Time-of-flight imaging
Course announcements

• Homework assignment 6 is due on December 11\textsuperscript{th}.
  - Do not leave for last minute, you won’t have time to complete it.

• Make sure to sign up for a project checkpoint meeting.
  - Sign up spreadsheet available on Piazza.

• We will have another reading group this Friday, please suggest topics.
15-468/668/868 Physics-based rendering, Spring 2021
Forward graphics (a.k.a. rendering)

digital scene specification
(geometry, materials, optics, light sources)

physically-accurate rendering

photorealistic simulated image
Physics-based rendering

- **Efficiency challenge**: how do we sample the most important of all the possible photon paths in scattering?

- **Generality challenge**: how do we model all the different sensing technologies used in this expedition?

Monte Carlo rendering:
- randomly sample photons: path₁, path₂, ..., pathₙ
- approximate image as:

\[
\text{Image} \approx \sum_{n} \frac{f(\text{path}_n)}{\text{prob}(\text{path}_n)}
\]
LANGEVIN MONTE CARLO RENDERING WITH GRADIENT-BASED ADAPTATION

Prior

Ours

10x acceleration
Prior

20x acceleration

Ours
Speckle and memory effect

- Speckle: noise-like pattern
- What real laser images look like
- What standard rendered images look like
- Projected speckle image
- Laser beam
- Scattering volume

8
Inverse graphics

digital scene specification
(geometry, materials, camera, light sources)

physically-accurate inverse rendering

photorealistic synthetic image
Differentiable rendering

Original image | Derivative image
Original image | Derivative image
Original image | Derivative image
Original image | Derivative image
Differentiable rendering this year

\[ I = \int_{\Omega} f(\vec{x}) \, d\mu(\vec{x}) \]

Forward rendering

\[ \int_{\Omega} \frac{d}{d\pi} f(\vec{x}) \, d\mu(\vec{x}) + \int_{\partial\Omega} g(\vec{x}) \, d\mu'(\vec{x}) \]

Reynolds theorem

Differentiable rendering

Original light path

Boundary light path
Suite of differentiable rendering algorithms

**Unidirectional estimator**
- Interior: unidirectional path tracing
- Boundary: unidirectional sampling of subpaths

**Bidirectional estimator**
- Interior: bidirectional path tracing
- Boundary: bidirectional sampling of subpaths
Seeing through diffusers

source and sensor

thin diffuser

occluded object

reconstruction evolution
Non-invasive tomography

camera
thick smoke cloud
simulated camera measurements
reconstructed cloud volume
slice through the cloud
Non-invasive tumor imaging

finding shape and location of a tumor 3 mm below the skin using a single above-the-skin photograph
Physics-aware learning

\[ \pi = (\text{Physics})^{-1}(\text{Img}) \]

\[ \text{Img} = \text{Physics}(\pi) \]

needs to be differentiable for training with backpropagation

force input and output images to be the same
Overview of today’s lecture

• Introduction to time-of-flight (ToF) imaging.
• Impulse ToF imaging and single-photon avalanche diodes.
• Continuous-wave ToF imaging.
• Epipolar continuous-wave ToF imaging.
• Interferometric ToF imaging.
Slide credits

A lot of these slides were adapted from:

- Mohit Gupta (Wisconsin).
- Supreeth Achar (Google, formerly CMU).
Introduction to time-of-flight (ToF) imaging
Time-of-flight (ToF) imaging
Time-of-flight (ToF) imaging

- Conventional imaging: Measure all photons together regardless of time of travel.
- Time-of-flight imaging: Measure photons separately based on time of travel.
Time-of-flight imaging in nature

echolocation using sound-wave time-of-flight
Time-of-flight applications: depth sensing
Time-of-flight applications: non-line-of-sight imaging

wall

hidden object

camera

what depth our camera sees

what a regular camera sees

wall

hidden object

camera

what depth our camera sees

what a regular camera sees
Time-of-flight applications: seeing inside objects

camera

thick smoke cloud

what a regular camera sees

what our camera sees

a slice through the cloud
Time-of-flight applications: light-in-flight visualization

video at $10^{12}$ frames per second

video at $10^{15}$ frames per second
## Time-of-flight imaging technologies

<table>
<thead>
<tr>
<th>Temporal resolution</th>
<th>interferometry</th>
<th>streak cameras</th>
<th>single-photon avalanche diodes</th>
<th>time-of-flight cameras</th>
<th>LIDAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 femtosecond</td>
<td>1 picosecond</td>
<td>100 picoseconds</td>
<td>1 nanosecond</td>
<td>10 nanoseconds</td>
<td>100 million fps</td>
</tr>
<tr>
<td>(10^{-15} secs)</td>
<td>(10^{-12} secs)</td>
<td>(10^{-10} secs)</td>
<td>(10^{-9} secs)</td>
<td>(10^{-8} secs)</td>
<td></td>
</tr>
<tr>
<td>quadrillion fps</td>
<td>trillion fps</td>
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<td>billion fps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 micron</td>
<td>1 millimeter</td>
<td>10 centimeters</td>
<td>1 meter</td>
<td>10 meters</td>
<td></td>
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<tr>
<td>(10^{-6} meters)</td>
<td>(10^{-3} meters)</td>
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**Continuous-wave ToF**
- 1 femtosecond (10^{-15} secs)
- 1 micron (10^{-6} meters)
- quadrillion fps

**Impulse ToF**
- 1 picosecond (10^{-12} secs)
- 1 millimeter (10^{-3} meters)
- trillion fps

**ToF cameras**
- 100 picoseconds (10^{-10} secs)
- 10 centimeters (10^{-1} meters)
- 10 billion fps

**LIDAR**
- 1 nanosecond (10^{-9} secs)
- 1 meter (10^{-0} meters)
- 100 million fps

**Avalanche diodes**
- 100 picoseconds (10^{-10} secs)
- 10 centimeters (10^{-1} meters)
- 10 billion fps

**Billion fps**
- 1 nanosecond (10^{-9} secs)
- 1 meter (10^{-0} meters)

**Trillion fps**
- 1 picosecond (10^{-12} secs)
- 1 millimeter (10^{-3} meters)
- 10 billion fps
Impulse ToF imaging and single-photon avalanche diodes
Impulse time-of-flight imaging

How can we infer depth from this?

\[ \text{depth} = \frac{c}{2\tau} \]
Indirect paths are nuisance for depth sensing (“multi-path interference”).
Indirect paths are very informative for other time-of-flight applications.
Two types of time-of-flight imaging

- **Range imaging**: Measuring only first returning photons (e.g., LIDAR).
- **Transient imaging**: Measuring entire transient (e.g., SPAD).

**Transient I(t): Time-resolved radiance distribution**

- **Source**: Emitted light pulse
- **Sensor**: Received light
- **Direct path**: Radiance
- **Indirect paths**: Indirect radiance
How exactly is the transient formed?

Depends on the kind of sensor we use.
- Here we will examine only photodiodes.
How exactly is the transient formed?

Depends on the kind of sensor we use.

- Here we will examine only photodiodes.

**Avalanche photodiode (APD):**

- Current is roughly proportional to number of photons.
- One photon produces tiny current.
How exactly is the transient formed?

 Depends on the kind of sensor we use.
 • Here we will examine only photodiodes.

Avalanche photodiode (APD):
 • Current is roughly proportional to number of photons.
 • One photon produces tiny current.

Single-photon avalanche diode (SPAD):
 • One photon produces huge current.
 • Requires multiple low power pulses, so that one photon returns from each.
Geiger-mode impulse time-of-flight imaging

From each received pulse, one photon saturates the SPAD.

- The SPAD records only photon arrival times, no intensity.
- Additional electronics maintain a histogram of arrival times over multiple pulses.
Geiger-mode impulse time-of-flight imaging

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Geiger-mode impulse time-of-flight imaging

From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?

- The SPAD records only photon arrival times, no intensity.
- Additional electronics maintain a histogram of arrival times over multiple pulses
From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?

- Photons earlier in the transient have a higher probability of being detected than photons later in the transient.
- As a result, histogram of photon detections underestimates later parts of the transient.
- This effect is called *pile-up* and is very severe under strong light conditions.
What hardware do we need for impulse ToF?

Expensive lasers
[short (picosecond) and powerful (mega joules) light pulses]

High speed and high dynamic range sensors
[single-photon sensitivity]

Expensive syncing and photon-counting electronics
[picosecond time resolution]
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- **ToF**
  - Continuous-wave ToF
  - Impulse ToF
Continuous-wave ToF imaging
Continuous-wave (CW) time-of-flight imaging

- Source
- Continuous temporally modulated light
- Scene
- Sensor
- Sinusoid
- Intensity
- Time
- Emitted light
Continuous-wave (CW) time-of-flight imaging

Source → continuous temporally modulated light → scene

Sensor → received light

Phase-shift ∝ travel distance (depth)

Intensity

Time

Emission light

Received light
Measuring phase shift

\[ L_{\text{rec}}(t) = O_{\text{rec}} + A_{\text{rec}} \cos(\omega t - \phi_{\text{rec}}) \]
Measuring phase shift: direct

Low Signal-to-Noise-Ratio
Measuring phase shift: correlation

Correlation: \( L = \int E(t) \times L_{\text{rec}}(t) \, dt \)

- **L**: measured brightness
- **E(t)**: exposure function
- **L_{\text{rec}}(t)**: received light

Graph showing intensity over time with large exposure time (milliseconds).
Measuring phase shift: correlation

Correlation 1: High Signal-to-Noise-Ratio

Real Time Capture

Correlation 2: $I_2 = \int E_2(t) \times L_{rec}(t) \, dt$

Correlation 3: $I_3 = \int E_3(t) \times L_{rec}(t) \, dt$

\[ \begin{align*}
\text{depth} &\quad \varphi_{rec} \\
\text{offset} &\quad O_{rec} \\
\text{amplitude} &\quad A_{rec}
\end{align*} \]
Phase ambiguity

A sensor source

Different Scene Depths Have Same Phase
- Also known as “phase wrapping”.

\[ \varphi(B) = \varphi(A) \]

frequency 1

sensor
Phase ambiguity

Unambiguous Depth Range: $R_{\text{unambiguous}} = \frac{1}{2\omega}$

How can we resolve the phase ambiguity?
Disambiguating phase

Compute phases at two different frequencies
Implementation: two-well architectures

- approximate sinusoid with a square pulse
- store photons in different wells depending on whether they arrive at 1 or 0
- take difference between two wells
Some examples

light source
(bank of laser diodes)

sensor
(PMD CamBoard Nano)

(only second generation of Kinect uses CW ToF)
Multi-path interference

ToF depth reconstruction
mean error = 86.6 mm
Transmitting imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

- Range imaging: Measuring only first returning photons (e.g., LIDAR).
- Transient imaging: Measuring entire transient (e.g., SPAD).

Transient I(t): Time-resolved radiance distribution
Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

- Each measurement we capture is of the form:

\[ I(\omega) = \int \sin(\omega t) \cdot I(t) dt \]
Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

- Each measurement we capture is of the form:

\[ I(\omega) = \int \sin(\omega t) \cdot I(t) dt \]

We can do transient imaging by taking measurements at multiple frequencies \( \omega \), then doing an inverse Fourier transform.
Interferometric ToF imaging
Tiny scenes

1 cm

Δt

0.5 cm

Δt

Δt \sim 10^{-3} \text{ ps}

toy cup
Interferometry example

- Light source
- Scene
- Beam splitter
- Regular camera
- Light source
Interferometry example

Continuous wave

Light source

Beamsplitter

Regular camera

Scene
Michelson interferometer

- **Continuous wave**
- **Light source**
- **Beam splitter**
- **Regular camera**
- **Scene**
- **Mirror**
- **Interference**
Optical coherence tomography

- Continuous wave light source
- Beam splitter
- Regular camera
- Translation stage
- Mirror
- Scene
- Interference: $I(\tau)$
Temporal coherence

Temporal coherence length (a.k.a. time resolution)

Temporal coherence interference

$I(\tau) =$

Mirrors' same length

Temporal coherence length

Temporal coherence interference

Mirror position

Correlation

Real field

Light source

Wavelength

Temporal coherence

Correlation

Mirror position

Real field
Temporal coherence length

- **Bandwidth**: broadband
- **Correlation bandwidth**: 25 nm
- **Pathlength resolution**: $\Delta t \sim 10 \mu m$

- **Superluminescent diode**
- **Monochromatic**
- **Very broadband supercontinuum laser**
Optical setup

Light source:
- Superluminescent diode
- Supercontinuum laser
- Broadband LED
- Sodium lamp

Components:
- Collimating lens
- Beamsplitter
- Filter wheel
- Camera + imaging lens
- Translating mirror
- Scene
Some transient images

centimeter-sized objects
Material properties

birefringence
dispersion
scattering
Gummy bear and diffuse corner

2 cm

diffuser
diffuser

pathlength
($\Delta \tau = 10 \mu m$)

dark frame
	surface reflections
	paths through gummy bear

very highly scattered paths
Chess knight and mirror

3 cm

mirror
diffuser

pathlength ($\Delta \tau = 10 \mu m$)
surface reflection
mirror-object
object-mirror
mirror-object-mirror
Subsurface scattering

1 cm pathlength ($\Delta \tau = 10 \mu m$)

- Surface reflection
- Subsurface scattering
- Paths transmitted through ground glass
- Diffuse-diffuse reflections

Zirconia coating, ground glass plate
White jade

3 cm

exquisite white jade

time (10^{-15} seconds)

specular reflections

low-order scattering

mid-order scattering

high-order scattering

[TOG 2015]
Dispersion

- cropped frame
- mirror diffuser
glass slab
- refractive index $\eta$ (wavelength)

1 cm

$\Delta t \sim \text{ns}$

$\Delta t \sim 10^{-3} \text{ ns}$
Dispersion

1 cm

plastic bead

mirror diffuser

pathlength $(\Delta \tau = 10 \, \mu m)$

facets changing color

surface reflections

surface-wall reflections

rainbow
Visualizing dispersion

- White light
- Rainbow
- Glass
- Camera
- $10^{-15}$ sec

- What a regular camera sees
- What our camera sees
Visualizing photoelasticity

detail under polarized light

low resolution $\Delta \tau = 1 \text{ mm}$

high resolution $\Delta \tau = 10 \mu\text{m}$
Toy cup

full transient

measured depth
direct-only transient

one-bounce reflections

Pixel

Pathlength

caustics

specular interreflections

one-bounce reflections

Pixel

Pathlength
Depth scanning

Depth resolution $\Delta \tau \sim 10 \, \mu m$

- coin
- potato
- soap carving
- gummy bear and diffusers

Dimensions:
- Coin: 2.5 cm
- Potato: 1 cm
- Soap carving: 1.5 cm
- Gummy bear: 3 cm
References

Basic reading:
  this tutorial provides an overview of many of the topics covered in this lecture, with a focus on continuous-wave ToF imaging.
  a great review paper for ToF imaging.
  the paper that introduced the idea of transient imaging to the computational imaging community, and an explanation of how streak cameras work.
  a standard reference on continuous-wave ToF sensors.
  three papers showing how continuous-wave ToF sensors can be used for transient imaging.
  a more recent paper that provides nice insights into how continuous-wave ToF works, as well as a way to deal with MPI.
  a very early paper showing visualization of light-in-flight, i.e., transient imaging.
  the paper introducing optical coherence tomography.
  the paper showing how interferometry can be used for time-of-flight imaging.
  the paper describing how SPADs can be used for ToF imaging.
  a paper explaining the operation of SPADs in a more accessible manner to computer science backgrounds.
• Heide et al., “Sub-picosecond photon-efficient 3D imaging using single-photon sensors,”
  three papers discussing the pile-up issue and proposing ways to overcome it.
  a keynote discussing advantages and current state of SPAD LiDAR technology.

Additional reading:
  the first two papers showing how ToF imaging can be used for looking around the corner.