Photographic optics

15-463, 15-663, 15-862
Computational Photography
Fall 2022, Lecture 4

http://graphics.cs.cmu.edu/courses/15-463

Course announcements

- Homework 2 will be posted tonight.
- Materials from Friday’s reading group available on Piazza.
- If you want to attend the computational imaging lab meetings, see details on Piazza.

Overview of today’s lecture

- Leftover from previous lecture.
- Paraxial optics.
- Ray transfer matrix analysis.
- Aberrations and compound lenses.
- Lens designations.
- Filters.
- Prisms.
- DSLR and mirrorless cameras.

Slide credits

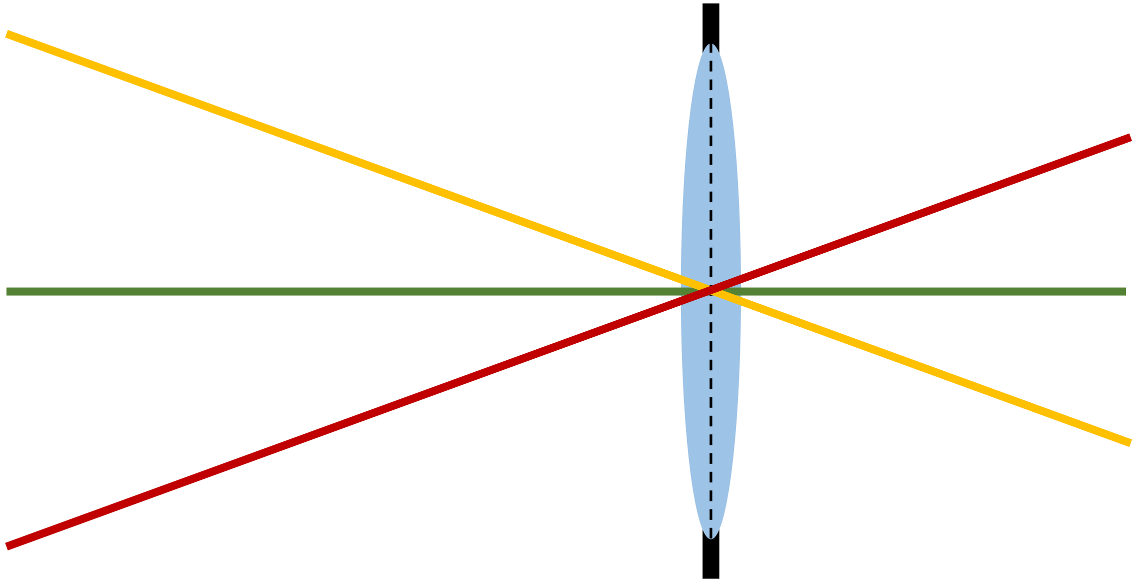
Many of these slides were adapted from:

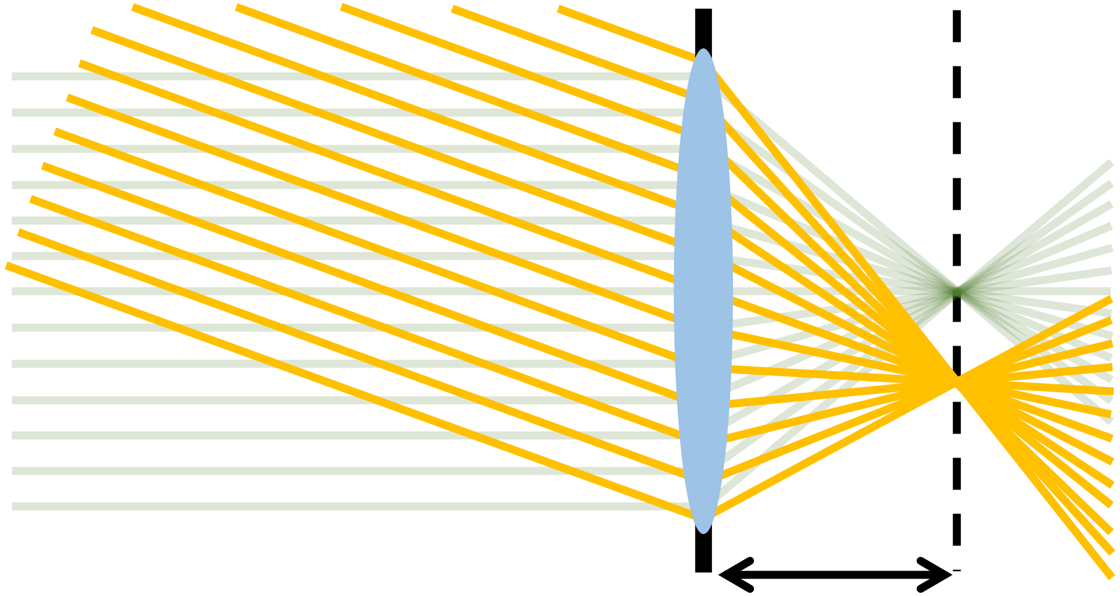
- Kris Kitani (15-463, Fall 2016).
- Fredo Durand (MIT).
- Marc Levoy (Stanford).
- Gordon Wetzstein (Stanford).

Paraxial optics

Thin lens model

Simplification of geometric optics for well-designed lenses.





Two assumptions:

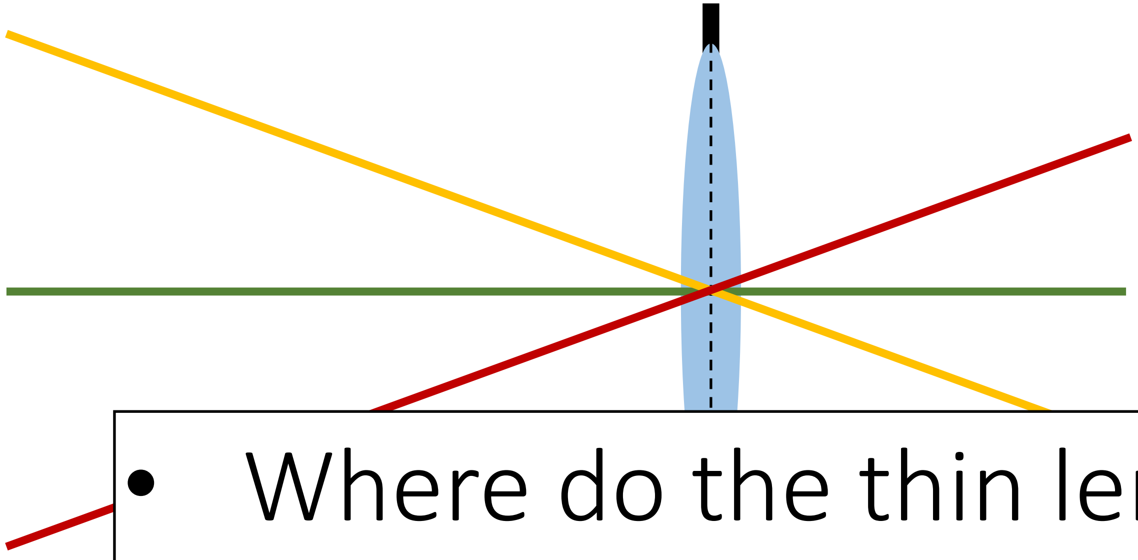
1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.

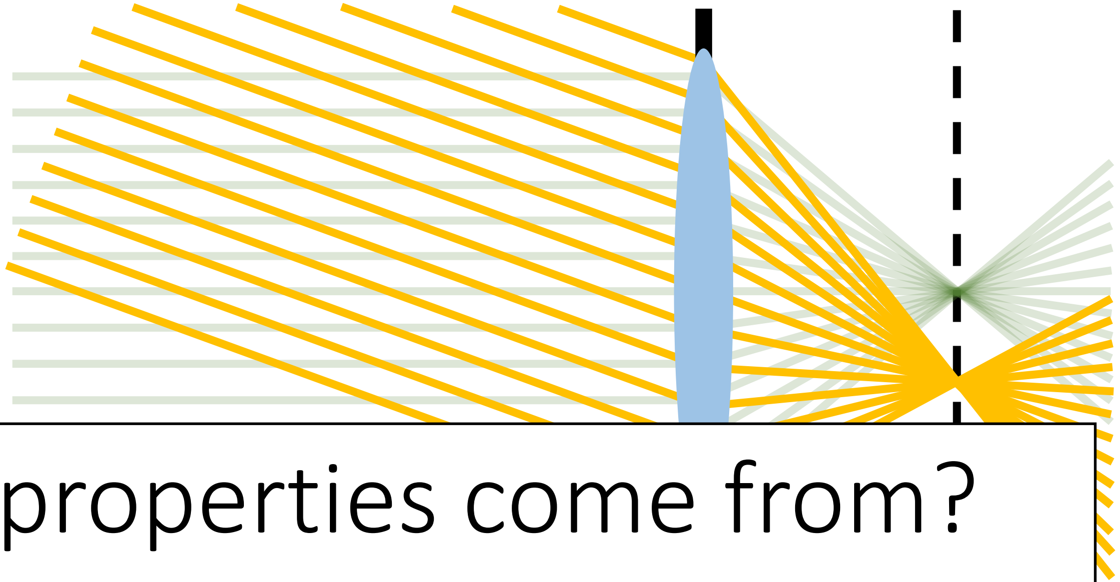
$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

$$m = \frac{S' - f}{f}$$

Thin lens model

Simplification of geometric optics for well-designed lenses.





- Where do the thin lens properties come from?
- What determines the focal length of a thin lens?

Two assumptions:

1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.

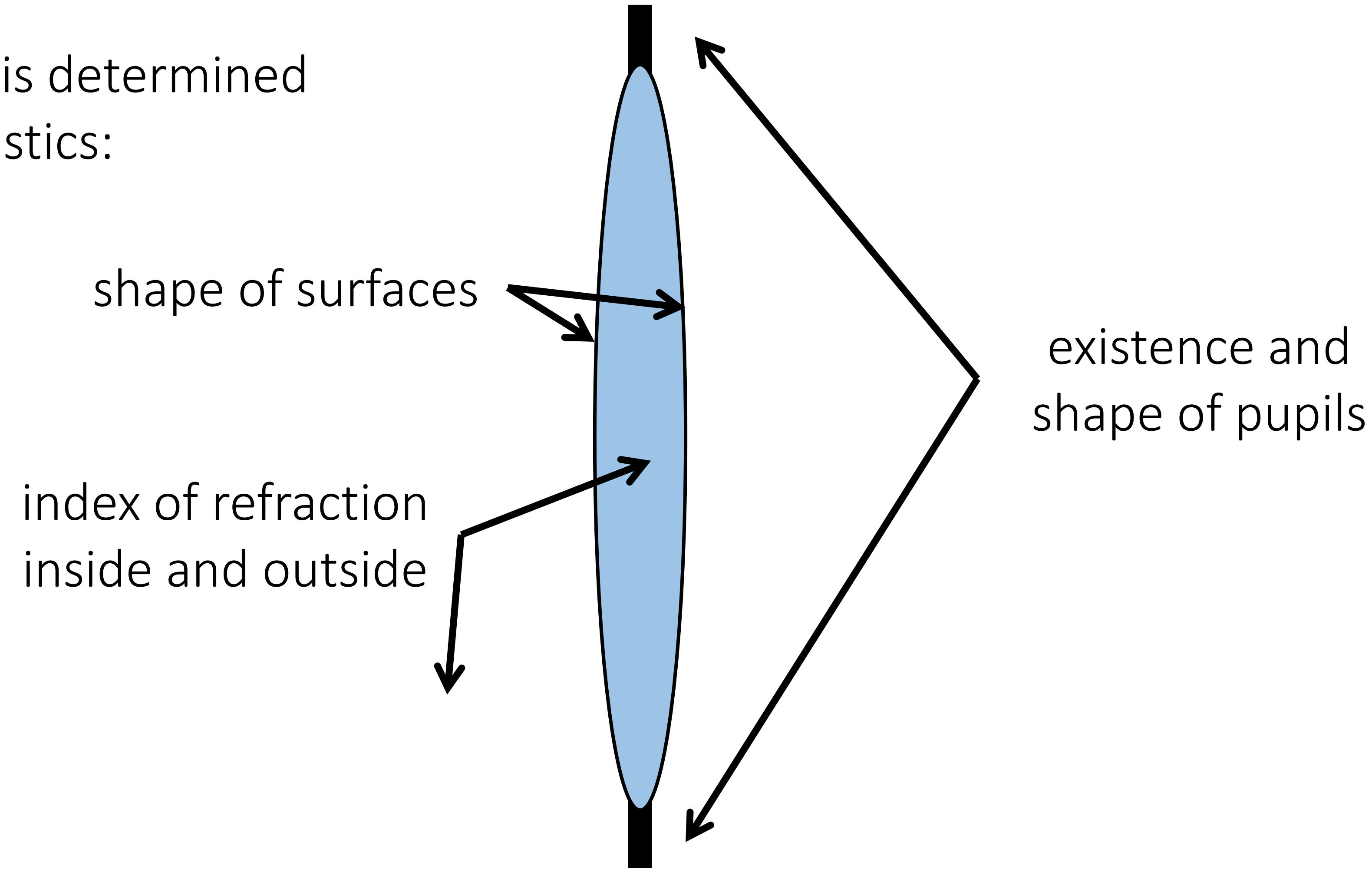
$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

$$m = \frac{S' - f}{f}$$

Real lenses

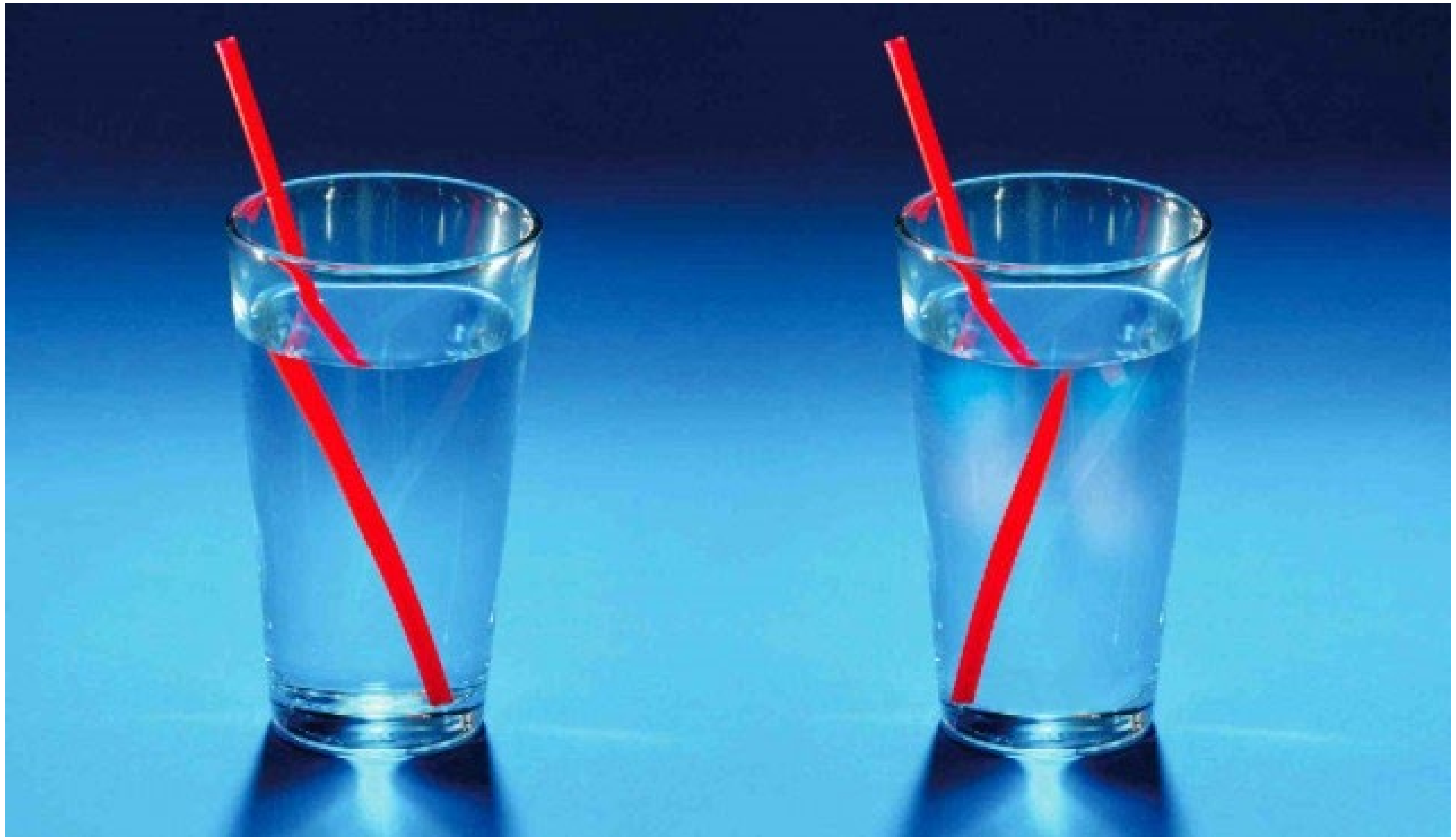
We will first consider the case of a system with an individual lens element

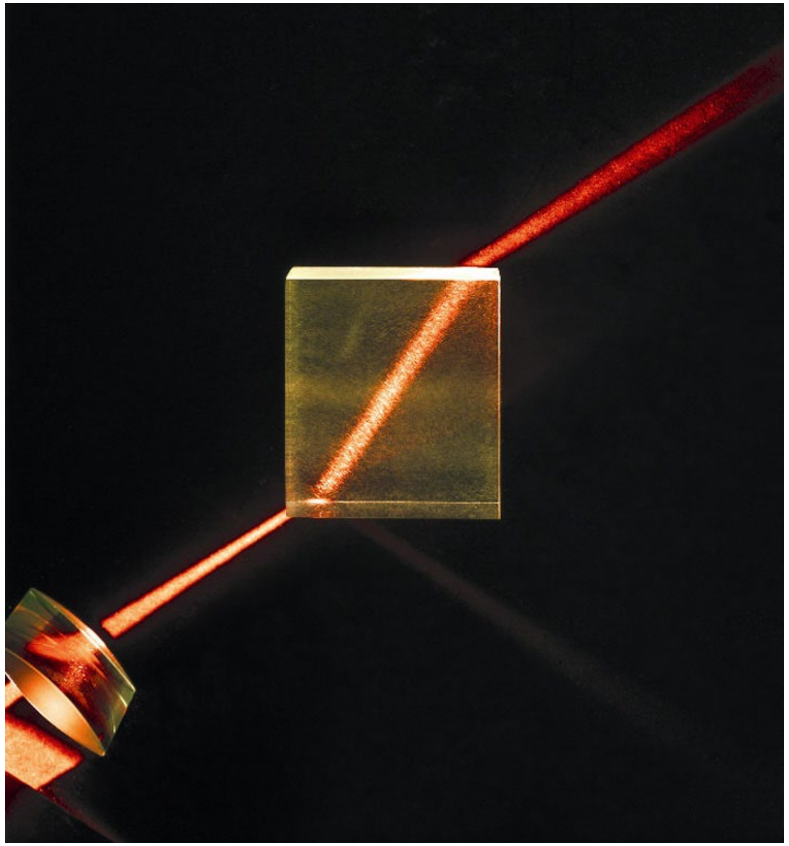
The lens’ behavior is determined by three characteristics:



Refraction

Refraction is the bending of rays of light when they cross optical interfaces (i.e., surfaces where the index of refraction changes).





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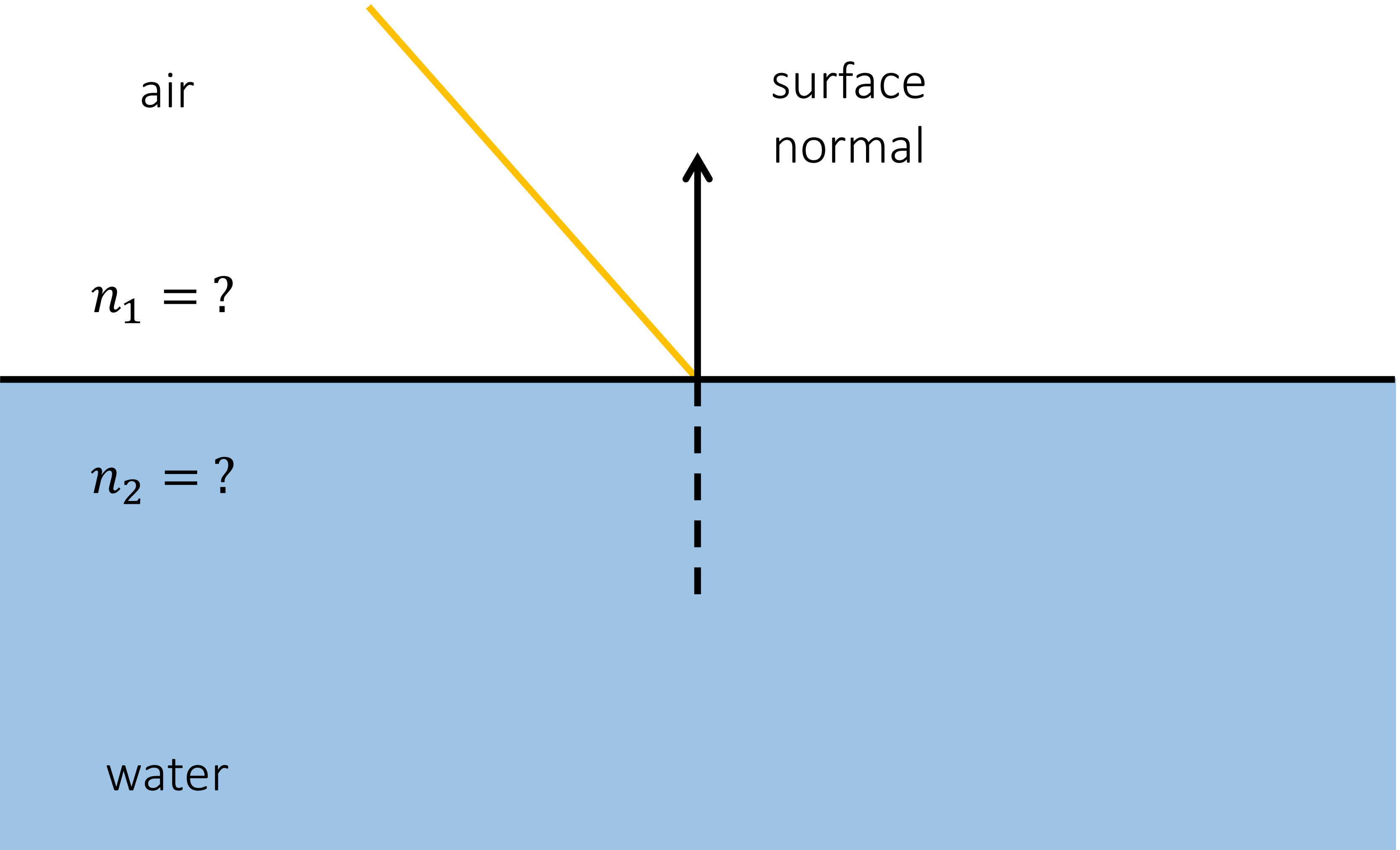

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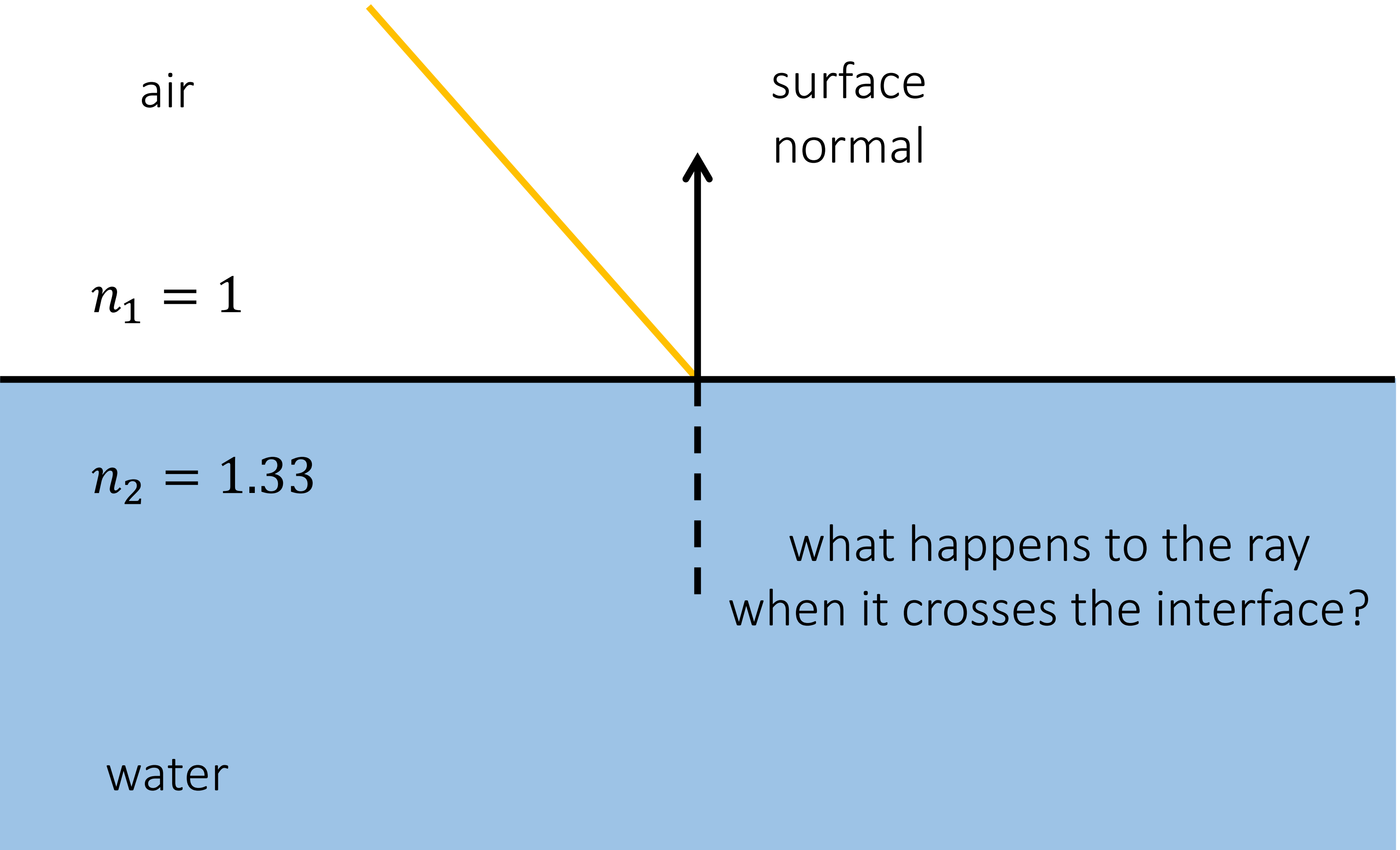

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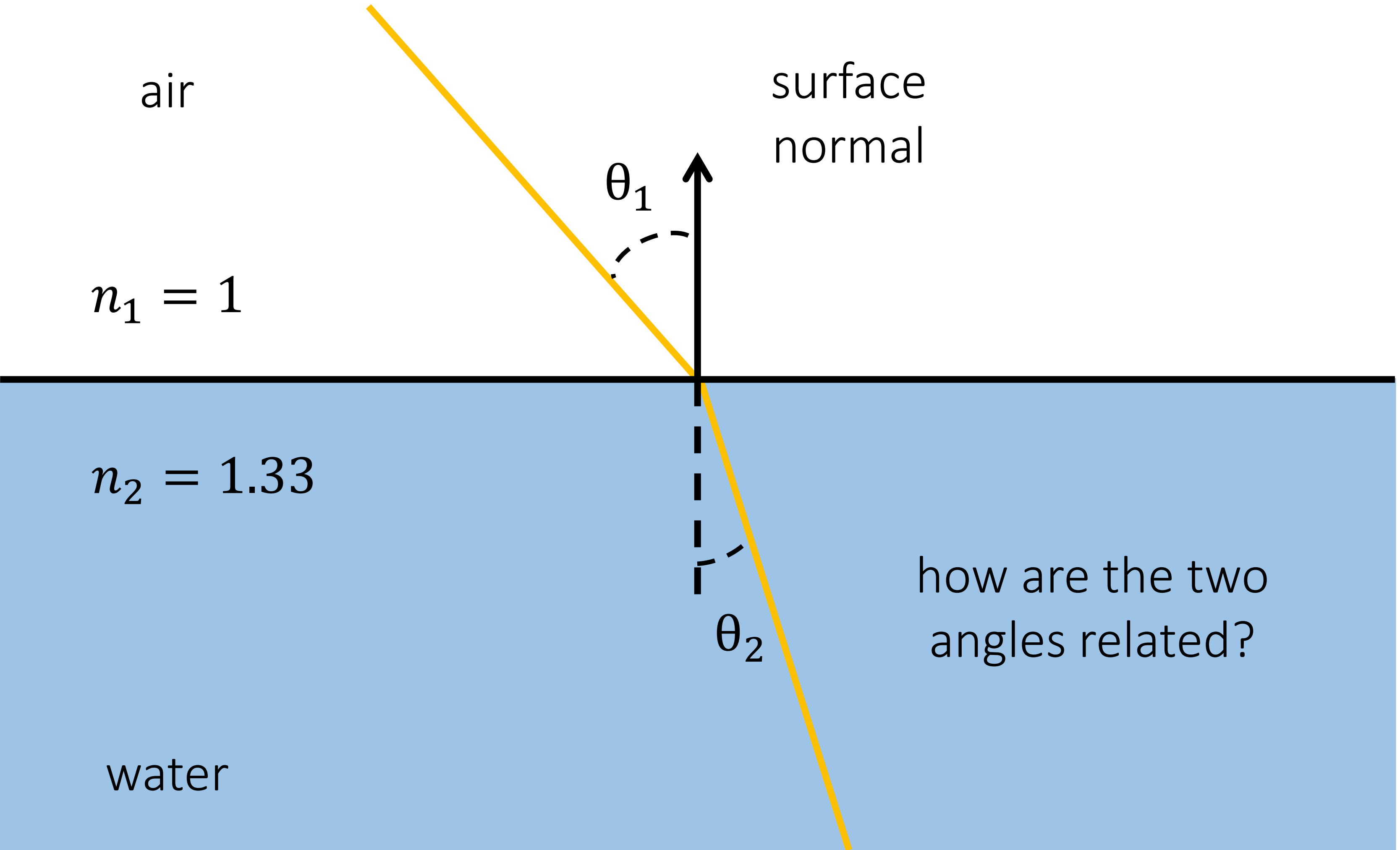

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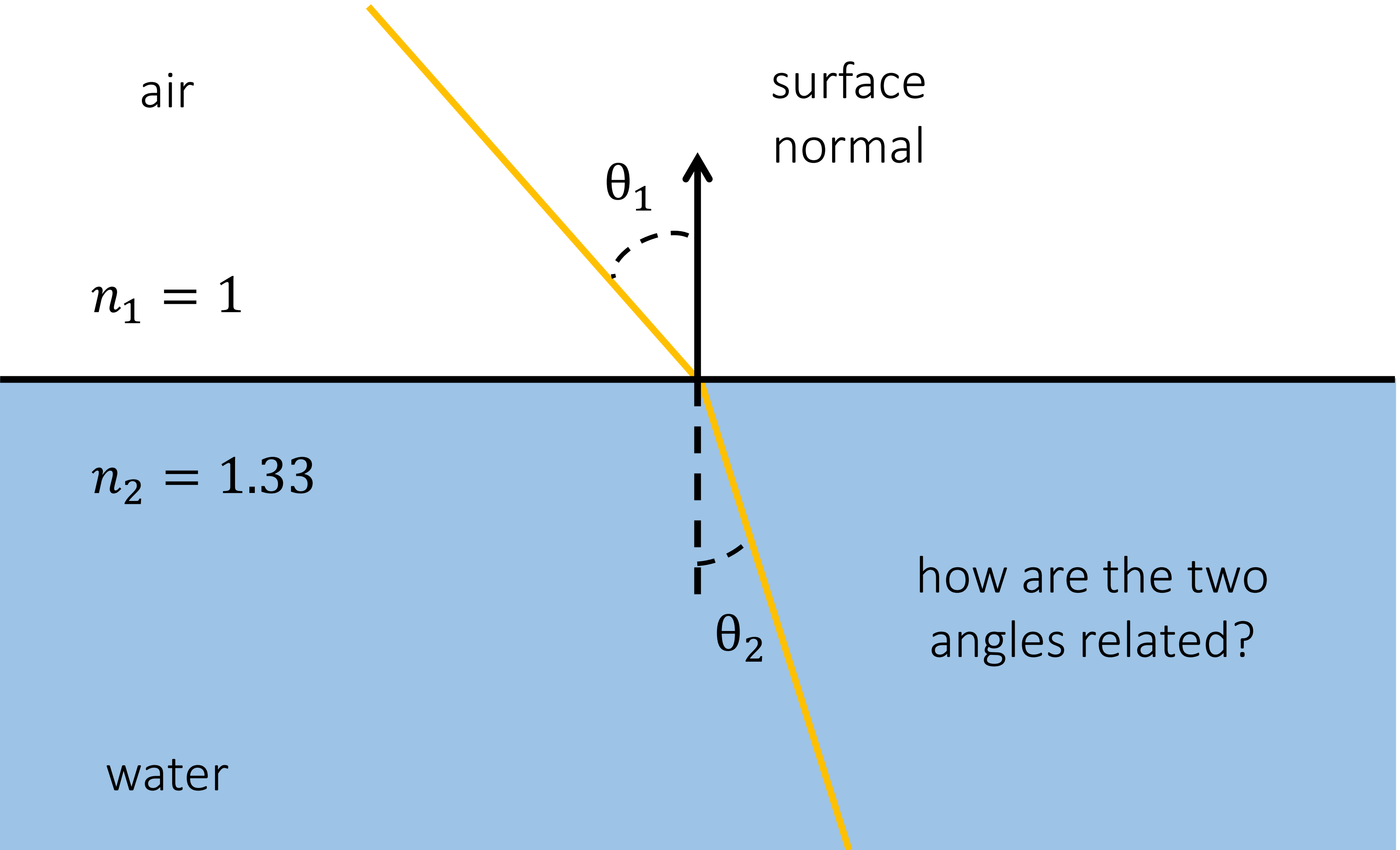


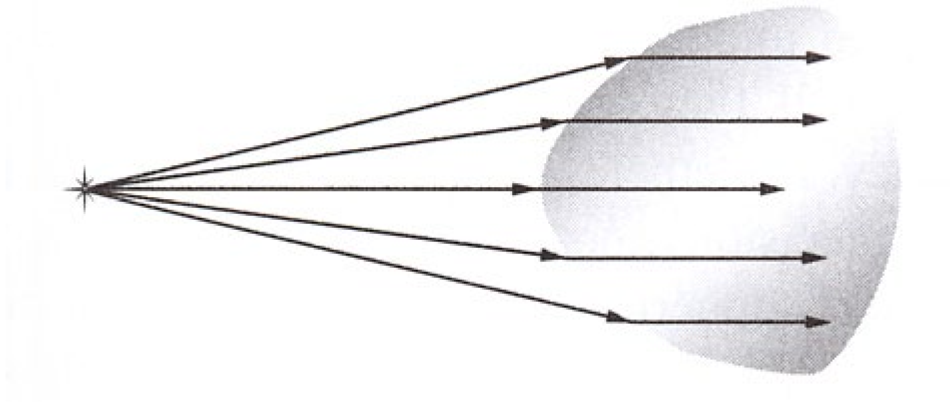
Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

How do we prove Snell's law?

Refraction at interfaces of complicated shapes

What shape should an interface have to make parallel rays converge to a point?



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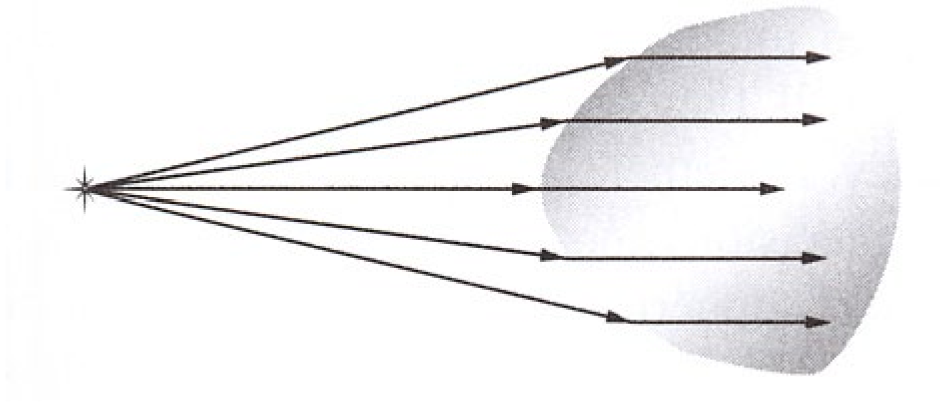
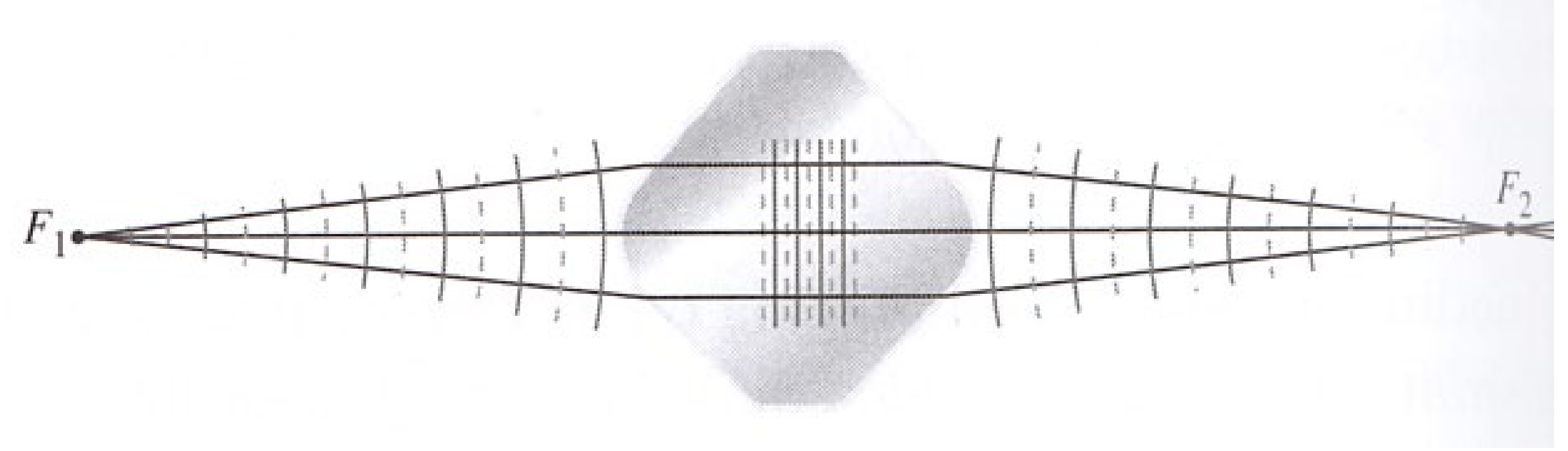


Single hyperbolic interface:
point to parallel rays

Double hyperbolic interface:
point to point rays



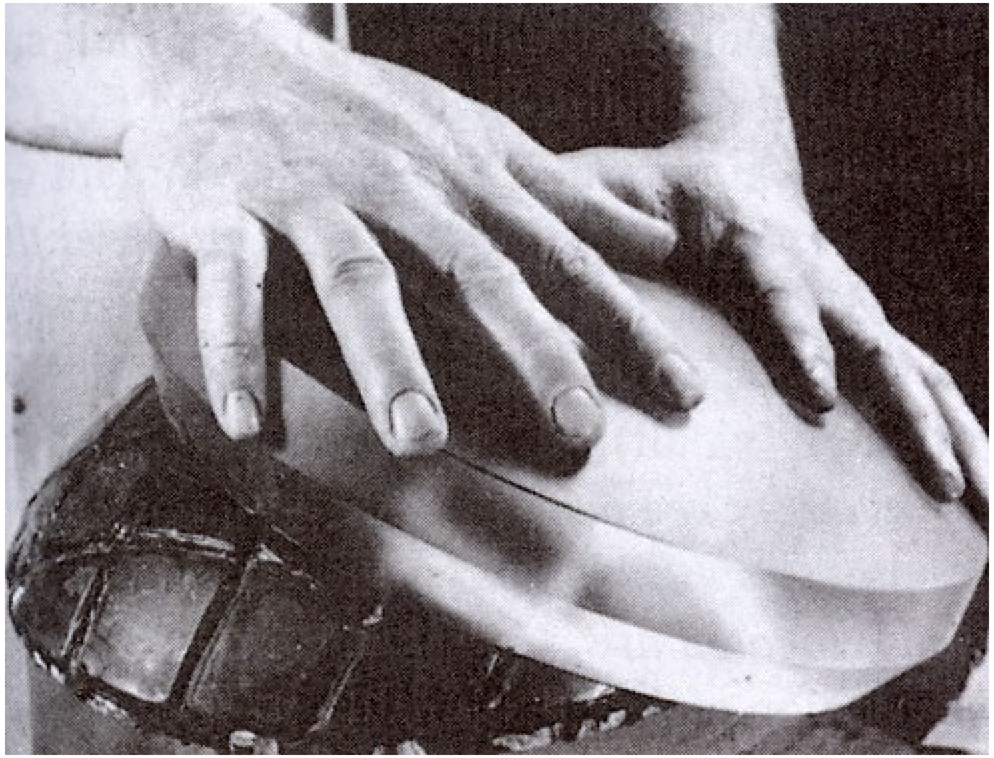
Therefore, lenses should also have hyperbolic shapes.

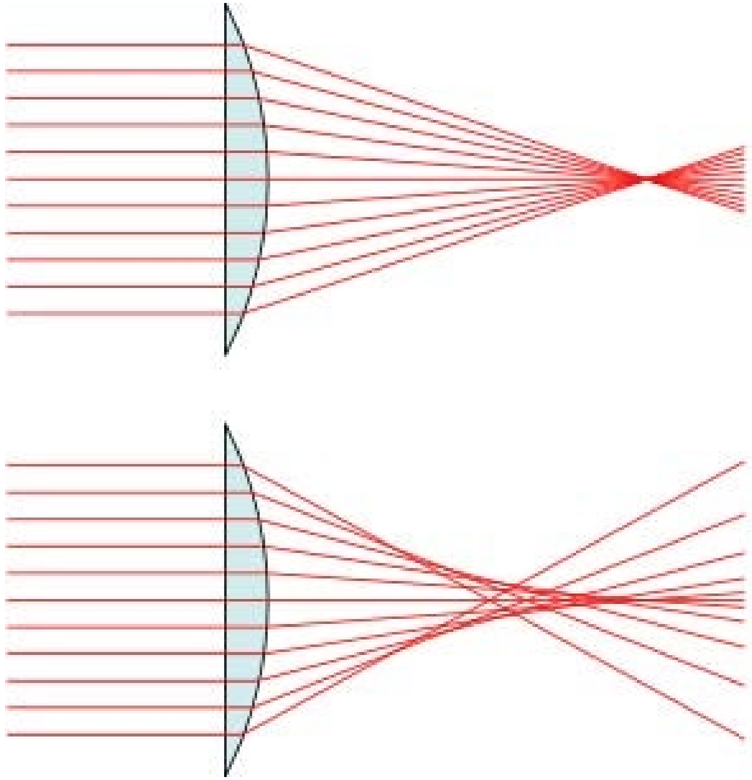
(Note: conics have different reflective and refractive properties.)

Spherical lenses

In practice, lenses are often made to have spherical interfaces for ease of fabrication.

- Two roughly fitting curved surfaces ground together will eventually become spherical.





Spherical lenses don’t bring parallel rays to a point.

- This is called *spherical aberration*.
- Approximately axial (i.e., paraxial) rays behave better.

Paraxial approximation

Assume angles are small. Then:

$$\sin\theta \simeq \theta \qquad \cos\theta \simeq 1 \qquad \tan\theta \simeq \theta$$

Where do these approximations come from?

Paraxial approximation (a.k.a. first-order optics)

Assume angles are small. Then:

$$\sin\theta \simeq \theta \qquad \cos\theta \simeq 1 \qquad \tan\theta \simeq \theta$$

Where do these approximations come from?

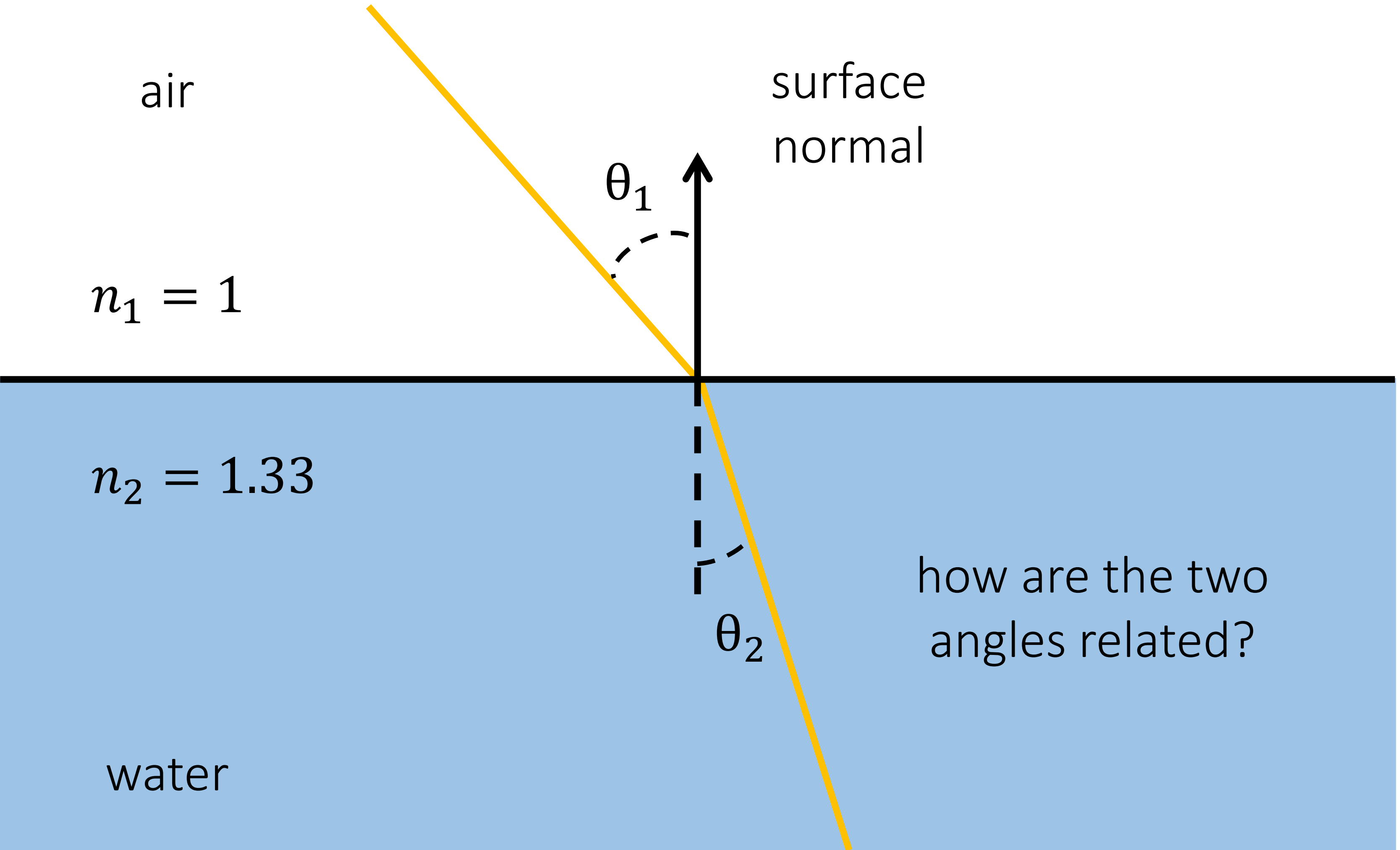
- First-order expansions of sin and cos functions.

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots$$

Refraction

Refraction is the bending of rays of light when they cross optical interfaces (i.e., surfaces where the index of refraction changes)



Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

How is Snell's law simplified under paraxial approximation?

Refraction

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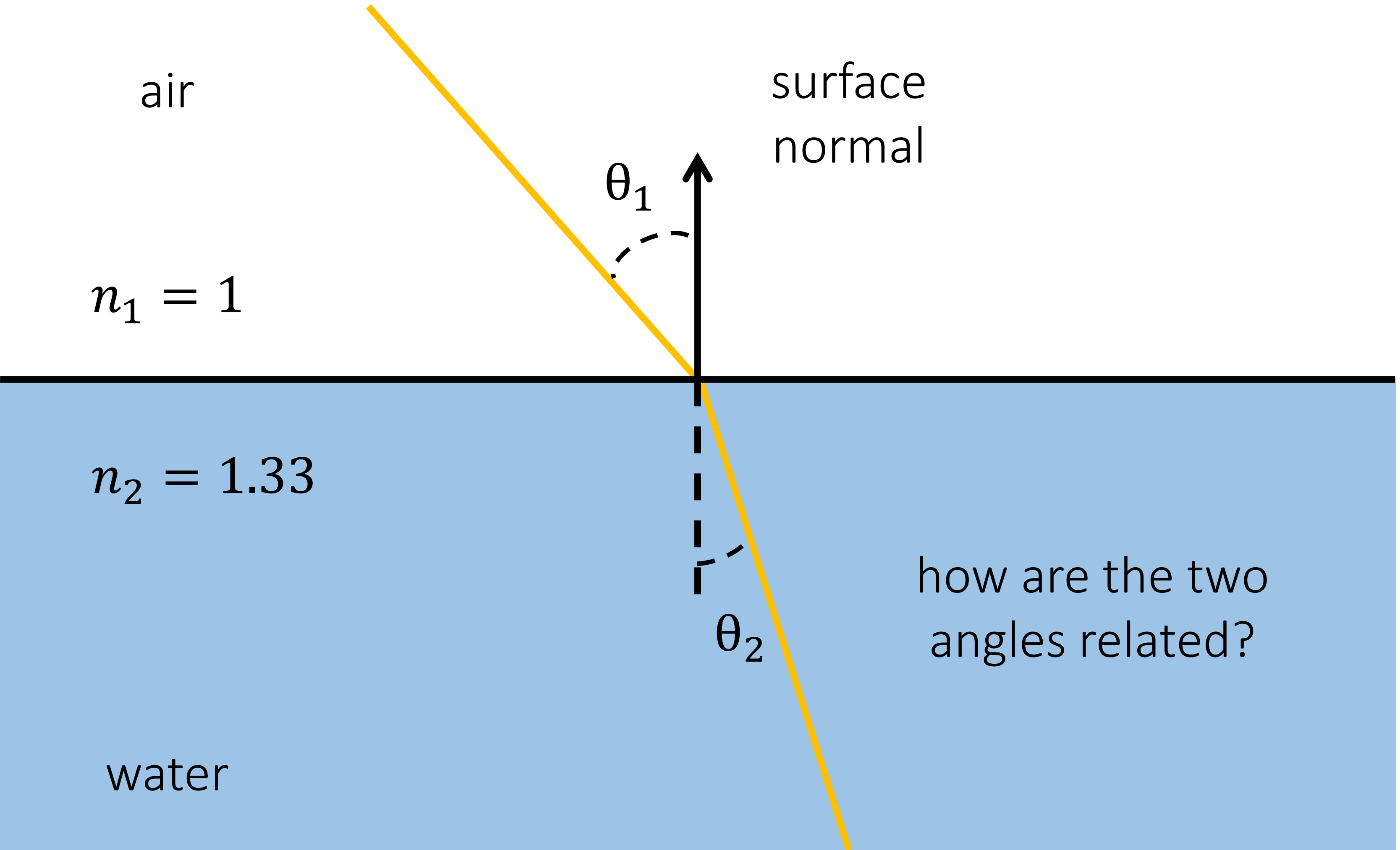

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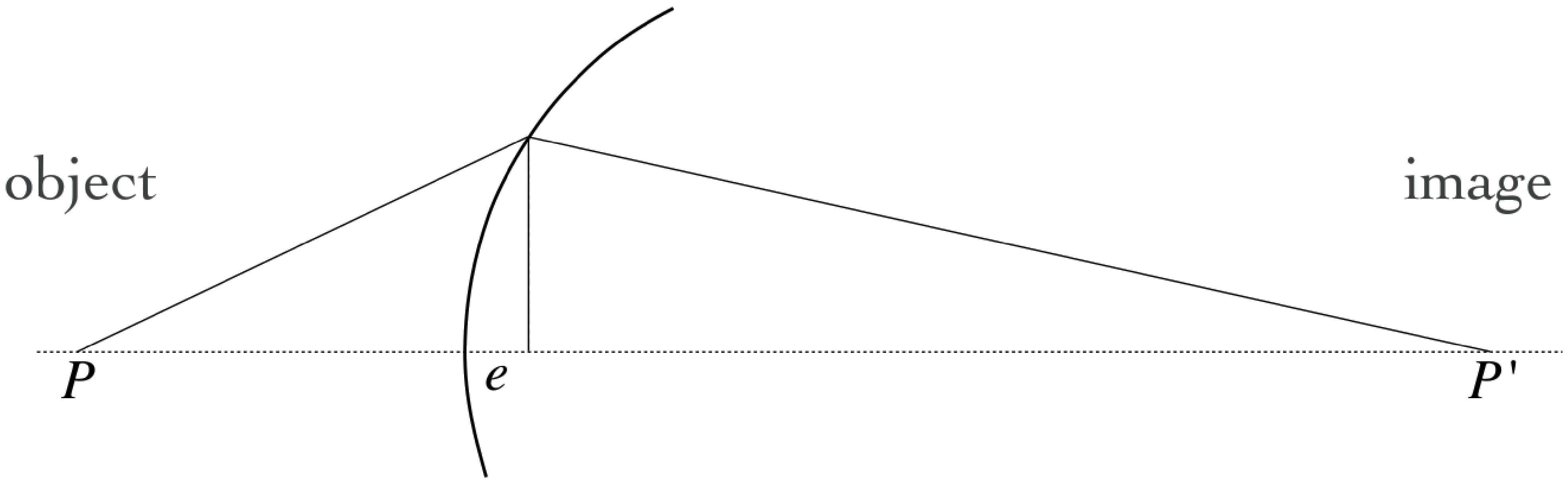
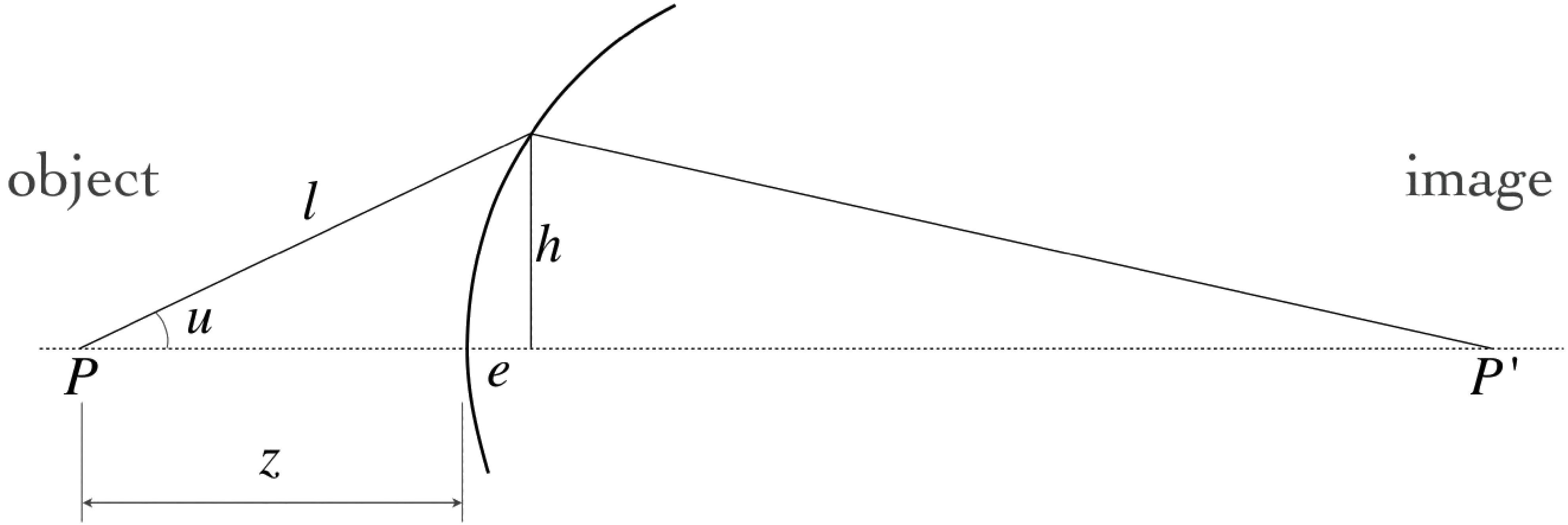
Paraxial focusing



spherical lens surface with radius r

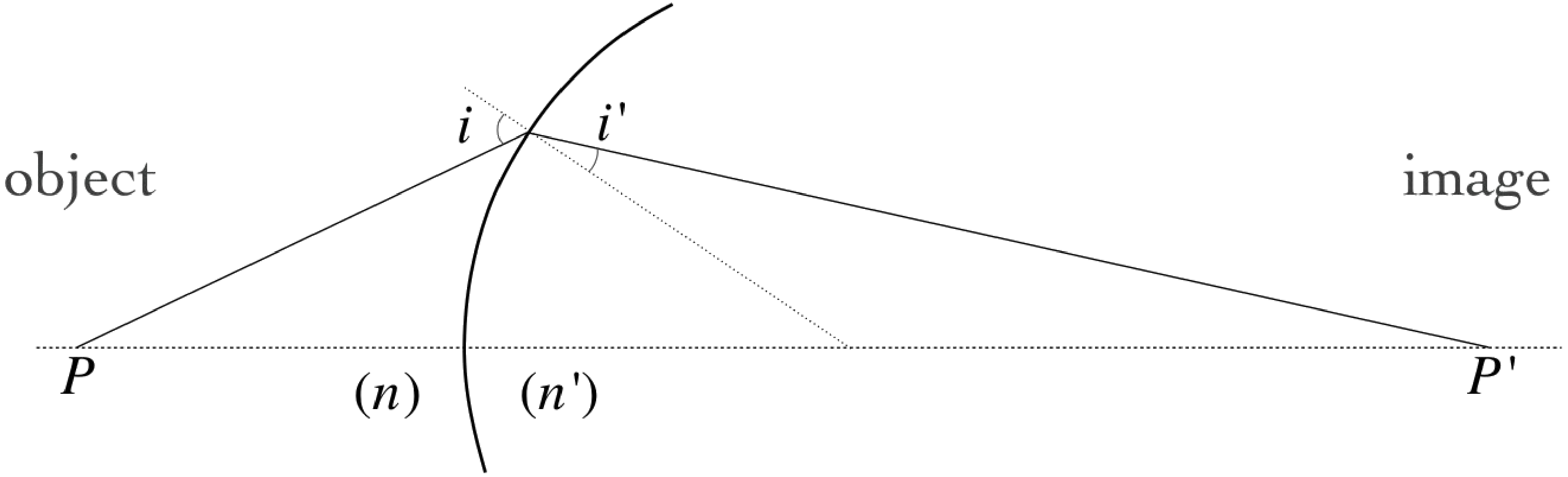
Paraxial focusing



- assume $e \approx 0$
- assume $\sin u = h/l \approx u$ (for u in radians)
- assume $\cos u \approx z/l \approx 1$

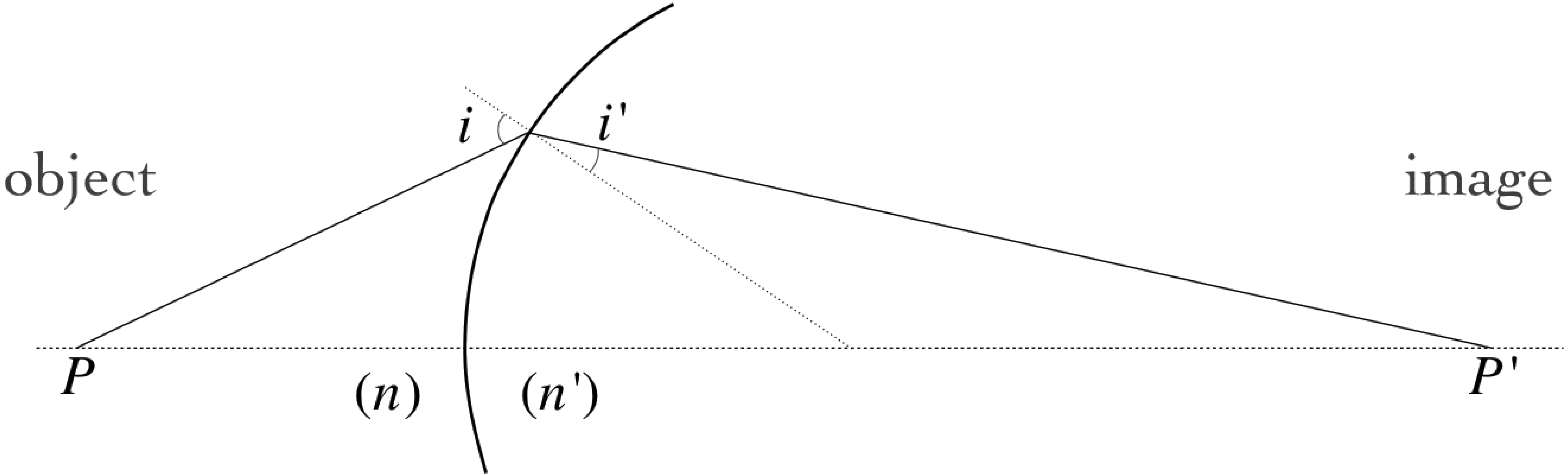
Where do these two equations come from?

Paraxial focusing



How can we relate angles i and i’ ?

Paraxial focusing



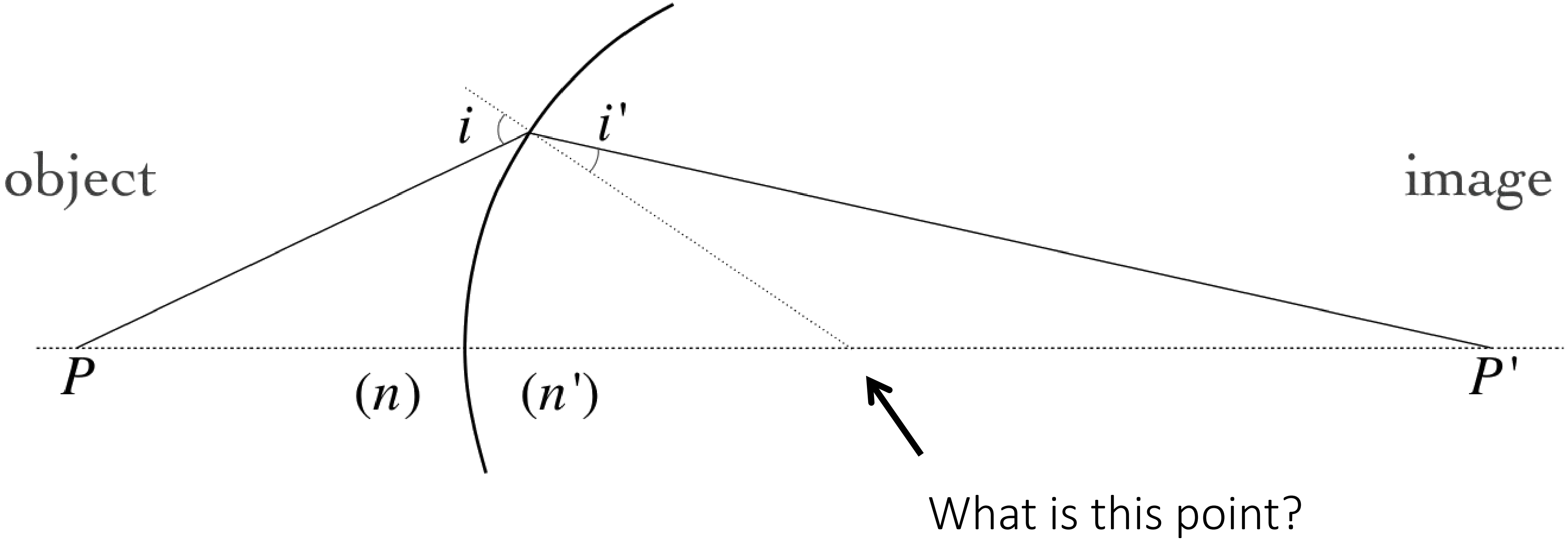
Snell's law:

$$n \sin i = n' \sin i'$$

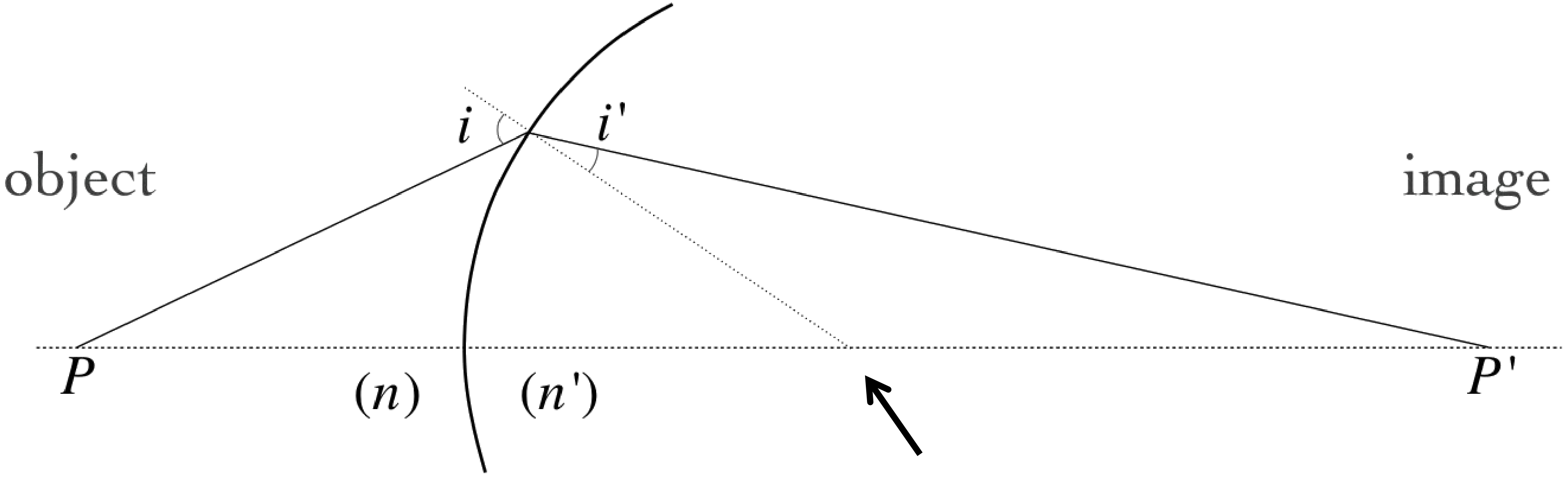
paraxial approximation:

$$n\, i \approx n'\, i'$$

Paraxial focusing



Paraxial focusing

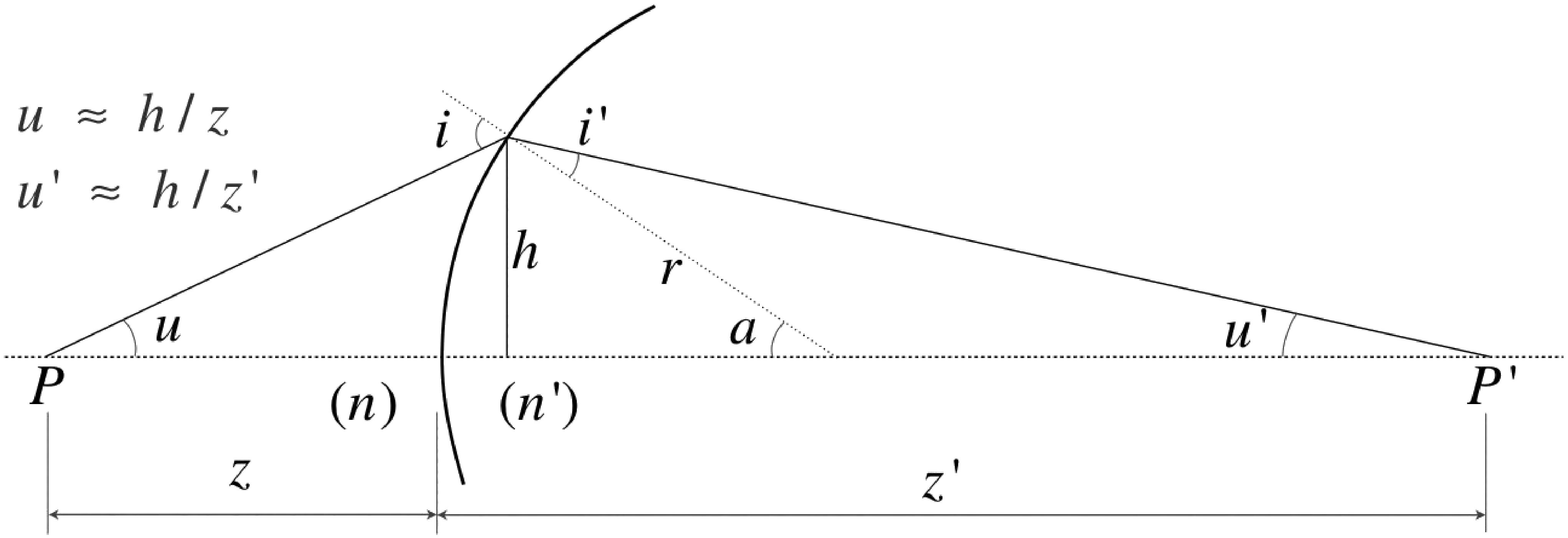


Center of spherical surface

Paraxial focusing

$$u \approx h / z$$

$$u' \approx h / z'$$



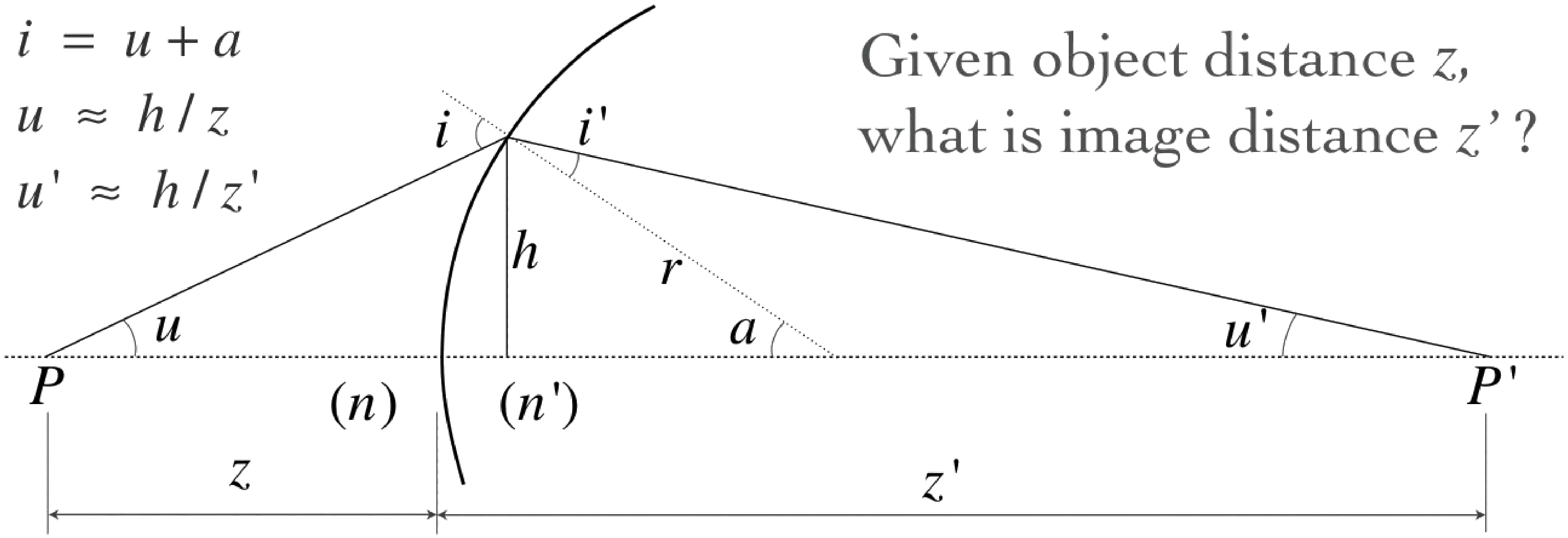
$$n\, i \approx n'\, i'$$

What is angle i equal to?

Paraxial focusing

$$i = u + a$$
$$u \approx h / z$$
$$u' \approx h / z'$$

Given object distance z, what is image distance z'?

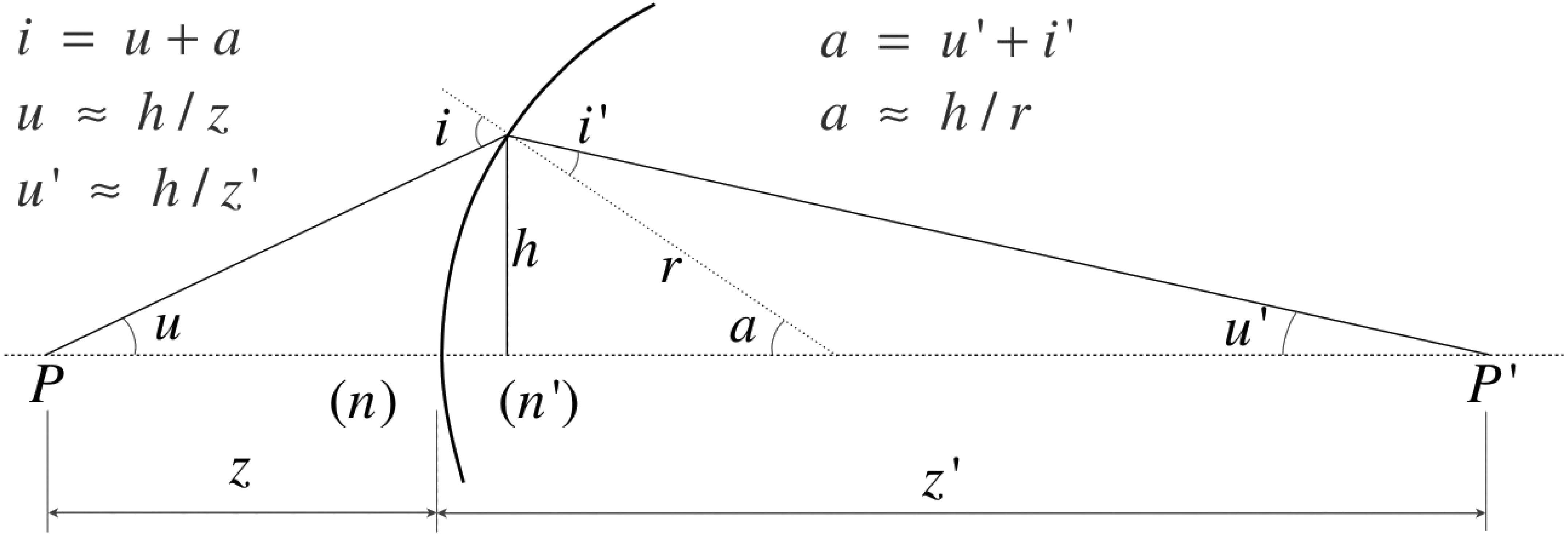


$$n\, i \approx n'\, i'$$

Paraxial focusing

$$i = u + a$$
$$u \approx h / z$$
$$u' \approx h / z'$$

$$a = u' + i'$$
$$a \approx h / r$$



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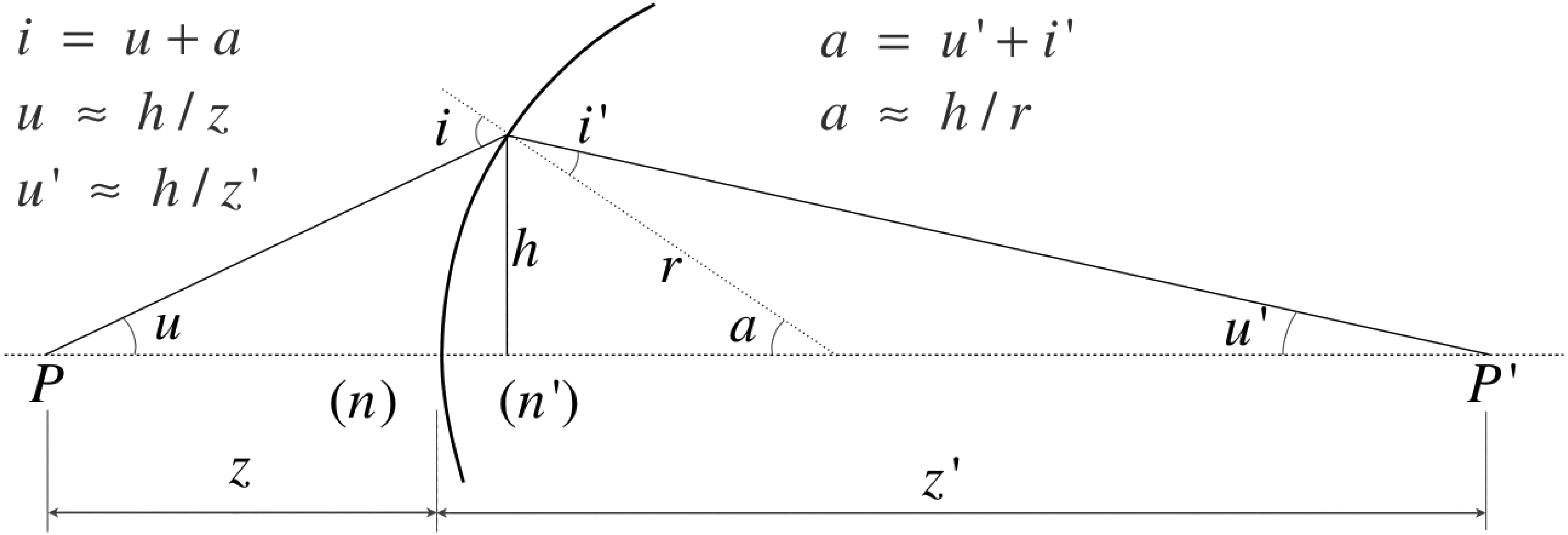


$$n\, i \approx n'\, i'$$

$$n\,(u + a) \approx n'\,(a - u')$$

$$n\,(h / z + h / r) \approx n'\,(h / r - h / z')$$

What does this last equation imply?

$$n / z + n / r \approx n' / r - n' / z'$$

Paraxial focusing

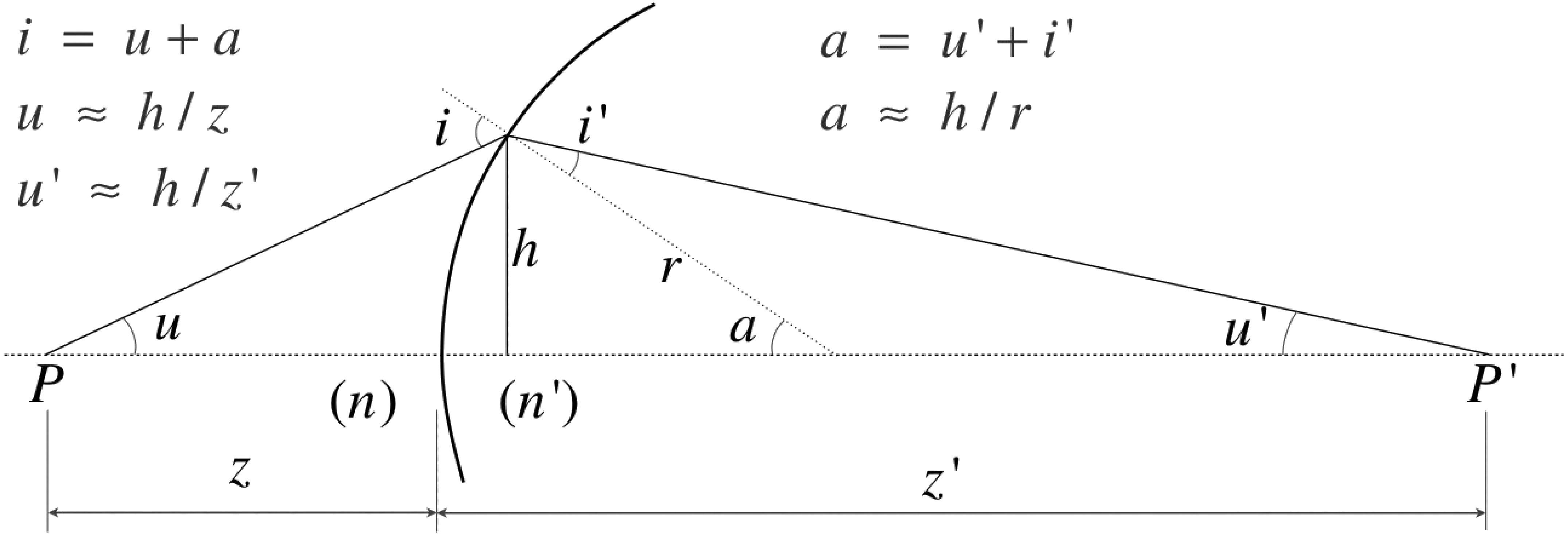
$i = u + a$

$u \approx h / z$

$u' \approx h / z'$

$a = u' + i'$

$a \approx h / r$



$n\, i \approx n'\, i'$

$n\,(u + a) \approx n'\,(a - u')$

$n\,(h / z + h / r) \approx n'\,(h / r - h / z')$

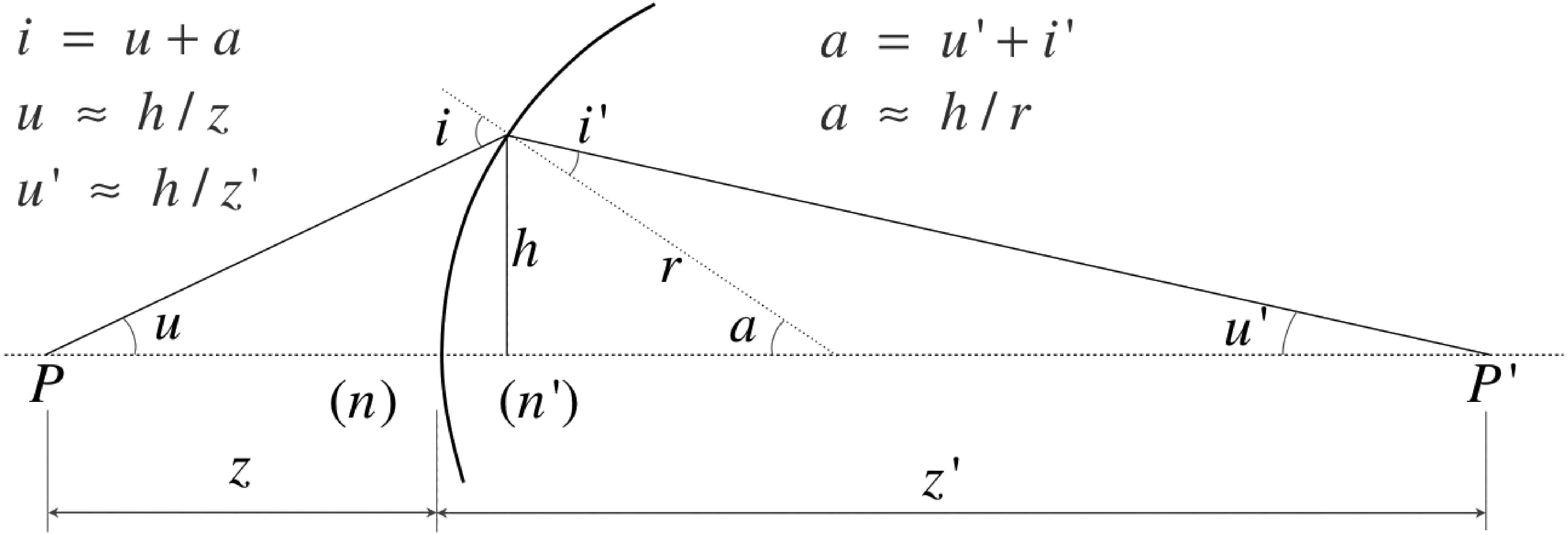
$n / z + n / r \approx n' / r - n' / z'$

h has cancelled out, so any ray from P will focus at P’.

Paraxial focusing

$$i = u + a$$
$$u \approx h / z$$
$$u' \approx h / z'$$

$$a = u' + i'$$
$$a \approx h / r$$



$$n\, i \approx n'\, i'$$

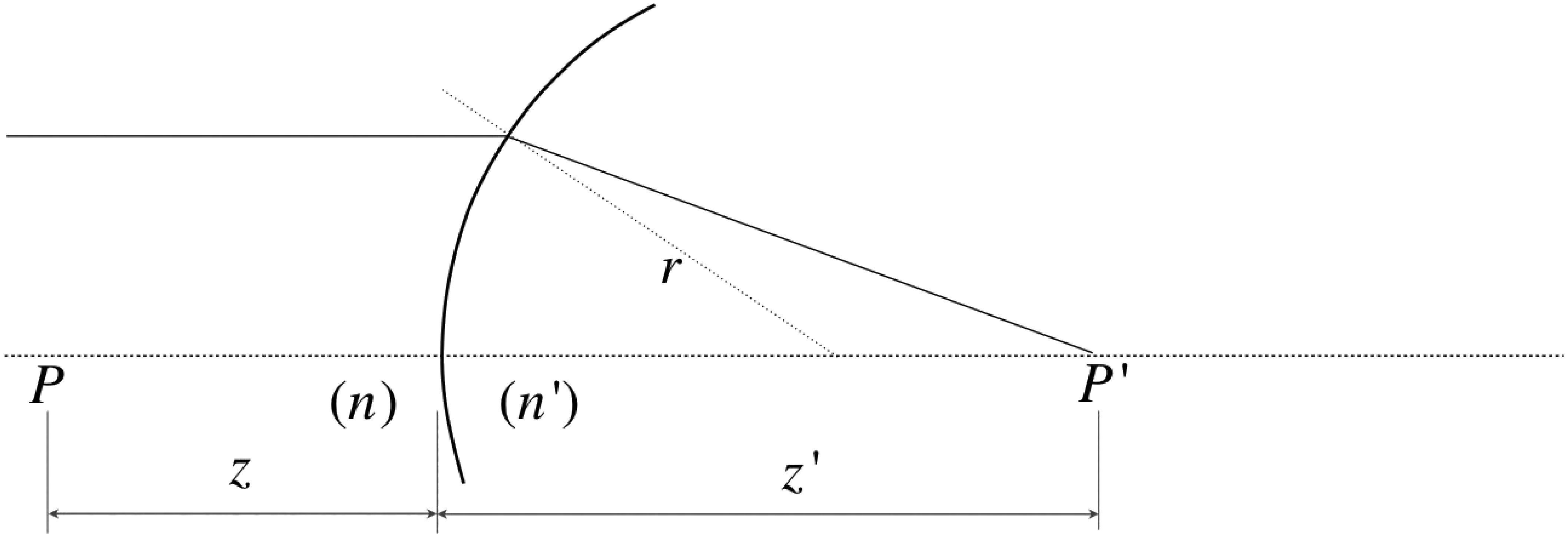
$$n\,(u + a) \approx n'\,(a - u')$$

$$n\,(h / z + h / r) \approx n'\,(h / r - h / z')$$

$$n / z + n / r \approx n' / r - n' / z'$$

What happens as z tends to infinity?

Focal length



What happens if z is ∞ ?

$$n / z + n / r \approx n' / r - n' / z'$$

$$n / r \approx n' / r - n' / z'$$

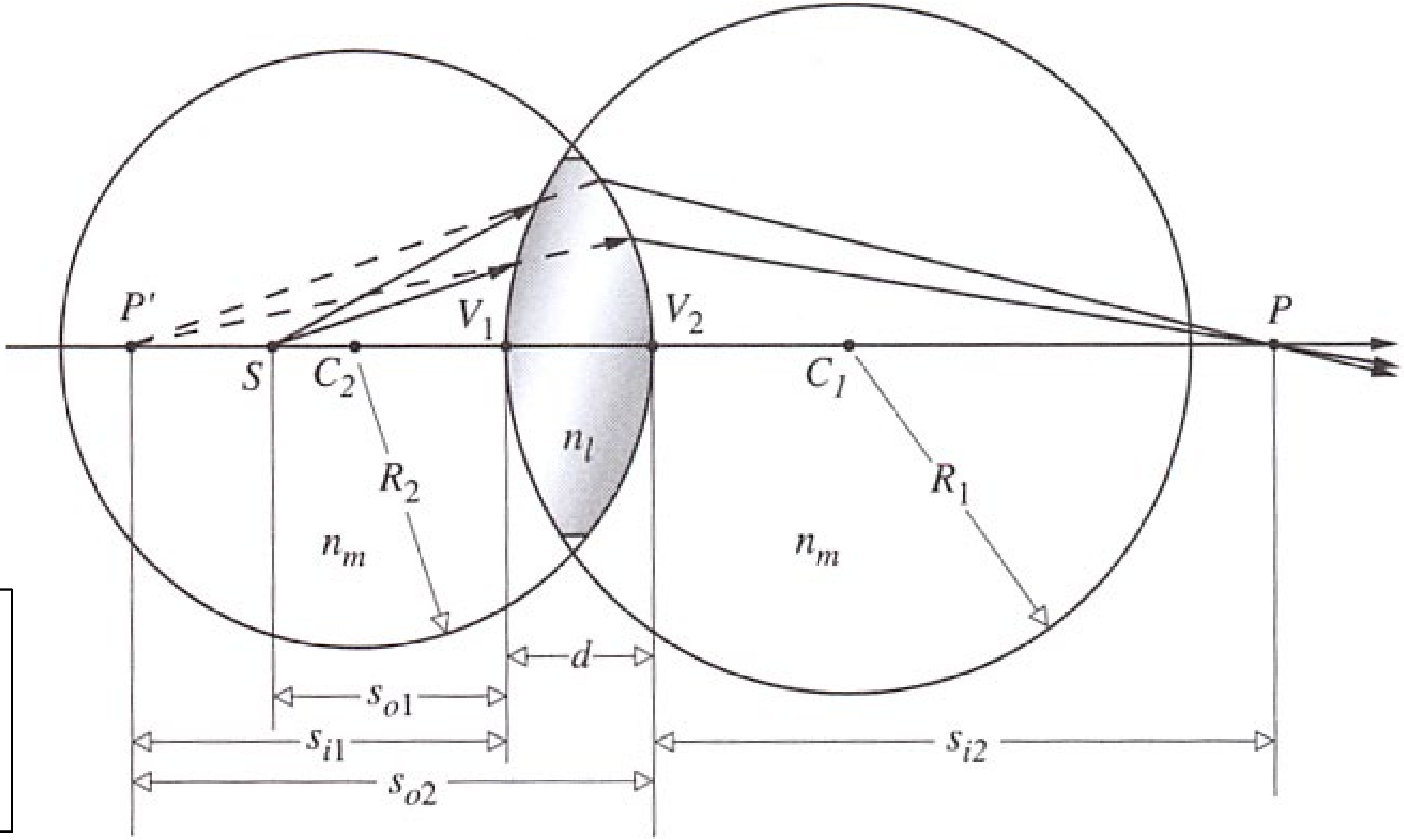
$$z' \approx (r\, n') / (n' - n)$$

- $f \triangleq$ focal length = z'

Thin lens

Using similar derivations, we can extend these results to two spherical interfaces.

- We obtain a spherical lens in air.



- Thin lens approximation: d close to zero.
- Under this approximation, we obtain the *lensmaker's equation*.

$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Gaussian lens formula

- Starting from the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

- and recalling that as object distance s_o is moved to infinity, image distance s_i becomes focal length f_i , we get

$$\frac{1}{f_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

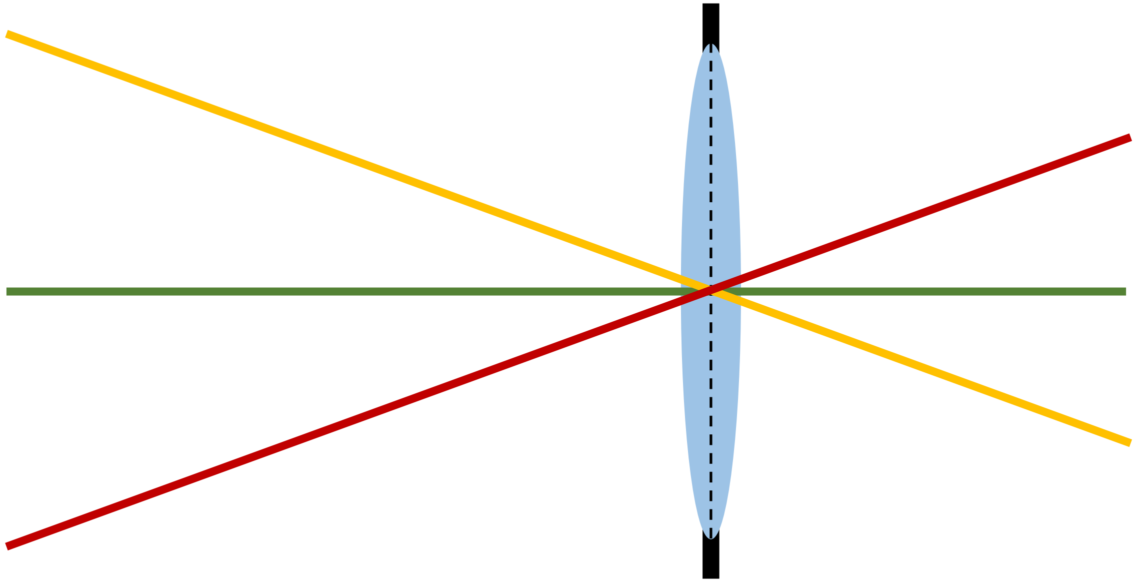
- Equating these two, we get the Gaussian lens formula

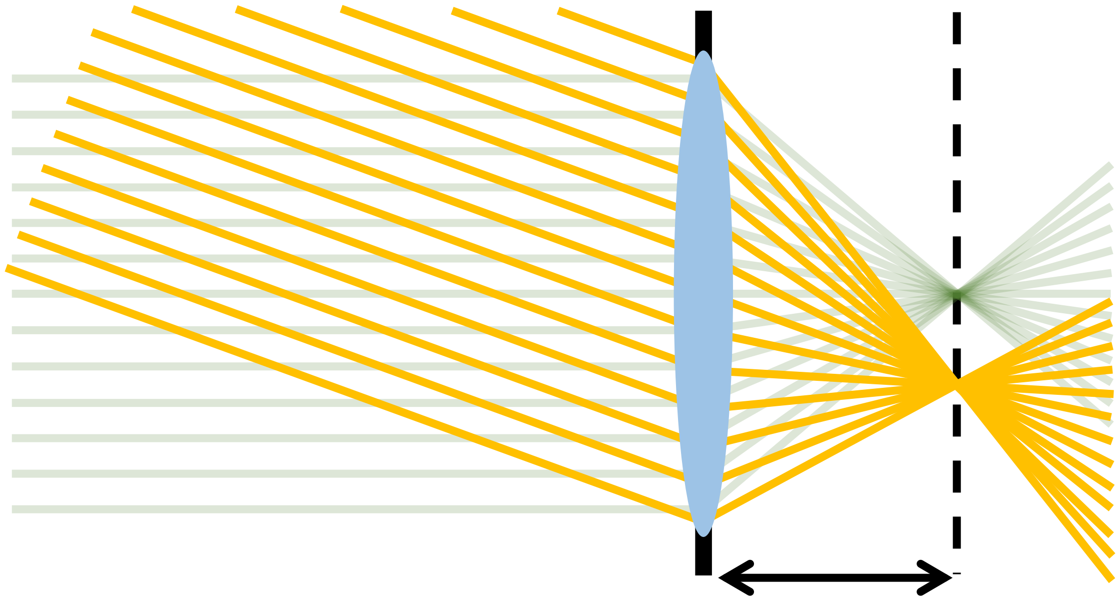
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}.$$

Looks familiar?

Thin lens model

Simplification of geometric optics for well-designed lenses.





focal length f

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

$$m = \frac{S' - f}{f}$$

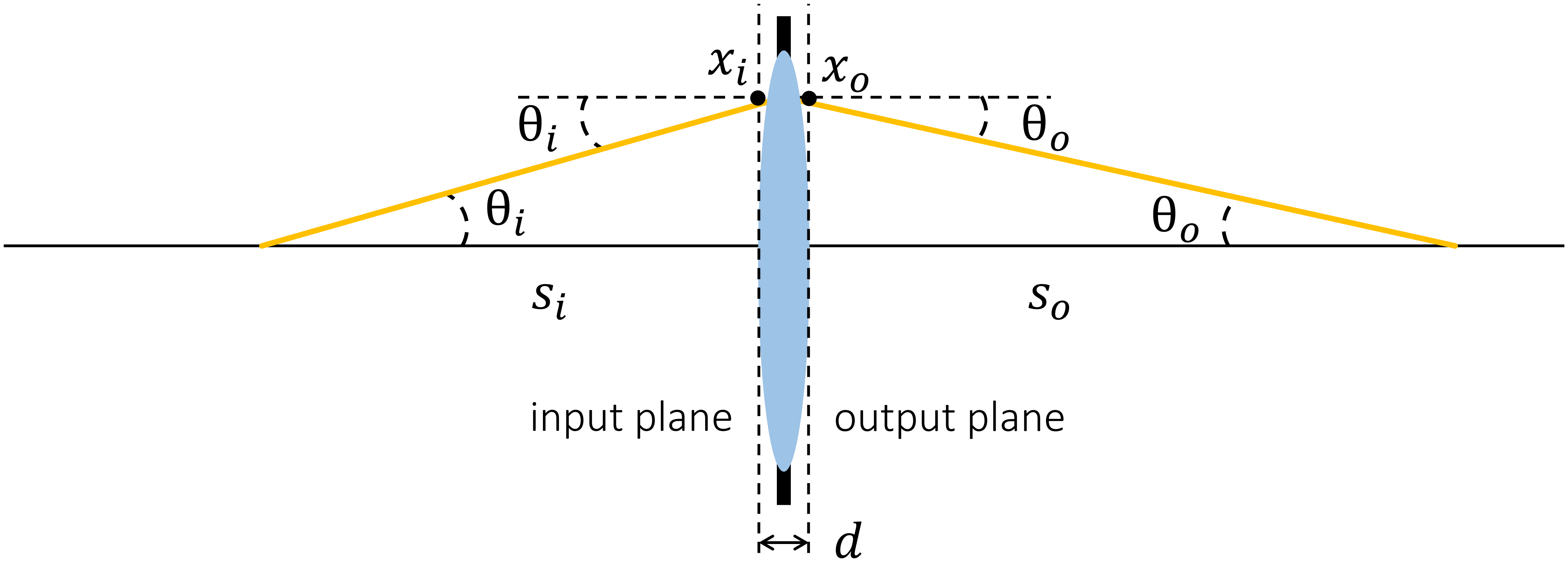
Two assumptions:

1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.

Same result as what we obtained last time using ray tracing assumptions.

Ray transfer matrix analysis

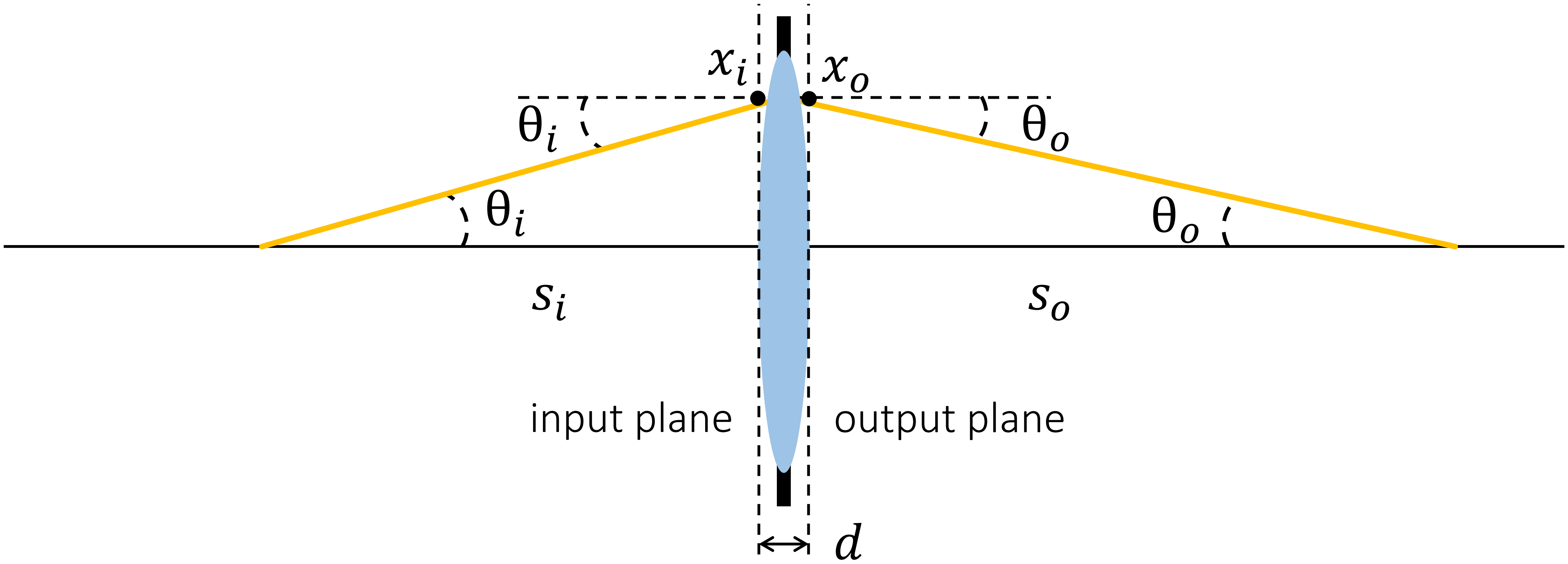
Let’s look at thin lenses (yet) again



Assumptions:

- Paraxial → ?
- Thin lens → ?

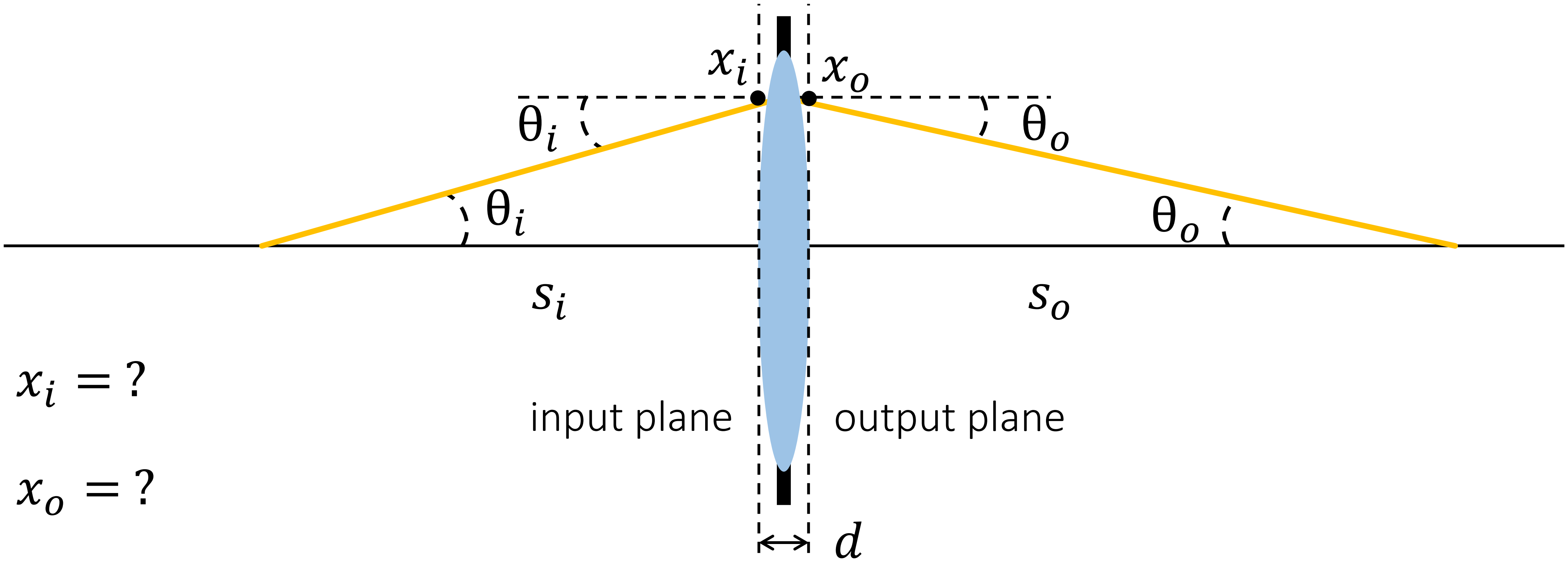
Let’s look at thin lenses (yet) again



Assumptions:

- Paraxial → angles θ are small, thus first-order approximations for sinθ, cosθ, and tanθ apply.
- Thin lens → width of lens is negligible (d ≃ 0) relative to distances s.

Let’s look at thin lenses (yet) again

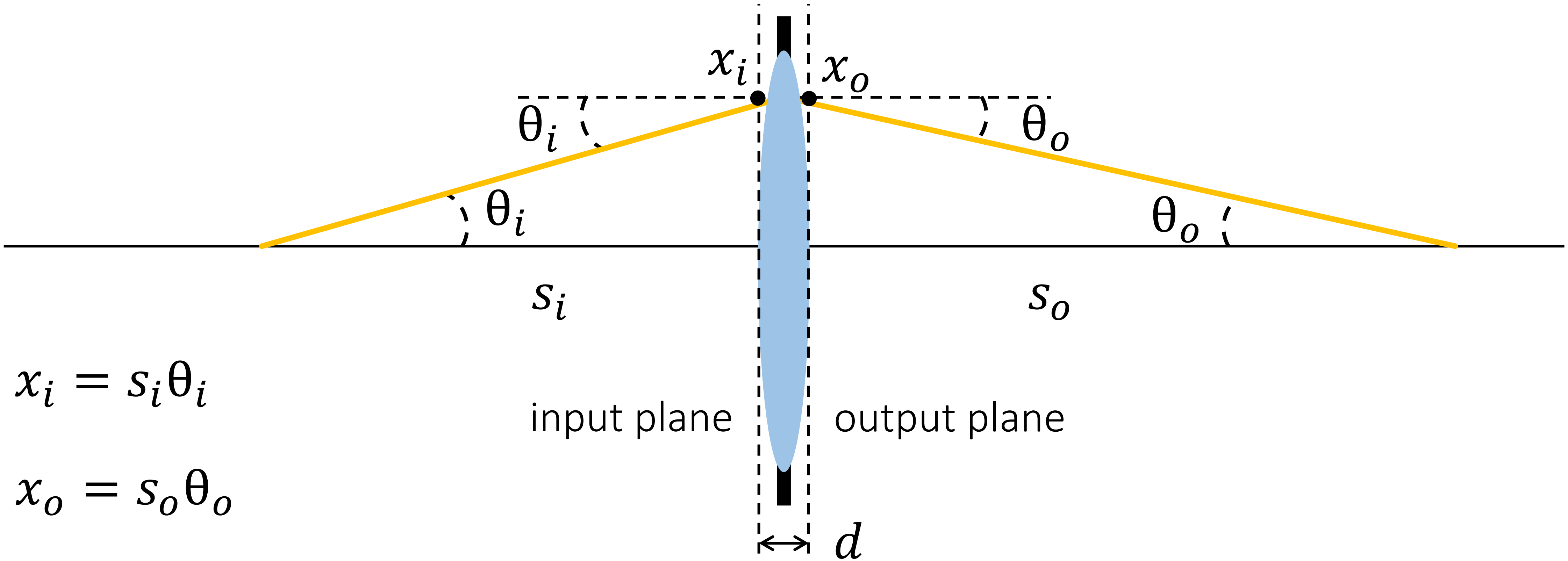


$x_i = ?$

$x_o = ?$

$$\frac{1}{s_o} + \frac{1}{s_i} = ?$$

Let’s look at thin lenses (yet) again



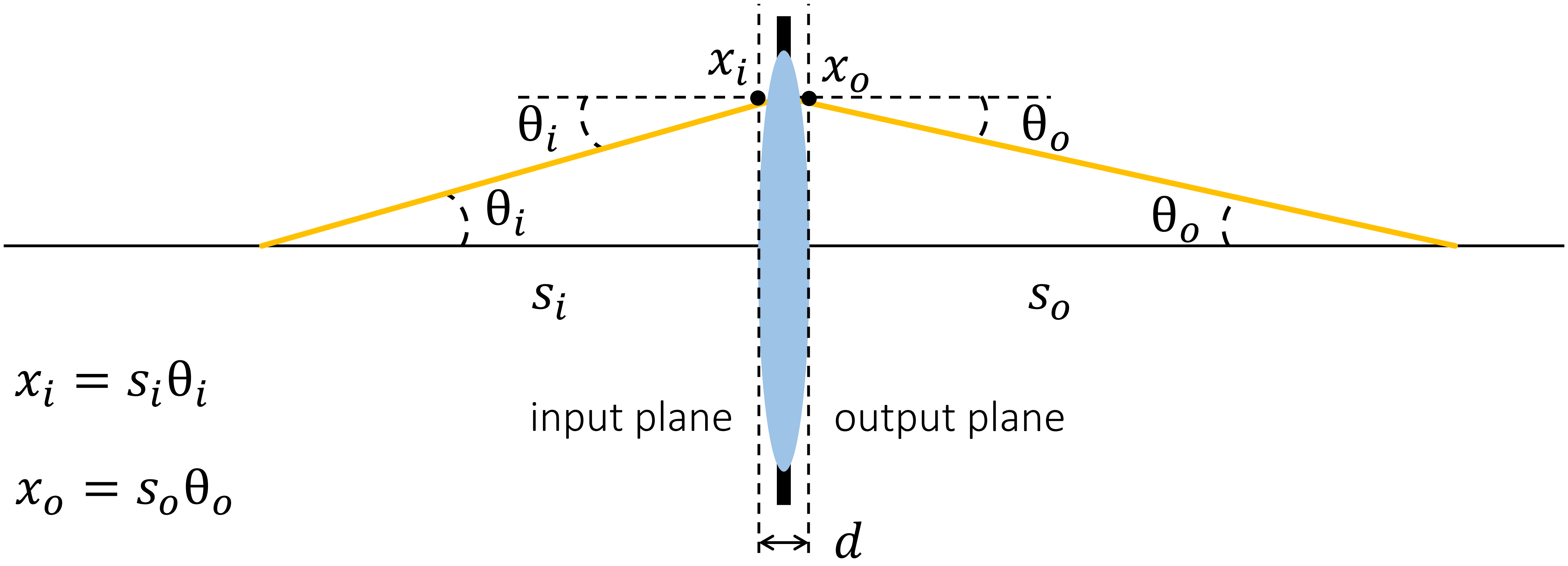
$x_i = s_i \theta_i$

$x_o = s_o \theta_o$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$d \simeq 0 \Rightarrow x_i = x_o$

Let's look at thin lenses (yet) again



$$x_i = s_i\theta_i$$

$$x_o = s_o\theta_o$$

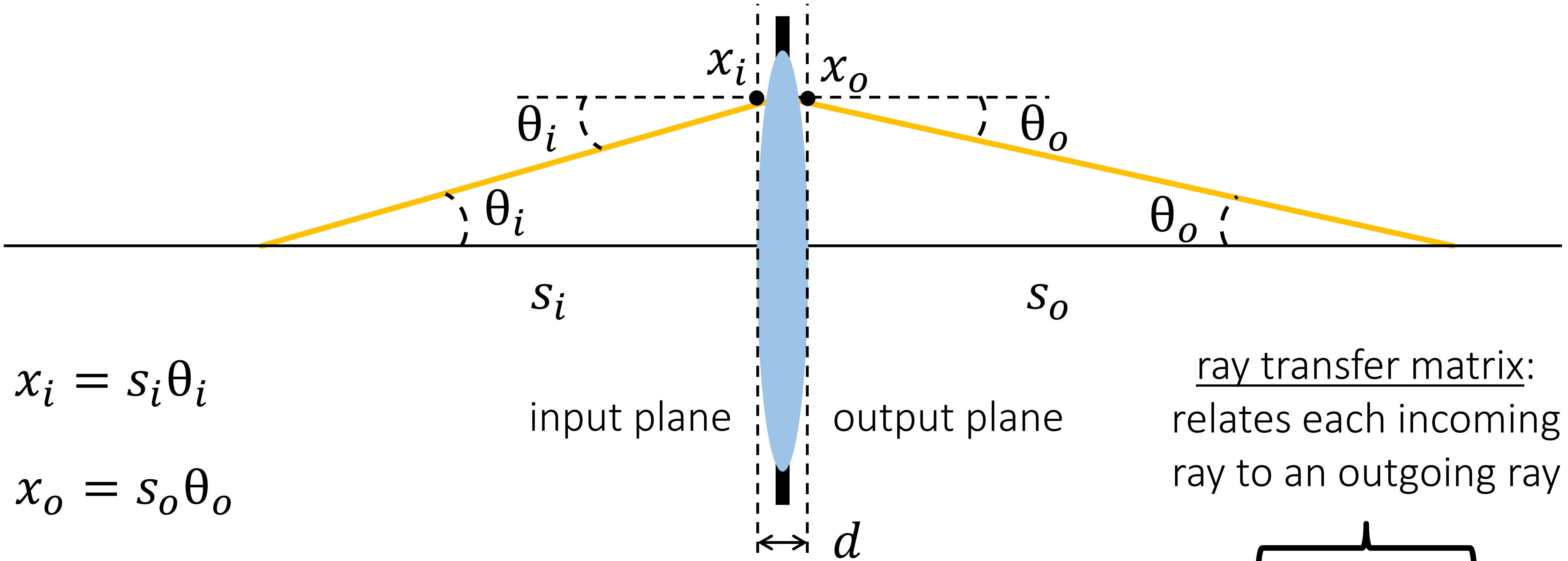
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$d \simeq 0 \Rightarrow x_i = x_o$$

Putting it all together, we can write:

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

Let's look at thin lenses (yet) again



$$x_i = s_i \theta_i$$

$$x_o = s_o \theta_o$$

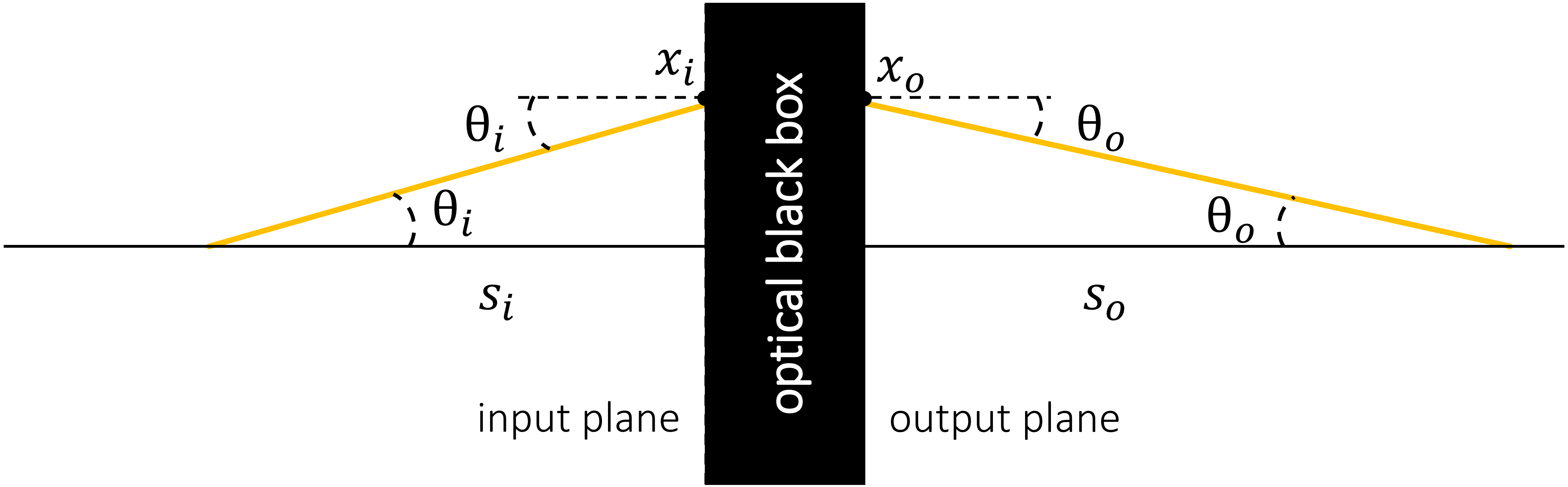
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$d \simeq 0 \Rightarrow x_i = x_o$$

Putting it all together, we can write:

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}}^{\substack{\text{ray transfer matrix: relates each incoming} \\ \text{ray to an outgoing ray}}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

Ray transfer matrix analysis

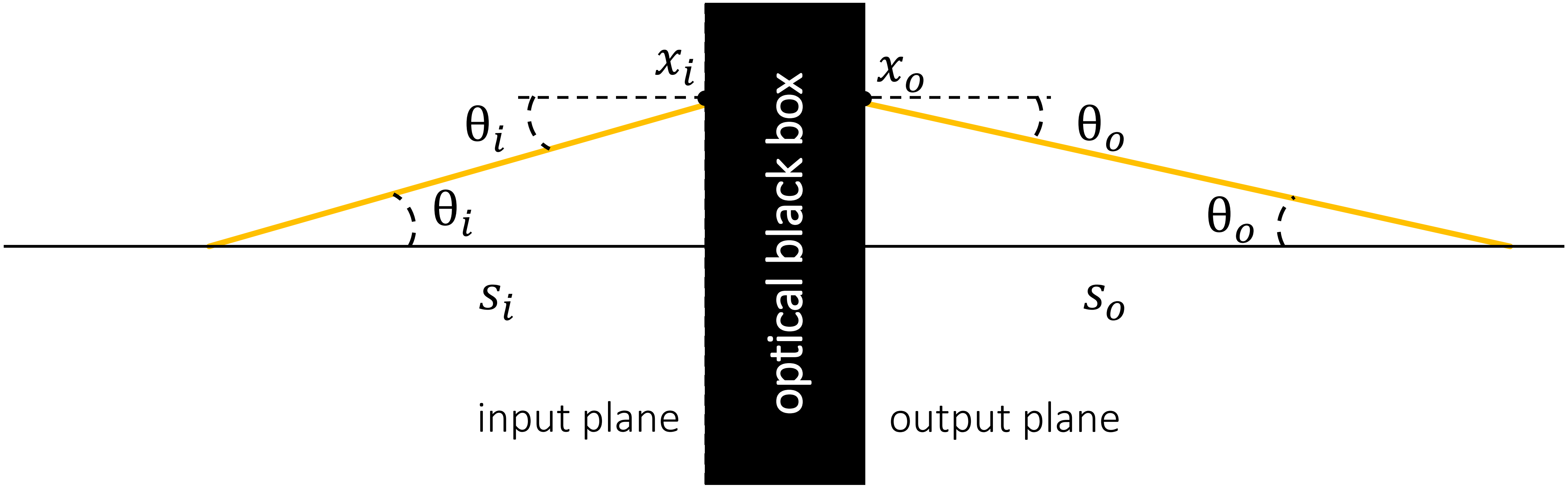


Every optical system implements a (generally non-linear) ray mapping of the form:

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} f(x_i, \theta_i) \\ g(x_i, \theta_i) \end{bmatrix}$$

How do we go from here to a ray transfer matrix?

Ray transfer matrix analysis



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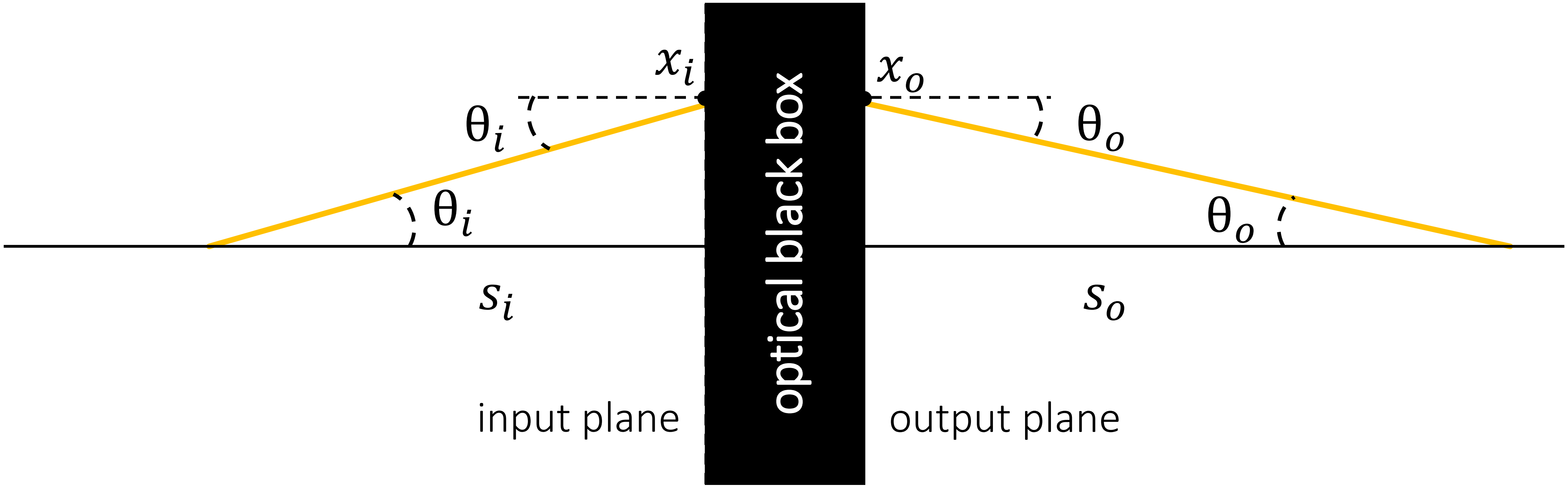
How do we go from here to a ray transfer matrix?

- Paraxial approximation: Use first-order approximation around axial ray.

Ray transfer matrix analysis



Under paraxial approximation:

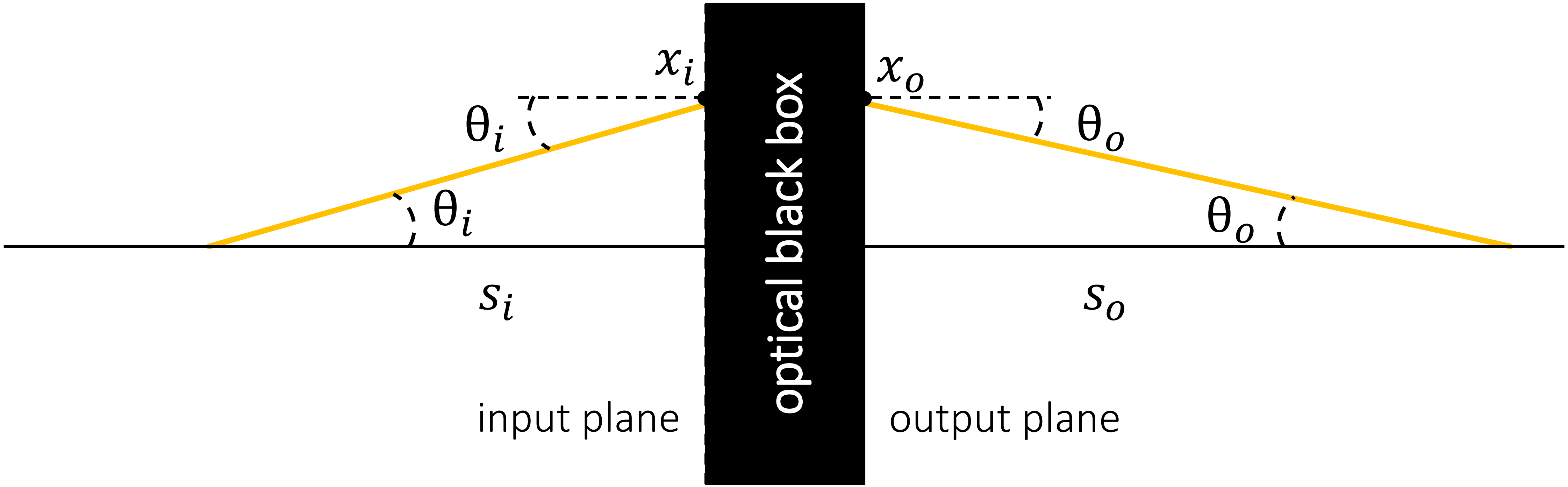
$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} f(x_i, \theta_i) \\ g(x_i, \theta_i) \end{bmatrix} \simeq \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

where

$A = ?$ $B = ?$

$C = ?$ $D = ?$

Ray transfer matrix analysis



Under paraxial approximation:

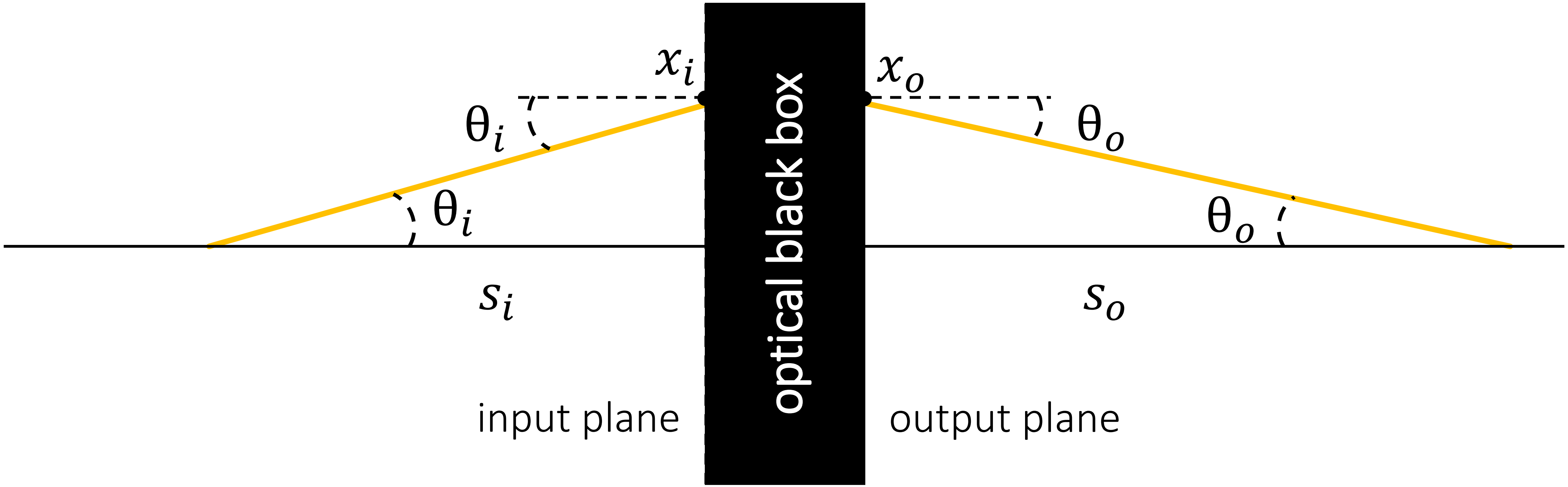
$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} f(x_i, \theta_i) \\ g(x_i, \theta_i) \end{bmatrix} \simeq \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

where

$$A = \left. \frac{\partial f}{\partial x_i} \right|_{x_i = \theta_i = 0} \qquad B = ?$$

$$C = ? \qquad D = ?$$

Ray transfer matrix analysis



Under paraxial approximation:

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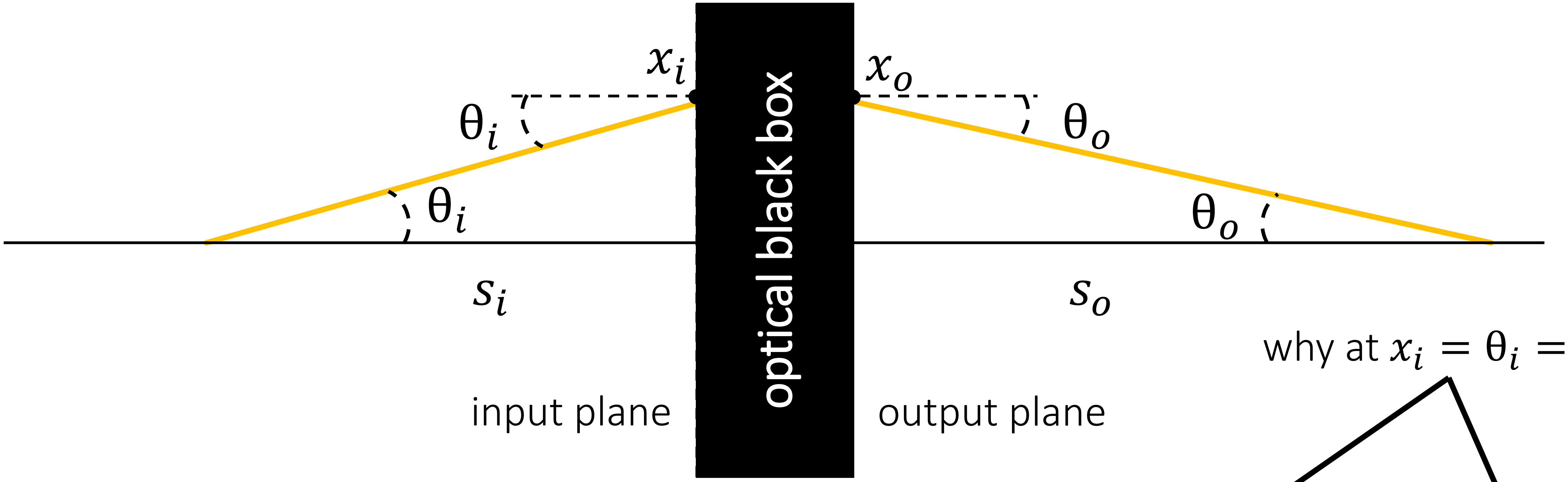
where

$$A = \left.\frac{\partial f}{\partial x_i}\right|_{x_i=\theta_i=0} \qquad B = \left.\frac{\partial f}{\partial \theta_i}\right|_{x_i=\theta_i=0}$$

$$C = ? \qquad D = ?$$

Ray transfer matrix analysis



why at $x_i = \theta_i = 0$?

Under paraxial approximation:

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} f(x_i, \theta_i) \\ g(x_i, \theta_i) \end{bmatrix} \simeq \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\text{definition of ray transfer matrix, a.k.a. ABCD matrix}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

where

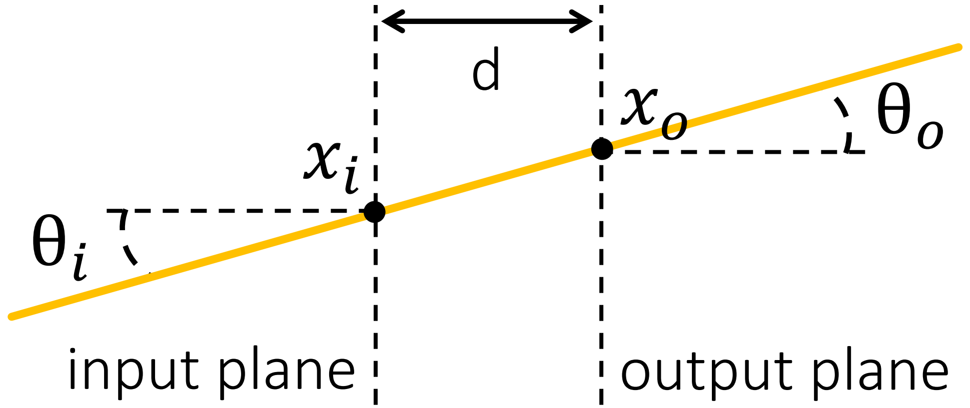
$$A = \left. \frac{\partial f}{\partial x_i} \right|_{x_i = \theta_i = 0} \qquad B = \left. \frac{\partial f}{\partial \theta_i} \right|_{x_i = \theta_i = 0}$$

$$C = \left. \frac{\partial g}{\partial x_i} \right|_{x_i = \theta_i = 0} \qquad D = \left. \frac{\partial g}{\partial \theta_i} \right|_{x_i = \theta_i = 0}$$

definition of <u>ray transfer matrix</u>, a.k.a. <u>ABCD matrix</u>

What is the ABCD matrix of…

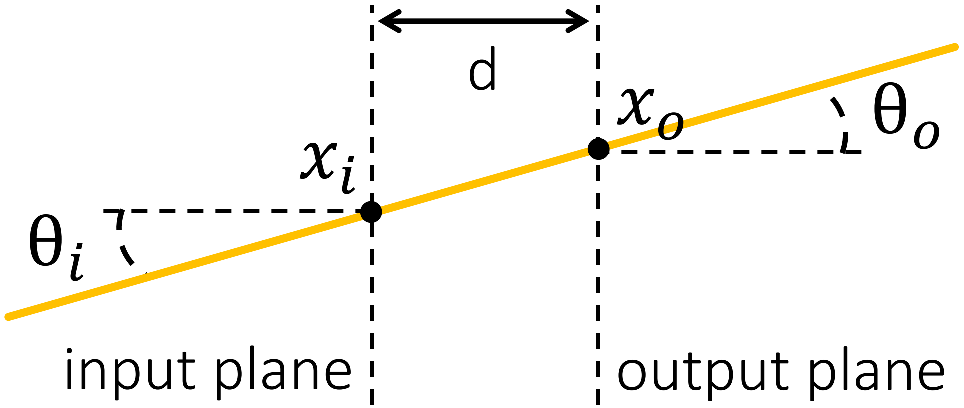
- free space propagation?



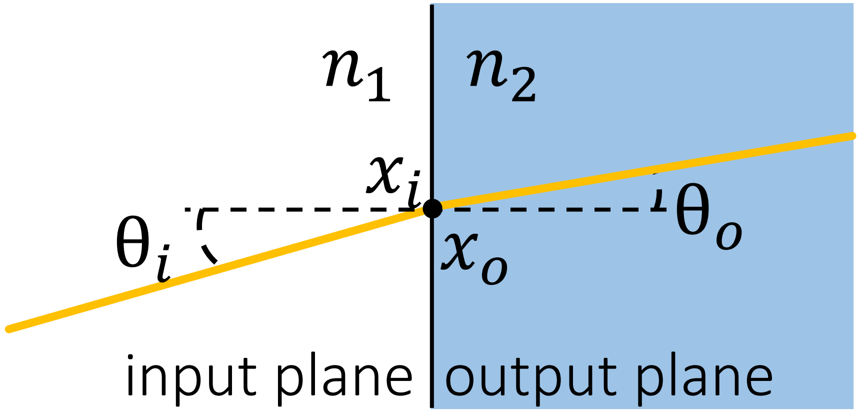
What is the ABCD matrix of…

- free space propagation?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$



- planar refractive interface?



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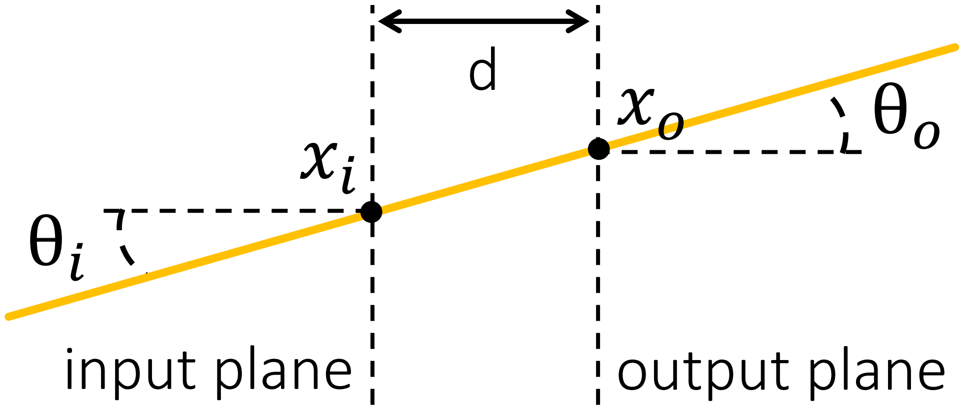
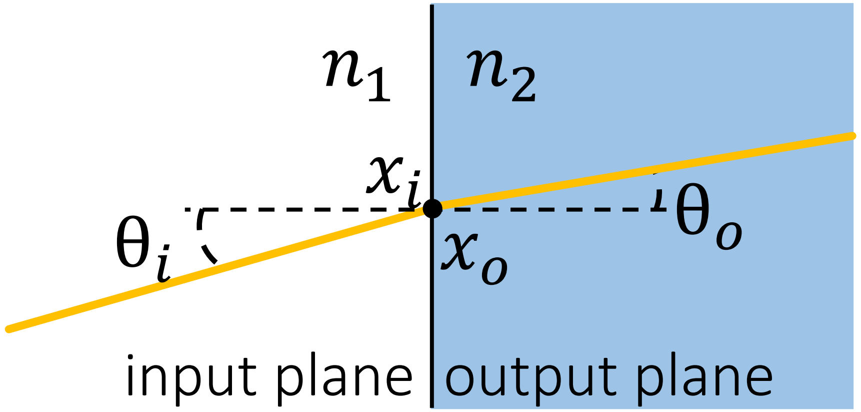
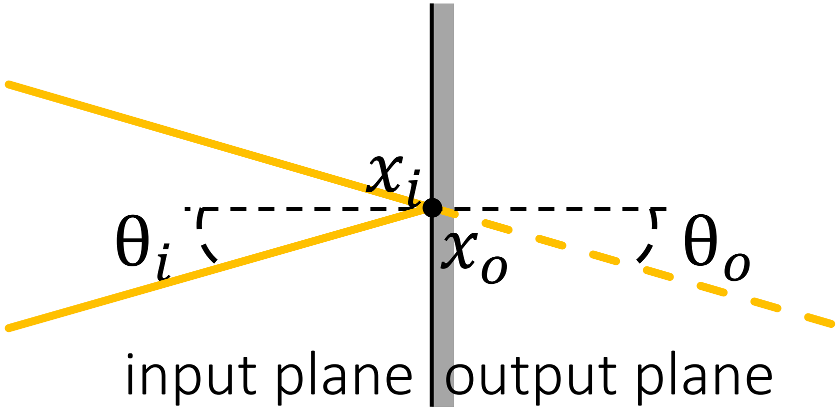


- planar refractive interface?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$



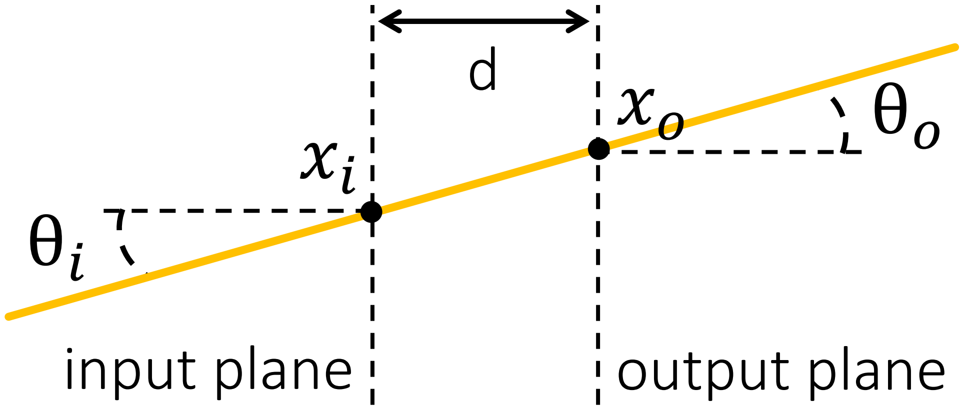
- planar mirror?



What is the ABCD matrix of…

- free space propagation?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$



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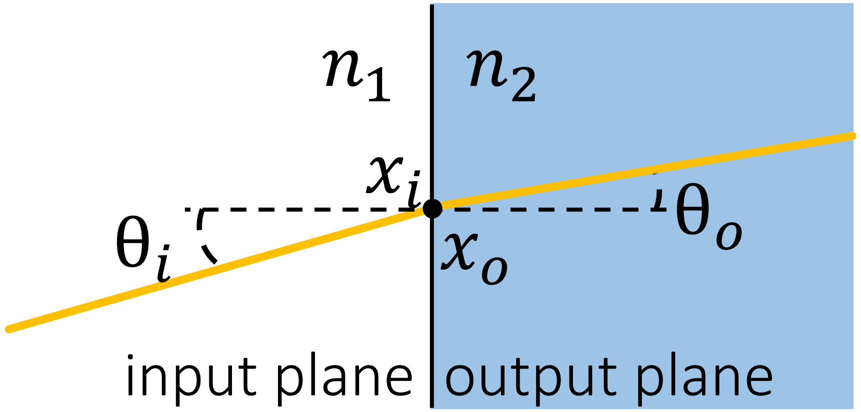

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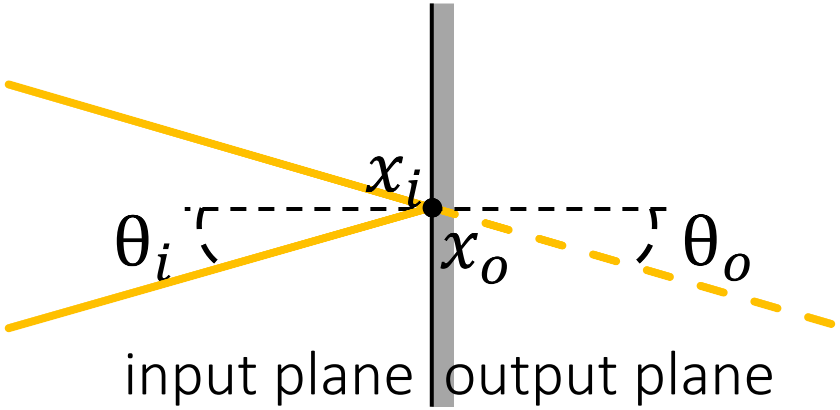
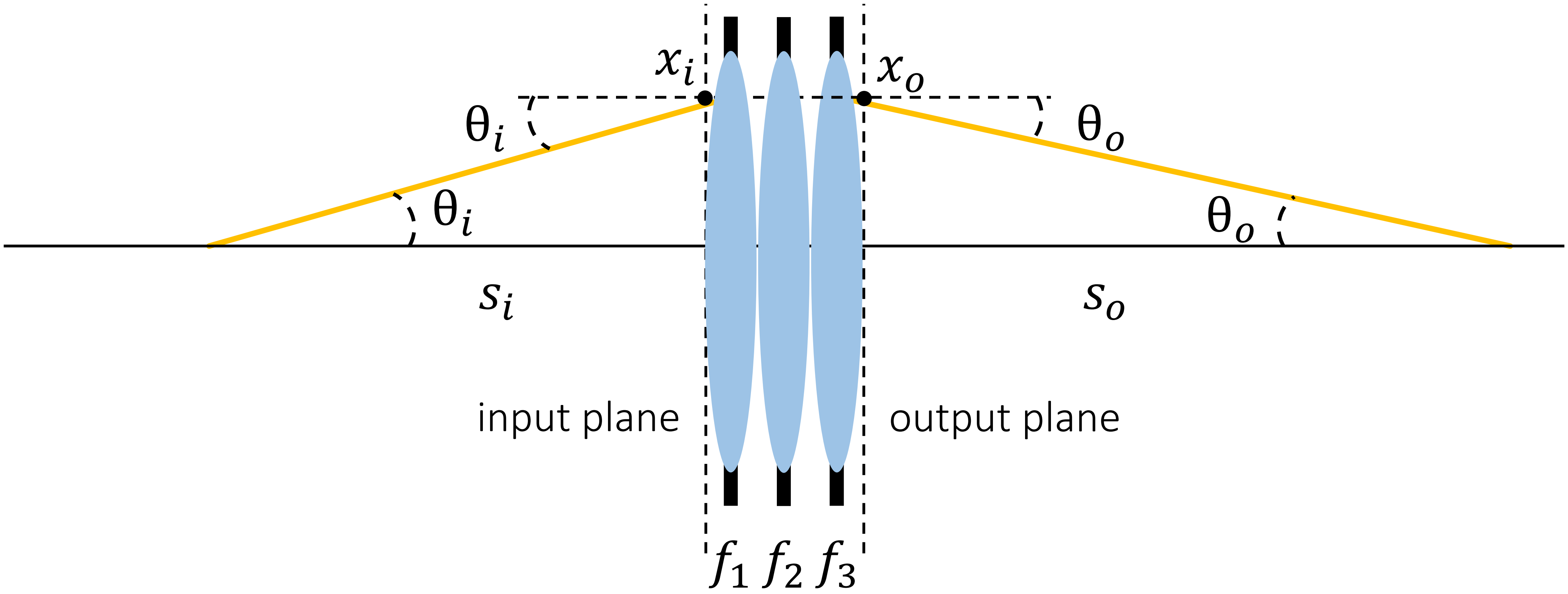


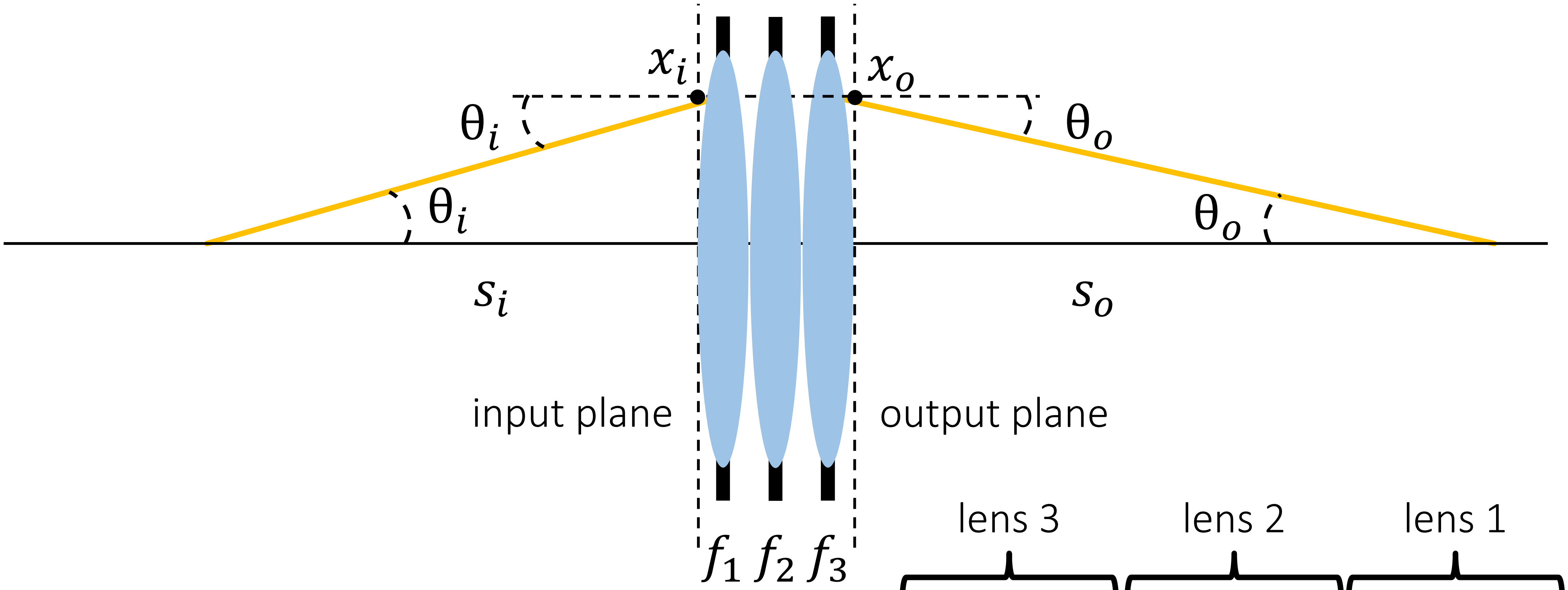
Cascaded optical systems



Let's say we stack together three lenses.

- What is the total ray transfer matrix?

Cascaded optical systems



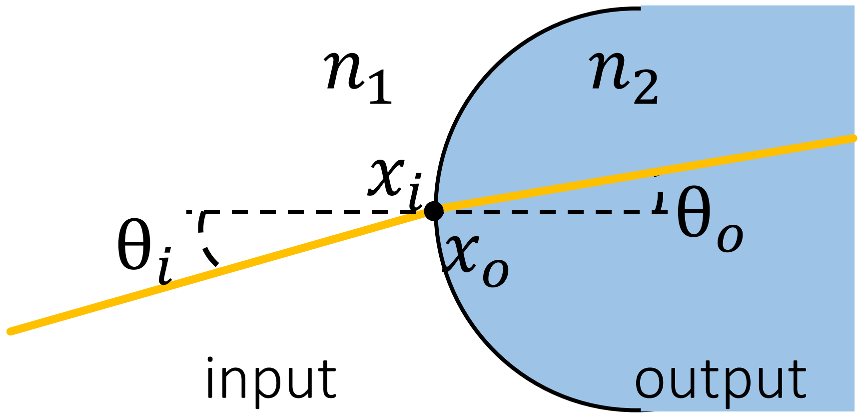
Let's say we stack together three lenses.

- What is the total ray transfer matrix?
- Notice the matrix ordering.

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_3} & 1 \end{bmatrix}}^{\text{lens 3}} \overbrace{\begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_2} & 1 \end{bmatrix}}^{\text{lens 2}} \overbrace{\begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_1} & 1 \end{bmatrix}}^{\text{lens 1}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

What is the ABCD matrix of…

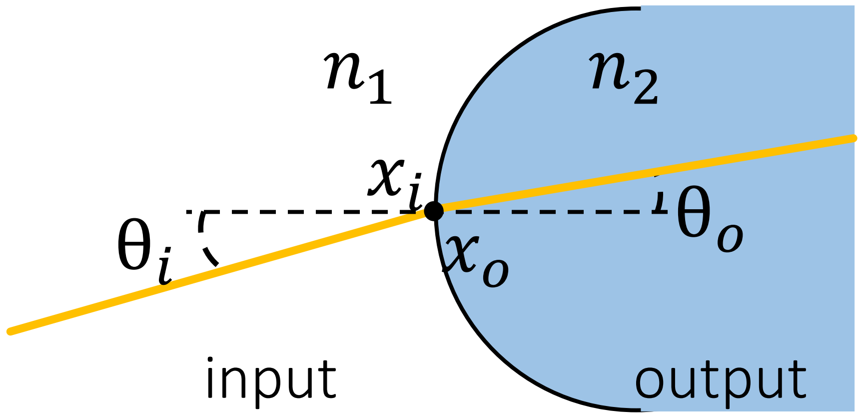
- spherical refractive interface?



What is the ABCD matrix of...

- spherical refractive interface?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right)/R & \frac{n_1}{n_2} \end{bmatrix}$$

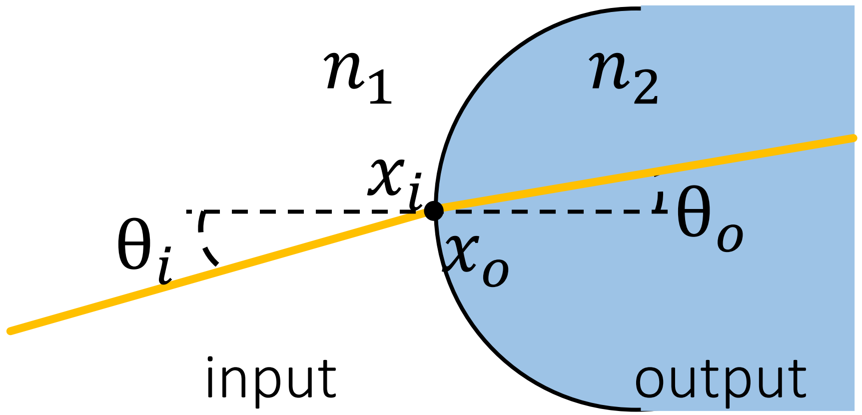


- two spherical interfaces?

What is the ABCD matrix of…

- spherical refractive interface?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right)/R & \frac{n_1}{n_2} \end{bmatrix}$$



- two spherical interfaces?

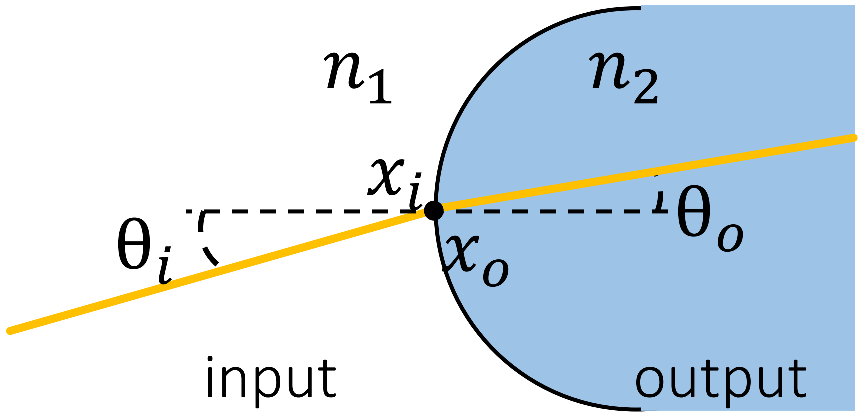
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \left(\frac{n_2}{n_1} - 1\right)/R_2 & \frac{n_2}{n_1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right)/R_1 & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \boxed{\left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} & 1 \end{bmatrix}$$

What is this?

What is the ABCD matrix of…

- spherical refractive interface?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right)/R & \frac{n_1}{n_2} \end{bmatrix}$$



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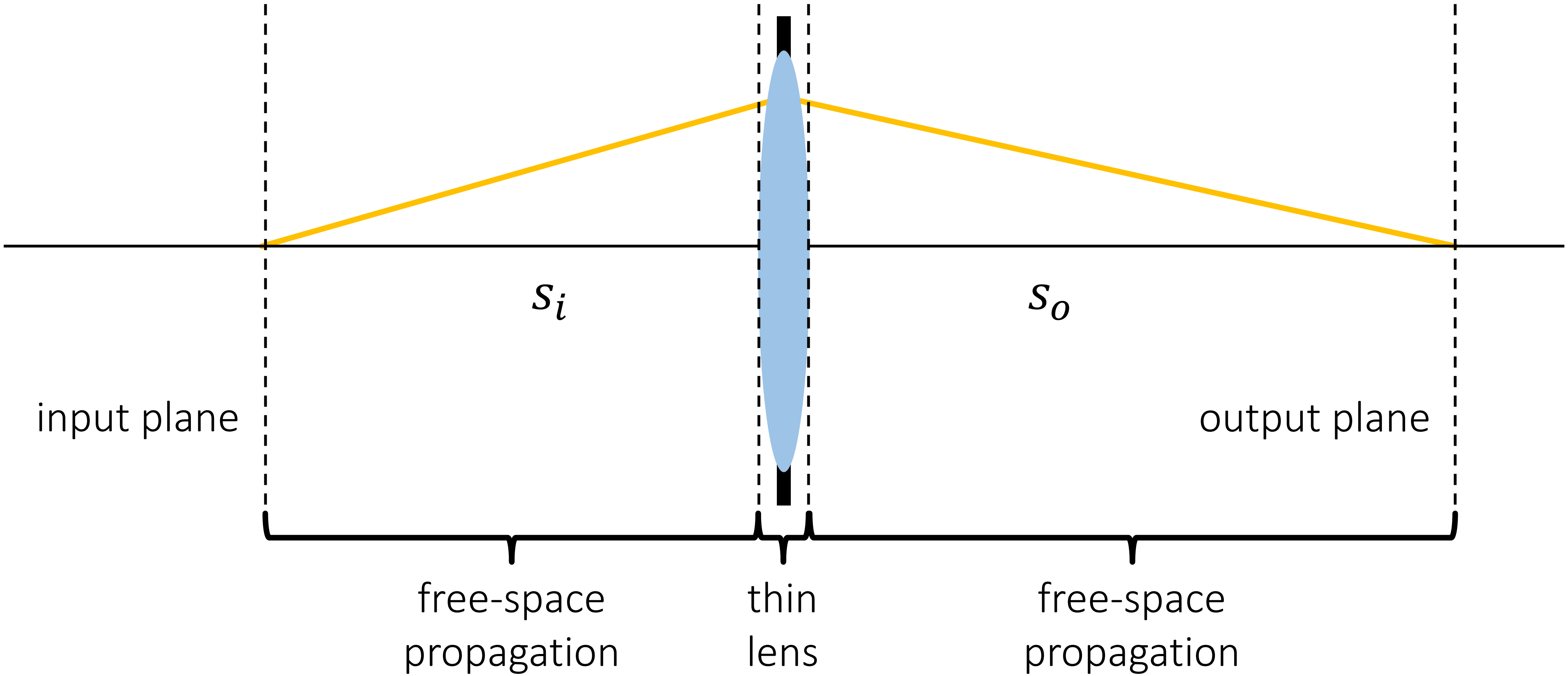
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \left(\frac{n_2}{n_1} - 1\right)/R_2 & \frac{n_2}{n_1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right)/R_1 & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \boxed{\left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} & 1 \end{bmatrix}$$

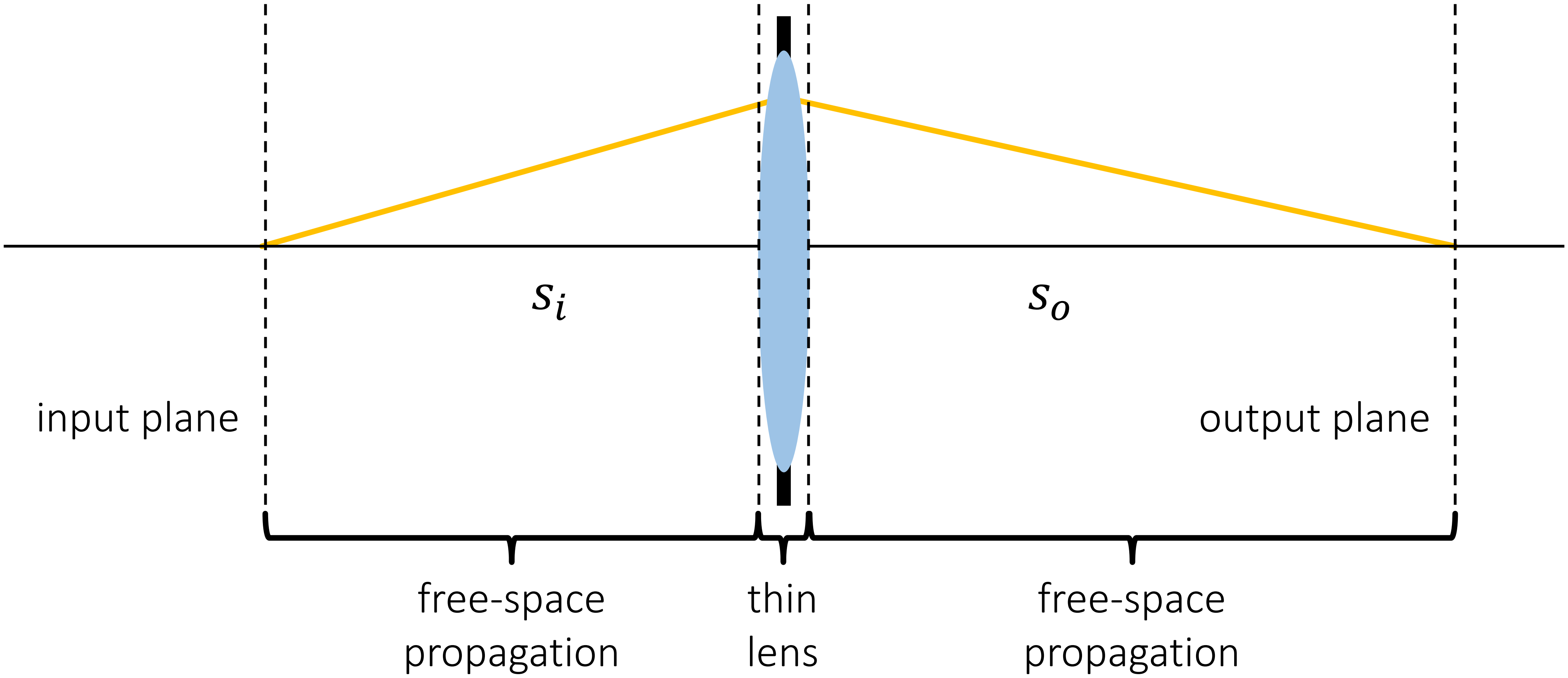
We rederived the lensmaker's equation!

Let's revisit focusing



What are the individual
ray transfer matrices?

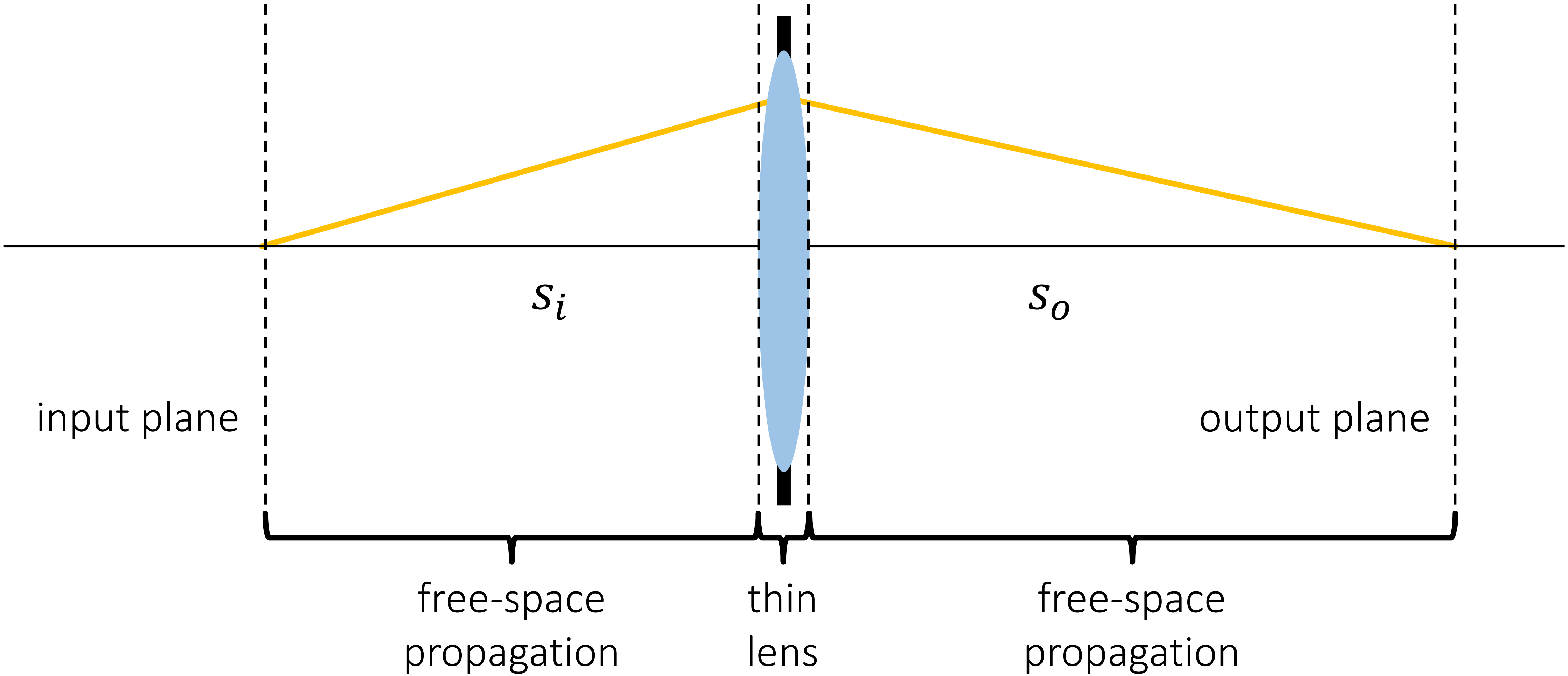
Let’s revisit focusing



What is the total ray transfer matrix?

$$\begin{bmatrix} 1 & s_i \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & s_o \\ 0 & 1 \end{bmatrix}$$

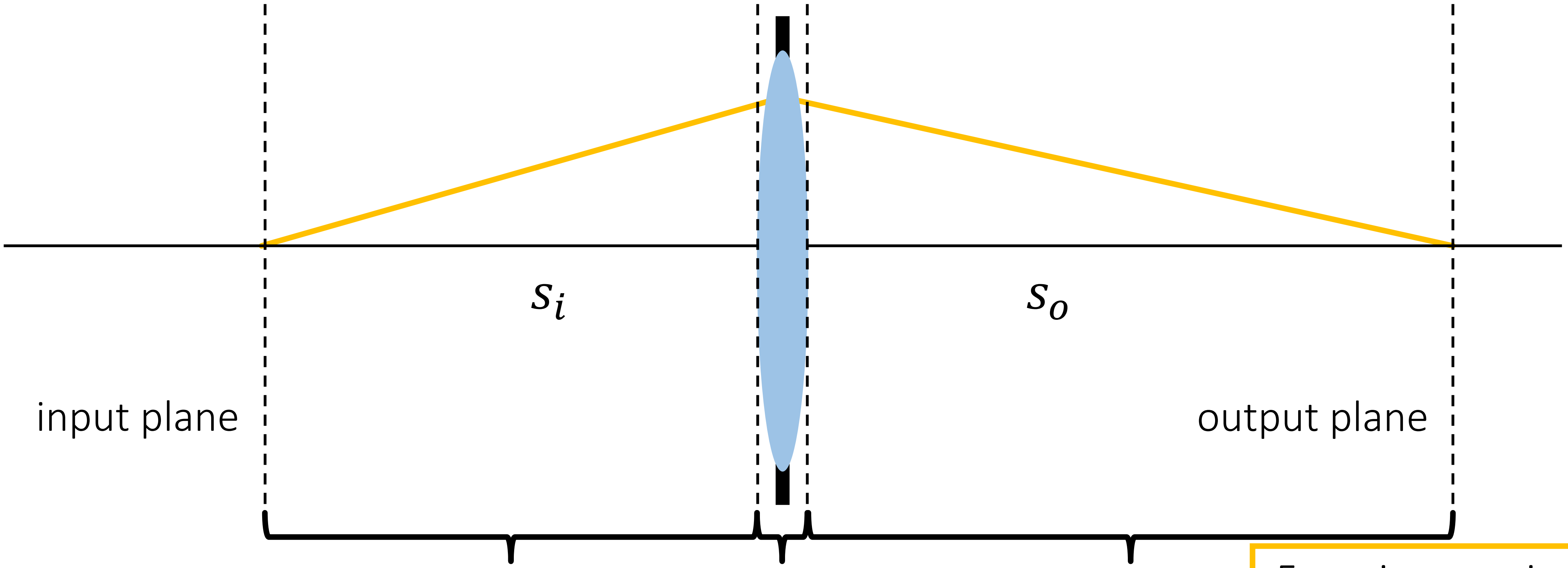
Let’s revisit focusing



What condition enables focusing?

$$\begin{bmatrix} 1 & s_o \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & s_i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - s_o/f & s_i + s_o - (s_i s_o)/f \\ -1/f & 1 - s_i/f \end{bmatrix}$$

Let's revisit focusing

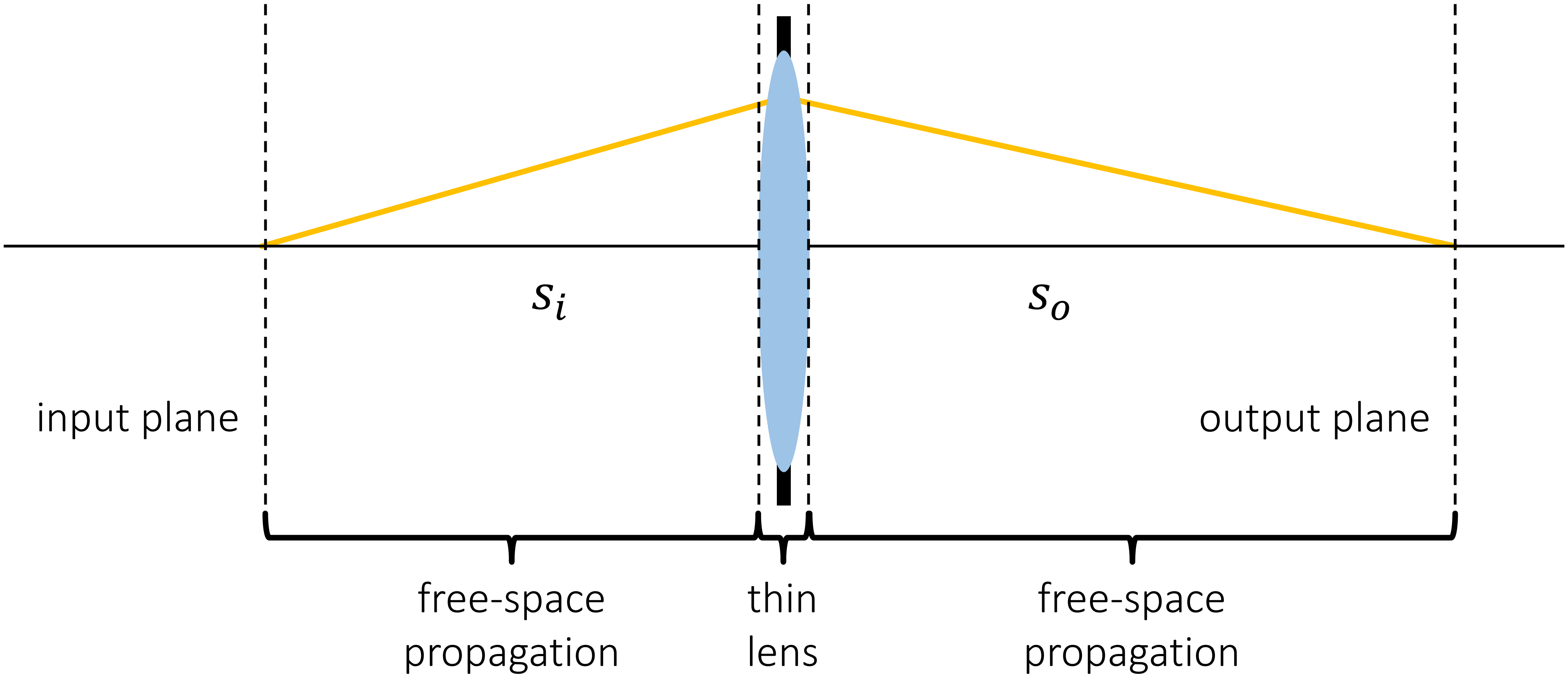


Focusing requires B to be zero. Why?

What condition enables focusing?

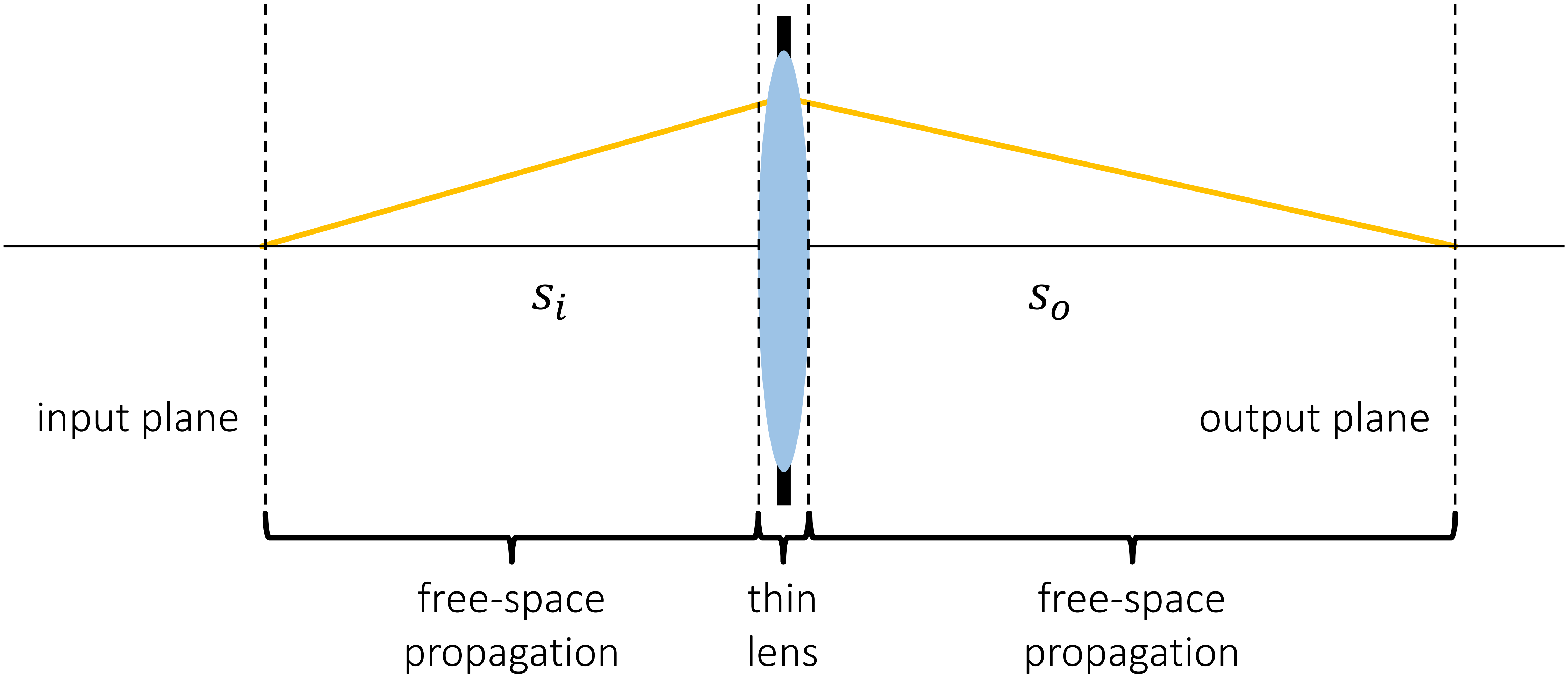
$$\begin{bmatrix} 1 & s_o \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & s_i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - s_o/f & \overbrace{s_i + s_o - (s_i s_o)/f} \\ -1/f & 1 - s_i/f \end{bmatrix}$$

Let's revisit focusing



To have focus: $s_i + s_o - \frac{s_i s_o}{f} = 0 \Rightarrow \frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$ What is this?

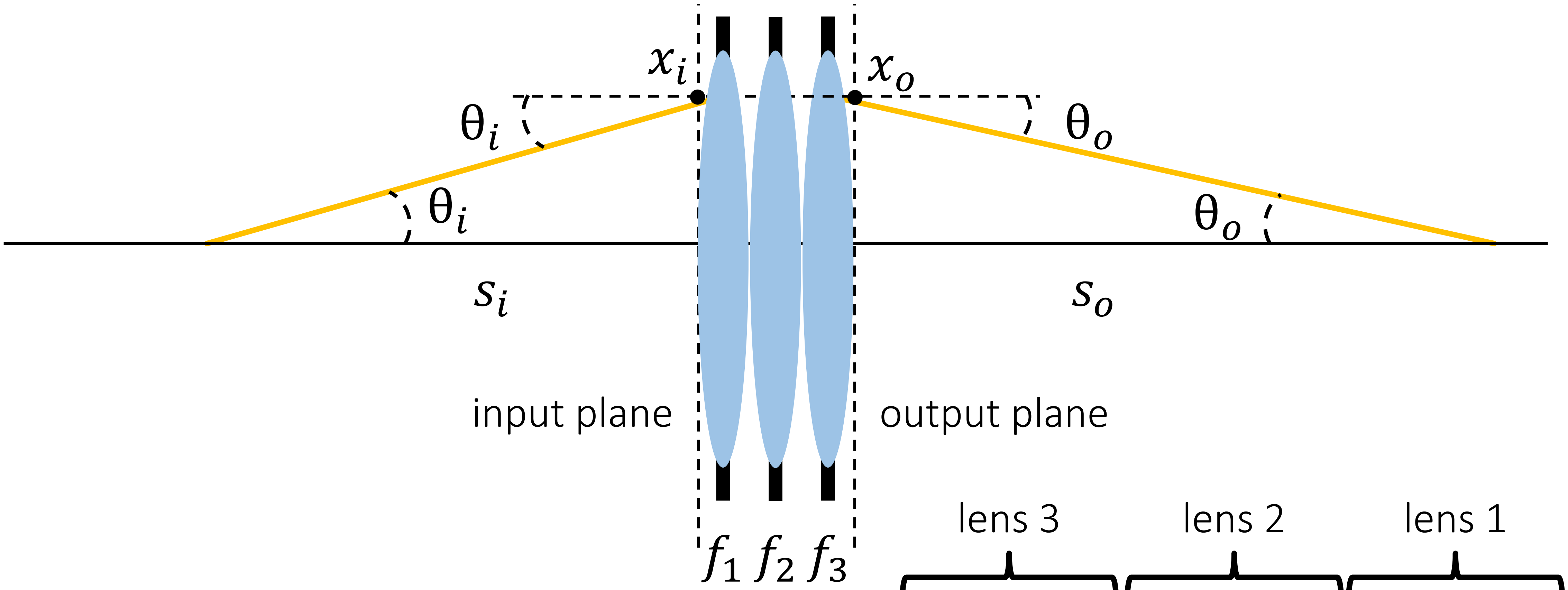
Let’s revisit focusing



To have focus: $$s_i + s_o - \frac{s_i s_o}{f} = 0 \Rightarrow \frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

We rederived the Gaussian lens formula!

Cascaded optical systems

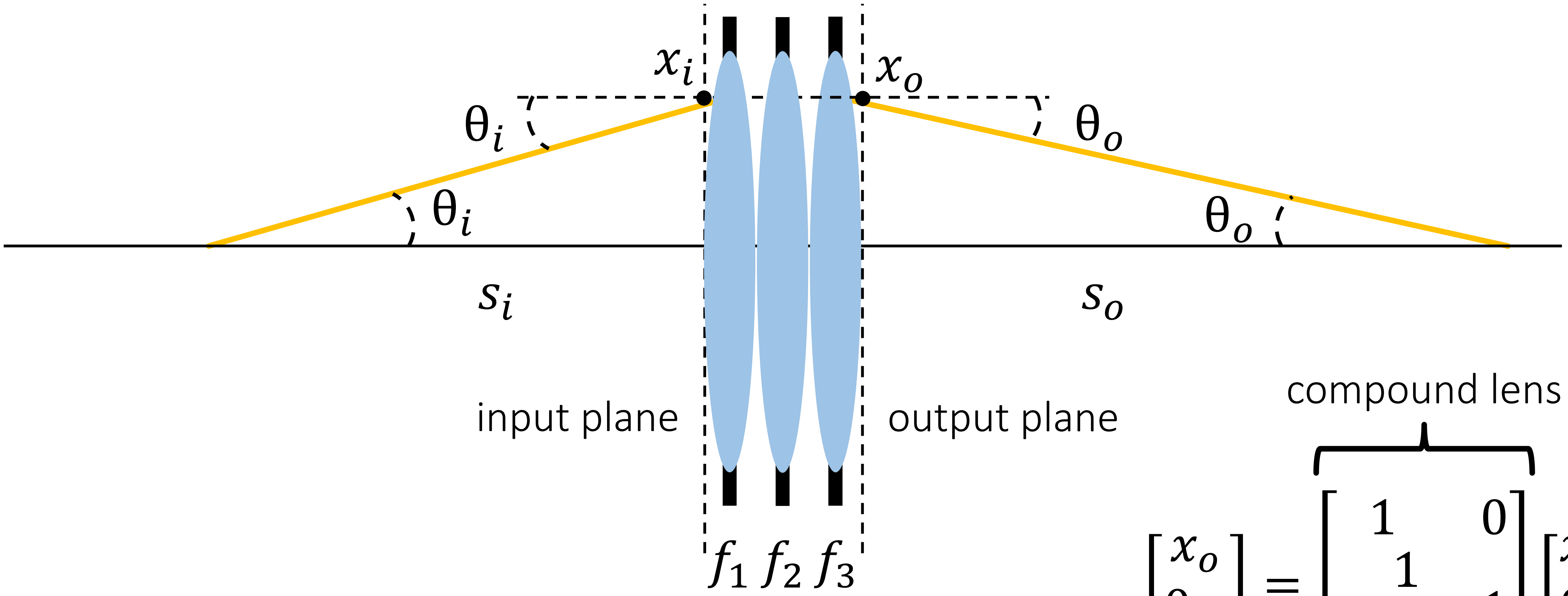


Let's say we stack together three lenses.

- What is the total ray transfer matrix?
- Notice the matrix ordering.

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_3} & 1 \end{bmatrix}}_{\text{lens 3}} \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix}}_{\text{lens 2}} \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}}_{\text{lens 1}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

Cascaded optical systems



compound lens

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_e} & 1 \end{bmatrix}}_{\text{compound lens}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

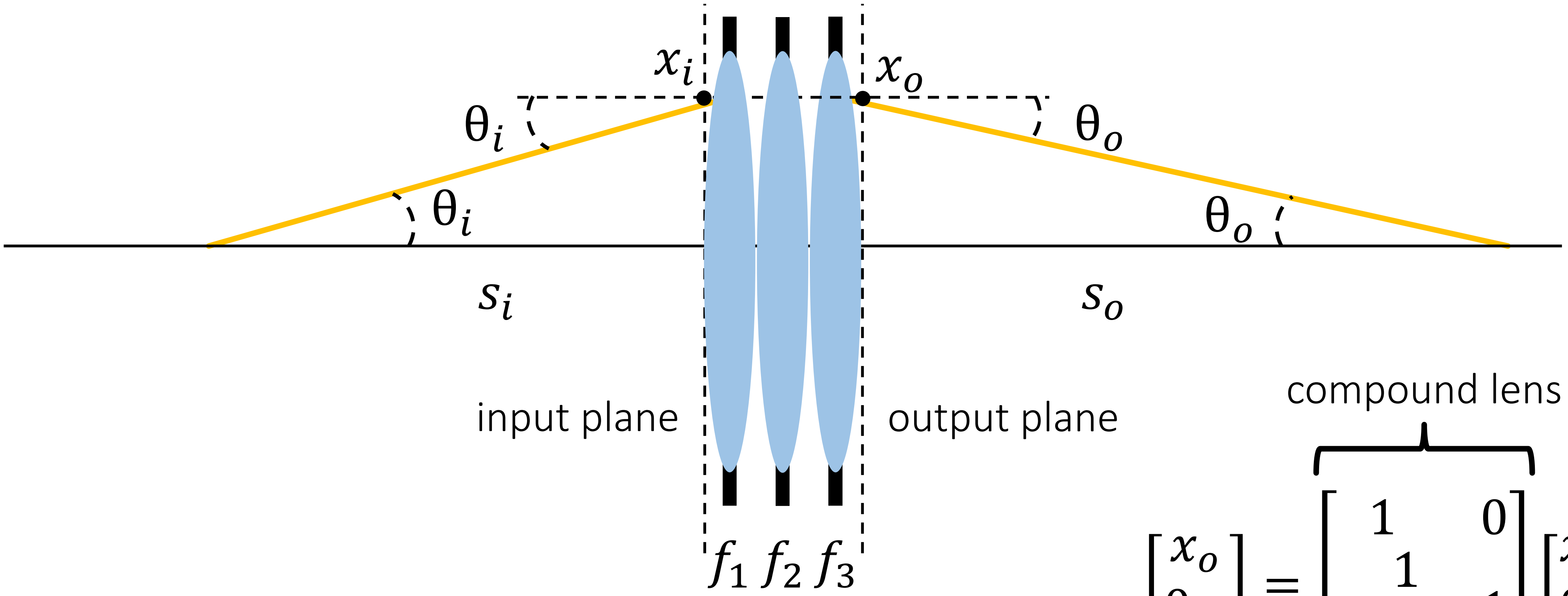
effective focal length

$$f_e = \frac{f_1 f_2 f_3}{f_1 f_2 + f_2 f_3 + f_3 f_1}$$

Let's say we stack together three lenses.

- What is the total ray transfer matrix?
- Notice the matrix ordering.

Cascaded optical systems



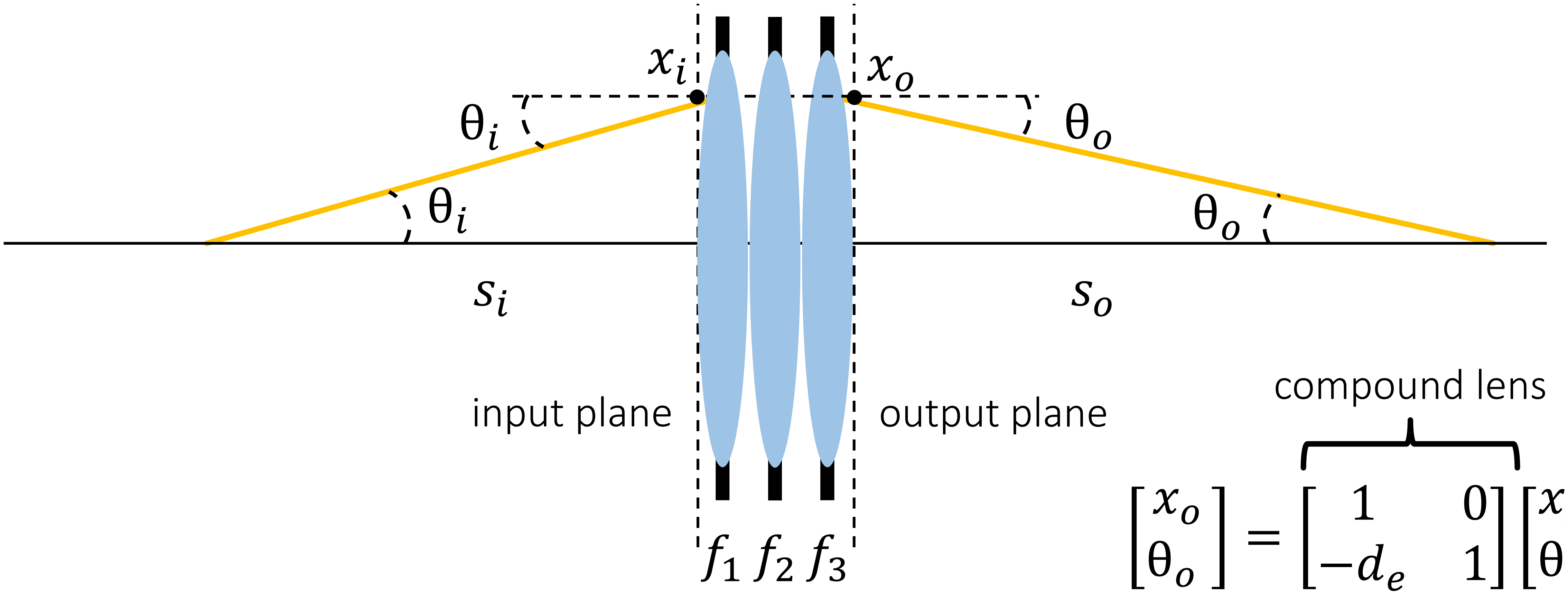
Note: Lens ABCD matrix involves *inverse* focal length.

compound lens

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f_e} & 1 \end{bmatrix}}^{\text{compound lens}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Cascaded optical systems



compound lens

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -d_e & 1 \end{bmatrix}}_{\text{compound lens}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

$$d_e = d_1 + d_2 + d_3$$

Note: Lens ABCD matrix involves *inverse* focal length.

- We call the inverse focal length the *diopter*.
- Often used as a unit of inverse length (e.g., for depth).
- Diopter lengths are additive.

Ray transfer matrix analysis

- Also known as ABCD matrix analysis (from the form of the ray transfer matrix).
- Any optical system, no matter how complicated, can be described by its ray transfer matrix.
- A cascaded optical system has a ray transfer matrix that is the product of the ray transfer matrices of its components.
- All of the above hold assuming paraxial rays, no aberrations, and no diffraction (geometric optics).

Graphics perspective on ray transfer matrix analysis

- How can I use ray transfer matrix analysis to make ray tracing faster?
- How can I use ray transfer matrix analysis to make Monte Carlo rendering faster?

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Principles and Applications of Pencil Tracing

Mikio Shinya Tokiichiro Takahashi
Seiichiro Naito
NTT Electrical Communications Laboratories
3-9-11, Midori-cho, Musashino-shi
Tokyo 180, Japan

Abstract

Pencil tracing, a new approach to ray tracing, is introduced for faster image synthesis with more physical fidelity. The paraxial approximation theory for efficiently tracing a pencil of rays is described and analysis of its errors is conducted to insure the accuracy required for pencil tracing. The paraxial approximation is formulated from a 4×4 matrix (a system matrix) that provides the basis for pencil tracing and a variety of ray tracing techniques, such as beam tracing, ray tracing with cones, ray-object intersection tolerance, and a lighting model for reflection and refraction. In the error analysis, functions that estimate approximation errors and determine a constraint on the spread angle of a pencil are given.

The theory results in the following fast ray tracing algorithms; ray tracing using a system matrix, ray interpolation, and extended 'beam tracing' using a 'generalized perspective transform'. Some experiments are described to show their advantages. A lighting model is also developed to calculate the illuminance for refracted and reflected light.

point, there have been problems such as high computational cost and aliasing. Many attempts have been made to tackle those problems, and some of them have produced good results by tracing a *pencil* [1] (or bundle) of rays, instead of an individual ray. However, as the methods lack sufficient mathematical bases, they are limited to specific applications.

Heckbert proposed a method called 'beam tracing'[2] which works well for reflecting polygonal objects. His method uses a pencil to be traced by introducing affine transformations in an object space. Unfortunately, the method finds only limited applications because of the way in which it approximates refractions. Moreover, since an error estimation method has not been proposed for guaranteeing the image accuracy, the accuracy cannot be controlled.

Amanatides proposed a 'ray tracing with cones' technique for anti-aliasing, fuzzy shadows, and dull reflections[3], where a conic pencil is traced. However, it failed to present a general equation for characterizing the spread-angle change of a conic pencil through an optical system. Such an equation is also required for the calculation of

A Frequency Analysis of Light Transport

Frédo Durand, MIT-CSAIL
Nicolas Holzschuch, ARTIS* GRAVIR/IMAG-INRIA
Cyril Soler, ARTIS* GRAVIR/IMAG-INRIA
Eric Chan, MIT-CSAIL
François X. Sillion, ARTIS* GRAVIR/IMAG-INRIA

Abstract

We present a signal-processing framework for light transport. We study the frequency content of radiance and how it is altered by phenomena such as shading, occlusion, and transport. This extends previous work that considered either spatial or angular dimensions, and it offers a comprehensive treatment of both space and angle.

We show that occlusion, a multiplication in the primal, amounts in the Fourier domain to a convolution by the spectrum of the blocker. Propagation corresponds to a shear in the space-angle frequency domain, while reflection on curved objects performs a different shear along the angular frequency axis. As shown by previous work, reflection is a convolution in the primal and therefore a multiplication in the Fourier domain. Our work shows how the spatial components of lighting are affected by this angular convolution.

Our framework predicts the characteristics of interactions such as caustics and the disappearance of the shadows of small features. Predictions on the frequency content can then be used to control sampling rates for rendering. Other potential applications include precomputed radiance transfer and inverse rendering.

Keywords: Light transport, Fourier analysis, signal processing

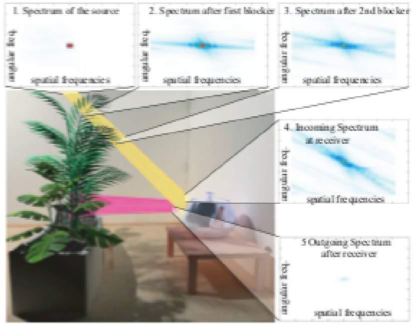


Figure 1: Space-angle frequency spectra of the radiance function measured in a 3D scene. We focus on the neighborhood of a ray path and measure the spectrum of a 4D light field at different steps, which we summarize as 2D plots that include only the radial components of the spatial and angular dimensions. Notice how the blockers result in higher spatial frequency and how transport in free space transfers these spatial frequencies to the angular domain. Aliasing is present in the visualized spectra due to the resolution challenge of manipulating 4D light fields.

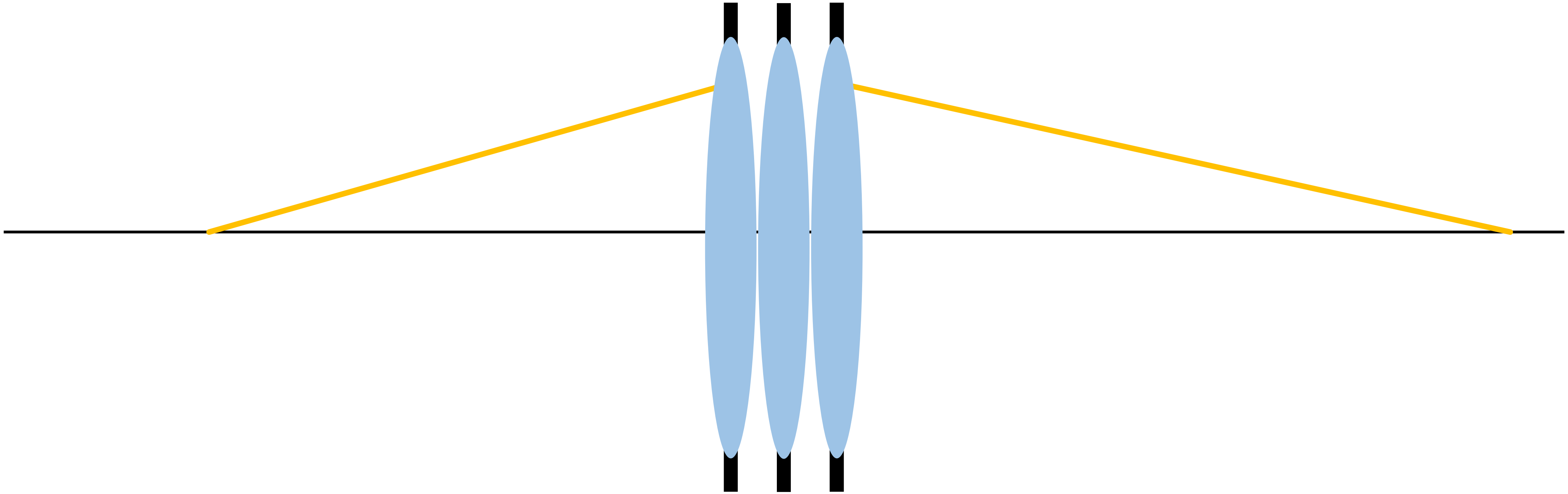
1 Introduction

Light in a scene is transported, occluded, and filtered by its complex interaction with objects. By the time it reaches our eyes, radiance is an intricate function, and simulating or analyzing it is challenging.

Frequency analysis of the radiance function is particularly interesting for many applications, including forward and inverse rendering. The effect of local interactions on the frequency content of radiance has previously been described in a limited context. For instance, it is well-known that diffuse reflection creates smooth (low-frequency) light distributions, while occlusion and hard shadows

This paper presents a theoretical framework for characterizing light transport in terms of frequency content. We seek a deep understanding of the frequency content of the radiance function in a scene and how it is affected by phenomena such as occlusion, reflection, and propagation in space (Fig. 1). We first present the two-dimensional case for simplicity of exposition. Then we show that it extends well to 3D because we only consider local neighborhoods

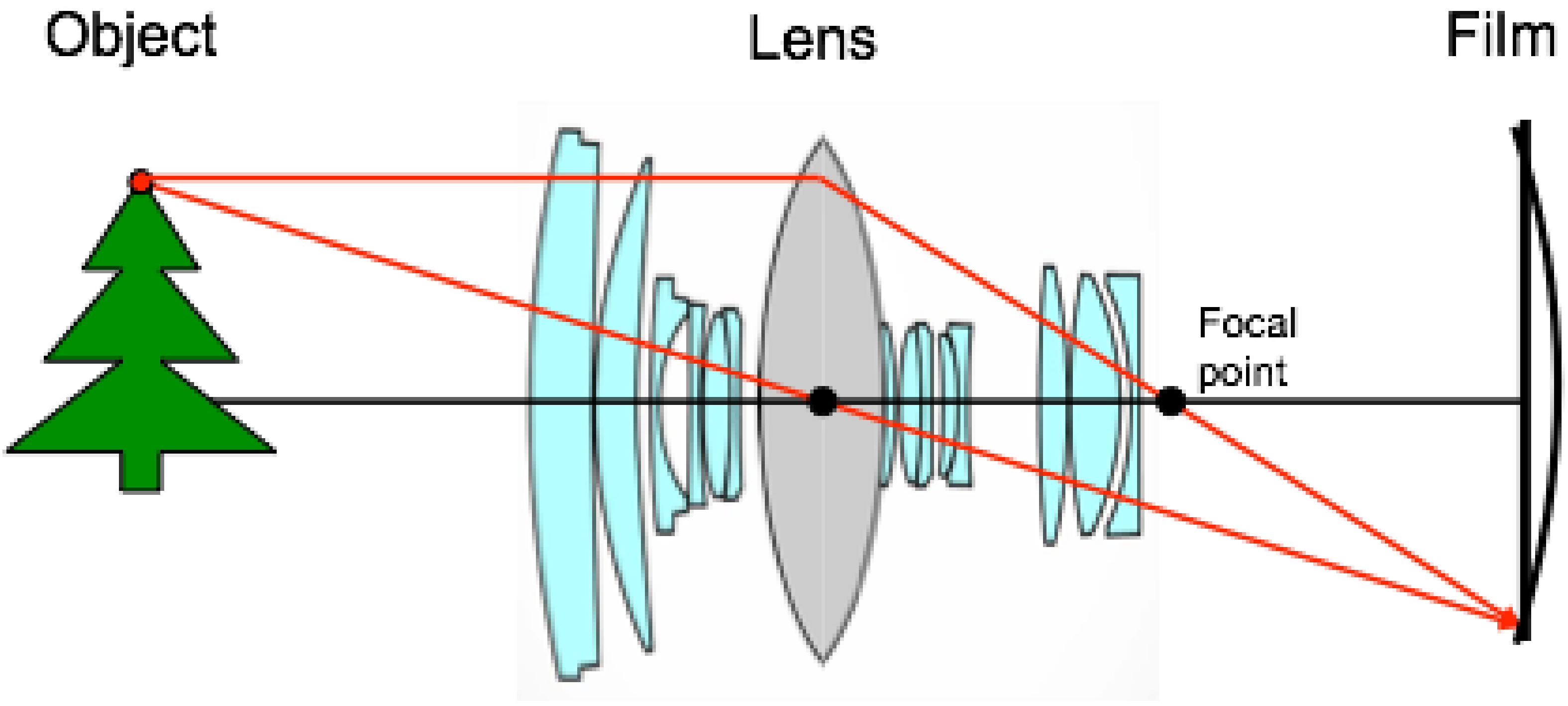
Why would we ever stack together multiple lenses?



Compound lenses and aberrations

Thin lenses are a fiction

The thin lens model assumes that the lens has no thickness, but this is rarely true…



- Even though we have multiple lenses, the entire optical system can be (paraxially) described using a single thin lens of some equivalent focal length and aperture number.
- Where and what exactly this lens is is difficult to determine.

To make real lenses behave like ideal thin lenses, we have to use combinations of multiple lens elements (compound lenses).

Thin lenses are a fiction

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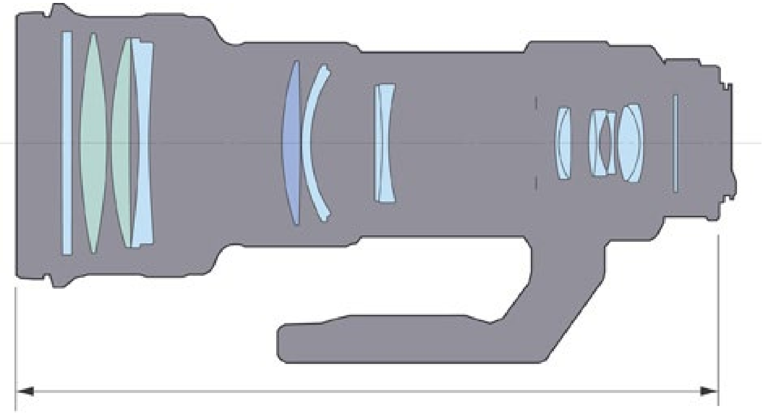

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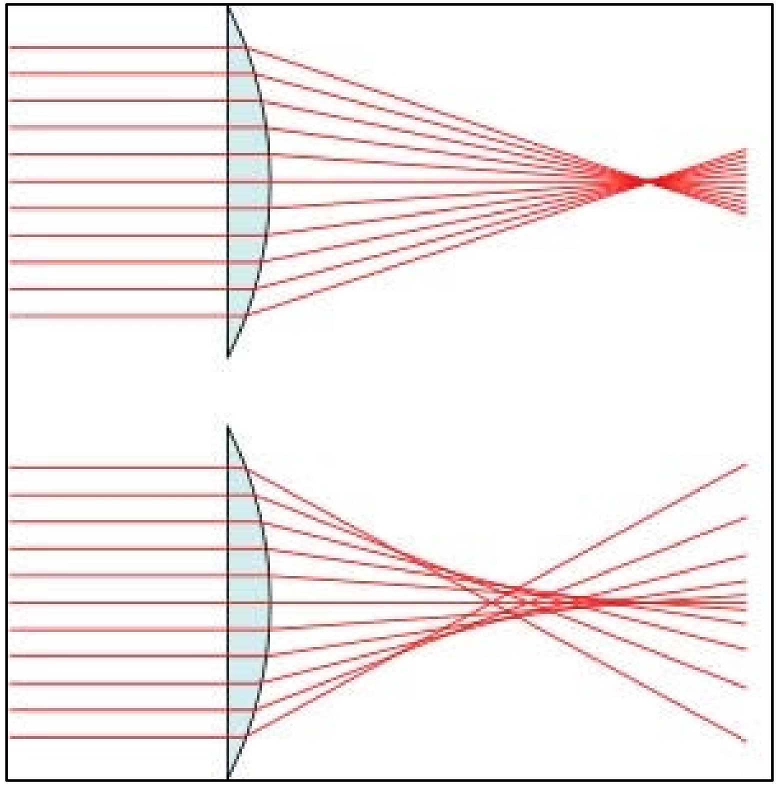
Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

- Example: spherical aberration.

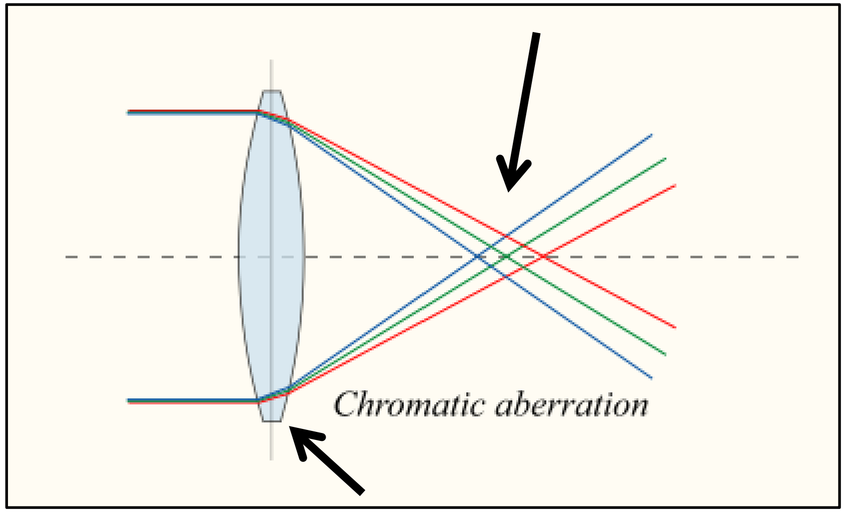


Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

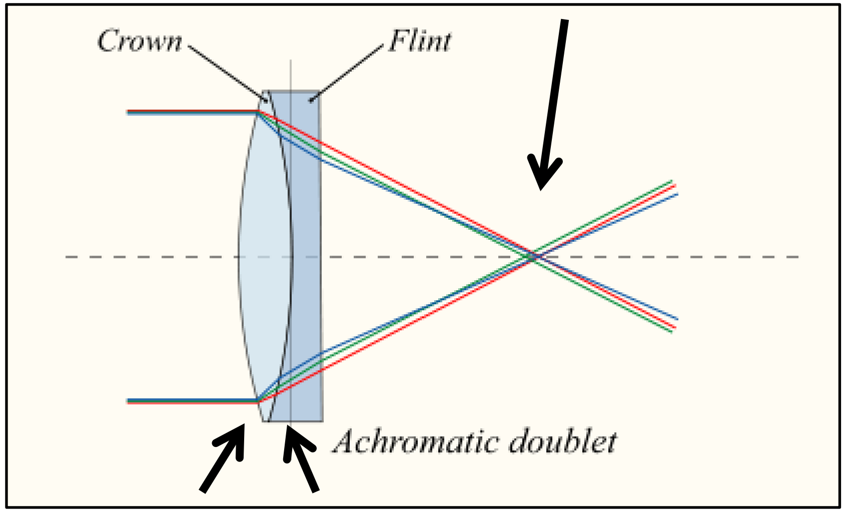
- Example: chromatic aberration.

focal length shifts with wavelength



glass has dispersion (refractive index changes with wavelength)

one lens cancels out dispersion of other



glasses of different refractive index

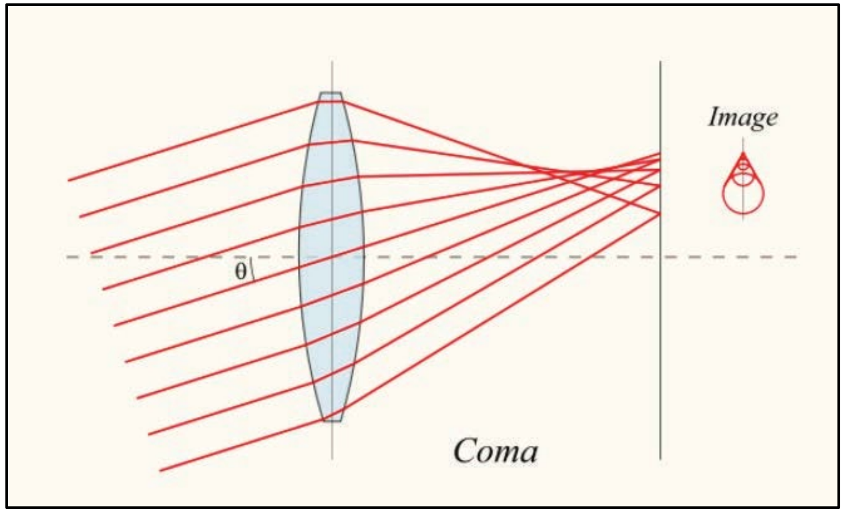
Using a doublet (two-element compound lens), we can reduce chromatic aberration.

Chromatic aberration examples

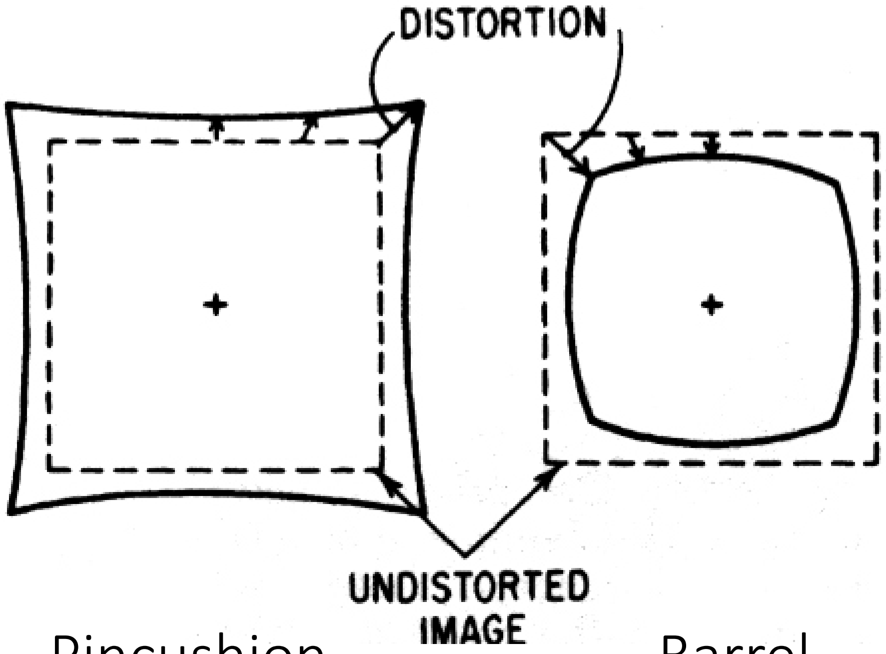
Oblique aberrations

These appear only as we move further from the center of the field of view.

- Contrast with spherical and chromatic, which appear everywhere.
- Many other examples (astigmatism, field curvature, etc.).



Coma



Pincushion Barrel

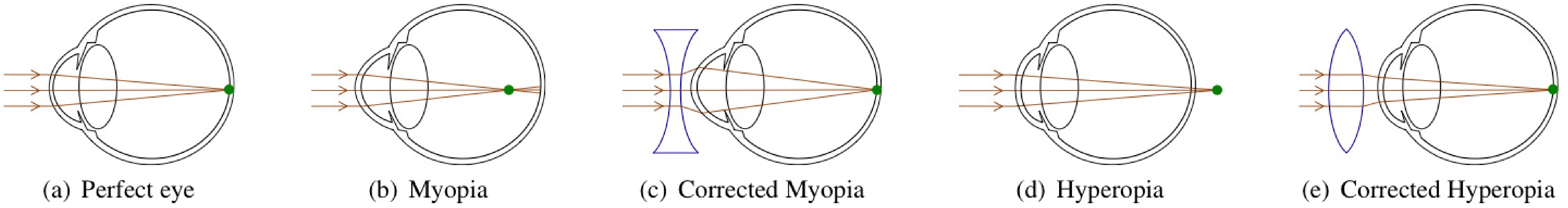
Distortion example

Why do we wear glasses?

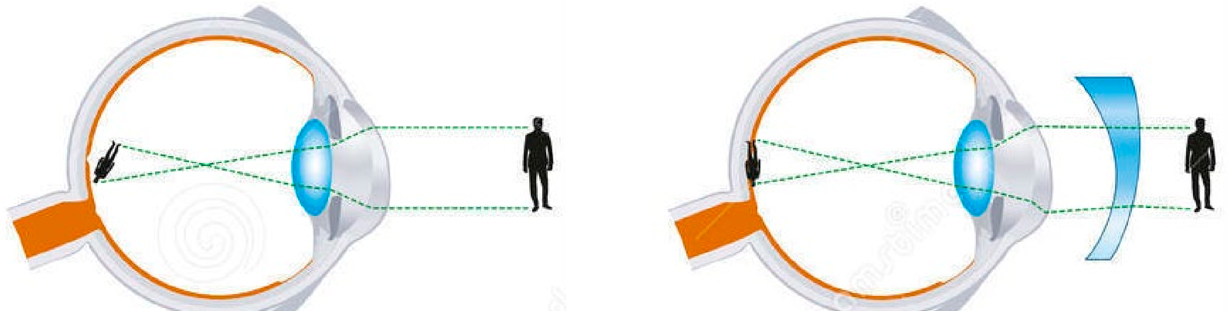
Why do we wear glasses?

We turn our eye into a compound lens to:

- Fix incorrect lens-retina placement.



- Correct lens aberrations.

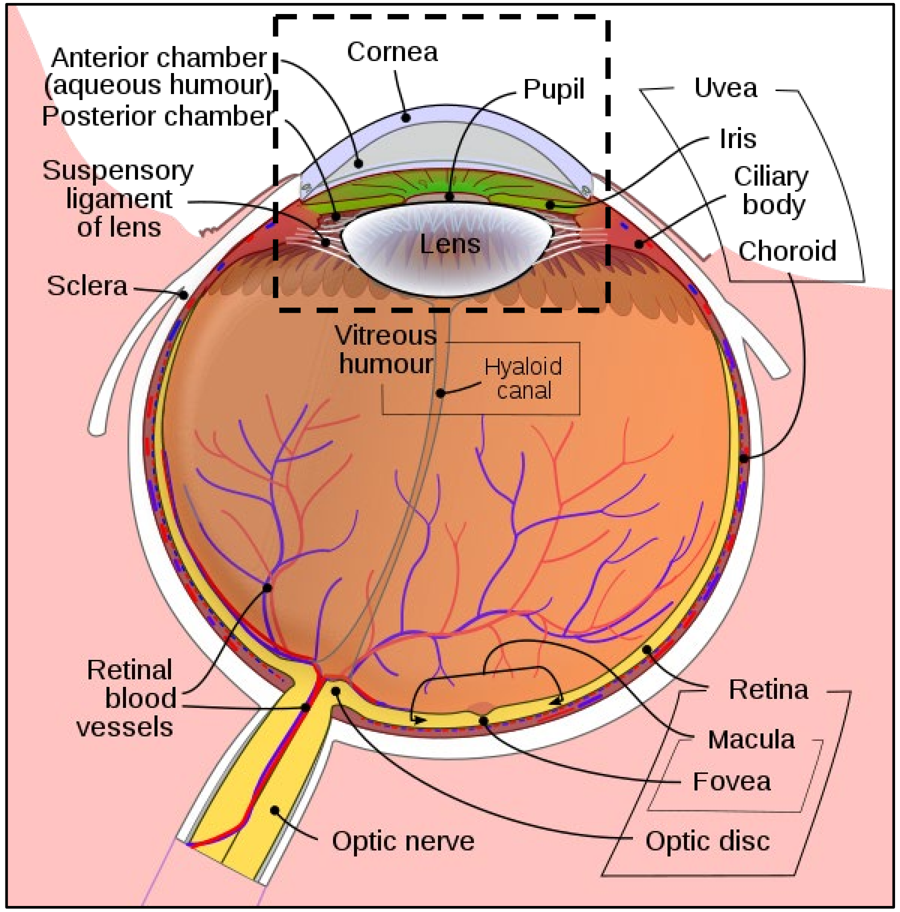


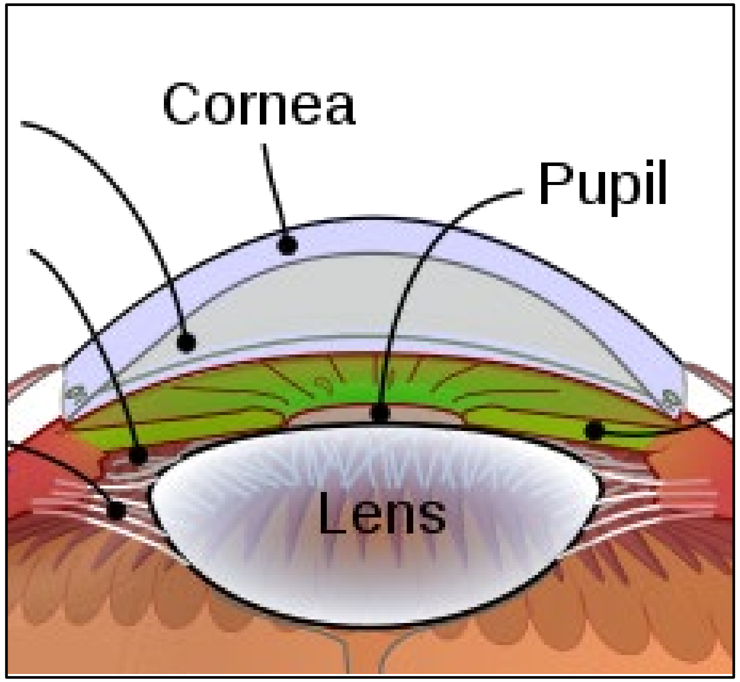
astigmatism

The human eye is already a compound lens

As the human eye is a liquid lens, and water has dispersion, it has chromatic aberration.

- The combined cornea, anterior chamber, and crystalline lens form an achromatic doublet.
- Our brain further reduces *perceived* aberration by “cleverly” processing LMS cone responses.





A costly aberration

Hubble telescope originally suffered from severe spherical aberration.

- COSTAR mission inserted optics to correct the aberration.

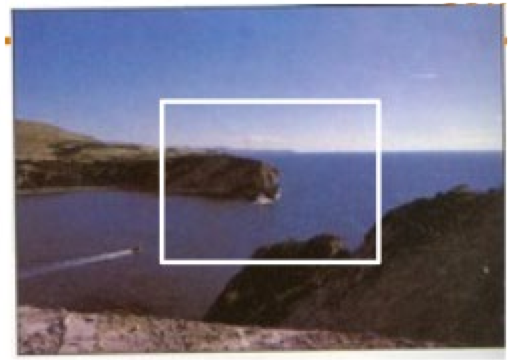
Lens designations

Designation based on field of view

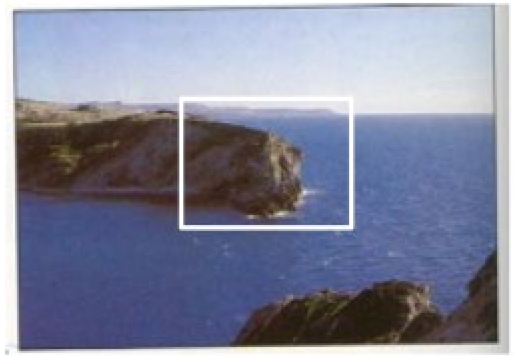
What focal lengths go to what category depends on sensor size.

- Here we assume full frame sensor (same as 35 mm film).
- Even then, there are no well-defined ranges for each category.

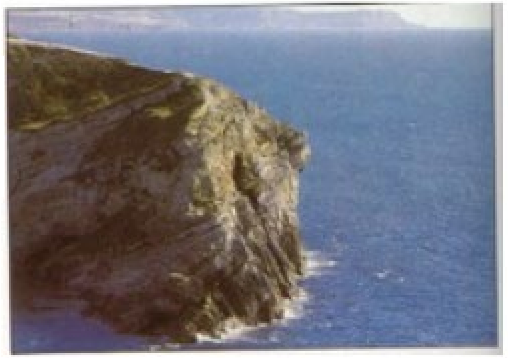
wide-angle f = 25 mm



mid-range f = 50 mm



telephoto f = 135 mm



Wide-angle lenses

Lenses with focal length 35 mm or smaller.

They tend to have large and curvy frontal elements.

Wide-angle lenses

Ultra-wide lenses can get impractically wide…

Fish-eye lens: can produce (near) hemispherical field of view.

Telephoto lenses

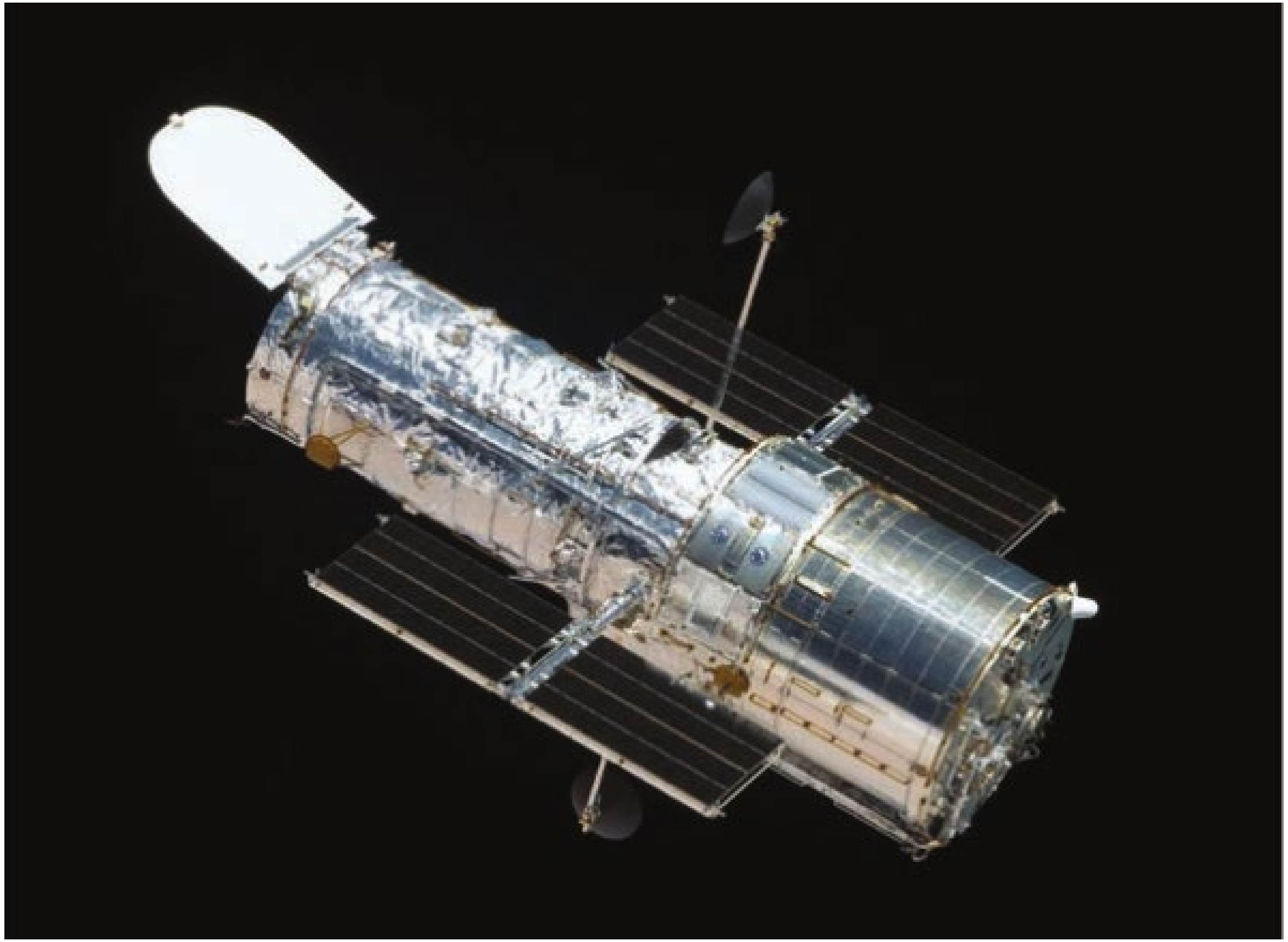
Lenses with focal length 85 mm or larger.

Technically speaking, “telephoto” refers to a specific lens design, not a focal length range. But that design is mostly useful for long focal lengths, so it has also come to mean any lens with such a focal length.

Telephotos can get very big…

Telephoto lenses

- What is this?
- What is its focal length?



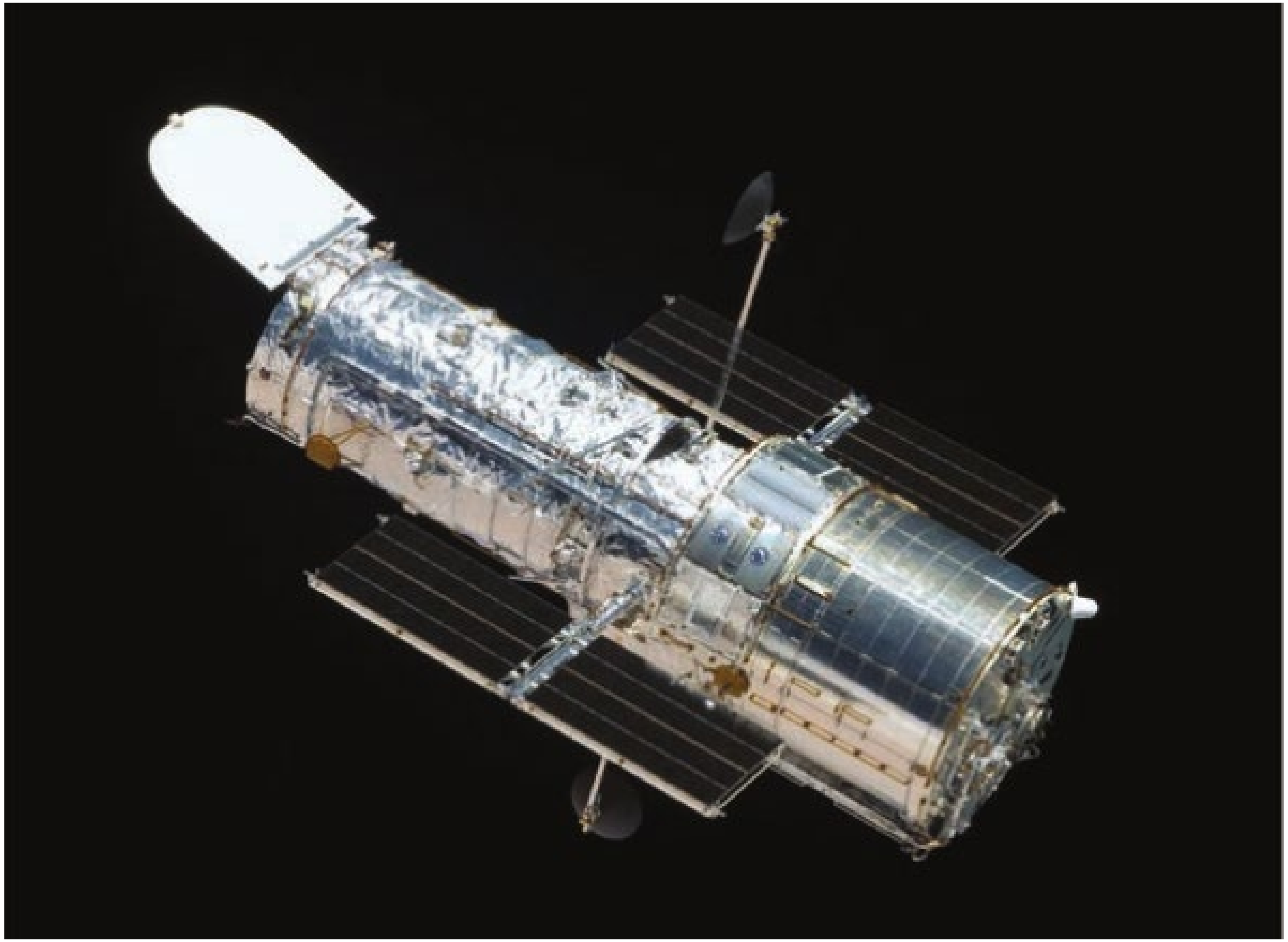
Telephotos can get very big…

Telephoto lenses

- What is this?
- What is its focal length?

About 57 meters.

Telephotos can get very big…



Prime vs zoom lenses

focus ring:
changes focus
distance

single focal
length

Prime lens: fixed focal length

available focal
length range

focus ring:
changes focus
distance

zoom ring:
changes focal
length

Zoom lens: variable focal length

Why use prime lenses and not always use the more versatile zoom lenses?

Prime vs zoom lenses

focus ring: changes focus distance

single focal length

available focal length range

focus ring: changes focus distance

zoom ring: changes focal length

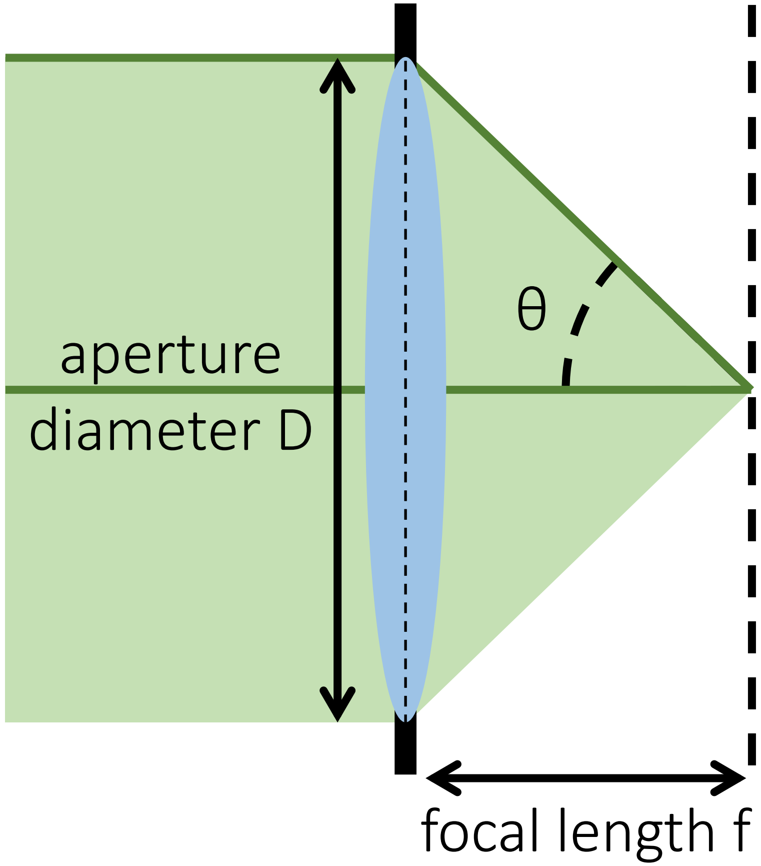
Prime lens: fixed focal length

Zoom lens: variable focal length

Why use prime lenses and not always use the more versatile zoom lenses?

- Zoom lenses have larger aberrations due to the need to cover multiple focal lengths.

Numerical aperture and f-number

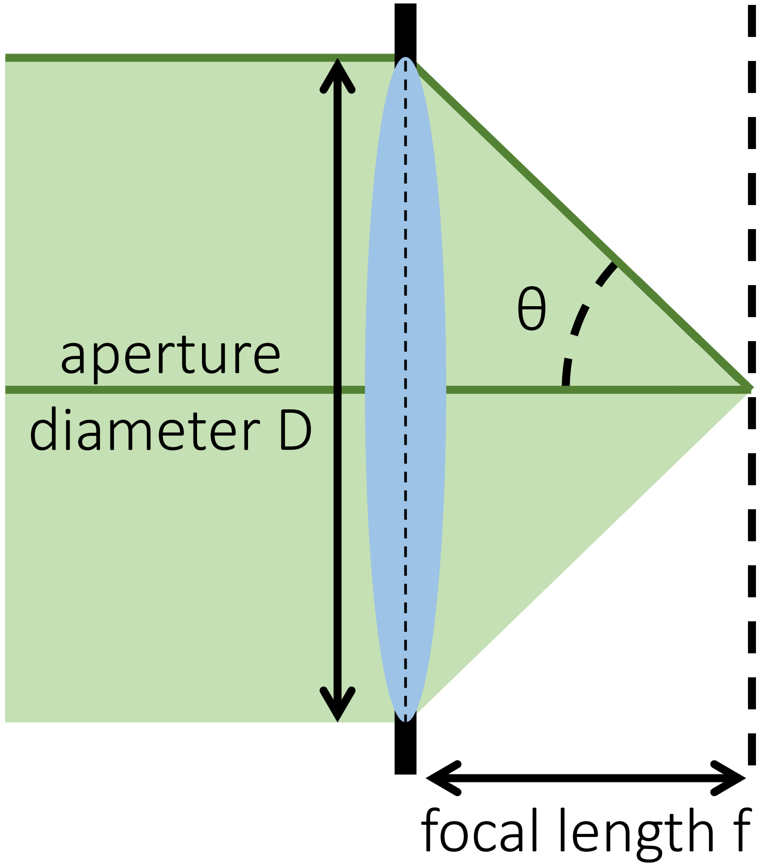


Numerical aperture (NA): sine of half-angle of entering light cone.

- Varies with focus settings, we consider NA at infinity focus.
- A larger NA means a larger aperture.

$$\mathrm{NA} \equiv \sin\theta$$

Numerical aperture and f-number



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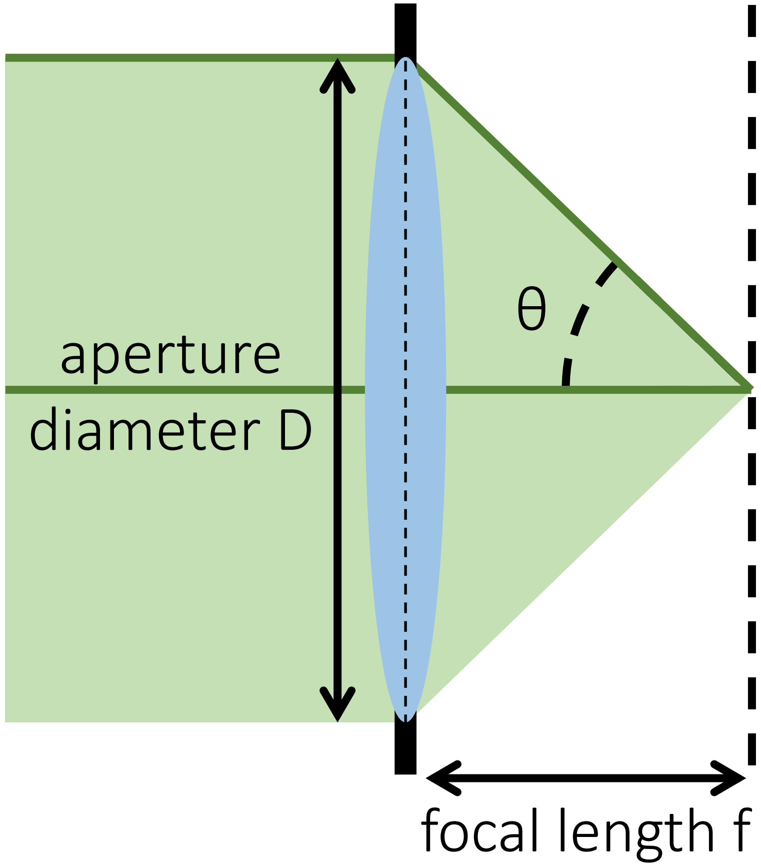
F-number (f/): ratio of focal length and aperture diameter.

- Independent of focus setting (at least for ideal lenses).
- A larger f/ means a smaller aperture.

$$\mathrm{f}/ \equiv \frac{f}{D}$$

How are the two related under paraxial approximation?

Numerical aperture and f-number



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How are the two related under paraxial approximation?

$$\mathrm{NA} = \sin\theta \approx \tan\theta = \frac{D}{2f} = \frac{1}{2\mathrm{f/}}$$

Aperture size

Most lenses have variable aperture size.

- F-number notation: “f/1.4” means $f/=1.4$.
- Usually aperture sizes available at steps of one-half or one-third stops.
- Older lenses have separate manual aperture ring.
- Modern lenses control the aperture through a dial on the camera body (“gelded” lenses).

f/1.4 f/2.8 f/4 f/8 f/16

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f/1.4 f/2.8 f/4 f/8 f/16

Reminder: A “stop” is a change in camera settings that changes amount of light by a factor of 2.

- If the current aperture is at f/4, what is the f-number one stop up and one stop down?

Lens speed

- A *fast* lens is one that has a large *maximum aperture*, or a small *minimum f-number*.
- The "speed" of a lens is its minimum f-number.

lens speed

Why does this zoom lens has more than one lens speeds?

Lens speed

- A *fast* lens is one that has a large *maximum aperture*, or a small *minimum f-number*.
- The "speed" of a lens is its minimum f-number.

lens speed

Why does this zoom lens has more than one lens speeds?

- The max aperture size varies as the focal length (zoom) varies.

Fastest possible lenses

What is the speed of the fastest possible lens?

Fastest possible lenses

What is the speed of the fastest possible lens?

- From paraxial approximation, fastest lens is f/0.5.
- In consumer photography, fastest lenses are f/0.9 – f/0.95.

Fast lenses tend to be bulky and expensive.

Leica Noctilux 50mm f/0.95
(price tag: > $10,000)

Fastest lens ever made?

Zeiss 50 mm f / 0.7 Planar lens

- Originally developed for NASA’s Apollo missions.
- Stanley Kubrick somehow got to use the lens to shoot Barry Lyndon under only candlelight.

Fastest lens ever made?

Zeiss 50 mm f / 0.7 Planar lens

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- Stanley Kubrick somehow got to use the lens to shoot Barry Lyndon under only candlelight.

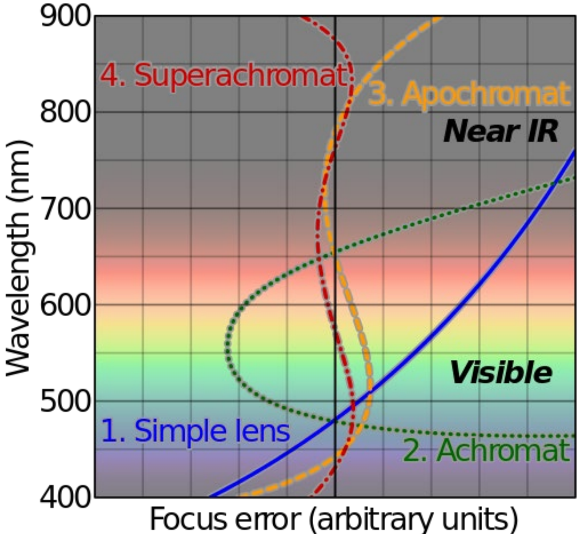
Other kinds of lens designations

Macro lens: can achieve very large magnifications (typically at least 1:1).

- Lens body allows effective lens plane to be placed far away from sensor.
- Macro photography: extremely close-up photography.

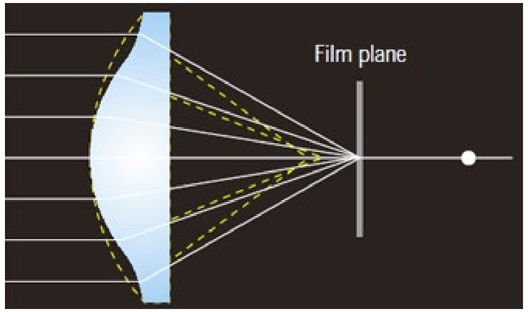
Achromatic or apochromatic lens: corrected for chromatic aberration.

- Achromatic: two wavelengths have same focus.
- Apochromatic (better): three wavelengths have same focus.
- Often done by inserting elements made from low-dispersion glass.
- Expensive.



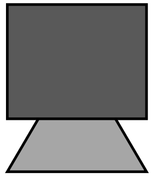
Aspherical lens: manufactured to have special (non-spherical) shape that reduces aberrations.

- Expensive, often only 1-2 elements in a compound lens are aspherical.

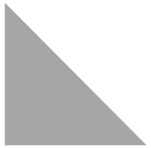


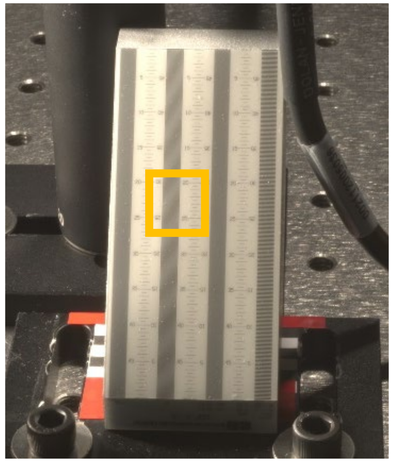
Other kinds of lens designations

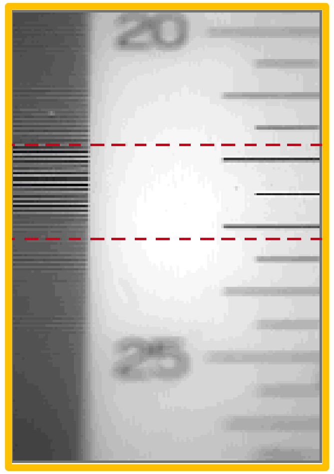
hyperspectral camera



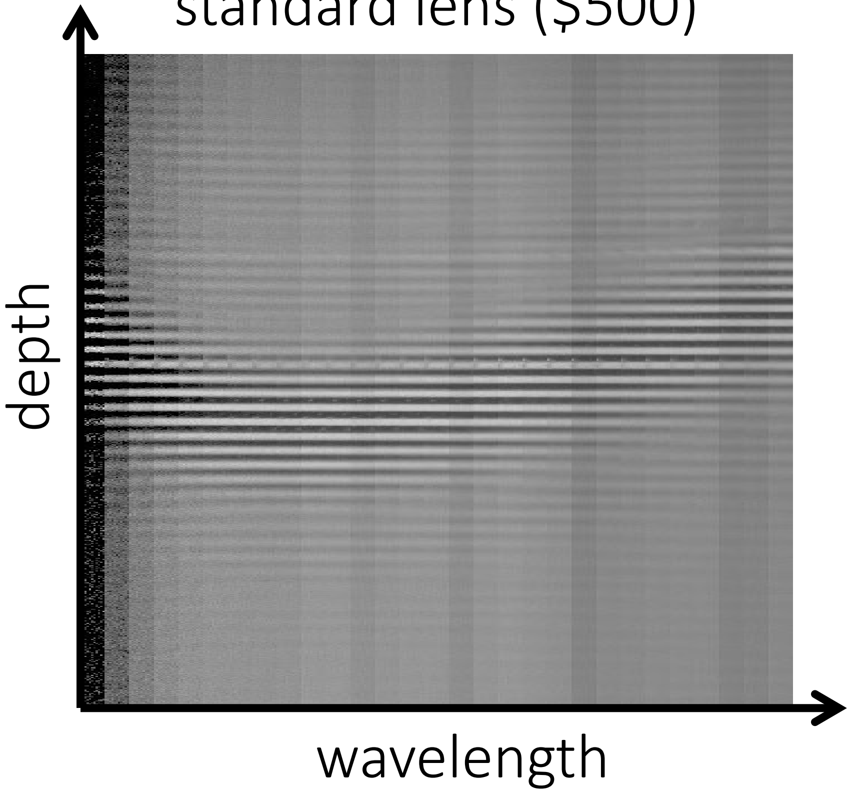
depth-of-field target



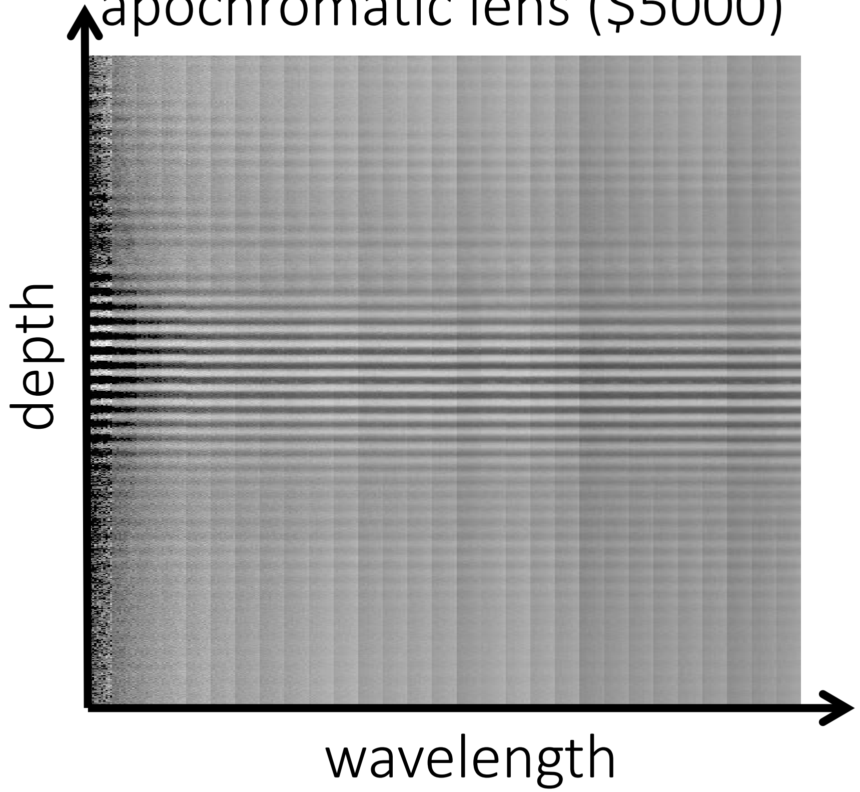




standard lens ($500)



apochromatic lens ($5000)

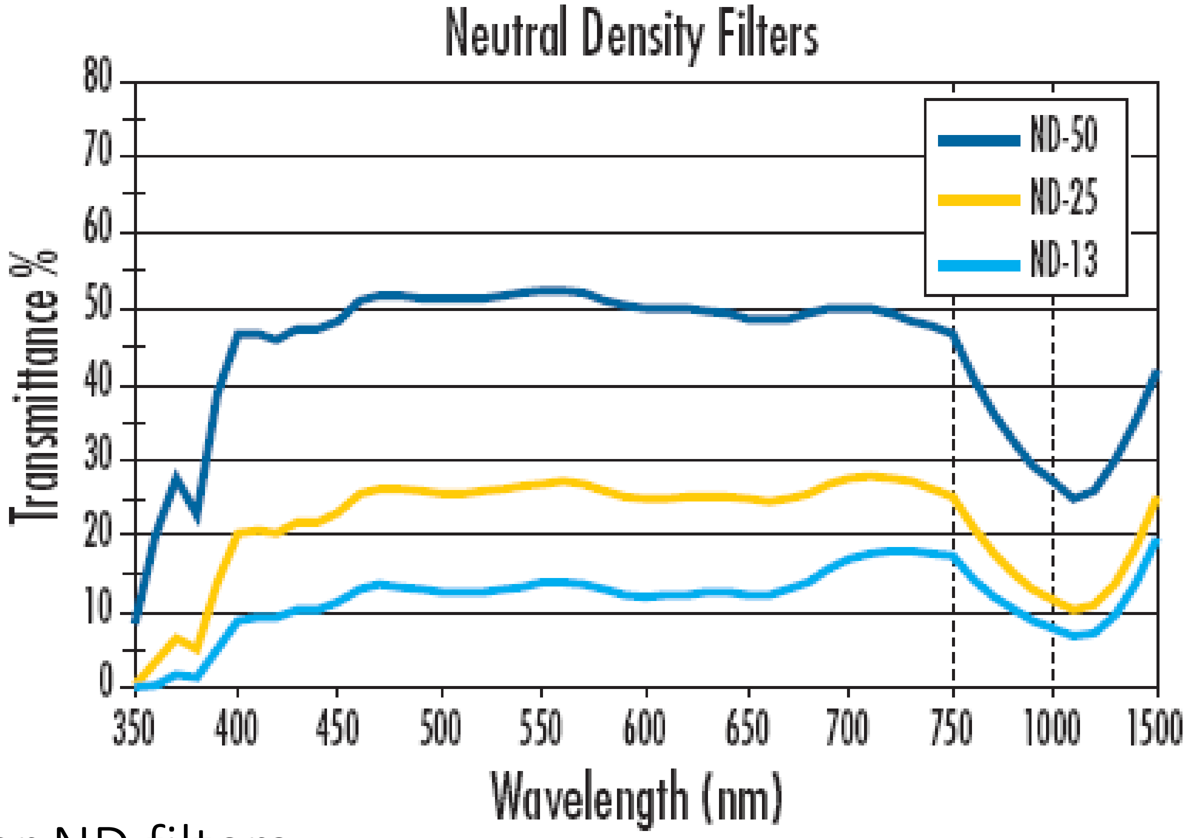


Filters

Neutral density (ND) filters

Alternative way to control exposure:

- (Approximately) spectrally flat from 400-700 nm.
- Homogeneous glass that blocks by absorption or by reflection



Often characterized by *optical density* (OD):

- Transmittance = 10^(-optical density) * 100.
- Optical density is additive as you stack together ND filters.

Graduated neutral-density filters

Variable optical density, from too high to too low/zero.

soft edge

hard edge

What are these filters useful for?

Graduated neutral-density filters

Useful in scenes with parts of very different brightness.

- Common scenario: Sky – horizon – ground.

Polarizing filters (or polarizers)

Most commonly circular polarizers.

- Same principle as polarizing sunglasses.

What are these filters useful for?

Polarizing filters (or polarizers)

Reduce sky light

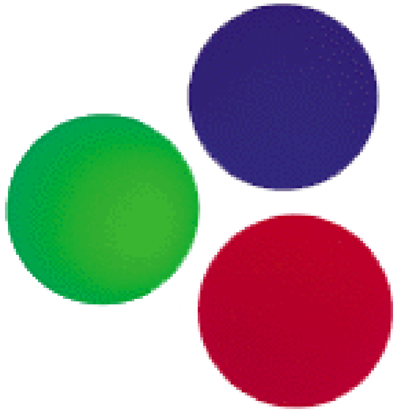
Reduce haze

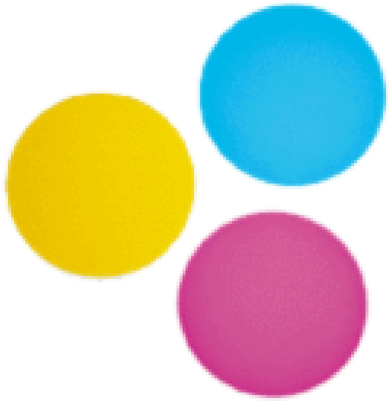
Polarizing filters (or polarizers)

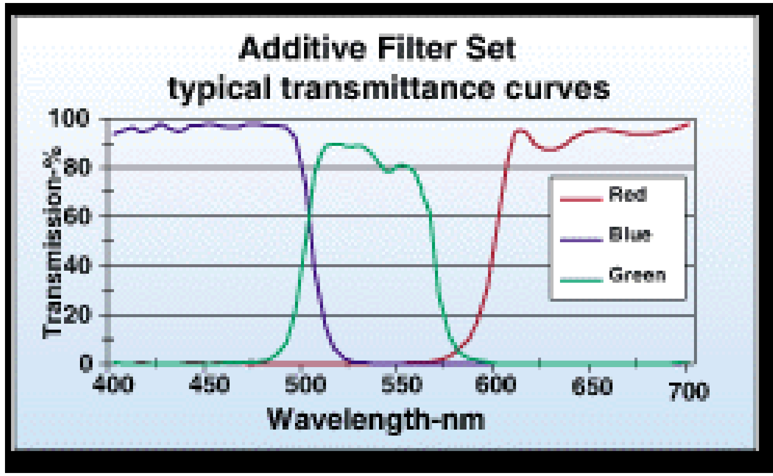
Reduce direct reflections

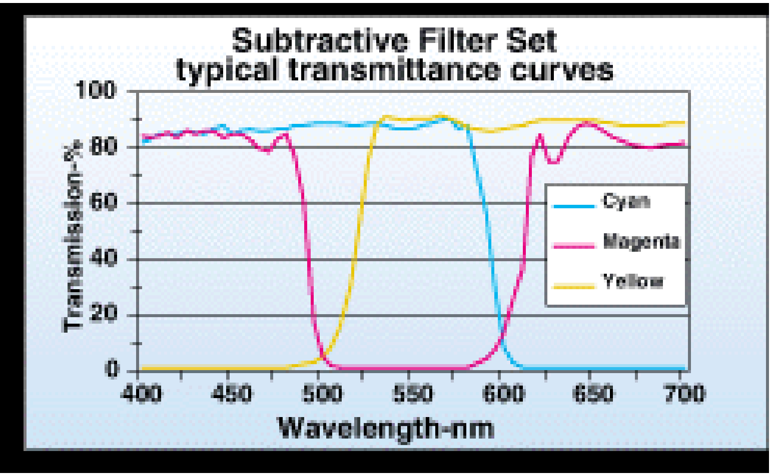
Spectral (color) filters

Mostly used for scientific applications or under very special lighting settings.



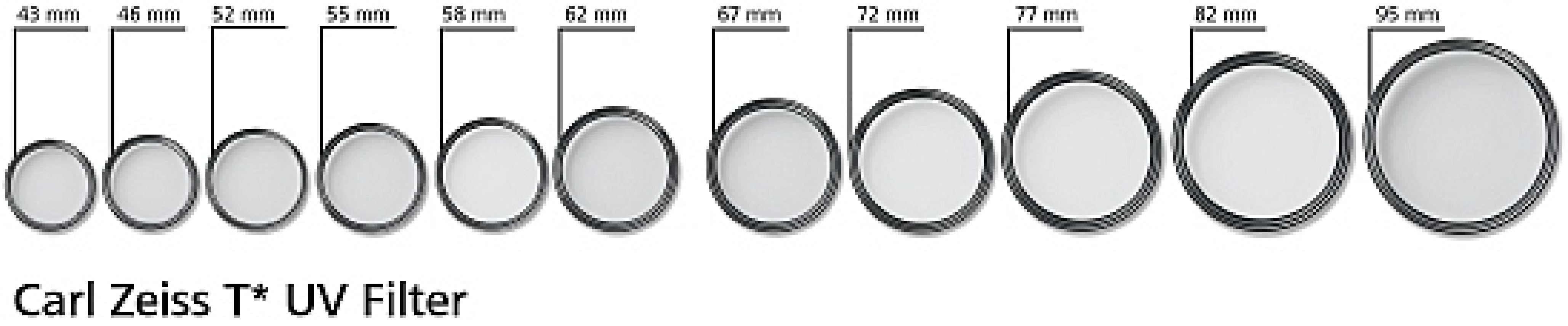


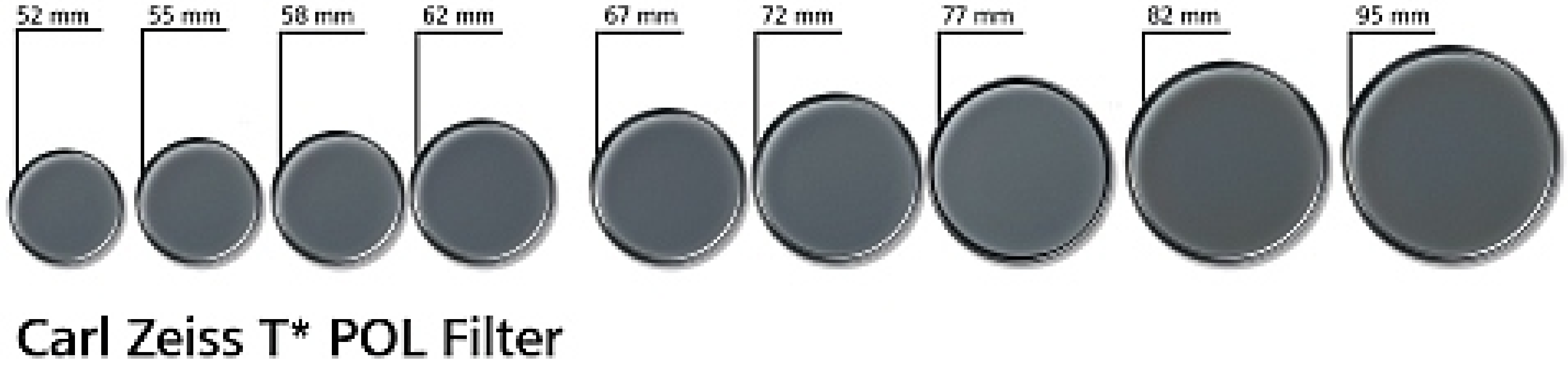




A note on filter sizes

Each individual filter is often offered in a variety of sizes, ranging from 30 mm to 100 mm





A note on filter sizes

The filter size you need to use is determined by the lens you are using.

- You can find the filter size marking in the front of the lens.
- You can avoid having to buy dozens of filters by using step-up and step-down rings.

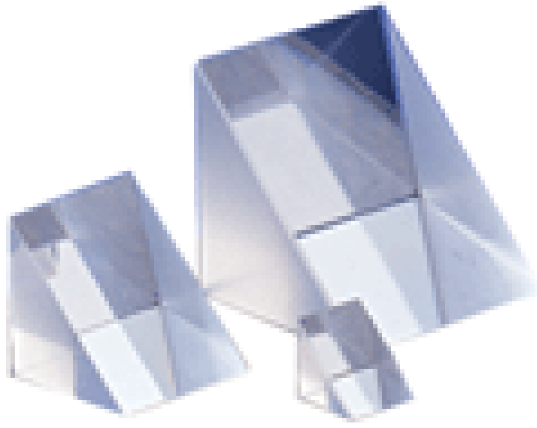
filter size marking

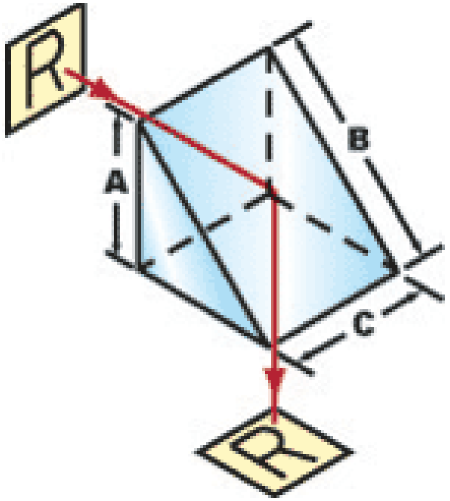
Prisms

Prisms

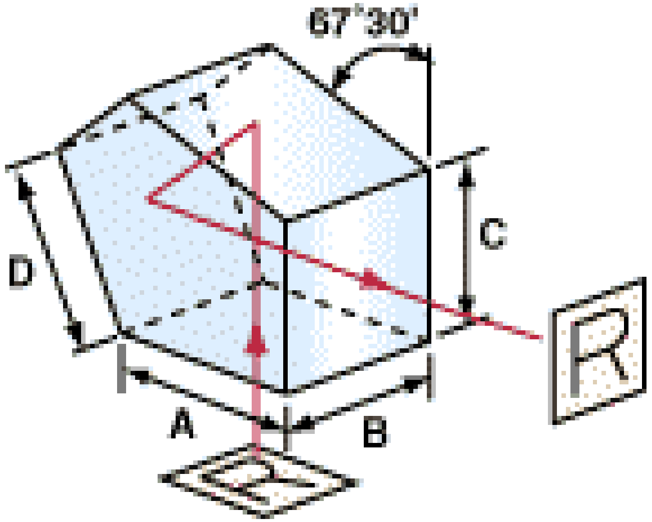
Many different types of prisms that produce different types of reflections.

right-angle prism





pentaprism

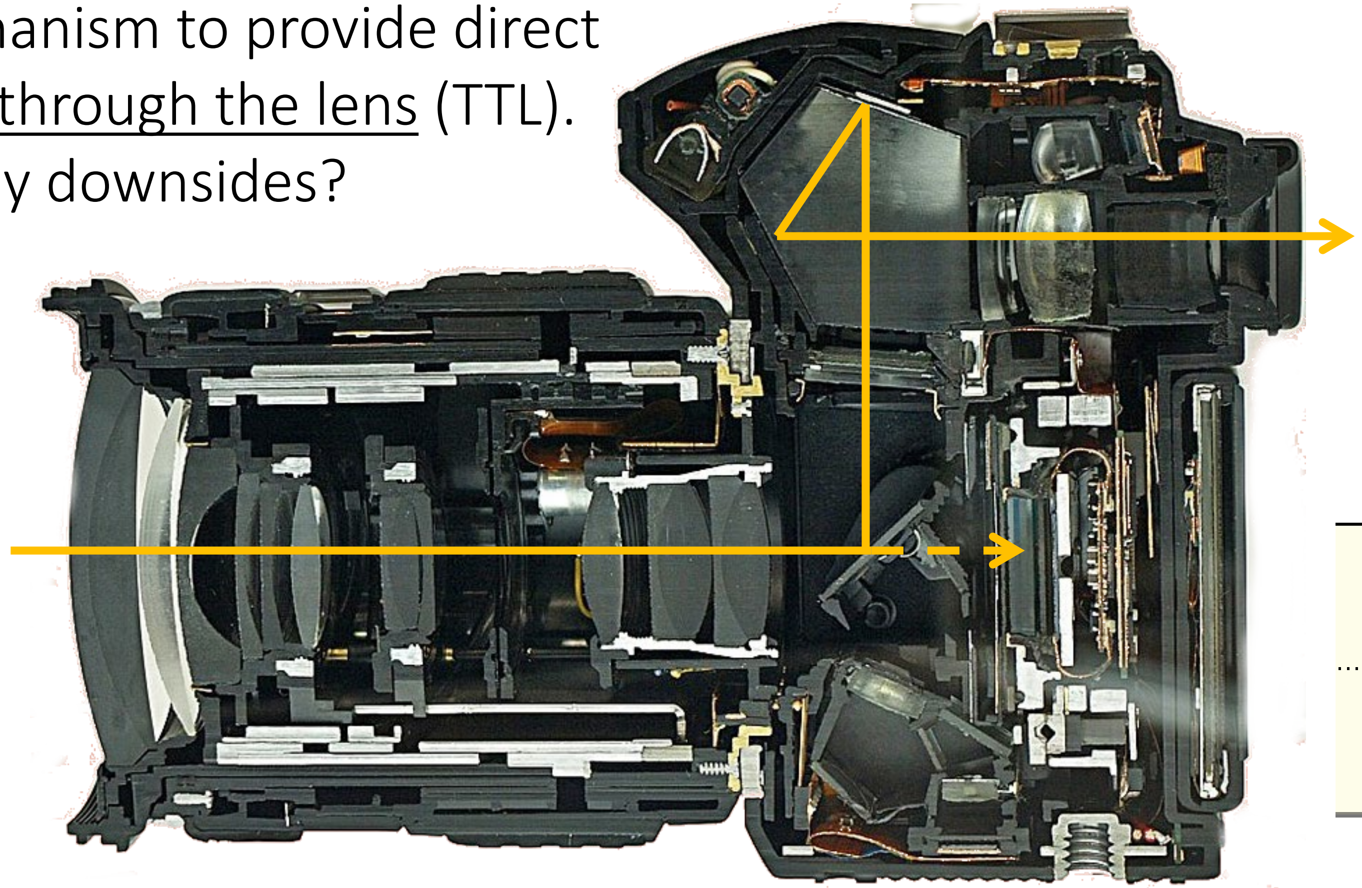


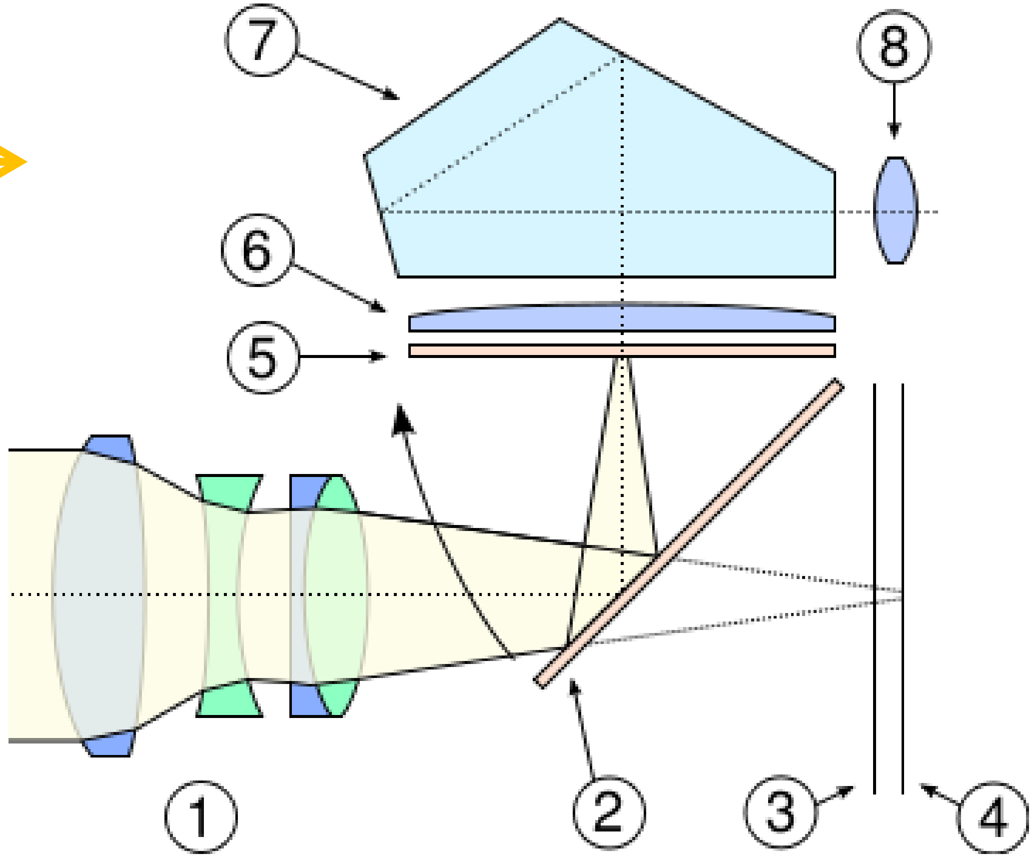
Do you know of any use of pentaprisms in photography?

Single Lens Reflex (SLR) cameras

Mechanism to provide direct view through the lens (TTL).

- Any downsides?

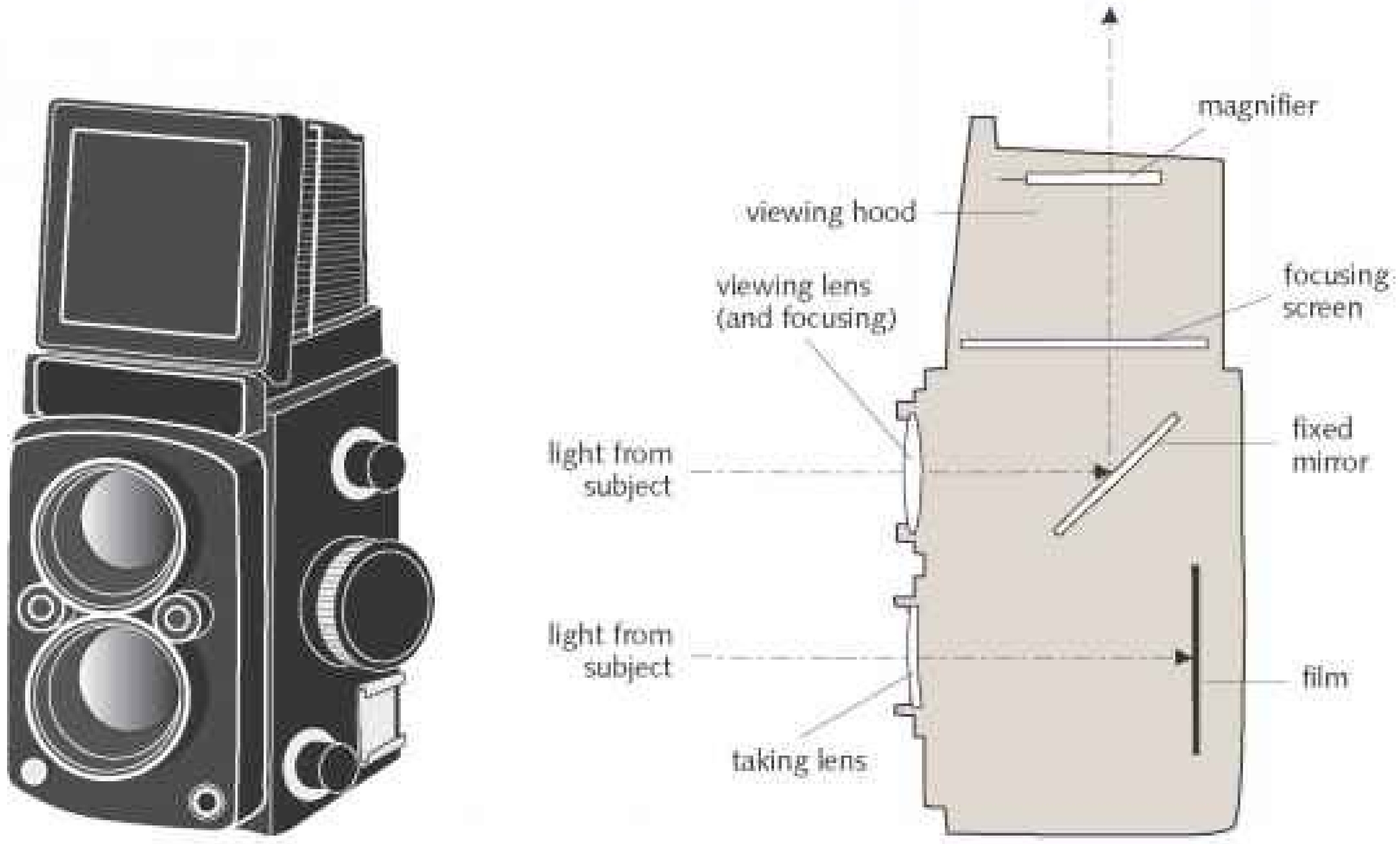




1 - Front-mount Lens
2 - Reflex Mirror at 45 degree angle
3 - Focal Plane Shutter
4 - Film or Sensor
5 - Focusing Screen
6 - Condenser Lens
7 - Optical Glass Pentaprism (or Pentamirror)
8 - Eyepiece (a.k.a. viewfinder)

Older types of cameras: twin lens reflex (TLR)

Older types of cameras: twin lens reflex (TLR)

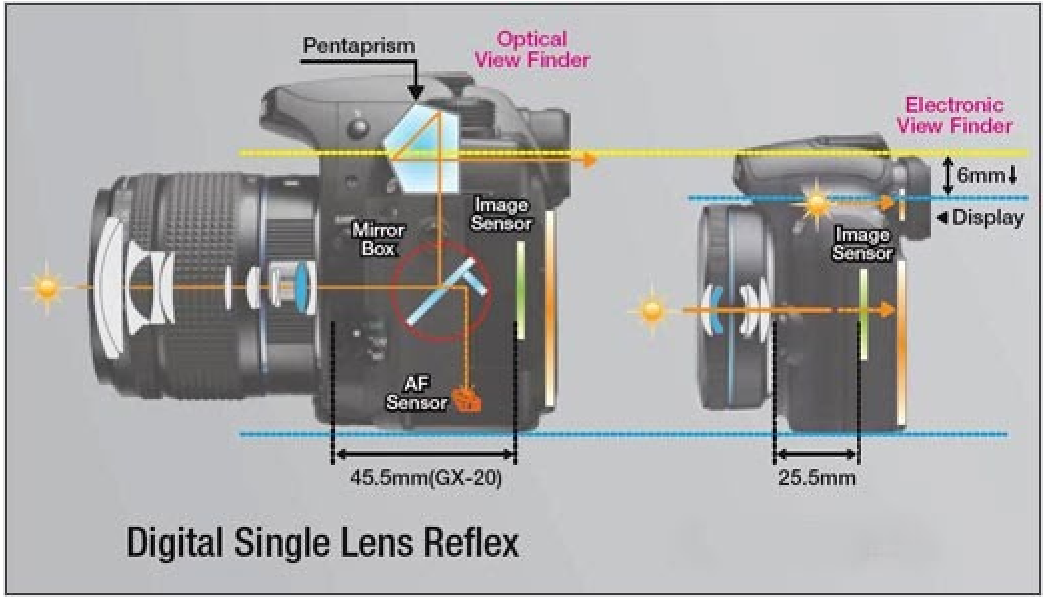


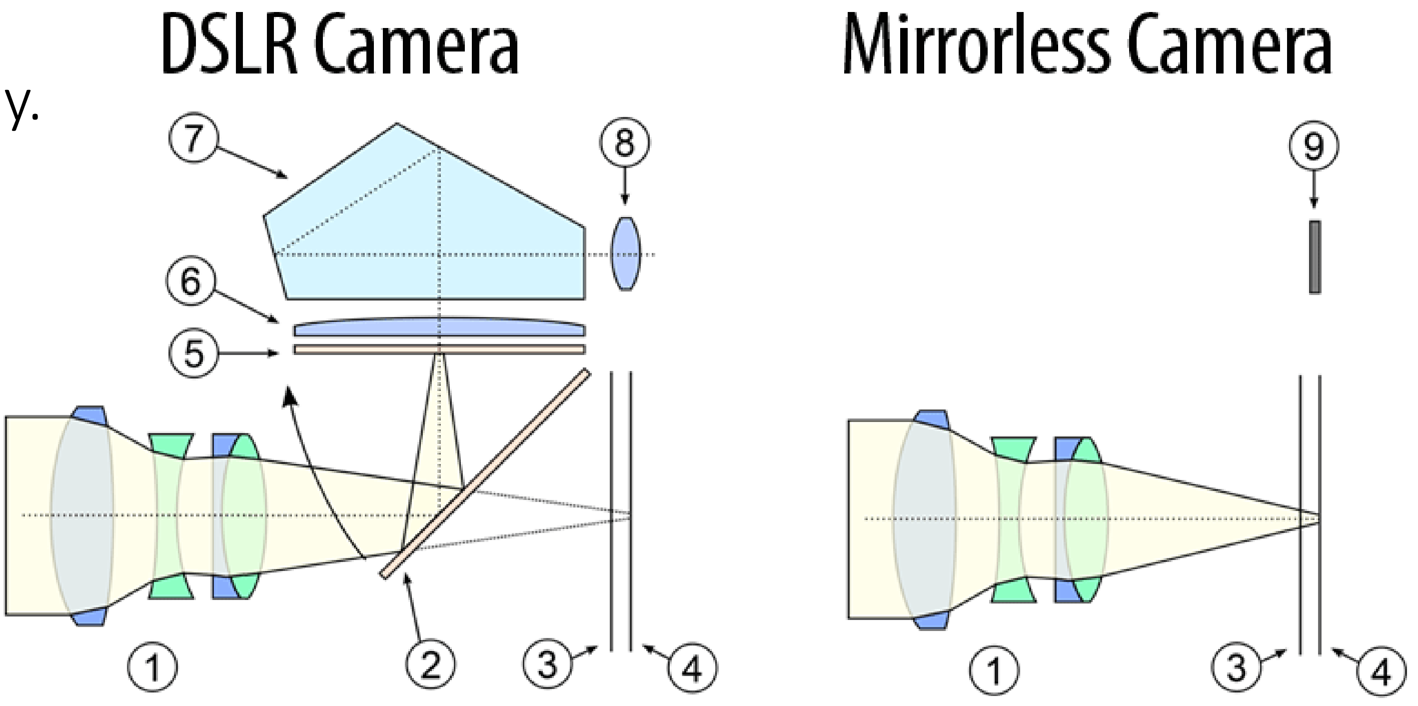
A twin-lens-reflex camera has two lenses—one on top of the other. Looking down onto a focusing screen, you view and focus your subject through the top lens. But when you press the shutter button, the bottom lens takes the picture. Twin-lens-reflex cameras take medium-format 120 roll film.

SLR versus mirrorless

Mirrorless cameras used to be mostly point-and-shoot, but are quickly becoming the dominant choice for high-end photography.

- What are some pros and cons of mirrorless compared to SLR?





1 - Front-mount Lens
2 - Reflex Mirror at 45 degrees
3 - Focal Plane Shutter
4 - Film or Sensor
5 - Focusing Screen
6 - Condenser Lens
7 - Optical Glass Pentaprism
8 - Eyepiece (a.k.a. viewfinder)

References

Basic reading:

- Szeliski textbook, Section 2.2.3.
- Pedrotti et al., "Introduction to Optics," Cambridge University Press 2017.
  This is a well-known general textbook on optics. Chapters 2 and 3 have the best (in my opinion) explanation of paraxial optics and ray transfer matrix analysis among common optics textbooks.

Additional reading:

- Hecht, "Optics," Pearson 2016.
  Probably the most commonly used optics textbook, also covers paraxial and ray transfer matrix analysis.
- London and Upton, "Photography," Pearson 2013.
  A great book on photography, discussing in detail many of the issues addressed in this lecture.
- Ray, "Applied Photographic Optics," Focal Press 2002.
  Another nice book covering everything about photographic optics.
- Mountford, "Refraction properties of conics," The Mathematical Gazette 1984.
  A small note on the refractive properties of conics, which seem to be much less known than their reflective ones.
- Shinya et al., "Principles and applications of pencil tracing," SIGGRAPH 1987.
- Durand et al., "A frequency analysis of light transport," SIGGRAPH 2005.
  Two graphics papers discussing ray transfer matrix analysis from the point of view of rendering.
- Navarro, "The Optical Design of the Human Eye: a Critical Review," Journal of Optometry 2009.
- Tang and Kutulakos, "What Does an Aberrated Photo Tell Us about the Lens and the Scene?," ICCP 2013.
- Hullin et al., "Polynomial Optics: A Construction Kit for Efficient Ray-Tracing of Lens Systems," Computer Graphics Forum 2012.
  Two papers discussing the modeling of aberrations and limitations of paraxial optics.
- Sun et al., "Lens Factory: Automatic Lens Generation Using Off-the-shelf Components," arXiv 2015.
  A discussion on lens design from a graphics-oriented point of view.
- Durand, "The DSLR will probably die. Are mirrorless the future of large standalone cameras?", http://www.thecomputationalphotographer.com/2018/10/the-dslr-will-probably-die-are-mirrorless-the-future-of-large-standalone-cameras/
  A great blog post by Fredo Durand, discussing the relative merits of DSLR and mirrorless cameras.