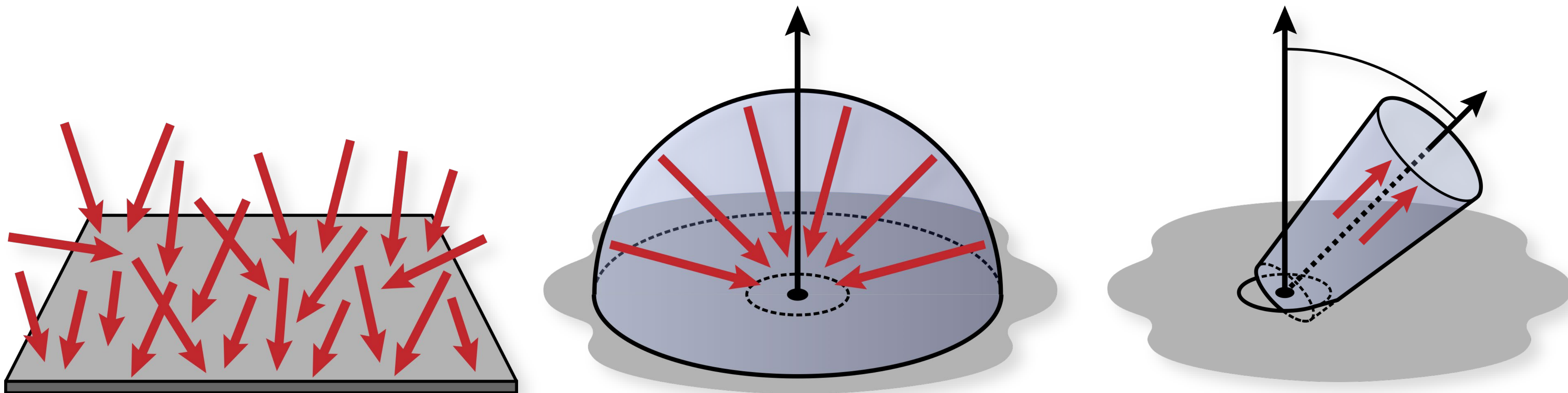


Radiometry and reflectance



Course announcements

- Remember make-up lecture on Friday.

Overview of today's lecture

- Radiometric quantities.
- A little bit about color.
- Reflectance equation.
- Standard reflectance functions.

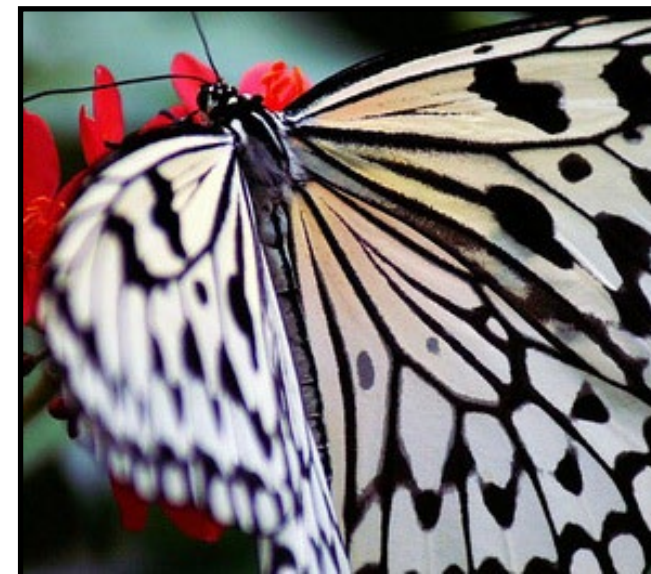
Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).
- Todd Zickler (Harvard).
- Srinivasa Narasimhan (CMU).

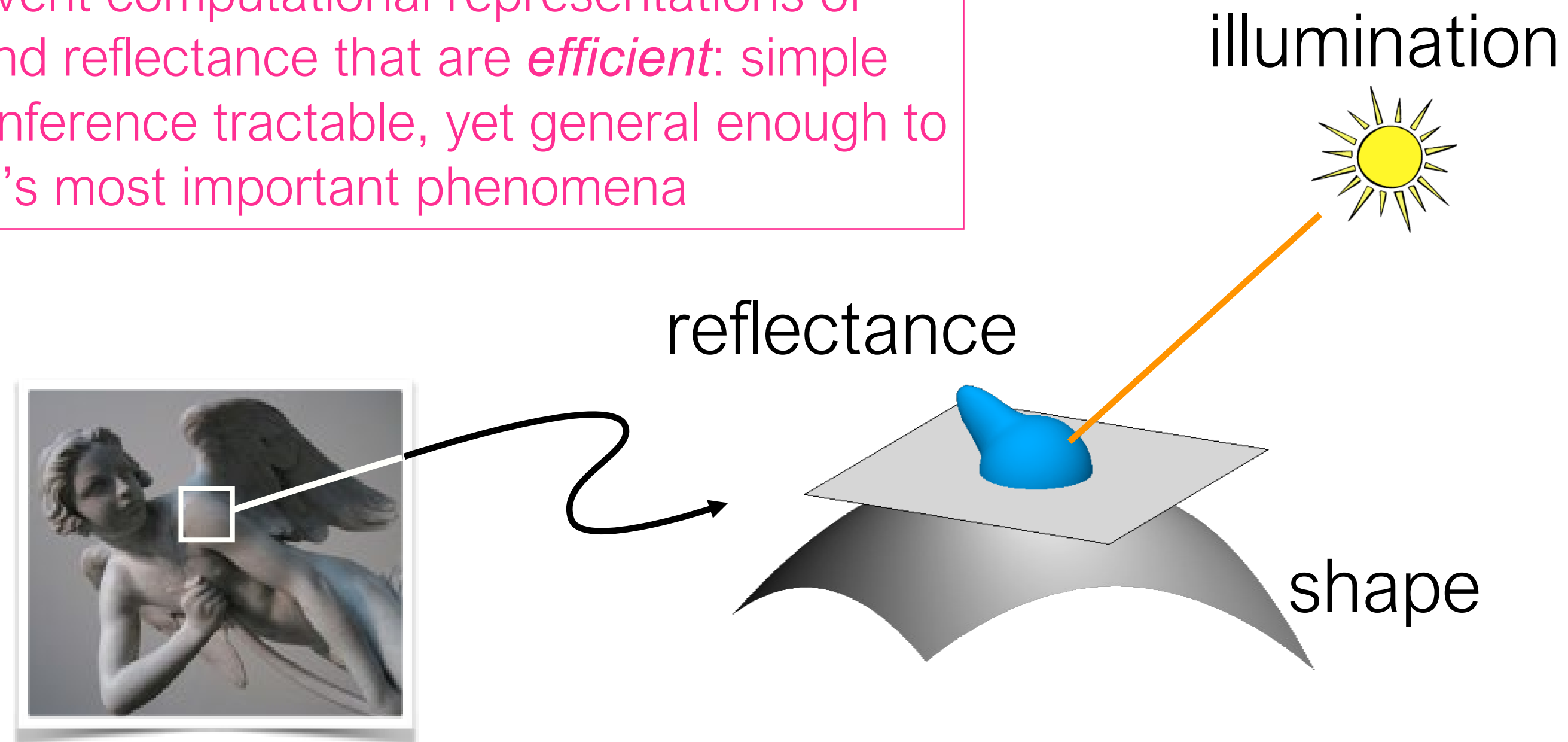
Appearance

Appearance



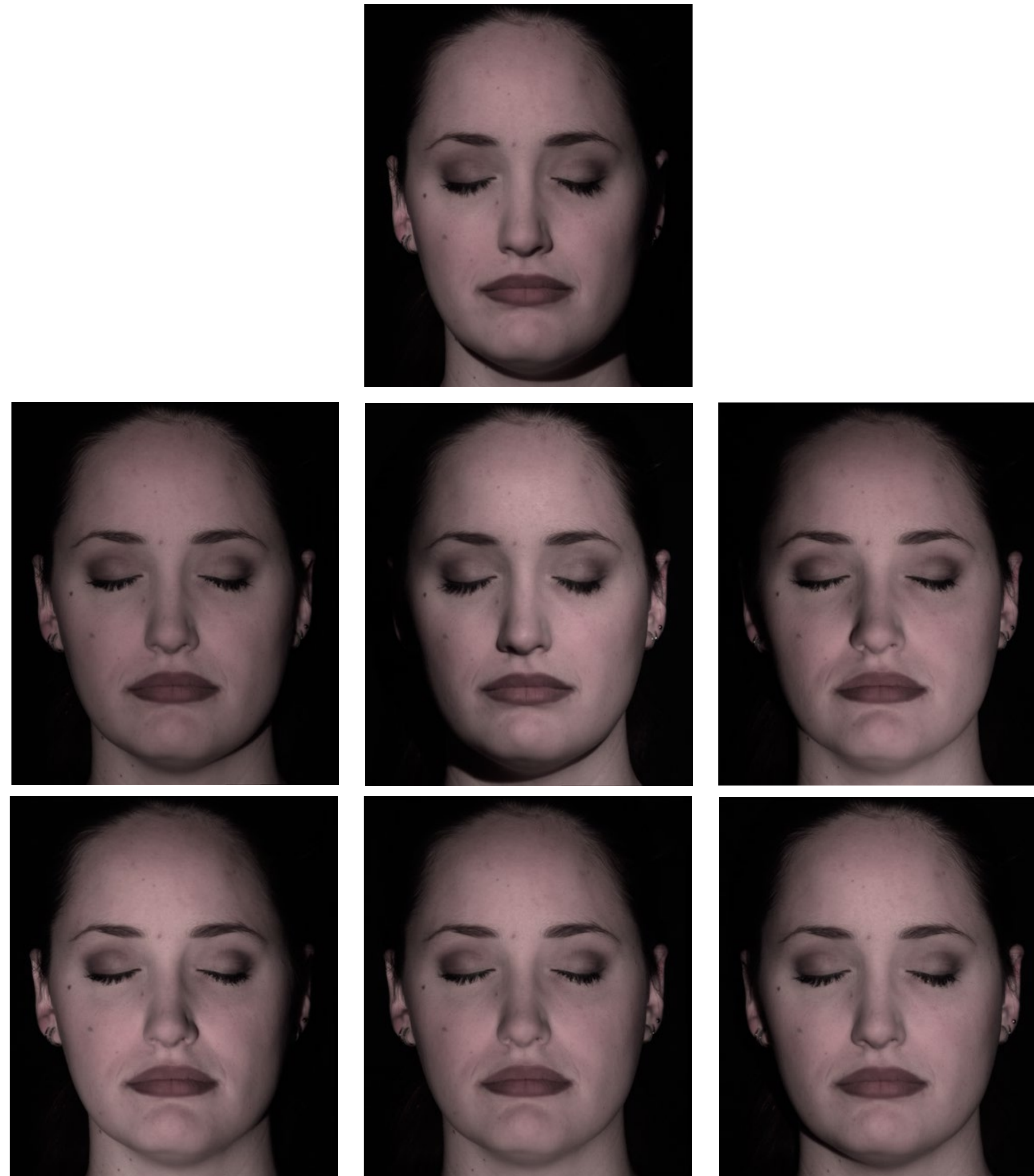
“Physics-based” computer vision (a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are *efficient*: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena



I $\longrightarrow$ shape, illumination, reflectance

Example application: Photometric Stereo



Quantifying Light

Assumptions

Light sources, reflectance spectra, sensor sensitivity modeled separately at each wavelength

Geometric/ray optics

No polarization

No fluorescence, phosphorescence, ...

Radiometry

Radiometry studies the measurement of electromagnetic radiation, including visible light.



Radiometry

Assume light consists of photons with:

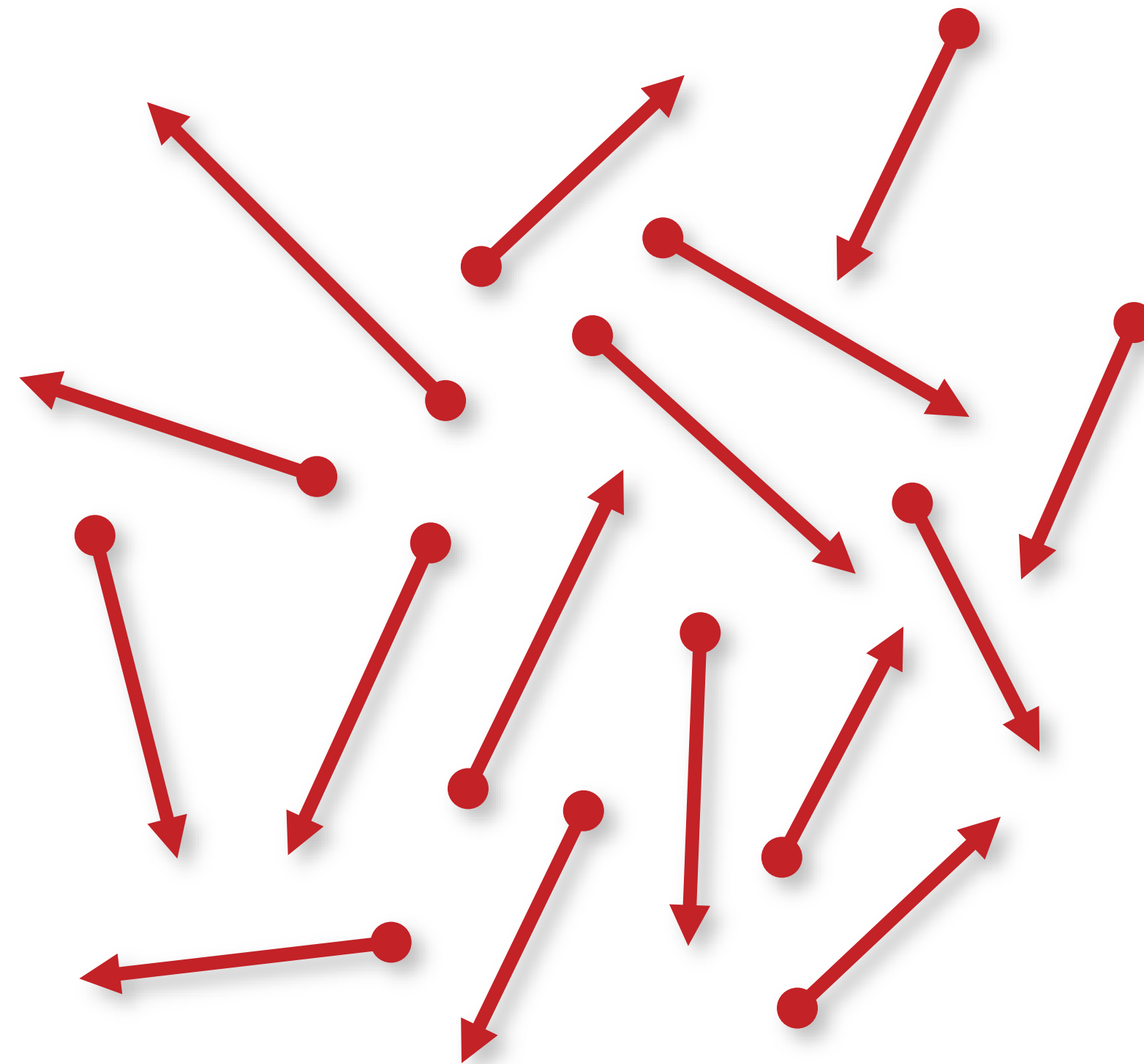
- $\mathbf{x}$: Position
- $\vec{\omega}$: Direction of travel
- λ : Wavelength

Each photon has an energy of: $\frac{h c}{\lambda}$

- $h \approx 6.63 \times 10^{-34} \text{ m}^2 \text{ kg/s}$: Planck's constant
- $c = 299,792,458 \text{ m/s}$: speed of light in vacuum
- Unit of energy, Joule: $\left[\text{J} = \text{kg m}^2 / \text{s}^2 \right]$

Radiometry

How do we measure the energy flow?



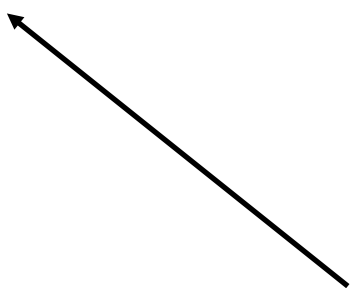
Measuring energy means “counting photons”

Radiometry

Basic quantities (depend on wavelength)

- flux Φ
- irradiance E
- radiosity B
- intensity I
- radiance L

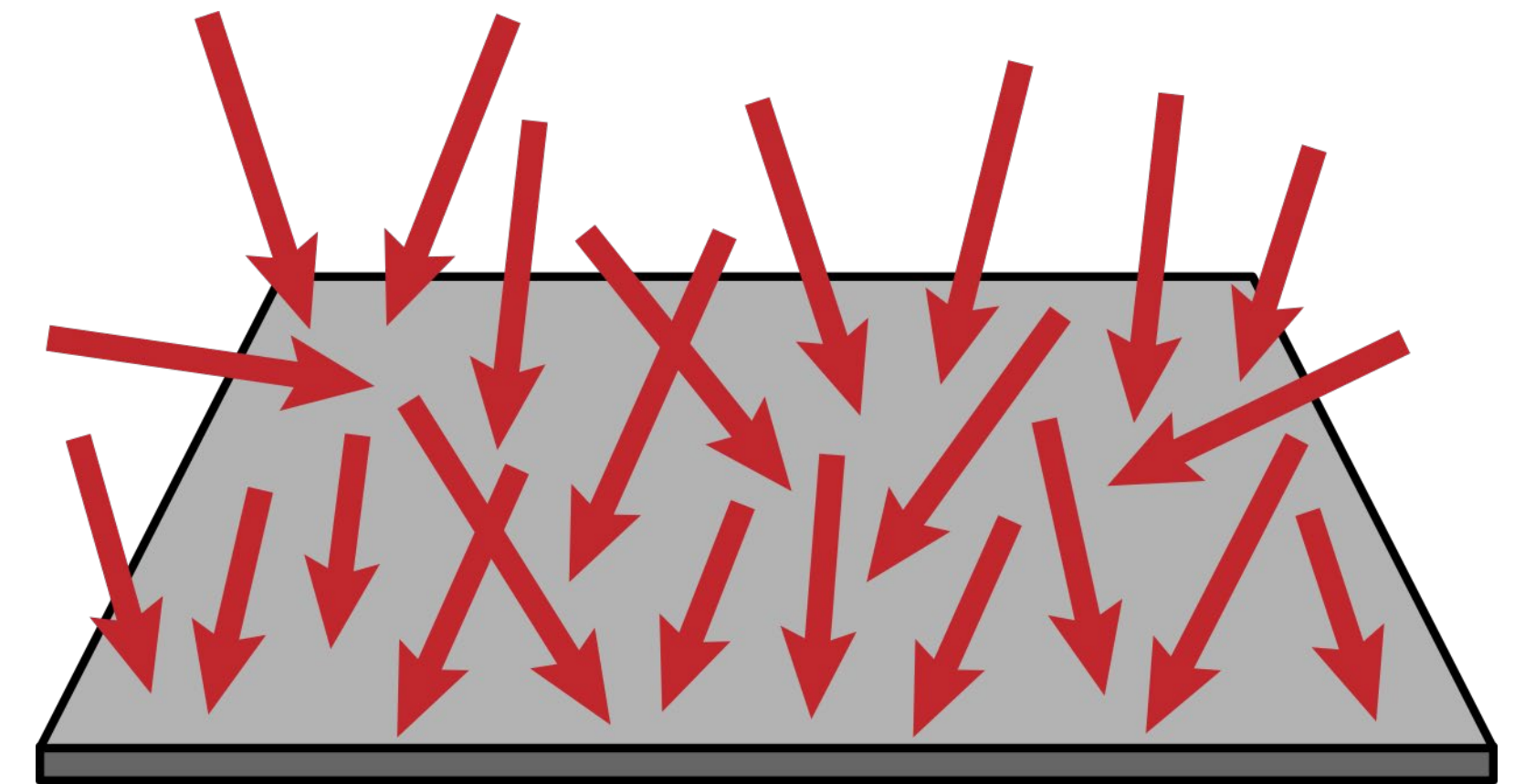
will be the most important quantity for us



Flux (Radiant Flux, Power)

total amount of radiant energy passing through surface or space
per unit time

$$\Phi(A) \quad \left[\frac{\text{J}}{\text{s}} = \text{W} \right]$$



examples:

- number of photons hitting a wall per second
- number of photons leaving a lightbulb per second (how do we quantify this exactly?)

Irradiance

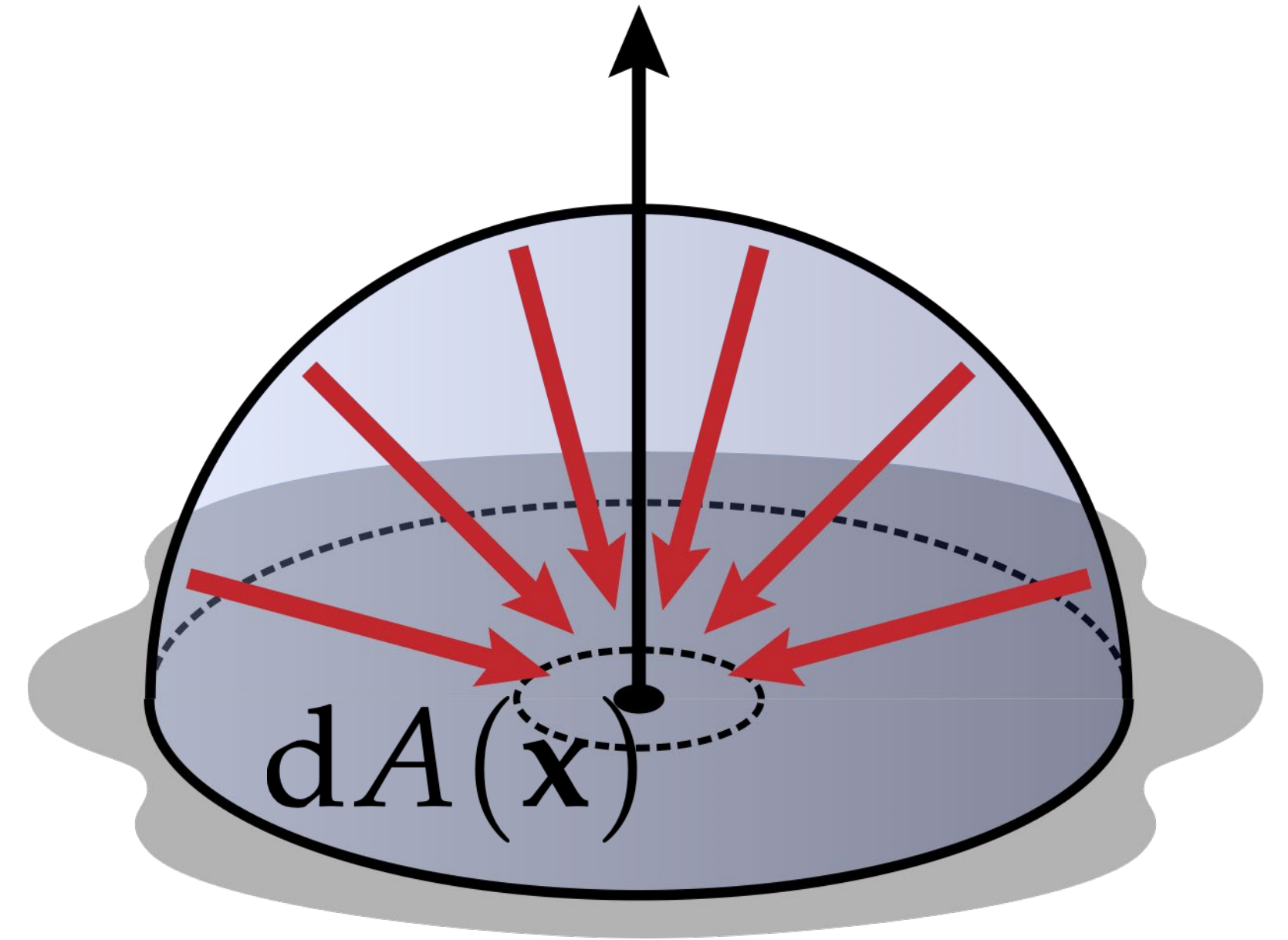
area density of flux

flux per unit area **arriving** at a surface

$$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \quad \left[\frac{\text{W}}{\text{m}^2} \right]$$

example:

- number of photons **hitting** a small patch of a wall per second,
divided by size of patch



Radiosity (Radiant Exitance)

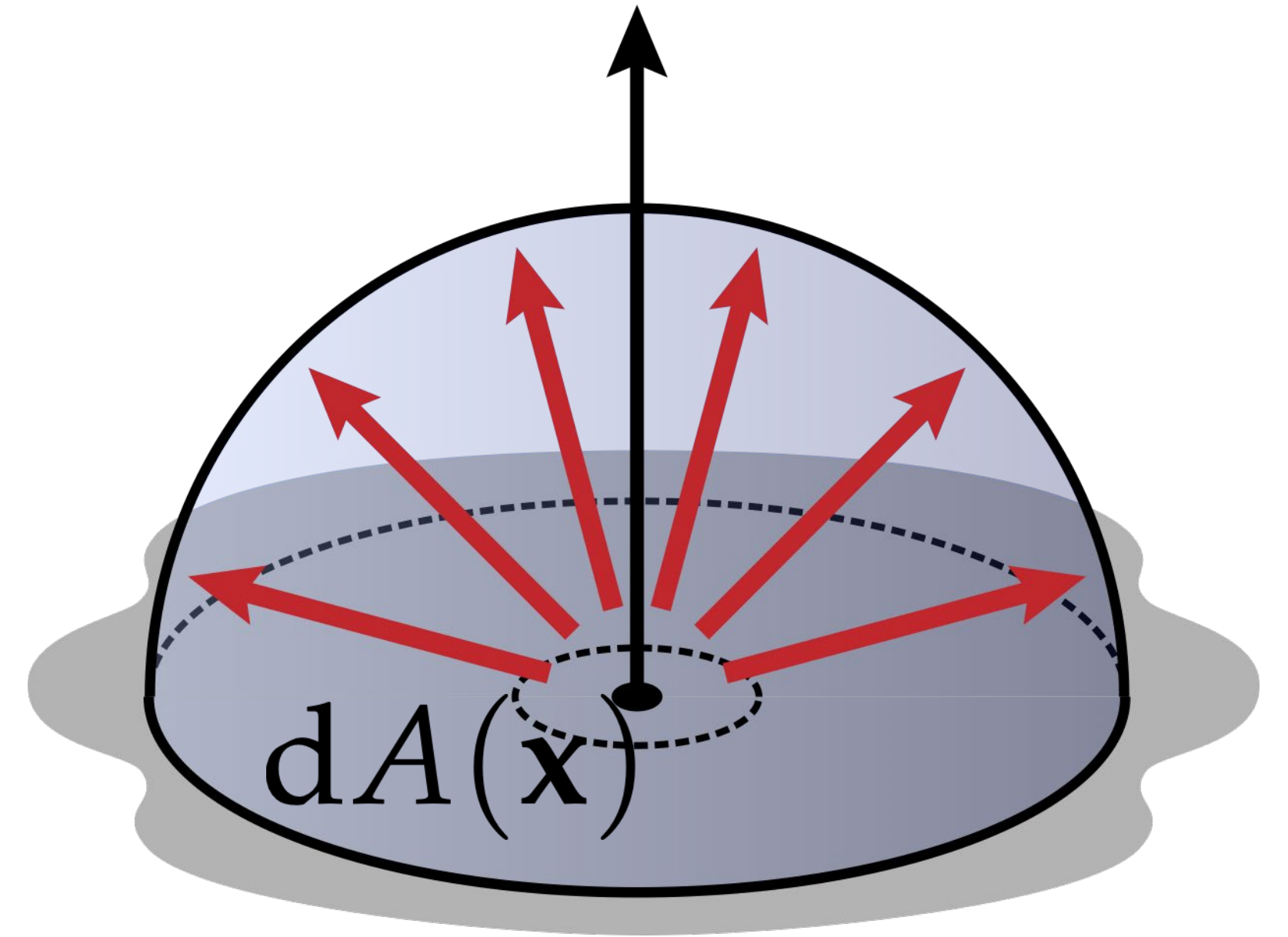
area density of flux

flux per unit area **leaving** a surface

$$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \quad \left[\frac{\text{W}}{\text{m}^2} \right]$$

example:

- number of photons **reflecting off** a small patch of a wall per second,
divided by size of patch

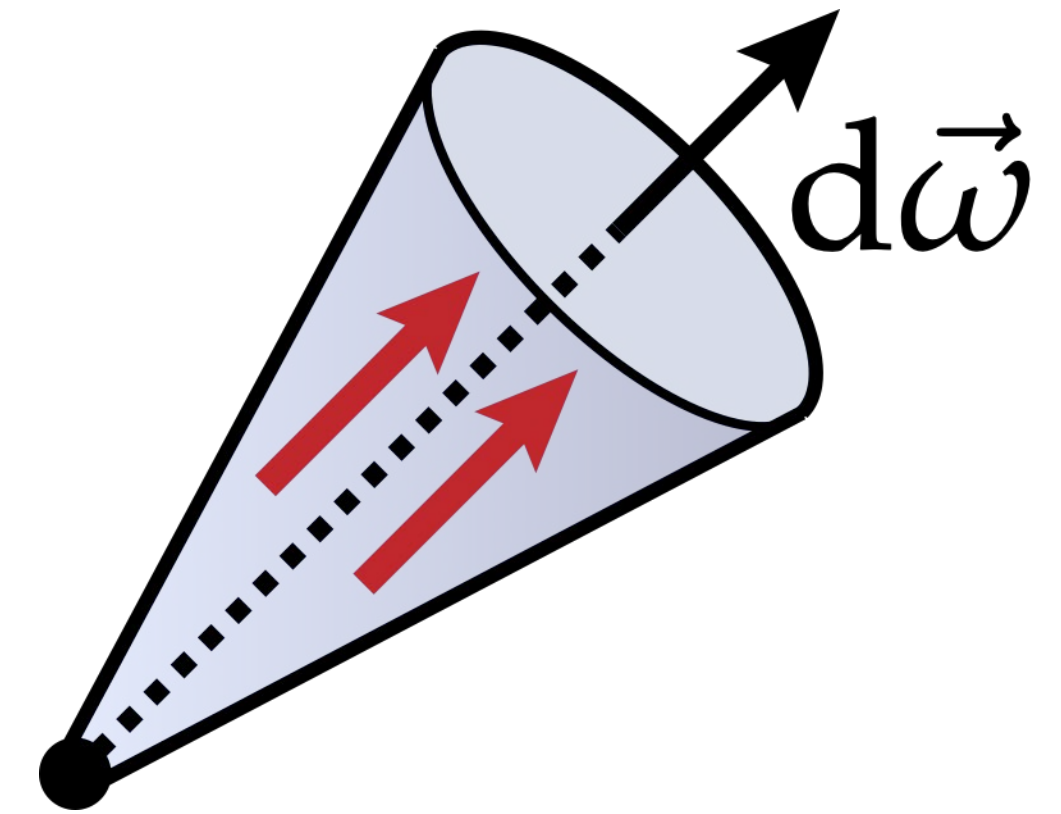


Radiant Intensity

directional density of flux

power (flux) per solid angle

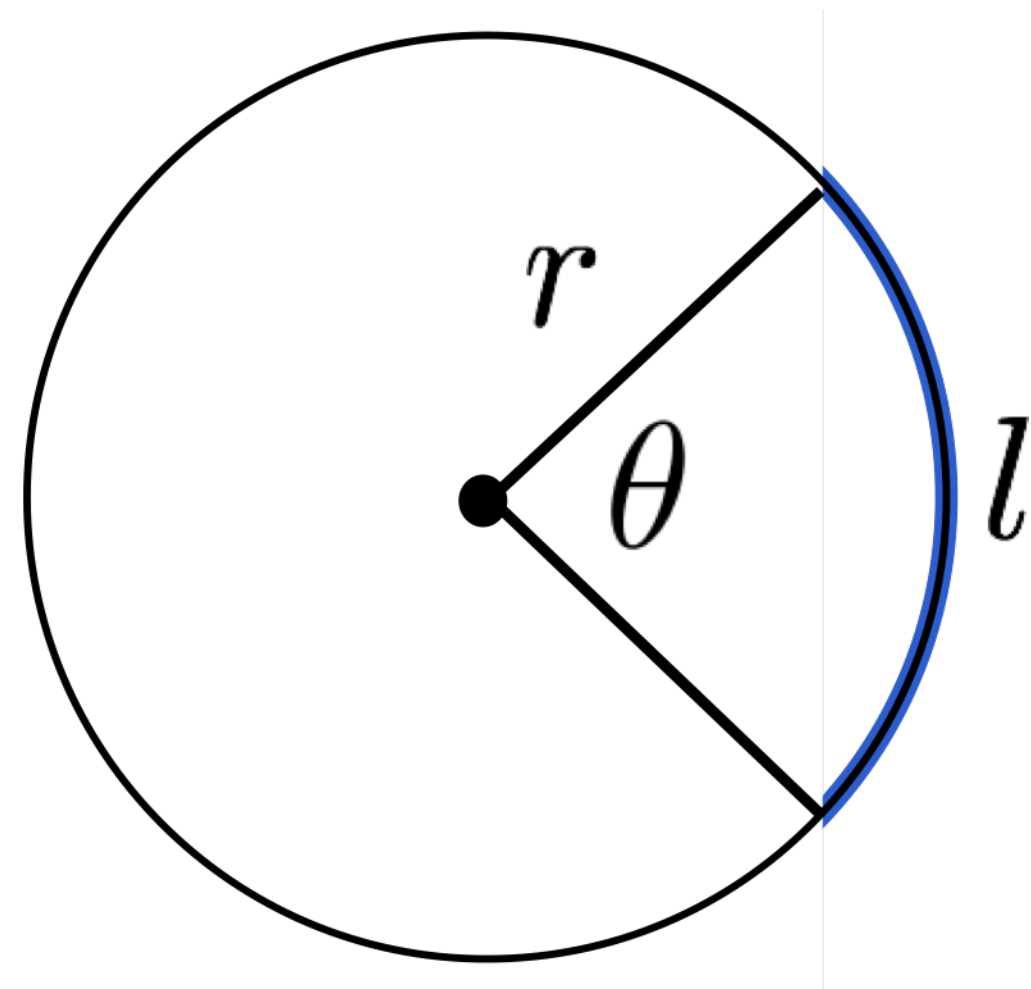
$$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \quad \left[\frac{\text{W}}{\text{sr}} \right]$$



Solid Angle

Angle

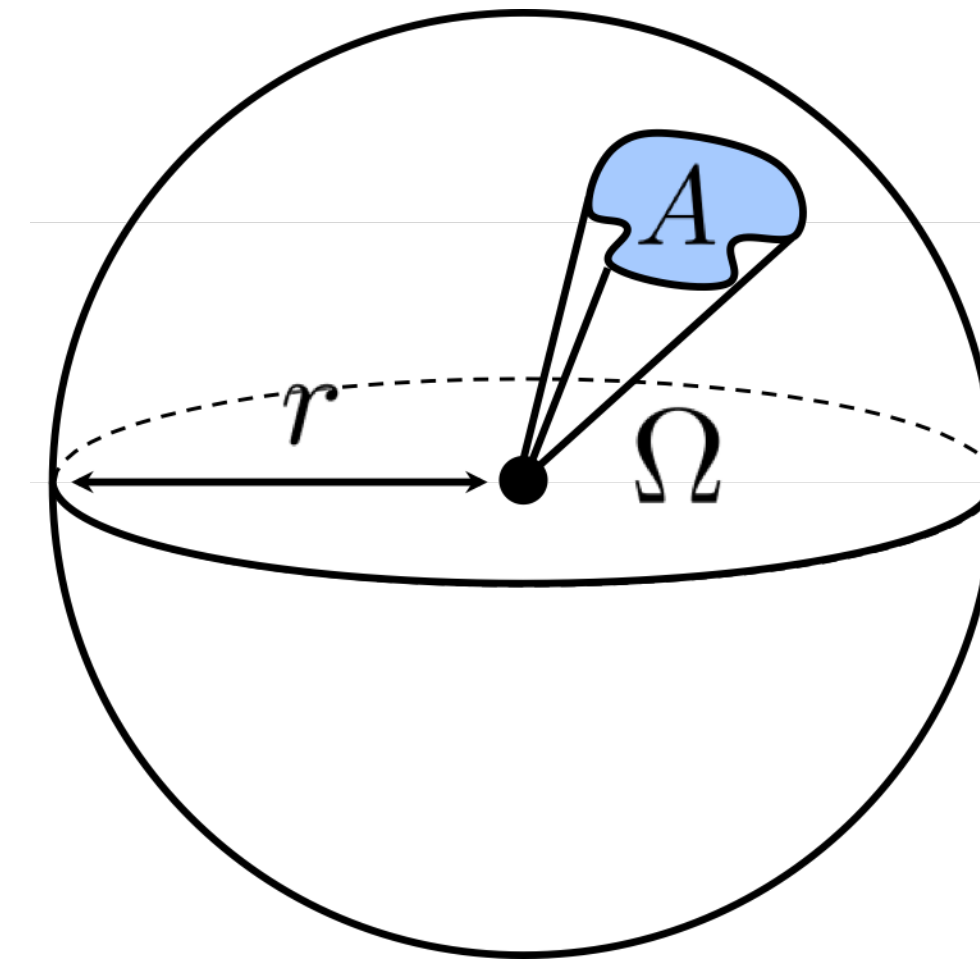
- circle: 2π radians



$$\theta = \frac{l}{r}$$

Solid angle

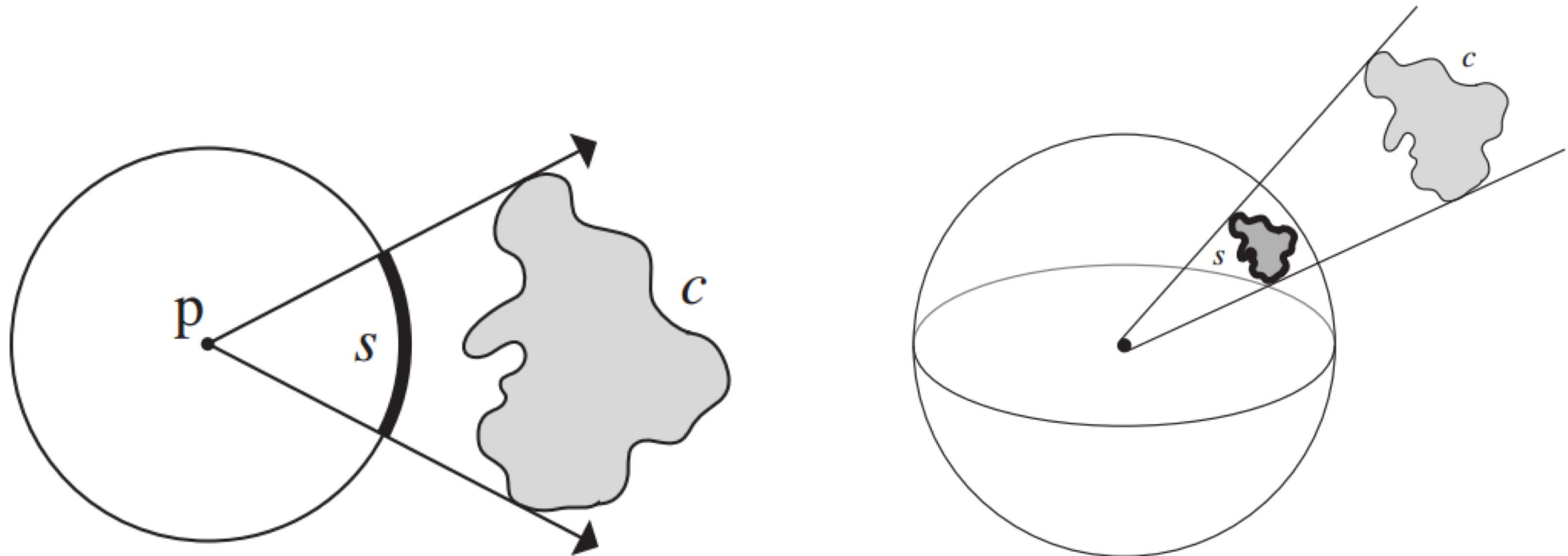
- sphere: 4π steradians



$$\Omega = \frac{A}{r^2}$$

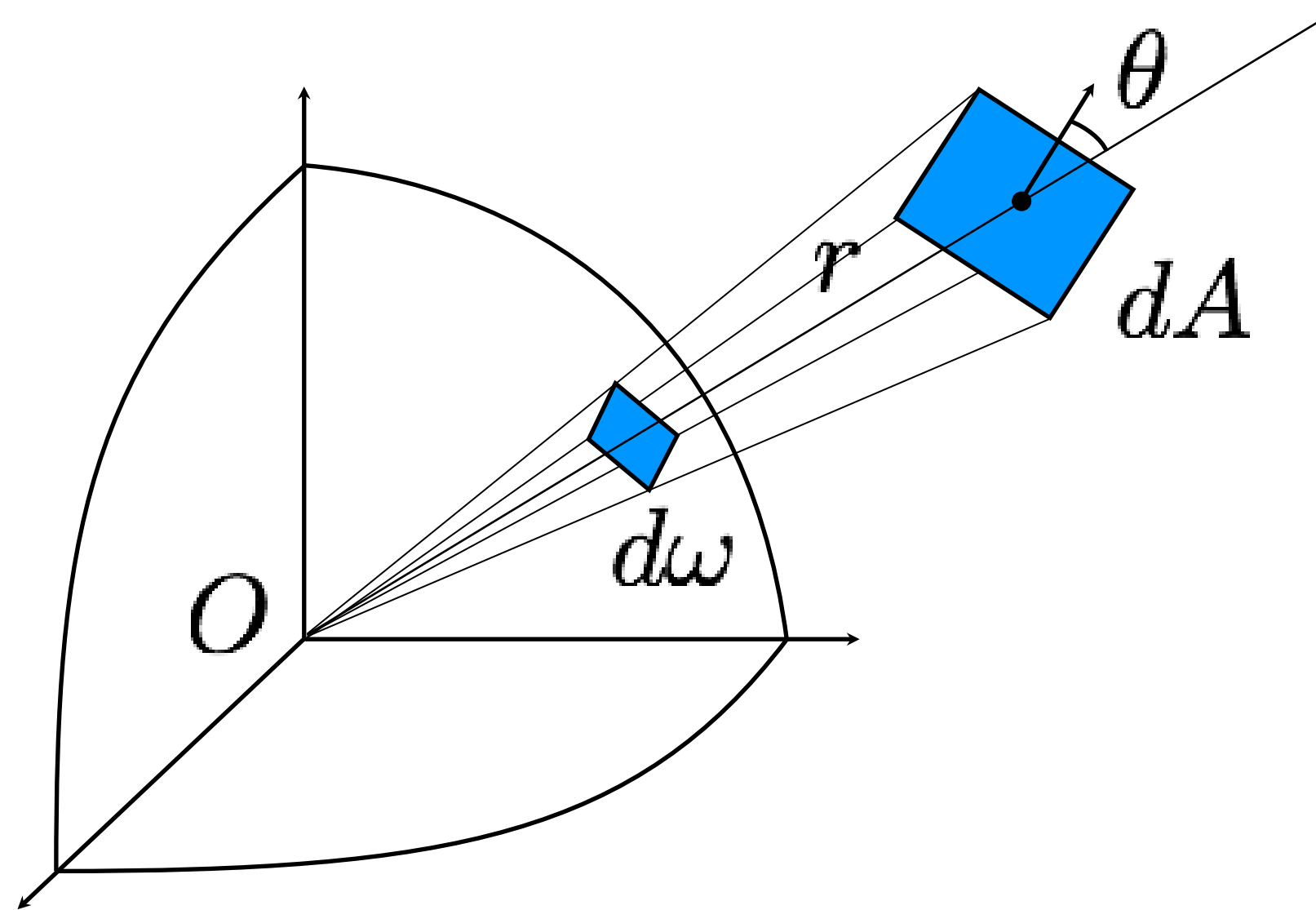
Subtended (Solid) Angle

Length/area of object's *projection* onto a unit circle/sphere



Solid angle

The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

One can show:

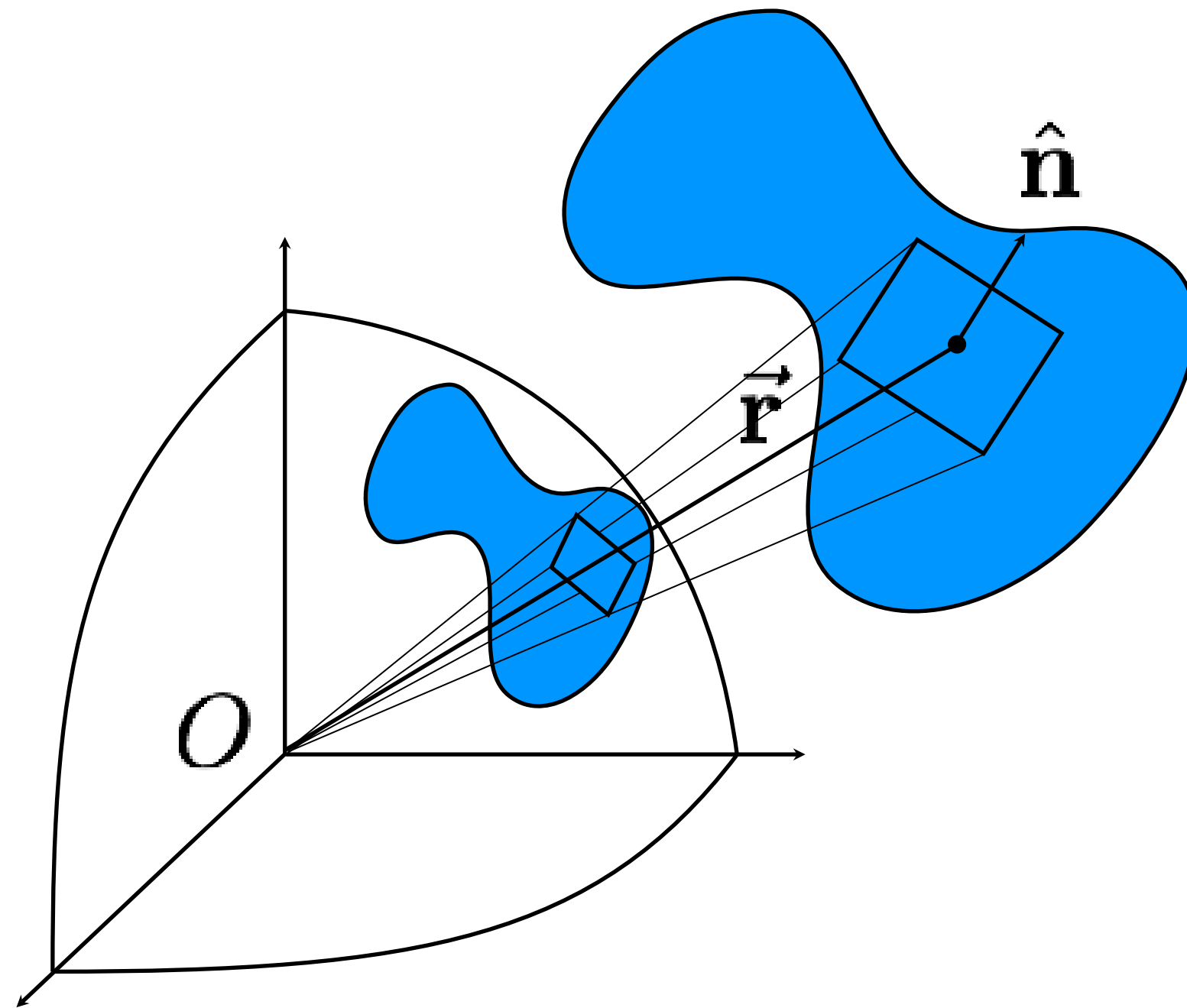
“surface foreshortening”

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Units: steradians [sr]

Solid angle

To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{r} \cdot \hat{n} dS}{|\vec{r}|^3}$$

One can show:

“surface foreshortening”

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Units: steradians [sr]

Radiant Intensity

directional density of flux

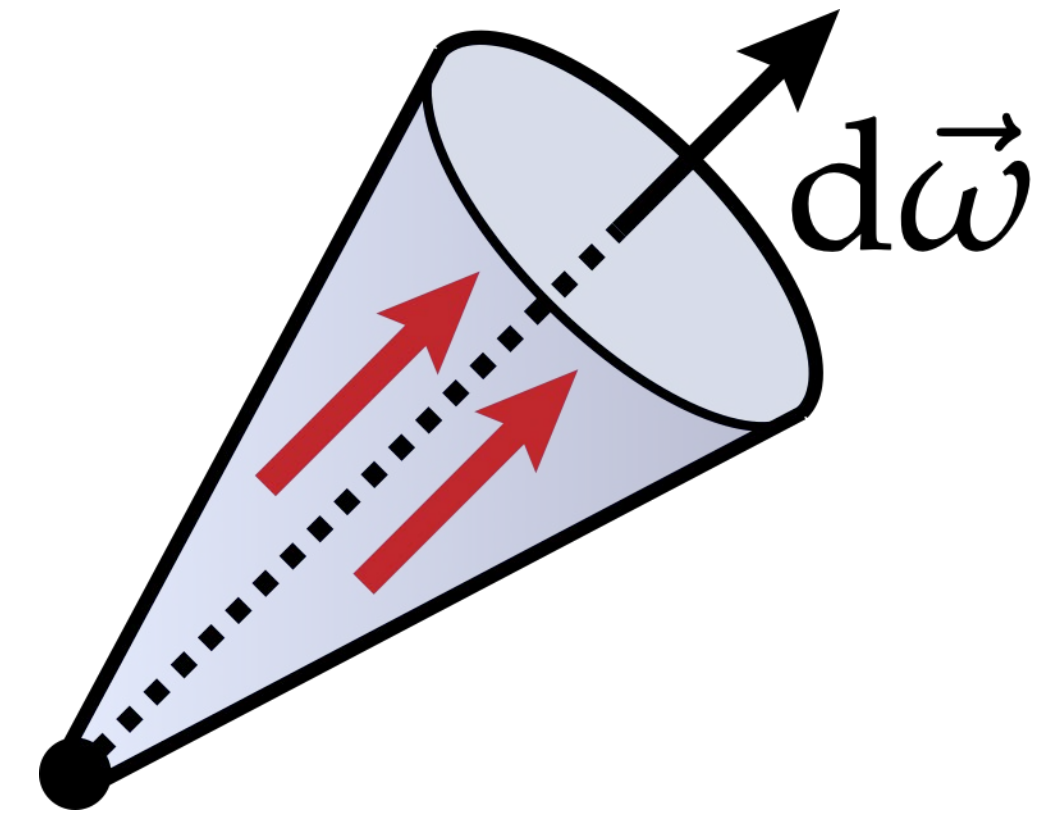
power (flux) per solid angle

$$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \quad \left[\frac{\text{W}}{\text{sr}} \right]$$

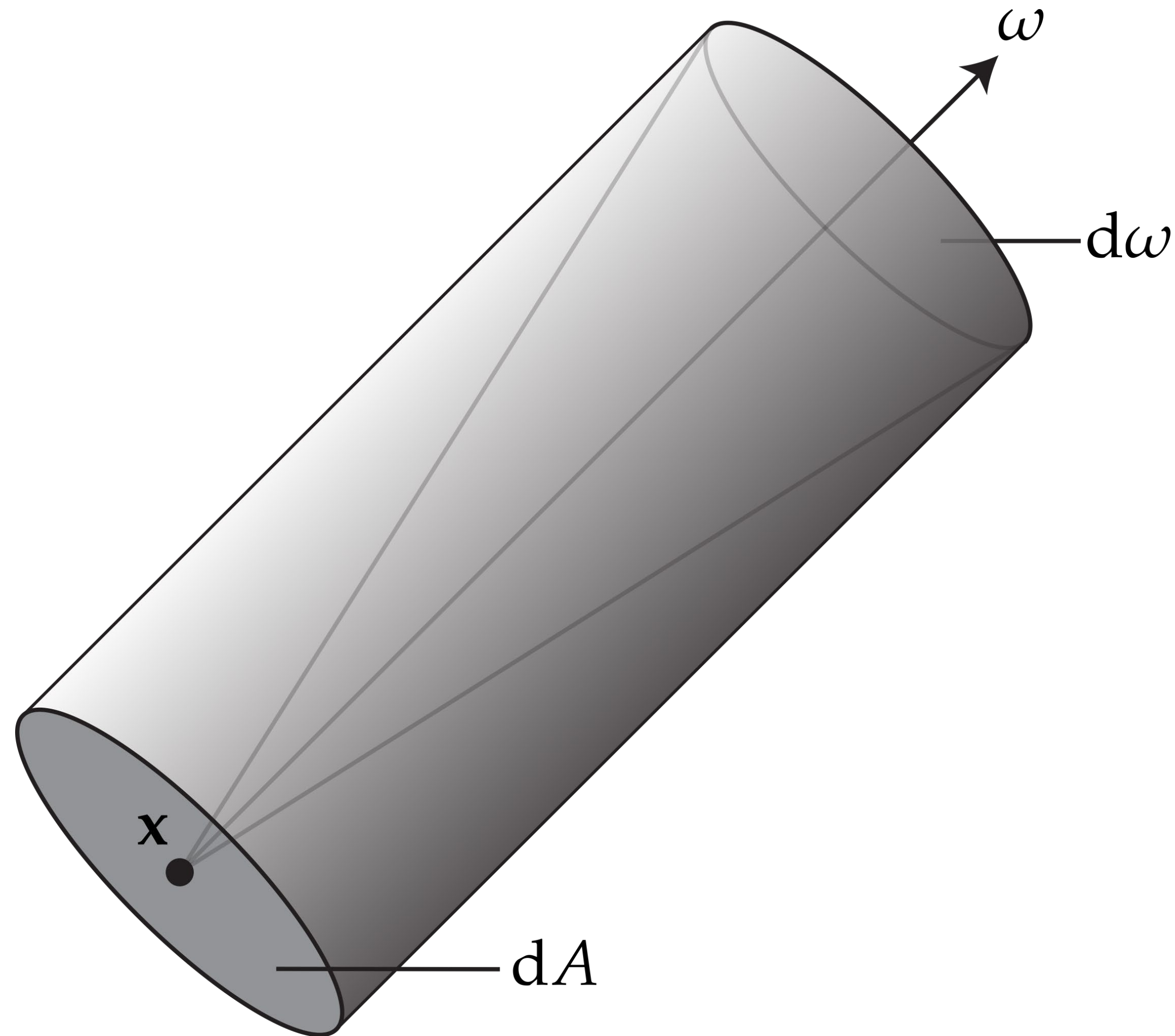
$$\Phi = \int_{S^2} I(\vec{\omega}) d\vec{\omega}$$

example: $\Phi = 4\pi I$ (for an isotropic point source)

- power per unit solid angle emanating from a point source



A hypothetical measurement device

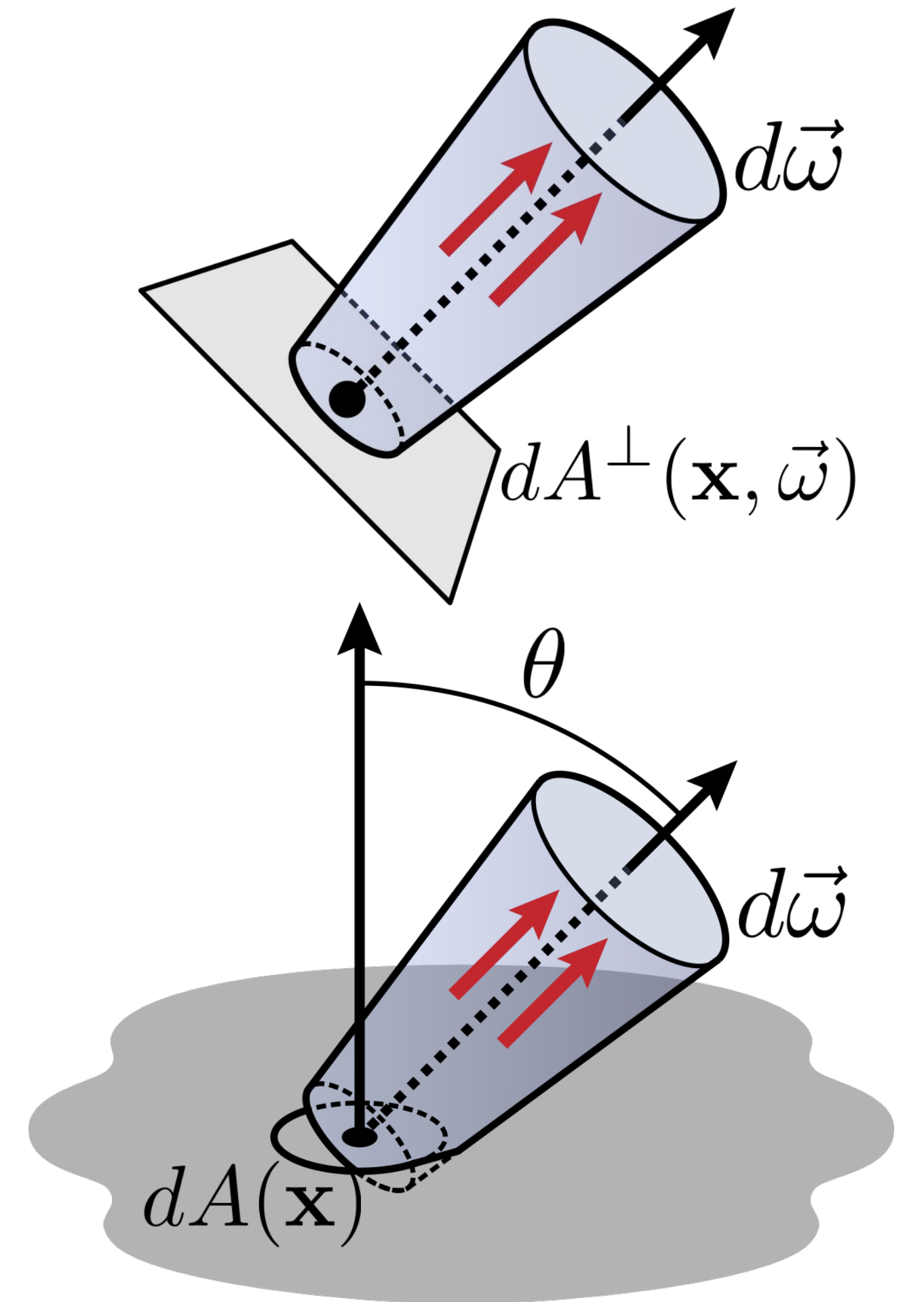


Radiance

flux density per unit solid angle, per *perpendicular* unit area

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2 \Phi(A)}{d\vec{\omega} dA^\perp(\mathbf{x}, \vec{\omega})} \left[\frac{W}{m^2 sr} \right]$$

$$= \frac{d^2 \Phi(A)}{d\vec{\omega} dA(\mathbf{x}) \cos \theta}$$



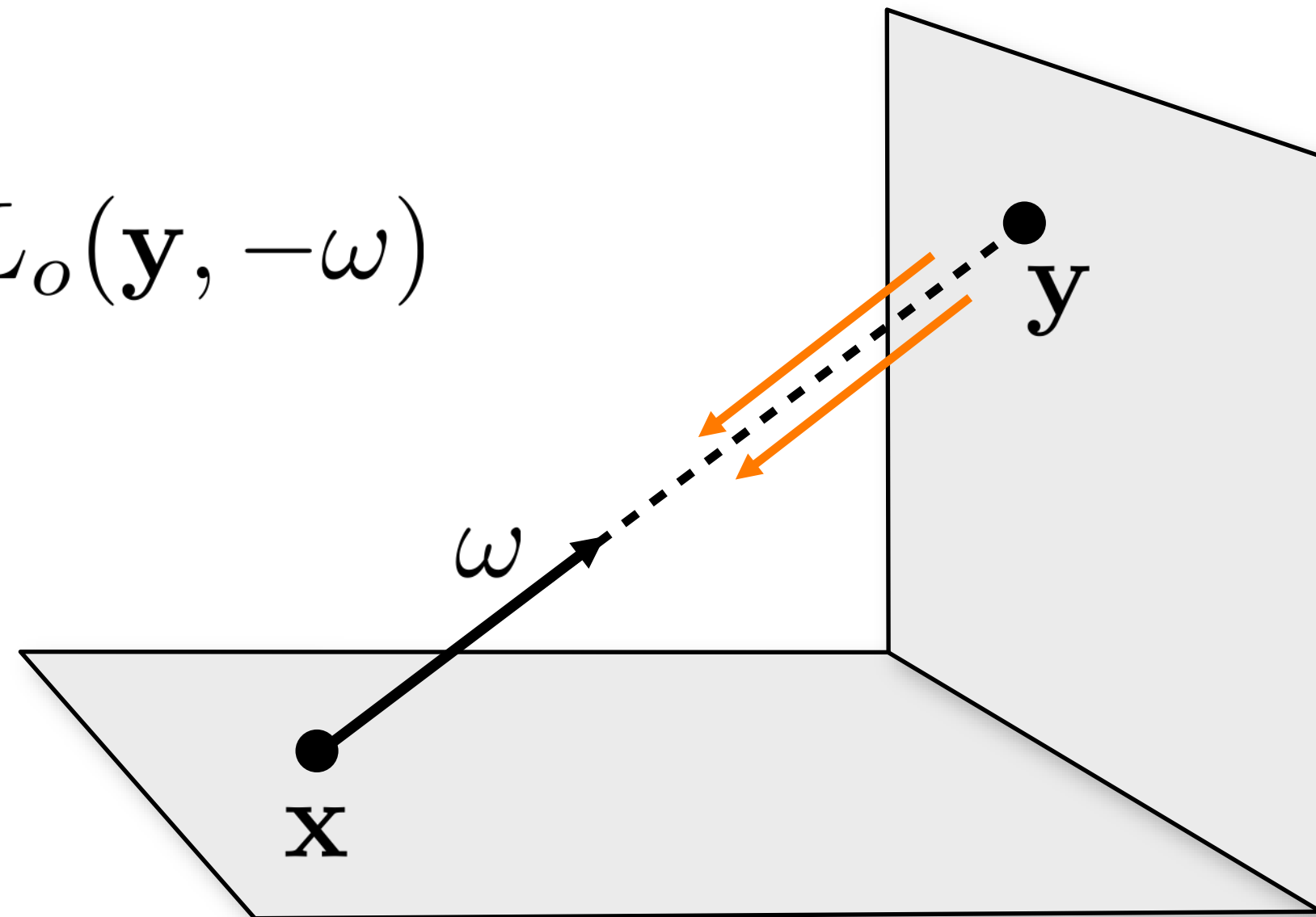
Radiance

fundamental quantity for vision and graphics

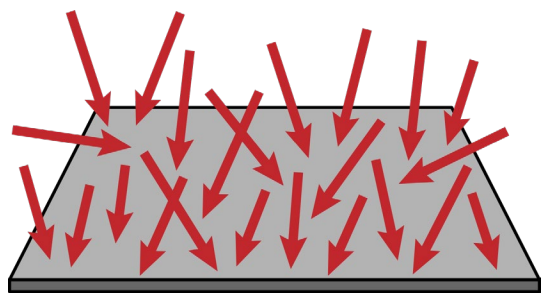
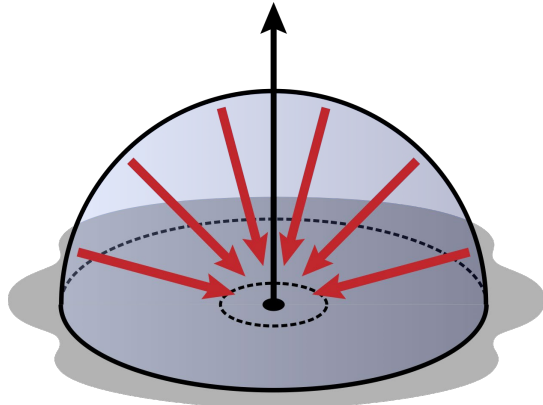
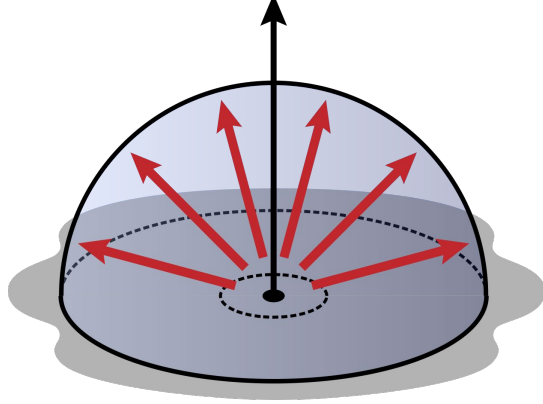
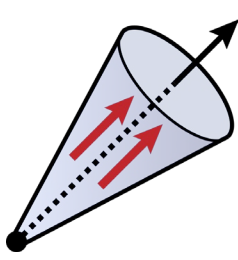
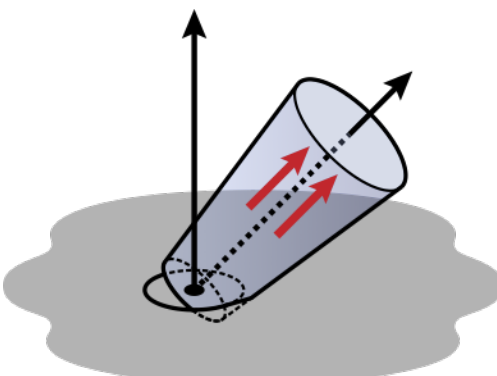
remains constant along a ray (*in vacuum only!*)

incident radiance L_i at one point can be expressed as outgoing radiance L_o at another point

$$L_i(\mathbf{x}, \omega) = L_o(\mathbf{y}, -\omega)$$



Overview of Quantities

• flux:	$\Phi(A)$	$\left[\frac{J}{s} = W \right]$	
• irradiance:	$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• radiosity:	$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• intensity:	$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$	$\left[\frac{W}{sr} \right]$	
• radiance:	$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x})d\vec{\omega}}$	$\left[\frac{W}{m^2 sr} \right]$	

Radiance

expressing *irradiance* in terms of radiance:

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x})d\vec{\omega}} \quad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$L(\mathbf{x}, \vec{\omega}) = \frac{dE(\mathbf{x})}{\cos\theta d\vec{\omega}}$$

$$L(\mathbf{x}, \vec{\omega}) \cos\theta d\vec{\omega} = dE(\mathbf{x})$$

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta d\vec{\omega} = E(\mathbf{x})$$

Integrate cosine-weighted
radiance over hemisphere

Radiance

expressing *irradiance* in terms of radiance:

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos \theta \, d\vec{\omega} = E(\mathbf{x})$$

expressing *flux* in terms of radiance:

$$\int_A E(\mathbf{x}) \, dA(\mathbf{x}) = \Phi(A) \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$\int_A \int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos \theta \, d\vec{\omega} dA(\mathbf{x}) = \Phi(A)$$

Integrate cosine-weighted radiance
over hemisphere and area

Radiance

Allows computing the radiant flux measured by *any* sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA$$

Cameras measure integrals of radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to (integrals of) radiance.

- “Processed” images (like PNG and JPEG) are not linear radiance measurements!!

Computing spherical integrals

Express function using spherical coordinates:

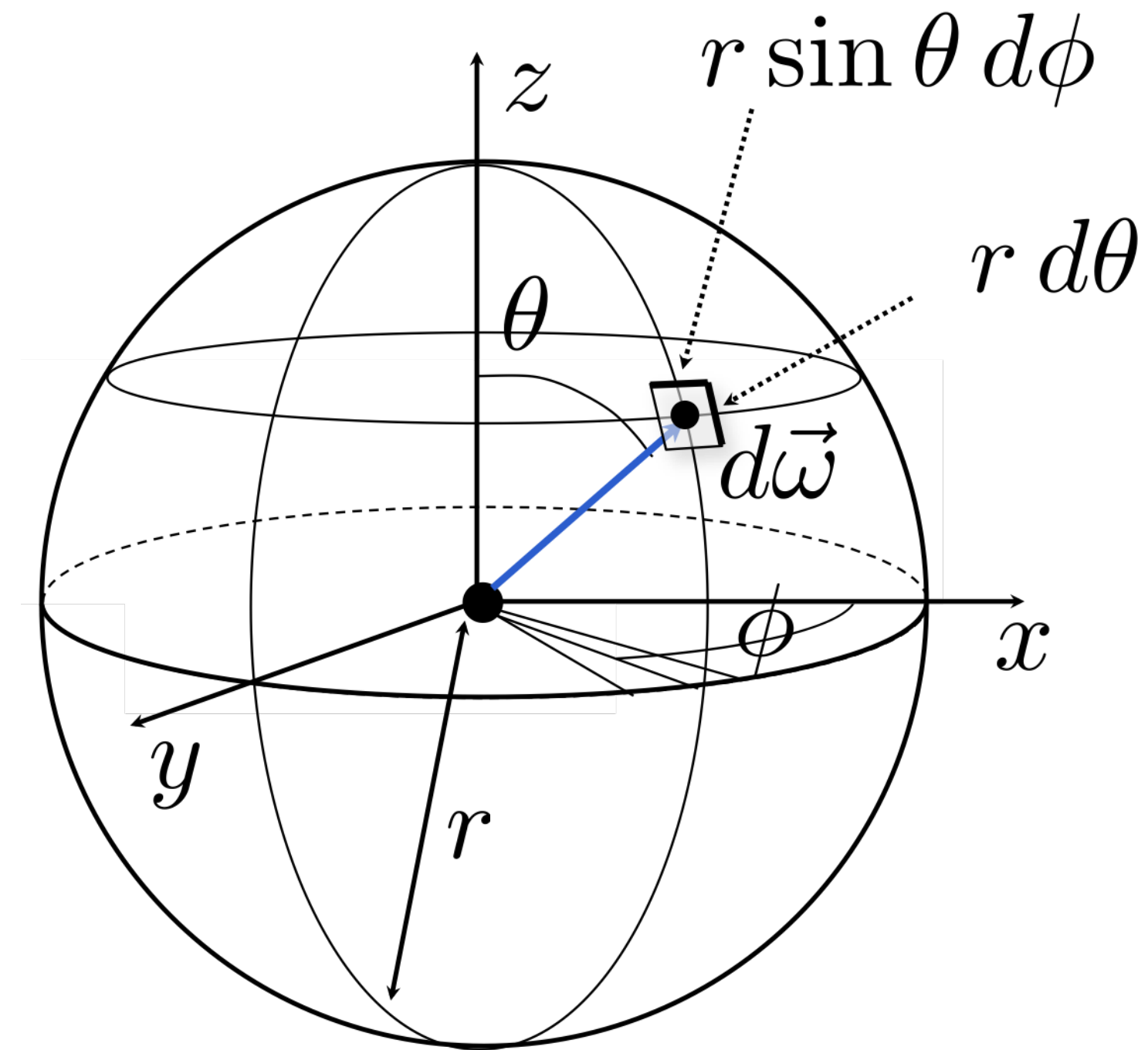
$$\int_0^{2\pi} \int_0^\pi f(\theta, \phi) \, d\theta \, d\phi \quad ?$$

Warning: this is not correct!

Differential Solid Angle

Differential area on the unit sphere around direction

$\vec{\omega}$

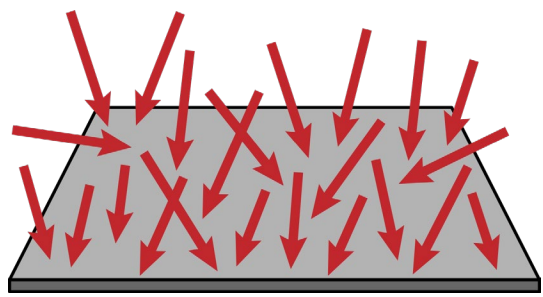
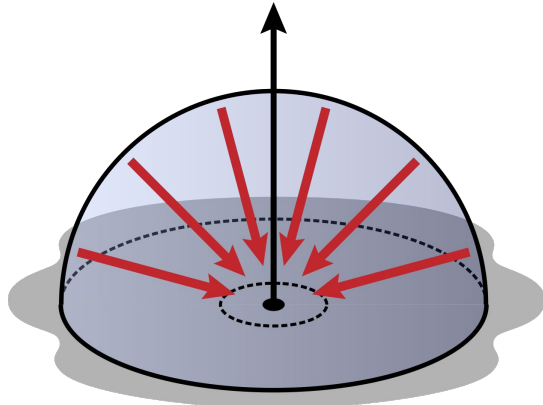
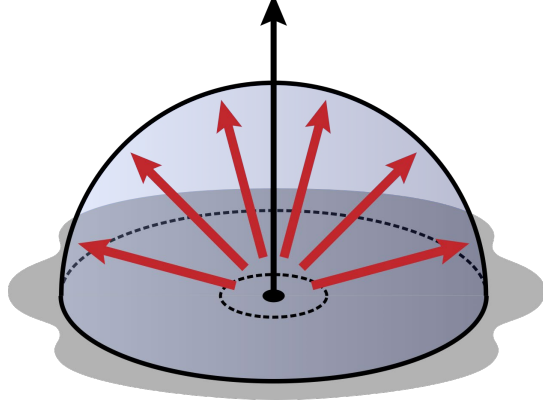
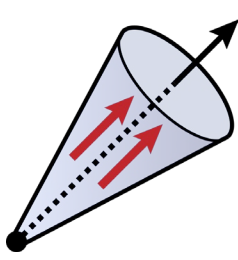
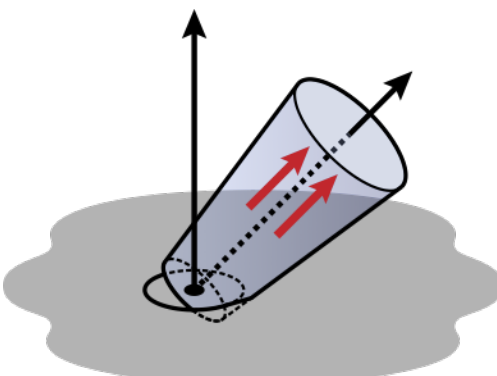


$$dA = (r d\theta)(r \sin \theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi = 4\pi$$

Overview of Quantities

• flux:	$\Phi(A)$	$\left[\frac{J}{s} = W \right]$	
• irradiance:	$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• radiosity:	$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$	$\left[\frac{W}{m^2} \right]$	
• intensity:	$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$	$\left[\frac{W}{sr} \right]$	
• radiance:	$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos \theta dA(\mathbf{x}) d\vec{\omega}}$	$\left[\frac{W}{m^2 sr} \right]$	

All of these quantities can be a function of wavelength!

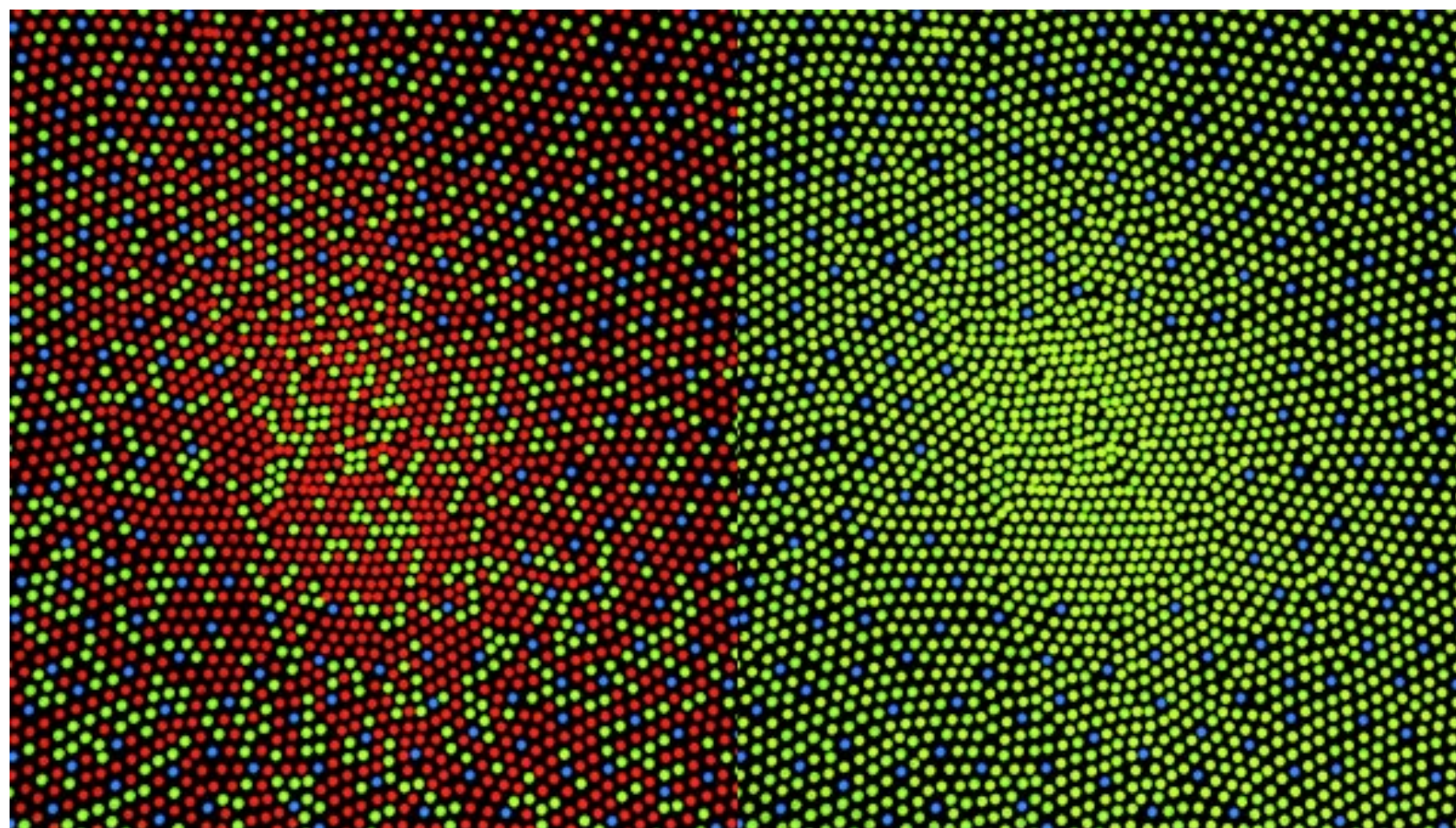
Handling color

- *Any* light sensor (digital or not) has different sensitivity to different wavelengths.
- This is described by the sensor's *spectral sensitivity function* (SSF).
- When measuring some incident *spectral* flux, the sensor produces a *scalar color* response:

$$\begin{array}{c} \text{sensor} \\ \text{response} \end{array} \longrightarrow R = \int_{\lambda} \begin{array}{c} \text{spectral flux} \\ \downarrow \\ \Phi(\lambda) \end{array} \begin{array}{c} \text{sensor SSF} \\ \downarrow \\ f(\lambda) \end{array} d\lambda$$

Handling color – the human eye

- The human eye is a collection of light sensors called cone cells.
- There are three types of cells with different spectral sensitivity functions.
- Human color perception is three-dimensional (*tristimulus color*).

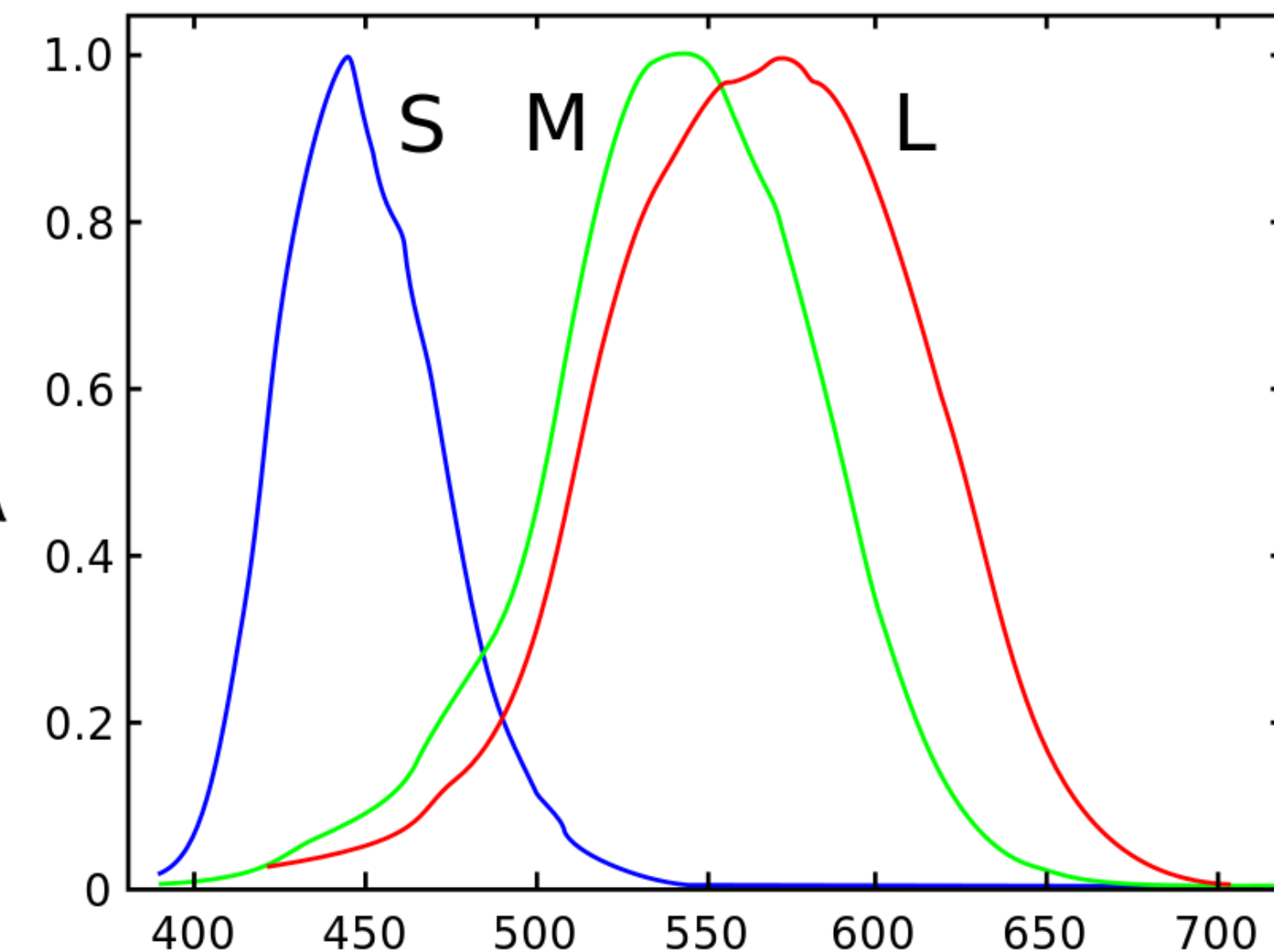


cone distribution
for normal vision
(64% L, 32% M)

“short” $S = \int_{\lambda} \Phi(\lambda) S(\lambda) d\lambda$

“medium” $M = \int_{\lambda} \Phi(\lambda) M(\lambda) d\lambda$

“long” $L = \int_{\lambda} \Phi(\lambda) L(\lambda) d\lambda$

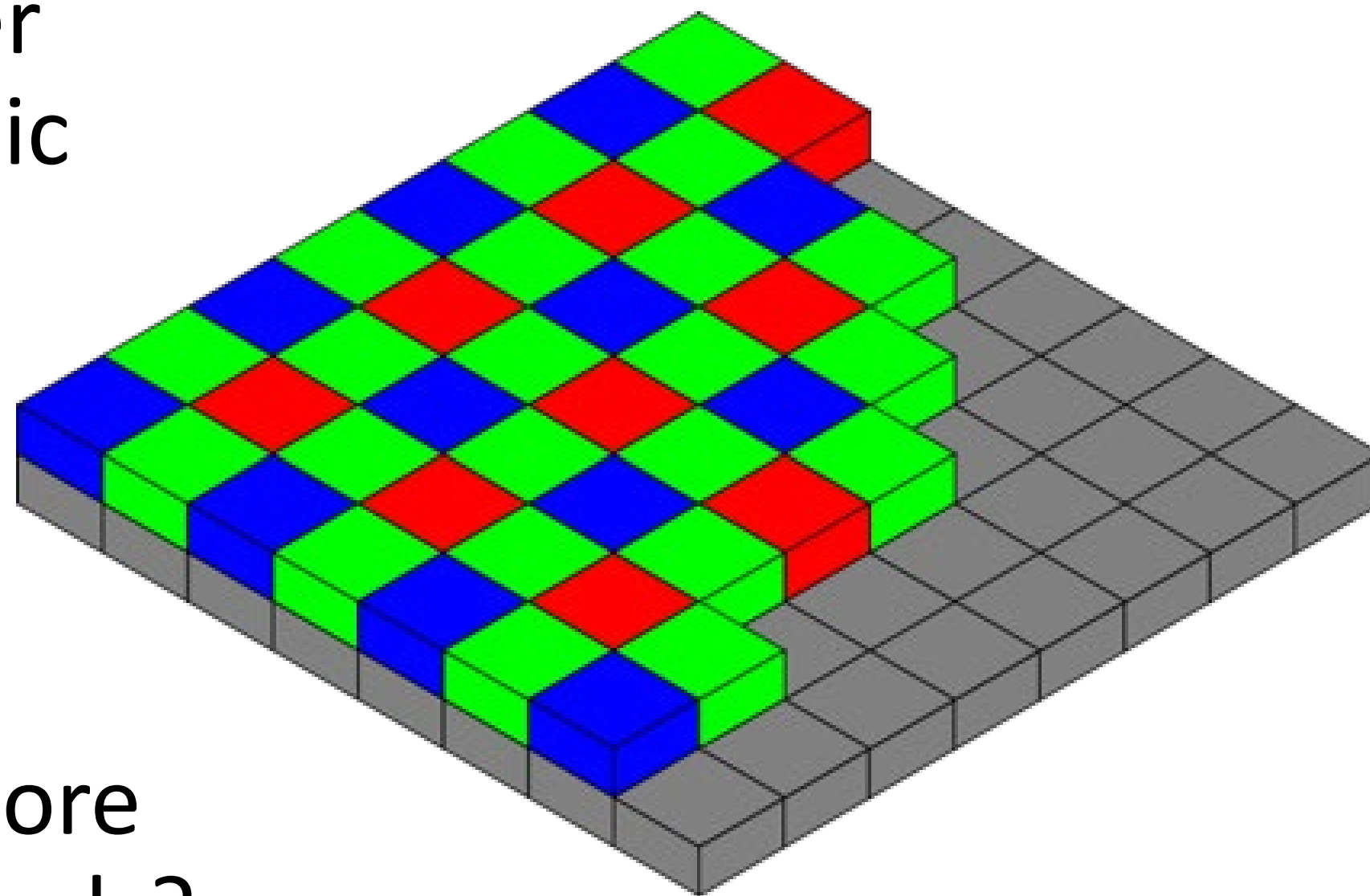


Handling color – photography

Two design choices:

- What spectral sensitivity functions $f(\lambda)$ to use for each color filter?
- How to spatially arrange (“mosaic”) different color filters

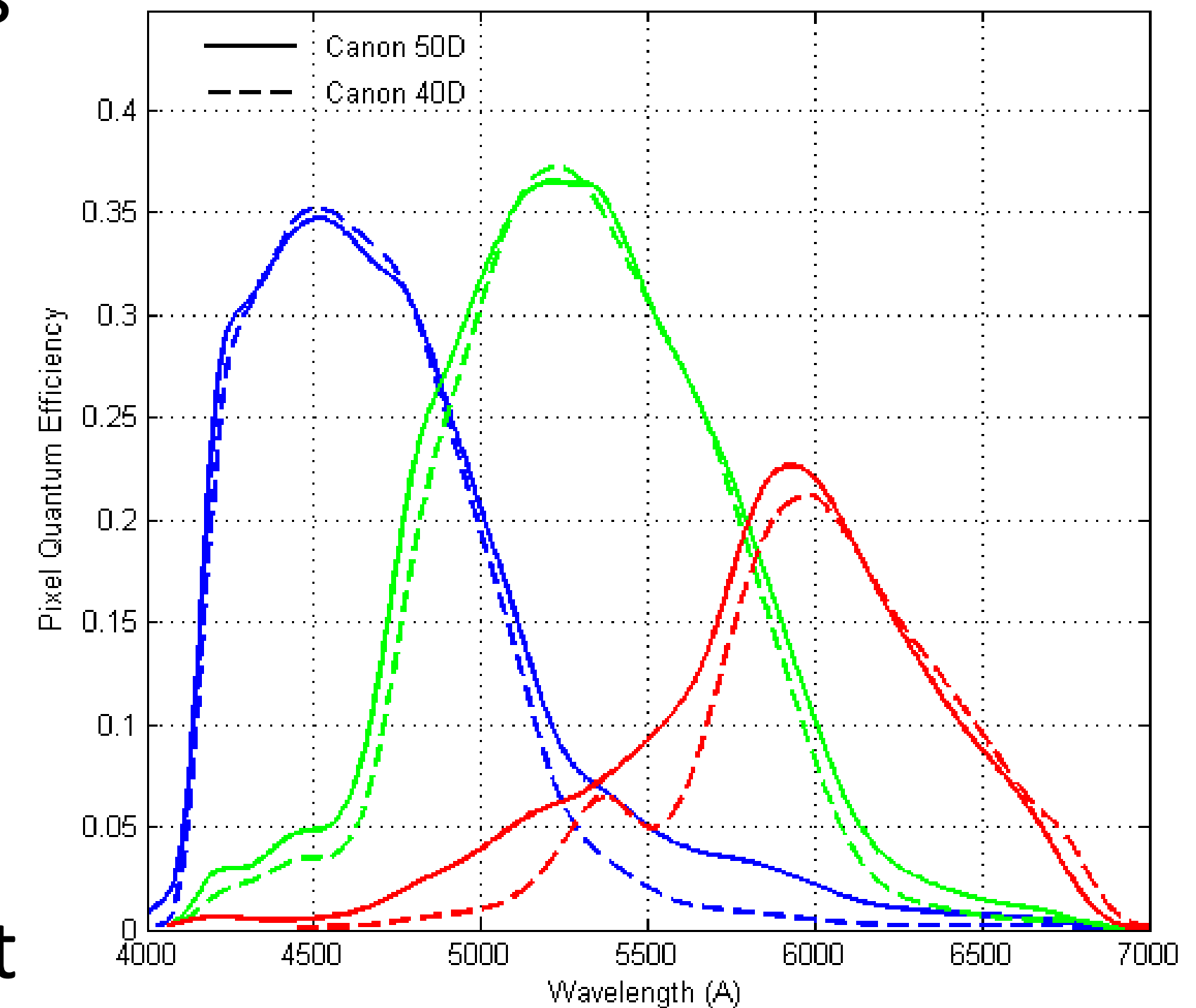
Bayer
mosaic



Why more
green pixels?

SSF for
Canon 50D

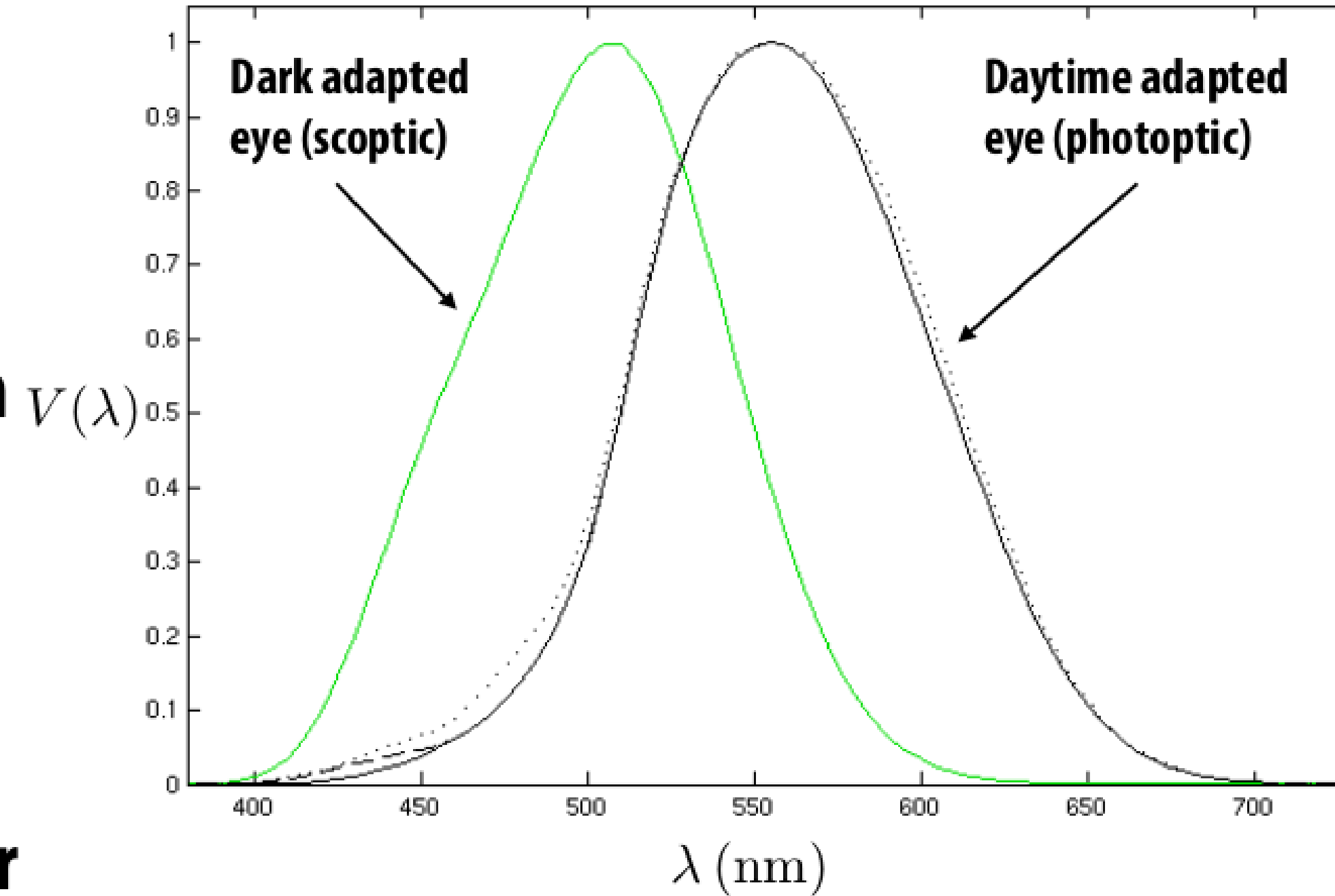
Generally do not
match human LMS.



$f(\lambda)$

Radiometry versus photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system to electromagnetic radiation
- Luminance (Y) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye's luminous efficacy curve, e.g.:



$$Y(p, \omega) = \int_0^\infty L(p, \omega, \lambda) V(\lambda) d\lambda$$

Radiometry versus photometry

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance (incoming) Radiosity (outgoing)	Illuminance (incoming) Luminosity (outgoing)
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Radiometry versus photometry

Photometry	MKS	CGS	British
Luminous Energy	Talbot	Talbot	Talbot
Luminous Power	Lumen	Lumen	Lumen
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit, Apostlib, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela	Candela	Candela

Modern LED light

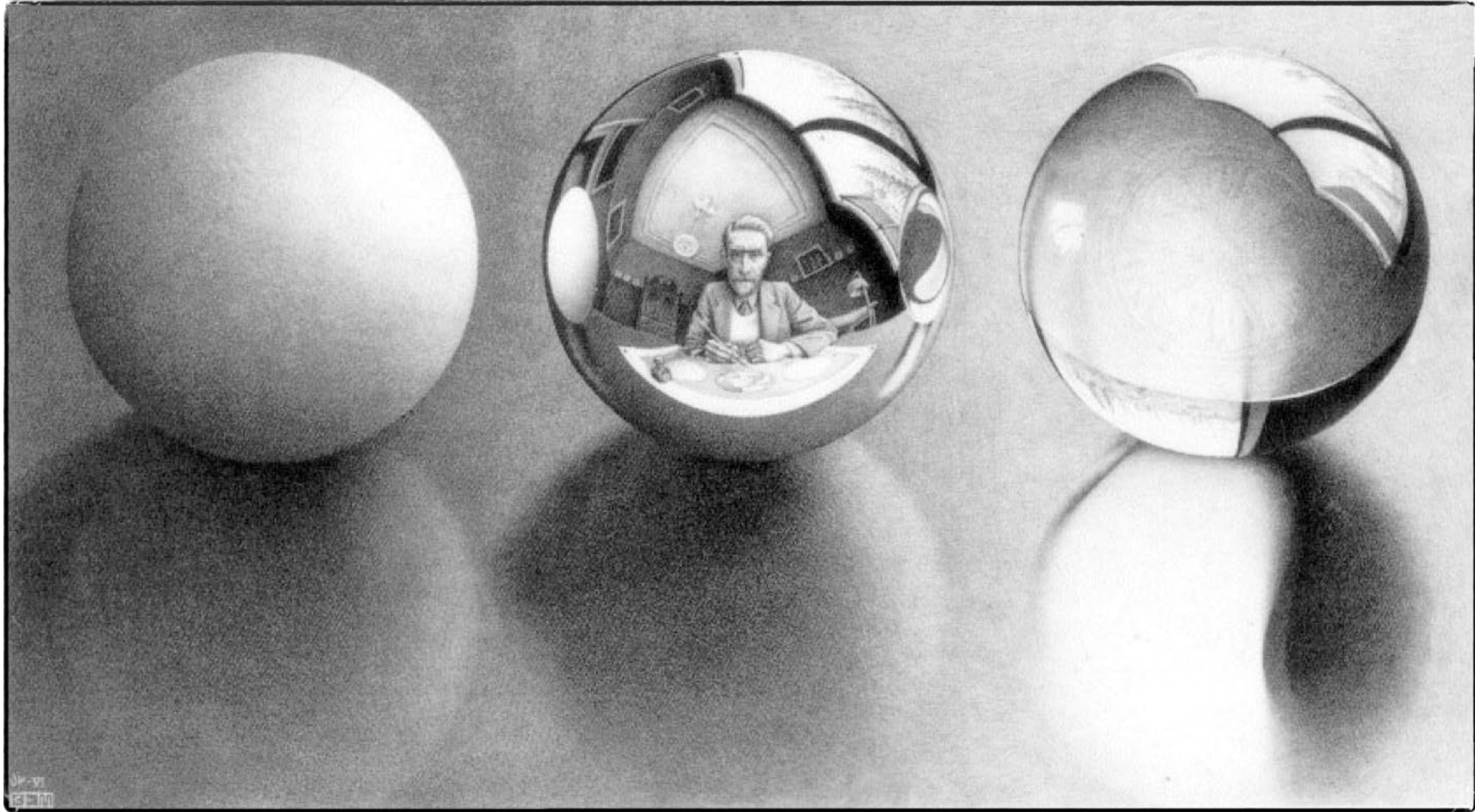
Input power: 11 W

**Output: 815 lumens
(~ 80 lumens / Watt)**

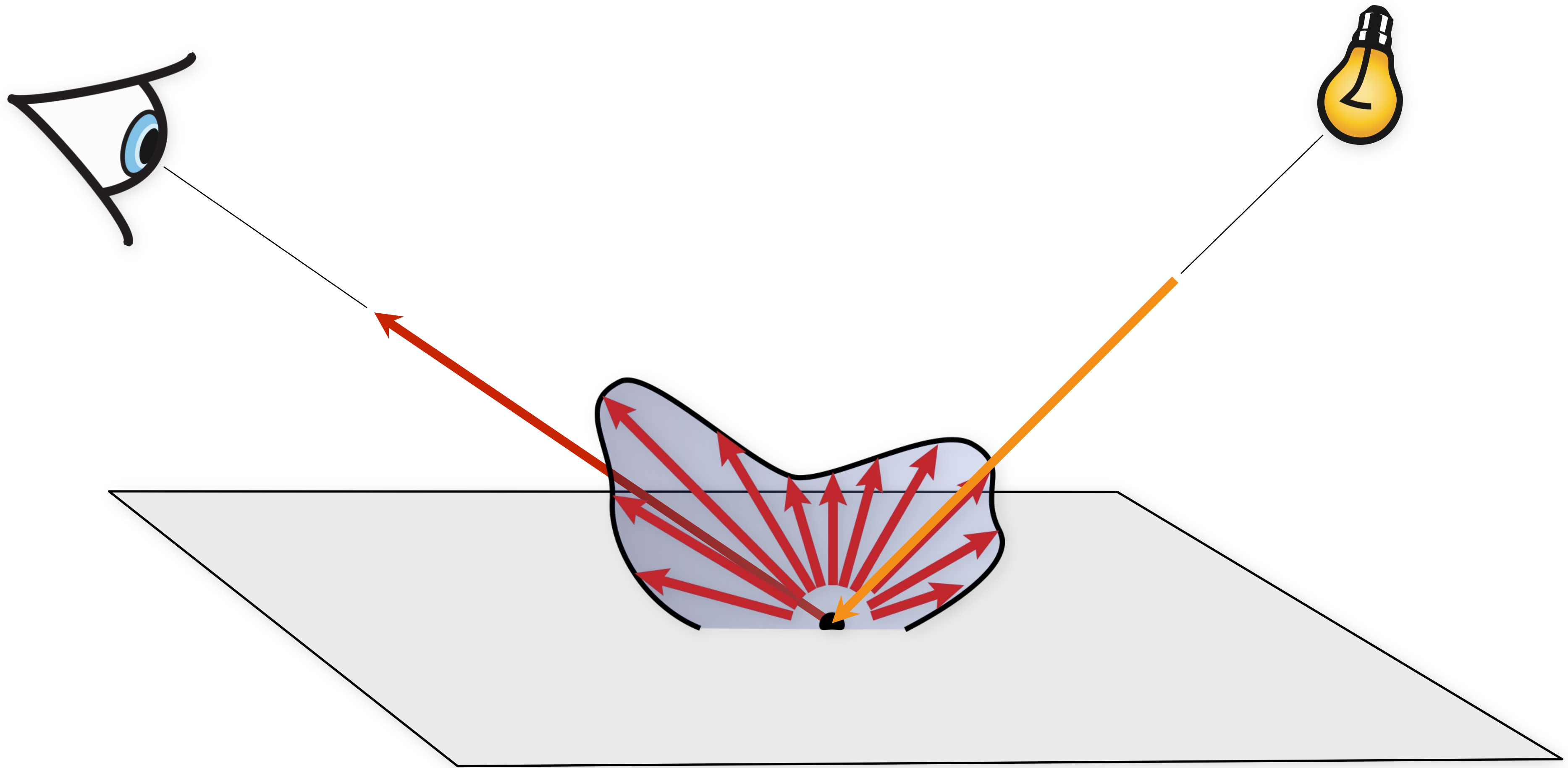
**Incandescent bulbs:
~15 lumens / Watt)**



Reflection equation



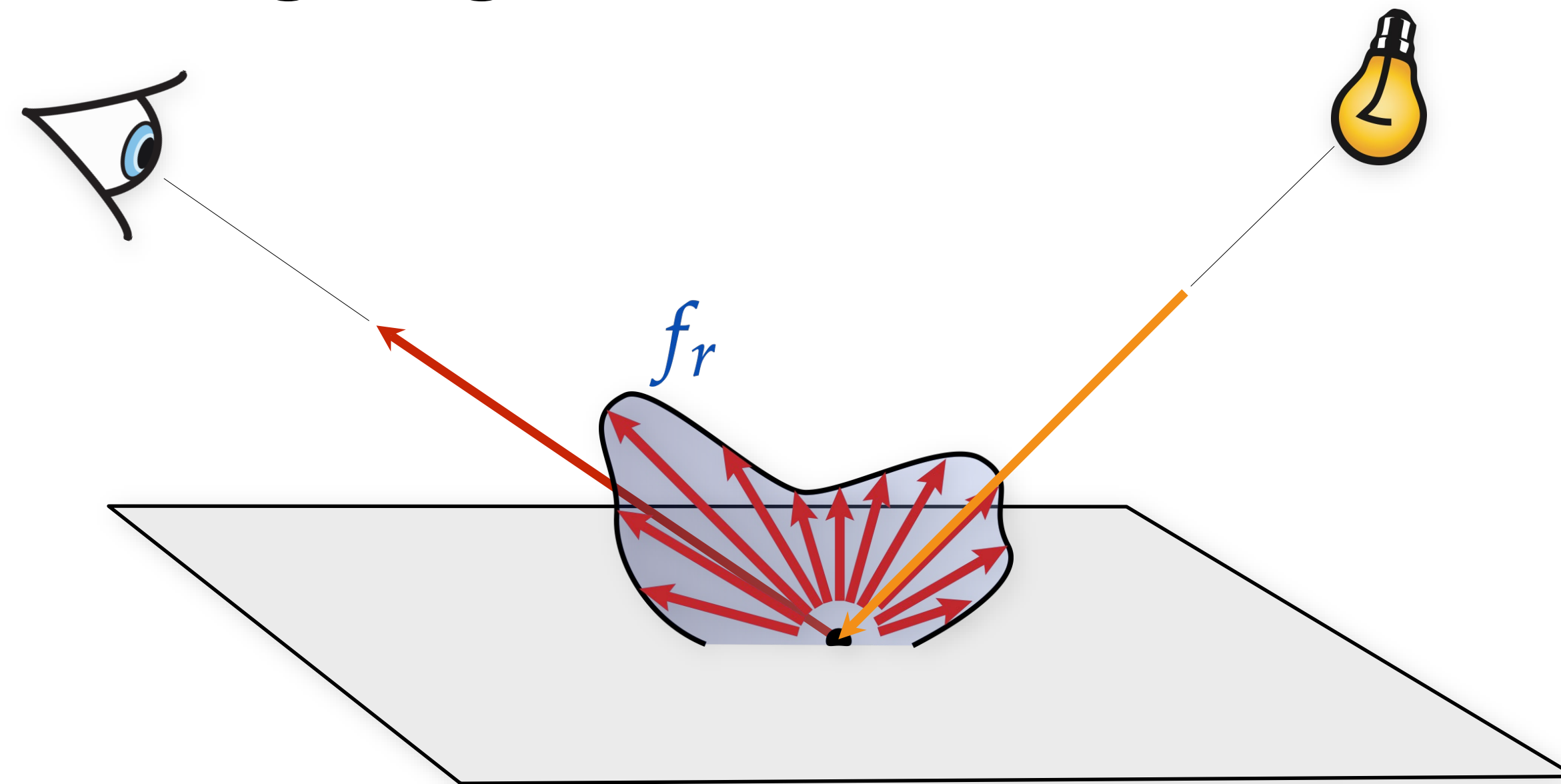
Light-Material Interactions



The BRDF

Bidirectional Reflectance Distribution Function

- how much light gets scattered from **one direction** into **each other direction**
- formally: ratio of outgoing *radiance* to incident *irradiance*

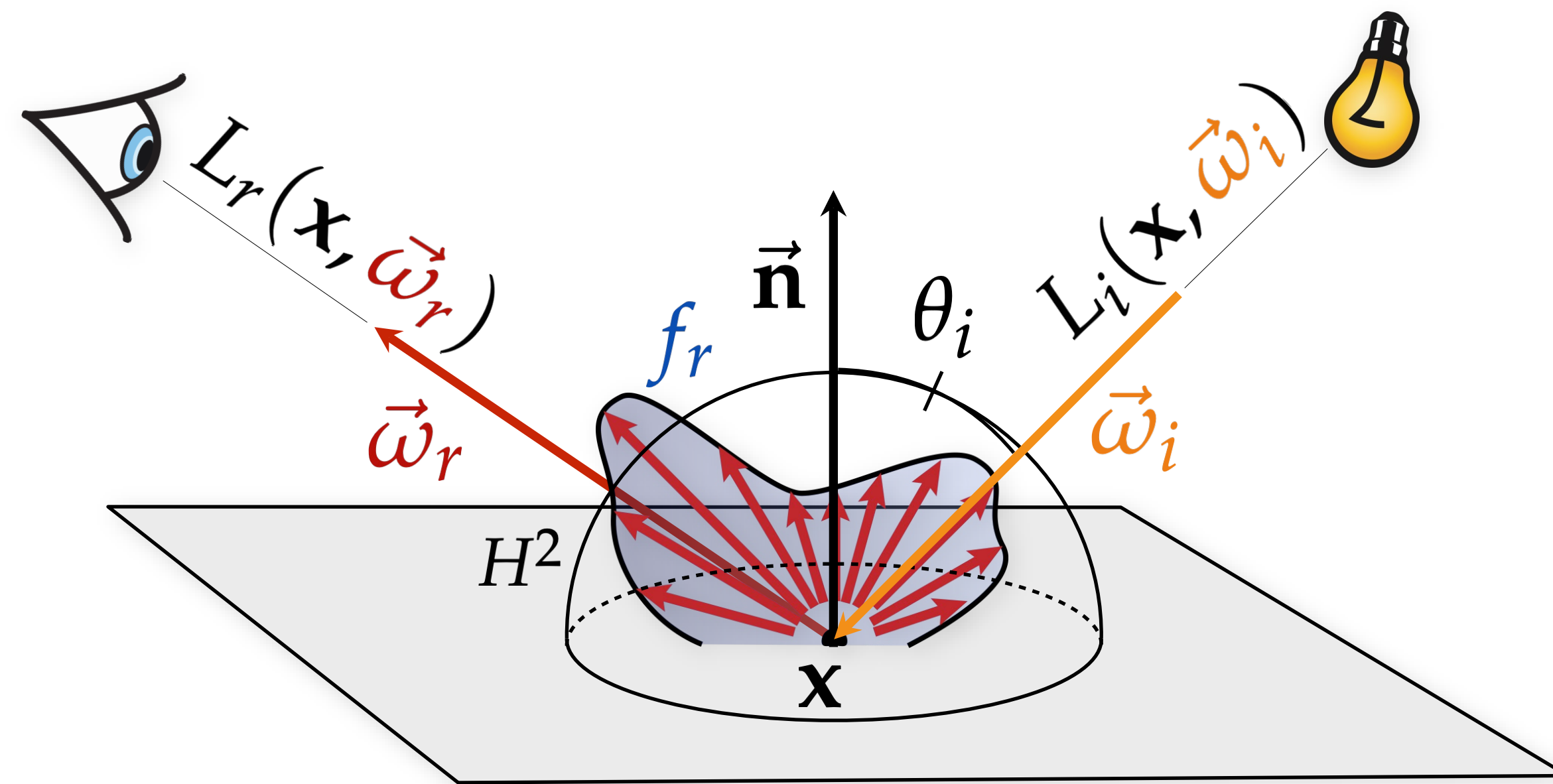


The reflection equation

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

Where does the cosine come from?



This describes a local illumination model

Motivation

Motivation



BRDF Properties

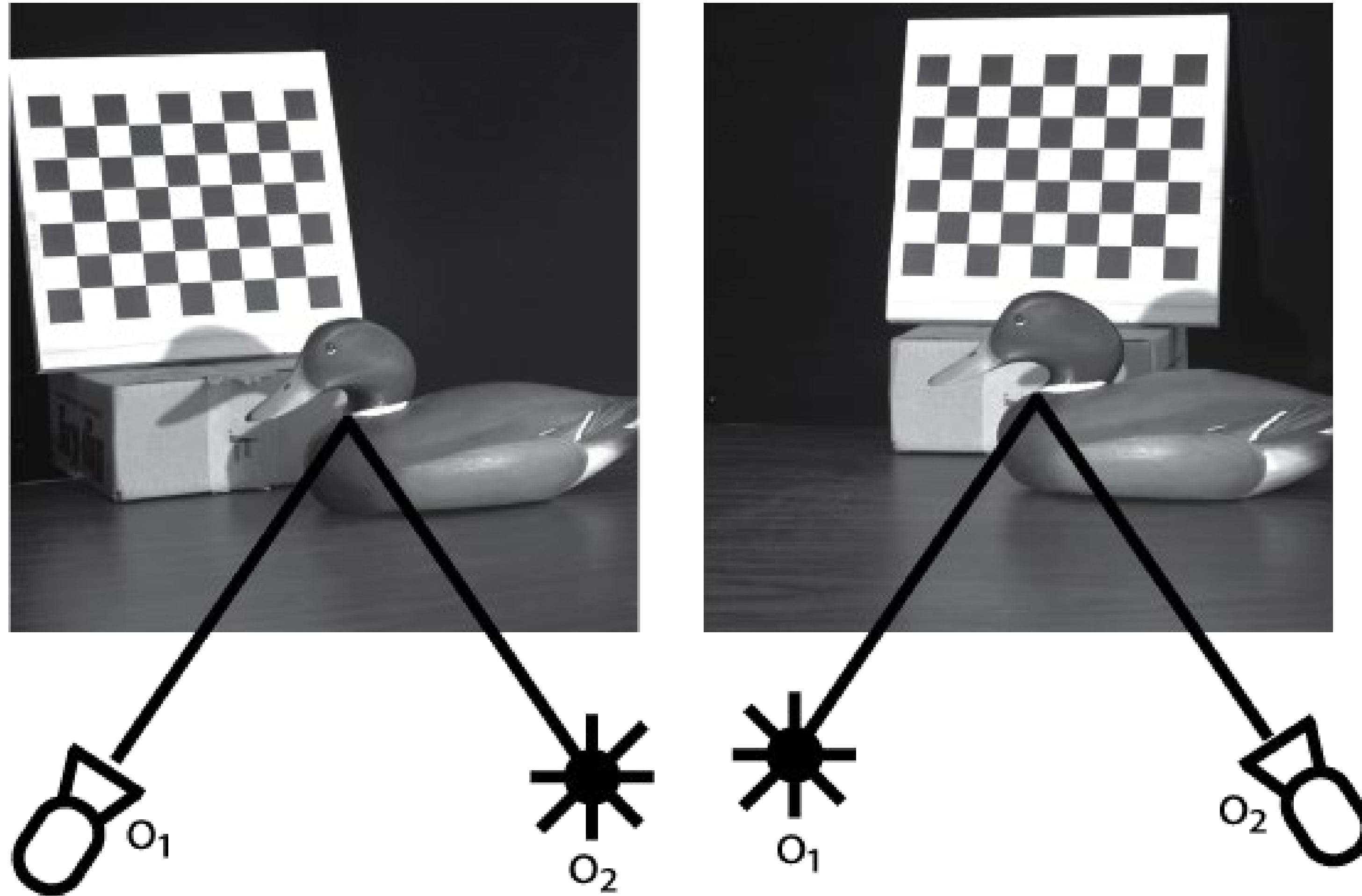
Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_i \, d\vec{\omega}_i \leq 1, \quad \forall \vec{\omega}_r$$

Where does the
cosine come from?

Helmholtz Reciprocity



BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos \theta_i \, d\vec{\omega}_i \leq 1, \quad \forall \vec{\omega}_r$$

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$

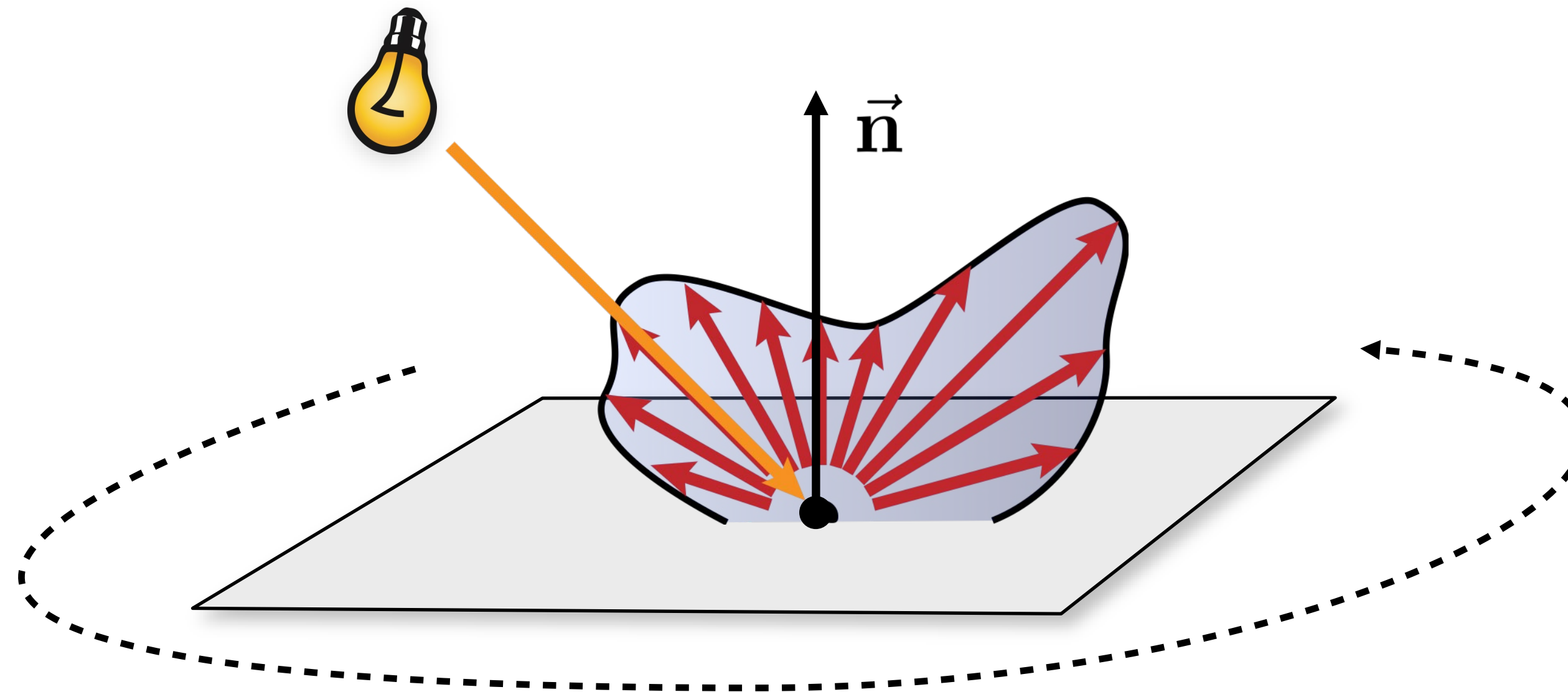
$$f_r(\mathbf{x}, \vec{\omega}_i \leftrightarrow \vec{\omega}_r)$$

BRDFs Properties

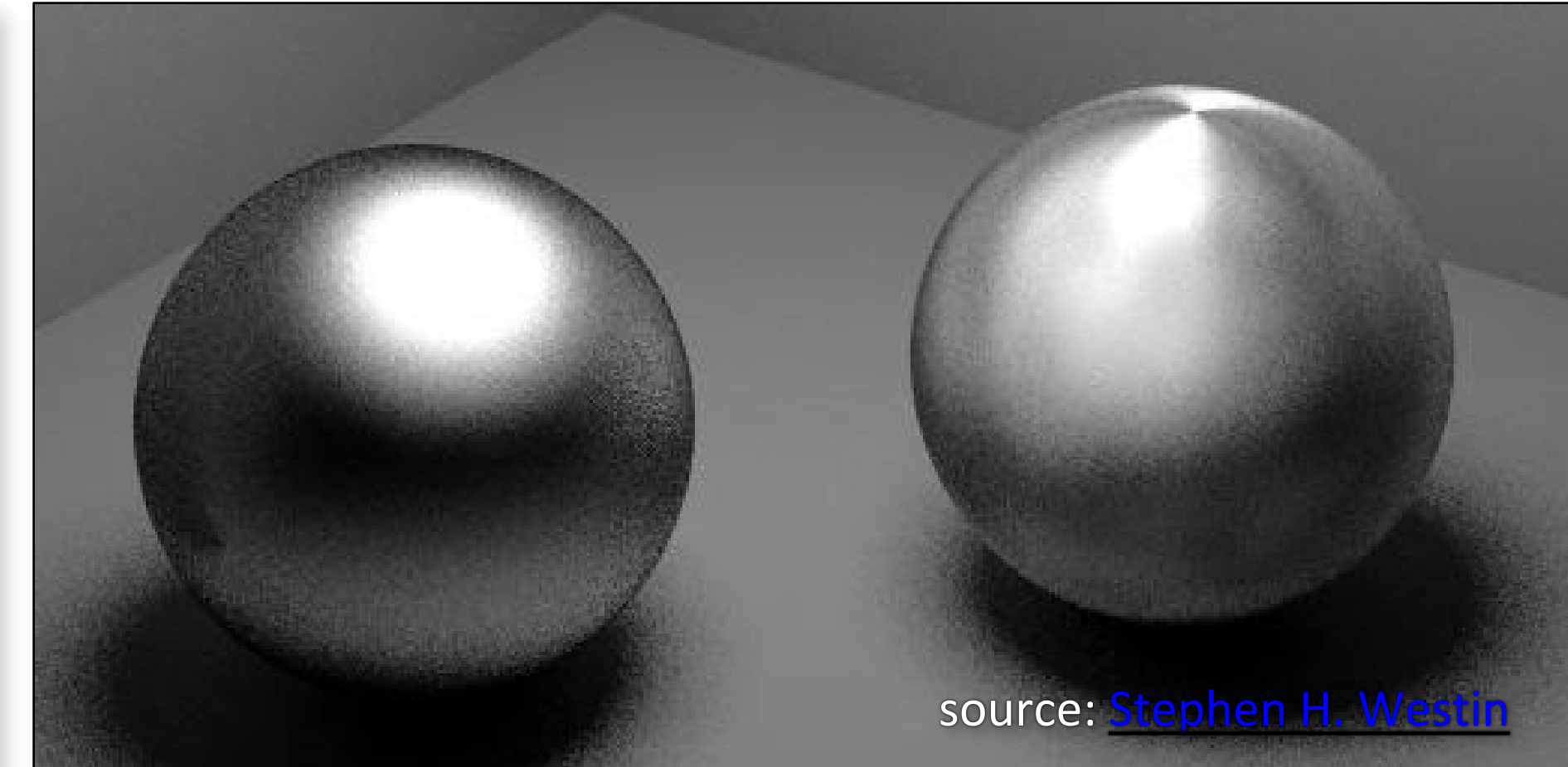
If the BRDF is unchanged as the material is rotated around the normal, then it is ***isotropic***, otherwise it is ***anisotropic***.

Isotropic BRDFs are functions of just 3 variables

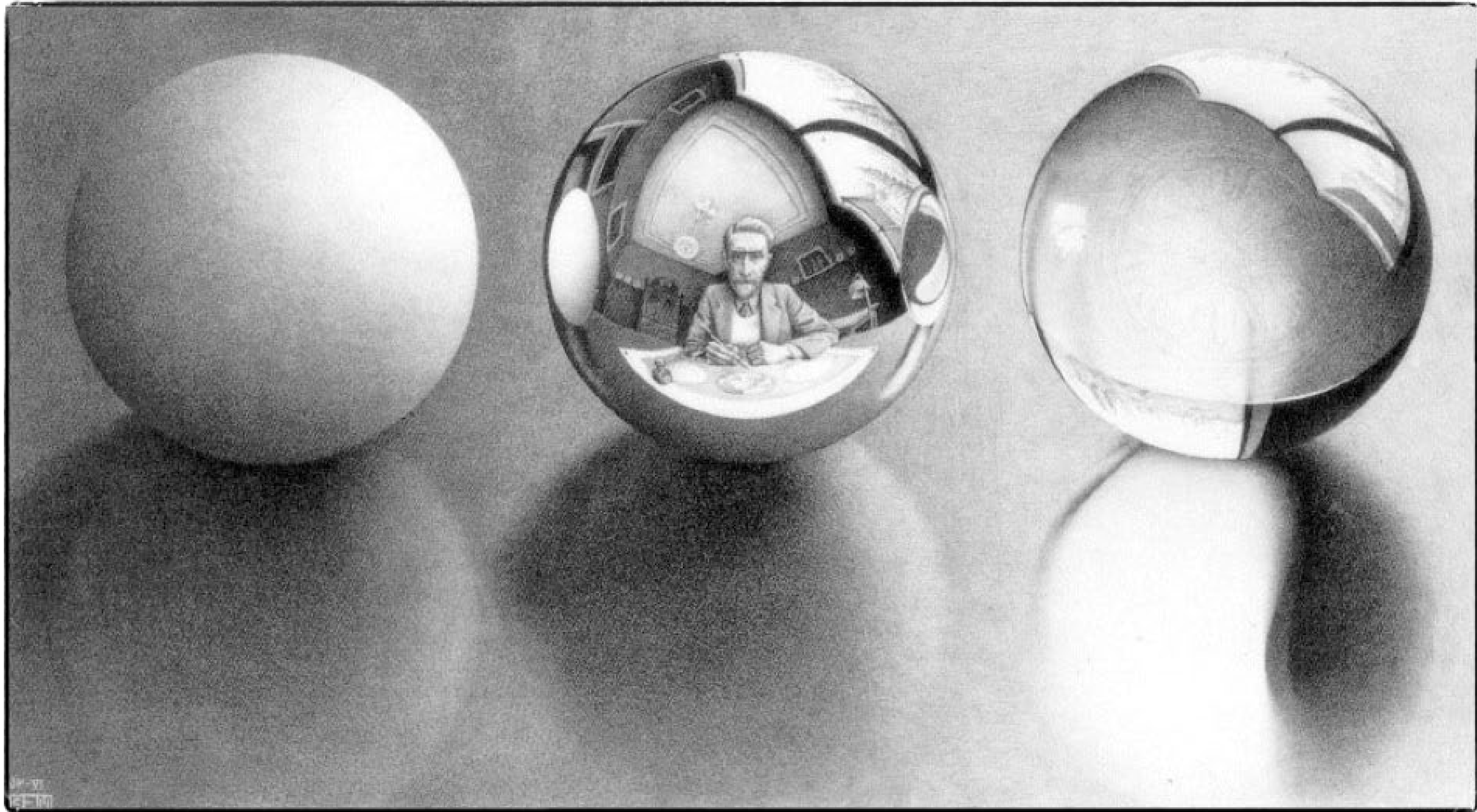
$$(\theta_i, \theta_r, \Delta\phi)$$



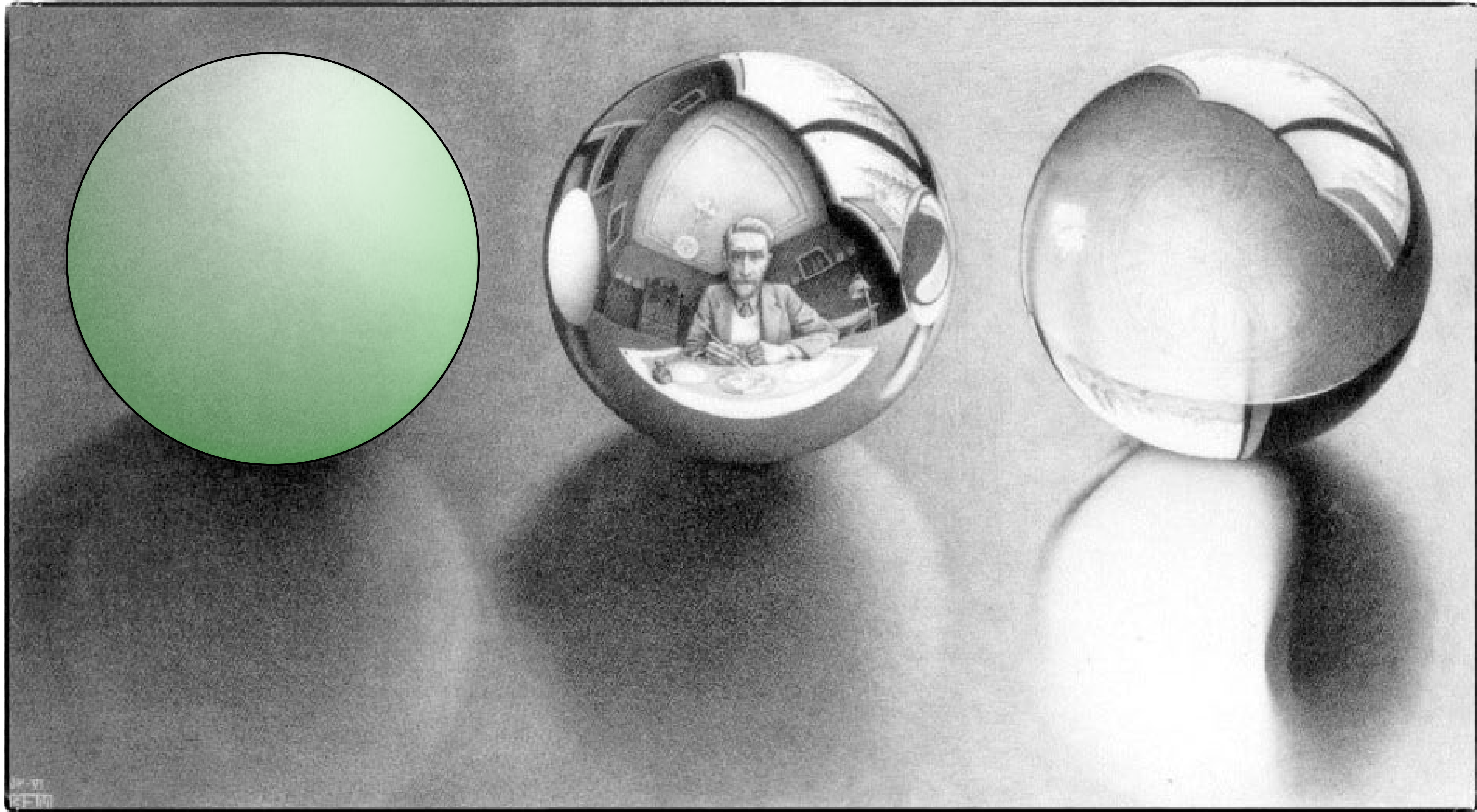
Isotropic vs Anisotropic Reflection



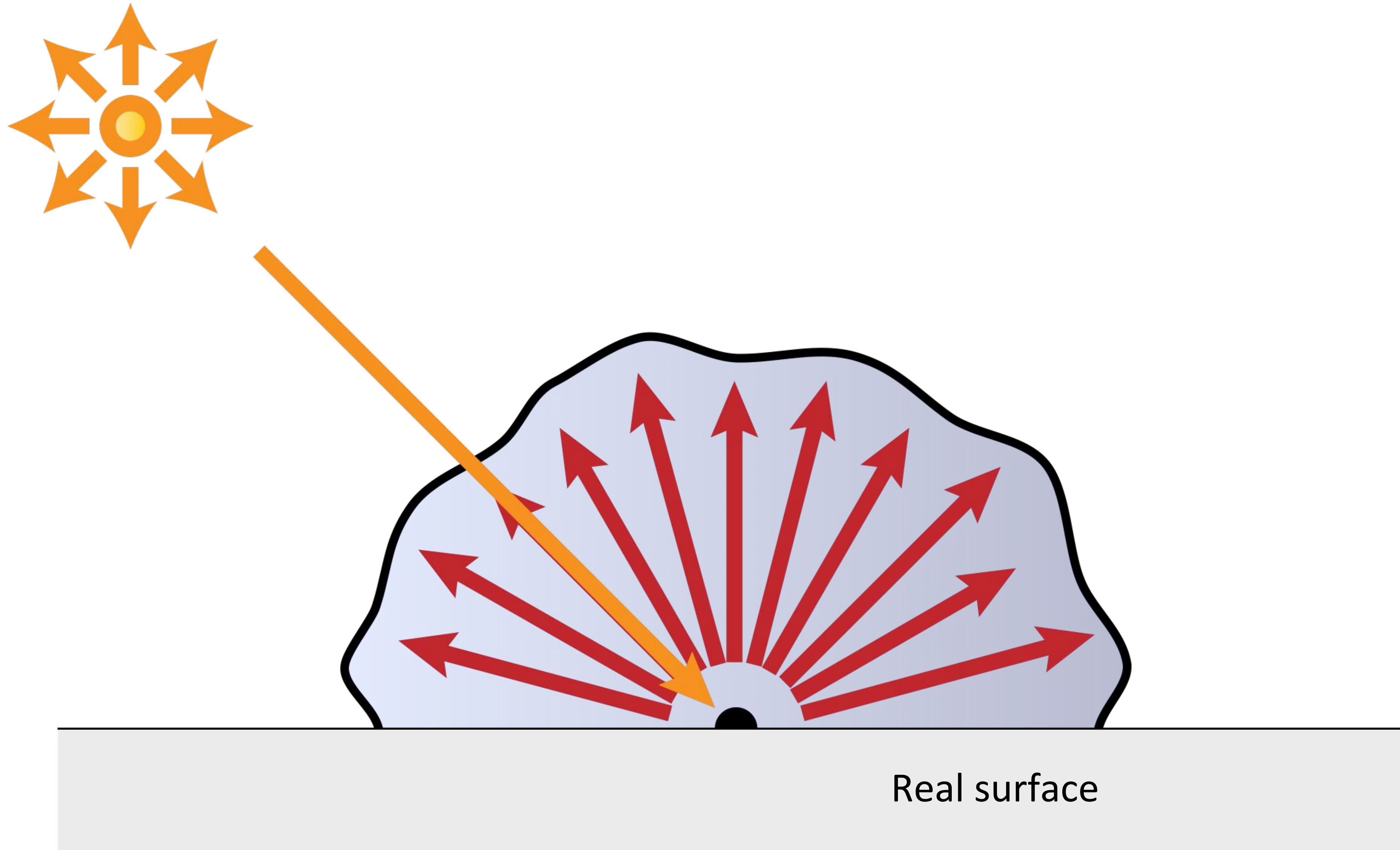
Idealized materials



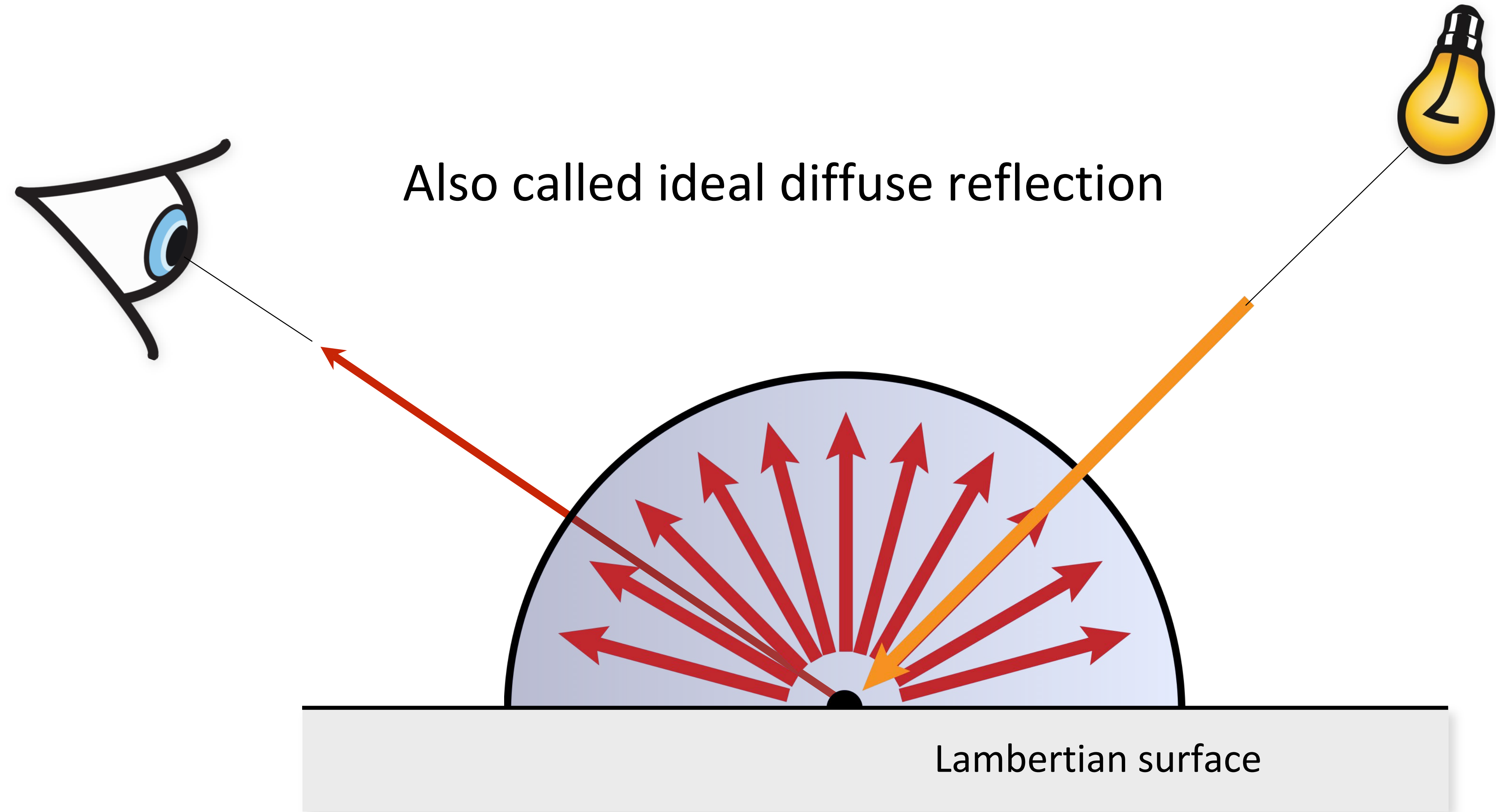
Diffuse reflection



Diffuse reflection



Lambertian reflection



BRDF for ideal diffuse reflection?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

Scatters light equal in all directions
BRDF is a constant

Ideal Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

Note: we can
drop ω_r

$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r E(\mathbf{x})$$

If *all* incoming light is reflected:

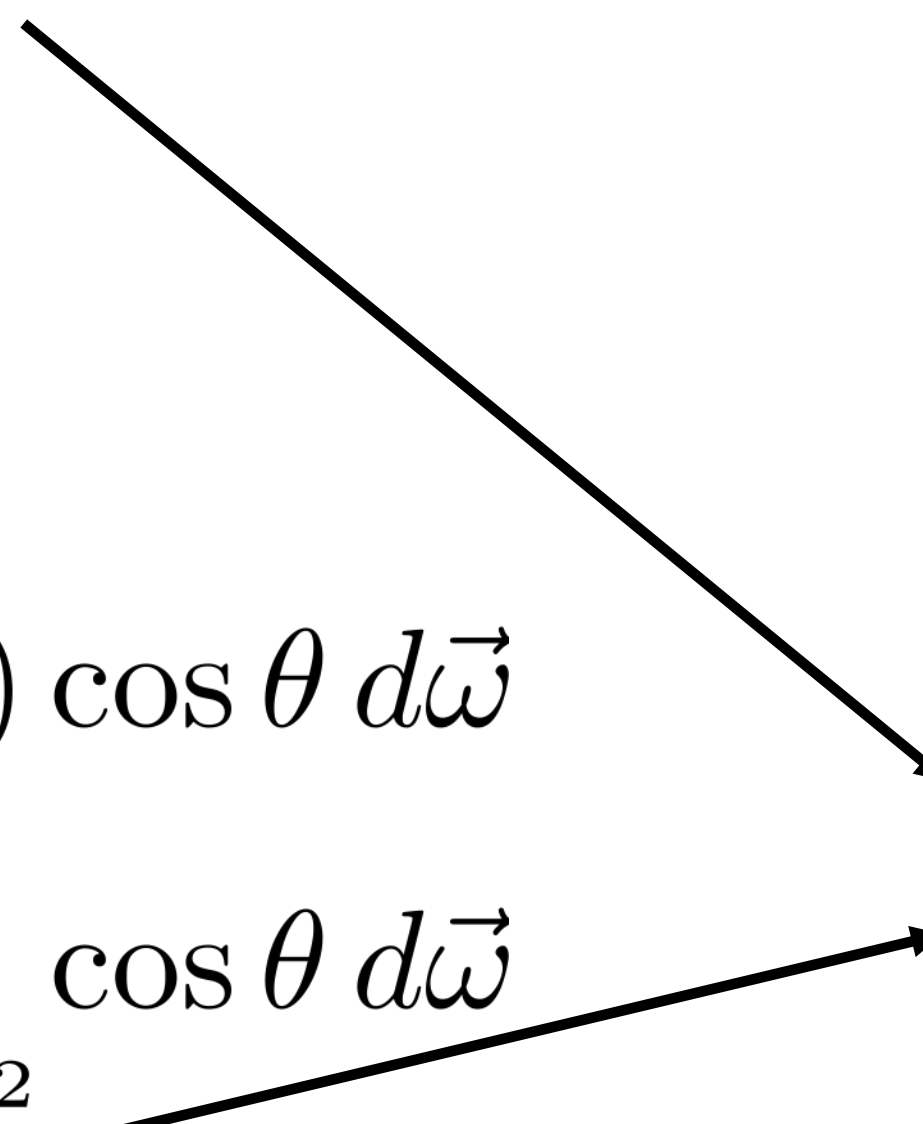
$$E(\mathbf{x}) = B(\mathbf{x})$$

$$E(\mathbf{x}) = \int_{H^2} L_r(\mathbf{x}) \cos \theta d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \int_{H^2} \cos \theta d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \pi$$

Note: can also be
derived from energy
conservation

$$f_r = \frac{1}{\pi}$$


Diffuse BRDF

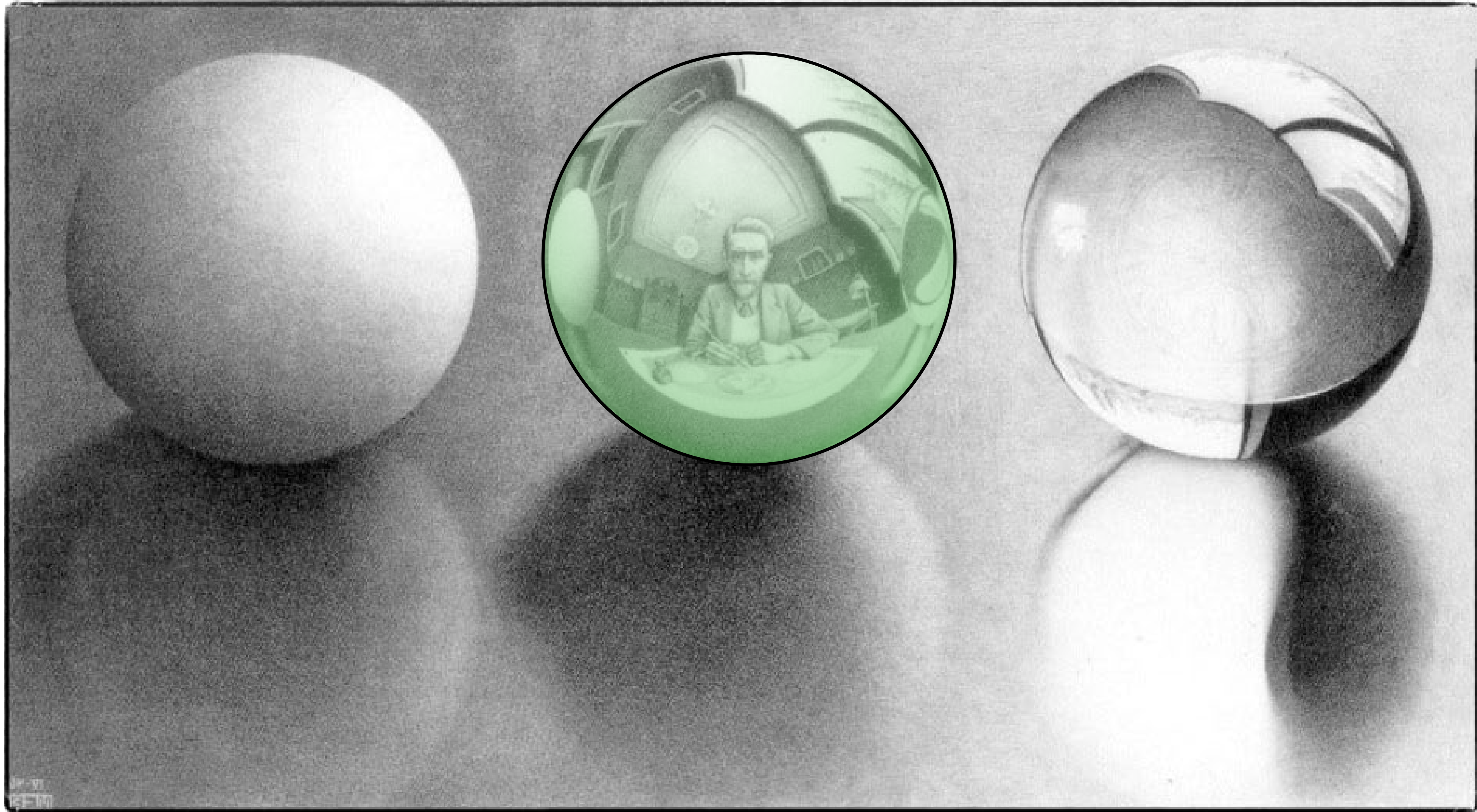
For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

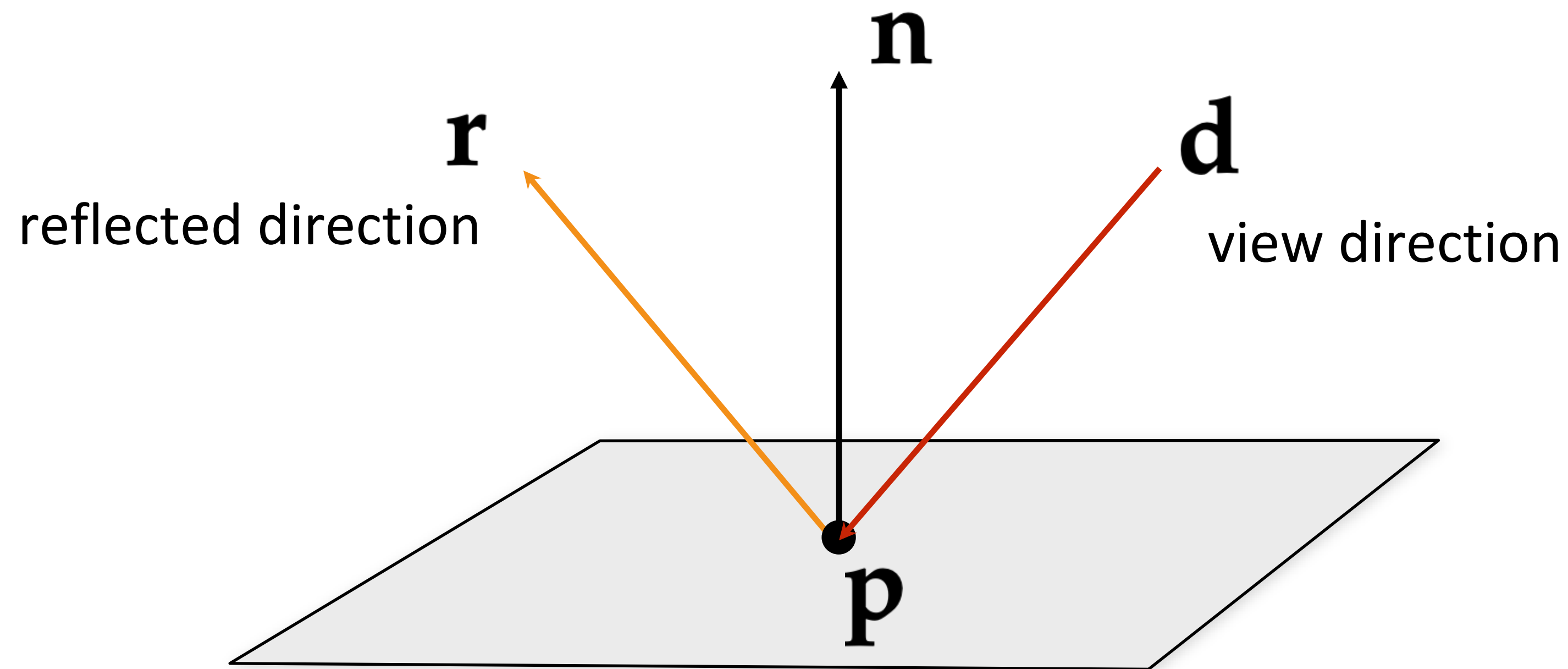
ρ : Diffuse reflectance (albedo) [0..1)

Specular/Mirror reflection



Mirror reflection

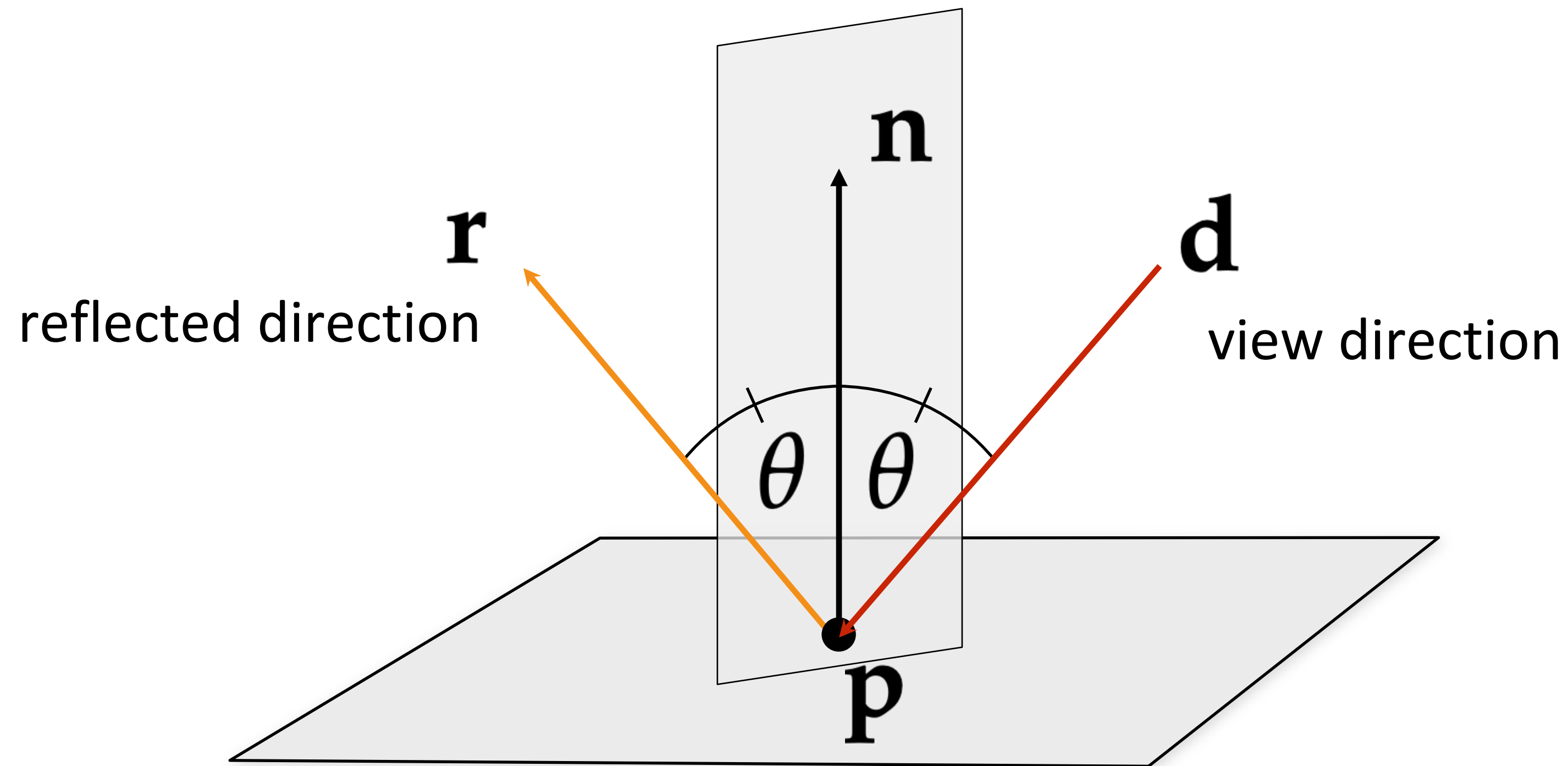
Assume $\mathbf{n}$ is unit length



What two properties defined reflection direction?

Mirror reflection

Assume $\mathbf{n}$ is unit length

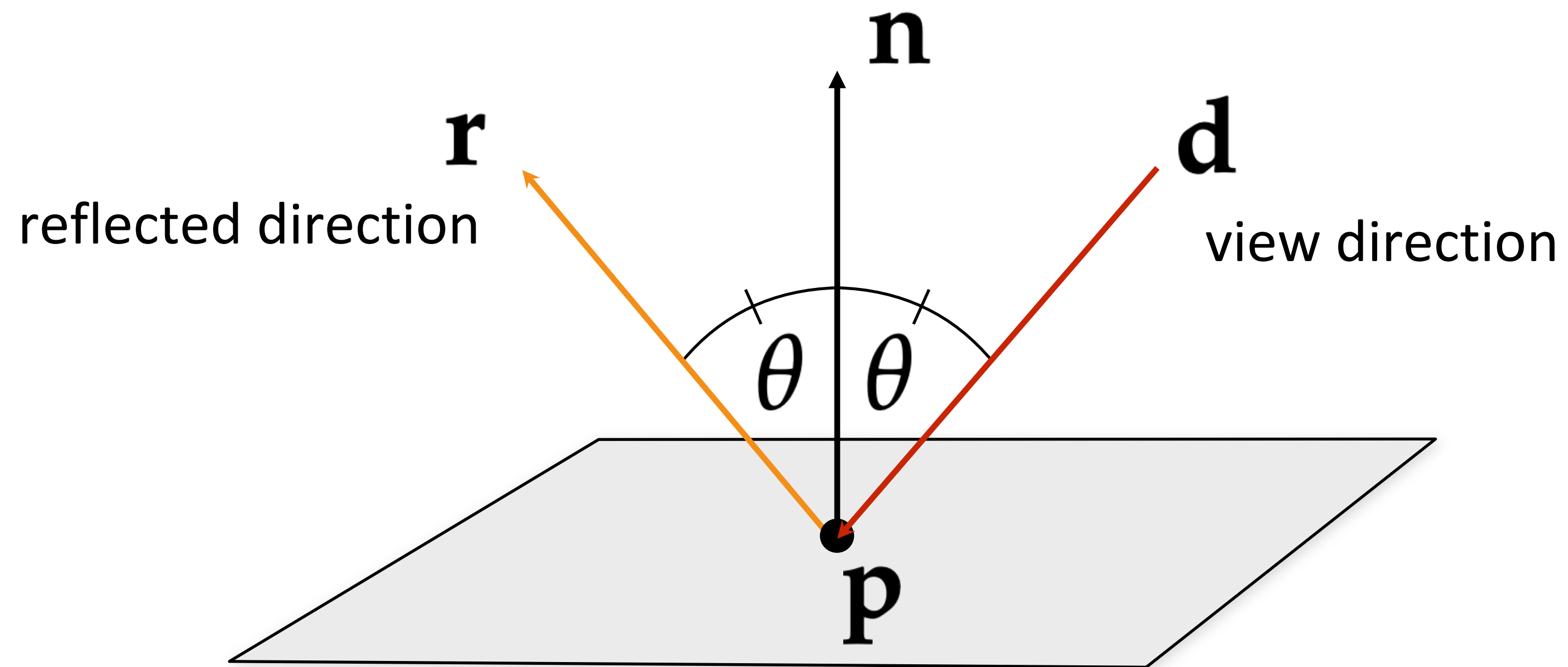


What two properties defined reflection direction?

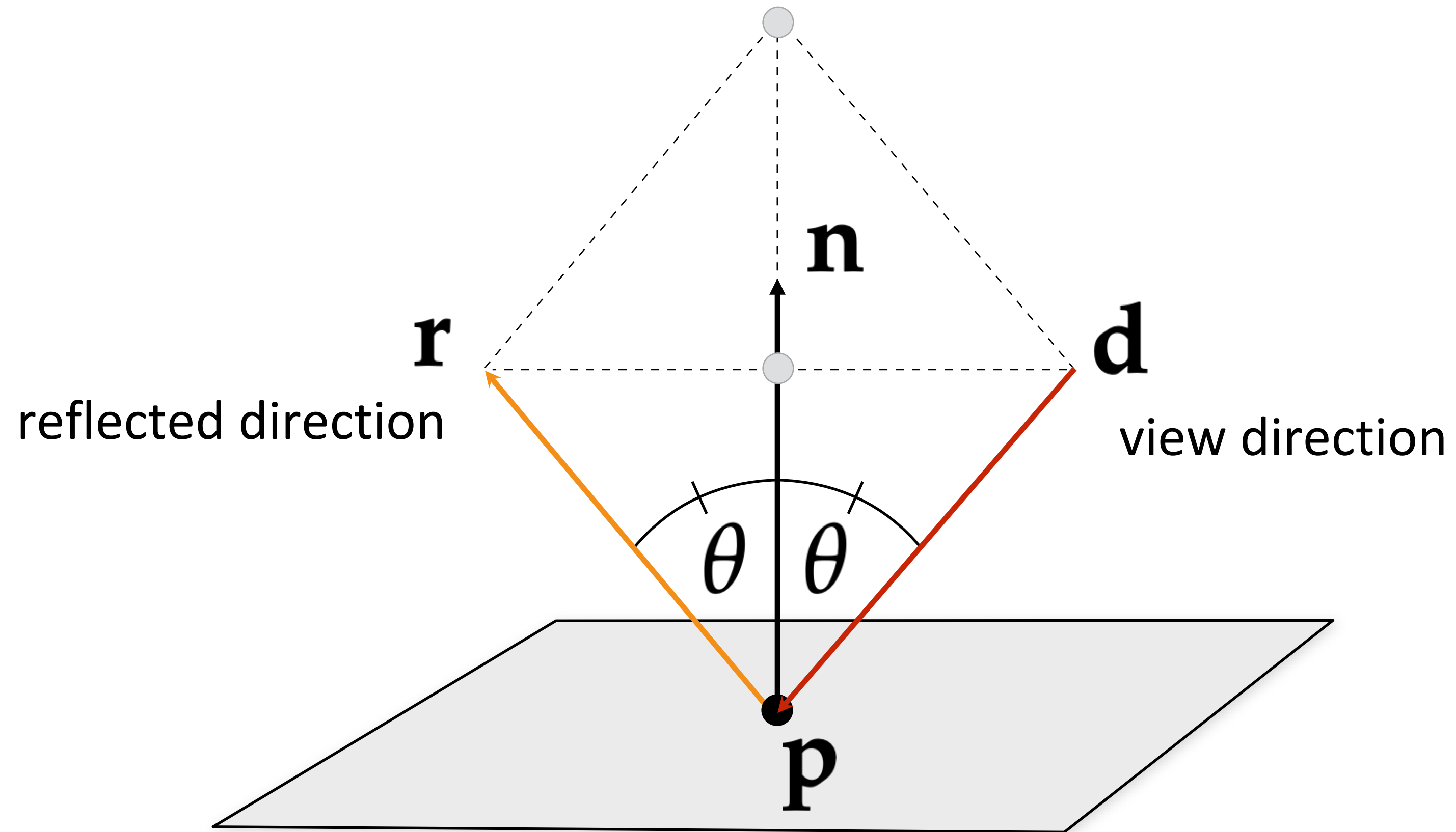
- co-planar view direction, reflected direction, and normal direction
- equal angles between normal-view directions, and normal-reflected directions

Mirror reflection

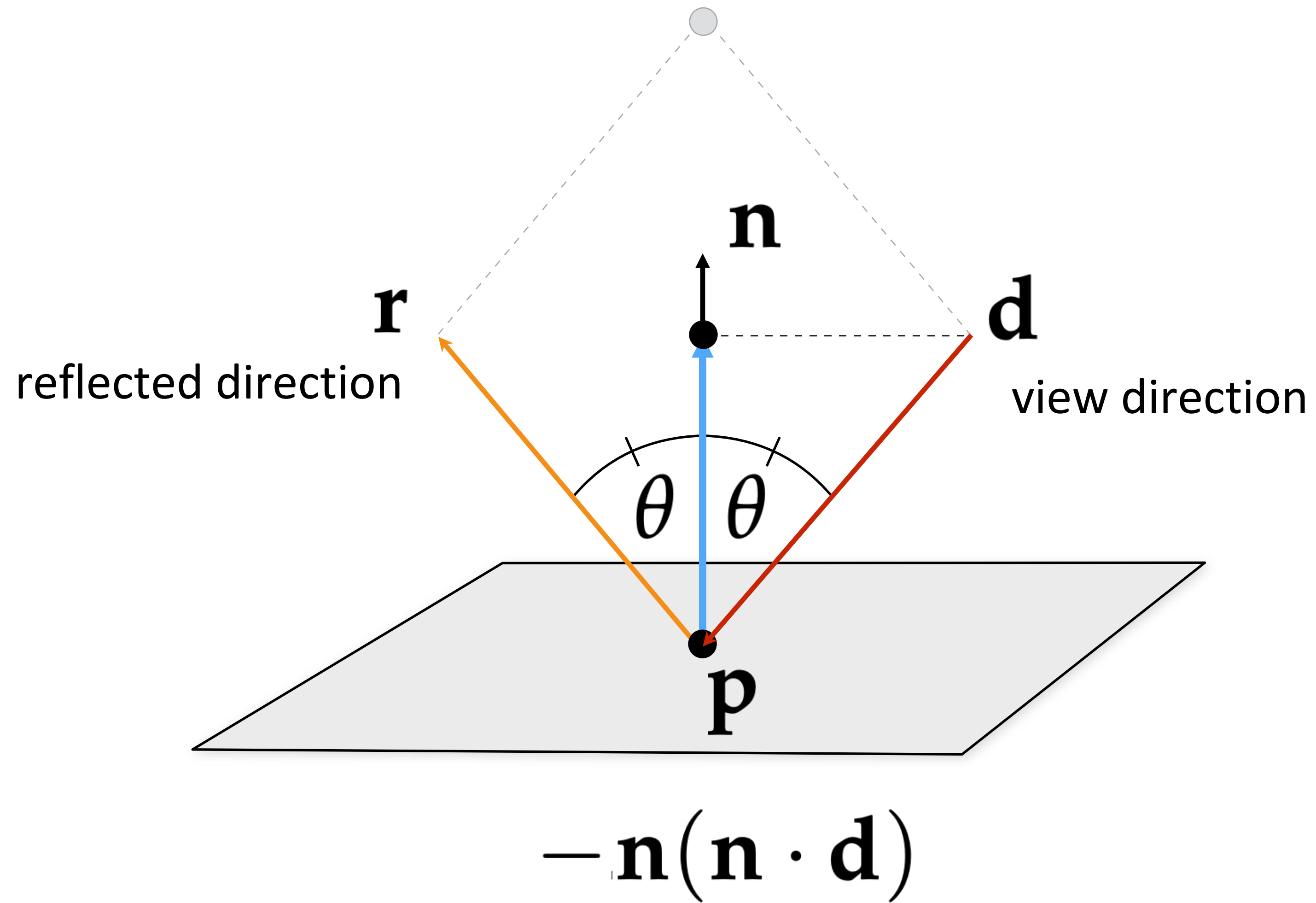
Assume $\mathbf{n}$ is unit length



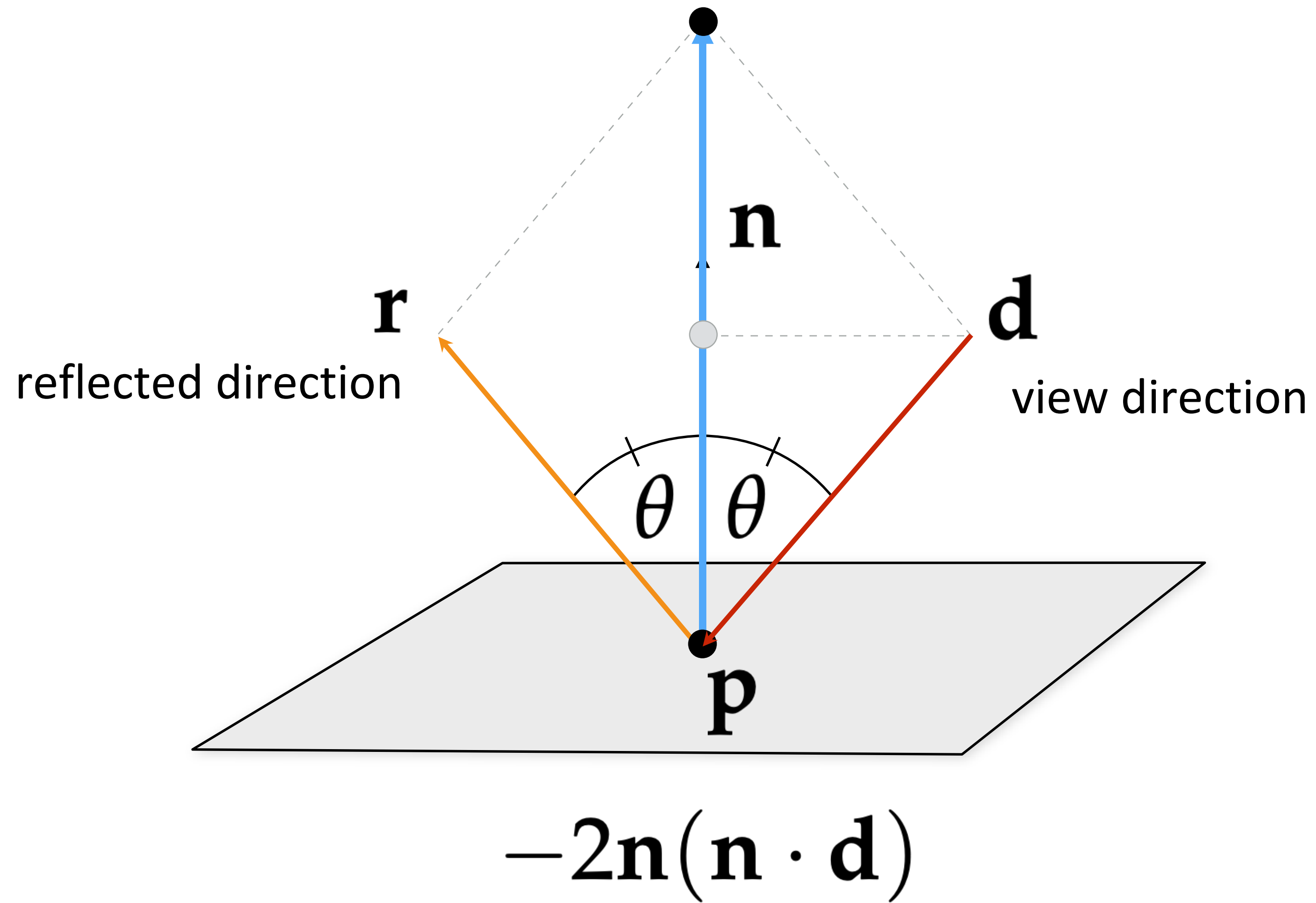
Mirror reflection



Mirror reflection

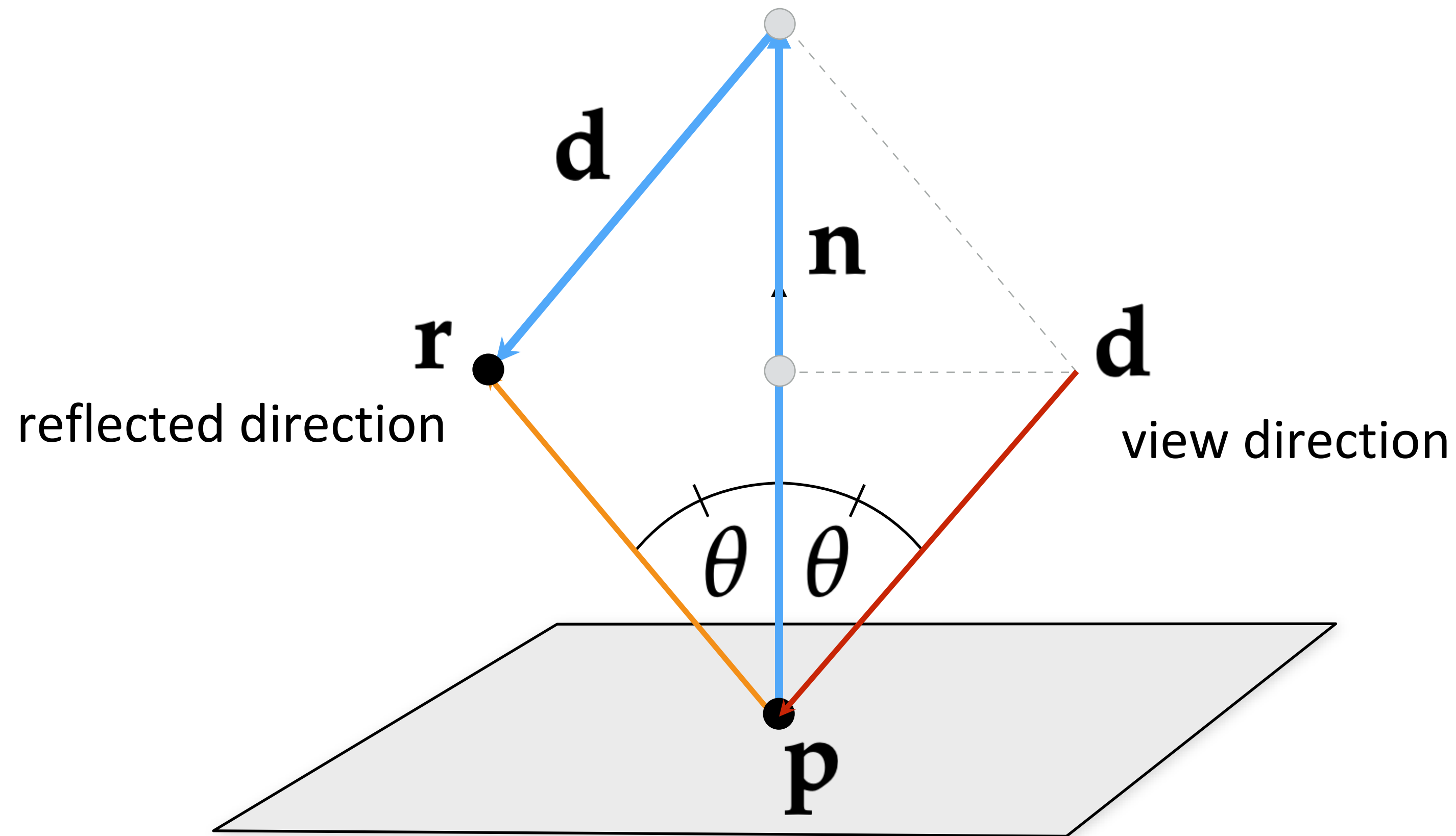


Mirror reflection



Mirror reflection

Assumes $\mathbf{n}$ is unit length



$$\mathbf{r} = -2\mathbf{n}(\mathbf{n} \cdot \mathbf{d}) + \mathbf{d}$$

Specular BRDF?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

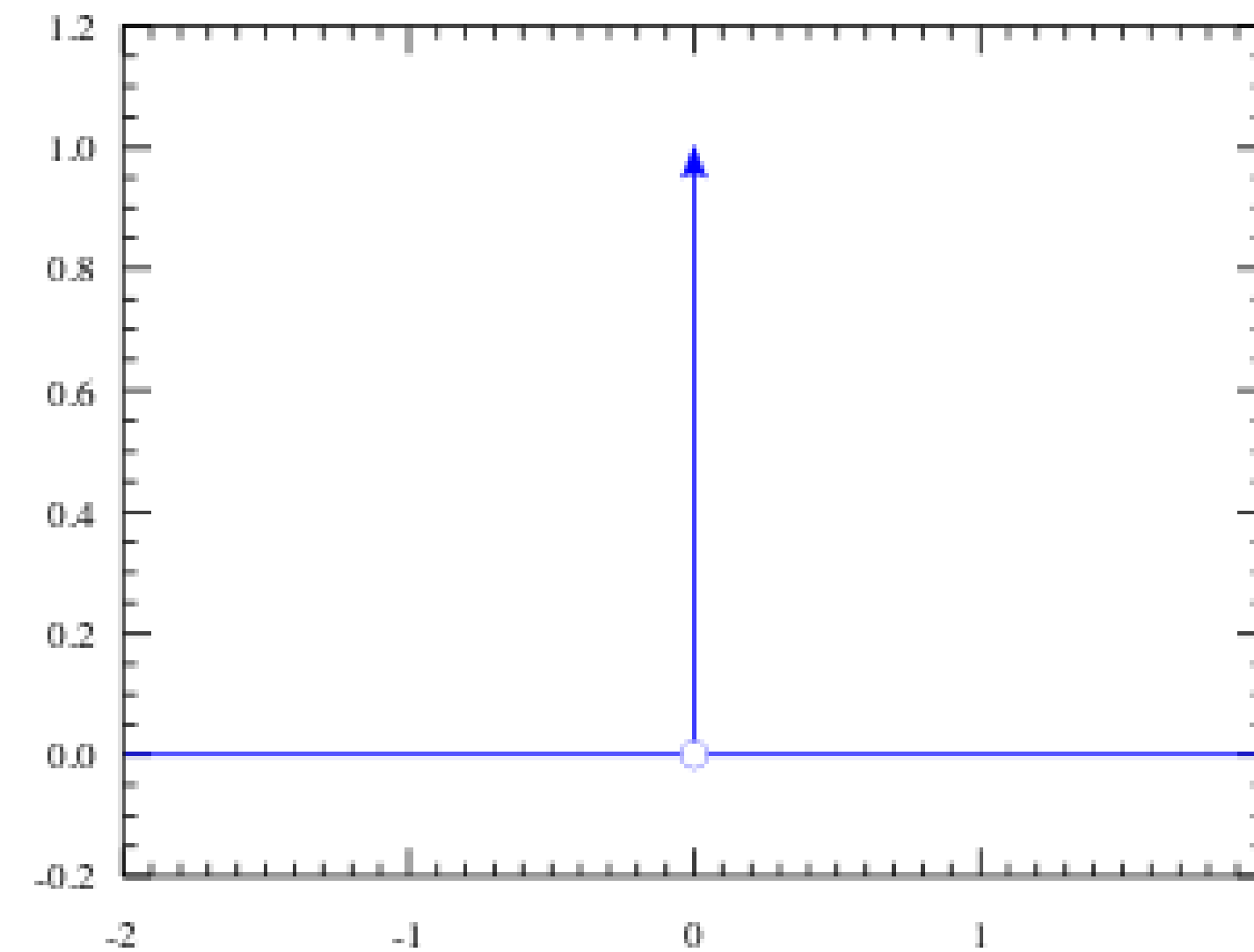
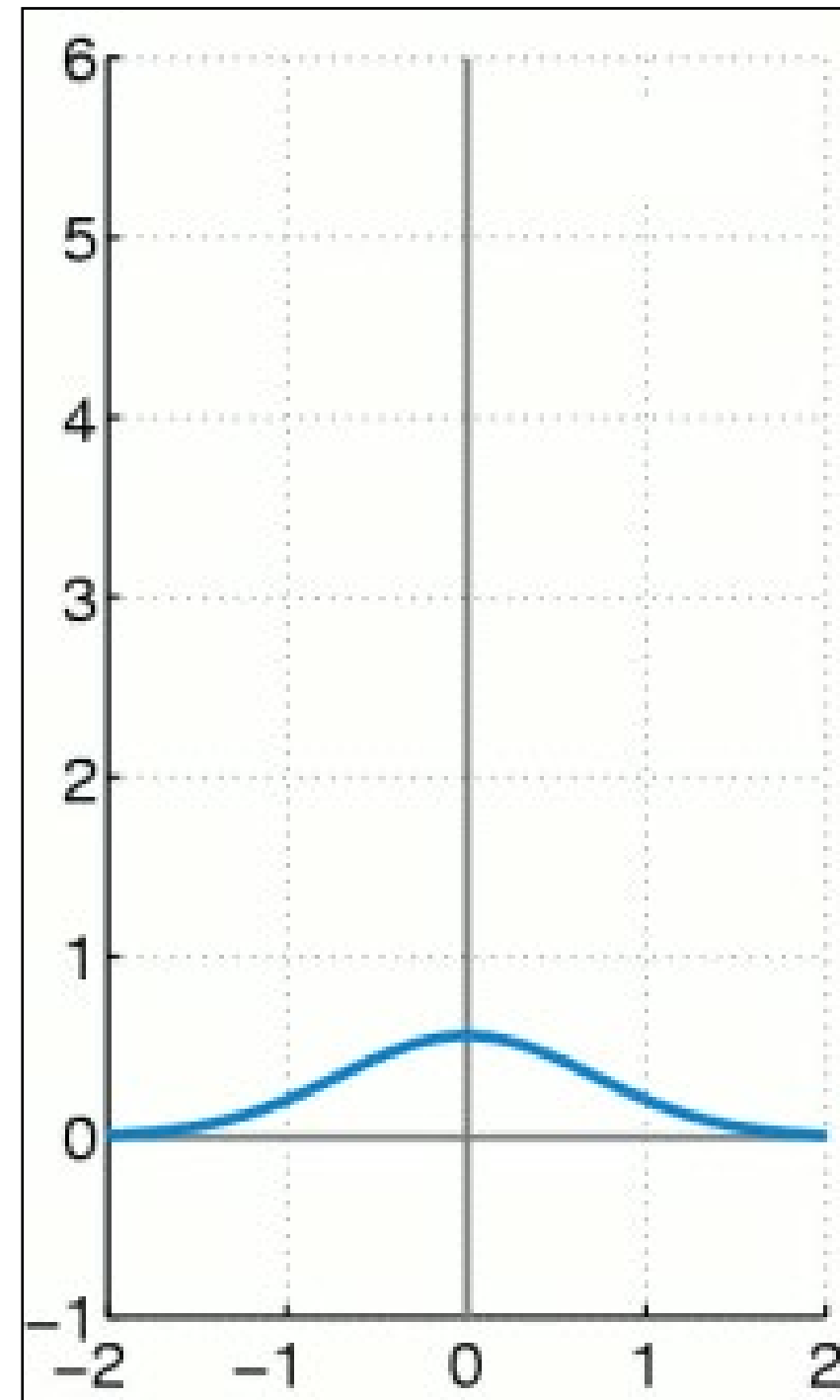
Scatters all light into one (or two) directions

Contains a Dirac delta

Integral drops out

What is the BRDF for specular reflection/refraction?

Dirac delta functions



$$\int_{-\infty}^{\infty} f(x) \delta(x - a) \, dx = f(a)$$

Note: careful when performing changes of variables in Dirac delta functions!

BRDF of Ideal Specular Reflection

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i$$

What is the BRDF for specular reflection?

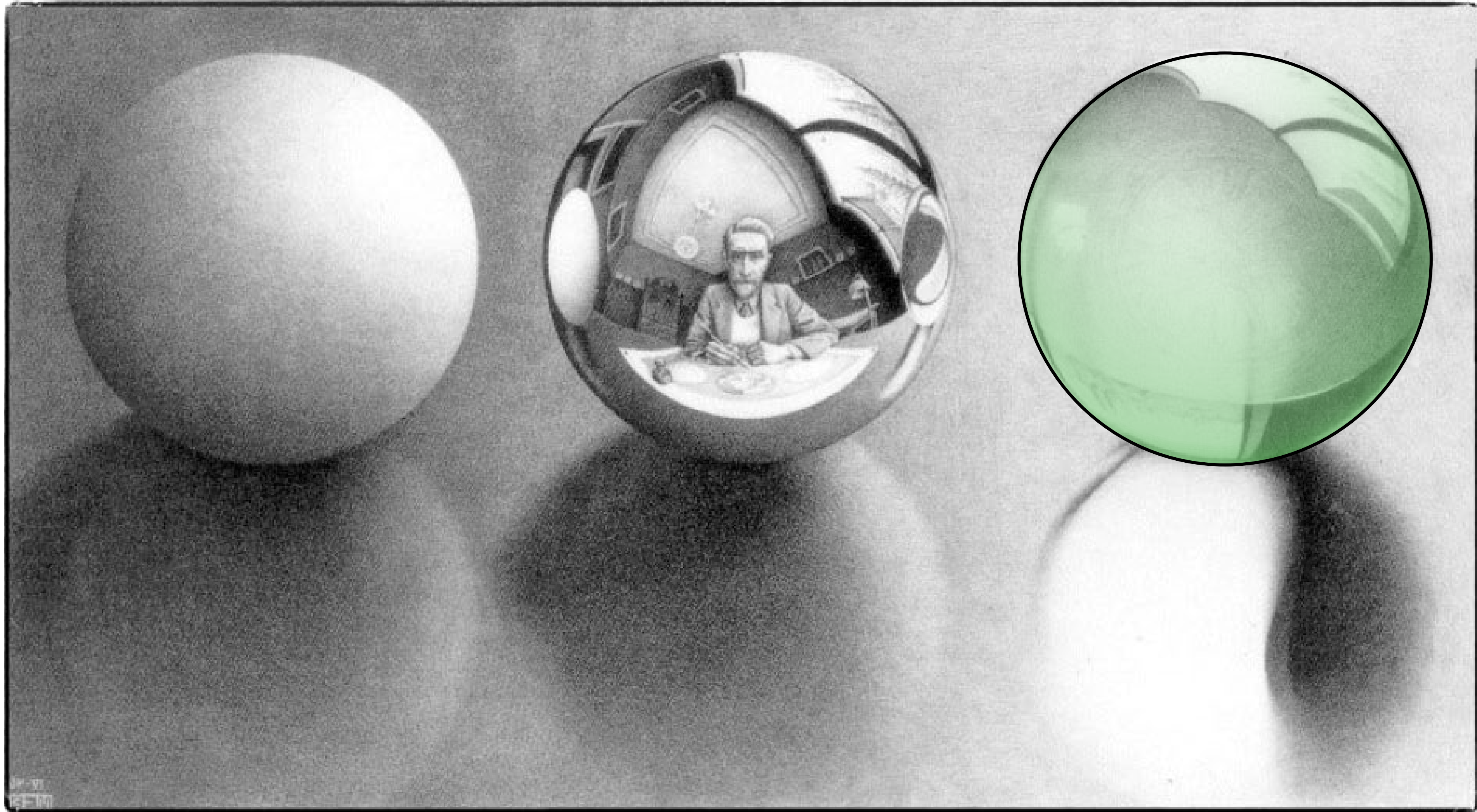
Diagram illustrating the BRDF for ideal specular reflection:

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_i - R(\vec{\omega}_r, \vec{\mathbf{n}}))}{\cos \theta_i}$$

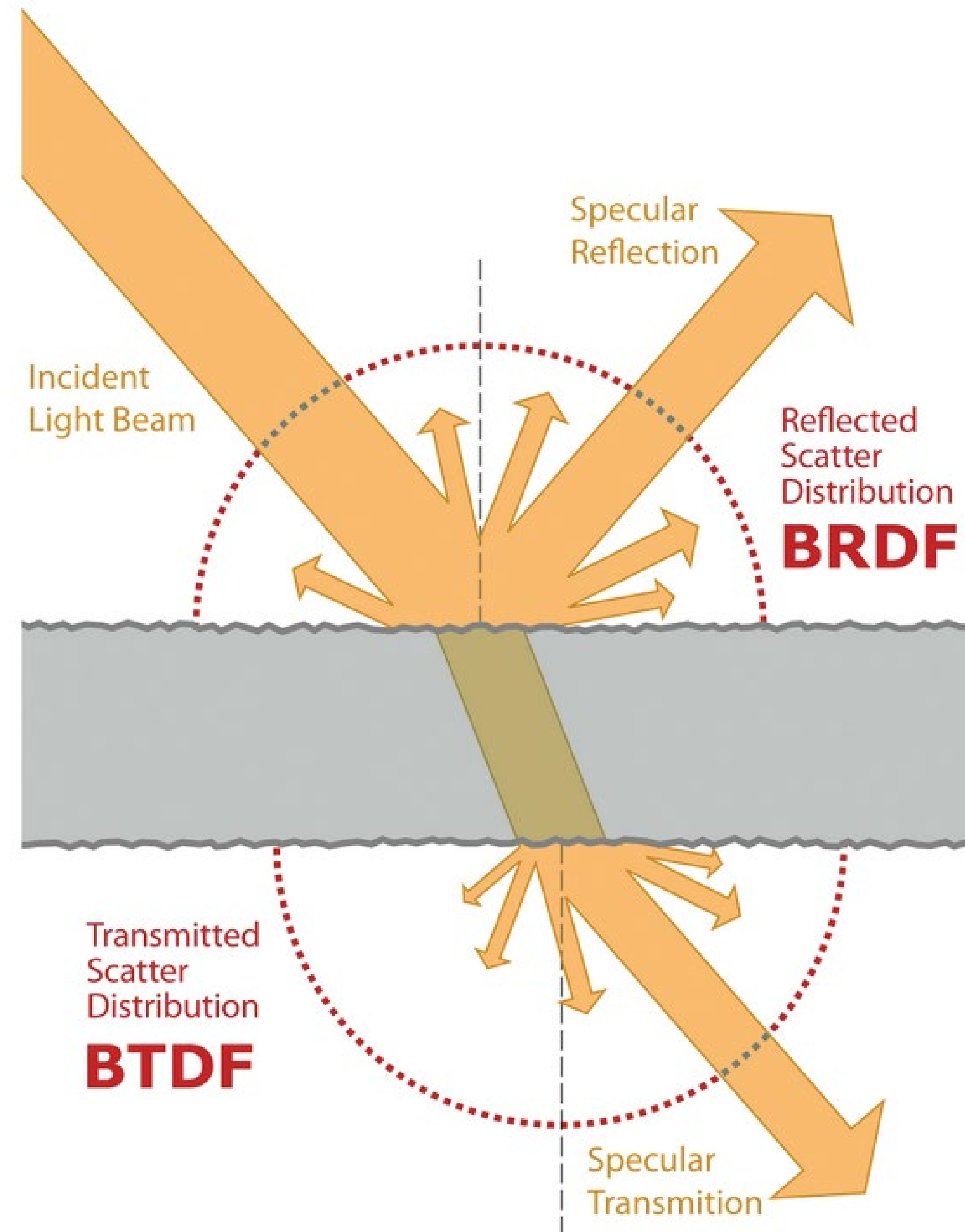
Annotations:

- Fresnel reflection (points to $F_r(\vec{\omega}_i)$)
- Dirac delta (points to $\delta(\vec{\omega}_i - R(\vec{\omega}_r, \vec{\mathbf{n}}))$)
- Reflection function (flips about normal) (points to $R(\vec{\omega}_r, \vec{\mathbf{n}})$)
- to cancel the cosine term in the reflection equation (Fresnel eqs. account for it) (points to $\cos \theta_i$)

Specular refraction



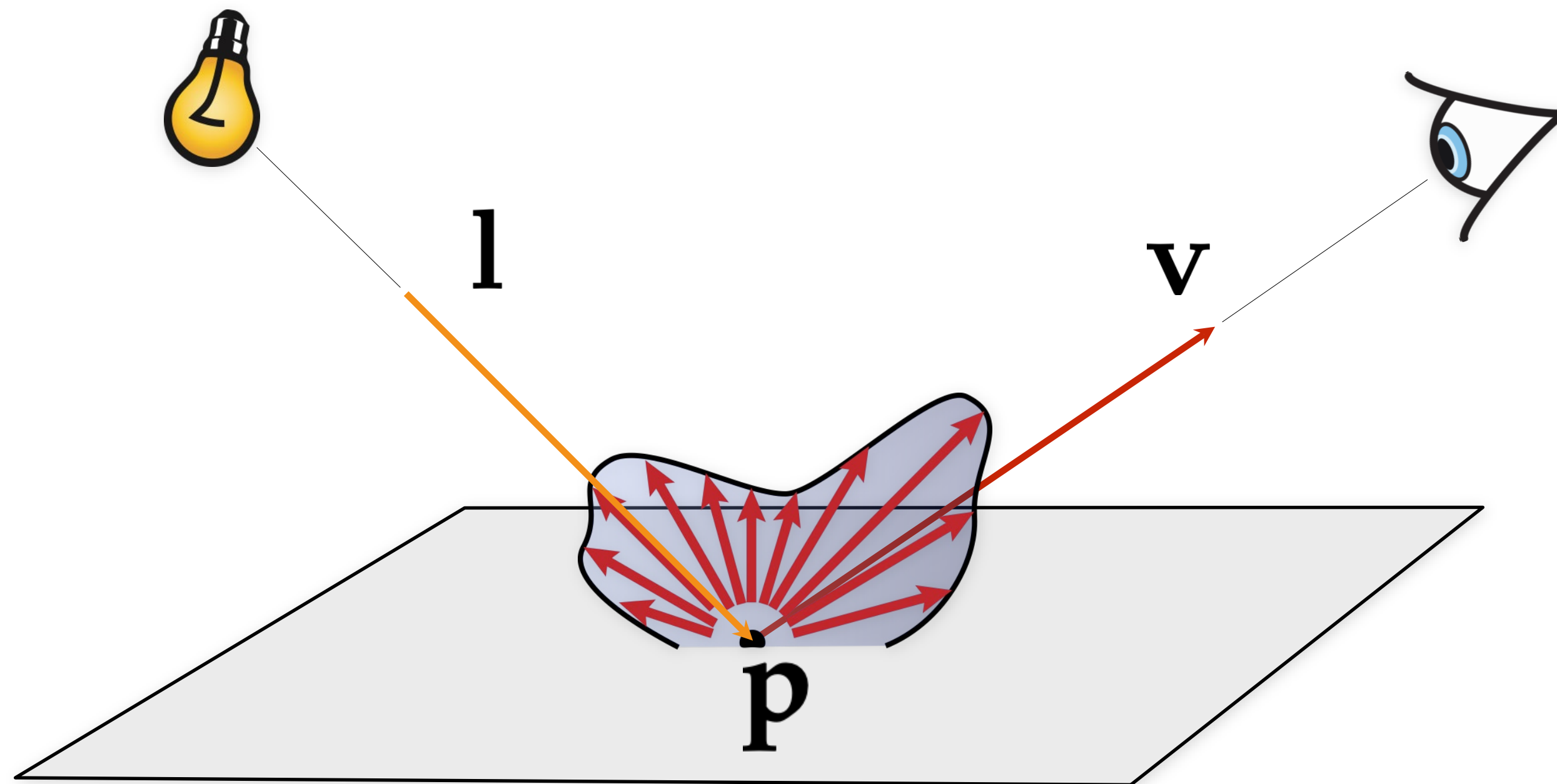
Reflection vs. Refraction



The BSDF

Bidirectional Scattering Distribution Function

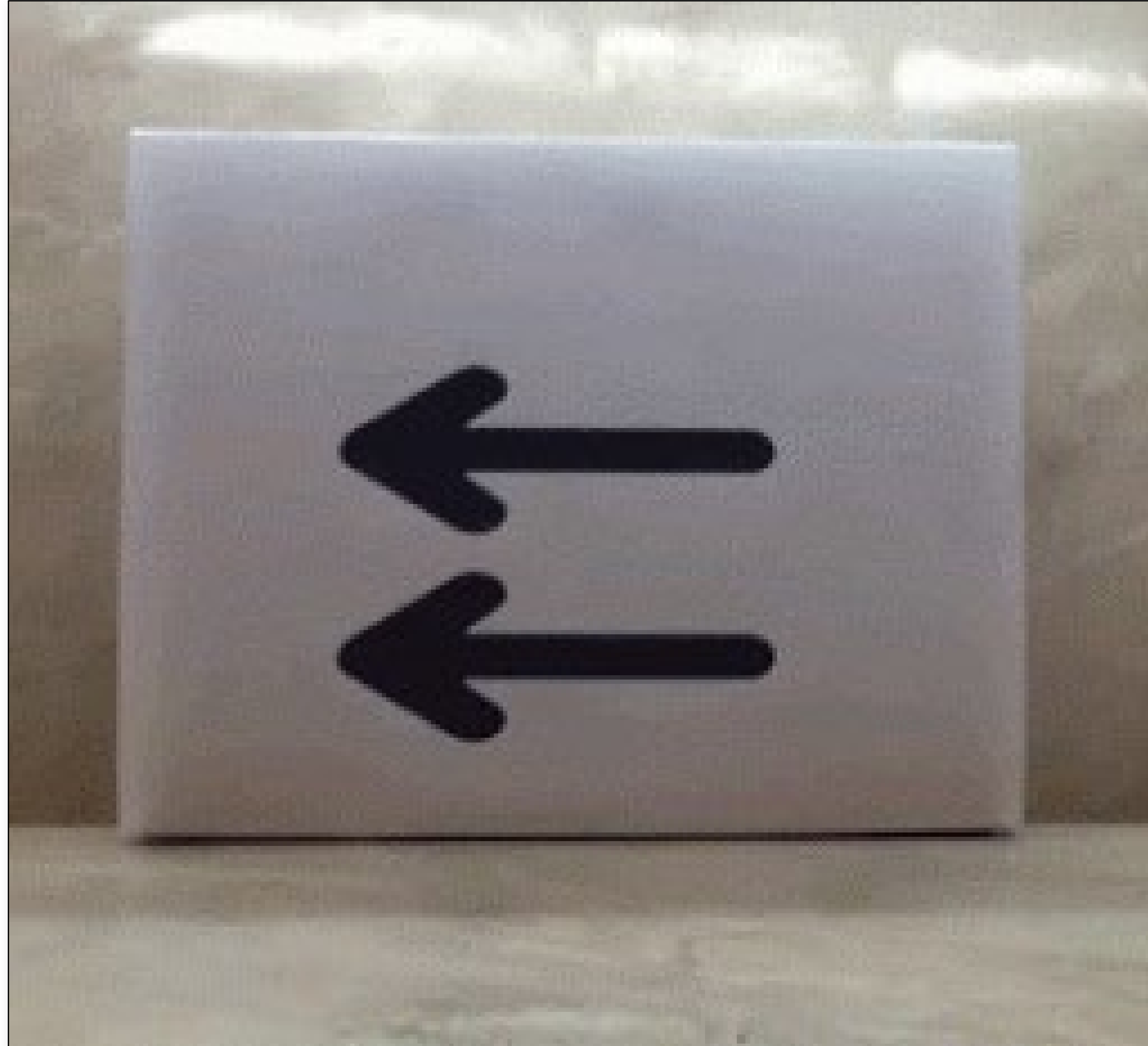
- informally: how much the material scatters light coming from one direction $\mathbf{l}$ into some other direction $\mathbf{v}$, at each point $\mathbf{p}$



Refraction



Refraction



Index of Refraction

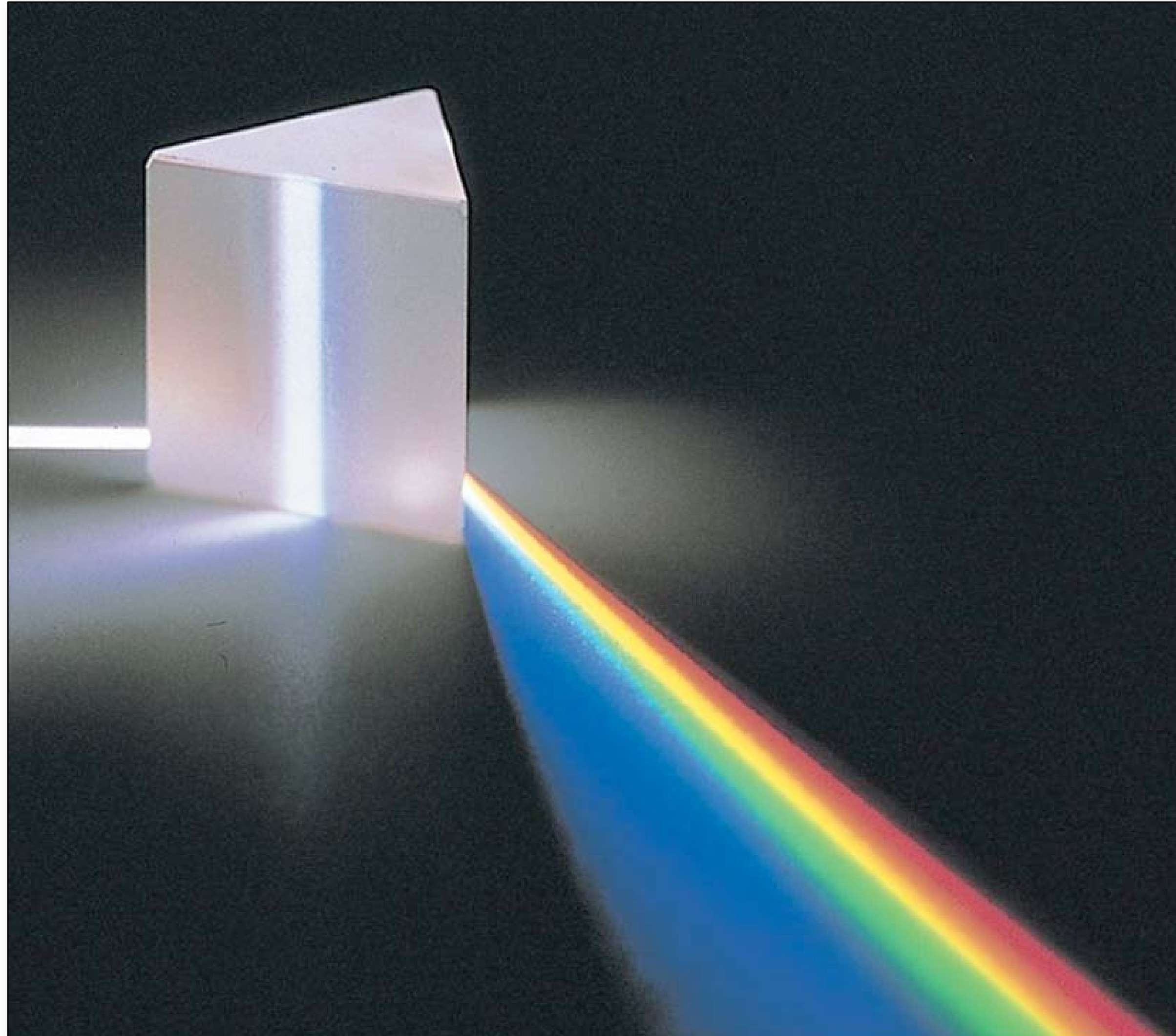
Speed of light in vacuum / speed of light in medium

Some values of	
Vacuum	1
Air at STP	1.00029
Ice	1.31
Water	1.33
Crown glass	1.52 - 1.65
Diamond	2.417

η

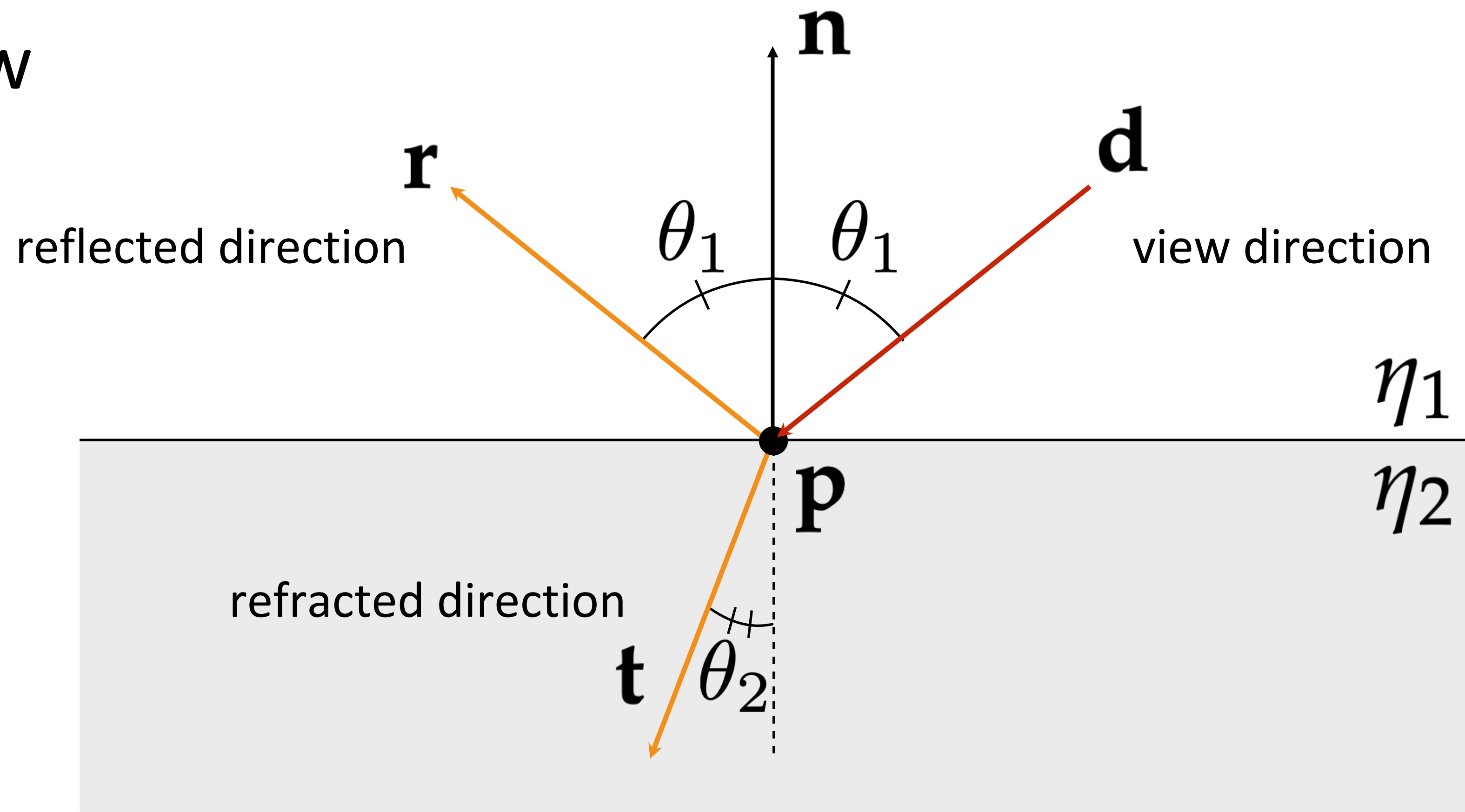
These are actually wavelength dependent!

Dispersion



Specular transmission/refraction

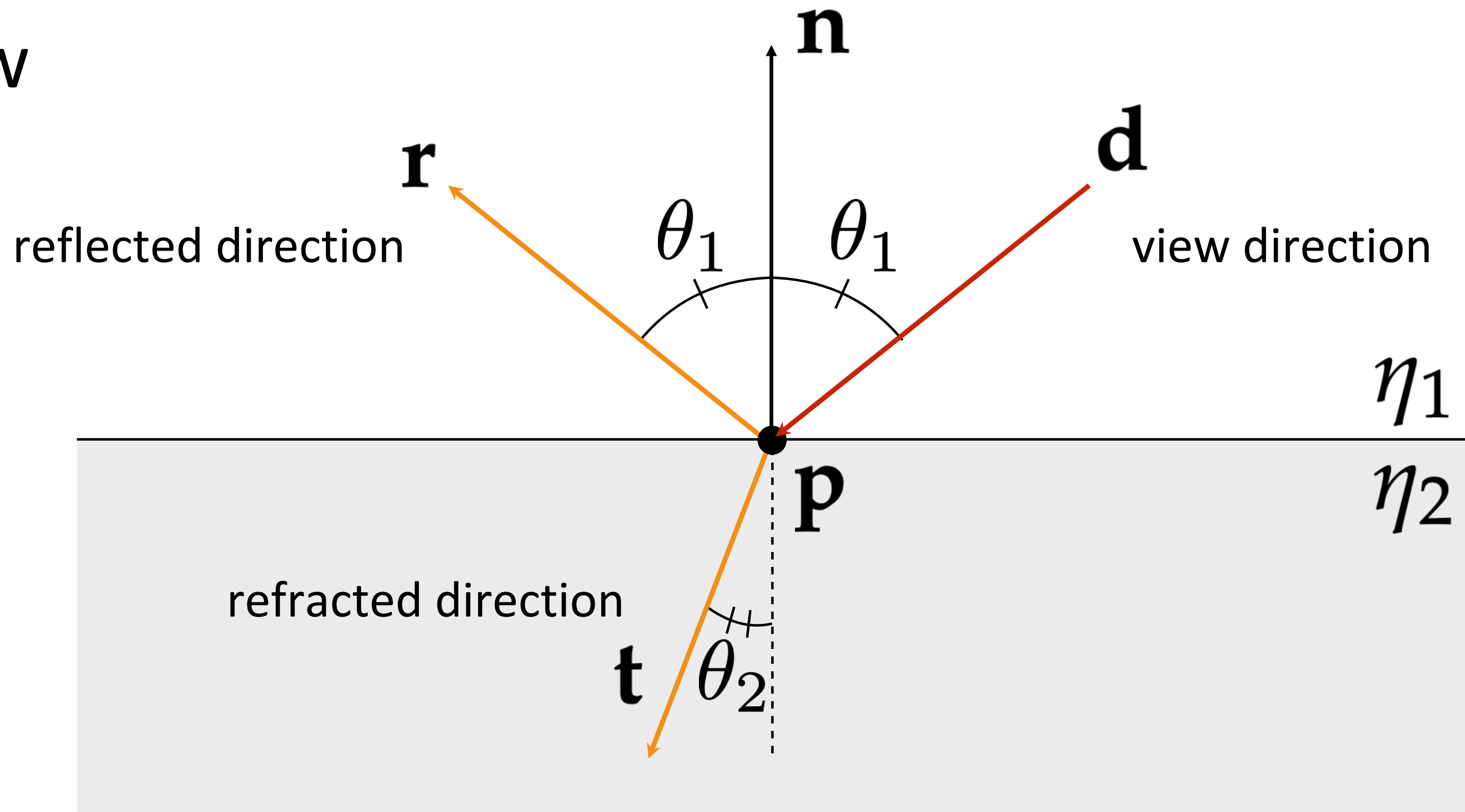
Snell's law



$$\eta_1 \sin \theta_1 = \eta_2 \sin \theta_2$$

Specular transmission/refraction

Snell's law



$$\mathbf{t} = \eta_1/\eta_2 (\mathbf{d} - (\mathbf{d} \cdot \mathbf{n}) \mathbf{n}) - \mathbf{n} \sqrt{1 - \eta_1^2/\eta_2^2 (1 - (\mathbf{d} \cdot \mathbf{n})^2)}$$

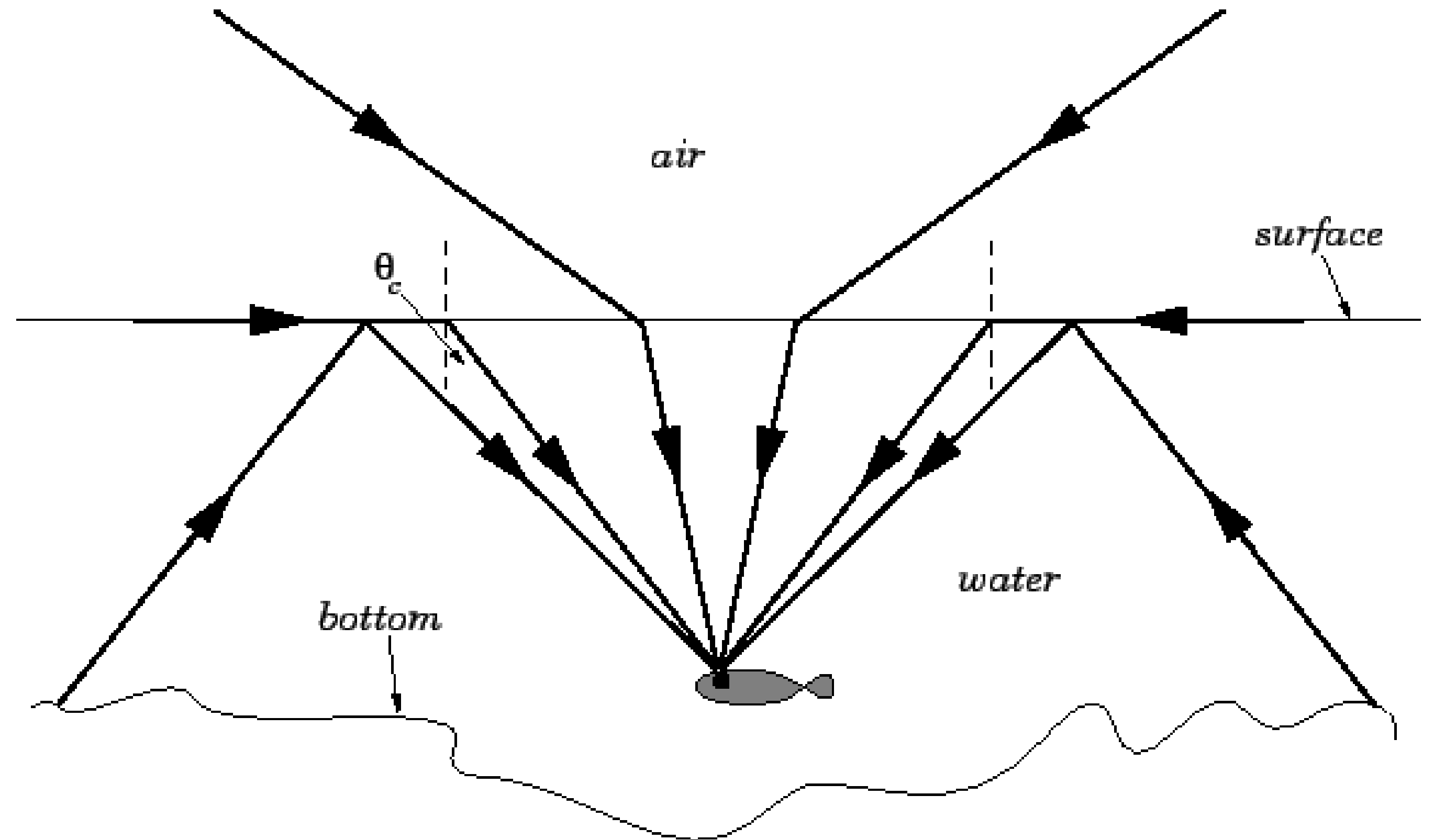
What is this dark circle?



What is this dark circle?

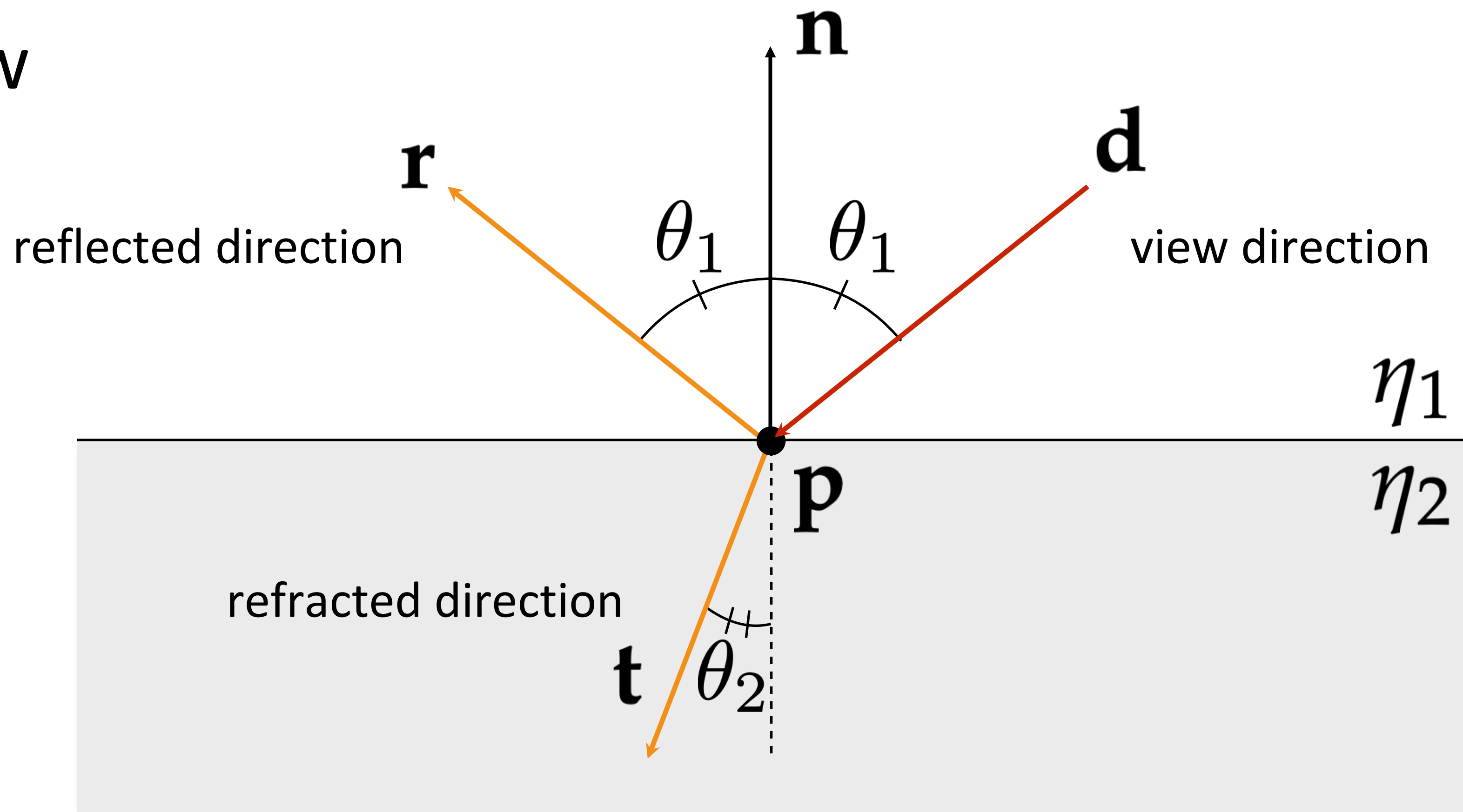


Called
“Snell's window”
Caused by total
internal reflection



Recall...

Snell's law



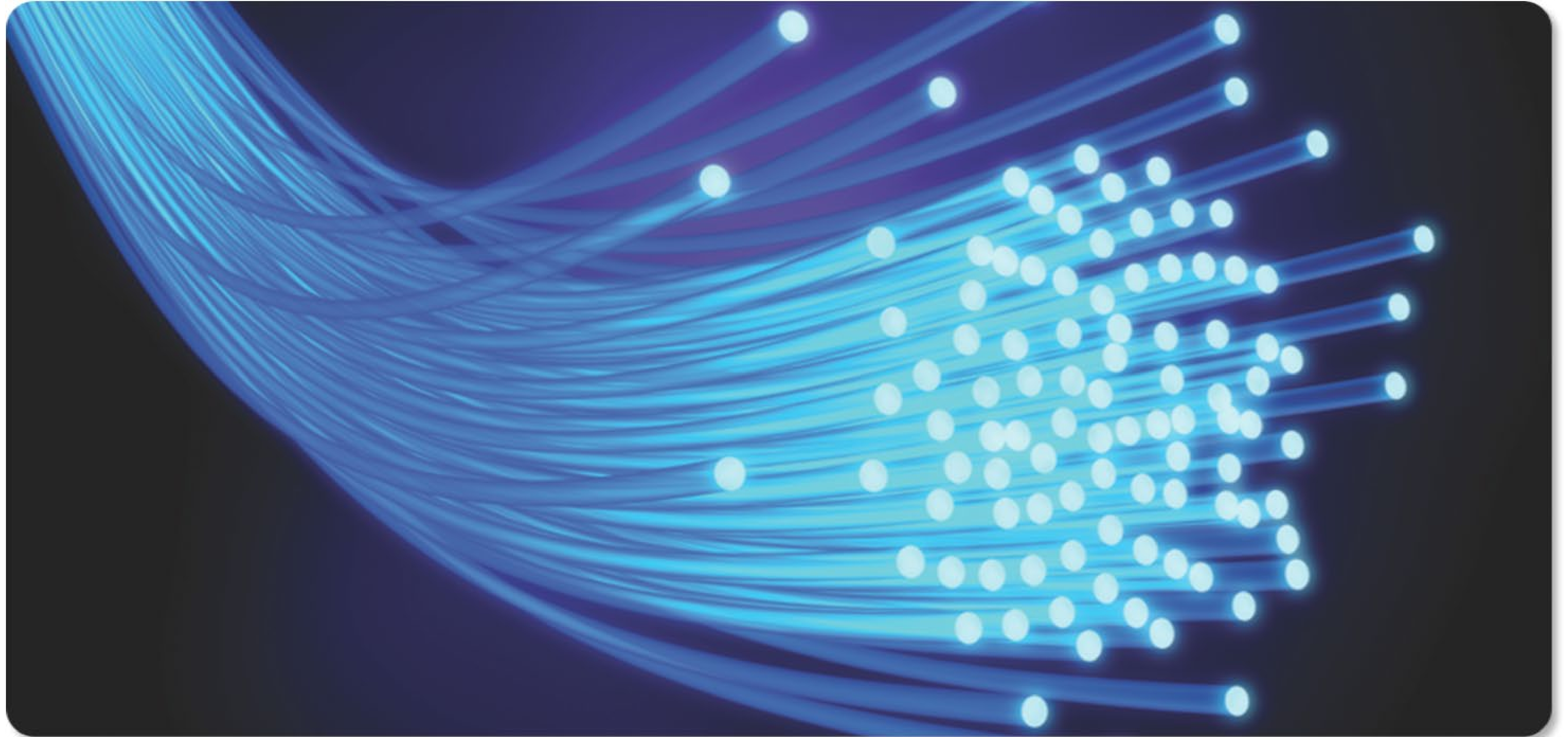
$$\mathbf{t} = \eta_1/\eta_2 (\mathbf{d} - (\mathbf{d} \cdot \mathbf{n}) \mathbf{n}) - \mathbf{n} \sqrt{1 - \eta_1^2/\eta_2^2 (1 - (\mathbf{d} \cdot \mathbf{n})^2)}$$

Can only happen when the ray starts in the higher index medium

Total Internal Reflection



Total Internal Reflection



Total Internal Reflection



BTDF of Ideal Specular Refraction

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i$$

What is the BTDF for specular refraction?

The diagram shows the BTDF equation for ideal specular refraction with several annotations:

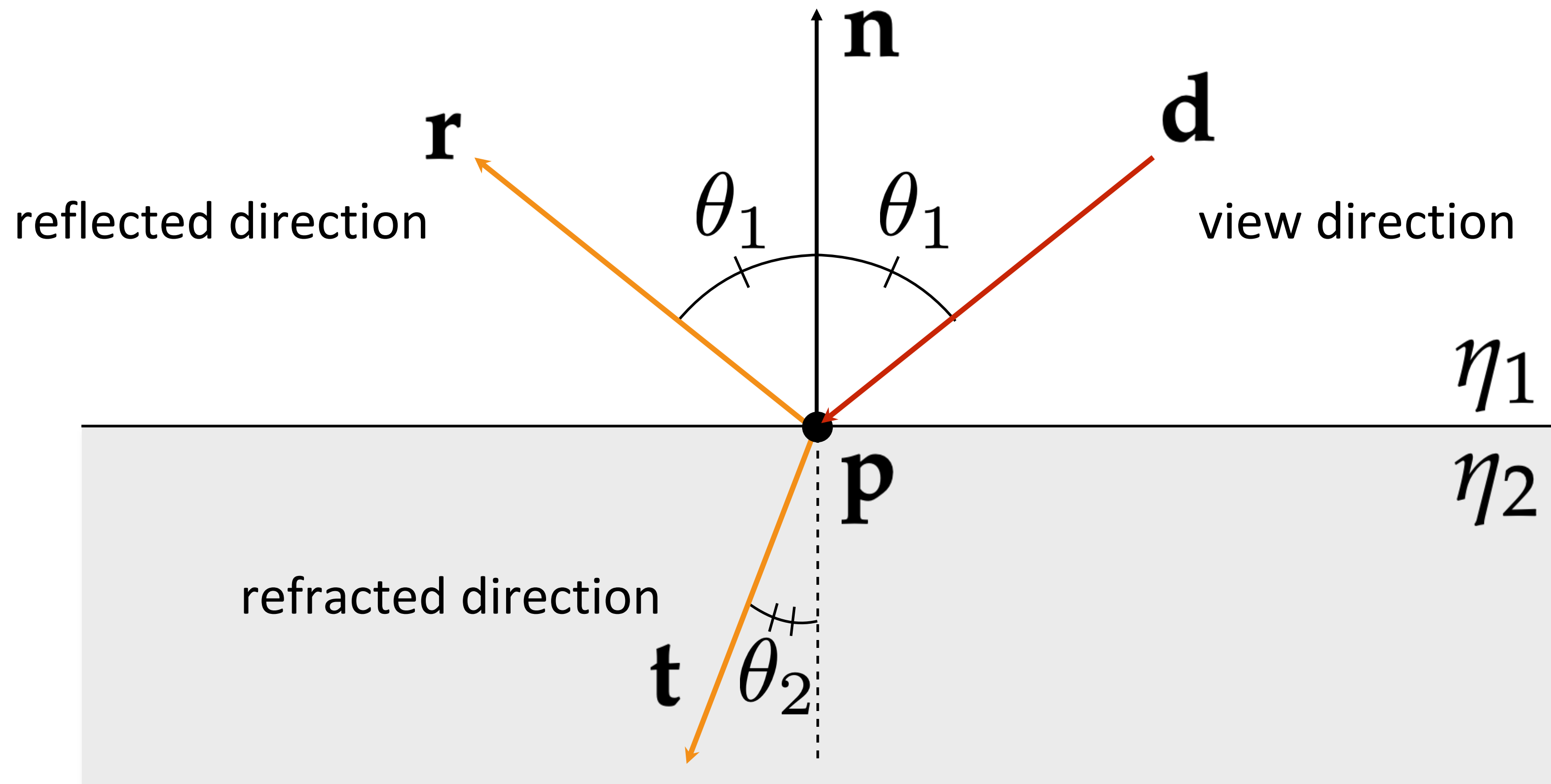
- Fresnel reflection**: An arrow points to the term $F_r(\vec{\omega}_i)$ in the equation.
- Dirac delta**: An arrow points to the $\delta(\vec{\omega}_i - T(\vec{\omega}_r, \vec{\mathbf{n}}))$ term.
- Refraction function**: An arrow points to the $T(\vec{\omega}_r, \vec{\mathbf{n}})$ term.
- to cancel the cosine term in the reflection equation (Fresnel eqs. account for it)**: An arrow points to the $\cos \theta_i$ term in the denominator.

$$f_t(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \boxed{\frac{\eta_1^2}{\eta_2^2}} (1 - F_r(\vec{\omega}_i)) \frac{\delta(\vec{\omega}_i - T(\vec{\omega}_r, \vec{\mathbf{n}}))}{\cos \theta_i}$$

Reflection vs. Refraction

How much light is reflected vs. refracted?

- in reality determined by “Fresnel equations”



Fresnel Equations

Reflection and refraction from smooth *dielectric* (e.g. glass) surfaces

Reflection from *conducting* (e.g. metal) surfaces

Derived from Maxwell equations

Involves polarization of the wave

Fresnel Equations for Dielectrics

Reflection of light polarized parallel and perpendicular to the plane of refraction

$$\rho_{||} = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2}$$

$$\rho_{\perp} = \frac{\eta_1 \cos \theta_1 - \eta_2 \cos \theta_2}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2}$$

$$\text{reflected: } F_r = \frac{1}{2} \left(\rho_{||}^2 + \rho_{\perp}^2 \right)$$

$$\text{refracted: } F_t = 1 - F_r$$

What's happening in this photo?



Polarizing Filter



Polarization



Without Polarizer



With Polarizing Filter

Polarization



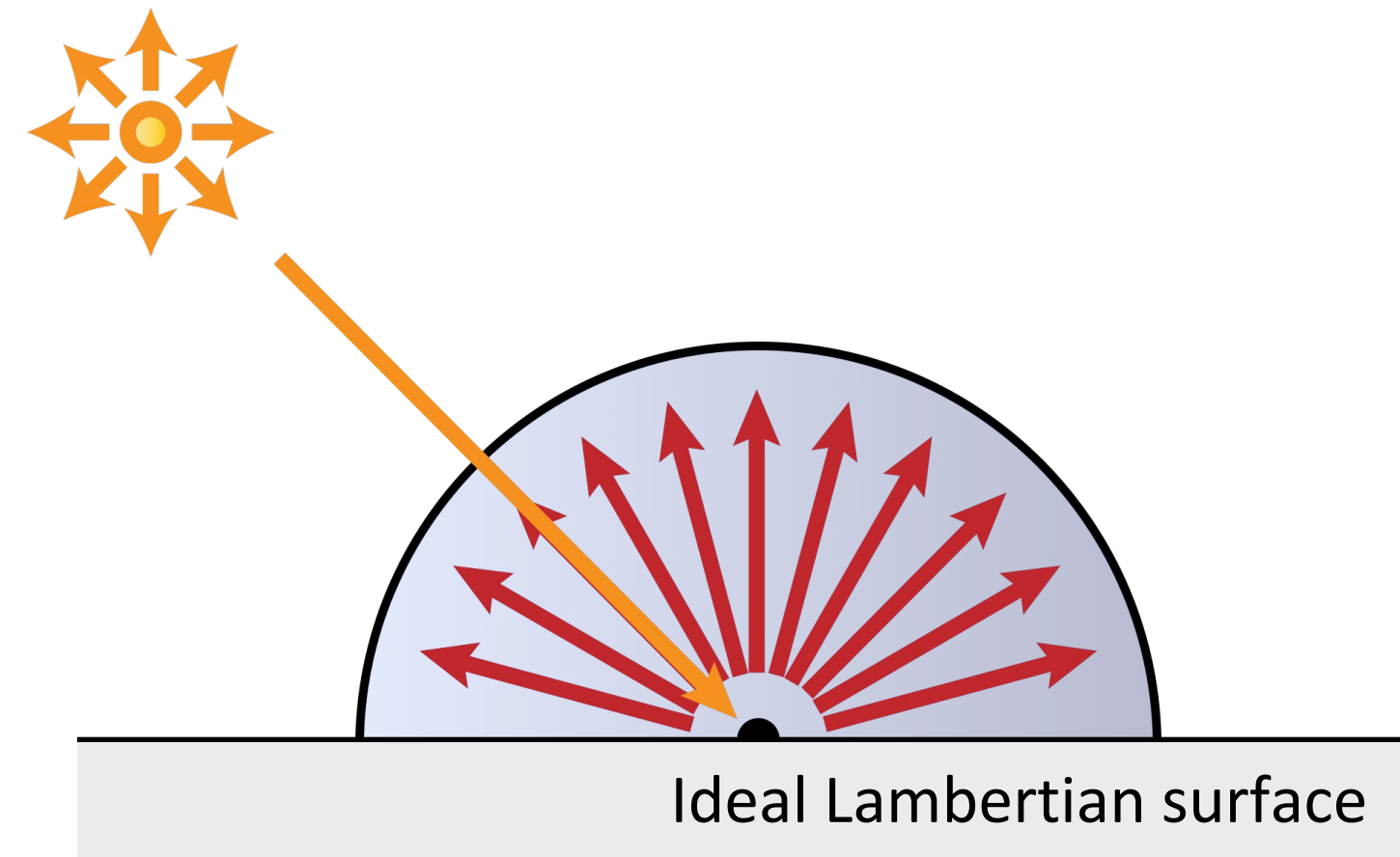
Without Polarizer



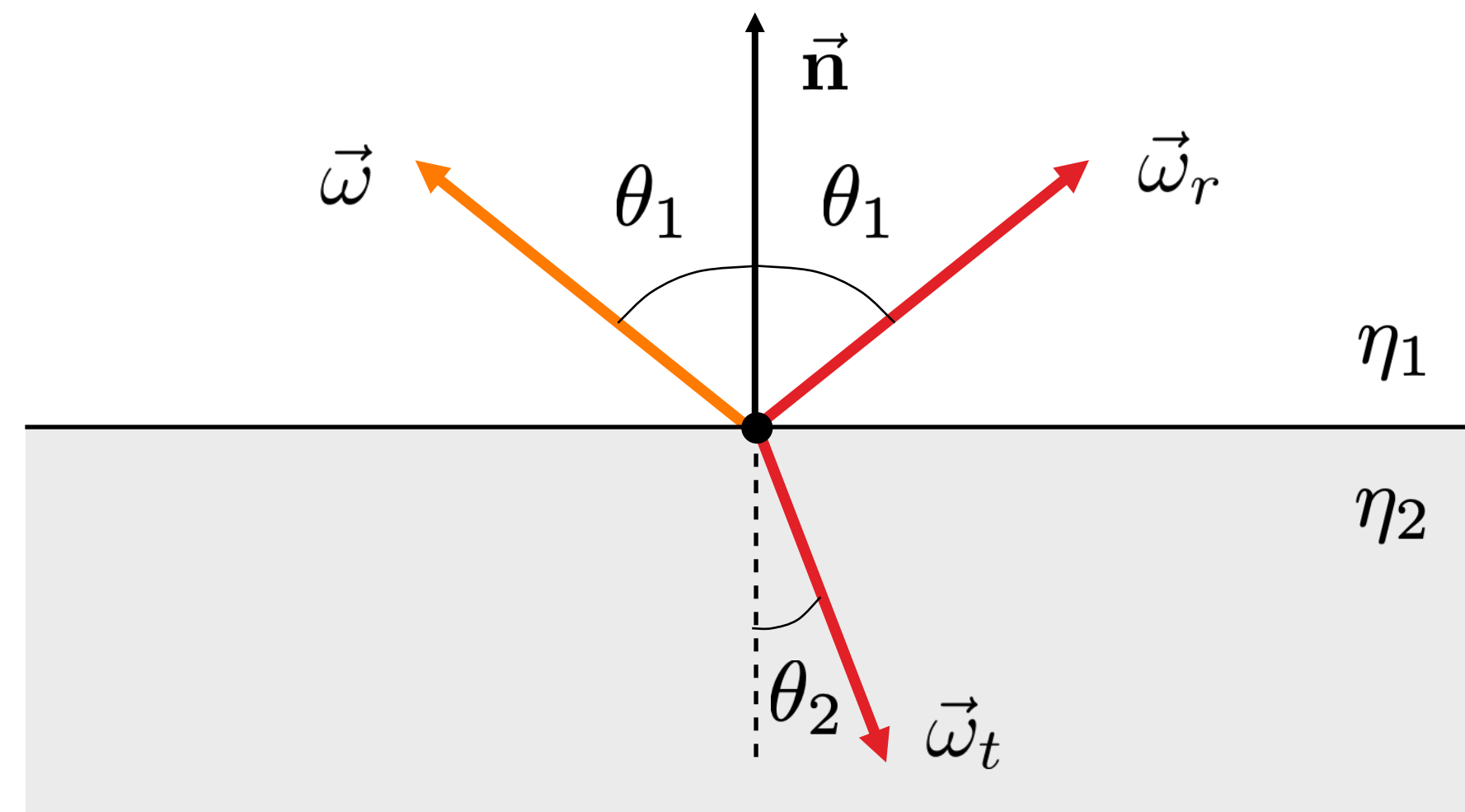
With Polarizing Filter

So Far: Idealized BRDF Models

Diffuse

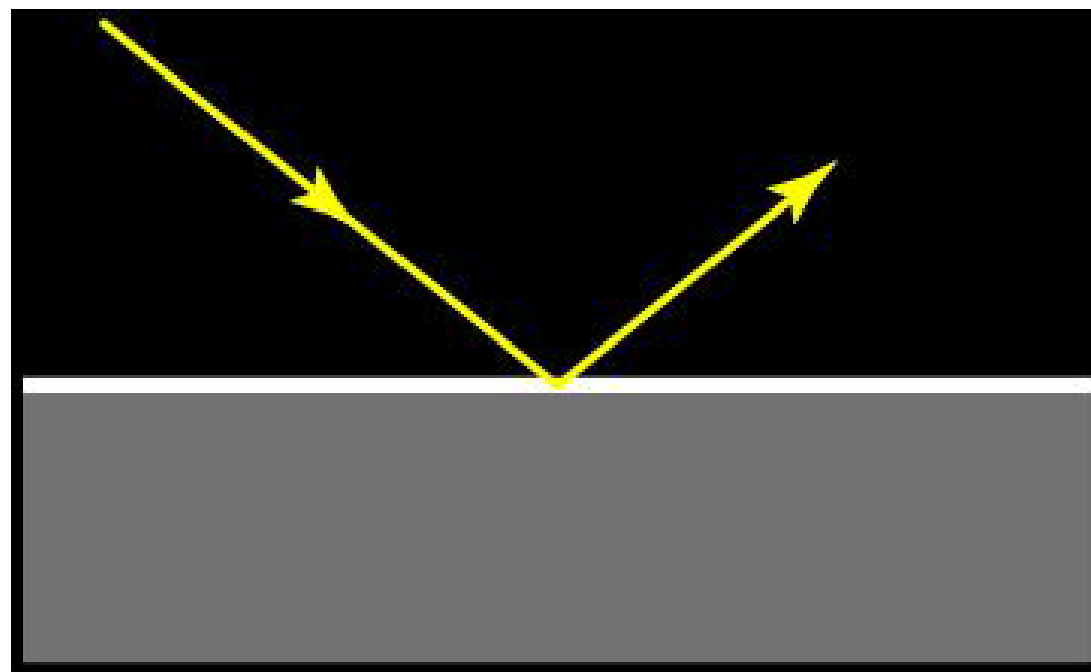


Specular Reflection and Refraction

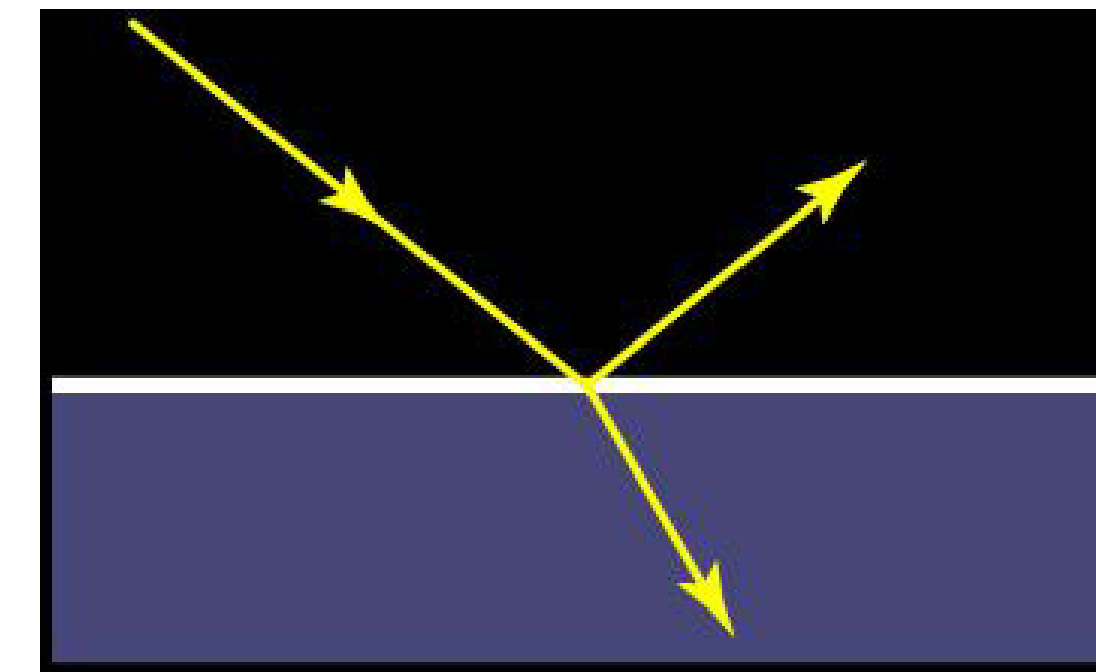


Real-world materials

Metals

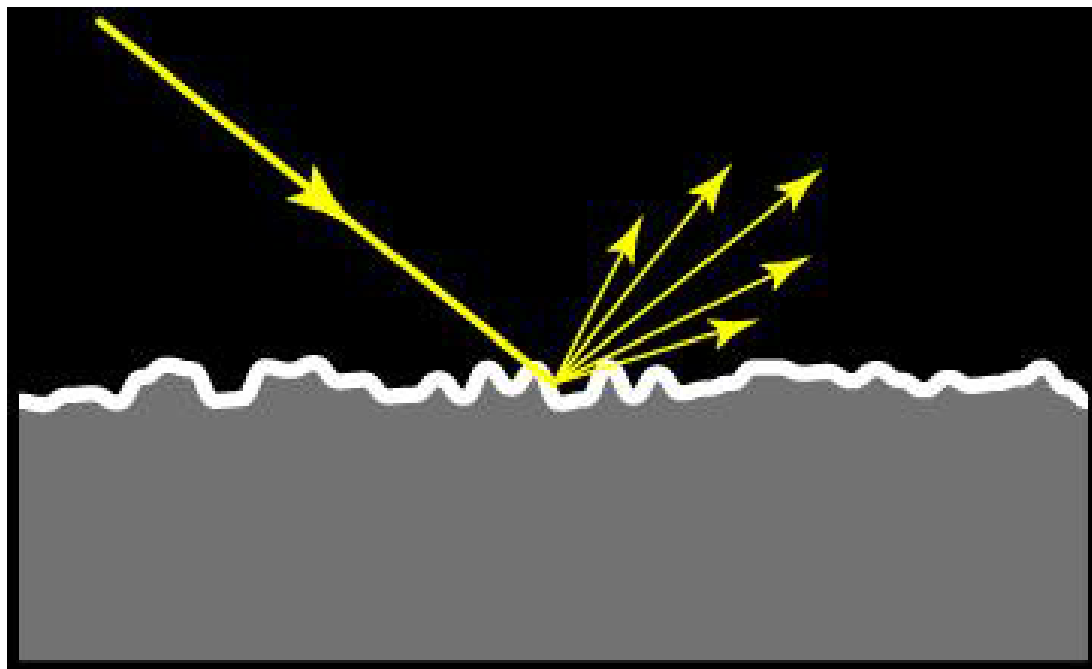


Dielectric

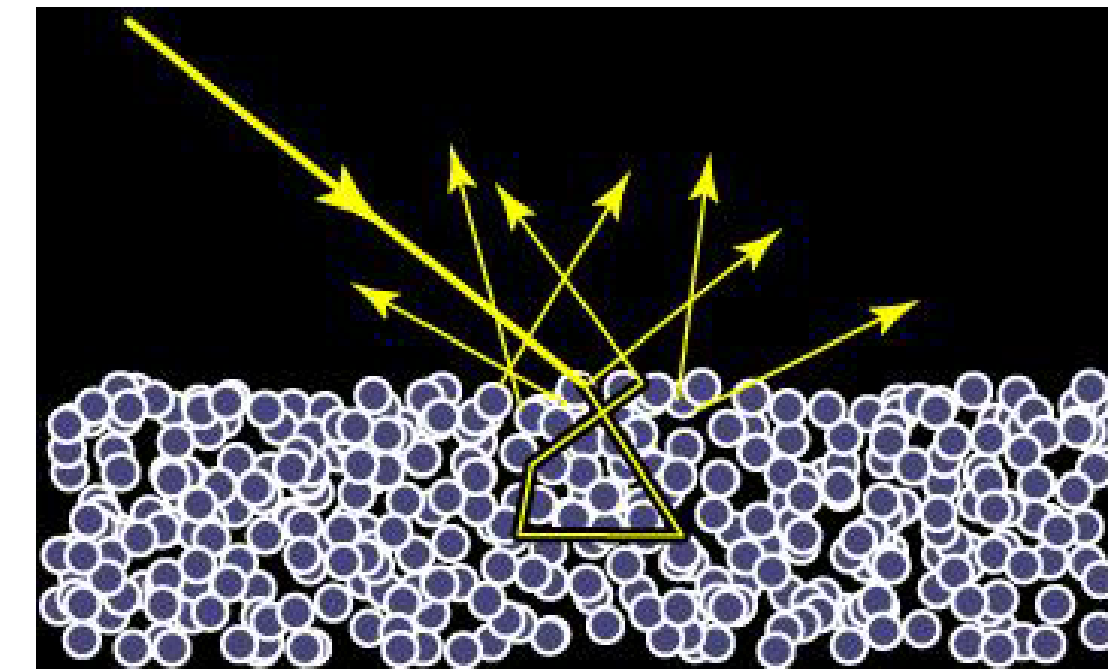


Real-world materials

Metals



Dielectric



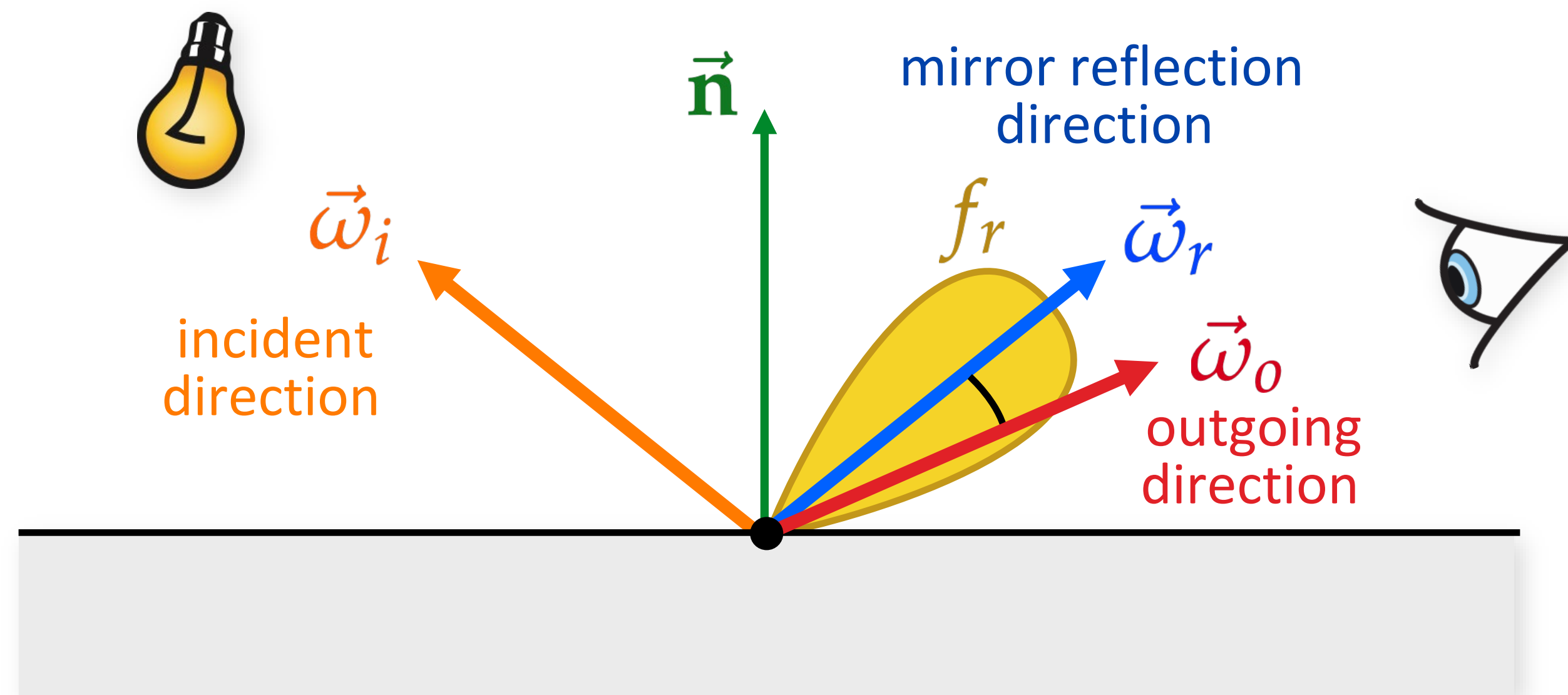
Real materials are more complex

Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_r \cdot \vec{\omega}_o)^e$$

$$\vec{\omega}_r = (2\vec{n}(\vec{n} \cdot \vec{\omega}_i) - \vec{\omega}_i)$$

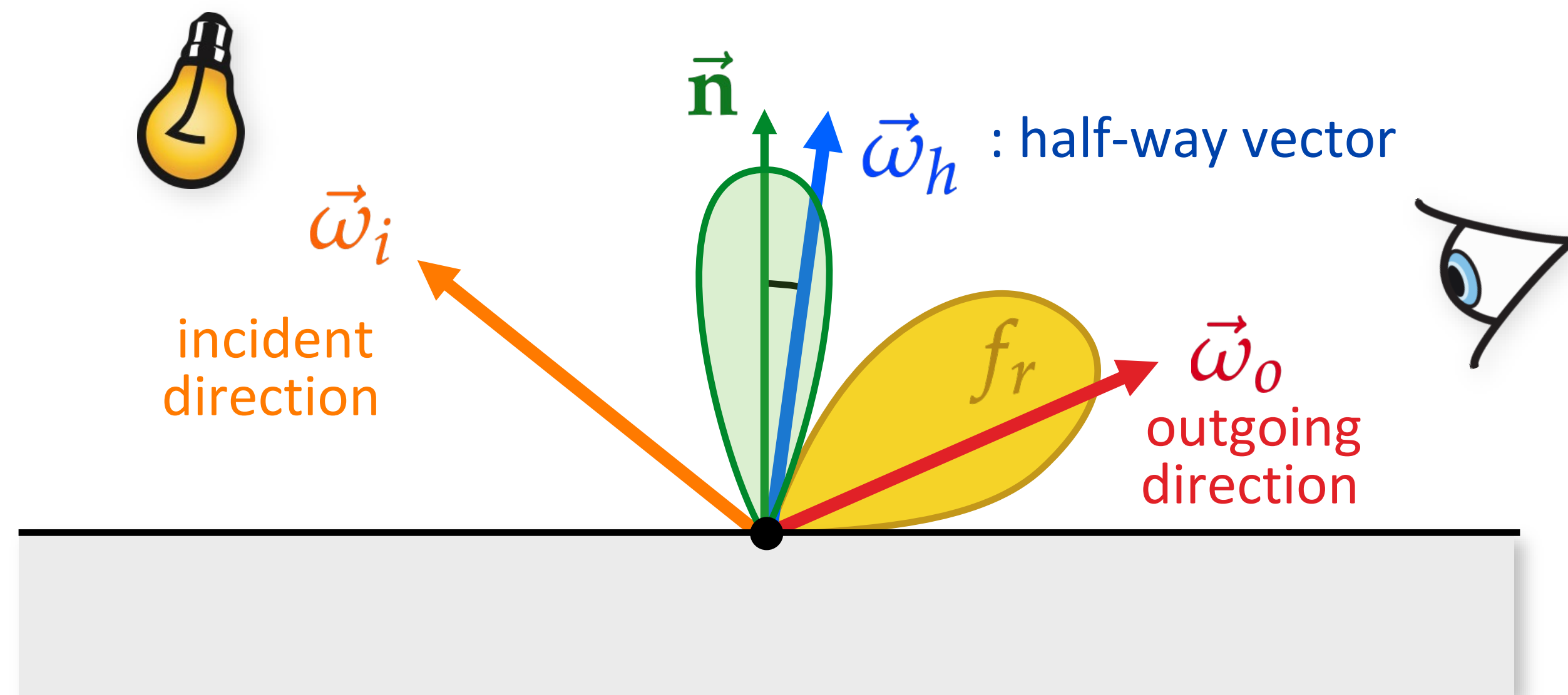


Blinn-Phong BRDF

Distribution of normals instead of reflection directions

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e + 2}{2\pi} (\vec{\omega}_h \cdot \vec{n})^e$$

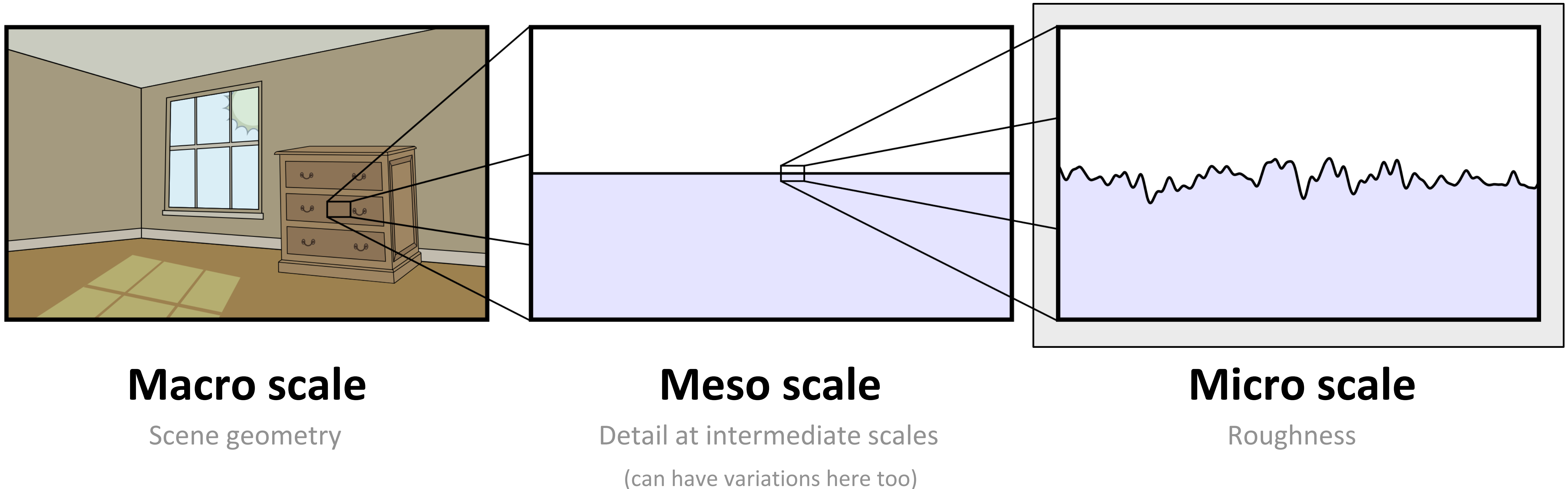
$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$



Microfacet Theory

Key idea:

- transition from individual interactions to statistical averages

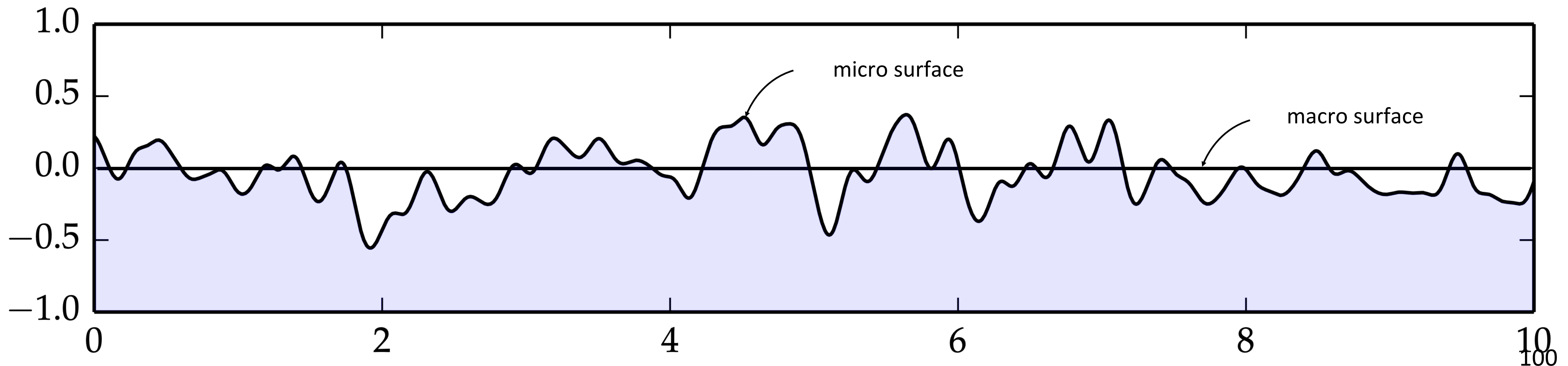


Microfacet Theory

Assume surface consists of tiny facets

Assume that the differential area being viewed/illuminated is relatively large compared to the size of microfacets

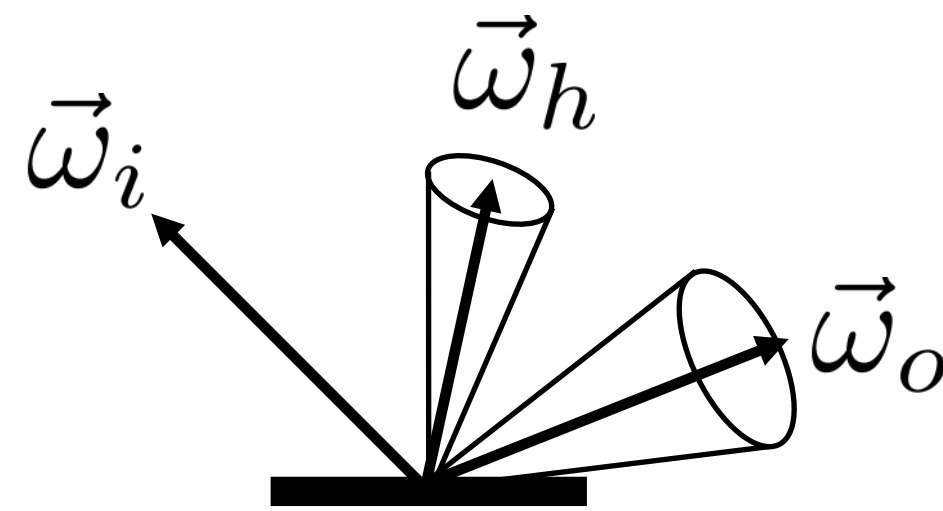
A facet can be perfectly specular or diffuse



General Microfacet Model

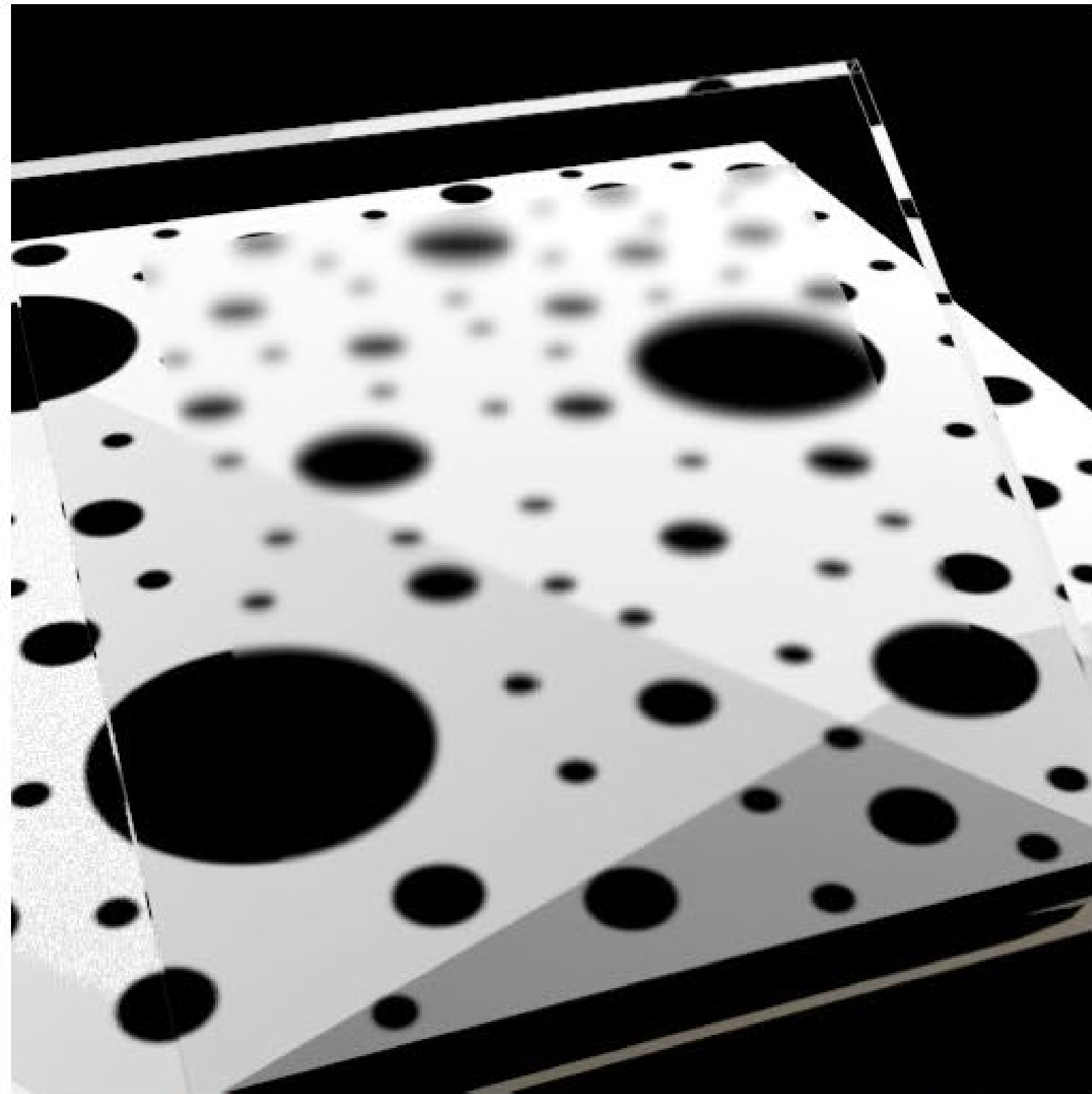
Fresnel coefficient Microfacet distribution Shadowing/masking

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4 |(\vec{\omega}_i \cdot \vec{n})(\vec{\omega}_o \cdot \vec{n})|}$$

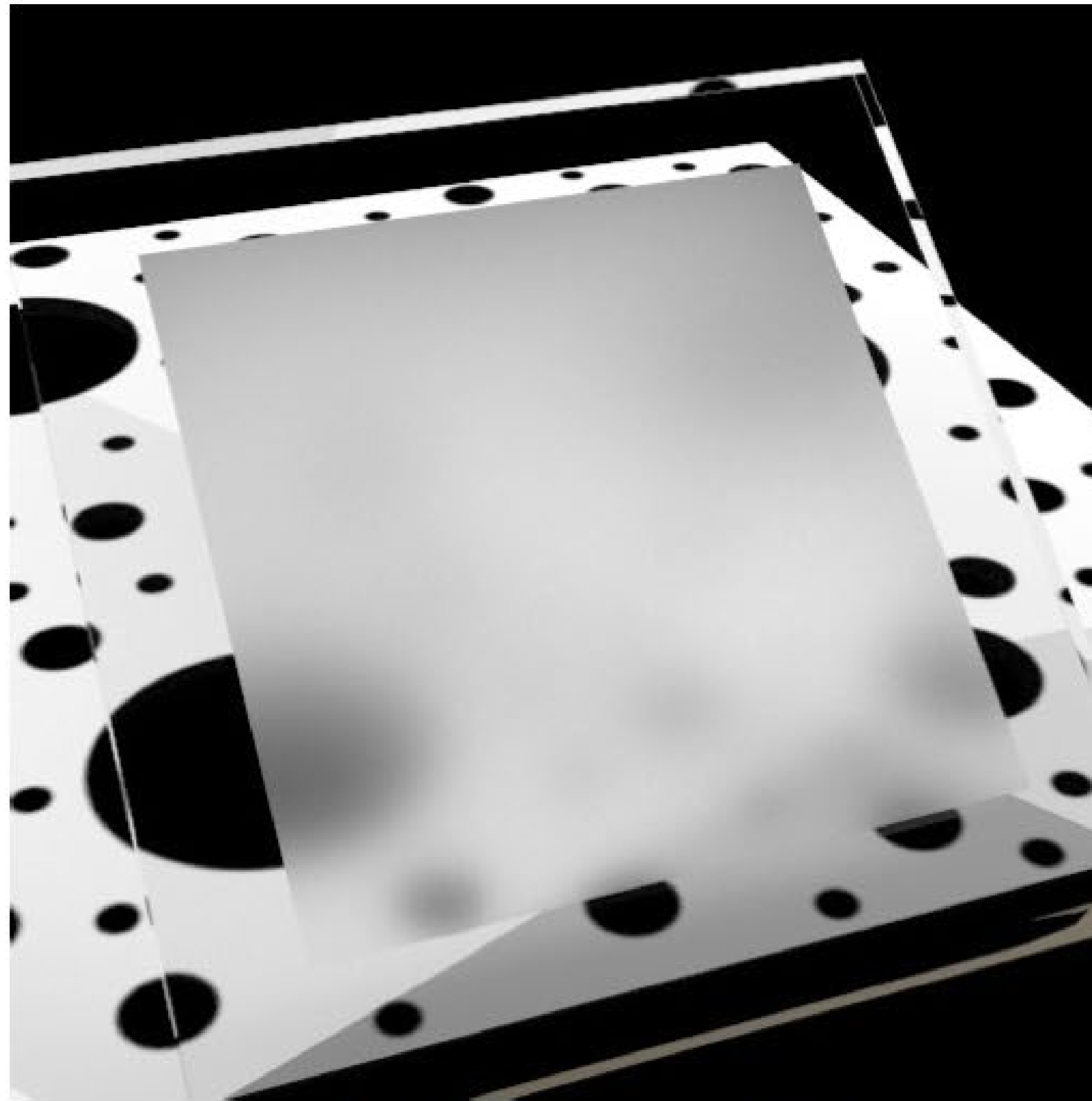


$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$

GGX and Beckmann



anti-glare (Beckman, $\alpha_b = 0.023$)

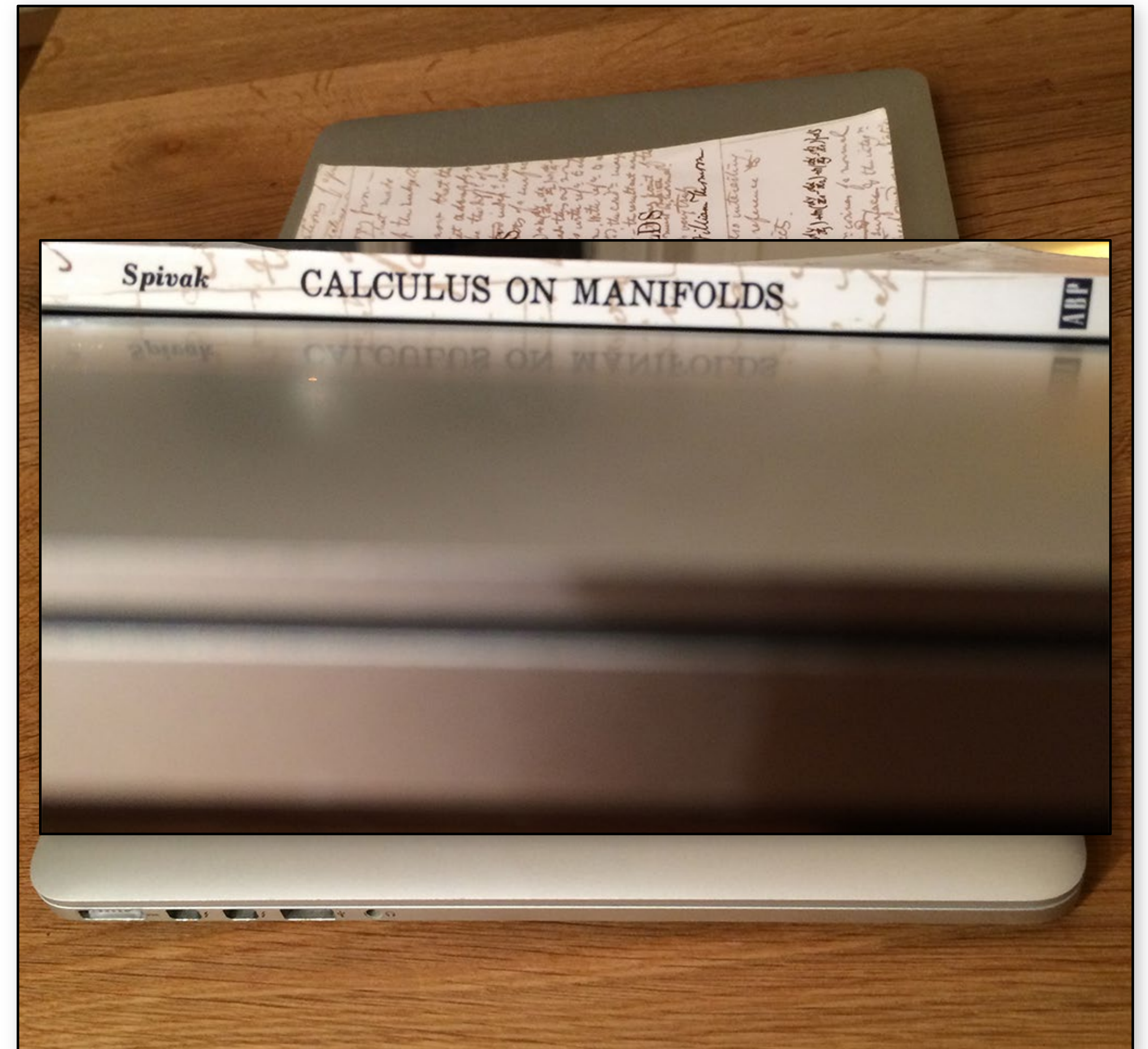


ground (GGX, $\alpha_g = 0.394$)

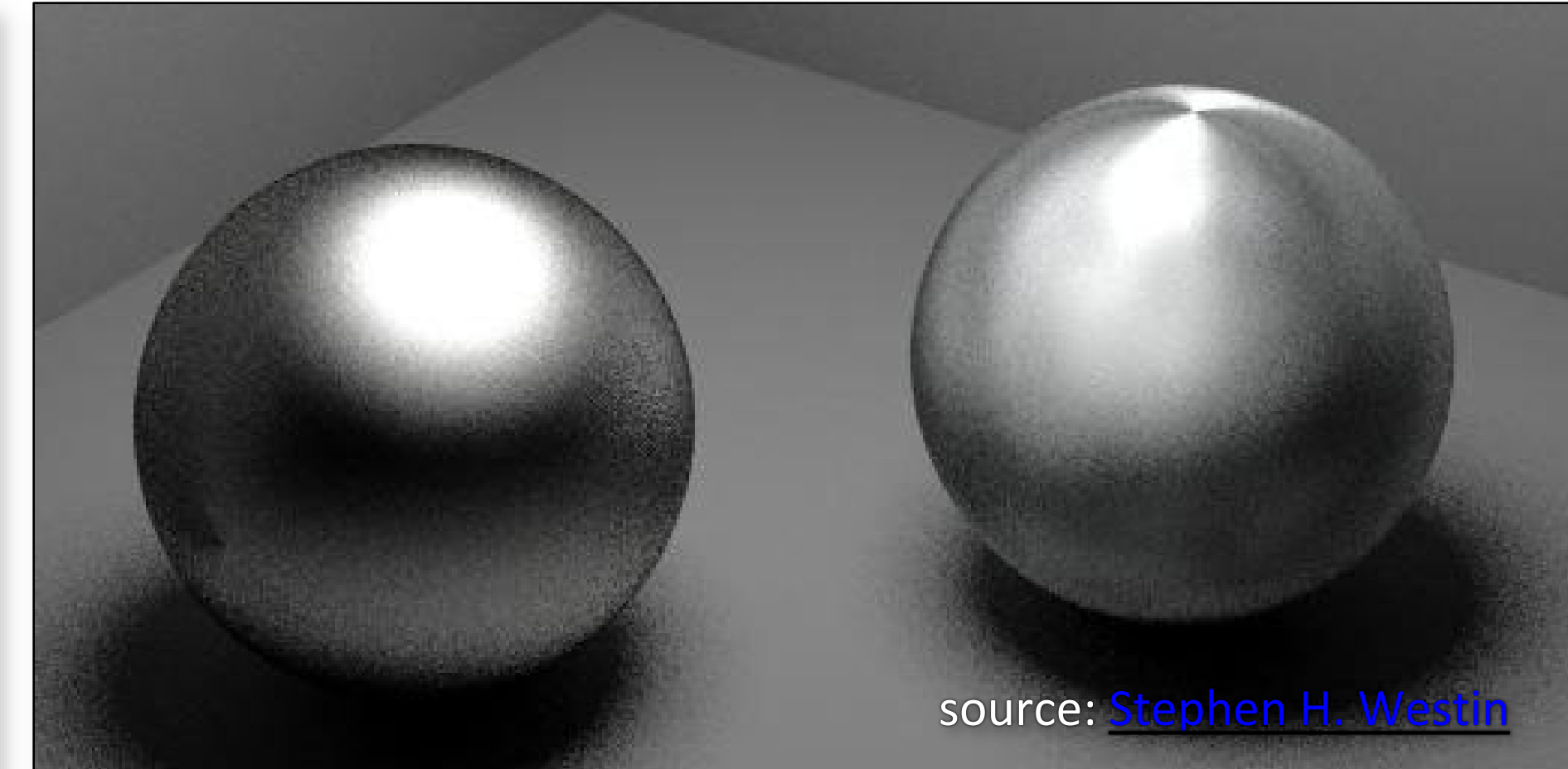


etched (GGX, $\alpha_g = 0.553$)

Interesting grazing angle behavior



Extension: Anisotropic Reflection



The Oren-Nayar Model

Same concept as the microfacet models, but assumes that the facets are diffuse

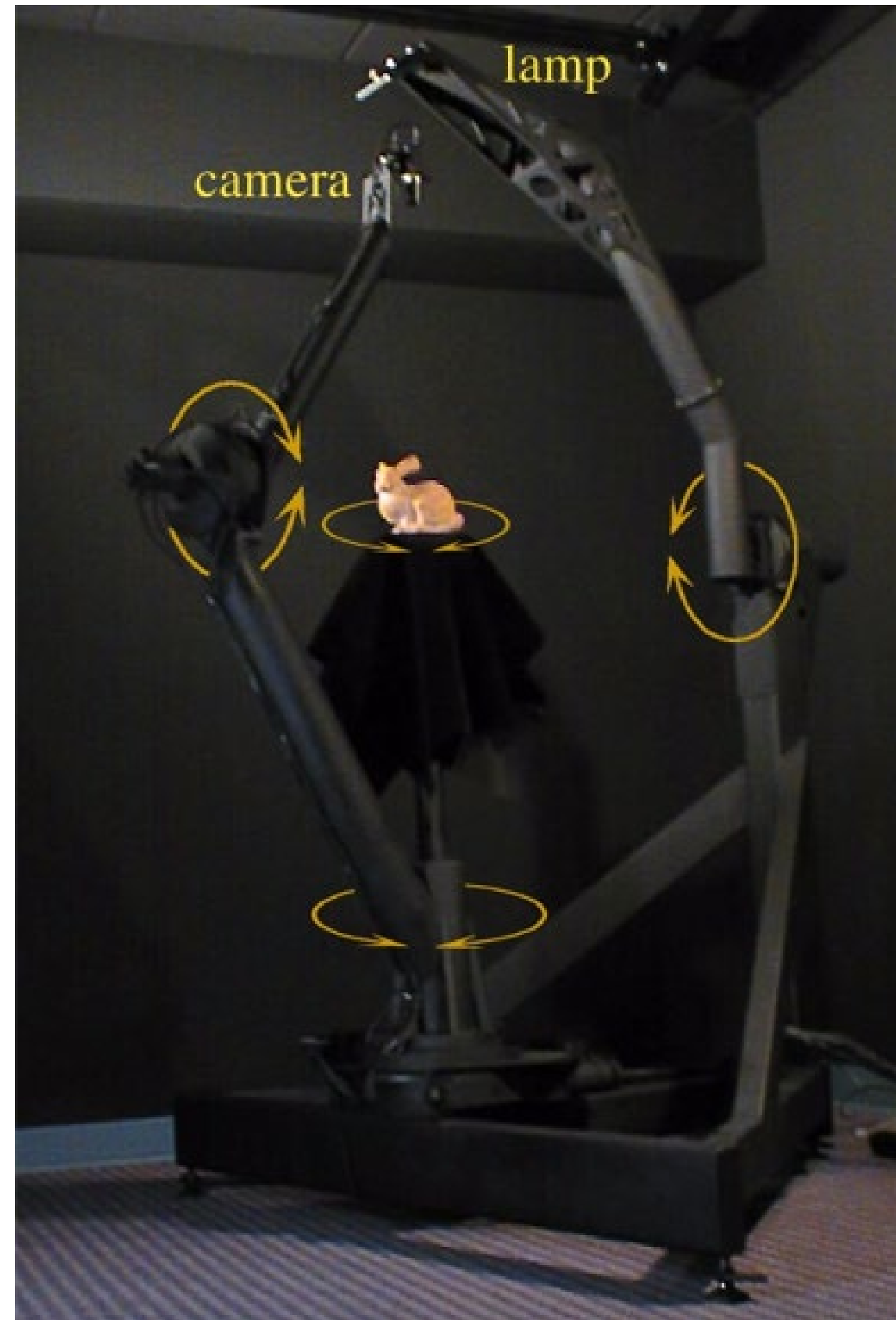
Shadowing/masking + interreflections

No analytic solution; fitted approximation

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{\rho}{\pi} (A + B \max(0, \cos(\phi_i - \phi_o)) \sin \alpha \tan \beta)$$
$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \qquad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09}$$
$$\alpha = \max(\theta_i, \theta_o) \qquad \beta = \min(\theta_i, \theta_o)$$

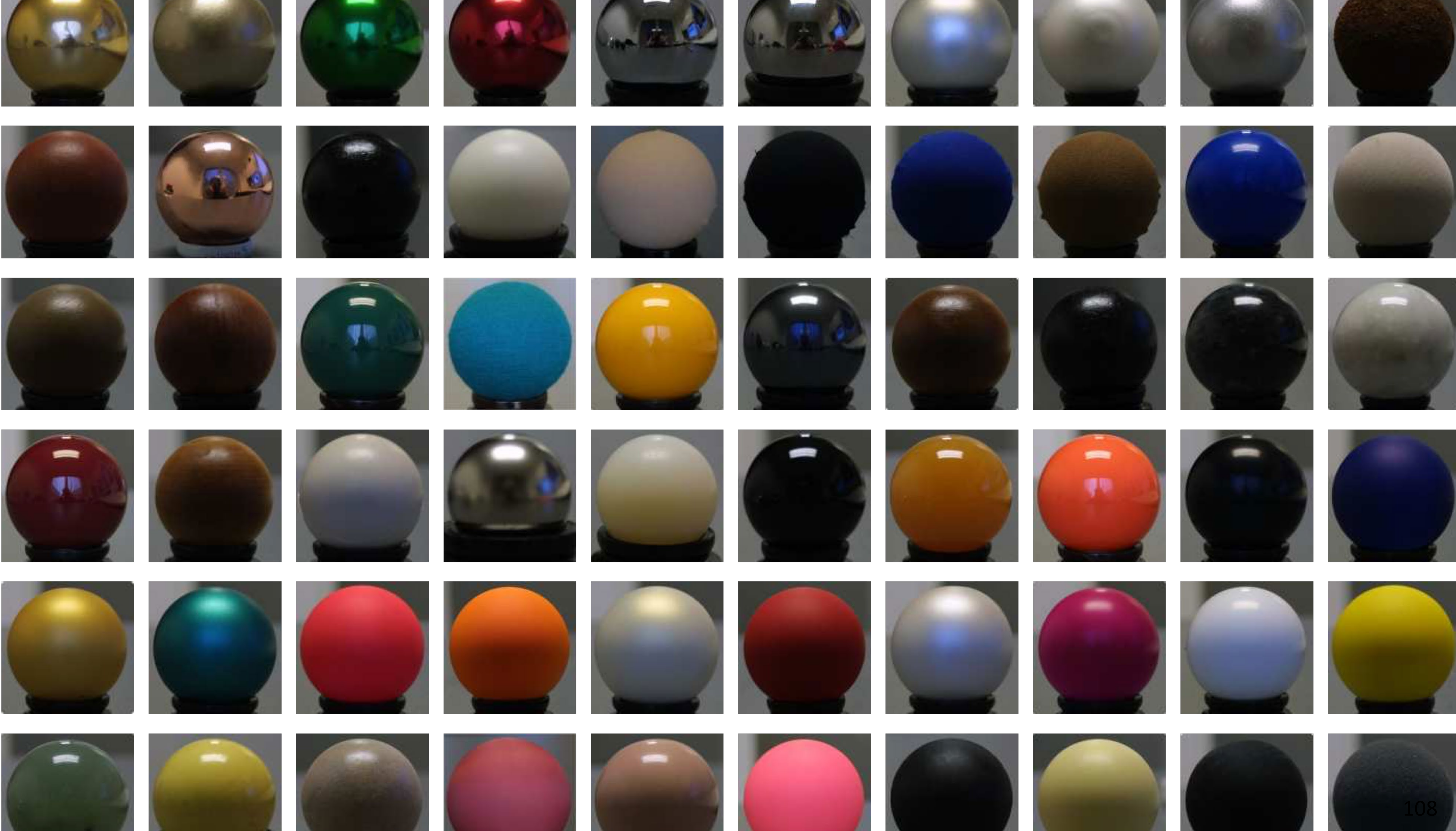
Ideal Lambertian is just a special case ($\sigma = 0$)

Measuring BRDFs



Measuring BRDFs







The MERL Database

"A Data-Driven Reflectance Model"

Wojciech Matusik, Hanspeter Pfister, Matt Brand and Leonard McMillan.

ACM Transactions on Graphics 22, 3(2003), 759-769.

- <http://www.merl.com/brdf/>