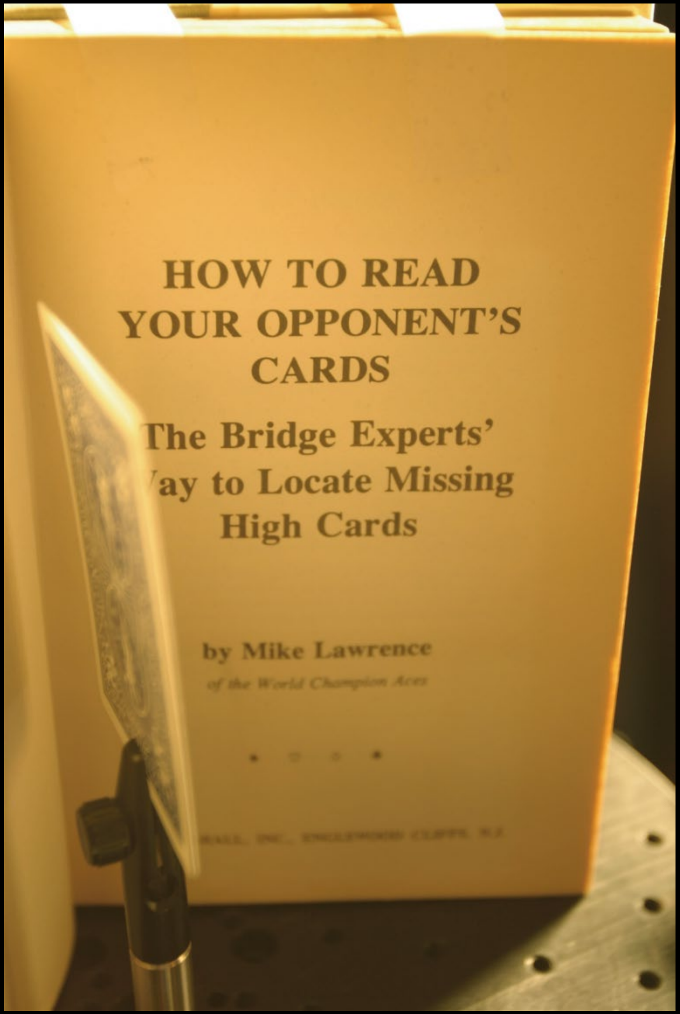
Light transport matrices



15-463, 15-663, 15-862
Computational Photography
Fall 2021, Lecture 21

http://graphics.cs.cmu.edu/courses/15-463

Course announcements

- Homework assignment 6 is due on Sunday, December 5th.
  - Any questions?

- Course evaluation surveys:
  - FCEs: https://cmu.smartevals.com/
  - TA evaluations: https://www.ugrad.cs.cmu.edu/ta/F21/feedback/
  - End-of-semester survey:

https://docs.google.com/forms/d/e/1FAIpQLSc6eXw_tbcxIF7-fm882V_7g80Q-l_34oObplTMCqOUgMcEkw/viewform

- Final project logistics posted on course website.
  - Make sure to read the details.

Overview of today’s lecture

- The light transport matrix.
- Image-based relighting.
- Optical computing using the light transport matrix.
- Dual photography.
- Light transport probing and epipolar imaging.

Slide credits

These slides were directly adapted from:

- Matt O’Toole (CMU).

The light transport matrix

How do these three images relate to each other?

the superposition principle


photo with lights 1 & 2 turned on

=


photo with light 1 turned on

+


photo with light 2 turned on

photo taken under two light sources =
sum of photos taken under each source individually

the superposition principle

photo taken under two light sources =
sum of photos taken under each source individually

the superposition principle

why is the error not exactly zero?

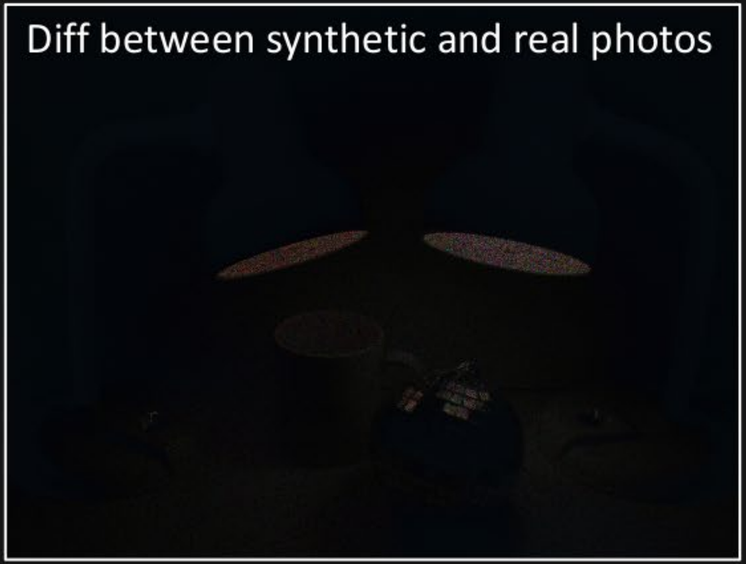


photo taken under two light sources =
sum of photos taken under each source individually

image-based relighting


Relit photo

=


photo with light 1 turned on


photo with light 2 turned on

image-based relighting


Relit photo

=


photo with light 1 turned on

Weight 1

x 1

+


photo with light 2 turned on

Weight 2

x 1

image-based relighting

=

Weight 1

x 1

+

Weight 2

x 0

$=$

Weight 1

x $\boxed{\mathbf{l_1}}$ $+$

Weight 2

x $\boxed{\mathbf{l_2}}$

=

Weight 1 x $\boxed{\mathbf{l_1}}$ +

Weight 2 x $\boxed{\mathbf{l_2}}$

$$\mathbf{p} = \sum_{i=1}^{2} \mathbf{T}_i \times \mathbf{l}_i$$

=

Weight 1 x $\boxed{\mathbf{l_1}}$ +

Weight 2 x $\boxed{\mathbf{l_2}}$

$$n \quad \mathbf{p} = \sum_{i=1}^{2} \mathbf{T}_i \times \mathbf{l}_i$$

n pixel values

=

Weight 1

x $\boxed{\mathbf{l_1}}$

+

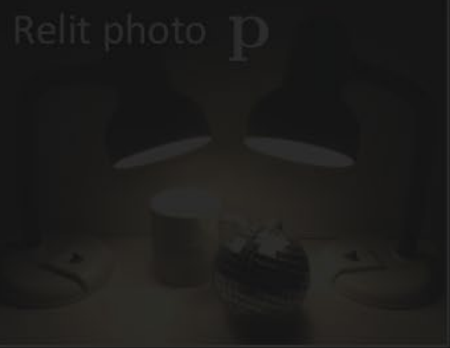
Weight 2

x $\boxed{\mathbf{l_2}}$

$$\mathbf{p} = \sum_{i=1}^{2} \mathbf{T}_i \times \mathbf{l}_i$$

n

n pixel values



=

Weight 1

x $\boxed{\mathbf{l}_1}$

+

Weight 2

x $\boxed{\mathbf{l}_2}$

$$\mathbf{p} = \sum_{i=1}^{2} \mathbf{T}_i \times \mathbf{l}_i$$

n

n pixel values

Contribution of the source

Relit photo $\mathbf{p}$

=

Weight 1

x $\boxed{\mathbf{l_1}}$ +

Weight 2

x $\boxed{\mathbf{l_2}}$

Number of controllable sources

Contribution of each source

$$\mathbf{p} = \sum_{i=1}^{2} \mathbf{T}_i \times \boxed{\mathbf{l}_i}$$

n

n pixel values

Relit photo $\mathbf{p}$

$=$

photo with light 1 turned on $\mathbf{T_1}$

Weight 1 x $\mathbf{l_1}$ $+$

photo with light 2 turned on $\mathbf{T_2}$

Weight 2 x $\mathbf{l_2}$

Number of controllable sources

Contribution of each source

$$n \quad \mathbf{p} = \sum_{i=1}^{2} \mathbf{T}_i \times \mathbf{l}_i$$

n pixel values

Relit photo $\mathbf{p}$

$=$ photo with light 1 turned on $\mathbf{T_1}$

Weight 1 x $\mathbf{l_1}$ $+$ photo with light 2 turned on $\mathbf{T_2}$

Weight 2 x $\mathbf{l_2}$

Number of controllable sources

Contribution of each source

$$\mathbf{p} = \sum_{i=1}^{2} \mathbf{T}_i \times \mathbf{l}_i$$

n

n pixel values

=

Weight 1

x $\boxed{\mathbf{l_1}}$ +

Weight 2

x $\boxed{\mathbf{l_2}}$

Number of controllable sources

Contribution of each source

$$\mathbf{p} = \sum_{i=1}^{m} \mathbf{T}_i \times \mathbf{l}_i$$

n

n pixel values

Relit photo $\mathbf{p}$

$=$

photo with light 1 turned on $\mathbf{T_1}$

Weight 1 x $\boxed{\mathbf{l_1}}$ $+$

photo with light 2 turned on $\mathbf{T_2}$

Weight 2 x $\boxed{\mathbf{l_2}}$

light transport matrix

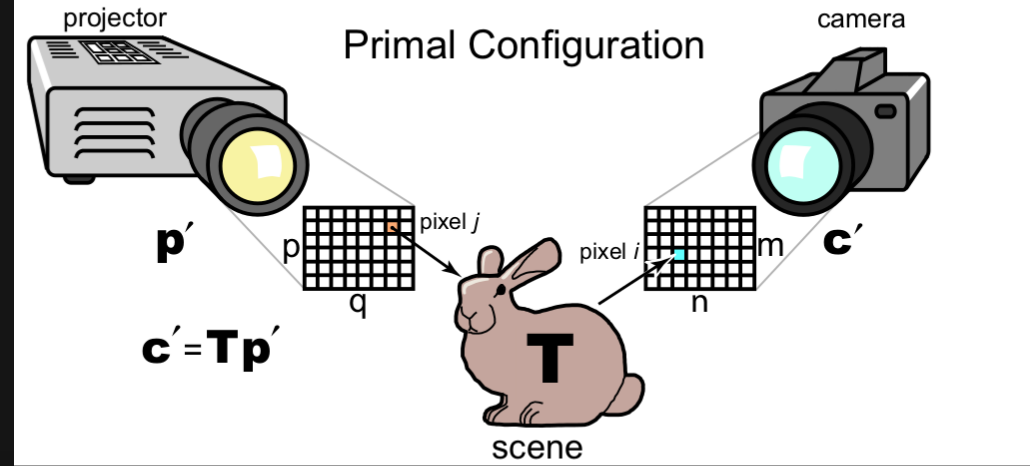
$$n \quad \mathbf{p} = \mathbf{T} \, \mathbf{l} \quad m$$

n pixel values

$n \times m$

Can you think of a case where we have a very large m?

Use a projector



$$\underbrace{\mathbf{p}}_{n} = \underbrace{\mathbf{T}}_{n \times m}\ \underbrace{\mathbf{l}}_{m}$$

n pixel values

$n \times m$

What does each row and column of T correspond to here?

Image-based relighting

Let’s say I have measured T.

- What does it mean to right-multiply it with some vector l?

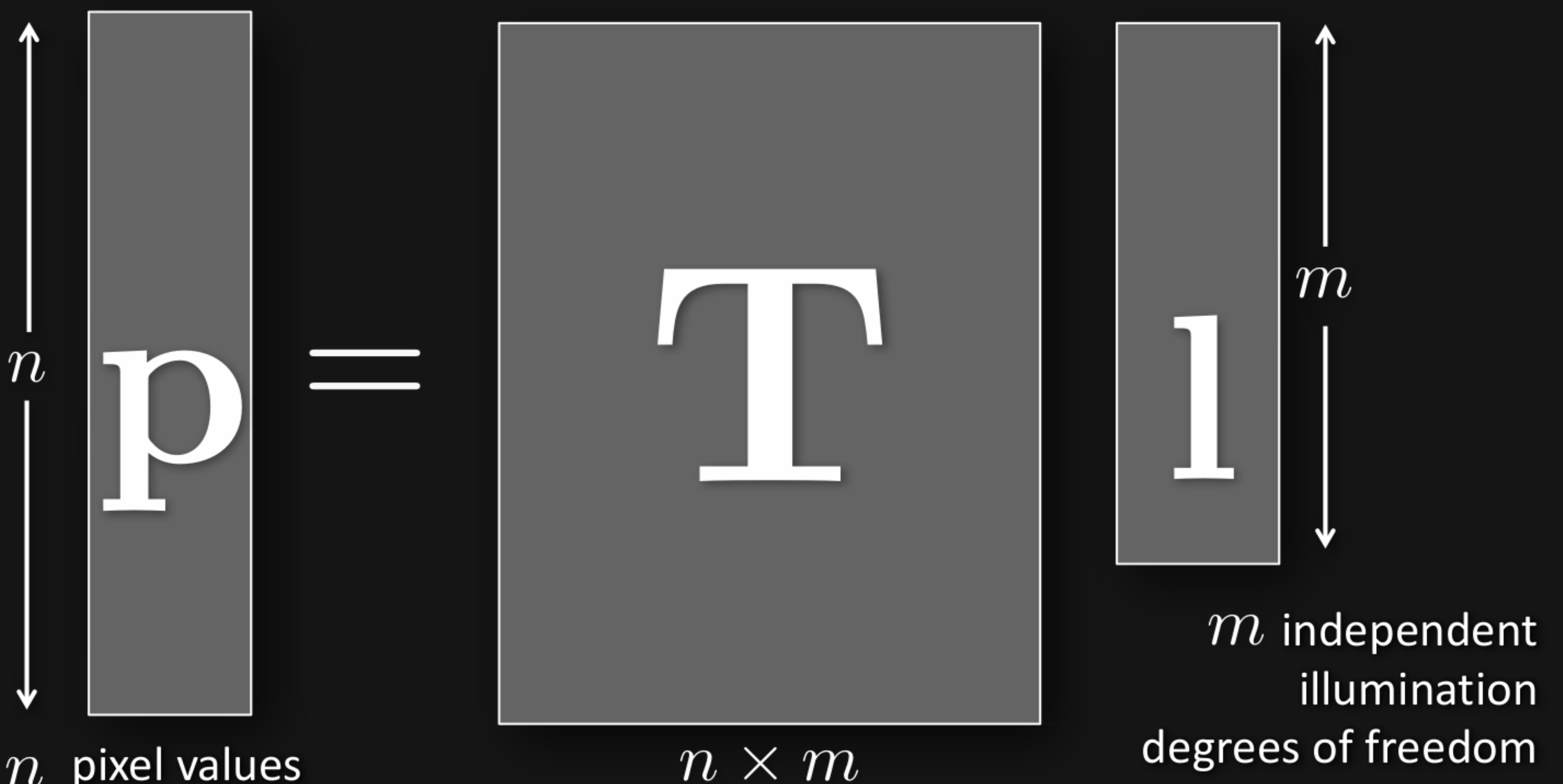


Image-based relighting: Use the measurements we already have of the scene (the pictures I took when measuring T) to simulate new illuminations of the scene.

$$\underbrace{\mathbf{p}}_{n} = \mathbf{T}\,\underbrace{\mathbf{l}}_{m}$$

n pixel values

$n \times m$

m independent illumination degrees of freedom

Acquiring the Reflectance Field [Debevec et al. 2000]

image-based rendering & relighting

Light stage 1.0

Reflectance field

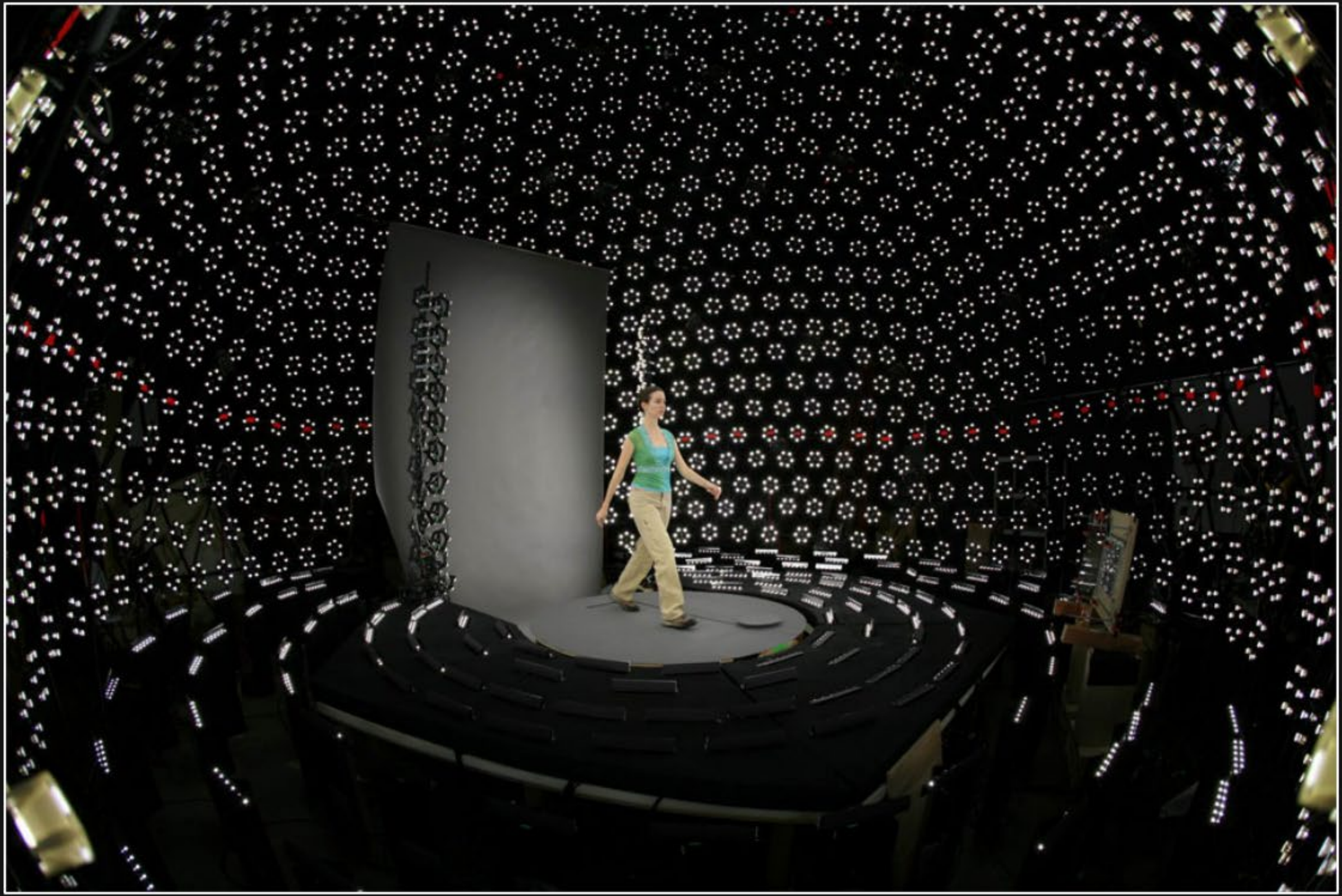
Acquiring the Reflectance Field

image-based rendering & relighting

Debevec et al, SIG 2000

Great demonstration: https://www.youtube.com/watch?v=mkzLLz1tXds

Acquiring the Reflectance Field



Light stage 6, Debevec et al., 2006

Optical computing using the light transport matrix

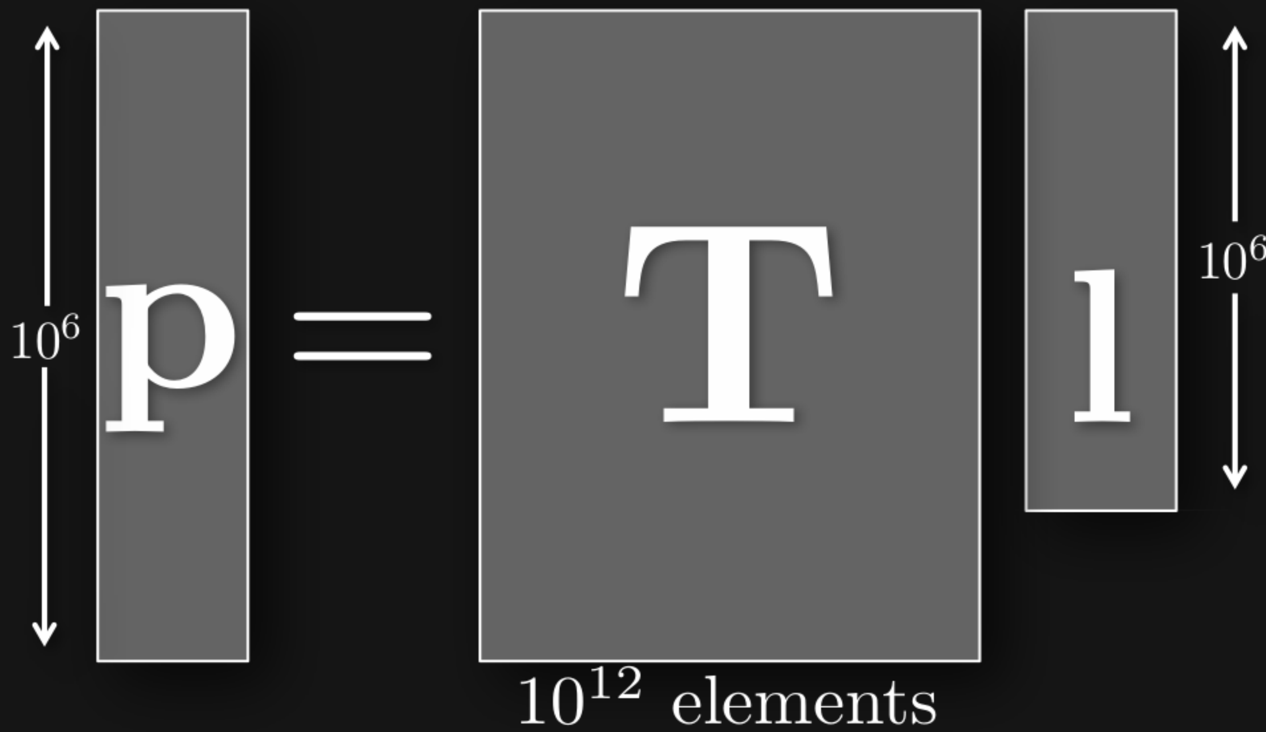
main difficulties

question: what are the challenges with analyzing $\mathbf{T}$?

$$\mathbf{p} = \mathbf{T}\,\mathbf{1}$$

main difficulties

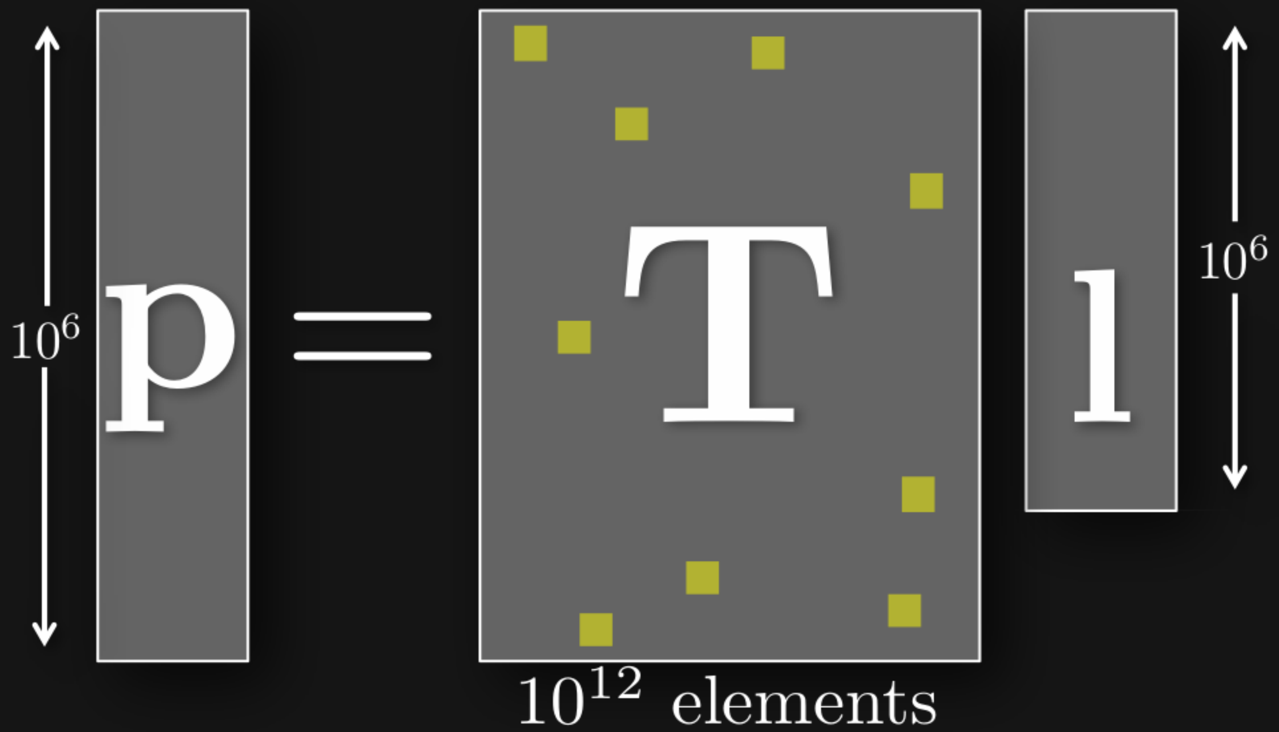
question: what are the challenges with analyzing $\mathbf{T}$?



- matrix can be extremely large

main difficulties

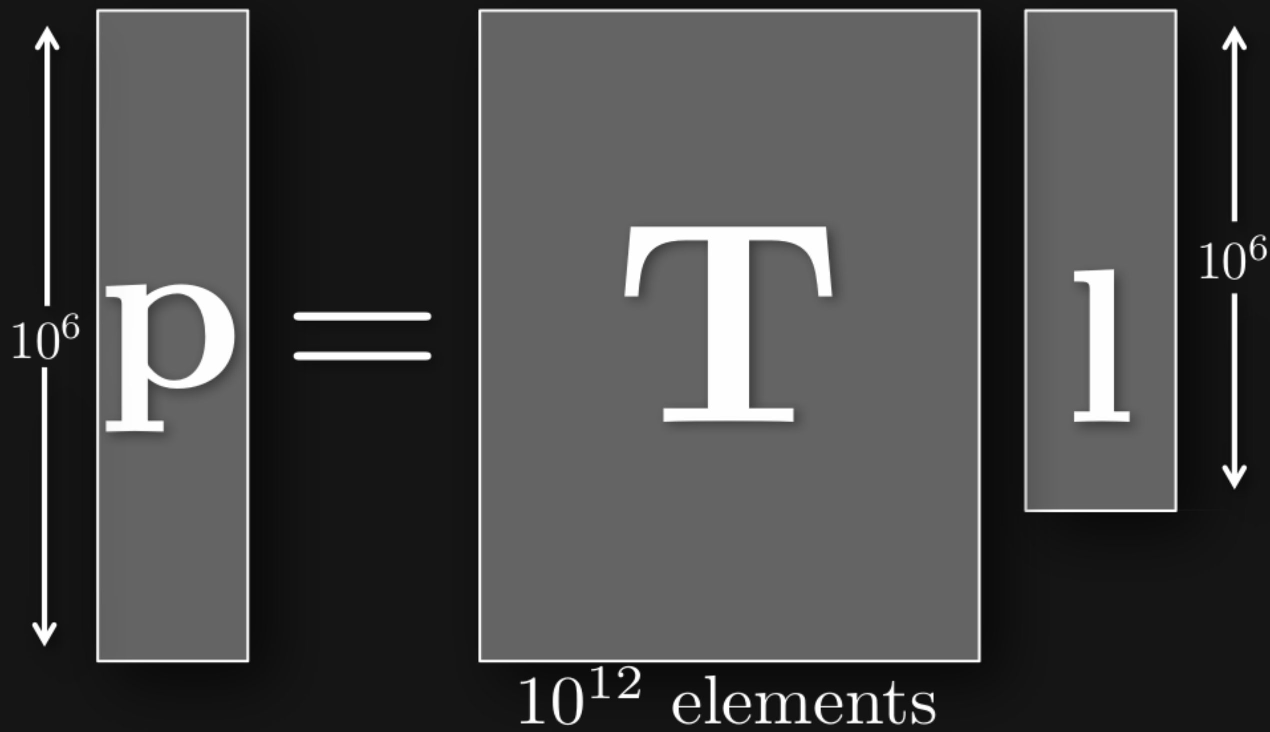
question: what are the challenges with analyzing $\mathbf{T}$?



- matrix can be extremely large
- elements not directly accessible

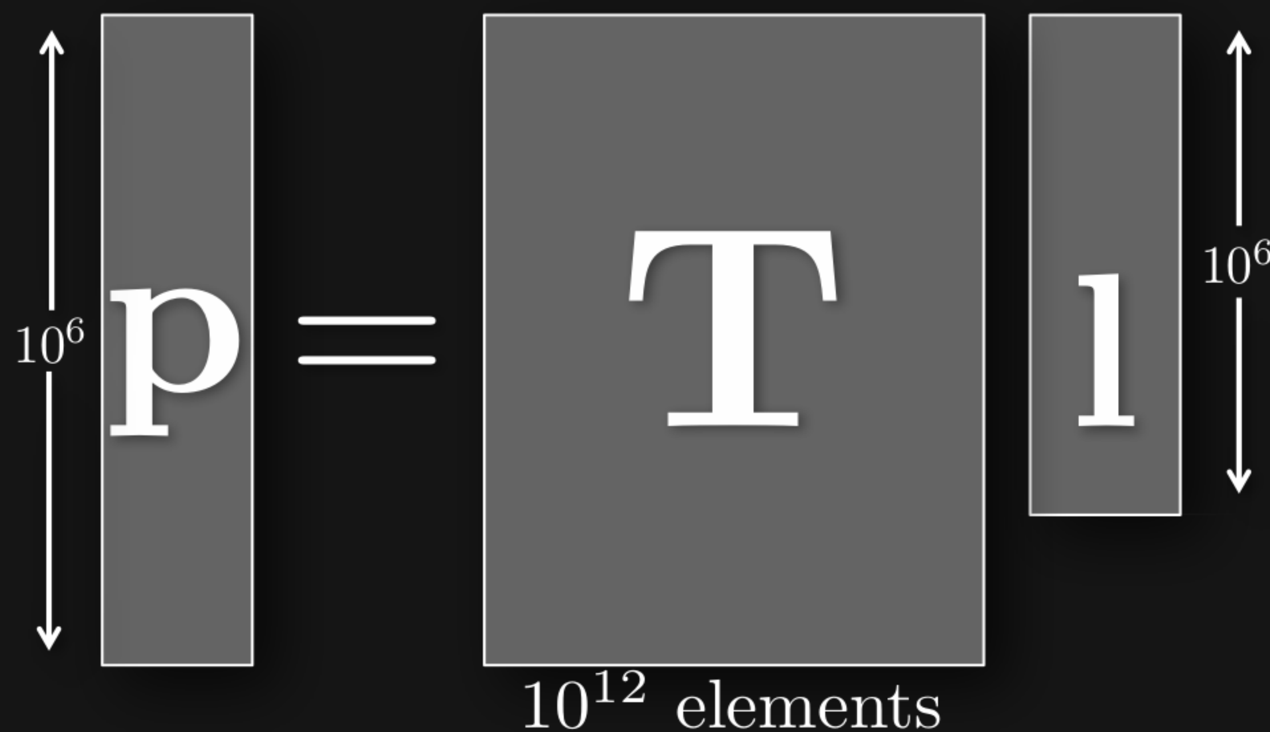
main difficulties

question: what are the challenges with analyzing $\mathbf{T}$?



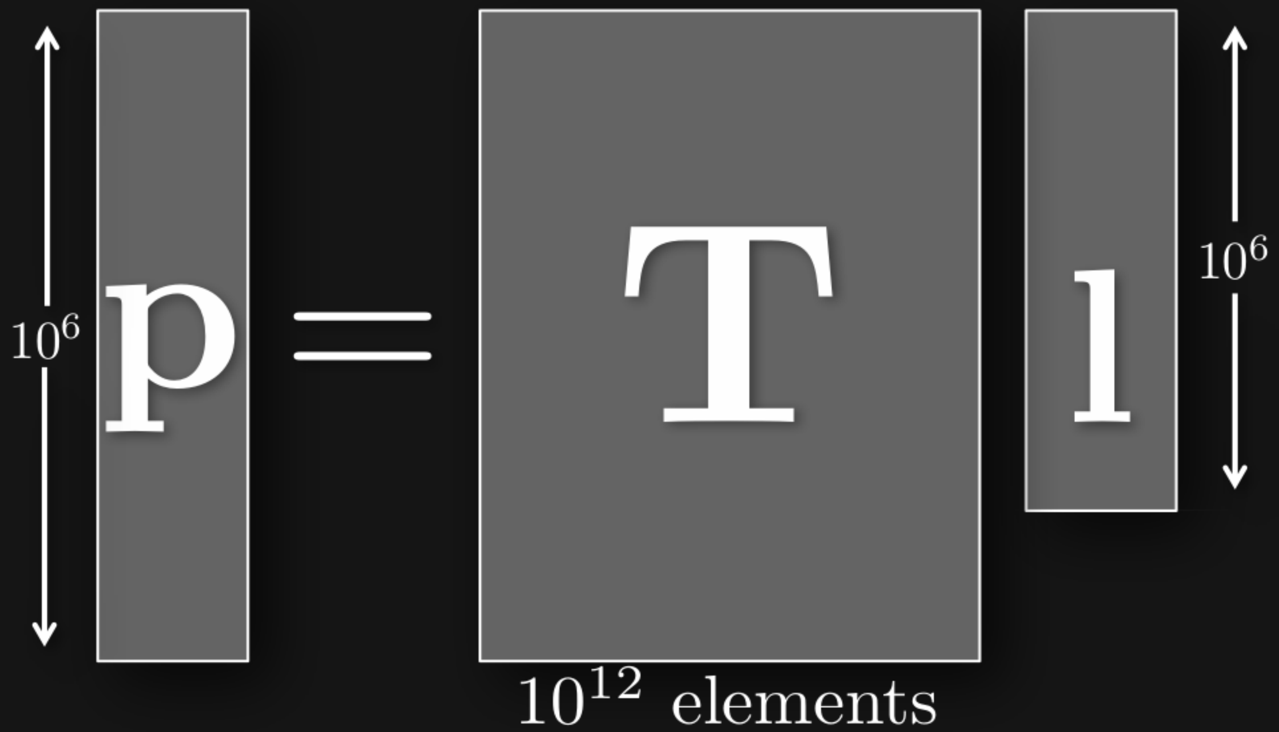
- matrix can be extremely large
- elements not directly accessible
- global structure poorly understood

How would you measure the light transport matrix T?



- matrix can be extremely large
- elements not directly accessible
- global structure poorly understood

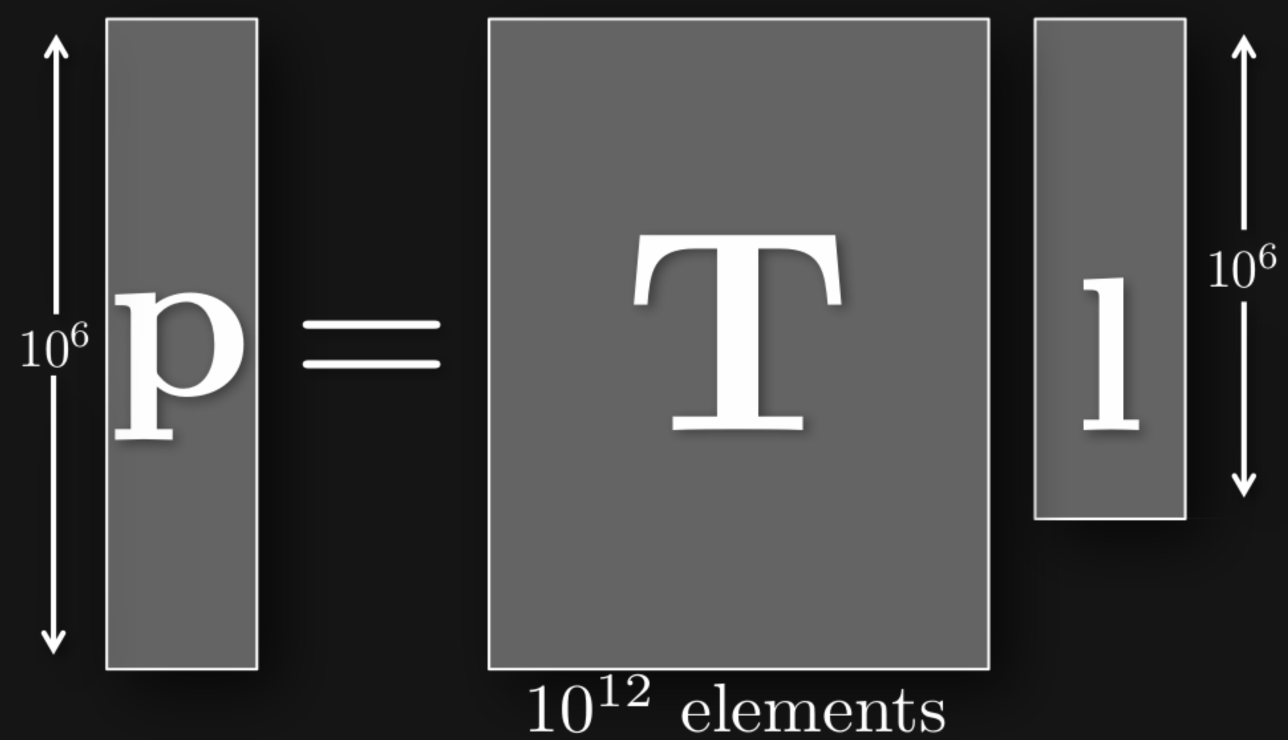
How would you measure the light transport matrix T?



Exhaustive/naïve approach: turn on projector pixels one at a time and take a photo for each of them.

- What does each photo correspond to in T?

How would you measure the light transport matrix T?



Exhaustive/naïve approach: turn on projector pixels one at a time and take a photo for each of them.

- How many photos do we need to capture?

computing with light

numerical algorithms implemented directly in optics

numerical domain

transport matrix

$$\mathbf{p} = \mathbf{T}\,\mathbf{l}$$

photo

illumination vector

$\Longleftrightarrow$

optical domain

computing with light

numerical algorithms implemented directly in optics

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$$\mathbf{p} = \mathbf{T}\,\mathbf{l}$$

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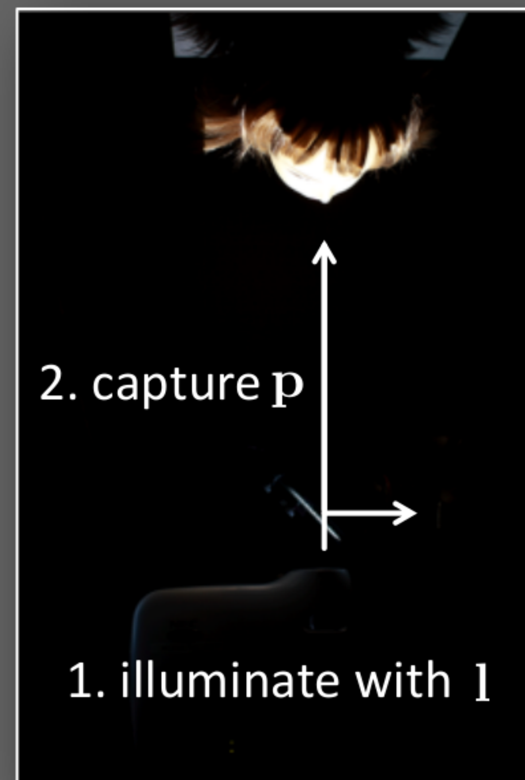


computing with light

numerical algorithms implemented directly in optics

numerical domain

function analyze($\mathbf{T}$)

...

for $i = 1$ to k {

...

$\boxed{\mathbf{p}_i = \mathbf{T}\mathbf{l}_i}$

...

$\boxed{\mathbf{d}_i = \mathbf{T}\mathbf{r}_i}$

...

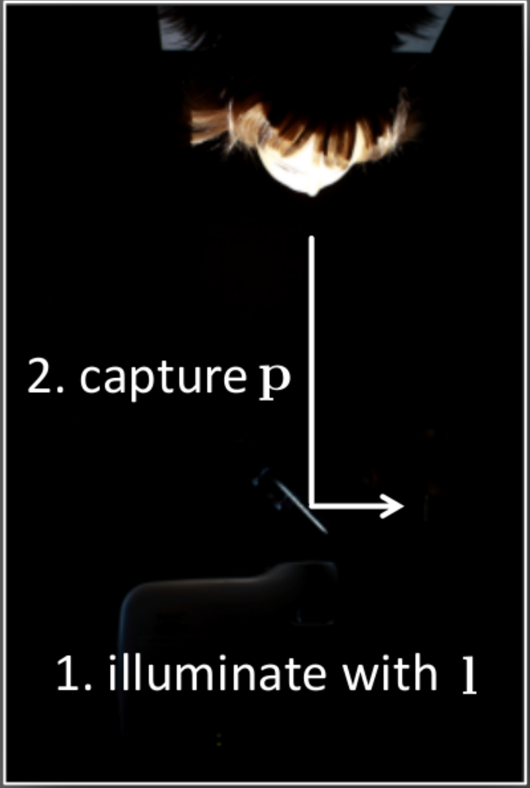
}

...

return result

$\Longleftrightarrow$

optical domain



computing with light

numerical algorithms implemented directly in optics

numerical domain

function analyze$\boxed{(\mathbf{T})}$

. . .

for $i = 1$ to k {

. . .

$\boxed{\mathbf{p}_i = \mathbf{T}\mathbf{l}_i}$

. . .

$\boxed{\mathbf{d}_i = \mathbf{T}\mathbf{r}_i}$

. . .

}

. . .

return result

$\Longleftrightarrow$

optical domain

function analyze$\boxed{()}$

. . .

for $i = 1$ to k {

. . .

$\boxed{\text{project } \mathbf{l}_i, \text{ capture } \mathbf{p}_i}$

. . .

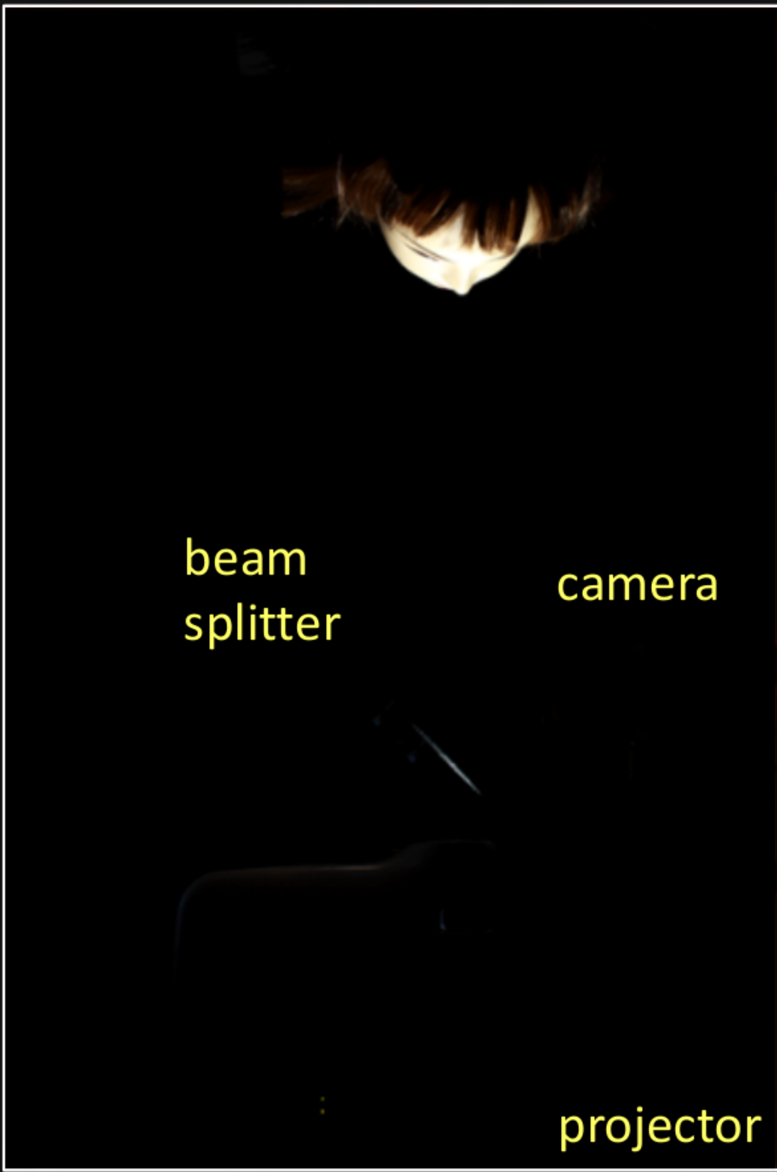
$\boxed{\text{project } \mathbf{r}_i, \text{ capture } \mathbf{d}_i}$

. . .

}

. . .

return result



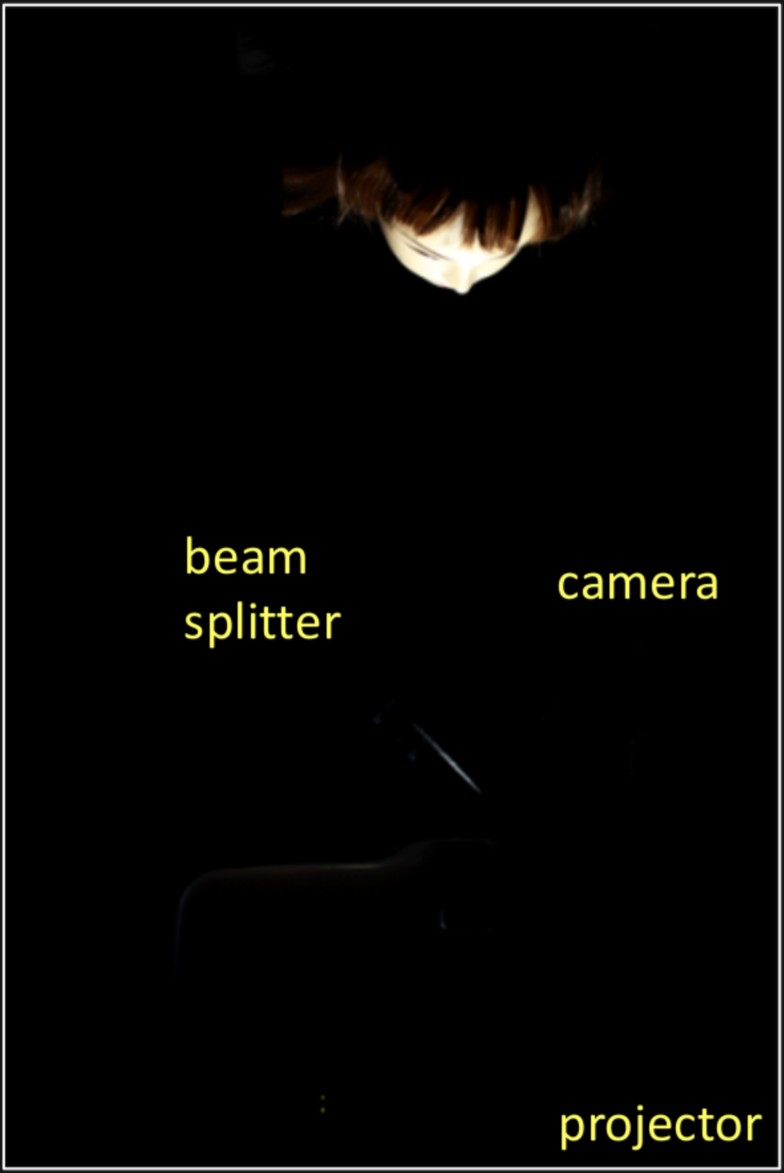
find an illumination pattern that
when projected onto scene,
we get the same photo back
(multiplied by a scalar)

project

capture

What do we call these patterns?

computing transport eigenvectors



eigenvector of a square matrix T

when projected onto scene,
we get the same photo back
(multiplied by a scalar)

project

capture

numerical goal

find $\mathbf{l}, \lambda$ such that

$$\mathbf{T}\mathbf{l} = \lambda\mathbf{l}$$

and λ is maximal

optical power iteration

goal: find principal eigenvector of $\mathbf{T}$

observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \ldots$

numerical domain

$$
\begin{aligned}
&\textbf{function } \mathrm{PowerIt}(\mathbf{T}) \\
&\boxed{\mathbf{l}_1 = \text{initial vector}} \\
&\textbf{for } i = 1 \text{ to } k \ \{ \\
&\quad \boxed{\mathbf{p}_i = \mathbf{T}\mathbf{l}_i} \\
&\quad \boxed{\mathbf{l}_{i+1} = \mathbf{p}_i / \|\mathbf{p}_i\|_2} \\
&\} \\
&\textbf{return } \mathbf{l}_{i+1}
\end{aligned}
$$

properties

- linear convergence [Trefethen and Bau 1997]
- eigenvalues must be distinct
- $\mathbf{l}_1$ cannot be orthogonal to principal eigenvector

optical power iteration

goal: find principal eigenvector of $\mathbf{T}$

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numerical domain		optical domain
function $\mathrm{PowerIt}(\mathbf{T})$		**function** $\mathrm{PowerIt}()$
$\mathbf{l}_1 = \text{initial vector}$		$\mathbf{l}_1 = \text{initial vector}$
for $i = 1$ to k {	$\Longleftrightarrow$	**for** $i = 1$ to k {
$\boxed{\mathbf{p}_i = \mathbf{T}\mathbf{l}_i}$		$\boxed{\text{project } \mathbf{l}_i, \text{ capture } \mathbf{p}_i}$
$\mathbf{l}_{i+1} = \mathbf{p}_i / \lVert \mathbf{p}_i \rVert_2$		$\mathbf{l}_{i+1} = \mathbf{p}_i / \lVert \mathbf{p}_i \rVert_2$
}		}
return $\mathbf{l}_{i+1}$		**return** $\mathbf{l}_{i+1}$

optical power iteration

goal: find principal eigenvector of $\mathbf{T}$

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numerical domain

function PowerIt($\mathbf{T}$)

$\mathbf{l}_1 = \text{initial vector}$

for $i = 1$ to k {

$\boxed{\mathbf{p}_i = \mathbf{T}\mathbf{l}_i}$

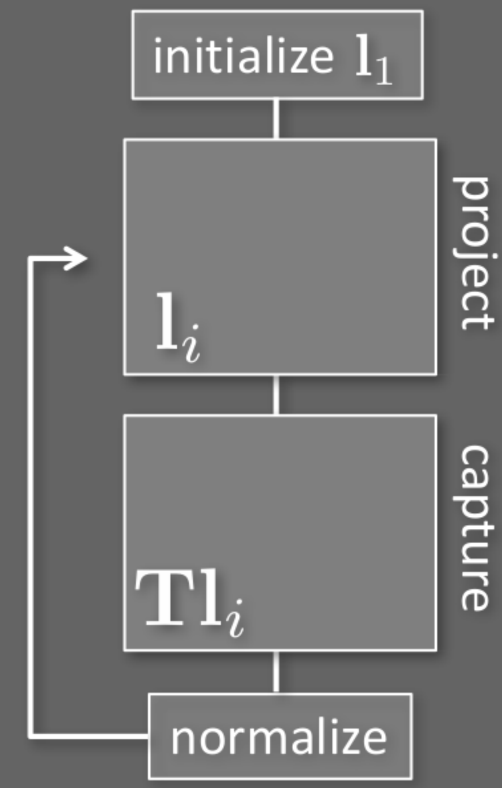
$\mathbf{l}_{i+1} = \mathbf{p}_i / \|\mathbf{p}_i\|_2$

}

return $\mathbf{l}_{i+1}$

$\Longleftrightarrow$

optical domain



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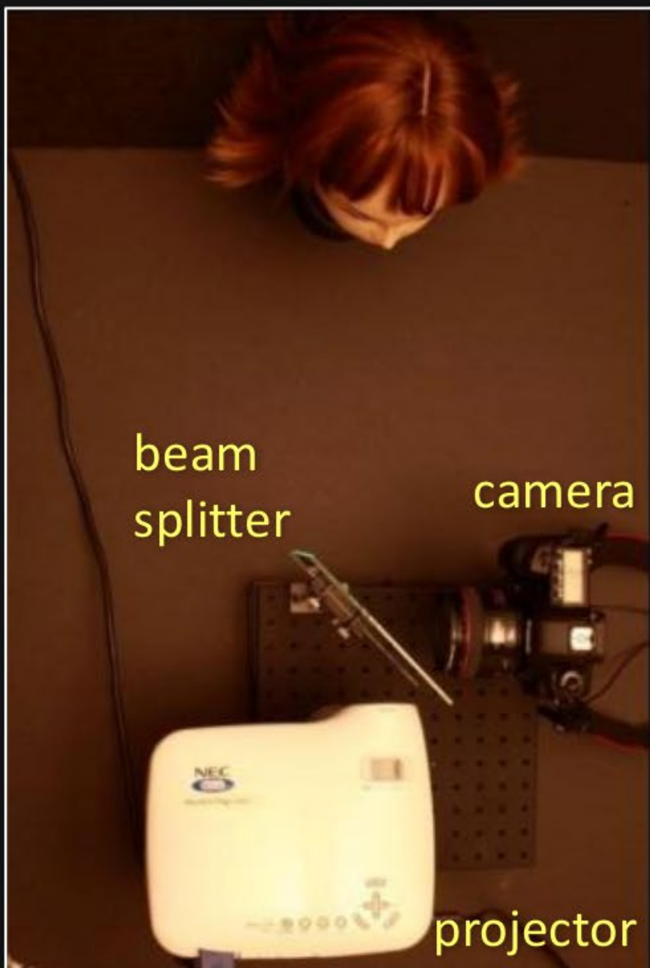
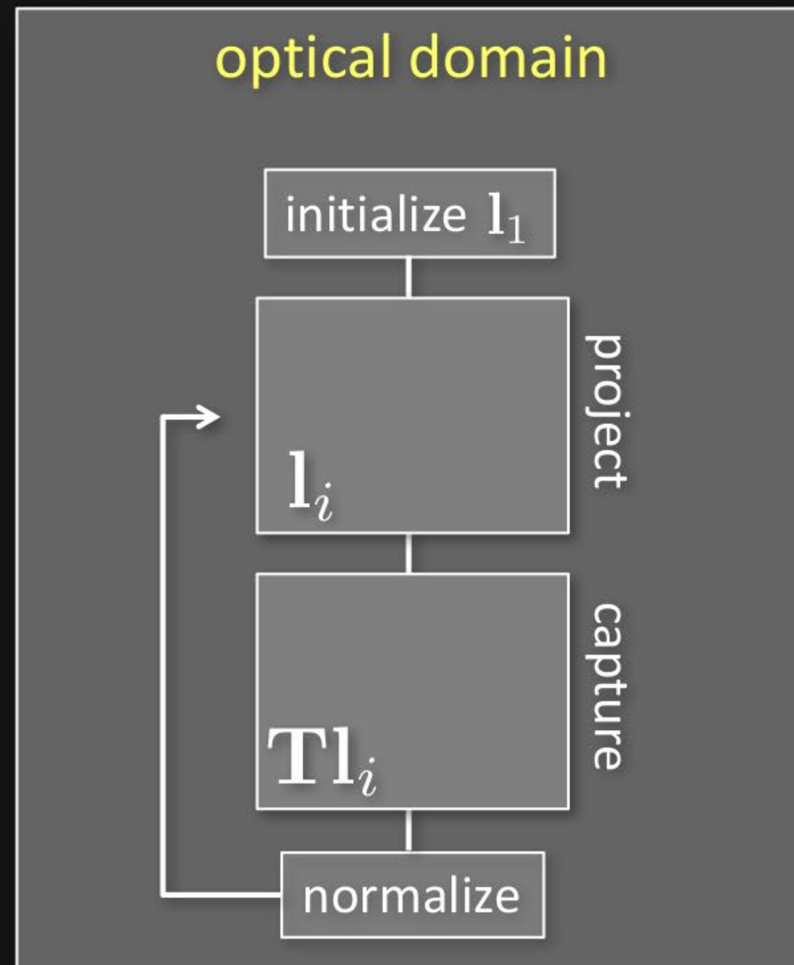
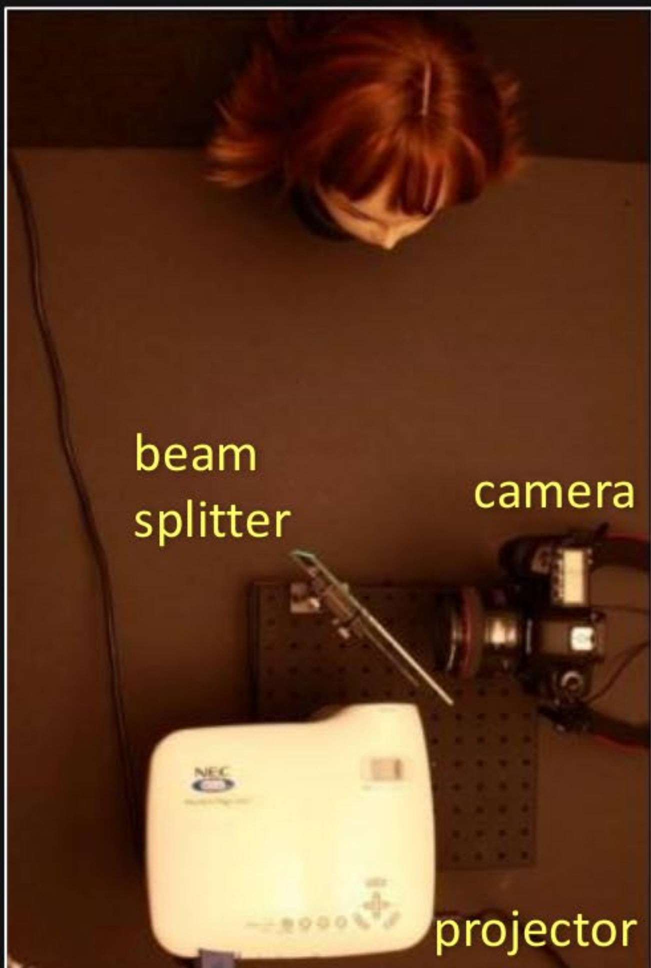


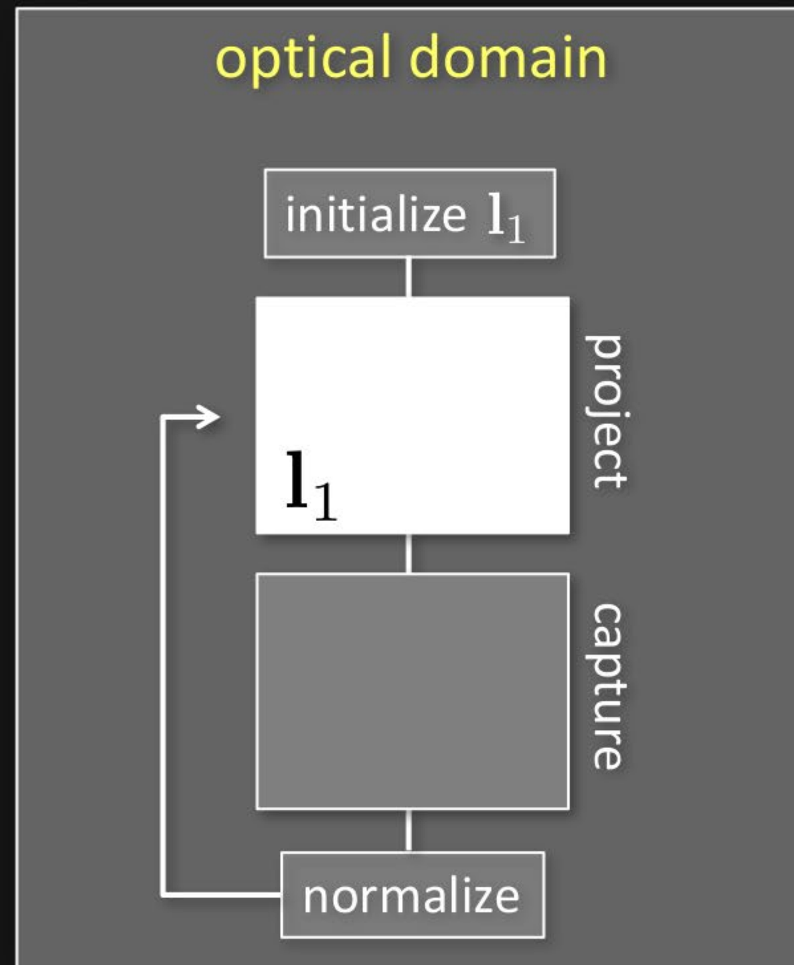


optical power iteration

goal: find principal eigenvector of $\mathbf{T}$

observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \ldots$





optical power iteration

goal: find principal eigenvector of $\mathbf{T}$

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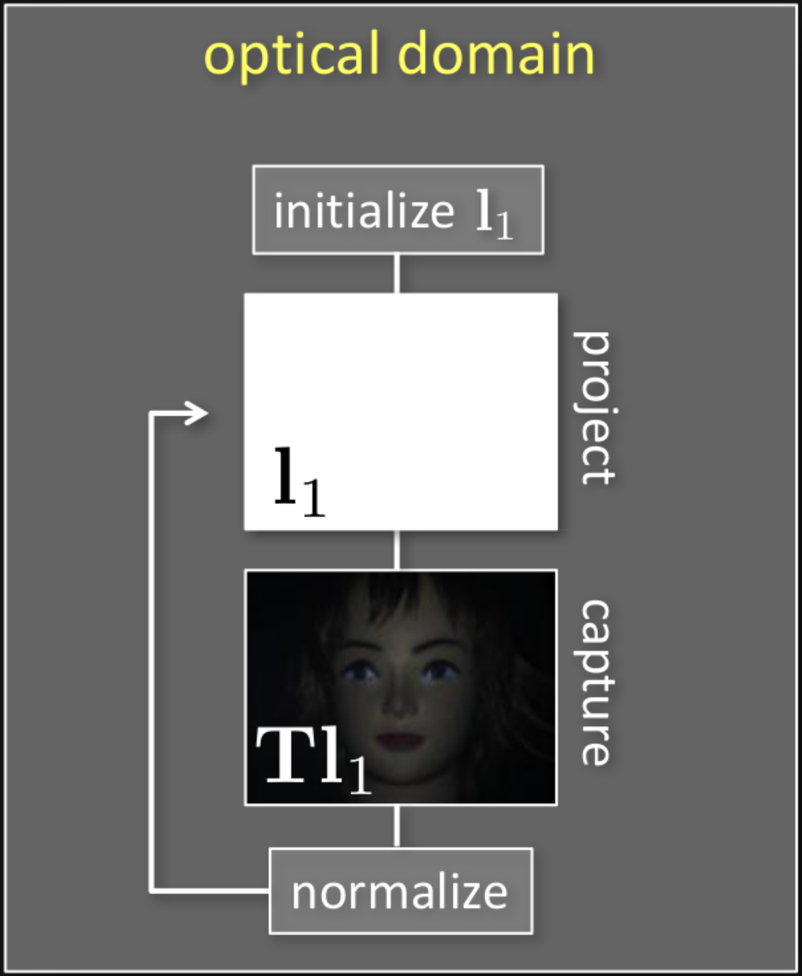

optical power iteration

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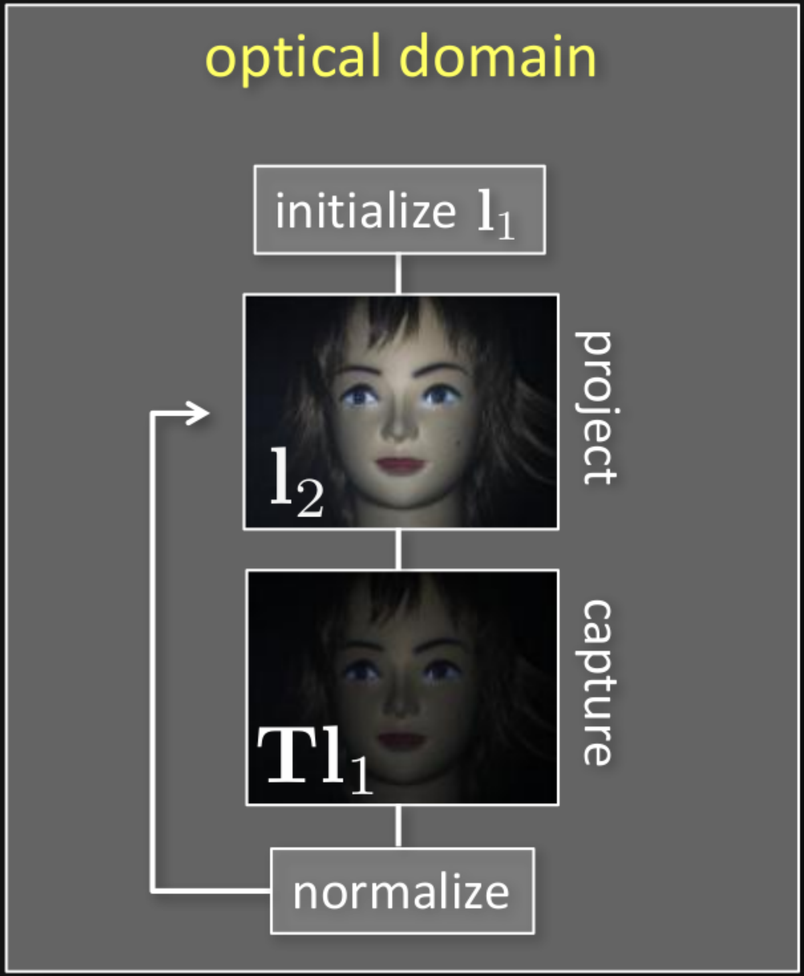

optical power iteration

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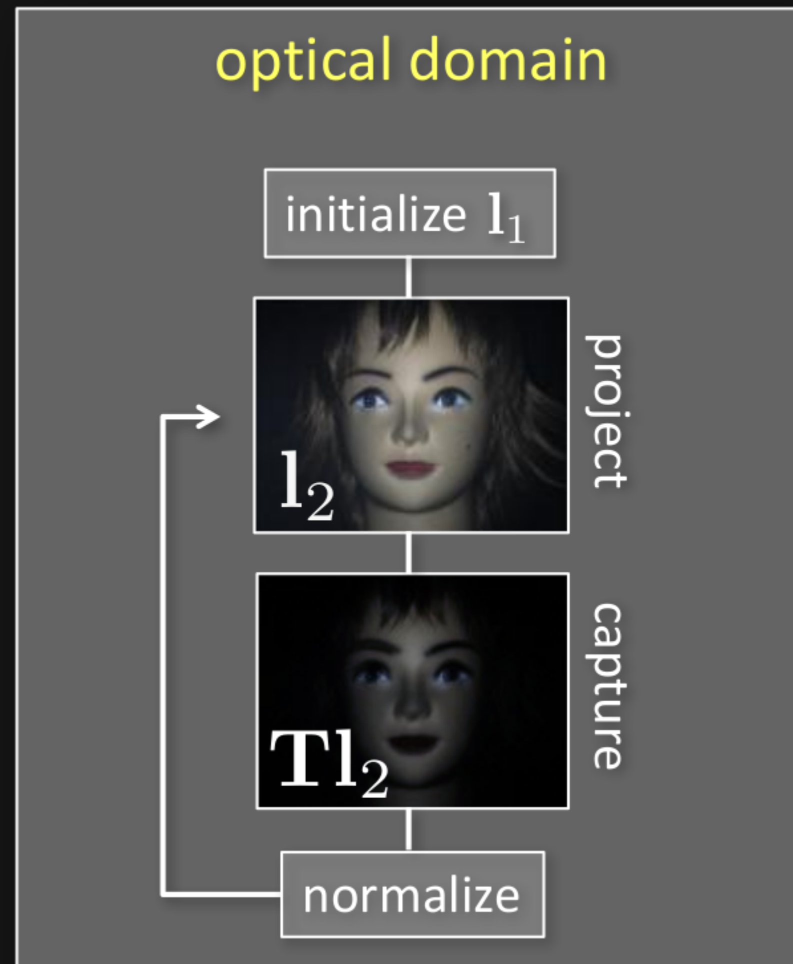

optical power iteration

goal: find principal eigenvector of $\mathbf{T}$

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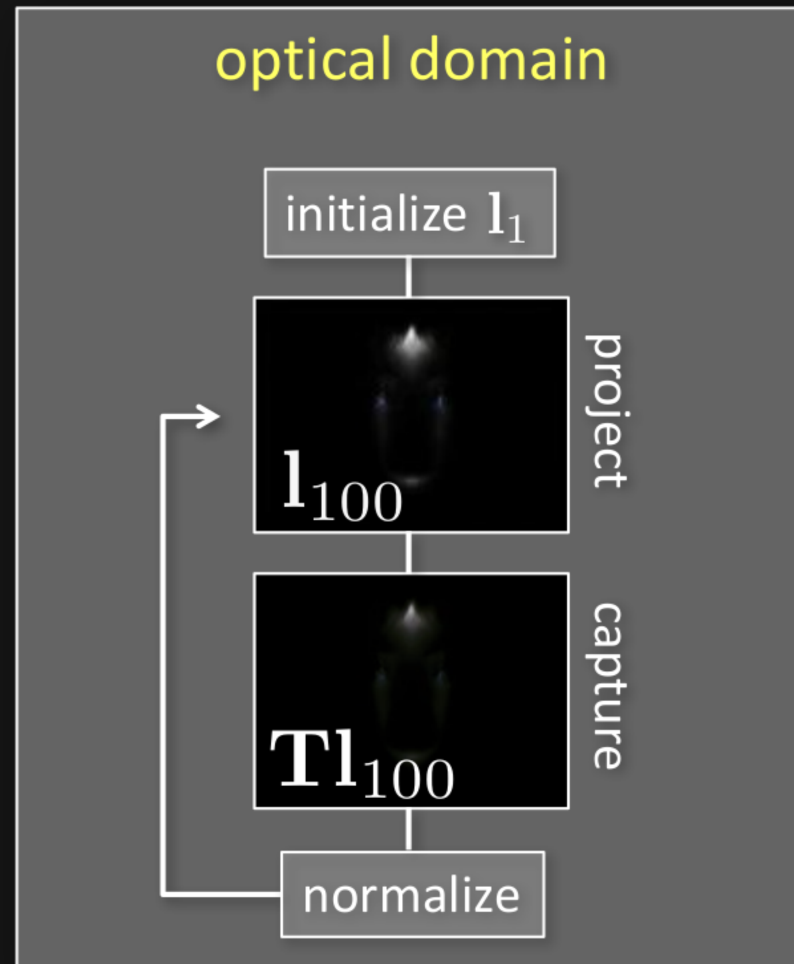

optical power iteration

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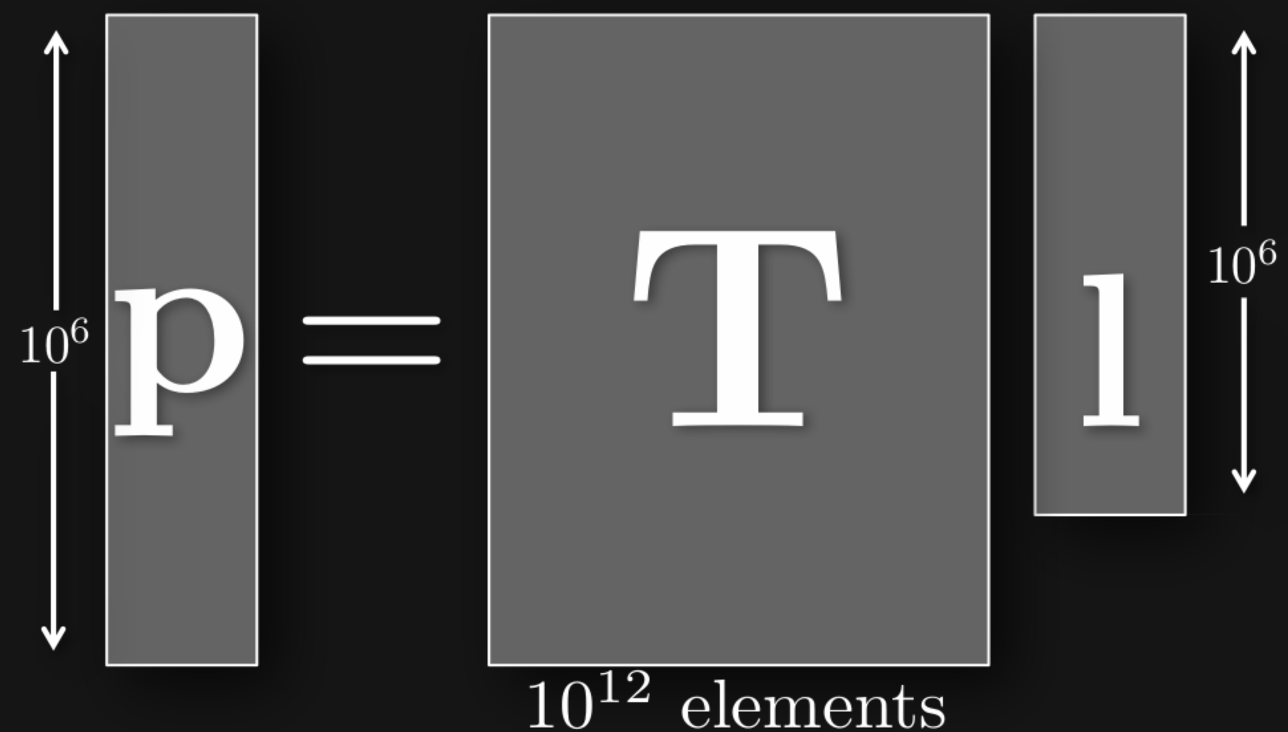
observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \ldots$

optical domain

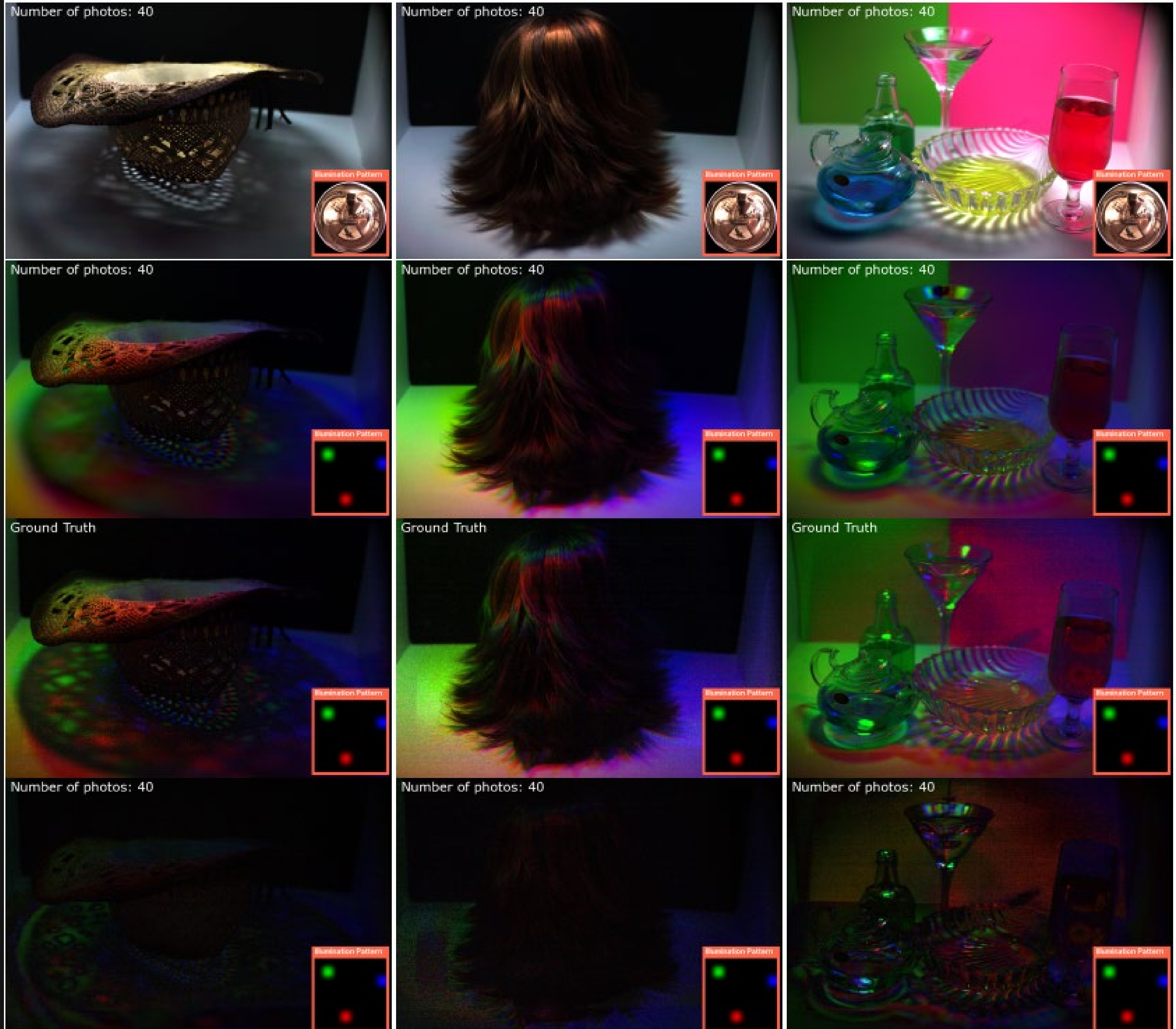
(approximate)
principal eigenvector

How would you measure the light transport matrix T?



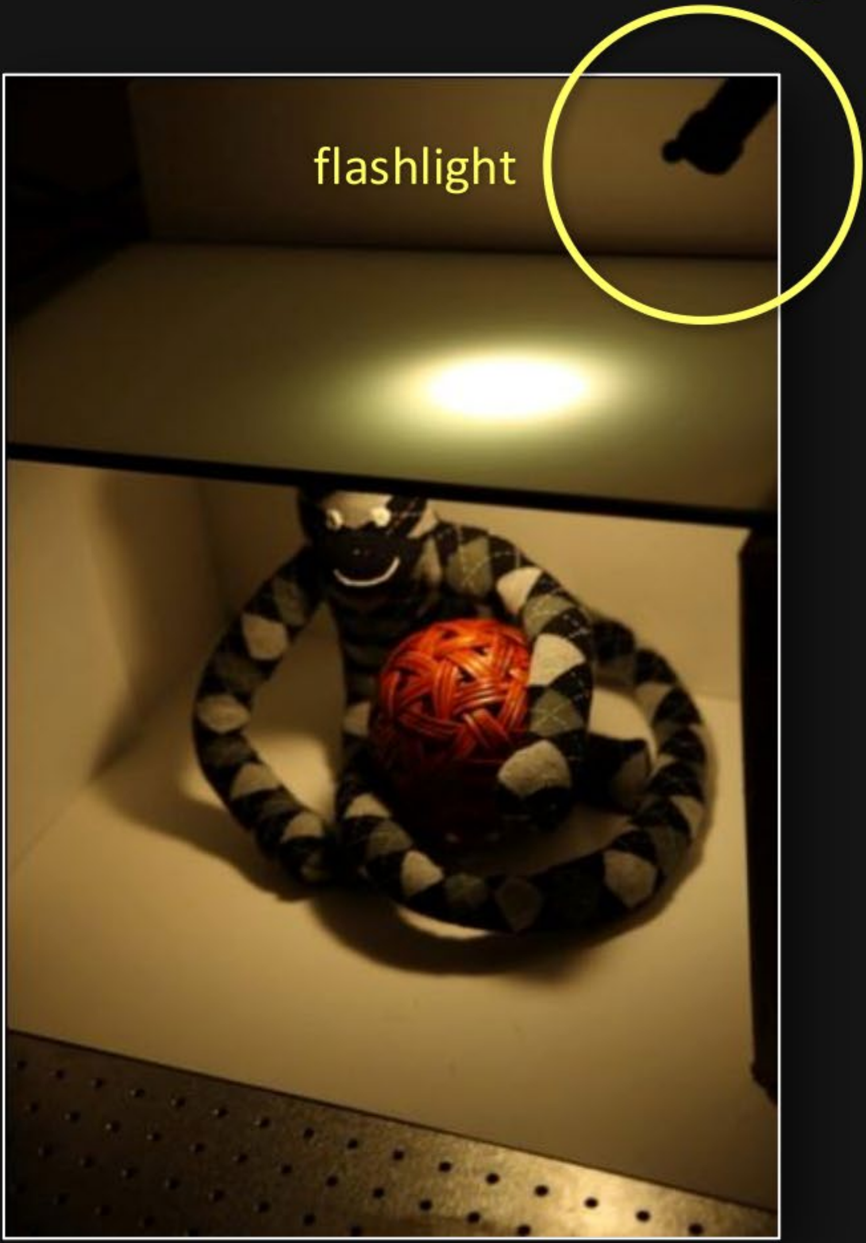
Alternative approach: use optical eigendecomposition to form a low-rank approximation to the light transport matrix.

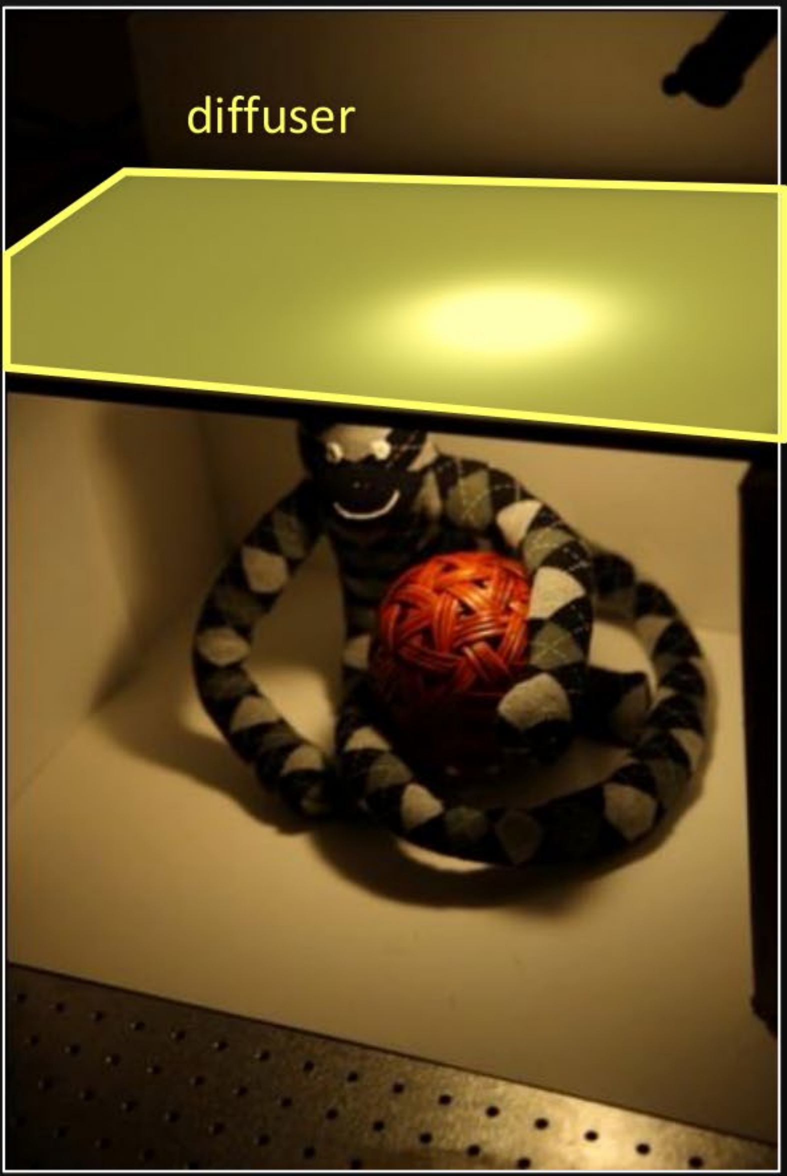
- How many photos do we need to capture?

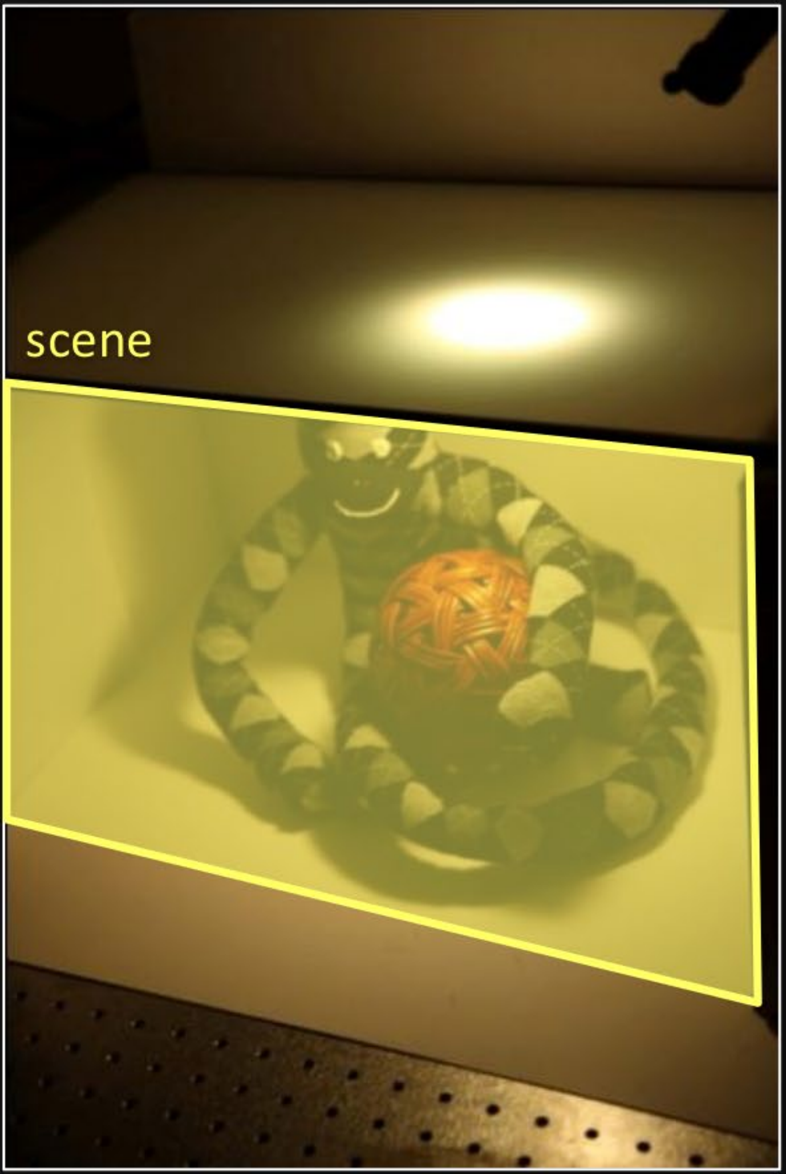


Number of photos: 40
Number of photos: 40
Number of photos: 40
Illumination Pattern
Number of photos: 40
Number of photos: 40
Number of photos: 40
Ground Truth
Ground Truth
Ground Truth
Number of photos: 40
Number of photos: 40
Number of photos: 40

Inverse transport


flashlight


diffuser

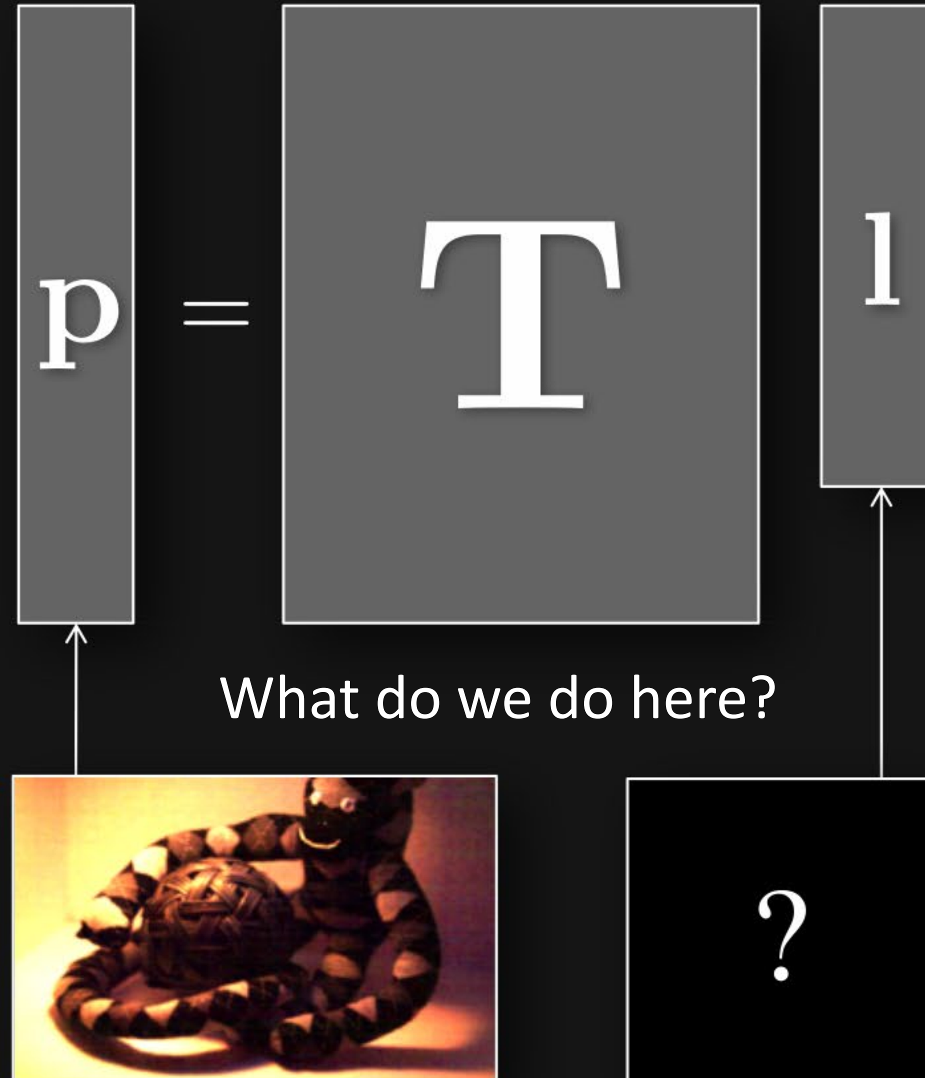

scene

input photo

How do you solve this problem if you know the light transport matrix T?

input photo

illumination



input photo

illumination

$$\mathbf{p} = \mathbf{T}\,\mathbf{l}$$

What if T is not invertible?

input photo

?

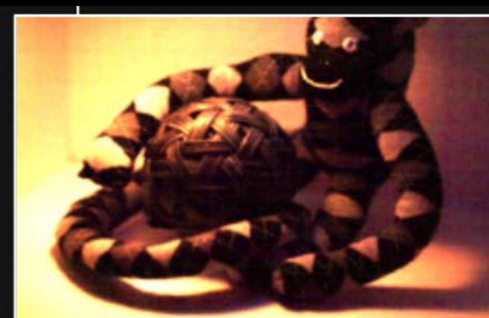
illumination

numerical goal

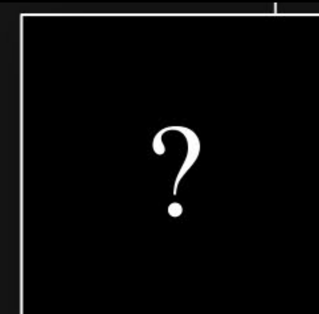
given photo $\mathbf{p}$, find illumination $\mathbf{l}$ that minimizes

$$\left\| \mathbf{T} \, \mathbf{l} - \mathbf{p} \right\|_2$$

How do you usually solve for l when T is large?



input photo



illumination

Reminder: gradient descent

Given the loss function:

$$E(f) = \|Gf - v\|^2$$

Minimize by iteratively computing:

$$f^{i+1} = f^{i} - \eta^{i} r^{i}, \quad r^{i} = v - Af^{i}, \ \eta^{i} = \frac{\left(r^{i}\right)^{T} r^{i}}{(r^{i})^{T} A r^{i}} \qquad \text{for i} = 0, 1, \ldots, \text{N}$$

Is this cheaper than the pseudo-inverse approach?

- We never need to compute A, only its products with vectors f, r.
- Vectors f, r are images.
- Because A is the *Laplacian matrix*, these matrix-vector products can be efficiently computed using *convolutions* with the *Laplacian kernel*.

Gradient descent in this case

Given the loss function:

$$E(f) = \|Gf - v\|^2$$

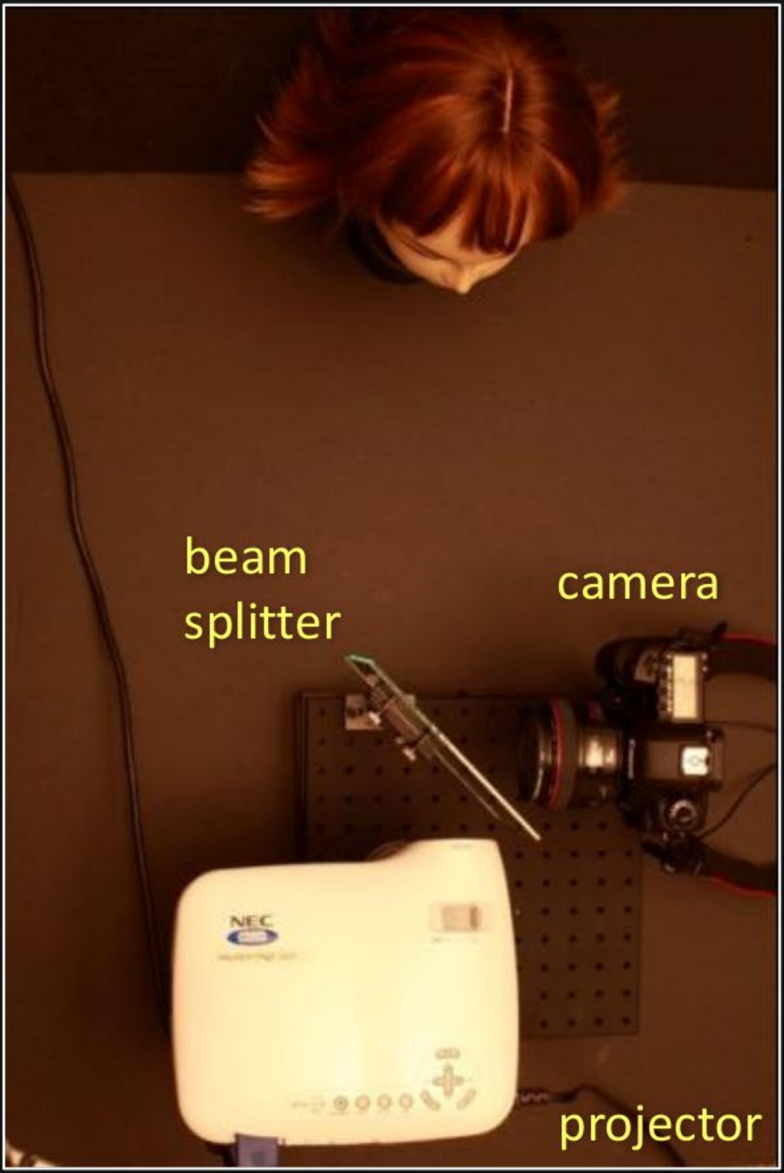
Minimize by iteratively computing:

$$f^{i+1} = f^i - \eta^i r^i, \quad r^i = v - Af^i, \ \ \eta^i = \frac{\left(r^i\right)^T r^i}{(r^i)^T A r^i} \qquad \text{for i} = 0, 1, \ldots, \text{N}$$

Is this cheaper than the pseudo-inverse approach?

- <u>We never need to compute A, only its products with vectors f, r.</u>
- ~~Vectors f, r are images.~~ What are f, r in this case?
- ~~Because A is the *Laplacian matrix*, these matrix-vector products can be efficiently computed using *convolutions* with the *Laplacian kernel*.~~ How do we compute matrix-vector products efficiently in this case?

inverting light transport



numerical goal

given photo $\mathbf{p}$, find illumination $\mathbf{l}$ that minimizes

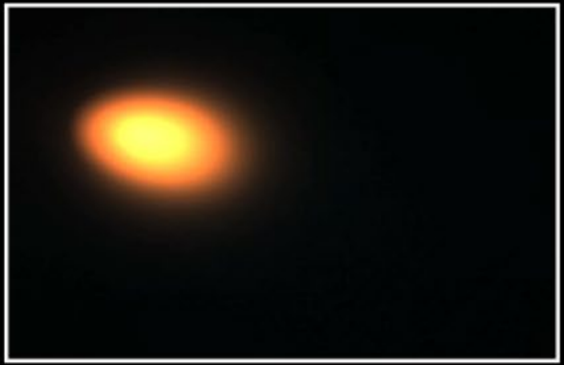
$$\left\| \mathbf{T}\,\mathbf{l} - \mathbf{p} \right\|_2$$

remarks

- $\mathbf{T}$ low-rank or high-rank
- $\mathbf{T}$ unknown & not acquired
- illumination sequence will be specific to input photo

inverting light transport

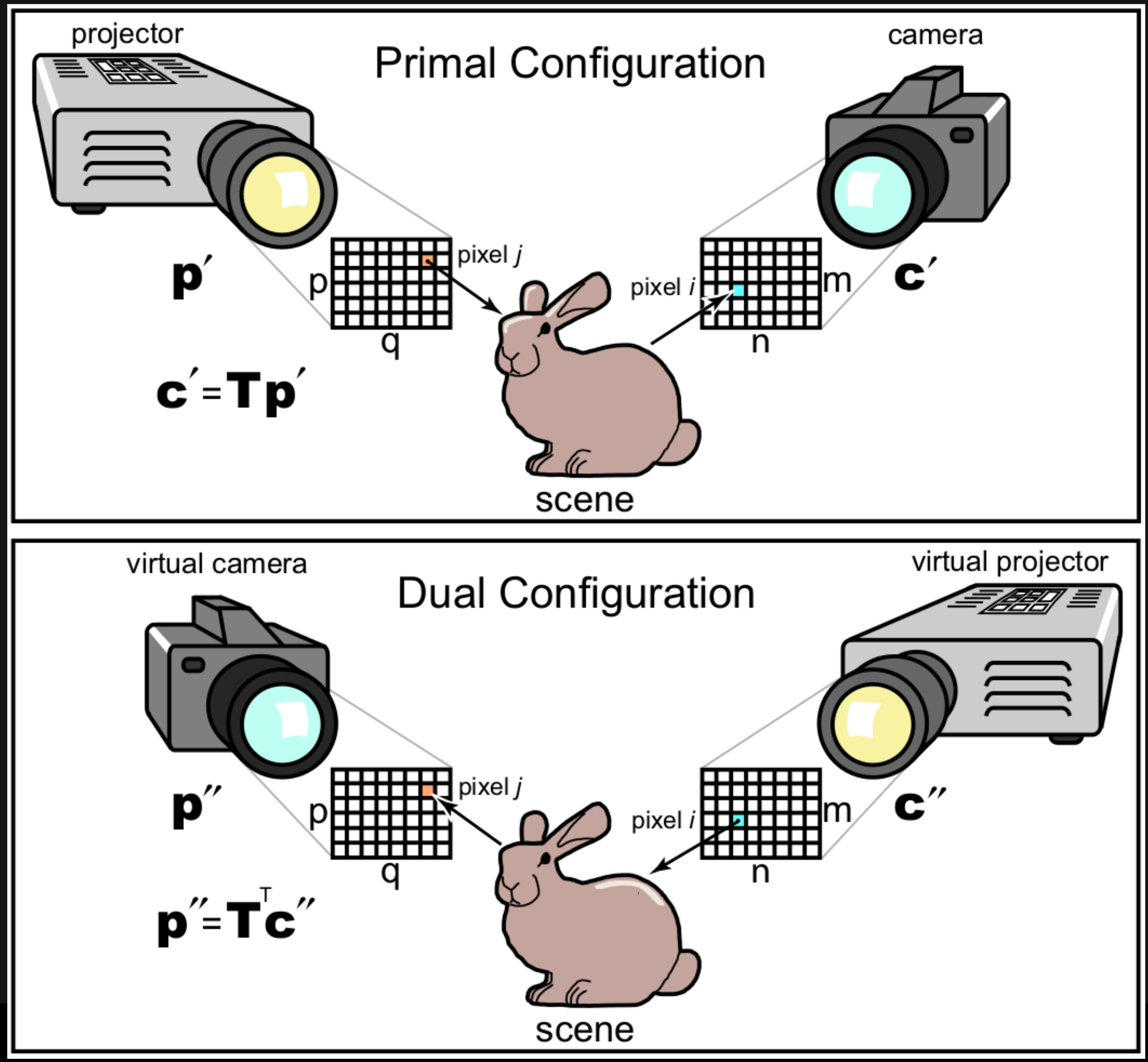
input photo



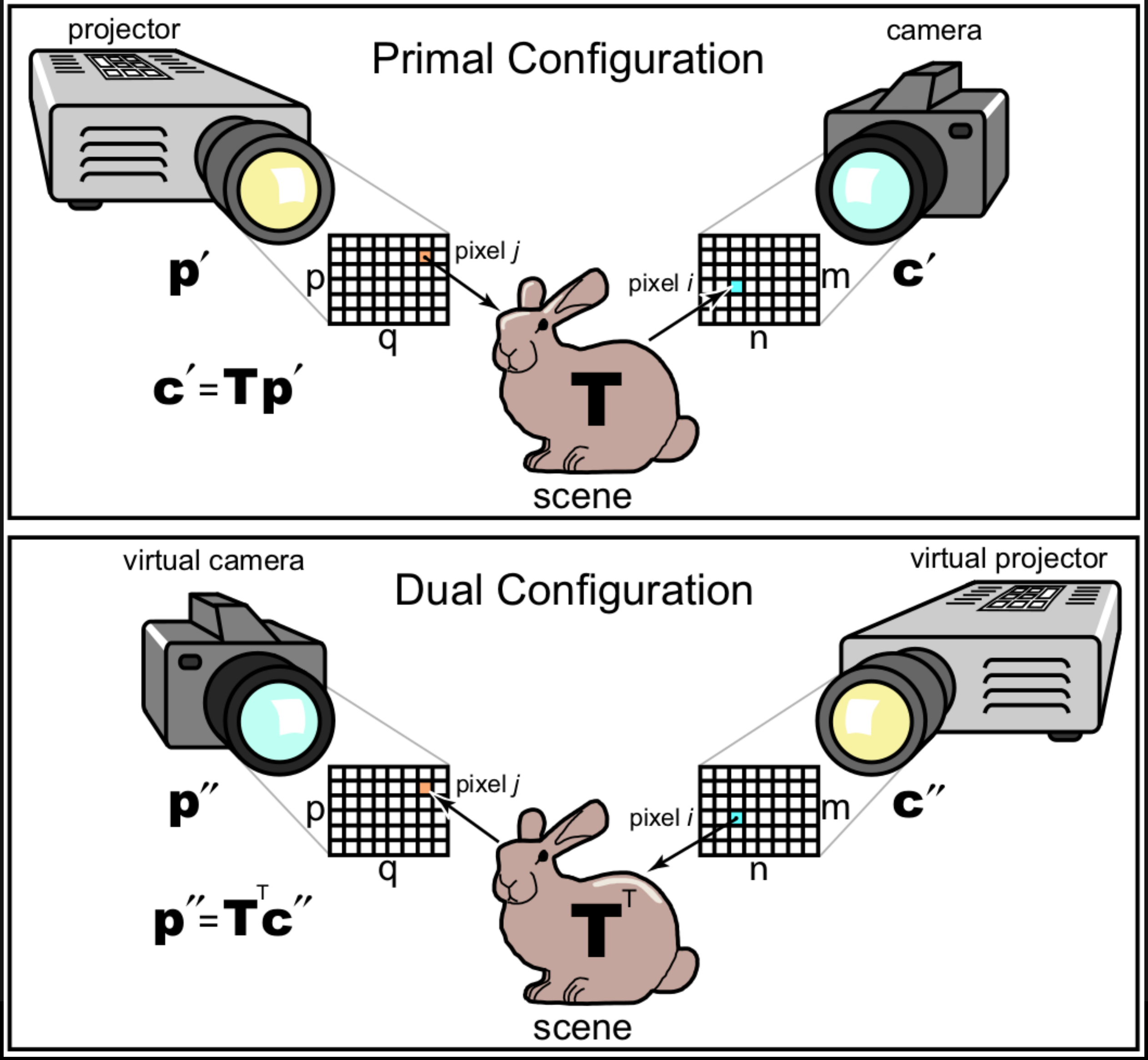
actual illumination

Dual photography

How do the light transport matrices for these two scenes relate to each other?



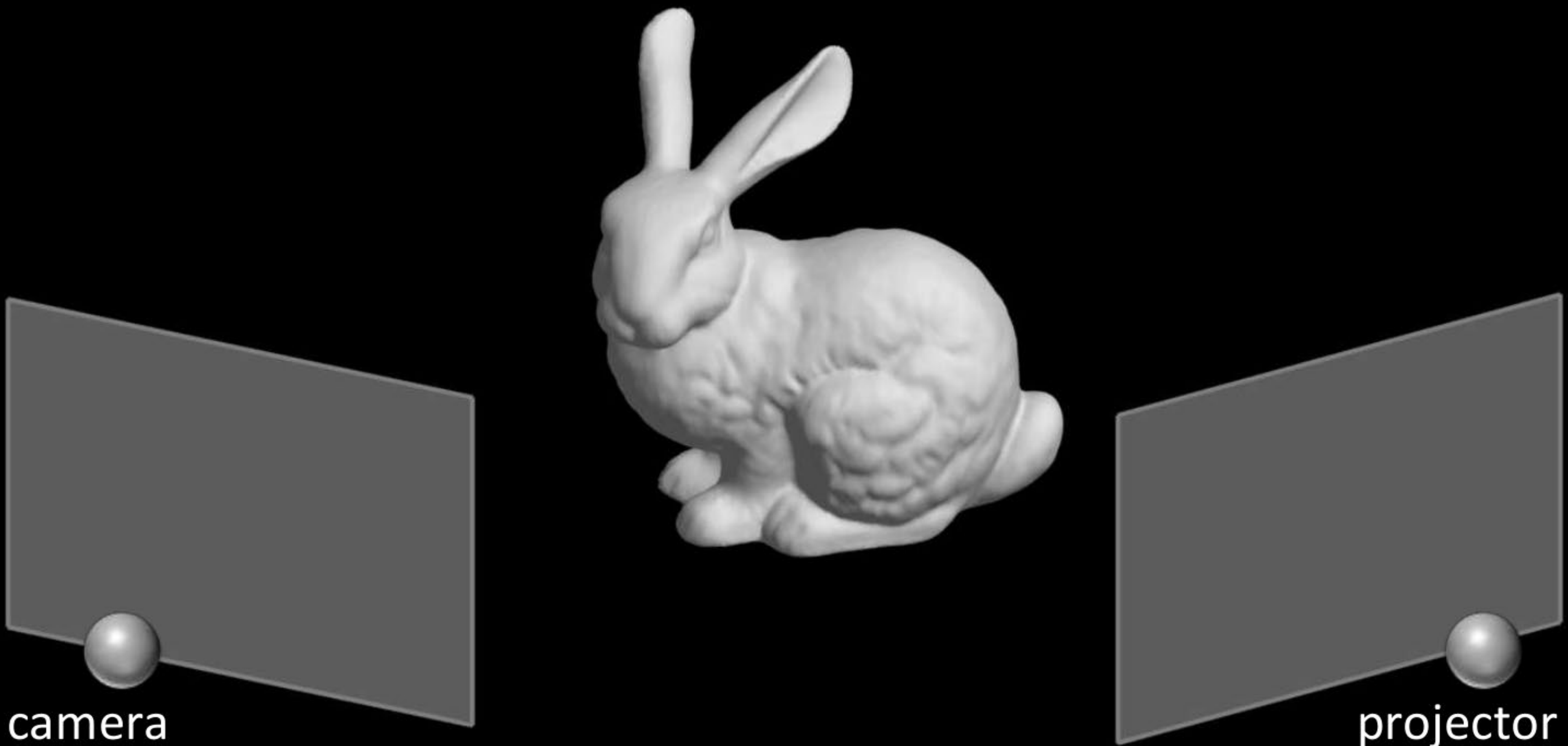
Helmholtz reciprocity: The two matrices are the transpose of each other.



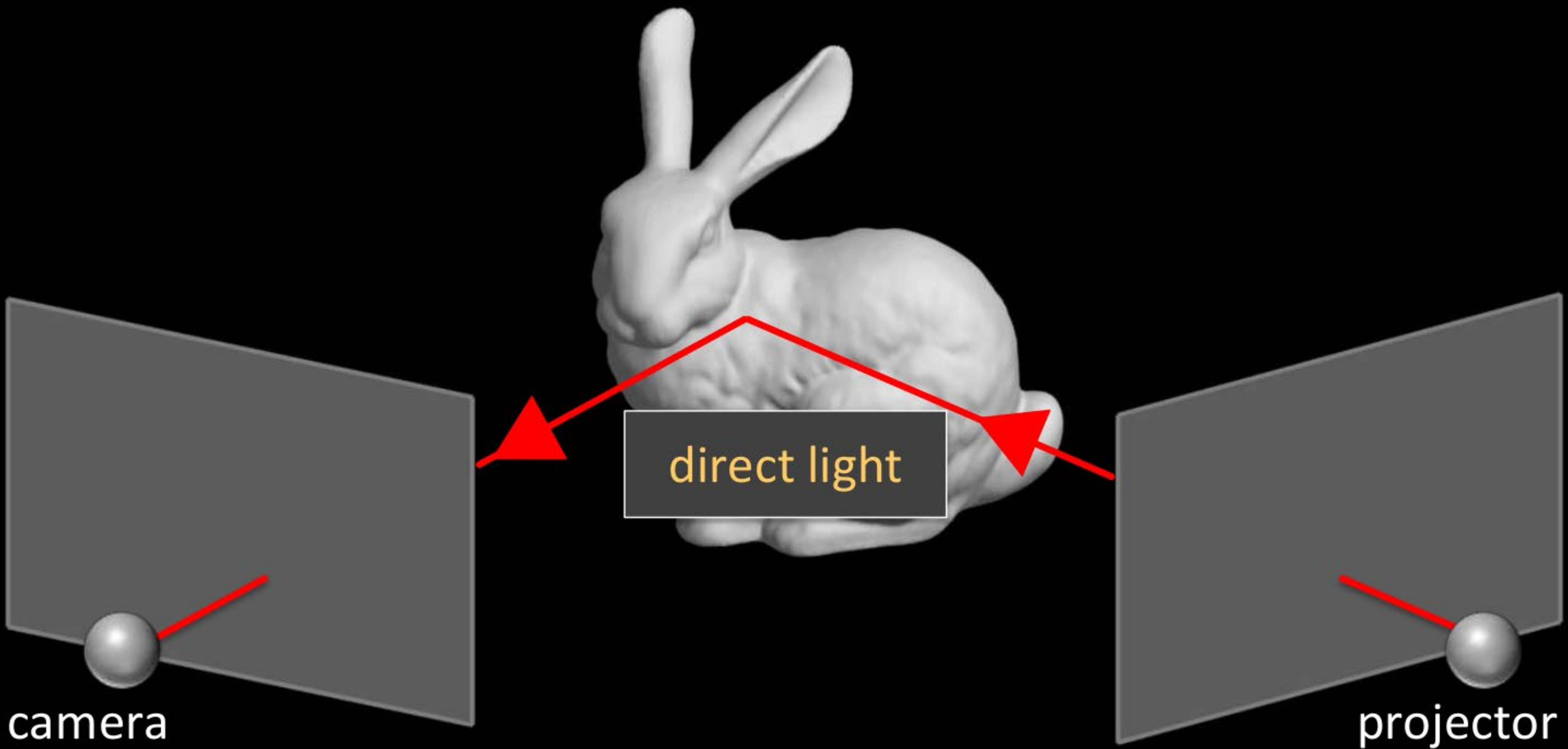
Great demonstration: https://www.youtube.com/watch?v=eV58Ko3iFuI

Direct-global separation using epipolar probing

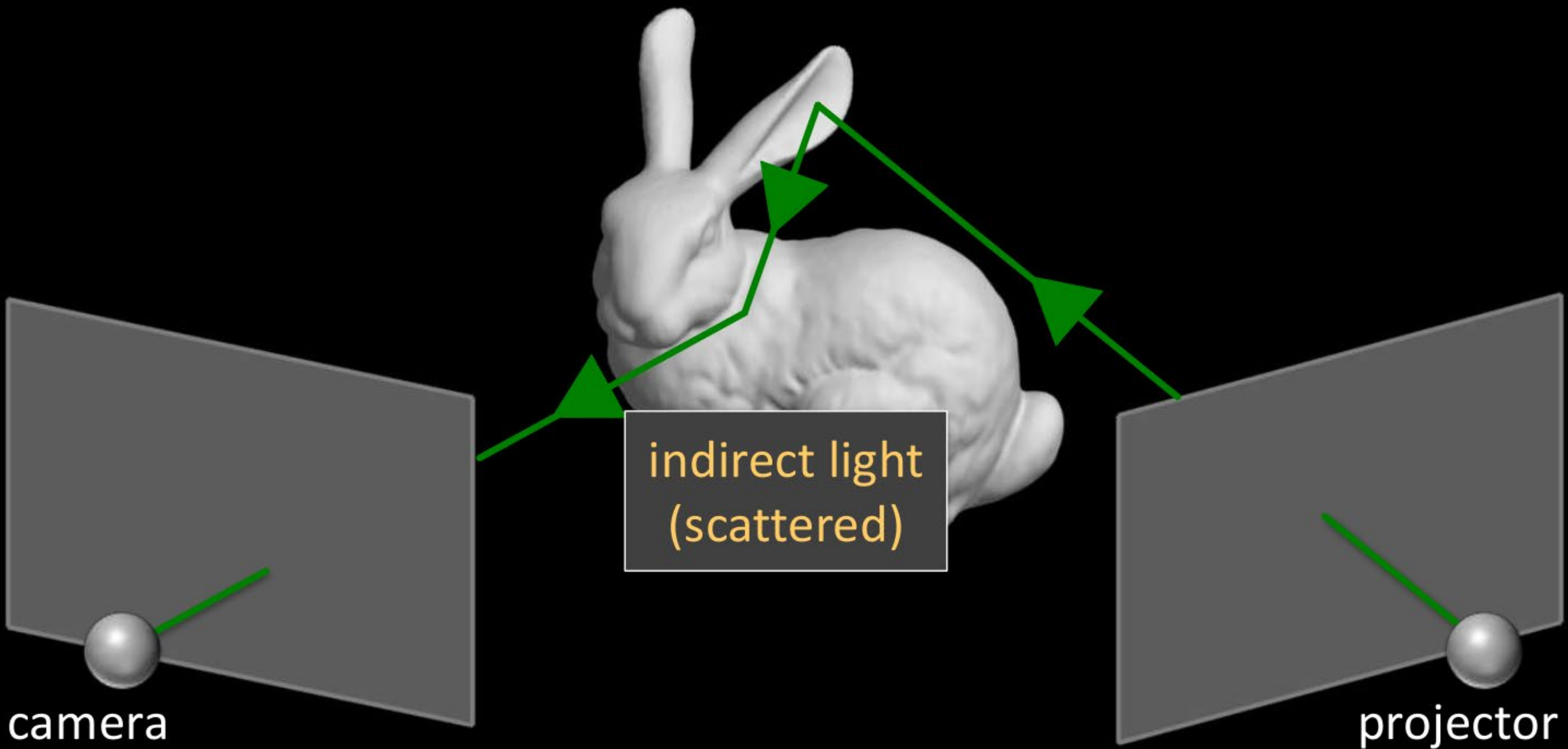
basic light paths



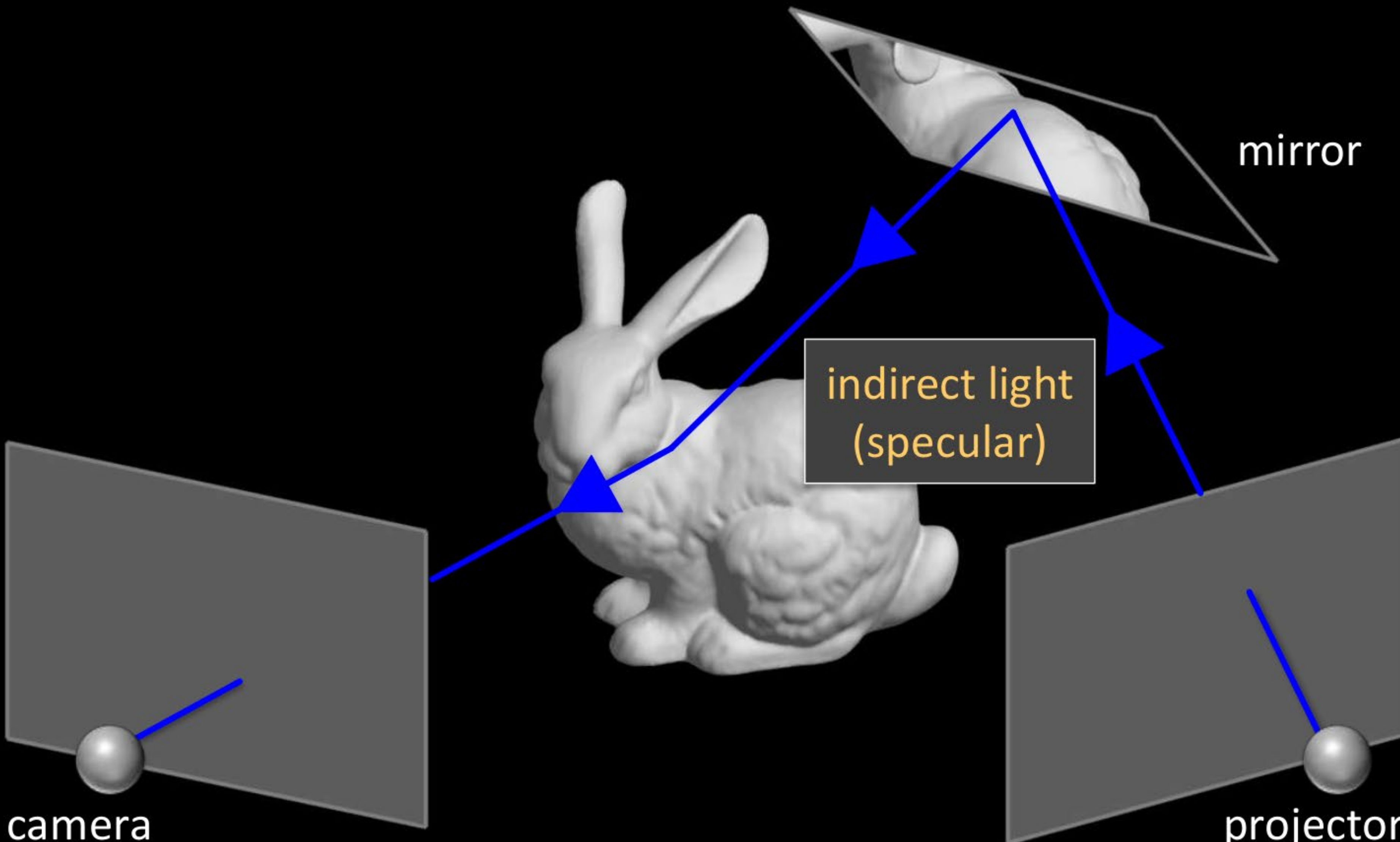
basic light paths



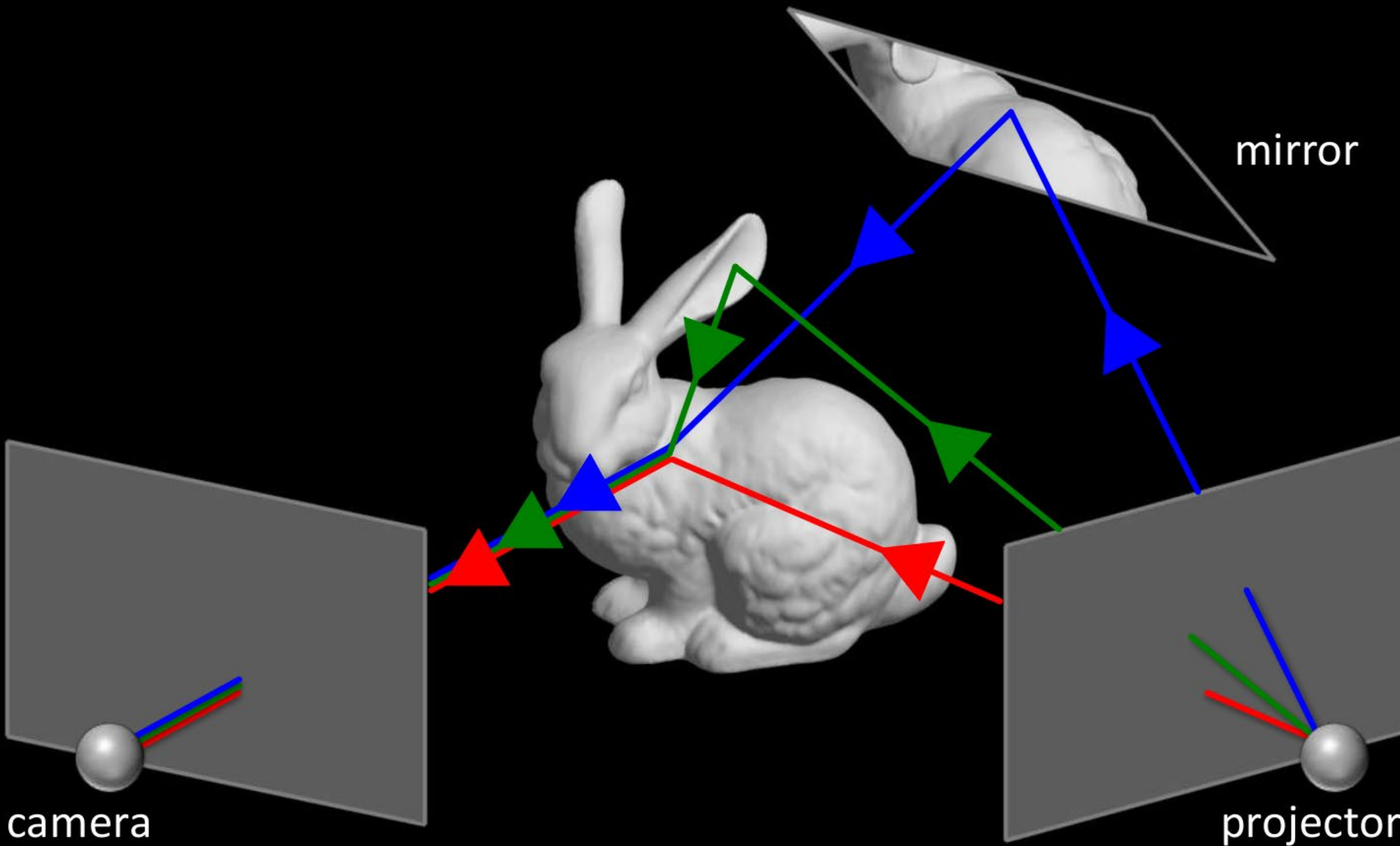
basic light paths



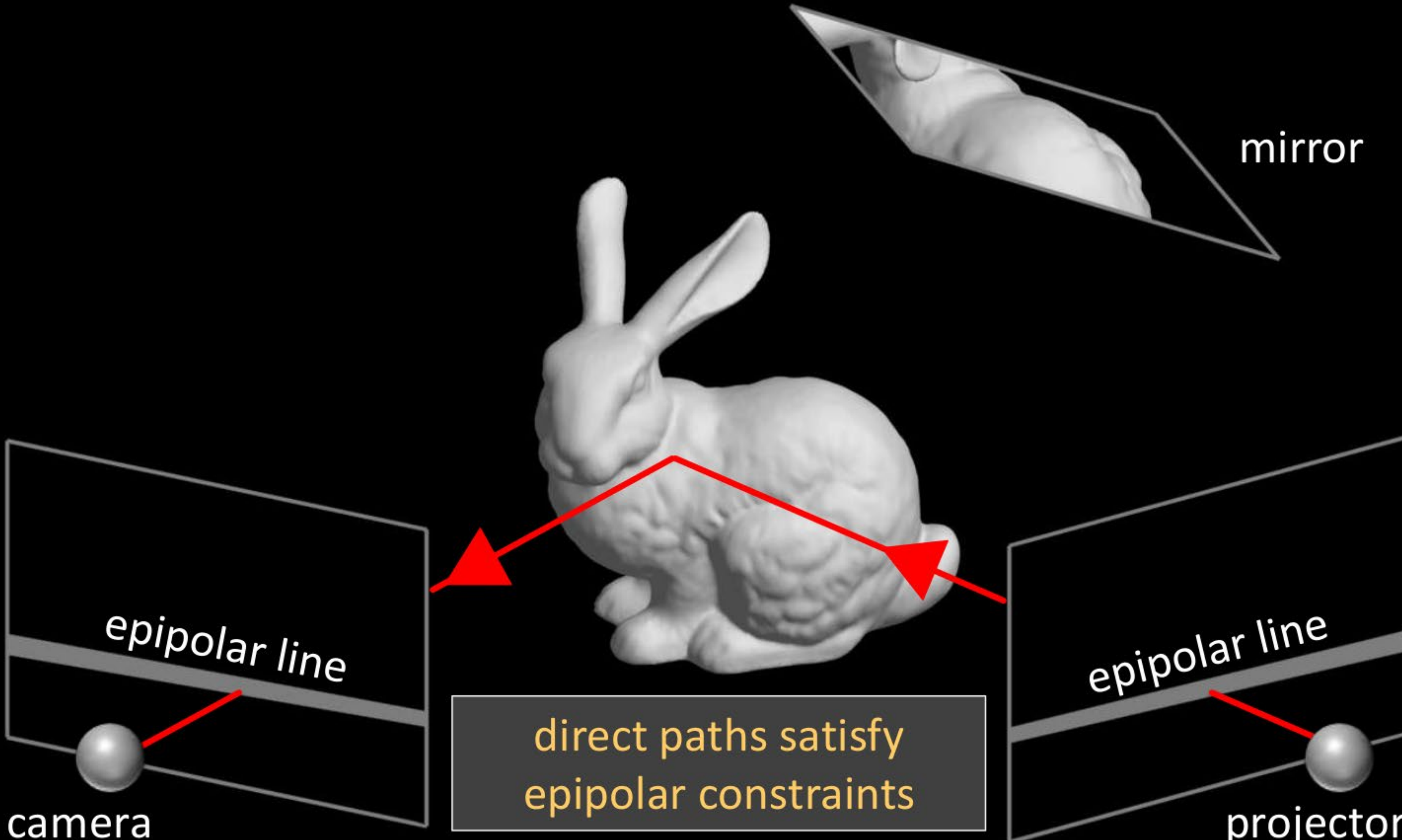
basic light paths



basic light paths

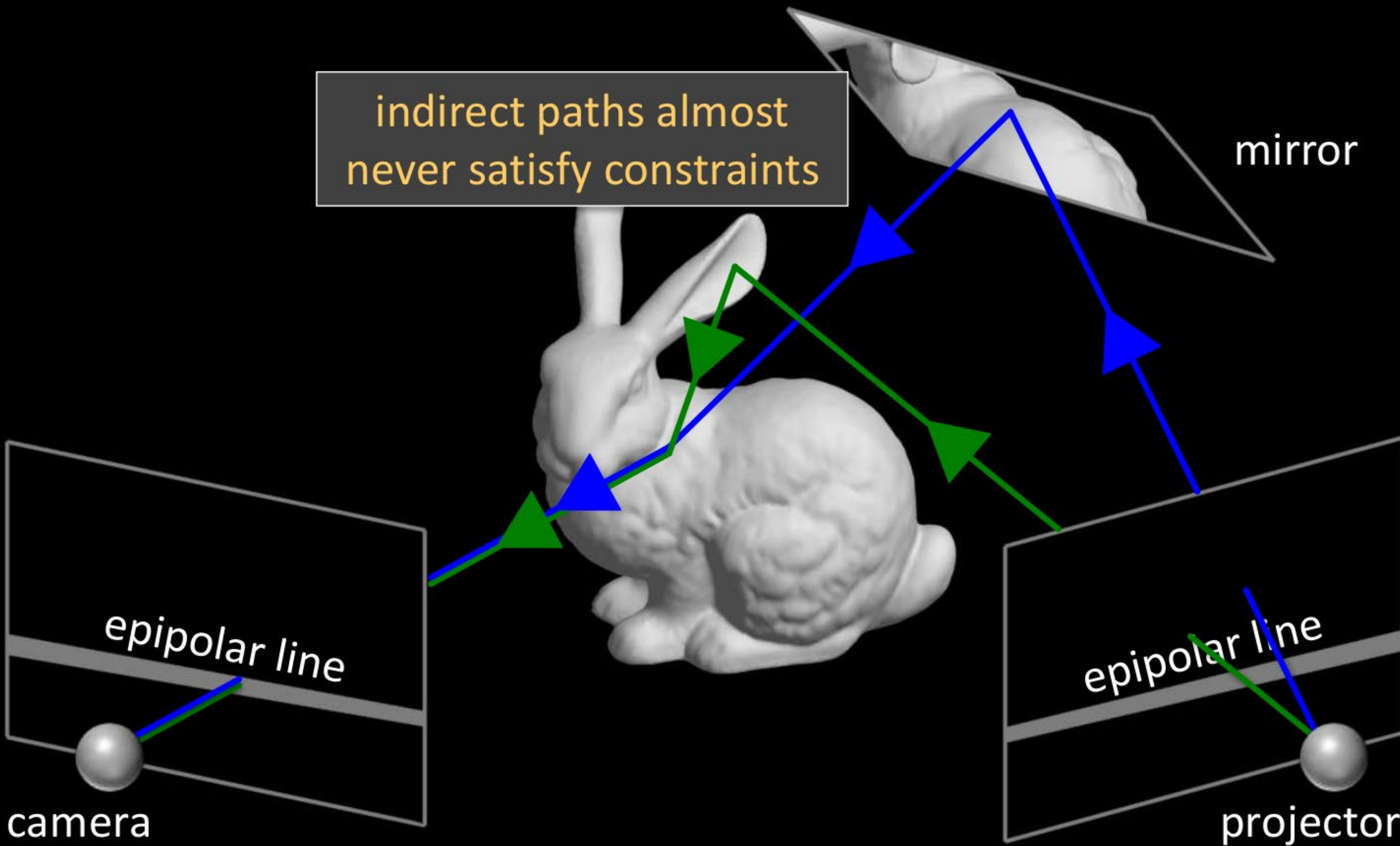


epipolar constraint & light transport

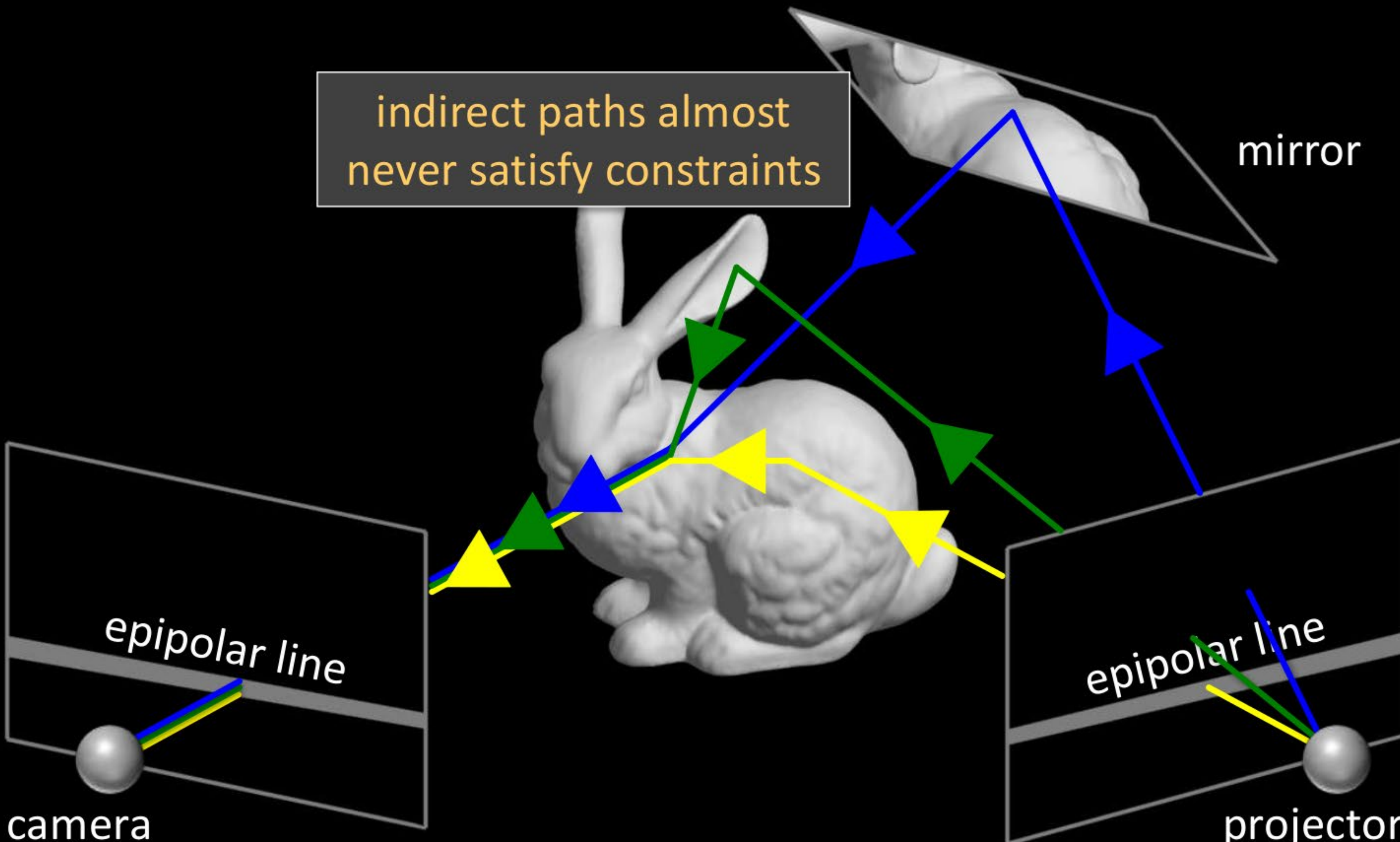


direct paths satisfy
epipolar constraints

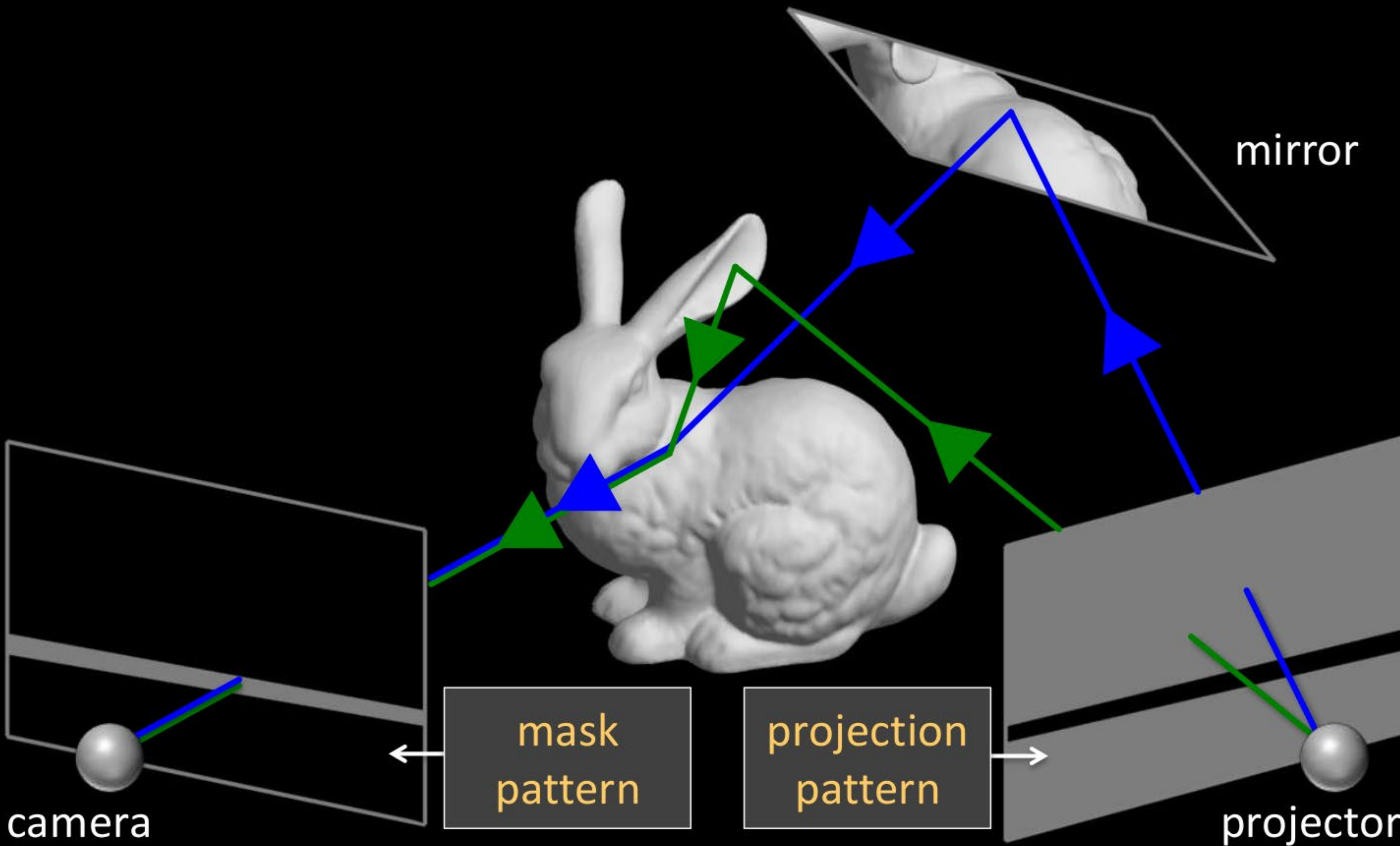
epipolar constraint & light transport



epipolar constraint & light transport



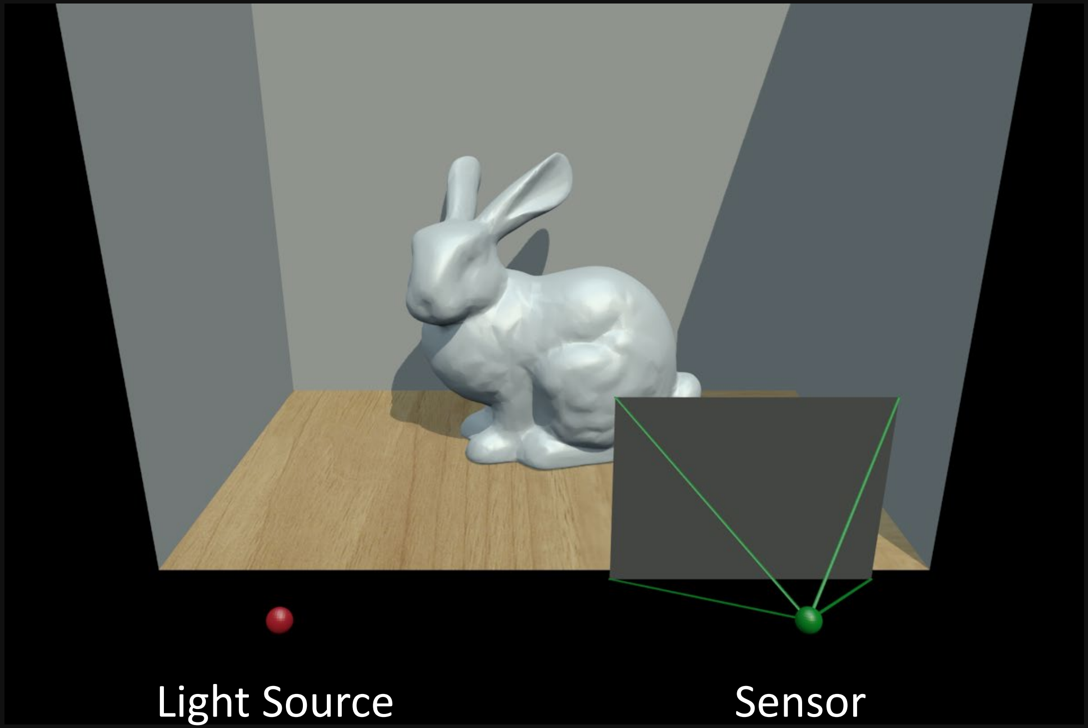
blocking epipolar paths with patterns & masks



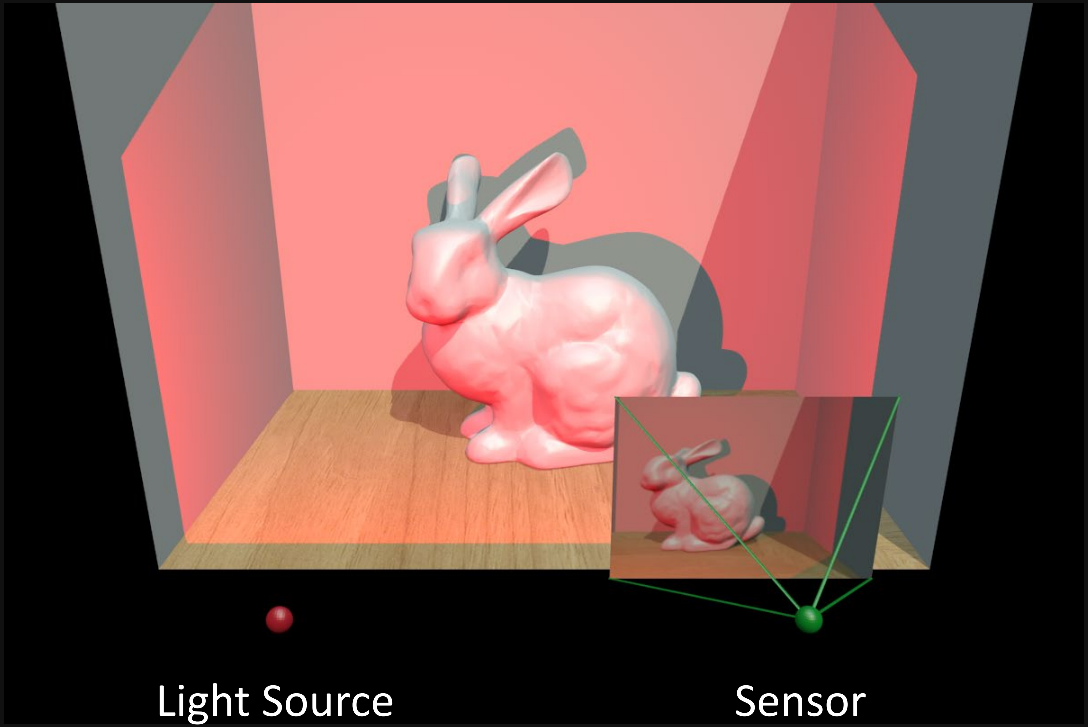
Energy-efficient epipolar imaging

Energy-efficient transport parsing

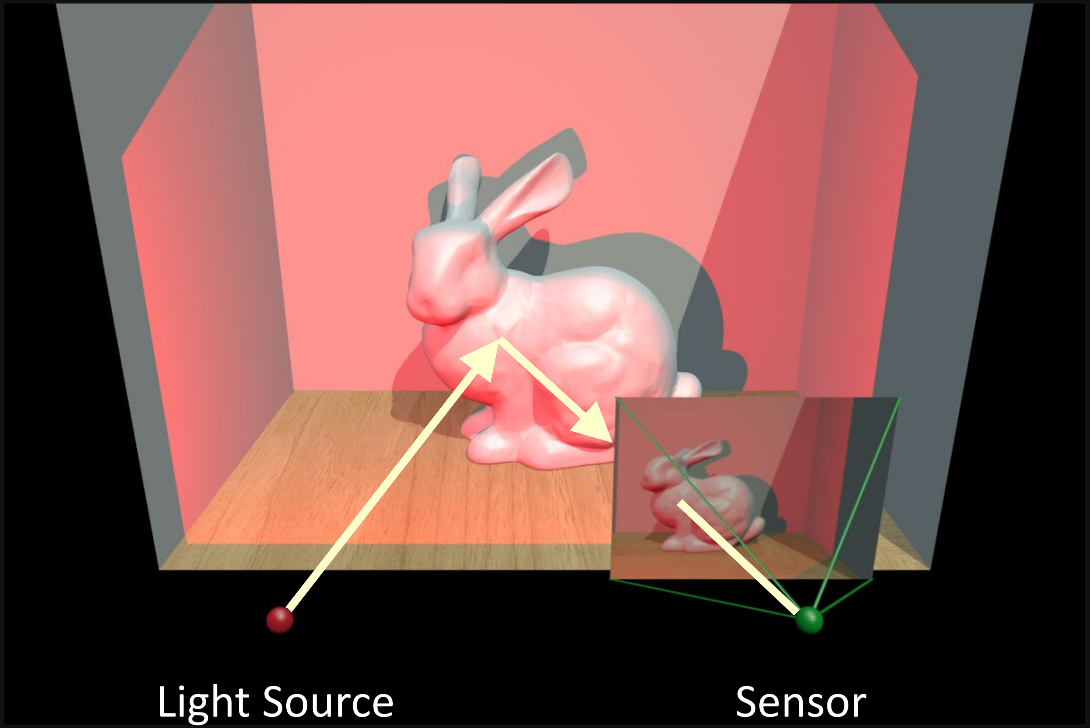
Regular Imaging



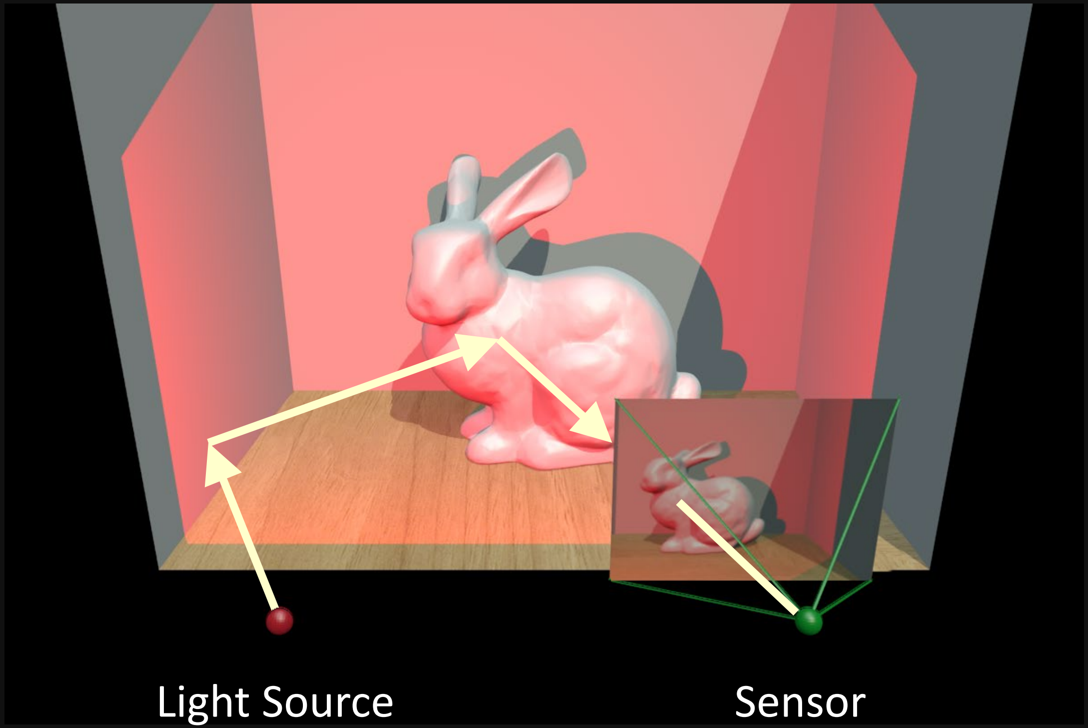
Regular Imaging



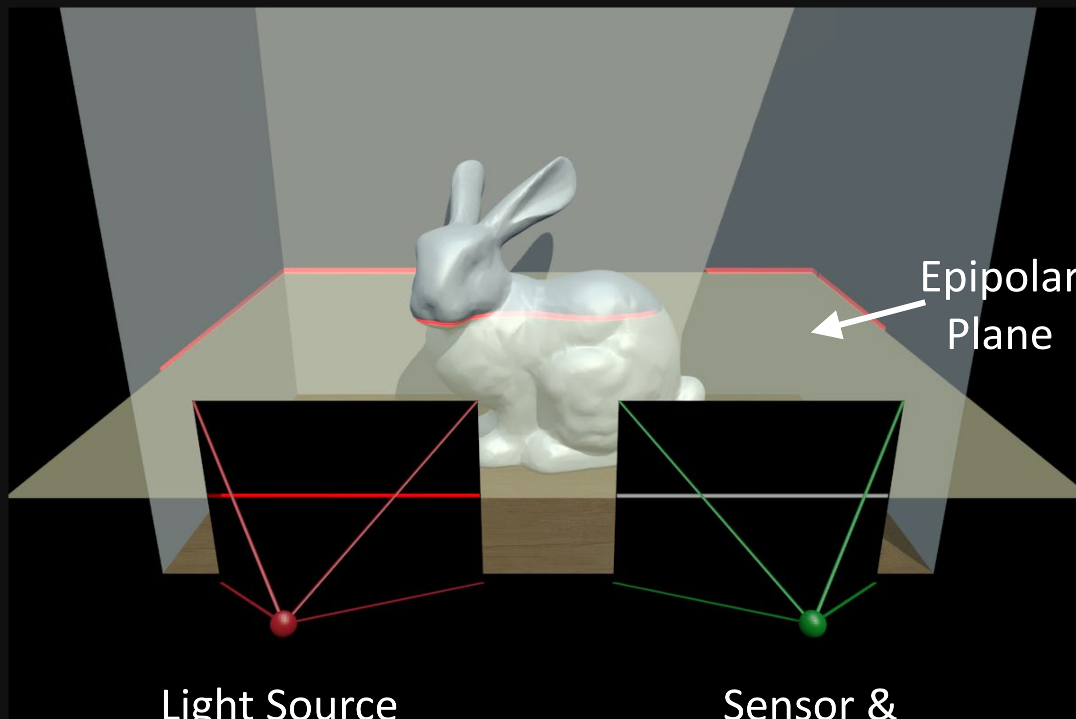
Regular Imaging



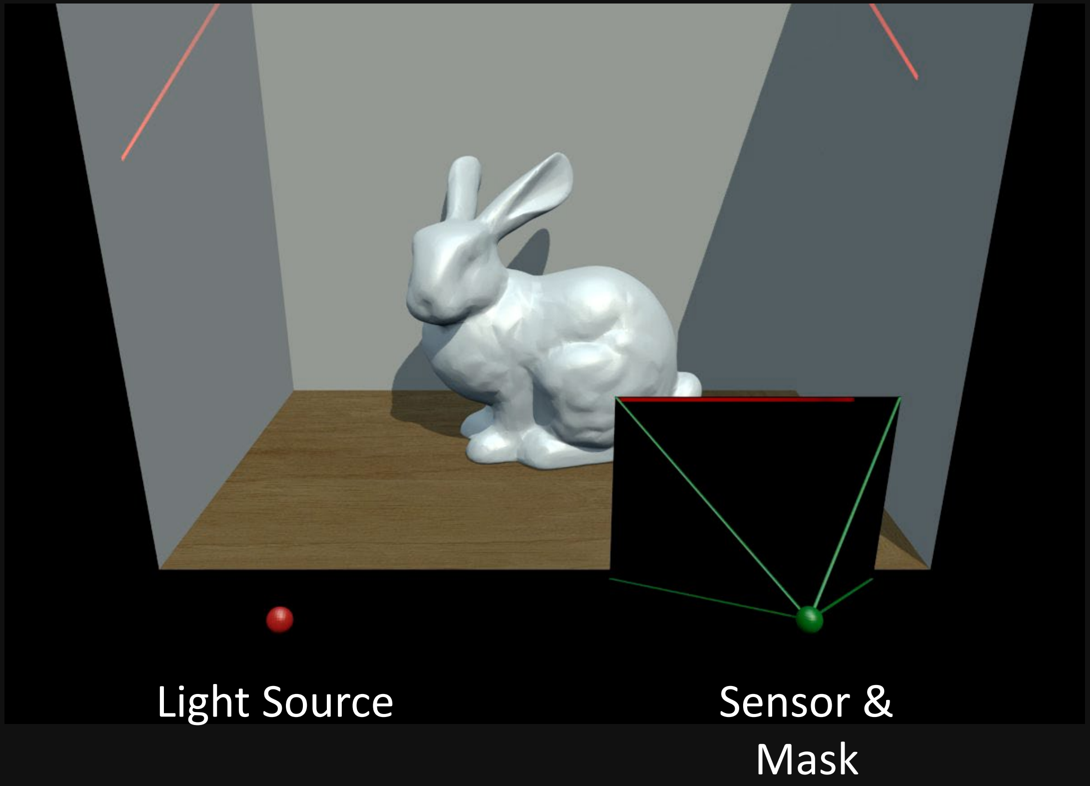
Regular Imaging



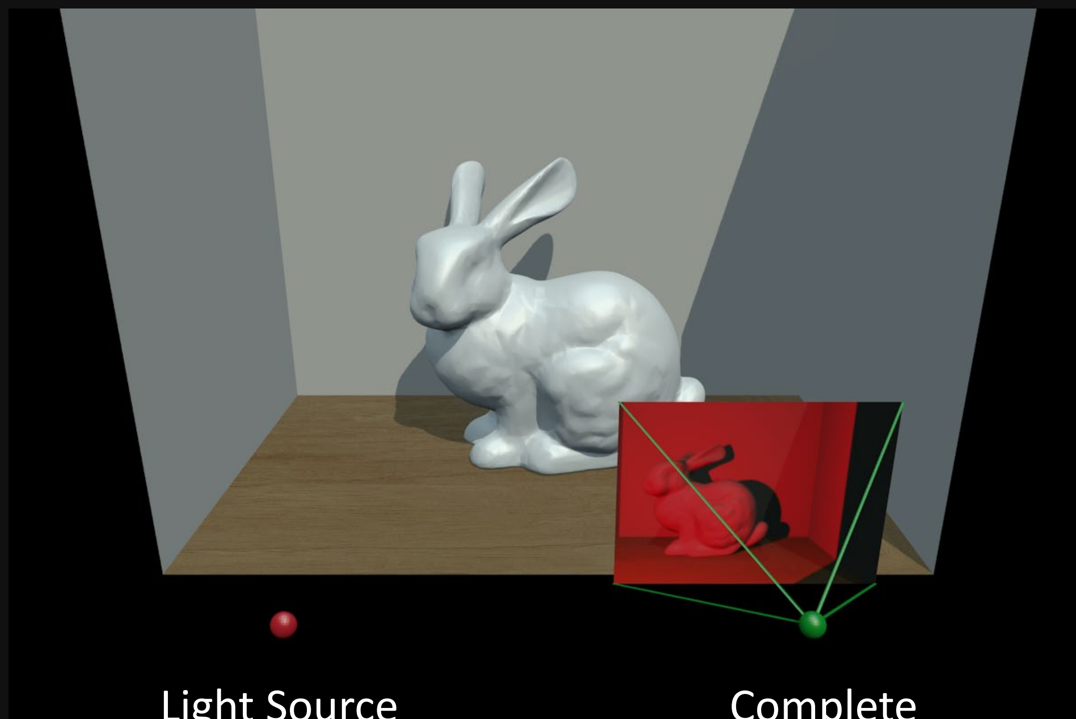
Epipolar Imaging



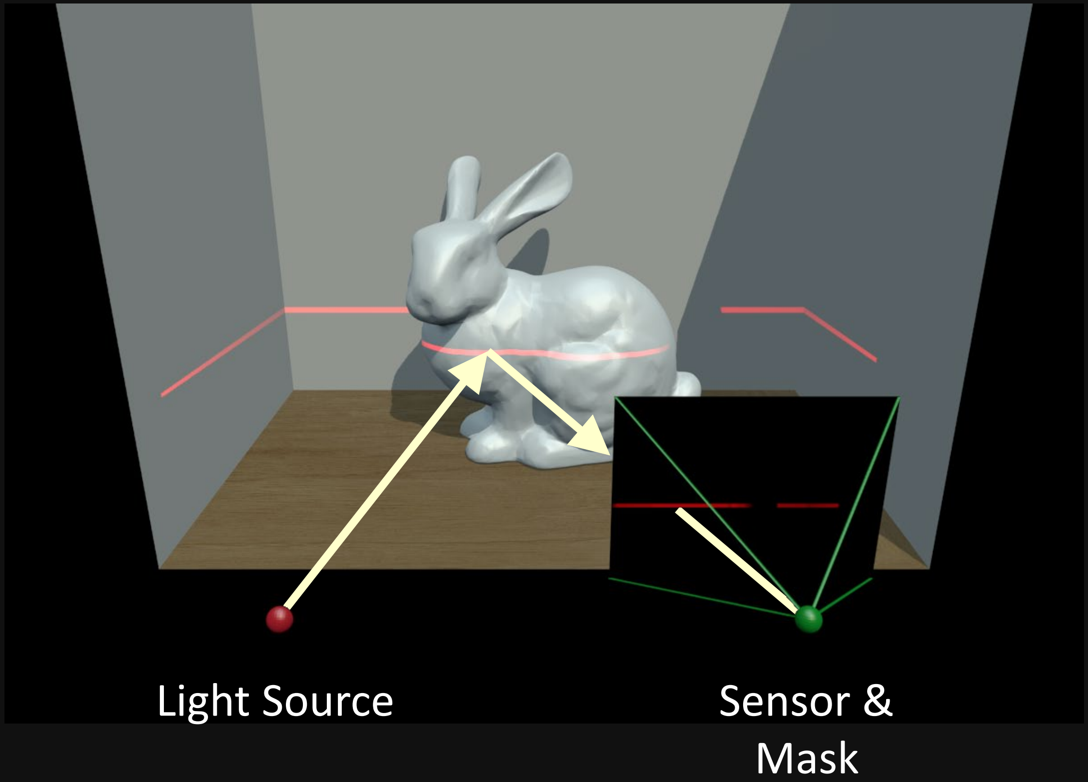
Epipolar Imaging



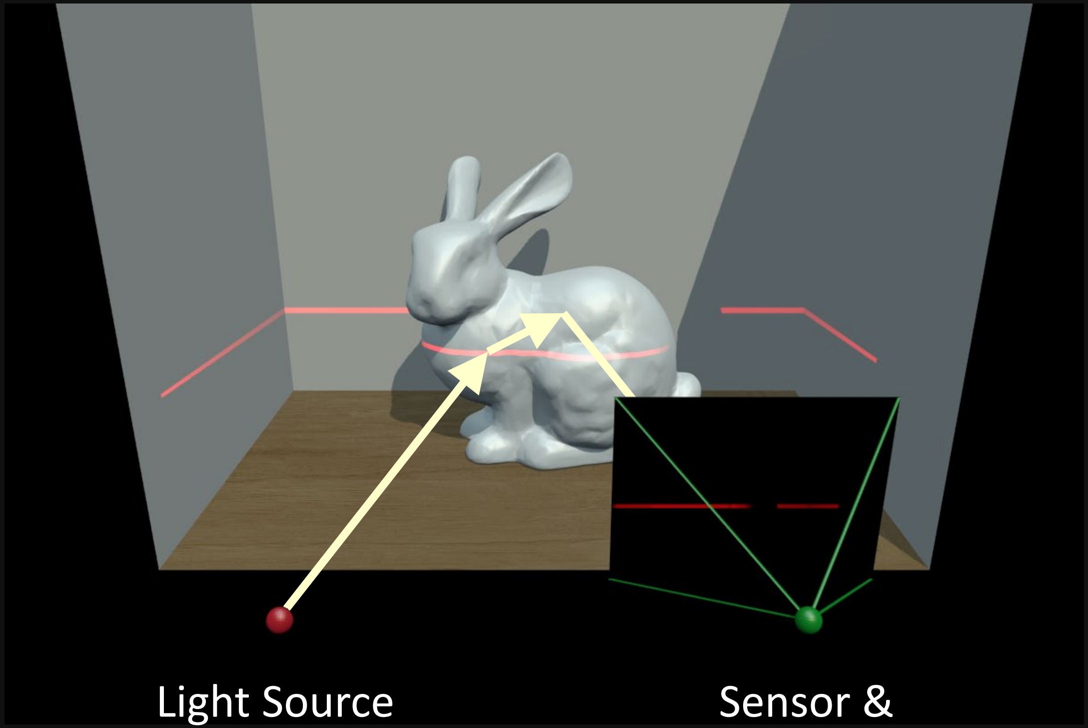
Epipolar Imaging



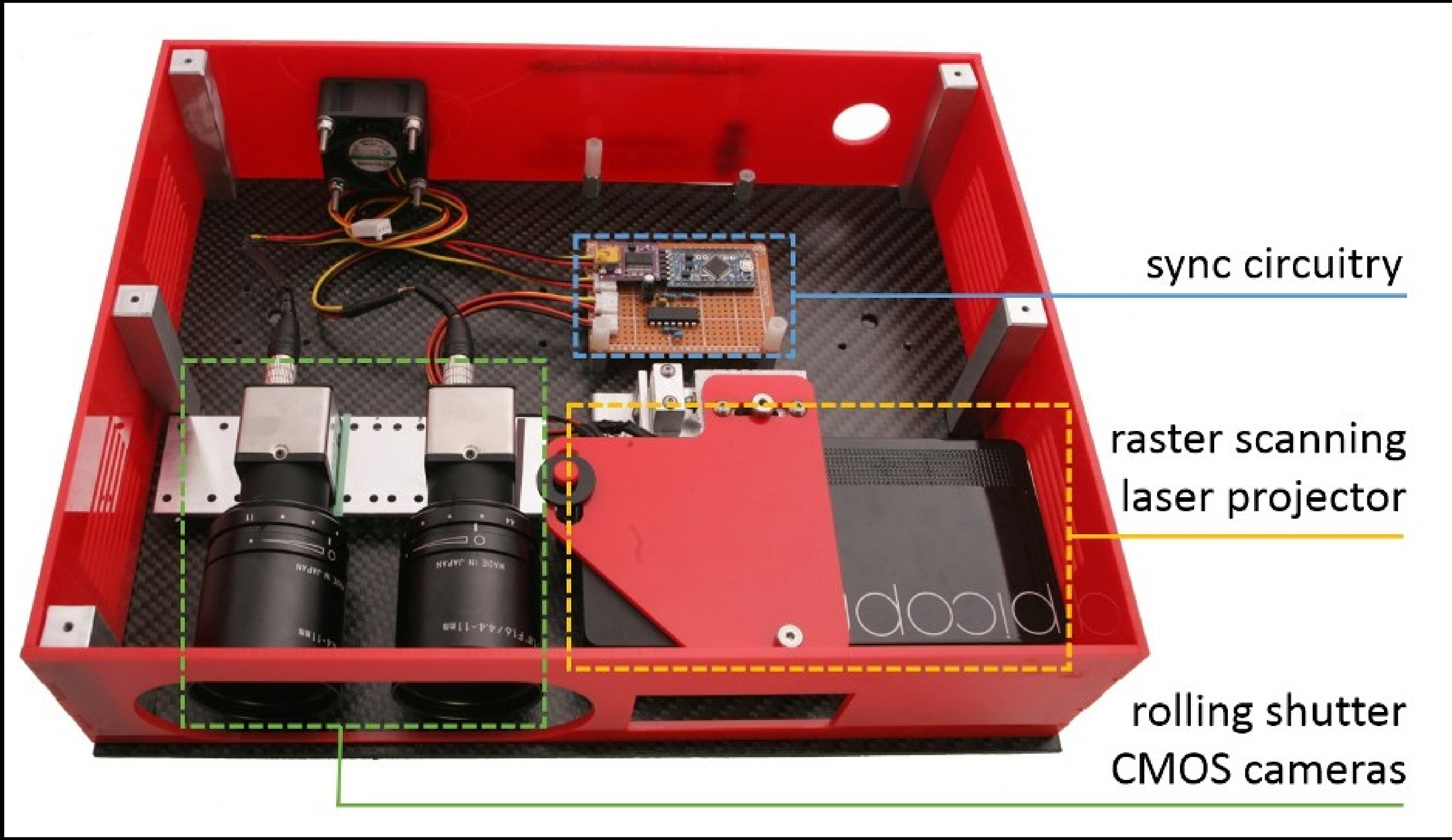
Epipolar Imaging



Epipolar Imaging



Energy-efficient transport parsing



all paths

planar (mostly direct)

non-planar (always indirect)

a great deal of indirect transport occurs in many common LOS scenes all paths

planar (mostly direct)

non-planar (always indirect)

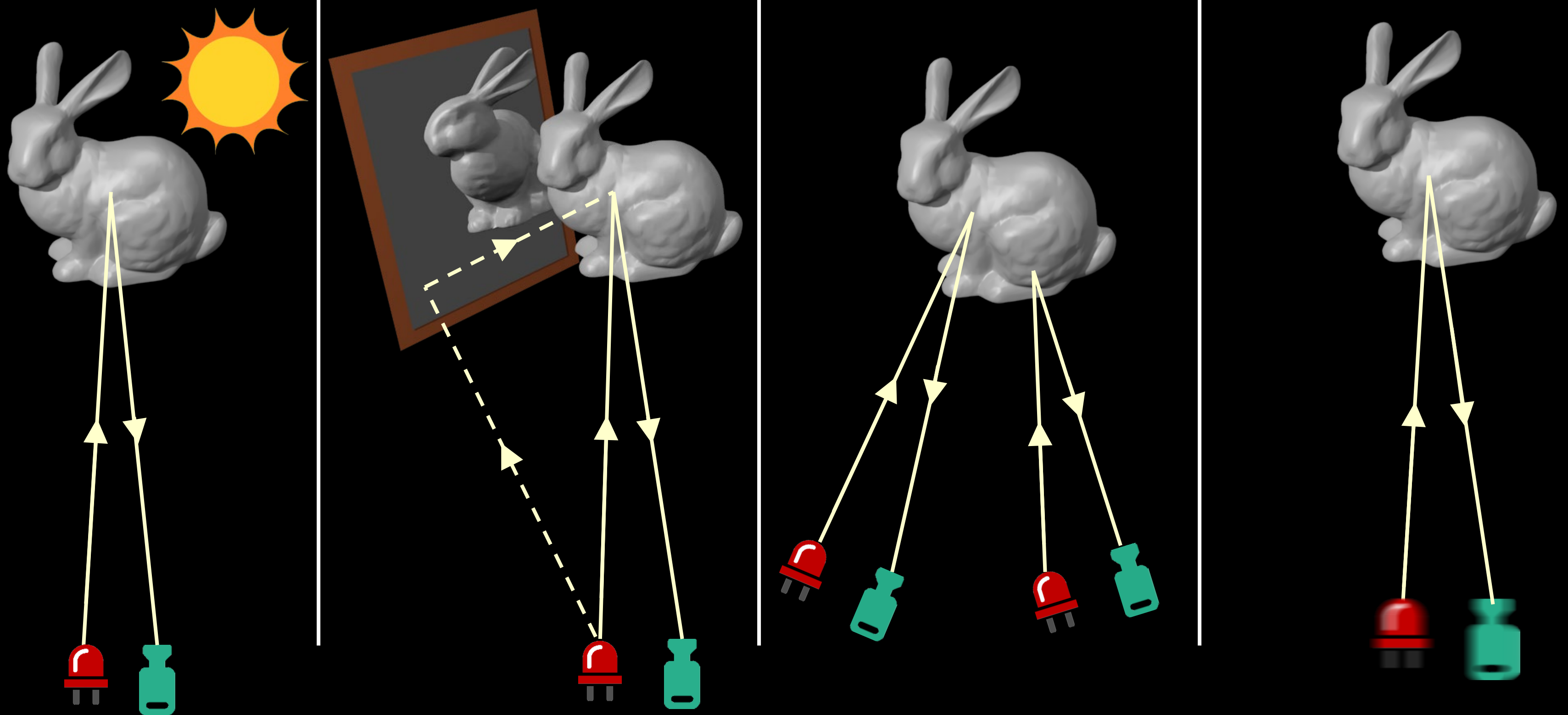
Benefits of Epipolar ToF Imaging

Ambient Light

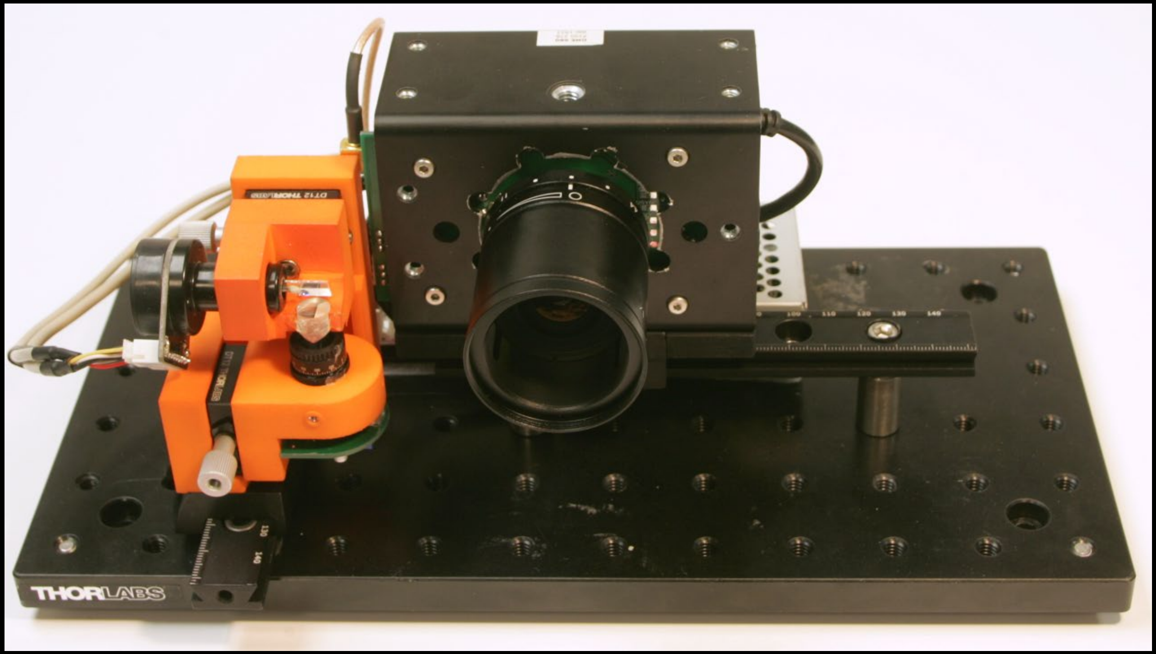
Multi-Path Interference

Multi-Device Interference

Camera Motion

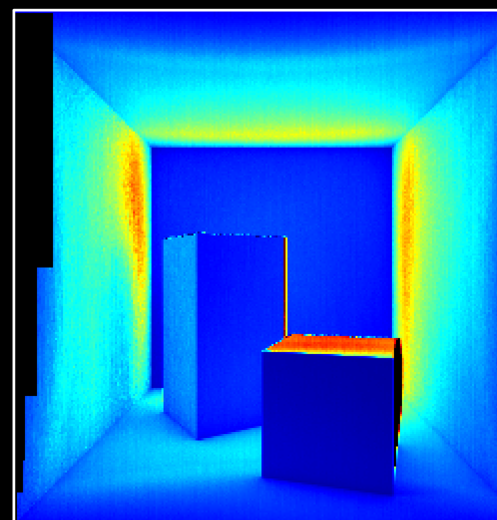


Epipolar ToF Prototype

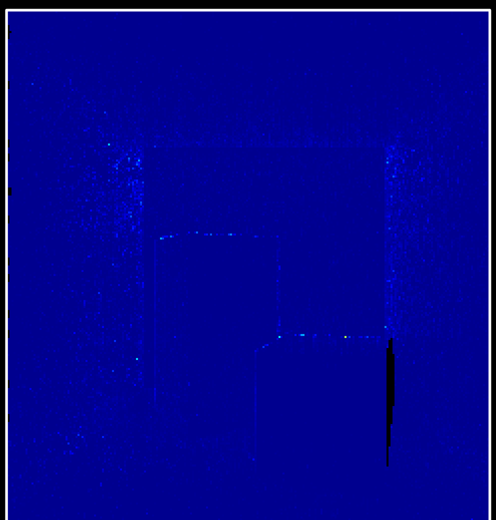


Epipolar ToF and Global Illumination

Depth Errors (in meters)



Regular ToF @ 30MHz

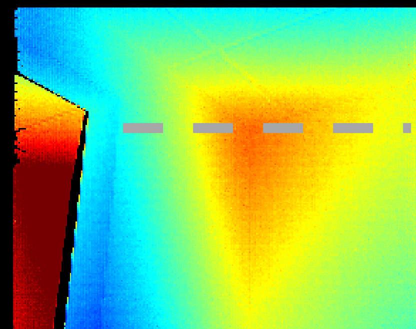


Epipolar ToF @ 30MHz

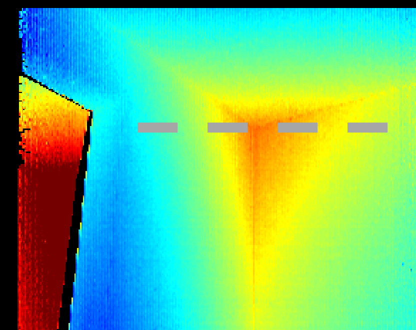
Epipolar ToF and Global Illumination

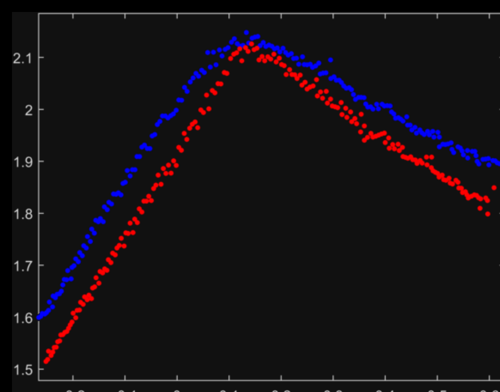

Corner of Room

Regular ToF



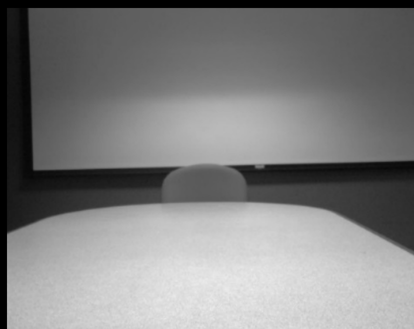
Epipolar ToF





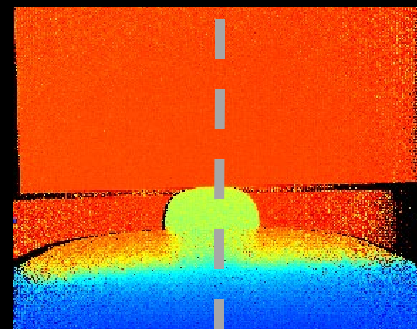
- Regular ToF
- Epipolar ToF

Epipolar ToF and Global Illumination

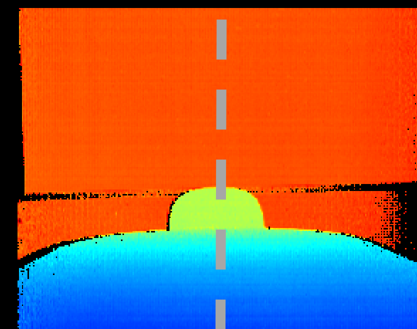


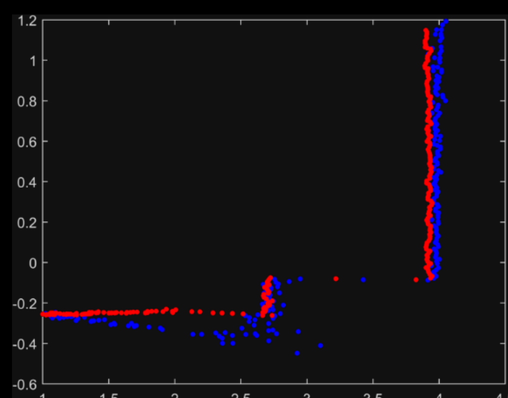
Conference Room

Regular ToF



Epipolar ToF



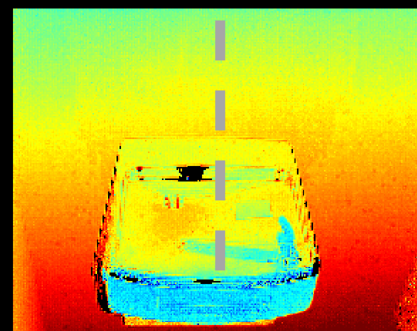


- Regular ToF
- Epipolar ToF

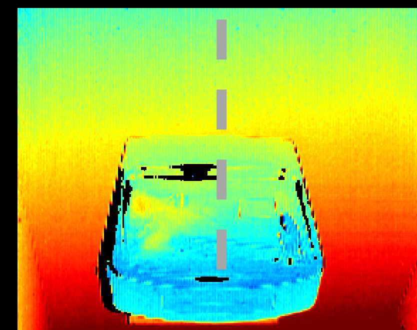
Epipolar ToF and Global Illumination

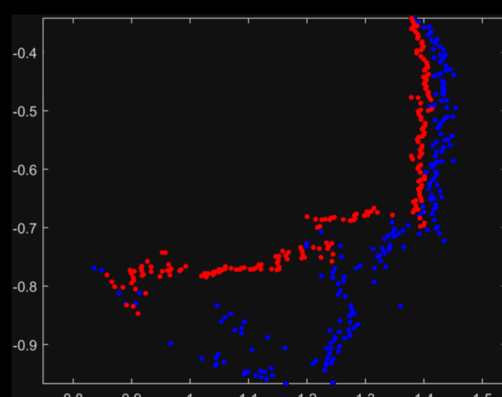
Water Fountain

Regular ToF



Epipolar ToF

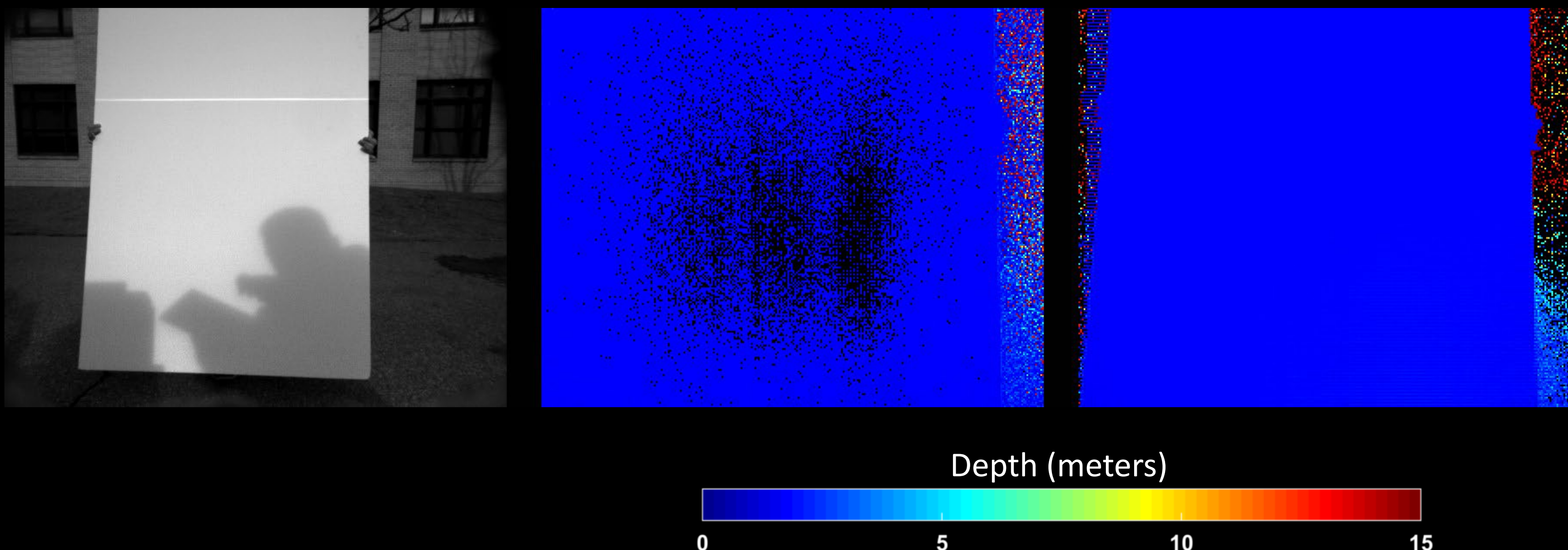




- Regular ToF
- Epipolar ToF

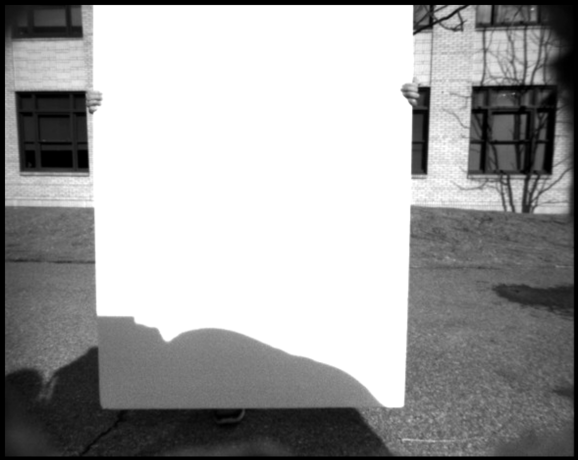
Outdoors (Cloudy – 10 kilolux)

Scene | Regular ToF | Epipolar ToF

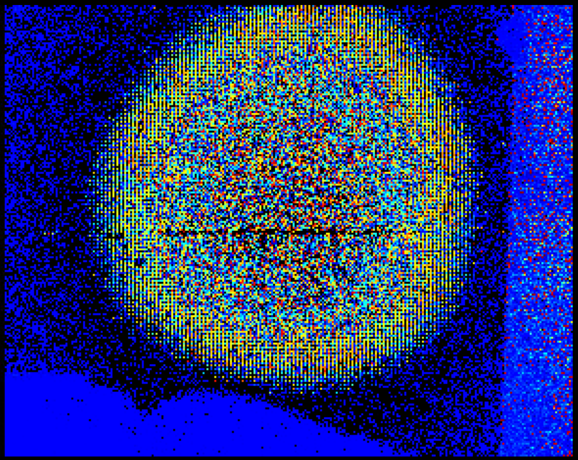


Outdoors (Sunny – 70 kilolux)

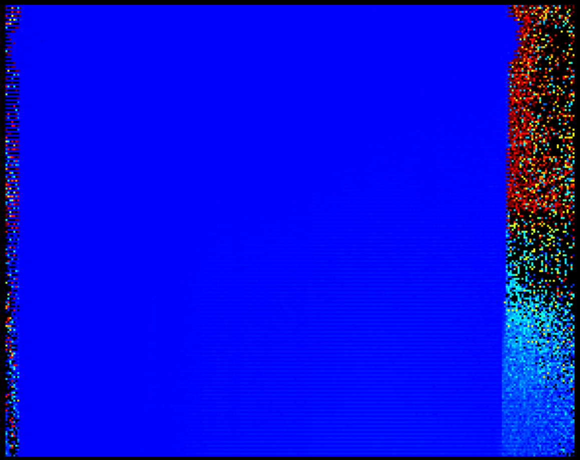
Scene

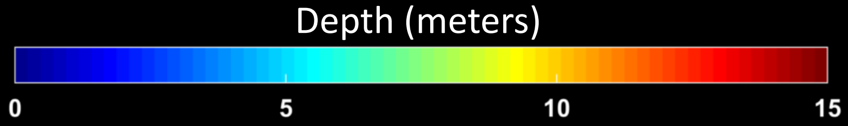


Regular ToF

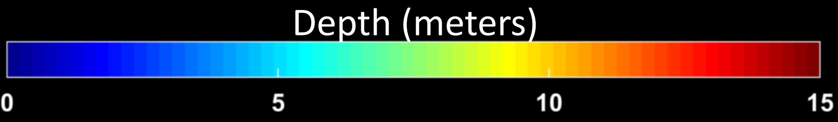


Epipolar ToF





Outdoors (Sunny – 70 kilolux)



References

Basic reading:

- Sloan et al., “Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments,” SIGGRAPH 2002.
- Ng et al., “All-frequency shadows using non-linear wavelet lighting approximation,” SIGGRAPH 2003.
- Seitz et al., “A theory of inverse light transport,” ICCV 2005.

  These three papers all discuss the concept of light transport matrix in detail.
- Debevec et al., “Acquiring the reflectance field of a human face,” SIGGRAPH 2000.

  The paper on image-based relighting.
- O’Toole and Kutulakos, “Optical computing for fast light transport analysis,” SIGGRAPH Asia 2010.

  The paper on eigenanalysis and optical computing using light transport matrices.
- Sen et al., “Dual photography,” SIGGRAPH 2005.

  The dual photography paper.
- O’Toole et al., “Primal-dual coding to probe light transport,” SIGGRAPH 2012.
- O’Toole et al., “3d shape and indirect appearance by structured light transport,” CVPR 2014.

  These two papers introduce the concepts of light transport probing and epipolar probing, as well as explain how to use primal-dual coding to achieve them.
- O’Toole et al., “Homogeneous codes for energy-efficient illumination and imaging,” SIGGRAPH 2015.

  This paper shows how to efficiently implement epipolar imaging with a simple projector and camera.
- Achar et al., “Epipolar time-of-flight imaging,” SIGGRAPH 2017.

  This paper combines epipolar imaging and time-of-flight imaging.

Additional reading:

- Peers et al., “Compressive light transport sensing,” TOG 2009.
- Wang et al., “Kernel Nyström method for light transport,” SIGGRAPH 2009.

  These two papers discuss alternative ways for efficient acquisition of the light transport matrix, using assumptions on its algebraic structure.
- Durand et al., “A frequency analysis of light transport,” SIGGRAPH 2005.
- Mahajan et al., “A theory of locally low dimensional light transport,” SIGGRAPH 2007.
- Reddy et al., “Frequency-space decomposition and acquisition of light transport under spatially varying illumination,” ECCV 2012.

  These papers more formally discuss the notion of light transport frequency, how it relates to light transport matrix rank, and the frequency/rank characteristics of different light transport effects (specular versus diffuse reflections, hard versus smooth shadows).