Time-of-flight imaging
Course announcements

- Homework assignment 6 is due on Sunday, December 5th.
  - Do not leave for last minute, you won’t have time to complete it.

- Sign up for optional final project checkpoint meeting.
  - Sign up spreadsheet available on Piazza.

- Vote on Piazza for optional extra lecture on Thursday or Friday.
Overview of today’s lecture

• Introduction to time-of-flight (ToF) imaging.
• Impulse ToF imaging and single-photon avalanche diodes.
• Continuous-wave ToF imaging.
• Interferometric ToF imaging.
A lot of these slides were adapted from:

- Mohit Gupta (Wisconsin).
- Supreeth Achar (Google, formerly CMU).
Introduction to time-of-flight (ToF) imaging
Time-of-flight (ToF) imaging
Time-of-flight (ToF) imaging

- Conventional imaging: Measure all photons together regardless of time of travel.
- Time-of-flight imaging: Measure photons separately based on time of travel.
Time-of-flight imaging in nature

echolocation using sound-wave time-of-flight
Time-of-flight applications: depth sensing
Time-of-flight applications: non-line-of-sight imaging

- **Wall**: The surface that the camera is facing.
- **Hidden Object**: An object that is not visible to a regular camera but can be detected by a TOF camera.
- **Camera**: The device that captures images.
- **What a Regular Camera Sees**: The standard view captured by a regular camera.
- **What Shape Our Camera Sees**: The shape of the hidden object as seen by the TOF camera.
- **What Depth Our Camera Sees**: The depth information of the hidden object as captured by the TOF camera.

**Time-of-Flight Imaging**

- TOF cameras use the time it takes for a light pulse to travel to an object and back to determine the distance to that object.
- This is particularly useful in scenarios where the object is not visible to a regular camera, such as behind a wall or around a corner.

**Applications**

- **Security and Surveillance**: TOF cameras can help in identifying objects that are hidden from view.
- **Robotics**: TOF cameras provide depth information, which is crucial for robotic navigation and object recognition.
- **Augmented Reality**: TOF cameras can be used to create a more accurate and interactive AR environment.

**Advantages**

- TOF cameras provide depth information, which is not available from regular cameras.
- They can operate in areas where regular cameras cannot function due to light conditions or object occlusion.
Time-of-flight applications: seeing inside objects

- Camera
- Thick smoke cloud
- What a regular camera sees
- What our camera sees
- A slice through the cloud
Time-of-flight applications: light-in-flight visualization

video at $10^{12}$ frames per second

video at $10^{15}$ frames per second
# Time-of-flight imaging technologies

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- **ToF**:
  - continuous-wave ToF
  - impulse ToF

- **ToF**

- **Impulse**
  - 1 femtosecond (10^{-15} secs)
  - 1 picosecond (10^{-12} secs)
  - 100 picoseconds (10^{-10} secs)
  - 1 nanosecond (10^{-9} secs)
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- **Frame rate**
  - Quadrillion fps
  - Trillion fps
  - 10 billion fps
  - Billion fps
  - 100 million fps
Impulse ToF imaging and single-photon avalanche diodes
Impulse time-of-flight imaging

How can we infer depth from this?

\[ depth = \frac{c}{2\tau} \]
Impulse time-of-flight imaging

- Indirect paths are nuisance for depth sensing ("multi-path interference").
- Indirect paths are very informative for other time-of-flight applications.
Two types of time-of-flight imaging

- Range imaging: Measuring only first returning photons (e.g., LIDAR).
- Transient imaging: Measuring entire transient (e.g., SPAD).

Transient $I(t)$: Time-resolved radiance distribution
How exactly is the transient formed?

Depends on the kind of sensor we use.
• Here we will examine only photodiodes.
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• Here we will examine only photodiodes.

Avalanche photodiode (APD):

• Current is roughly proportional to number of photons.
• One photon produces tiny current.
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Depends on the kind of sensor we use.

• Here we will examine only photodiodes.

Avalanche photodiode (APD):

• Current is roughly proportional to number of photons.
• One photon produces tiny current.

Single-photon avalanche diode (SPAD):

• One photon produces huge current.
• Requires multiple low power pulses, so that one photon returns from each.
Geiger-mode impulse time-of-flight imaging

From each received pulse, one photon saturates the SPAD.

- The SPAD records only photon arrival times, no intensity.
- Additional electronics maintain a histogram of arrival times over multiple pulses.
Geiger-mode impulse time-of-flight imaging

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From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?

- The SPAD records only photon arrival times, no intensity.
- Additional electronics maintain a histogram of arrival times over multiple pulses.
From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?

- Photons earlier in the transient have a higher probability of being detected than photons later in the transient.
- As a result, histogram of photon detections underestimates later parts of the transient.
- This effect is called pile-up and is very severe under strong light conditions.
What hardware do we need for impulse ToF?

- Expensive lasers [short (picosecond) and powerful (mega joules) light pulses]
- High speed and high dynamic range sensors [single-photon sensitivity]
- Expensive syncing and photon-counting electronics [picosecond time resolution]
## Time-of-flight imaging technologies

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Continuous-wave ToF imaging
Continuous-wave (CW) time-of-flight imaging

Source emits modulated light, which is reflected back to the sensor. The intensity of the reflected light is plotted over time to form a sinusoidal pattern.
Continuous-wave (CW) time-of-flight imaging

- Source emits continuous temporally modulated light.
- Light travels to the scene.
- Sensor receives the light.

Phase-shift $\propto$ travel distance (depth)

Intensity and time graph showing emitted light and received light.
Measuring phase shift

\[ L_{\text{rec}}(t) = O_{\text{rec}} + A_{\text{rec}} \cos(\omega t - \phi_{\text{rec}}) \]
Measuring phase shift: direct

Three Impulse Samples

Short Exposure Time (nanoseconds)

received light

Low Signal-to-Noise-Ratio
Measuring phase shift: correlation

\[ L = \int E(t) \times L_{rec}(t) \, dt \]

- Measured brightness
- Exposure function
- Received light
Measuring phase shift: correlation

Correlation 1: \[ I_1 = \int E_1(t) \times L_{rec}(t) \, dt \]

Correlation 2: \[ I_2 = \int E_2(t) \times L_{rec}(t) \, dt \]

Correlation 3: \[ I_3 = \int E_3(t) \times L_{rec}(t) \, dt \]

- **High Signal-to-Noise-Ratio**
- **Real Time Capture**
- **Depth** \( \phi_{rec} \)
- **Offset** \( O_{rec} \)
- **Amplitude** \( A_{rec} \)
Different Scene Depths Have Same Phase

- Also known as “phase wrapping”.

Phase ambiguity

\[ \varphi(B) = \varphi(A) \]

Sensor

Source

Frequency 1
Phase ambiguity

Unambiguous Depth Range: $R_{\text{unambiguous}} = \frac{1}{2\omega}$

How can we resolve the phase ambiguity?
Disambiguating phase

Compute phases at two different frequencies
Implementation: two-well architectures

- approximate sinusoid with a square pulse
- store photons in different wells depending on whether they arrive at 1 or 0
- take difference between two wells
Some examples

light source
(bank of laser diodes)

sensor
(PMD CamBoard Nano)

(only second generation of Kinect uses CW ToF)
Multi-path interference

2.0 meters

wall

interreflections

sensor

5.0 meters

2.0 meters

ToF depth reconstruction
mean error = 86.6 mm

ground truth
Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

• Range imaging: Measuring only first returning photons (e.g., LIDAR).
• Transient imaging: Measuring entire transient (e.g., SPAD).

Transient $I(t)$: Time-resolved radiance distribution
Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

- Each measurement we capture is of the form:
  \[ I(\omega) = \int \sin(\omega t) \cdot I(t) dt \]

Transient \( I(t) \): Time-resolved radiance distribution
Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

• Each measurement we capture is of the form:

  \[ I(\omega) = \int \sin(\omega t) \cdot I(t) dt \]

  We can do transient imaging by taking measurements at multiple frequencies \( \omega \), then doing an inverse Fourier transform.
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| impulse ToF                                            |                |                |                                |                        | Te
Tiny scenes

玩具杯子

$\Delta t \sim 10^{-3}$ ps

$\Delta t \sim \text{ps}$
Interferometry example

- Light source
- Beamsplitter
- Regular camera
- Scene
Interferometry example

- Continuous wave
- Light source
- Beamsplitter
- Regular camera
- Scene
Michelson interferometer

- Continuous wave light source
- Beamsplitter
- Regular camera
- Scene
- Mirror
- Interference
Optical coherence tomography

- Continuous wave light source
- Beamsplitter
- Regular camera
- Translation stage
- Interference pattern

\[ I(\tau) \]
Temporal coherence

Temporal coherence length (a.k.a. time resolution)

wavelength

real field

mirror position

correlation

I(τ) = I(τ) interference
Temporal coherence length

bandwidth

25 nm

correlation

pathlength resolution $\Delta t \sim 10 \mu m$

broadband

superluminescent diode

very broadband

monochromatic

supercontinuum laser
Optical setup

light source

- superluminescent diode
- supercontinuum laser
- broadband LED
- sodium lamp

beamsplitter

collimating lens

filter wheel

translating mirror

scene

camera + imaging lens
Some transient images

centimeter-sized objects

mirror
diffuser
diffuser
diffuser
Material properties

birefringence

dispersion

scattering
Gummy bear and diffuse corner

- **diffuser** diffuser

2 cm
dark frame
surface reflections
paths through gummy bear
very highly scattered paths

pathlength ($\Delta \tau = 10 \, \mu m$)
Chess knight and mirror

3 cm

pathlength ($\Delta \tau = 10 \mu m$)
Subsurface scattering

- Surface reflection
- Subsurface scattering
- Zirconia coating
- Ground glass plate
- 1 cm pathlength ($\Delta \tau = 10 \mu m$)
- Paths transmitted through ground glass
- Diffuse-diffuse reflections
White jade

3 cm

exquisite white jade

specular reflections

time (10^{-15} seconds)

low-order scattering

mid-order scattering

high-order scattering

[TOG 2015]
Dispersion

1 cm

cropped frame

glass slab

mirror
diffuser

refractive index \( \eta(\text{wavelength}) \)

\[ \Delta t \sim \text{ns} \]

\[ \Delta t \sim 10^{-3} \text{ ns} \]
Dispersion

1 cm

- mirror diffuser
- plastic bead

pathlength ($\Delta \tau = 10 \mu m$)

- facets changing color
- surface reflections
- surface-wall reflections
- rainbow
Visualizing dispersion

- Glass
- $10^{-15}$ sec
- Camera
- White light
- Rainbow
- What a regular camera sees
- What our camera sees
Visualizing photoelasticity

detail under polarized light

low resolution $\Delta \tau = 1 \text{ mm}$

high resolution $\Delta \tau = 10 \, \mu\text{m}$
Toy cup

full transient

pixel

direct-only transient

measured depth

pathlength

coustic

one-bounce reflections

specular interreflections

pixel

one-bounce reflections
Depth scanning

depth resolution $\Delta \tau \sim 10 \, \mu m$

- coin
- gnocchi
- soap carving
- gummy bear and diffusers

Measurements:
- 2.5 cm
- 1 cm
- 1.5 cm
- 3 cm
References

Basic reading:
  this tutorial provides an overview of many of the topics covered in this lecture, with a focus on continuous-wave ToF imaging.
  a great review paper for ToF imaging.
  the paper that introduced the idea of transient imaging to the computational imaging community, and an explanation of how streak cameras work.
  a standard reference on continuous-wave ToF sensors.
  three papers showing how continuous-wave ToF sensors can be used for transient imaging.
  a more recent paper that provides nice insights into how continuous-wave ToF works, as well as a way to deal with MPI.
  a very early paper showing visualization of light-in-flight, i.e., transient imaging.
  the paper introducing optical coherence tomography.
  the paper showing how interferometry can be used for time-of-flight imaging.
  the paper describing how SPADs can be used for ToF imaging.
  a paper explaining the operation of SPADs in a more accessible manner to computer science backgrounds.
- Heide et al., “Sub-picosecond photon-efficient 3D imaging using single-photon sensors,”
  three papers discussing the pile-up issue and proposing ways to overcome it.
  a keynote discussing advantages and current state of SPAD LiDAR technology.

Additional reading:
  the first two papers showing how ToF imaging can be used for looking around the corner.