Pinholes and lenses
Course announcements

• Homework 1 is out.
  - Due September 13\textsuperscript{th}.
  - Make sure to sign up for a camera if you need one.
  - Drop by Yannis’ office to pick up cameras any time.

• Office hours for the semester:
  - Tiancheng: Tuesdays 5-7 pm, Smith Hall graphics lounge.
  - Yannis: Fridays 5-7 pm, Smith Hall graphics lounge.
Overview of today’s lecture

- Leftover from lecture 2: thoughts on the image processing pipeline.
- Some motivational imaging experiments.
- Pinhole camera.
- Accidental pinholes.
- The thin lens model.
- Lens camera and pinhole camera.
- Perspective.
- Field of view.
- Orthographic camera and telecentric lenses.
Many of these slides were adapted from:

- Kris Kitani (15-463, Fall 2016).
- Fredo Durand (MIT).
- Gordon Wetzstein (Stanford).
The modern photography pipeline

- Optics and optical controls (lectures 2-3, 11-20)
- Sensor, analog front-end, and color filter array (lectures 2, 23)
- In-camera image processing pipeline (lecture 2)

Post-capture processing (lectures 5-10)
Some motivational imaging experiments
Let’s say we have a sensor...

digital sensor (CCD or CMOS)
... and an object we like to photograph

What would an image taken like this look like?
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

All scene points contribute to all sensor pixels

What does the image on the sensor look like?

digital sensor (CCD or CMOS)

real-world object
Bare-sensor imaging

All scene points contribute to all sensor pixels
What can we do to make our image look better?

real-world object
digital sensor (CCD or CMOS)
Let’s add something to this scene

What would an image taken like this look like?
Pinhole imaging

most rays are blocked

one makes it through

digital sensor (CCD or CMOS)

real-world object
Pinhole imaging

Most rays are blocked, one makes it through.

Real-world object

Digital sensor (CCD or CMOS)
Each scene point contributes to only one sensor pixel
Pinhole imaging

real-world object

copy of real-world object (inverted and scaled)
Pinhole camera
Pinhole camera a.k.a. camera obscura
Chinese philosopher Mozi (470 to 390 BC)  

Greek philosopher Aristotle (384 to 322 BC)
Pinhole camera terms

- real-world object
- barrier (diaphragm)
- pinhole (aperture)
- digital sensor (CCD or CMOS)
Pinhole camera terms

- real-world object
- barrier (diaphragm)
- pinhole (aperture)
- camera center (center of projection)
- image plane
- digital sensor (CCD or CMOS)
Focal length

real-world object

focal length $f$
What happens as we change the focal length?

real-world object

focal length 0.5 f
Focal length

What happens as we change the focal length?

real-world object

focal length 0.5 f
Focal length

What happens as we change the focal length?

object projection is half the size

real-world object

focal length 0.5 f
Pinhole size

Ideal pinhole has infinitesimally small size
- In practice that is impossible.
Pinhole size

What happens as we change the pinhole diameter?

real-world object

pinhole diameter
Pinhole size

What happens as we change the pinhole diameter?
What happens as we change the pinhole diameter?
Pinhole size

What happens as we change the pinhole diameter?

object projection becomes blurrier

real-world object
Pinhole size

What happens as we change the pinhole diameter?

Will the image keep getting sharper the smaller we make the pinhole?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
Diffraction limit

Diffraction pattern = Fourier transform of the pinhole.
• Smaller pinhole means bigger Fourier spectrum.
• Smaller pinhole means more diffraction.
What about light efficiency?

• What is the effect of doubling the pinhole diameter?
• What is the effect of doubling the focal length?
What about light efficiency?

- 2x pinhole diameter $\rightarrow$ 4x light
- 2x focal length $\rightarrow$ $\frac{1}{4}$x light
Some terminology notes

A “stop” is a change in camera settings that changes amount of light by a factor of 2

The “f-number” is the ratio: focal length / pinhole diameter
Accidental pinholes
What does this image say about the world outside?
Accidental pinhole camera

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MIT
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Accidental pinhole camera

window is an aperture

projected pattern on the wall

upside down

window with smaller gap

view outside window
Accidental pinspeck camera

![Accidental pinspeck camera diagram](image-url)
Pinhole camera trade-off

Small (ideal) pinhole:
1. Image is sharp.
2. Signal-to-noise ratio is low.
Large pinhole:
1. Image is blurry.
2. Signal-to-noise ratio is high.

Can we get best of both worlds?
Almost, by using lenses

Lenses map “bundles” of rays from points on the scene to the sensor.

How does this mapping work exactly?
Lens (very) basics
What is a lens?

A piece of glass manufactured to have a specific shape
What is a lens?

A piece of glass manufactured to have a specific shape

- shape of surfaces (usually spherical)
- type of glass
- focal length $f$

Focal length is determined by the lens’ shape and material

Aperture limiting the extent of the lens

Focal plane
The lens on your camera
Aperture size

Most lenses have apertures of variable size.

- The size of the aperture is expressed as the “f-number”: The bigger this number, the smaller the aperture.

You can see the aperture by removing the lens and looking inside it.

- f / 1.4
- f / 2.8
- f / 4
- f / 8
- f / 16
How does a lens work?

Lenses are designed so that their refraction makes light rays bend in a very specific way.
Refraction

Refraction is the bending of rays of light when they move from one material to another.
The thin lens model
Thin lens model

Simplification of geometric optics for well-designed lenses.

Two assumptions:
1. Rays passing through lens center are unaffected.
Thin lens model

Simplification of geometric optics for well-designed lenses.

Two assumptions:
1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.
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Two assumptions:
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2. Parallel rays converge to a single point located on focal plane.
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?
Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.

object distance $D$  focal length $f$
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.
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2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.

Focusing property:
1. Rays emitted from a point on one side converge to a point on the other side.
Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
   a. Trace their parallel through lens center.
   b. Connect on focal plane.

Focusing property:
1. Rays emitted from a point on one side converge to a point on the other side.
2. Bundles emitted from a plane parallel to the lens converge on a common plane.
Gaussian lens formula

How can we relate scene-space \((D, y)\) and image space \((D', y')\) quantities?
Gaussian lens formula

How can we relate scene-space \((D, y)\) and image space \((D', y')\) quantities?

Use similar triangles
Gaussian lens formula

How can we relate scene-space (D, y) and image space (D’, y’) quantities?

\[ \frac{y}{y'} = ? \]

Use similar triangles
How can we relate scene-space \((D, y)\) and image space \((D', y')\) quantities?

\[
\frac{y}{y'} = \frac{D}{D'}
\]

Use similar triangles.
Gaussian lens formula

How can we relate scene-space \((D, y)\) and image space \((D', y')\) quantities?

\[
\frac{y}{y'} = \frac{D}{D'}
\]

\[
\frac{y}{y'} = ?
\]

Use similar triangles
How can we relate scene-space \((D, y)\) and image space \((D', y')\) quantities?

\[
\frac{y}{y'} = \frac{D}{D'}
\]

\[
\frac{y}{y'} = \frac{f}{D' - f}
\]
**Gaussian lens formula**

How can we relate scene-space \((D, y)\) and image space \((D', y')\) quantities?

Use similar triangles
- We call \(m = y / y'\) the magnification

\[
m = \frac{f}{D' - f}
\]

\[
\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}
\]
Special focus distances

D’ = f, D = ?, m = ?

\[ m = \frac{f}{D' - f} \]

\[ \frac{1}{D'} + \frac{1}{D} = \frac{1}{f} \]
Special focus distances

\[ D' = f, \ D = \infty, \ m = \infty \rightarrow \text{infinity focus (parallel rays)} \]

\[
m = \frac{f}{D' - f}
\]

\[
\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}
\]

\[ D' = D = ?, \ m = ? \]
Special focus distances

\[ D' = f, \quad D = \infty, \quad m = \infty \rightarrow \text{infinity focus (parallel rays)} \]

\[ m = \frac{f}{D' - f} \]

\[ \frac{1}{D'} + \frac{1}{D} = \frac{1}{f} \]

\[ D' = 2f, \quad m = 1 \rightarrow \text{object is reproduced in real-life size} \]

\[ D = 2f \]

\[ D' = 2f \]
Do we lose anything by using a lens?

By using a lens we simultaneously achieve:
1. Image is sharp.
2. Signal-to-noise ratio is high.
Defocus

What happens if we don’t place the sensor at the focus distance?
Defocus

What happens if we don’t place the sensor at the focus distance?

We get a blurry image. This is called defocus.
- Defocus never happens with a pinhole camera.
Defocus

Can’t we just move the sensor to the correct distance?
Defocus

Can’t we just move the sensor to the correct distance?

objects at one depth are in focus

objects at all other depths are out of focus

Unless our scene is just one plane, part of it will always be out of focus.
Change of focus for different depths
How do I control what is in focus?
How do I control what is in focus?

I change the distance between the sensor and the lens plane in focus.
How do I control what is in focus?

I change the distance between the sensor and the lens

• What happens to plane in focus?

move lens further away from sensor
How do I control what is in focus?

I change the distance between the sensor and the lens

- What happens to plane in focus? → It moves closer.
The lens on your camera

Focus ring: controls distance of lens from sensor
Sequence of images at different focus settings
Lens camera and pinhole camera
The lens camera
The pinhole camera
Central rays propagate in the same way for both models!
Describing both lens and pinhole cameras

We can derive properties and descriptions that hold for both camera models if:

- We use only central rays.
- We assume the lens camera is in focus.
Important difference: focal length

In a pinhole camera, focal length is distance between aperture and sensor.
Important difference: focal length

In a lens camera, focal length is distance where parallel rays intersect.
Describing both lens and pinhole cameras

We can derive properties and descriptions that hold for both camera models if:

- We use only central rays.
- We assume the lens camera is in focus.
- We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.
Field of view
What happens as you take a closer look?
Field of view

also known as angle of view

Note: here I drew a lens, but I could have just as well drawn a pinhole
Field of view

The part of the in-focus plane that gets mapped on the sensor

- What happens to field of view as we focus closer?
Field of view

The part of the in-focus plane that gets mapped on the sensor

- What happens to field of view as we focus closer? $\rightarrow$ It becomes smaller.

move lens further away from sensor
Field of view also depends on sensor size

- What happens to field of view when we reduce sensor size? → It decreases.
Field of view also depends on sensor size

- “Full frame” corresponds to standard film size.
- Digital sensors come in smaller formats due to manufacturing limitations (now mostly overcome).
- Lenses are often described in terms of field of view on film instead of focal length.
- These descriptions are invalid when not using full-frame sensor.
Crop factor

How much field of view is cropped when using a sensor smaller than full frame.
Magnification and perspective
The pinhole camera

real-world object

camera center

focal length $f$

image plane
The (rearranged) pinhole camera

real-world object

image plane

camera center

camera center

focal length f
The 2D view of the (rearranged) pinhole camera

What is the equation for image coordinate $x$ in terms of $X$?
The 2D view of the (rearranged) pinhole camera image plane

$$\begin{bmatrix} X & Y & Z \end{bmatrix}^T \mapsto \begin{bmatrix} fX/Z & fY/Z \end{bmatrix}^T$$

magnification changes with depth
Magnification depends on depth

\[
\frac{y}{y'} = \frac{f}{D' - f}
\]

\[
\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}
\]

What happens to image size as depth increases?

object distance D

object height y

focal length f

image height y'

focus distance D'
Forced perspective
The Ames room illusion
The Ames room illusion
The arrow illusion

Prof. Kokichi Sugihara has many other amazing illusions involving perspective distortion, check them out on YouTube or on his website:

http://www.isc.meiji.ac.jp/~kokichis/
Zooming
Zooming means changing the focal length

Very different process from refocusing

focal length $f$ changes

lens-sensor distance remains the same
• What happens to field of view when we focus closer? → It decreases.
• What happens to field of view when we increase lens focal length?
• When we increase lens focal length, field of view decreases (we “zoom in”).
Field of view

1000 mm: 2.5°
500 mm: 5°
350 mm: 7.5°
250 mm: 10°
135 mm: 18°
85 mm: 29°
50 mm: 43°
35 mm: 63°
28 mm: 75°
8 mm: 180°
Field of view

Increasing the lens focal length is similar to cropping

Is this effect identical to cropping?

f = 25 mm

f = 50 mm

f = 135 mm
The lens on your camera

Focus ring: controls distance of lens from sensor

Zoom ring: controls focal length of lens
Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:
1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

When you turn the zoom ring to decrease the focal length of the lens:
1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).
Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:
1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

When you turn the zoom ring to decrease the focal length of the lens:
1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).

We can use both focus and zoom to cancel out their effects.
Magnification depends on depth

What happens as we change the focal length?

real-world object

depth Z

depth 2 Z
Magnification depends on focal length

real-world object

focal length $f$
focal length $2f$

focal length $f$
focal length $2f$
1. Set focal length to half
What if…

1. Set focal length to half
2. Set depth to half

Is this the same image as the one I had at focal length $2f$ and distance $2Z$?
Perspective distortion

long focal length  mid focal length  short focal length
Perspective distortion
What is the best focal length for portraits?

That’s like asking which is better, vi or emacs...

long focal length  mid focal length  short focal length
Vertigo effect

Named after Alfred Hitchcock’s movie
• also known as “dolly zoom”
Vertigo effect

How would you create this effect?
Orthographic camera and telecentric lenses
What if…

real-world object

Continue increasing Z and f while maintaining same magnification?

\[ f \to \infty \text{ and } \frac{f}{Z} = \text{constant} \]
The 2D view of the (rearranged) pinhole camera

The magnification changes with depth.

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}^T \mapsto \begin{bmatrix}
fX/Z \\
fY/Z
\end{bmatrix}^T
\]
Orthographic vs pinhole camera

$$[X \ Y \ Z]^T \rightarrow [fX \ fY]^T$$

Magnification does not change with depth

$$[X \ Y \ Z]^T \rightarrow [fX/Z \ fY/Z]^T$$

Magnification changes with depth
How can we implement such a camera with lenses?
Telecentric lens

Place a pinhole at focal length, so that only rays parallel to primary ray pass through.
Telecentric lens

Place a pinhole at focal length, so that only rays parallel to primary ray pass through.

Magnification independent of object depth.

Magnification depends only on sensor-lens distance.
Regular vs telecentric lens

regular lens
telecentric lens
References

Basic reading:
• Szeliski textbook, Section 2.1.5, 2.2.3.
• Pedrotti, Pedrotti, and Pedrotti, Introduction to Optics.
  Chapters 2 and 3 have a detailed overview of basic geometric optics and lenses.

Additional reading:
  Chapter 6 of this book is a very thorough treatment of camera models.
  The standard reference on Fourier optics, chapter 4 covers aperture diffraction.
  A great book covering everything about photographic optics.
• Torralba and Freeman, “Accidental Pinhole and Pinspeck Cameras,” CVPR 2012.
  The eponymous paper discussed in the slides.