

# Dealing with indirect illumination



15-463, 15-663, 15-862  
Computational Photography  
Fall 2023, Lecture 18

# Course announcements

- Homework assignment 6 is due on Friday, December 8<sup>th</sup>.
  - Do not leave for last minute, you won't have time to complete it.
- Make-up lecture: Thursday 12/7 at 11 – 12:30 in NSH 4305.

# Overview of today's lecture

- Direct and indirect illumination.
- Direct-indirect separation using high-frequency illumination.
- Back to structured light.
- Back to time-of-flight imaging.
- Direct-indirect separation using epipolar constraints.

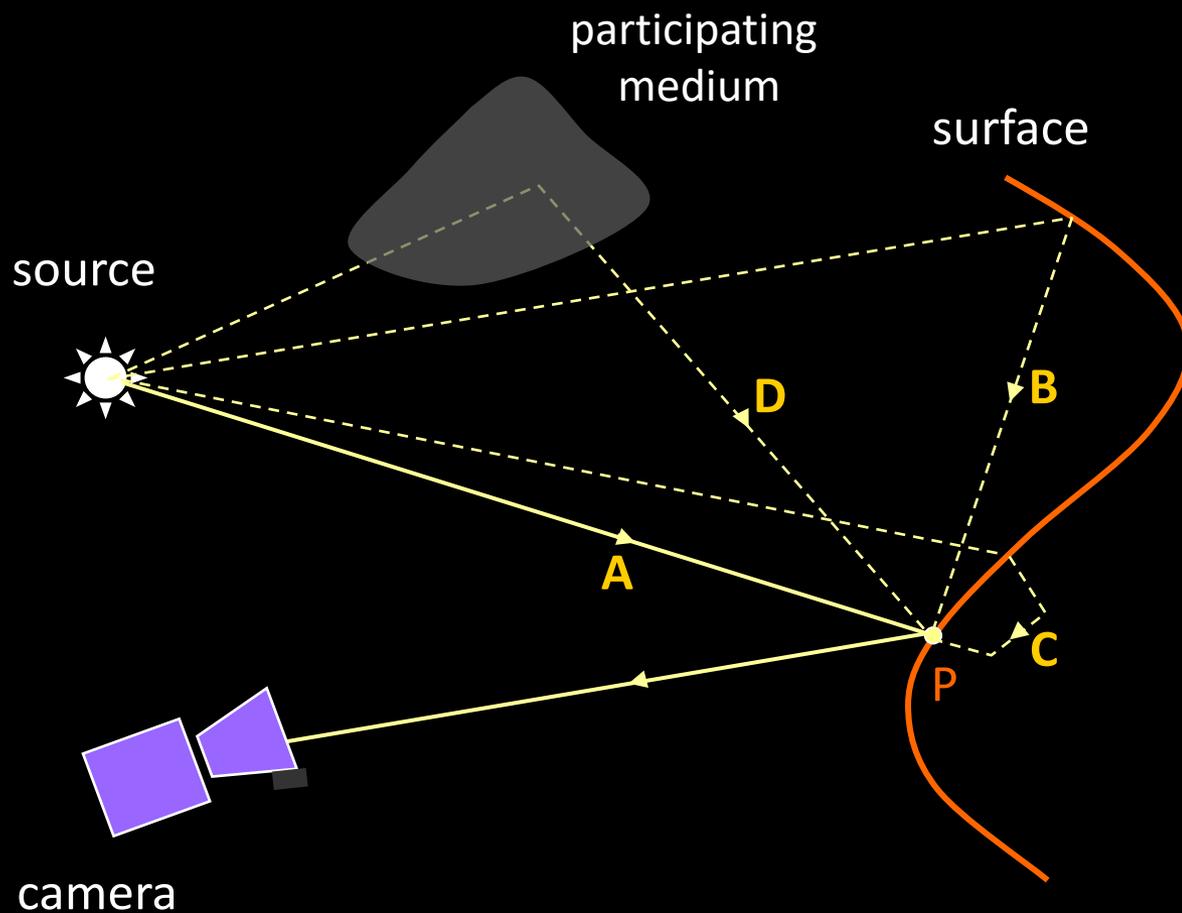
# Slide credits

These slides were directly adapted from:

- Shree Nayar (Columbia).
- Matthew O'Toole (CMU).
- Supreeth Achar (Google, formerly CMU).
- Mohit Gupta (Wisconsin).

# Direct and indirect illumination

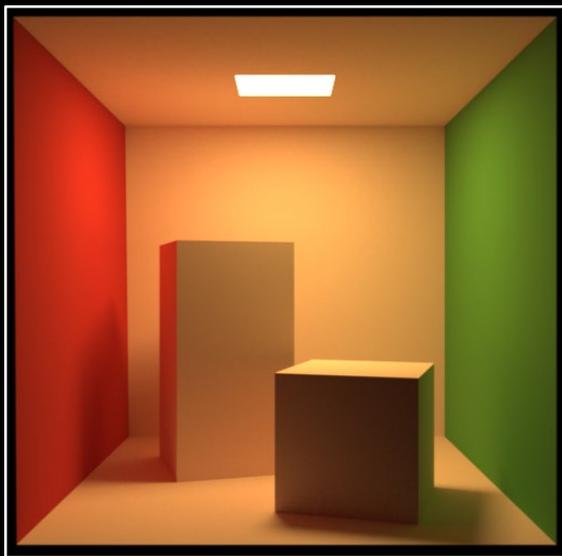
# Direct and Indirect Illumination



- A : Direct
- B : Interreflection
- C : Subsurface
- D : Volumetric

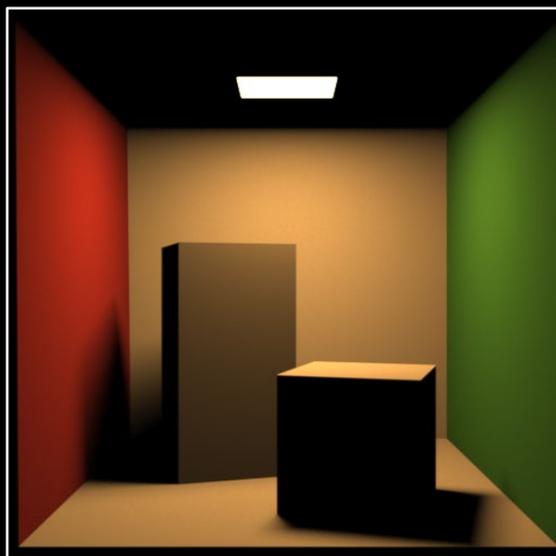
# Easy to separate in a renderer

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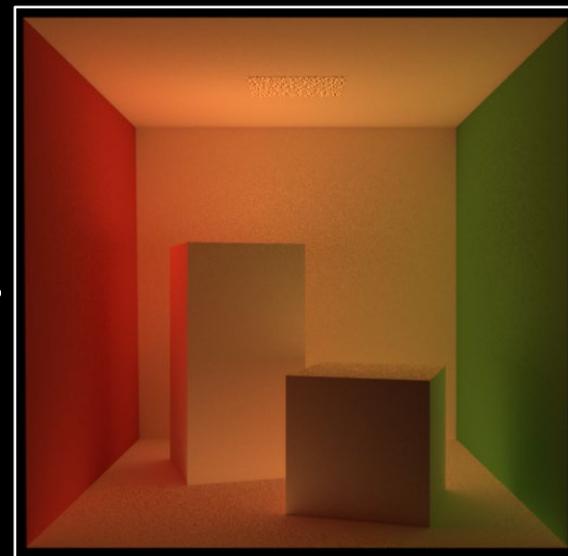
full image

=



direct

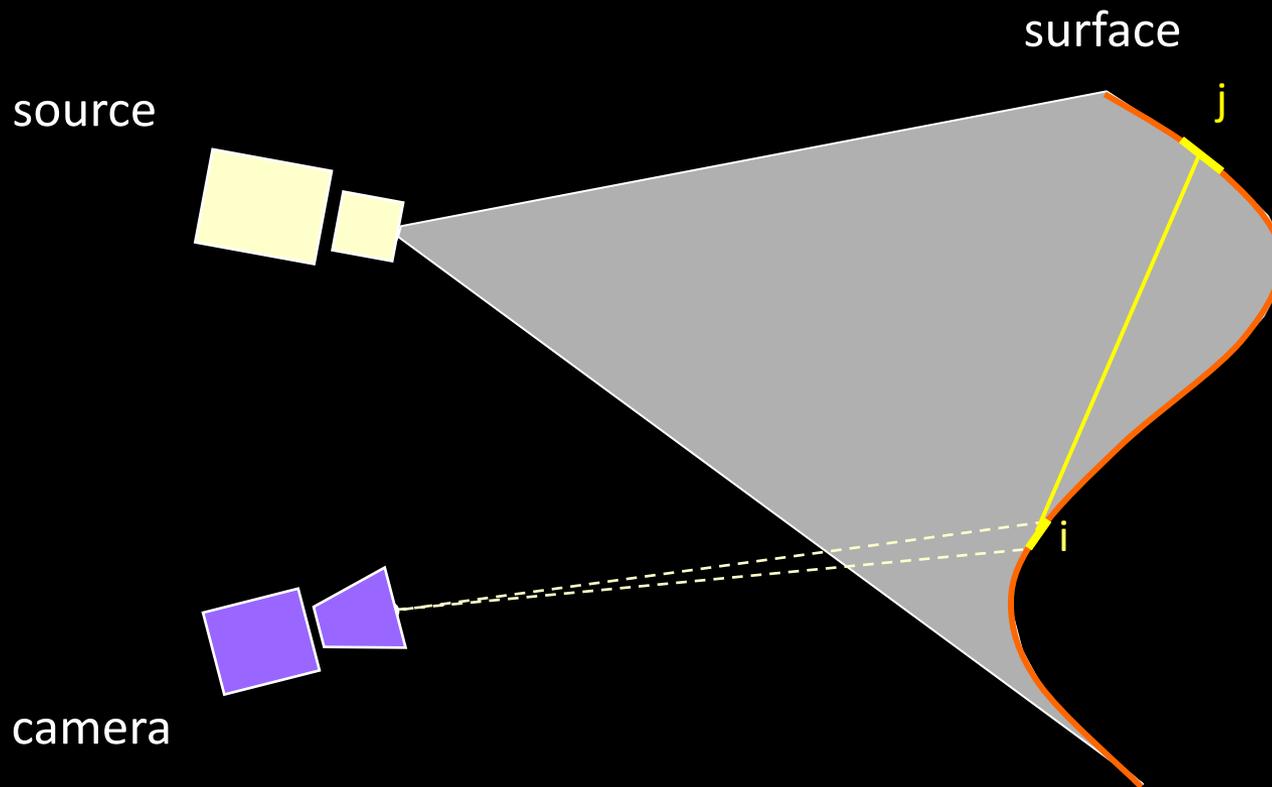
+



indirect

Direct-indirect separation using high-frequency illumination

# Direct and indirect components: interreflections



$$L[c, i] = L_d[c, i] + L_g[c, i]$$

radiance

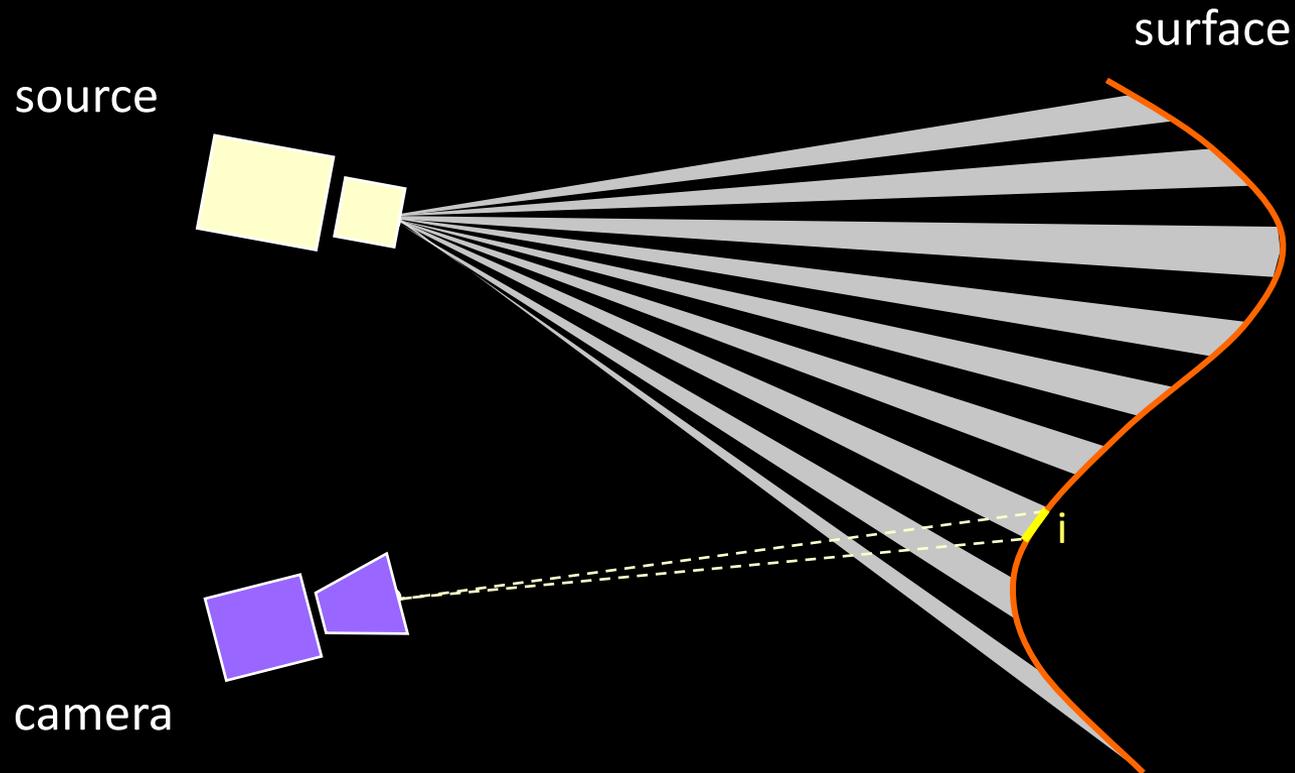
direct

indirect

$$L_g[c, i] = \sum_P A[i, j] L[i, j]$$

BRDF and geometry

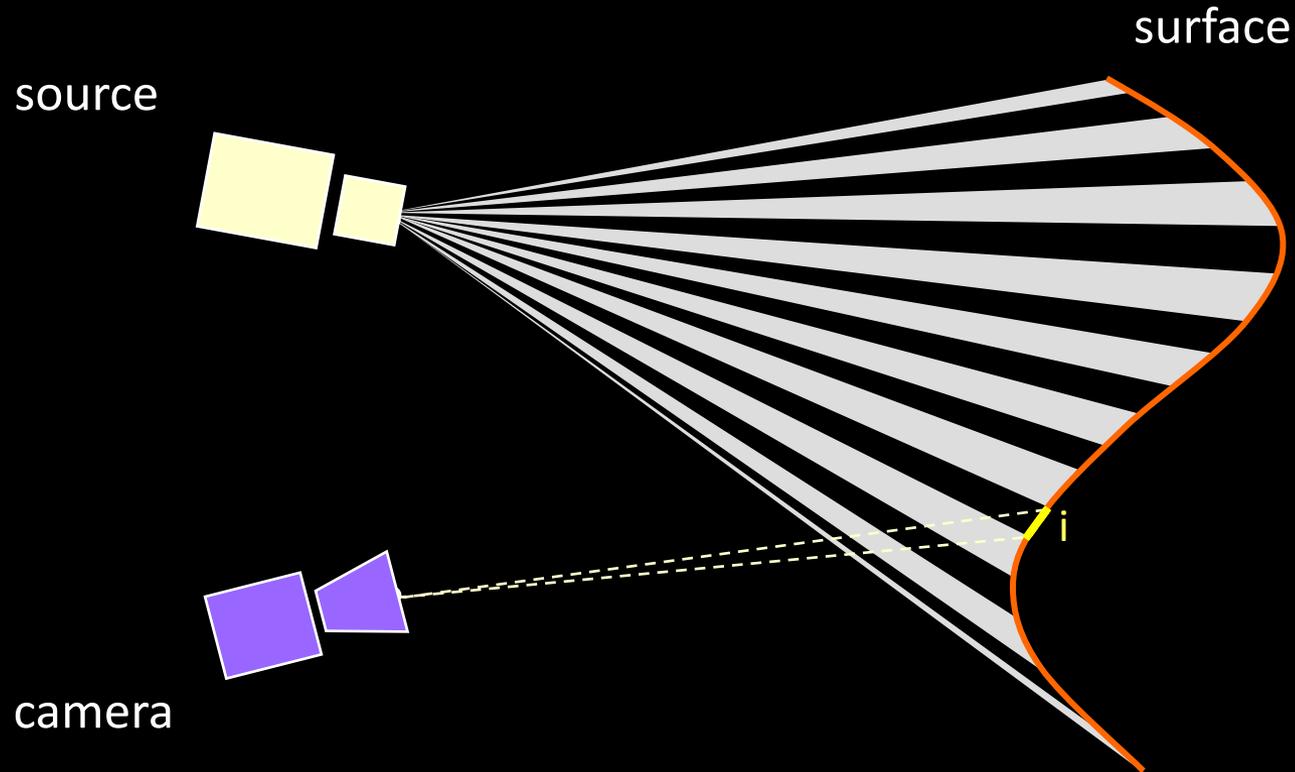
# High Frequency Illumination Pattern



$$L^+[c, i] = L_d[c, i] + \alpha L_g[c, i]$$

fraction of activated source elements

# High Frequency Illumination Pattern



$$L^+[c, i] = L_d[c, i] + \alpha L_g[c, i]$$

$$L^-[c, i] = (1 - \alpha) L_g[c, i]$$

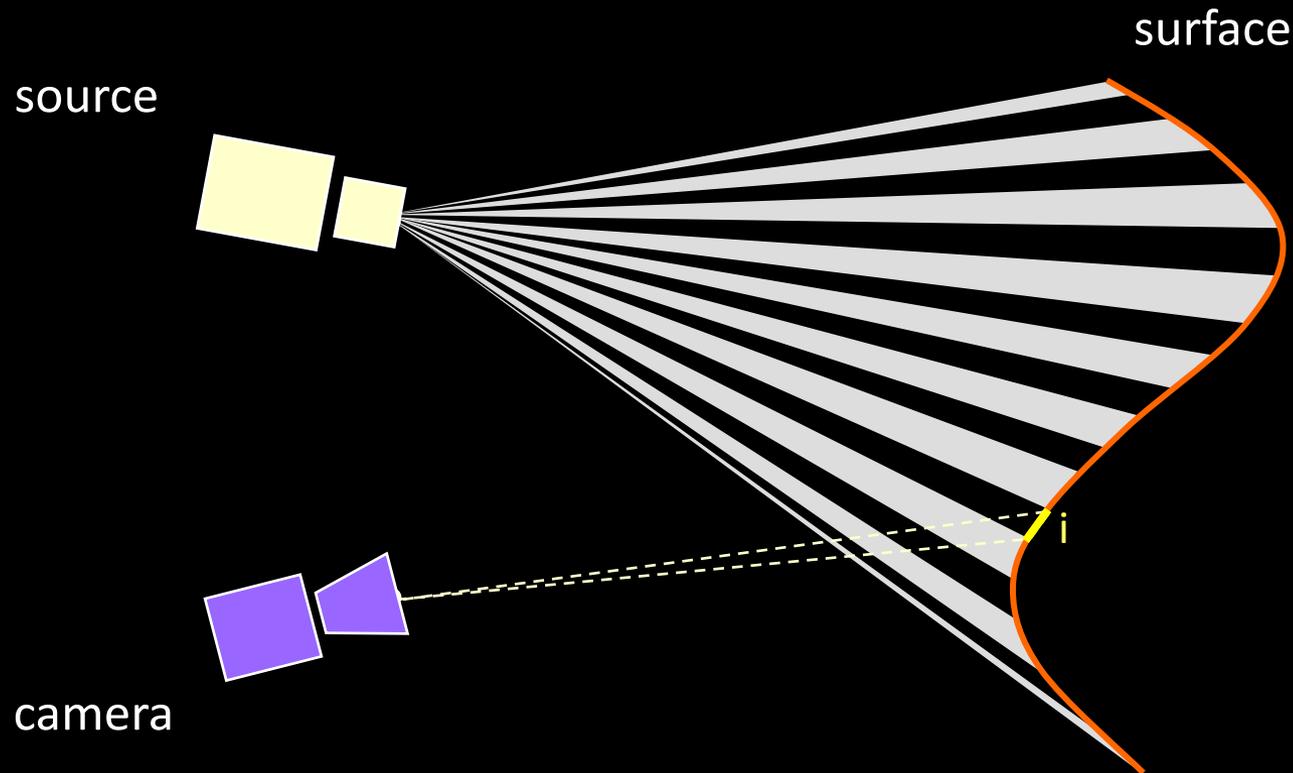
fraction of activated source elements

# Important insight:

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- Indirect illumination is approximately invariant to high-frequency lighting.
- You can think of indirect illumination effects as a low-pass filter.

# High Frequency Illumination Pattern



$$L^+[c, i] = L_d[c, i] + \alpha L_g[c, i]$$

$$\bar{L}^-[c, i] = (1 - \alpha) L_g[c, i]$$

What does approximate invariance mean in this case?

# Separation from Two Images

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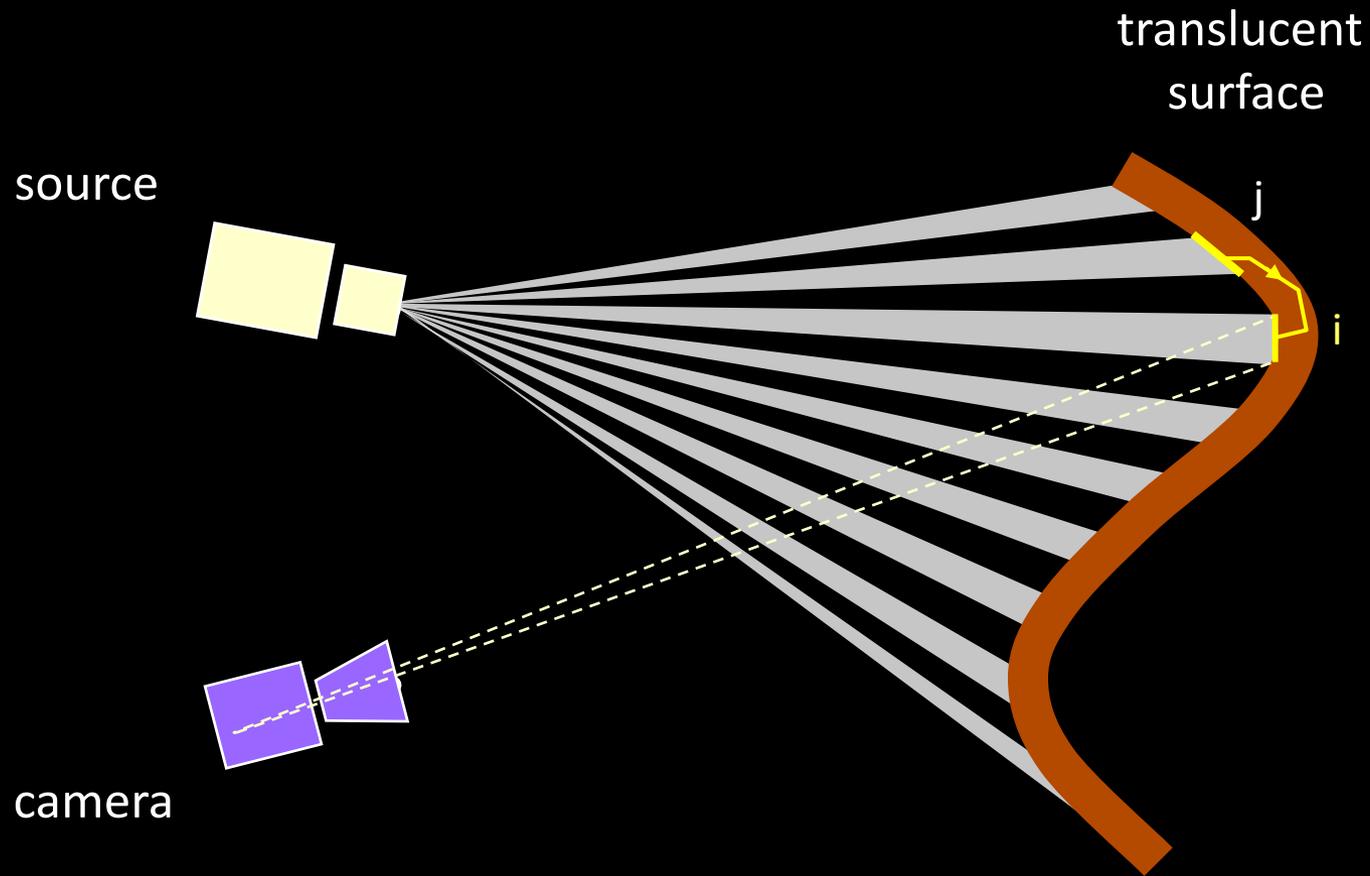
$$\alpha = \frac{1}{2}:$$

$$L_d = L_{\max} - L_{\min}, \quad L_g = 2L_{\min}$$

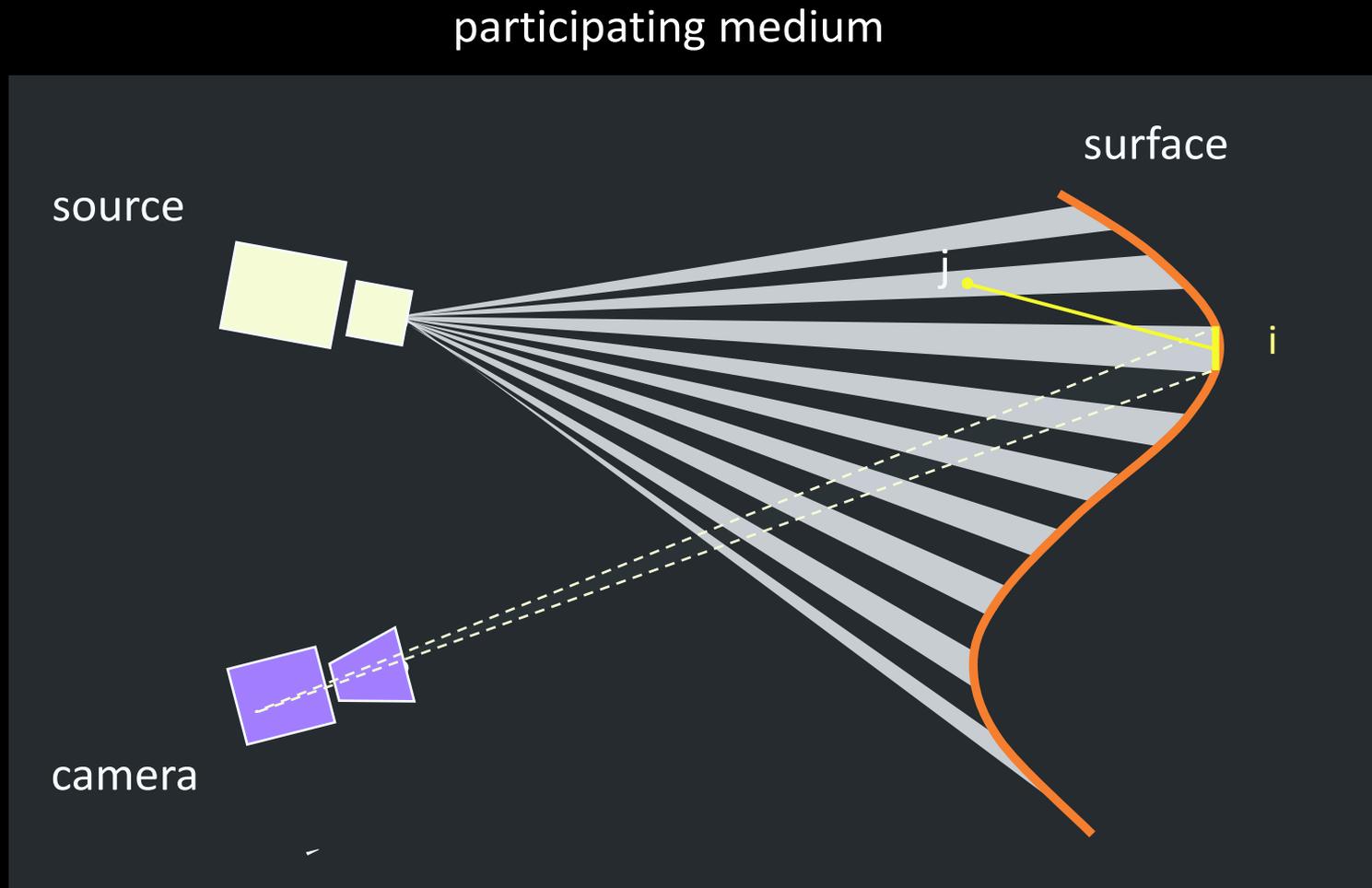
direct

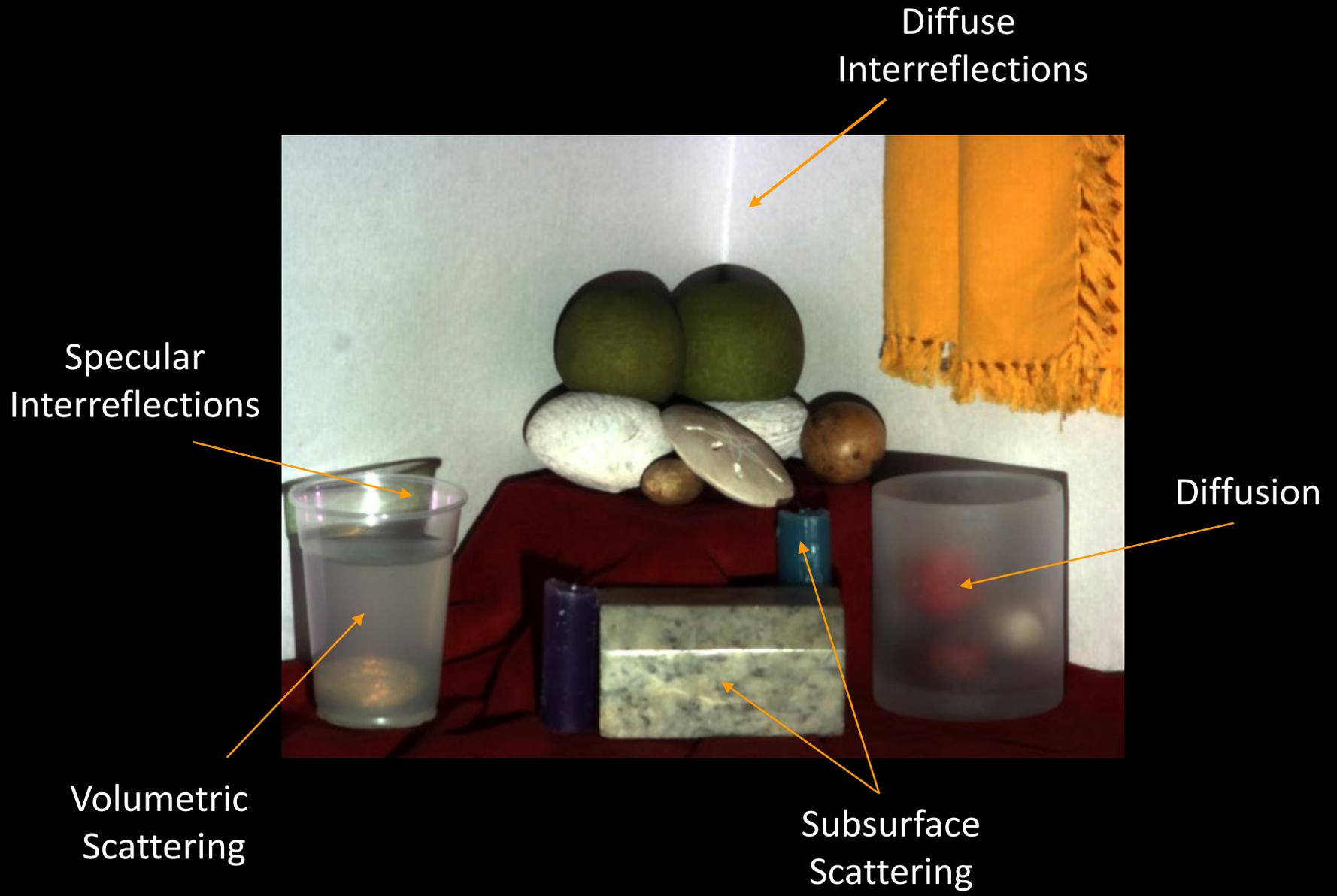
indirect

# Other Indirect Effects: Subsurface Scattering



# Other Indirect Effects: Volumetric Scattering





# Scene



Direct

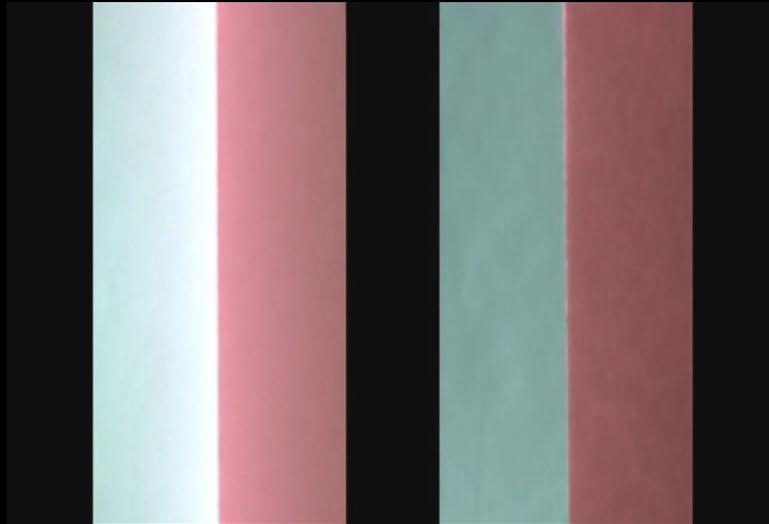


Indirect

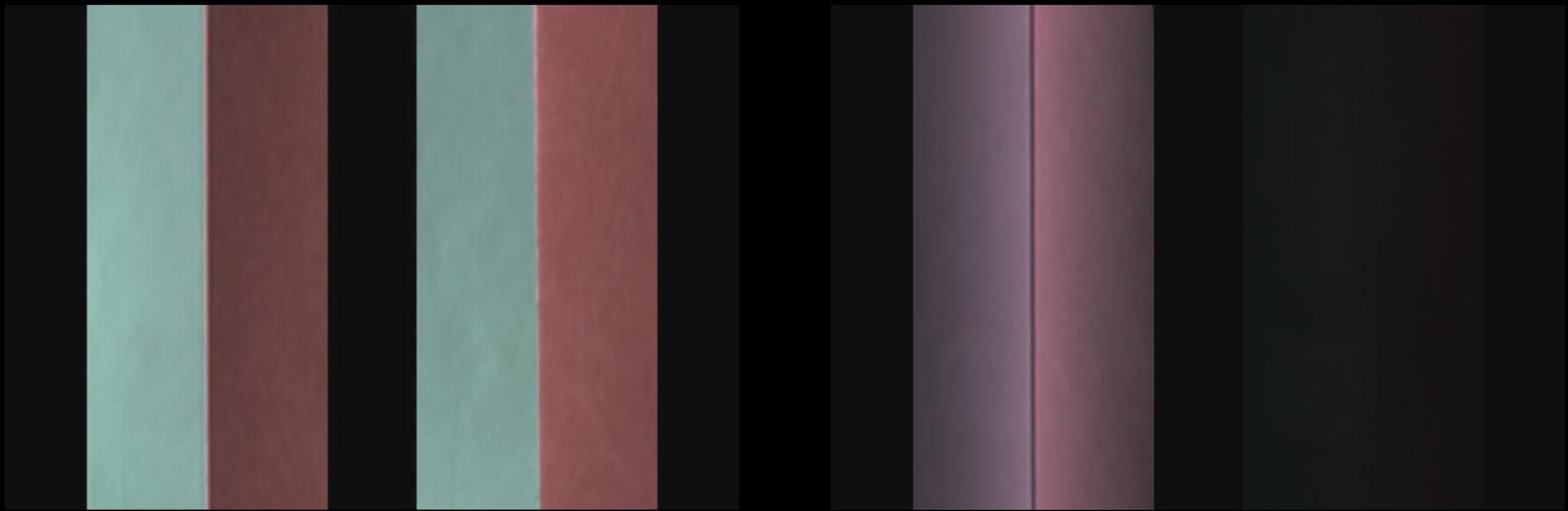
# V-Grooves: Diffuse Interreflections

concave

convex



**Psychophysics:**  
Gilchrist 79, Bloj et al. 04



Direct

Indirect

Real World Examples:

Can You Guess the Images?

# Eggs: Diffuse Interreflections

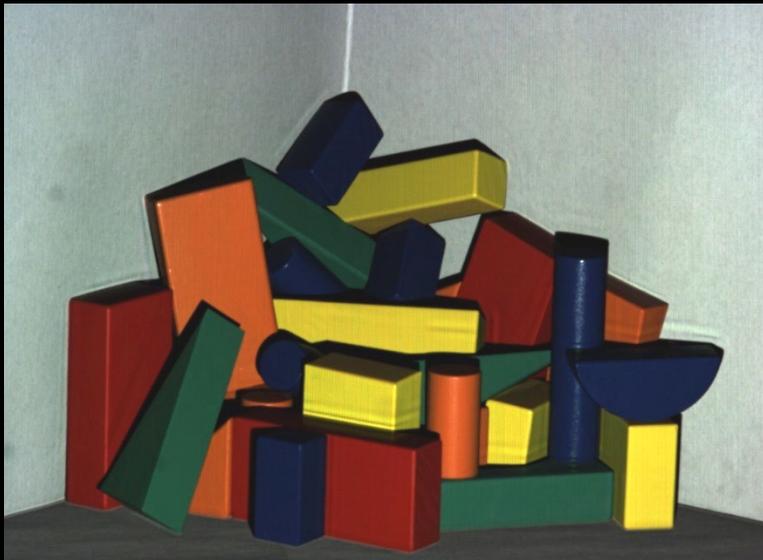
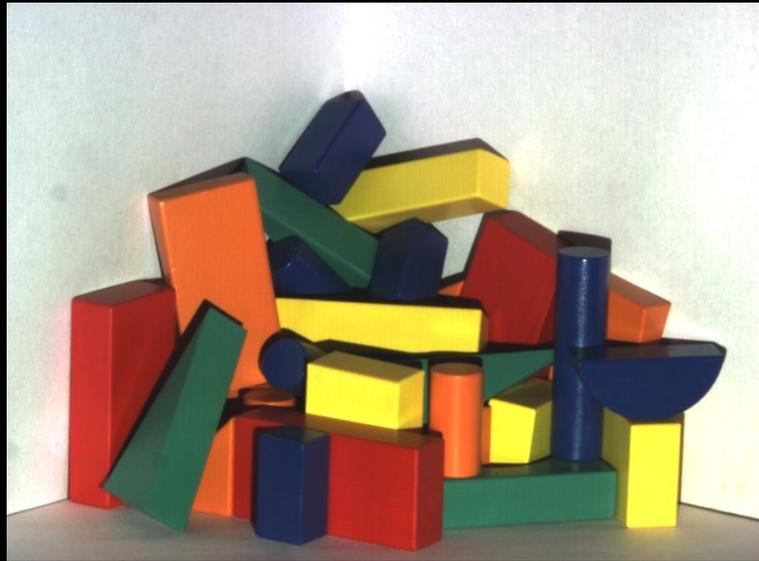


Direct



Indirect

# Wooden Blocks: Specular Interreflections

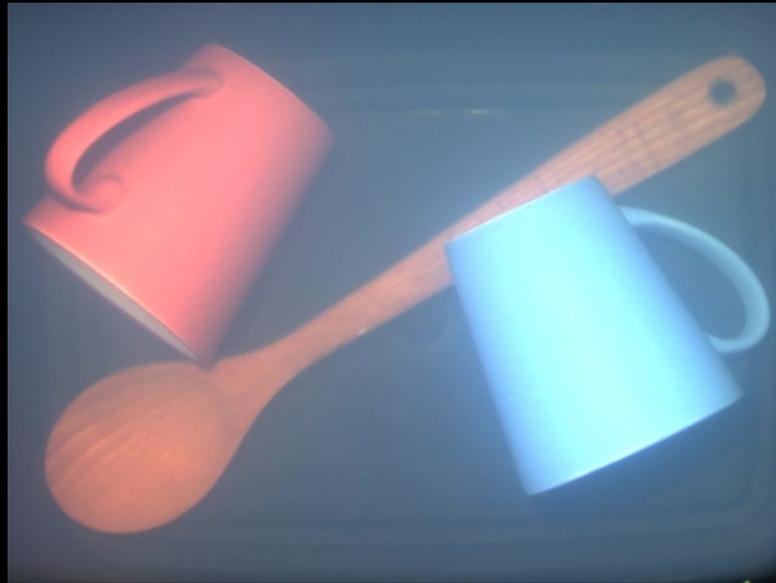


Direct



Indirect

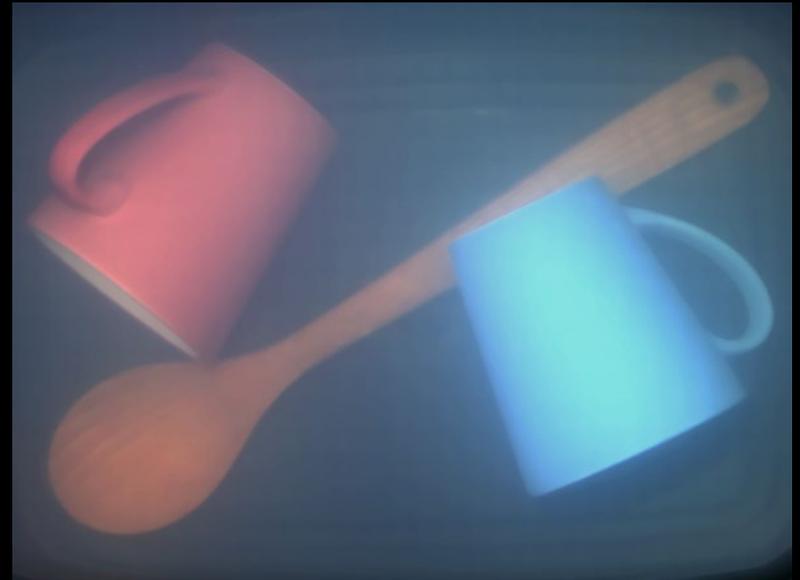
# Kitchen Sink: Volumetric Scattering



**Volumetric Scattering:**  
Chandrasekar 50, Ishimaru 78



Direct



Indirect

# Peppers: Subsurface Scattering



Direct



Indirect

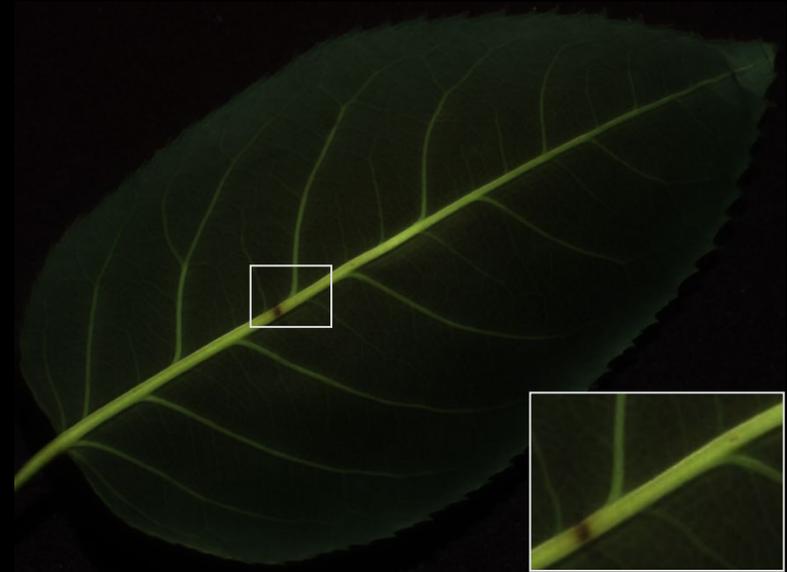
# Tea Rose Leaf



Leaf Anatomy: Purves et al. 03

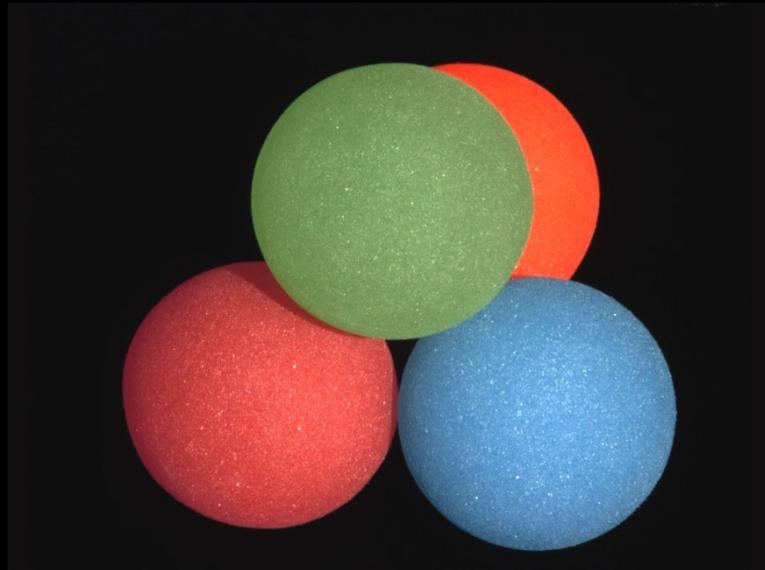


Direct

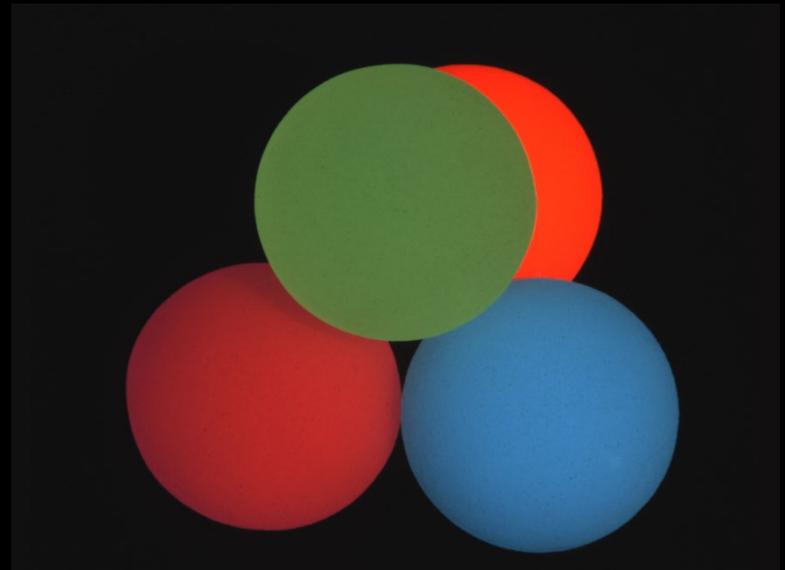


Indirect

# Translucent Rubber Balls



Direct

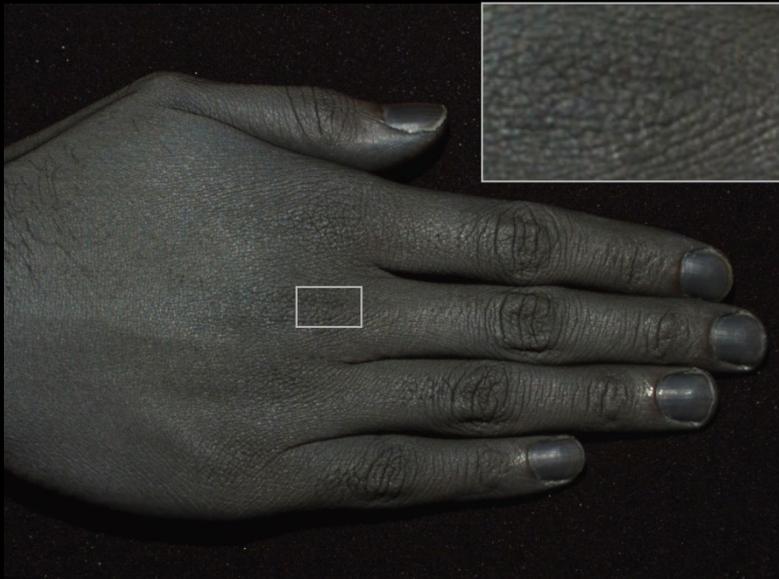


Indirect

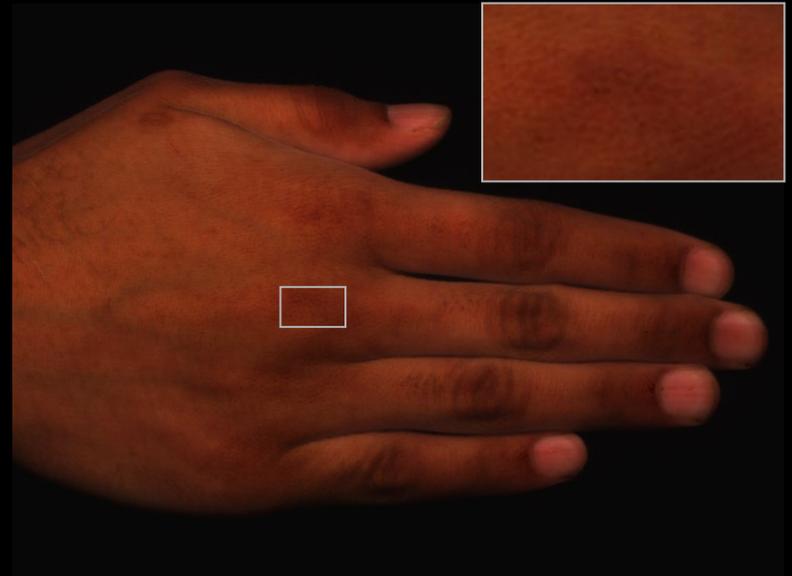
# Hand



**Skin:** Hanrahan and Krueger 93,  
Uchida 96, Haro 01, Jensen et al. 01,  
Igarashi et al. 05, Weyrich et al. 05



Direct

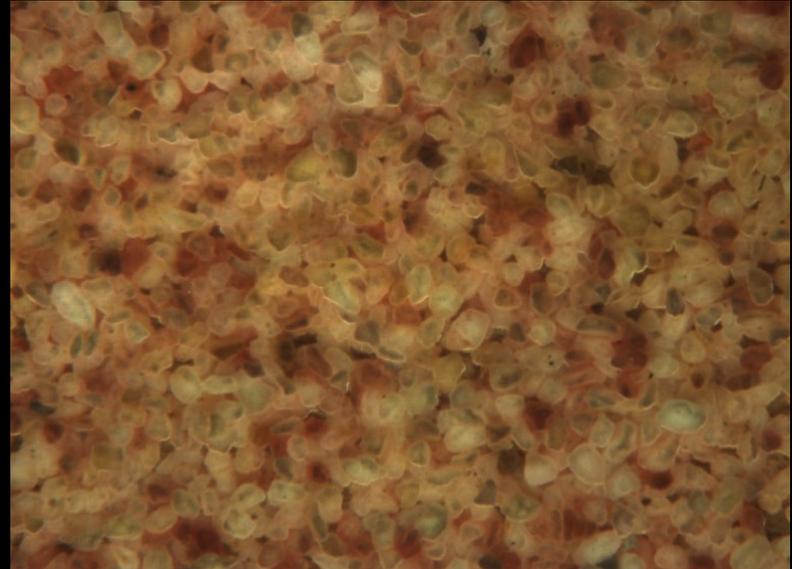


Indirect

# Pebbles: 3D Texture



Direct



Indirect

# Pink Carnation



Spectral Bleeding: Funt et al. 91

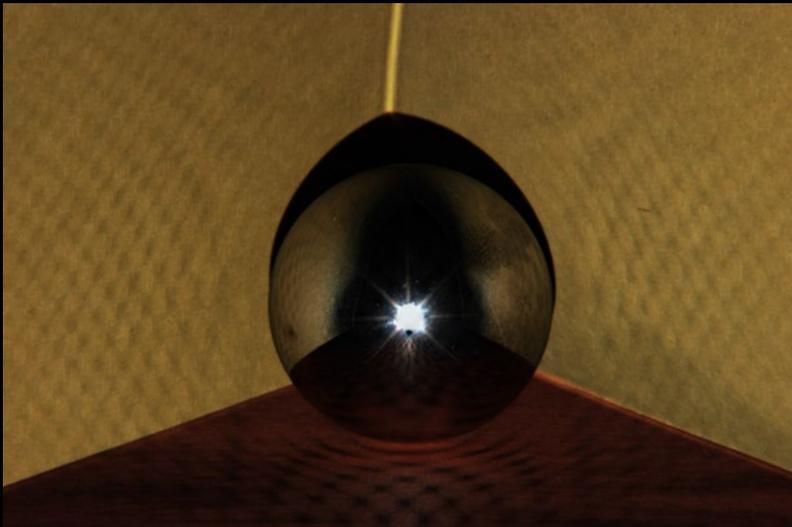
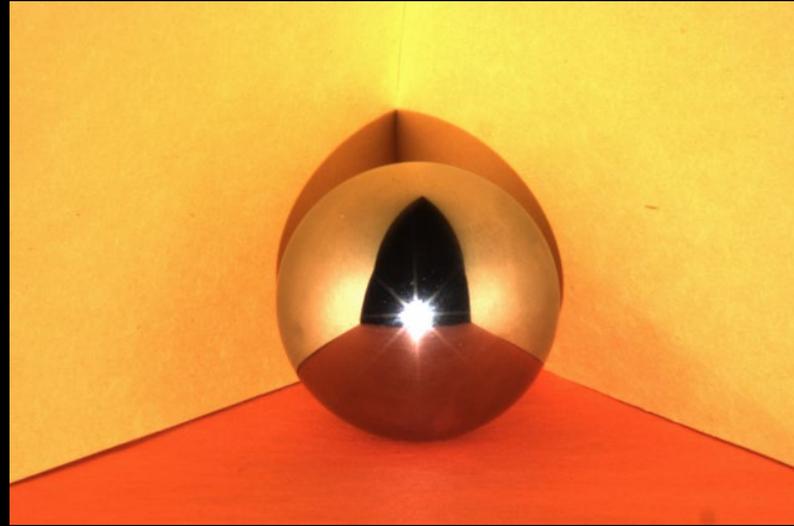


Direct

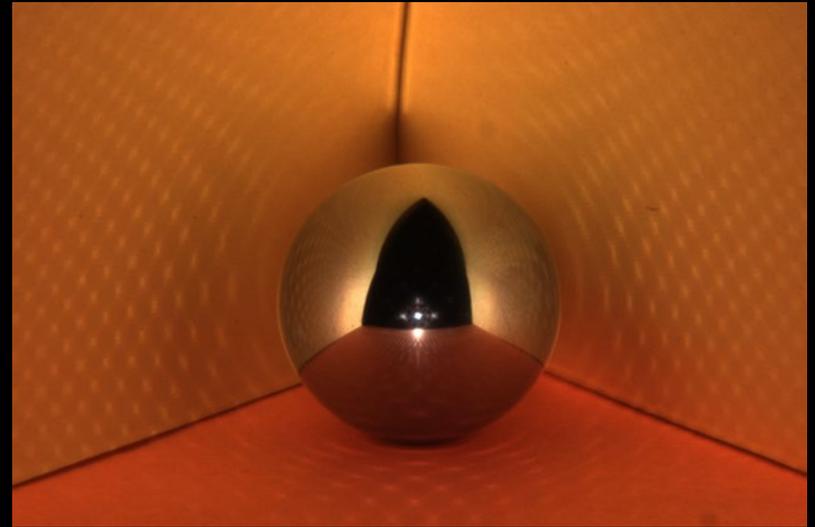


Indirect

# Mirror Ball: Failure Case



Direct



Indirect

# Building Corner



Stick



Shadow

3D from Shadows:  
Bouguet and Perona 99

$$L_d = L_{\max} - L_{\min} , \quad L_g = L_{\min}$$

direct

indirect

# Building Corner



Direct



Indirect

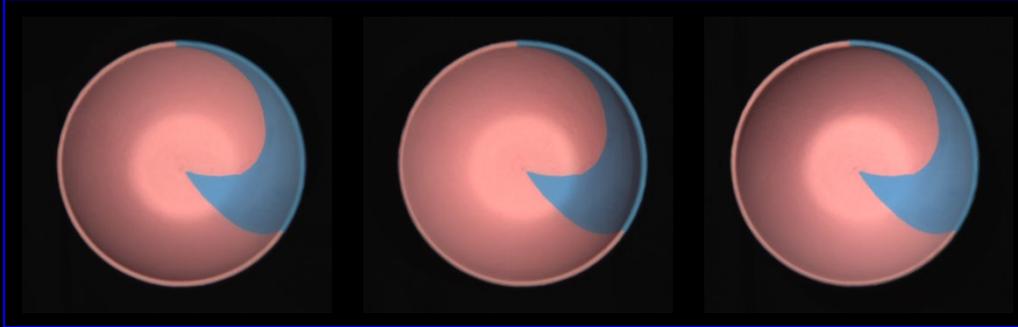
# Photometric Stereo using Direct Images

Source 1

Source 2

Source 3

Bowl



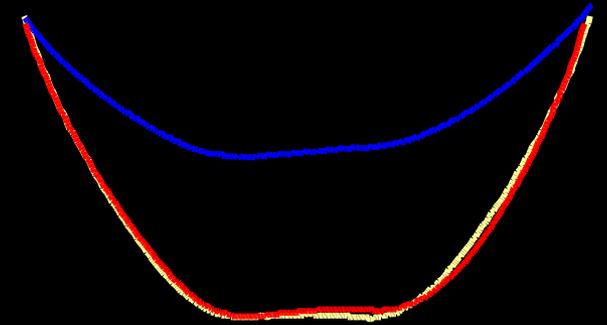
Indirect



Direct



Shape



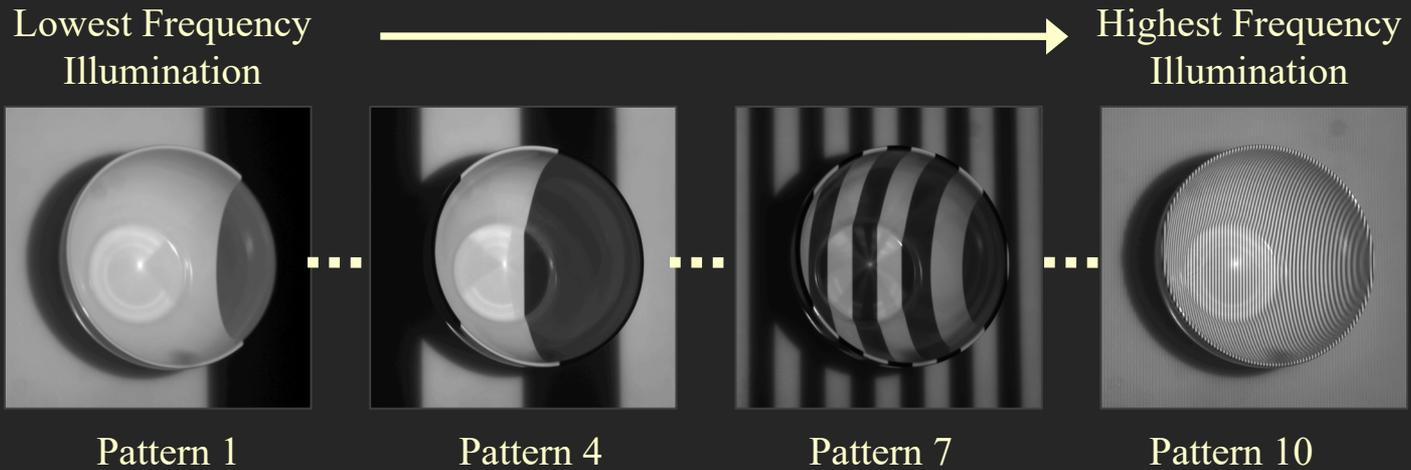
# Application to structured light

Why is indirect illumination a problem?

## Bowl on a Marble Slab

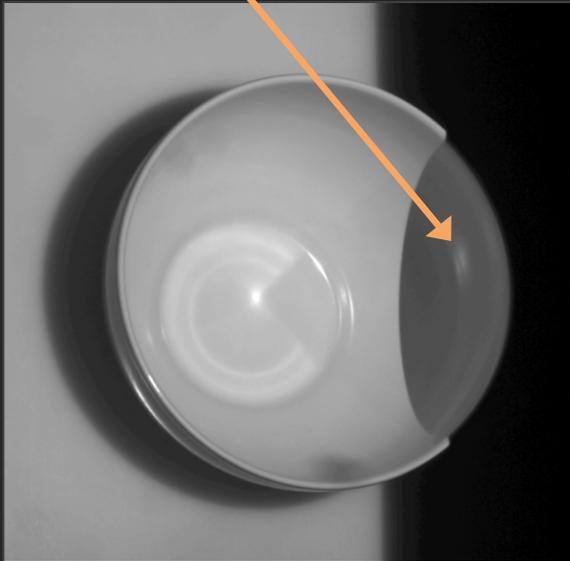


# Captured images under conventional Gray codes



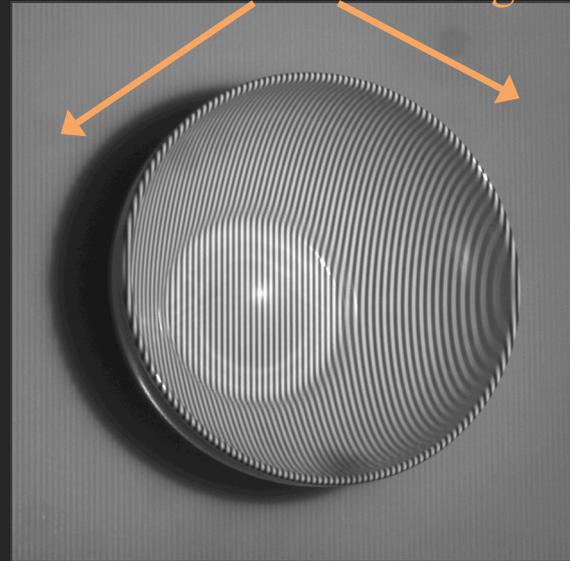
## Issues due to indirect illumination effects

Strong Inter-reflections



Low-frequency pattern

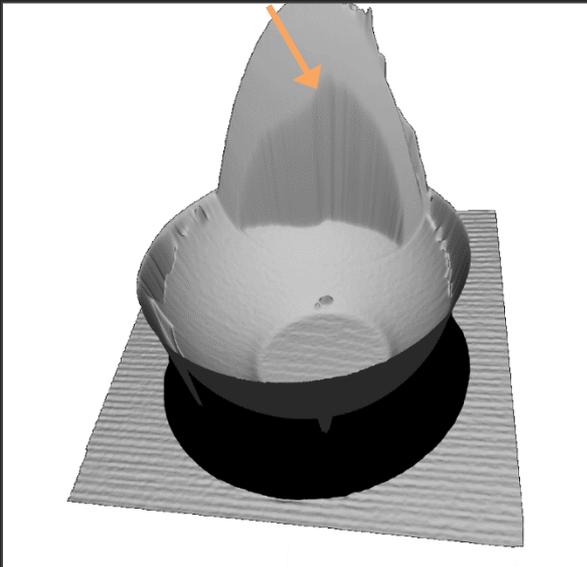
Blurring due to  
Sub-surface Scattering



High-frequency pattern

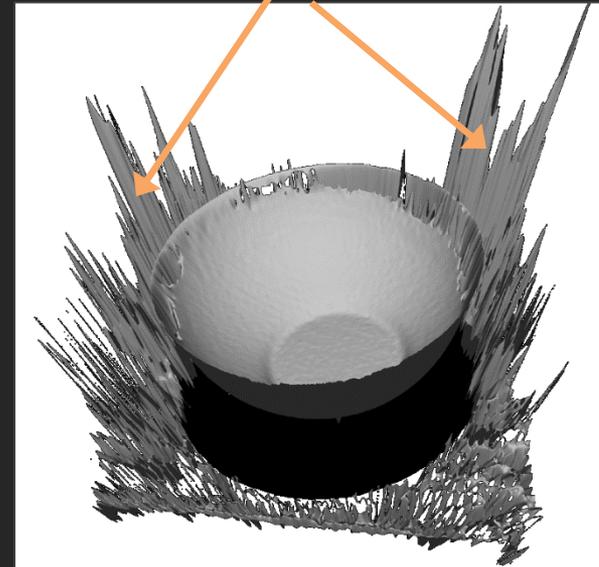
## 3D Visualizations: State of the Art

Errors due to  
interreflections



Conventional Gray  
(11 images)

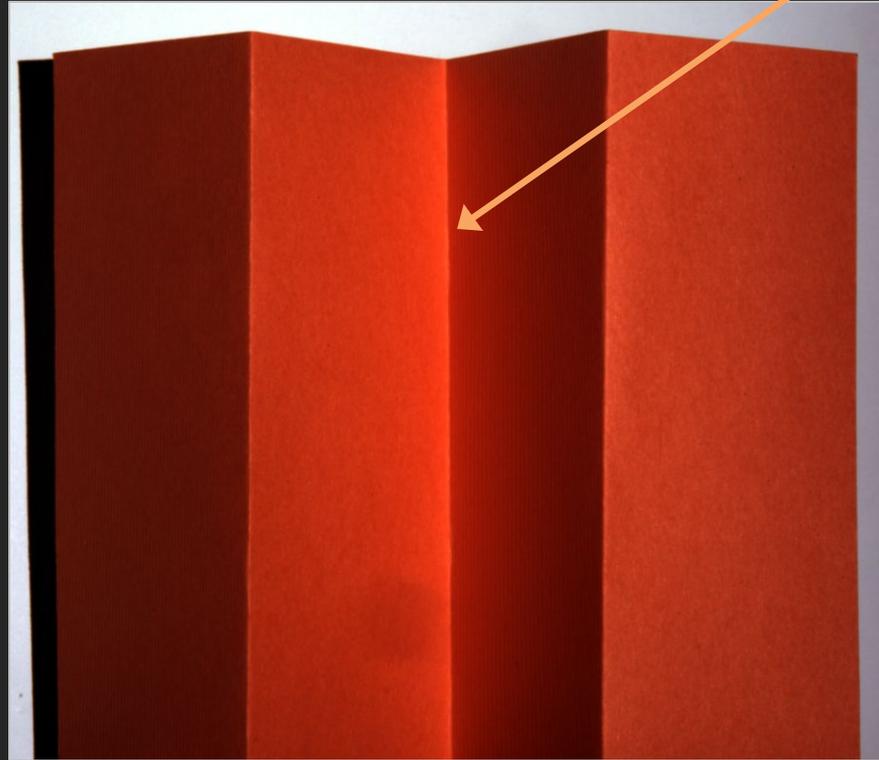
Errors due to  
sub-surface scattering



Modulated Phase-Shifting  
(162 images)

# V-Groove Scene

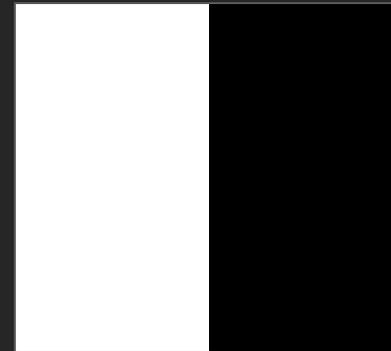
Inter-reflections



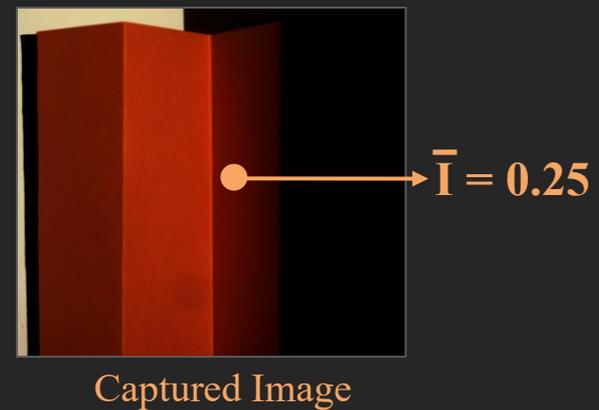
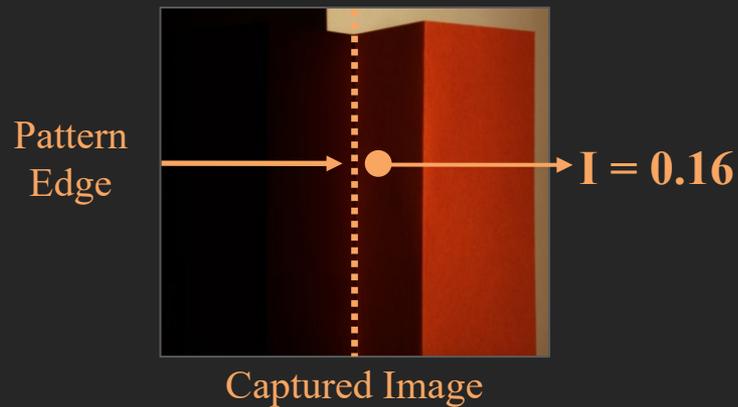
# Conventional Gray codes



Low frequency pattern



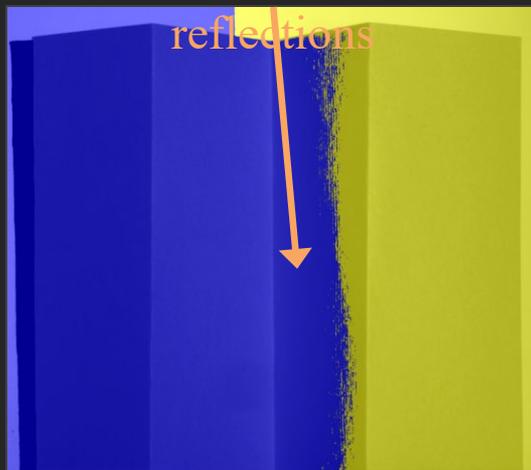
Inverse Pattern



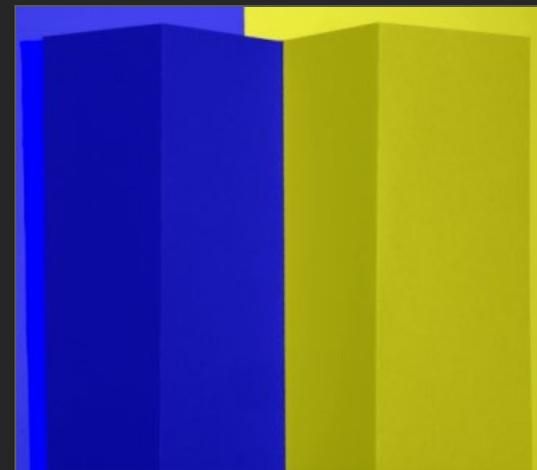
# Binarization error

Errors due to inter-

reflections



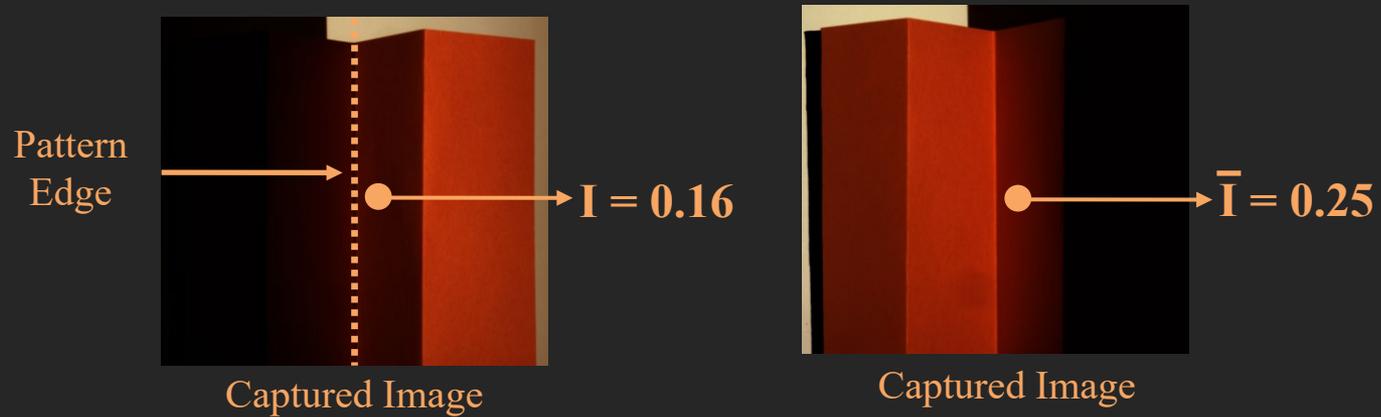
Incorrect Binarization



Ground-truth Binarization

■ One (illuminated)    ■ Zero (not-illuminated)

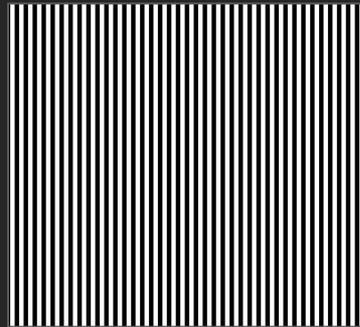
## Low-frequency patterns



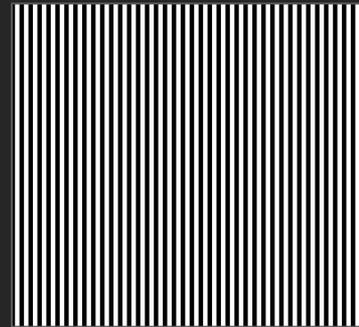
$$I = \text{Direct} + \alpha \cdot \text{Indirect} \quad \bar{I} = (1 - \alpha) \cdot \text{Indirect}$$

$$\alpha \approx 0, \text{ Direct} < \text{Indirect} \Rightarrow I < \bar{I}$$

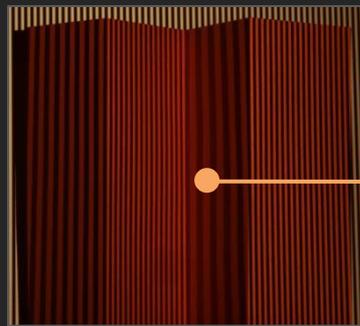
# High-frequency patterns



Pattern

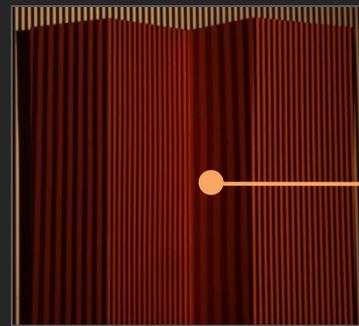


Inverse Pattern



Captured Image

$I = 0.25$



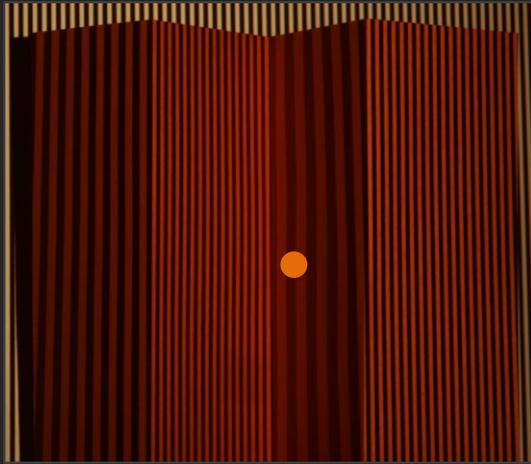
Captured Image

$\bar{I} = 0.16$

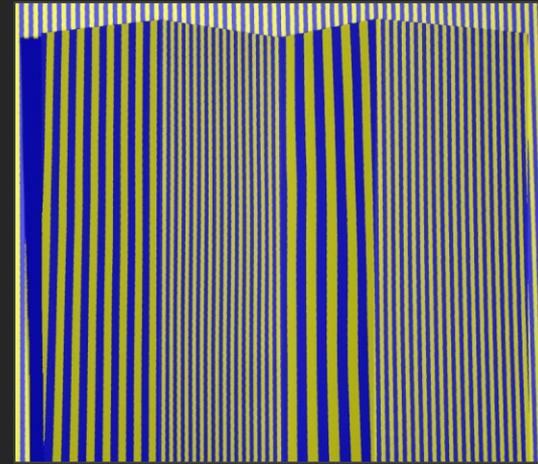
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$$I = \text{Direct} + 0.5 \text{ Indirect} > \bar{I} = 0.5 \text{ Indirect}$$

# High-frequency Patterns are Decoded Correctly



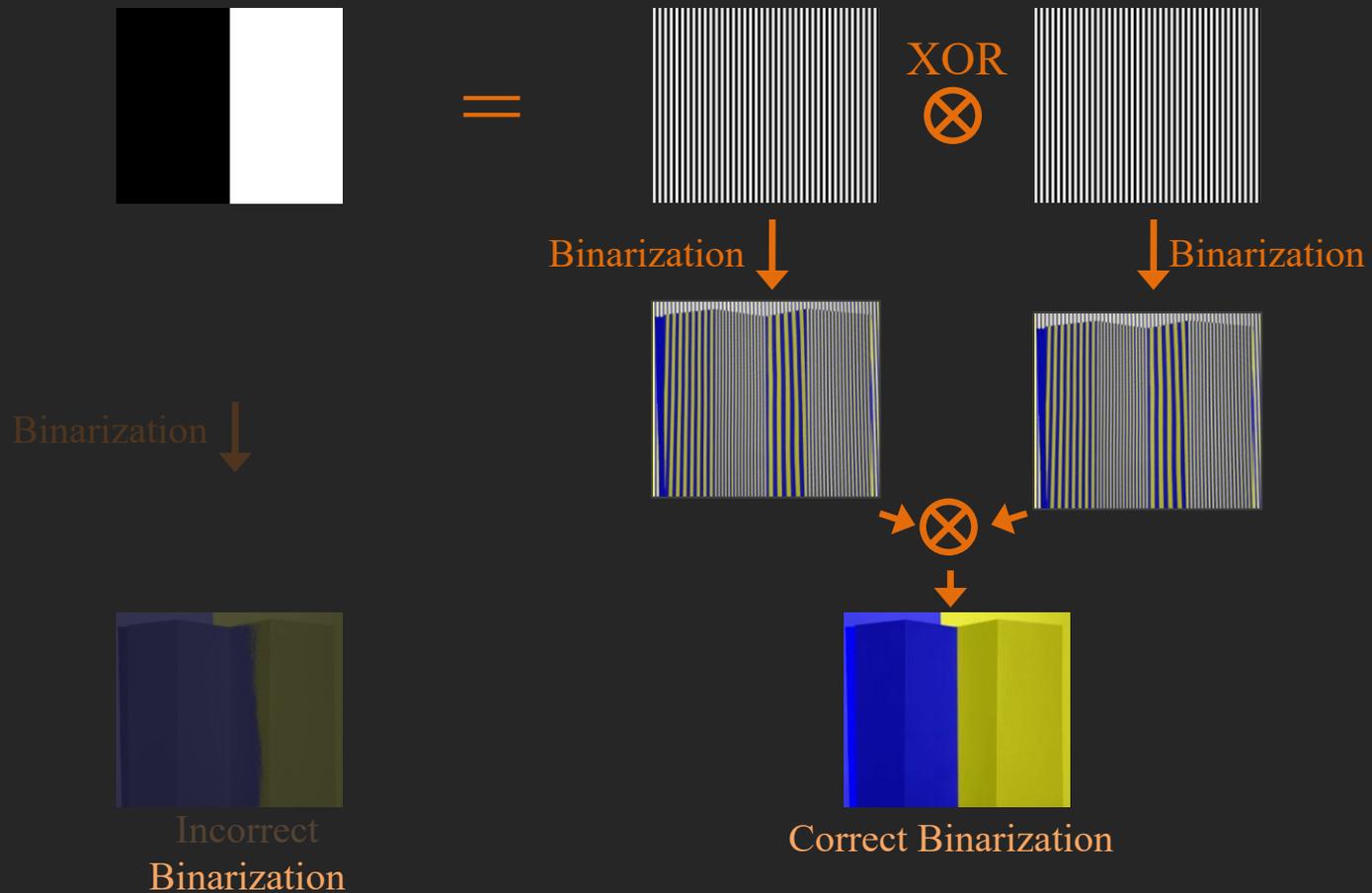
Captured Image



Binary Decoding

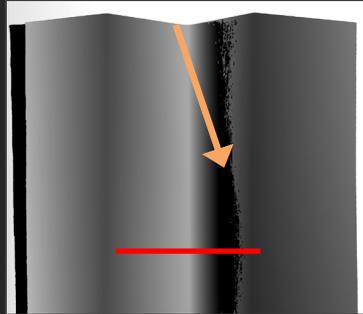
# Logical Coding and Decoding

# Logical Coding and Decoding

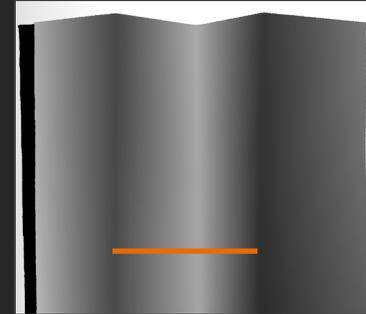


# Depth Map Comparison

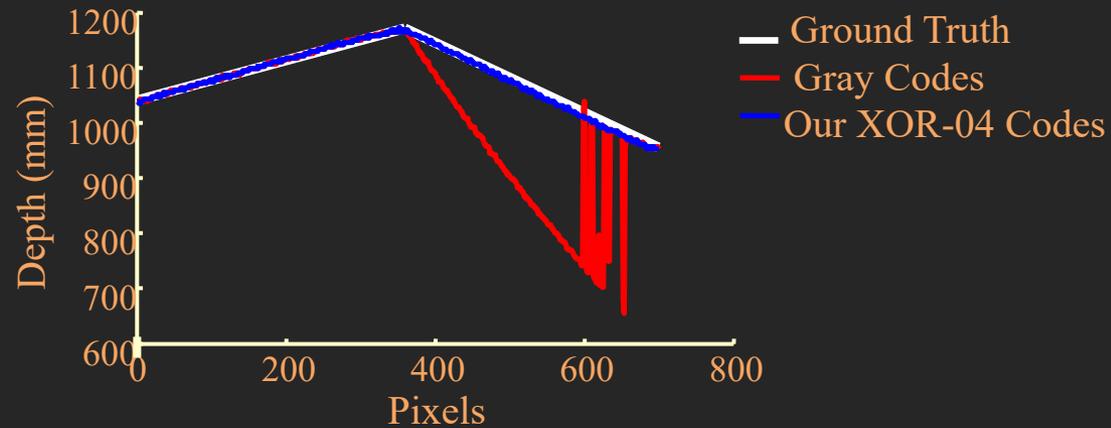
Errors due to Inter-reflections



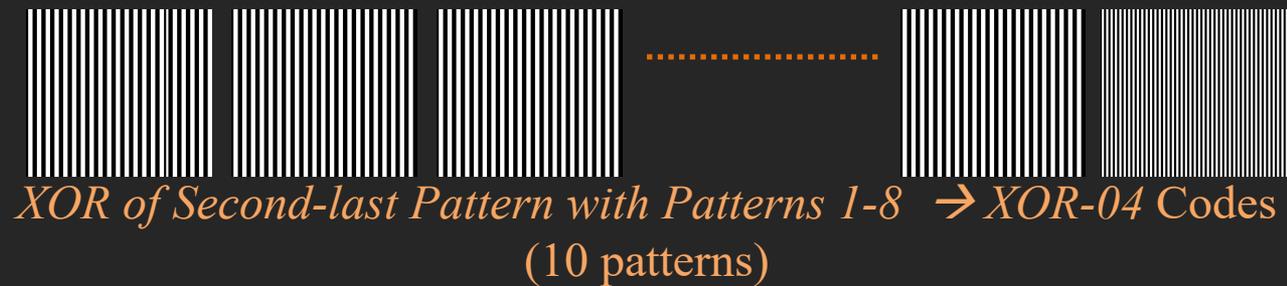
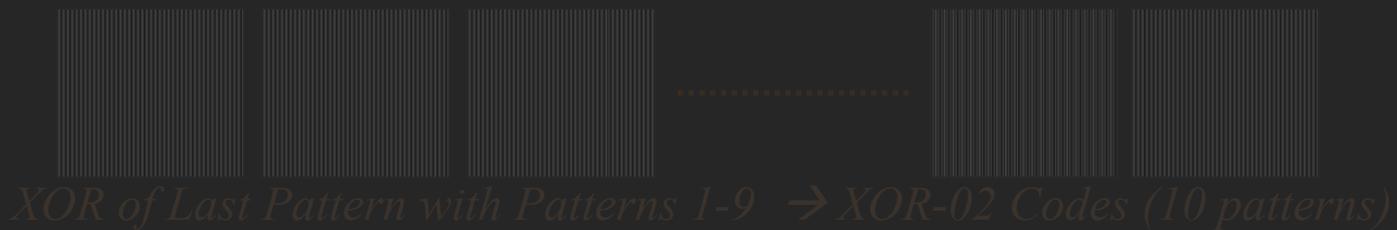
Conventional Gray Codes (11 images)



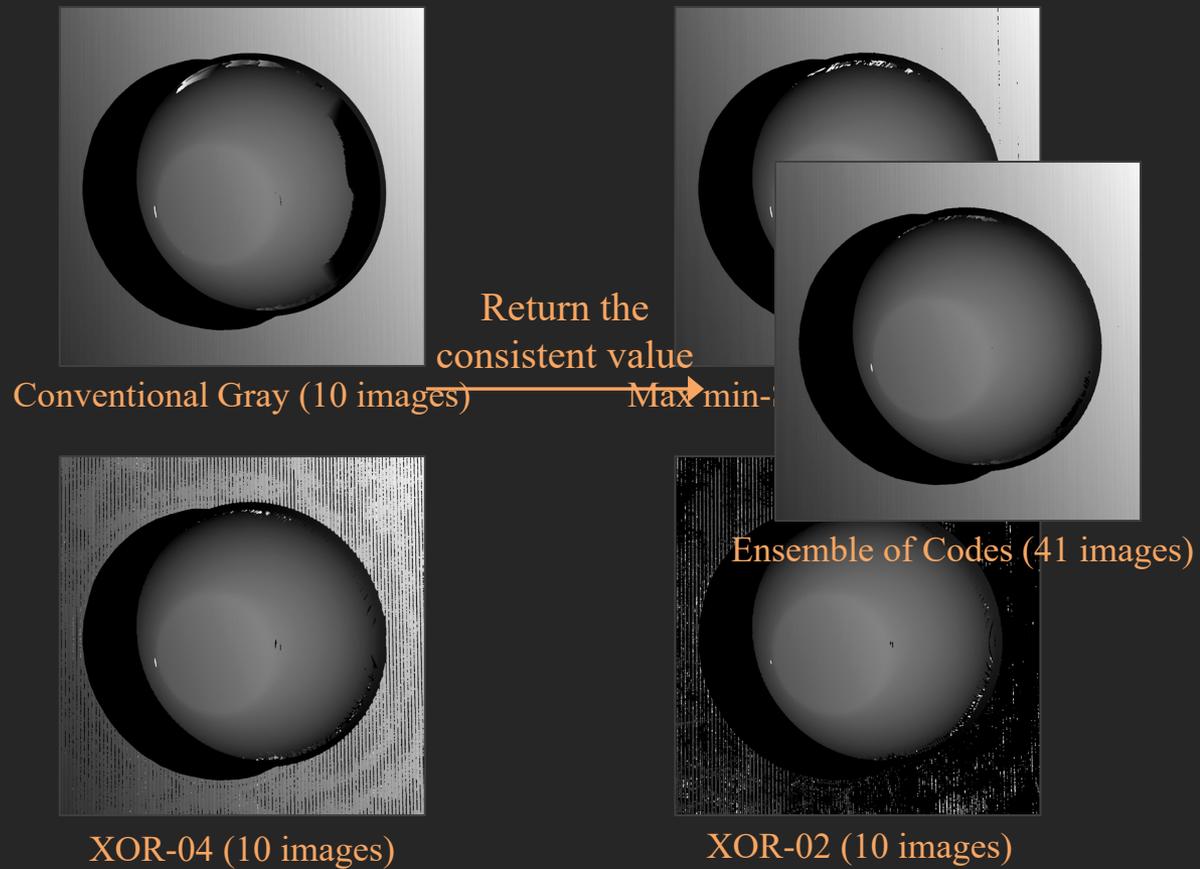
Our XOR-04 Codes (11 images)



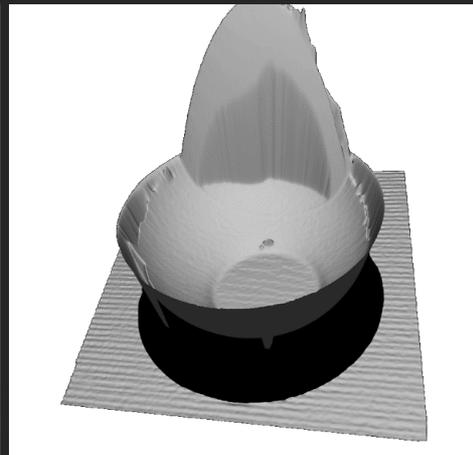
# Making the Logical XOR Codes



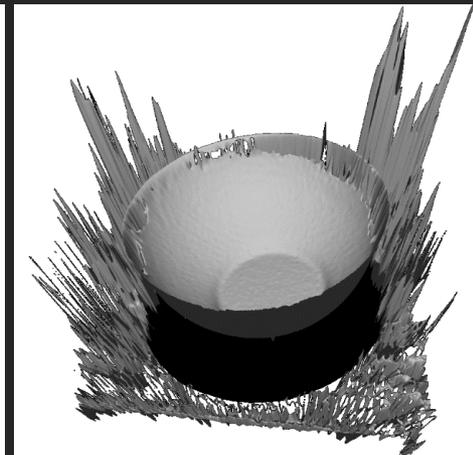
# Ensemble of Codes for General Scenes



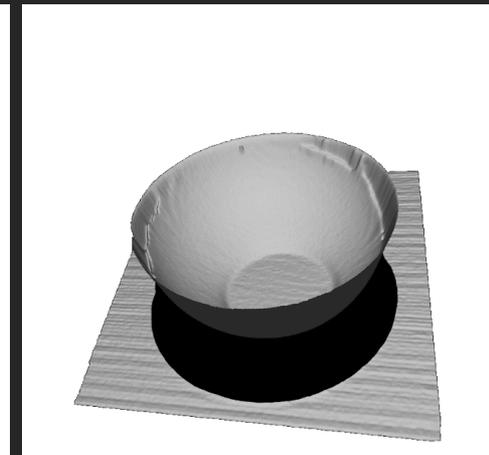
# Shape Comparison



Conventional Gray  
(11 images)



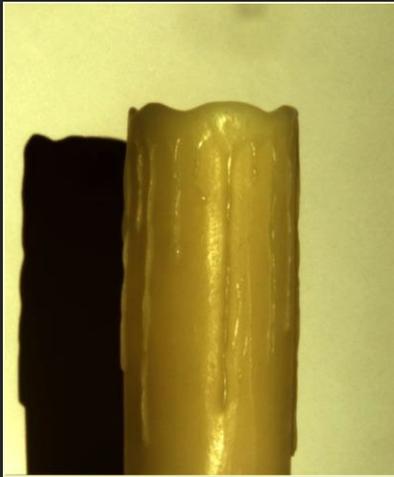
Modulated Phase-Shifting  
(162 images)



Our Technique  
(41 images)

# Translucent Wax Candle

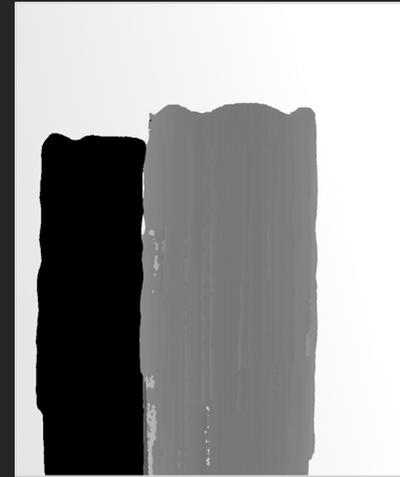
Errors due to strong  
sub-surface scattering



Scene



Modulated Phase-  
Shifting (162 images)



Our Ensemble Codes  
(41 images)

# Translucent Wax Object

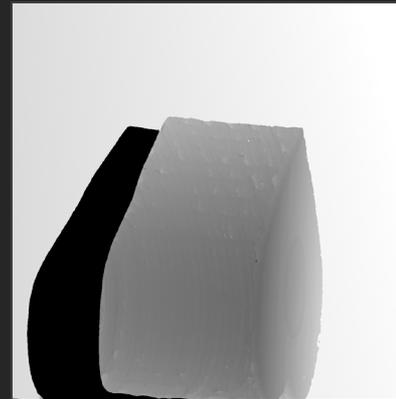
Errors due to strong  
sub-surface scattering



Scene



Modulated Phase-  
Shifting (162 images)



Our Ensemble Codes  
(41 images)

## Depth-Map Comparison



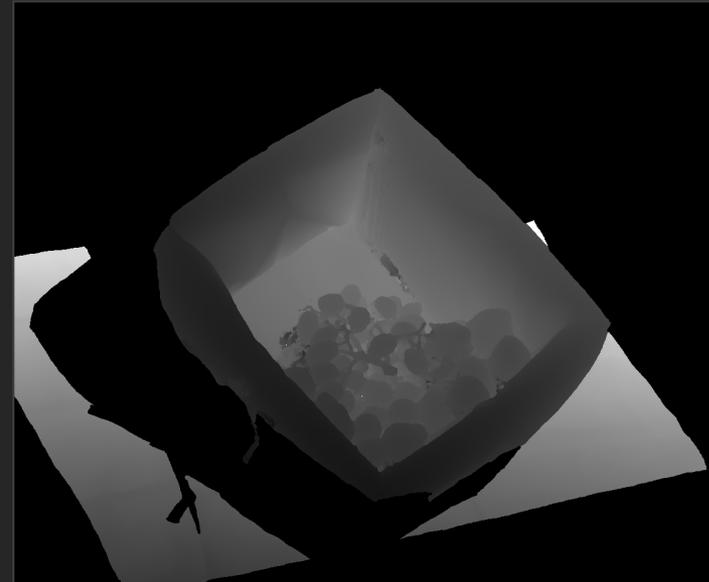
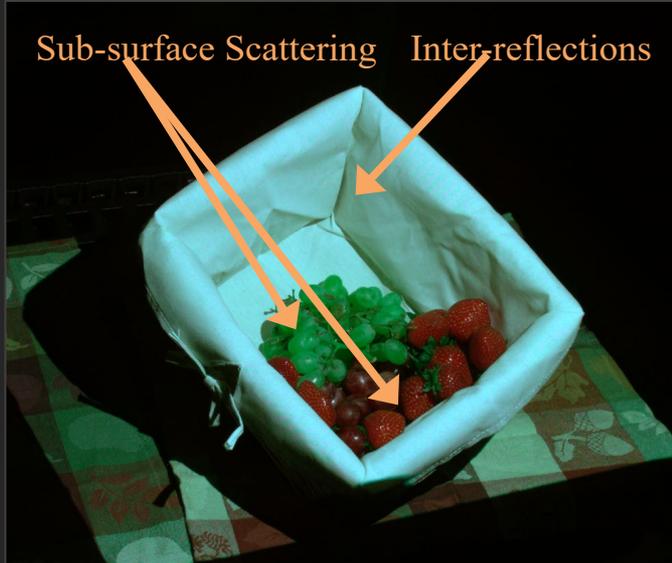
Regular Gray Codes (11 images)



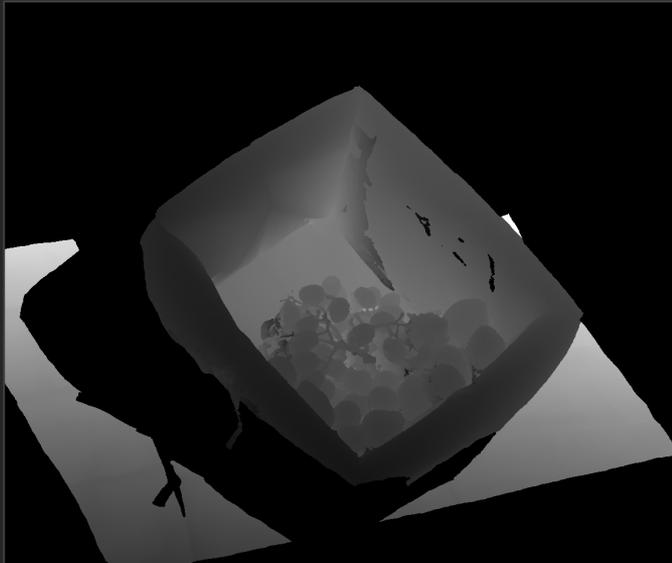
Our Ensemble Codes (41 images)

# Fruit Basket: Multiple Effects

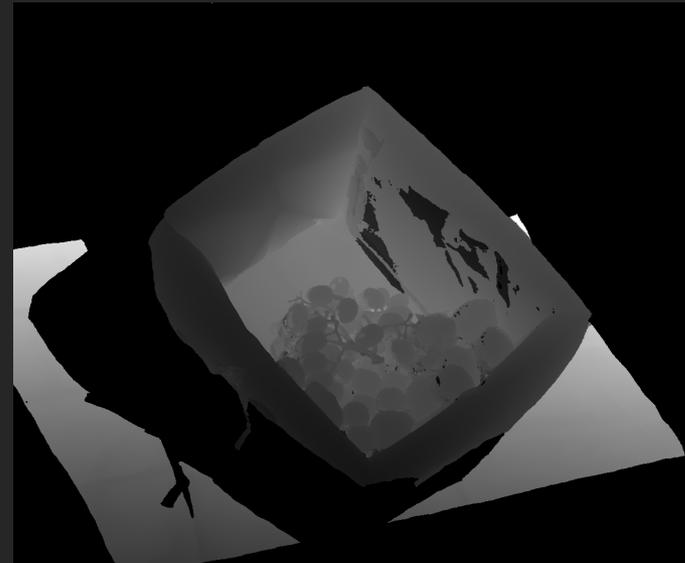
# Depth-maps with previous state of the art



Our Ensemble Codes (41 images)

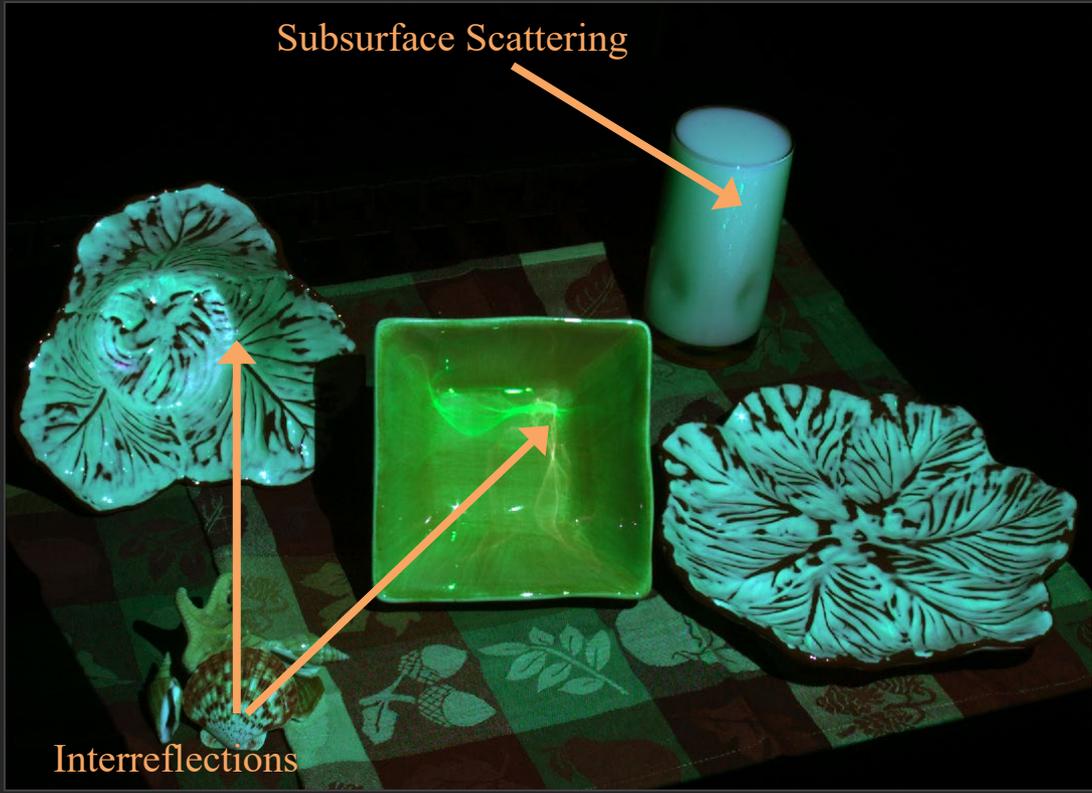


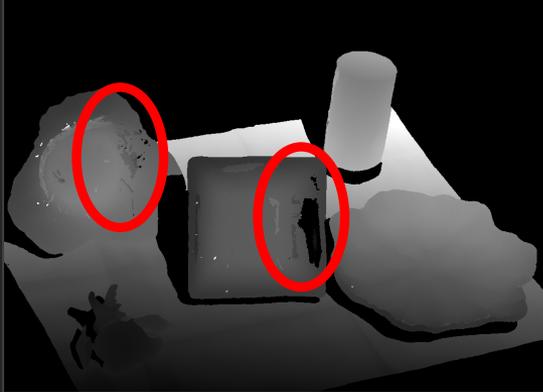
Regular Gray (11 images)



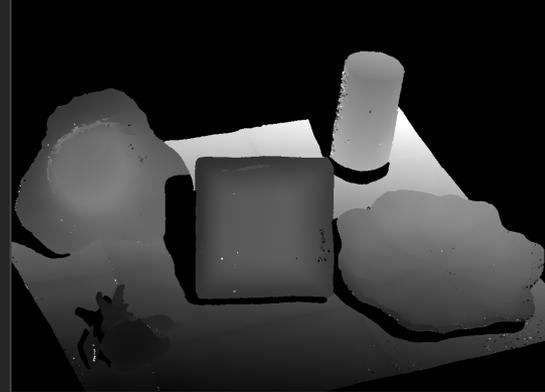
Phase-Shifting (18 images)

# Bowls and Milk: Multiple Effects

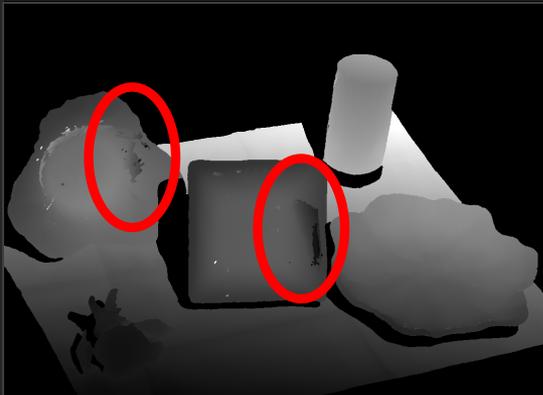




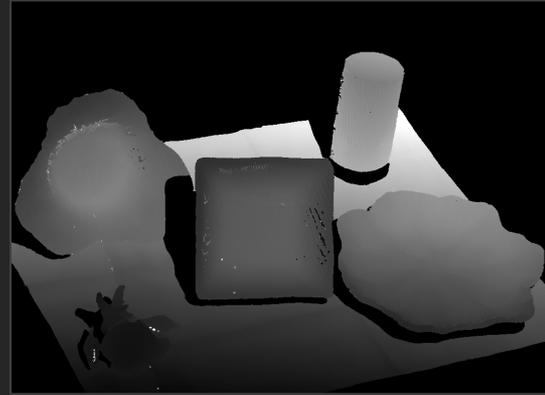
Phase-Shifting (18 images)



Modulated Phase-Shifting (162 images)

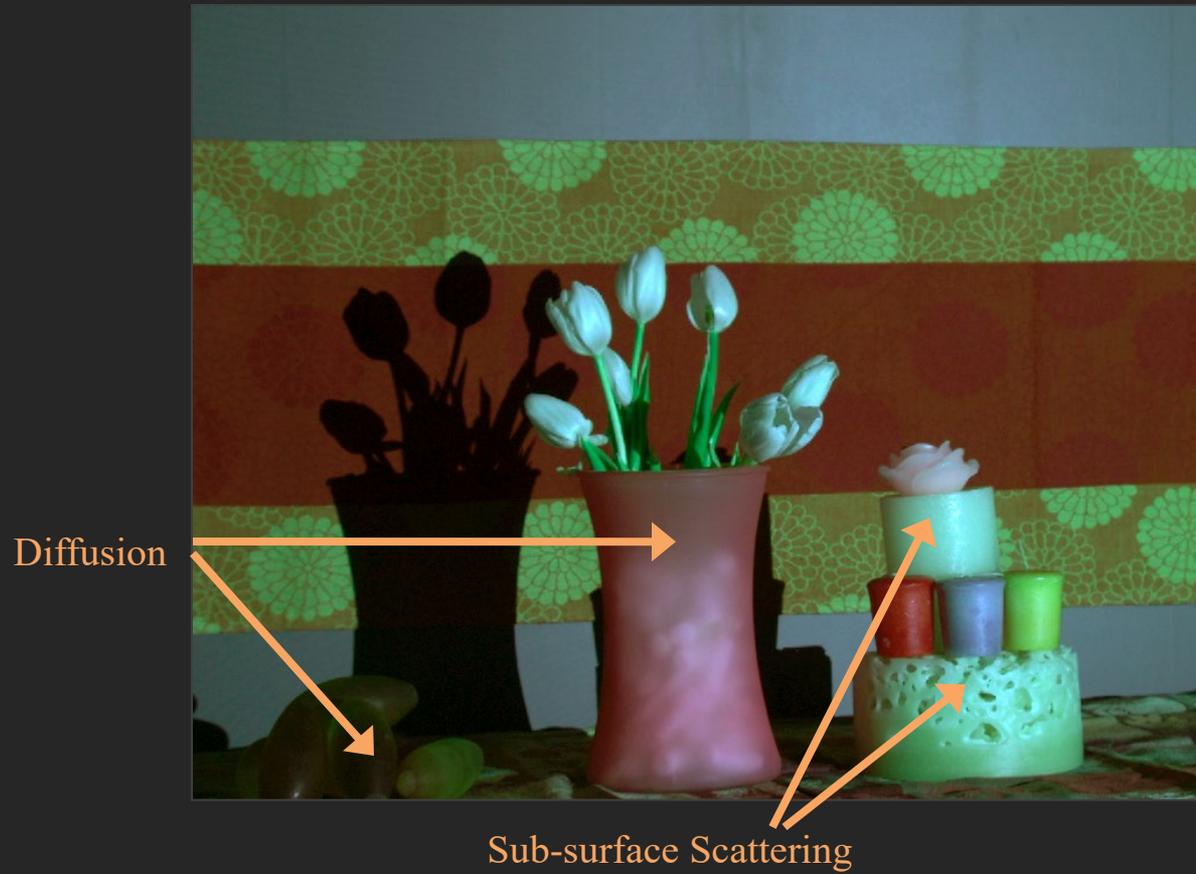


Regular Gray Codes (11 images)

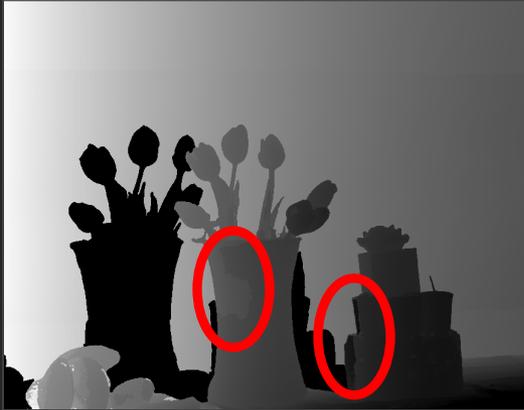


Our XOR Codes (11 images)

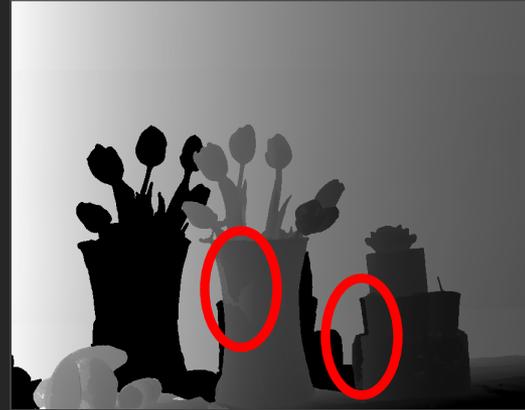
# Flower-Vase



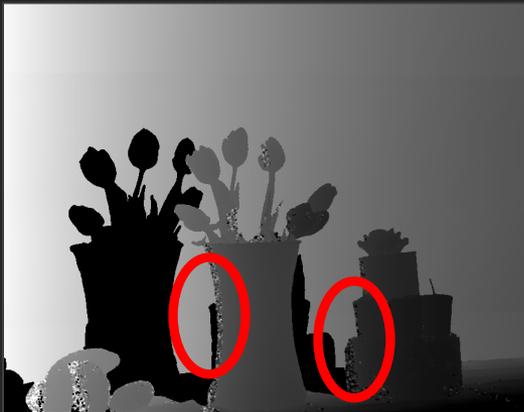
# Comparison



Phase-Shifting (18 images)



Regular Gray Code (11 images)



Modulated Phase-Shifting (162 images)

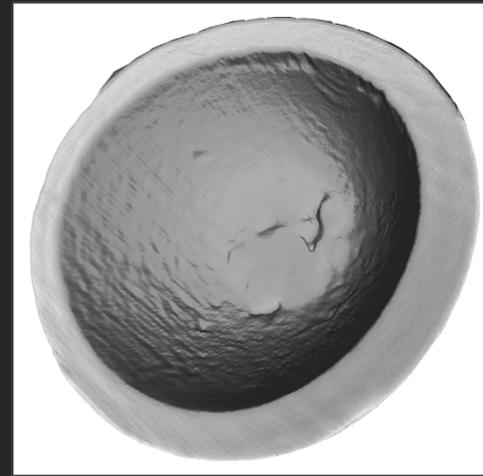


Our Ensemble Codes (41 images)

## Multiple Indirect Illumination Effects



Wax Bowl

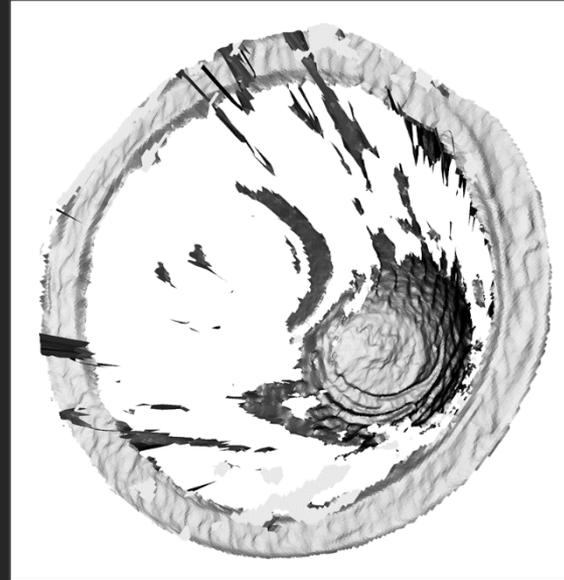


Shape Using Ensemble Codes

## Multiple Indirect Illumination Effects



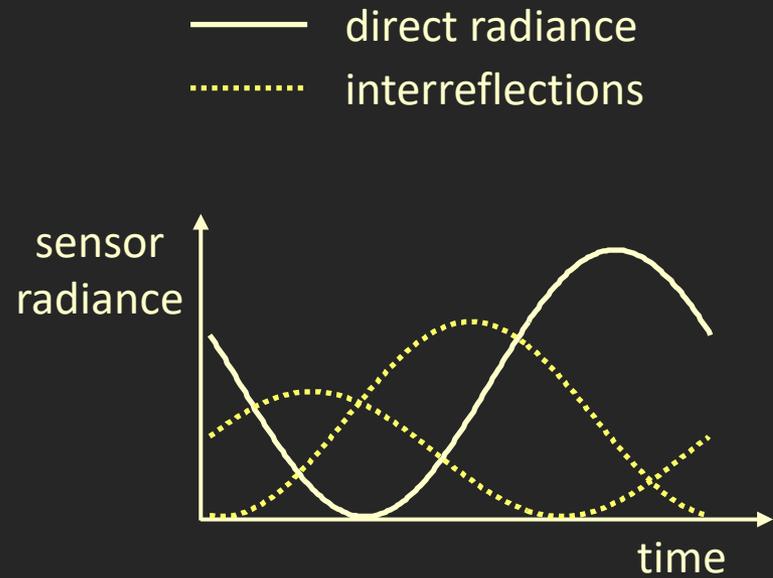
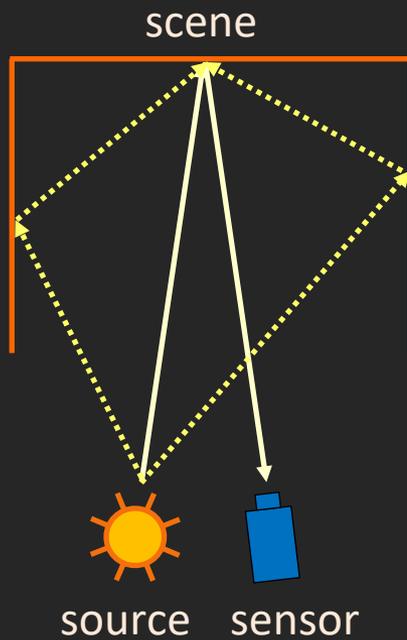
Deep Wax Container



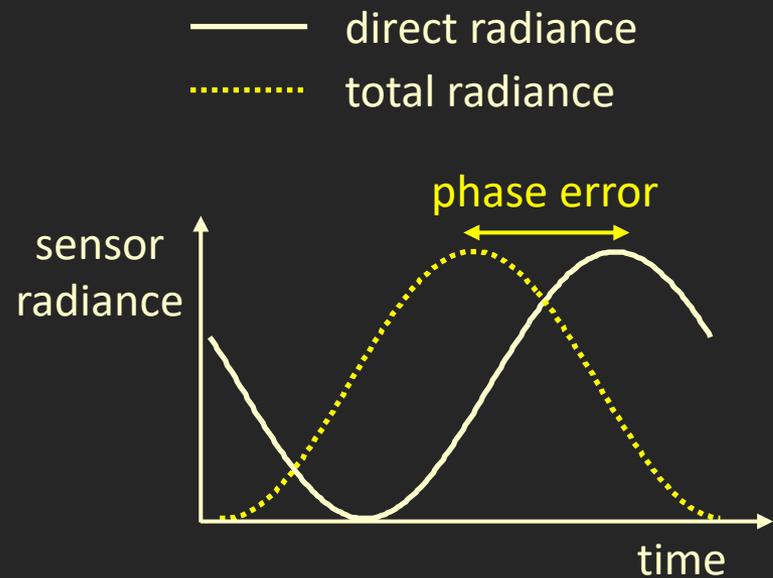
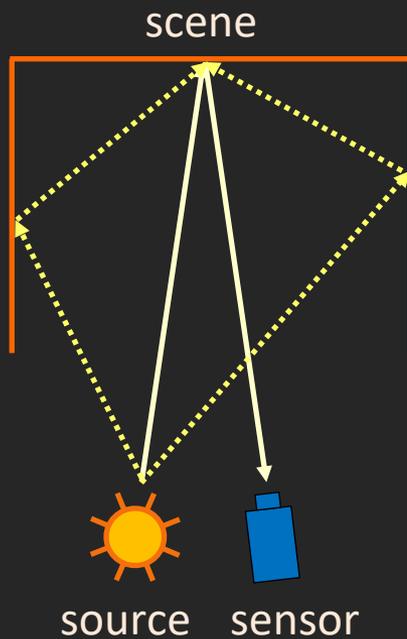
Shape Using Ensemble Codes

Application to time-of-flight imaging

# Interreflections and ToF Imaging

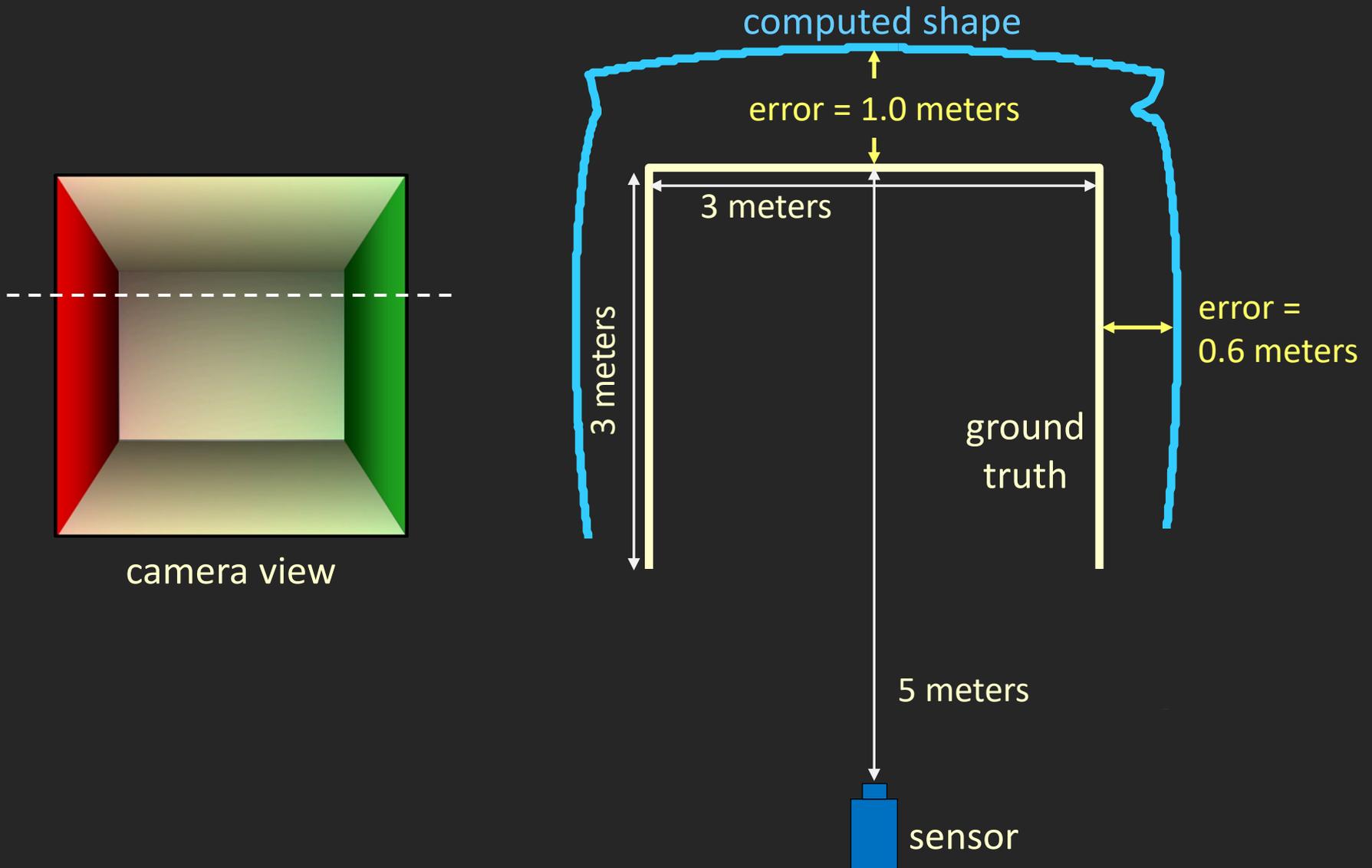


# Interreflections and ToF Imaging

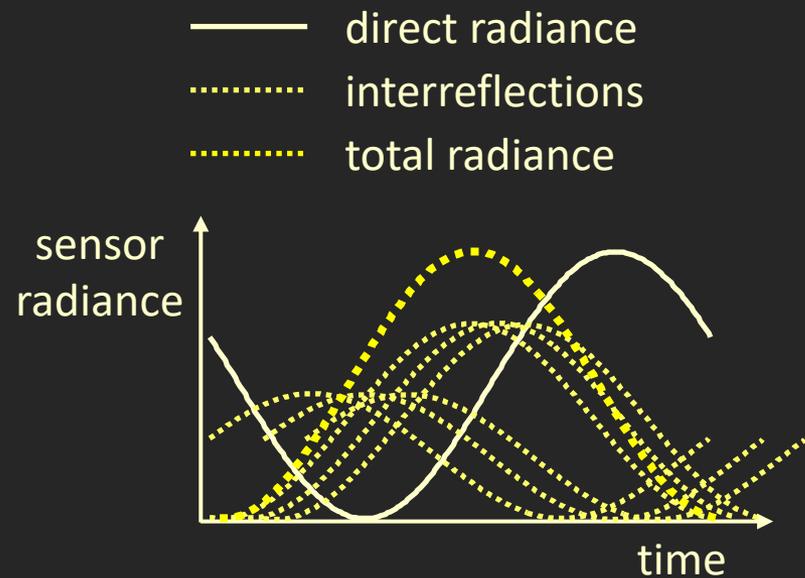
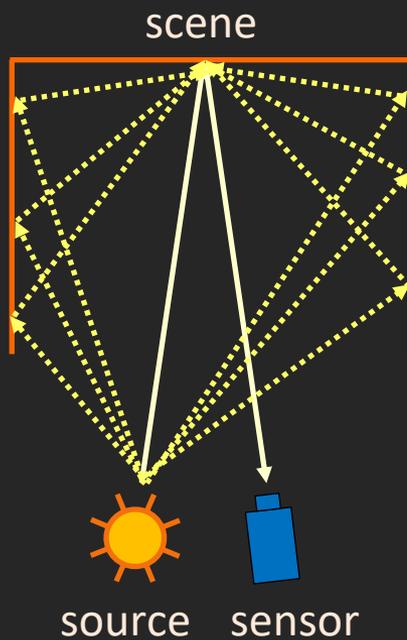


Interreflections Produce Incorrect Phase

# Errors in Shape Recovery

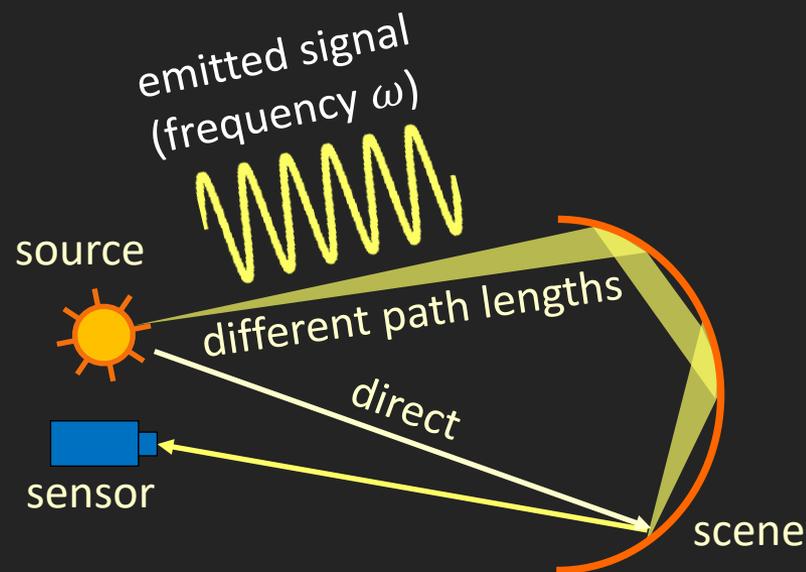


# Multipath Interference: Existing Work

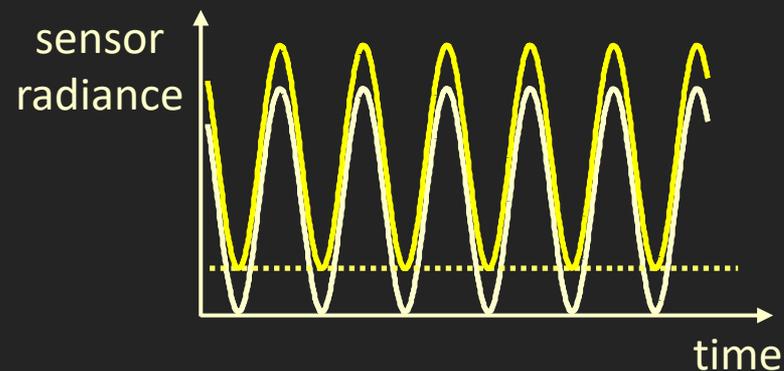


How To Separate Different Components?

# Interreflections vs. Modulation Frequency

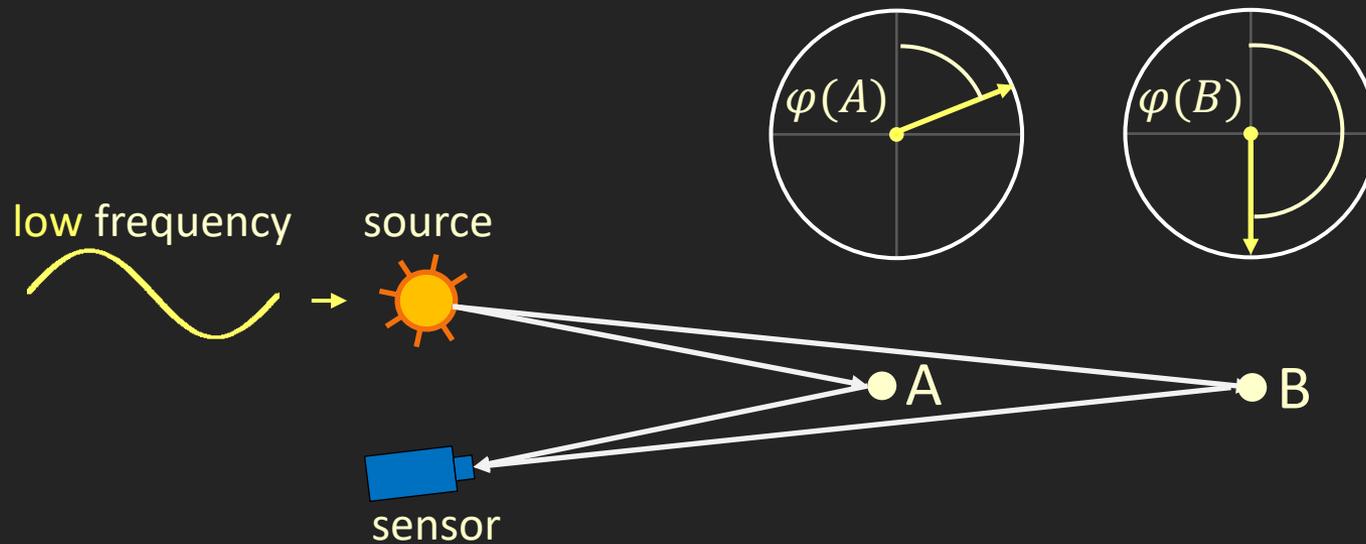


— total radiance  
 — direct radiance  
 ..... interreflection



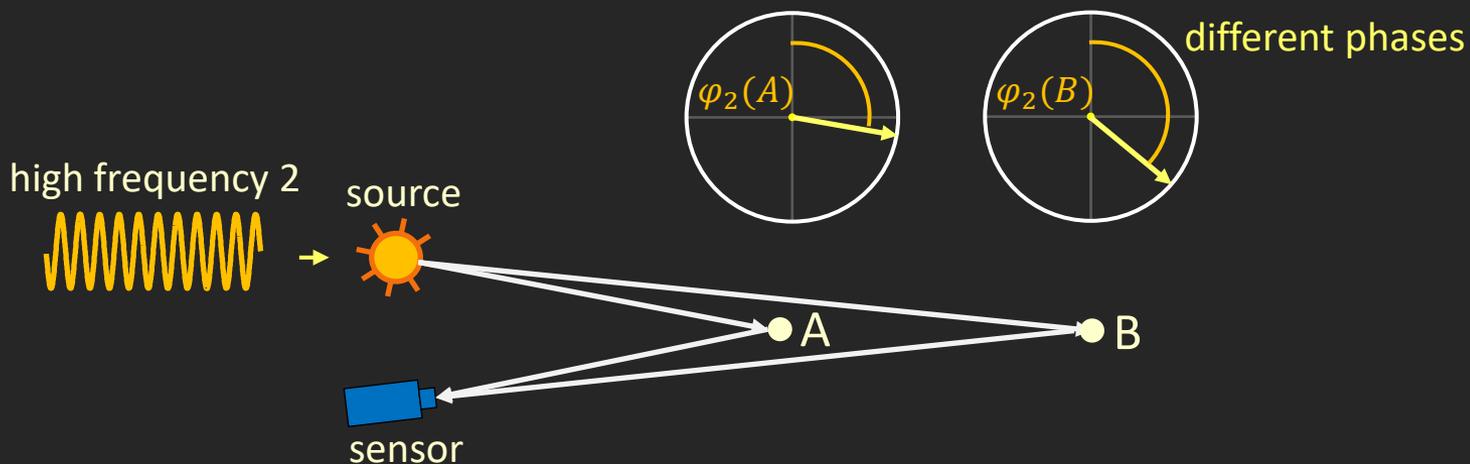
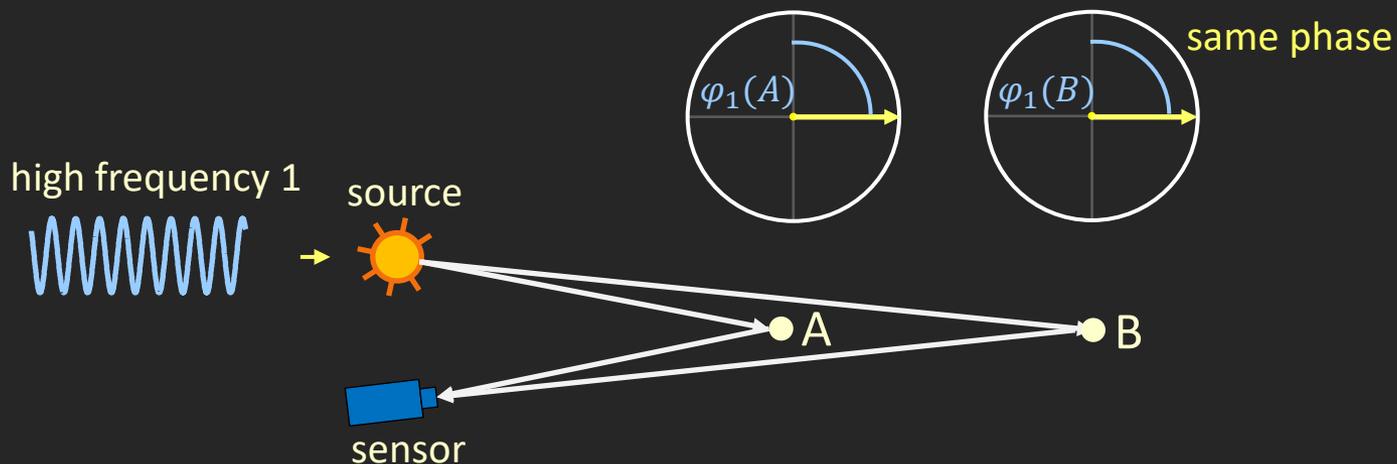
For High Temporal Frequency  
 Interreflections Do Not Affect Phase

# Phase Ambiguity



Unambiguous Depth Range:  $R_{unambiguous} = \frac{1}{2\omega}$

# Disambiguating Phase

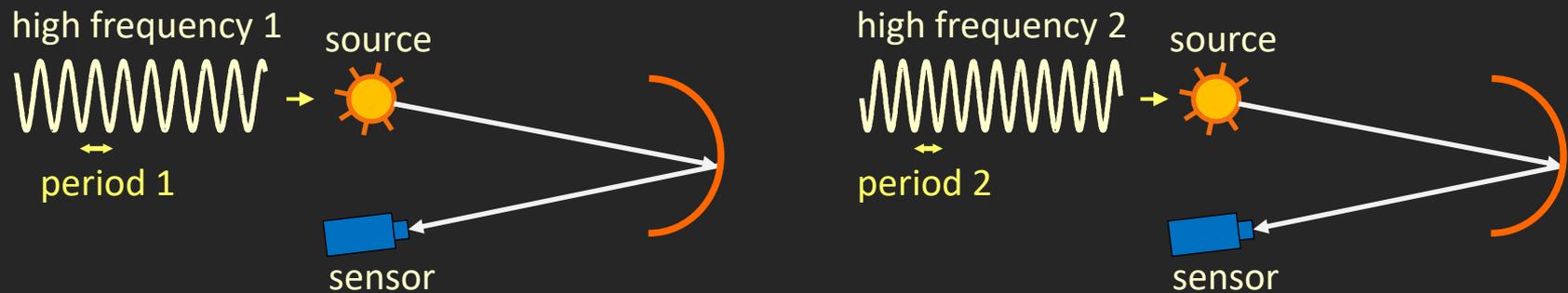


Compute Phases at Two High Frequencies

[Jongenelen *et al.* 2010, 2011]

# Micro Time-of-Flight Imaging

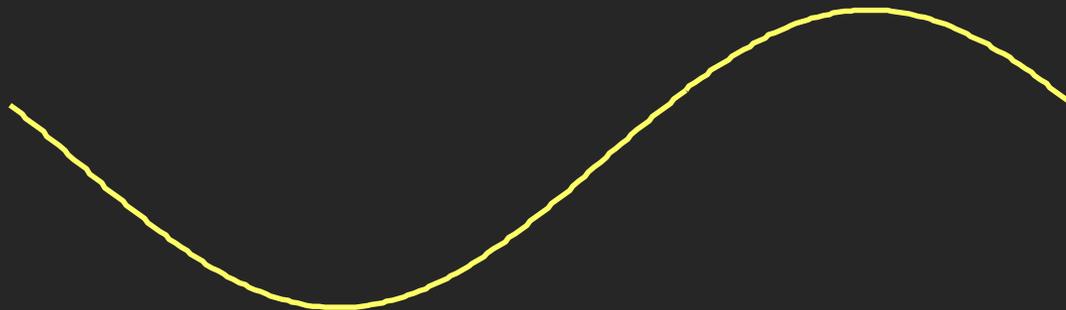
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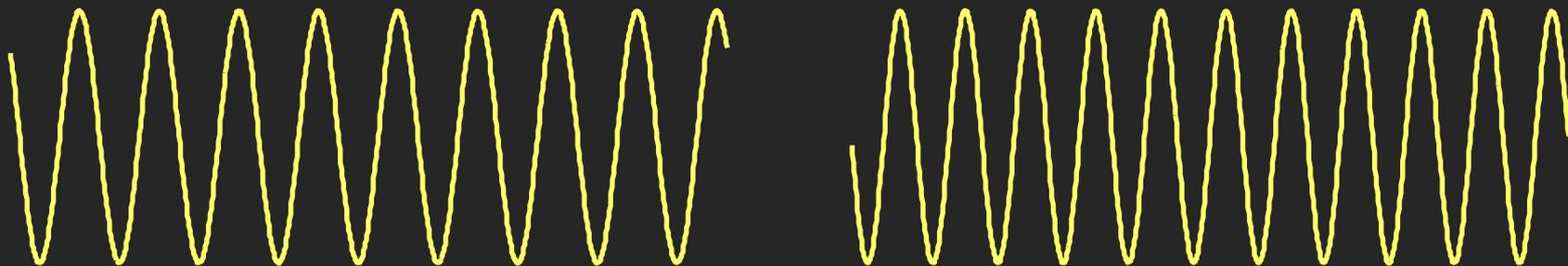
Modulation Signals With Micro (Small) Periods

# Conventional vs. Micro ToF Imaging

---

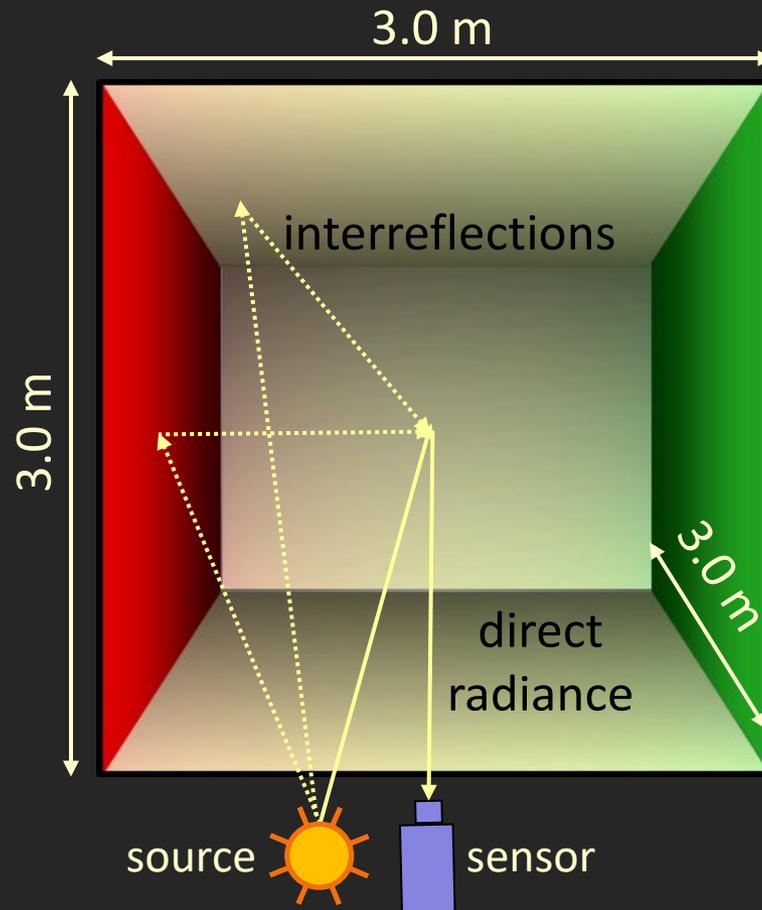


Conventional ToF Shifting: One Low Frequency  
Three Measurements



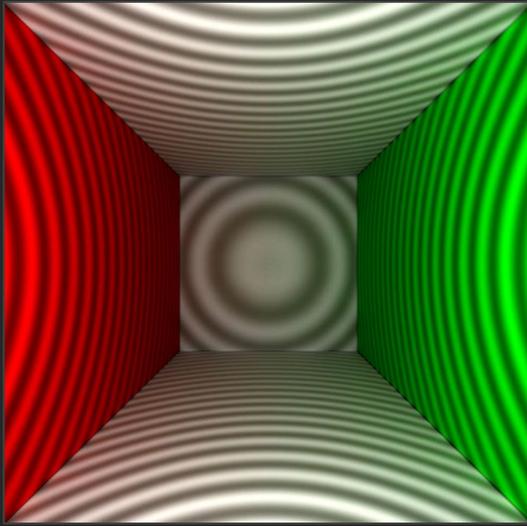
Micro ToF Shifting: Two High Frequencies  
Four Measurements

# Simulations: Cornell Box

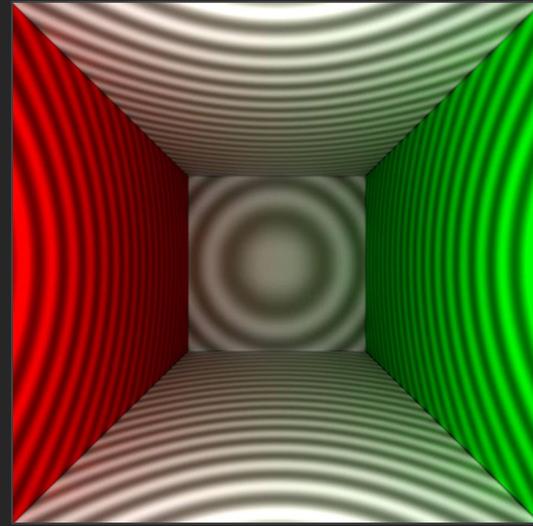


# Cornell Box: Input Images

---

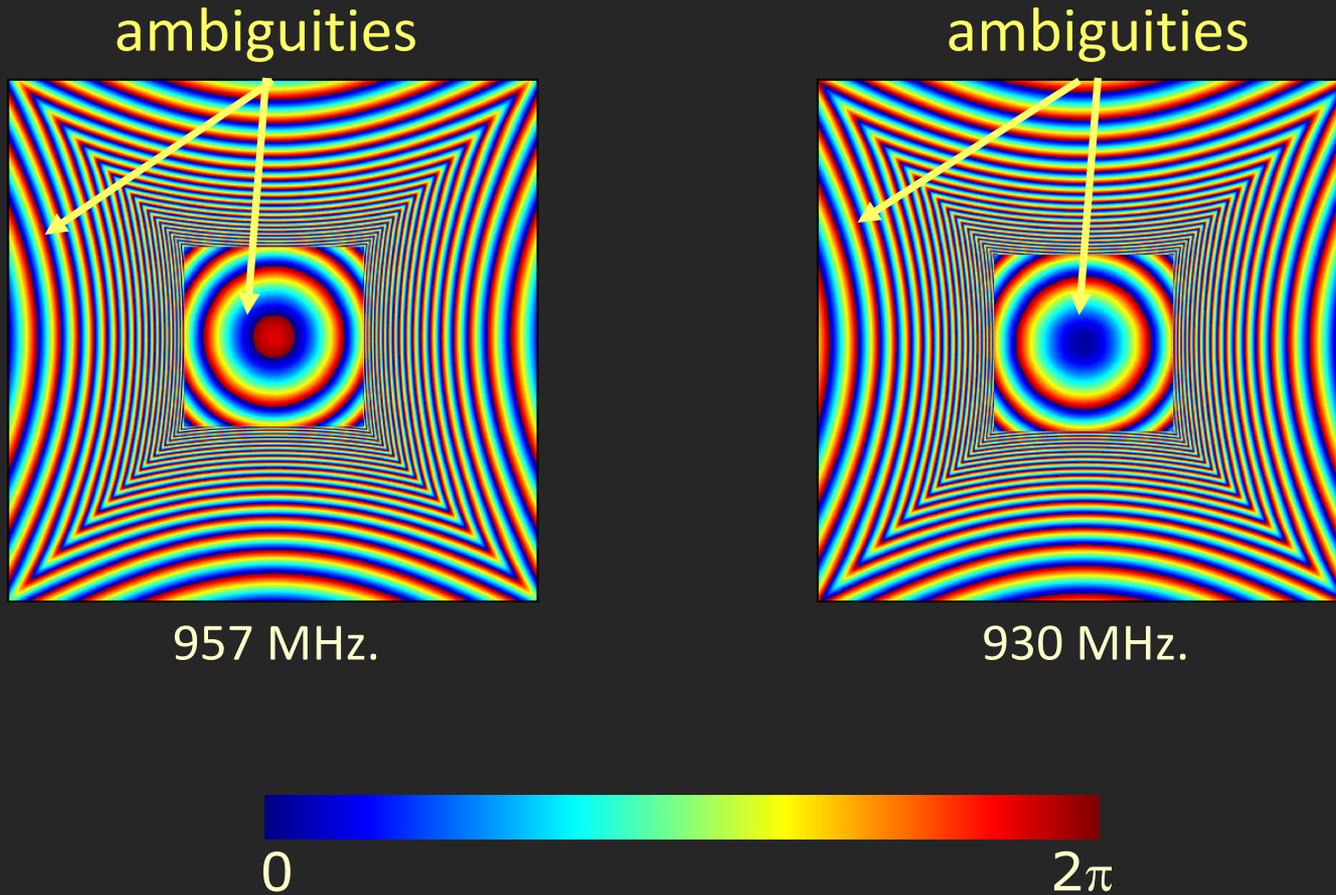


957 MHz.

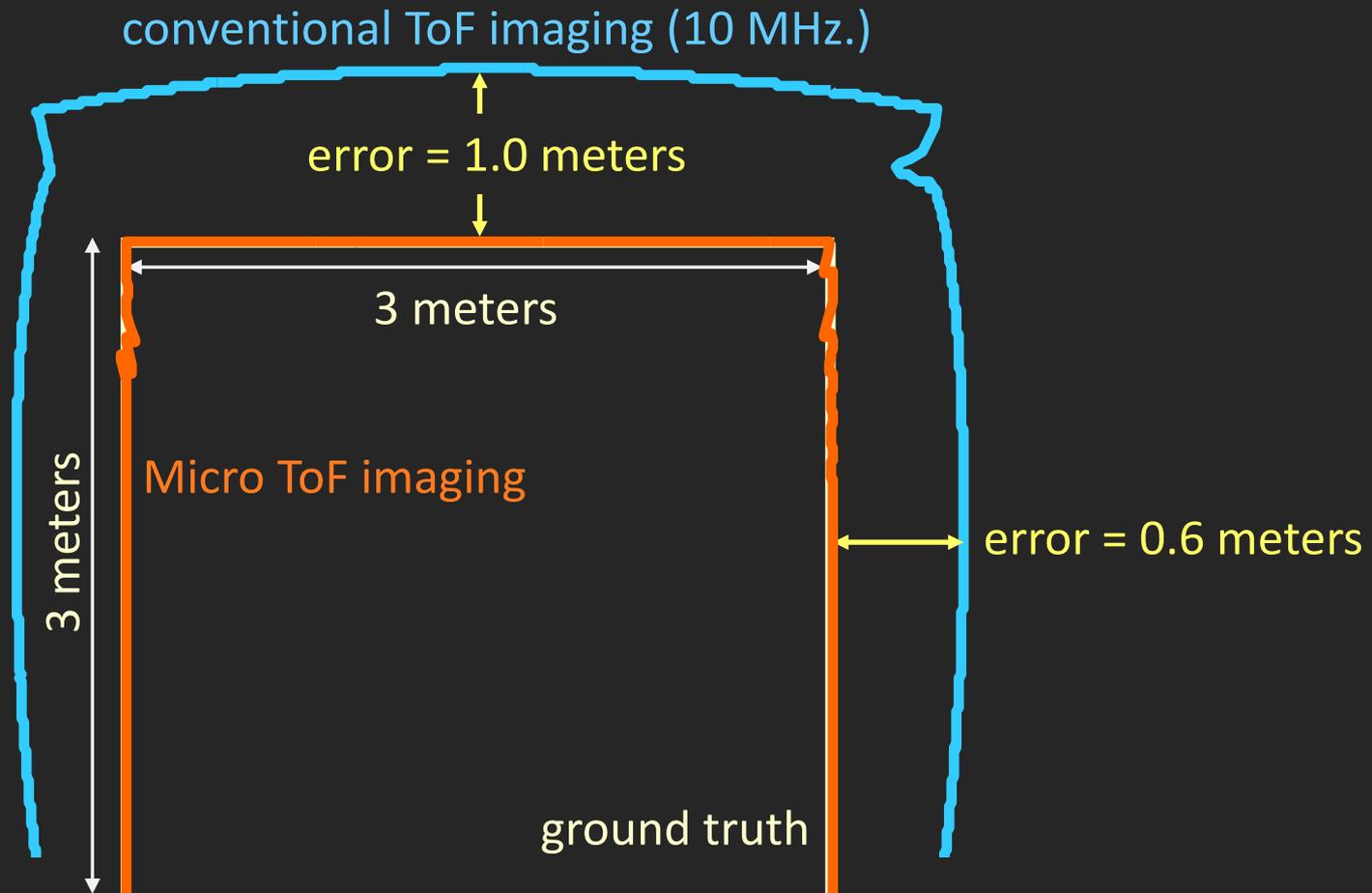


930 MHz.

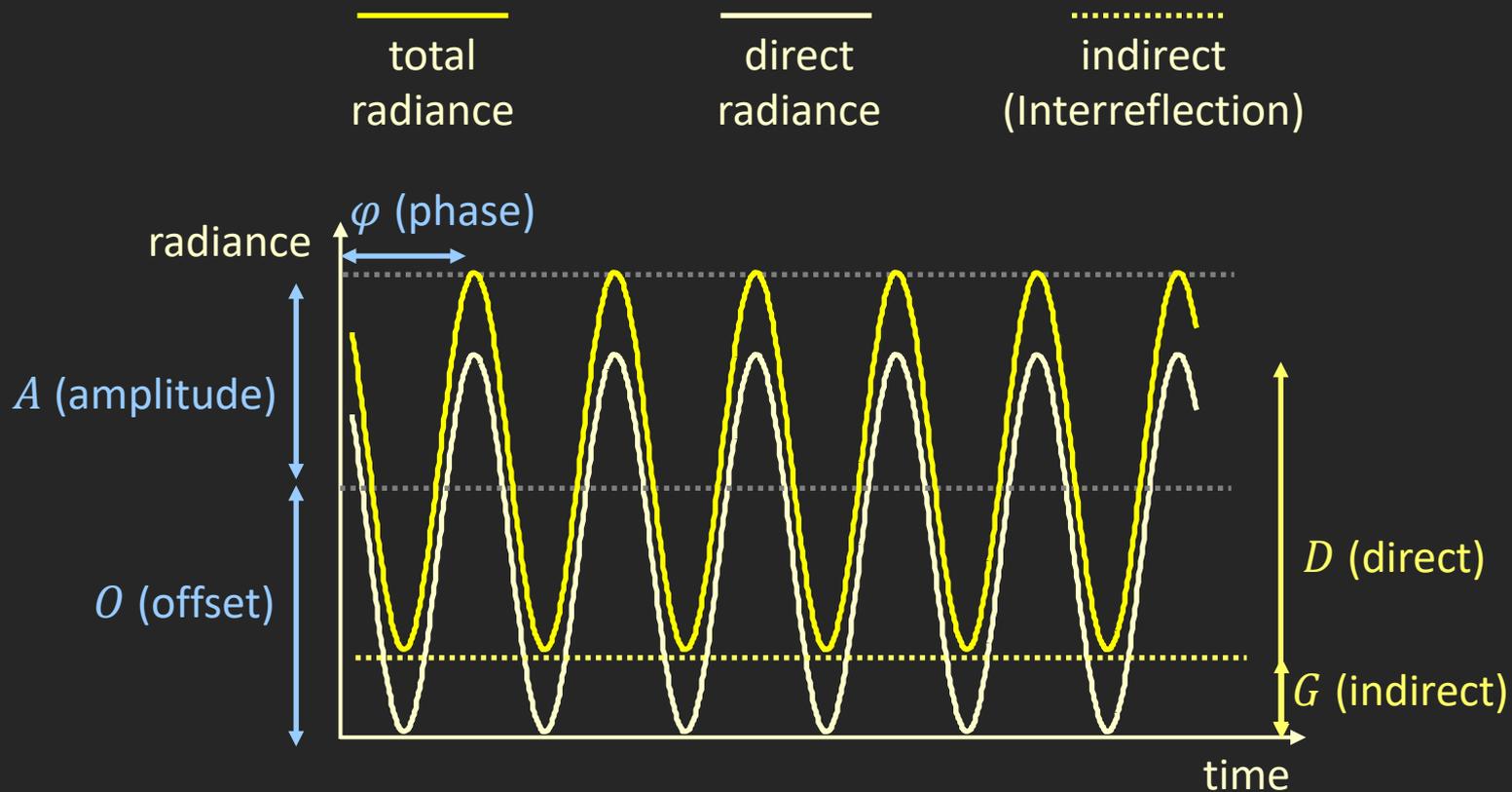
# Cornell Box: Phase Maps



# Cornell Box: Shape Comparison



# Direct-Indirect Separation

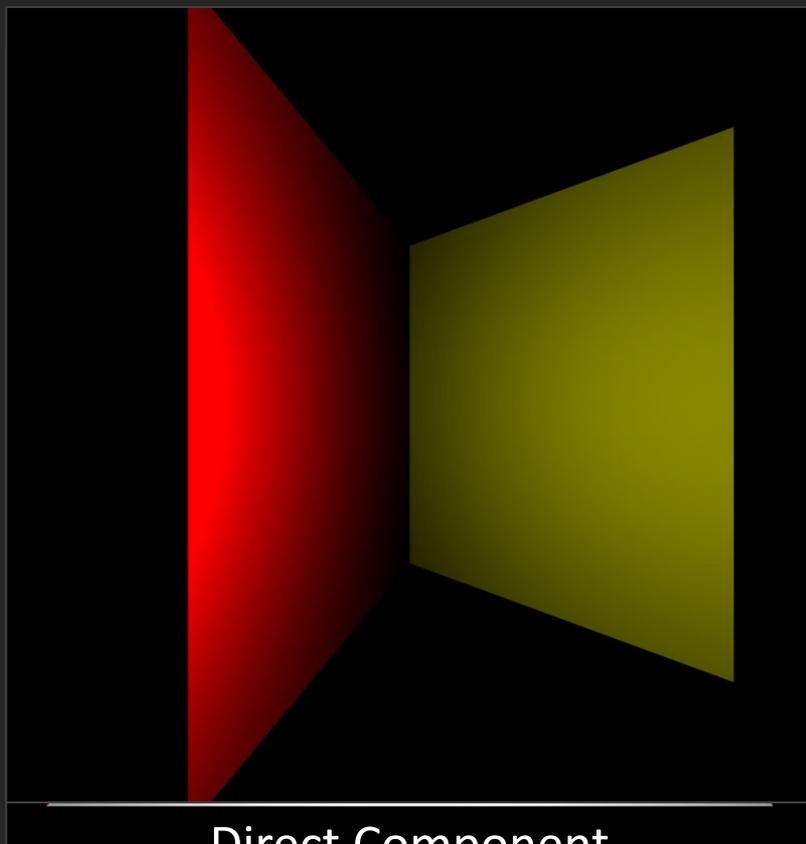


$$D = 2A \quad G = O - A$$

Direct-Indirect Separation Using Three Measurements

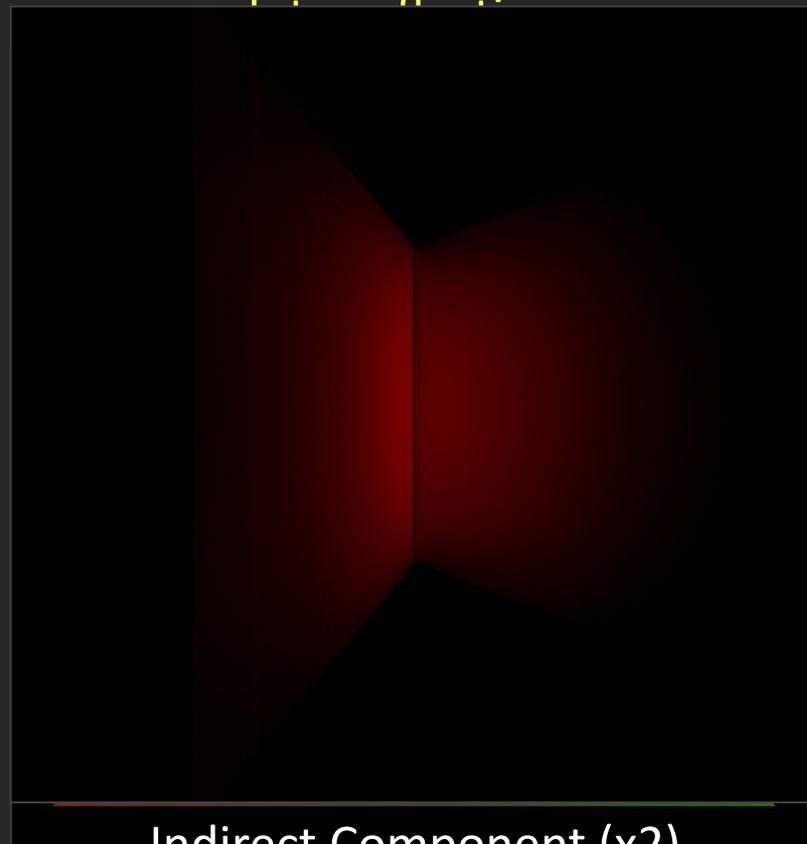
# Direct-Indirect Separation

---



Direct Component  
Direct Component

Color Bleeding due to



Indirect Component (x2)  
Indirect Component (x2)

# Experimental Setup

light source  
(bank of laser diodes)

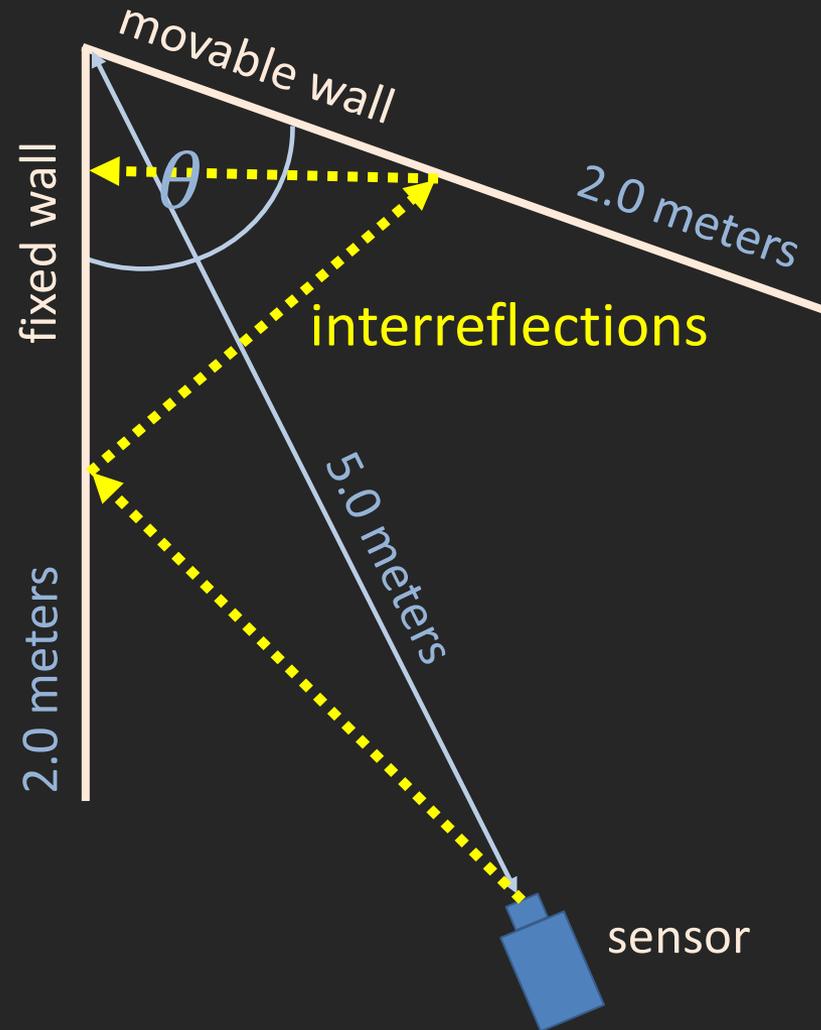


sensor  
(PMD CamBoard Nano)



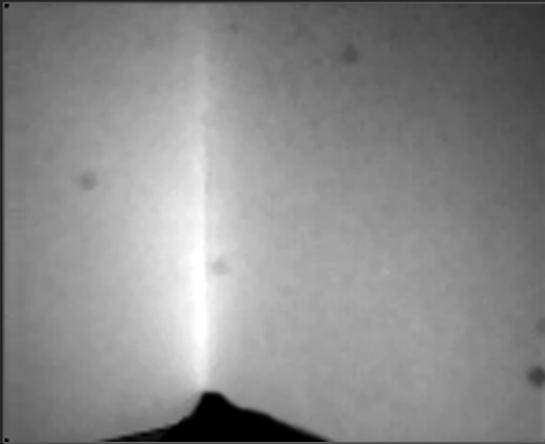
Maximum System Modulation Frequency = 125 MHz.

# Experiments: V-Groove

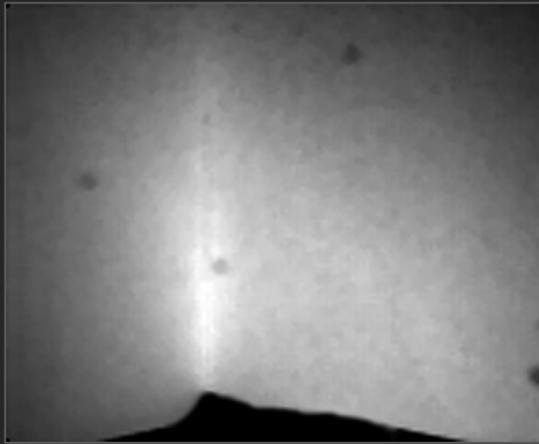


# Scene Images Captured By PMD Sensor

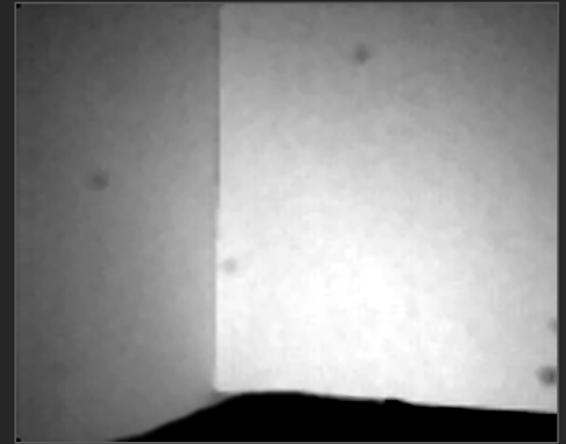
---



apex angle = 45°



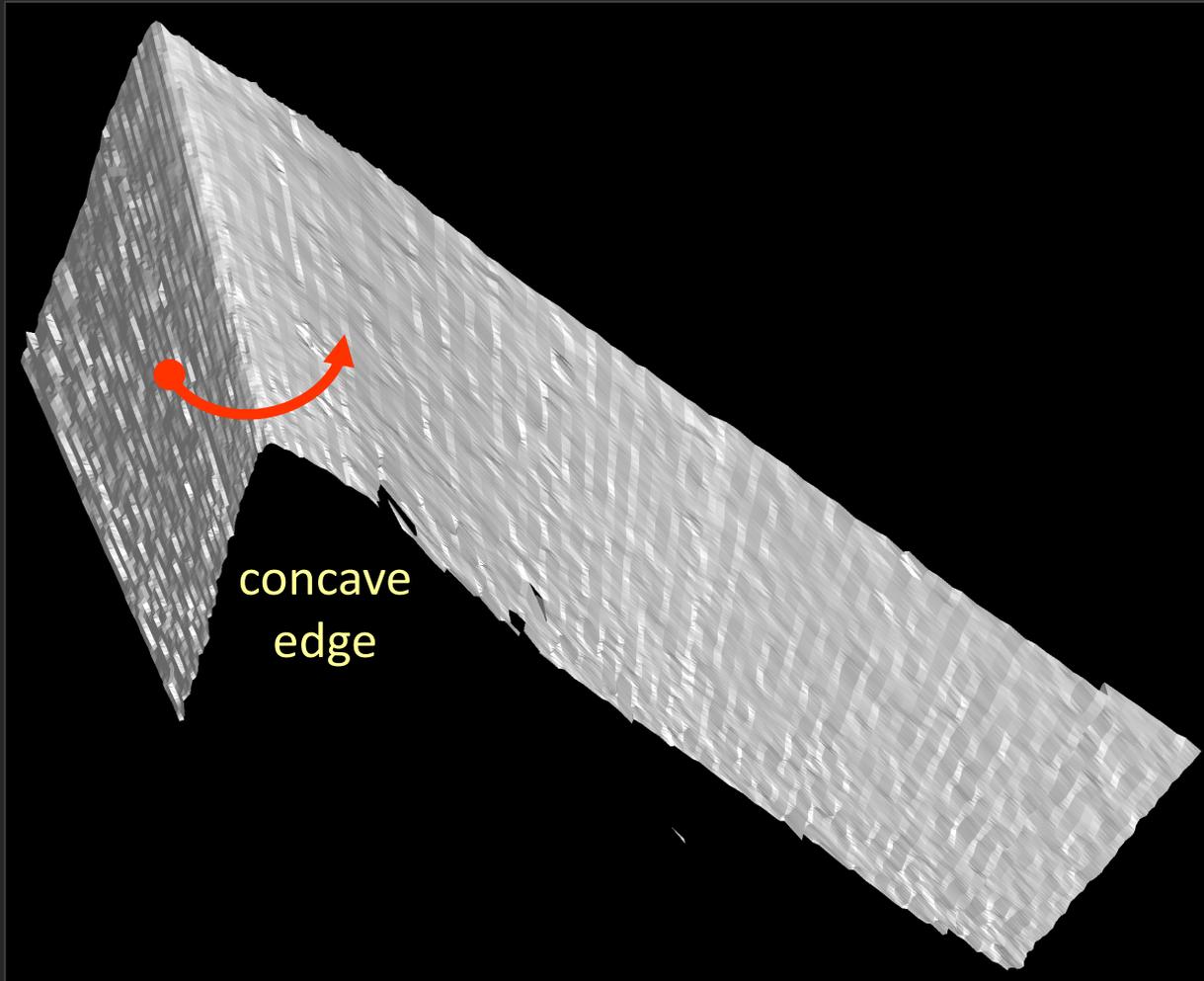
apex angle = 60°



apex angle = 90°

image resolution = 120 x 165

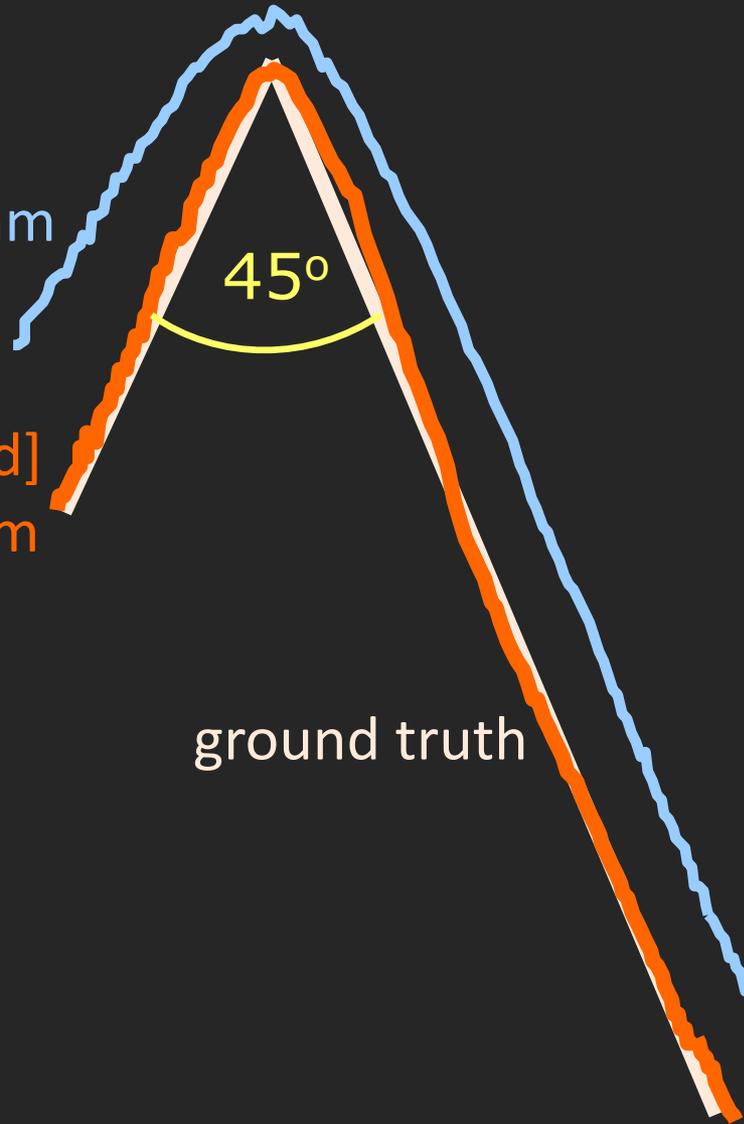
# Reconstructed Shape using Micro ToF



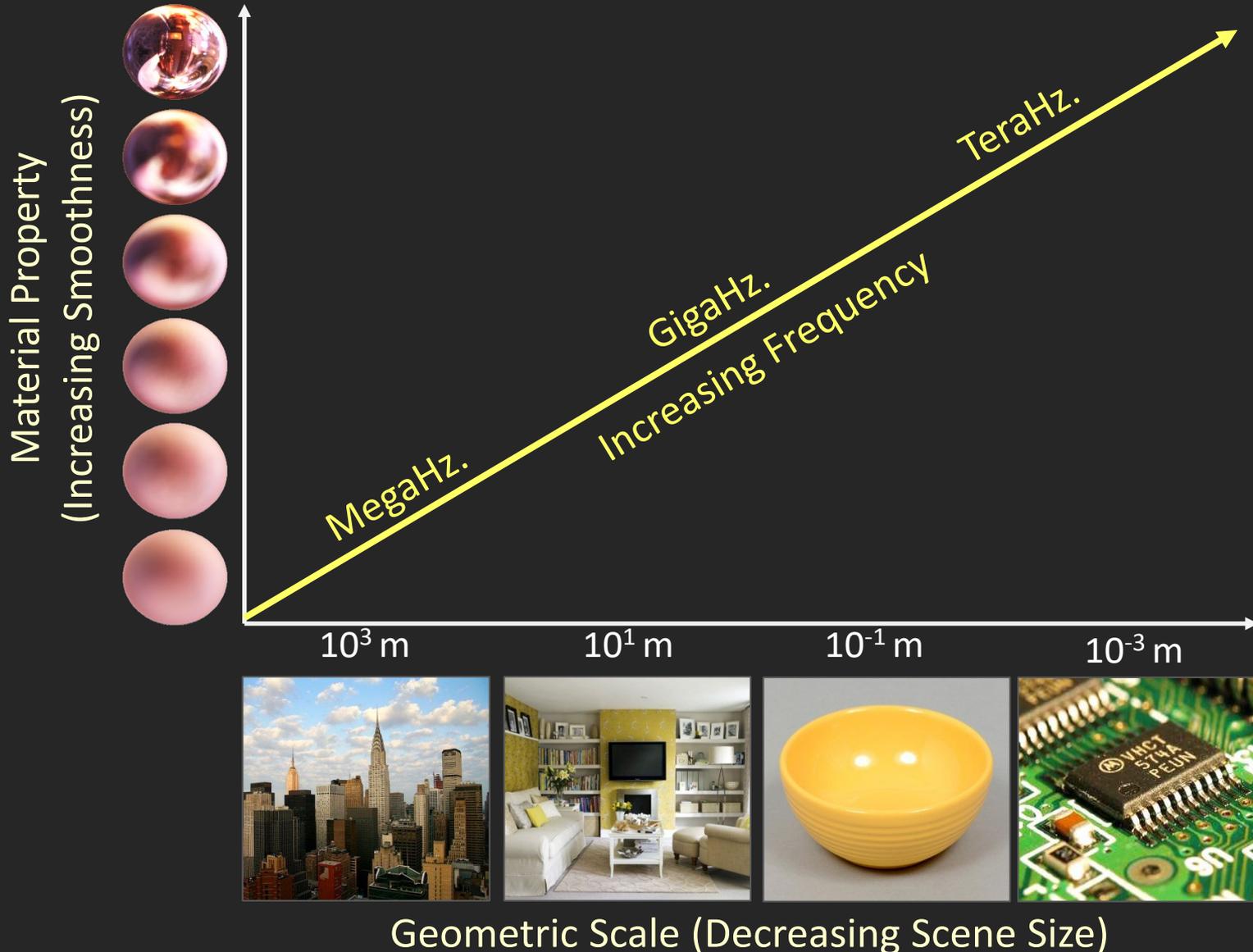
# Shape Comparisons

conventional ToF  
mean error = 86.6 mm

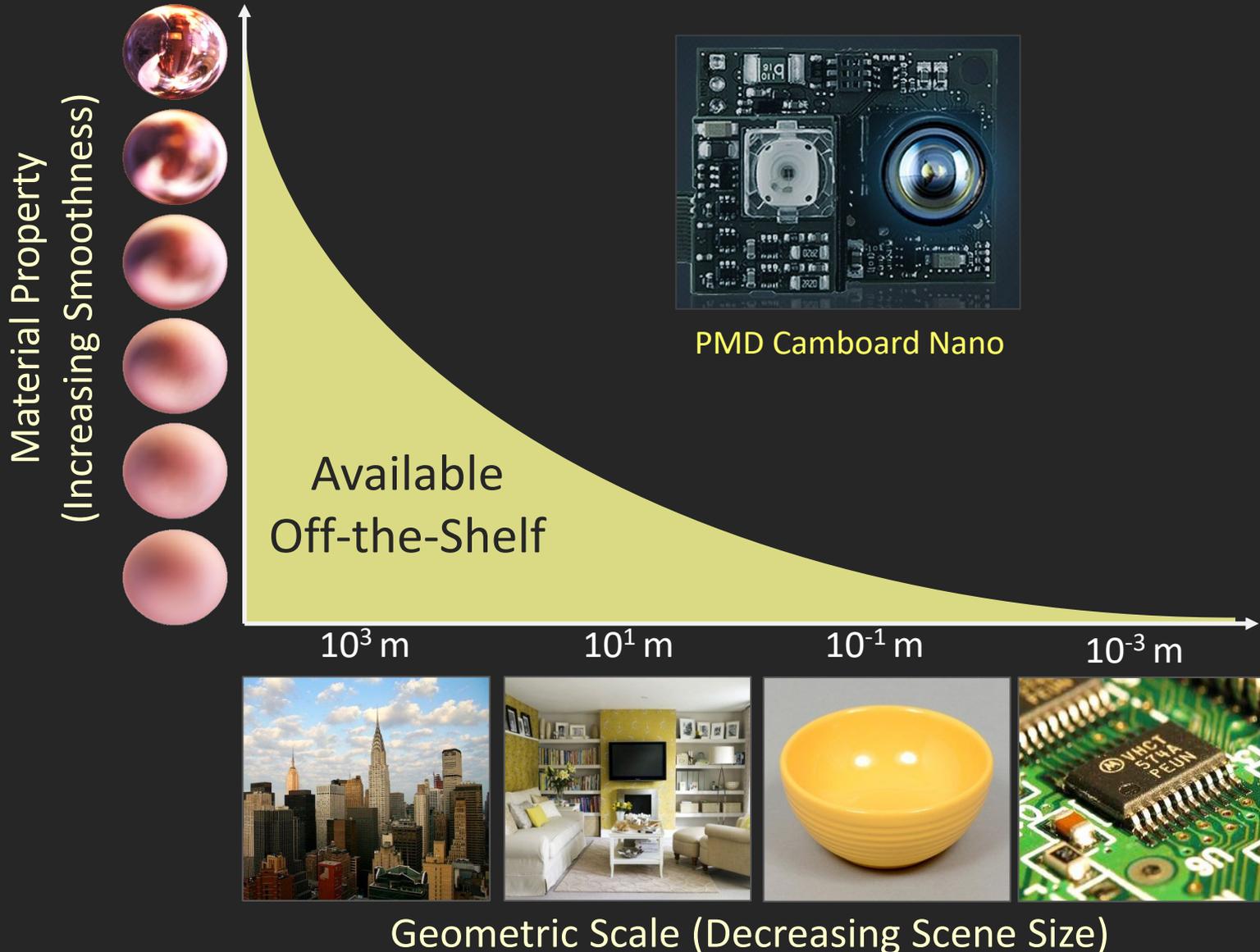
Micro ToF [proposed]  
mean error = 2.8 mm



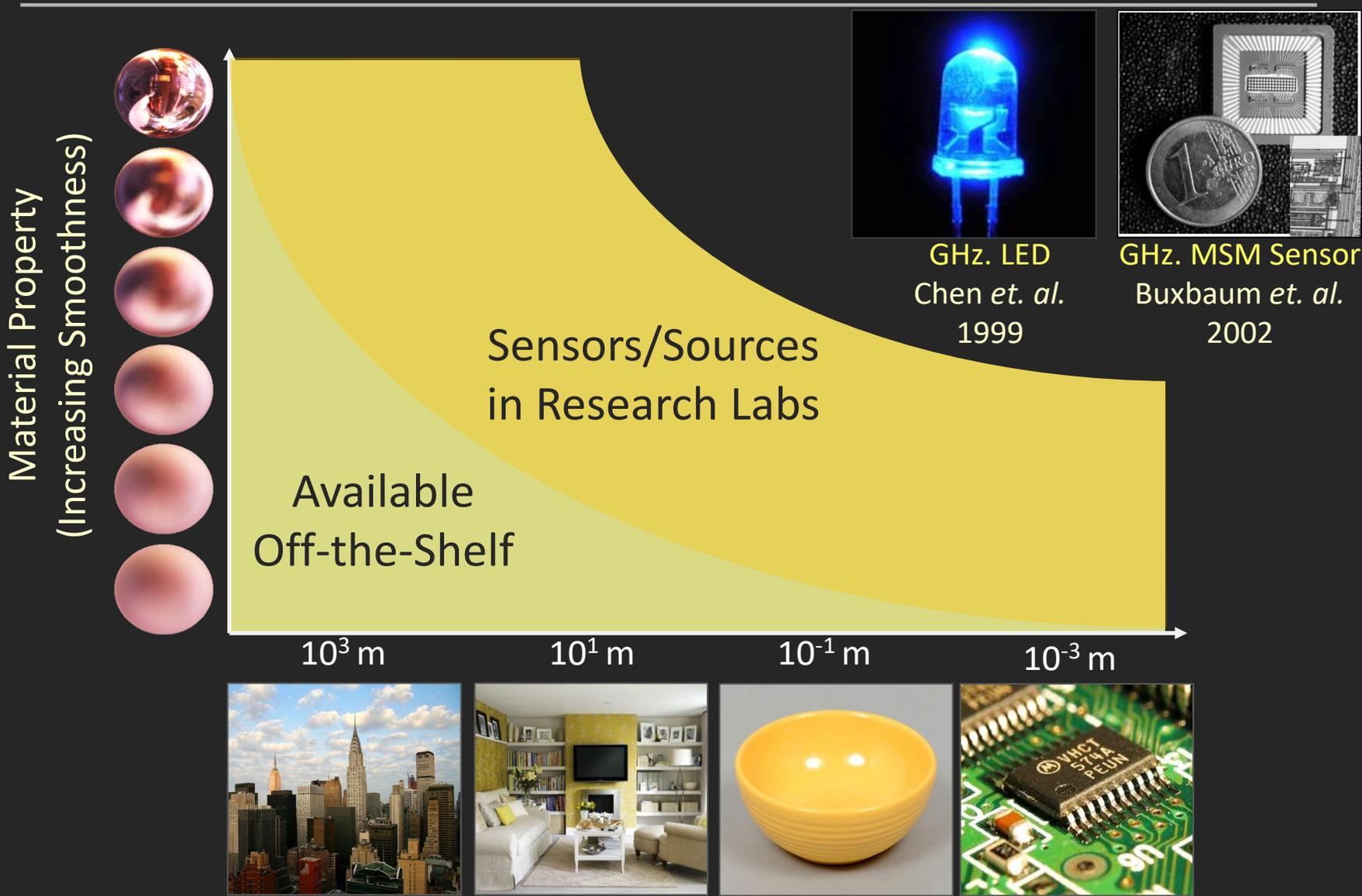
# How High Should The Frequency Be?



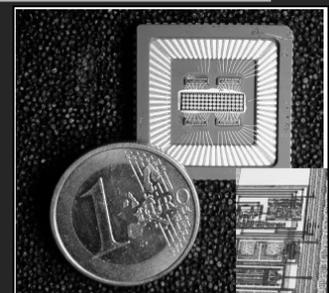
# Technology (Devices) Required



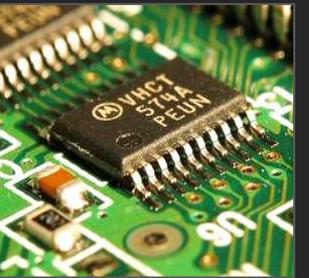
# Technology (Devices) Required



GHz. LED  
Chen *et. al.*  
1999

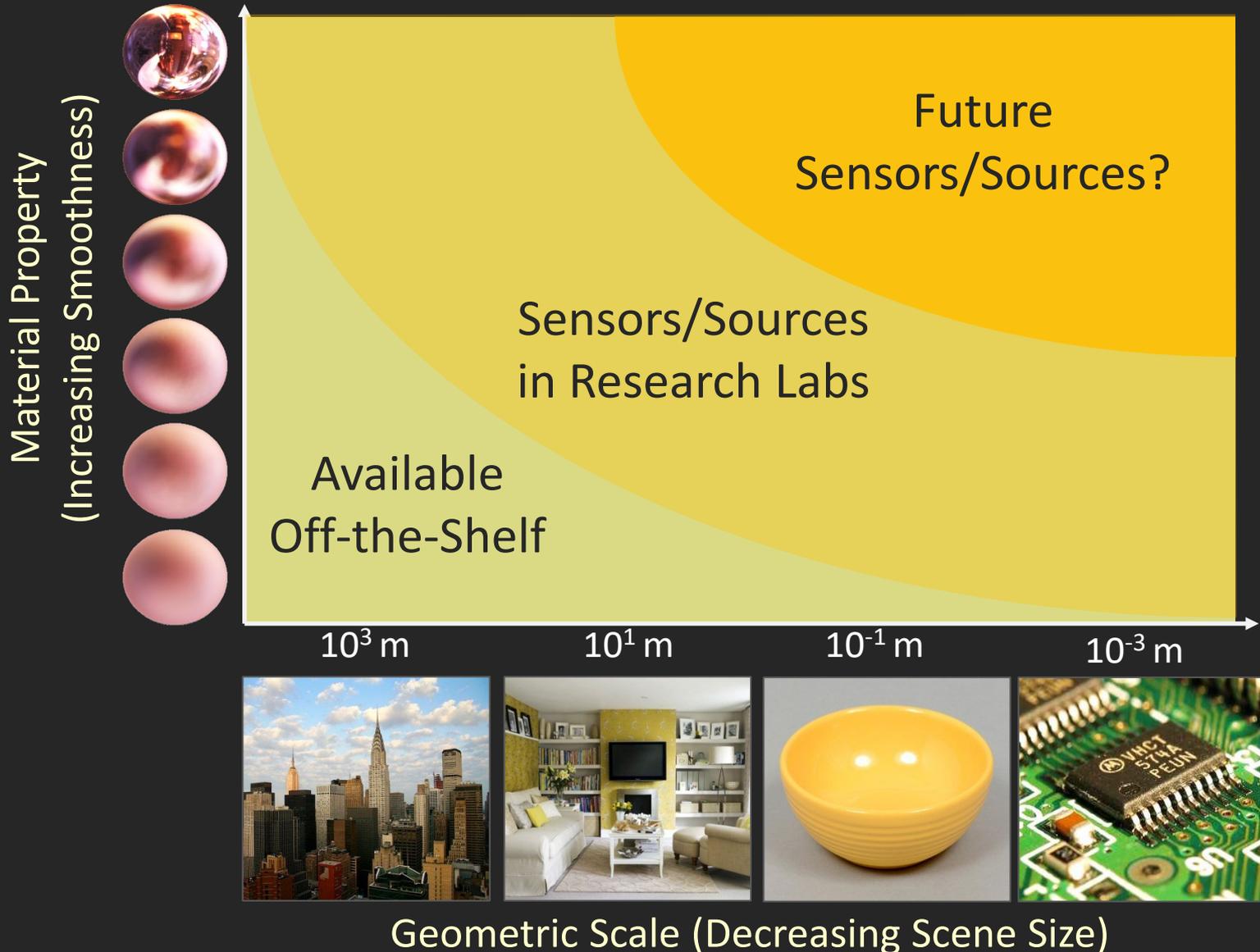


GHz. MSM Sensor  
Buxbaum *et. al.*  
2002



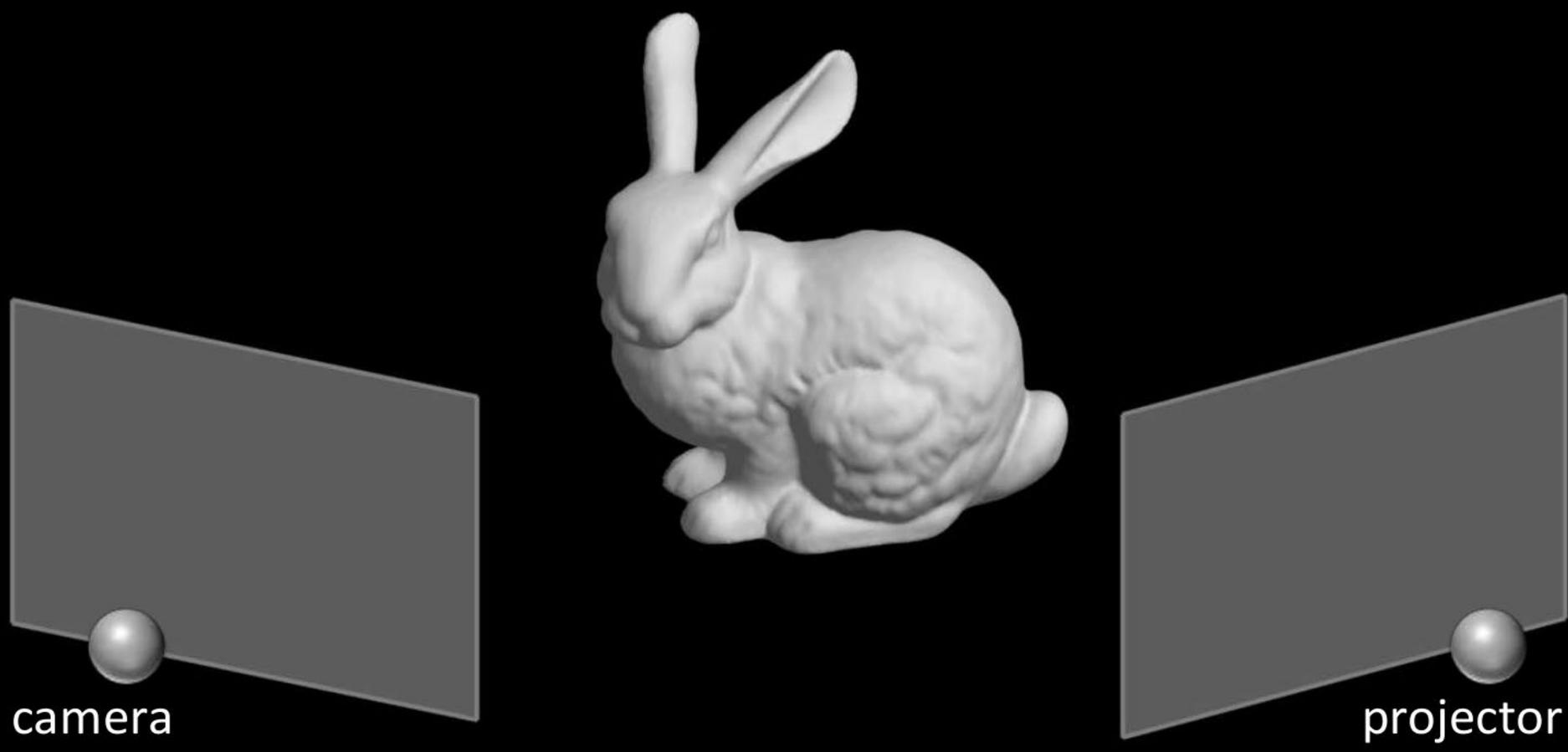
Geometric Scale (Decreasing Scene Size)

# Technology (Devices) Required



# Direct-indirect separation using epipolar probing

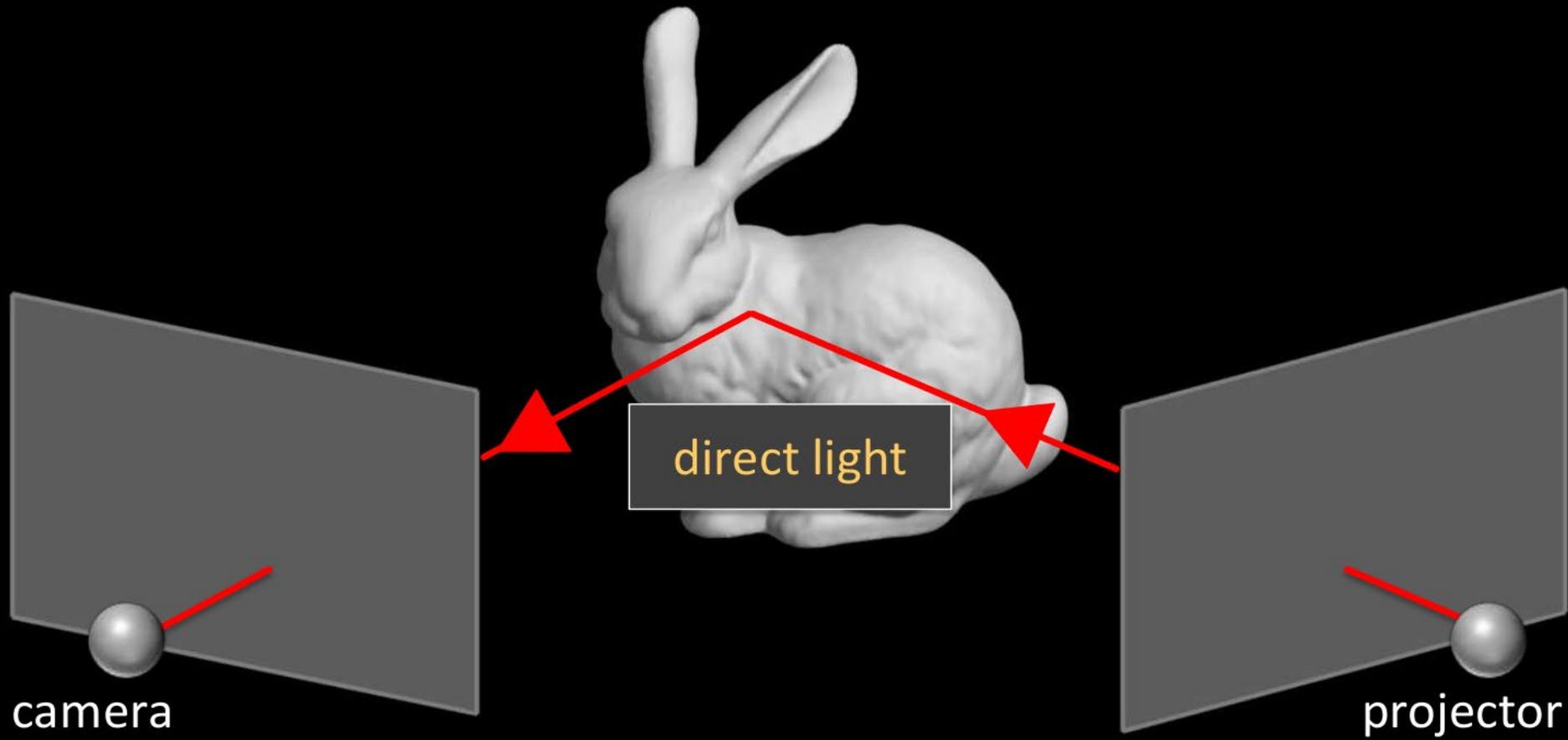
# basic light paths



camera

projector

# basic light paths

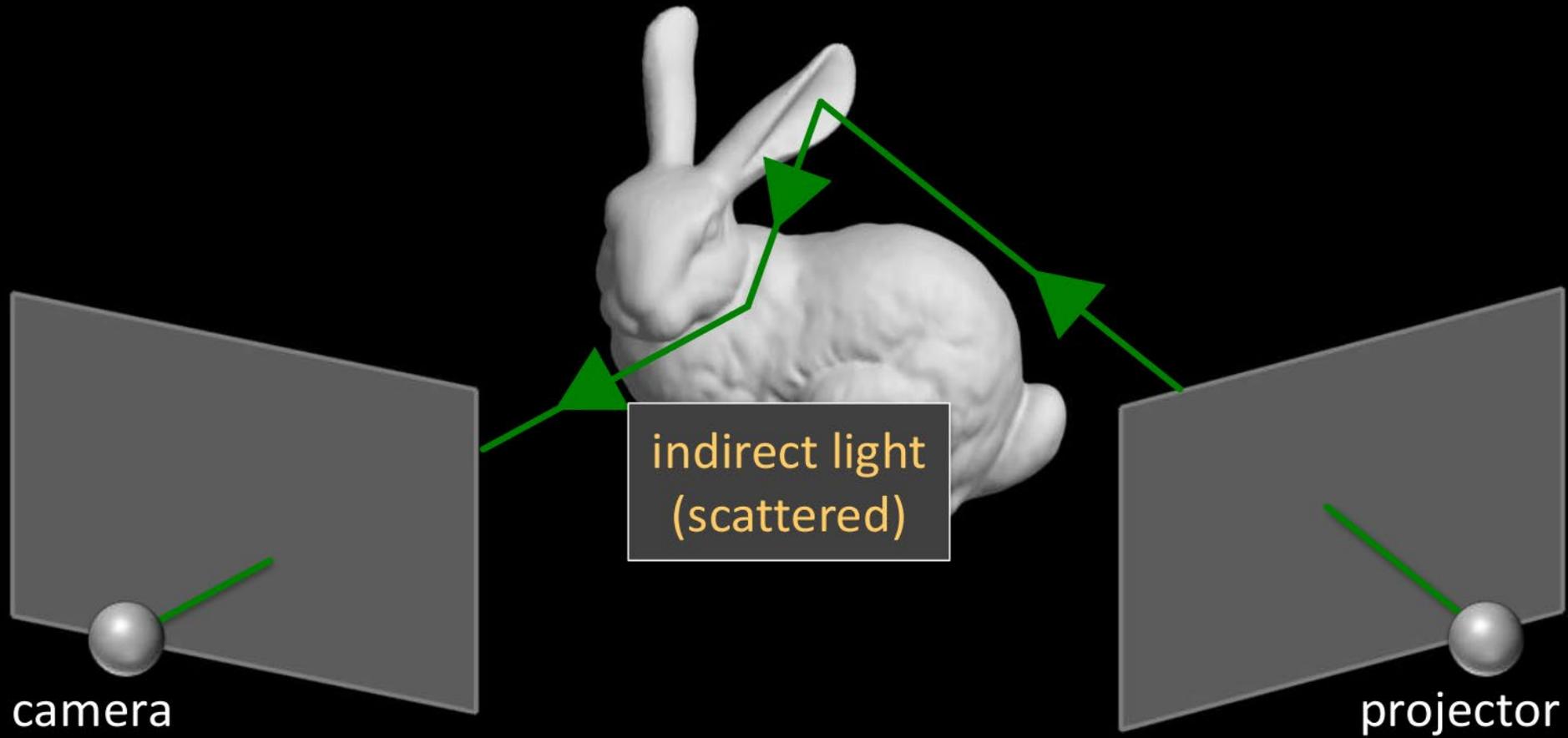


camera

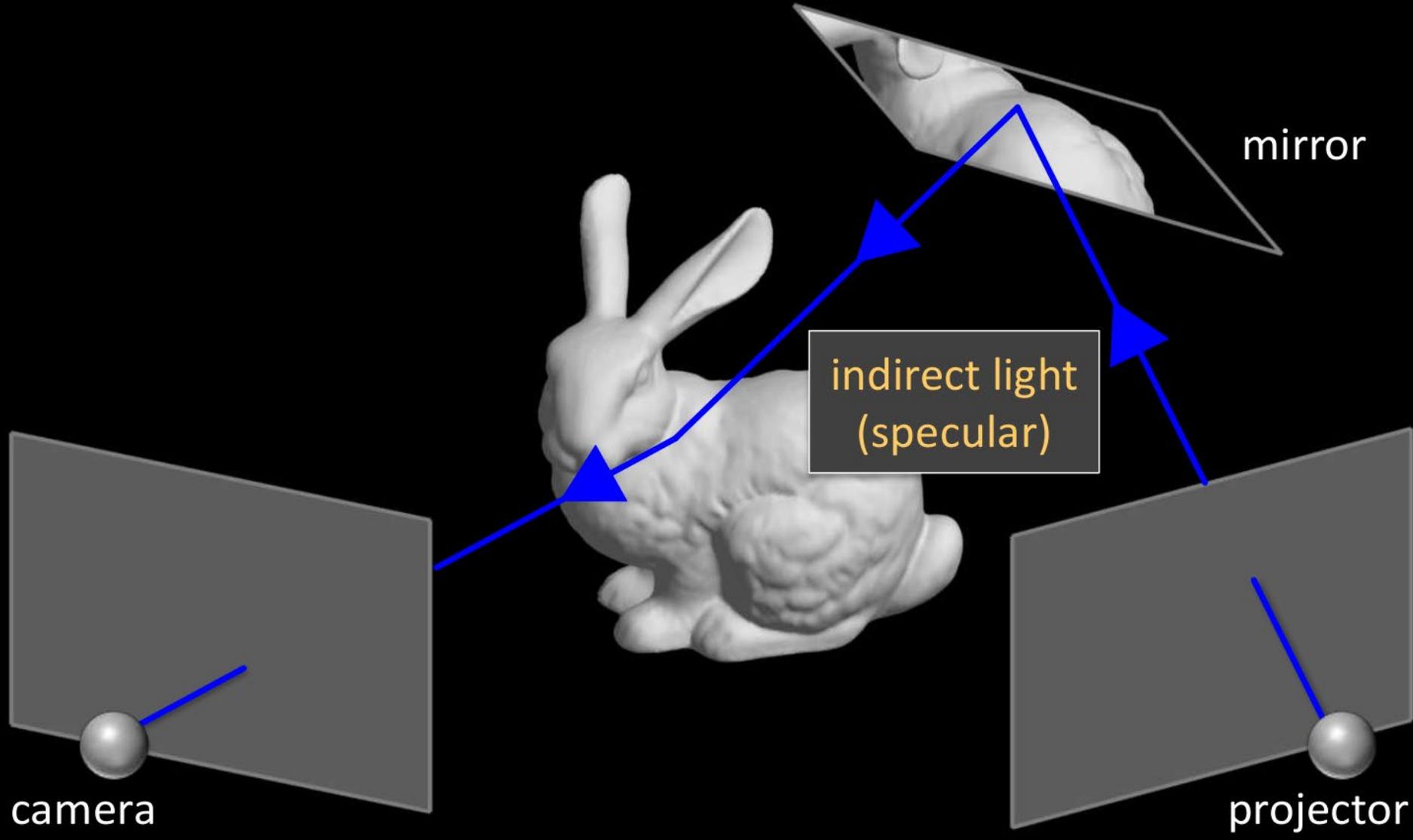
projector

direct light

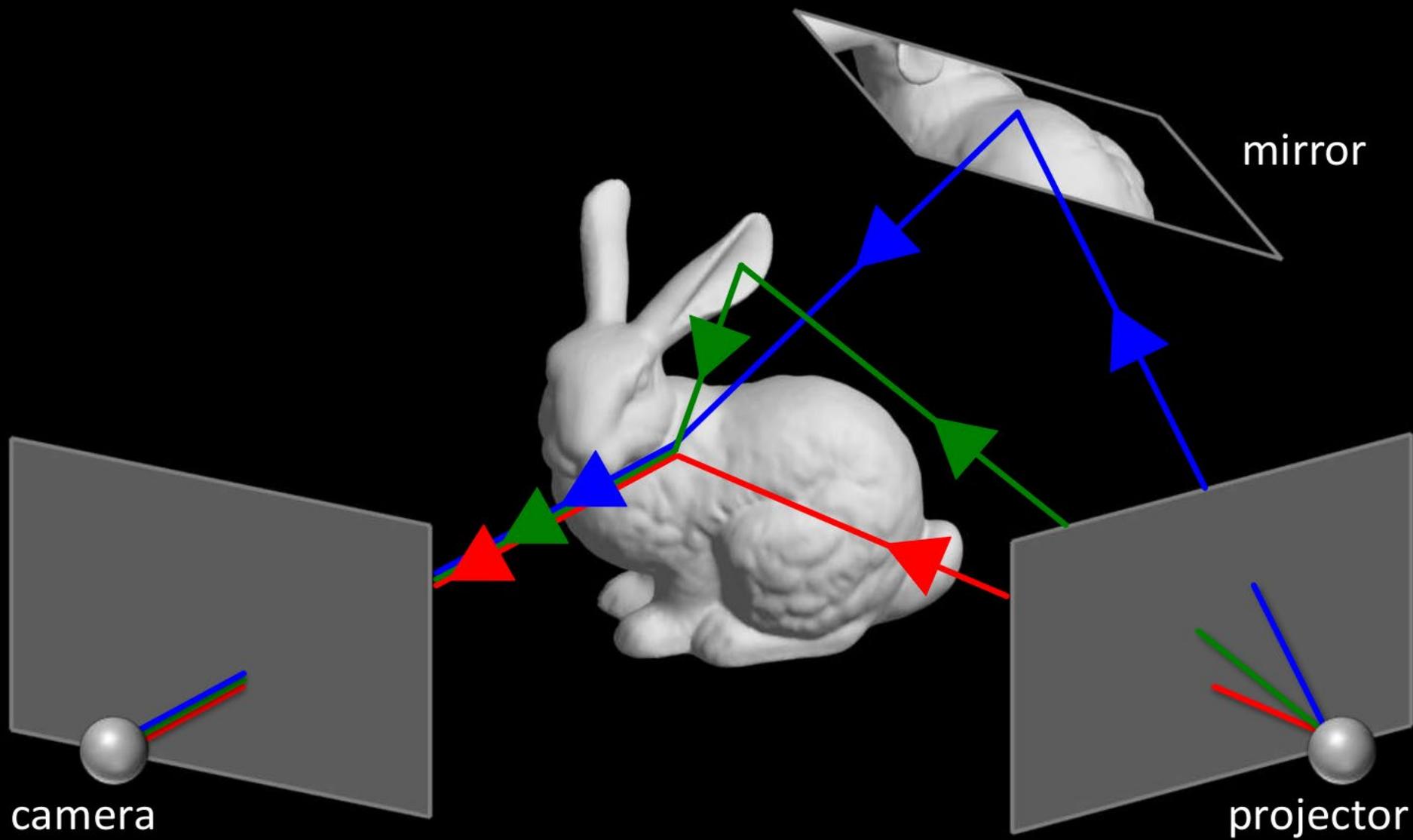
# basic light paths



# basic light paths



# basic light paths

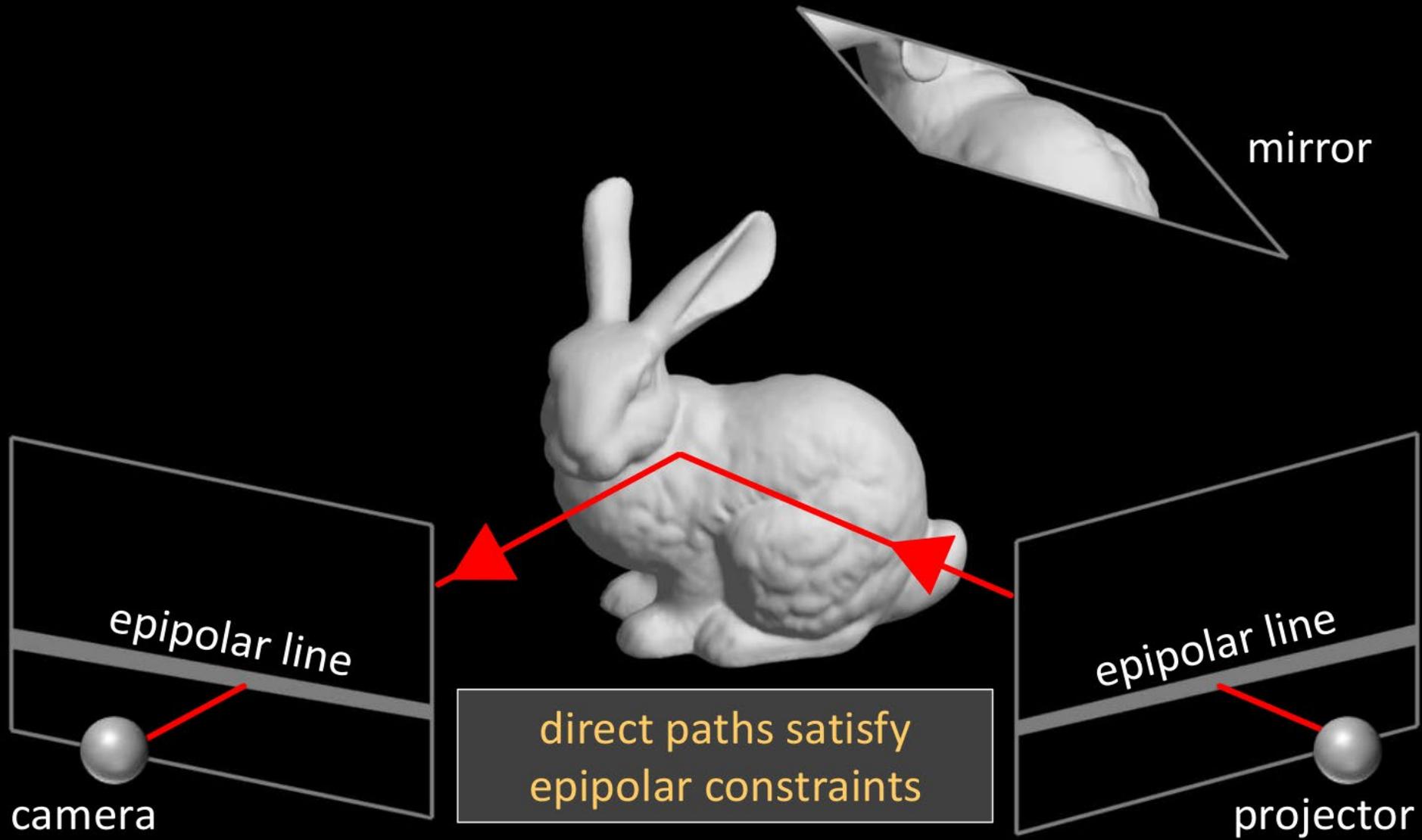


camera

mirror

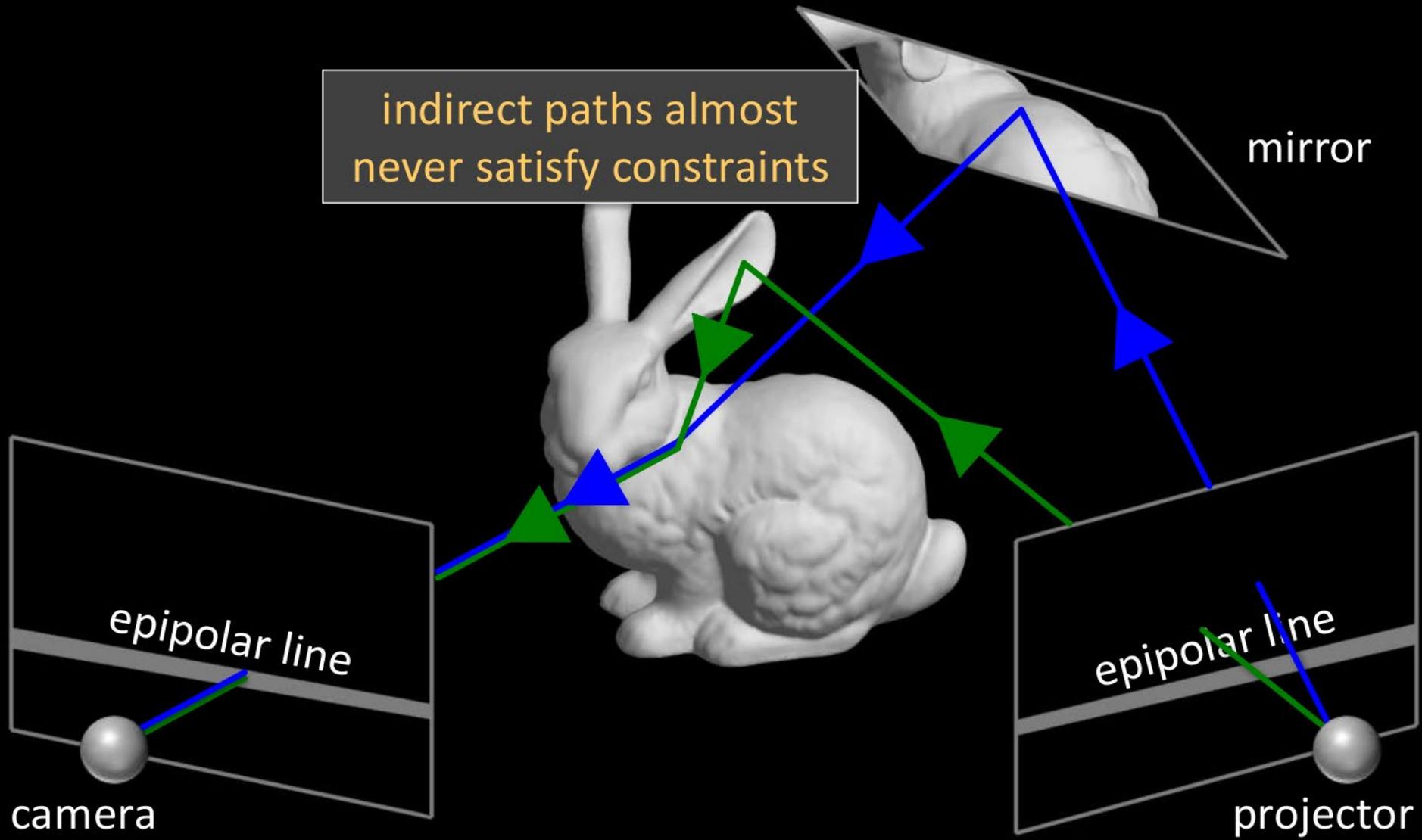
projector

# epipolar constraint & light transport



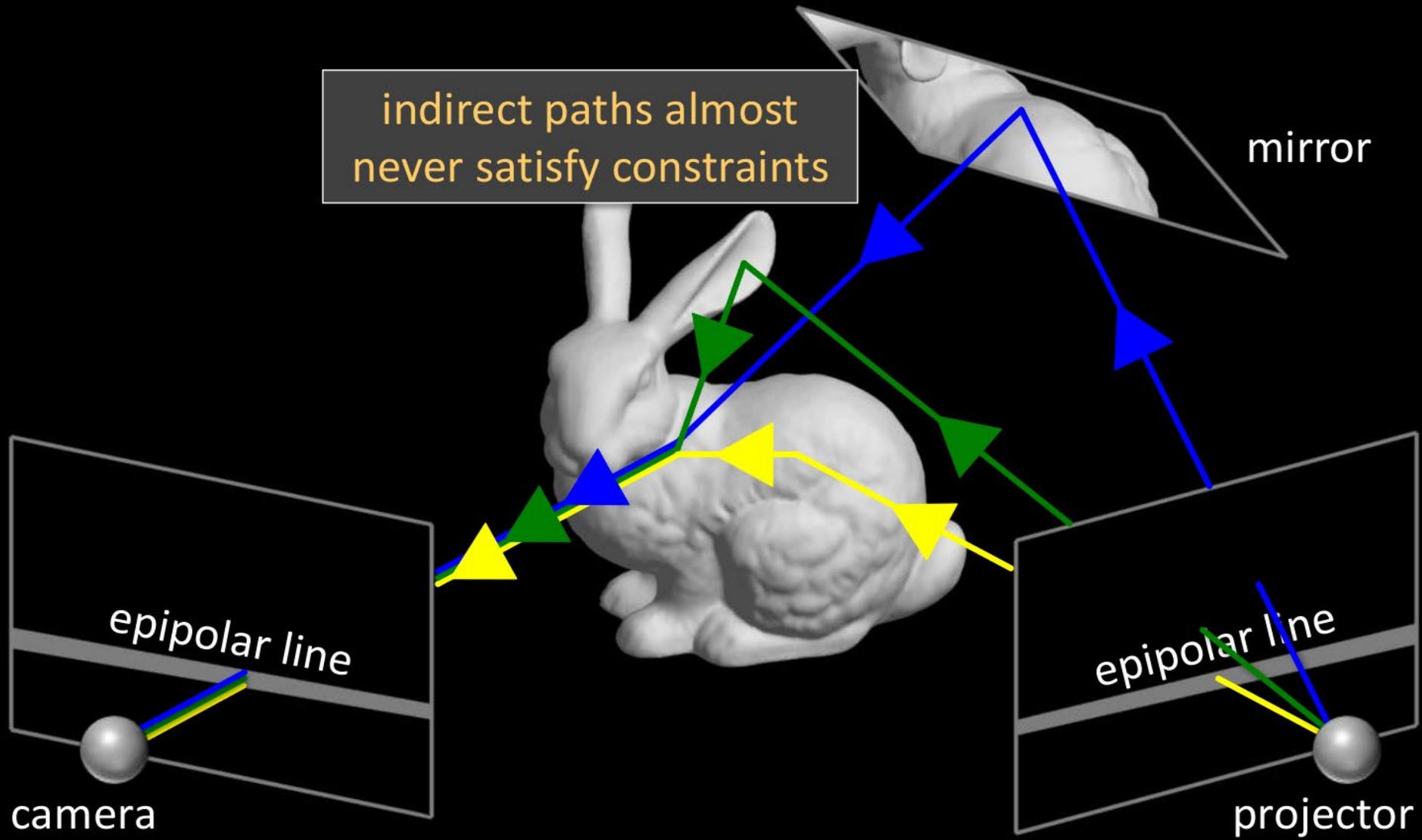
# epipolar constraint & light transport

indirect paths almost  
never satisfy constraints

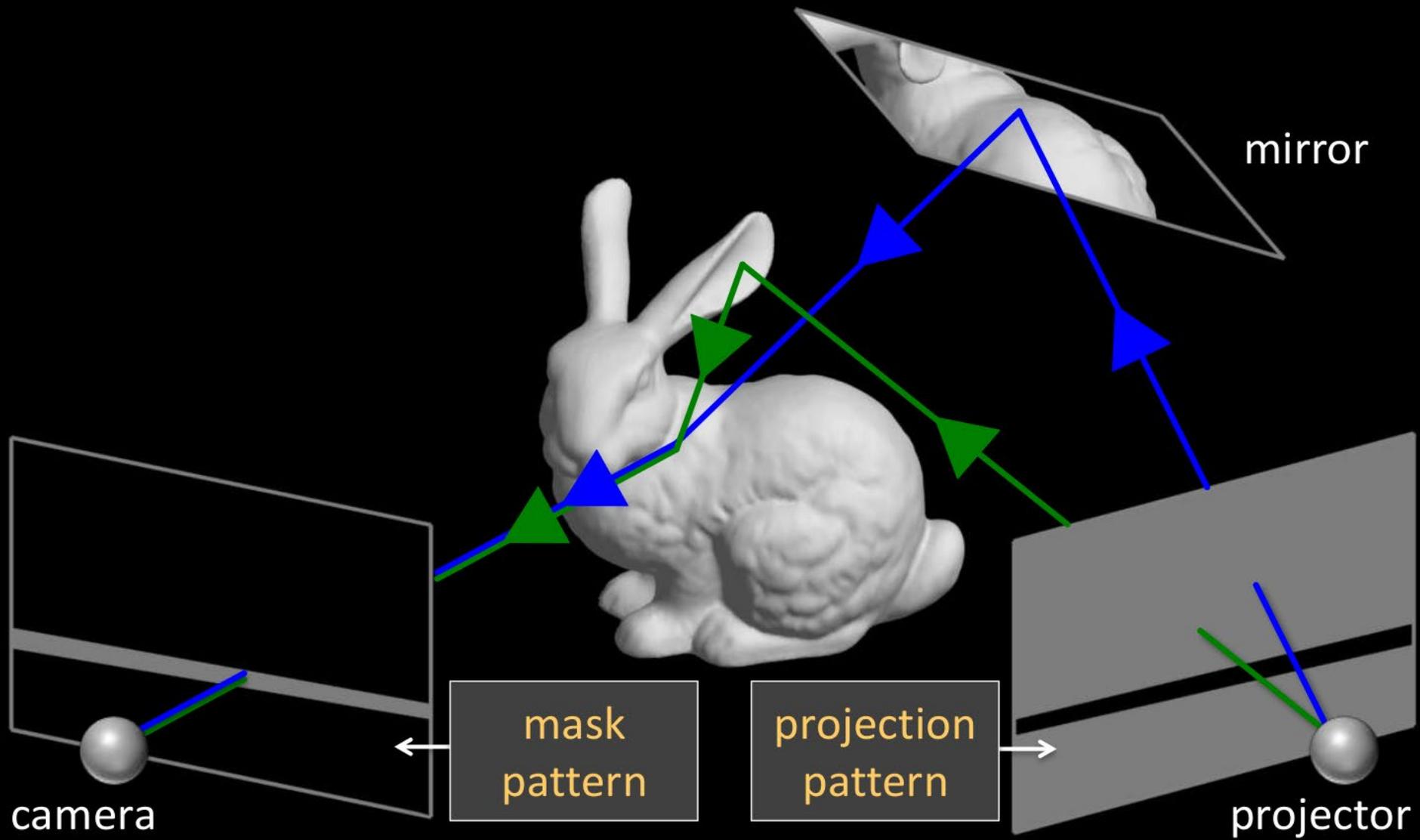


# epipolar constraint & light transport

indirect paths almost  
never satisfy constraints



# blocking epipolar paths with patterns & masks















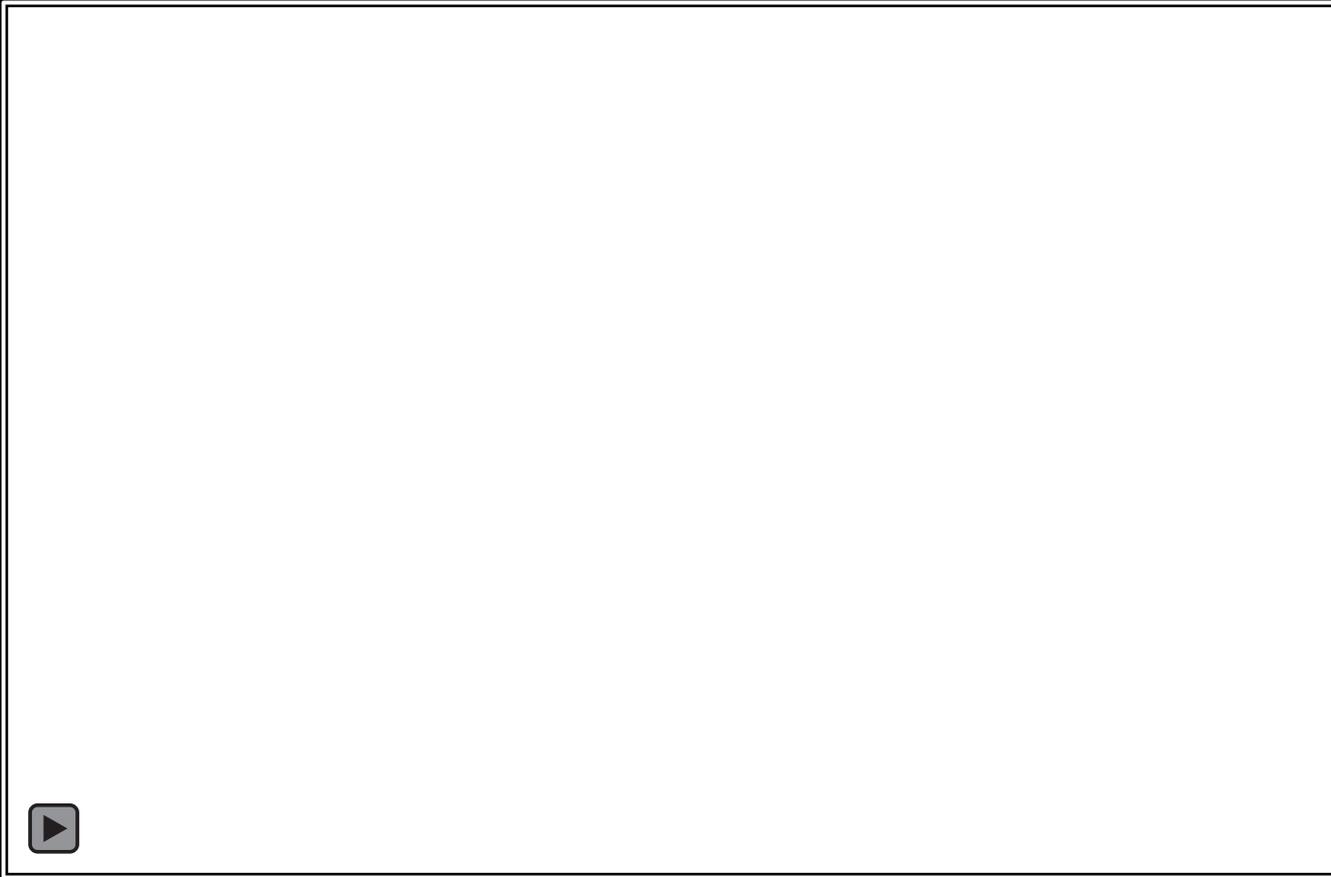




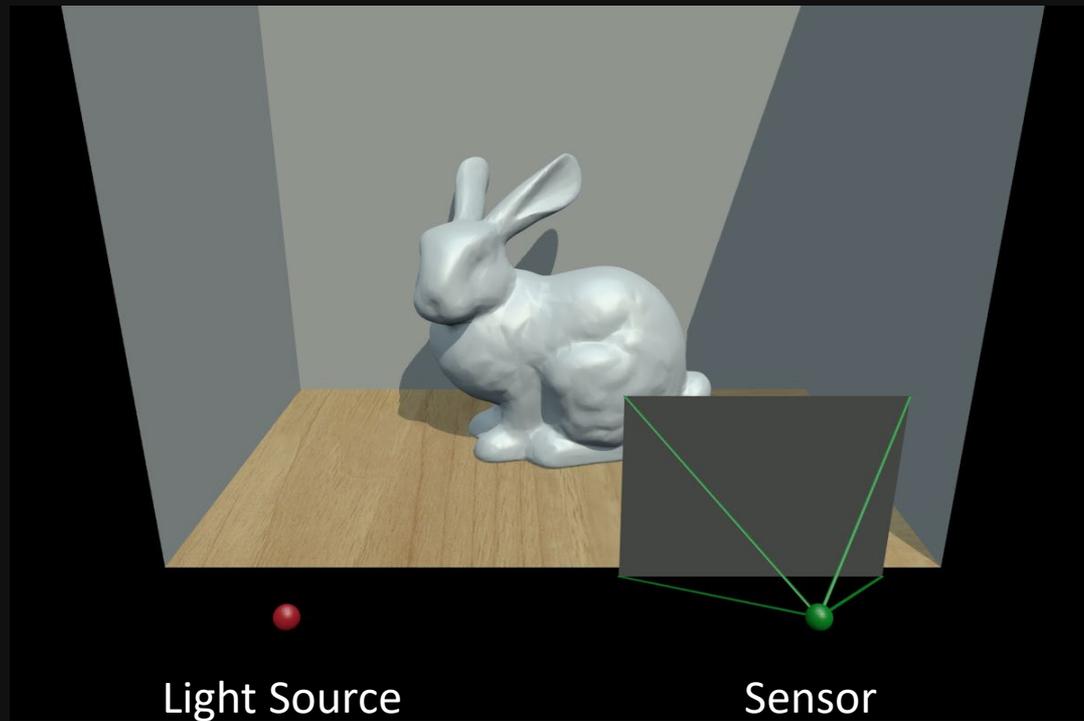


# Energy-efficient epipolar imaging

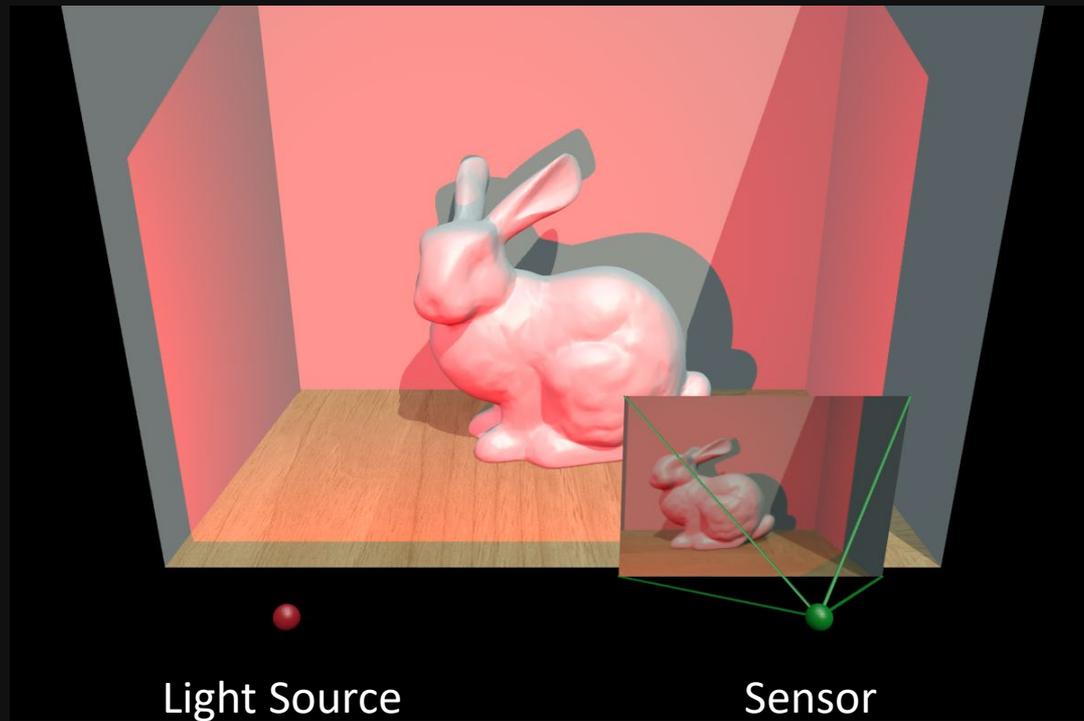
# Energy-efficient transport parsing



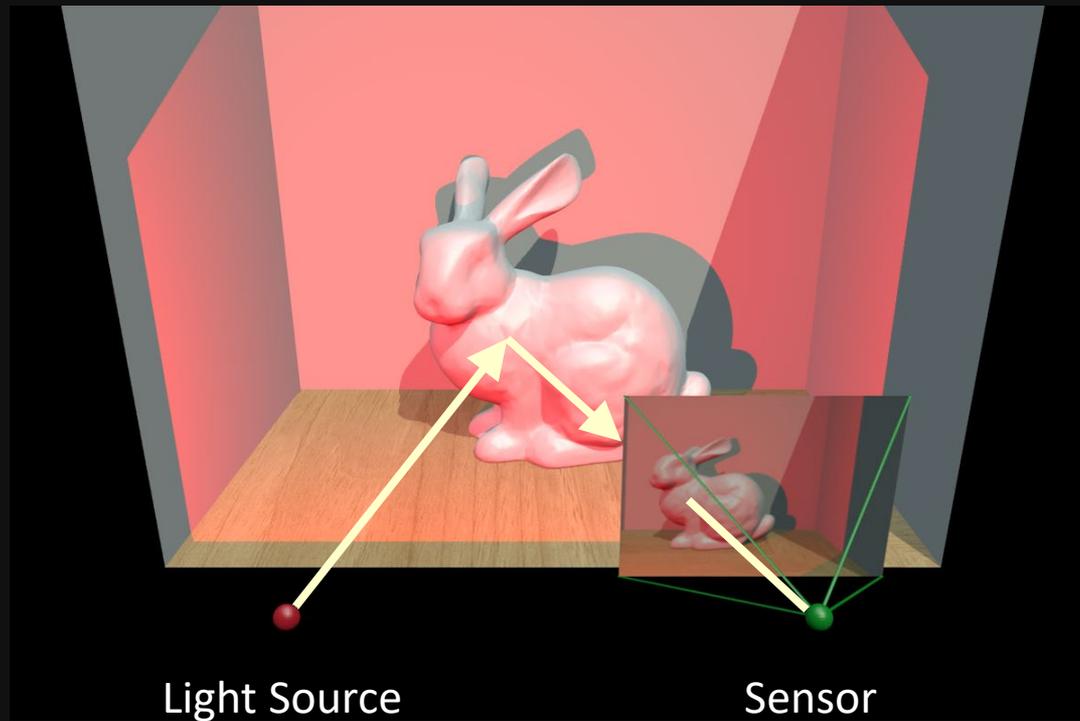
# Regular Imaging



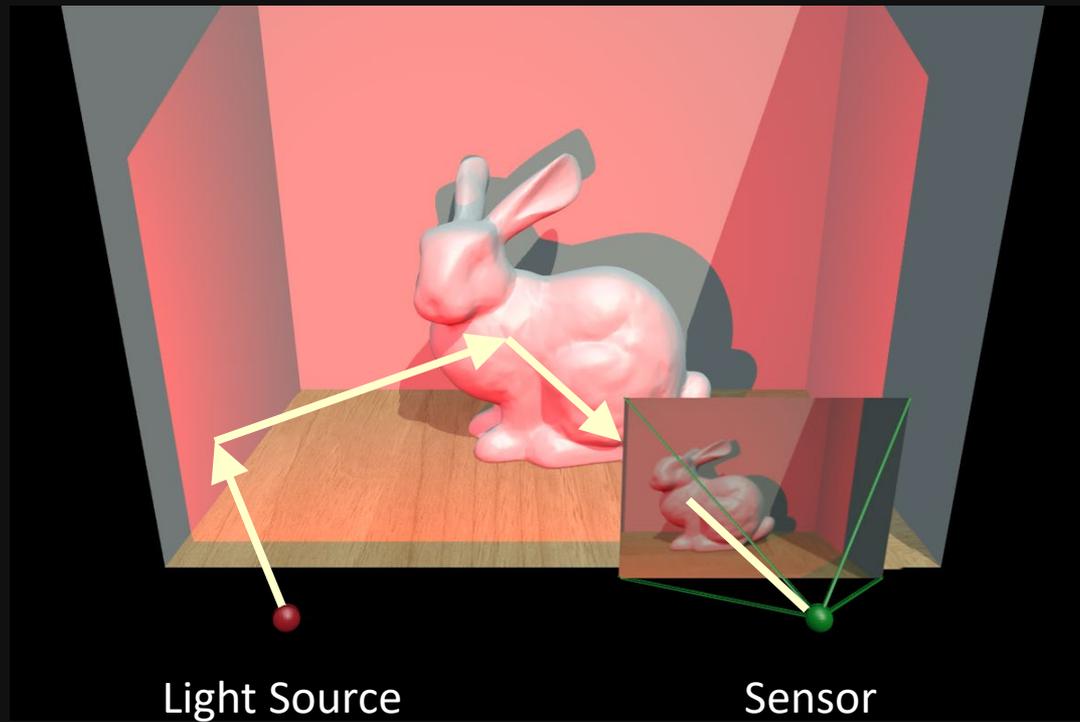
# Regular Imaging



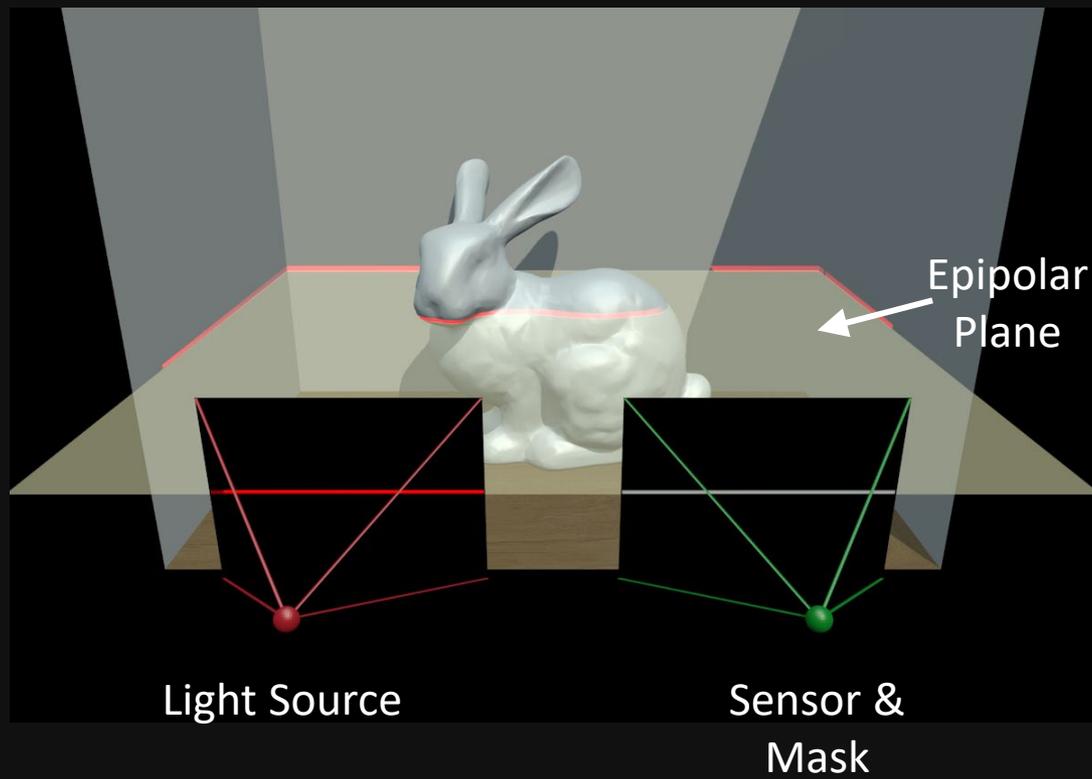
# Regular Imaging



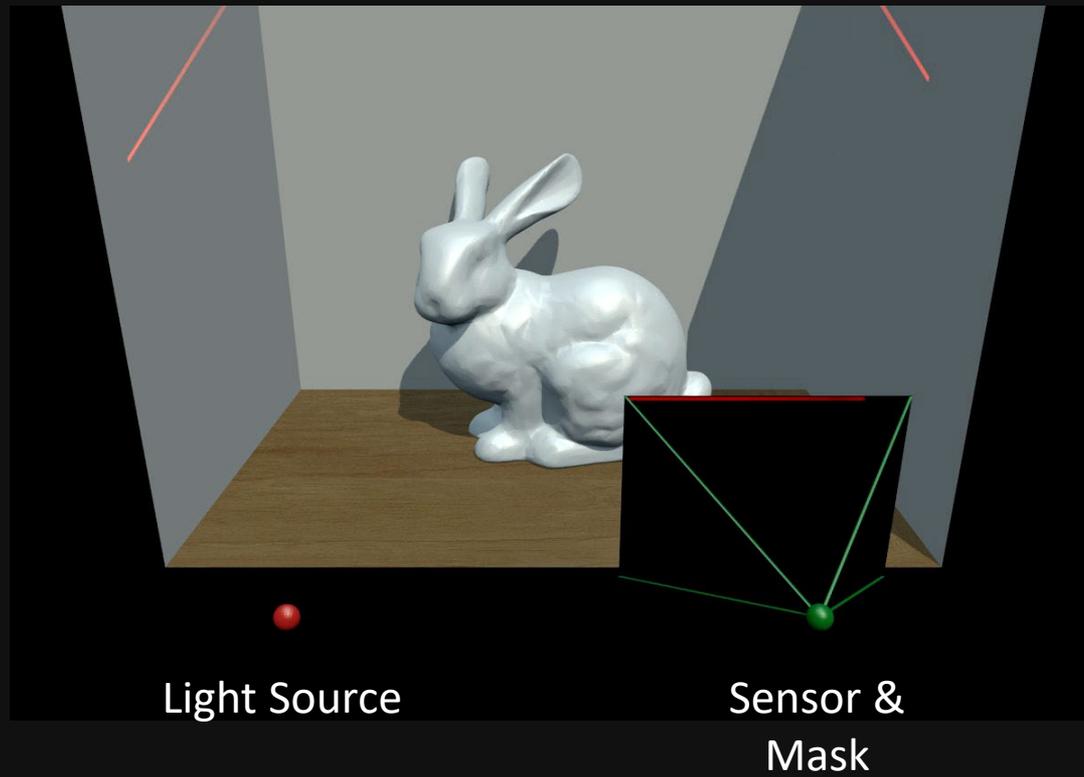
# Regular Imaging



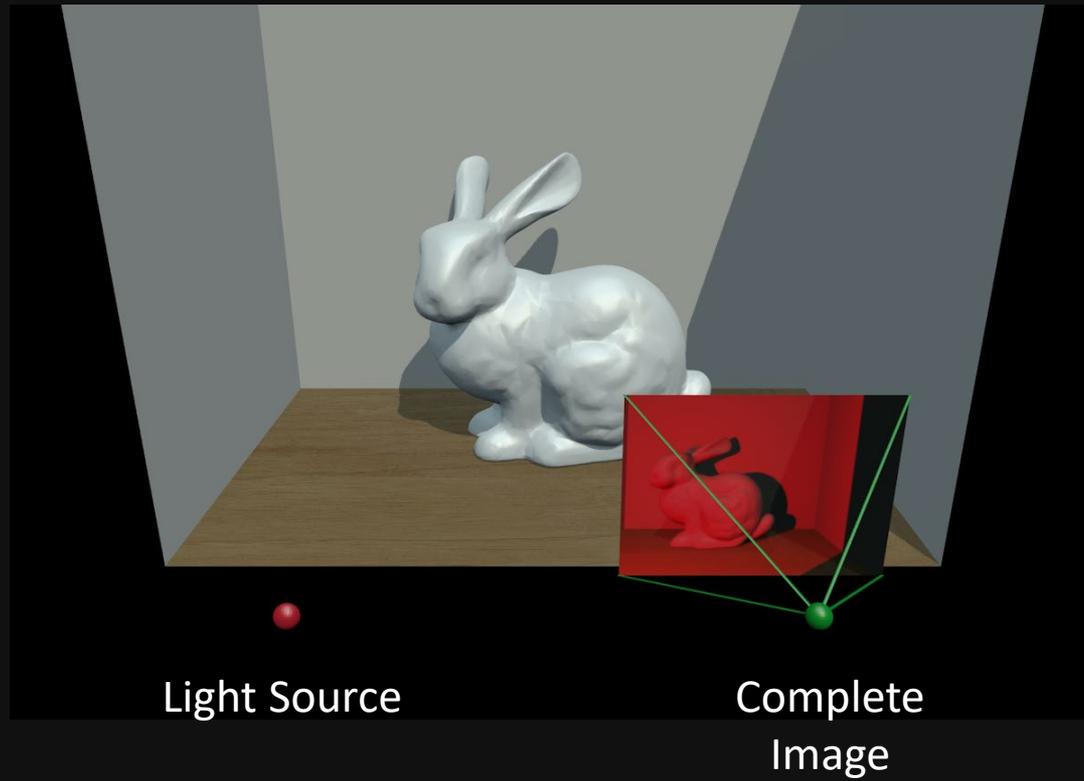
# Epipolar Imaging



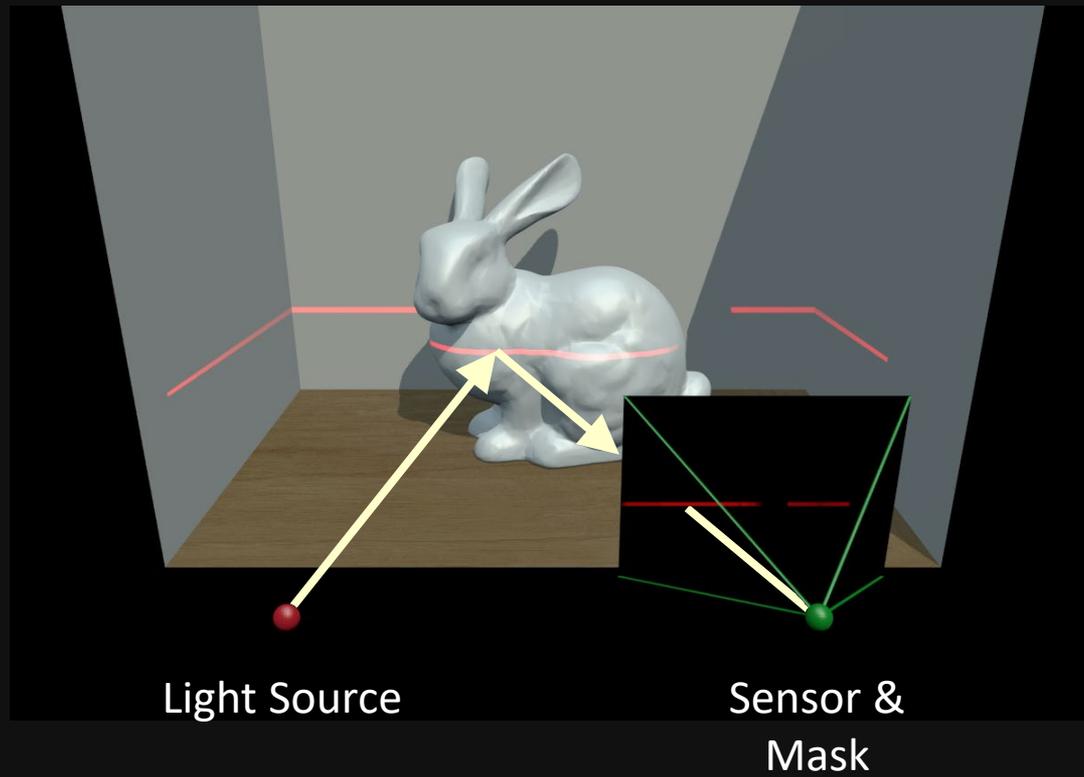
# Epipolar Imaging



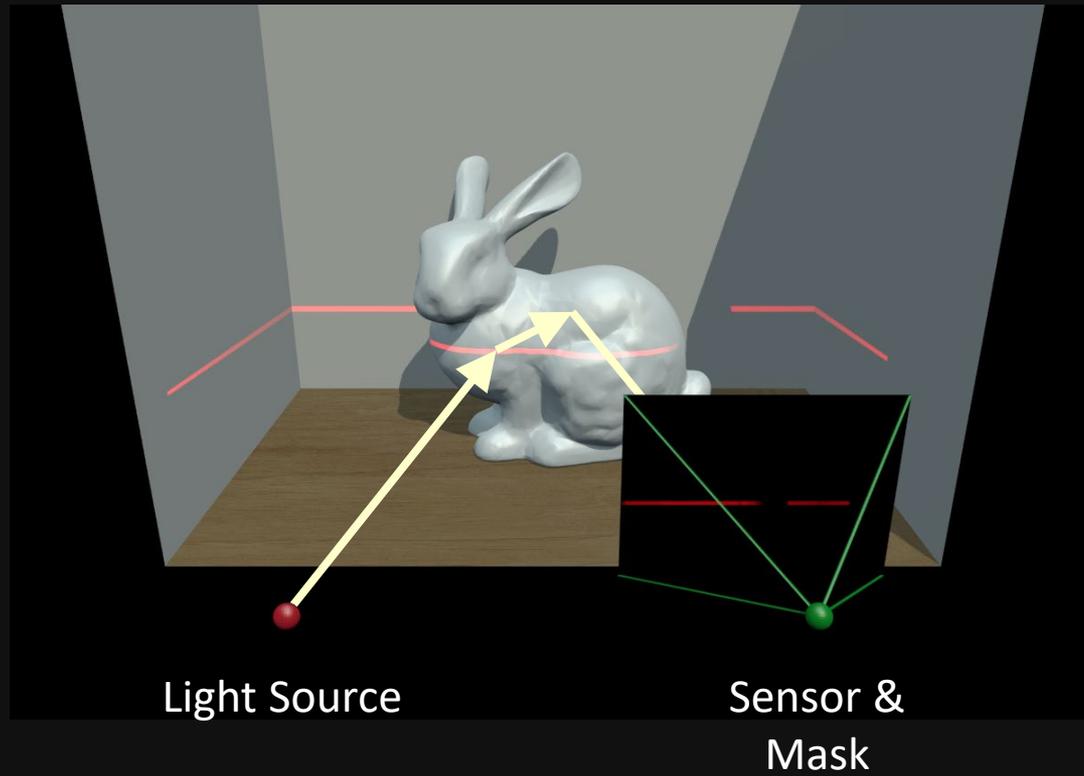
# Epipolar Imaging



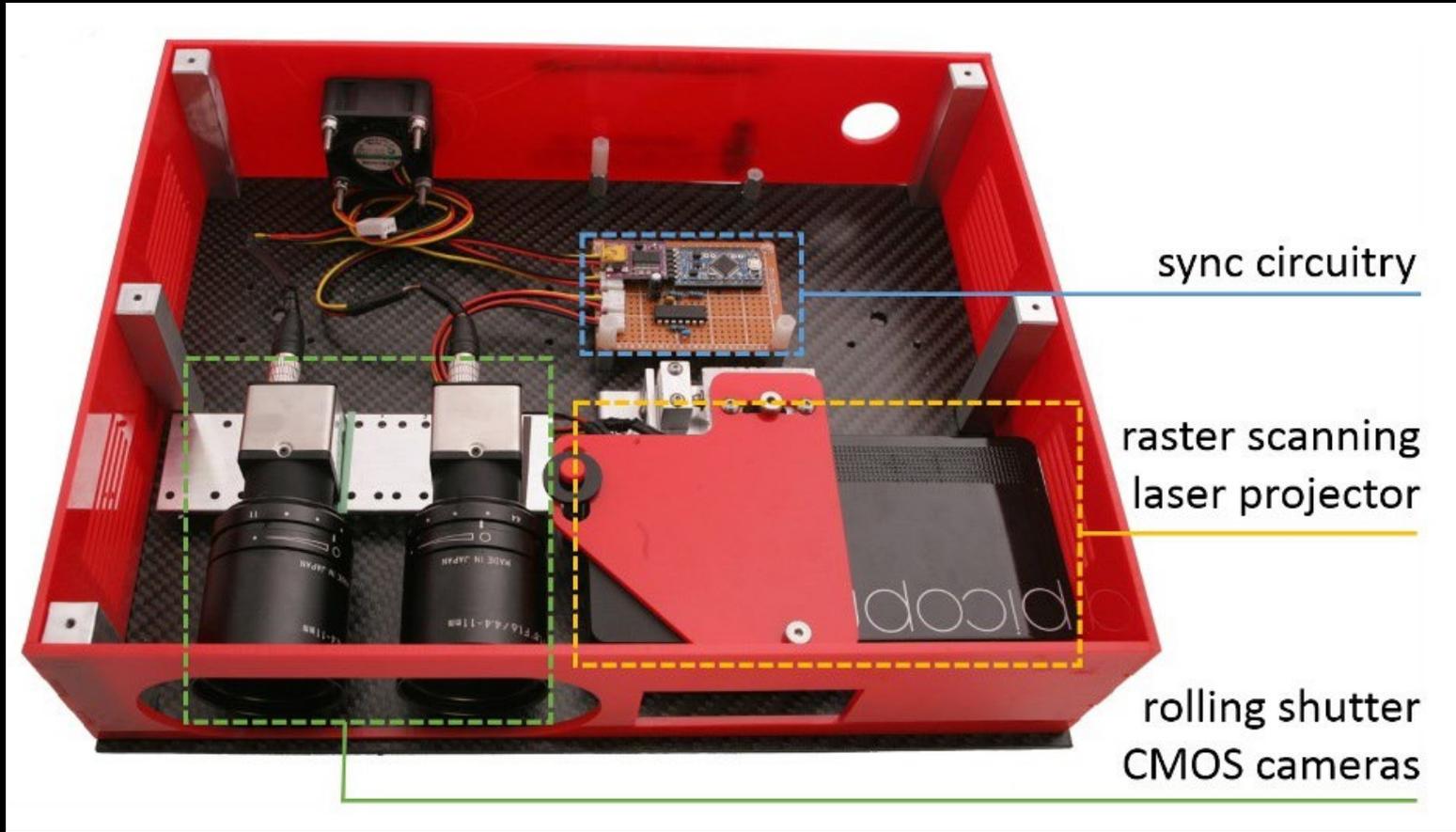
# Epipolar Imaging



# Epipolar Imaging



# Energy-efficient transport parsing



all paths

planar (mostly direct)

non-planar (always indirect)



all paths

```
graph TD; A[all paths] --> B[planar (mostly direct)]; A --> C[non-planar (always indirect)];
```

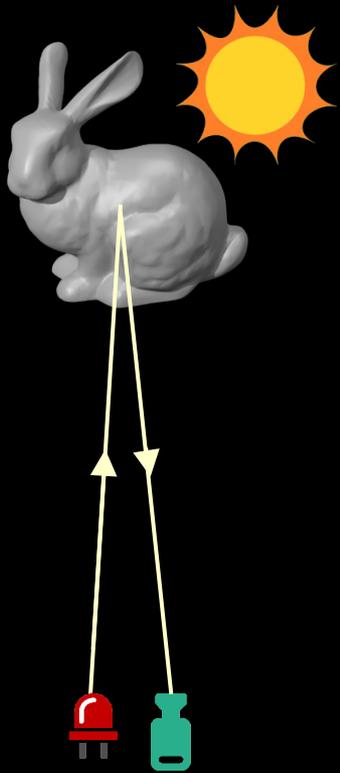
planar (mostly direct)

non-planar (always indirect)

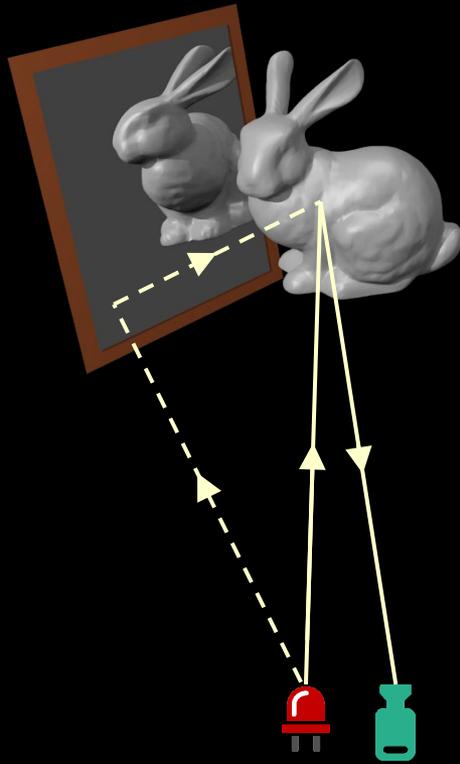


# Benefits of Epipolar ToF Imaging

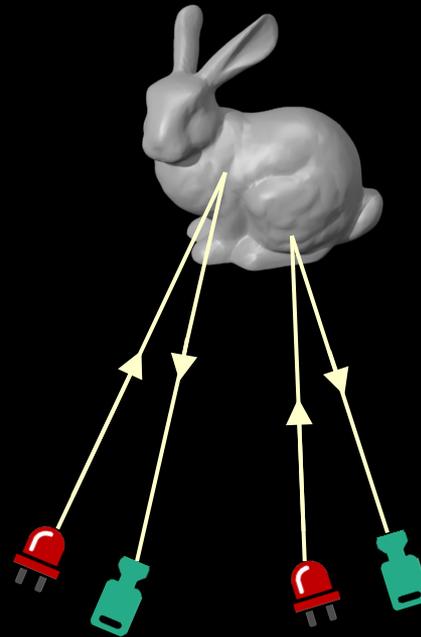
Ambient Light



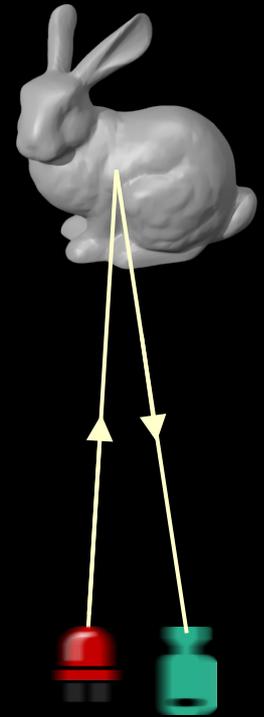
Multi-Path Interference



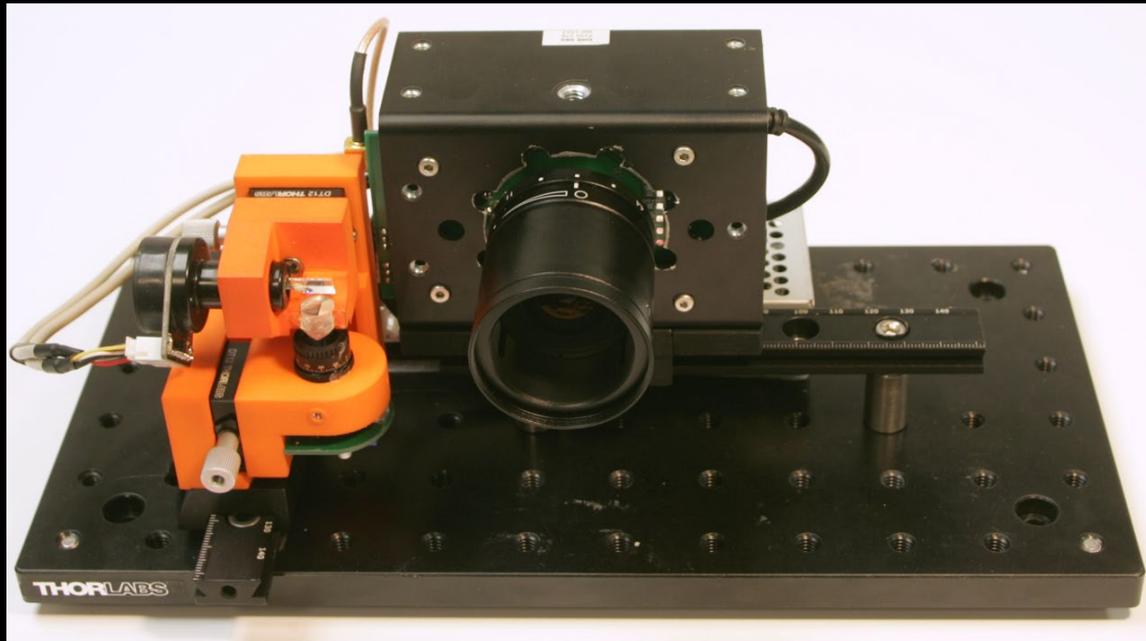
Multi-Device Interference



Camera Motion



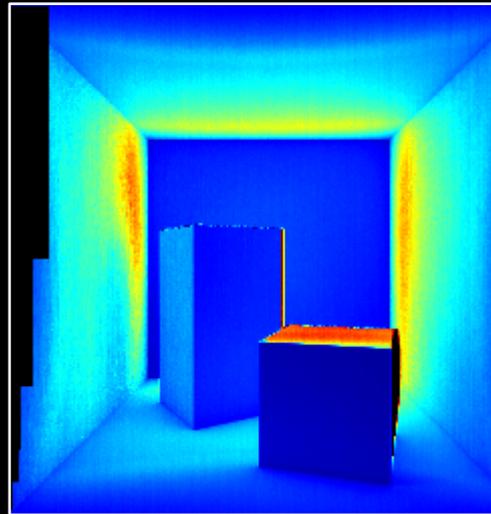
# Epipolar ToF Prototype



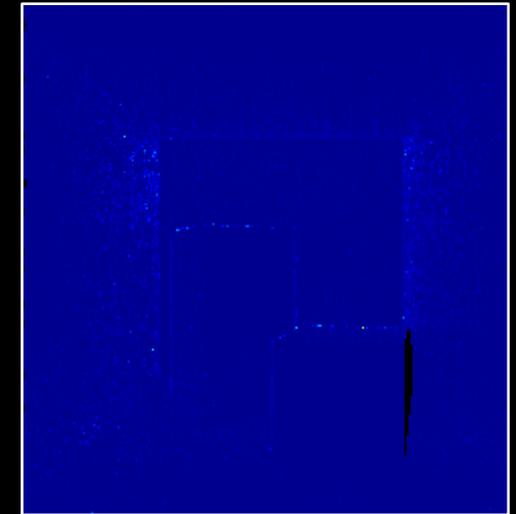
# Epipolar ToF and Global Illumination



Depth Errors (in meters)



Regular ToF @ 30MHz



Epipolar ToF @ 30MHz

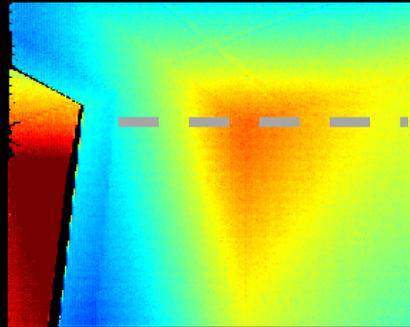


# Epipolar ToF and Global Illumination

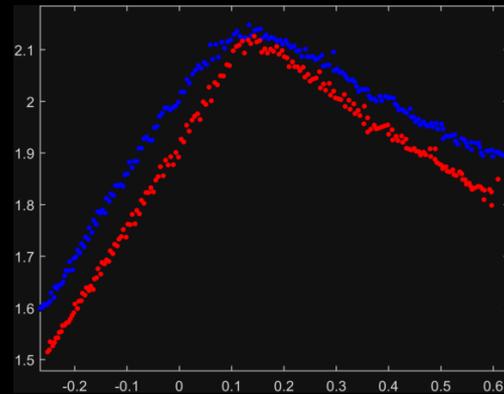
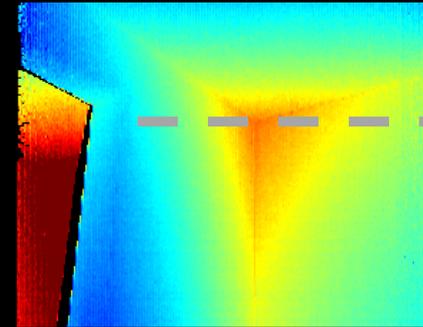


Corner of Room

Regular ToF

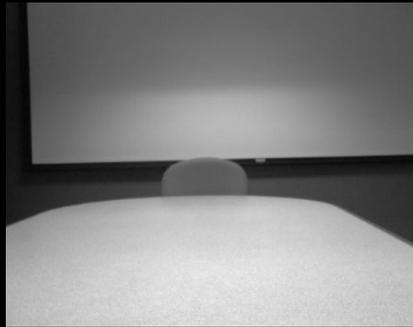


Epipolar ToF



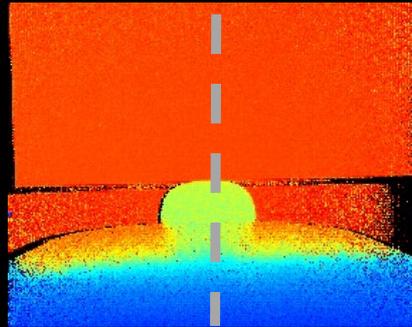
- Regular ToF
- Epipolar ToF

# Epipolar ToF and Global Illumination

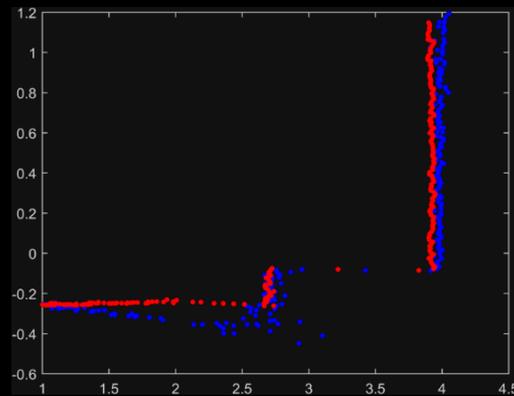
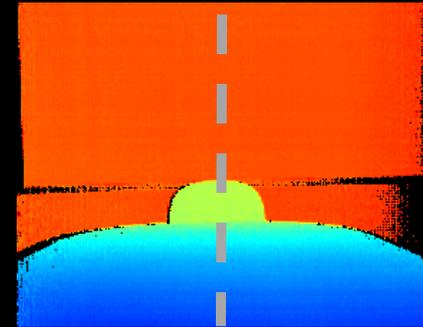


Conference Room

Regular ToF



Epipolar ToF



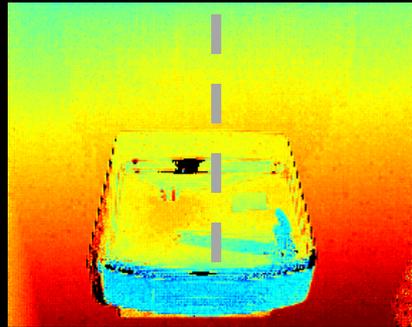
- Regular ToF
- Epipolar ToF

# Epipolar ToF and Global Illumination

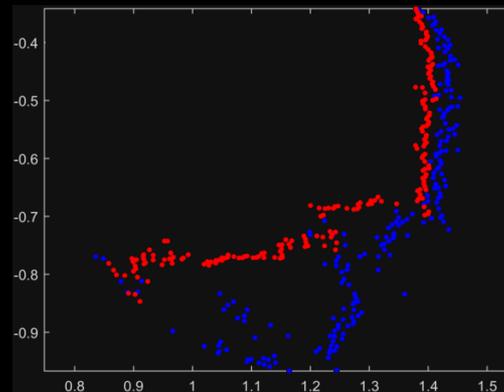
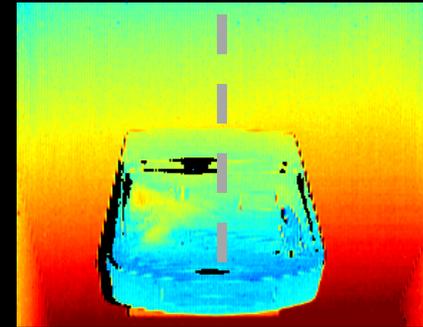


Water Fountain

Regular ToF



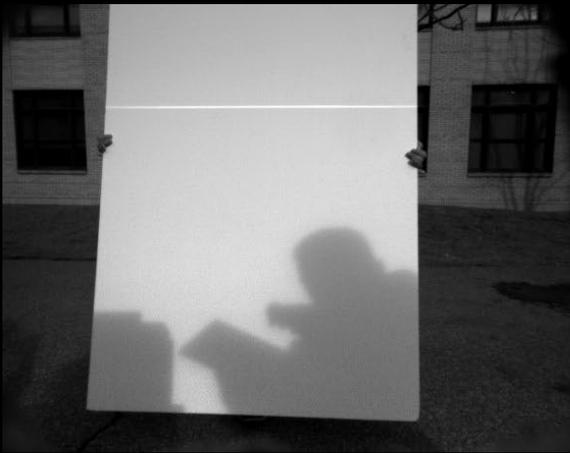
Epipolar ToF



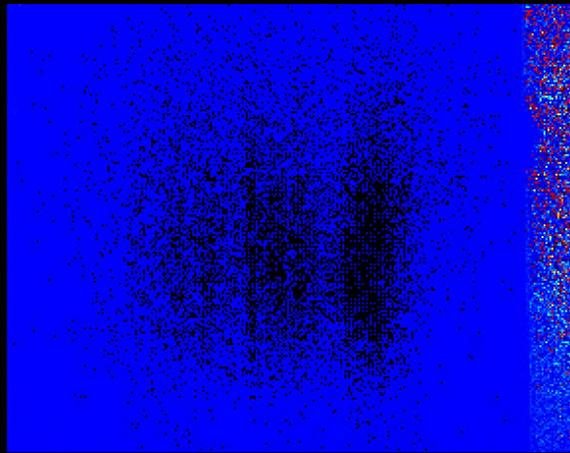
- Regular ToF
- Epipolar ToF

# Outdoors (Cloudy – 10 kilolux)

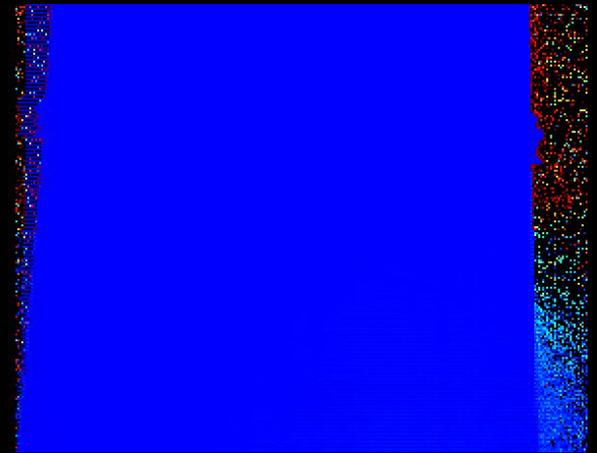
Scene



Regular ToF



Epipolar ToF

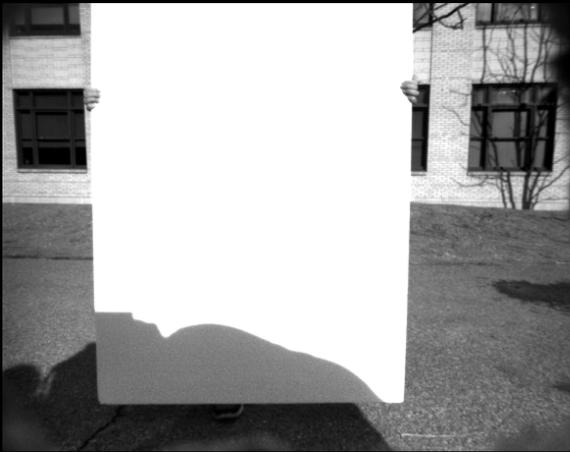


Depth (meters)

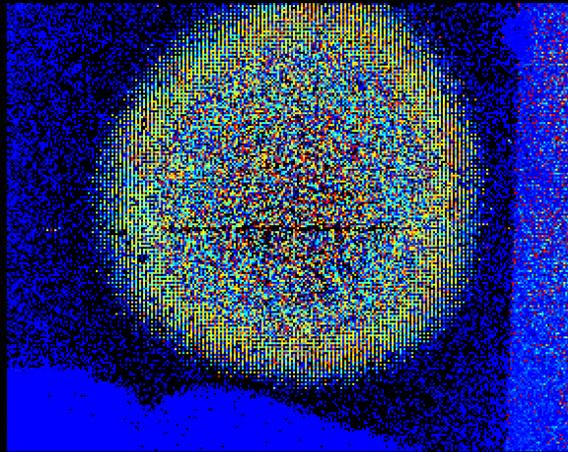


# Outdoors (Sunny – 70 kilolux)

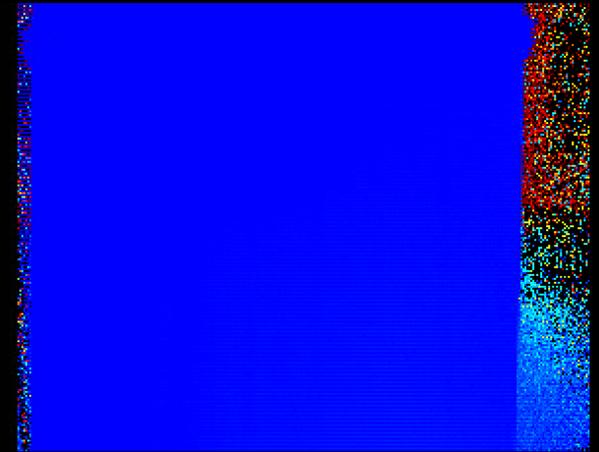
Scene



Regular ToF



Epipolar ToF



Depth (meters)



# Outdoors (Sunny – 70 kilolux)



# References

## Basic reading:

- Nayar et al., “Fast separation of direct and indirect components of a scene using high frequency illumination,” SIGGRAPH 2004.  
The paper on separation of direct and indirect illumination using high-frequency illumination.
- Gupta et al., “A Practical Approach to 3D Scanning in the Presence of Interreflections, Subsurface Scattering and Defocus,” IJCV 2013.  
The paper on using XOR codes to deal with indirect illumination in structured light 3D.
- Gupta et al., “Phasor imaging: A generalization of correlation-based time-of-flight imaging,” TOG 2015.  
The paper on using high-frequency modulation to deal with interreflections and MPI in CW-ToF imaging.
- O’Toole et al., “Primal-dual coding to probe light transport,” SIGGRAPH 2012.
- O’Toole et al., “3d shape and indirect appearance by structured light transport,” CVPR 2014.  
These two papers introduce the concepts of light transport probing and epipolar probing, as well as explain how to use primal-dual coding to achieve them.
- O’Toole et al., “Homogeneous codes for energy-efficient illumination and imaging,” SIGGRAPH 2015.  
This paper shows how to efficiently implement epipolar imaging with a simple projector and camera.
- Achar et al., “Epipolar time-of-flight imaging,” SIGGRAPH 2017.  
This paper combines epipolar imaging and time-of-flight imaging.

## Additional reading:

- Seitz et al., “A theory of inverse light transport,” ICCV 2005.  
This early paper shows a way to *exactly* decompose light transport by number of bounces, under certain assumptions for the imaged scene.
- Chandraker et al., “On the duality of forward and inverse light transport,” PAMI 2011.
- Reddy et al., “Frequency-space decomposition and acquisition of light transport under spatially varying illumination,” ECCV 2012.  
These two papers have additional analysis about the relationship between direct and indirect illumination and illumination frequency.
- Durand et al., “A frequency analysis of light transport,” SIGGRAPH 2005.  
This paper more formally discusses the notion of light transport frequency, and the frequency characteristics of different light transport effects (specular versus diffuse reflections, hard versus smooth shadows).