These colors can be extremely misleading depending on the file origin and the display you are using.

CIE xy (chromaticity)

\[ x = \frac{X}{X + Y + Z} \]

\[ y = \frac{Y}{X + Y + Z} \]

\((X, Y, Z) \leftrightarrow (x, y, Y)\)
What does the boundary of the chromaticity diagram correspond to?
Color gamuts

We can compare color spaces by looking at what parts of the chromaticity space they can reproduce with their primaries.

But why would a color space not be able to reproduce all of the chromaticity space?
We can compare color spaces by looking at what parts of the chromaticity space they can reproduce with their primaries.

But why would a color space not be able to reproduce all of the chromaticity space?

- Many colors require negative weights to be reproduced, which are not realizable.
sRGB color gamut:
• What are the three triangle corners?
• What is the interior of the triangle?
• What is the exterior of the triangle?
Color gamuts

sRGB color gamut

sRGB impossible colors

sRGB realizable colors

sRGB color primaries
Color gamuts

Gamuts of various common industrial RGB spaces

What is this?
The problem with RGBs visualized in chromaticity space

RGB values have no meaning if the primaries between devices are not the same!
Color gamuts

- Can we create an RGB color space that reproduces the entire chromaticity diagram?
- What would be the pros and cons of such a color space?
- What devices would you use it for?
Chromaticity diagrams can be misleading

Different gamuts may compare very differently when seen in full 3D retinal color space.
Some take-home messages about color spaces

Analytic: Retinal color is three numbers formed by taking the dot product of a power spectral distribution with three color matching/sensitivity functions.

Synthetic: Retinal color is three numbers formed by assigning weights to three color primaries to match the perception of a power spectral distribution.

Fundamental problem: Analysis spectrum (camera, eyes) cannot be the same as synthesis one (display) - impossible to encode all possible colors without something becoming negative
• CIE XYZ only needs positive coordinates, but need primaries with negative light.
• RGB must use physical (non-negative) primaries, but needs negative coordinates for some colors.

Problem with current practice: Many different RGB color spaces used by different devices, without clarity of what exactly space a set of RGB color values are in.
• Huge problem for color reproduction from one device to another.
See for yourself

Images of the same scene captured using 3 different cameras with identical settings, supposedly in sRGB space.
Color calibration and homography estimation
Color calibration

Apply linear scaling and translation to RGB vectors in the image:

\[ c' = M \cdot c + b \]

transformed RGB vector \hspace{1cm} \text{original RGB vector}

What are the dimensions of each quantity in this equation?
Color calibration

Apply linear scaling and translation to RGB vectors in the image:

\[ c' = M \cdot c + b \]

transformed RGB vector  original RGB vector

What are the dimensions of each quantity in this equation?

How do we decide what transformed vectors to map to?
Using (again) a color checker

Calibration chart can be used for:
1. color calibration
2. radiometric calibration (i.e., response curve) using the bottom row

Color patches manufactured to have pre-calibrated XYZ coordinates.
Using (again) a color checker

1. Color calibration
2. Radiometric calibration (i.e., response curve) using the bottom row

Color patches manufactured to have pre-calibrated XYZ coordinates.

Can we use any color chart image for color calibration?

Calibration chart can be used for:
1. Color calibration
2. Radiometric calibration (i.e., response curve) using the bottom row
Using (again) a color checker

Color patches manufactured to have pre-calibrated XYZ coordinates.

Can we use any color chart image for color calibration?
- It needs to be a *linear* image!
- Do radiometric calibration first.

Calibration chart can be used for:
1. color calibration
2. radiometric calibration (i.e., response curve) using the bottom row
Color calibration

Apply linear scaling and translation to RGB vectors in the image:

\[ c' = M \cdot c + b \]

What are the dimensions of each quantity in this equation?

How do we decide what transformed vectors to map to?

How do we solve for matrix M and vector b?
Color calibration

Apply linear scaling and translation to RGB vectors in the image:

\[ c' = \begin{bmatrix} M & b \end{bmatrix} \begin{bmatrix} c \\ 1 \end{bmatrix} \]
Color calibration

Apply linear scaling and translation to RGB vectors in the image:

\[ c' = \begin{bmatrix} M & b \end{bmatrix} \begin{bmatrix} c \\ T \end{bmatrix} \]
Color calibration

Apply an affine transform to homogeneous RGB vectors in the image:

$$c' = T \cdot C$$

- heterogeneous transformed RGB vector
- homogeneous original RGB vector

How do we solve for an affine transformation?
Determining the affine transform matrix

Write out linear equation for each color vector correspondence:

\[ c' = T \cdot C \quad \text{or} \quad \begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \\ t_5 & t_6 & t_7 & t_8 \\ t_9 & t_{10} & t_{11} & t_{12} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \\ 1 \end{bmatrix} \]
Determining the affine transform matrix

Rearrange into an equation involving a vectorized form of $T$:

$$
\begin{bmatrix}
    r' \\
    g' \\
    b'
\end{bmatrix} = \begin{bmatrix}
    r & g & b & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & r & g & b & 1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r & g & b \\
\end{bmatrix} \begin{bmatrix}
    t_1 \\
    t_2 \\
    t_3 \\
    t_4 \\
    t_5 \\
    t_6 \\
    t_7 \\
    t_8 \\
    t_9 \\
    t_{10} \\
    t_{11} \\
    t_{12}
\end{bmatrix}
$$
Determining the affine transform matrix

Stack equations from multiple color vector correspondences:
Solving the linear system

Convert the system to a linear least-squares problem:

\[ E_{\text{LSS}} = \|Ax - b\|^2 \]

Expand the error:

\[ E_{\text{LSS}} = x^\top (A^\top A)x - 2x^\top (A^\top b) + \|b\|^2 \]

Minimize the error:

Set derivative to 0 \[ (A^\top A)x = A^\top b \]

Solve for \( x \)

\[ x = (A^\top A)^{-1} A^\top b \]

In Matlab:

\[ x = A \backslash b \]

Note: You almost never want to compute the inverse of a matrix.
An example

original

color-corrected
Quick note

If you cannot do calibration, take a look at the image’s EXIF data (if available).

Often contains information about tone reproduction curve and color space.
Color profiling for displays

Exactly analogous procedure for figuring out the color space of a display.

Note: In displays, color *calibration* refers to changing the display’s primaries so that colors are shown differently. This is a completely separate procedure from color profiling.

Note also the discrepancy in terminology between cameras and displays.
Non-linear color spaces
A few important linear color spaces

- LMS color space
- CIE RGB color space
- CIE XYZ color space

What about non-linear color spaces?

- sRGB color space
CIE xy (chromaticity)

\[ x = \frac{X}{X + Y + Z} \]
\[ y = \frac{Y}{X + Y + Z} \]

\((X, Y, Z) \leftrightarrow (x, y, Y)\)

CIE xyY is a non-linear color space.
Uniform color spaces

Find map $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that perceptual distance can be well approximated using Euclidean distance:

$$d(\hat{c}, \hat{c}') \approx \|F(\hat{c}) - F(\hat{c}')\|_2$$
MacAdam ellipses

Areas in chromaticity space of imperceptible change:
- They are ellipses instead of circles.
- They change scale and direction in different parts of the chromaticity space.
MacAdam ellipses

Note: MacAdam ellipses are almost always shown at 10x scale for visualization. In reality, the areas of imperceptible difference are much smaller.
The Lab (aka L*ab, aka L*a*b*) color space

The L* component of lightness is defined as

\[ L^* = 116 f \left( \frac{Y}{Y_n} \right), \] (2.105)

where \( Y_n \) is the luminance value for nominal white (Fairchild 2005) and

\[ f(t) = \begin{cases} 
  t^{1/3} & t > \delta^3 \\
  t/(3\delta^2) + 2\delta/3 & \text{else},
\end{cases} \] (2.106)

is a finite-slope approximation to the cube root with \( \delta = 6/29 \). The resulting 0 . . . 100 scale roughly measures equal amounts of lightness perceptibility.

In a similar fashion, the a* and b* components are defined as

\[ a^* = 500 \left[ f \left( \frac{X}{X_n} \right) - f \left( \frac{Y}{Y_n} \right) \right] \quad \text{and} \quad b^* = 200 \left[ f \left( \frac{Y}{Y_n} \right) - f \left( \frac{Z}{Z_n} \right) \right], \] (2.107)

where again, \( (X_n, Y_n, Z_n) \) is the measured white point. Figure 2.32i–k show the L*a*b* representation for a sample color image.
The Lab (aka L*ab, aka L*a*b*) color space

The L* component of lightness is defined as

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\]

where again, \((X_n, Y_n, Z_n)\) is the measured white point. Figure 2.32i–k show the L*a*b* representation for a sample color image.
Perceived vs measured brightness by human eye

Human-eye response (measured brightness) is linear.

However, human-eye perception (perceived brightness) is non-linear:
- More sensitive to dark tones.
- Approximately a Gamma function.
The Lab (aka L*ab, aka L*a*b*) color space
Hue, saturation, and value

Do not use color space HSV! Use \( \text{LCh} \):
- \( \text{L}^* \) for “value”.
- \( C = \sqrt{a^2 + b^2} \) for “saturation” (chroma).
- \( h = \text{atan}(b / a) \) for “hue”.
How could you make an image like this from a color image?
How could you make an image like this from a color image?

Zero saturation

Higher saturation

Control saturation with red-pass filter

Easier to do color processing in HSV

http://en.wikipedia.org/wiki/Schindler%27s_List
Some thoughts about color reproduction
The image processing pipeline

The sequence of image processing operations applied by the camera’s image signal processor (ISP) to convert a RAW image into a “conventional” image.

RAW image (mosaiced, linear, 12-bit) -> analog front-end

- denoising
- CFA demosaicing
- white balance

color transforms -> tone reproduction

- compression

final RGB image (non-linear, 8-bit)
Color reproduction notes

To properly reproduce the color of an image file, you need to?
Color reproduction notes

To properly reproduce the color of an image file, you need to convert it from the color space it was stored in, to a reference color space, and then to the color space of your display.

On the camera side:
• If the file is RAW, it *often* has EXIF tags with information about the RGB color space corresponding to the camera’s color sensitivity functions.
• If the file is not RAW, you *may* be lucky and still find accurate information in the EXIF tags about what color space the image was converted in during processing.
• If there is no such information and you own the camera that shot the image, then you can do color calibration for the camera.
• If all of the above fails, assume sRGB.

On the display side:
• If you own a high-end display, it likely has accurate color profiles provided by the manufacturer.
• If not, you can use a spectrometer to do color profiling (not color calibration).
• Make sure your viewer does not automatically do color transformations.

Be careful to account for any gamma correction!

Amazing resource for color management and photography: [https://ninedegreesbelow.com/](https://ninedegreesbelow.com/)
How do you convert an image to grayscale?
How do you convert an image to grayscale?

First, you need to answer two questions:

1) Is your image linear or non-linear?
   • If the image is linear (RAW, HDR, or otherwise radiometrically calibrated), skip this step.
   • If the image is nonlinear (PNG, JPEG, etc.), you must undo the tone reproduction curve.
     i. If you can afford to do radiometric calibration, do that.
     ii. If your image has EXIF tags, check there about the tone reproduction curve.
     iii. If your image is tagged as non-linear sRGB, use the inverse of the sRGB tone reproduction curve.
     iv. If none of the above, assume sRGB and do as in (iii).

2) What is the color space of your image?
   • If it came from an original RAW file, read the color transform matrix from there (e.g., dcraw).
   • If not, you need to figure out the color space.
     i. If you can afford to do color calibration, use that.
     ii. If your image has EXIF tags, check there about the color space.
     iii. If your image is tagged as non-linear sRGB, use the color transform matrix for linear sRGB.
     iv. If none of the above, assume sRGB and do (iii).

With this information in hand:
   • Transform your image into the XYZ color space.
   • Extract the Y channel.
   • If you want brightness instead of luminance, apply the Lab brightness non-linearity.
References

Basic reading:
• Szeliski textbook, Section 2.3.2, 3.1.2
• Gortler, “Foundations of 3D Computer Graphics,” MIT Press 2012. Chapter 19 of this book has a great coverage of color spaces and the theory we discussed in class, it is available in PDF form from the CMU library.

Additional reading:
  All of the above books are great references on color photography, reproduction, and management. The book by Reinhard et al. is my go-to reference on color.
• Nine Degrees Below, https://ninedegreesbelow.com/ Amazing resource for color photography, reproduction, and management.
• Bruce Lindbloom’s website, http://brucelindbloom.com/ An online page with a lot of information about color transforms, adaptation, and so on.