Pinhole cameras

http://graphics.cs.cmu.edu/courses/15-463

Computational Photography
Fall 2017, Lecture 14
Course announcements

- Homework 4 is out.
  - Due October 26th.
  - Bilateral filter will take a very long time to run.
  - Make sure to sign up for a camera and a team.
  - Drop by Yannis’ office to pick up cameras any time.

- Yannis has extra office hours on Wednesday, 2-4pm.
  - You can come to ask questions about HW4 (e.g., “how do I use a DSLR camera?”).
  - You can come to ask questions about final project.

- Project ideas are due on Piazza on Friday 20th.
Overview of today’s lecture

- Some motivational imaging experiments.
- Pinhole camera.
- Accidental pinholes.
- Camera matrix.
- Perspective.
- Orthographic camera.
Slide credits

Most of these slides were adapted from:

• Kris Kitani (15-463, Fall 2016).

Some slides inspired from:

• Fredo Durand (MIT).
Some motivational imaging experiments
Let’s say we have a sensor...
... and an object we like to photograph

real-world object

digital sensor (CCD or CMOS)

What would an image taken like this look like?
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)
Bare-sensor imaging

All scene points contribute to all sensor pixels

What does the image on the sensor look like?
Bare-sensor imaging

All scene points contribute to all sensor pixels
Let’s add something to this scene

What would an image taken like this look like?
Pinhole imaging

real-world object

most rays are blocked

digital sensor (CCD or CMOS)

one makes it through
Pinhole imaging

real-world object

digital sensor (CCD or CMOS)

most rays are blocked

one makes it through
Pinhole imaging

Each scene point contributes to only one sensor pixel

What does the image on the sensor look like?
Pinhole imaging

real-world object

copy of real-world object (inverted and scaled)
Pinhole camera
Pinhole camera a.k.a. camera obscura
Chinese philosopher Mozi 
(470 to 390 BC)

Greek philosopher Aristotle 
(384 to 322 BC)
Pinhole camera terms

- real-world object
- barrier (diaphragm)
- pinhole (aperture)
- digital sensor (CCD or CMOS)
Pinhole camera terms

- real-world object
- barrier (diaphragm)
- pinhole (aperture)
- camera center (center of projection)
- image plane
- digital sensor (CCD or CMOS)
Focal length

real-world object

focal length $f$
What happens as we change the focal length?

real-world object

focal length 0.5 f
What happens as we change the focal length?
Focal length

What happens as we change the focal length?

real-world object

object projection is half the size

focal length 0.5 f
Pinhole size

Ideal pinhole has infinitesimally small size
- In practice that is impossible.
Pinhole size

What happens as we change the pinhole diameter?

real-world object

pinhole diameter
Pinhole size

What happens as we change the pinhole diameter?

real-world object
Pinhole size

What happens as we change the pinhole diameter?

real-world object
What happens as we change the pinhole diameter?

object projection becomes blurrier

real-world object
What happens as we change the pinhole diameter?

Will the image keep getting sharper the smaller we make the pinhole?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
Diffraction limit

A consequence of the wave nature of light

What do geometric optics predict will happen?

What do wave optics predict will happen?
Diffraction limit

Diffraction pattern = Fourier transform of the pinhole.
• Smaller pinhole means bigger Fourier spectrum.
• Smaller pinhole means more diffraction.
What about light efficiency?

- What is the effect of doubling the pinhole diameter?
- What is the effect of doubling the focal length?
What about light efficiency?

- 2x pinhole diameter → 4x light
- 2x focal length → ¼x light
Some terminology notes

A “stop” is a change in camera settings that changes amount of light by a factor of 2

The “f-number” is the ratio: focal length / pinhole diameter
Can we do better than pinhole imaging?

real-world object

barrier (diaphragm)

pinhole (aperture)

digital sensor (CCD or CMOS)
Accidental pinholes
What does this image say about the world outside?
Accidental pinhole camera
Accidental pinhole camera

window is an aperture

projected pattern on the wall

upside down

window with smaller gap

view outside window
Accidental pinspeck camera

a) Difference image
b) Difference upside down
c) True outdoor view
Camera matrix
The camera as a coordinate transformation

A camera is a mapping from:

- the 3D world

To:

- a 2D image

3D object

\[\text{3D to 2D transform (camera)}\]

2D image

\[\text{2D to 2D transform (image warping)}\]
The camera as a coordinate transformation

A camera is a mapping from:

the 3D world
to:

a 2D image

What are the dimensions of each variable?

\[ x = PX \]

homogeneous coordinates

2D image point  camera matrix  3D world point
The camera as a coordinate transformation

\[ x = PX \]

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= 
\begin{bmatrix}
p_1 & p_2 & p_3 & p_4 \\
p_5 & p_6 & p_7 & p_8 \\
p_9 & p_{10} & p_{11} & p_{12}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

- \( X, Y, Z \): Homogeneous image coordinates, 3 x 1
- \( P \): Camera matrix, 3 x 4
- \( X, Y, Z \): Homogeneous world coordinates, 4 x 1
The pinhole camera

real-world object

camera center

focal length $f$

image plane
The (rearranged) pinhole camera

real-world object

image plane

focal length $f$
camera center
The (rearranged) pinhole camera

What is the equation for image coordinate $x$ in terms of $X$?
The 2D view of the (rearranged) pinhole camera

What is the equation for image coordinate $x$ in terms of $X$?
The 2D view of the (rearranged) pinhole camera

\[
[X \ Y \ Z]^\top \mapsto \left[\frac{fX}{Z} \ \frac{fY}{Z}\right]^\top
\]
The (rearranged) pinhole camera

What is the camera matrix \( P \) for a pinhole camera?

\[
x = PX
\]
The pinhole camera matrix

Relationship from similar triangles:

\[
\begin{bmatrix}
X & Y & Z
\end{bmatrix} \to \begin{bmatrix}
fX/Z & fY/Z
\end{bmatrix}
\]

General camera model:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
p_1 & p_2 & p_3 & p_4 \\
p_5 & p_6 & p_7 & p_8 \\
p_9 & p_{10} & p_{11} & p_{12}
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

What does the pinhole camera projection look like?

\[
P = \begin{bmatrix}
\end{bmatrix}
\]
The pinhole camera matrix

Relationship from similar triangles:

$$[X \ Y \ Z]^\top \mapsto [fX/Z \ fY/Z]^\top$$

General camera model:

$$
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
p_1 & p_2 & p_3 & p_4 \\
p_5 & p_6 & p_7 & p_8 \\
p_9 & p_{10} & p_{11} & p_{12}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
$$

What does the pinhole camera projection look like?

$$P =
\begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}$$
Generalizing the camera matrix

Camera origin and image origin might be different

![Diagram showing image plane, camera coordinate system, and image coordinate system with vector p]

How does the camera matrix change?

\[
P = \begin{bmatrix}
    f & 0 & 0 & 0 \\
    0 & f & 0 & 0 \\
    0 & 0 & 1 & 0 \\
\end{bmatrix}
\]
Generalizing the camera matrix

Camera origin and image origin might be different

How does the camera matrix change?

\[
P = \begin{bmatrix}
  f & 0 & p_x & 0 \\
  0 & f & p_y & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]
Camera matrix decomposition

We can decompose the camera matrix like this:

\[ P = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

intrinsic (3 x 3)    extrinsic (3 x 4)

\[ P = K[I|0] \]

\[ K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \]

calibration matrix
Extrinsic camera parameters

We can decompose the camera matrix like this:

\[
P = \begin{bmatrix}
f & 0 & p_x \\
0 & f & p_y \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

intrinsic (3 x 3)   extrinsic (3 x 4)

assumes camera and world share the same coordinate system

What if world and camera coordinate systems are different?

![Diagram of camera and world coordinate systems](image)
Extrinsic camera parameters

We can decompose the camera matrix like this:

$$P = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

assumes camera and world share the same coordinate system

intrinsic (3 x 3)  extrinsic (3 x 4)

What if world and camera coordinate systems are different?

3D rotation and translation
Extrinsic camera parameters

We can decompose the camera matrix like this:

\[
P = \begin{bmatrix}
f & 0 & p_x \\
0 & f & p_y \\
0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

assumes camera and world share the same coordinate system

intrinsic (3 x 3)    extrinsic (3 x 4)

What if world and camera coordinate systems are different?

\[
\begin{bmatrix}
X_c \\
Y_c \\
Z_c \\
1
\end{bmatrix} = \begin{bmatrix}
R & -RC \\
0 & 1
\end{bmatrix} \begin{bmatrix}
X_w \\
Y_w \\
Z_w \\
1
\end{bmatrix}
\]
Extrinsic camera parameters

We can decompose the camera matrix like this:

\[
P = \begin{bmatrix}
f & 0 & p_x \\
0 & f & p_y \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\begin{array}{c|c}
R & -RC \\
\hline
0 & 1
\end{array}
\end{bmatrix}
\]

intrinsic (3 x 3) extrinsic (3 x 4)

What if world and camera coordinate systems are different?

\[
\begin{bmatrix}
X_c \\
Y_c \\
Z_c \\
1
\end{bmatrix}
= \begin{bmatrix}
\begin{array}{c|c}
R & -RC \\
\hline
0 & 1
\end{array}
\end{bmatrix}
\begin{bmatrix}
X_w \\
Y_w \\
Z_w \\
1
\end{bmatrix}
\]
General pinhole camera matrix

We can decompose the camera matrix like this:

\[ P = KR[I] - C \]

(translate first then rotate)

Another way to write the mapping:

\[ P = K[R|t] \]

where \( t = -RC \)

(rotate first then translate)
Recap

What is the size and meaning of each term in the camera matrix?

\[ P = \text{KR}[I] - \text{C} \]

Recap

What is the size and meaning of each term in the camera matrix?

\[ P = KR[I] - C \]

3x3 intrinsics

?  

?  

?
Recap

What is the size and meaning of each term in the camera matrix?

\[ P = KR[I] - C \]

- **3x3 intrinsics**
- **3x3 3D rotation**
- ?
- ?
Recap

What is the size and meaning of each term in the camera matrix?

\[ P = KR[I] - C \]

- **3x3 intrinsics**
- **3x3 3D rotation**
- **3x3 identity**
- ?

Recap
Recap

What is the size and meaning of each term in the camera matrix?

\[ P = KR[I| - C] \]

- 3x3 intrinsics
- 3x3 3D rotation
- 3x3 identity
- 3x1 3D translation
Perspective
Forced perspective
The Ames room illusion
The Ames room illusion
The 2D view of the (rearranged) pinhole camera image plane magnification changes with depth

\[
\begin{bmatrix} X & Y & Z \end{bmatrix}^T \rightarrow \begin{bmatrix} fX/Z & fY/Z \end{bmatrix}^T
\]
Magnification depends on depth

What happens as we change the focal length?

real-world object

depth $Z$

depth $2Z$
Magnification depends on focal length

real-world object

focal length $f$

focal length $2f$
1. Set focal length to half depth 2 \( Z \)

focal length \( f \)

focal length 2 \( f \)
What if...

1. Set focal length to half
2. Set depth to half

Is this the same image as the one I had at focal length 2f and distance 2Z?
Perspective distortion

long focal length
mid focal length
short focal length
Perspective distortion
What is the best focal length for portraits?

That’s like asking which is better, vi or emacs...

- long focal length
- mid focal length
- short focal length
Vertigo effect

Named after Alfred Hitchcock’s movie
• also known as “dolly zoom”
Vertigo effect

How would you create this effect?
Orthographic camera
What if...

Continue increasing $Z$ and $f$ while maintaining the same magnification?

$$f \to \infty \text{ and } \frac{f}{Z} = \text{constant}$$
Orthographic vs pinhole camera

\[ [X \ Y \ Z]^T \rightarrow [fX \ fY]^T \]

magnification does not change with depth

\[ [X \ Y \ Z]^T \rightarrow [fX/Z \ fY/Z]^T \]

magnification changes with depth

Image plane

\( y \)

\( f \)

\( Z \)

\( z \)
Orthographic vs pinhole camera

General pinhole camera:

\[ P = K R [I] - C \]

We also call these cameras:

Projective camera

General orthographic camera:

\[ P = K R [I] - C \]

Bottom row is always \([0 \ 0 \ 0 \ 1]\)

Affine camera

What is the rationale behind these names?
References

Basic reading:
• Szeliski textbook, Section 2.1.5.

Additional reading:
• Torralba and Freeman, “Accidental Pinhole and Pinspeck Cameras,” CVPR 2012. the eponymous paper discussed in the slides.