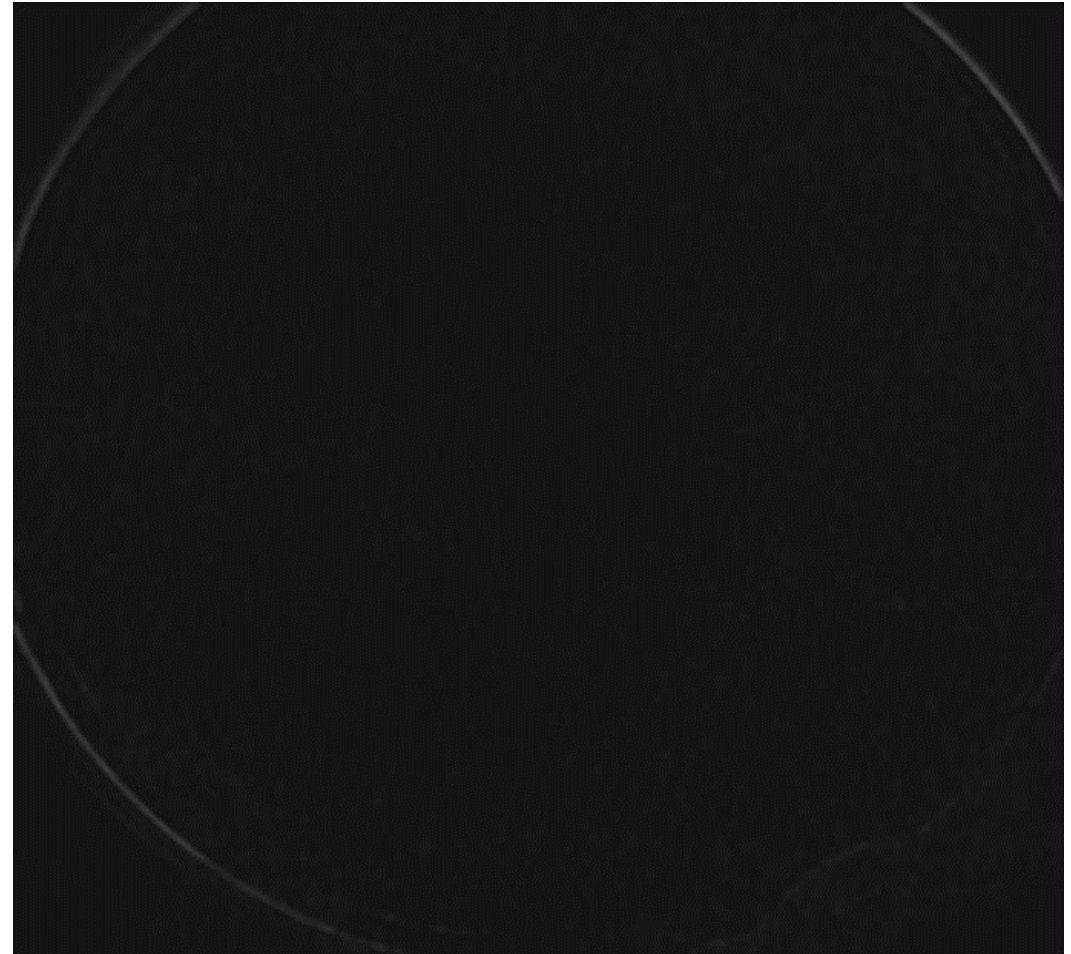
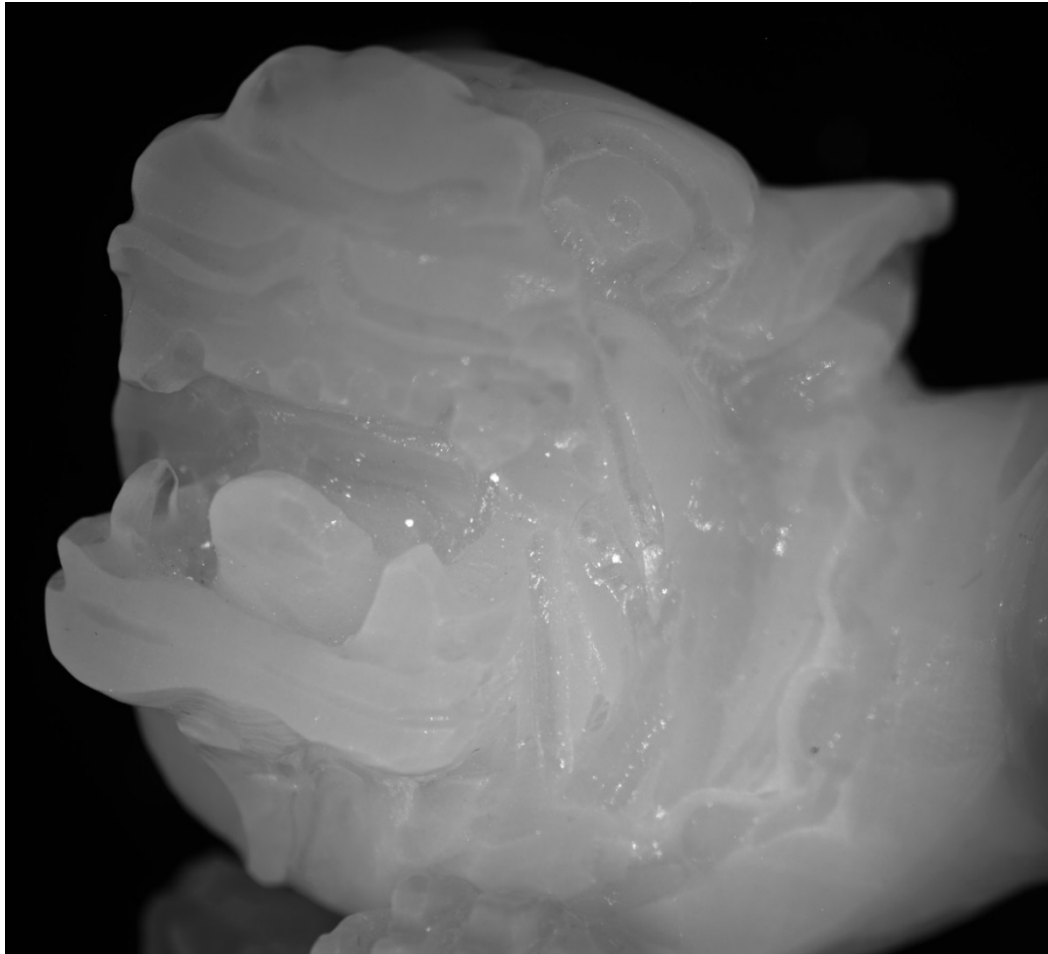


# Time-of-flight imaging



# Course announcements

- Homework assignment 6 is due on Sunday, December 5<sup>th</sup>.
  - Do not leave for last minute, you won't have time to complete it.
- Sign up for *optional* final project checkpoint meeting.
  - Sign up spreadsheet available on Piazza.
- Vote on Piazza for *optional* extra lecture on Thursday or Friday.

# Overview of today's lecture

- Introduction to time-of-flight (ToF) imaging.
- Impulse ToF imaging and single-photon avalanche diodes.
- Continuous-wave ToF imaging.
- Interferometric ToF imaging.

# Slide credits

A lot of these slides were adapted from:

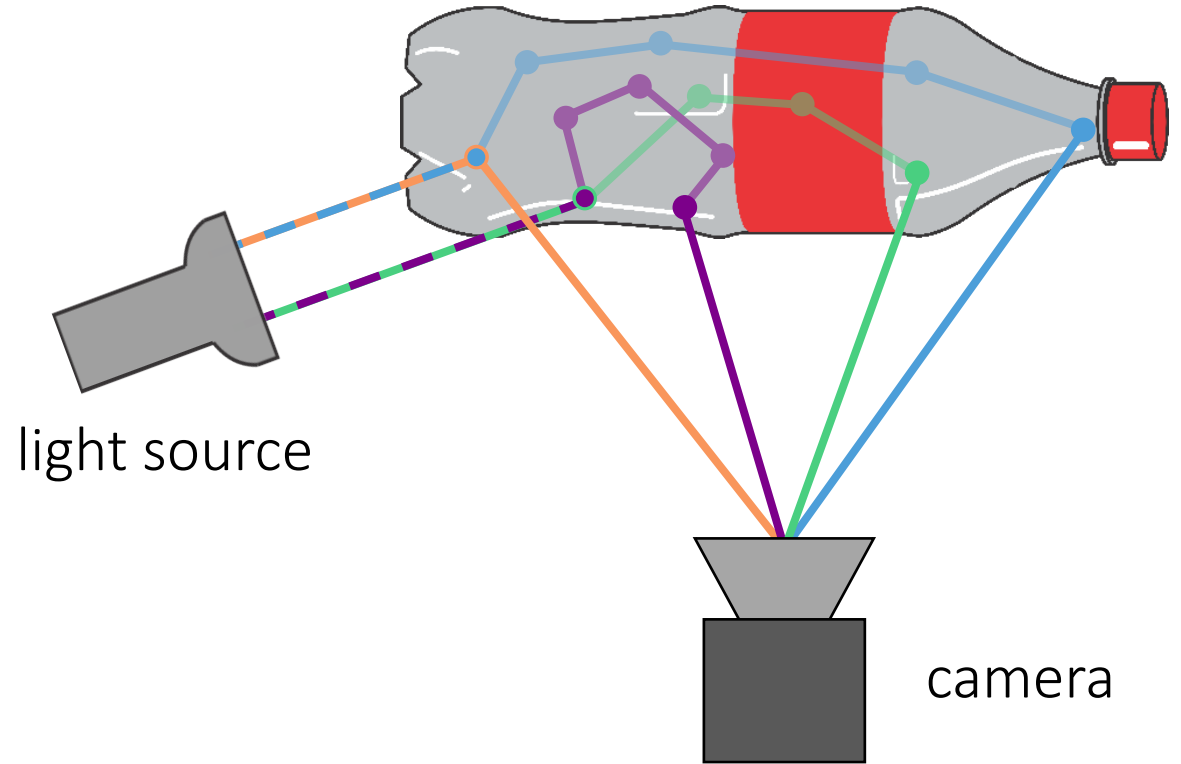
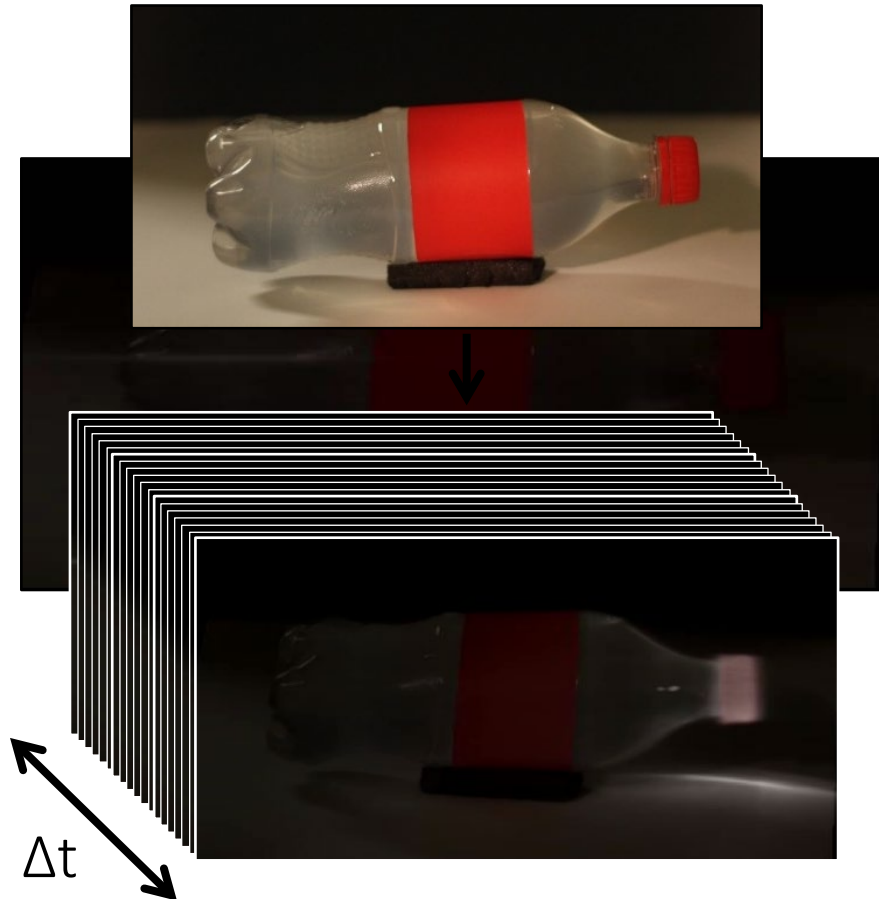
- Mohit Gupta (Wisconsin).
- Supreeth Achar (Google, formerly CMU).

# Introduction to time-of-flight (ToF) imaging

# Time-of-flight (ToF) imaging

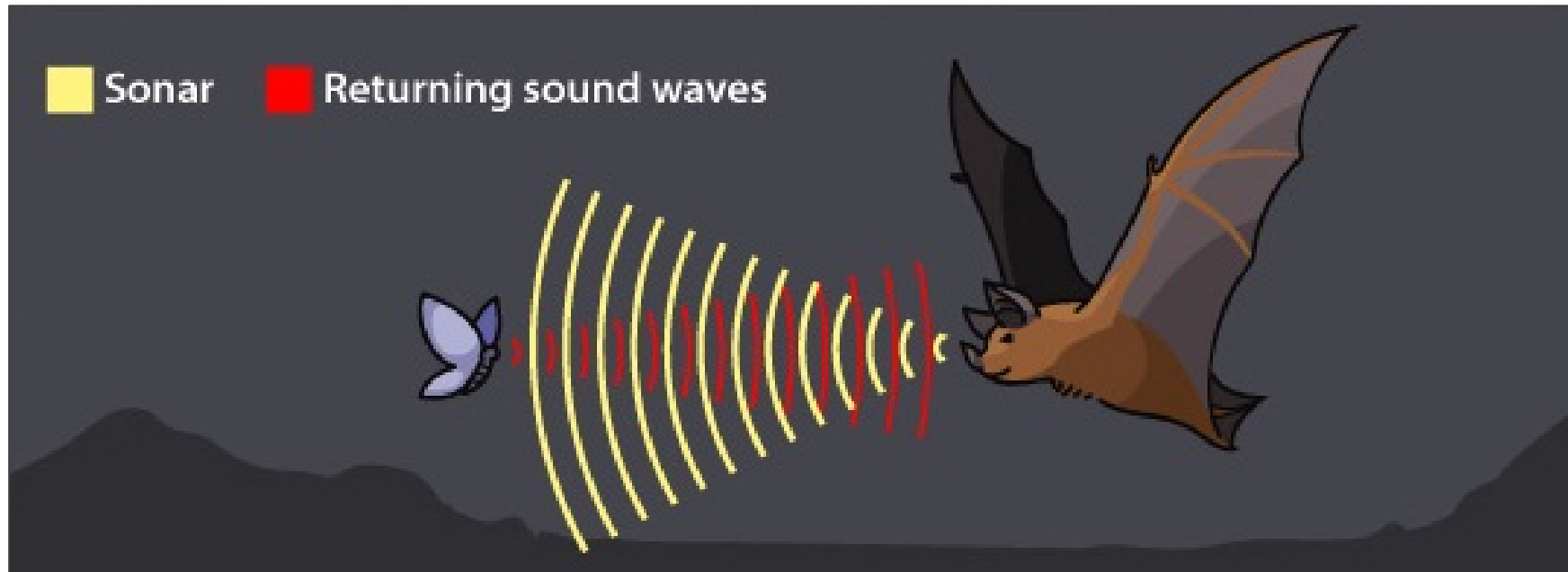


# Time-of-flight (ToF) imaging

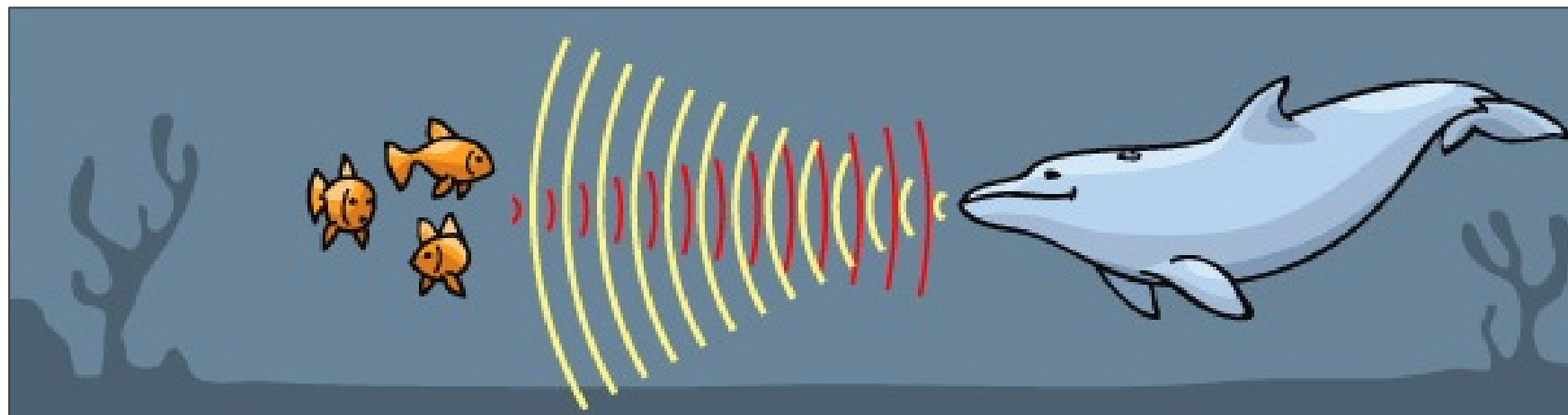


- Conventional imaging: Measure all photons together regardless of time of travel.
- Time-of-flight imaging: Measure photons separately based on time of travel.

# Time-of-flight imaging in nature

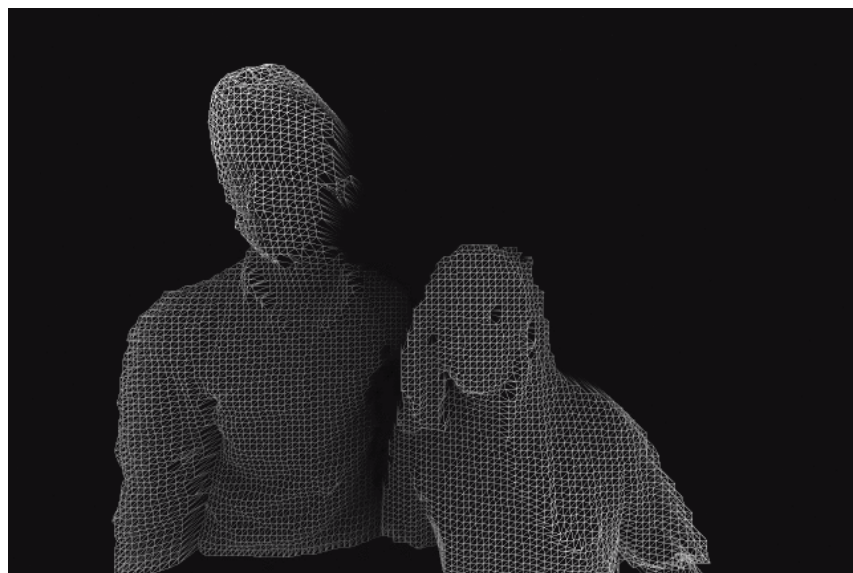


echolocation using sound-wave time-of-flight

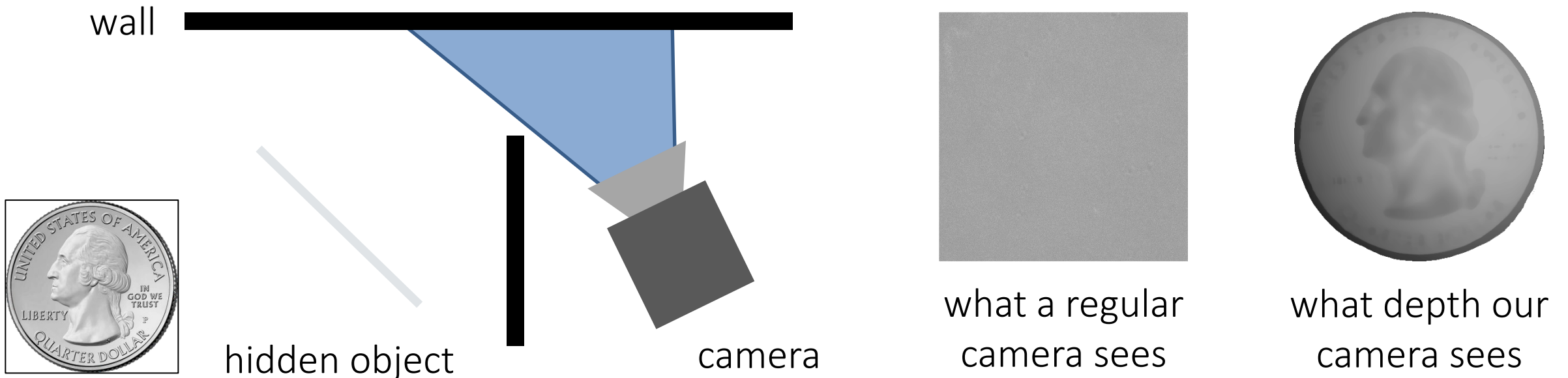
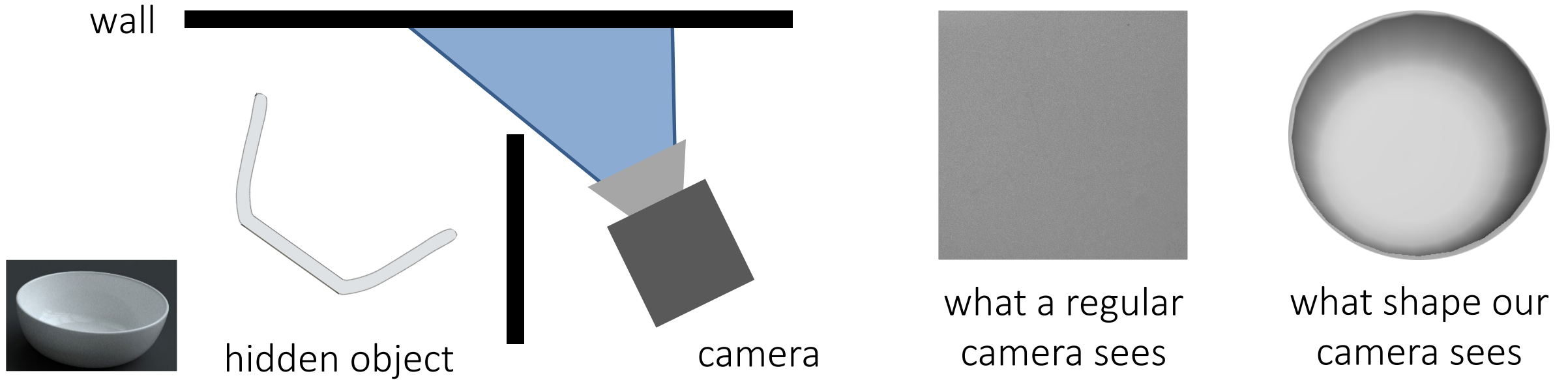




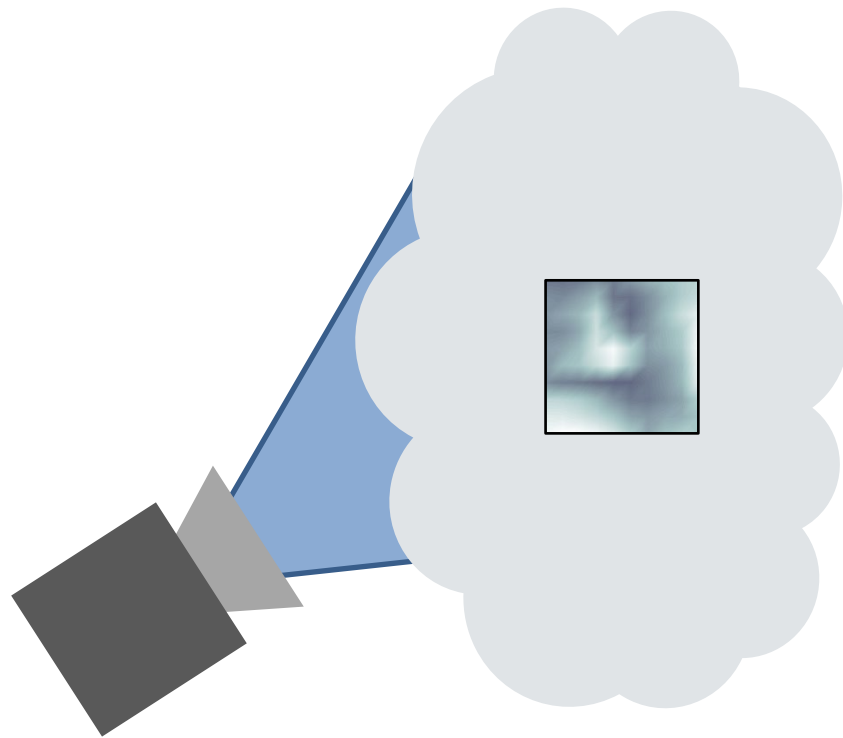
# Time-of-flight applications: depth sensing



# Time-of-flight applications: non-line-of-sight imaging

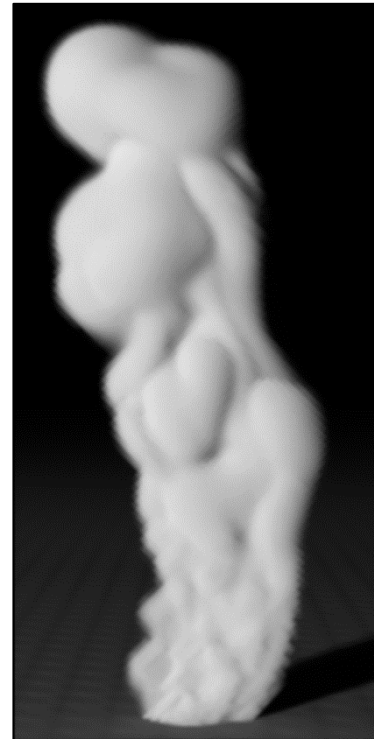


# Time-of-flight applications: seeing inside objects

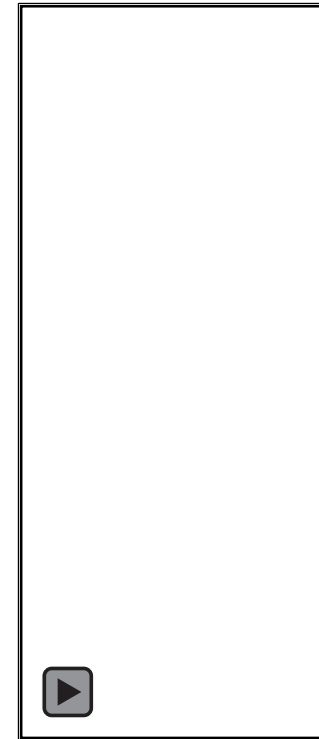


camera

thick smoke  
cloud



what a regular  
camera sees

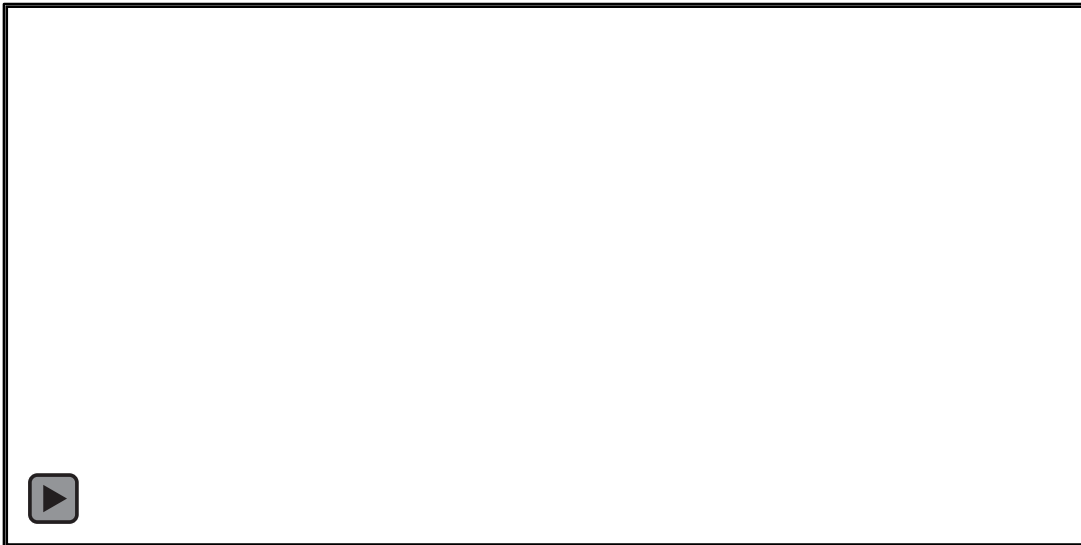
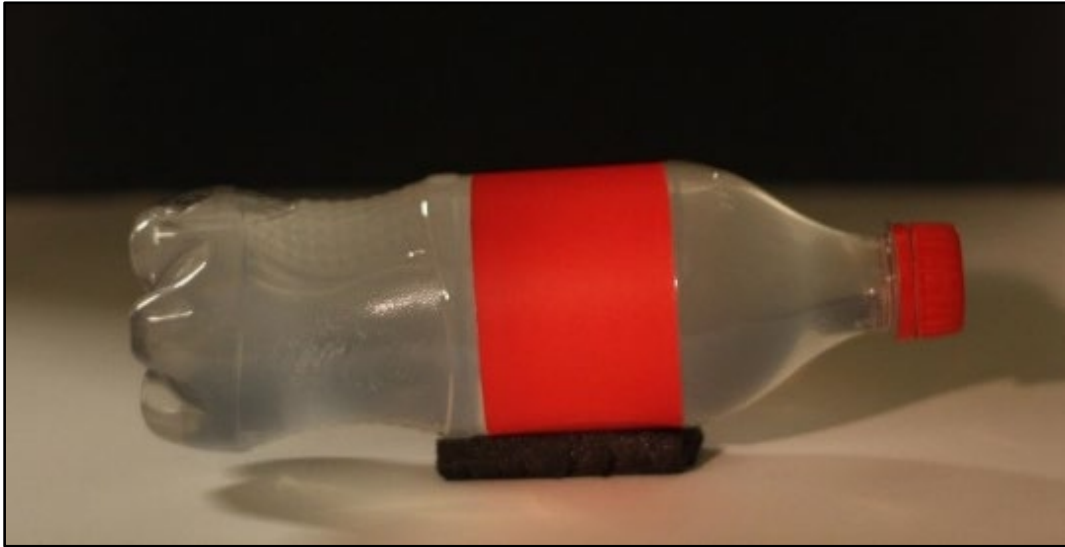


what our  
camera sees

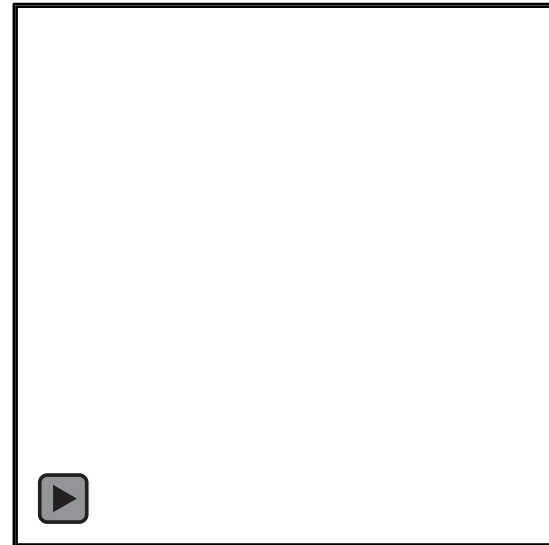


a slice through  
the cloud

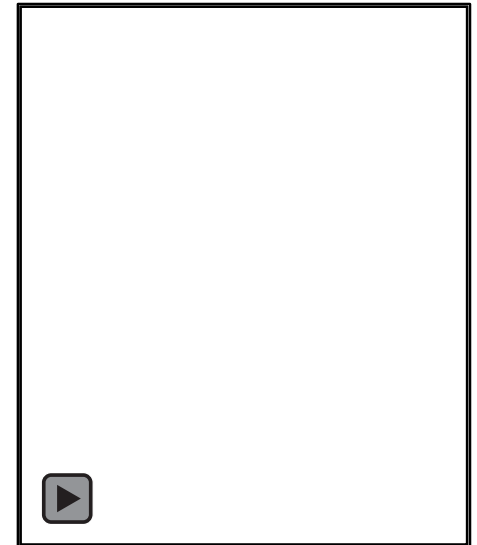
# Time-of-flight applications: light-in-flight visualization



video at  $10^{12}$  frames per second



video at  $10^{15}$  frames per second



# Time-of-flight imaging technologies

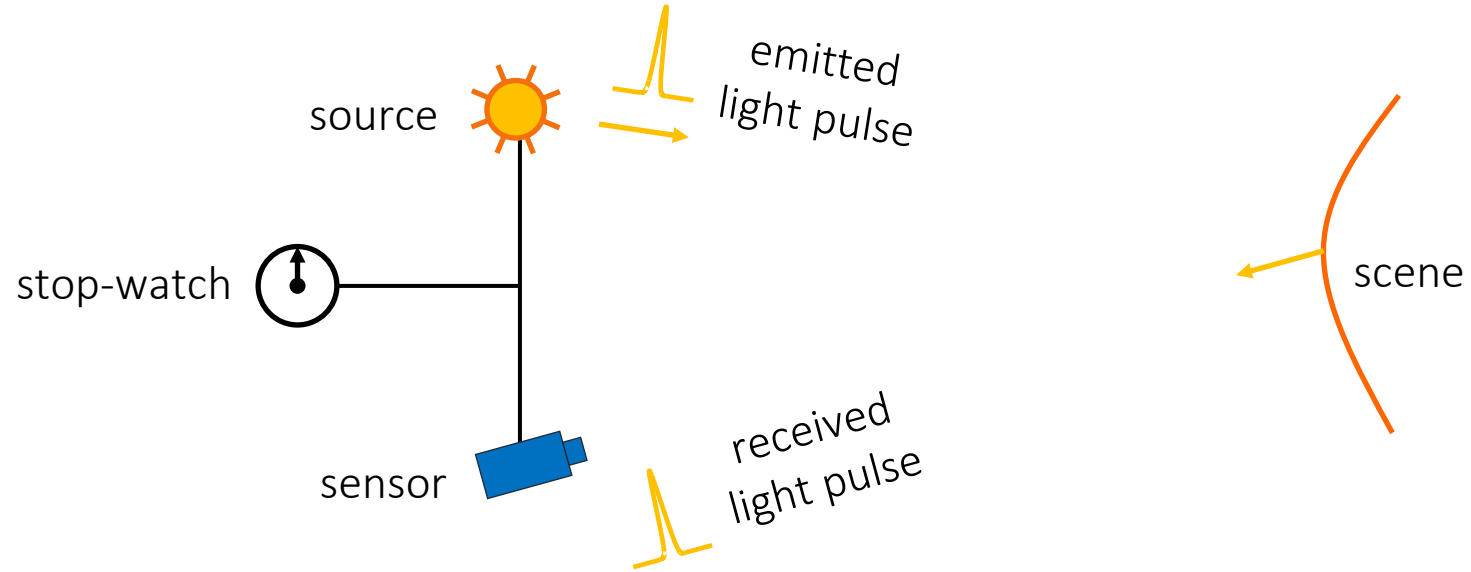
	interferometry	streak cameras	single-photon avalanche diodes	time-of-flight cameras	LIDAR
temporal resolution	1 femtosecond ( $10^{-15}$ secs)	1 picosecond ( $10^{-12}$ secs)	100 picoseconds ( $10^{-10}$ secs)	1 nanosecond ( $10^{-9}$ secs)	10 nanoseconds ( $10^{-8}$ secs)
frame rate	quadrillion fps	trillion fps	10 billion fps	billion fps	100 million fps
distance travelled	1 micron ( $10^{-6}$ meters)	1 millimeter ( $10^{-3}$ meters)	10 centimeters ( $10^{-1}$ meters)	1 meter ( $10^0$ meters)	10 meters ( $10^1$ meters)

# Time-of-flight imaging technologies

	interferometry	streak cameras	single-photon avalanche diodes	time-of-flight cameras	LIDAR
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	continuous-wave ToF	impulse ToF			

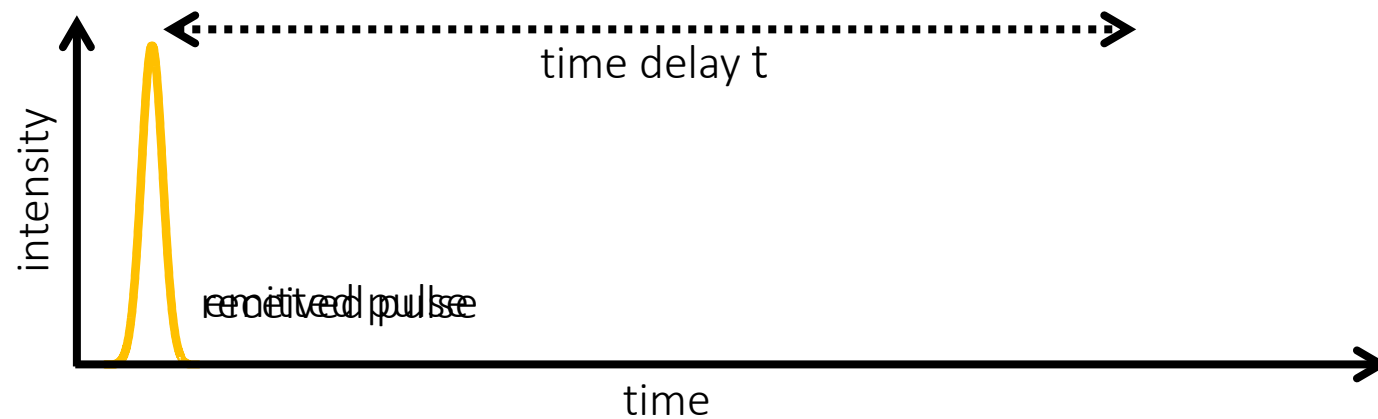
# Impulse ToF imaging and single-photon avalanche diodes

# Impulse time-of-flight imaging



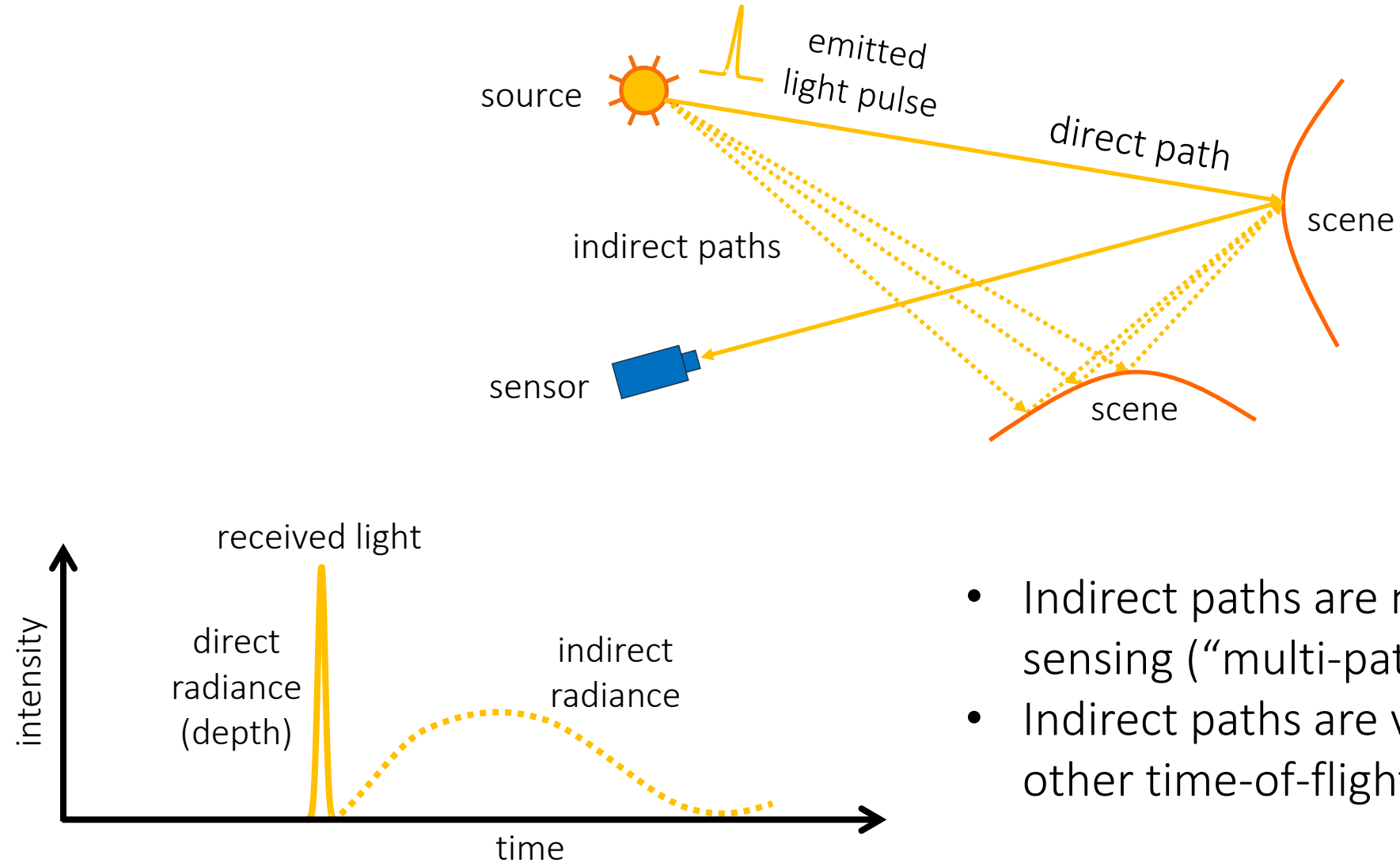
How can we infer depth from this?

$$\text{depth} = \frac{c}{2\tau}$$



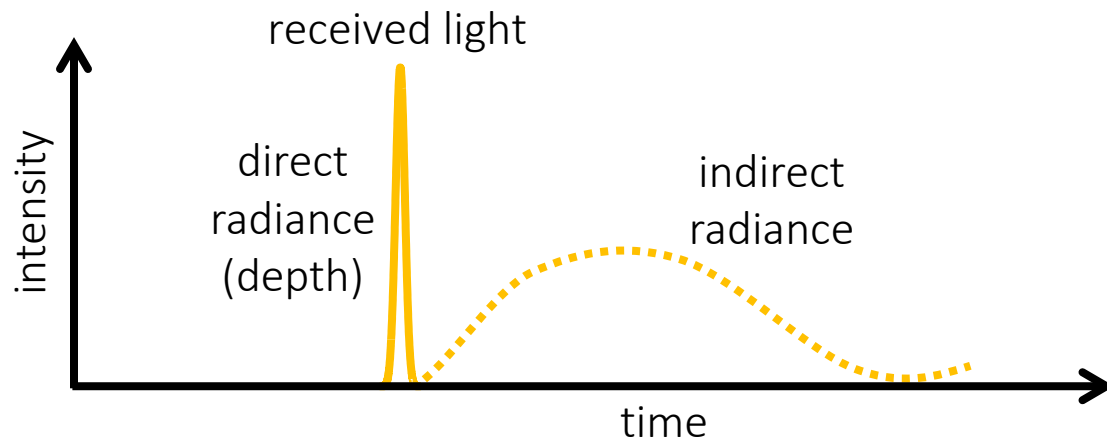
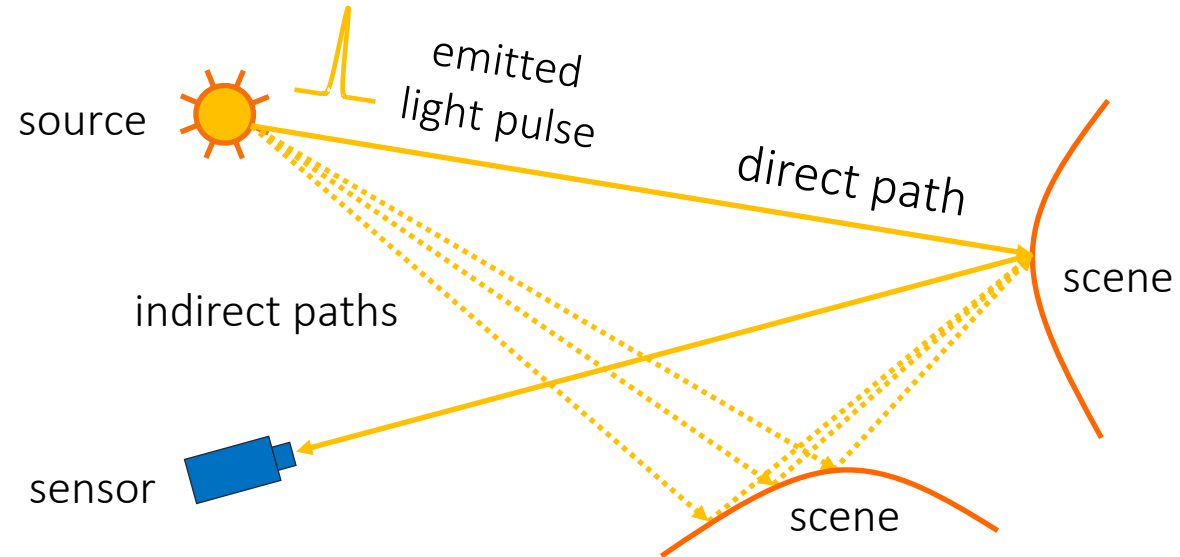


# Impulse time-of-flight imaging



- Indirect paths are nuisance for depth sensing (“multi-path interference”).
- Indirect paths are very informative for other time-of-flight applications.

# Two types of time-of-flight imaging



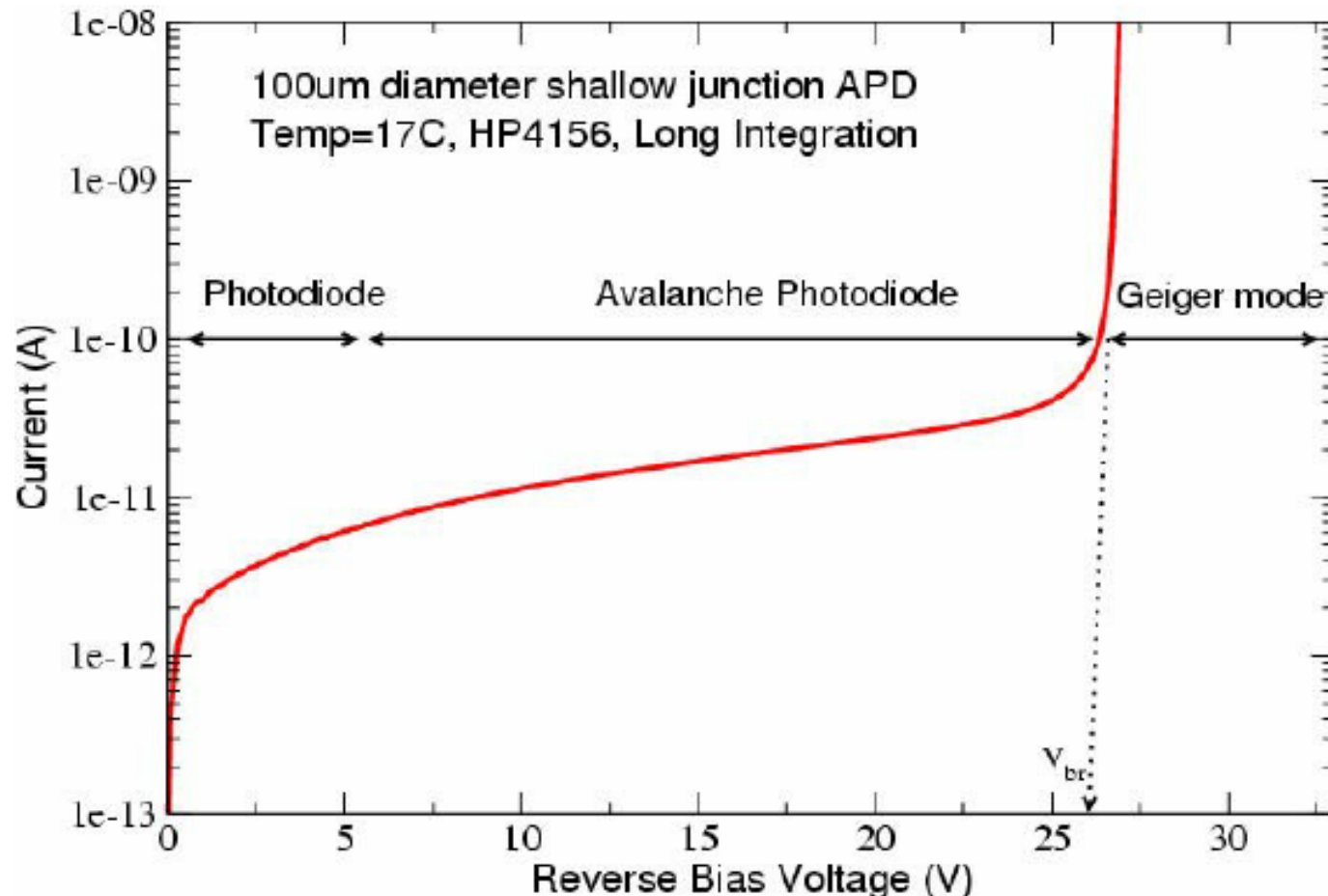
- Range imaging: Measuring only first returning photons (e.g., LIDAR).
- Transient imaging: Measuring entire transient (e.g., SPAD).

Transient  $I(t)$ : Time-resolved radiance distribution

# How exactly is the transient formed?

Depends on the kind of sensor we use.

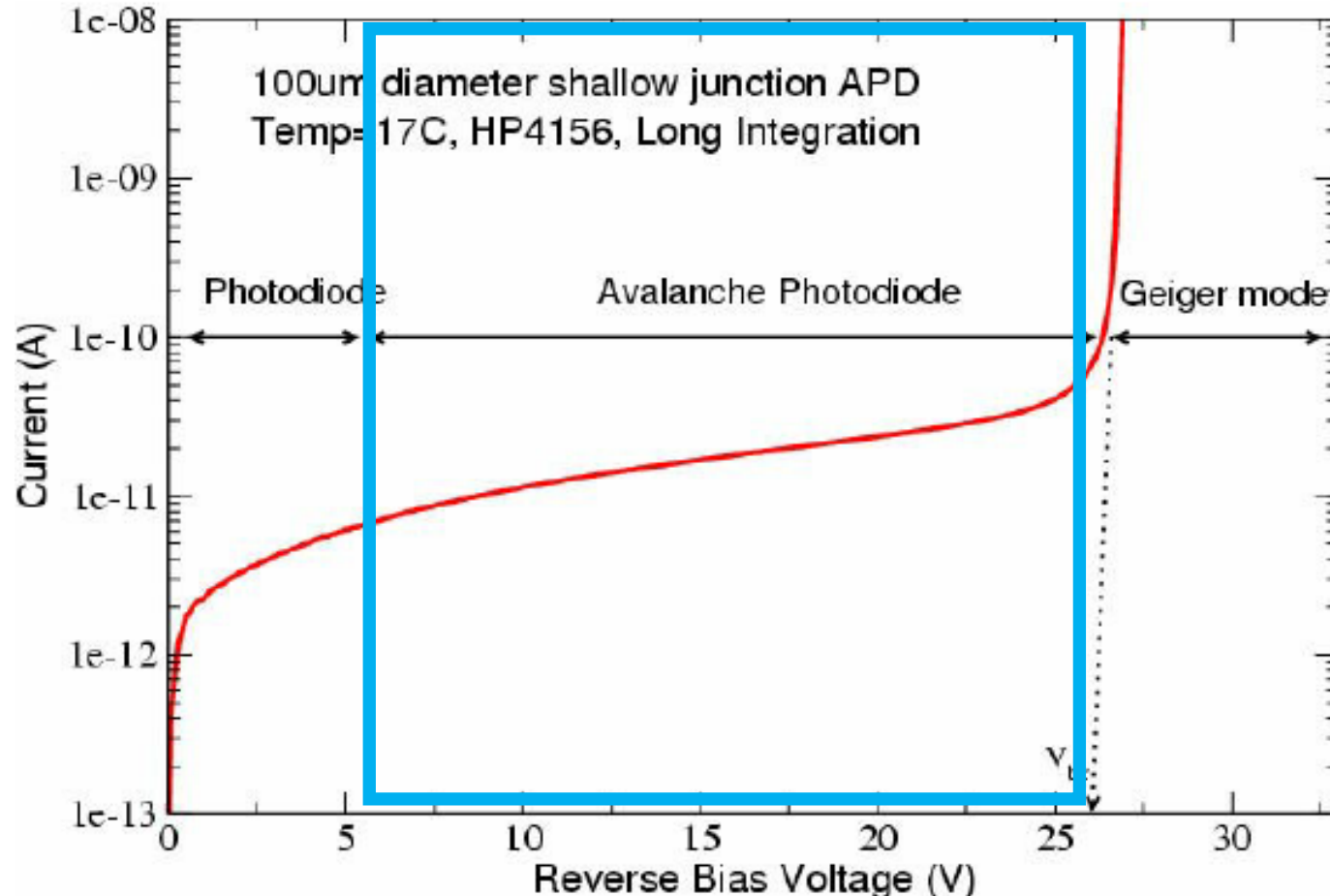
- Here we will examine only photodiodes.



# How exactly is the transient formed?

Depends on the kind of sensor we use.

- Here we will examine only photodiodes.



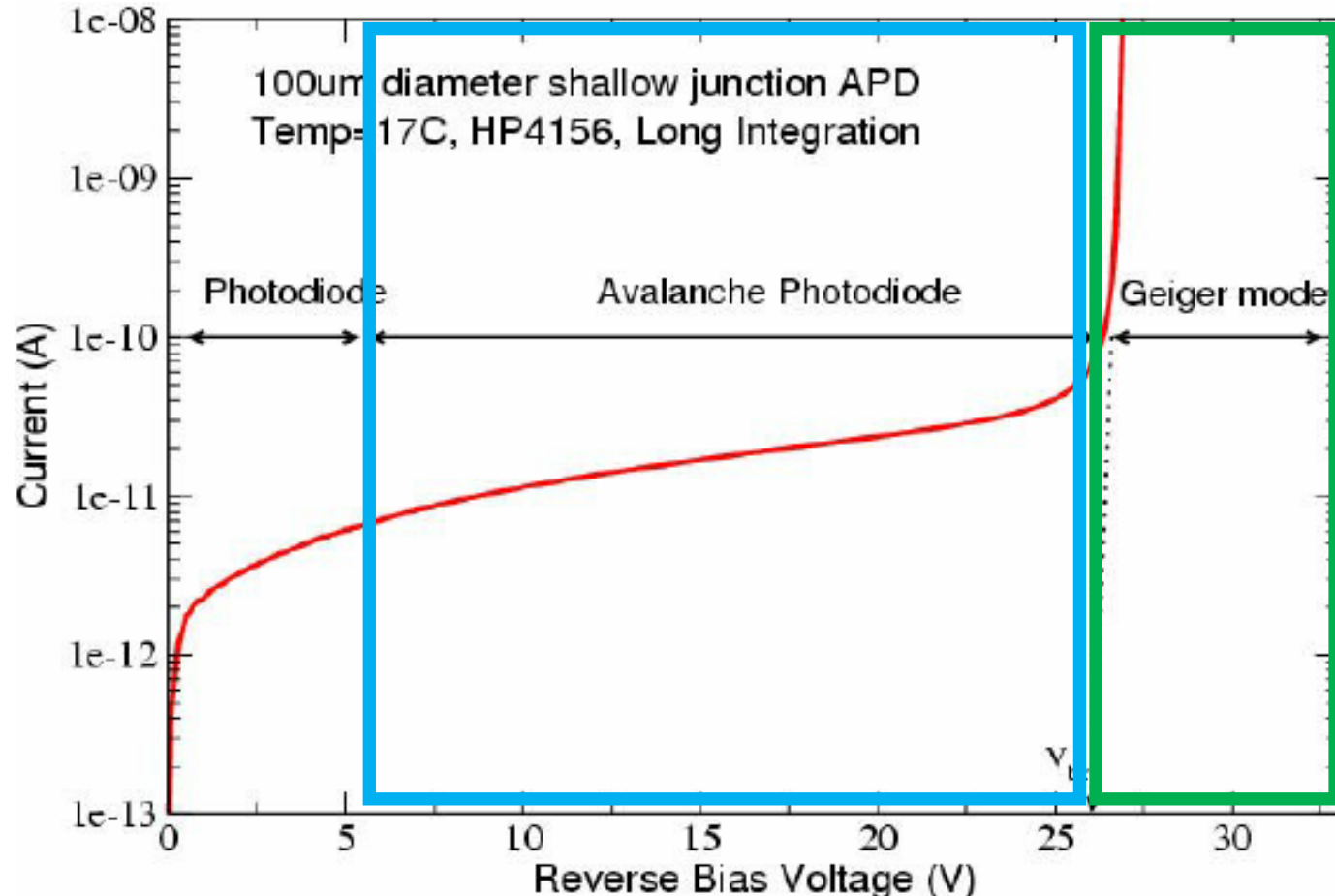
## Avalanche photodiode (APD):

- Current is roughly proportional to number of photons.
- One photon produces tiny current.

# How exactly is the transient formed?

Depends on the kind of sensor we use.

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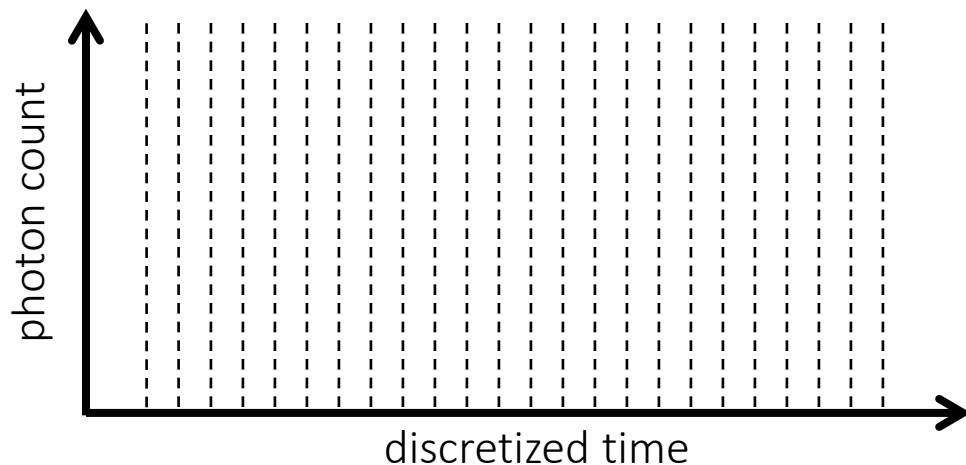
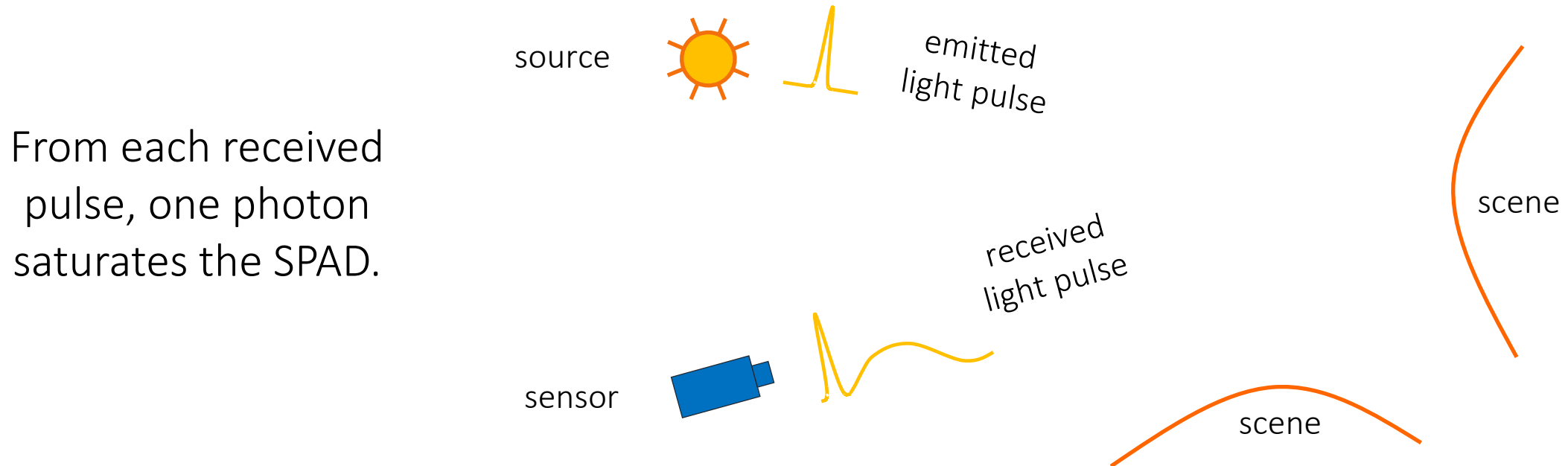
## Avalanche photodiode (APD):

- Current is roughly proportional to number of photons.
- One photon produces tiny current.

## Single-photon avalanche diode (SPAD):

- One photon produces huge current.
- Requires multiple low power pulses, so that one photon returns from each.

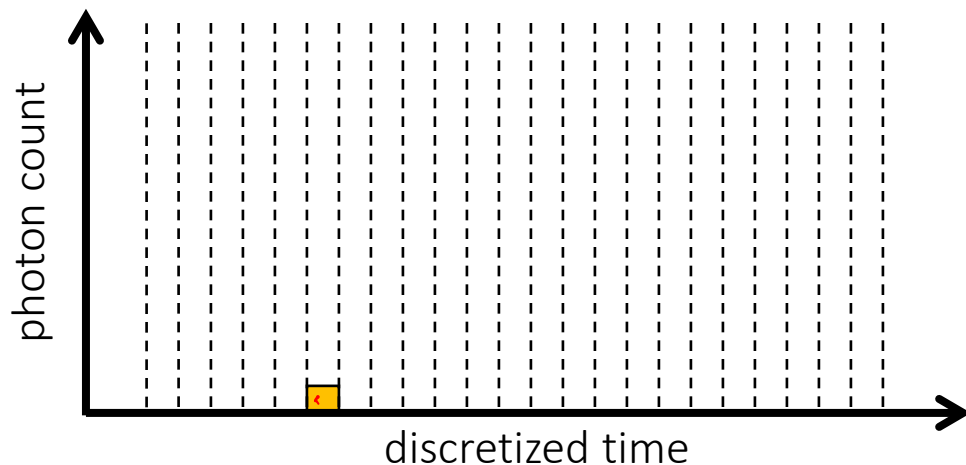
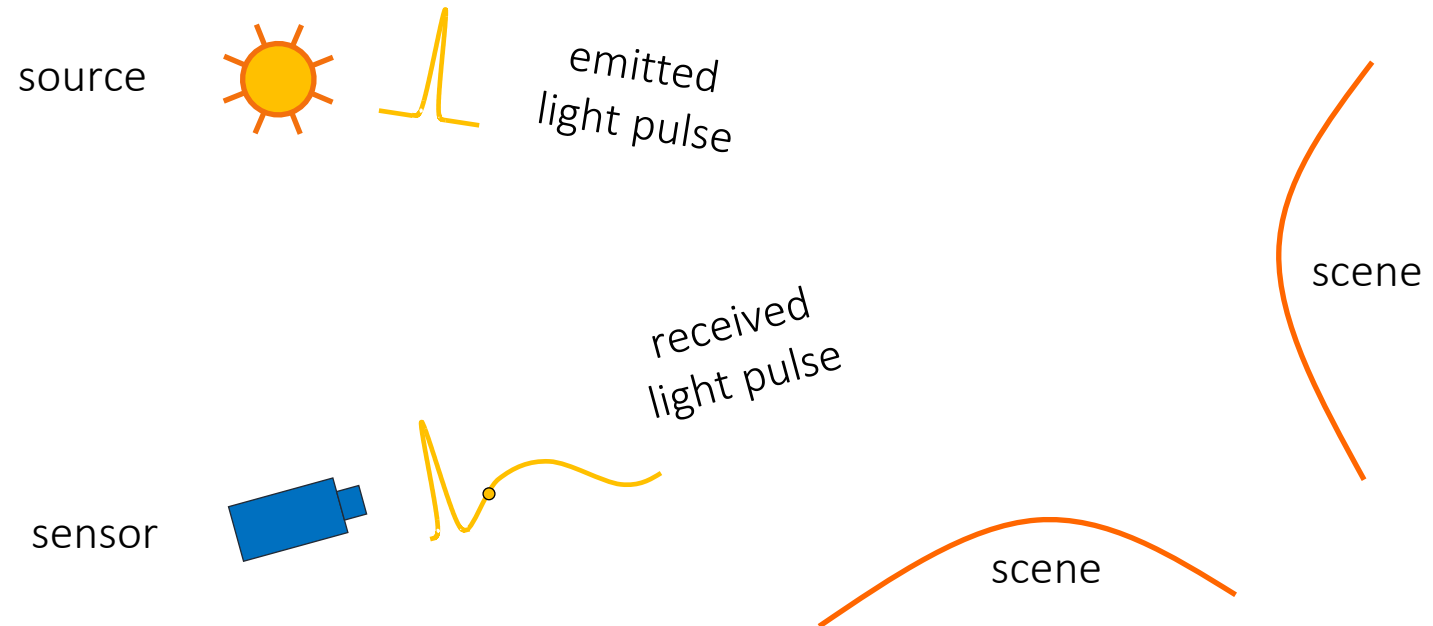
# Geiger-mode impulse time-of-flight imaging



- The SPAD records only photon arrival times, no intensity.
- Additional electronics maintain a histogram of arrival times over multiple pulses

# Geiger-mode impulse time-of-flight imaging

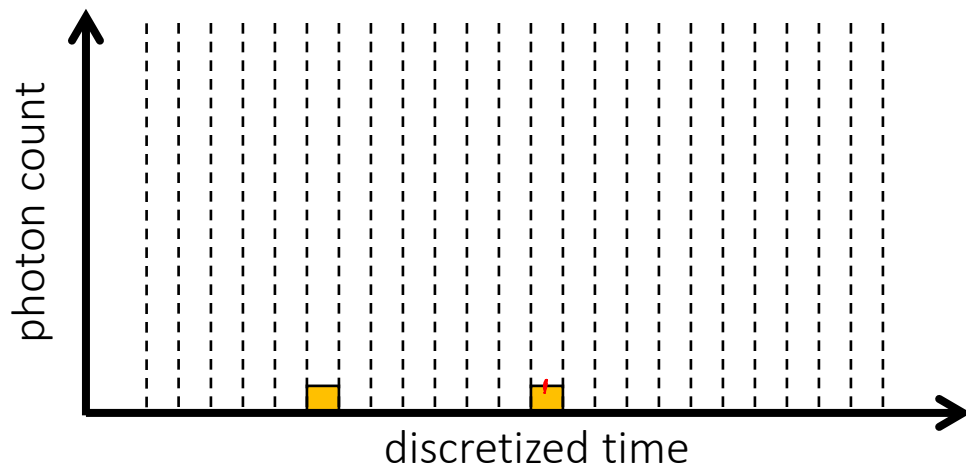
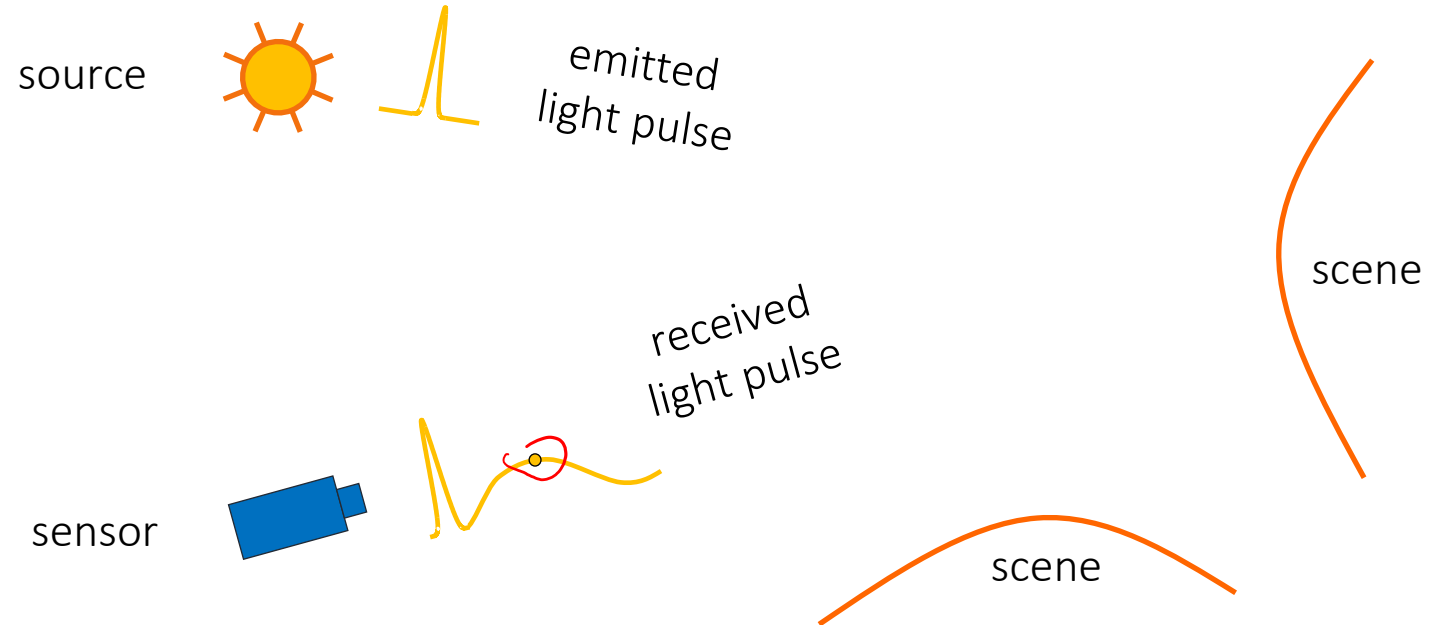
From each received pulse, one photon saturates the SPAD.



- The SPAD records only photon arrival times, no intensity.
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# Geiger-mode impulse time-of-flight imaging

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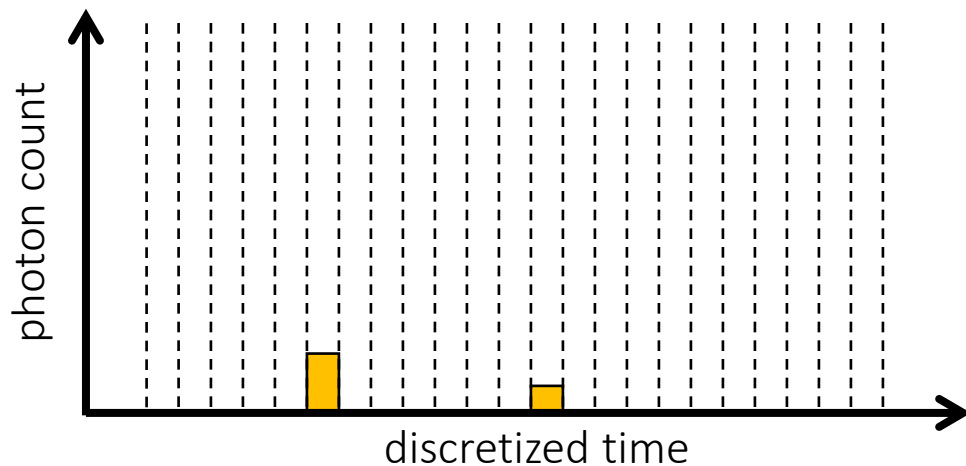
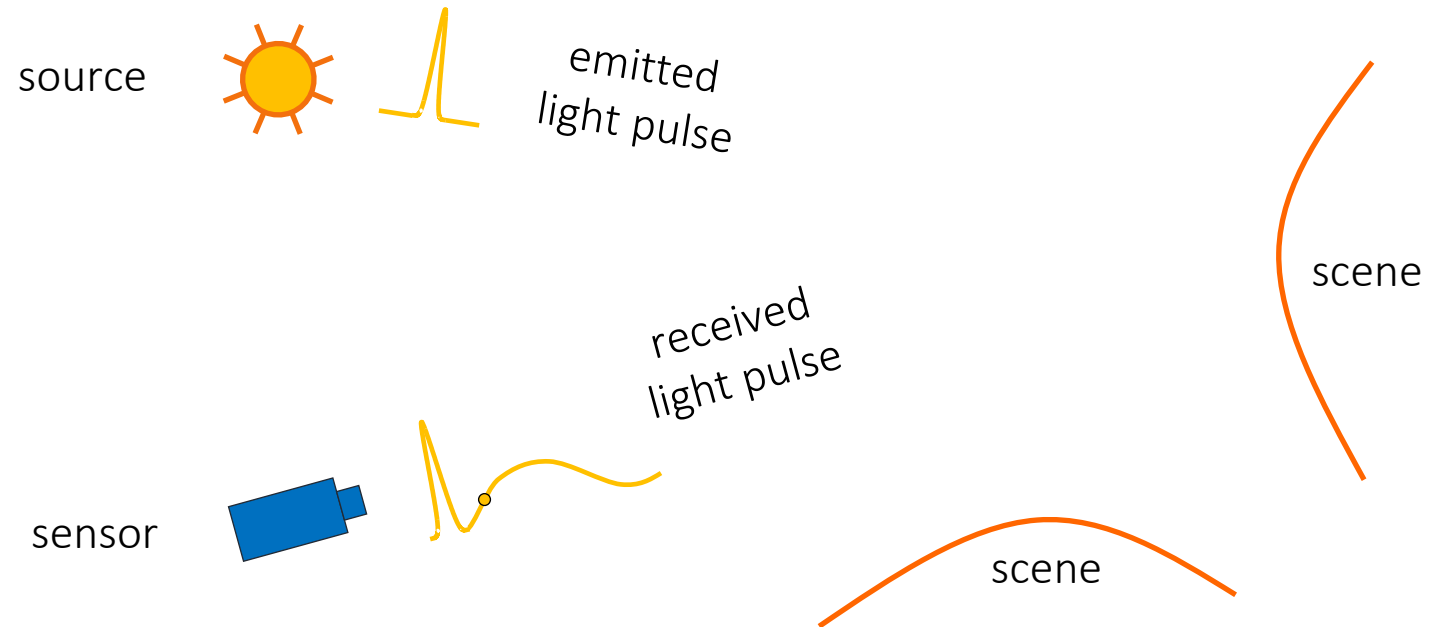


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# Geiger-mode impulse time-of-flight imaging

From each received pulse, one photon saturates the SPAD.

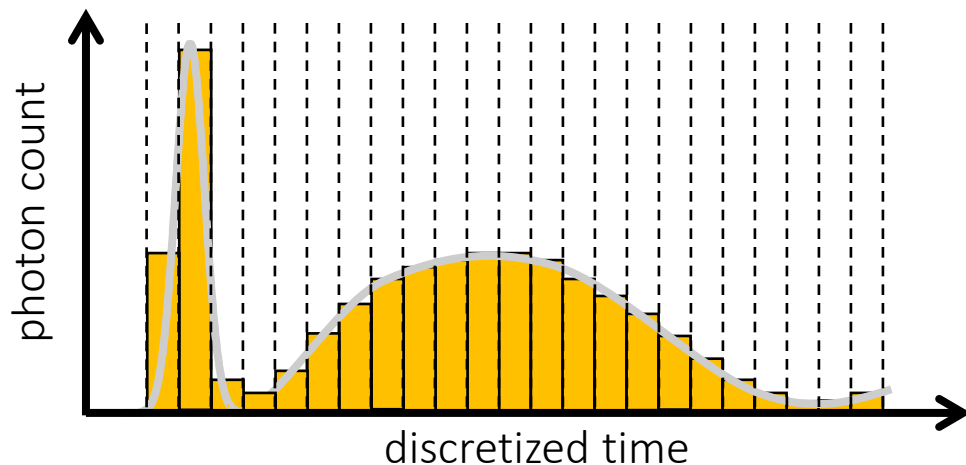
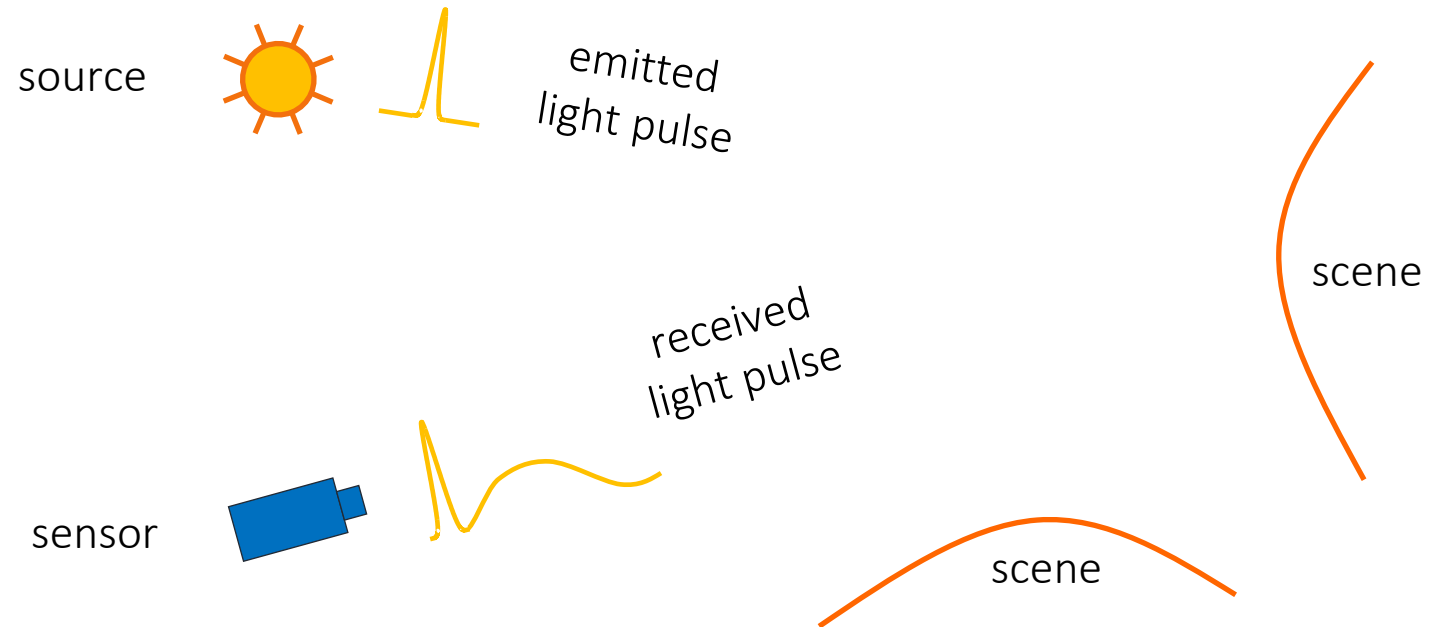


- The SPAD records only photon arrival times, no intensity.
- Additional electronics maintain a histogram of arrival times over multiple pulses

# Geiger-mode impulse time-of-flight imaging

From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?

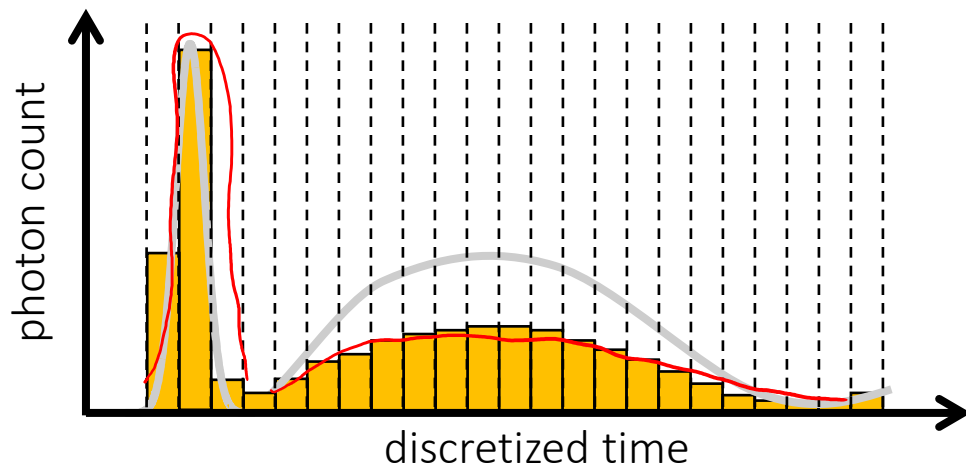
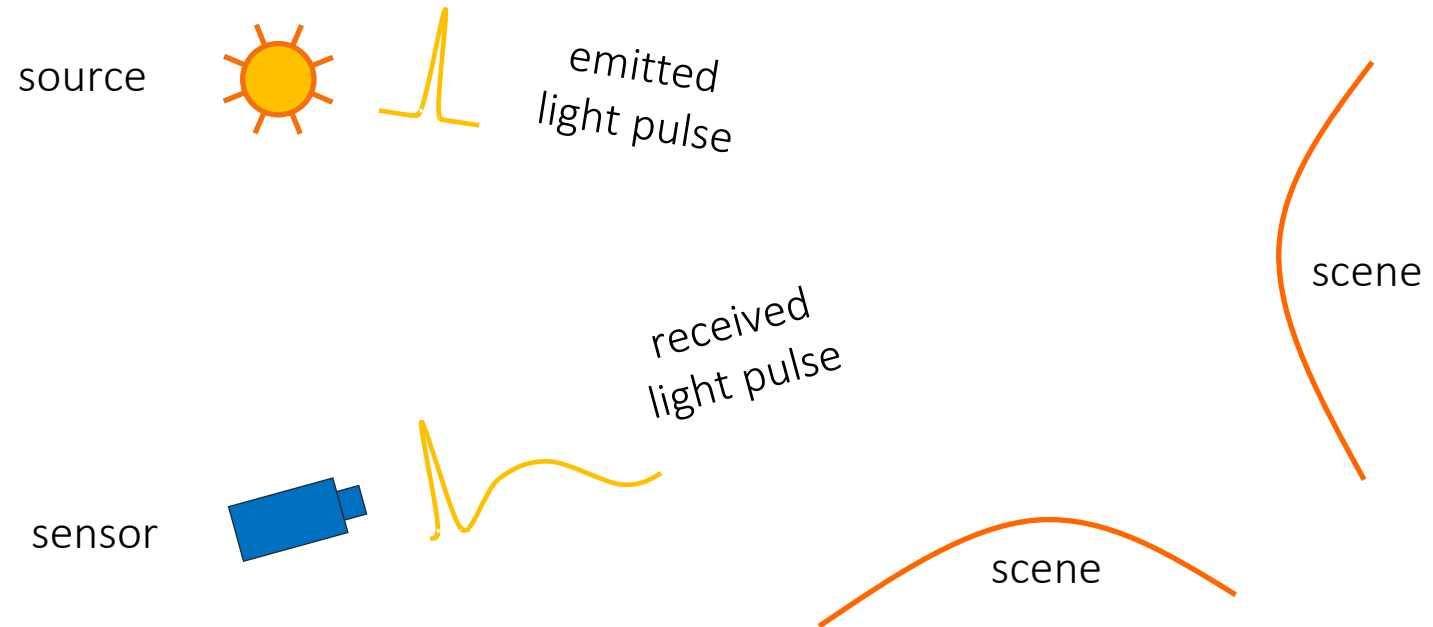


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# Geiger-mode impulse time-of-flight imaging

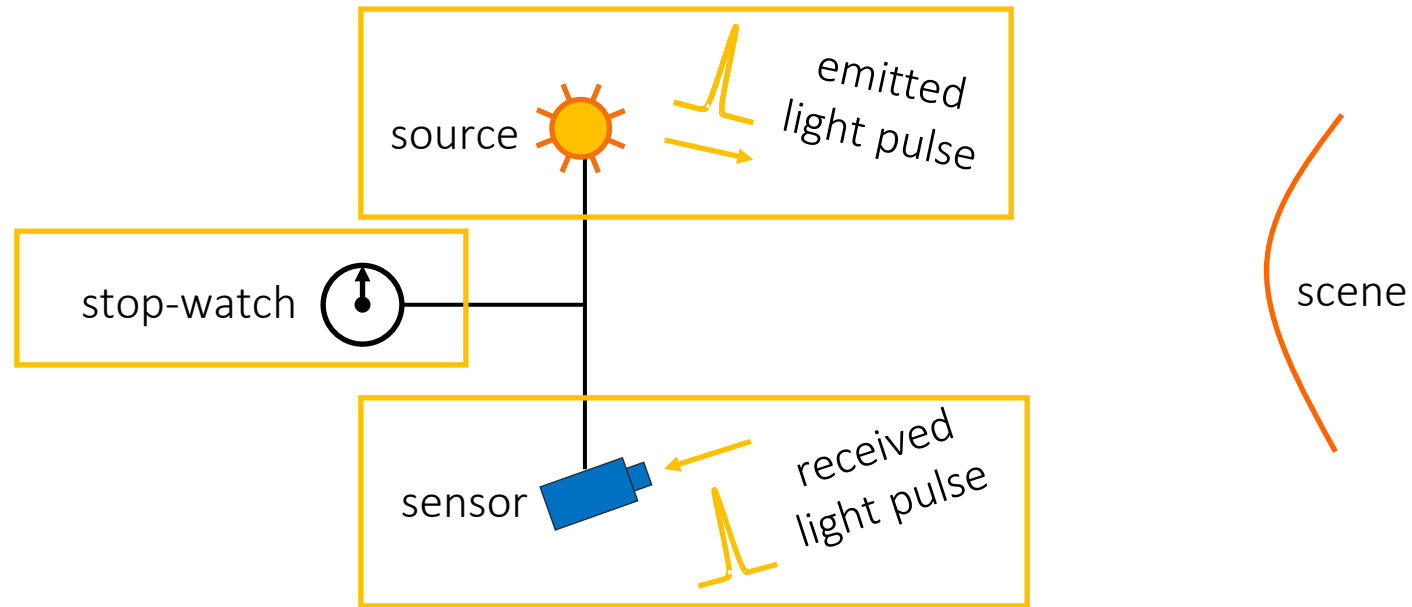
From each received pulse, one photon saturates the SPAD.

What determines which photon gets picked?



- Photons earlier in the transient have a higher probability of being detected than photons later in the transient.
- As a result, histogram of photon detections underestimates later parts of the transient.
- This effect is called *pile-up* and is very severe under strong light conditions.

# What hardware do we need for impulse ToF?



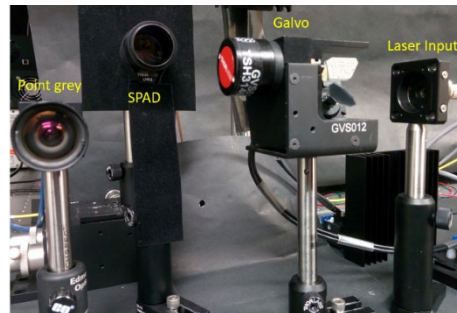
Expensive lasers

[short (picosecond) and powerful  
(mega joules) light pulses]



High speed and high

dynamic range sensors  
[single-photon sensitivity]



Expensive syncing and

photon-counting electronics  
[picosecond time resolution]

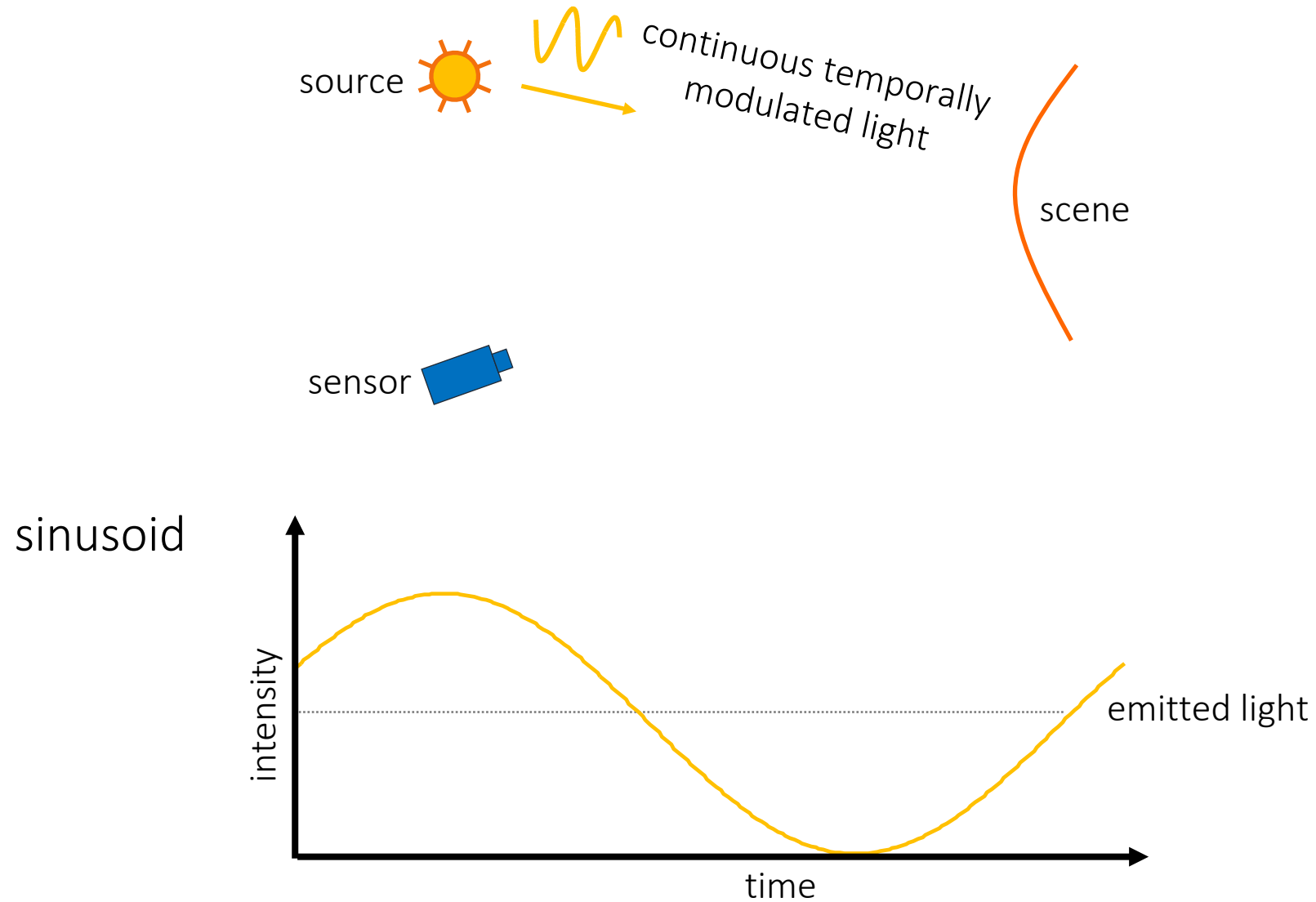


# Time-of-flight imaging technologies

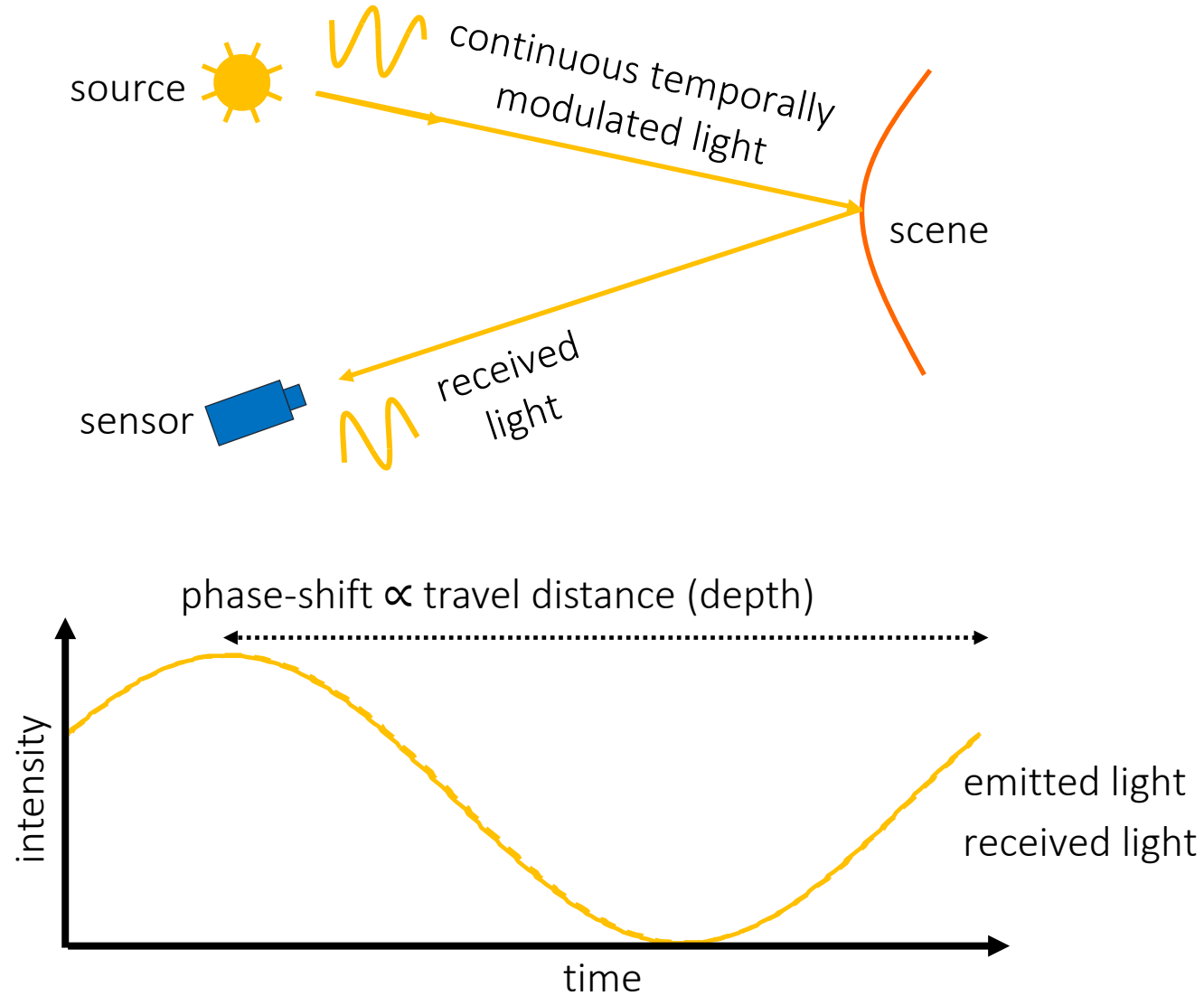
	interferometry	streak cameras	single-photon avalanche diodes	time-of-flight cameras	LIDAR
temporal resolution	1 femtosecond ( $10^{-15}$ secs)	1 picosecond ( $10^{-12}$ secs)	100 picoseconds ( $10^{-10}$ secs)	1 nanosecond ( $10^{-9}$ secs)	10 nanoseconds ( $10^{-8}$ secs)
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	continuous-wave ToF	impulse ToF			

# Continuous-wave ToF imaging

# Continuous-wave (CW) time-of-flight imaging

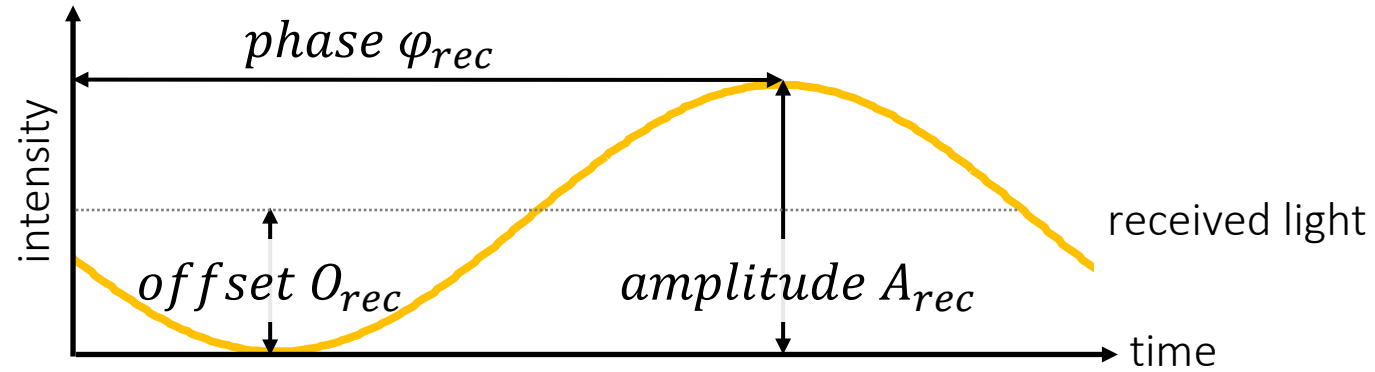


# Continuous-wave (CW) time-of-flight imaging





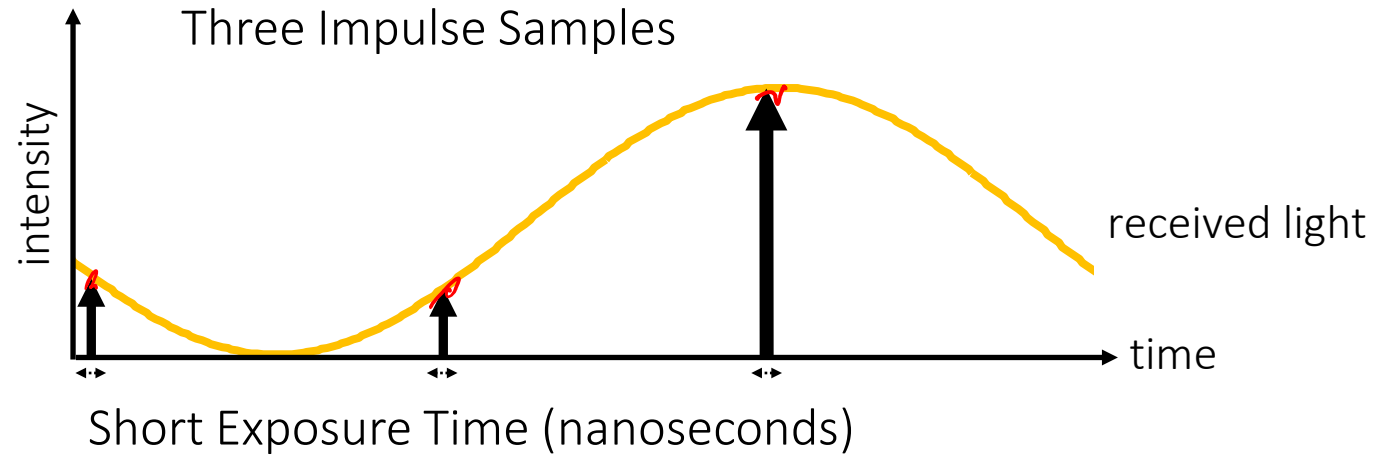
# Measuring phase shift



Three Unknowns

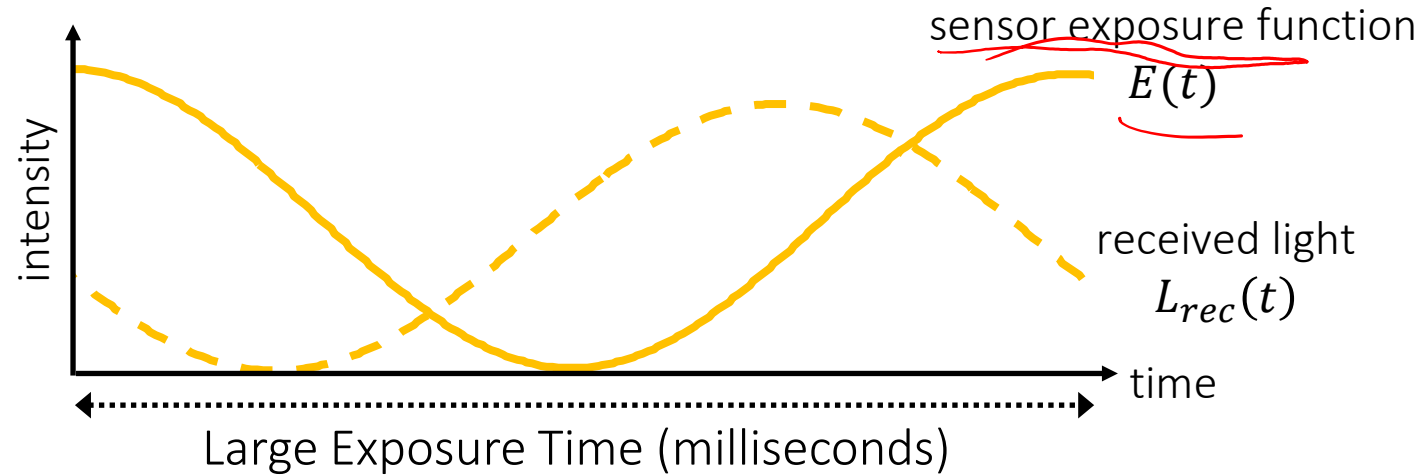
$$L_{rec}(t) = O_{rec} + A_{rec} \cos(\omega t - \phi_{rec})$$

# Measuring phase shift: direct



Low Signal-to-Noise-Ratio

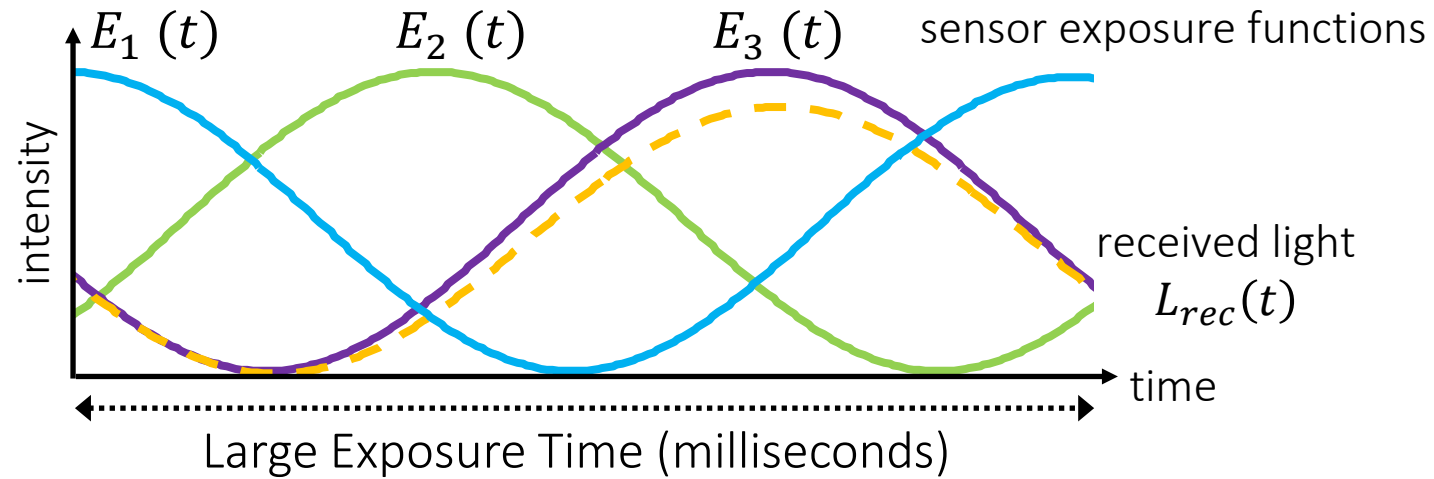
# Measuring phase shift: correlation



Correlation:  $I = \int E(t) \times L_{rec}(t) dt$

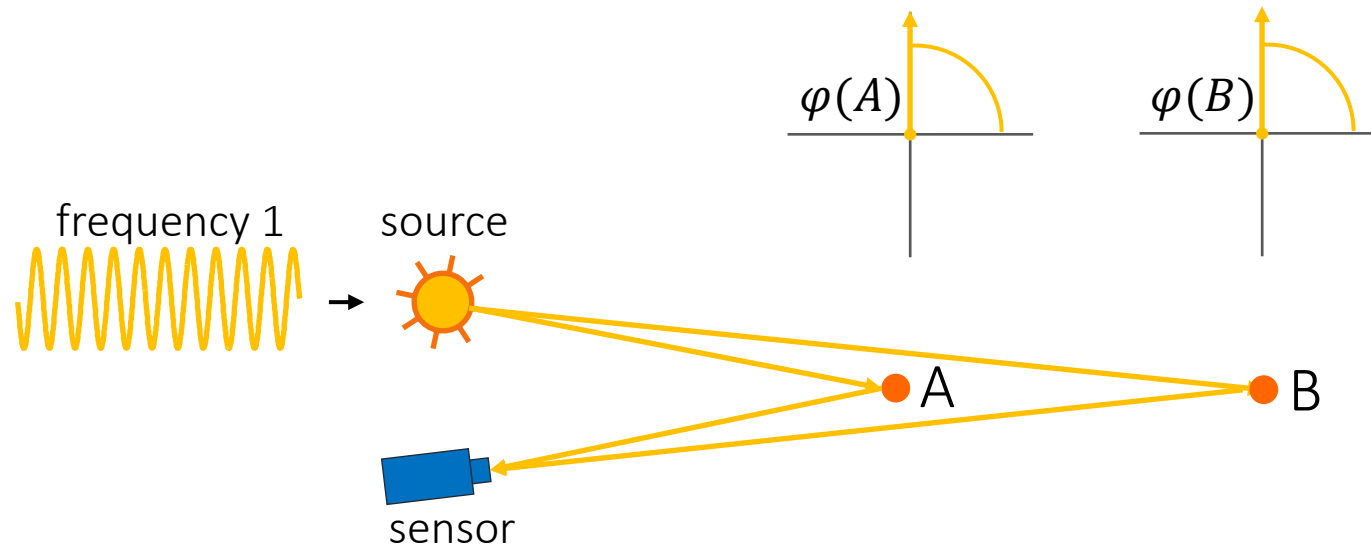
measured brightness      exposure function      received light

# Measuring phase shift: correlation



$$\begin{array}{l}
 \text{Correlation 1: } I_1 = \int E_1(t) \times L_{rec}(t) dt \\
 \text{Correlation 2: } I_2 = \int E_2(t) \times L_{rec}(t) dt \\
 \text{Correlation 3: } I_3 = \int E_3(t) \times L_{rec}(t) dt
 \end{array}
 \left. \begin{array}{l}
 \text{High Signal-to-Noise Ratio} \\
 \text{Real Time Capture}
 \end{array} \right\} \begin{array}{l}
 \text{depth} \\
 \boxed{\text{phase } \varphi_{rec}} \\
 \text{offset } O_{rec} \\
 \text{amplitude } A_{rec}
 \end{array}$$

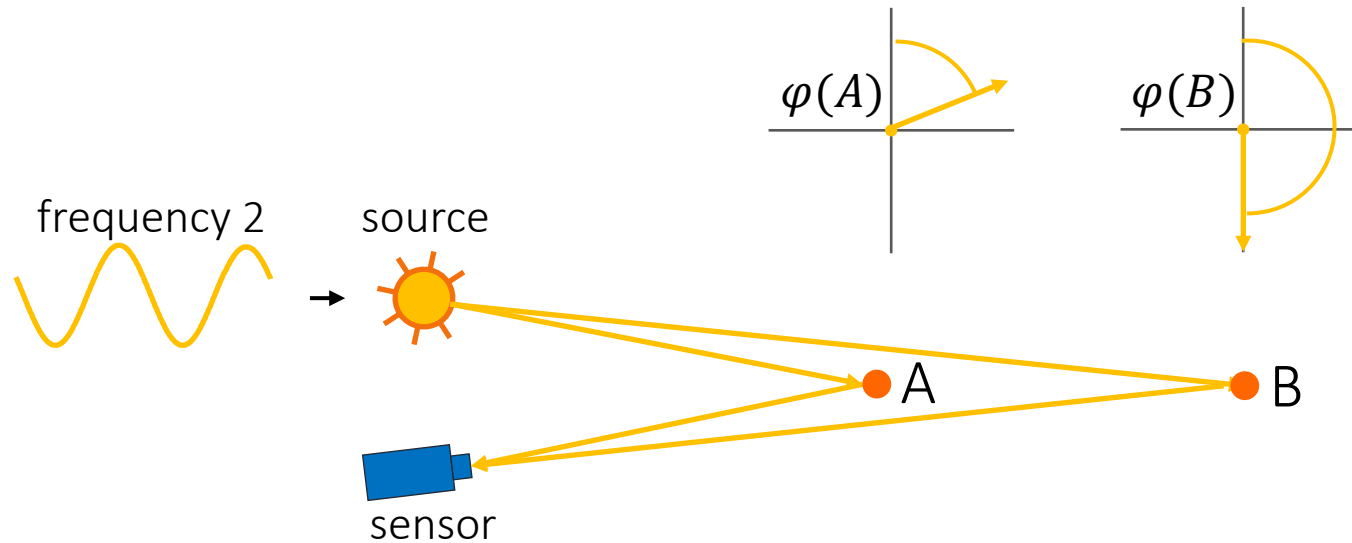
# Phase ambiguity



Different Scene Depths Have Same Phase

- Also known as “phase wrapping”.

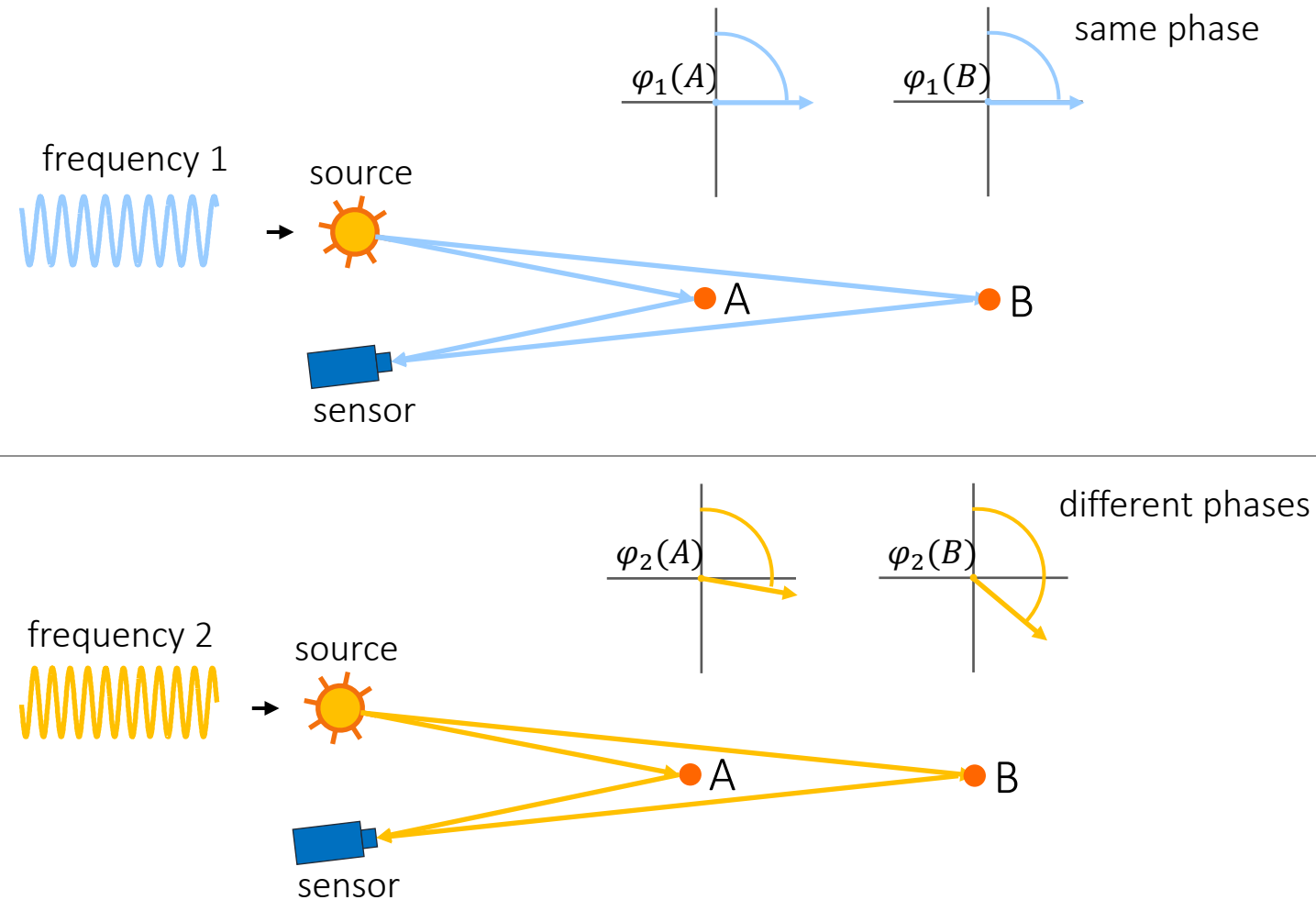
# Phase ambiguity



Unambiguous Depth Range:  $R_{unambiguous} = \frac{1}{2\omega}$

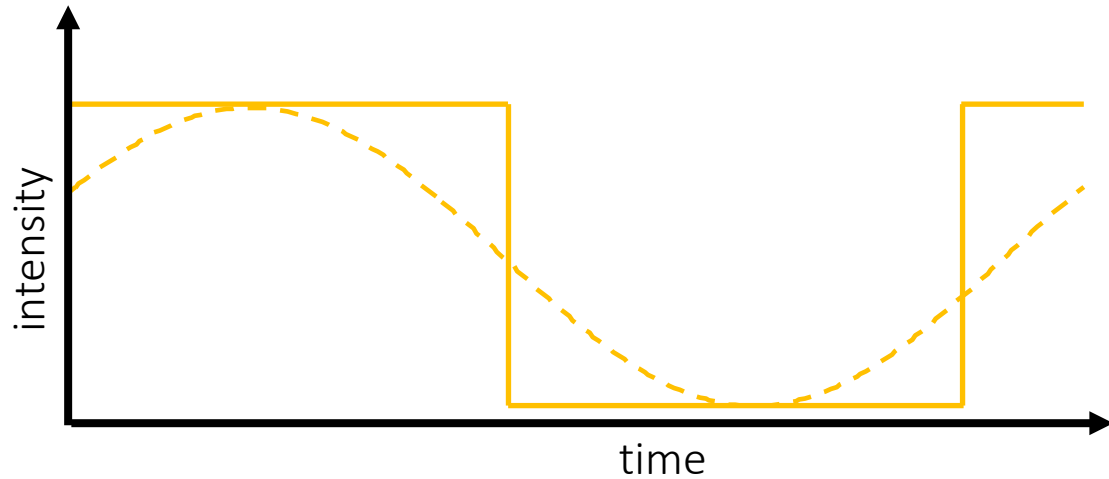
How can we resolve the phase ambiguity?

# Disambiguating phase

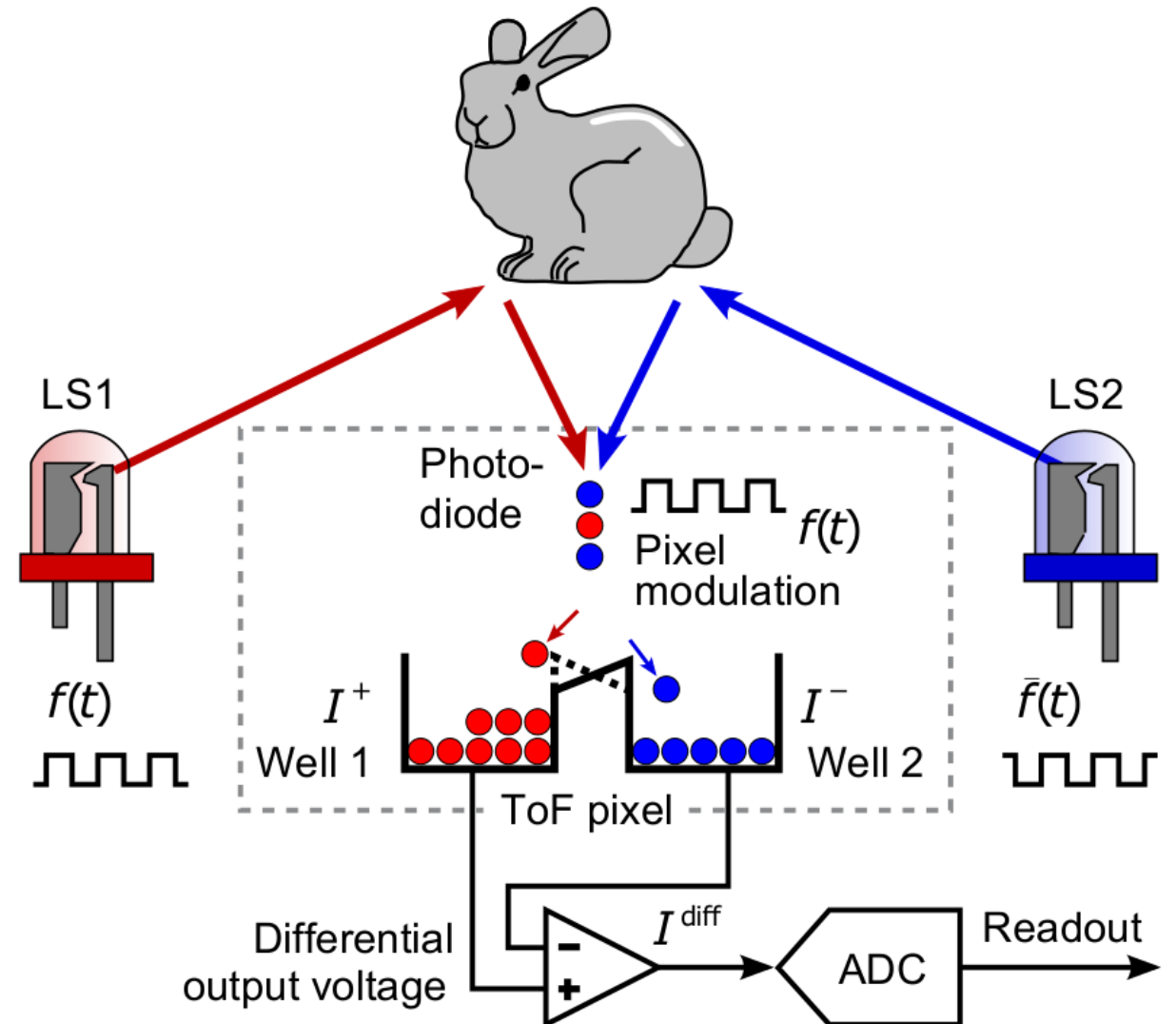


Compute phases at two different frequencies

# Implementation: two-well architectures



- approximate sinusoid with a square pulse
- store photons in different wells depending on whether they arrive at 1 or 0
- take difference between two wells





# Some examples

light source  
(bank of laser diodes)



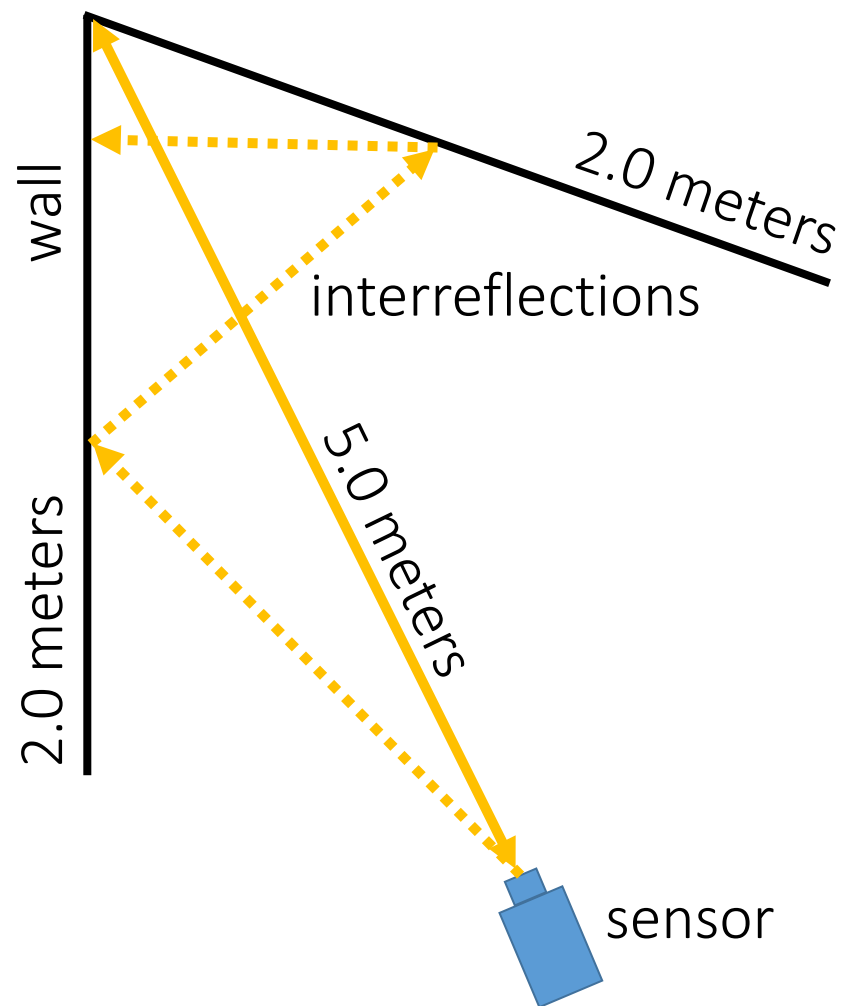
sensor  
(PMD CamBoard Nano)



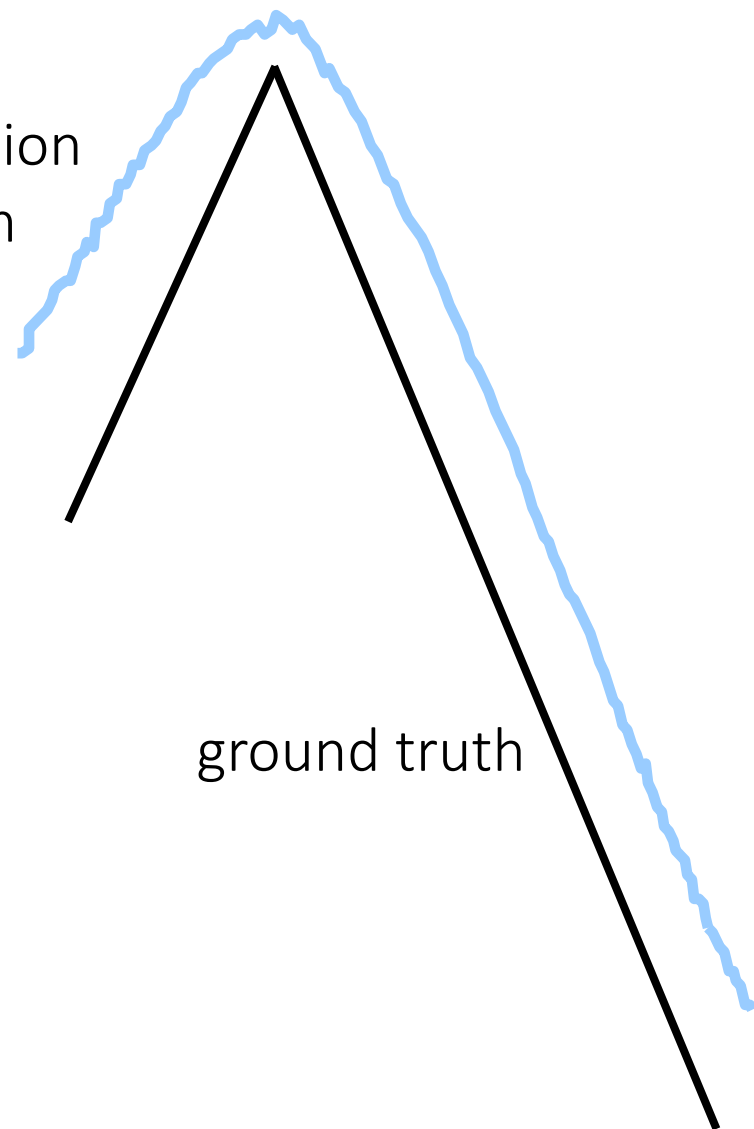
(only second generation of Kinect uses CW ToF)



# Multi-path interference

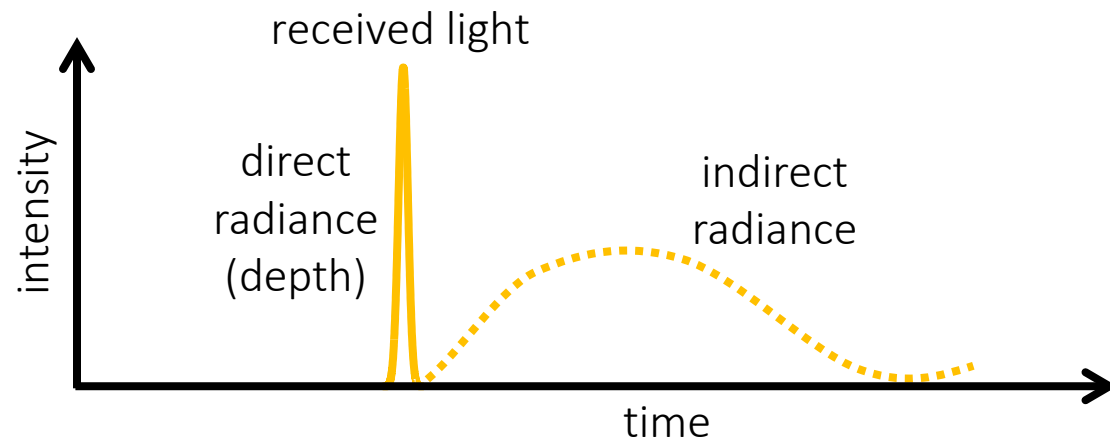
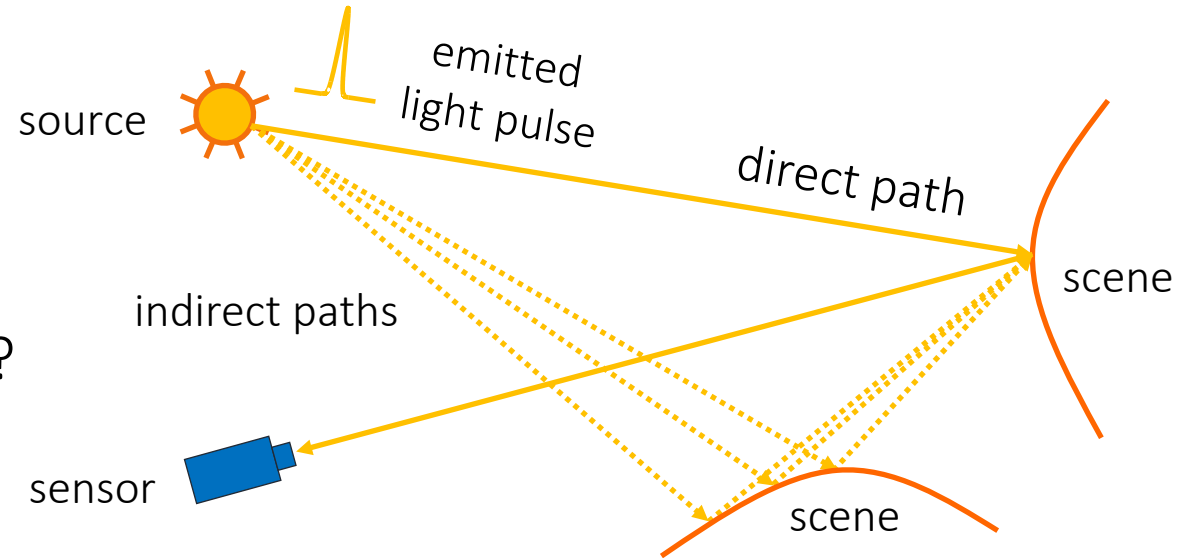


ToF depth reconstruction  
mean error = 86.6 mm



# Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?

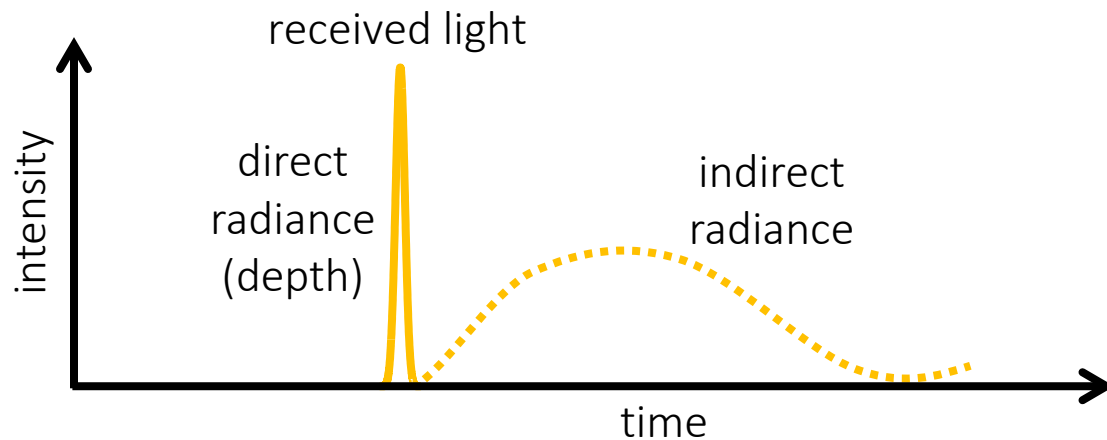
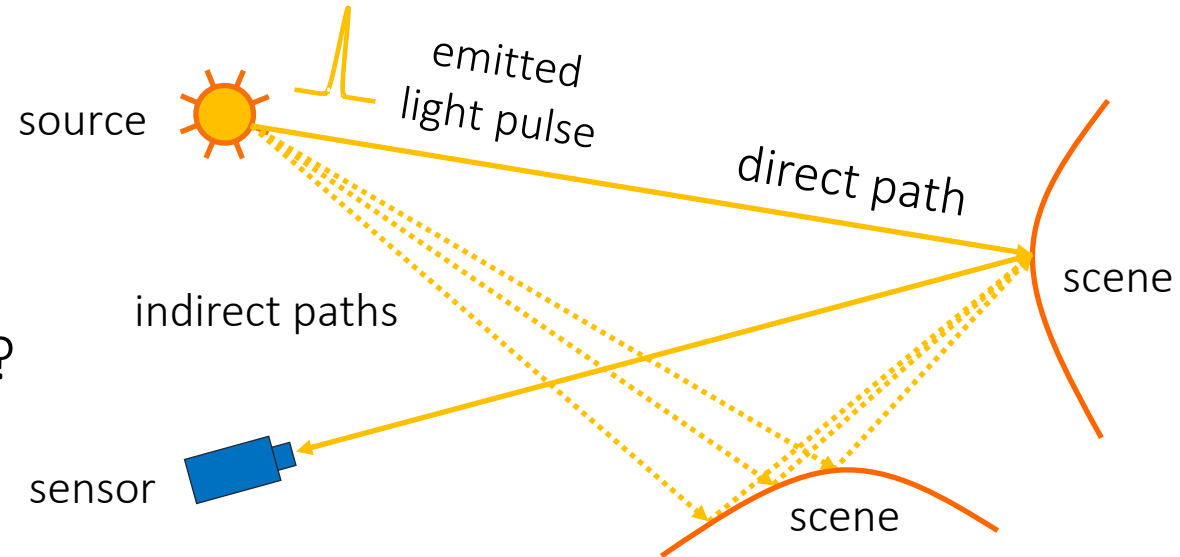


Transient  $I(t)$ : Time-resolved radiance distribution

- Range imaging: Measuring only first returning photons (e.g., LIDAR).
- Transient imaging: Measuring entire transient (e.g., SPAD).

# Transient imaging with continuous-wave ToF

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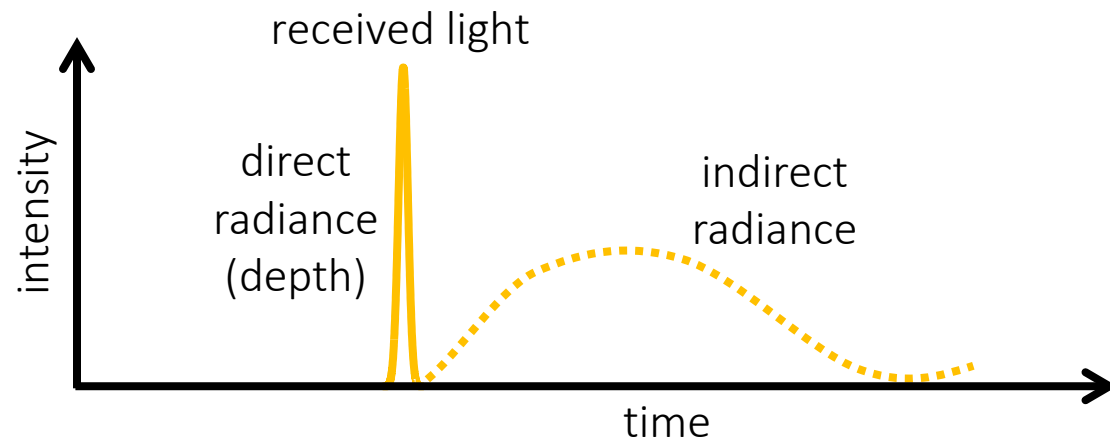
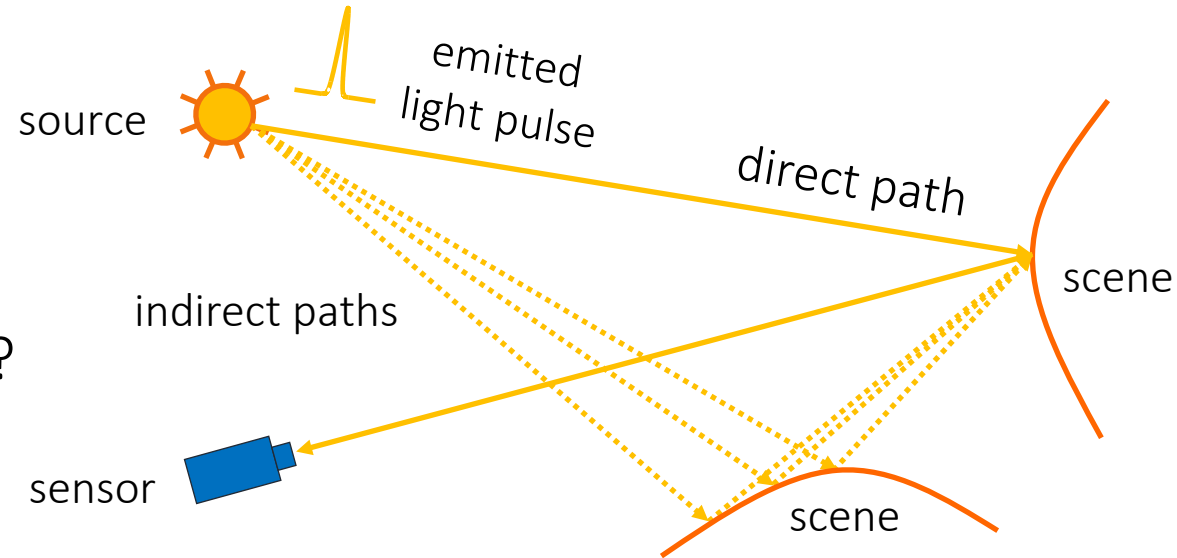
Transient  $I(t)$ : Time-resolved radiance distribution

- Each measurement we capture is of the form:

$$I(\omega) = \int \sin(\omega t) \cdot I(t) dt$$

# Transient imaging with continuous-wave ToF

How do we do transient imaging in the CW-ToF case?



Transient  $I(t)$ : Time-resolved radiance distribution

- Each measurement we capture is of the form:

$$I(\omega) = \int \sin(\omega t) \cdot I(t) dt$$

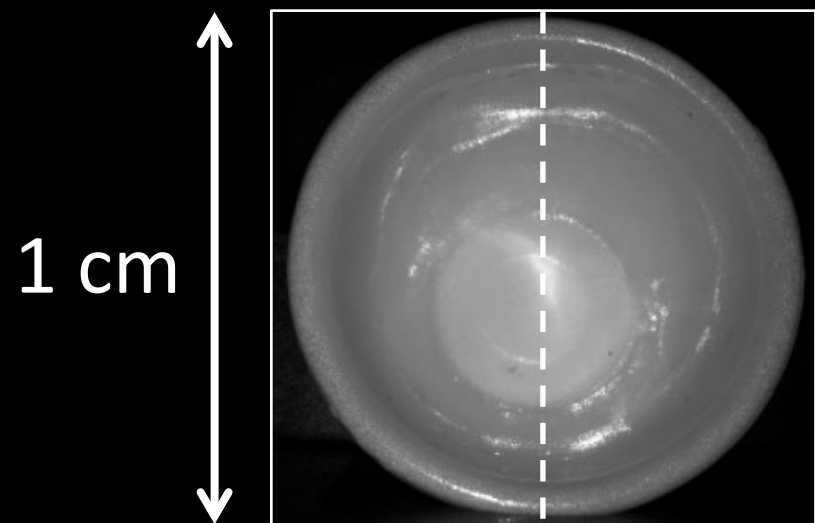
We can do transient imaging by taking measurements at multiple frequencies  $\omega$ , then doing an inverse Fourier transform

# Interferometric ToF imaging

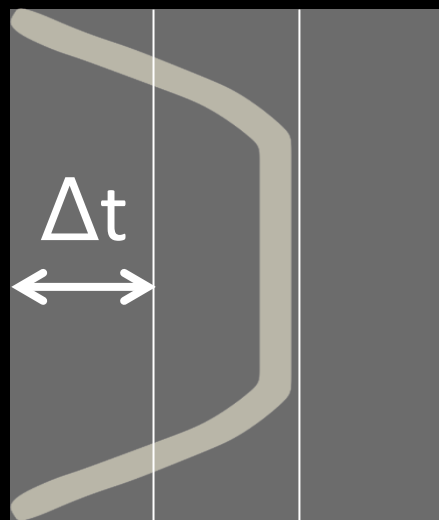
# Time-of-flight imaging technologies

	interferometry	streak cameras	single-photon avalanche diodes	time-of-flight cameras	LIDAR
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	continuous-wave ToF	impulse ToF			

# Tiny scenes



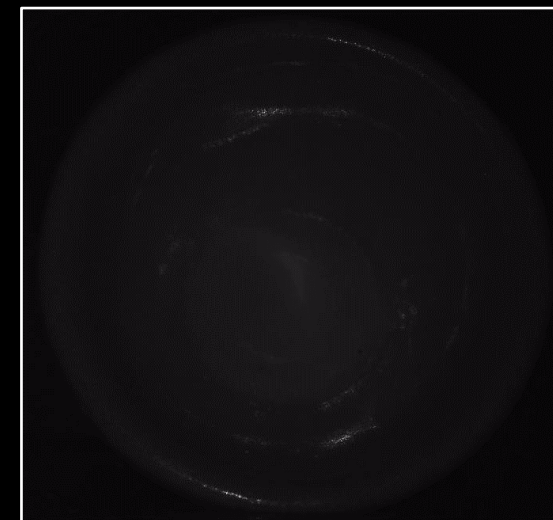
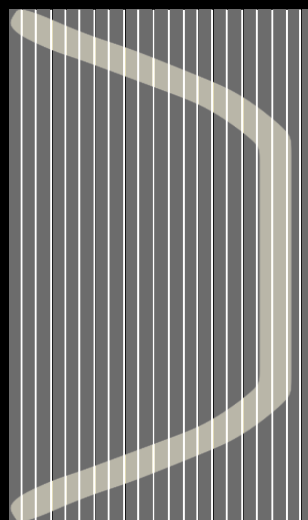
toy cup



0.5 cm



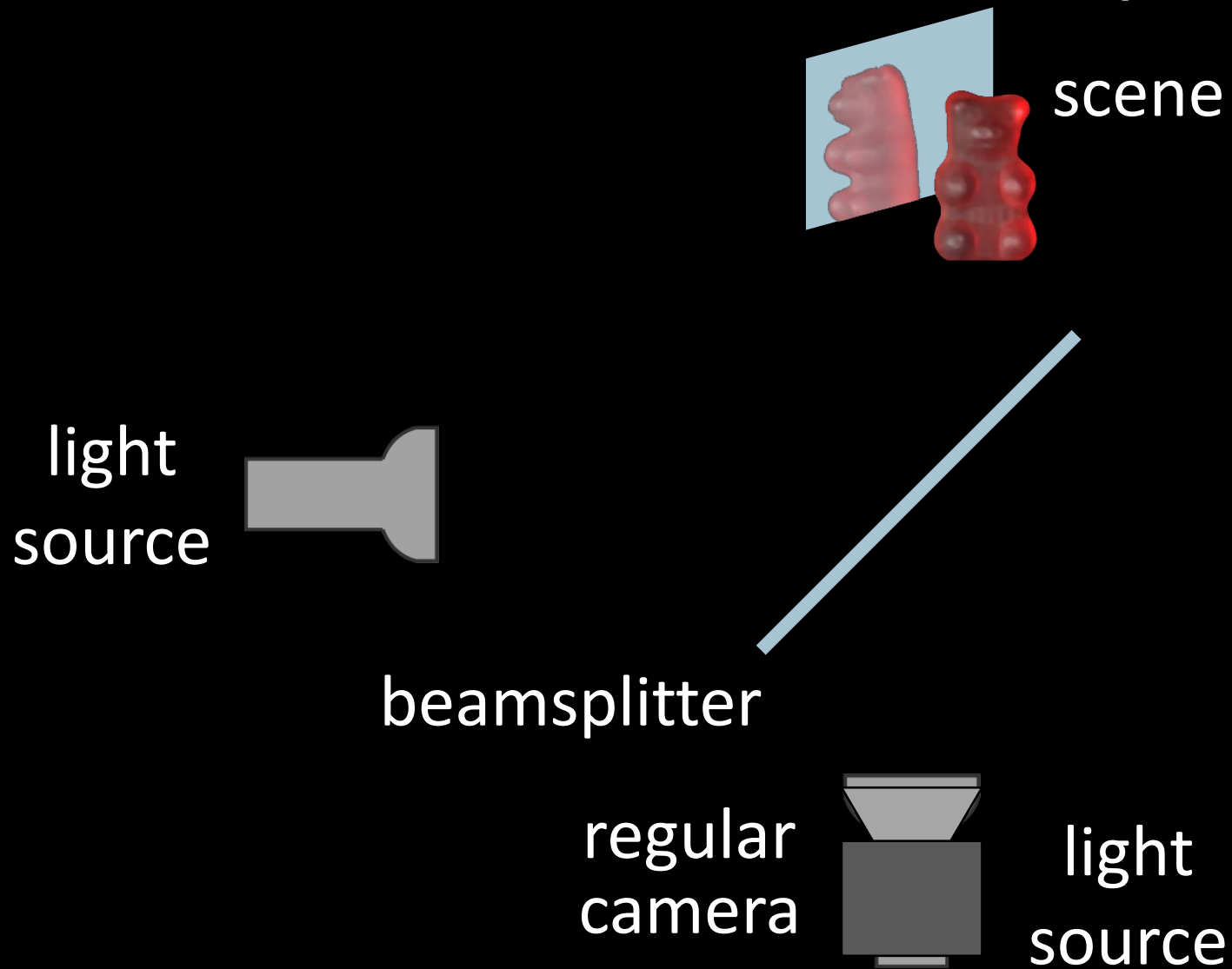
$\Delta t \sim \text{ps}$



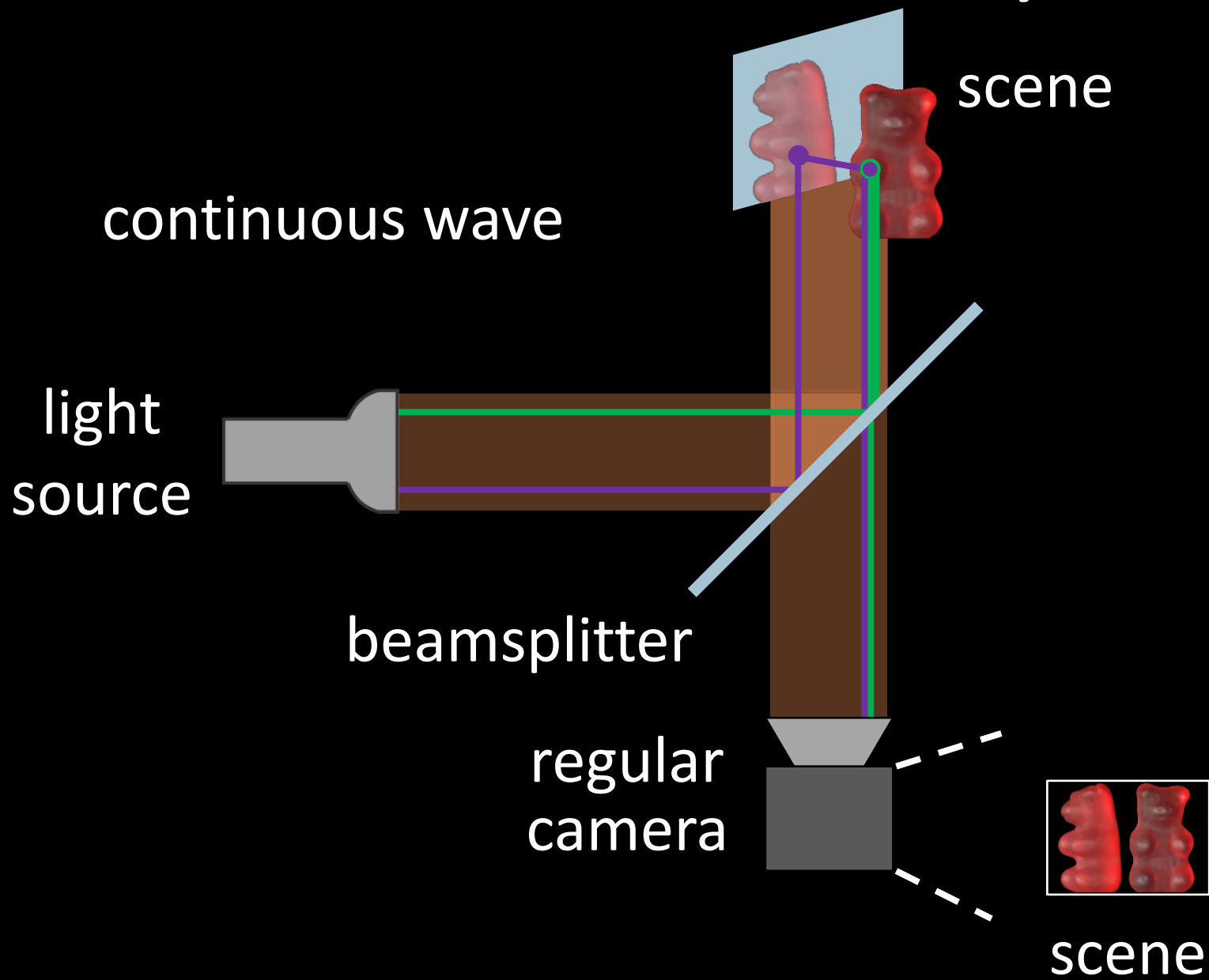
$\Delta t \sim 10^{-3} \text{ ps}$



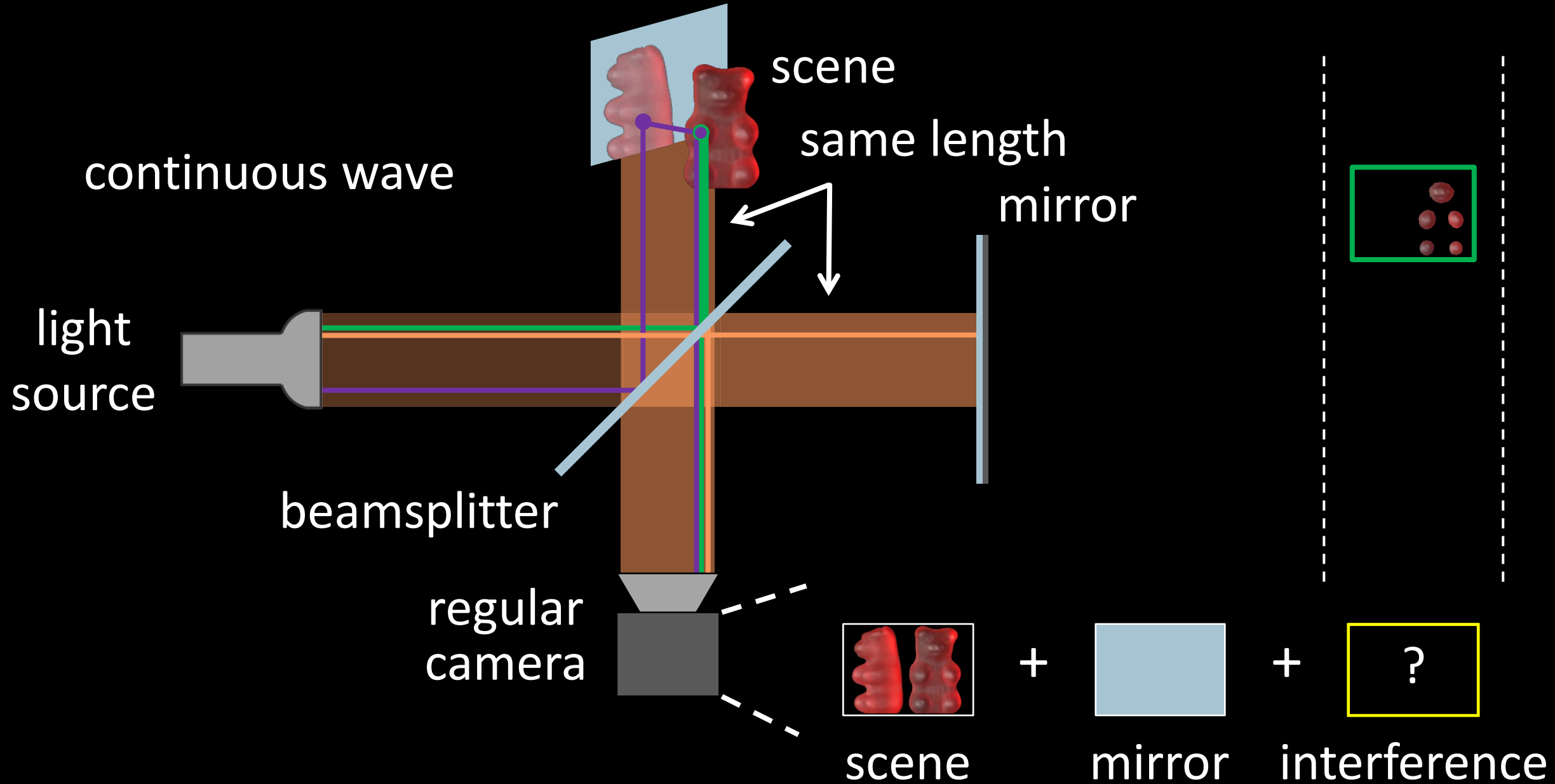
# Interferometry example



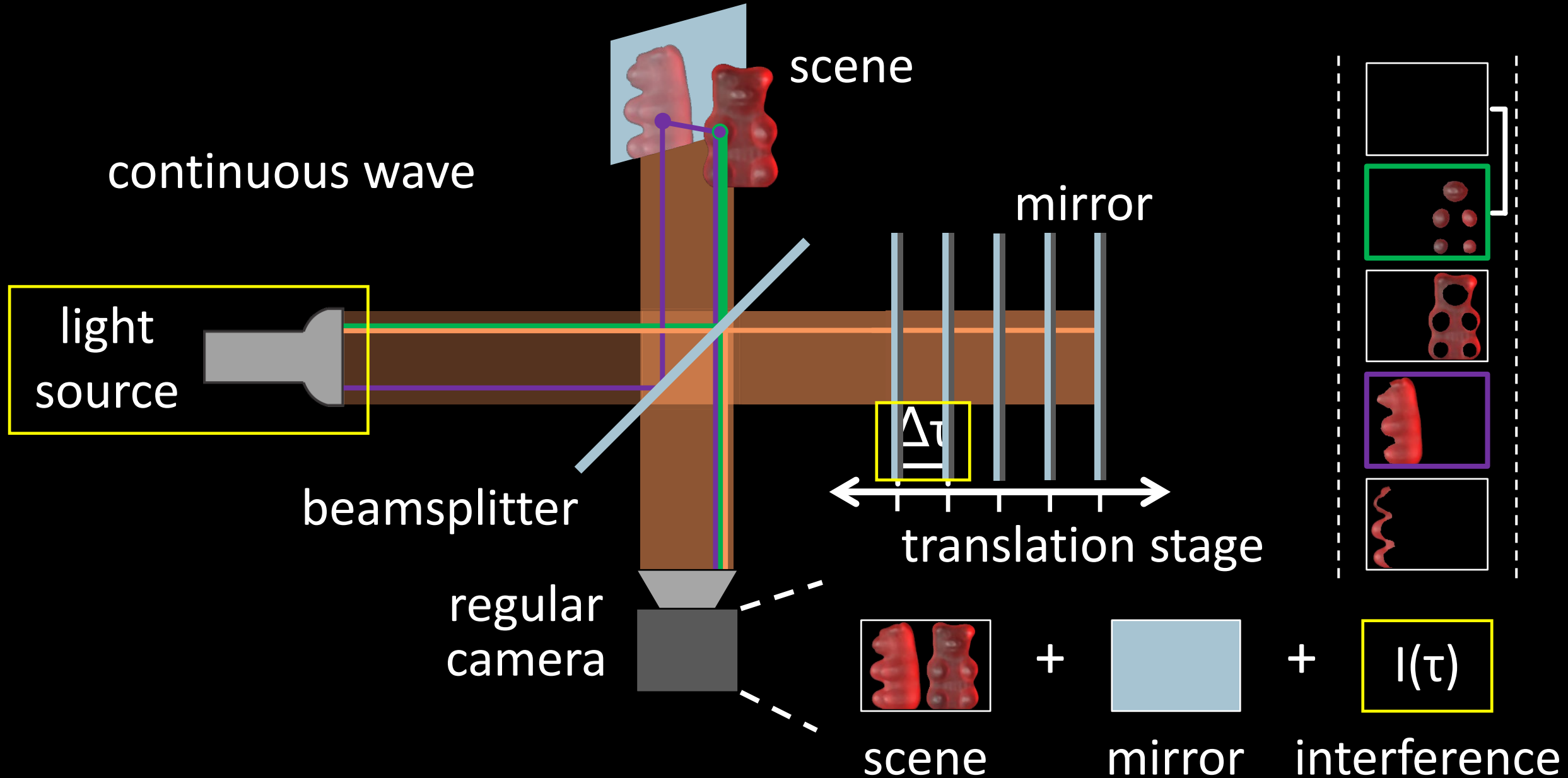
# Interferometry example



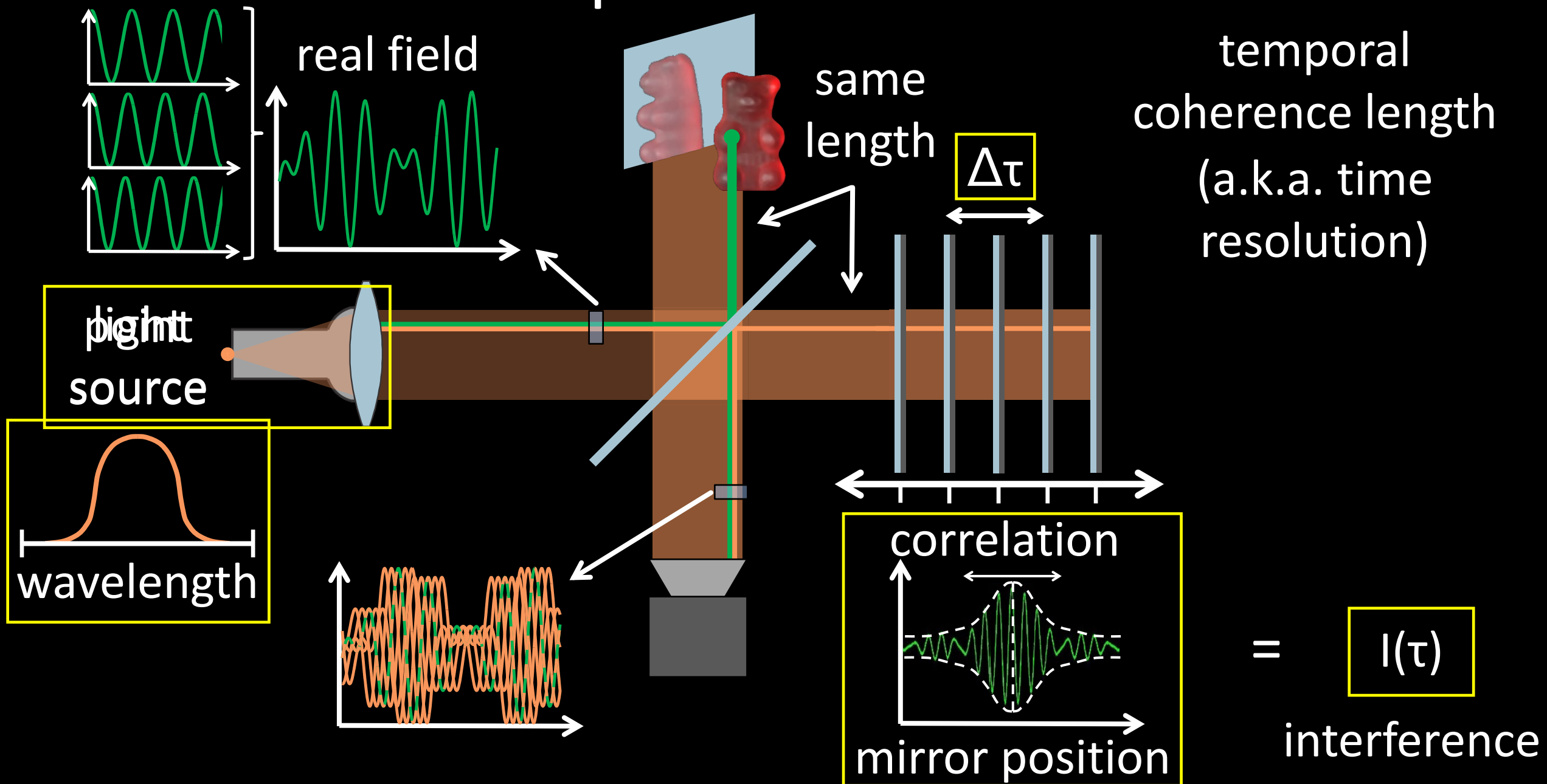
# Michelson interferometer



# Optical coherence tomography



# Temporal coherence



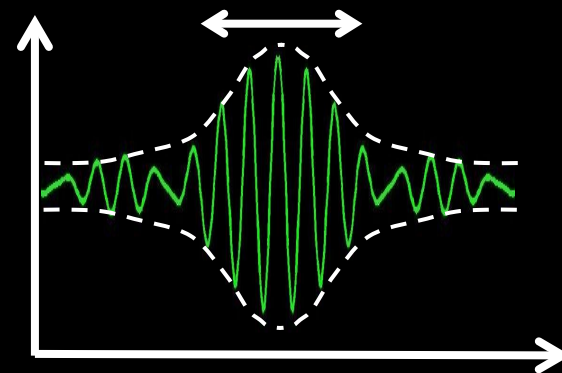
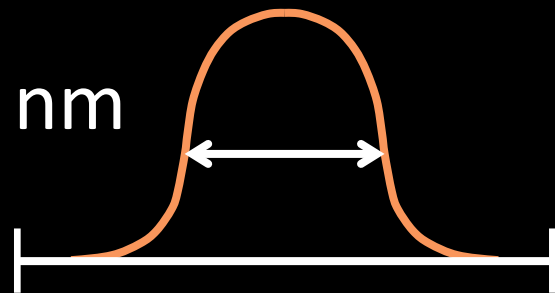
# Temporal coherence length

bandwidth

correlation

broadband

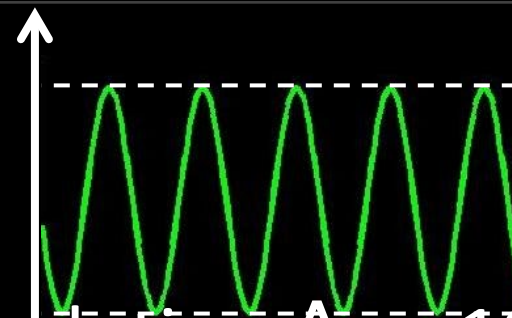
25 nm



photonic

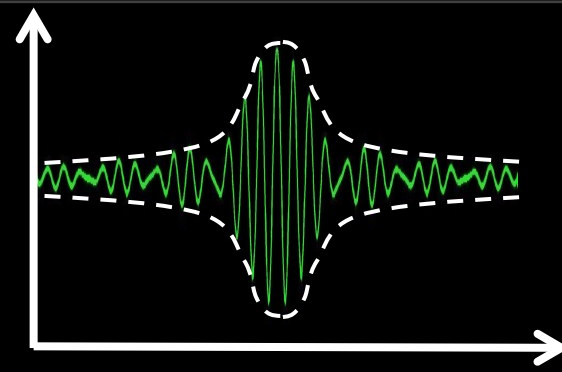
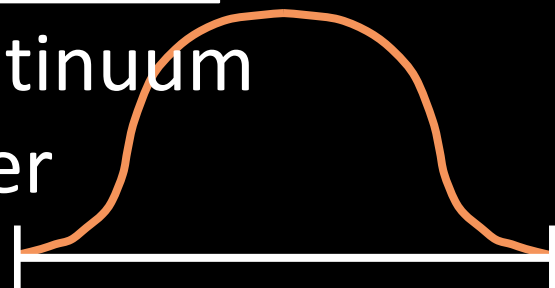


pathlength resolution  $\Delta\tau \sim 10 \mu\text{m}$



superluminescent  
broadband  
source

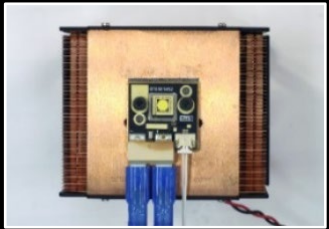
supercontinuum  
laser



# Optical setup



superluminescent diode    supercontinuum laser



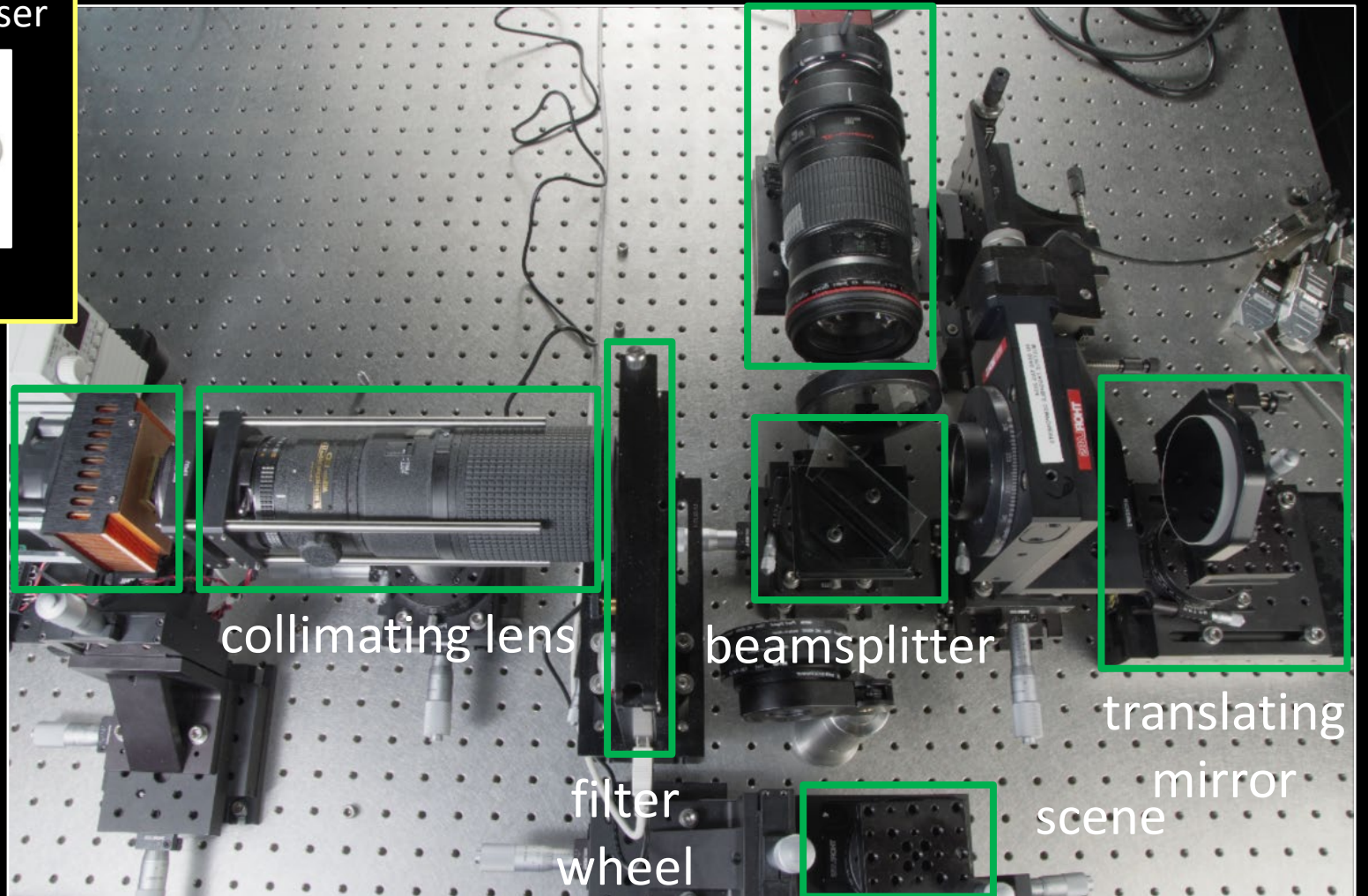
broadband LED



sodium lamp

light  
source

camera + imaging lens



collimating lens

beamsplitter

translating  
mirror

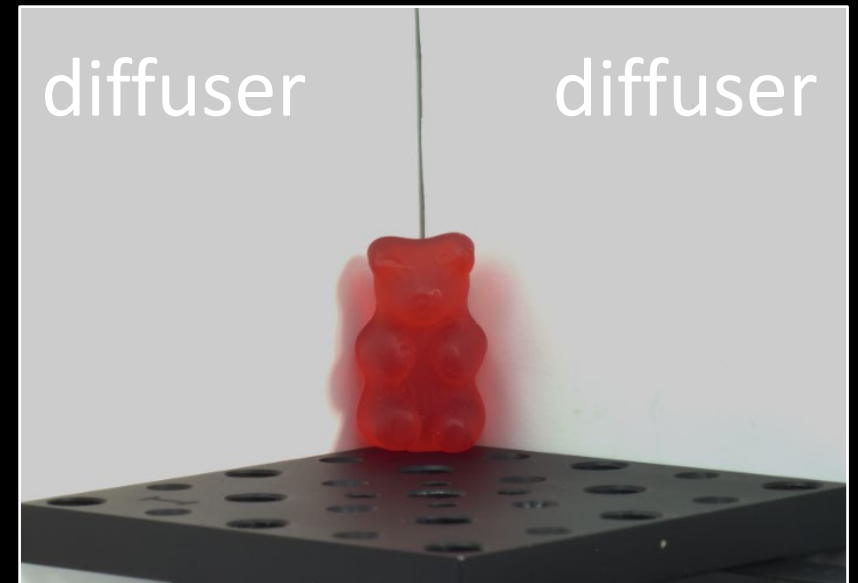
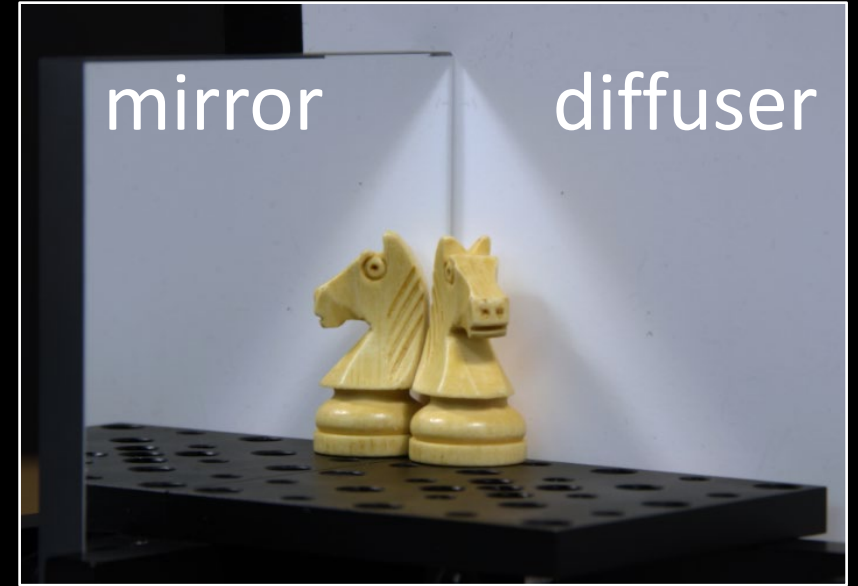
filter  
wheel

scene

# Some transient images

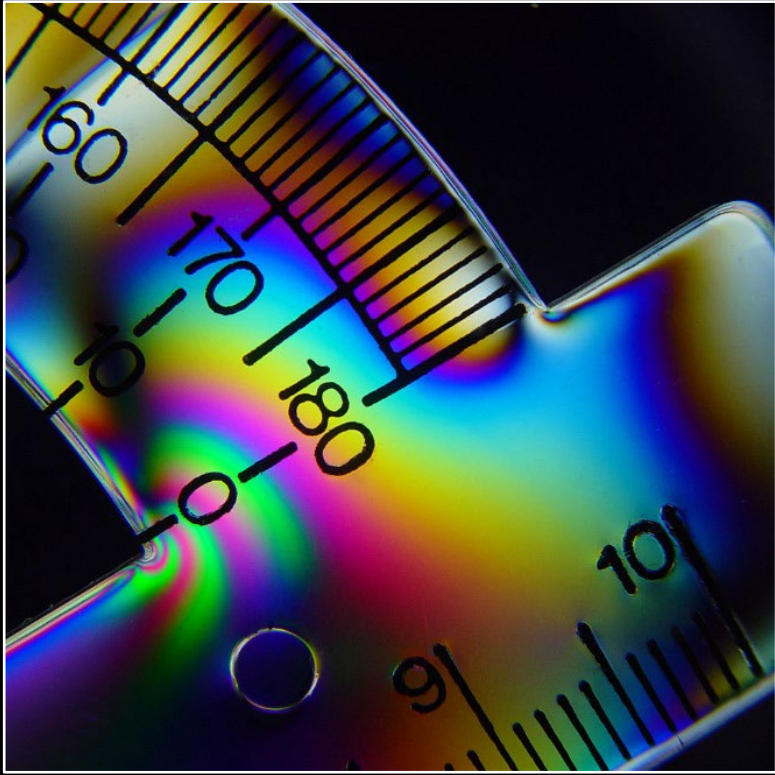


centimeter-sized objects





# Material properties



birefringence

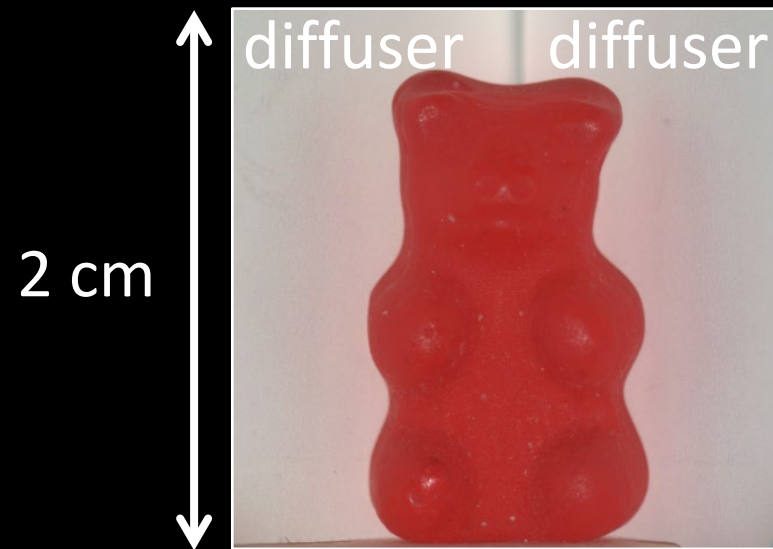


dispersion



scattering

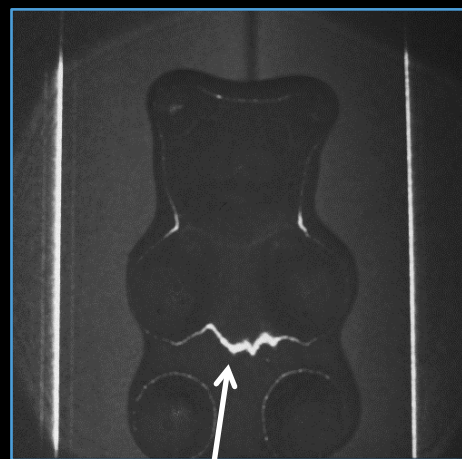
# Gummy bear and diffuse corner



pathlength  
( $\Delta\tau = 10 \mu\text{m}$ )



dark frame



surface reflections

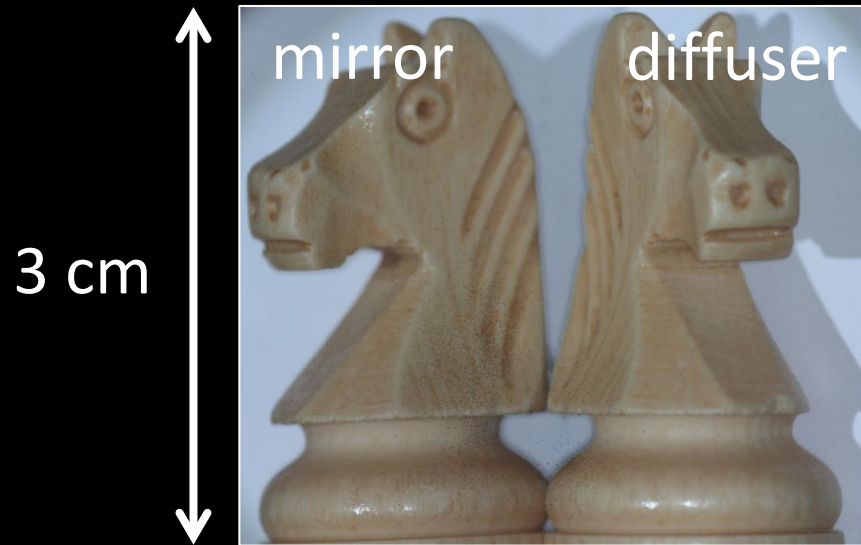


paths through  
gummy bear

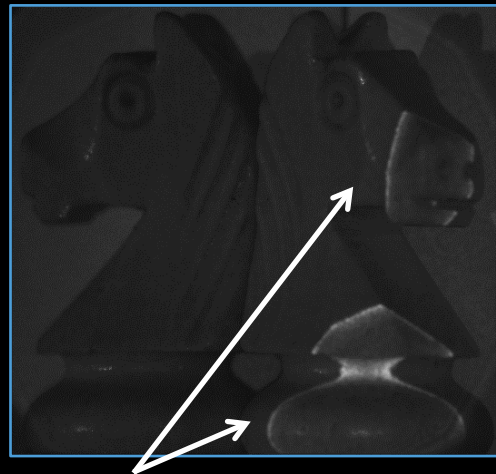


very highly  
scattered paths

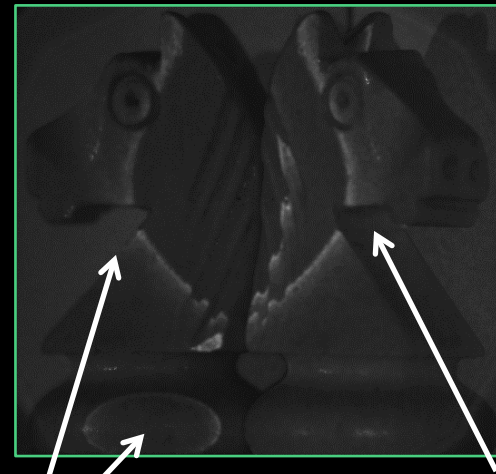
# Chess knight and mirror



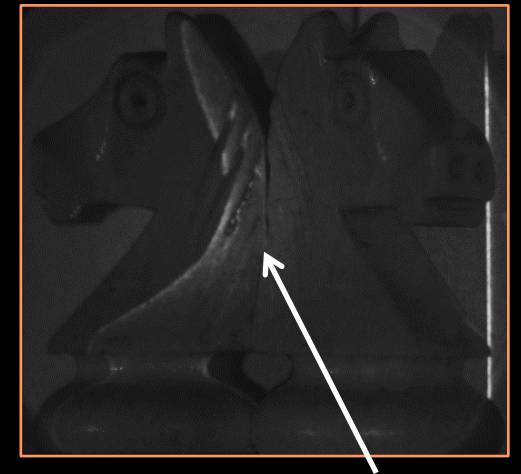
pathlength  
( $\Delta\tau = 10 \mu\text{m}$ )



surface reflection

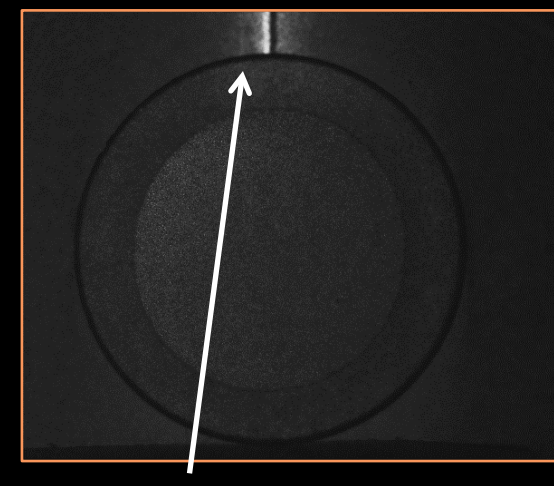
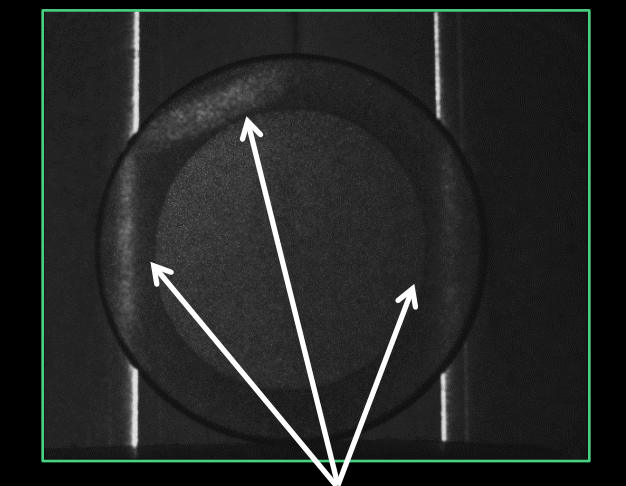
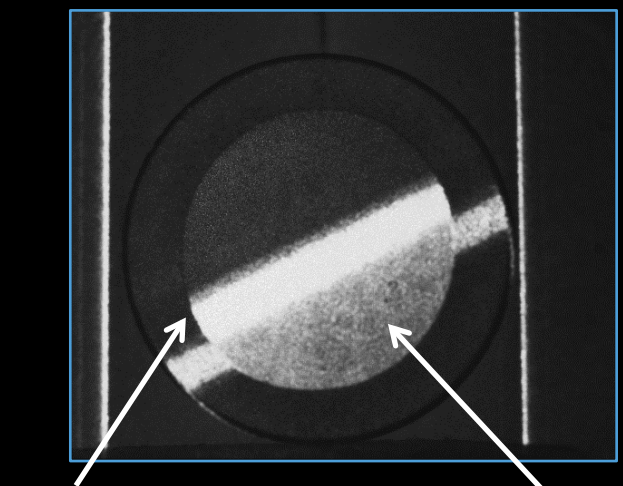
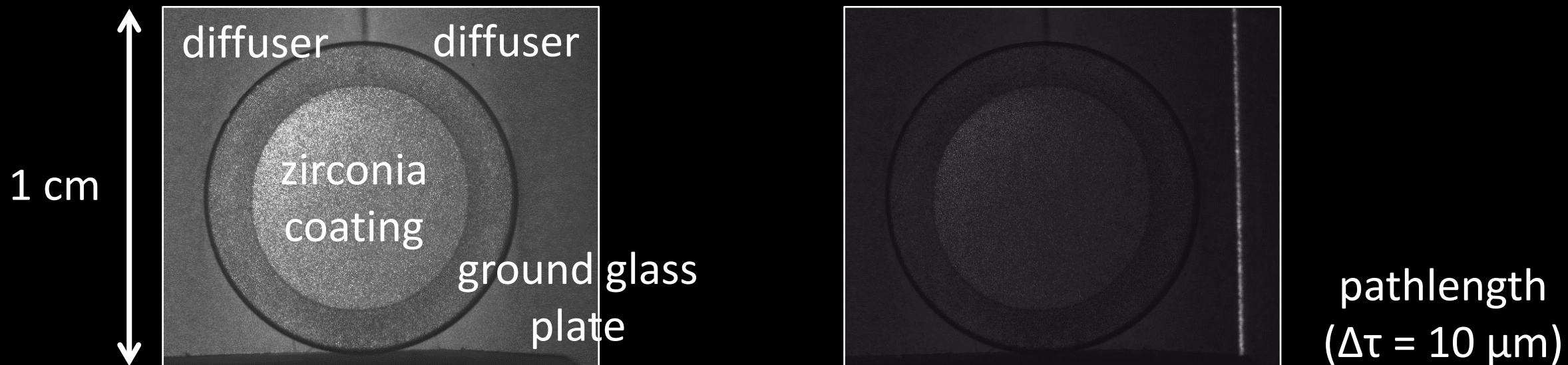


mirror-object  
object-mirror



mirror-object-mirror

# Subsurface scattering



surface reflection

subsurface scattering

paths transmitted through ground glass

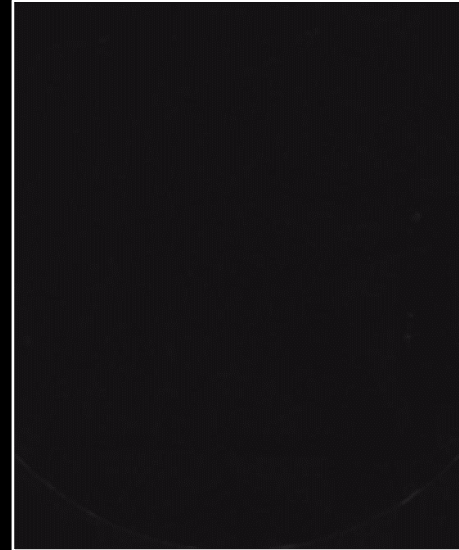
diffuse-diffuse reflections

# White jade

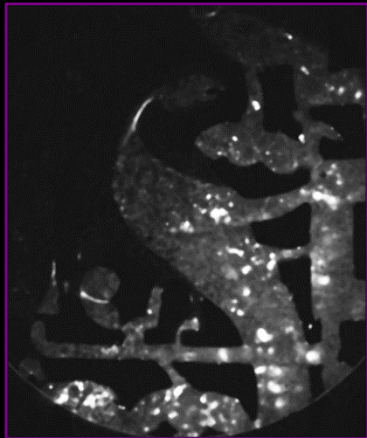
3 cm



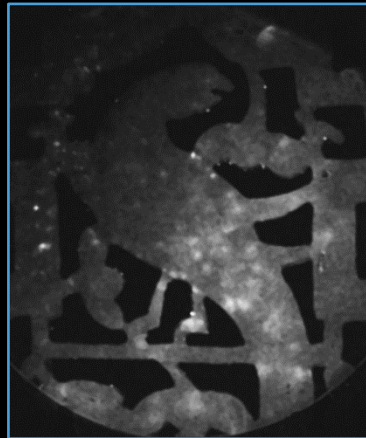
exquisite  
white jade



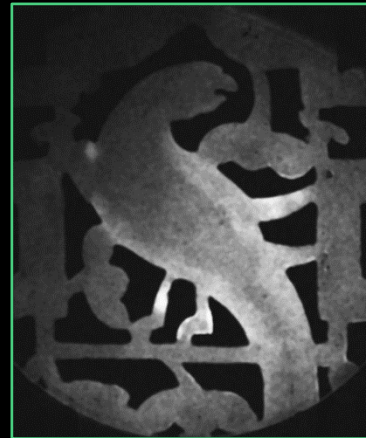
time ( $10^{-15}$  seconds)



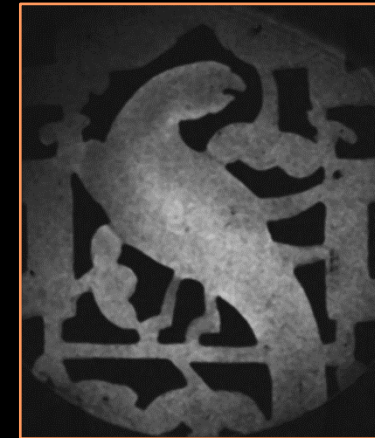
specular  
reflections



low-order  
scattering

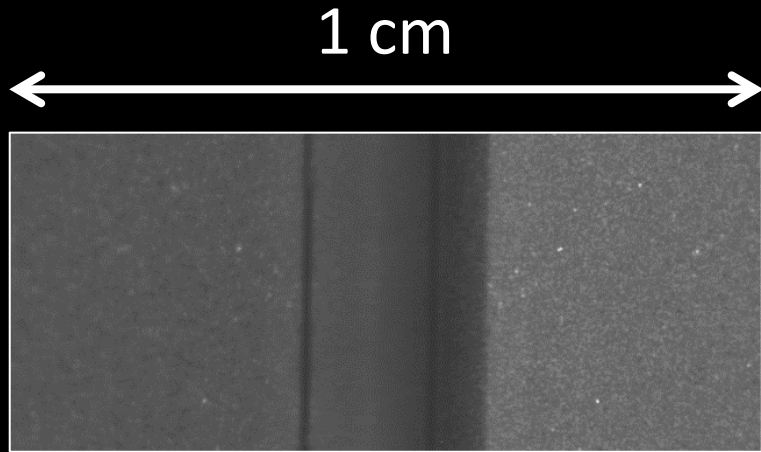


mid-order  
scattering

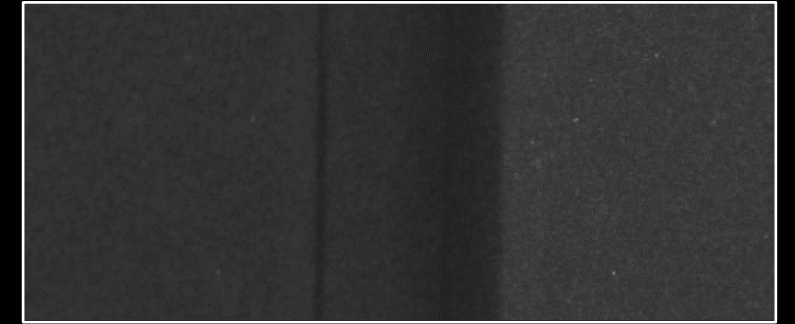
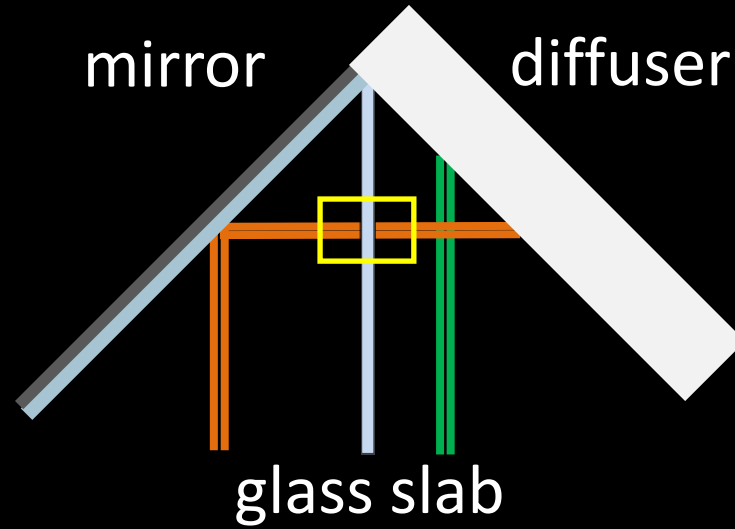


high-order scattering  
[TOG 2015]

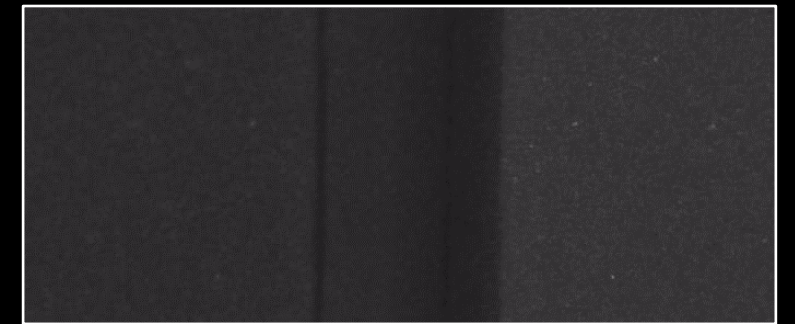
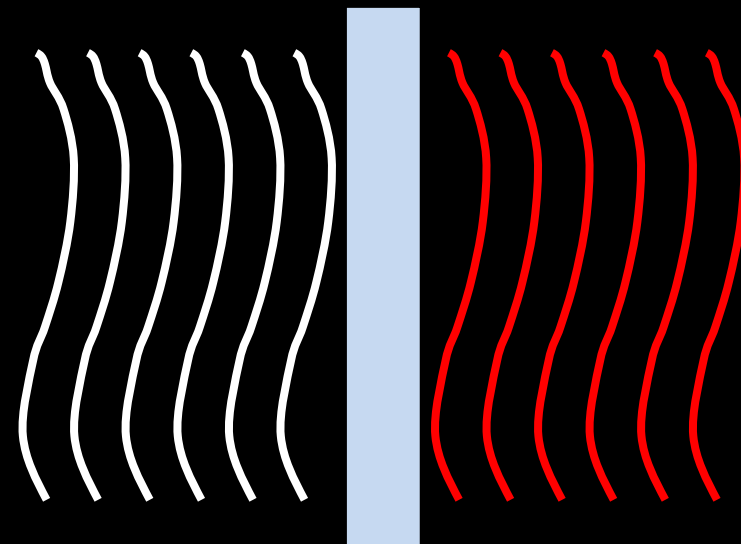
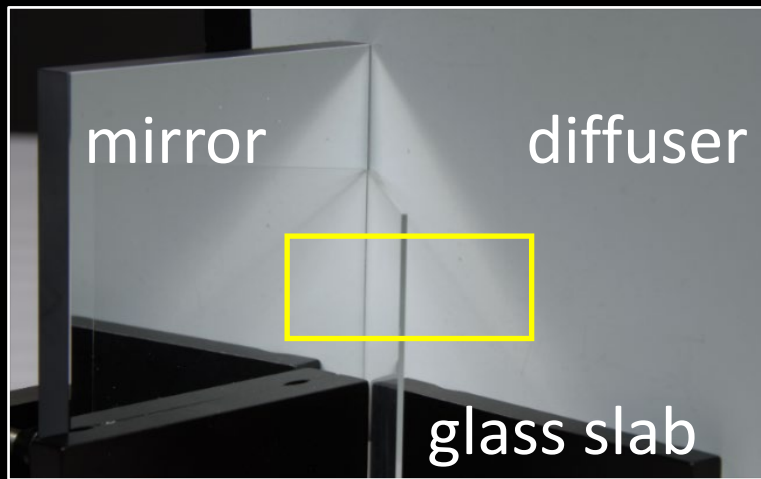
# Dispersion



cropped frame



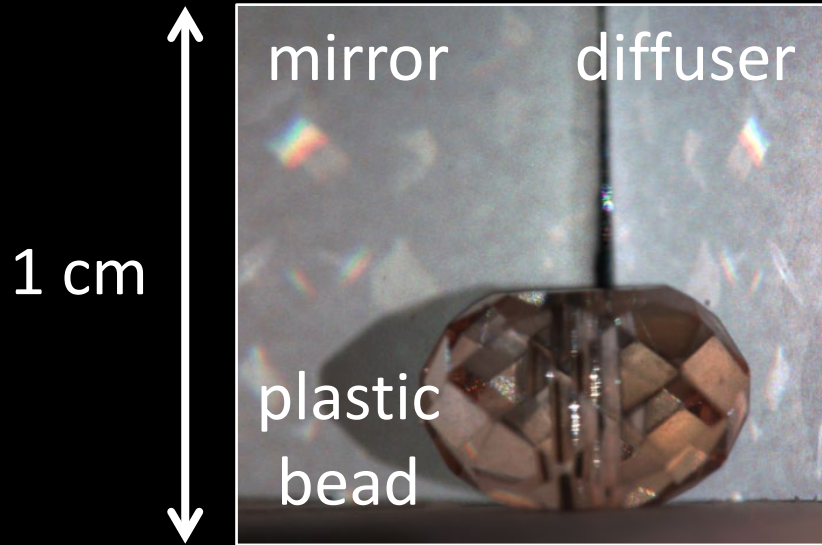
$\Delta t \sim \text{ns}$



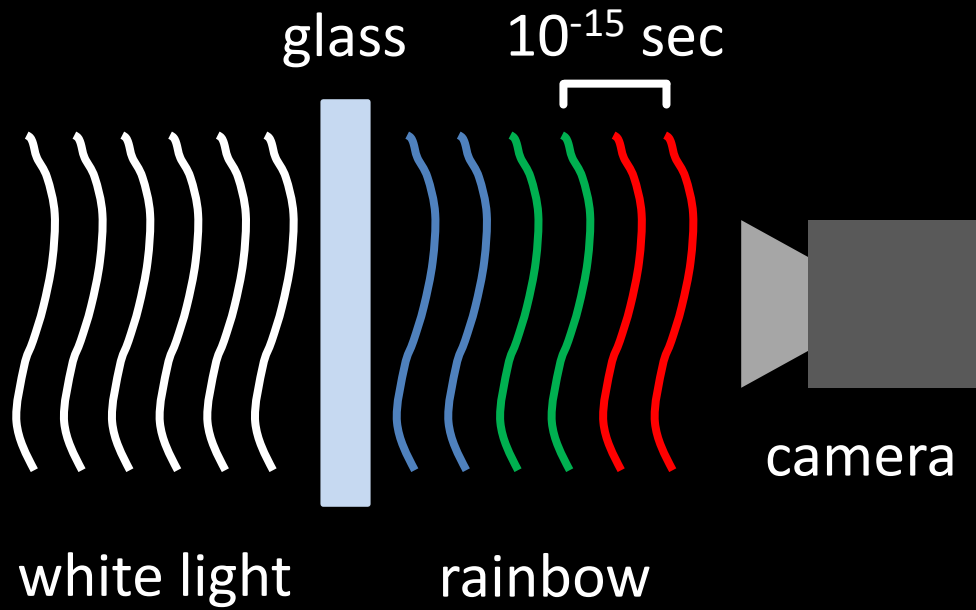
$\Delta t \sim 10^{-3} \text{ ns}$

refractive index  $\eta(\text{wavelength})$

# Dispersion



# Visualizing dispersion



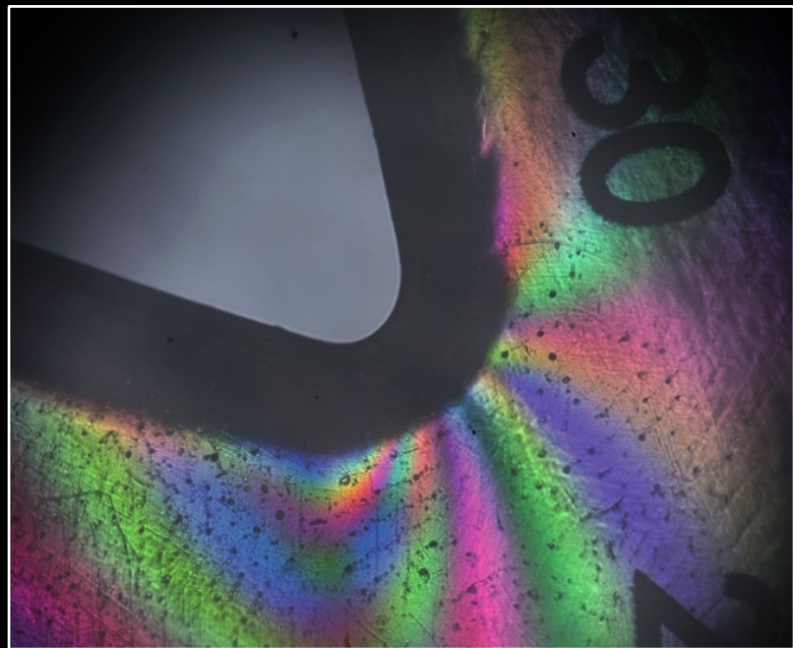
what a regular camera sees



what our camera sees



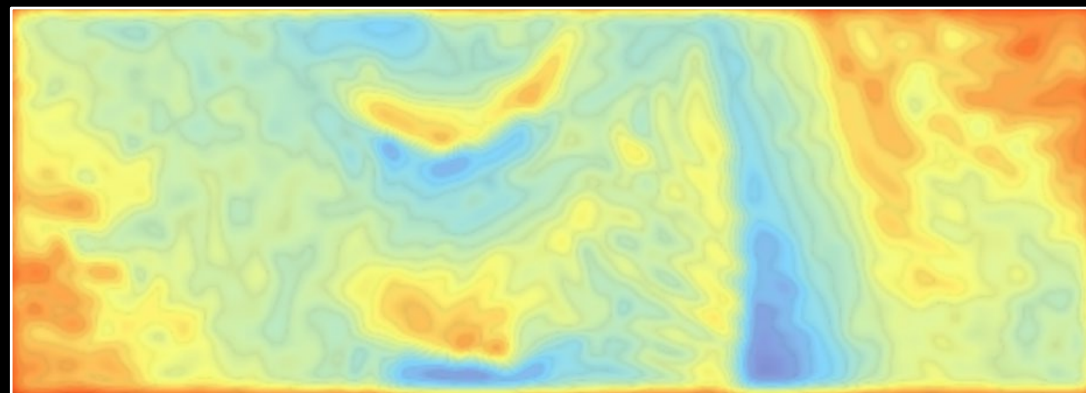
# Visualizing photoelasticity



detail under cross polarized light



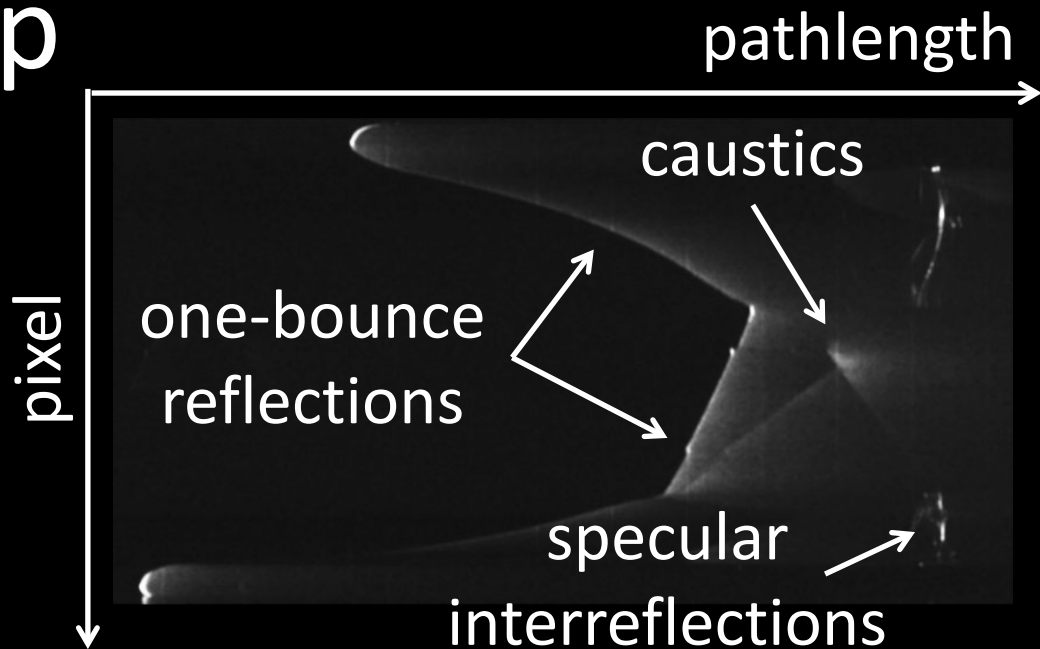
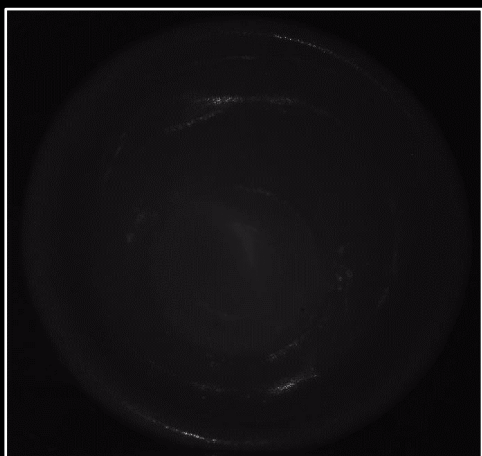
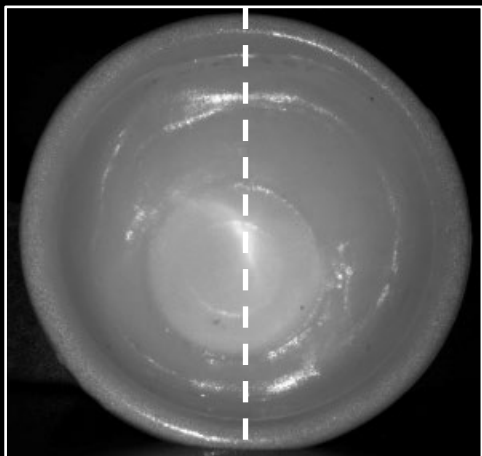
low resolution  $\Delta\tau = 1 \text{ mm}$



high resolution  $\Delta\tau = 10 \mu\text{m}$

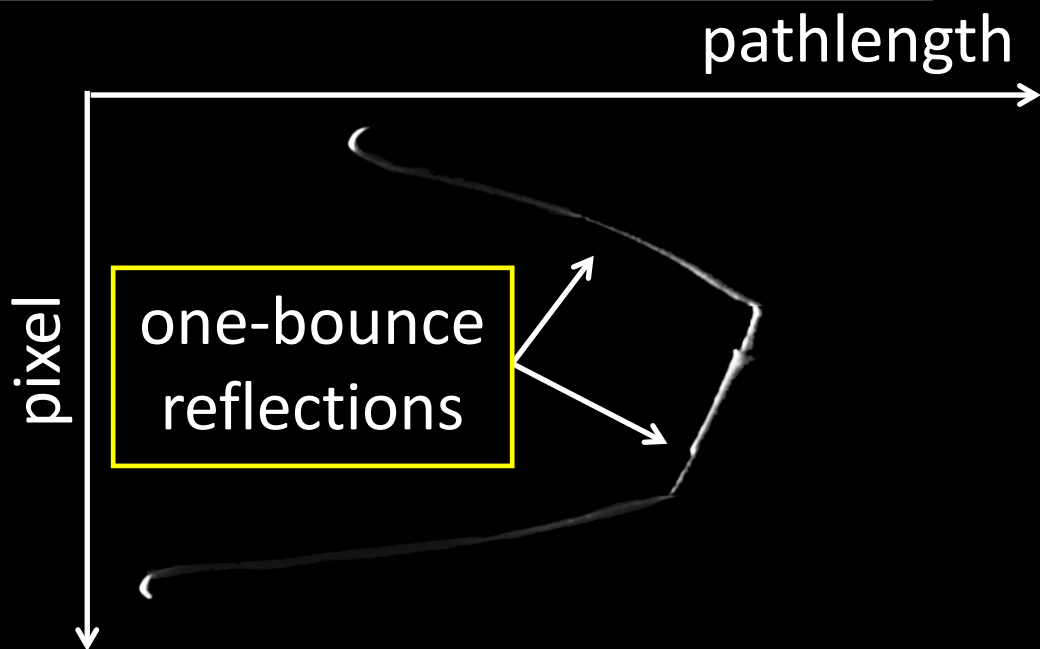
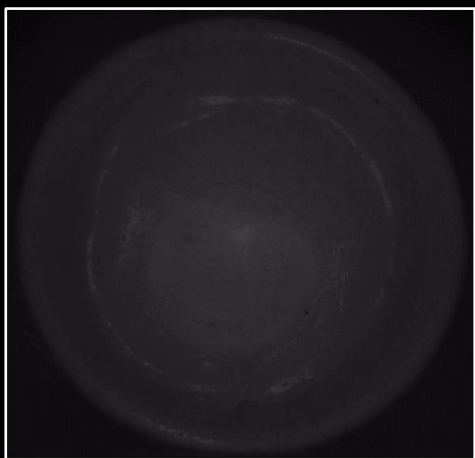
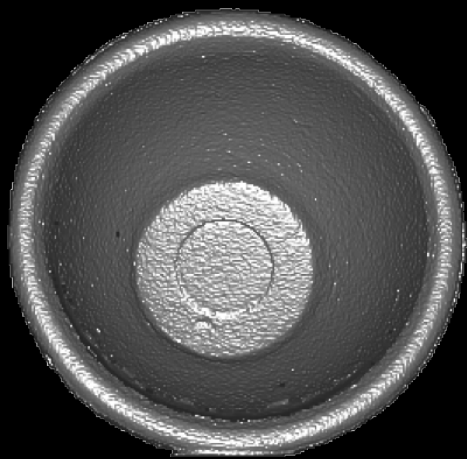
# Toy cup

full transient



measured depth

direct-only transient



# Depth scanning

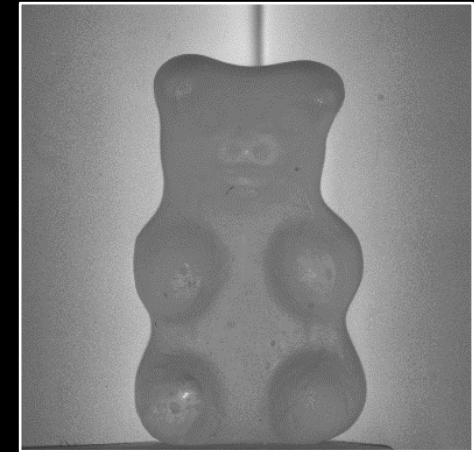
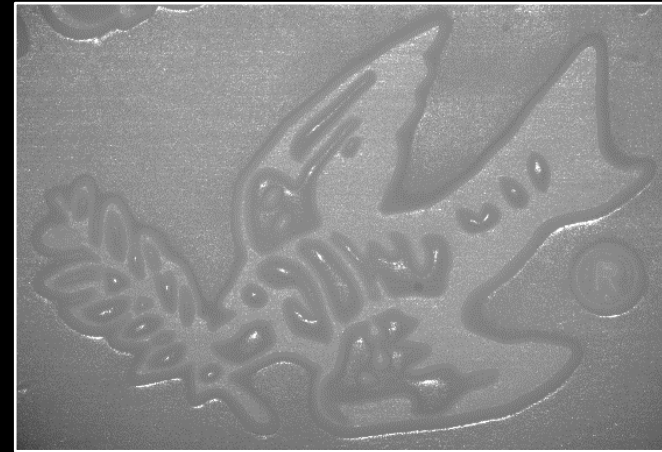
depth resolution  $\Delta\tau \sim 10 \mu\text{m}$

coin

gnocchi

soap carving

gummy bear  
and diffusers

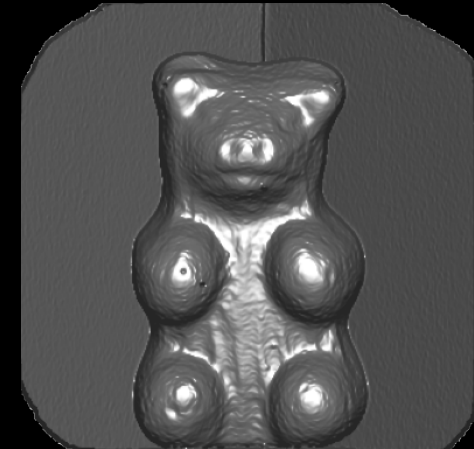
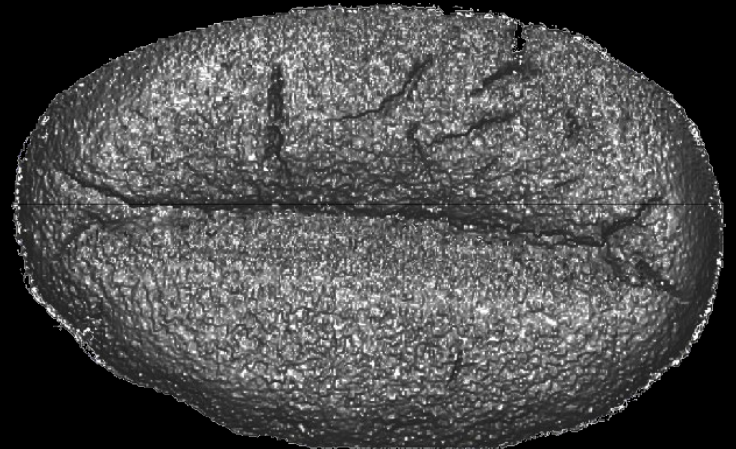


2.5 cm

1 cm

1.5 cm

3 cm



# References

## Basic reading:

- Gupta et al., “Computational Time-of-Flight,” ICCV 2015 tutorial, <http://web.media.mit.edu/~achoo/iccvtoftutorial/>  
this tutorial provides an overview of many of the topics covered in this lecture, with a focus on continuous-wave ToF imaging.
- Jarabo et al., “Recent Advances in Transient Imaging: A Computer Graphics and Vision Perspective,” Visual Informatics 2017  
a great review paper for ToF imaging.
- Velten et al., “Femto-photography: capturing and visualizing the propagation of light,” SIGGRAPH 2013, CACM 2016.  
the paper that introduced the idea of transient imaging to the computational imaging community, and an explanation of how streak cameras work.
- Lange et al., “Solid-state time-of-flight range camera,” JQE 2001.  
a standard reference on continuous-wave ToF sensors.
- Heide et al., “Low-budget transient imaging using photonic mixer devices,” SIGGRAPH 2013.
- Lin et al., “Fourier analysis on transient imaging with a multifrequency time-of-flight camera,” CVPR 2014.
- Peters et al., “Solving trigonometric moment problems for fast transient imaging,” SIGGRAPH 2015.  
three papers showing how continuous-wave ToF sensors can be used for transient imaging.
- Gupta et al., “Phasor imaging: A generalization of correlation-based time-of-flight imaging,” TOG 2015.  
a more recent paper that provides nice insights into how continuous-wave ToF works, as well as a way to deal with MPI.
- Abramson, “Light-in-flight recording by holography,” Optics Letters 1978.  
a very early paper showing visualization of light-in-flight, i.e., transient imaging.
- Huang et al., “Optical Coherence Tomography,” Science 1991.  
the paper introducing optical coherence tomography.
- Gkioulekas et al., “Micron-scale light transport decomposition using interferometry,” SIGGRAPH 2014.  
the paper showing how interferometry can be used for time-of-flight imaging.
- Gariepy et al., “Single-photon sensitive light-in-flight imaging,” Nature Communications 2015.  
the paper describing how SPADs can be used for ToF imaging.
- O’Toole et al., “Reconstructing Transient Images from Single-Photon Sensors,” CVPR 2017.  
a paper explaining the operation of SPADs in a more accessible manner to computer science backgrounds.
- Pediredla et al., “Signal processing based pile-up compensation for gated single-photon avalanche diodes,” 2018.
- Heide et al., “Sub-picosecond photon-efficient 3D imaging using single-photon sensors,”
- Gupta et al., “Photon-flooded single-photon 3d cameras,” CVPR 2019.  
three papers discussing the pile-up issue and proposing ways to overcome it.
- Mark Itzler, “Single-photon LiDAR imaging: from airborne to automotive platforms,” ICCP 2020 keynote, <https://www.youtube.com/watch?v=4tEfVr6fKgw>  
a keynote discussing advantages and current state of SPAD LiDAR technology.

## Additional reading:

- Kirmani et al., “Looking around the corner using ultrafast transient imaging,” ICCV 2009 and IJCV 2011.
- Velten et al., “Recovering three-dimensional shape around a corner using ultrafast time-of-flight imaging,” Nature Communications 2012.  
the first two papers showing how ToF imaging can be used for looking around the corner.