Radiometry and reflectance
Course announcements

• Homework 4 is due on October 25th.
  - Any questions?

• Project proposals due tonight.
  - One-day extension to make up for late feedback on project ideas.

• Mid-semester grades have been posted.
  - Based on average grades for HW1 – HW3.
  - Please let me know of any issues before 5pm today.
Overview of today’s lecture

- Appearance phenomena.
- Measuring light and radiometry.
- Reflectance and BRDF.
- Light sources.
Slide credits

Most of these slides were adapted from:

• Srinivasa Narasimhan (16-385, Spring 2014).
• Todd Zickler (Harvard University).
• Steven Gortler (Harvard University).
Appearance
Appearance
“Physics-based” computer vision (a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are **efficient**: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena

\[ I \xrightarrow{} \text{shape, illumination, reflectance} \]
Example application: Photometric Stereo
Why study the physics (optics) of the world?

Let's see some pictures!
Light and Shadows
Reflections
Refractions
Interreflections
Mies Courtyard House with Curved Elements
Scattering
More Complex Appearances
Measuring light and radiometry
Solid angle

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

Depends on:
- orientation of patch
- distance of patch
The *solid angle* subtended by a small surface patch with respect to point $O$ is the area of its central projection onto the unit sphere about $O$.

**Depends on:**
- orientation of patch
- distance of patch

One can show:

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Units: steradians [sr]
**Solid angle**

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

- Depending on:
  - orientation of patch
  - distance of patch

One can show:

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

Units: steradians [sr]
Solid angle

To calculate solid angle subtended by a surface \( S \) relative to \( O \) you must add up (integrate) contributions from all tiny patches (nasty integral)

\[
\Omega = \int \int_S \frac{\mathbf{r} \cdot \mathbf{n}}{|\mathbf{r}|^3} \, dS
\]

One can show: “surface foreshortening”

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]

Units: steradians \([\text{sr}]\)
Question

Suppose surface $S$ is a hemisphere centered at $O$. What is the solid angle it subtends?
Question

- Suppose surface $S$ is a hemisphere centered at $O$. What is the solid angle it subtends?

- Answer: $2\pi$ (area of sphere is $4\pi r^2$; area of unit sphere is $4\pi$; half of that is $2\pi$)
Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch \( X \) in directions within angular wedge \( W \).
- It measures radiant flux [watts = joules/sec]: rate of photons hitting sensor area.
- Measurement depends on sensor area \(|X|\).

\[
\Phi(W, X)
\]

*shown in 2D for clarity; imagine three dimensions*
Quantifying light: flux, irradiance, and radiance

- **Irradiance:**
  - A measure of incoming light that is independent of sensor area $|X|$
  - Units: watts per square meter [$W/m^2$]
Quantifying light: flux, irradiance, and radiance

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Quantifying light: flux, irradiance, and radiance

- **Irradiance:**
  A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter $[\text{W/m}^2]$ 
- Depends on sensor direction normal.

\[
\Phi(W, X) \div |X| = \lim_{X \to x} E_{\hat{n}}(W, x)
\]

- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit.
- In the literature, notations $n$ and $W$ are often omitted, and values are implied by context.
Quantifying light: flux, irradiance, and radiance

- **Radiance**: A measure of incoming light that is independent of sensor area $|X|$, orientation $n$, and wedge size (solid angle) $|W|$
- Units: watts per steradian per square meter $[W/(m^2\cdot sr)]$

\[
\frac{E_{\hat{n}}(W, x)}{|W|} \quad \text{or} \quad L_{\hat{n}}(\hat{\omega}, x)
\]

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction $\omega$
Quantifying light: flux, irradiance, and radiance

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$E_n(W, x)\over |W|$

$L_n(\hat{\omega}, x)$

$\cos \theta = {\Box/2 \over |X|/2}$

$\rightarrow \Box = |X| \cos \theta$

“foreshortened area”
Quantifying light: flux, irradiance, and radiance

- **Radiance:**
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|

- Units: watts per steradian per square meter [W/(m²·sr)]

\[
\frac{E_n(W, x)}{|W|}
\]

\[
L_n(\hat{\omega}, x)
\]

\[
L(\hat{\omega}, x)
\]
Quantifying light: flux, irradiance, and radiance

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- Units: watts per steradian per square meter $[W/(m^2\cdot sr)]$

To correct this, convert to measurement that would have been made if sensor was perpendicular to direction $\omega$.

```
\[ E_n(W, x) \]
\[ |W| \]

\[ L_n(\hat{\omega}, x) \]
\[ L(\hat{\omega}, x) \]
```

- Has correct units, but still depends on sensor orientation.
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction $\omega$. 

“foreshortened in the direction of travel”
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by any finite sensor
Quantifying light: flux, irradiance, and radiance

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  - Allows computing the radiant flux measured by any finite sensor

\[
\Phi(W, X) = \int_X \int_W L(\hat{\mathbf{w}}, x) \cos \theta \, d\omega \, dA
\]
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by any finite sensor
    \[ \Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta \, d\omega \, dA \]
  - Constant along a ray in free space
    \[ L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega}) \]
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by any finite sensor
    \[ \Phi(W, X) = \int_X \int_W L(\omega, x) \cos \theta d\omega dA \]
  - Constant along a ray in free space
    \[ L(\omega, x) = L(\omega, x + \omega) \]
  - A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.
    - “Processed” images (like PNG and JPEG) are not linear radiance measurements!!
Question

Most light sources, like a heated metal sheet, follow Lambert’s Law

\[ J(\hat{\omega}) = J_0 \langle \hat{\omega}, \hat{n} \rangle = J_0 \cos \theta \]

“Lambertian area source”

What is the radiance \( L(\hat{\omega}, x) \) of an infinitesimal patch [W/sr \cdot m^2]?
Question

Most light sources, like a heated metal sheet, follow Lambert’s Law.

\[ J(\hat{w}) = J_o \langle \hat{w}, \hat{n} \rangle = J_o \cos \theta \]

What is the radiance \( L(\hat{w}, x) \) of an infinitesimal patch [W/sr \( \cdot \) m\(^2\)]?

Answer: \[ L(\hat{w}, x) = J_o / |X| \] (independent of direction)
Most light sources, like a heated metal sheet, follow Lambert’s Law

$$J(\hat{\omega}) = J_o \langle \hat{\omega}, \hat{n} \rangle = J_o \cos \theta$$

"Lambertian area source"

What is the radiance $L(\hat{\omega}, x)$ of an infinitesimal patch [W/sr·m²]?

Answer: $L(\hat{\omega}, x) = J_o/|X|$ (independent of direction)

“Looks equally bright when viewed from any direction”
Appearance
“Physics-based” computer vision (a.k.a “inverse optics”)

\[ I \implies \text{shape, illumination, reflectance} \]
Reflectance and BRDF
Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)
Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
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  - does not depend on the size of the wedges (i.e. is intrinsic to the material)

\[
f_{x,\hat{n}}(W_{\text{in}}, \hat{\omega}_{\text{out}}) = \frac{L_{\text{out}}(x, \hat{\omega}_{\text{out}})}{E_{\hat{n}}(W_{\text{in}}, x)}
\]

- Notations $x$ and $n$ often implied by context and omitted;
- directions $\omega$ are expressed in local coordinate system defined by normal $n$ (and some chosen tangent vector)
- Units: $sr^{-1}$
- Called Bidirectional Reflectance Distribution Function (BRDF)
BRDF: Bidirectional Reflectance Distribution Function

Irradiance at Surface in direction \((\theta_i, \phi_i)\): 
\[ E_{\text{surface}}(\theta_i, \phi_i) \]

Radiance of Surface in direction \((\theta_r, \phi_r)\): 
\[ L_{\text{surface}}(\theta_r, \phi_r) \]

\[
\text{BRDF} : f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L_{\text{surface}}(\theta_r, \phi_r)}{E_{\text{surface}}(\theta_i, \phi_i)}
\]
Reflectance: BRDF

- Units: sr\(^{-1}\)
- Real-valued function defined on the double-hemisphere
- Has many useful properties
Important Properties of BRDFs

- Conservation of Energy:

\[
\forall \omega_{in}, \quad \int_{\Omega_{out}} f(\omega_{in}, \omega_{out}) \cos \theta_{out} d\omega_{out} \leq 1
\]

Why smaller than or equal?
Property: “Helmholtz reciprocity”

- **Helmholtz Reciprocity**: (follows from 2\textsuperscript{nd} Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

\[
f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) = f_r(\vec{\omega}_{out}, \vec{\omega}_{in})
\]
**Common assumption**: Isotropy

BRDF does not change when surface is rotated about the normal.

\[ f_r(\overrightarrow{\omega}_{in}, \cdot) \]

\[ 4D \rightarrow 3D \]

\[ f_r(\overrightarrow{\omega}_{in}, \overrightarrow{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables: \( f (\theta_i, \theta_r, \phi_i - \phi_r) \)

[Matusik et al., 2003]
Reflectance: BRDF

- Units: sr\(^{-1}\)
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for any configuration of lights and viewpoint

\[
L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos \theta_{\text{in}} d\hat{\omega}_{\text{in}}
\]

Why is there a cosine in the reflectance equation?
Derivation of the Reflectance Equation

From the definition of BRDF:

\[ L_{\text{surface}}^{\text{src}} (\theta_i, \phi_i) \]

\[ L_{\text{surface}} (\theta_r, \phi_r) = E_{\text{surface}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r) \]
Derivation of the Scene Radiance Equation

From the definition of BRDF:

\[
L_{\text{surface}} (\theta_r, \phi_r) = \frac{E_{\text{surface}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r)}{2\pi}
\]

Write Surface Irradiance in terms of Source Radiance:

\[
L_{\text{surface}} (\theta_r, \phi_r) = L_{\text{src}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i d\omega_i
\]

Integrate over entire hemisphere of possible source directions:

\[
L_{\text{surface}} (\theta_r, \phi_r) = \int \int L_{\text{src}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i d\omega_i
\]

Convert from solid angle to theta-phi representation:
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]

\[ S = \int_0^\pi \int_0^{2\pi} \sin \theta \, d\theta \, d\phi = 4\pi \]
BRDF

\[ f_r(\vec{\omega}_{\text{in}}, \cdot) \]

\[ f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) \]

Bi-directional Reflectance Distribution Function (BRDF)
BRDF
Lambertian (diffuse) BRDF: energy equally distributed in all directions

What does the BRDF equal in this case?

\[ f_r(\vec{\omega}_{in}, \cdot) \]

\[ f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)
Diffuse Reflection and Lambertian BRDF

- Surface appears equally bright from ALL directions! (independent of \( \vec{v} \))

- Lambertian BRDF is simply a constant: 
  \[
  f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}
  \]

- Most commonly used BRDF in Vision and Graphics!

source intensity \( I \)

incident direction \( s \)

normal \( n \)

viewing direction \( v \)

surface element

\( \theta_i \)

\( \rho_d \) albedo

\( \pi \)
BRDF

Specular BRDF: all energy concentrated in mirror direction

What does the BRDF equal in this case?
Specular Reflection and Mirror BRDF

- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).
- Mirror BRDF is simply a double-delta function:

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v)$$
Example Surfaces

Body Reflection:

- Diffuse Reflection
- Matte Appearance
- Non-Homogeneous Medium
- Clay, paper, etc

Surface Reflection:

- Specular Reflection
- Glossy Appearance
- Highlights
- Dominant for Metals

Many materials exhibit both Reflections:
BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere

\[ f_r(\vec{\omega}_{in}, \cdot) \]

\[ f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)
Trick for dielectrics (non-metals)

- BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components
- The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

\[ f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o) \]
Trick for dielectrics (non-metals)

• BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components

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\[ f(\omega_i, \omega_o) = f_d + f_s(\omega_i, \omega_o) \]

Often called the *dichromatic BRDF*:
• Diffuse term varies with wavelength, constant with polarization
• Specular term constant with wavelength, varies with polarization
Trick for dielectrics (non-metals)

- In this example, the two components were separated using linear polarizing filters on the camera and light source.
Trick for dielectrics (non-metals)

\[ I_{||} = \frac{1}{2} I_{\text{diffuse}} + I_{\text{specular}} \]

\[ I_{\perp} = \frac{1}{2} I_{\text{diffuse}} \]
Tabulated 4D BRDFs (hard to measure)

[Ngan et al., 2005]
Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D, 3D, or 4D) function
- Except for Lambertian, the BRDF is non-linear in these parameters
- Examples:

  Lambertian: \[ f(\omega_i, \omega_o) = \frac{a}{\pi} \]

  Phong: \[ f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c (2\langle \omega_i, n \rangle \langle \omega_o, n \rangle - \langle \omega_i, \omega_o \rangle) \]

  Blinn: \[ f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c b(\omega_i, \omega_o) \]

  Lafortune: \[ f(\omega_i, \omega_o) = \frac{a}{\pi} + b(-\omega_i^\top A \omega_o)^k \]

  Ward: \[ f(\omega_i, \omega_o) = \frac{a}{\pi} + \frac{b}{4\pi c^2 \sqrt{\langle n, \omega_i \rangle \langle n, \omega_o \rangle}} \exp \left( \frac{-\tan^2 b(\omega_i, \omega_o)}{c^2} \right) \]

Where do these constants come from?

\( \alpha \) is called the \textit{albedo}
Reflectance Models

Reflection: An Electromagnetic Phenomenon

Two approaches to derive Reflectance Models:

- Physical Optics (Wave Optics)
- Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models
But they are easier to use!
Reflectance that Require Wave Optics
Summary of some useful lighting models

• plenoptic function (function on 5D domain)
• far-field illumination (function on 2D domain)
• low-frequency far-field illumination (nine numbers)
• directional lighting (three numbers = direction and strength)
• point source (four numbers = location and strength)
Basic reading:
- Szeliski, Section 2.2.
- Gortler, Chapter 21.

This book by Steven Gortler has a great introduction to radiometry, reflectance, and their use for image formation.

Additional reading:
  These two thesis are foundational for modern computer graphics. Among other things, they include a thorough derivation (starting from wave optics and measure theory) of all radiometric quantities and associated integro-differential equations. You can also look at them if you are interested in physics-based rendering.
  A book discussing modeling and simulation of other appearance effects beyond single-bounce reflectance.
  A very thorough review of everything that has to do with modeling and measuring BRDFs.
  This paper has a great review of physics-based models for reflectance and refraction.
  This thesis introduced the largest measured dataset of 4D reflectances. It also provides detailed discussion of many topics relating to modelling reflectance.
  These two papers discuss the isotropy and other properties of common BRDFs, and how one can take advantage of them using alternative parameterizations.
  The paper introducing the dichromatic reflectance model.
  These three papers describe reflectance effects that can only be modeled using wave optics (and in particular diffraction).