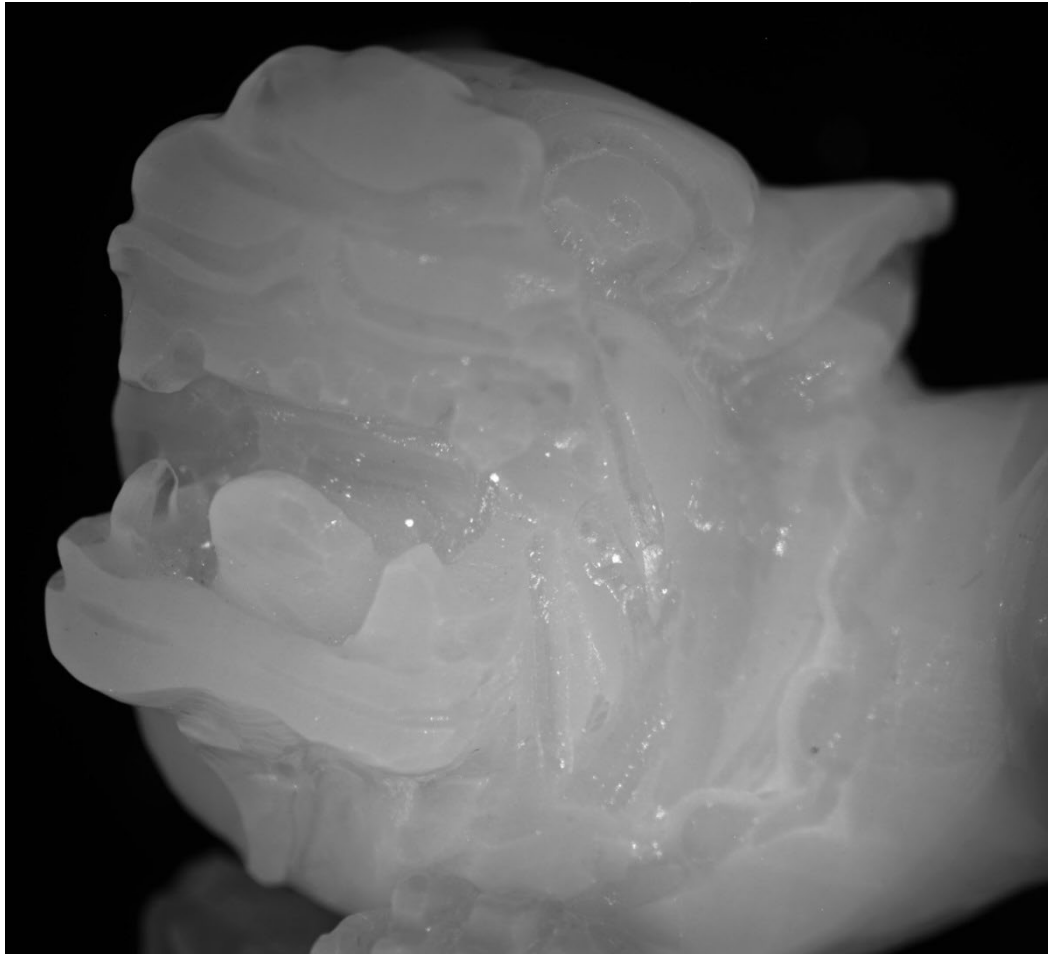


Time-of-flight imaging



15-463, 15-663, 15-862
Computational Photography
Fall 2023, Lecture 17

Course announcements

- Homework assignment 6 is due on Friday, December 8th.
 - Do not leave for last minute, you won't have time to complete it.
- Feedback for final project proposals posted.
- If your final project requires special equipment, sign up here:
https://docs.google.com/spreadsheets/d/1vbpppfjVQMJ7zGiaJEhxhVi4bW999GeJymeHxxL_0Pk/edit#gid=1109741985
- Vote for preferred date for make-up lecture on Slack.
- Two graphics talks this week—see Slack for details.

Overview of today's lecture

- Introduction to time-of-flight (ToF) imaging.
- Impulse ToF imaging and single-photon avalanche diodes.
- Continuous-wave ToF imaging.
- Interferometric ToF imaging.

Slide credits

A lot of these slides were adapted from:

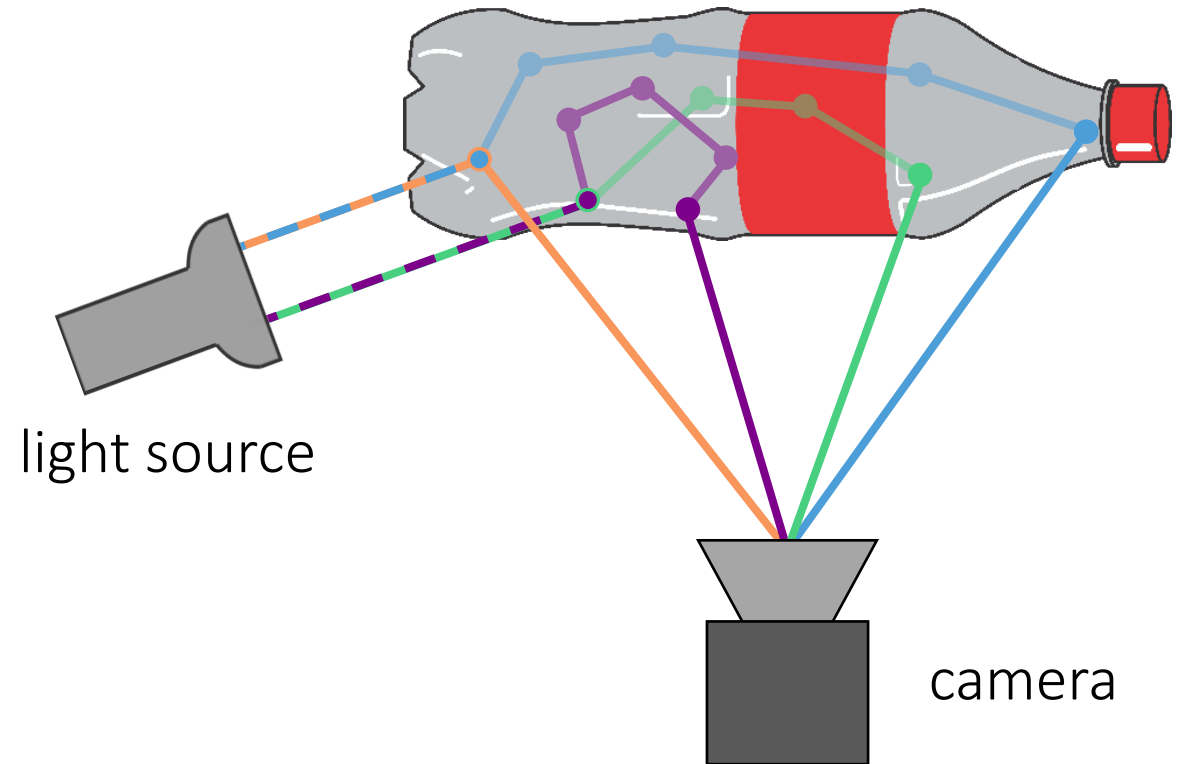
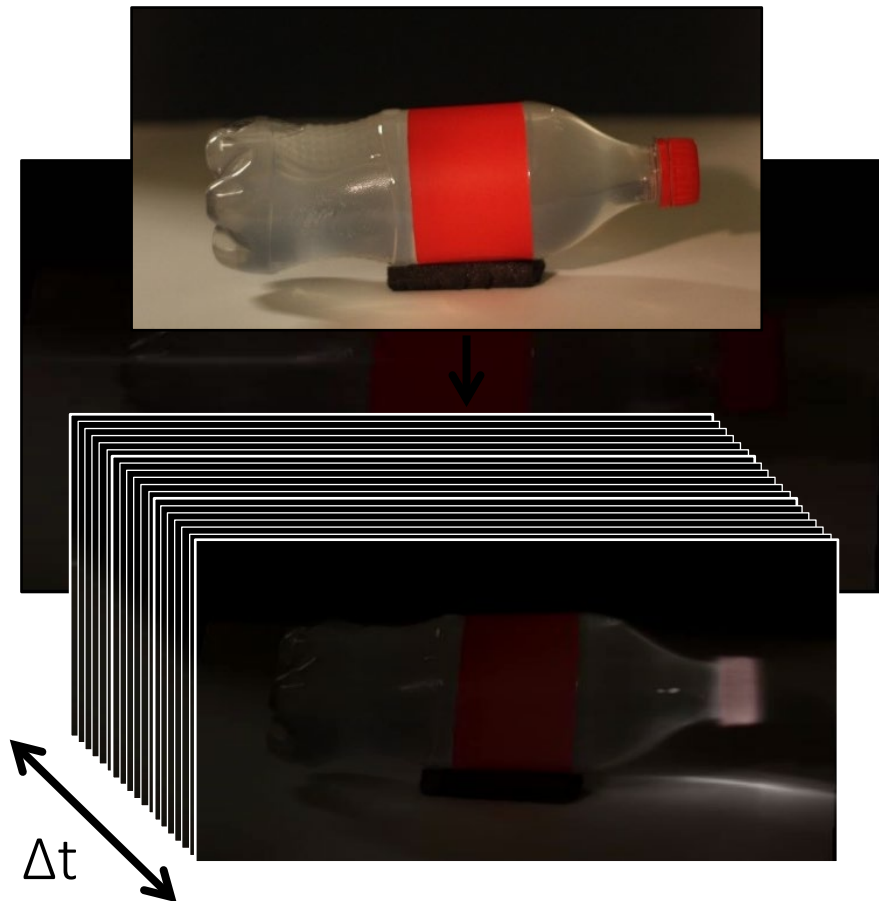
- Mohit Gupta (Wisconsin).
- Supreeth Achar (Google, formerly CMU).

Introduction to time-of-flight (ToF) imaging

Time-of-flight (ToF) imaging

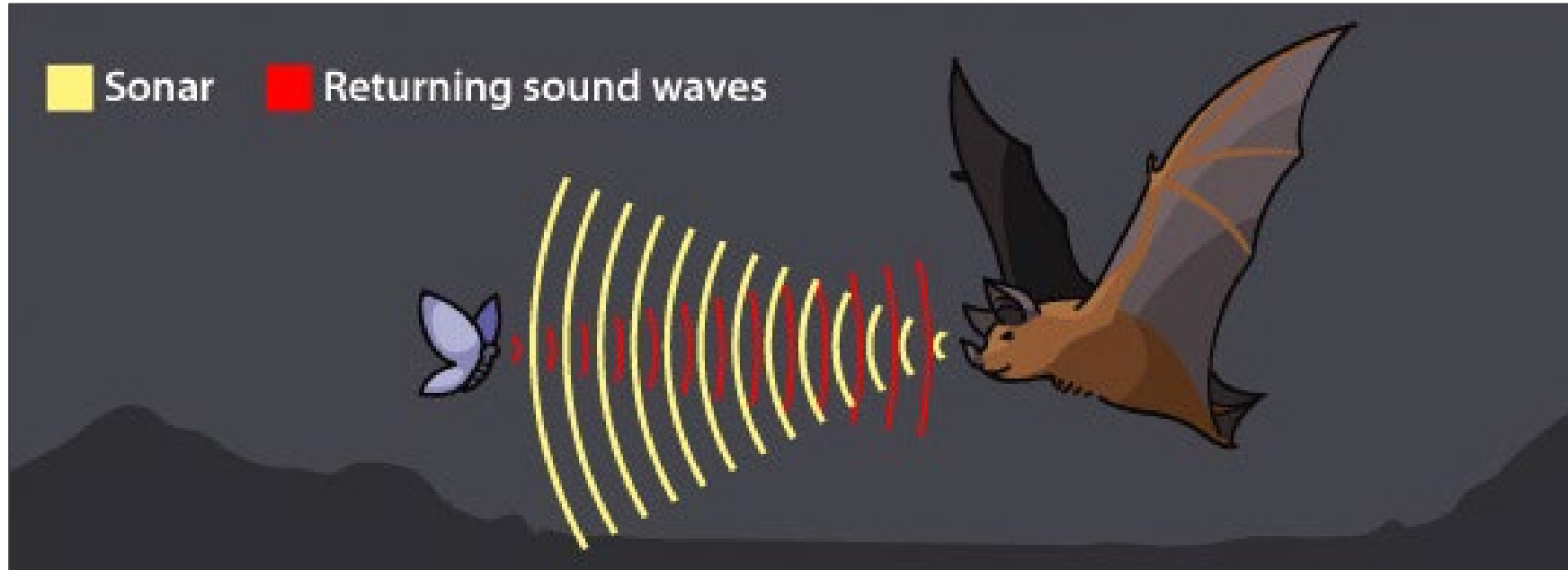


Time-of-flight (ToF) imaging

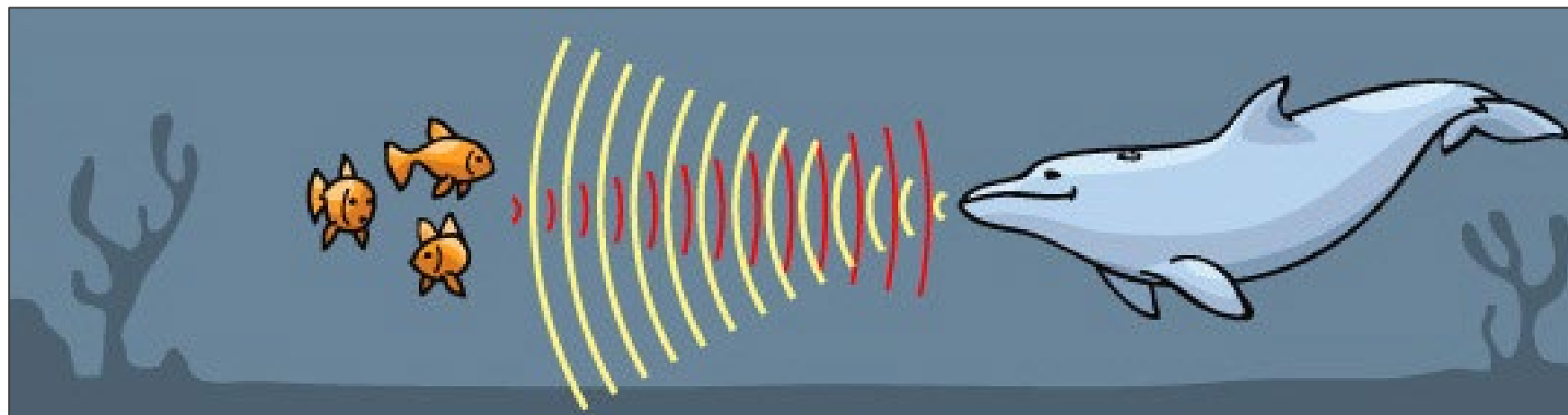


- Conventional imaging: Measure all photons together regardless of time of travel.
- Time-of-flight imaging: Measure photons separately based on time of travel.

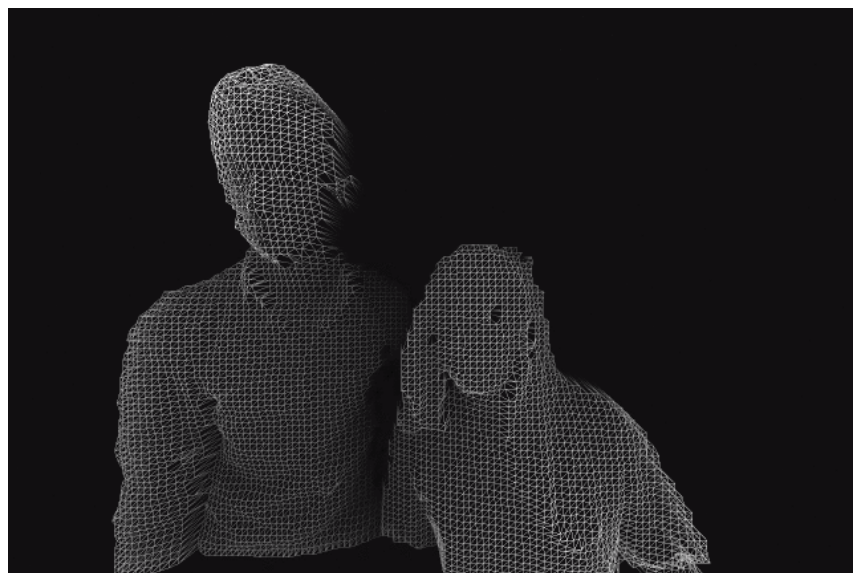
Time-of-flight imaging in nature



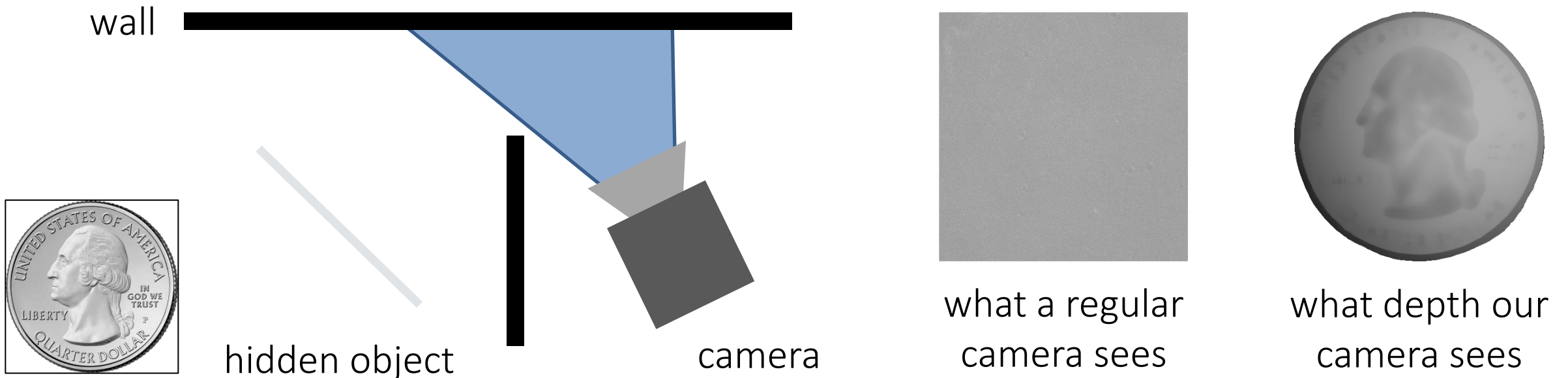
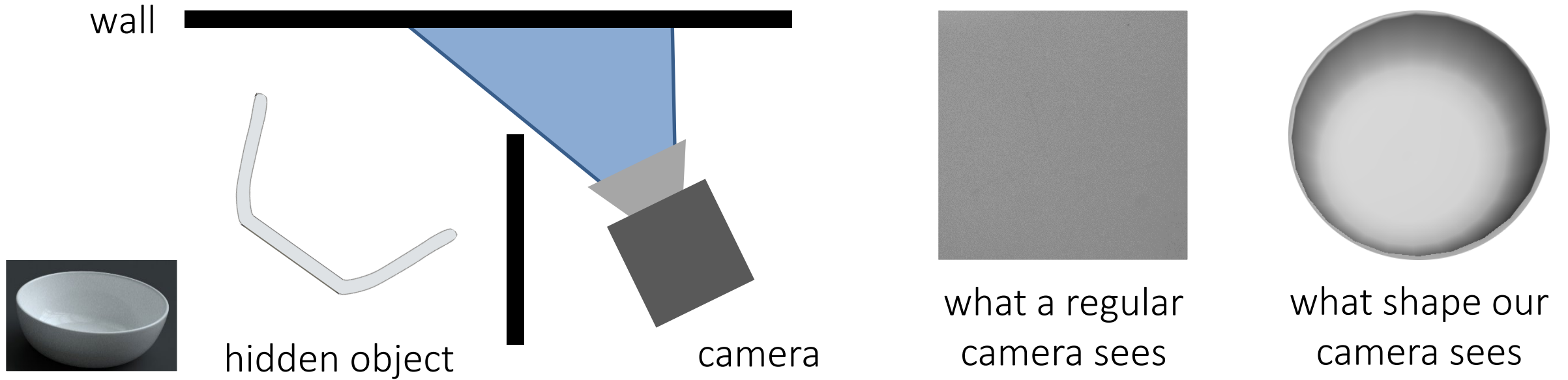
echolocation using sound-wave time-of-flight



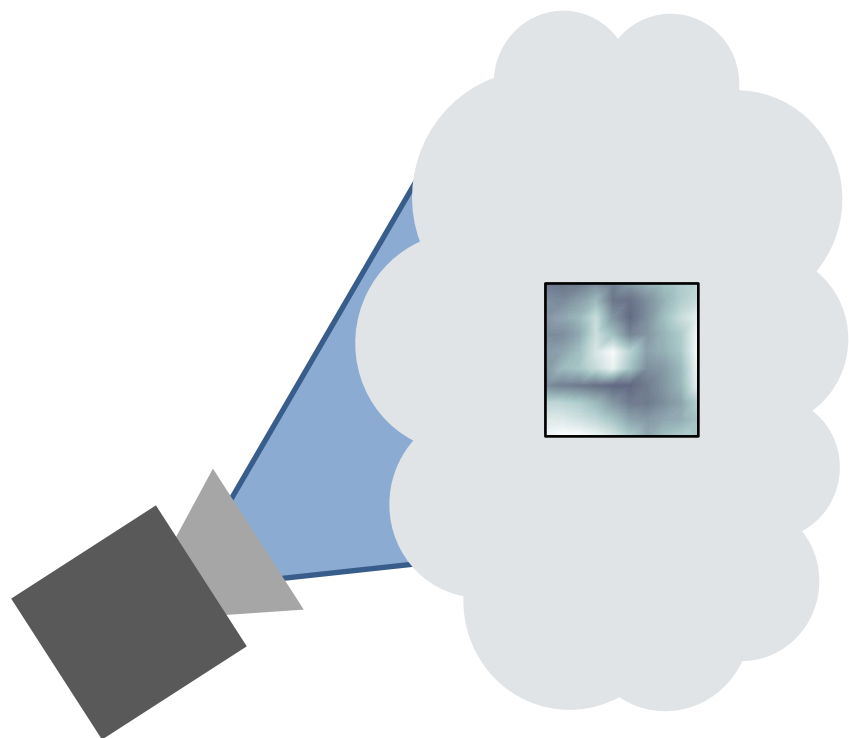
Time-of-flight applications: depth sensing



Time-of-flight applications: non-line-of-sight imaging



Time-of-flight applications: seeing inside objects

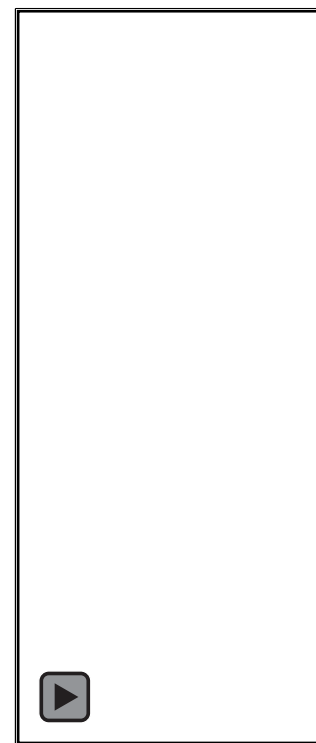


camera

thick smoke
cloud



what a regular
camera sees

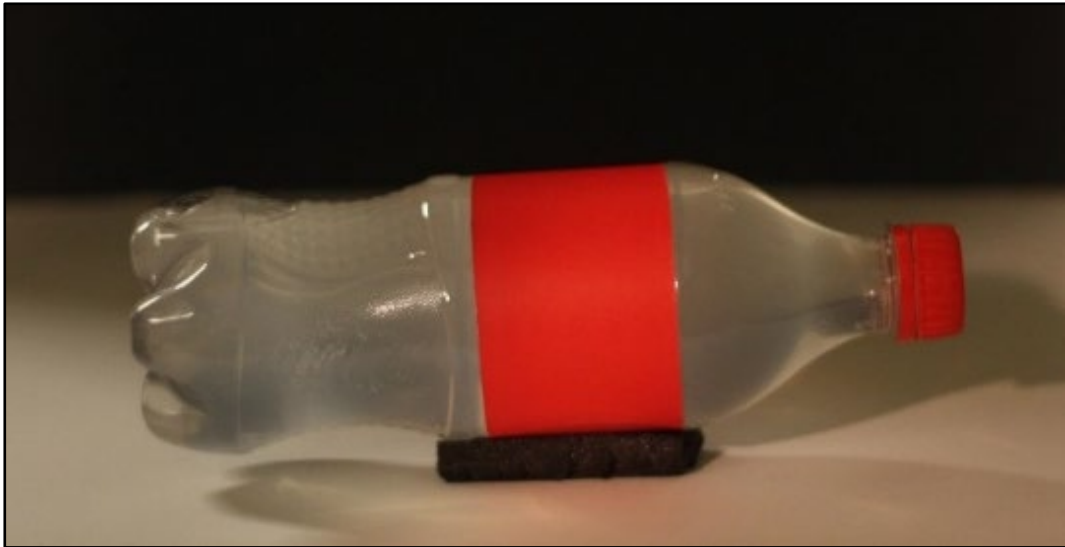


what our
camera sees

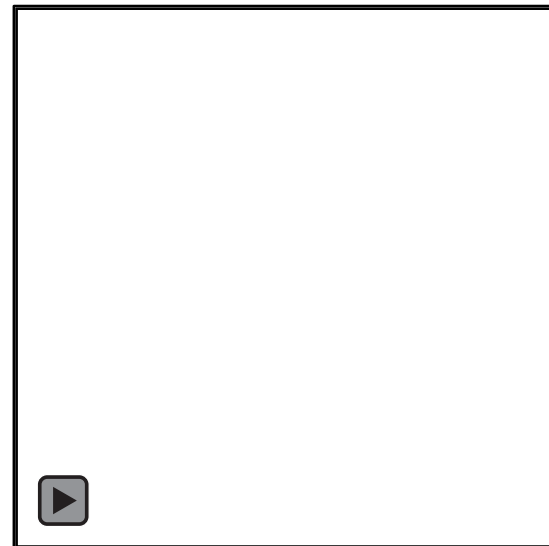


a slice through
the cloud

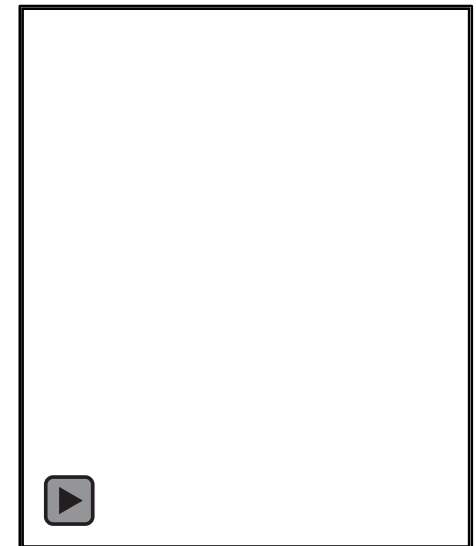
Time-of-flight applications: light-in-flight visualization



video at 10^{12} frames per second



video at 10^{15} frames per second



Time-of-flight imaging technologies

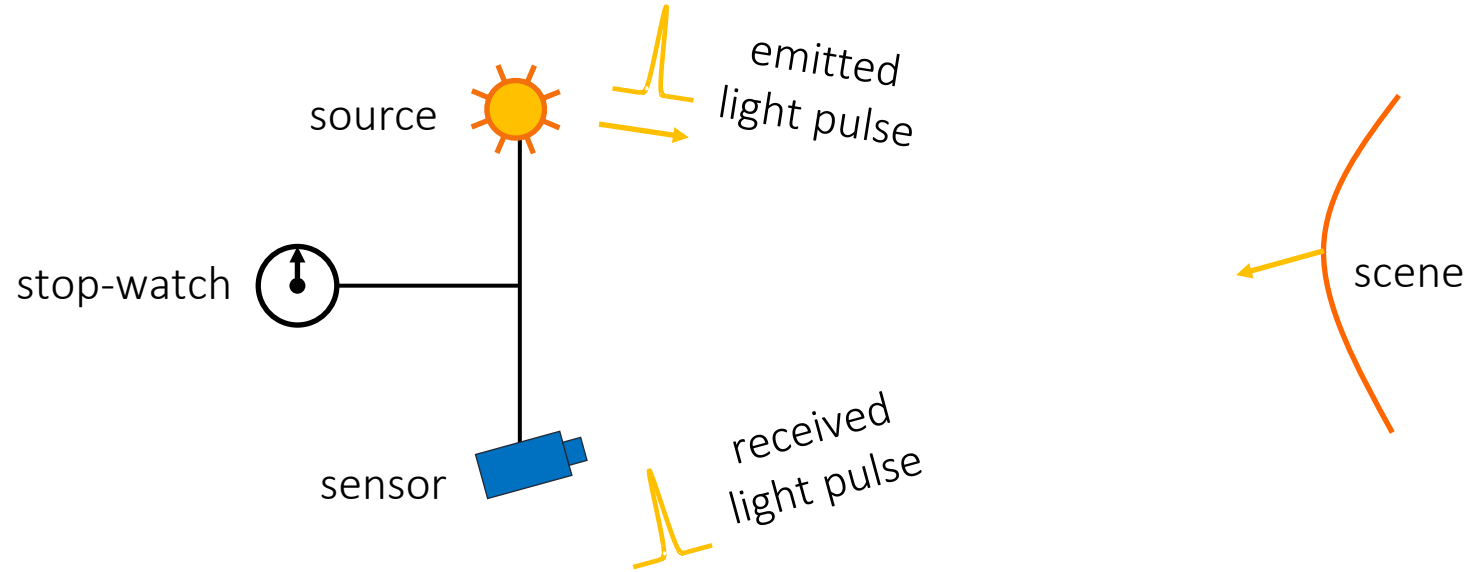
	interferometry	streak cameras	single-photon avalanche diodes	time-of-flight cameras	LIDAR
temporal resolution	1 femtosecond (10^{-15} secs)	1 picosecond (10^{-12} secs)	100 picoseconds (10^{-10} secs)	1 nanosecond (10^{-9} secs)	10 nanoseconds (10^{-8} secs)
frame rate	quadrillion fps	trillion fps	10 billion fps	billion fps	100 million fps
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Time-of-flight imaging technologies

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	continuous-wave ToF	impulse ToF			

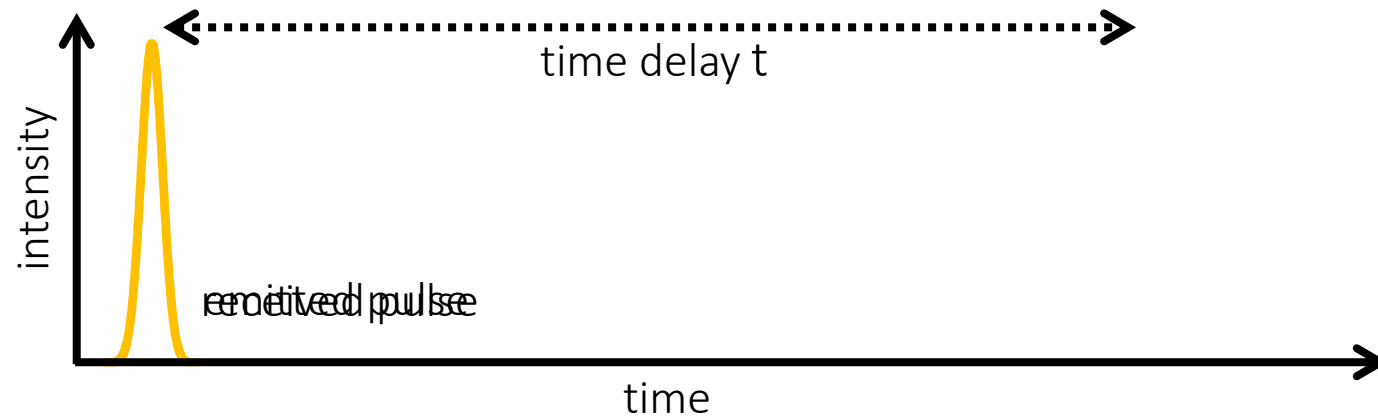
Impulse ToF imaging and single-photon avalanche diodes

Impulse time-of-flight imaging

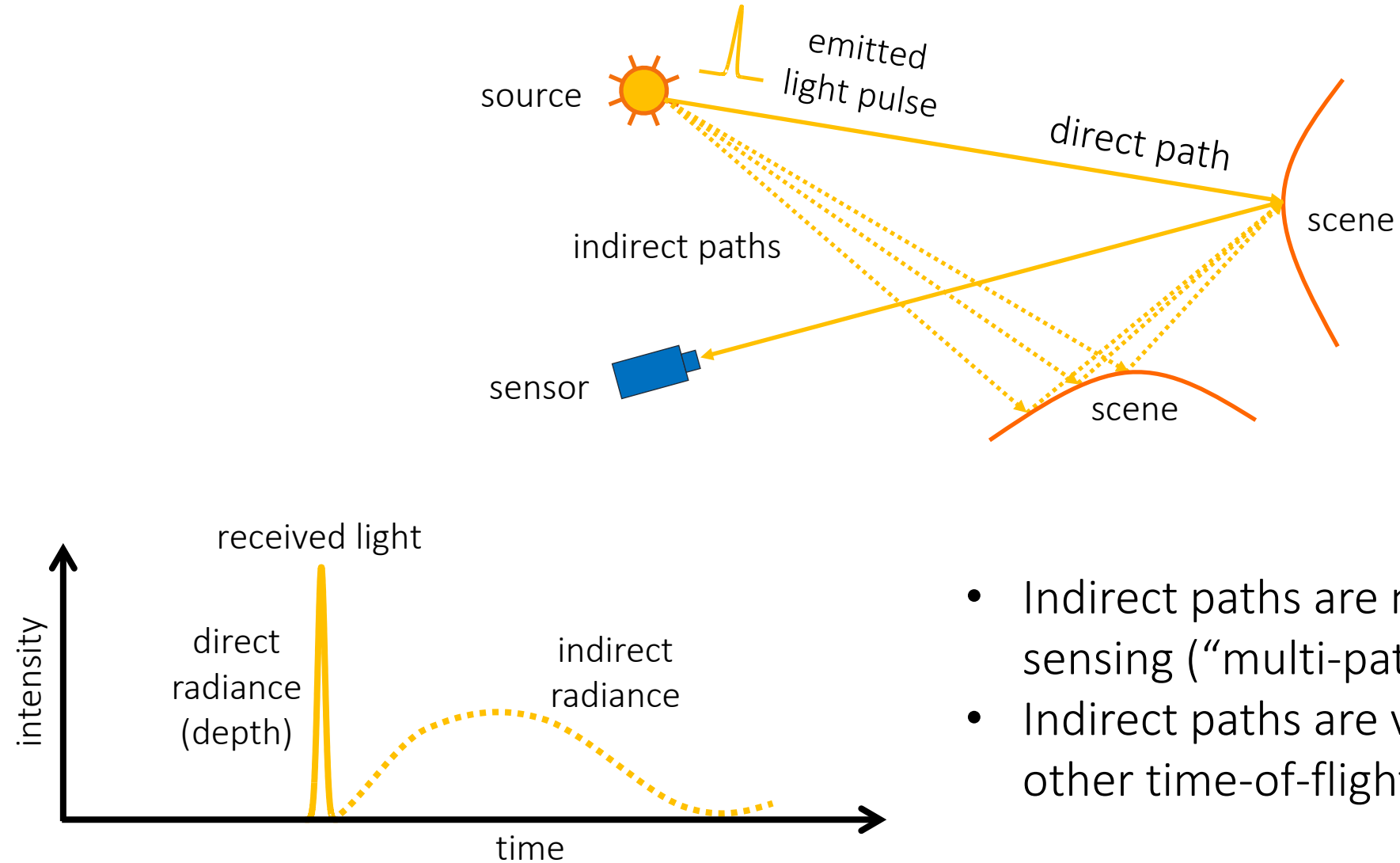


How can we infer depth from this?

$$\text{depth} = \frac{t}{2c}$$

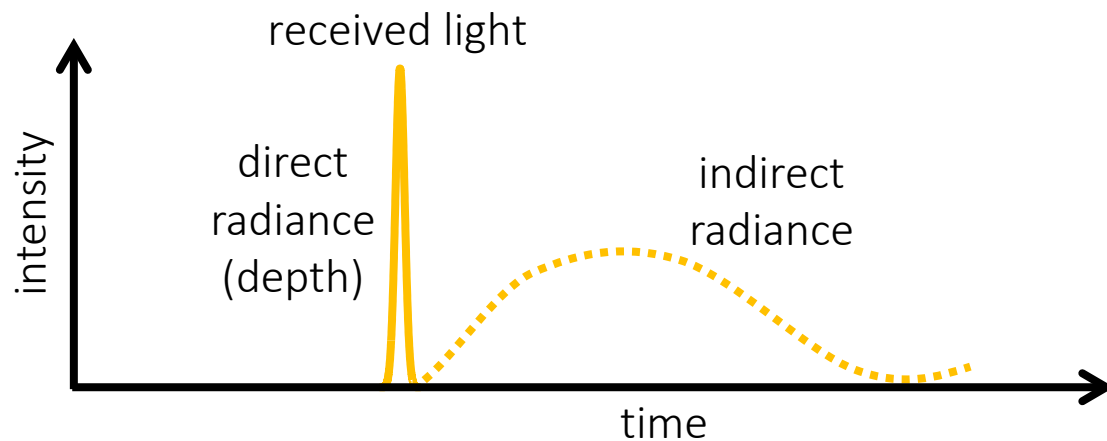
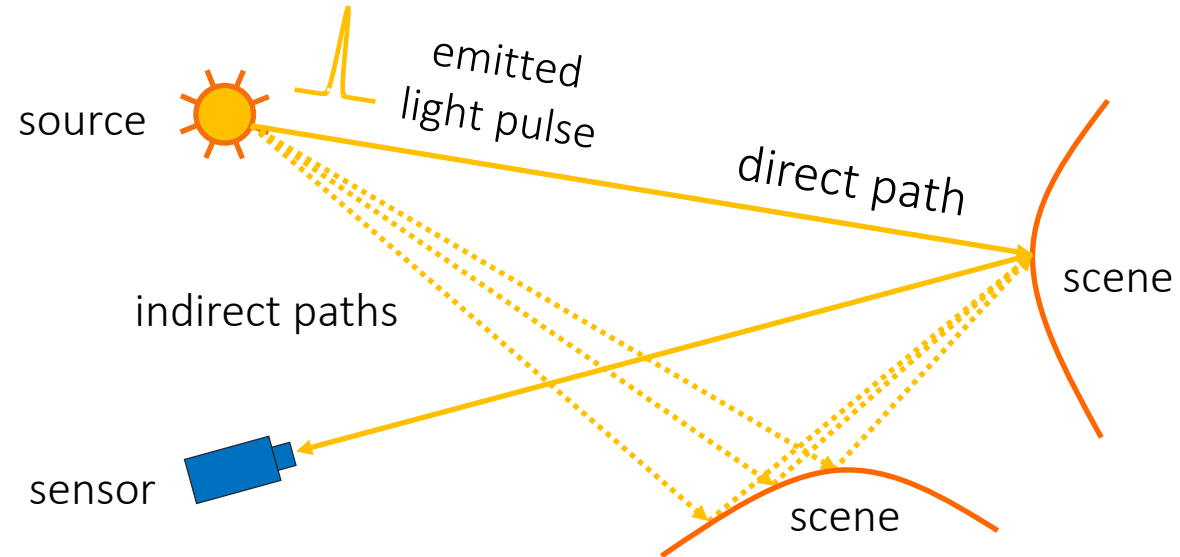


Impulse time-of-flight imaging



- Indirect paths are nuisance for depth sensing (“multi-path interference”).
- Indirect paths are very informative for other time-of-flight applications.

Two types of time-of-flight imaging



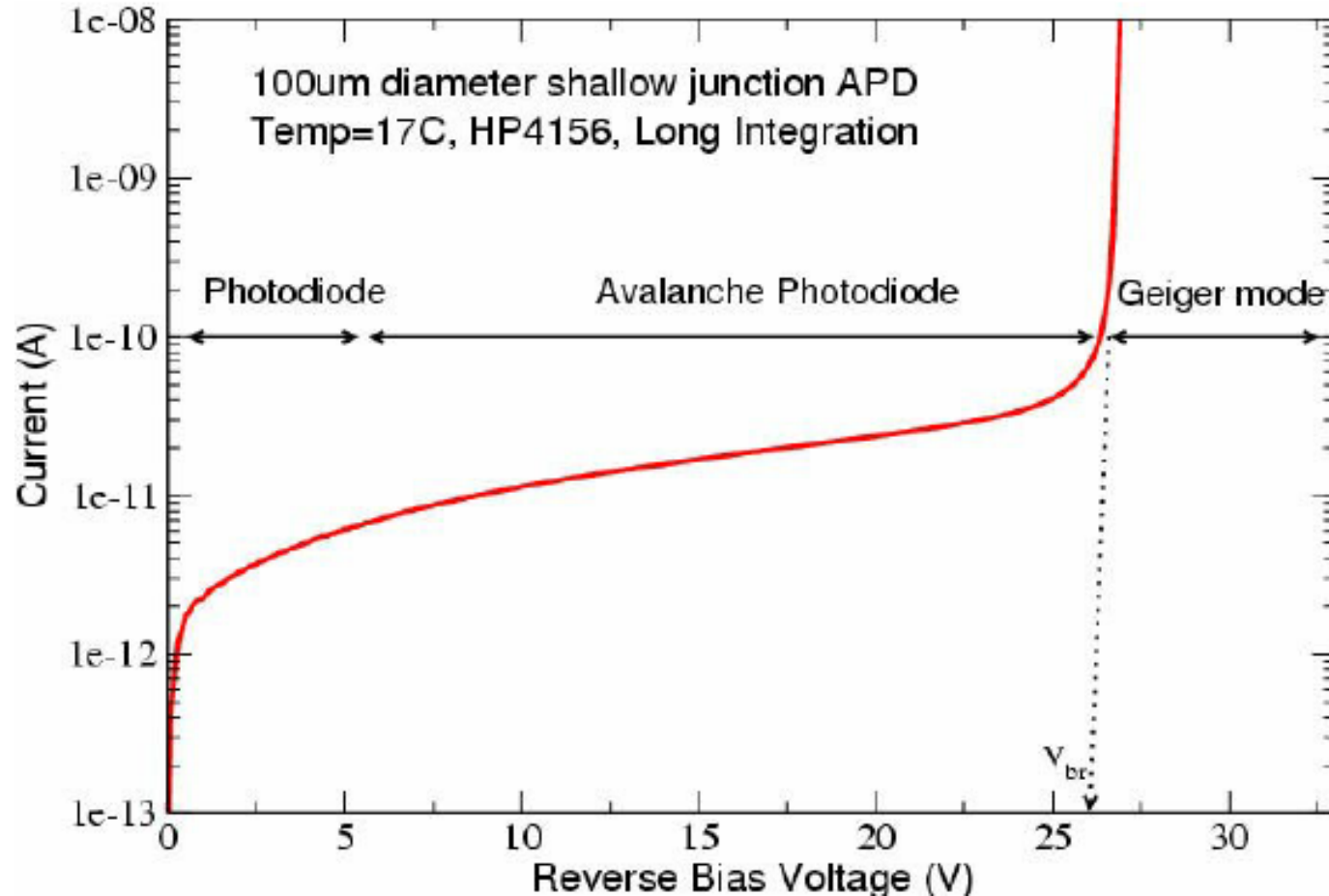
- Range imaging: Measuring only first returning photons (e.g., LIDAR).
- Transient imaging: Measuring entire transient (e.g., SPAD).

Transient $I(t)$: Time-resolved radiance distribution

How exactly is the transient formed?

Depends on the kind of sensor we use.

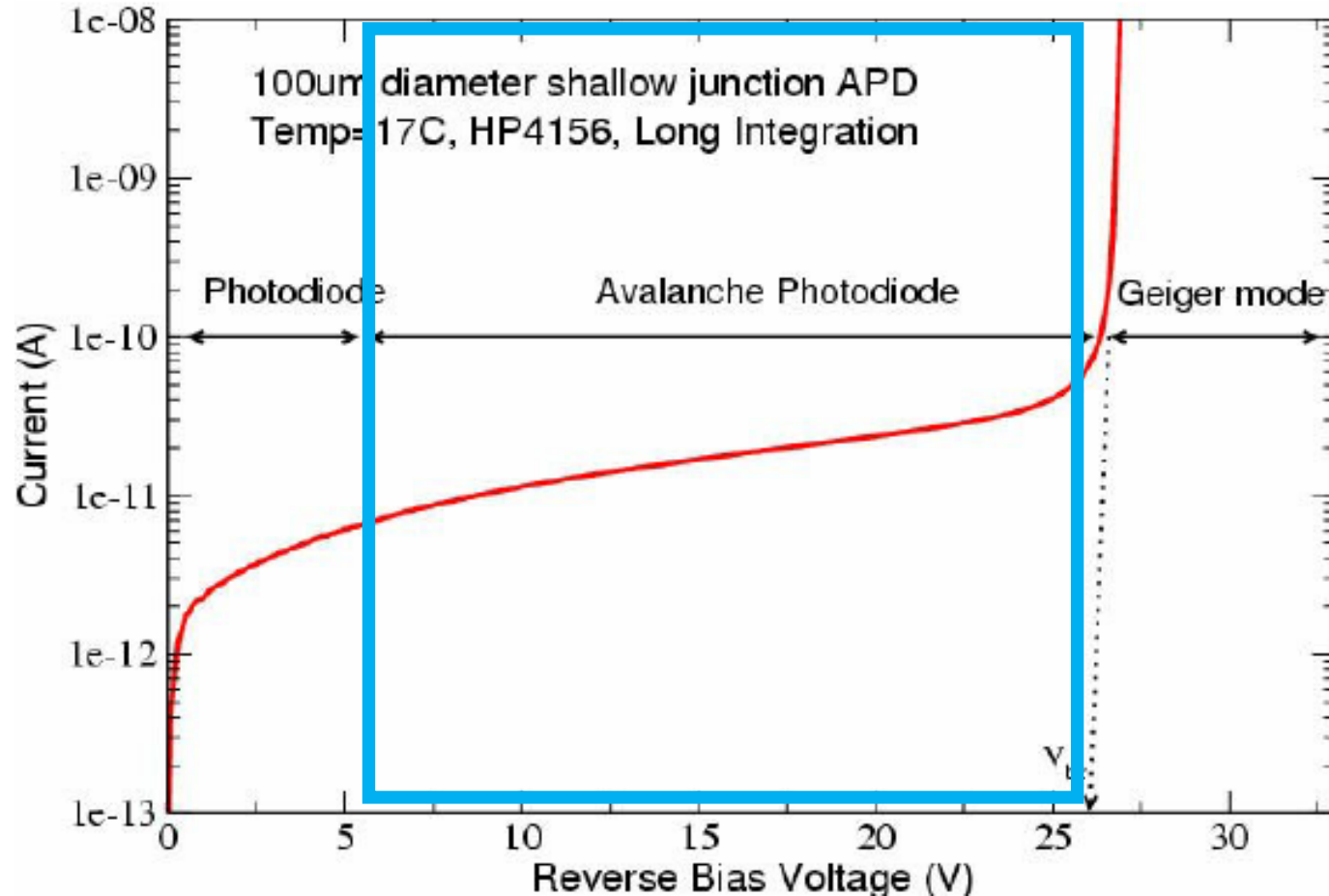
- Here we will examine only photodiodes.



How exactly is the transient formed?

Depends on the kind of sensor we use.

- Here we will examine only photodiodes.



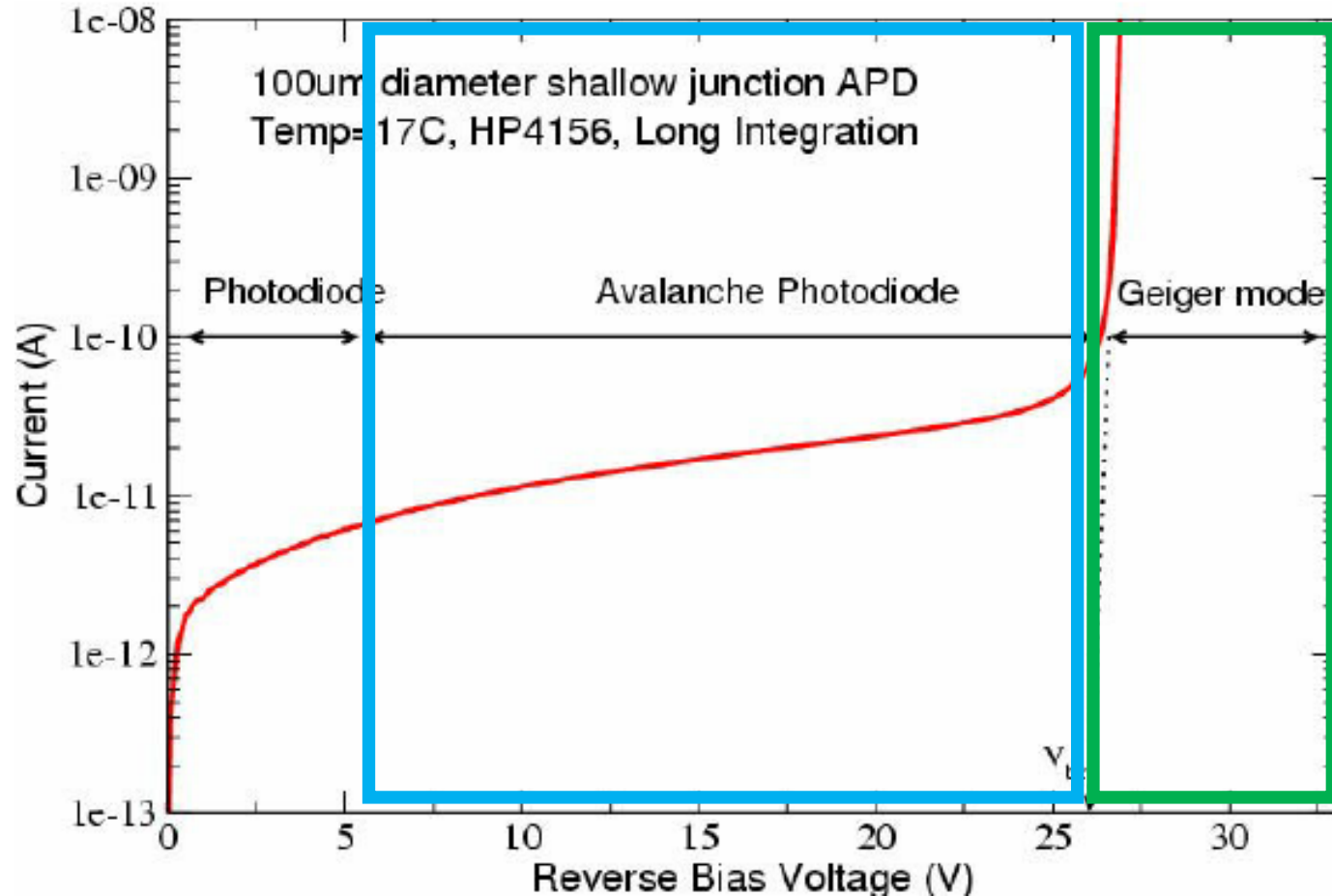
Avalanche photodiode (APD):

- Current is roughly proportional to number of photons.
- One photon produces tiny current.

How exactly is the transient formed?

Depends on the kind of sensor we use.

- Here we will examine only photodiodes.



Avalanche photodiode (APD):

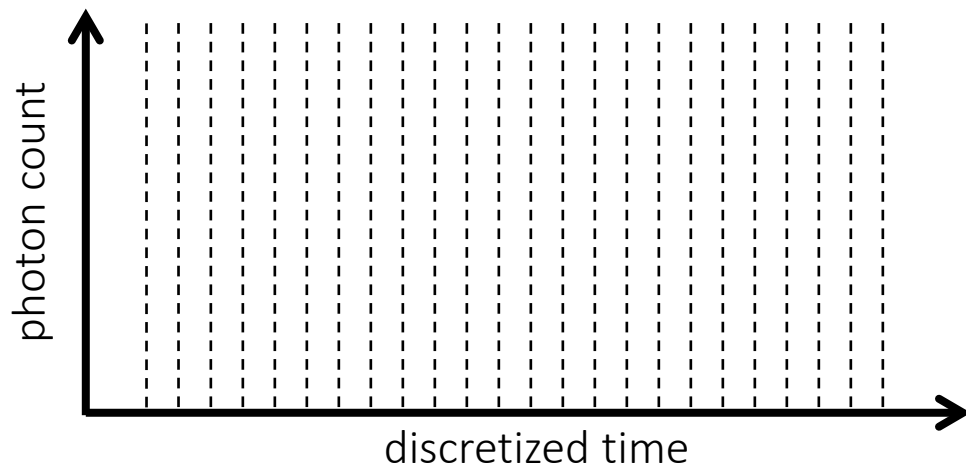
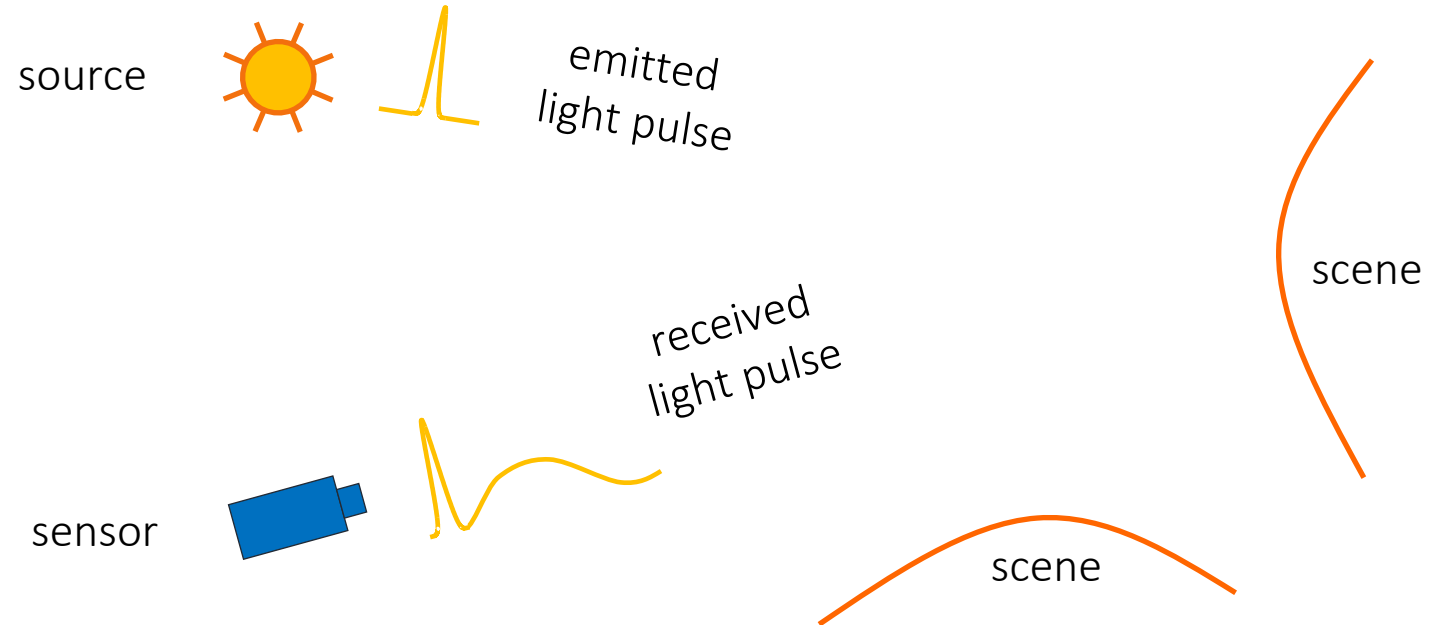
- Current is roughly proportional to number of photons.
- One photon produces tiny current.

Single-photon avalanche diode (SPAD):

- One photon produces huge current.
- Requires multiple low power pulses, so that one photon returns from each.

Geiger-mode impulse time-of-flight imaging

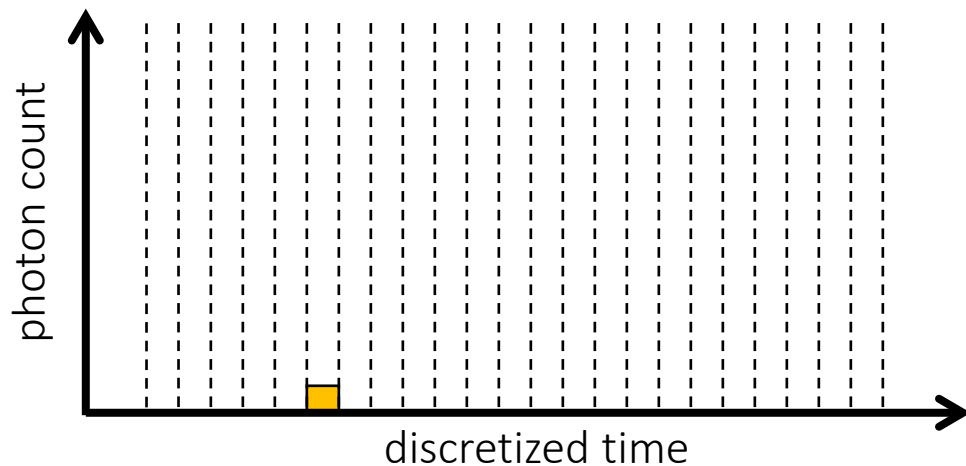
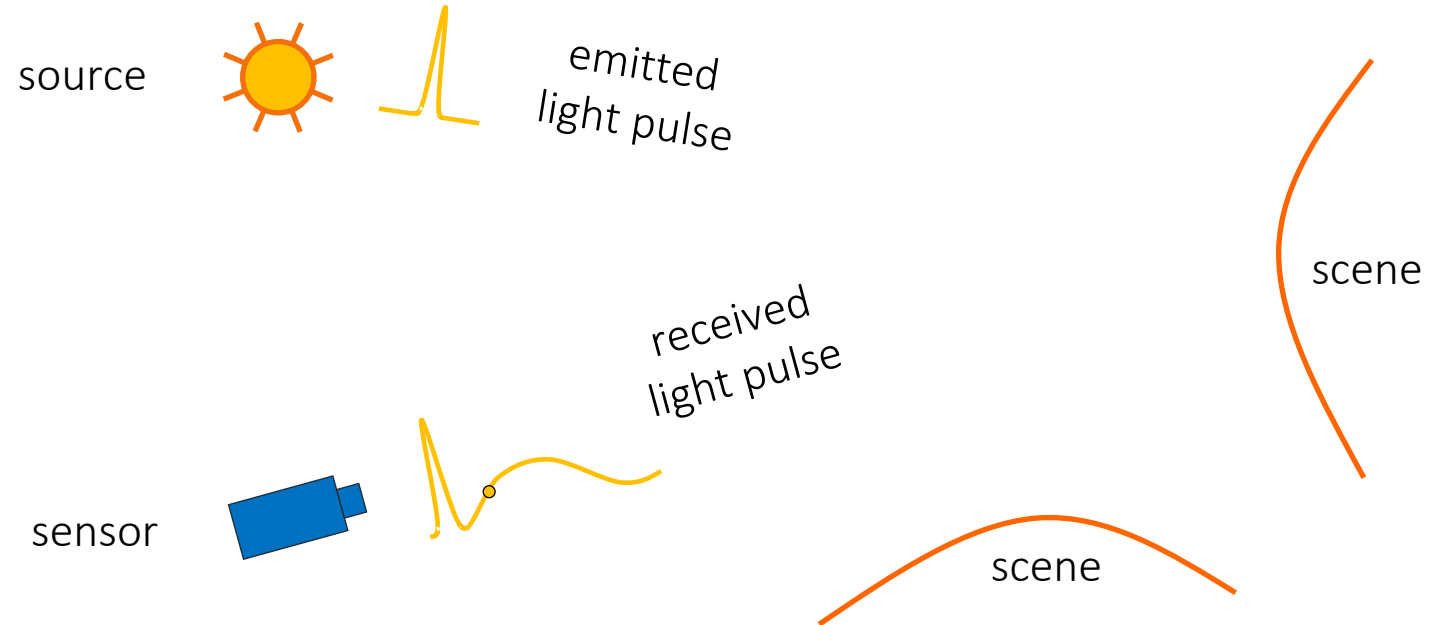
From each received pulse, one photon saturates the SPAD.



- The SPAD records only photon arrival times, no intensity.
- Additional electronics maintain a histogram of arrival times over multiple pulses

Geiger-mode impulse time-of-flight imaging

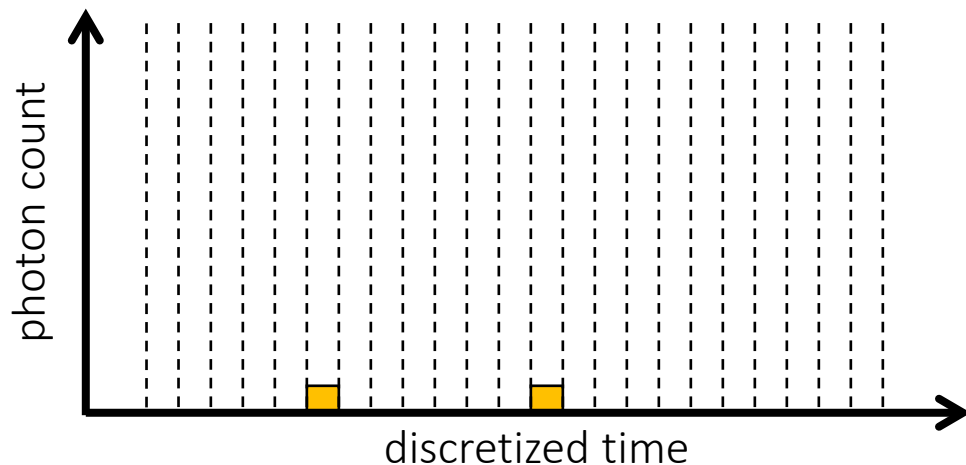
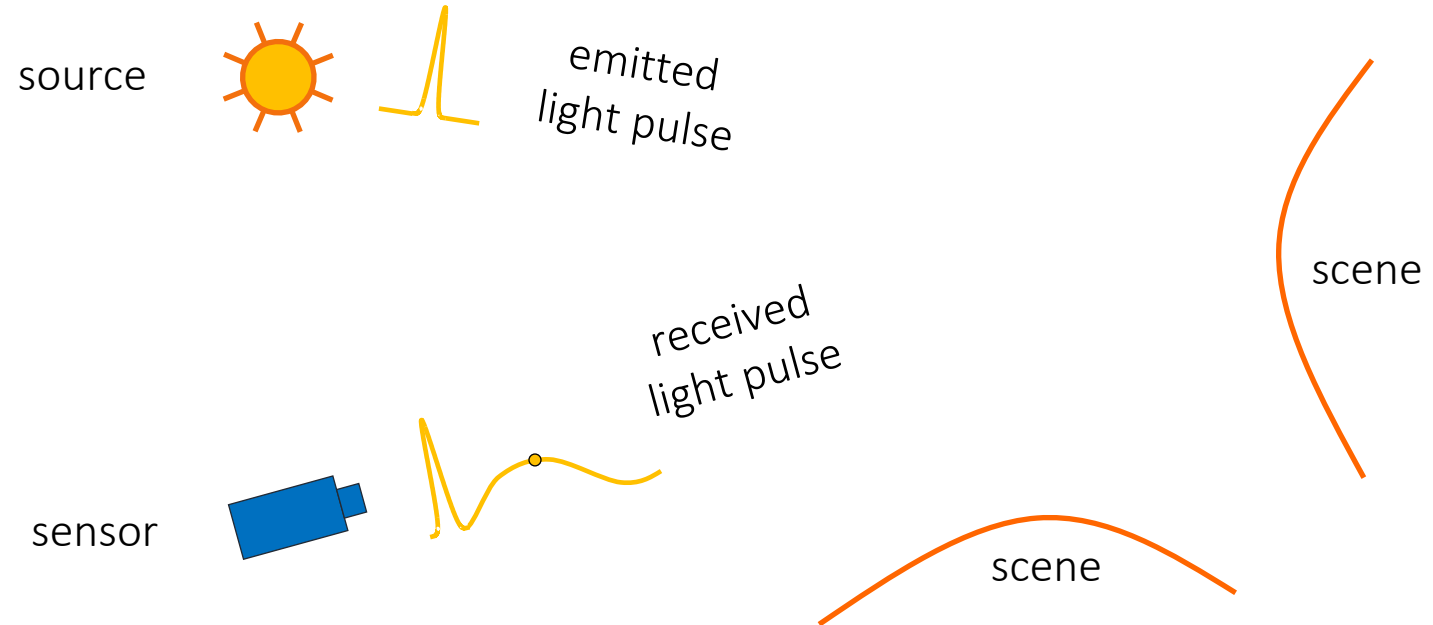
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Geiger-mode impulse time-of-flight imaging

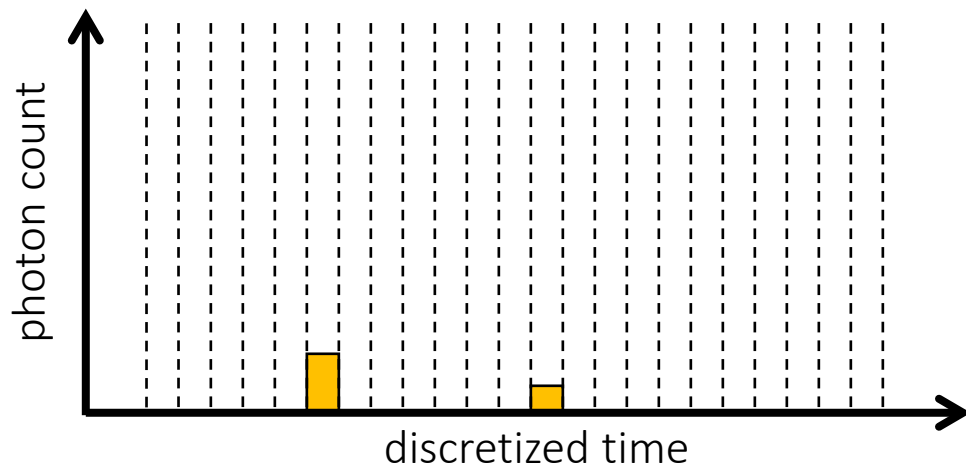
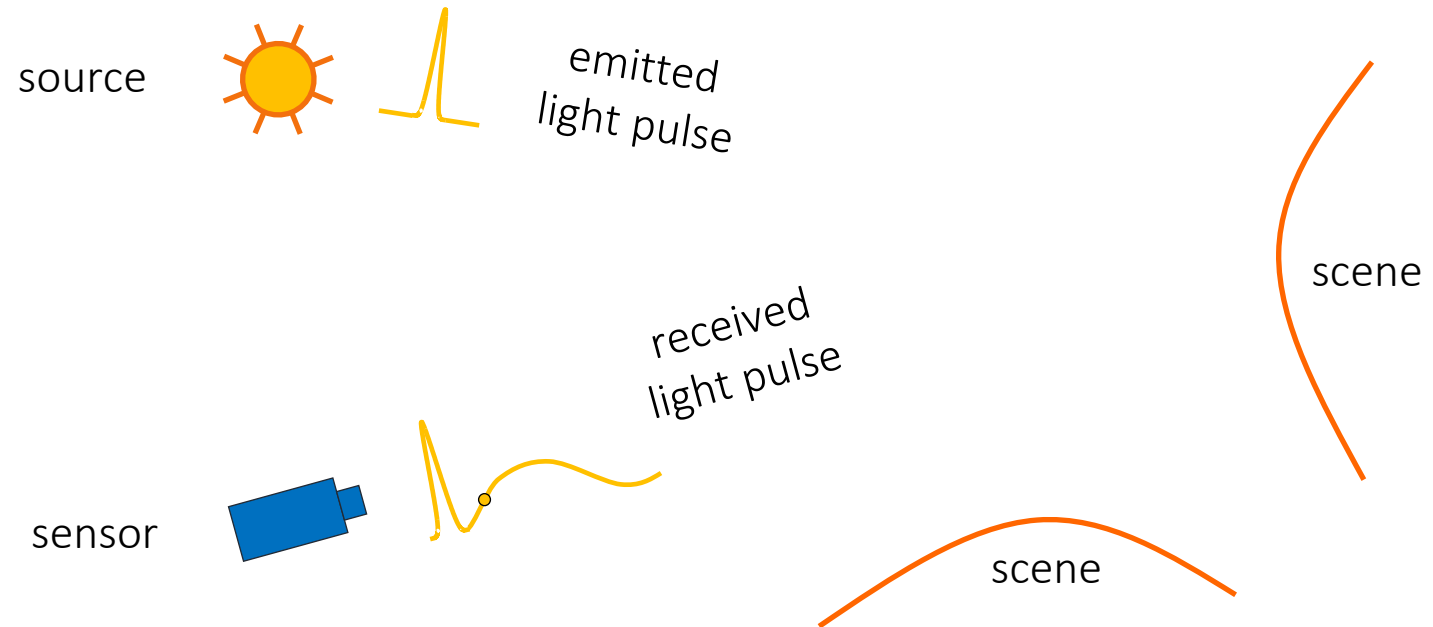
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Geiger-mode impulse time-of-flight imaging

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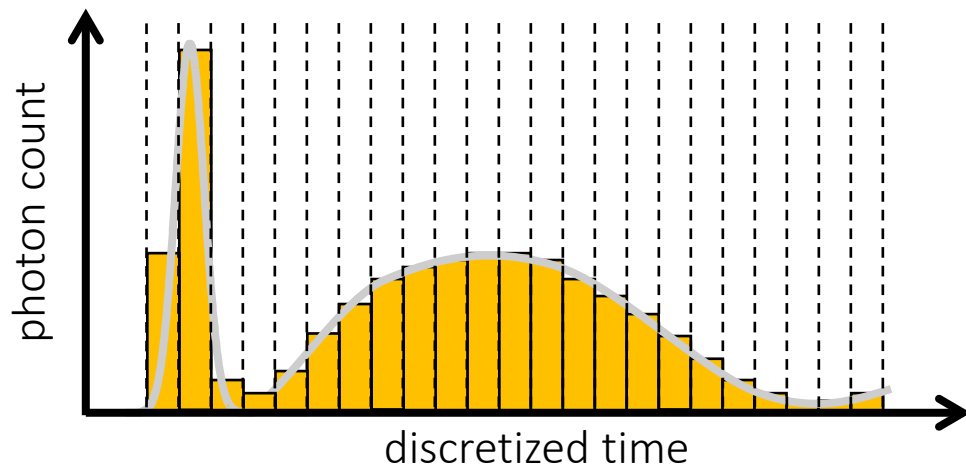
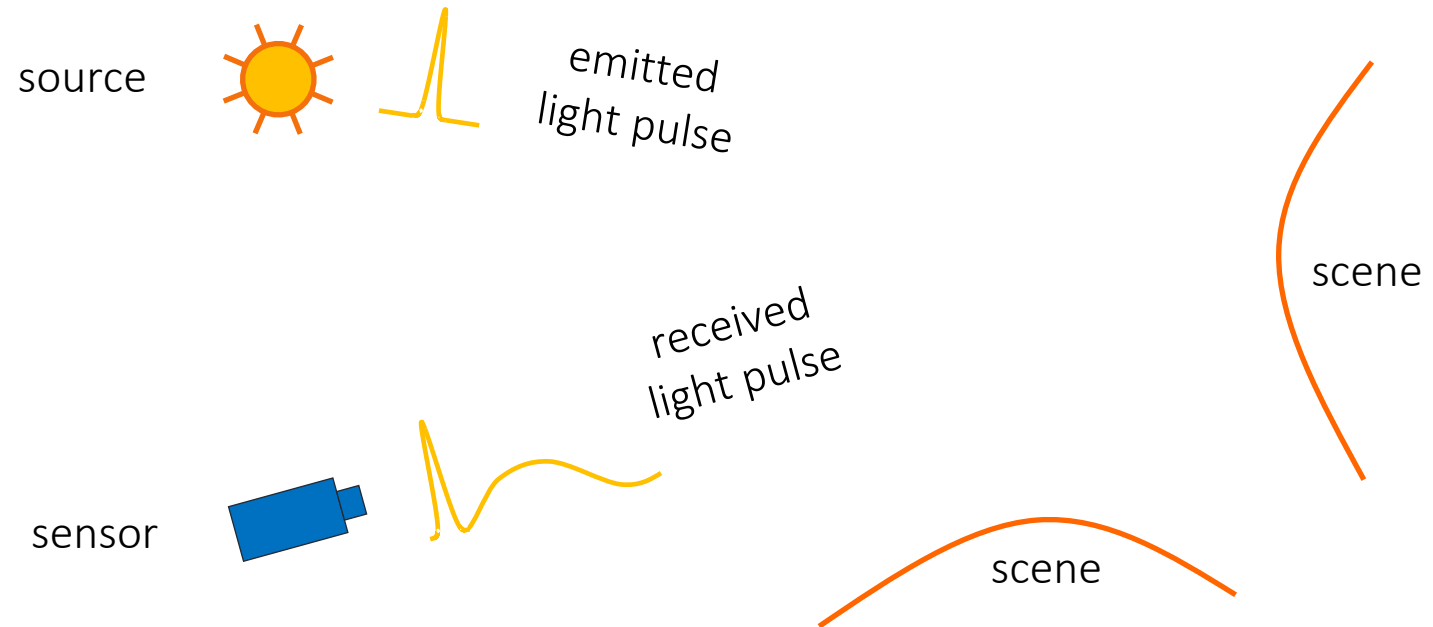


- The SPAD records only photon arrival times, no intensity.
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Geiger-mode impulse time-of-flight imaging

From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?

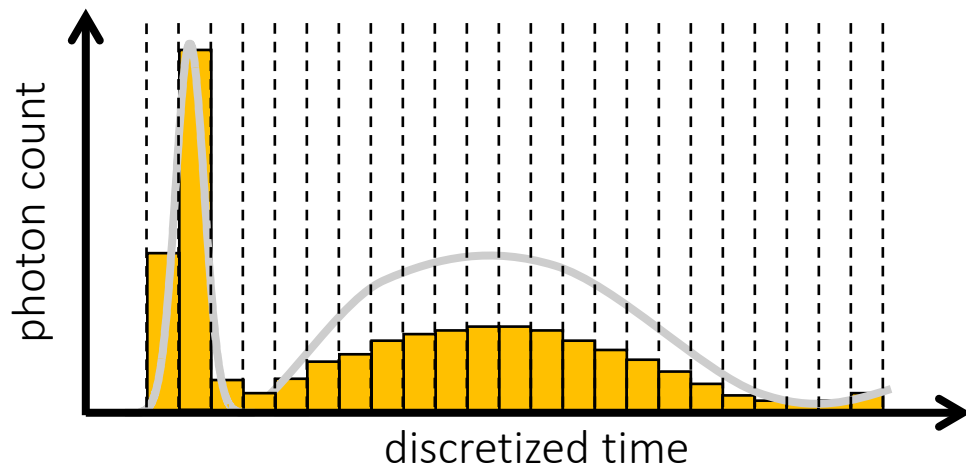
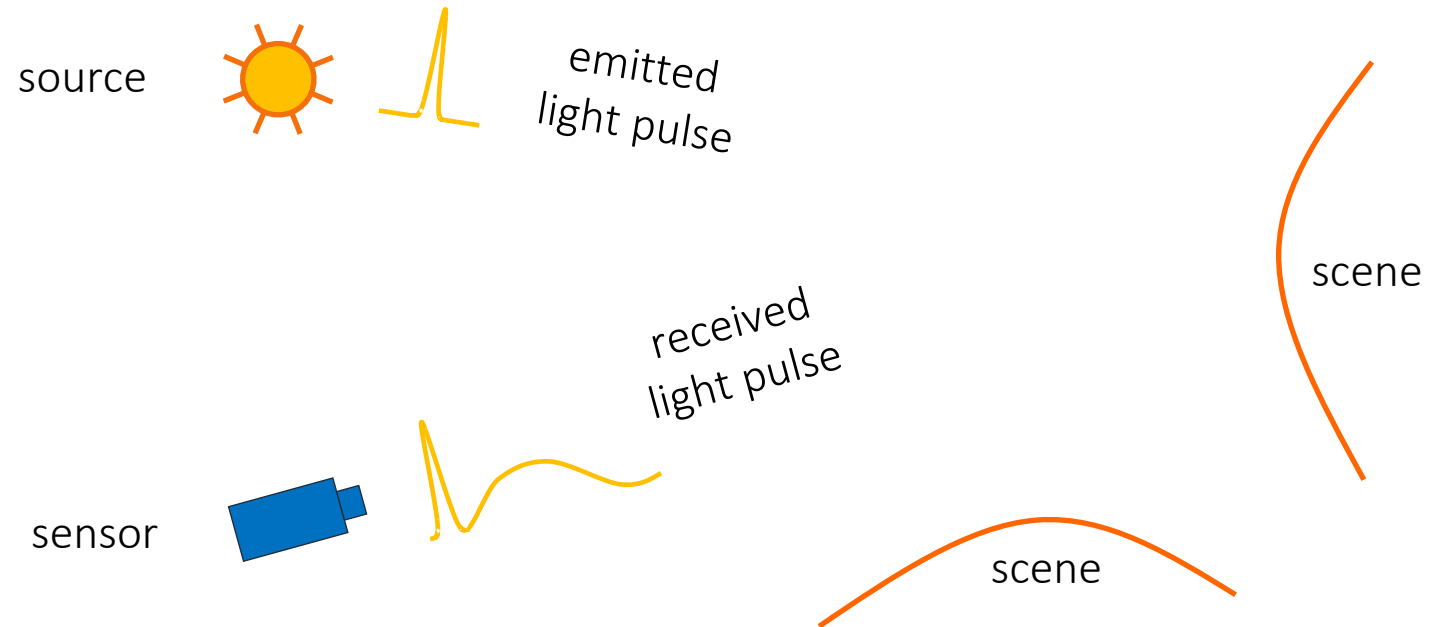


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Geiger-mode impulse time-of-flight imaging

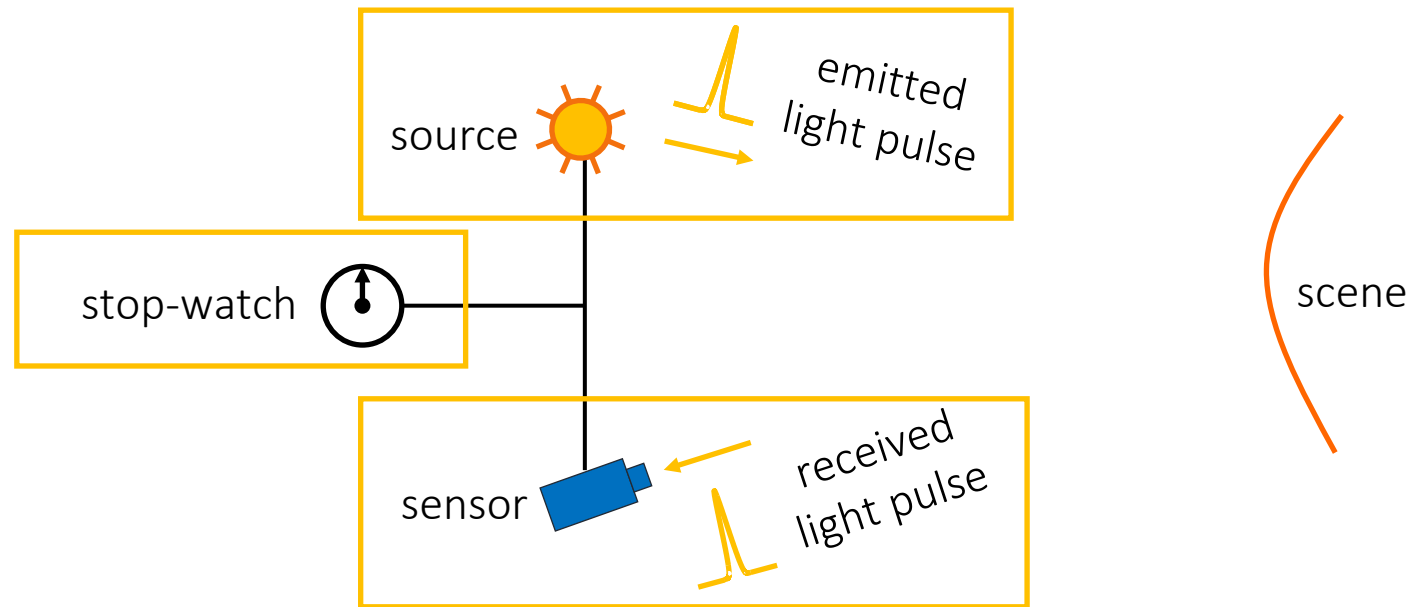
From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?



- Photons earlier in the transient have a higher probability of being detected than photons later in the transient.
- As a result, histogram of photon detections underestimates later parts of the transient.
- This effect is called *pile-up* and is very severe under strong light conditions.

What hardware do we need for impulse ToF?



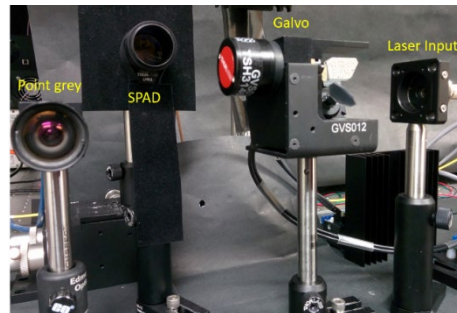
Expensive lasers

[short (picosecond) and powerful (mega joules) light pulses]



High speed and high

dynamic range sensors [single-photon sensitivity]



Expensive syncing and

photon-counting electronics [picosecond time resolution]

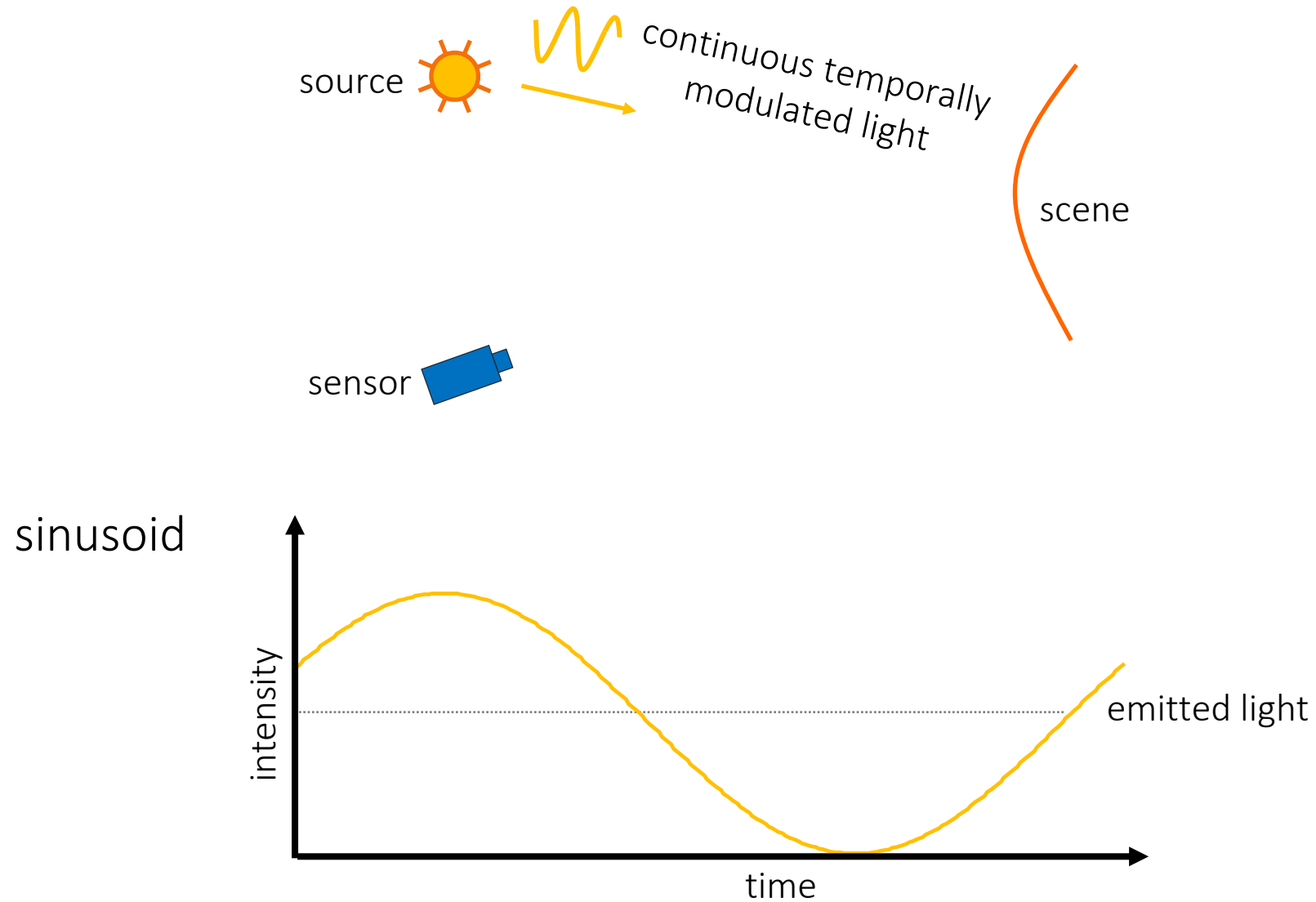


Time-of-flight imaging technologies

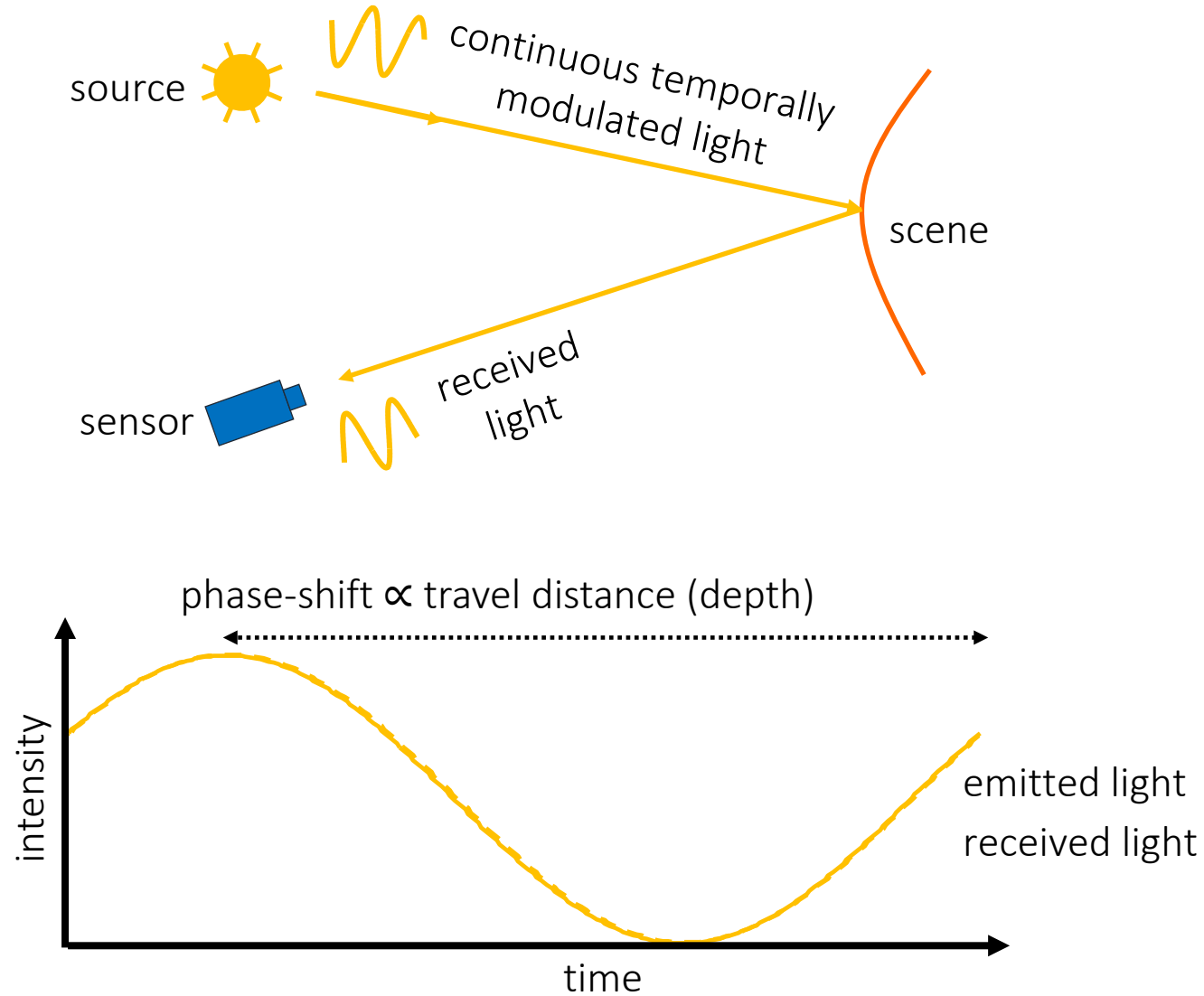
	interferometry	streak cameras	single-photon avalanche diodes	time-of-flight cameras	LIDAR
temporal resolution	1 femtosecond (10^{-15} secs)	1 picosecond (10^{-12} secs)	100 picoseconds (10^{-10} secs)	1 nanosecond (10^{-9} secs)	10 nanoseconds (10^{-8} secs)
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	continuous-wave ToF	impulse ToF			

Continuous-wave ToF imaging

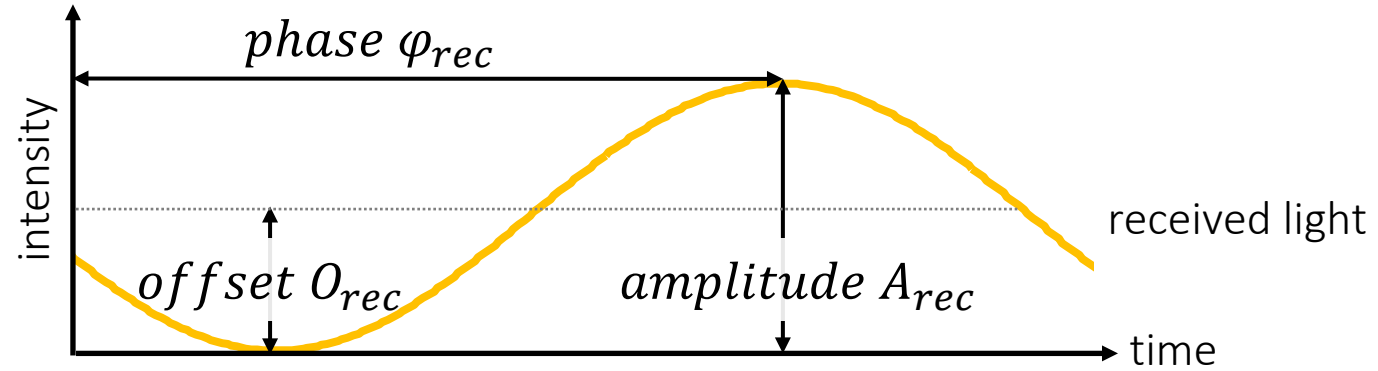
Continuous-wave (CW) time-of-flight imaging



Continuous-wave (CW) time-of-flight imaging



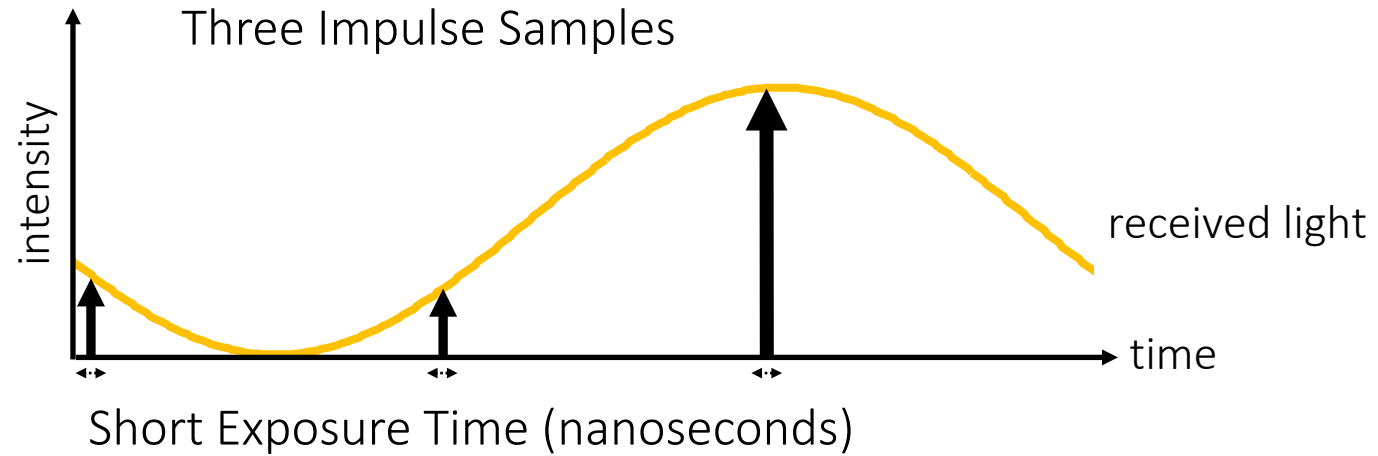
Measuring phase shift



Three Unknowns

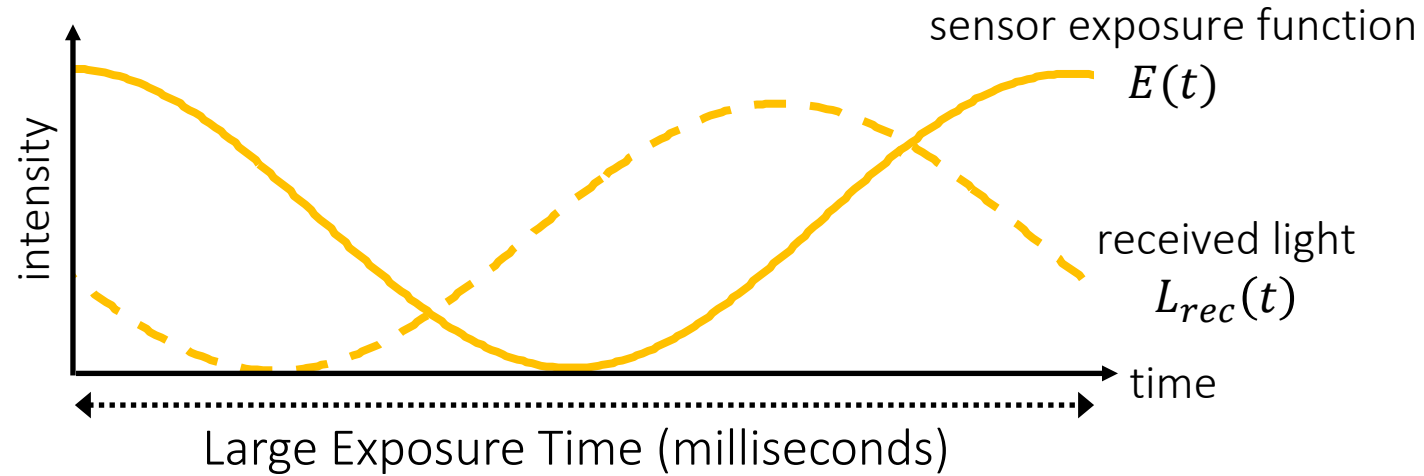
$$L_{rec}(t) = O_{rec} + A_{rec} \cos(\omega t - \phi_{rec})$$

Measuring phase shift: direct



Low Signal-to-Noise-Ratio

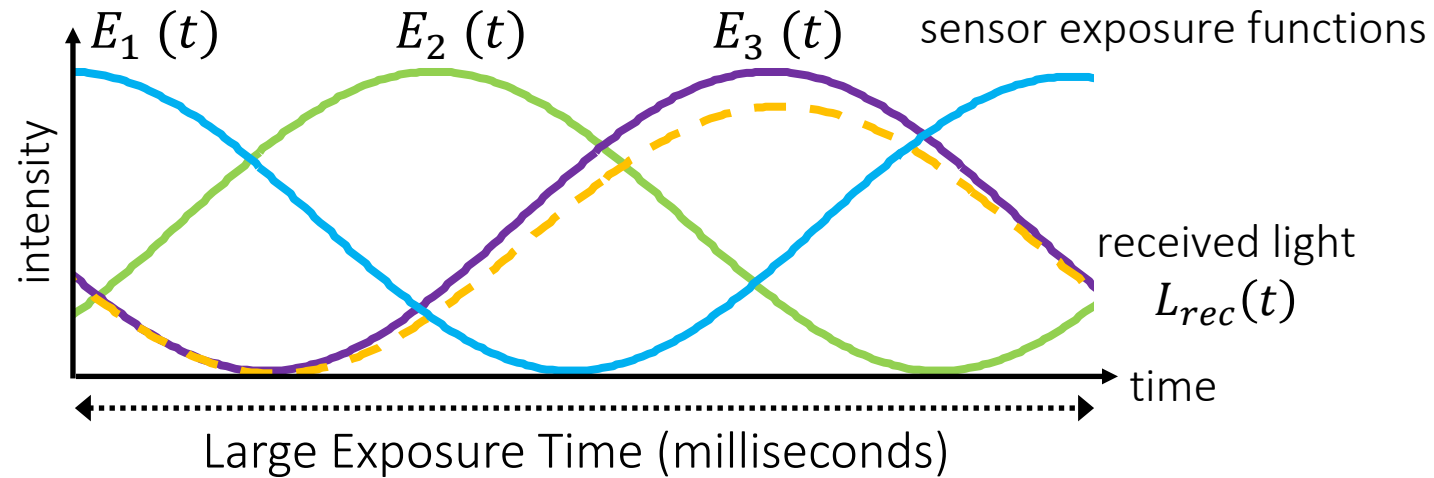
Measuring phase shift: correlation



Correlation: $I = \int E(t) \times L_{rec}(t) dt$

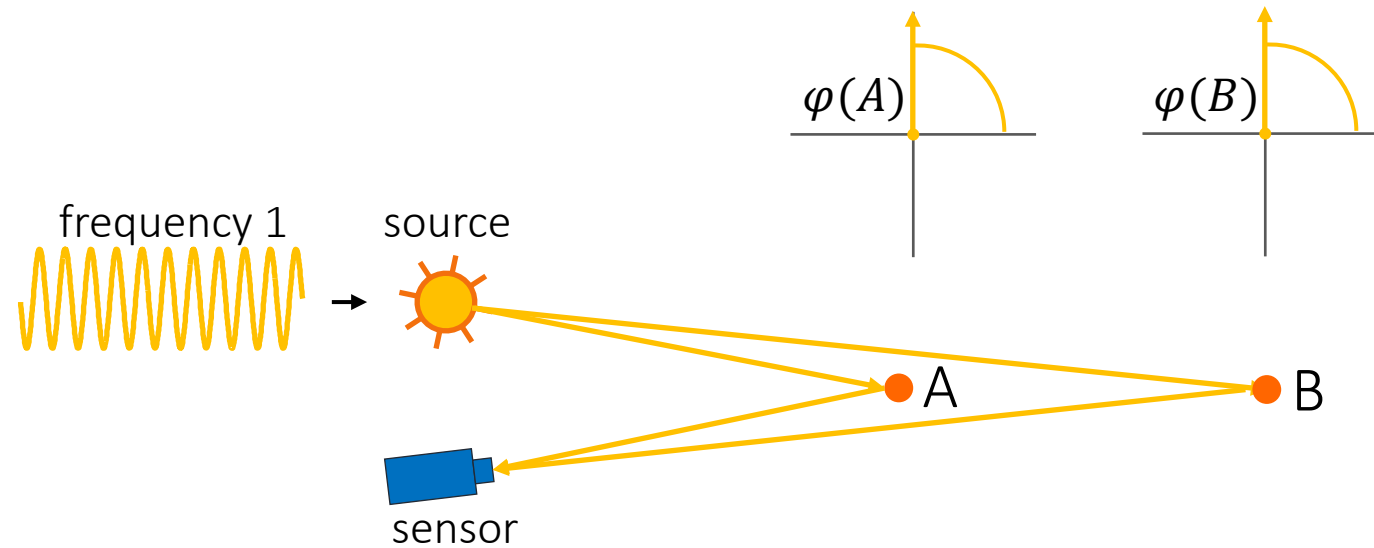
measured brightness exposure function received light

Measuring phase shift: correlation



$$\begin{array}{l}
 \text{Correlation 1: } I_1 = \int E_1(t) \times L_{rec}(t) dt \\
 \text{Correlation 2: } I_2 = \int E_2(t) \times L_{rec}(t) dt \\
 \text{Correlation 3: } I_3 = \int E_3(t) \times L_{rec}(t) dt
 \end{array}
 \left. \begin{array}{l}
 \text{High Signal-to-Noise Ratio} \\
 \text{Real Time Capture}
 \end{array} \right\} \begin{array}{l}
 \text{depth} \\
 \boxed{\text{phase } \varphi_{rec}} \\
 \text{offset } O_{rec} \\
 \text{amplitude } A_{rec}
 \end{array}$$

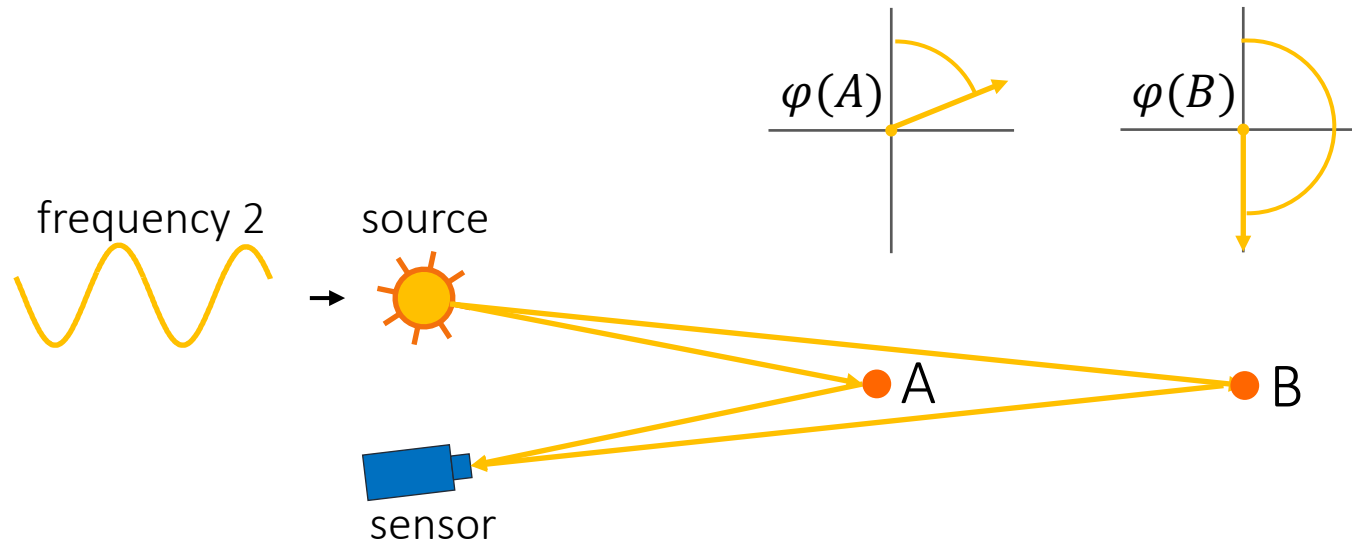
Phase ambiguity



Different Scene Depths Have Same Phase

- Also known as “phase wrapping”.

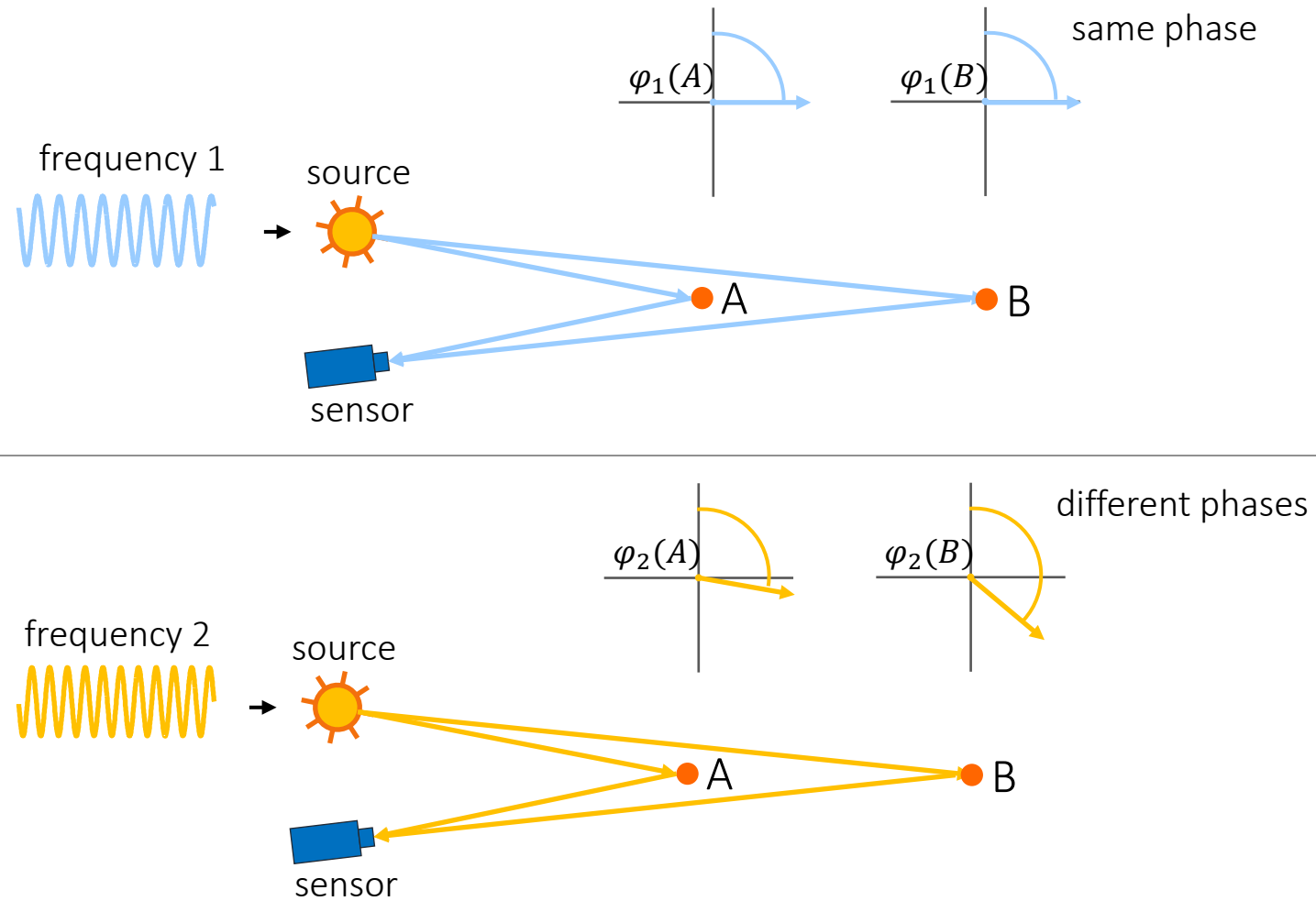
Phase ambiguity



Unambiguous Depth Range: $R_{unambiguous} = \frac{1}{2\omega}$

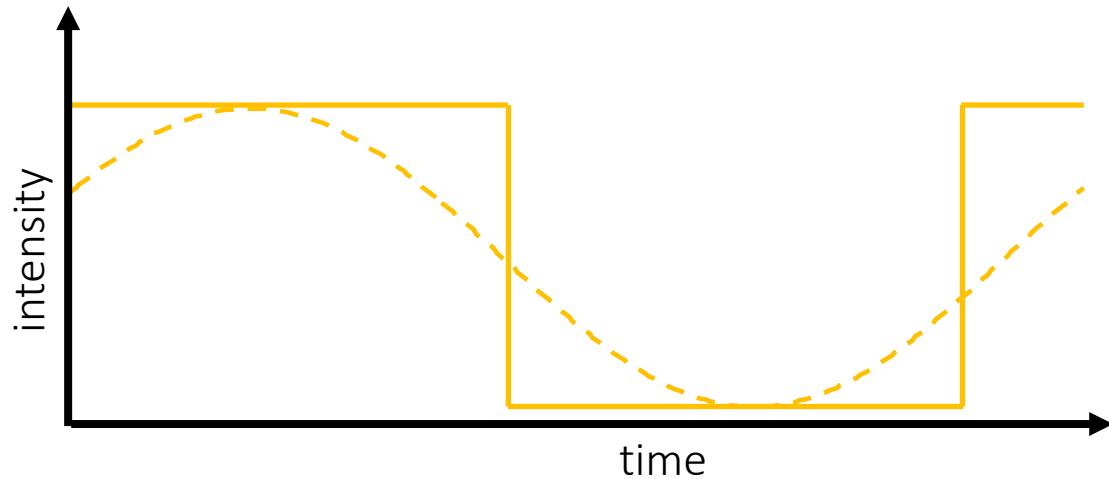
How can we resolve the phase ambiguity?

Disambiguating phase

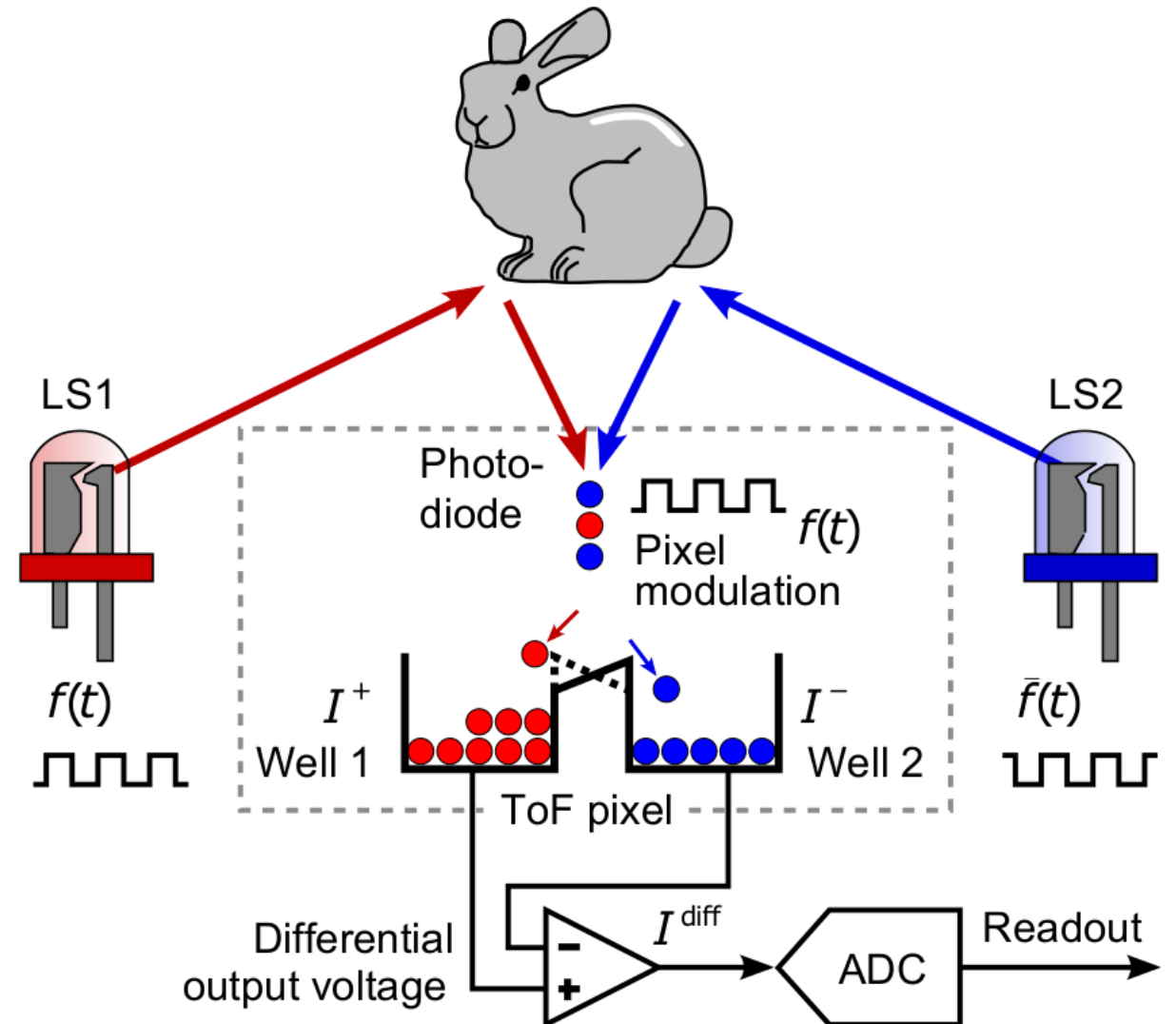


Compute phases at two different frequencies

Implementation: two-well architectures



- approximate sinusoid with a square pulse
- store photons in different wells depending on whether they arrive at 1 or 0
- take difference between two wells



Some examples

light source
(bank of laser diodes)



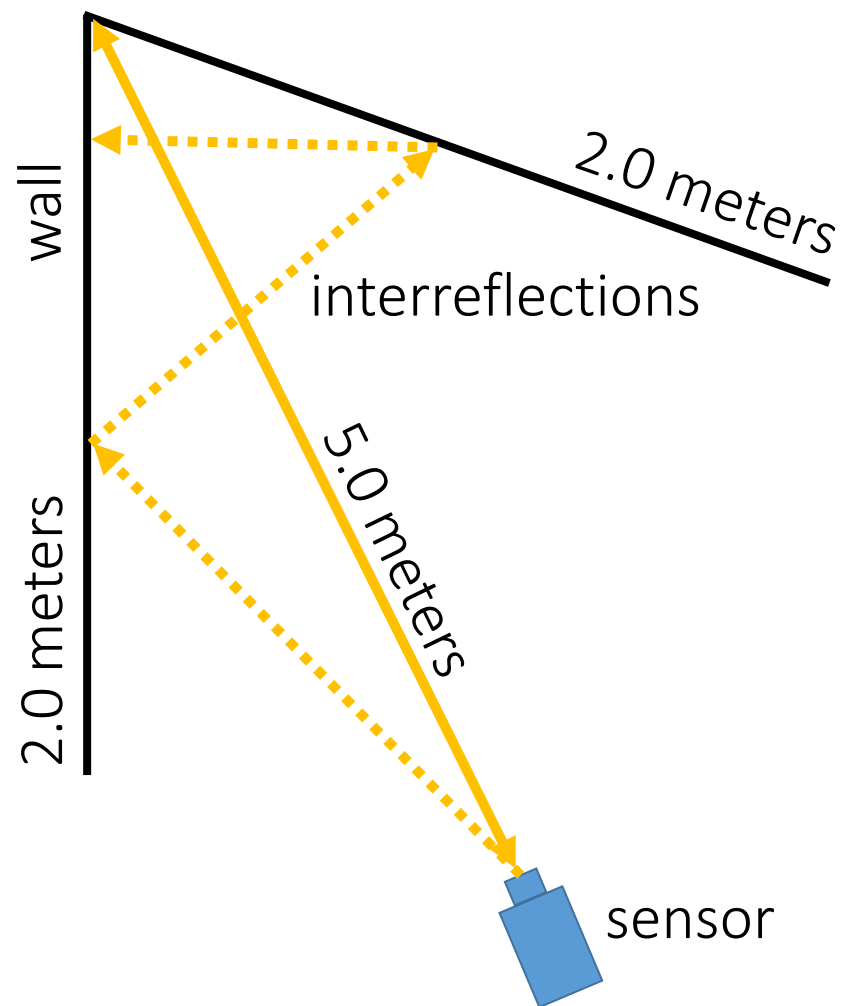
sensor
(PMD CamBoard Nano)



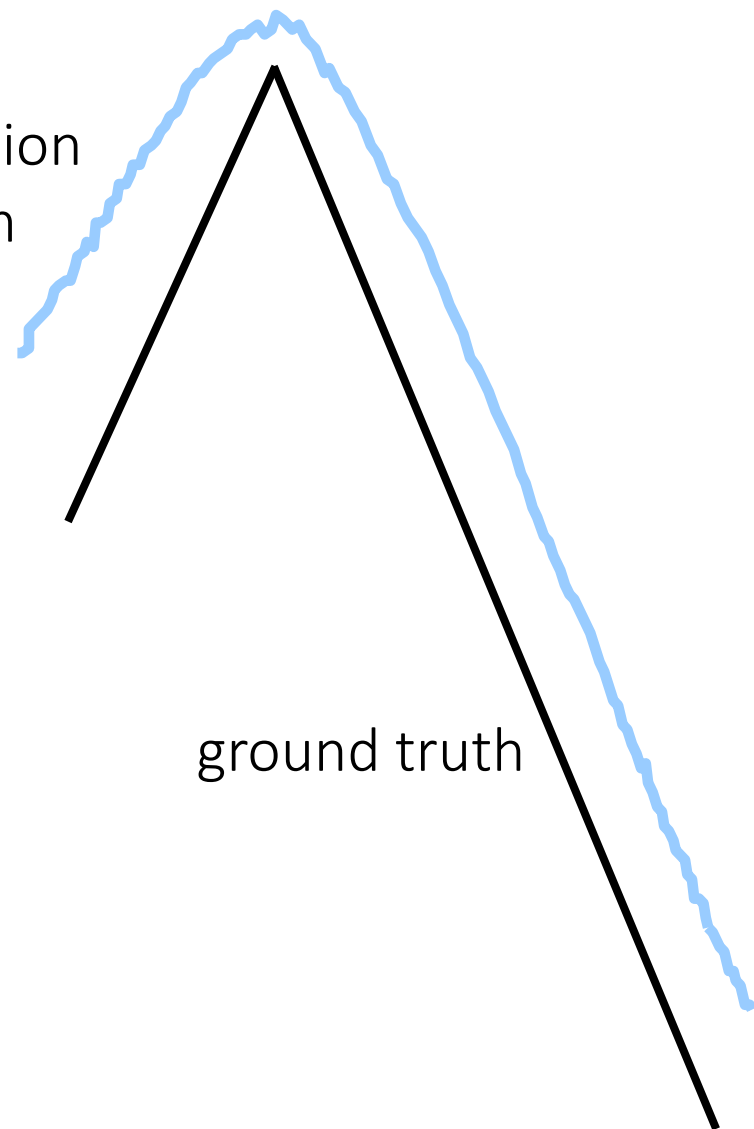
(only second generation of Kinect uses CW ToF)



Multi-path interference

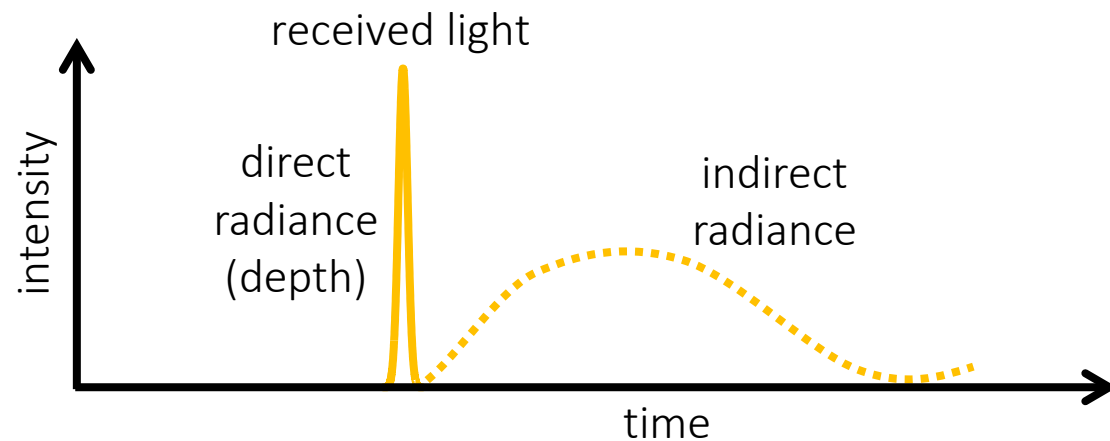
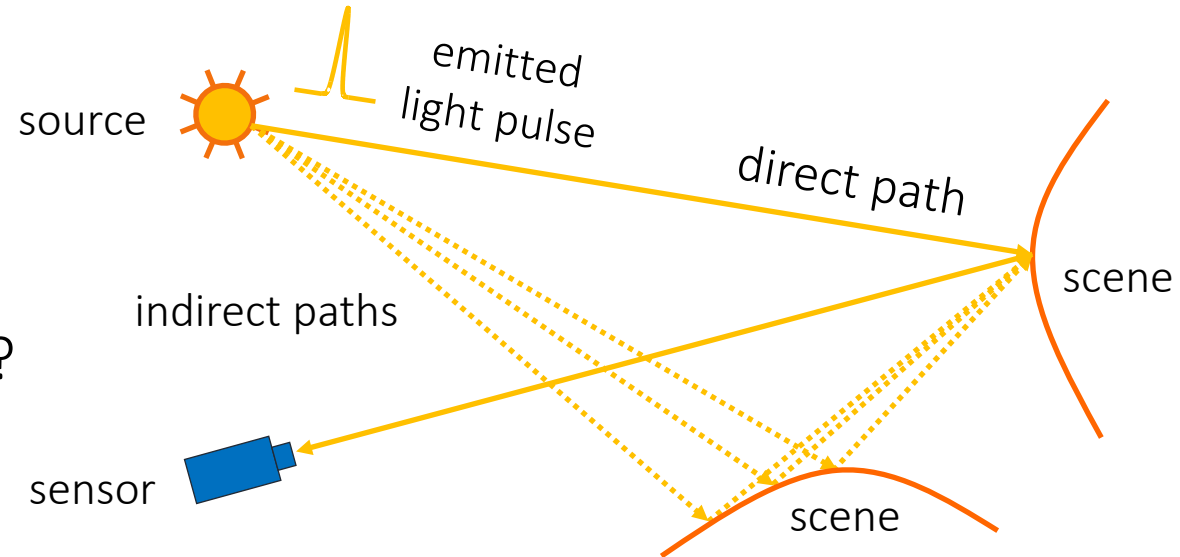


ToF depth reconstruction
mean error = 86.6 mm



Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

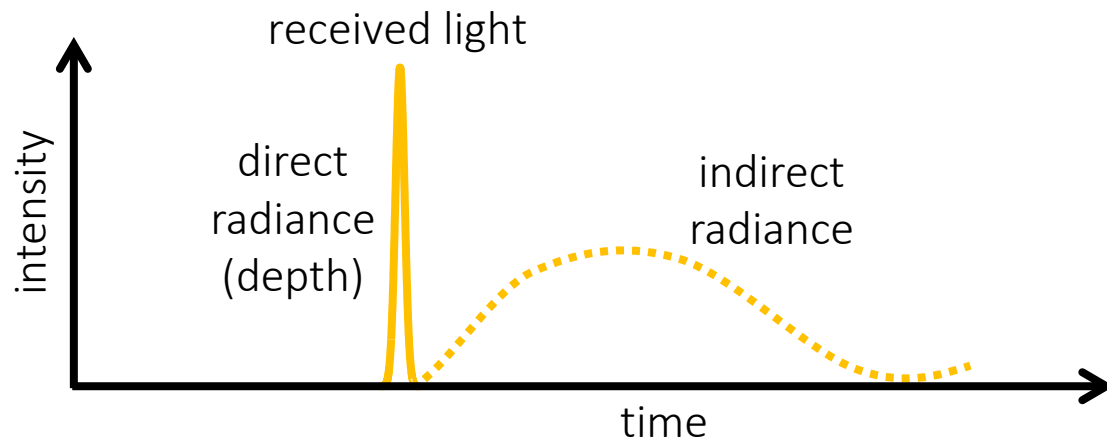
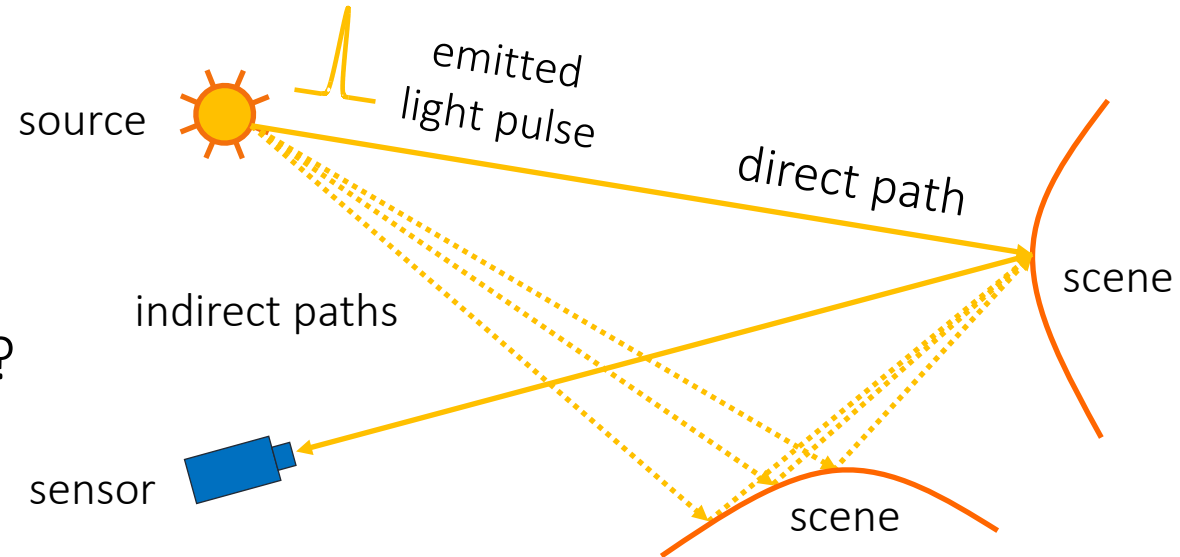


Transient $I(t)$: Time-resolved radiance distribution

- Range imaging: Measuring only first returning photons (e.g., LIDAR).
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Transient imaging with continuous-wave ToF

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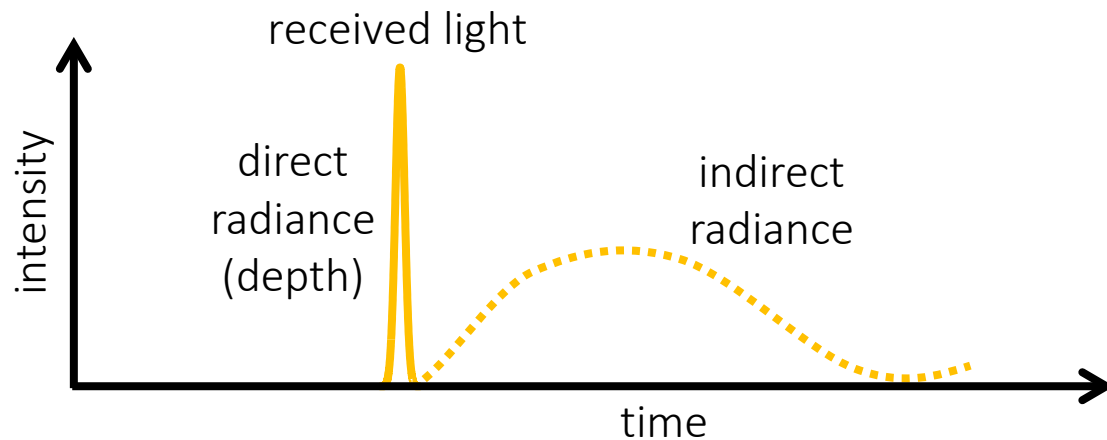
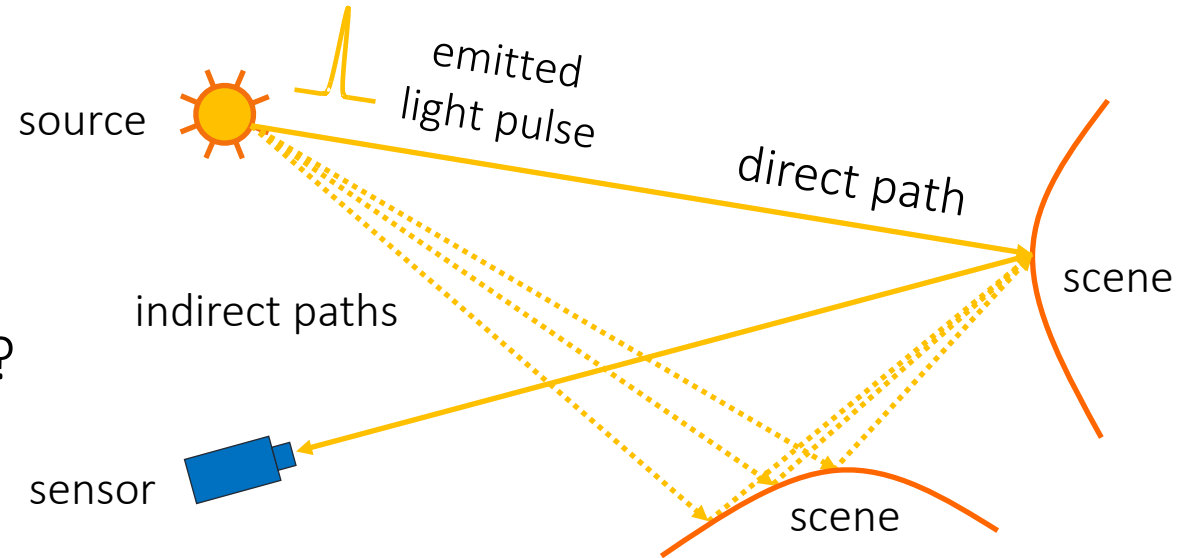
Transient $I(t)$: Time-resolved radiance distribution

- Each measurement we capture is of the form:

$$I(\omega) = \int \sin(\omega t) \cdot I(t) dt$$

Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?



Transient $I(t)$: Time-resolved radiance distribution

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$$I(\omega) = \int \sin(\omega t) \cdot I(t) dt$$

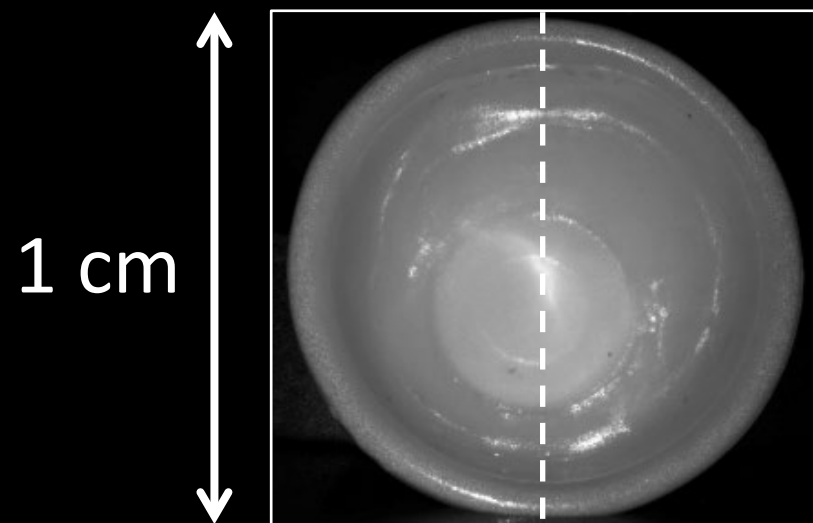
We can do transient imaging by taking measurements at multiple frequencies ω , then doing an inverse Fourier transform

Interferometric ToF imaging

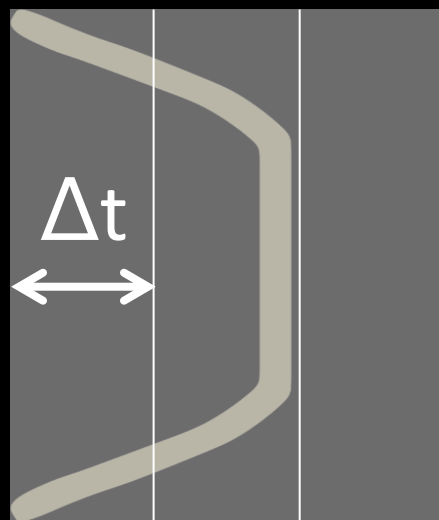
Time-of-flight imaging technologies

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	continuous-wave ToF	impulse ToF			

Tiny scenes



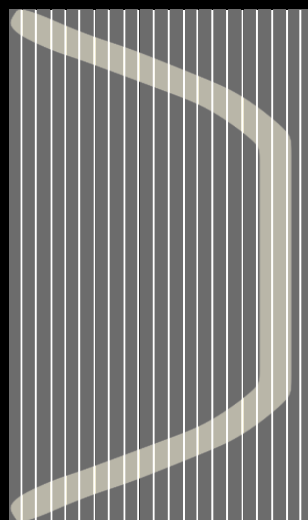
toy cup



0.5 cm

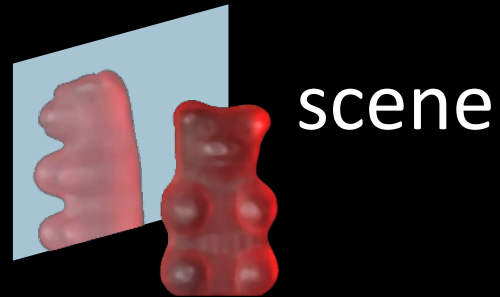


$\Delta t \sim \text{ps}$



$\Delta t \sim 10^{-3} \text{ ps}$

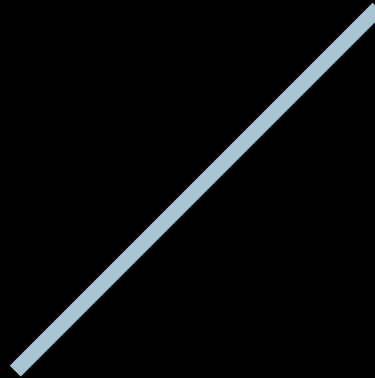
Interferometry example



light
source



beamsplitter

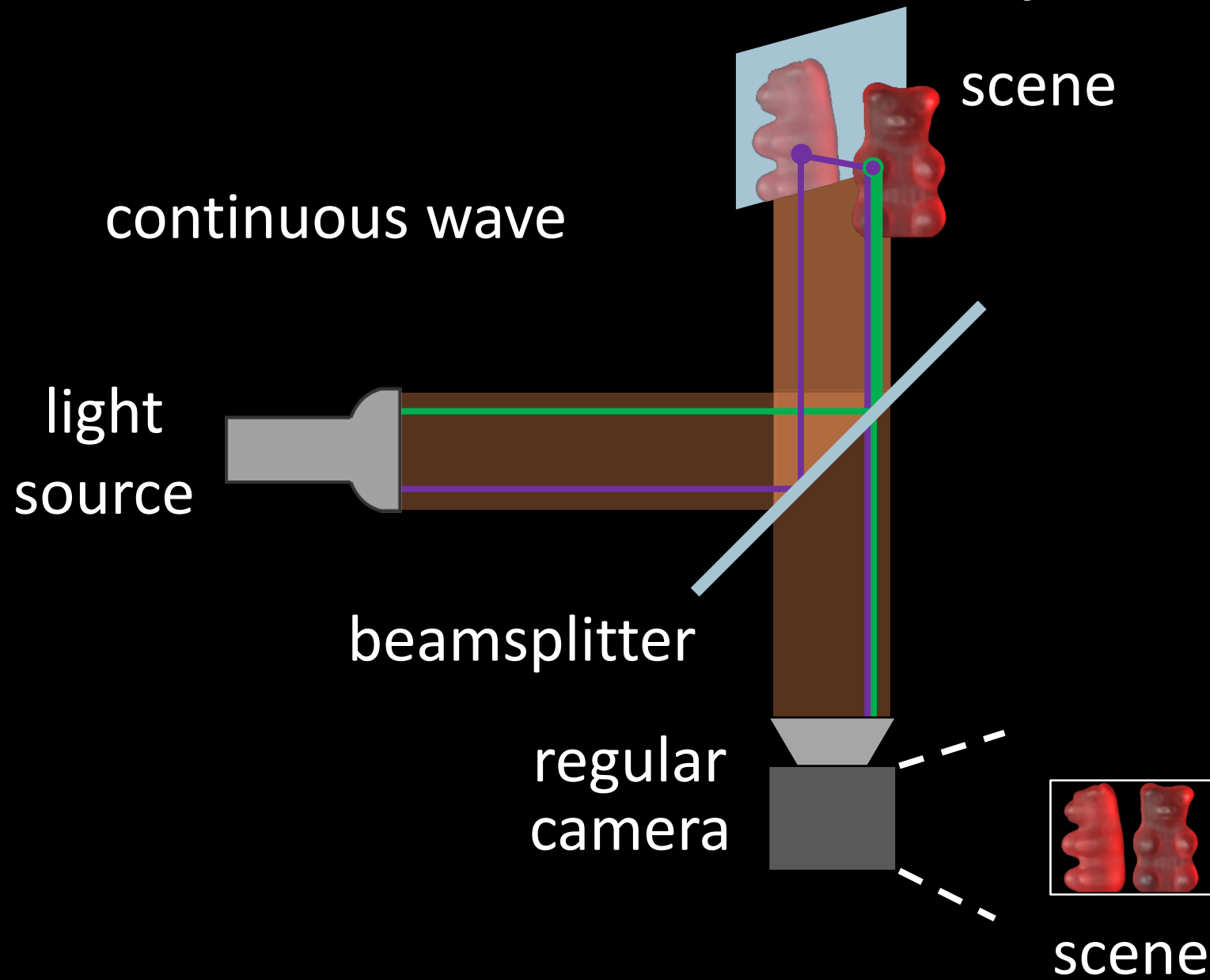


regular
camera

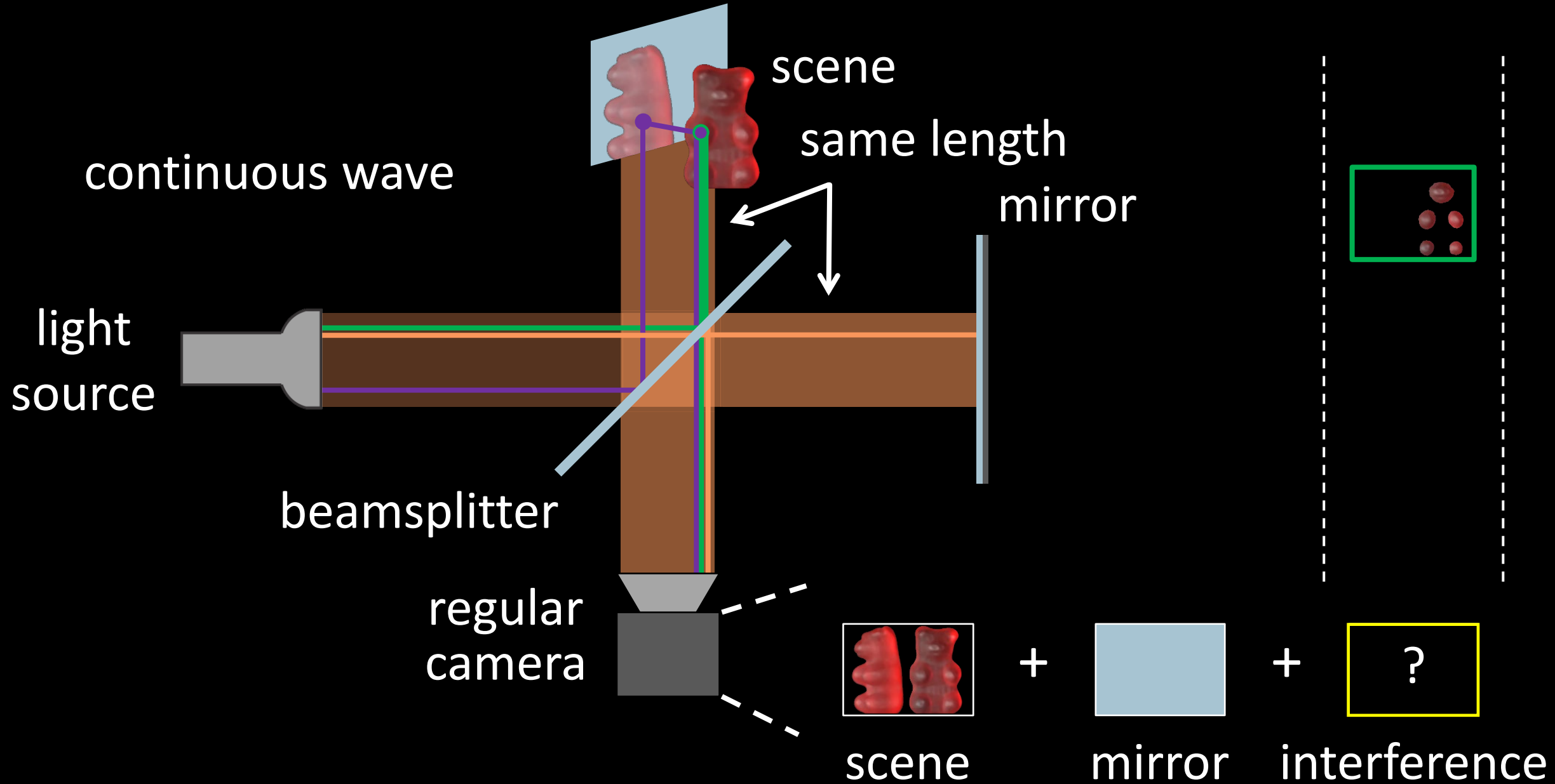


light
source

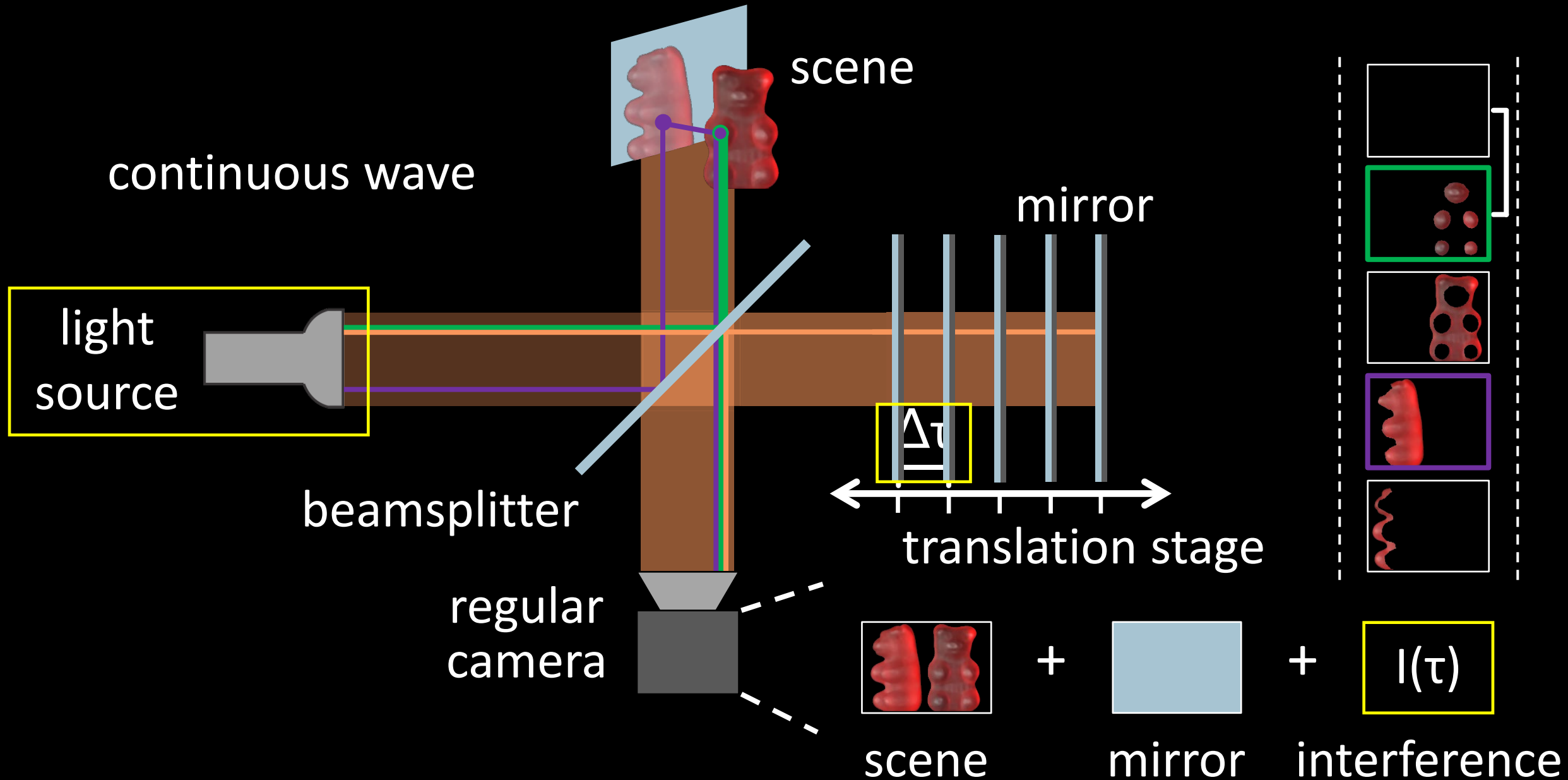
Interferometry example



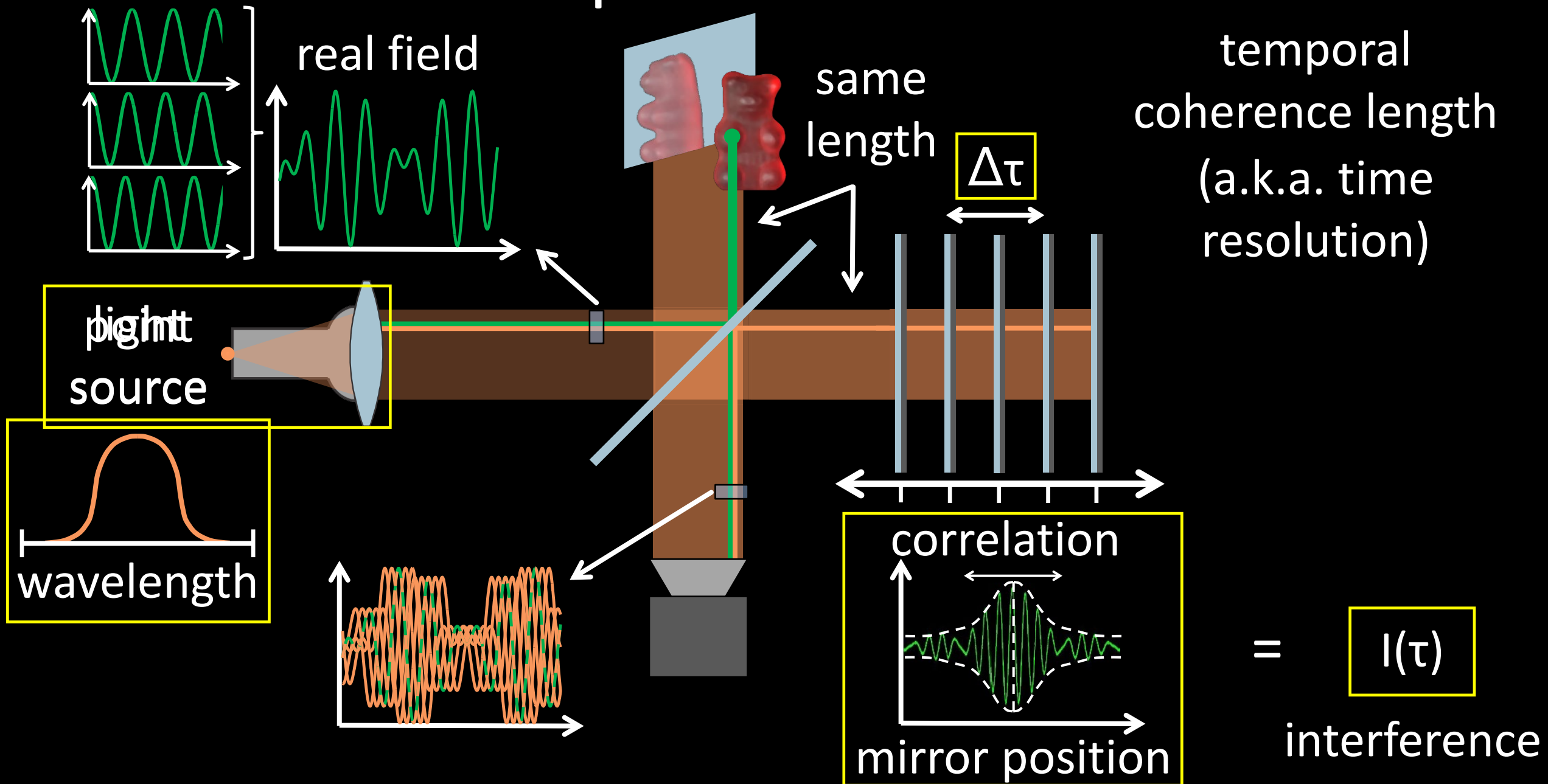
Michelson interferometer



Optical coherence tomography



Temporal coherence



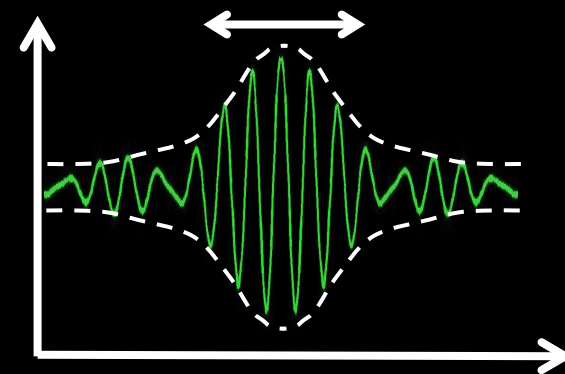
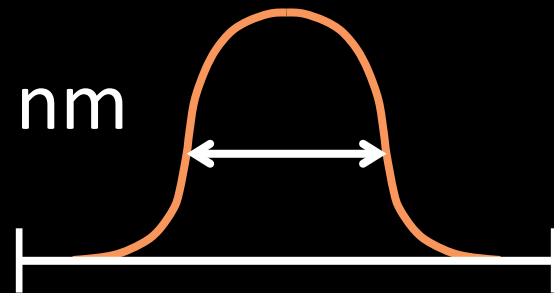
Temporal coherence length

bandwidth

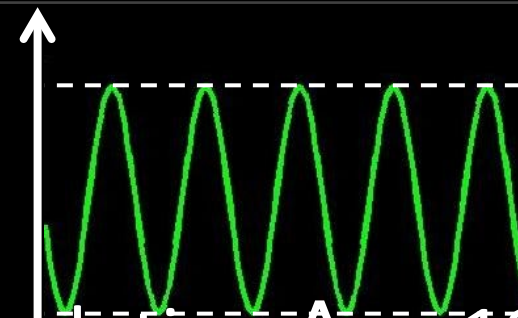
correlation

broadband

25 nm

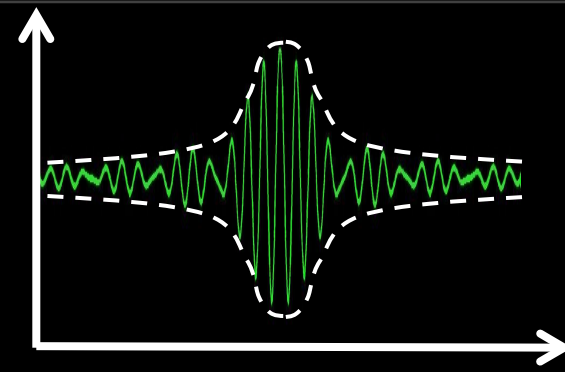
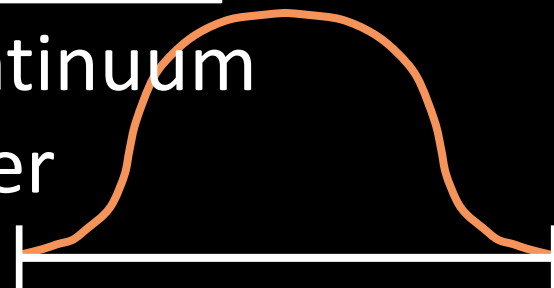


pathlength resolution $\Delta\tau \sim 10 \mu\text{m}$



superluminescent
broadband
diode

supercontinuum
laser



Optical setup



superluminescent diode supercontinuum laser



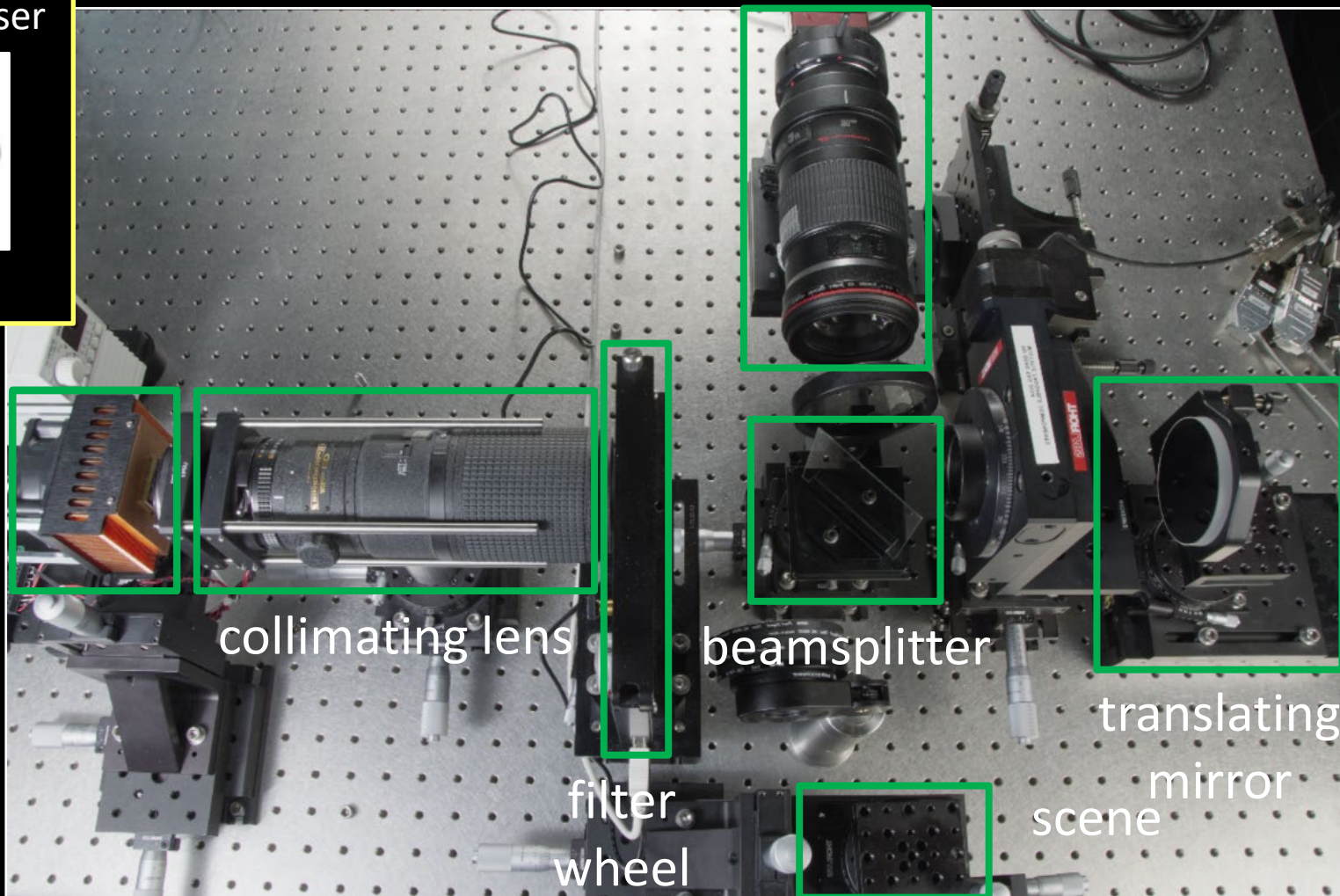
broadband LED



sodium lamp

light source

camera + imaging lens



collimating lens

beamsplitter

translating mirror

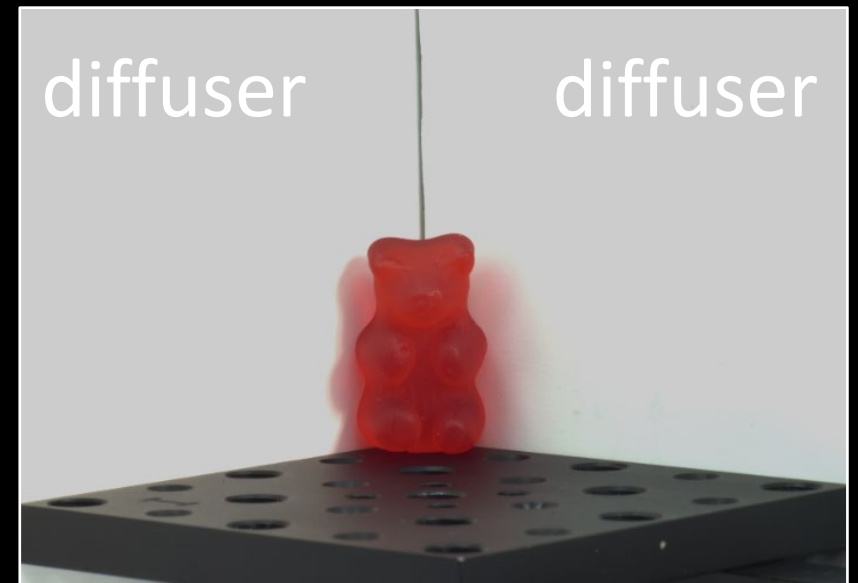
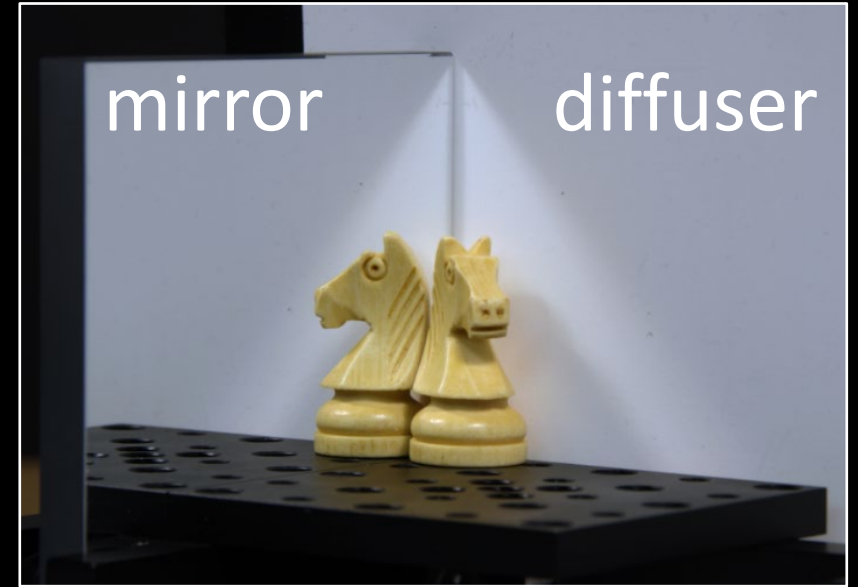
filter wheel

scene

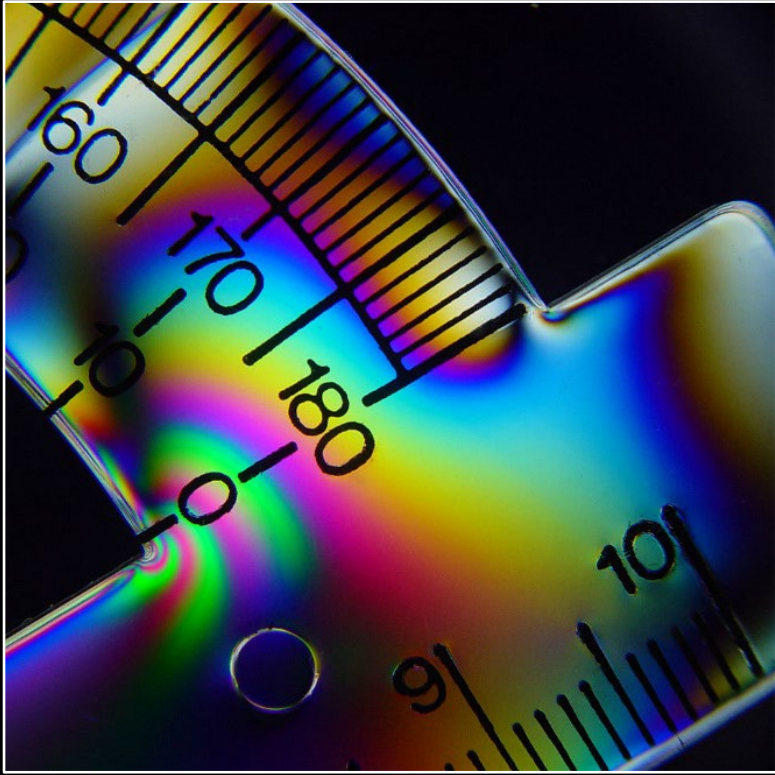
Some transient images



centimeter-sized objects



Material properties



birefringence

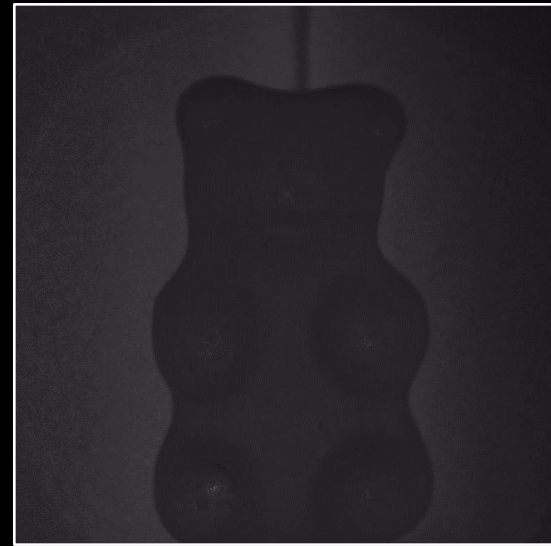
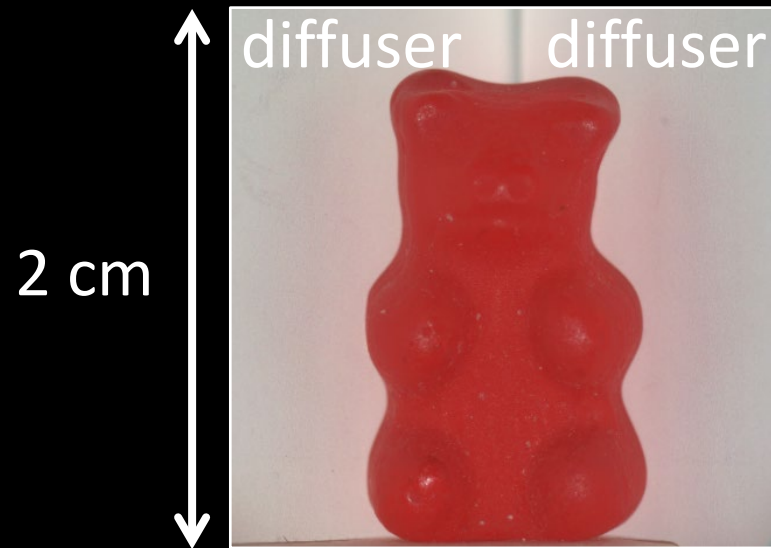


dispersion



scattering

Gummy bear and diffuse corner



pathlength
($\Delta\tau = 10 \mu\text{m}$)



dark frame



surface reflections

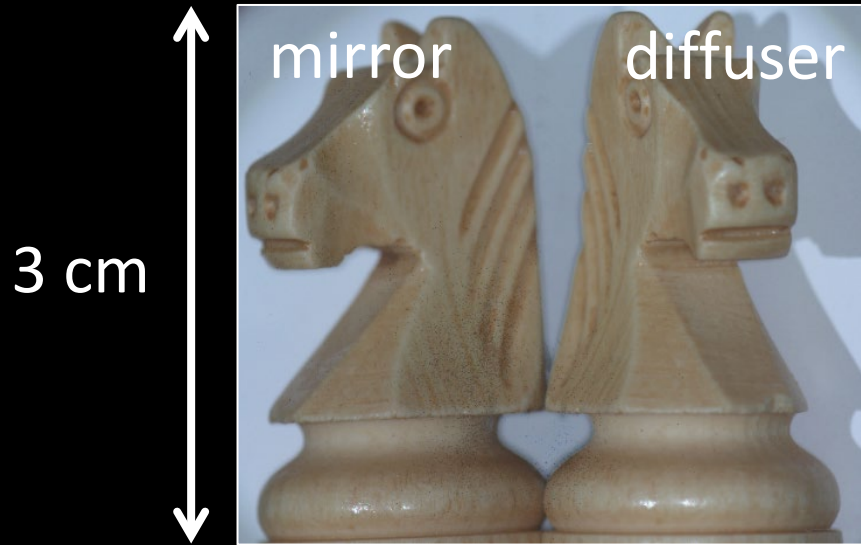


paths through
gummy bear

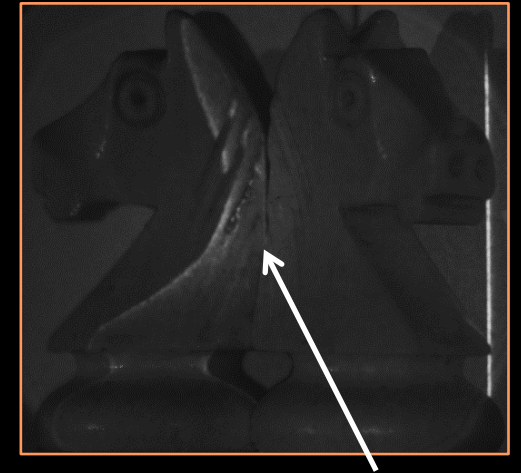
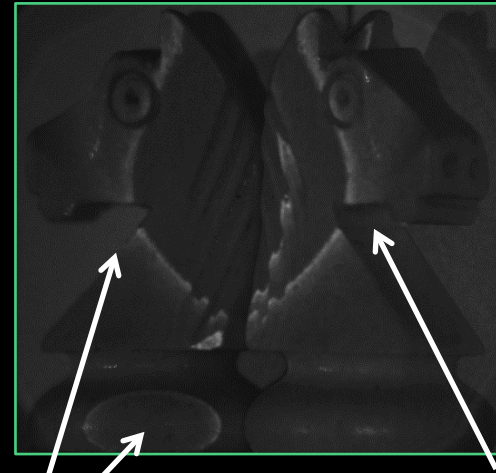
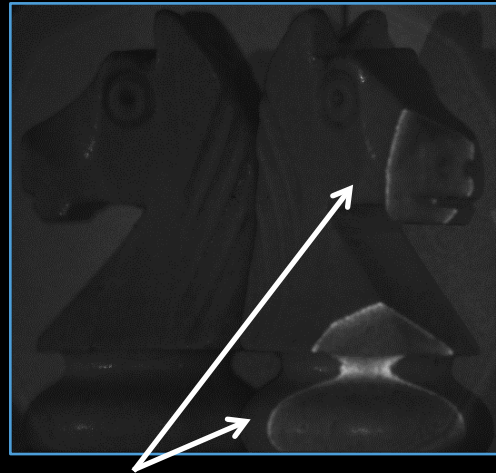


very highly
scattered paths

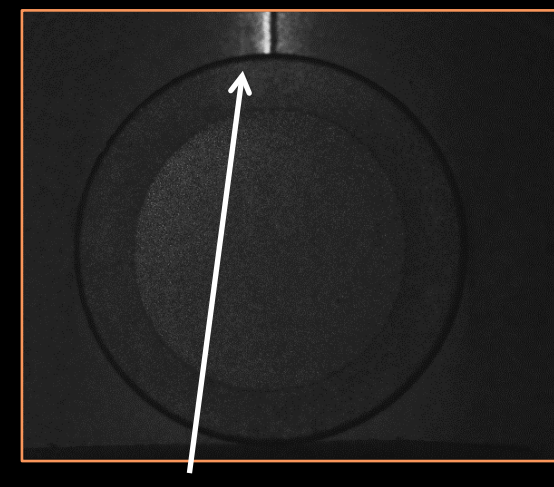
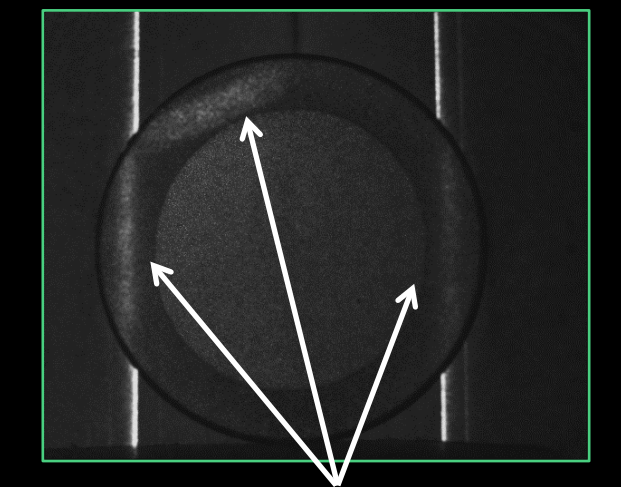
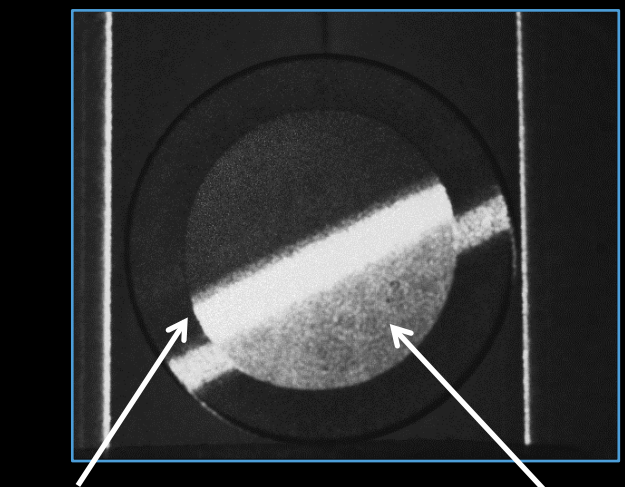
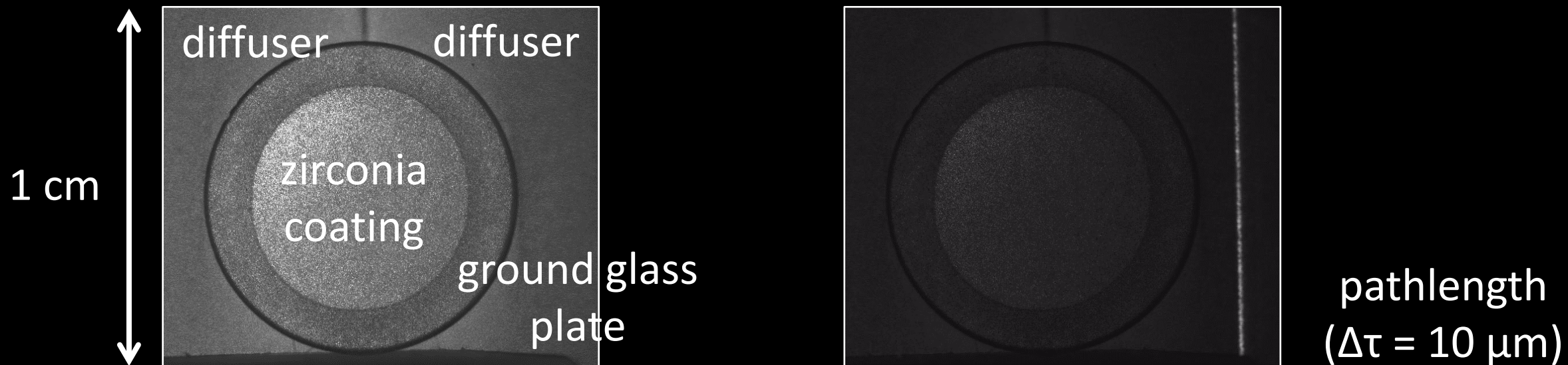
Chess knight and mirror



pathlength
($\Delta\tau = 10 \mu\text{m}$)



Subsurface scattering



White jade

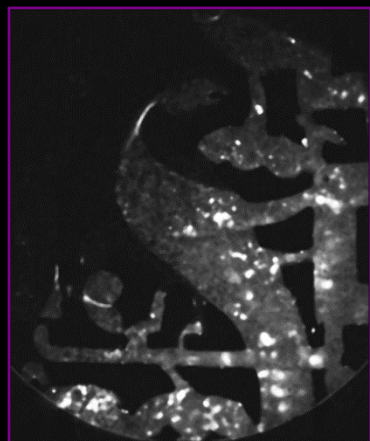
3 cm



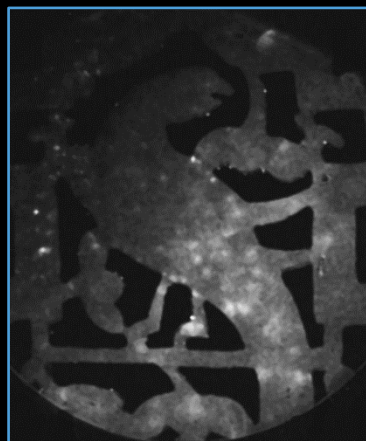
exquisite
white jade



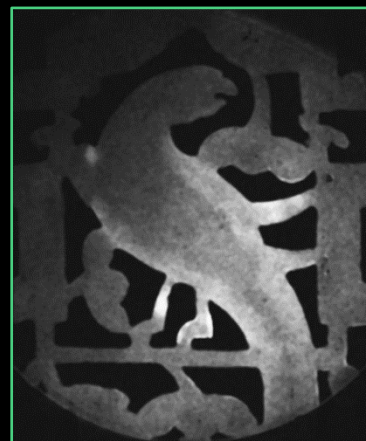
time (10^{-15} seconds)



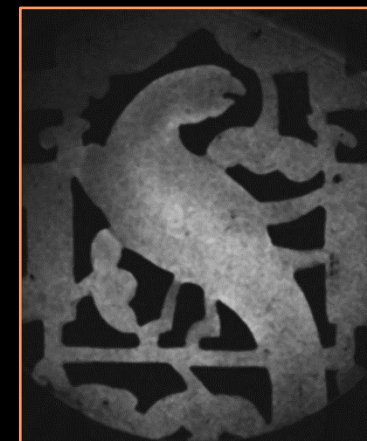
specular
reflections



low-order
scattering

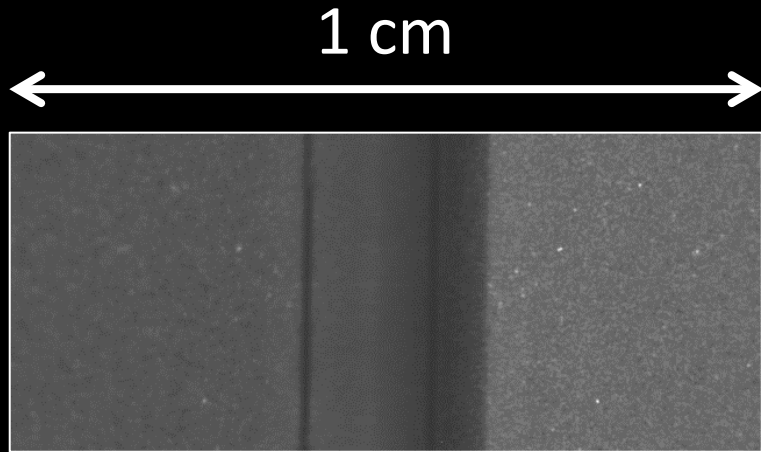


mid-order
scattering

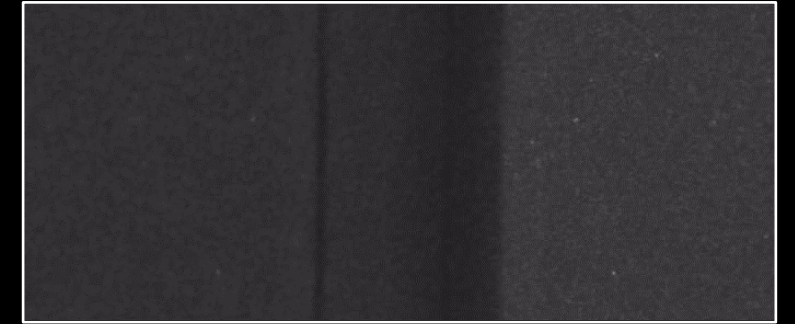
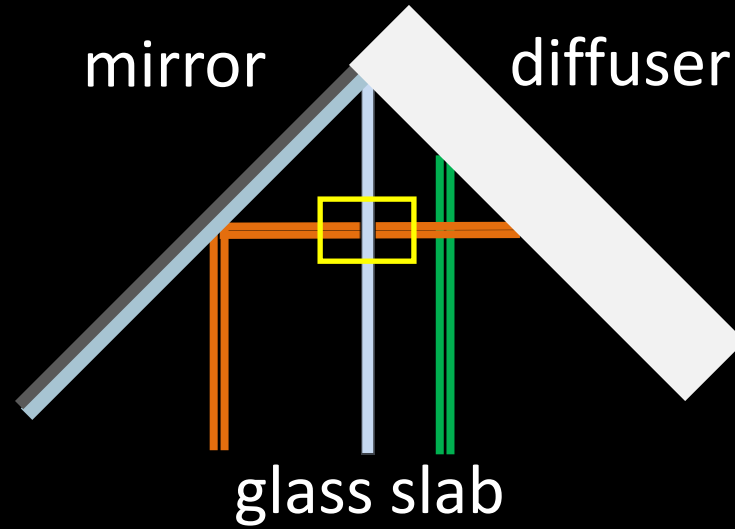


high-order scattering
[TOG 2015]

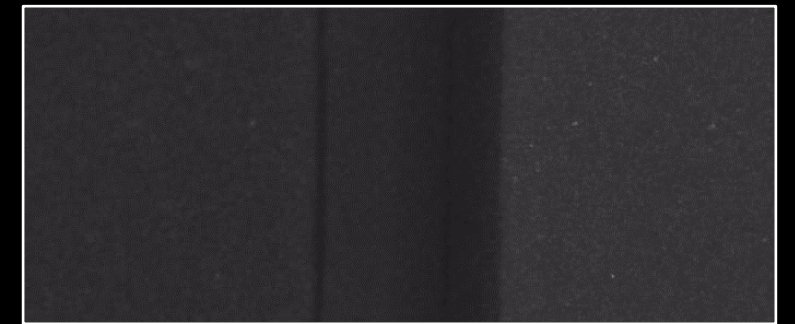
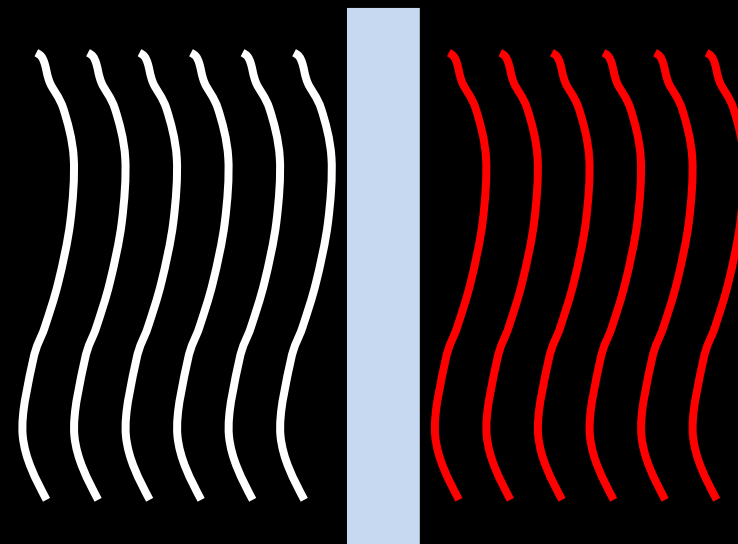
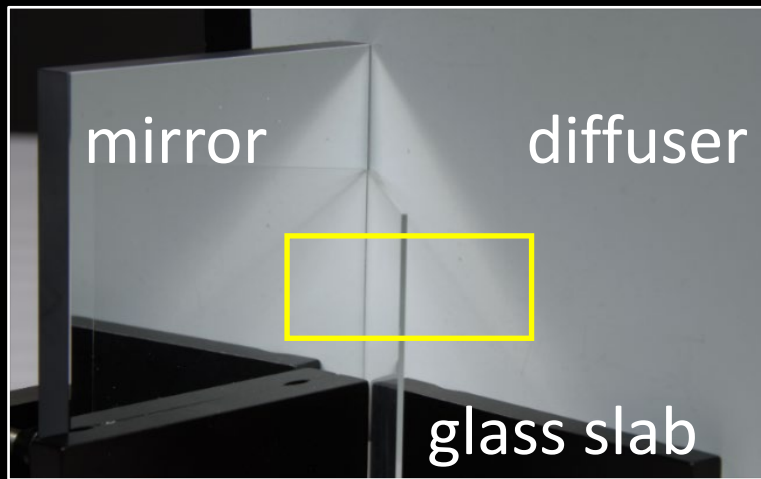
Dispersion



cropped frame



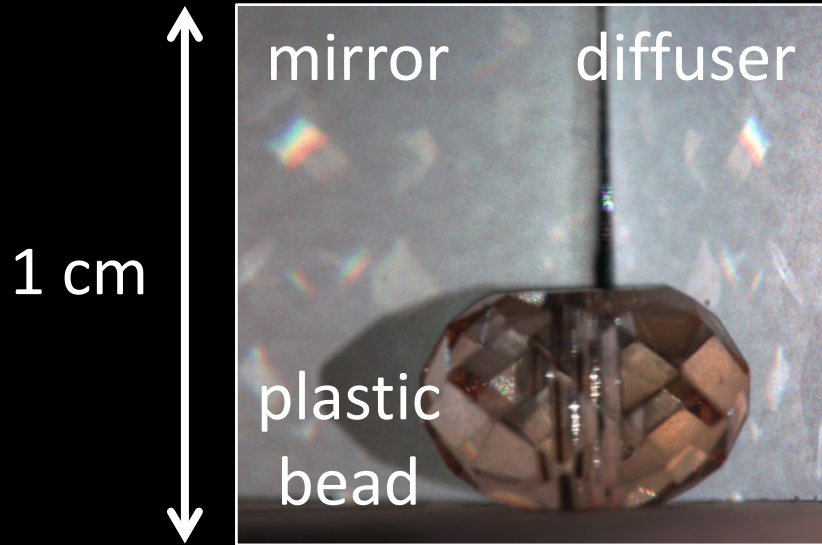
$\Delta t \sim \text{ns}$



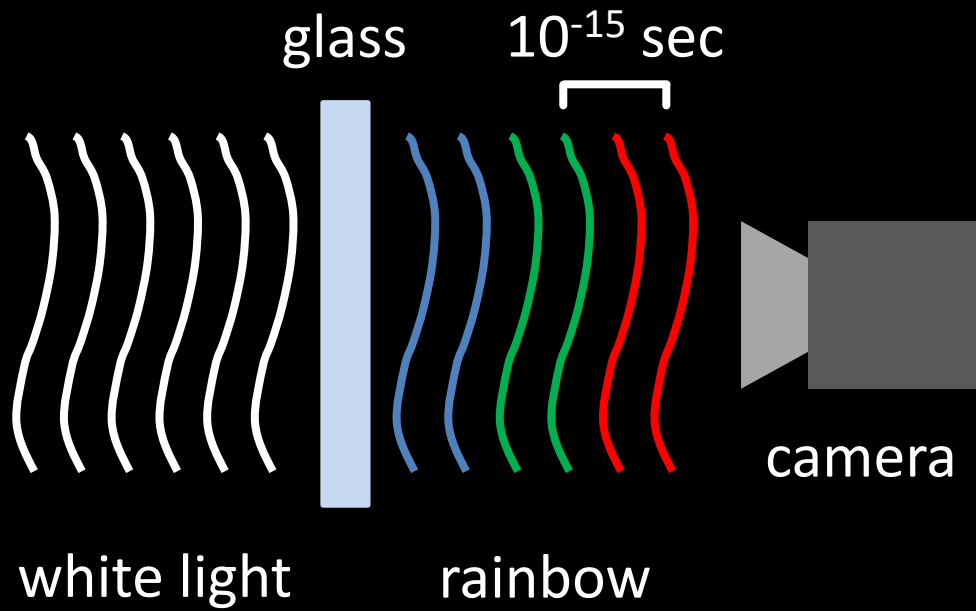
$\Delta t \sim 10^{-3} \text{ ns}$

refractive index $\eta(\text{wavelength})$

Dispersion



Visualizing dispersion

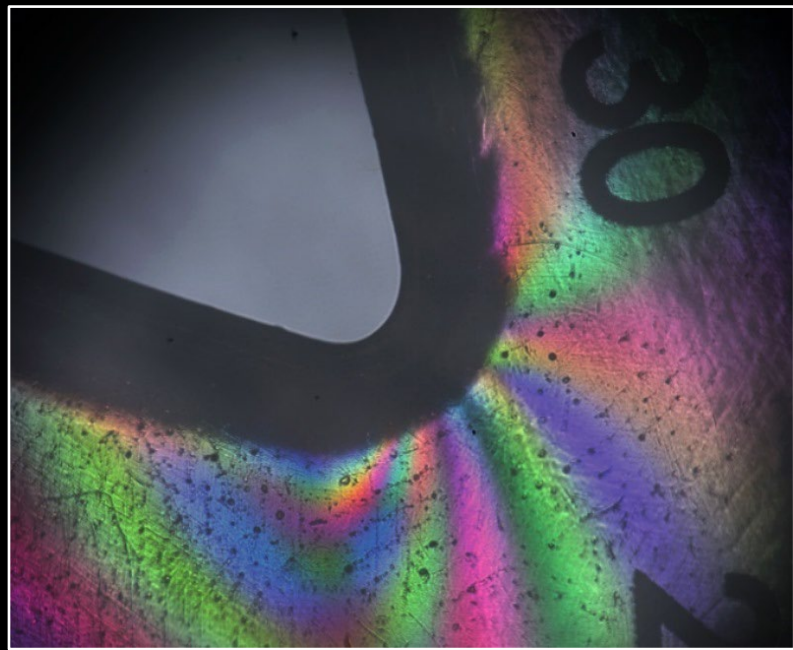


what a regular camera sees



what our camera sees

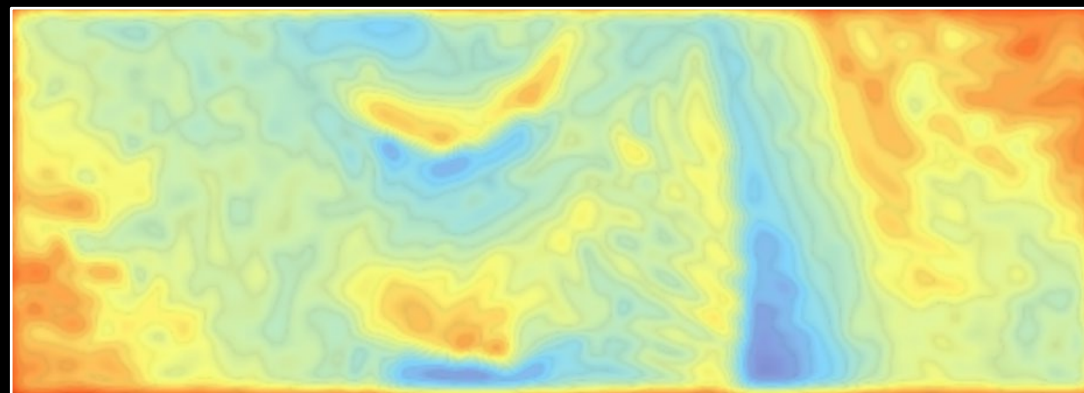
Visualizing photoelasticity



detail under cross-polarized light



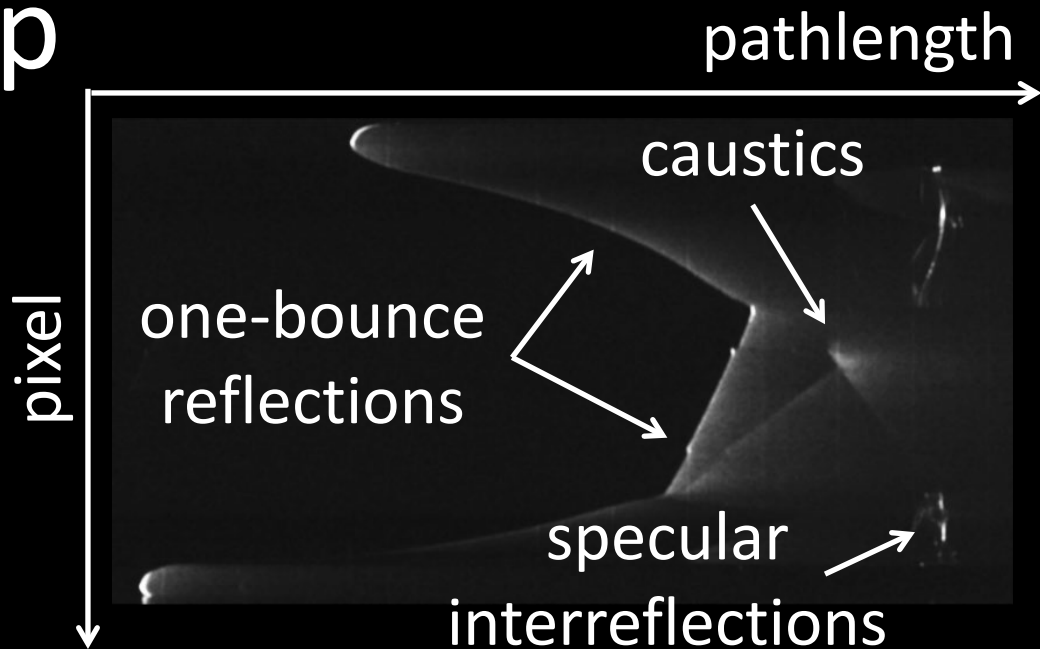
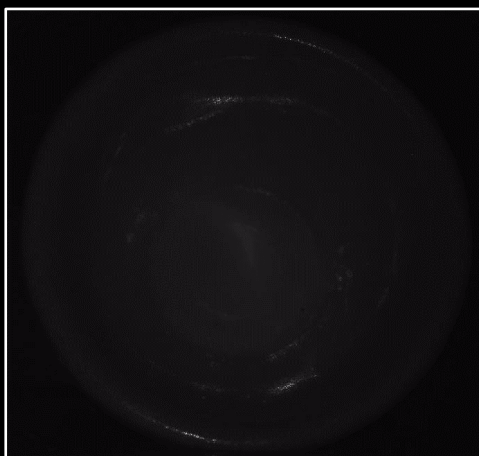
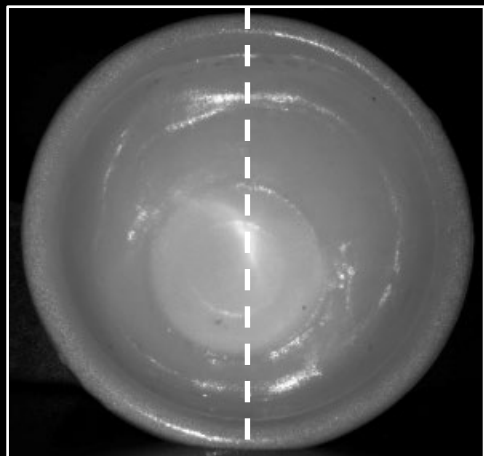
low resolution $\Delta\tau = 1 \text{ mm}$



high resolution $\Delta\tau = 10 \mu\text{m}$

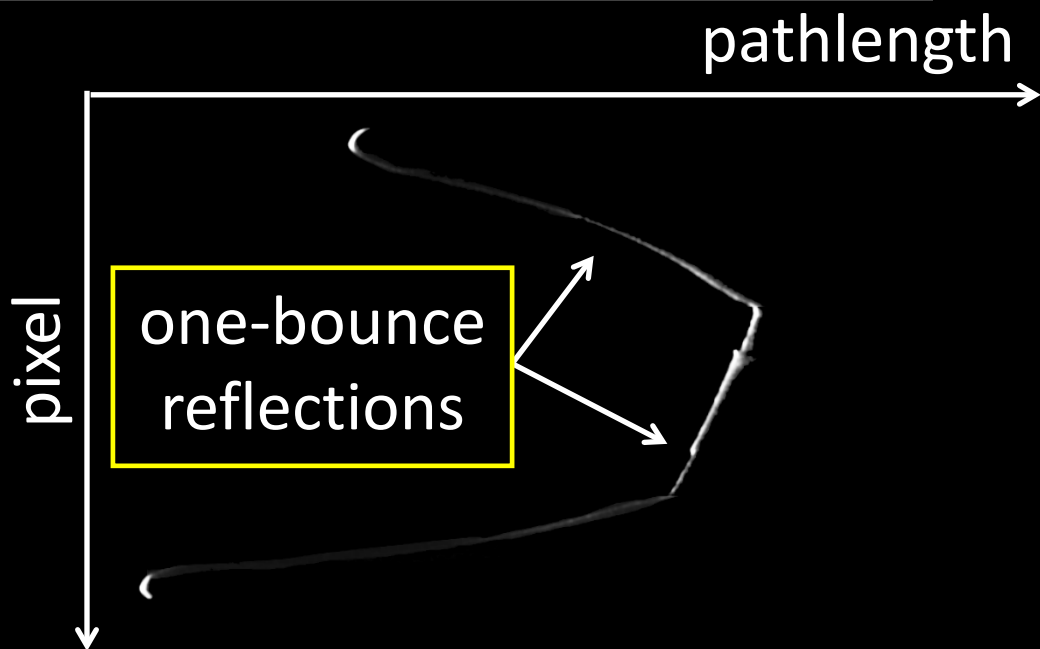
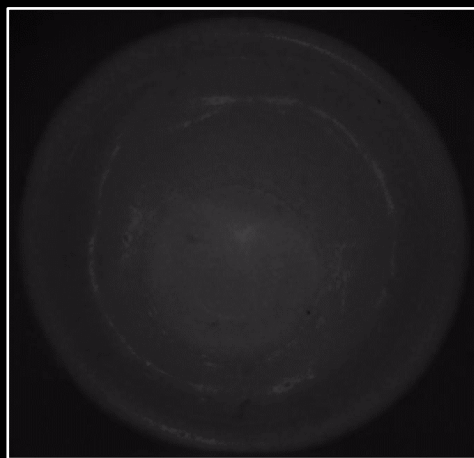
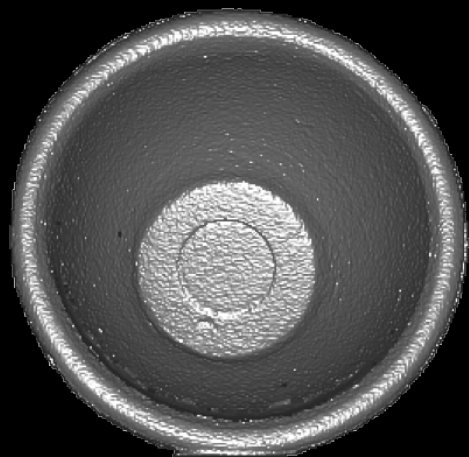
Toy cup

full transient



measured depth

direct-only transient



Depth scanning

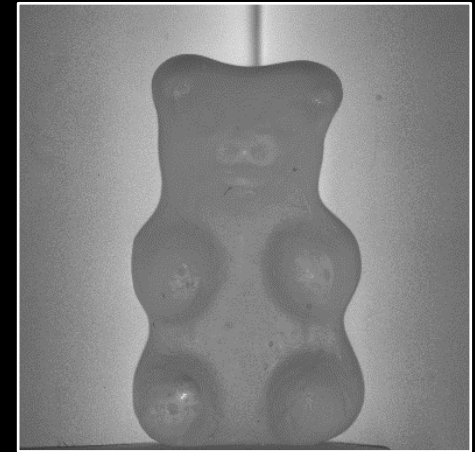
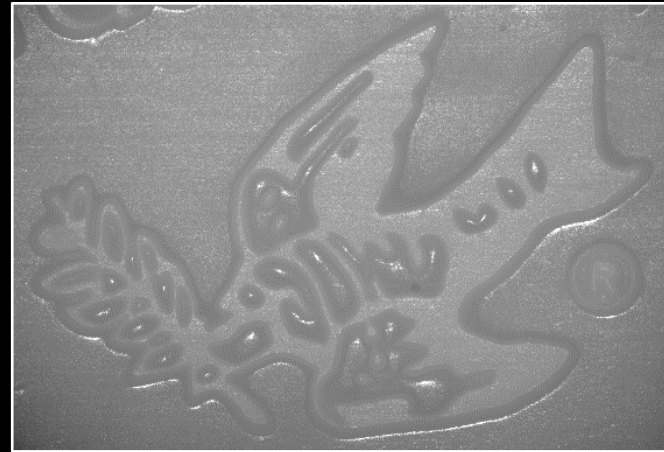
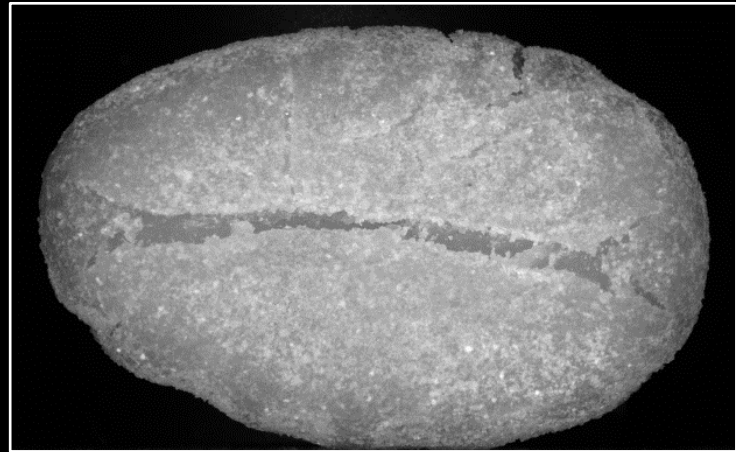
depth resolution $\Delta\tau \sim 10 \mu\text{m}$

coin

gnocchi

soap carving

gummy bear
and diffusers

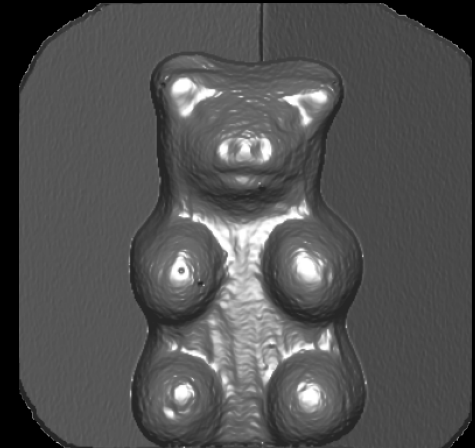


2.5 cm

1 cm

1.5 cm

3 cm



References

Basic reading:

- Gupta et al., “Computational Time-of-Flight,” ICCV 2015 tutorial, <http://web.media.mit.edu/~achoo/iccvtoftutorial/>
this tutorial provides an overview of many of the topics covered in this lecture, with a focus on continuous-wave ToF imaging.
- Jarabo et al., “Recent Advances in Transient Imaging: A Computer Graphics and Vision Perspective,” Visual Informatics 2017
a great review paper for ToF imaging.
- Velten et al., “Femto-photography: capturing and visualizing the propagation of light,” SIGGRAPH 2013, CACM 2016.
the paper that introduced the idea of transient imaging to the computational imaging community, and an explanation of how streak cameras work.
- Lange et al., “Solid-state time-of-flight range camera,” JQE 2001.
a standard reference on continuous-wave ToF sensors.
- Heide et al., “Low-budget transient imaging using photonic mixer devices,” SIGGRAPH 2013.
- Lin et al., “Fourier analysis on transient imaging with a multifrequency time-of-flight camera,” CVPR 2014.
- Peters et al., “Solving trigonometric moment problems for fast transient imaging,” SIGGRAPH 2015.
three papers showing how continuous-wave ToF sensors can be used for transient imaging.
- Gupta et al., “Phasor imaging: A generalization of correlation-based time-of-flight imaging,” TOG 2015.
a more recent paper that provides nice insights into how continuous-wave ToF works, as well as a way to deal with MPI.
- Abramson, “Light-in-flight recording by holography,” Optics Letters 1978.
a very early paper showing visualization of light-in-flight, i.e., transient imaging.
- Huang et al., “Optical Coherence Tomography,” Science 1991.
the paper introducing optical coherence tomography.
- Gkioulekas et al., “Micron-scale light transport decomposition using interferometry,” SIGGRAPH 2014.
the paper showing how interferometry can be used for time-of-flight imaging.
- Garipey et al., “Single-photon sensitive light-in-flight imaging,” Nature Communications 2015.
the paper describing how SPADs can be used for ToF imaging.
- O’Toole et al., “Reconstructing Transient Images from Single-Photon Sensors,” CVPR 2017.
a paper explaining the operation of SPADs in a more accessible manner to computer science backgrounds.
- Pediredla et al., “Signal processing based pile-up compensation for gated single-photon avalanche diodes,” 2018.
- Heide et al., “Sub-picosecond photon-efficient 3D imaging using single-photon sensors,”
- Gupta et al., “Photon-flooded single-photon 3d cameras,” CVPR 2019.
three papers discussing the pile-up issue and proposing ways to overcome it.
- Mark Itzler, “Single-photon LiDAR imaging: from airborne to automotive platforms,” ICCP 2020 keynote, <https://www.youtube.com/watch?v=4tEfVr6fKgw>
a keynote discussing advantages and current state of SPAD LiDAR technology.

Additional reading:

- Kirmani et al., “Looking around the corner using ultrafast transient imaging,” ICCV 2009 and IJCV 2011.
- Velten et al., “Recovering three-dimensional shape around a corner using ultrafast time-of-flight imaging,” Nature Communications 2012.
the first two papers showing how ToF imaging can be used for looking around the corner.