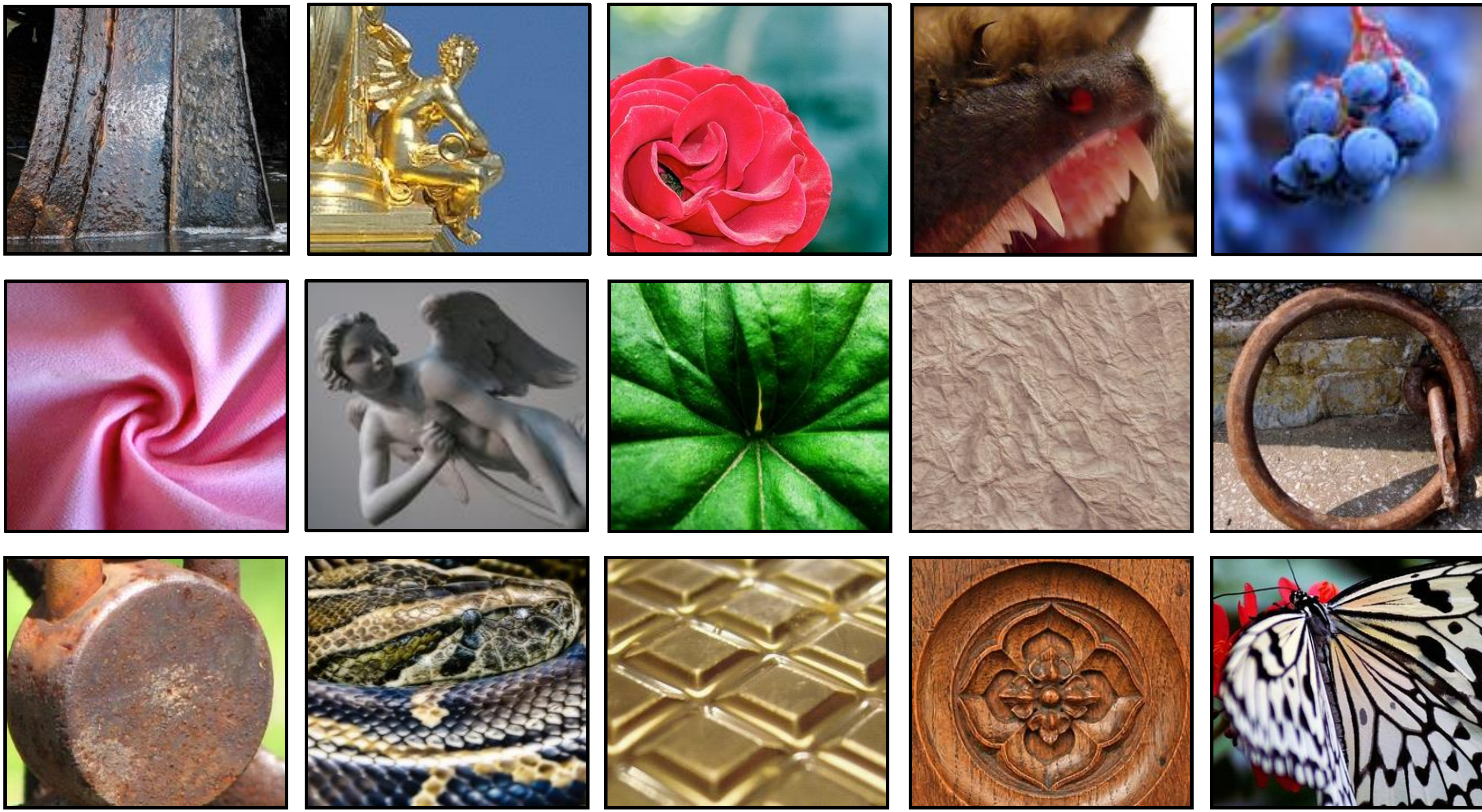
Radiometry and reflectance



15-463, 15-663, 15-862
Computational Photography
Fall 2019, Lecture 15

http://graphics.cs.cmu.edu/courses/15-463

Course announcements

- Homework 4 is due on October 25th.
  - Any questions?

- Project proposals due **tonight**.
  - One-day extension to make up for late feedback on project ideas.

- Mid-semester grades have been posted.
  - Based on average grades for HW1 – HW3.
  - Please let me know of any issues **before 5pm today.**

Overview of today's lecture

- Appearance phenomena.
- Measuring light and radiometry.
- Reflectance and BRDF.
- Light sources.

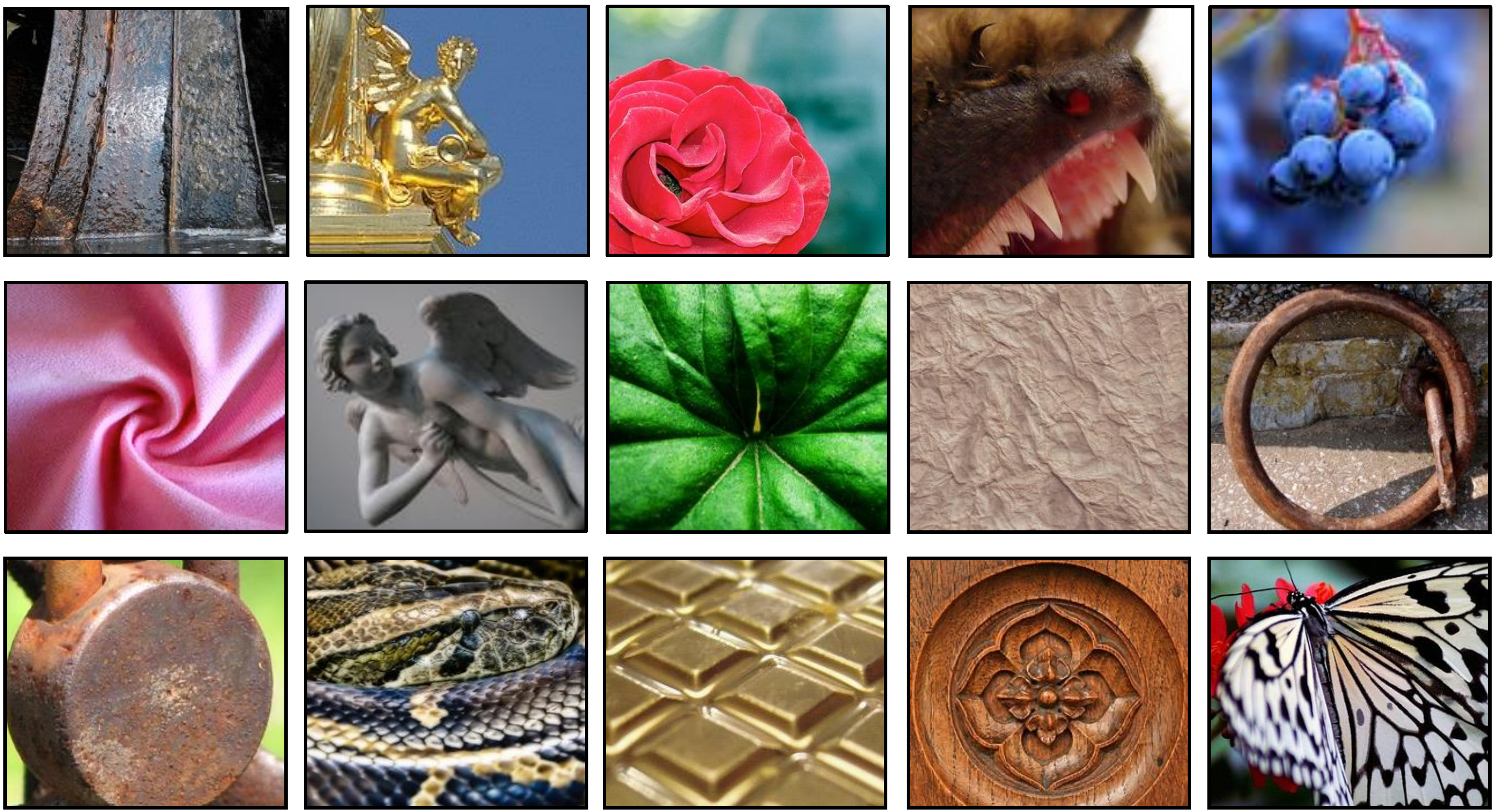
Slide credits

Most of these slides were adapted from:

- Srinivasa Narasimhan (16-385, Spring 2014).
- Todd Zickler (Harvard University).
- Steven Gortler (Harvard University).

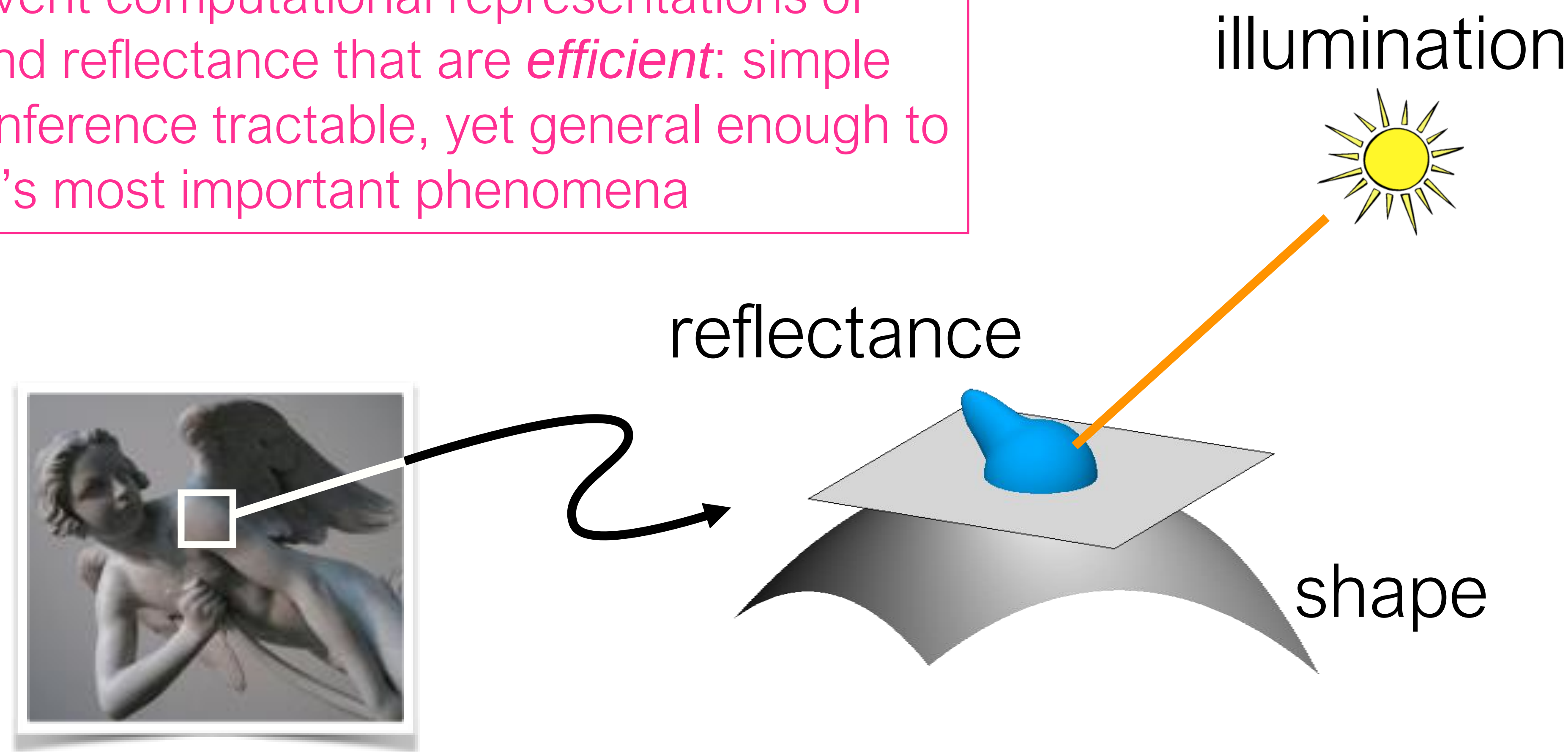
Appearance

Appearance



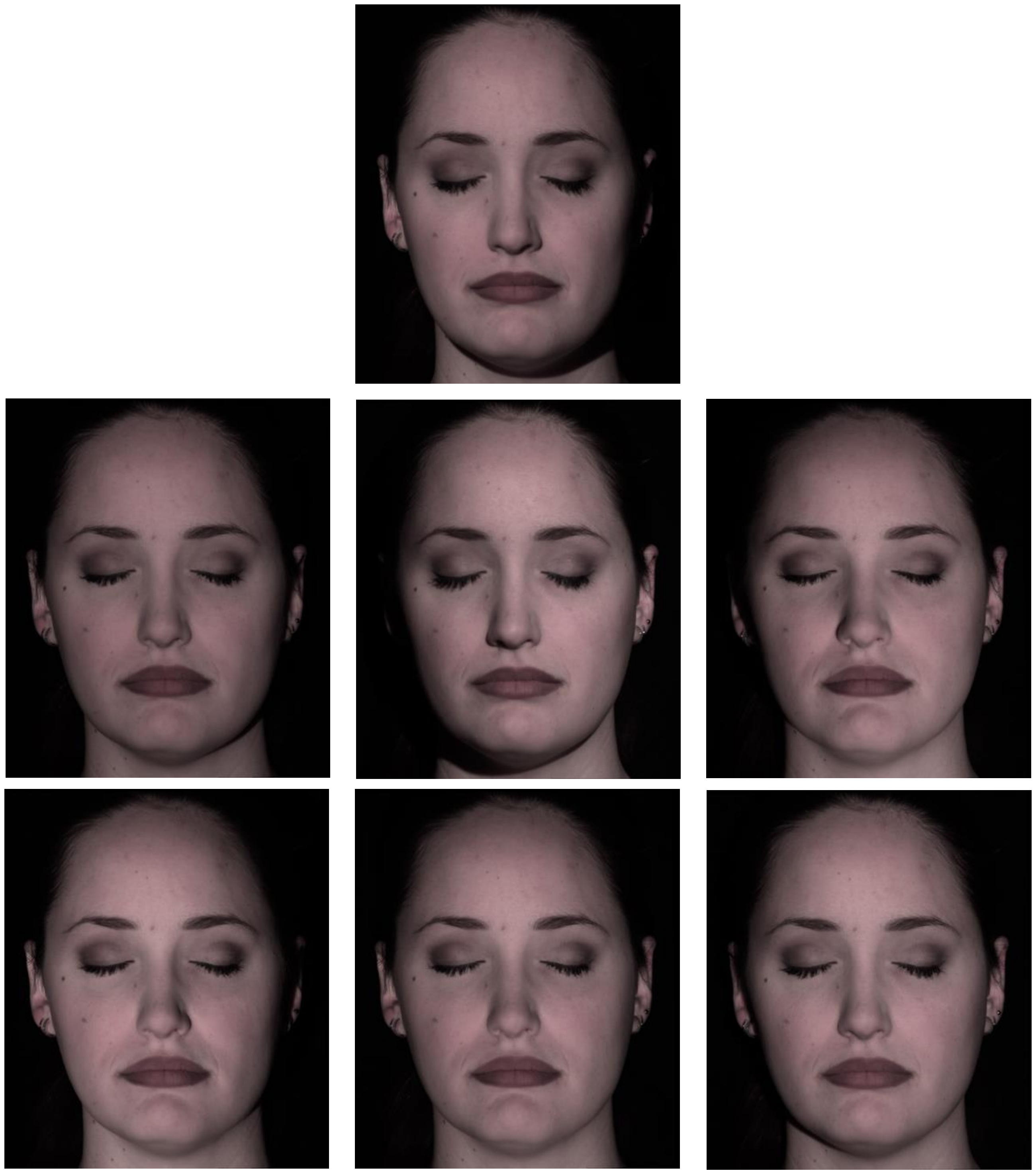
“Physics-based” computer vision (a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are ***efficient***: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena



$\mathbf{I} \Rightarrow$ shape, illumination, reflectance

Example application: Photometric Stereo



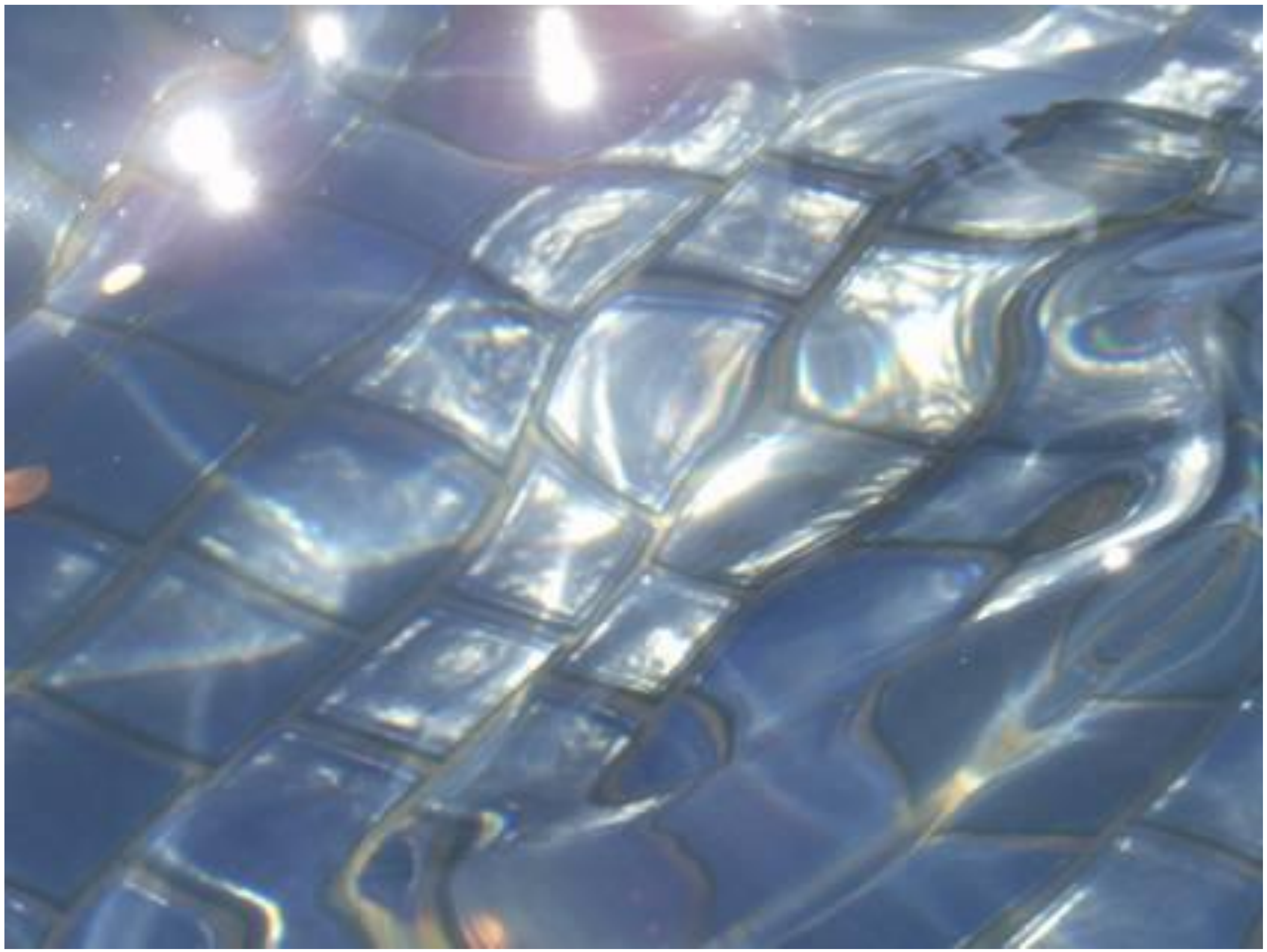
Why study the physics (optics) of the world?

Lets see some pictures!

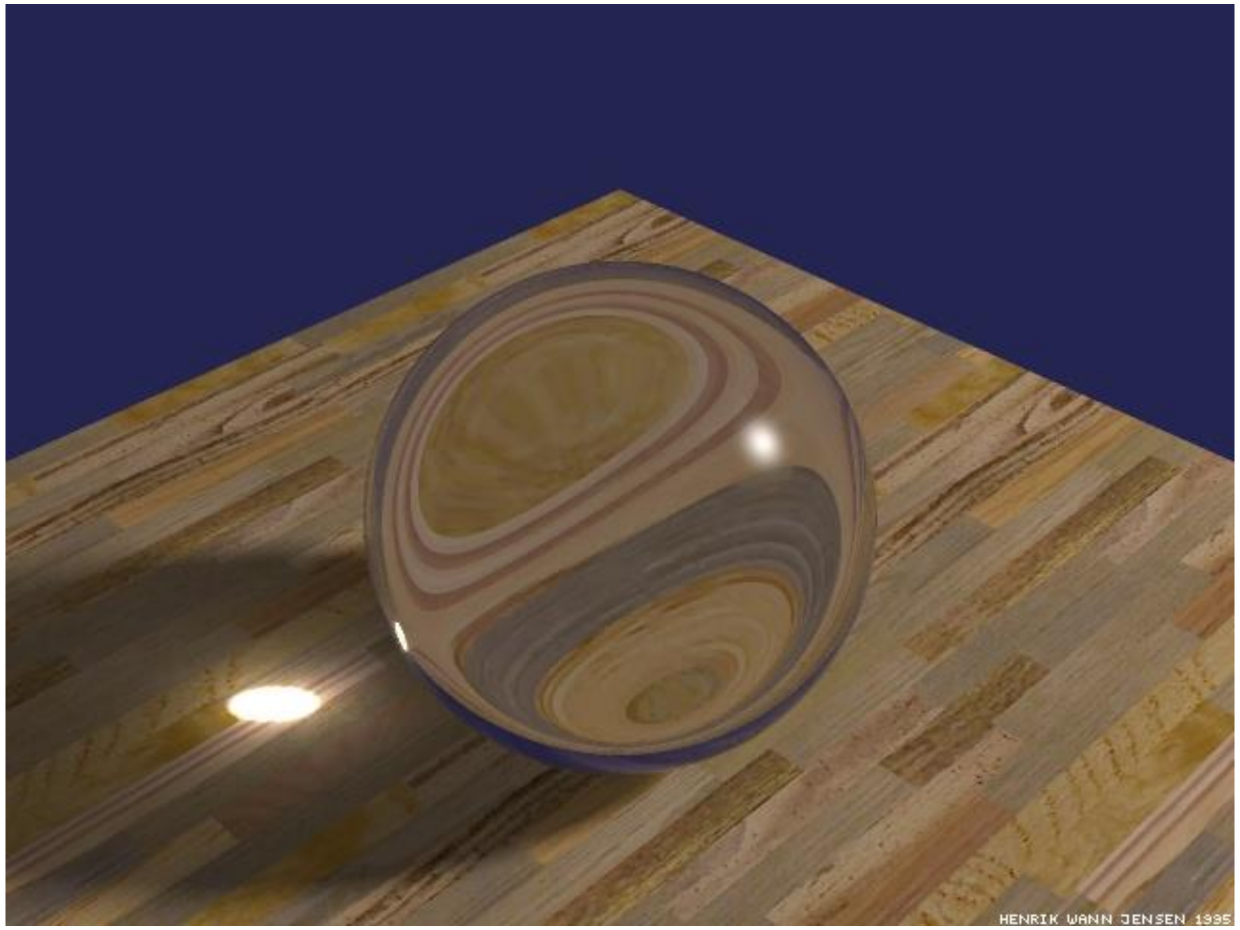
Light and Shadows


Reflections


Refractions



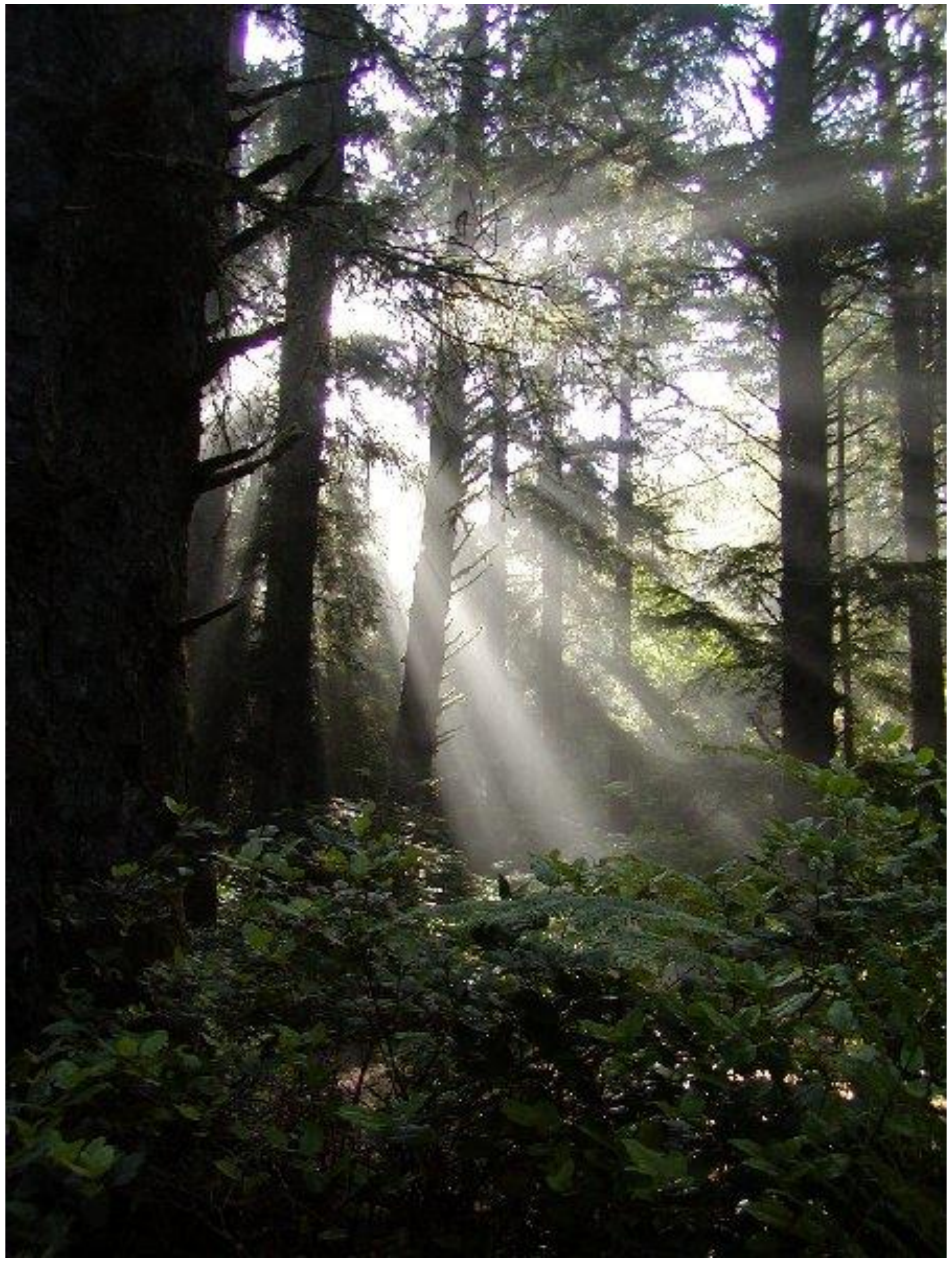

HENRIK WANN JENSEN 1995


HENRIK WANN JENSEN 1995

Interreflections


Mies Courtyard House with Curved Elements

Scattering

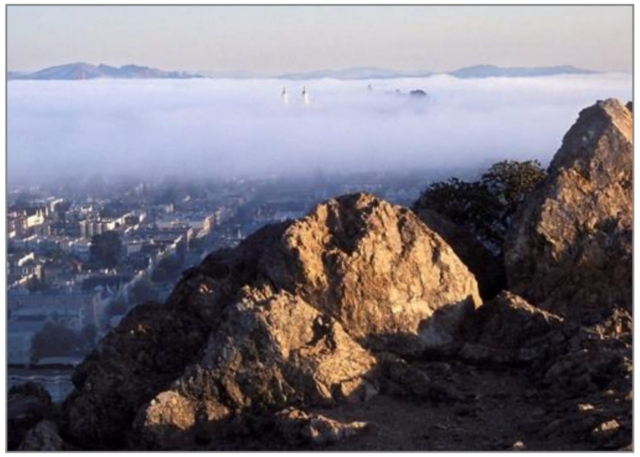


More Complex Appearances


RENDERED BY HENRIK WANN JENSEN - 2001


RENDERED BY HENRIK WANN JENSEN - 2001





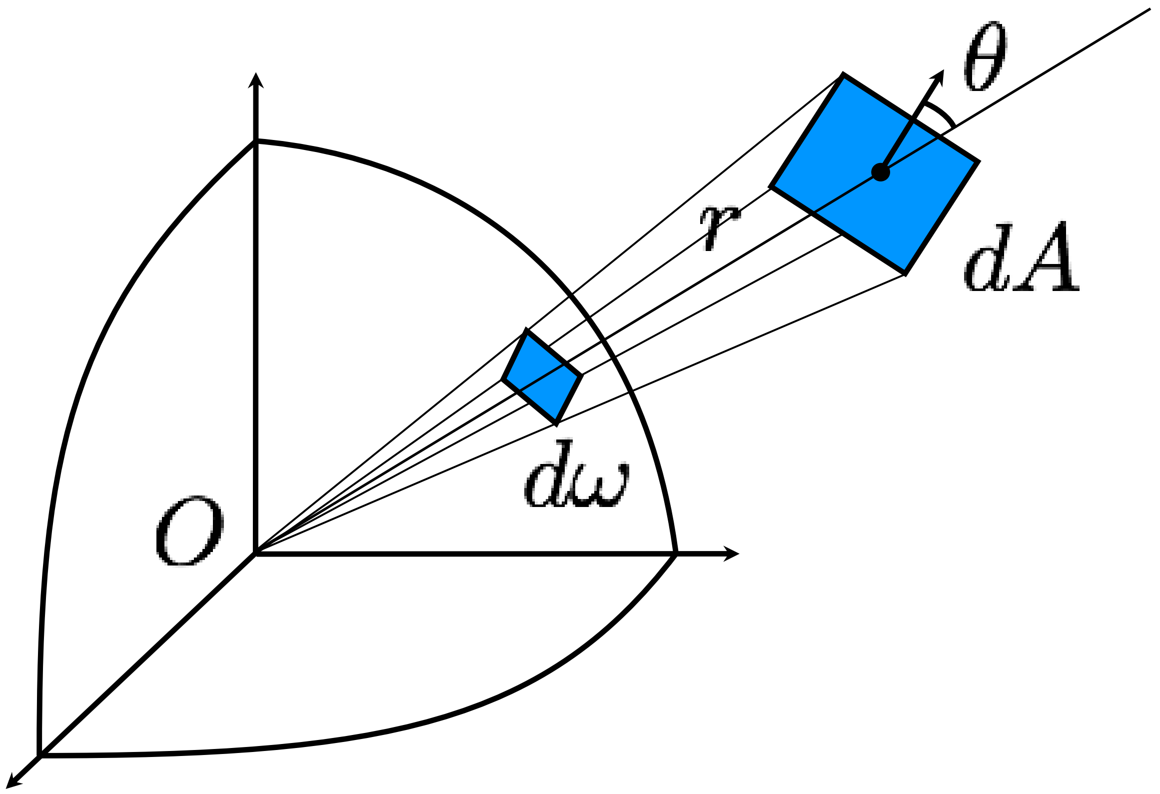




Measuring light and radiometry

Solid angle

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

Solid angle

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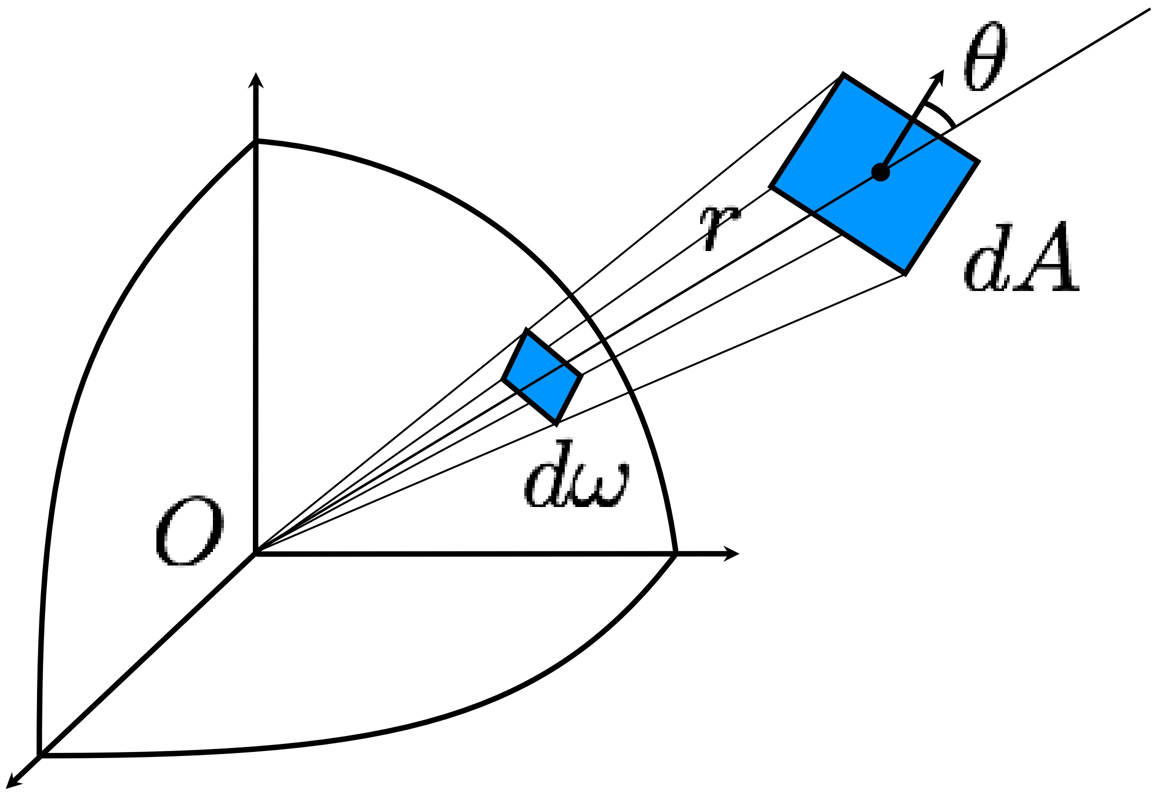

Depends on:

- orientation of patch
- distance of patch

One can show:

$$d\omega = \frac{dA\cos\theta}{r^2}$$

Units: steradians [sr]

Solid angle

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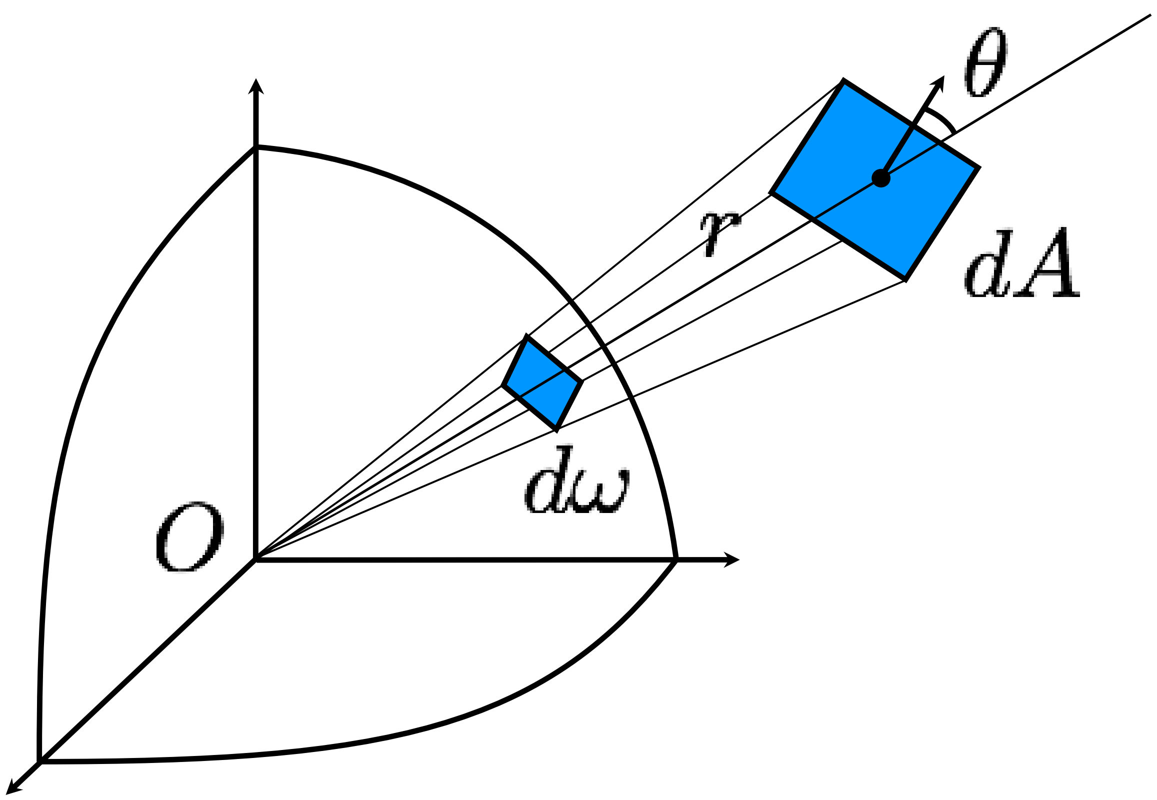


Depends on:

- orientation of patch
- distance of patch

One can show:

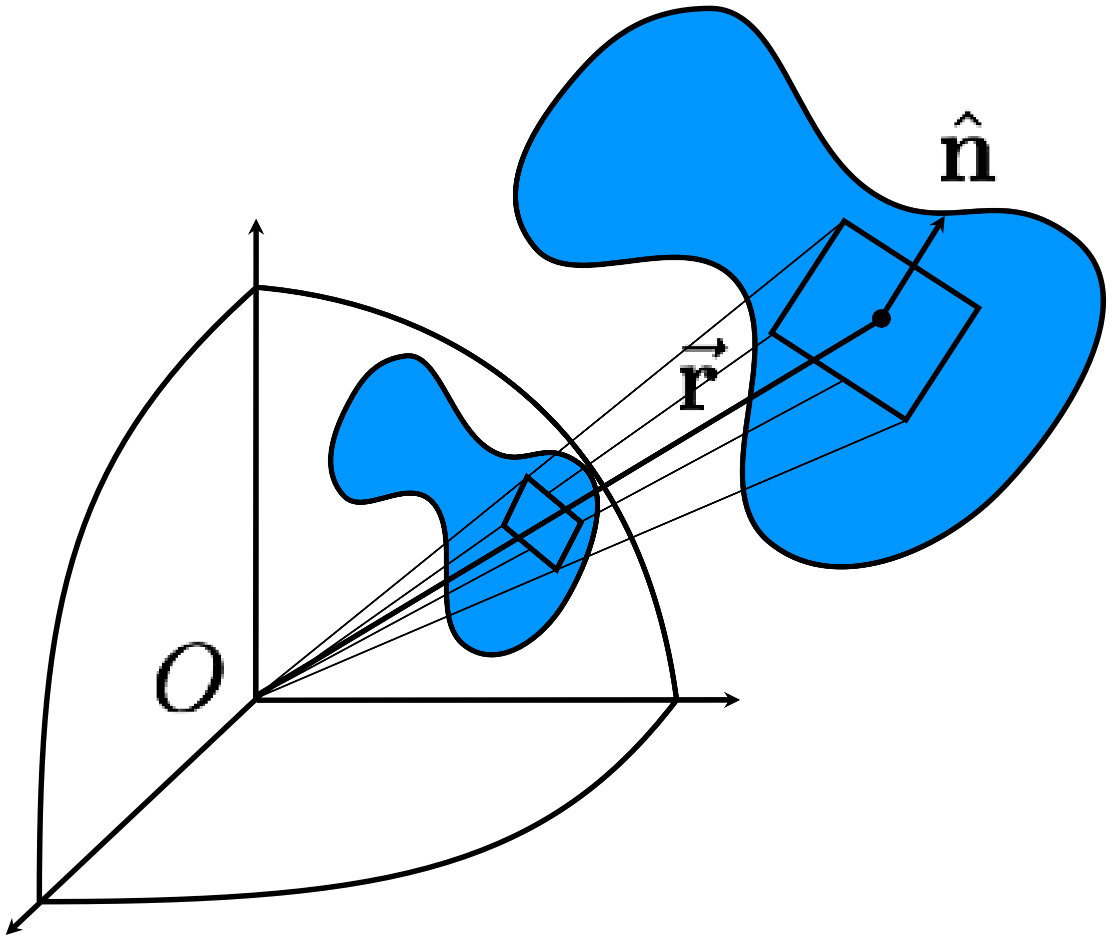
"surface foreshortening"

$$d\omega = \frac{dA\cos\theta}{r^2}$$

Units: steradians [sr]

Solid angle

- To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{\mathbf{r}} \cdot \hat{\mathbf{n}}\, dS}{|\vec{\mathbf{r}}|^3}$$

One can show:

“surface foreshortening”

$$d\omega = \frac{dA \cos\theta}{r^2}$$

Units: steradians [sr]

Question

- Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

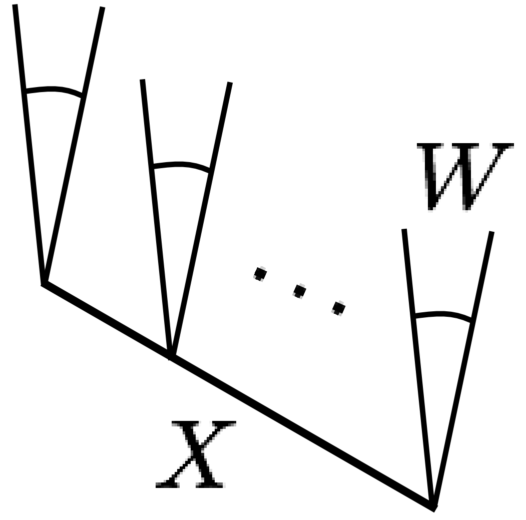
Question

- Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

- Answer: 2π (area of sphere is $4\pi r^2$; area of unit sphere is 4π; half of that is 2π)

Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W
- It measures *radiant flux* [watts = joules/sec]: rate of photons hitting sensor area
- Measurement depends on sensor area |X|

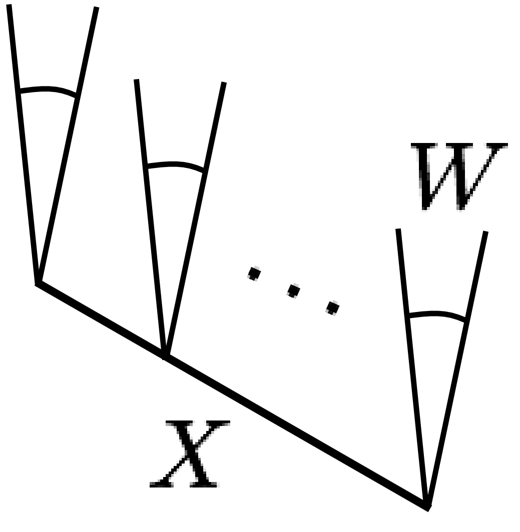


* shown in 2D for clarity; imagine three dimensions

radiant flux $\Phi(W, X)$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]



$$\frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

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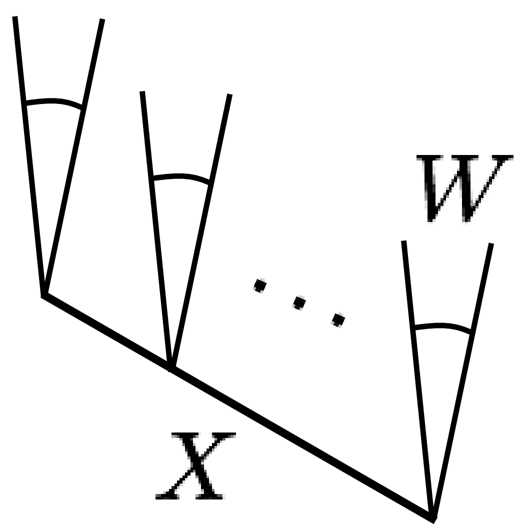
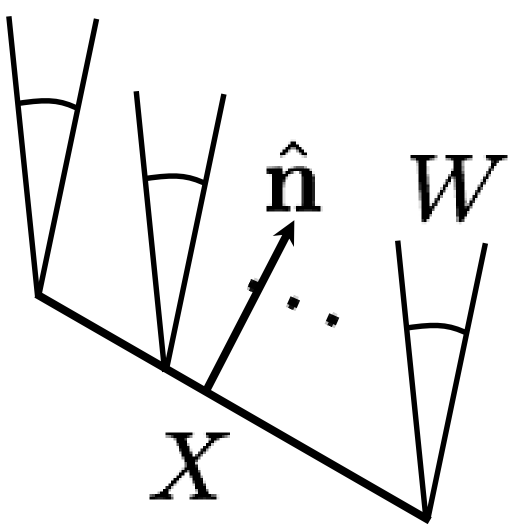


$$\lim_{X \longrightarrow x} \frac{\Phi(W, X)}{|X|}$$

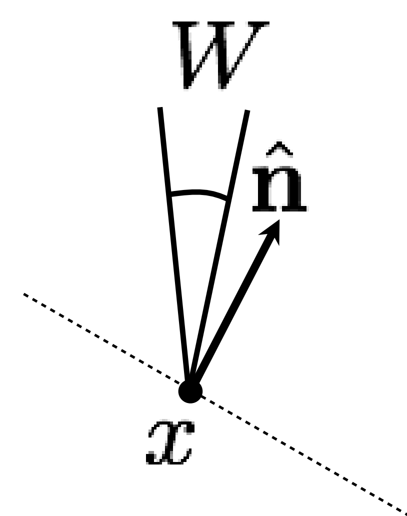
Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]
- Depends on sensor direction normal.



$$\frac{\Phi(W,X)}{|X|}$$

$$\lim_{X\longrightarrow x}$$

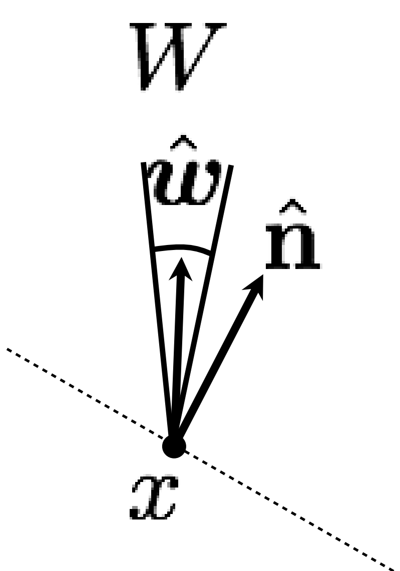


$$E_{\hat{\mathbf{n}}}(W,x)$$

- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit
- In the literature, notations n and W are often omitted, and values are implied by context

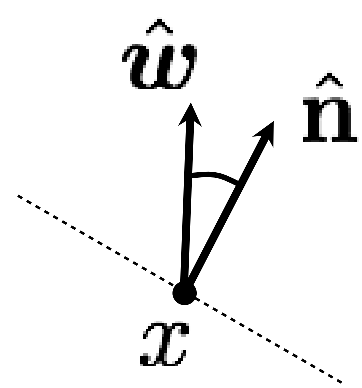
Quantifying light: flux, irradiance, and radiance

- *Radiance*:
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|
- Units: watts per steradian per square meter [W/(m^2·sr)]



$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|}$$

$$\lim_{W \longrightarrow \hat{\boldsymbol{\omega}}}$$



$$L_{\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}, x)$$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
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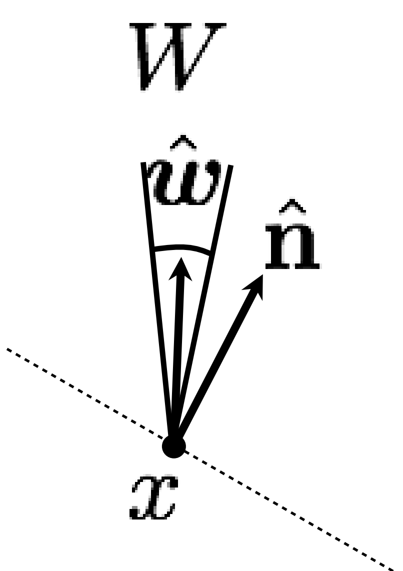
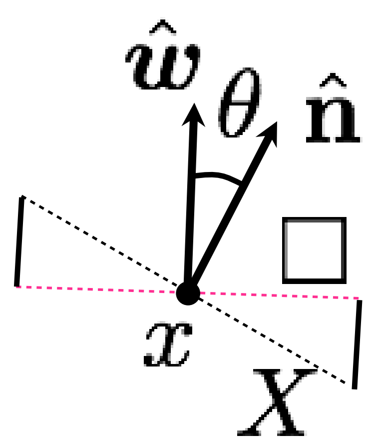


$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \quad \lim_{W \longrightarrow \hat{\omega}} \quad L_{\hat{\mathbf{n}}}(\hat{\omega}, x)$$



$$\cos\theta = \frac{\square/2}{|X|/2}$$

$$\longrightarrow \square = |X| \cos\theta$$

"foreshortened area"

- Has correct units, but still depends on sensor orientation
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Quantifying light: flux, irradiance, and radiance

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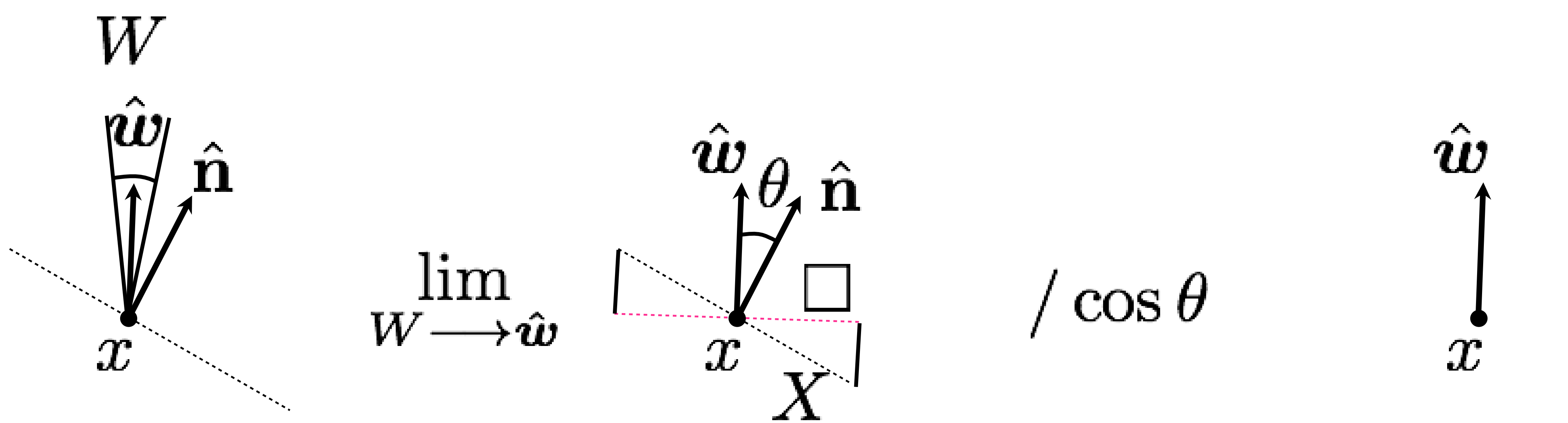

$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \qquad L_{\hat{\mathbf{n}}}(\hat{\omega}, x) \qquad L(\hat{\omega}, x)$$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|
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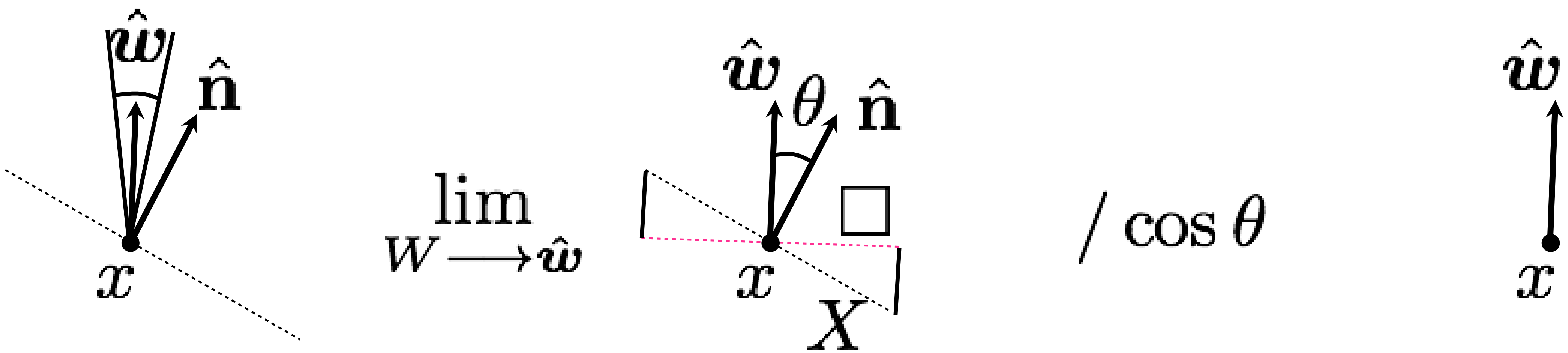
"foreshortened in the direction of travel"



$$\frac{E_{\hat{\mathbf{n}}}(W,x)}{|W|} \quad \lim_{W \longrightarrow \hat{\boldsymbol{w}}} \quad L_{\hat{\mathbf{n}}}(\hat{\omega},x) \quad / \cos\theta \quad L(\hat{\omega},x)$$

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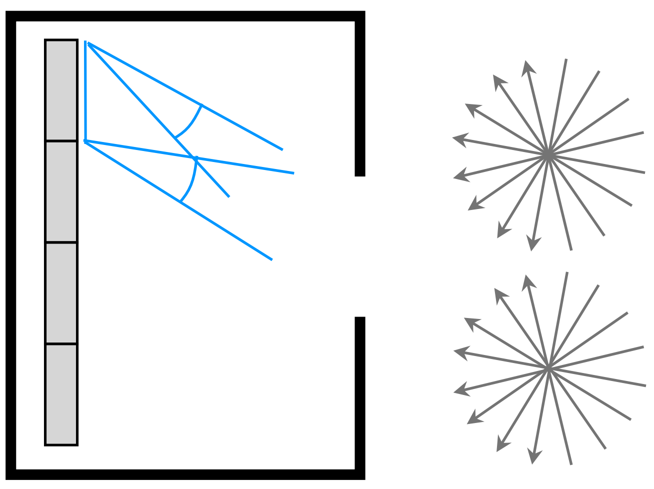
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor



Quantifying light: flux, irradiance, and radiance

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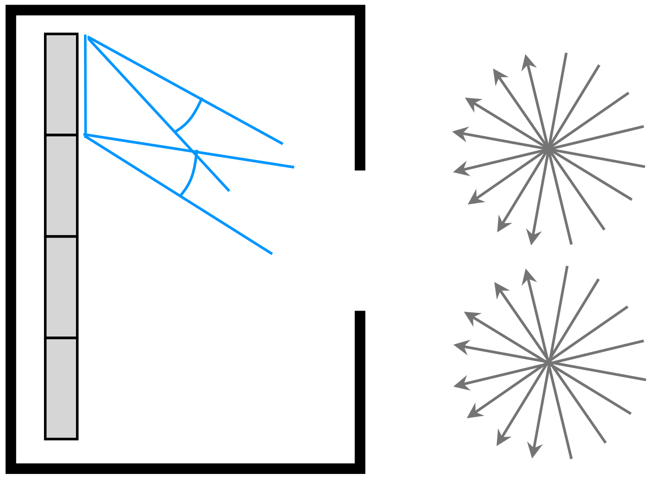
$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos\theta \, d\omega \, dA$$



Quantifying light: flux, irradiance, and radiance

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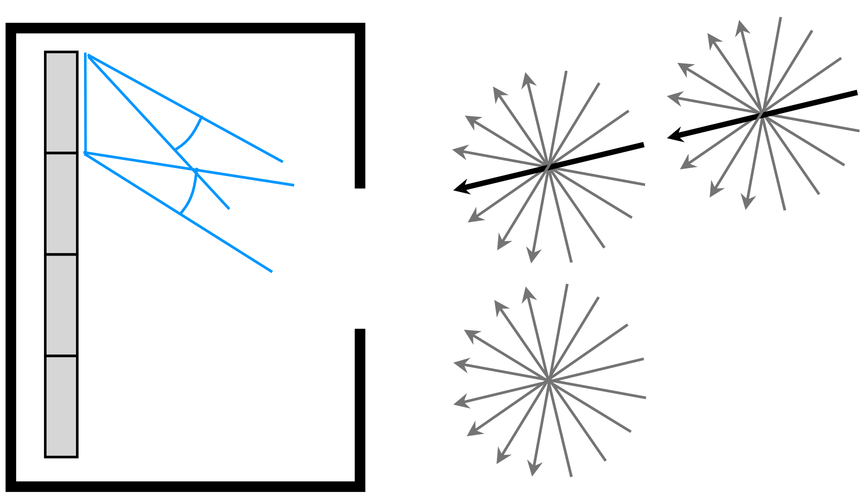
$$\Phi(W,X) = \int_X \int_W L(\hat{\boldsymbol{\omega}}, x)\cos\theta d\boldsymbol{\omega} dA$$

  - Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$



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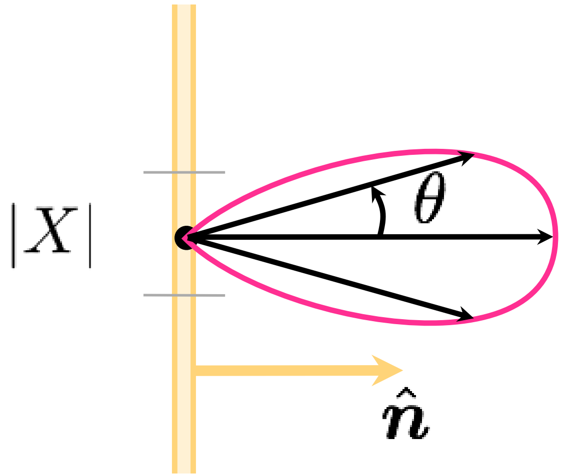
  - Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$

  - A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.
    - “Processed” images (like PNG and JPEG) are not linear radiance measurements!!

Question

- Most light sources, like a heated metal sheet, follow Lambert's Law



"Lambertian area source"

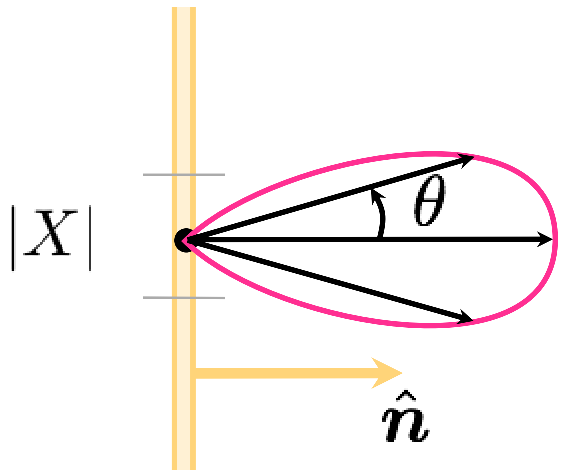
$$J(\hat{\boldsymbol{\omega}}) = J_o \langle \hat{\boldsymbol{\omega}}, \hat{\boldsymbol{n}} \rangle = J_o \cos\theta$$

radiant intensity [W/sr]

- What is the radiance $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x})$ of an infinitesimal patch [W/sr·m^2]?

Question

- Most light sources, like a heated metal sheet, follow Lambert's Law



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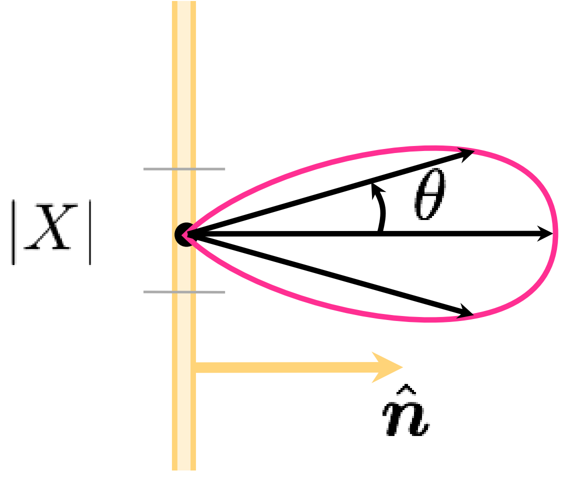
"Lambertian area source"

- What is the radiance $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x})$ of an infinitesimal patch [W/sr·m^2]?

Answer: $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x}) = J_o / |X|$ (independent of direction)

Question

- Most light sources, like a heated metal sheet, follow Lambert's Law



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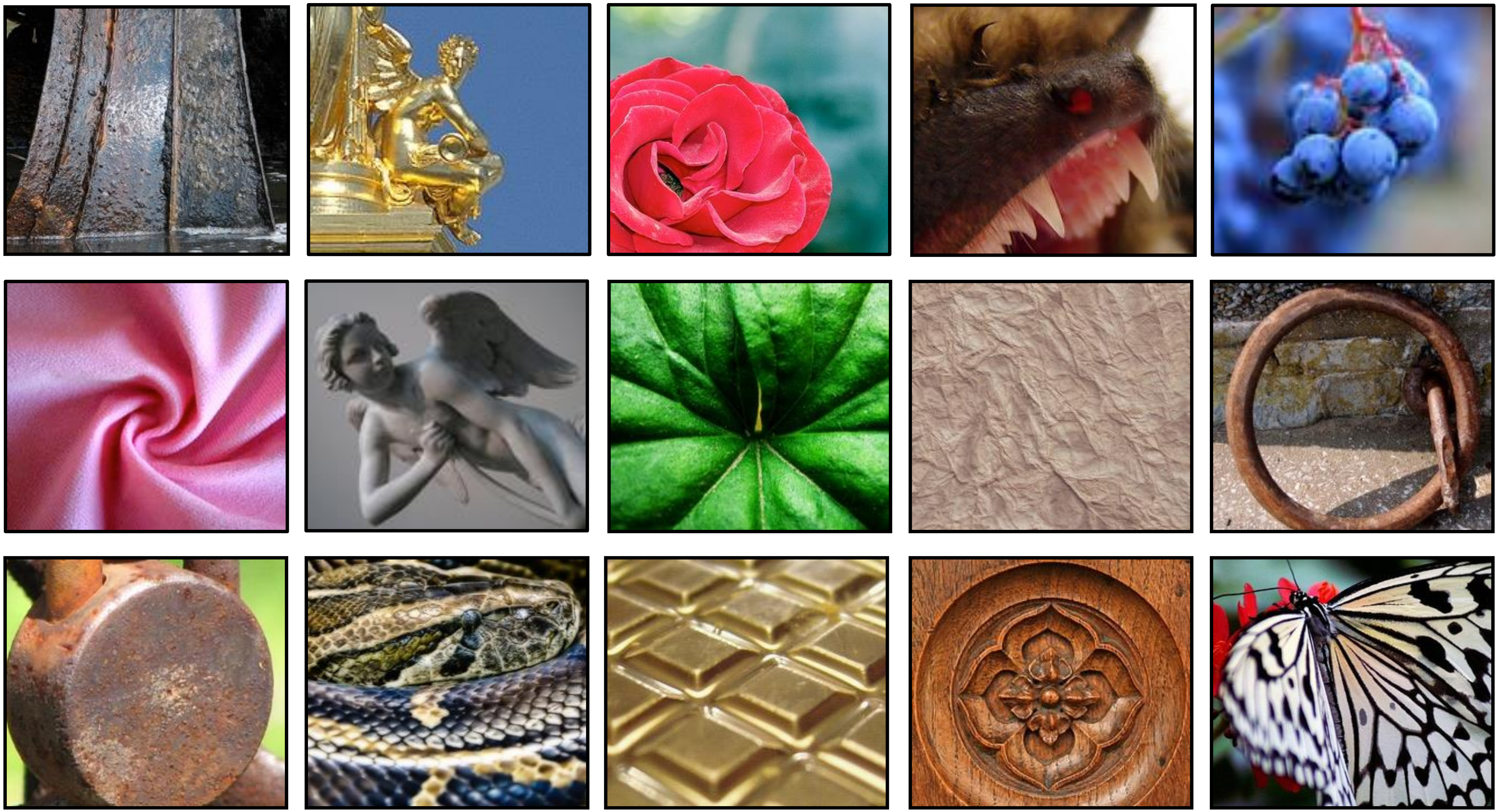
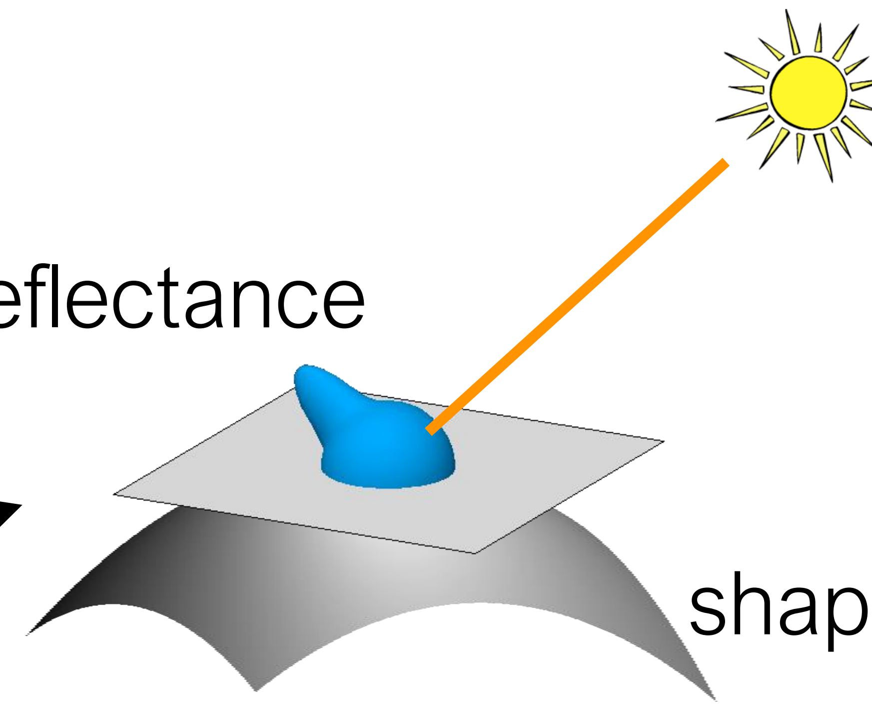
Answer: $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x}) = J_o/|X|$ (independent of direction)

"Looks equally bright when viewed from any direction"

Appearance



"Physics-based" computer vision
(a.k.a "inverse optics")



$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Reflectance and BRDF

Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)

Reflectance

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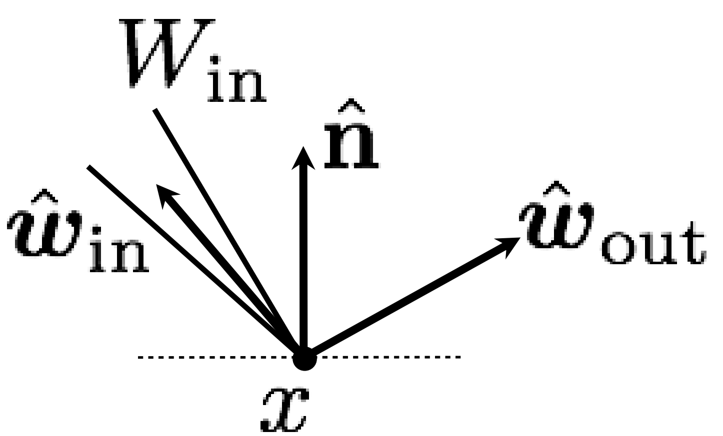


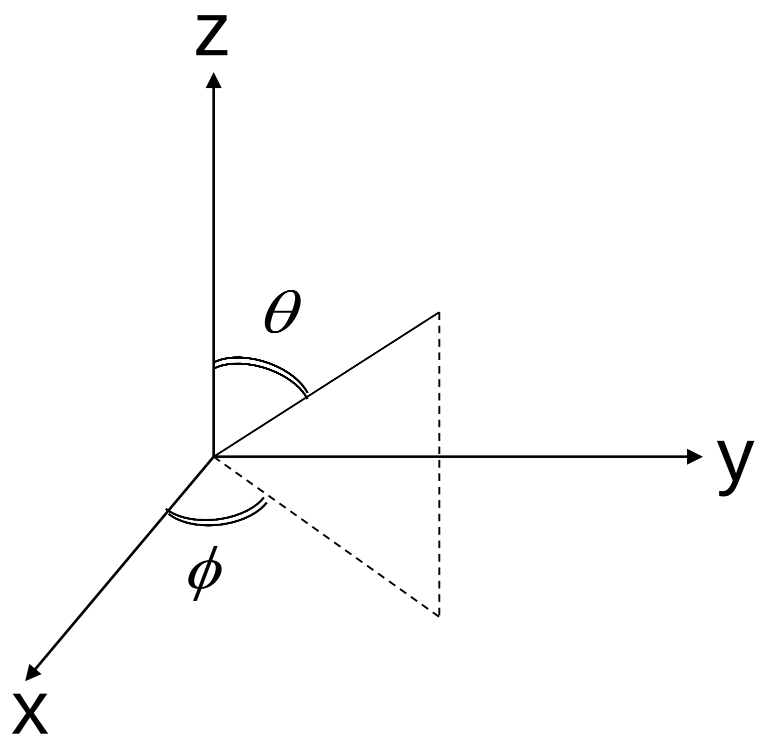
$$\lim_{W_{\text{in}} \longrightarrow \hat{\boldsymbol{w}}_{\text{in}}}$$

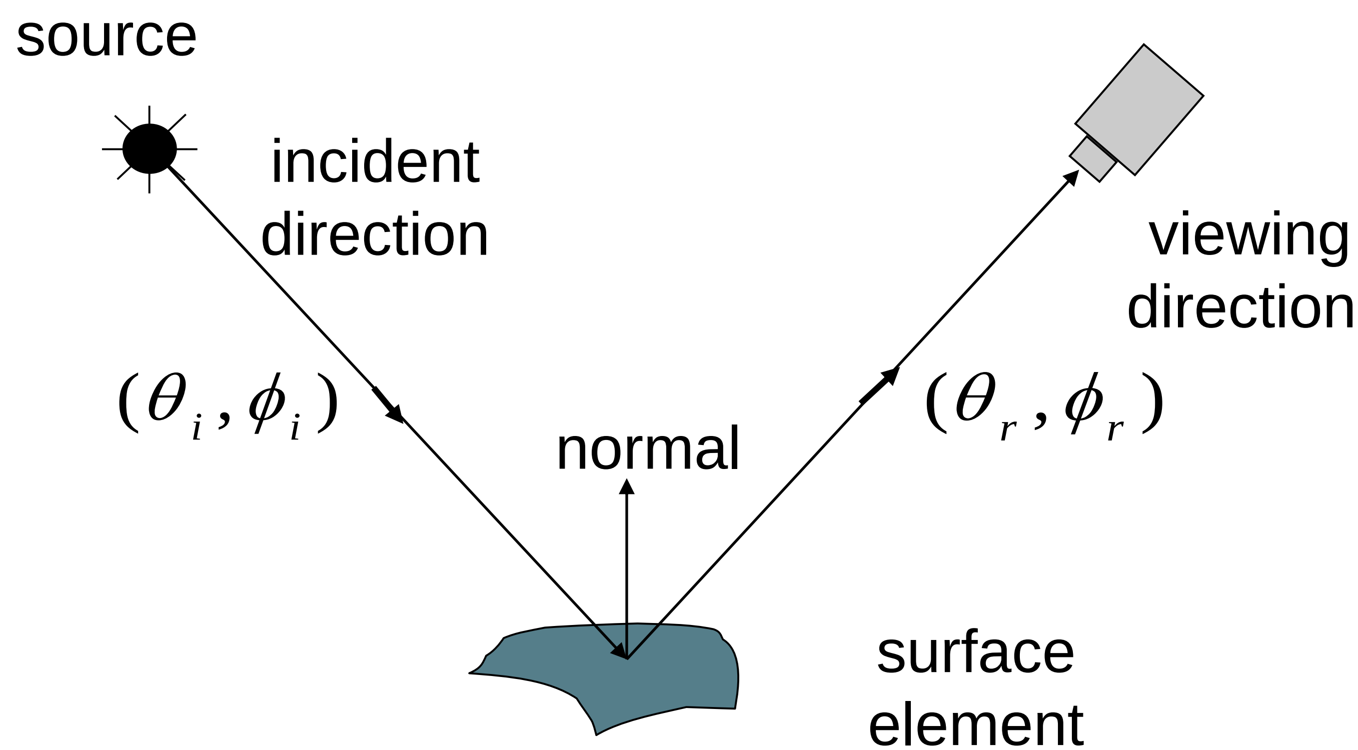
$$f_{x,\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}})$$

$$f_{x,\hat{\mathbf{n}}}(W_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) = \frac{L^{\text{out}}(x, \hat{\boldsymbol{\omega}}_{\text{out}})}{E^{\text{in}}_{\hat{\mathbf{n}}}(W_{\text{in}}, x)}$$

- Notations x and n often implied by context and omitted; directions \omega are expressed in local coordinate system defined by normal n (and some chosen tangent vector)
- Units: sr^{-1}
- Called Bidirectional Reflectance Distribution Function (BRDF)

BRDF: Bidirectional Reflectance Distribution Function





$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

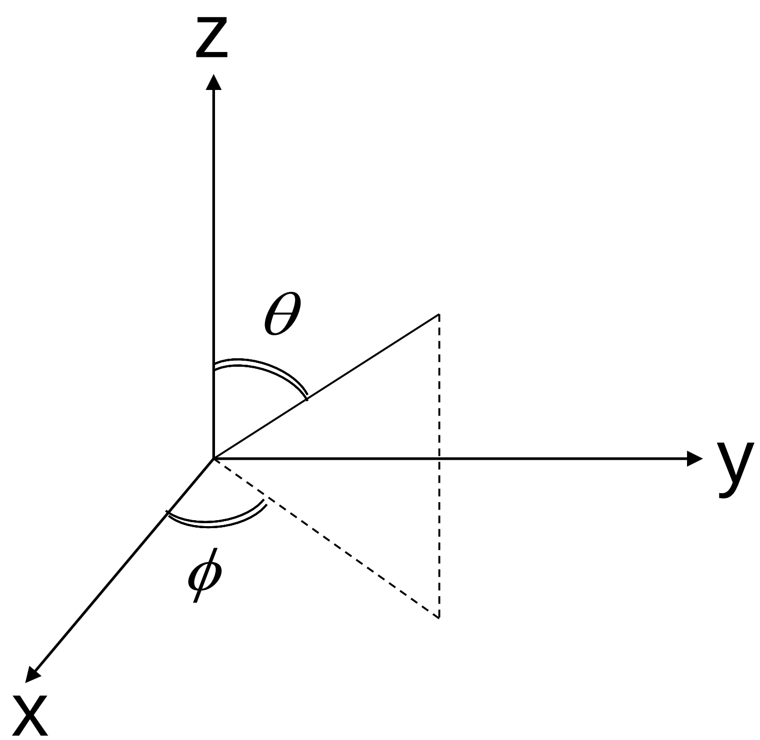
$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

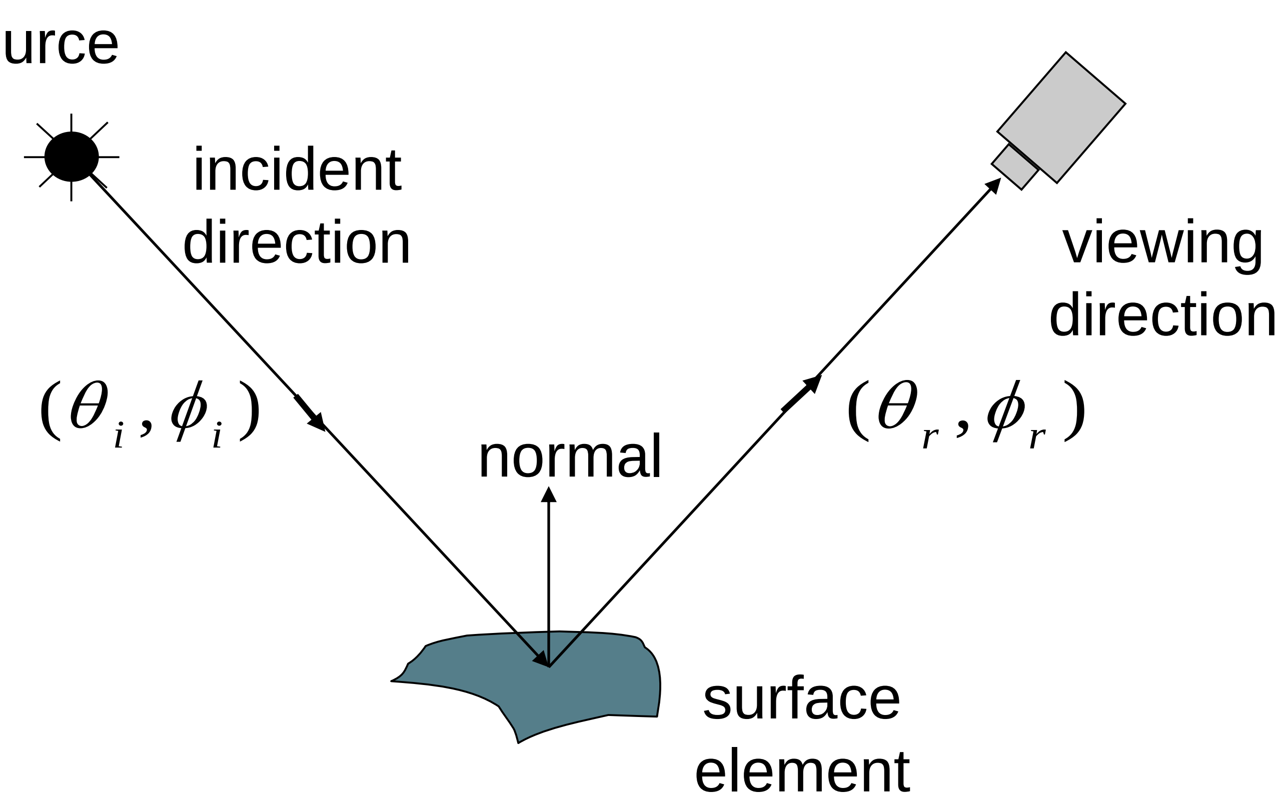
$$\text{BRDF}: f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

Reflectance: BRDF

- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties

Important Properties of BRDFs



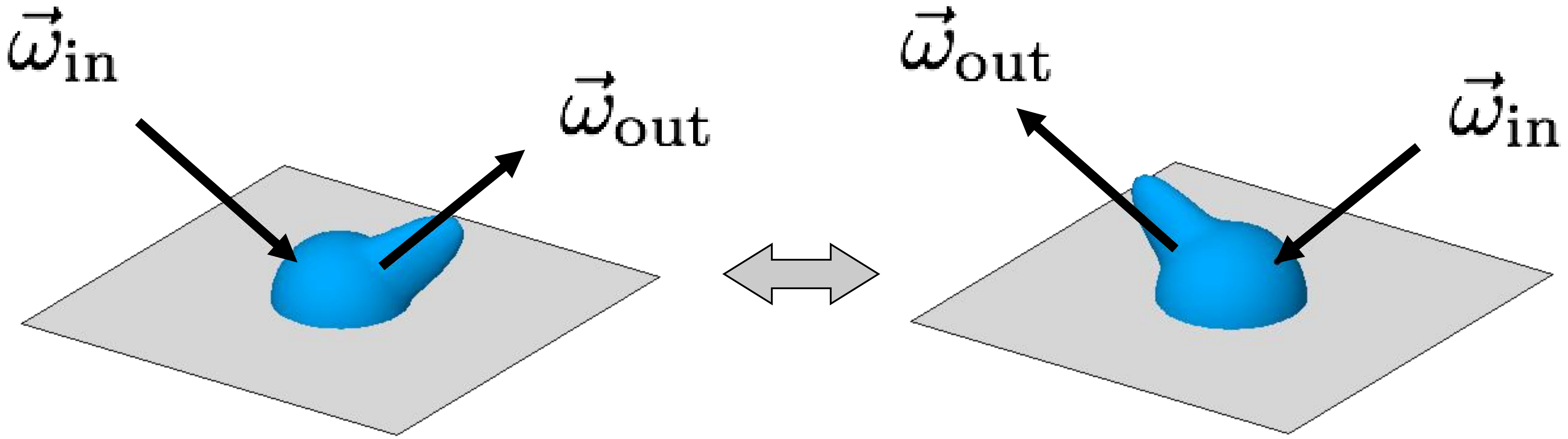


- Conservation of Energy:

$$\forall \hat{\boldsymbol{\omega}}_{\text{in}}, \quad \int_{\Omega_{\text{out}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) \cos\theta_{\text{out}} d\hat{\boldsymbol{\omega}}_{\text{out}} \leq 1$$

Why smaller than or equal?

Property: "Helmholtz reciprocity"

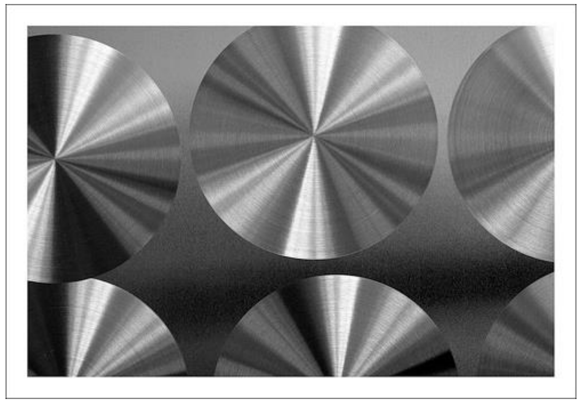


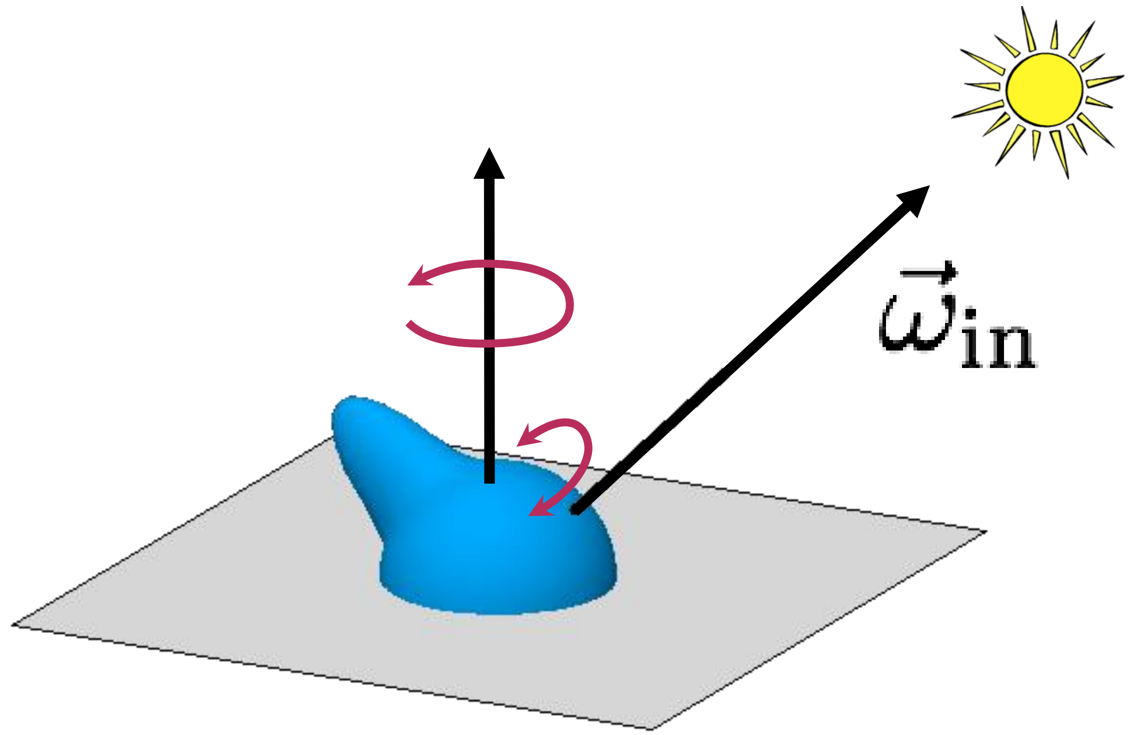
- Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}})$$

Common assumption: Isotropy





[Matusik et al., 2003]

BRDF does not change when surface is rotated about the normal.

$f_r(\vec{\omega}_{\text{in}}, \cdot)$

4D → 3D

$$f_r(\sout{\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables : $f\,(\theta_i, \theta_r, \phi_i - \phi_r)$

Reflectance: BRDF

- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for *any* configuration of lights and viewpoint

$$L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\omega}_{\text{in}}$$

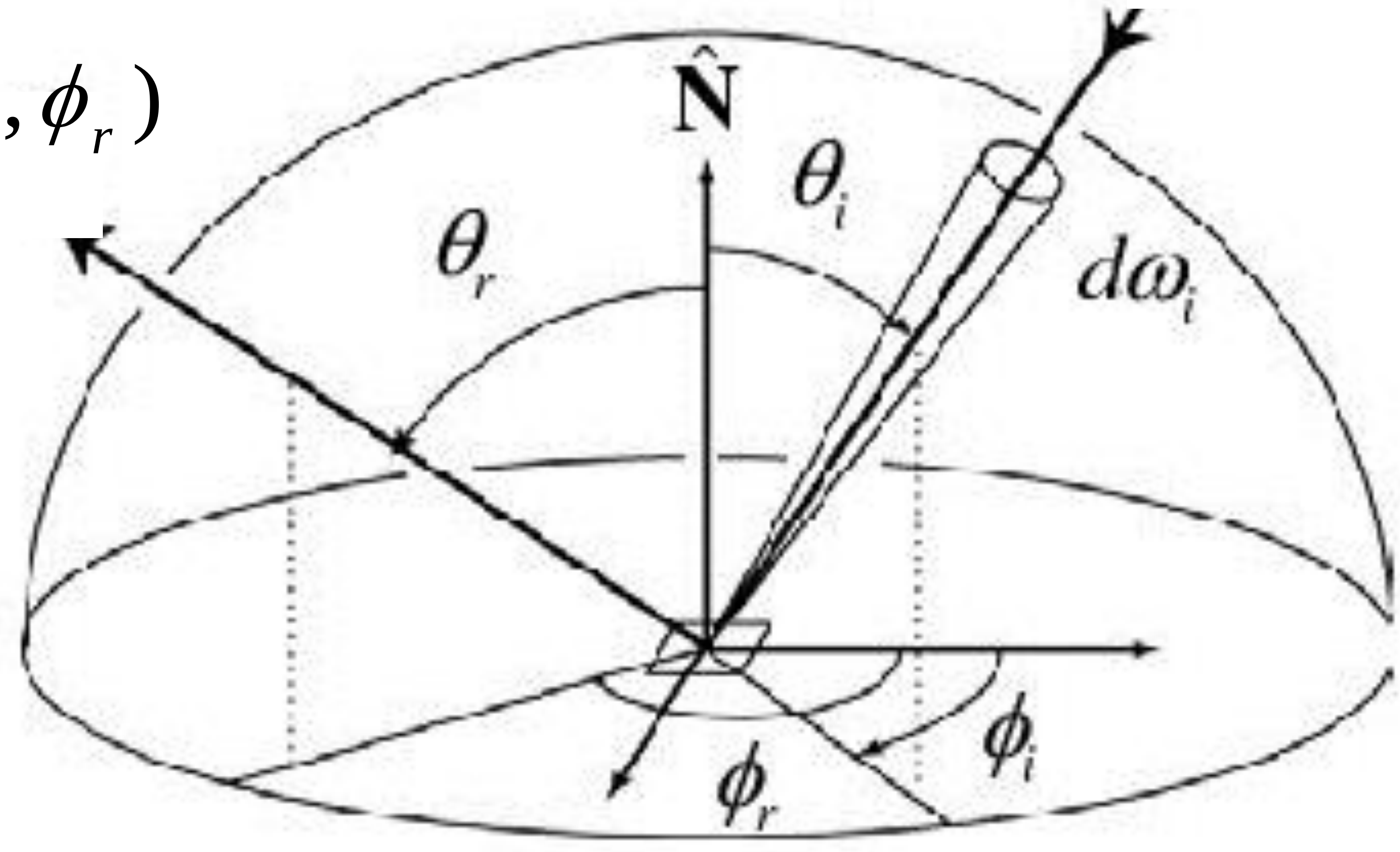
reflectance equation

Why is there a cosine in the reflectance equation?

Derivation of the Reflectance Equation

$L^{src}(\theta_i, \phi_i)$

$L^{surface}(\theta_r, \phi_r)$



From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) \quad = \quad E^{surface}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) = \underline{E^{surface}(\theta_i, \phi_i)}\, f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r, \phi_r) = \underline{L^{src}(\theta_i, \phi_i)}\, f(\theta_i, \phi_i; \theta_r, \phi_r)\, \underline{\cos\theta_i d\omega_i}$$

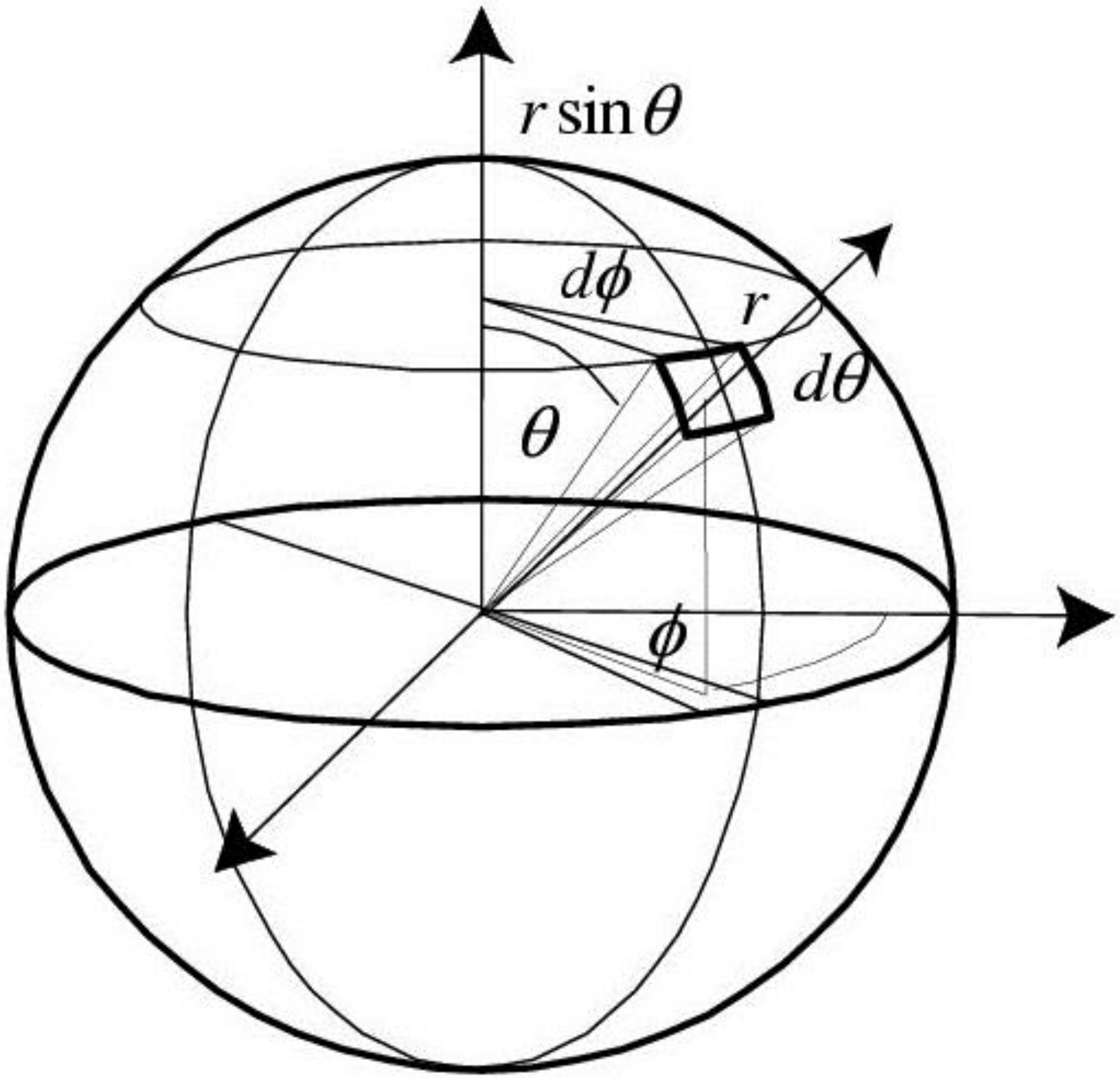
Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r, \phi_r) = \int_{2\pi} L^{src}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i\, \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L^{src}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i\, \underline{\sin\theta_i d\theta_i d\phi_i}$$

Differential Solid Angles

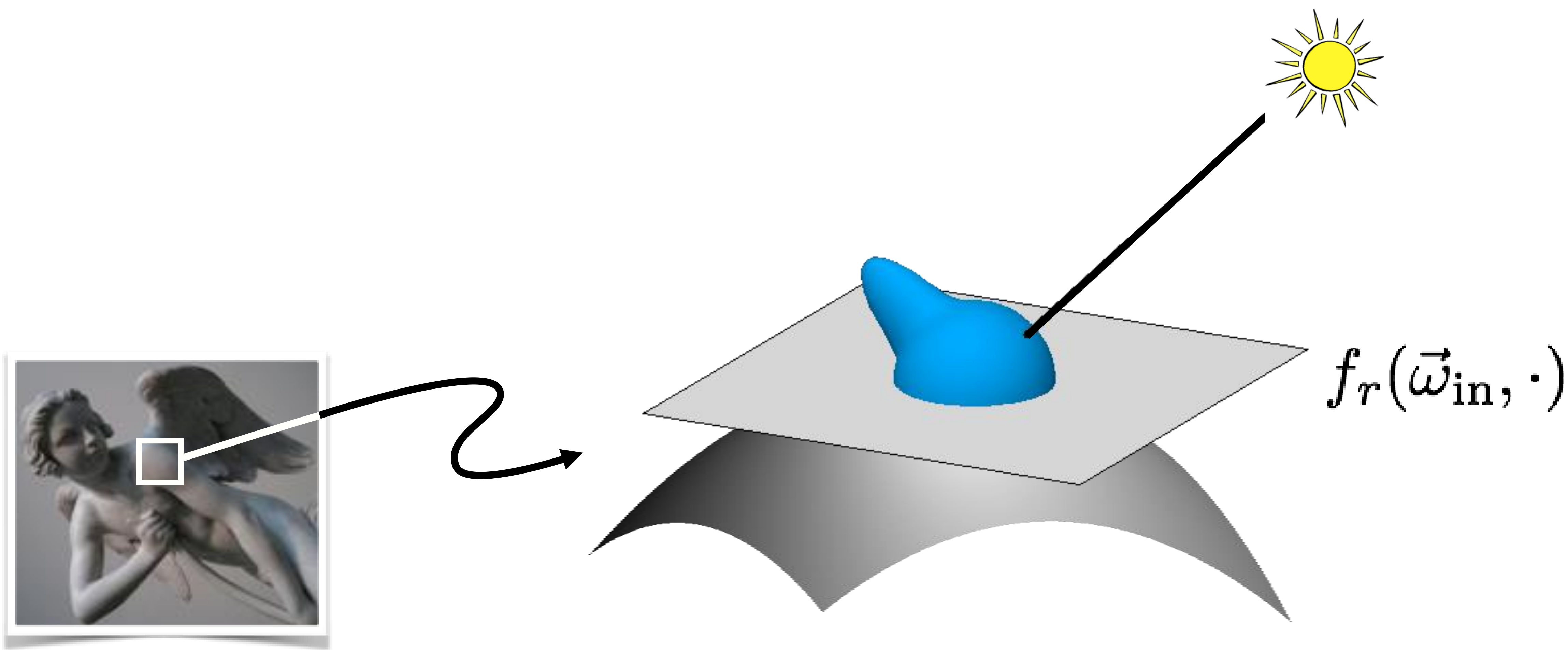


$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

BRDF



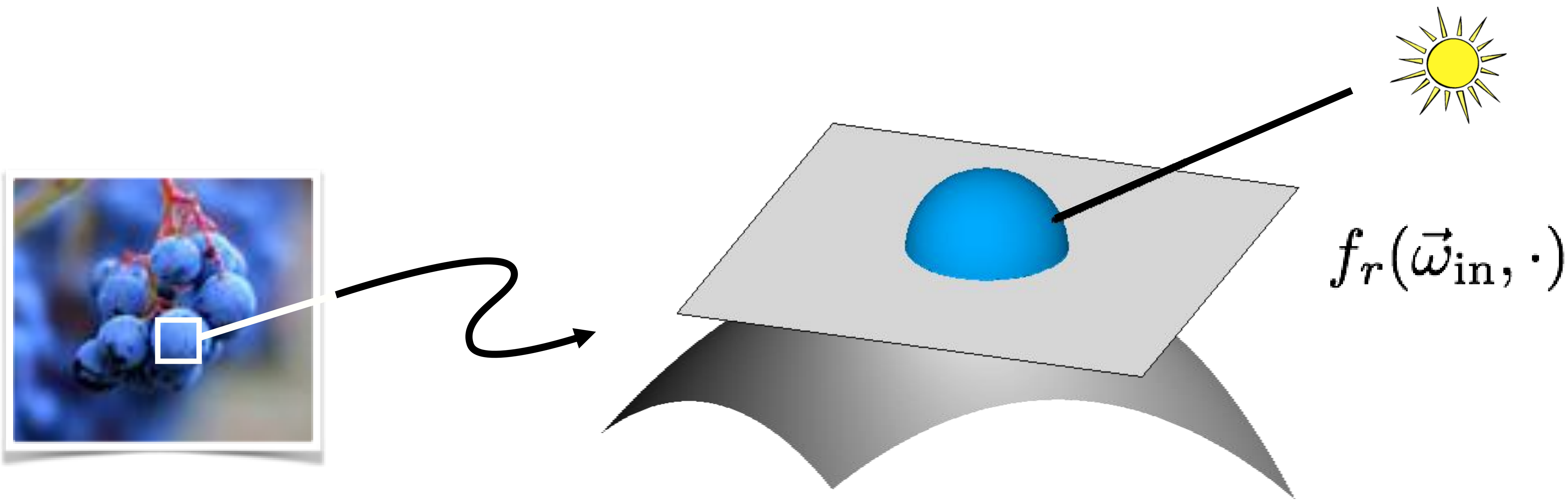
$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions

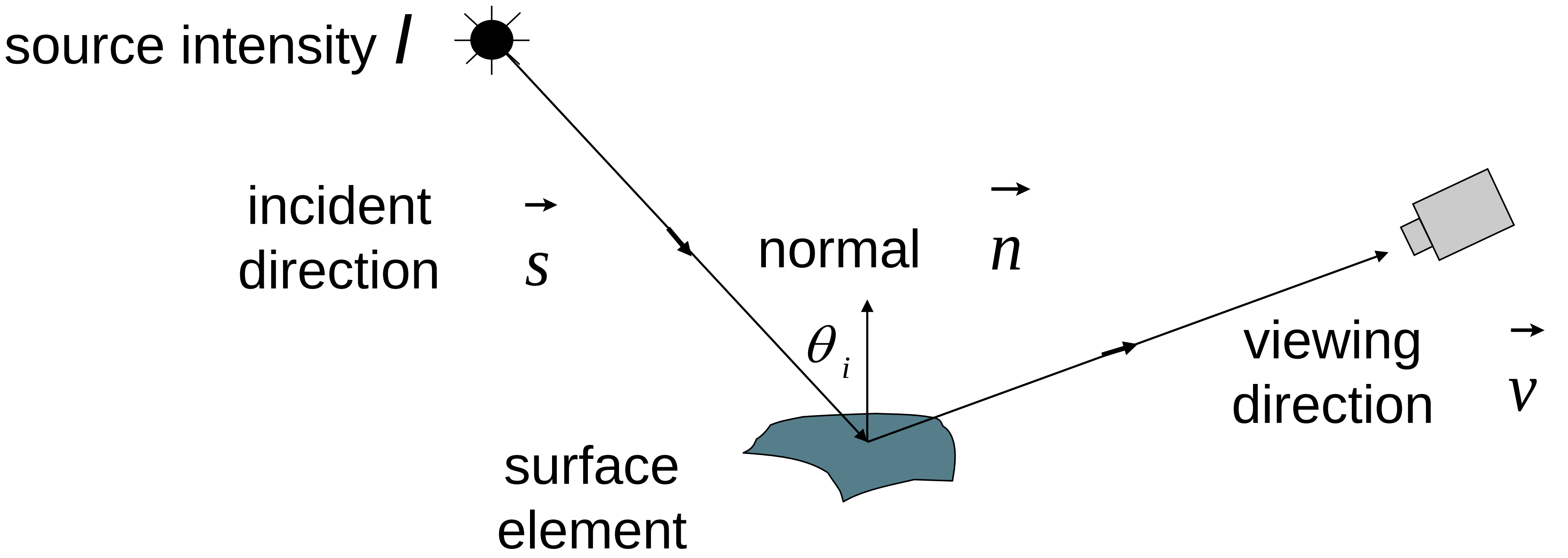
What does the BRDF equal in this case?



$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Diffuse Reflection and Lambertian BRDF

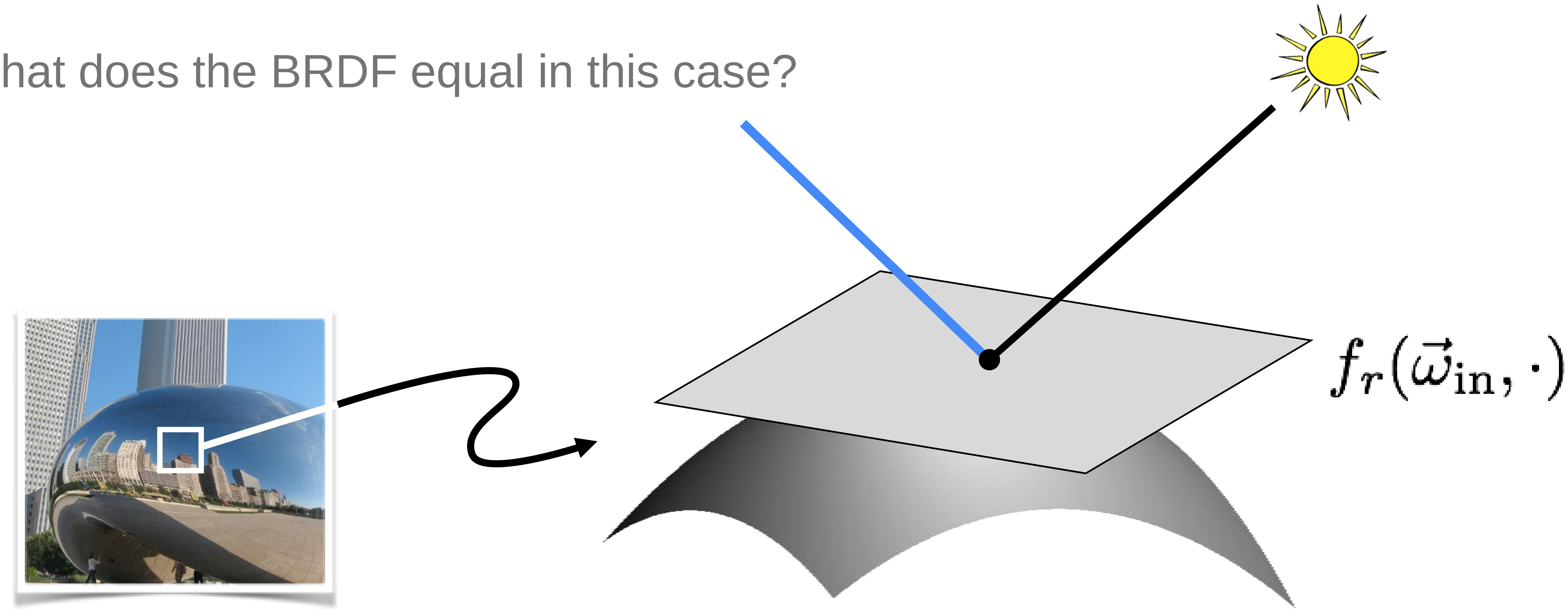


- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ (albedo: ρ_d)
- Most commonly used BRDF in Vision and Graphics!

BRDF

Specular BRDF: all energy concentrated in mirror direction

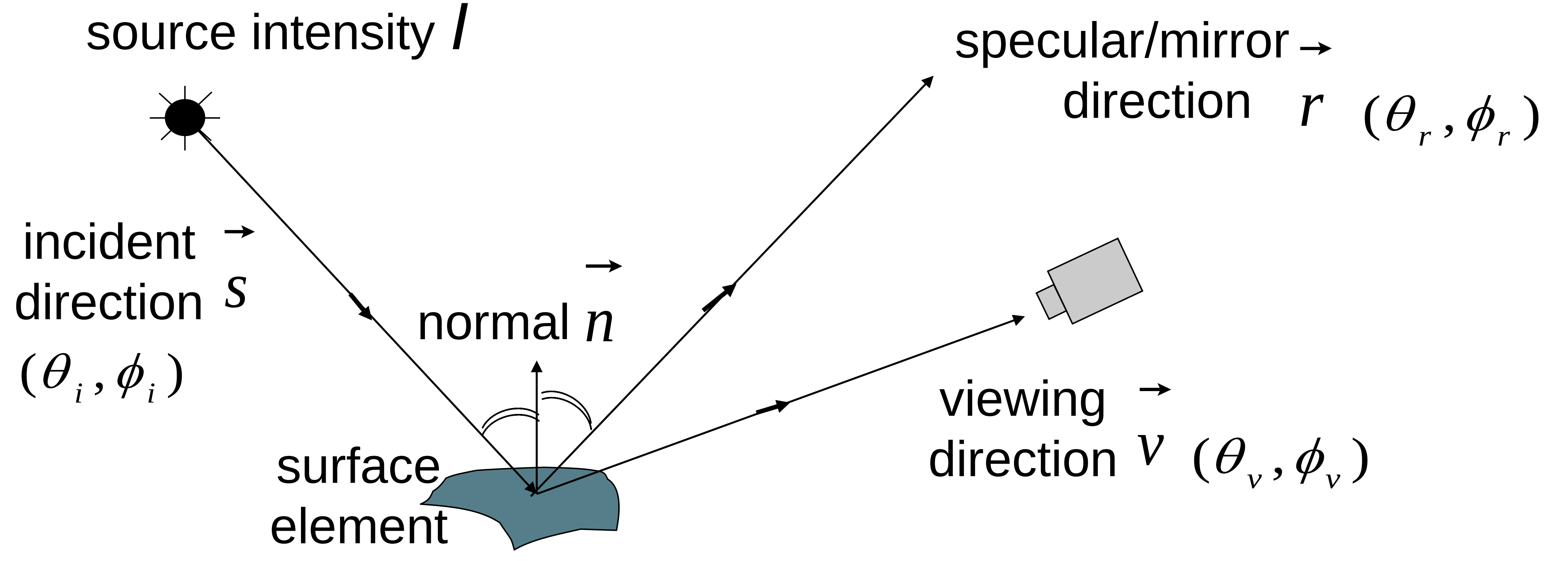
What does the BRDF equal in this case?



$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.

- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).

- Mirror BRDF is simply a double-delta function :

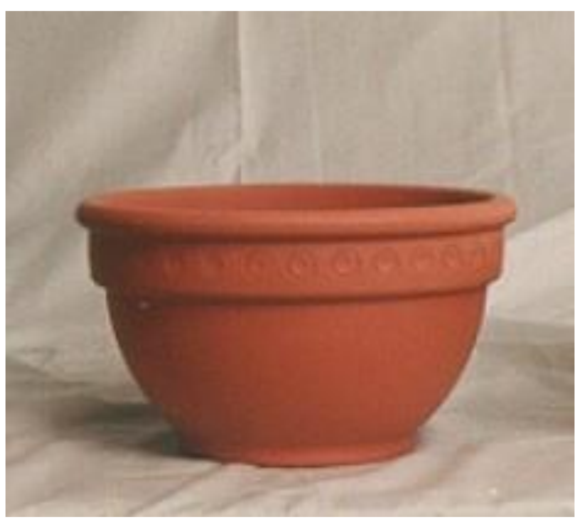
specular albedo

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \, \delta(\theta_i - \theta_v) \, \delta(\phi_i + \pi - \phi_v)$$

Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc



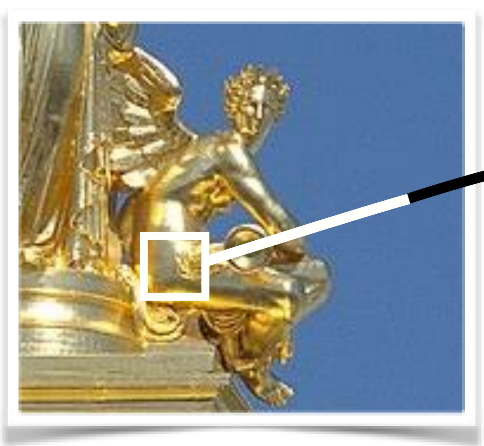
Surface Reflection:

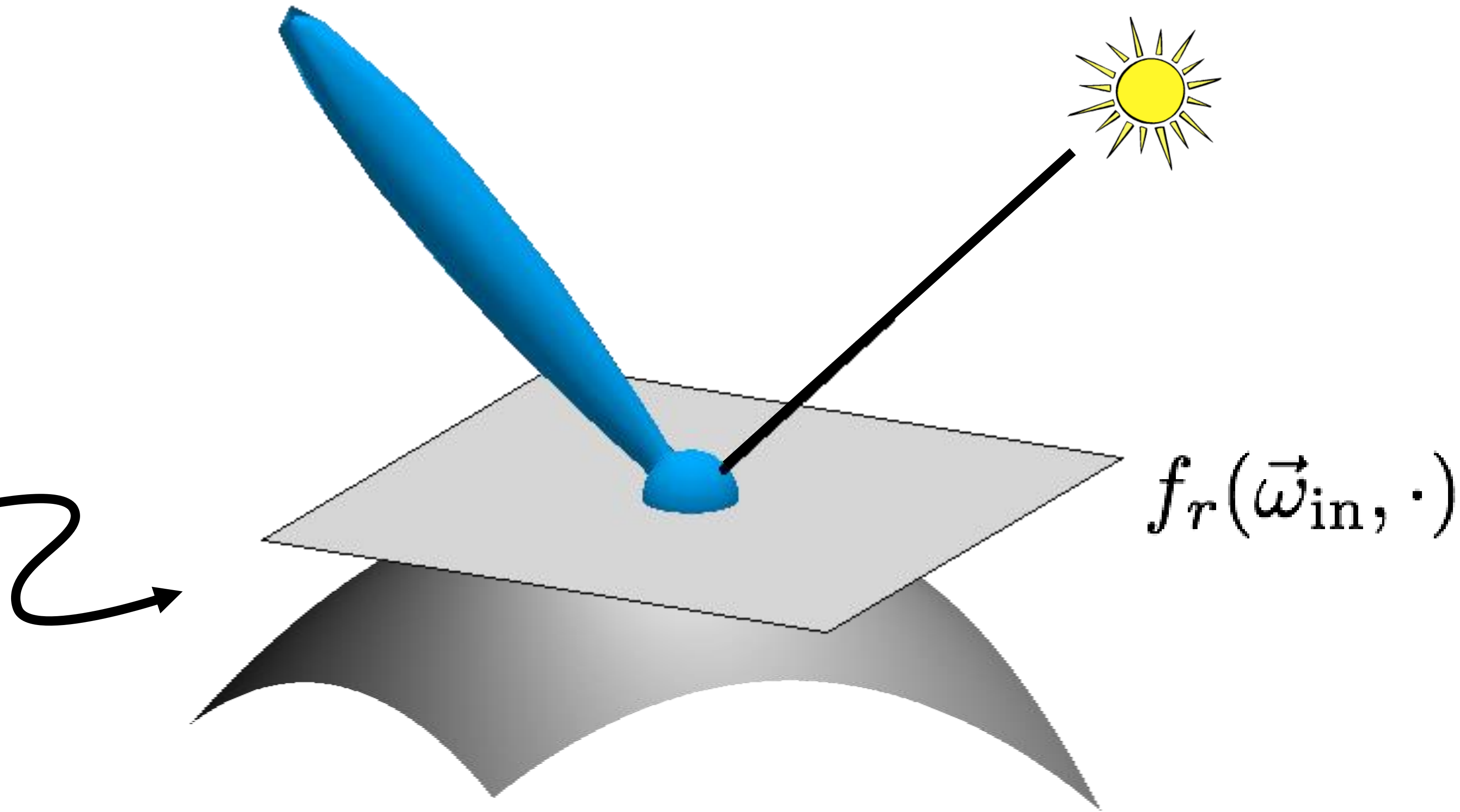
Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:

BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere





$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Trick for dielectrics (non-metals)

- BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components
- The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

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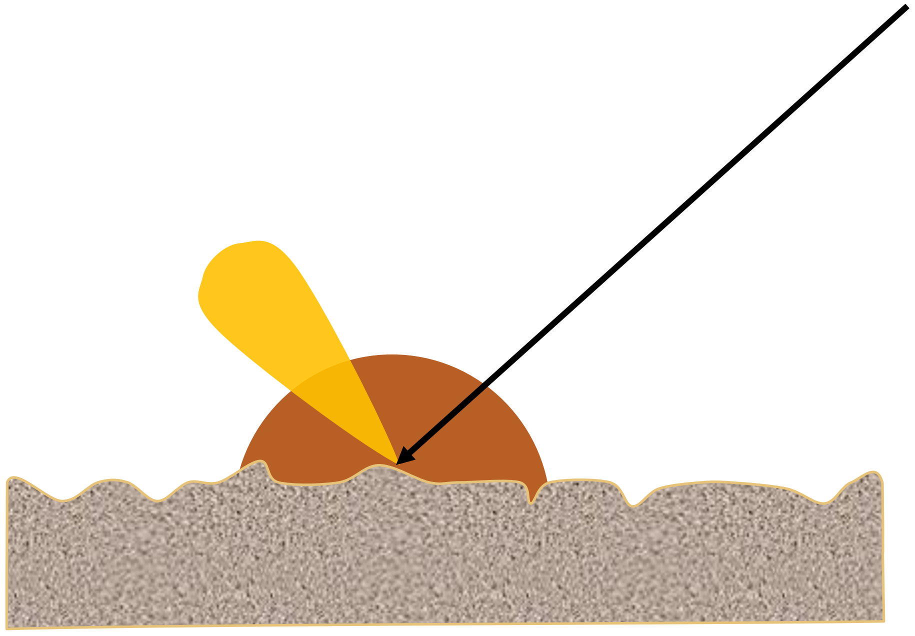
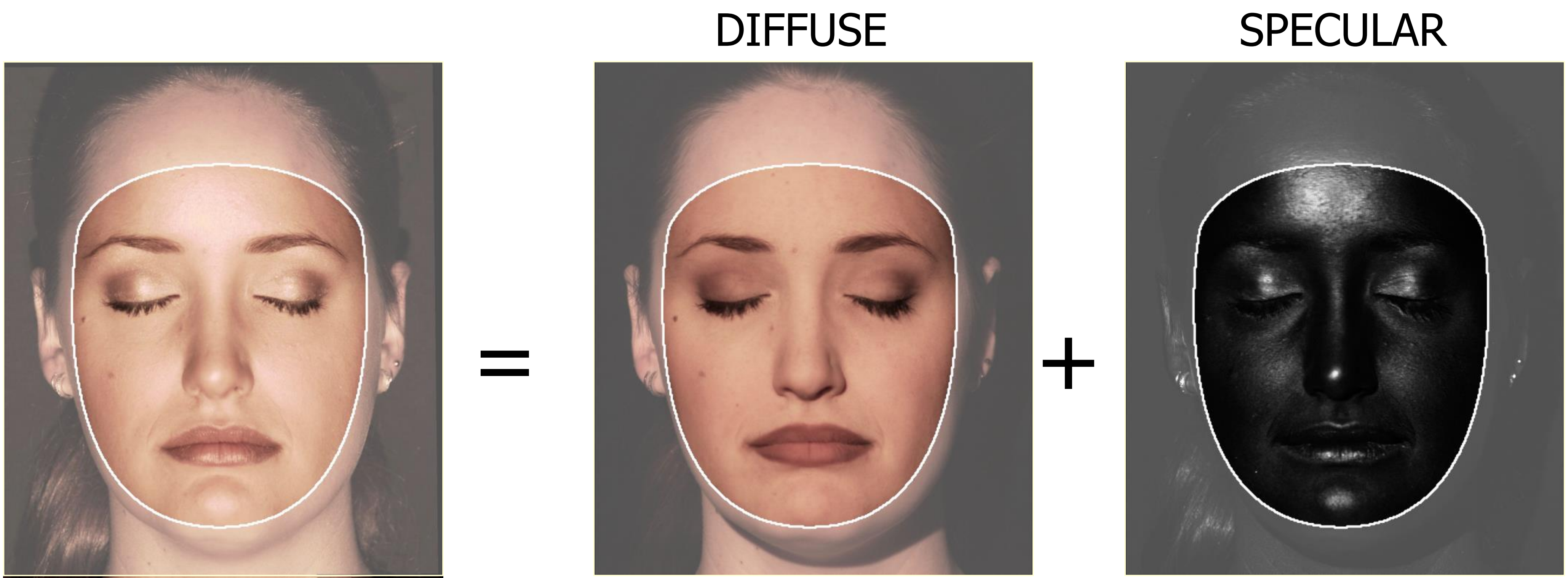
$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

Often called the *dichromatic BRDF*:

- Diffuse term varies with wavelength, constant with polarization
- Specular term constant with wavelength, varies with polarization

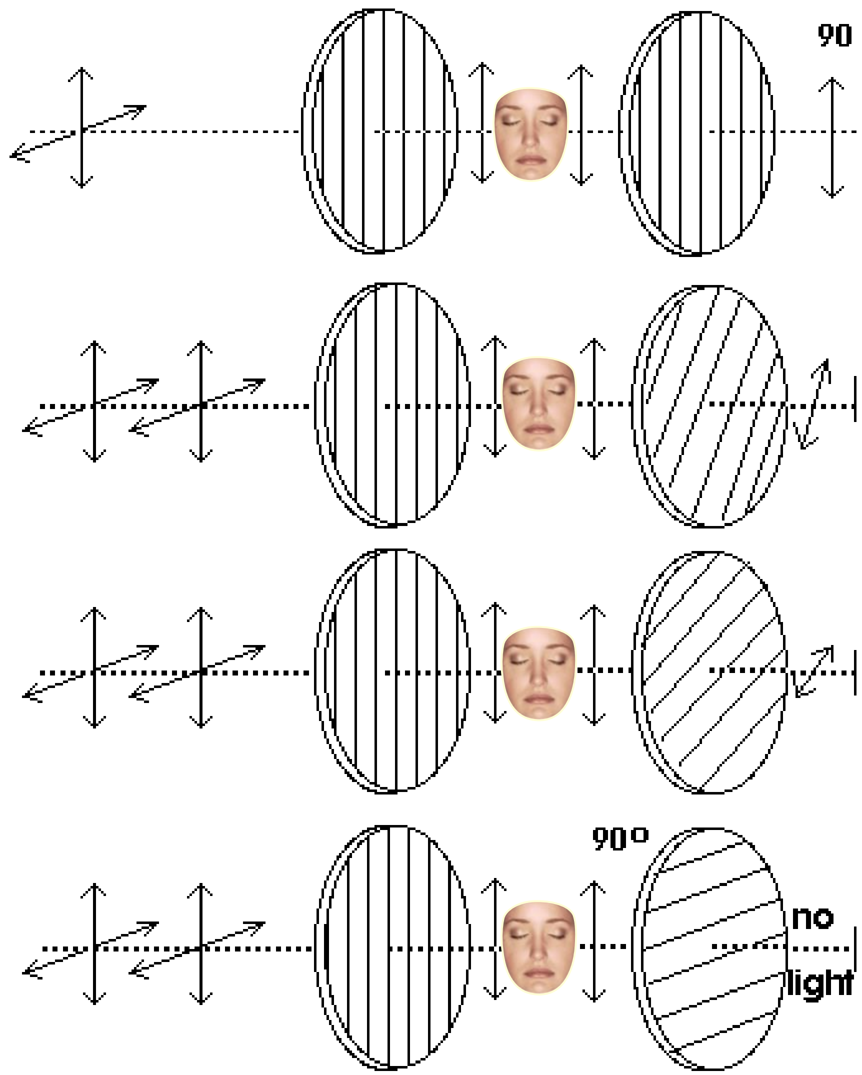


Trick for dielectrics (non-metals)



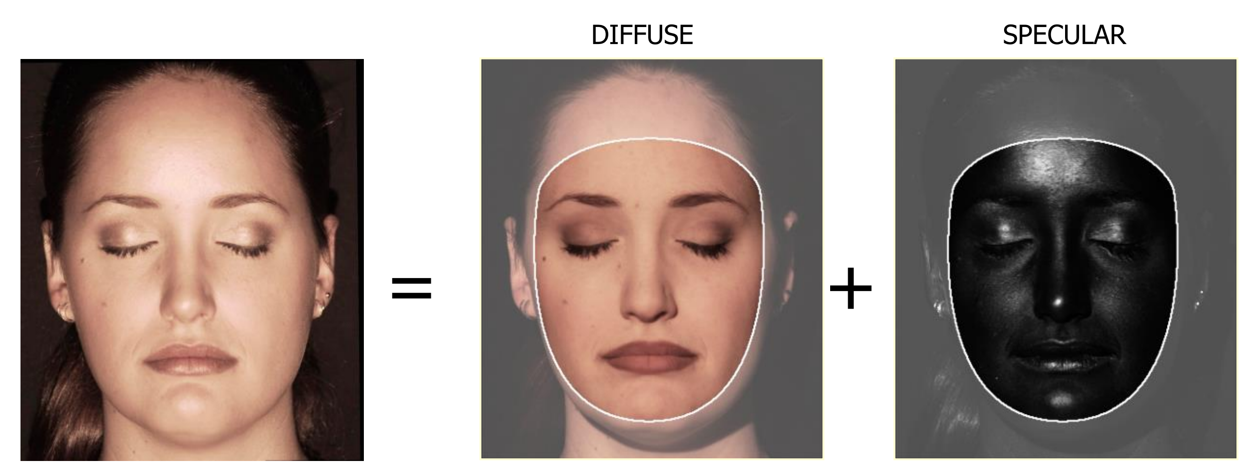
- In this example, the two components were separated using linear polarizing filters on the camera and light source.

Trick for dielectrics (non-metals)

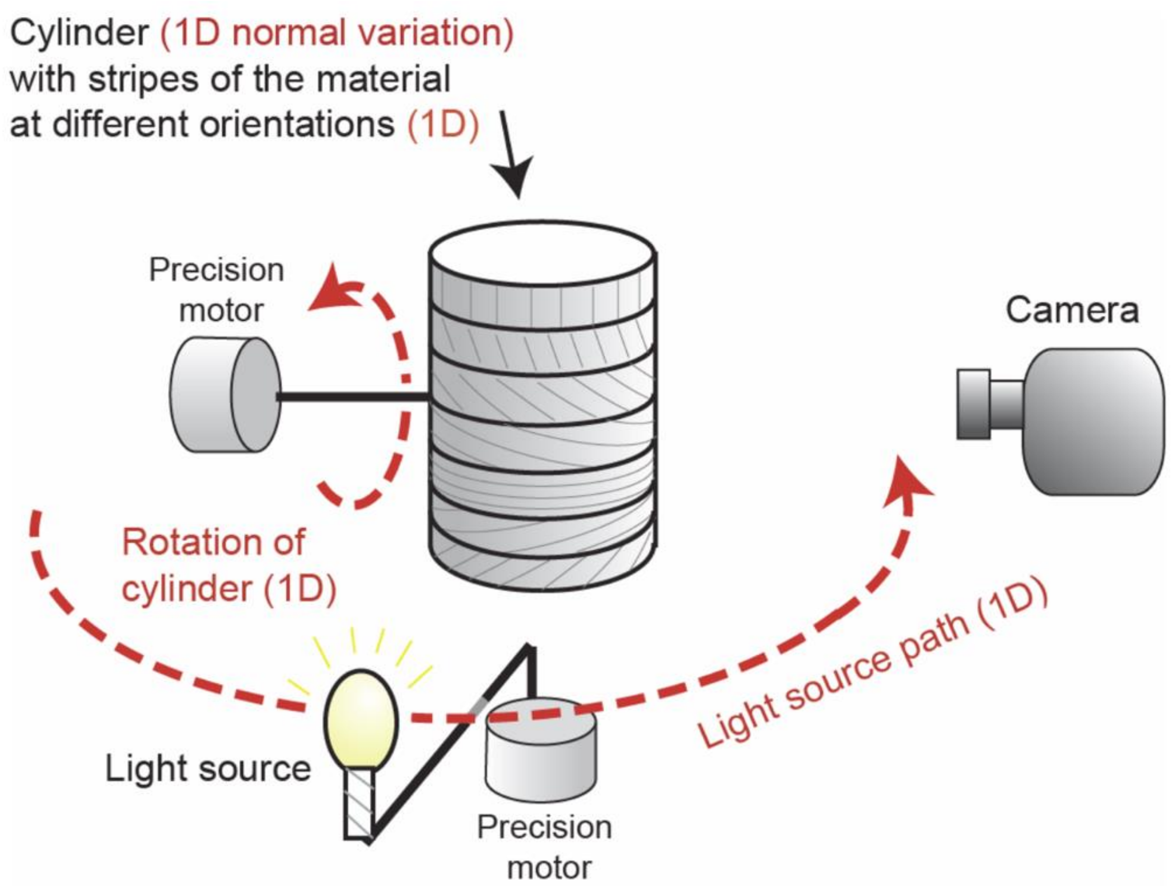


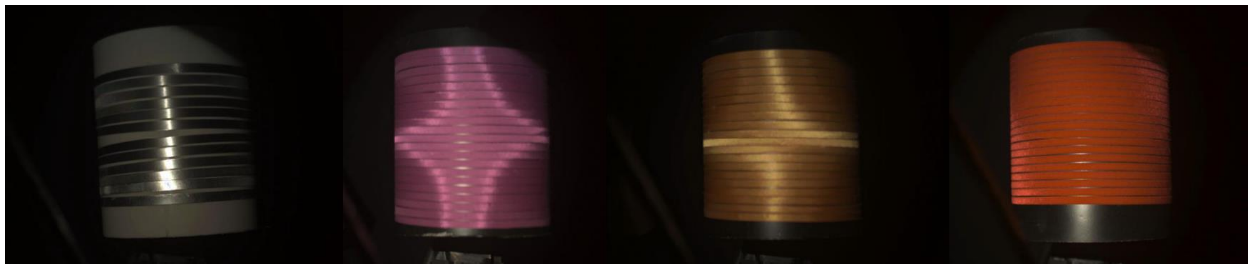
$$I_{\parallel} = \frac{1}{2} I_{\text{diffuse}} + I_{\text{specular}}$$

$$I_{\perp} = \frac{1}{2} I_{\text{diffuse}}$$

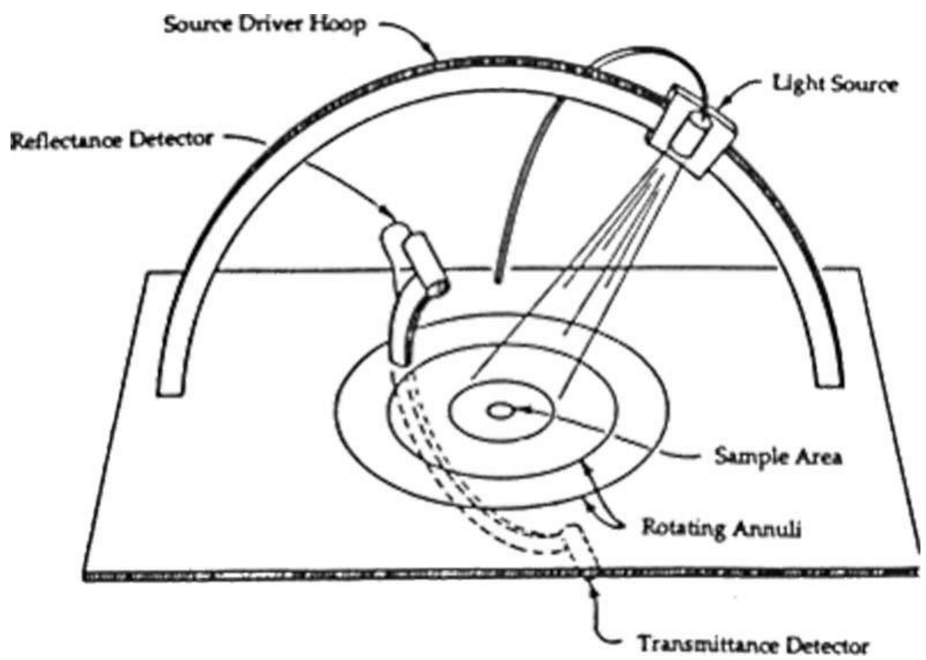


Tabulated 4D BRDFs (hard to measure)





[Ngan et al., 2005]



Gonioreflectometer

Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D,3D, or 4D) function
- Except for Lambertian, the BRDF is non-linear in these parameters
- Examples:

Where do these constants come from?

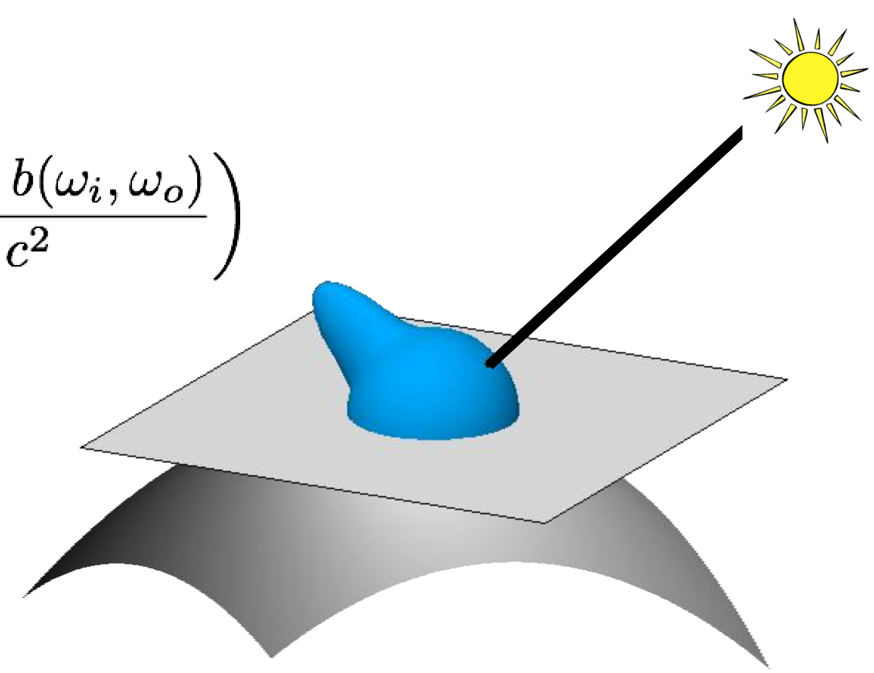
Lambertian: $$f(\omega_i, \omega_o) = \frac{a}{\pi}$$

Phong: $$f(\omega_i, \omega_o) = \frac{a}{\pi} + b\cos^c\left(2\langle \omega_i, n\rangle\langle \omega_o, n\rangle - \langle \omega_i, \omega_o\rangle\right)$$

Blinn: $$f(\omega_i, \omega_o) = \frac{a}{\pi} + b\cos^c b(\omega_i, \omega_o)$$

Lafortune: $$f(\omega_i, \omega_o) = \frac{a}{\pi} + b(-\omega_i^\top A\omega_o)^k$$

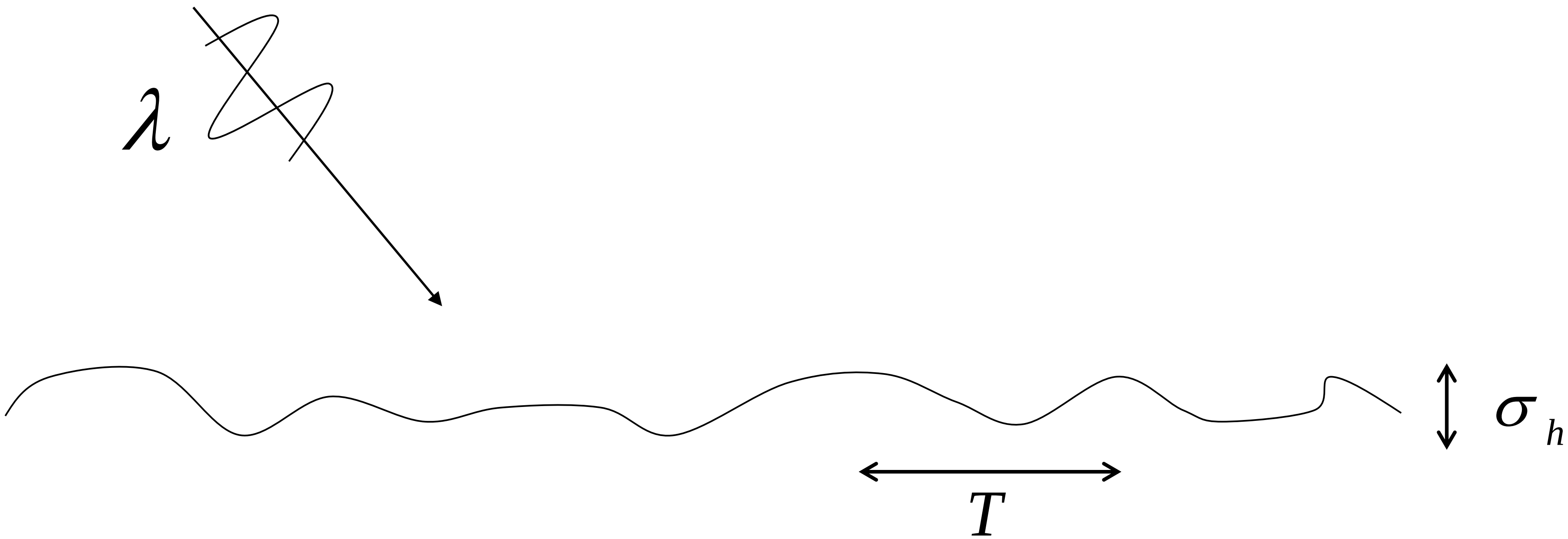
Ward: $$f(\omega_i, \omega_o) = \frac{a}{\pi} + \frac{b}{4\pi c^2\sqrt{\langle n, \omega_i\rangle\langle n, \omega_o\rangle}}\exp\left(\frac{-\tan^2 b(\omega_i, \omega_o)}{c^2}\right)$$



α is called the *albedo*

Reflectance Models

Reflection: An Electromagnetic Phenomenon

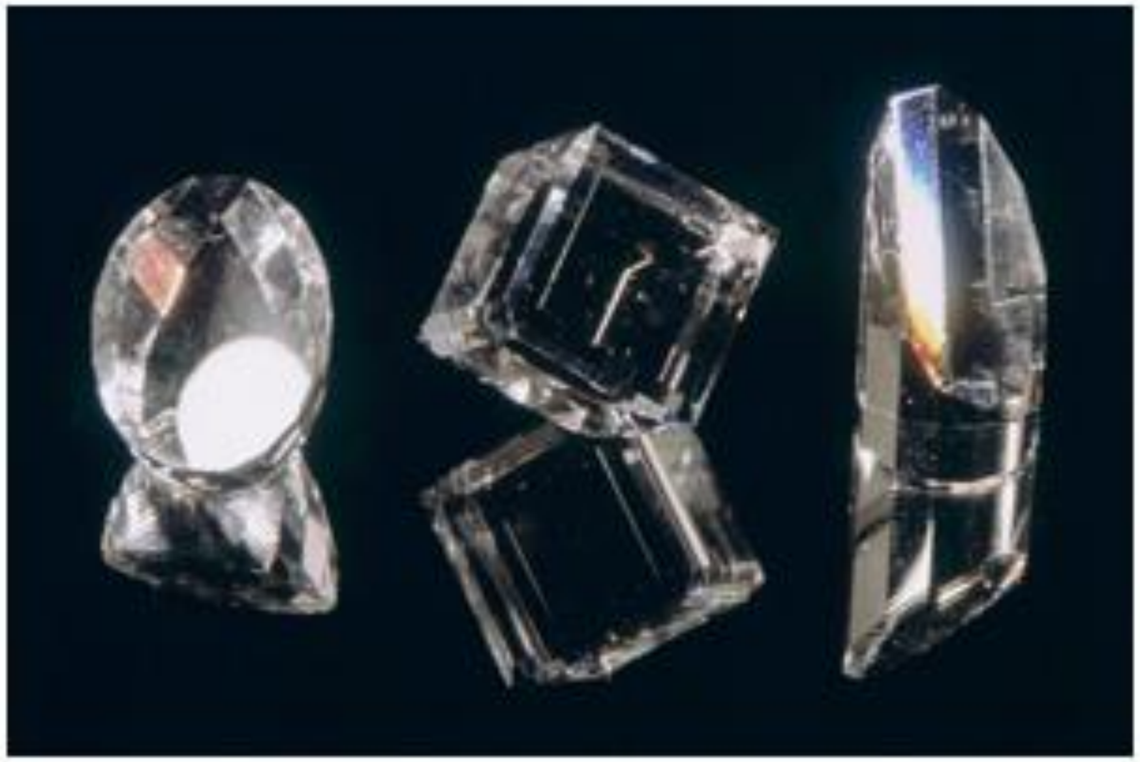


Two approaches to derive Reflectance Models:

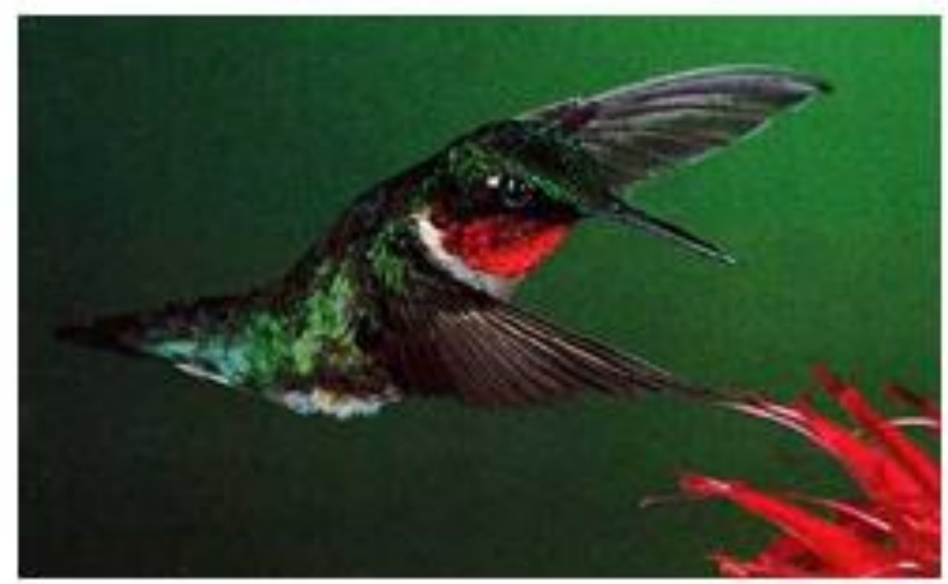
– Physical Optics (Wave Optics)
– Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models
But they are easier to use!

Reflectance that Require Wave Optics





Summary of some useful lighting models

- plenoptic function (function on 5D domain)
- far-field illumination (function on 2D domain)
- low-frequency far-field illumination (nine numbers)
- directional lighting (three numbers = direction and strength)
- point source (four numbers = location and strength)

References

Basic reading:

- Szeliski, Section 2.2.
- Gortler, Chapter 21.
  This book by Steven Gortler has a great *introduction* to radiometry, reflectance, and their use for image formation.

Additional reading:

- Arvo, “Analytic Methods for Simulated Light Transport,” Yale 1995.
- Veach, “Robust Monte Carlo Methods for Light Transport Simulation,” Stanford 1997.
  These two thesis are foundational for modern computer graphics. Among other things, they include a thorough derivation (starting from wave optics and measure theory) of all radiometric quantities and associated integro-differential equations. You can also look at them if you are interested in physics-based rendering.
- Dutre et al., “Advanced Global Illumination,” 2006.
  A book discussing modeling and simulation of other appearance effects beyond single-bounce reflectance.
- Weyrich et al., “Principles of Appearance Acquisition and Representation,” FTCGV 2009.
  A very thorough review of everything that has to do with modeling and measuring BRDFs.
- Walter et al., “Microfacet models for refraction through rough surfaces,” EGSR 2007.
  This paper has a great review of physics-based models for reflectance and refraction.
- Matusik, “A data-driven reflectance model,” MIT 2003.
  This thesis introduced the largest measured dataset of 4D reflectances. It also provides detailed discussion of many topics relating to modelling reflectance.
- Rusinkiewicz, “A New Change of Variables for Efficient BRDF Representation,” 1998.
- Romeiro and Zickler, “Inferring reflectance under real-world illumination,” Harvard TR 2010.
  These two papers discuss the isotropy and other properties of common BRDFs, and how one can take advantage of them using alternative parameterizations.
- Shafer, “Using color to separate reflection components,” 1984.
  The paper introducing the dichromatic reflectance model.
- Stam, “Diffraction Shaders,” SIGGRAPH 1999.
- Levin et al., “Fabricating BRDFs at high spatial resolution using wave optics,” SIGGRAPH 2013.
- Cuypers et al., “Reflectance model for diffraction,” TOG 2013.
  These three papers describe reflectance effects that can only be modeled using wave optics (and in particular diffraction).