

# Pinholes and lenses



15-463, 15-663, 15-862  
Computational Photography  
Fall 2017, Lecture 3

# Course announcements

- Homework 1 is out.
  - Due September 14<sup>th</sup>.
  - Make sure to sign up for a camera if you need one.
  - Drop by Yannis' office to pick up cameras any time.
- Office hours for the semester:
  - Alankar: Tuesdays 5-7 pm, Smith 220.
  - Yannis: Fridays 5-7 pm, Smith 225.
- Those on the waitlist: Class is currently capped, but some seats will likely become available after the drop deadline.

# Overview of today's lecture

- Leftover from lecture 2: thoughts on the image processing pipeline.
- Some motivational imaging experiments.
- Pinhole camera.
- Accidental pinholes.
- The thin lens model.
- Lens camera and pinhole camera.
- Perspective.
- Field of view.
- Orthographic camera and telecentric lenses.

# Slide credits

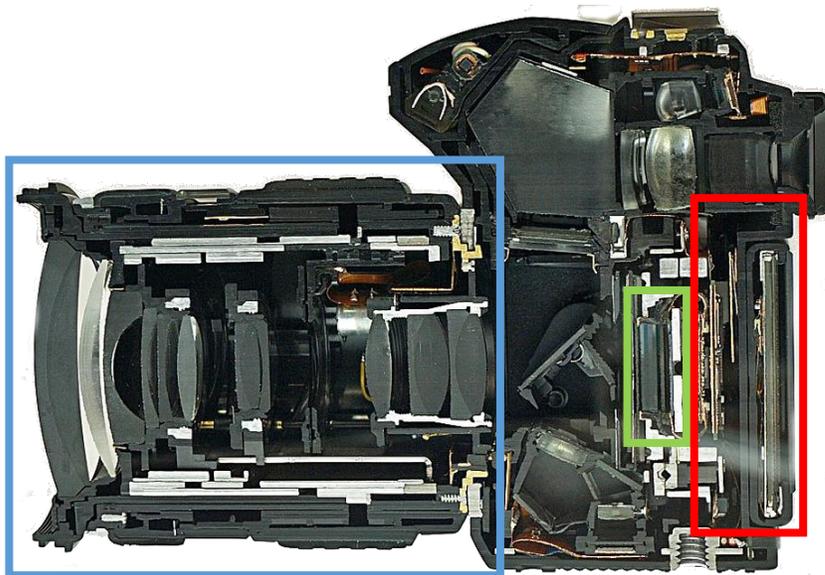
Many of these slides were adapted from:

- Kris Kitani (15-463, Fall 2016).
- Fredo Durand (MIT).
- Gordon Wetzstein (Stanford).

# The modern photography pipeline



post-capture processing  
(lectures 5-10)



optics and  
optical controls

(lectures 2-3, 11-20)



sensor, analog  
front-end, and  
color filter array

(lectures 2, 23)



in-camera image  
processing  
pipeline

(lecture 2)

Some motivational imaging experiments

Let's say we have a sensor...



digital sensor  
(CCD or CMOS)

... and an object we like to photograph

real-world  
object



digital sensor  
(CCD or CMOS)



What would an image taken like this look like?

# Bare-sensor imaging

real-world  
object



digital sensor  
(CCD or CMOS)

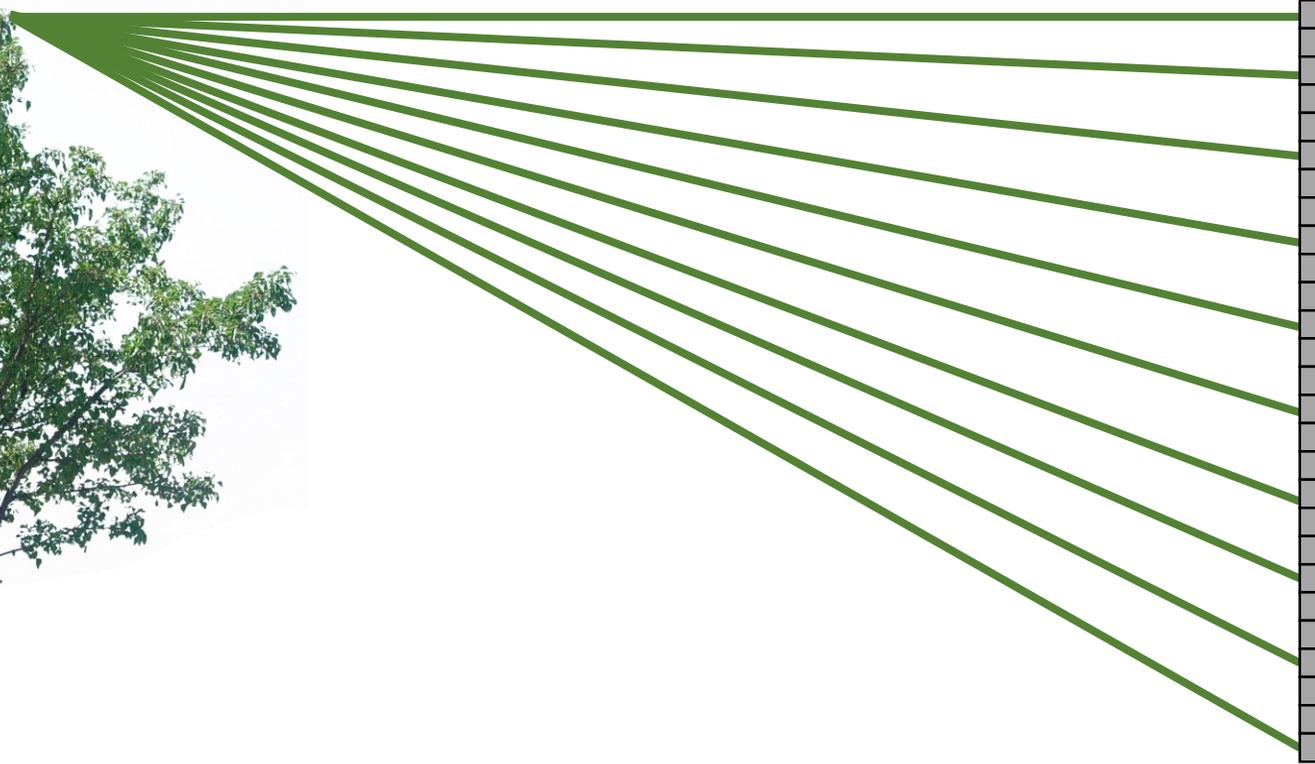


# Bare-sensor imaging

real-world  
object

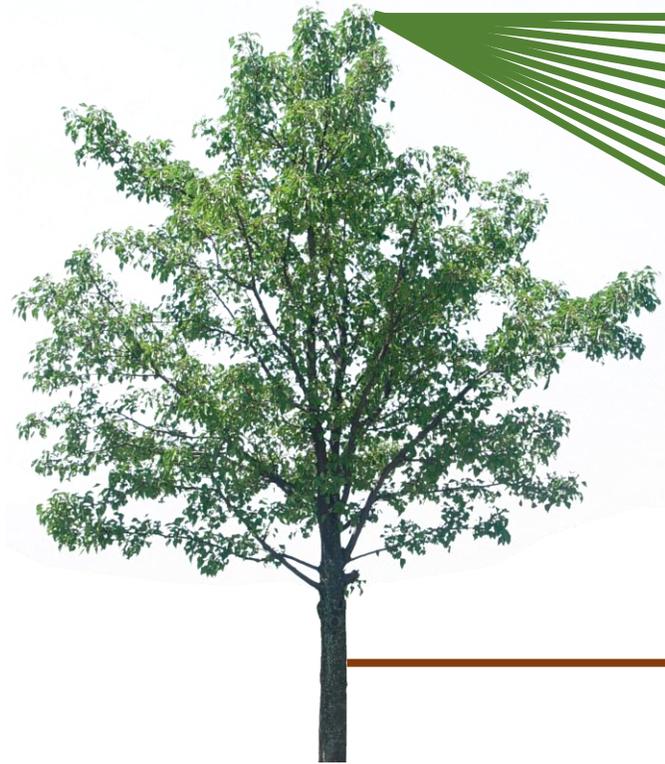


digital sensor  
(CCD or CMOS)

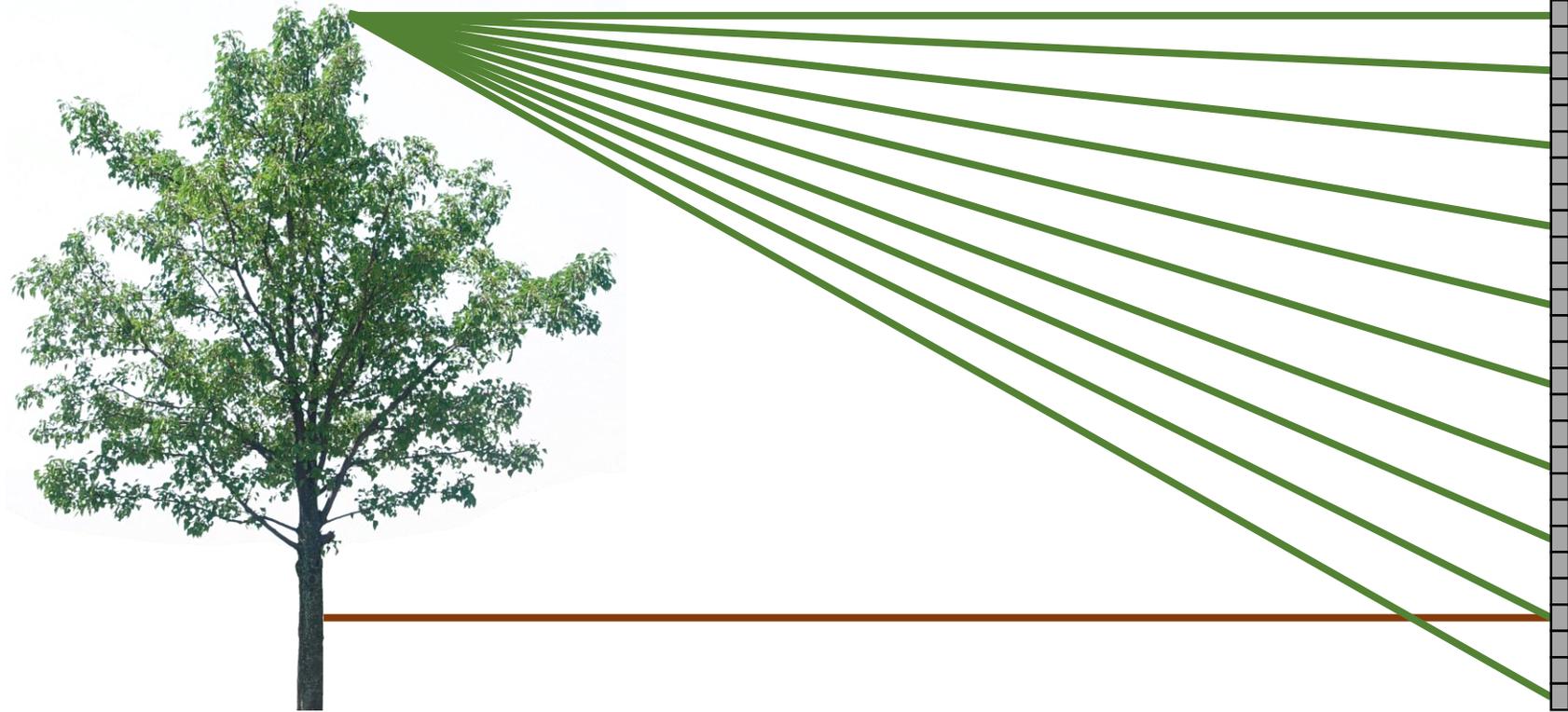


# Bare-sensor imaging

real-world  
object

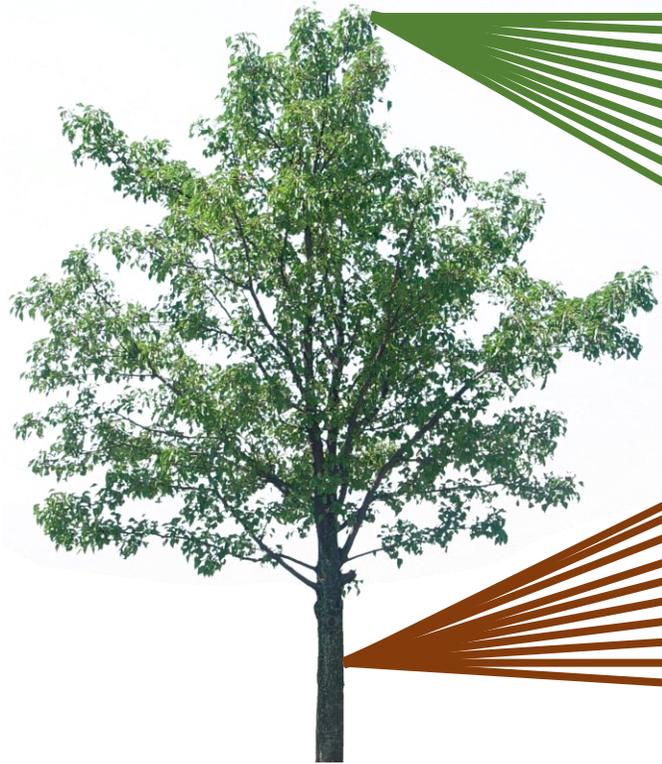


digital sensor  
(CCD or CMOS)



# Bare-sensor imaging

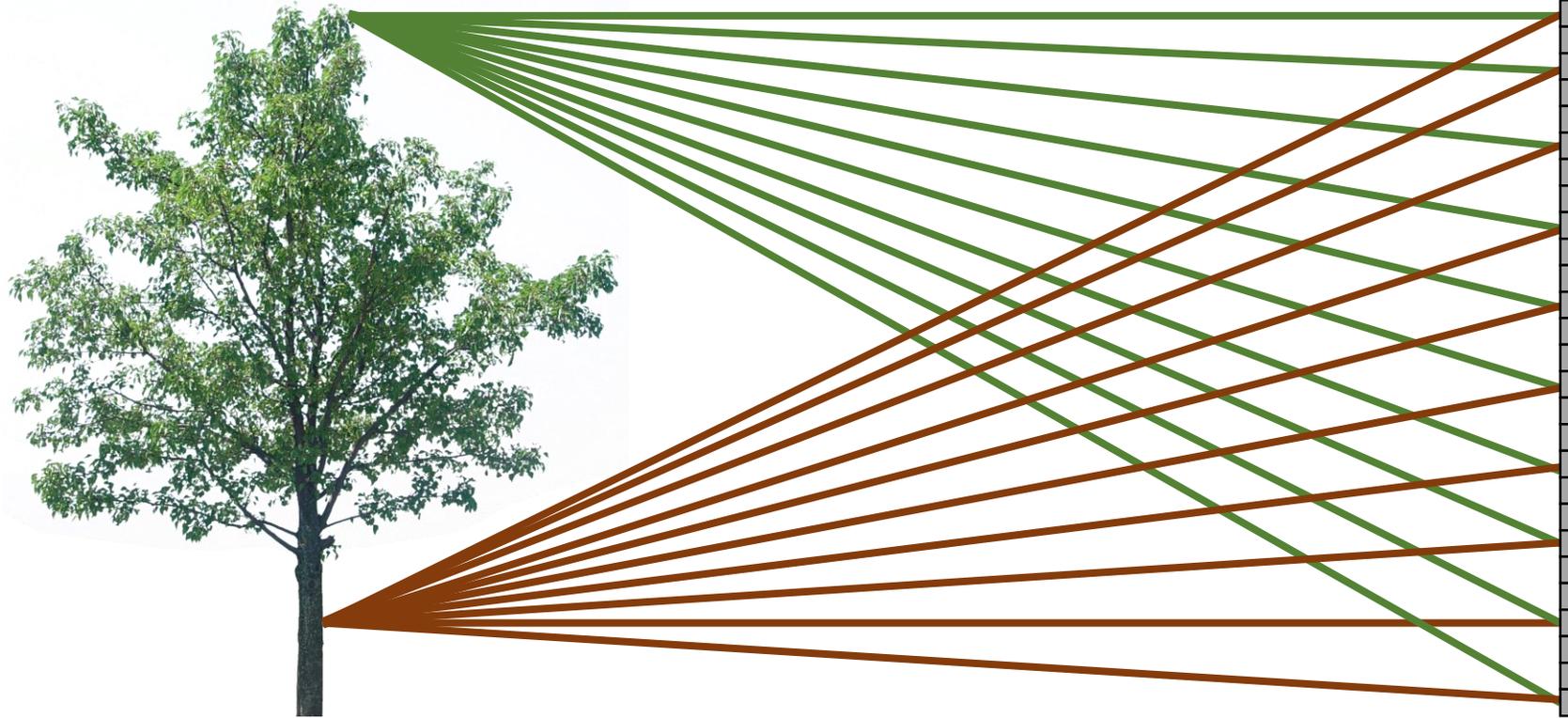
real-world  
object



digital sensor  
(CCD or CMOS)

What does the  
image on the  
sensor look like?

All scene points contribute to all sensor pixels



# Bare-sensor imaging



All scene points contribute to all sensor pixels

# What can we do to make our image look better?

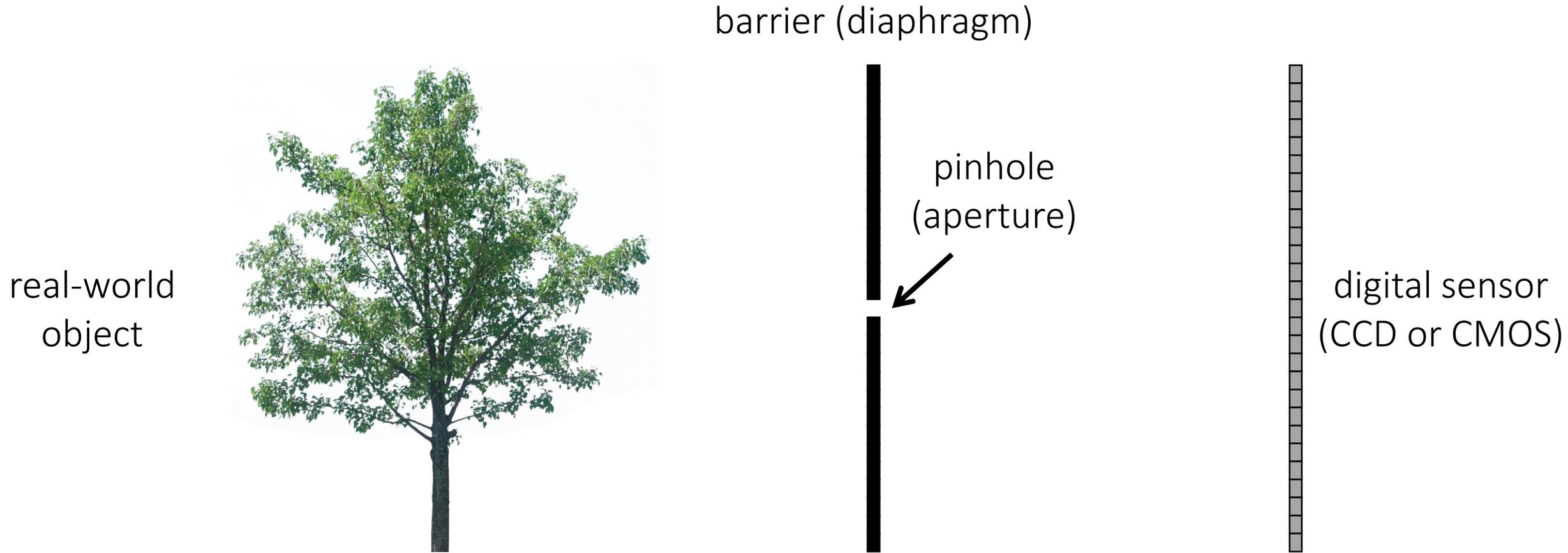
real-world  
object



digital sensor  
(CCD or CMOS)



# Let's add something to this scene



What would an image taken like this look like?

# Pinhole imaging

real-world  
object

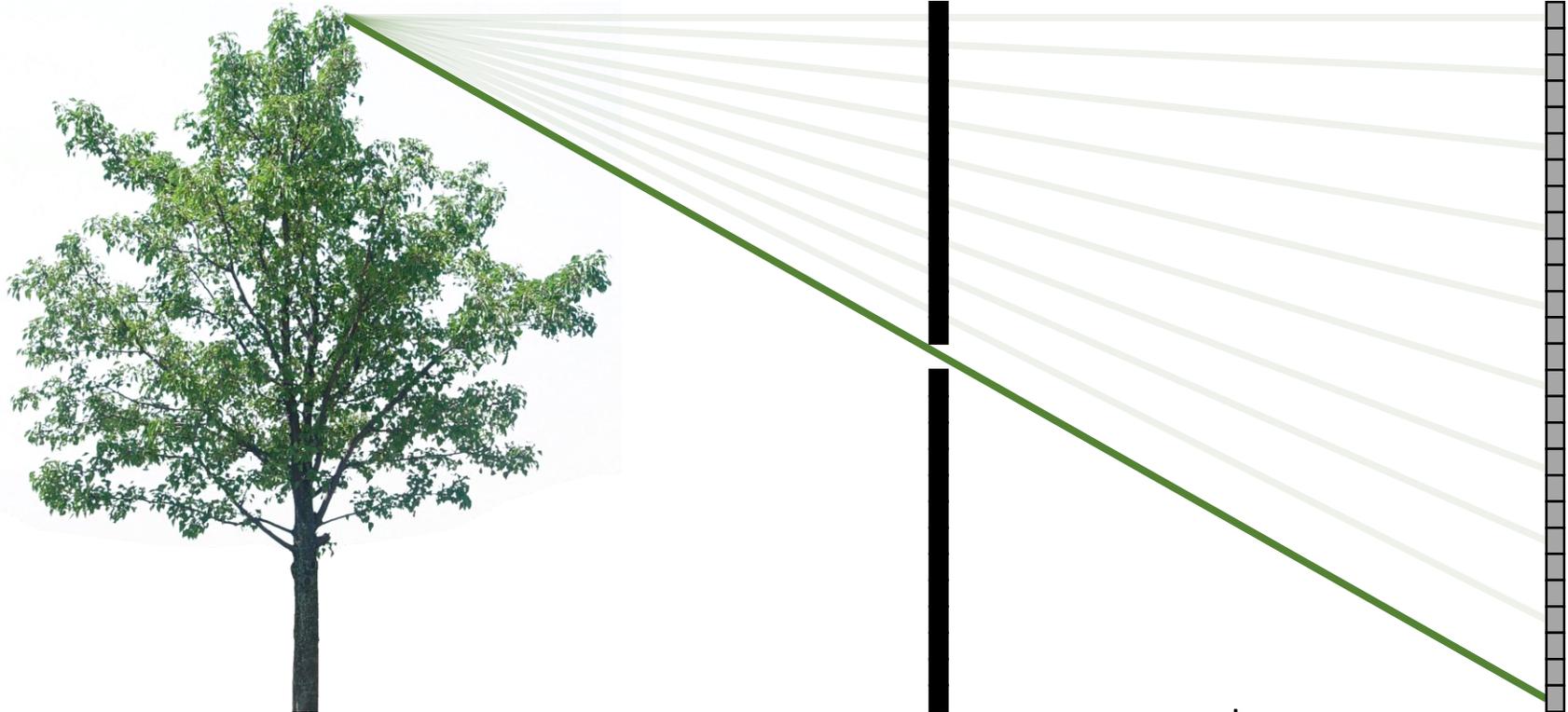


most rays  
are blocked



one makes  
it through

digital sensor  
(CCD or CMOS)



# Pinhole imaging

real-world  
object

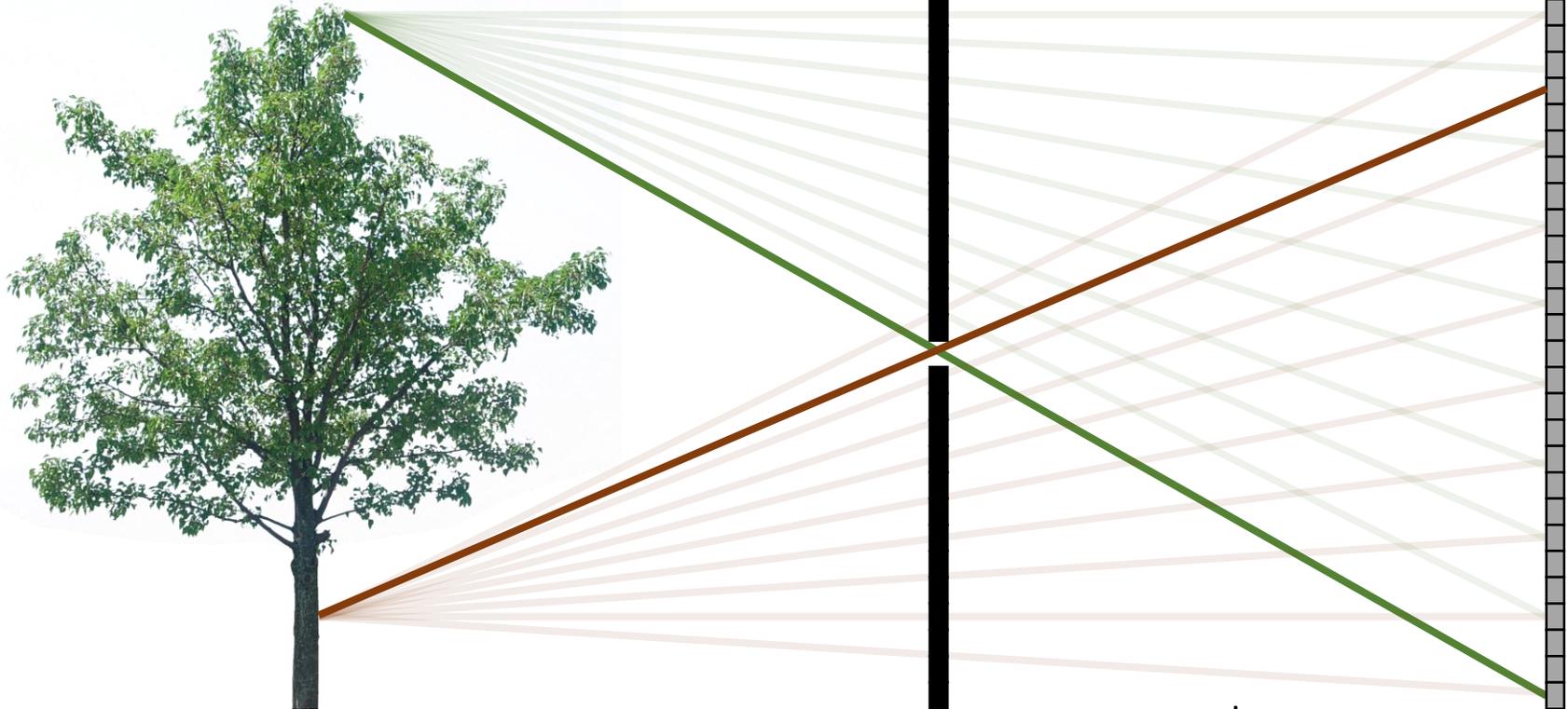


most rays  
are blocked



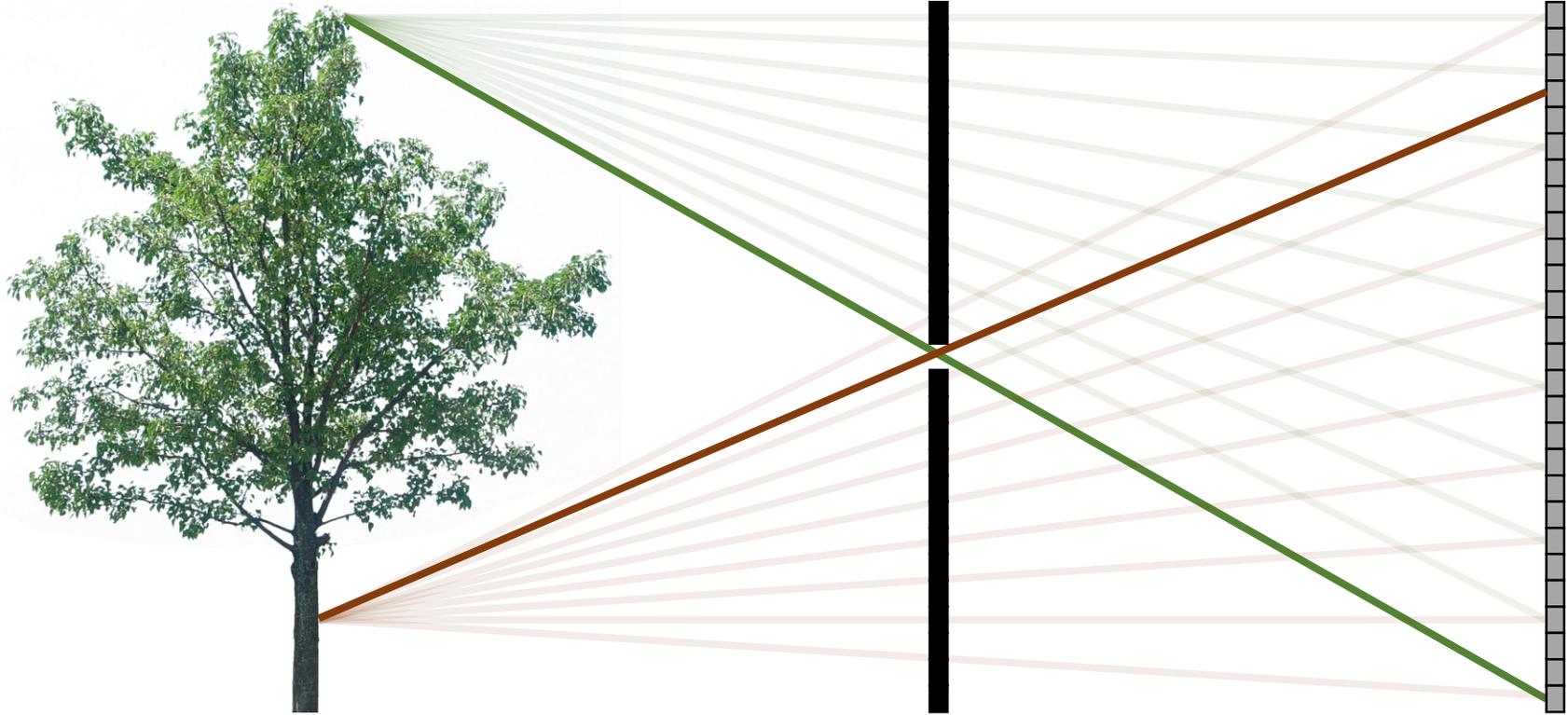
one makes  
it through

digital sensor  
(CCD or CMOS)



# Pinhole imaging

real-world  
object



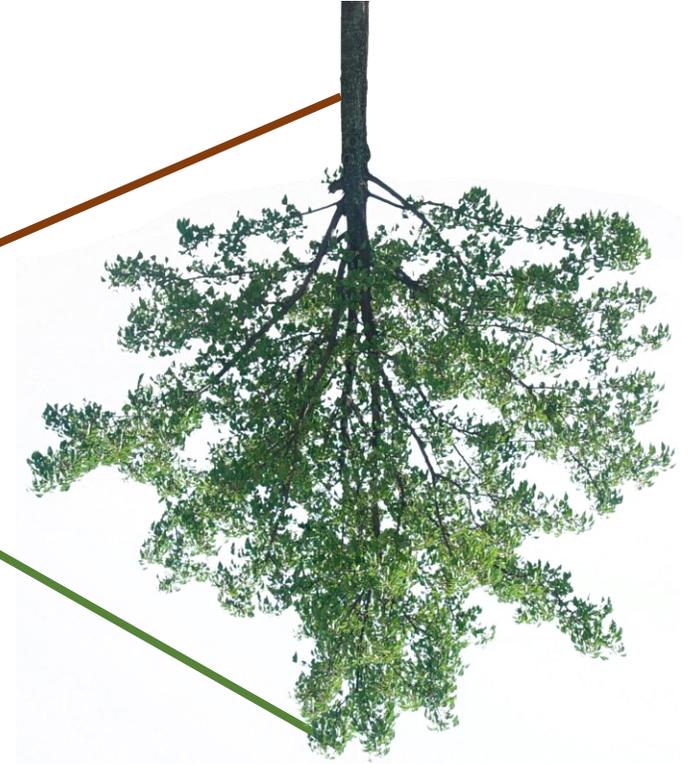
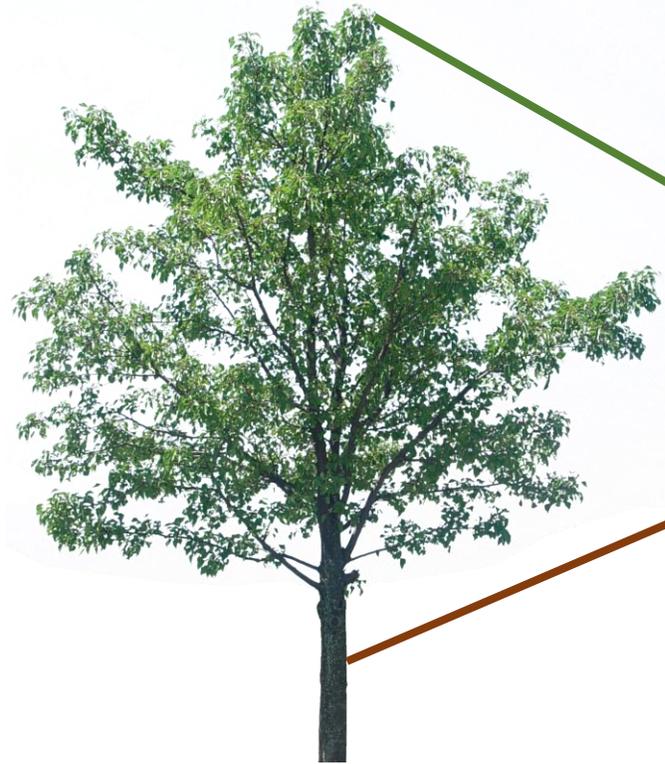
digital sensor  
(CCD or CMOS)

Each scene point contributes to only one sensor pixel

What does the  
image on the  
sensor look like?

# Pinhole imaging

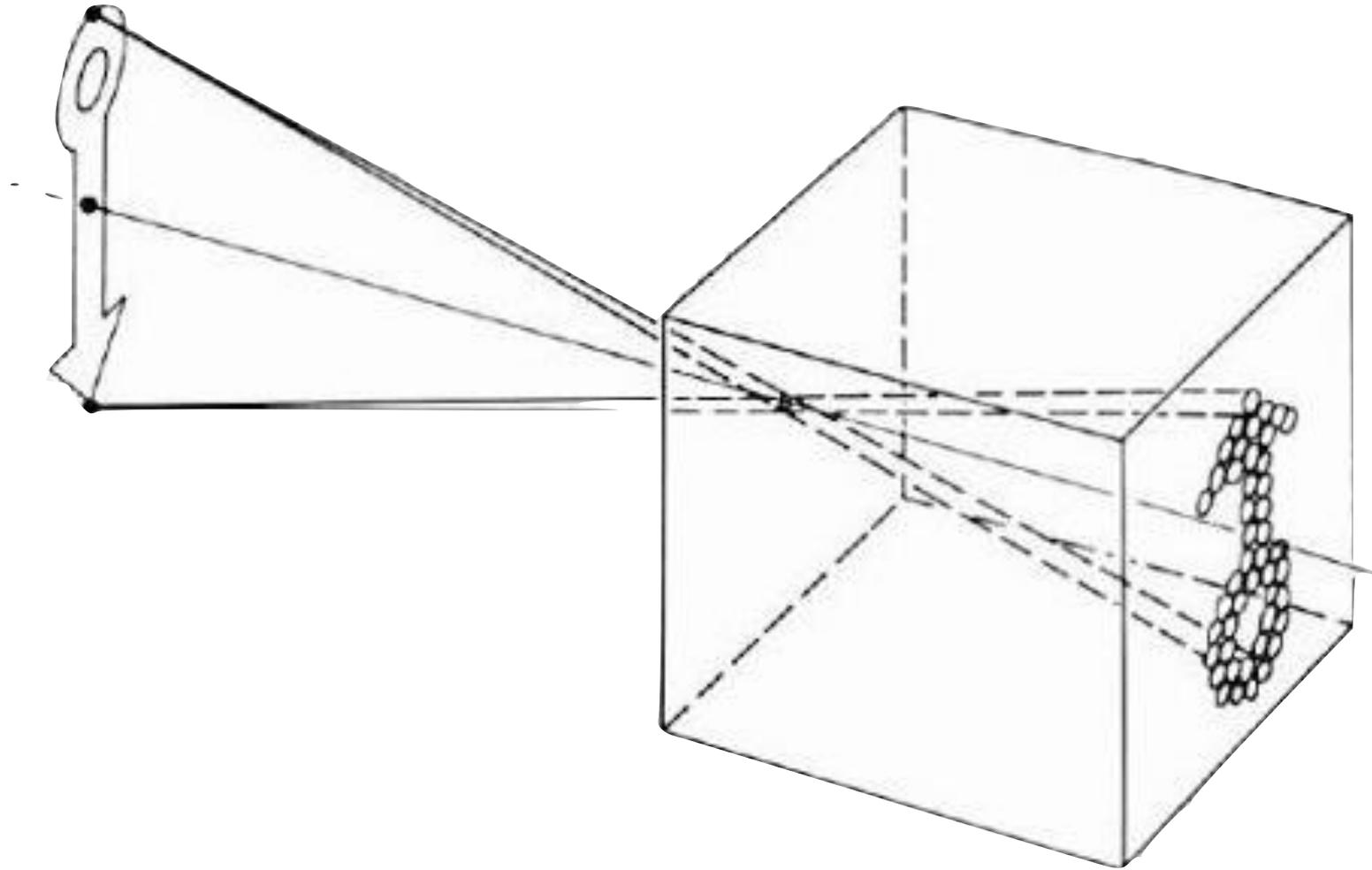
real-world  
object



copy of real-world object  
(inverted and scaled)

Pinhole camera

# Pinhole camera a.k.a. camera obscura



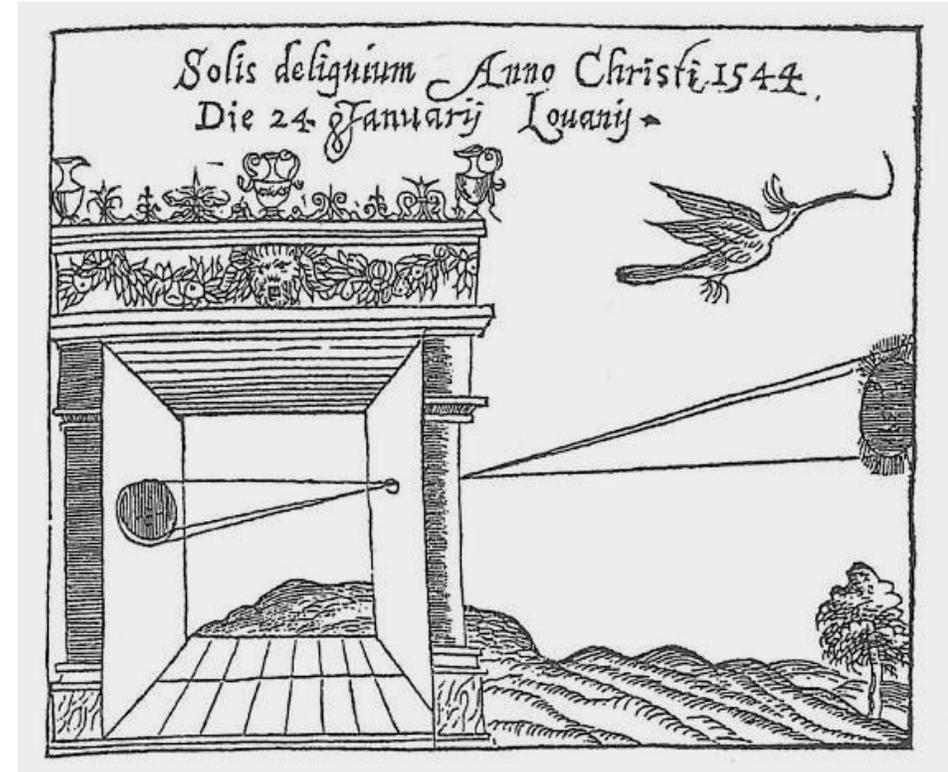
# Pinhole camera a.k.a. camera obscura

First mention ...



Chinese philosopher Mozi  
(470 to 390 BC)

First camera ...



Greek philosopher Aristotle  
(384 to 322 BC)

# Pinhole camera terms

real-world  
object



barrier (diaphragm)



pinhole  
(aperture)



digital sensor  
(CCD or CMOS)

# Pinhole camera terms

real-world  
object



barrier (diaphragm)



pinhole  
(aperture)



camera center  
(center of projection)

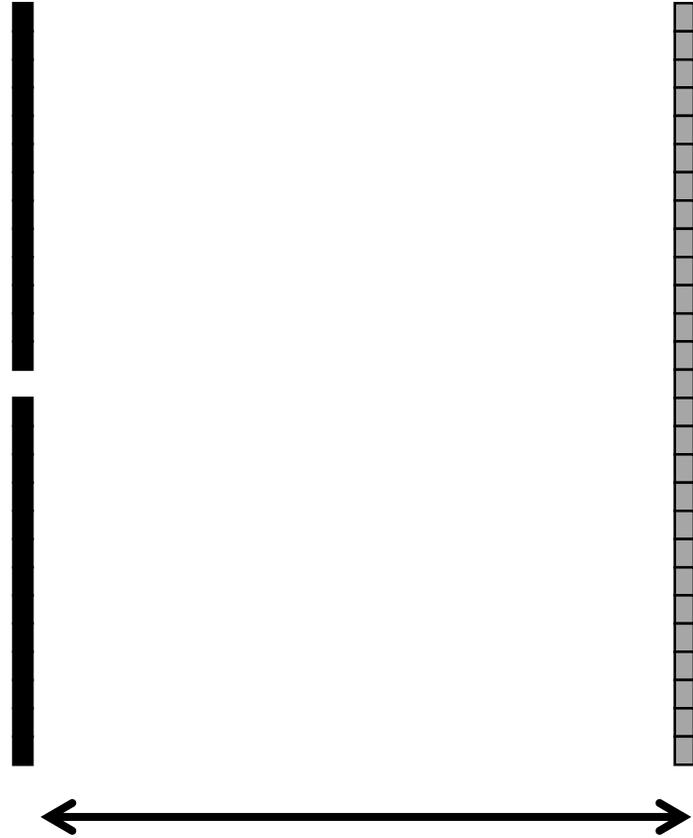
image plane



digital sensor  
(CCD or CMOS)

# Focal length

real-world  
object

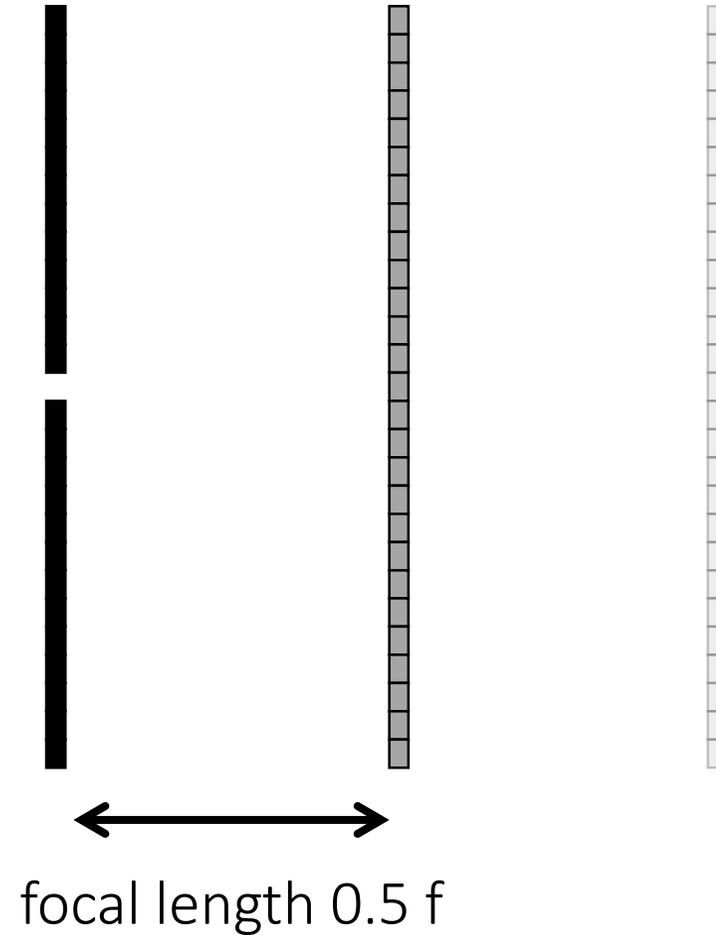


focal length  $f$

# Focal length

What happens as we change the focal length?

real-world  
object



# Focal length

What happens as we change the focal length?

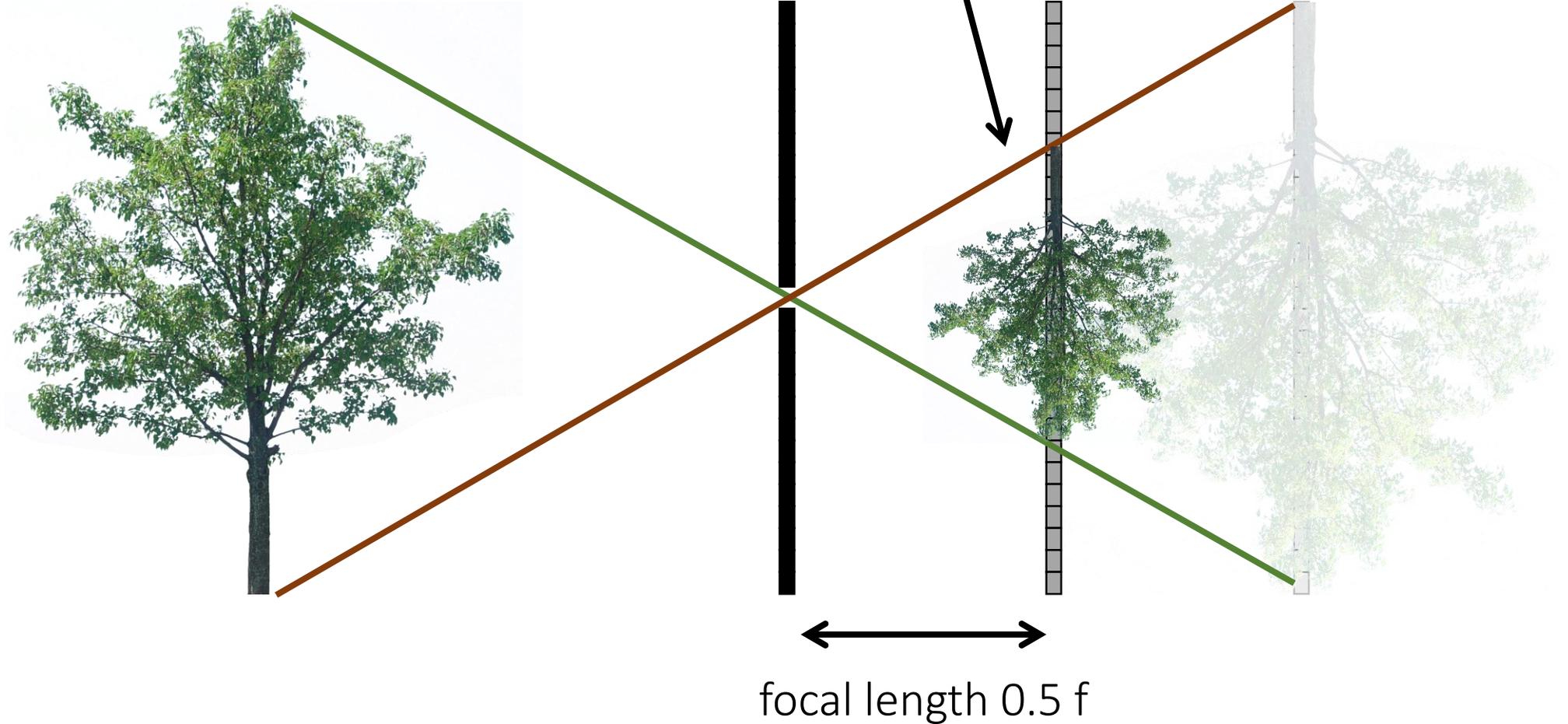


# Focal length

What happens as we change the focal length?

object projection is half the size

real-world  
object



# Pinhole size

real-world  
object



pinhole  
diameter



- Ideal pinhole has infinitesimally small size
- In practice that is impossible.

# Pinhole size

What happens as we change the pinhole diameter?

real-world  
object



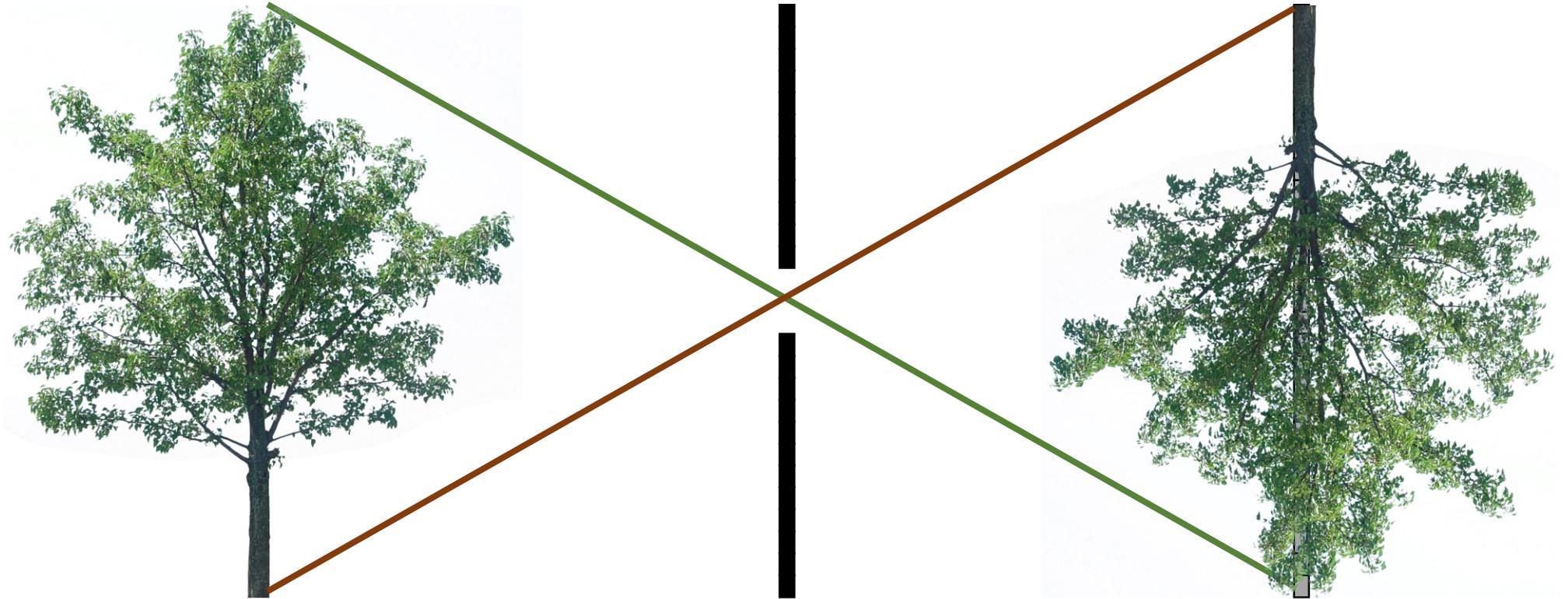
pinhole  
diameter



# Pinhole size

What happens as we change the pinhole diameter?

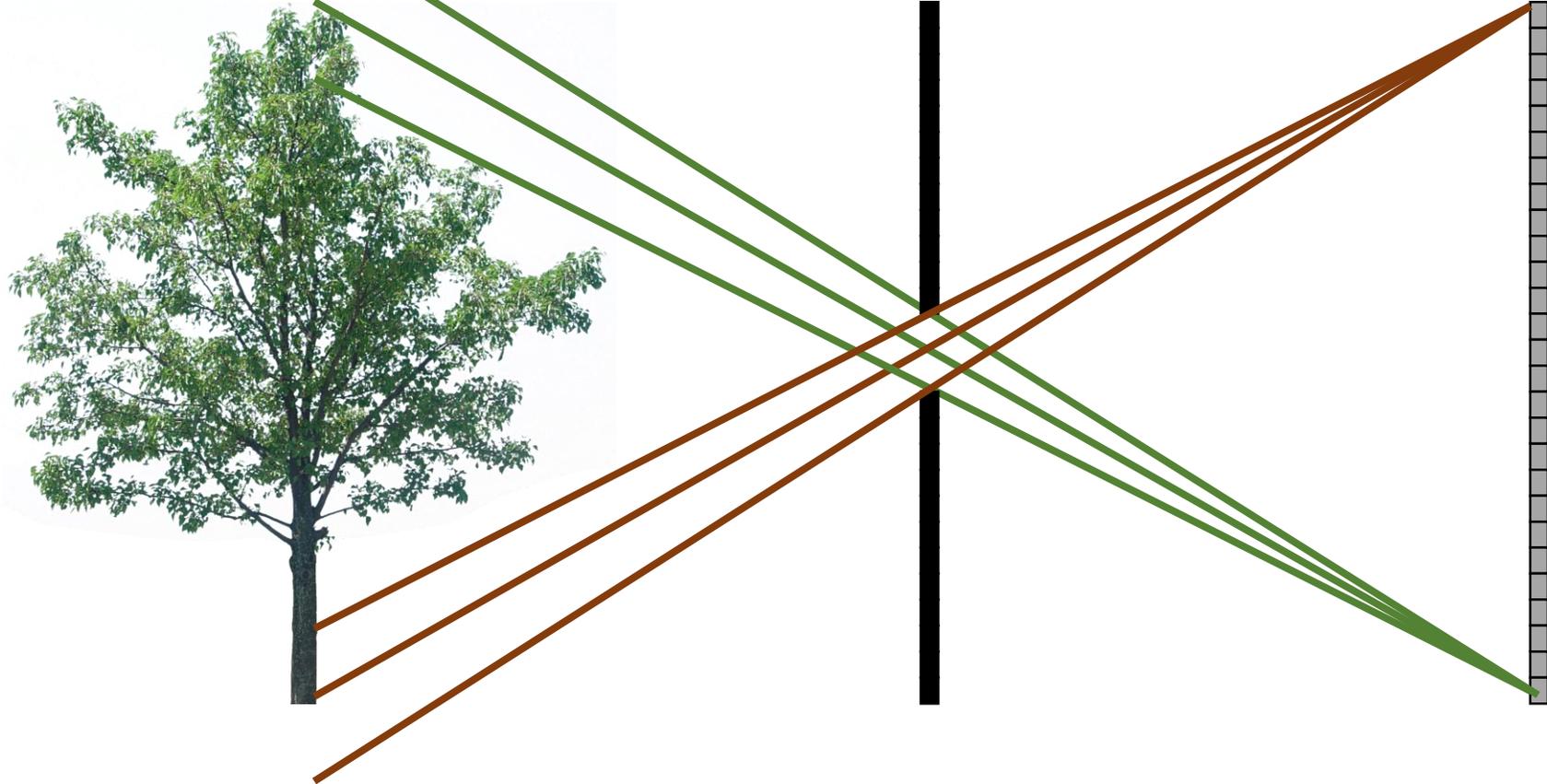
real-world  
object



# Pinhole size

What happens as we change the pinhole diameter?

real-world  
object

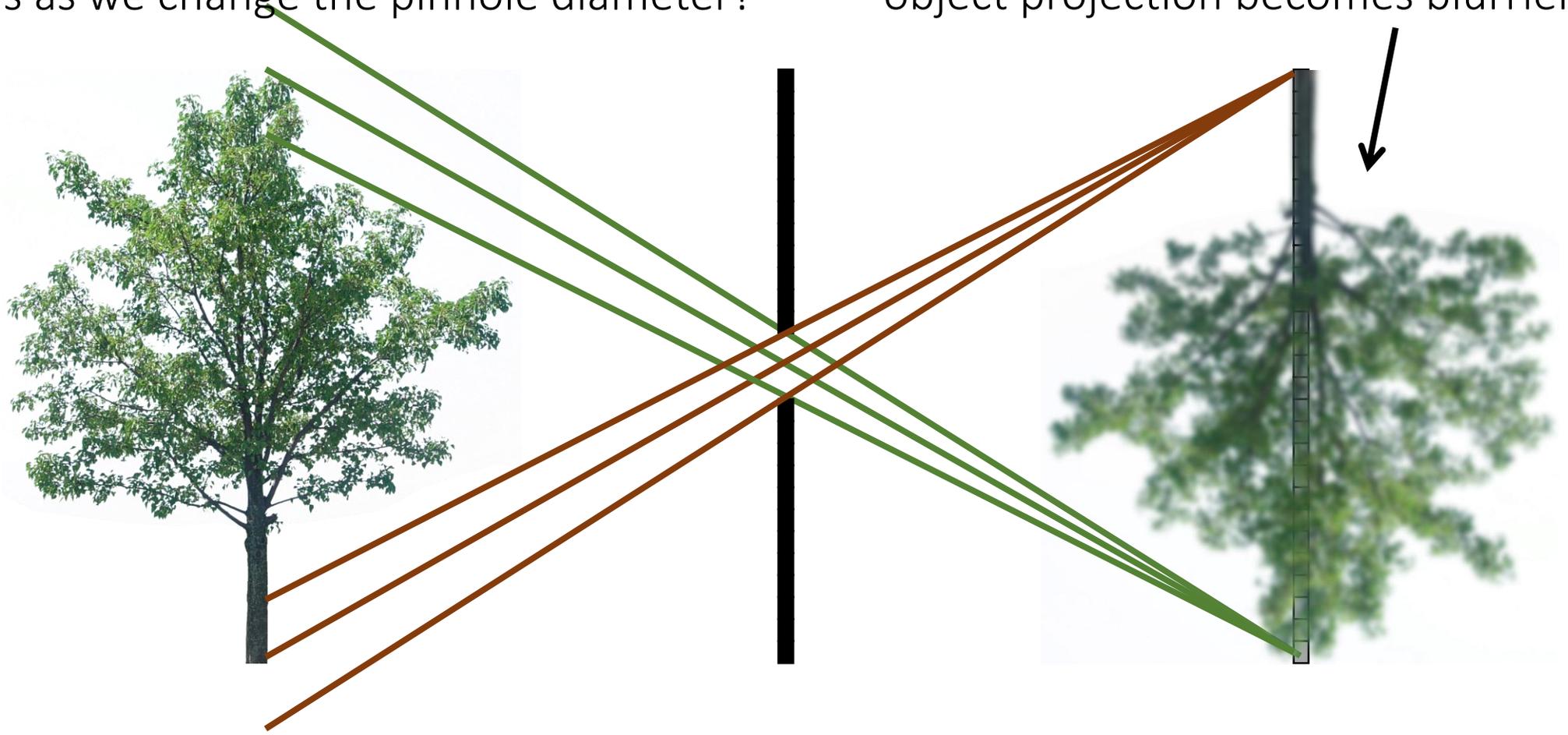
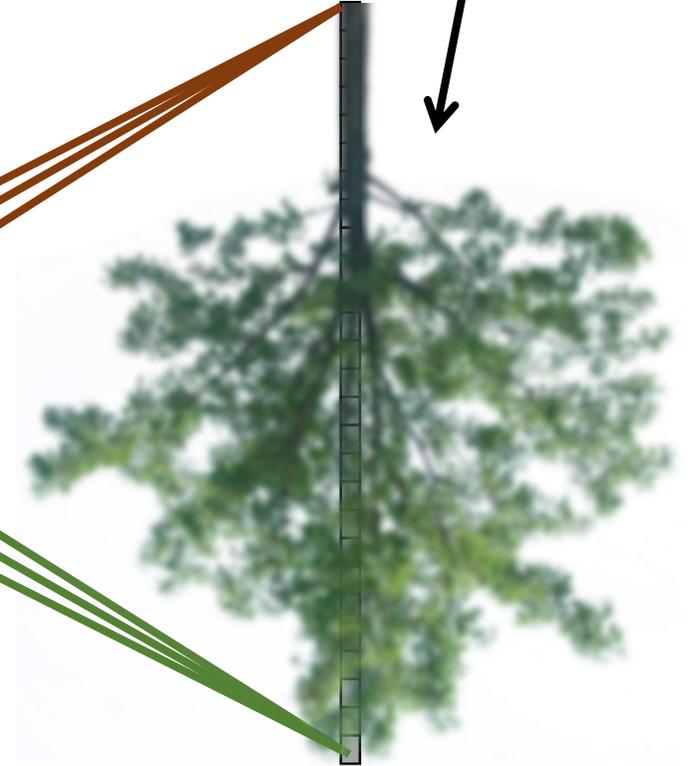
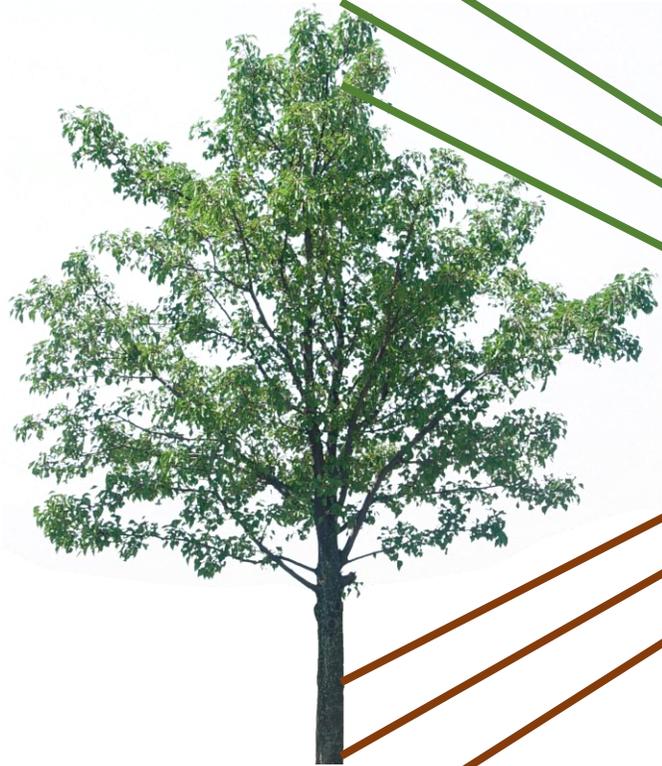


# Pinhole size

What happens as we change the pinhole diameter?

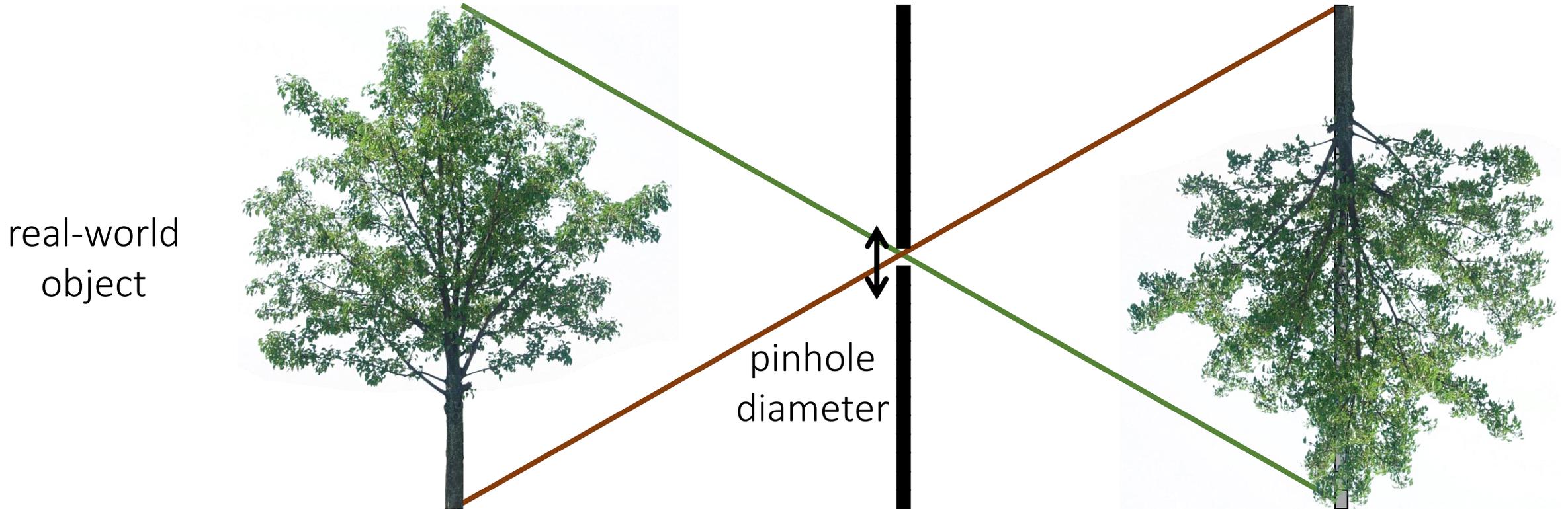
object projection becomes blurrier

real-world  
object



# Pinhole size

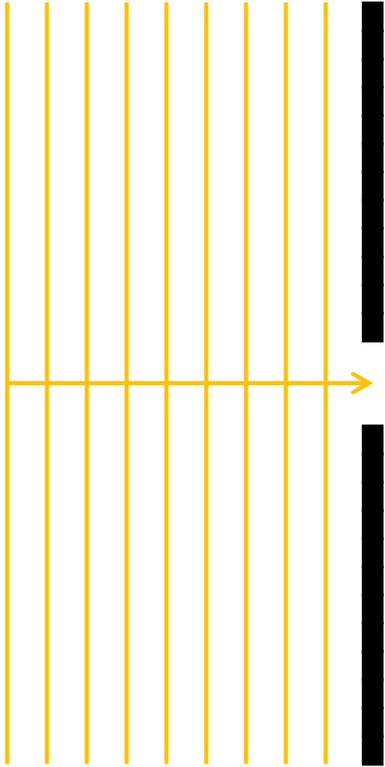
What happens as we change the pinhole diameter?



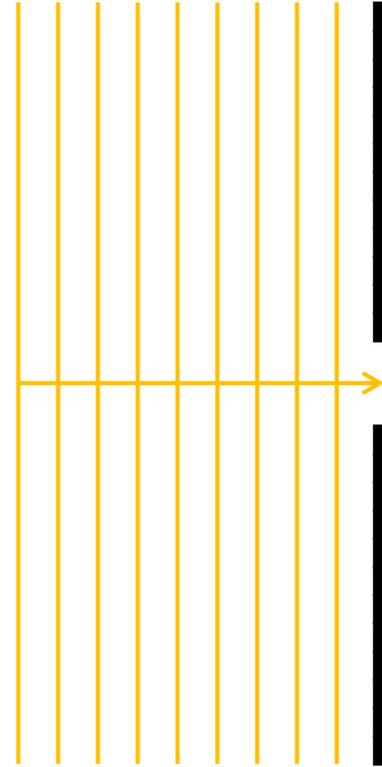
Will the image keep getting sharper the smaller we make the pinhole?

# Diffraction limit

A consequence of the wave nature of light



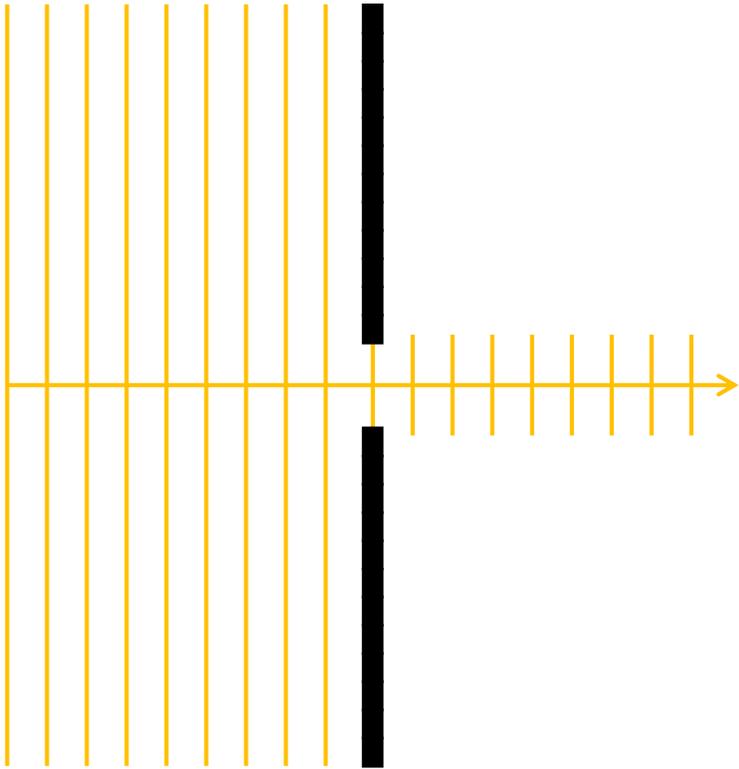
What do geometric optics predict will happen?



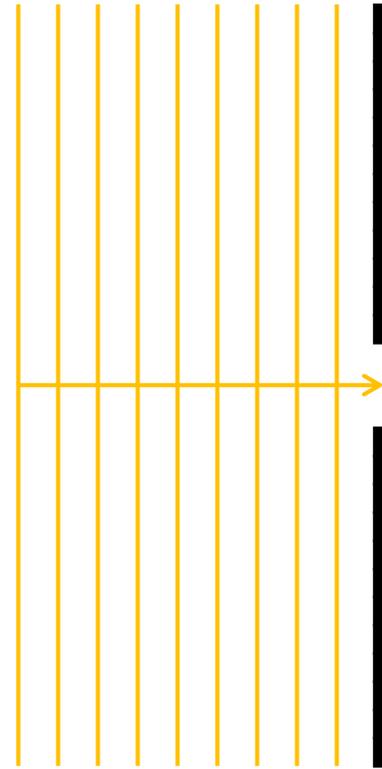
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# Diffraction limit

A consequence of the wave nature of light



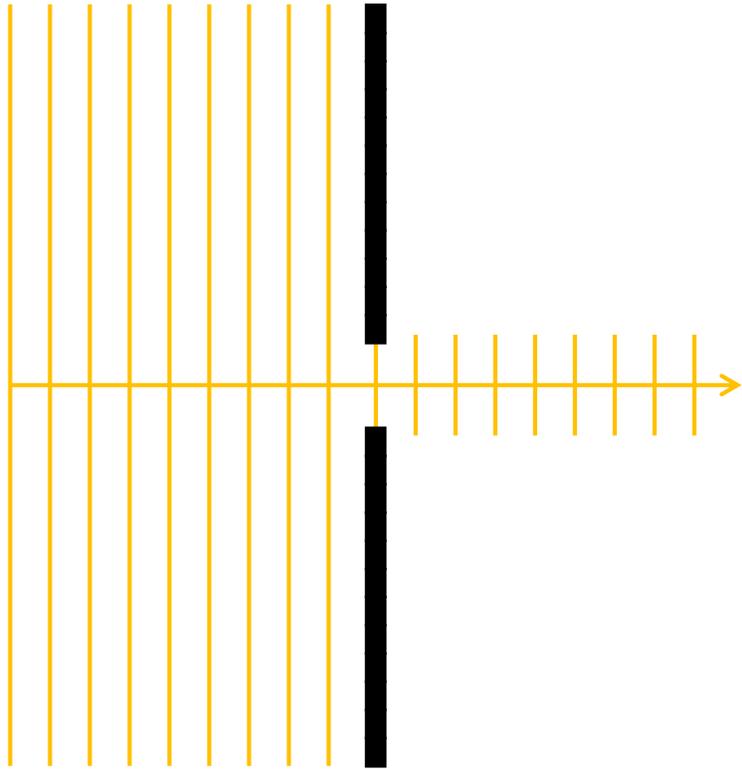
What do geometric optics predict will happen?



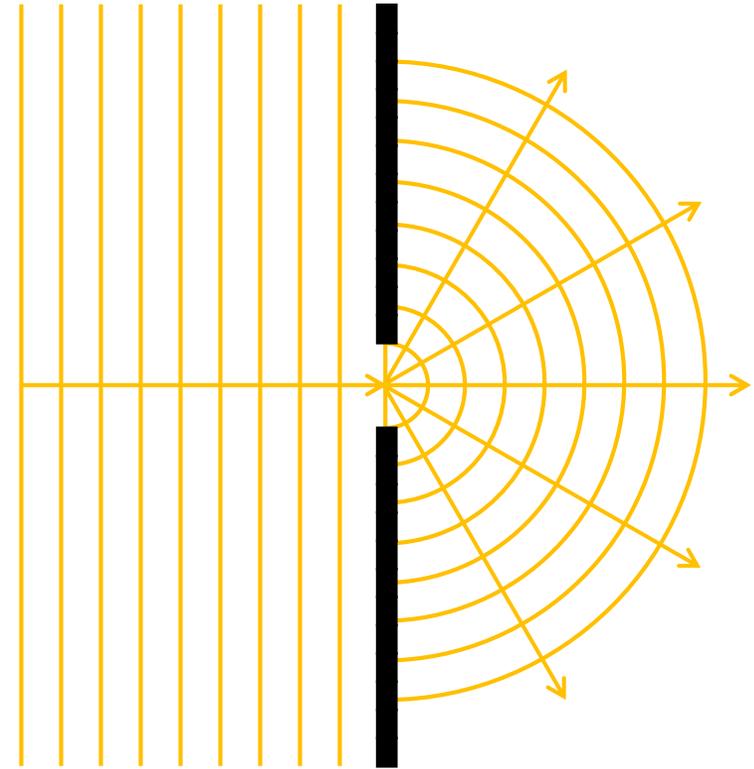
What do wave optics predict will happen?

# Diffraction limit

A consequence of the wave nature of light



What do geometric optics predict will happen?

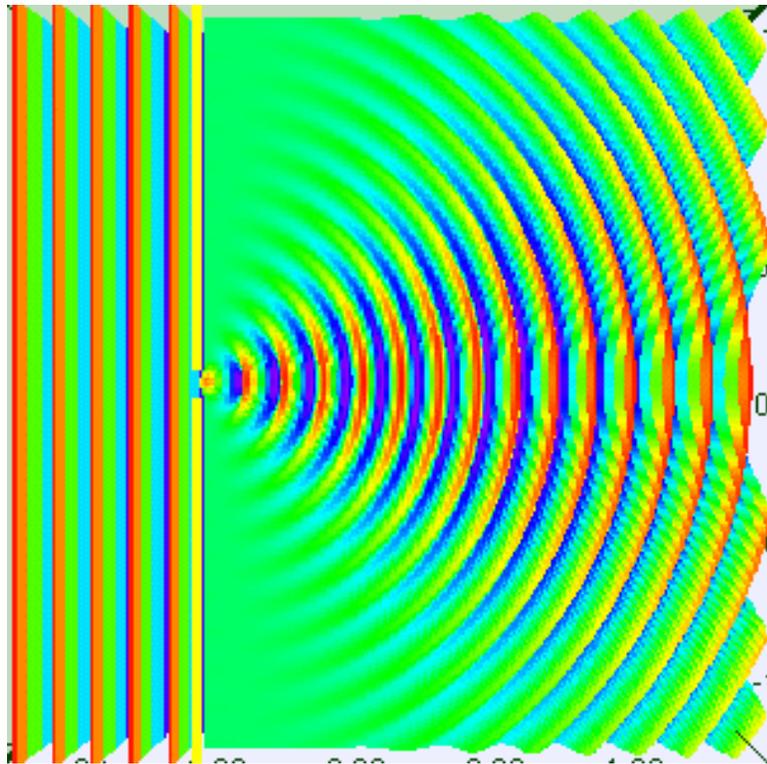


What do wave optics predict will happen?

# Diffraction limit

Diffraction pattern = Fourier transform of the pinhole.

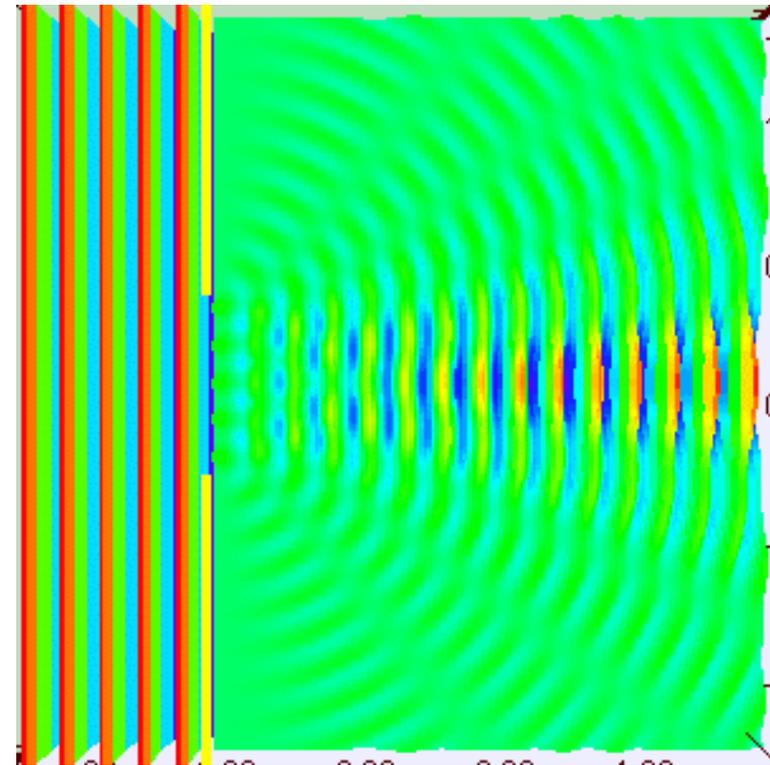
- Smaller pinhole means bigger Fourier spectrum.
- Smaller pinhole means more diffraction.



small pinhole



wide  
diffraction  
pattern



large pinhole



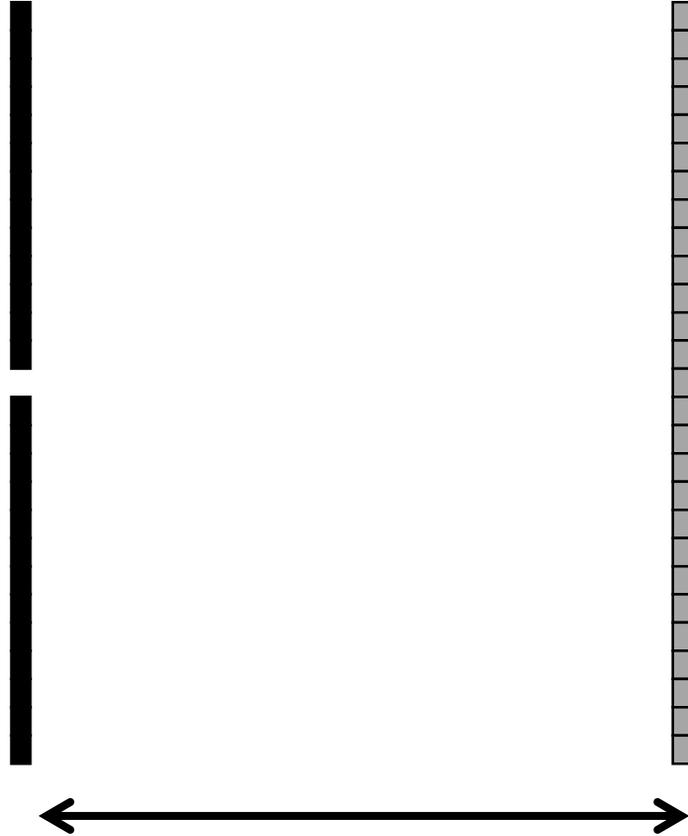
narrow  
diffraction  
pattern

# What about light efficiency?

real-world  
object



pinhole  
diameter



focal length  $f$

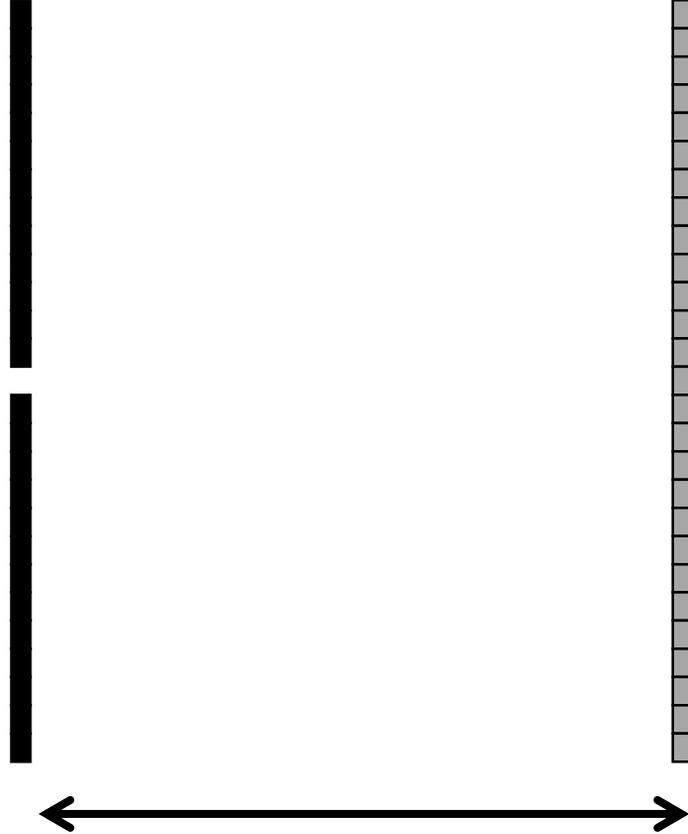
- What is the effect of doubling the pinhole diameter?
- What is the effect of doubling the focal length?

# What about light efficiency?

real-world  
object



pinhole  
diameter

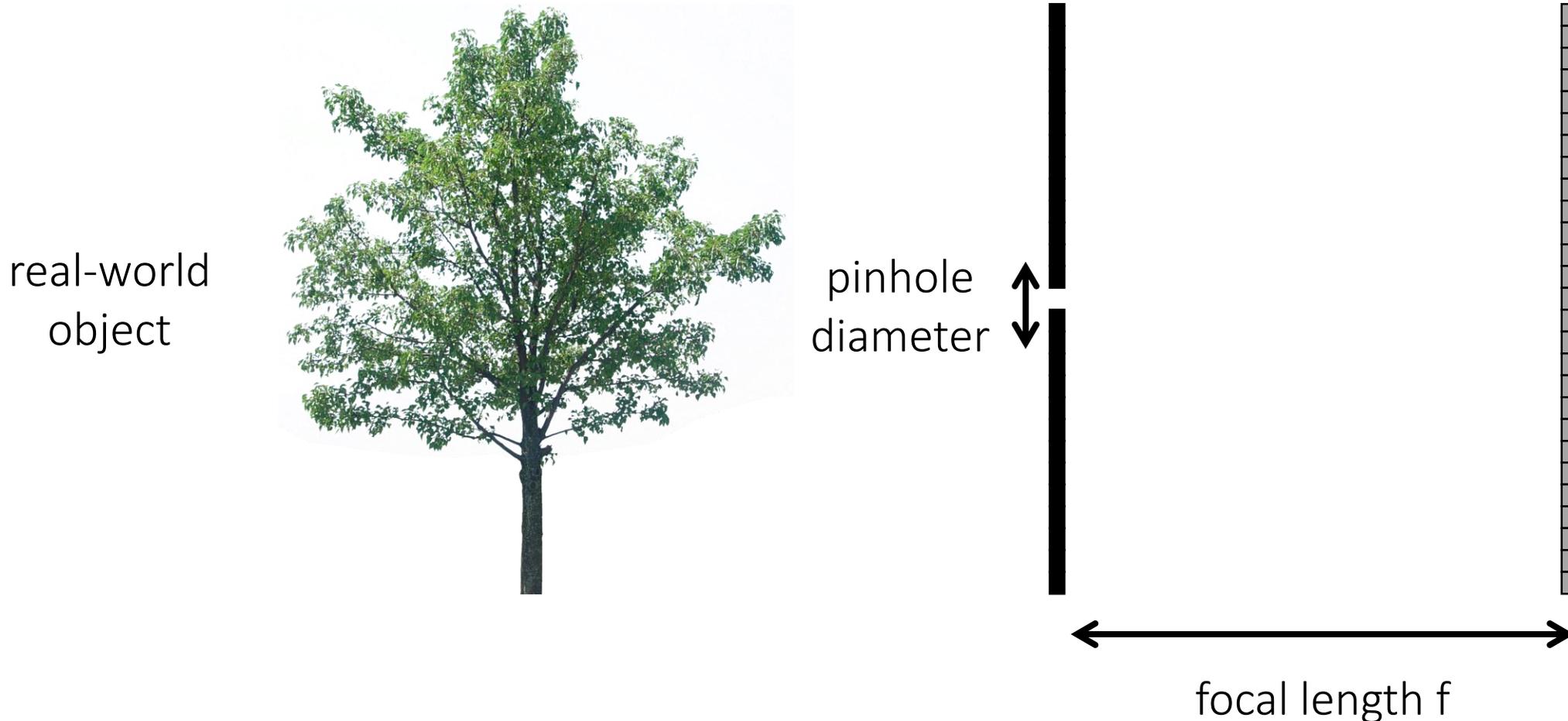


focal length  $f$

- 2x pinhole diameter  $\rightarrow$  4x light
- 2x focal length  $\rightarrow$   $\frac{1}{4}$ x light

# Some terminology notes

A “stop” is a change in camera settings that changes amount of light by a factor of 2



The “f-number” is the ratio: focal length / pinhole diameter

Accidental pinholes





What does this image say about the world outside?



# Accidental pinhole camera



Antonio Torralba, William T. Freeman  
Computer Science and Artificial Intelligence Laboratory (CSAIL)  
MIT  
torralba@mit.edu, billf@mit.edu

# Accidental pinhole camera

projected pattern on the wall



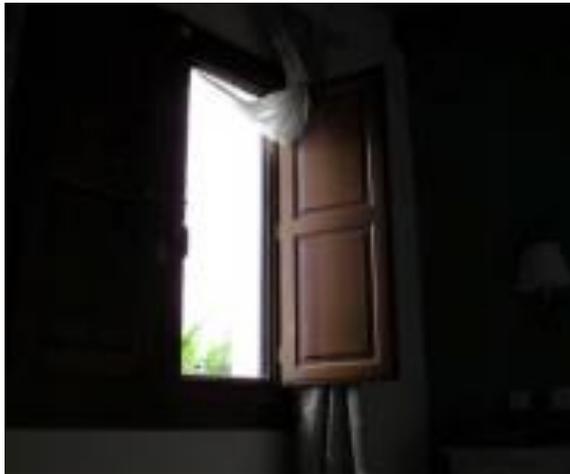
upside down



window with smaller gap

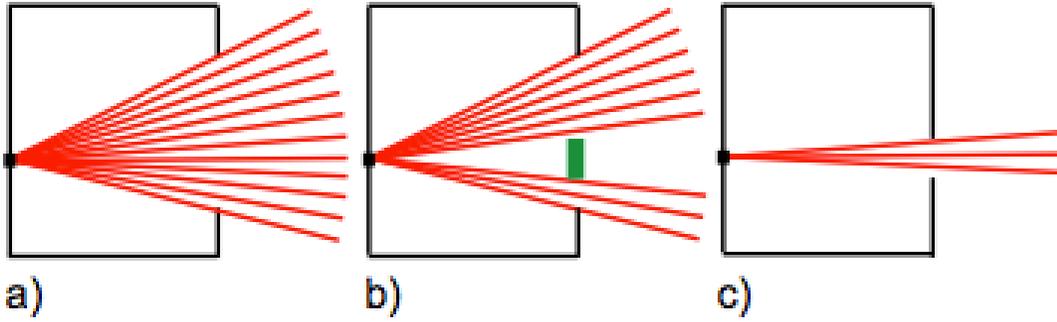


view outside window



window is an aperture

# Accidental pinspeck camera



a)



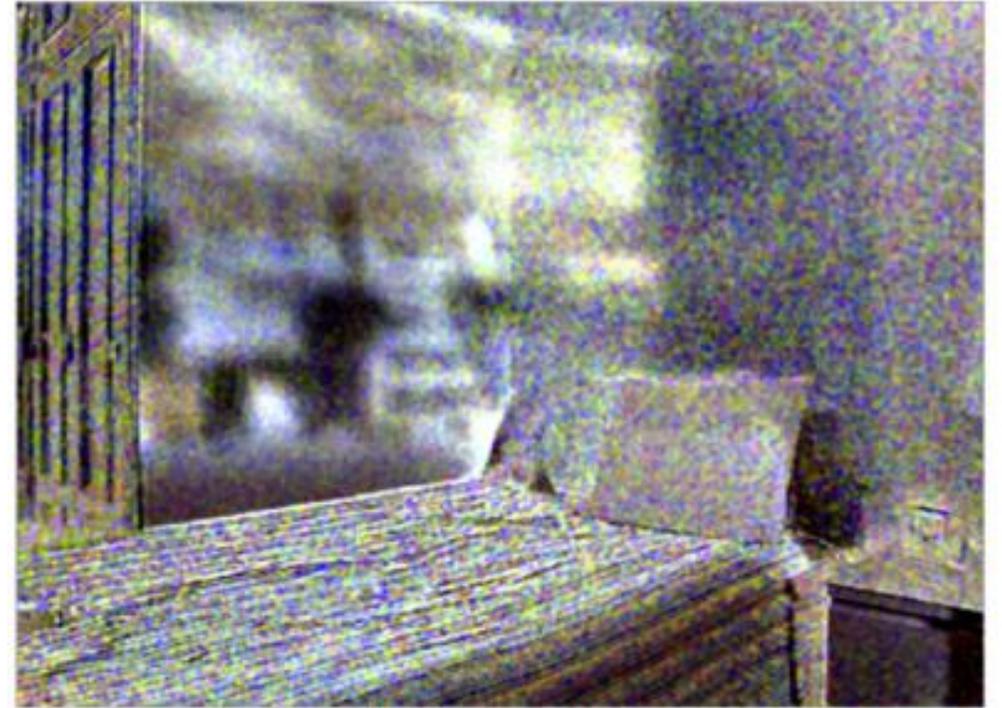
b)



c)



d)



a) Difference image

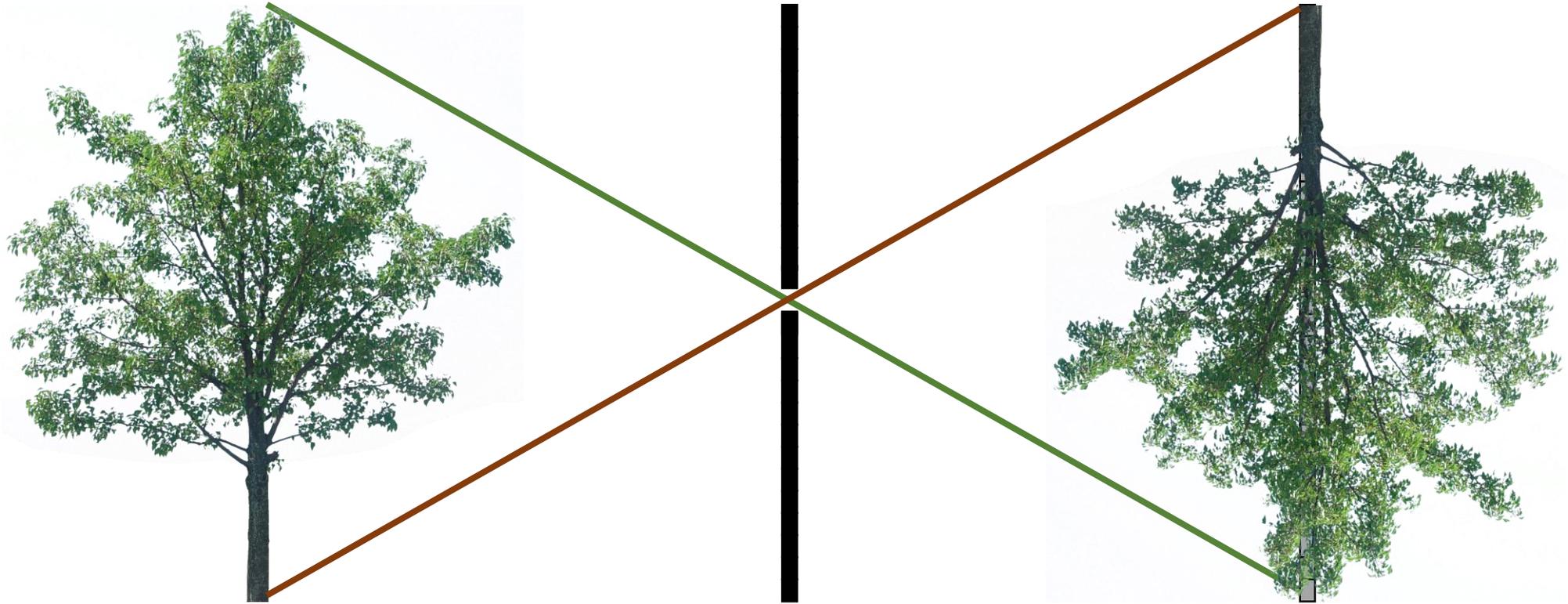


b) Difference upside down



c) True outdoor view

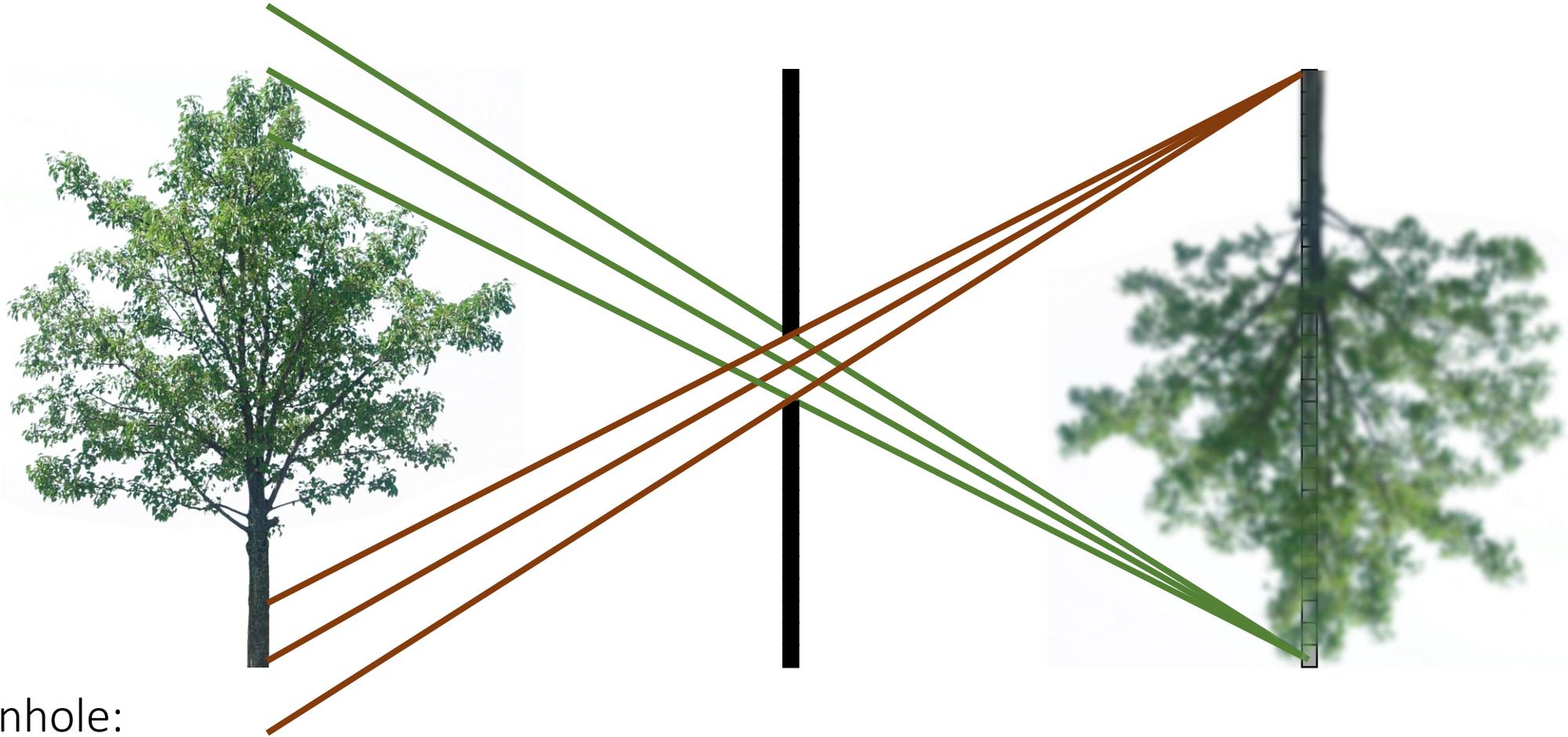
# Pinhole camera trade-off



Small (ideal) pinhole:

1. Image is sharp.
2. Signal-to-noise ratio is low.

# Pinhole camera trade-off

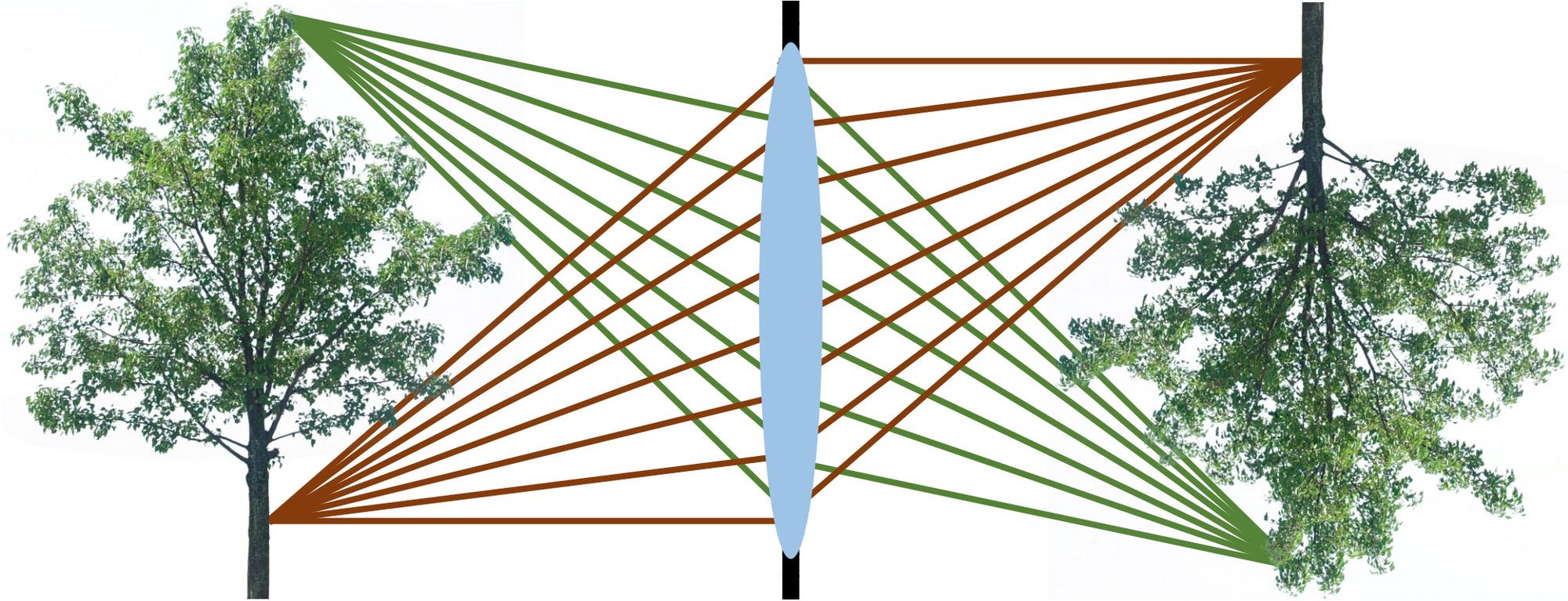


Large pinhole:

1. Image is blurry.
2. Signal-to-noise ratio is high.

Can we get best of both worlds?

# Almost, by using lenses



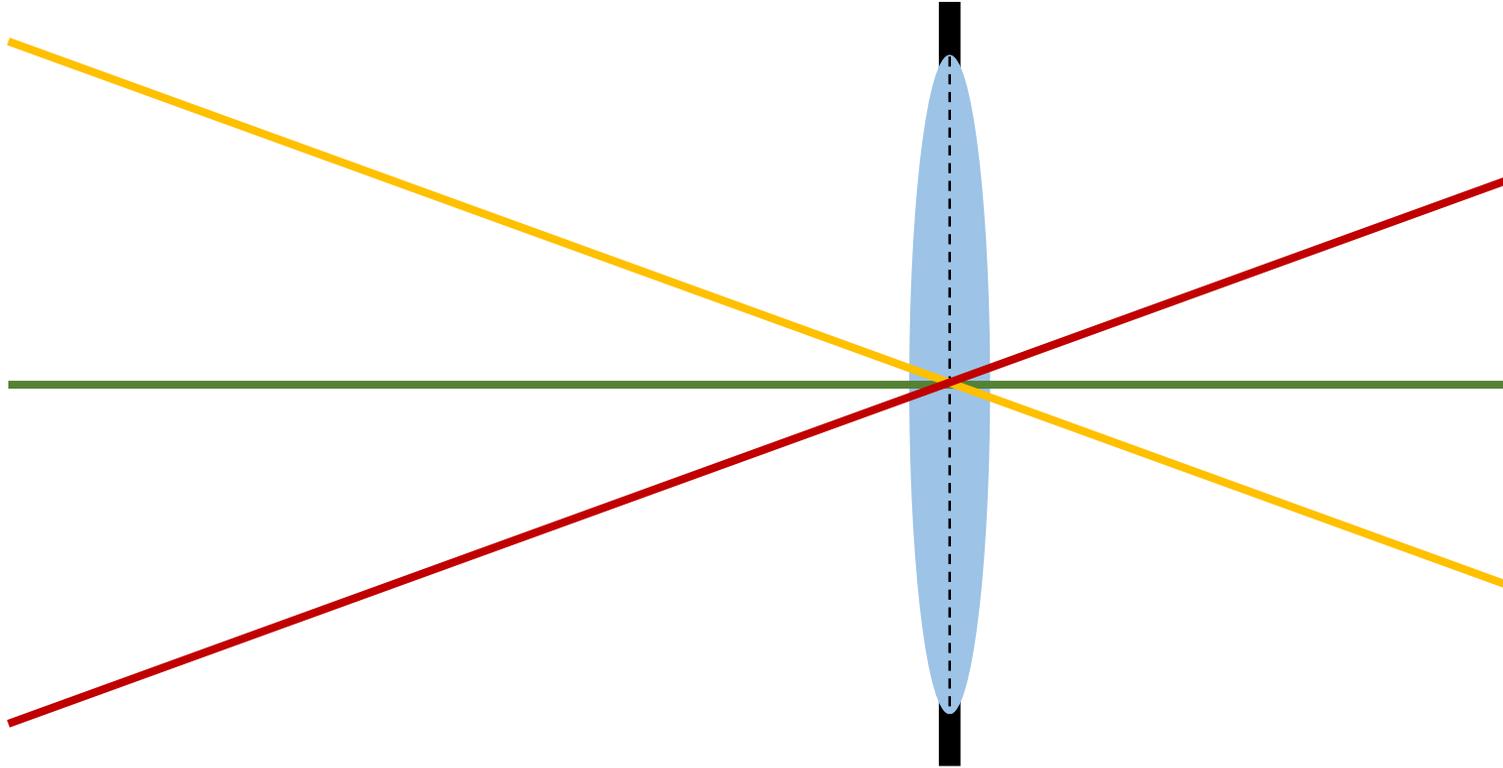
Lenses map “bundles” of rays from points on the scene to the sensor.

How does this mapping work exactly?

# The thin lens model

# Thin lens model

Simplification of geometric optics for well-designed lenses.

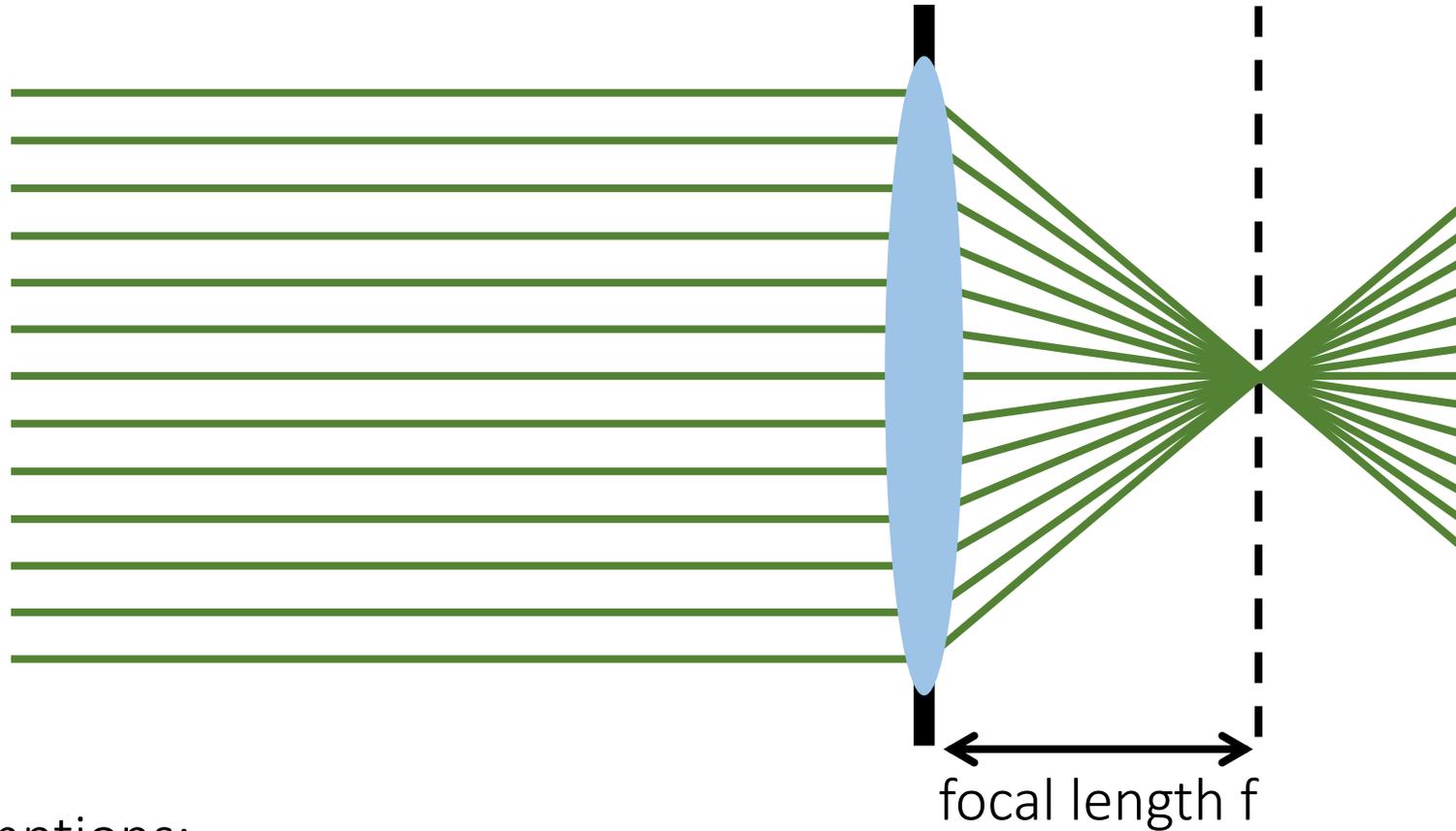


Two assumptions:

1. Rays passing through lens center are unaffected.

# Thin lens model

Simplification of geometric optics for well-designed lenses.

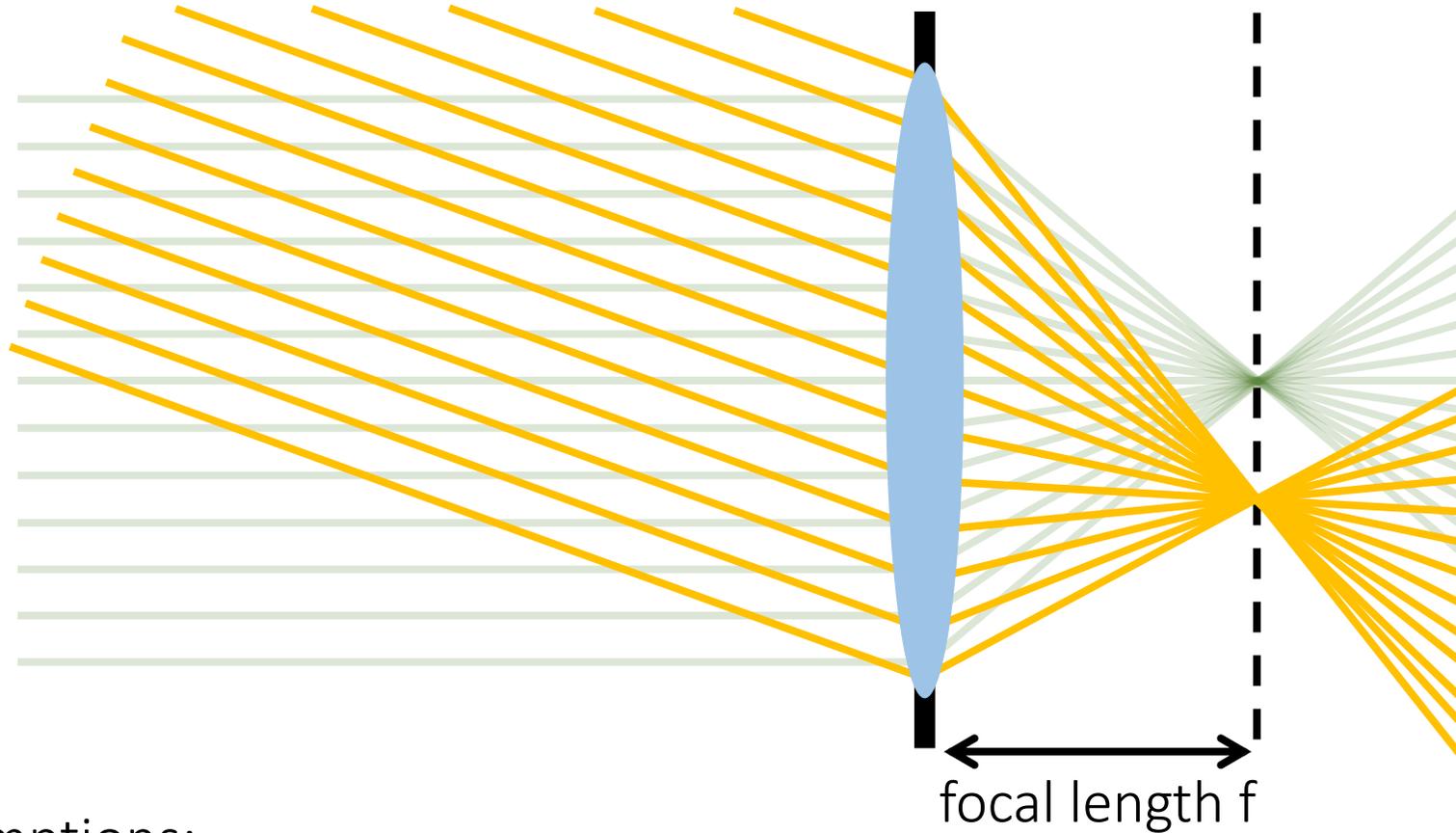


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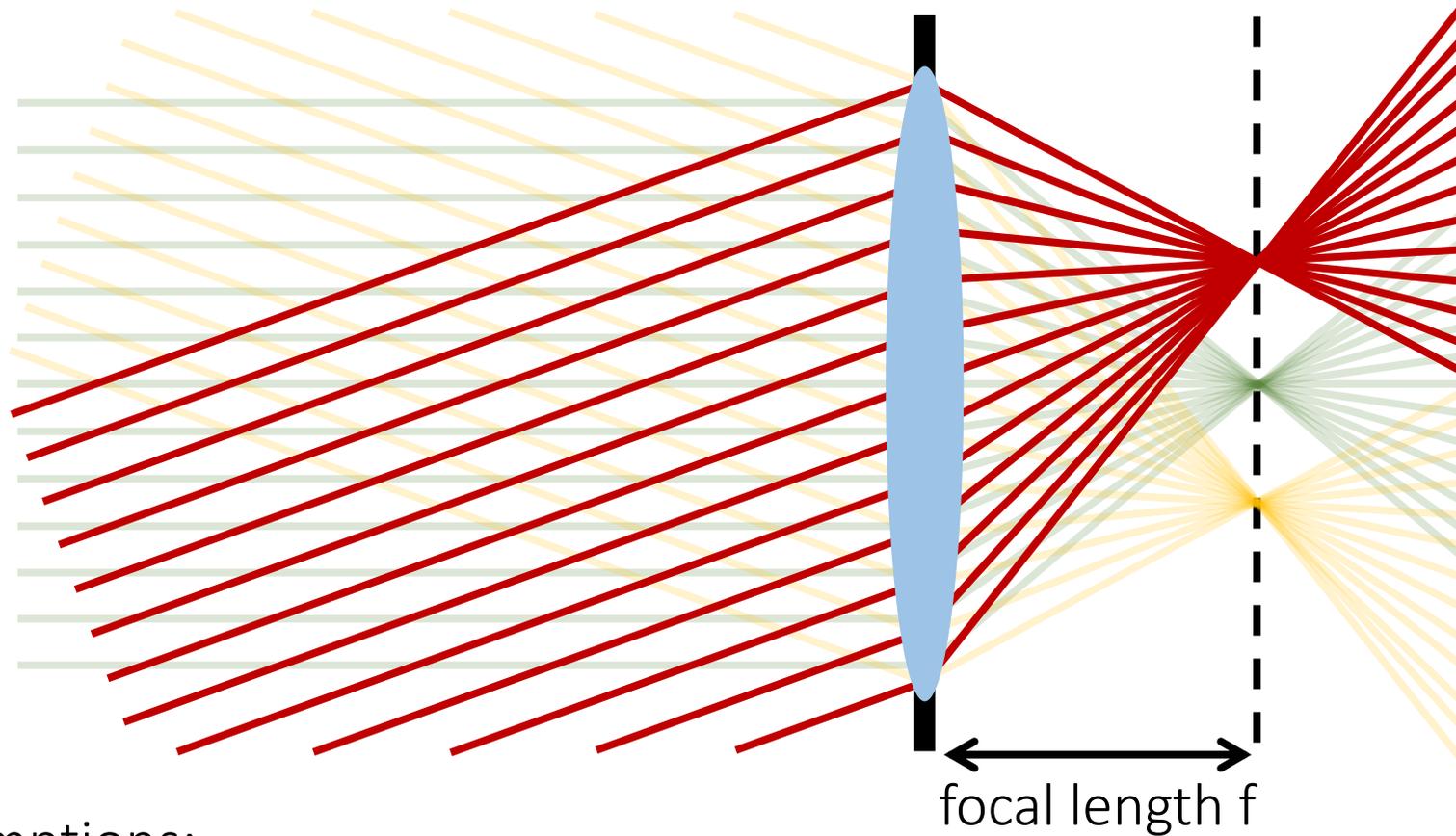


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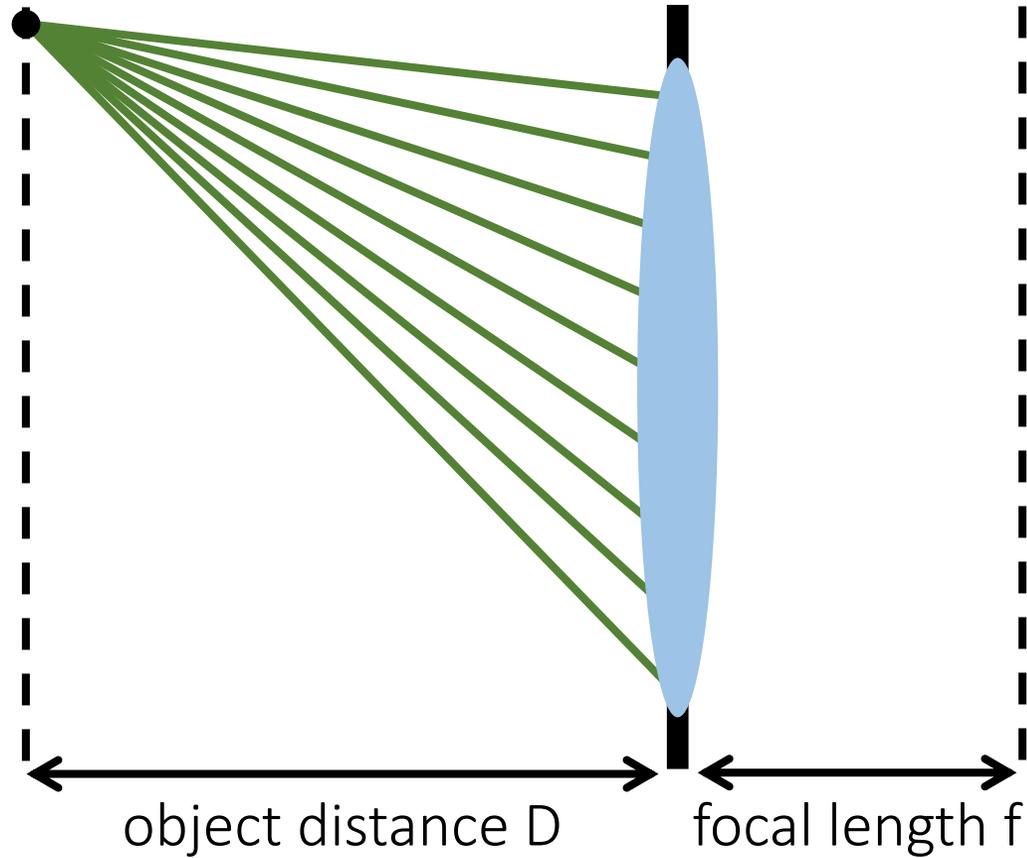


Two assumptions:

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# Tracing rays through a thin lens

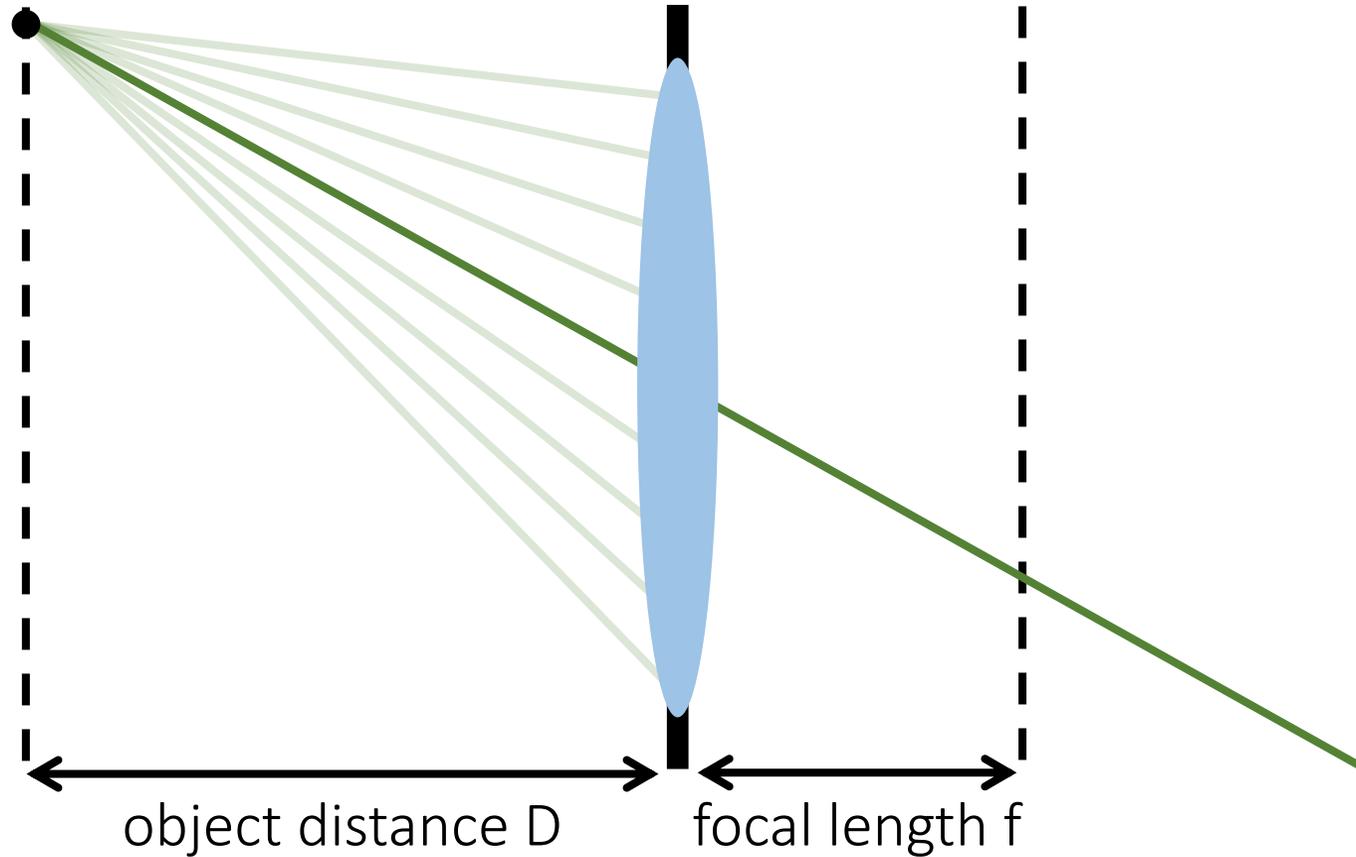
Consider an object emitting a bundle of rays. How do they propagate through the lens?



# Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

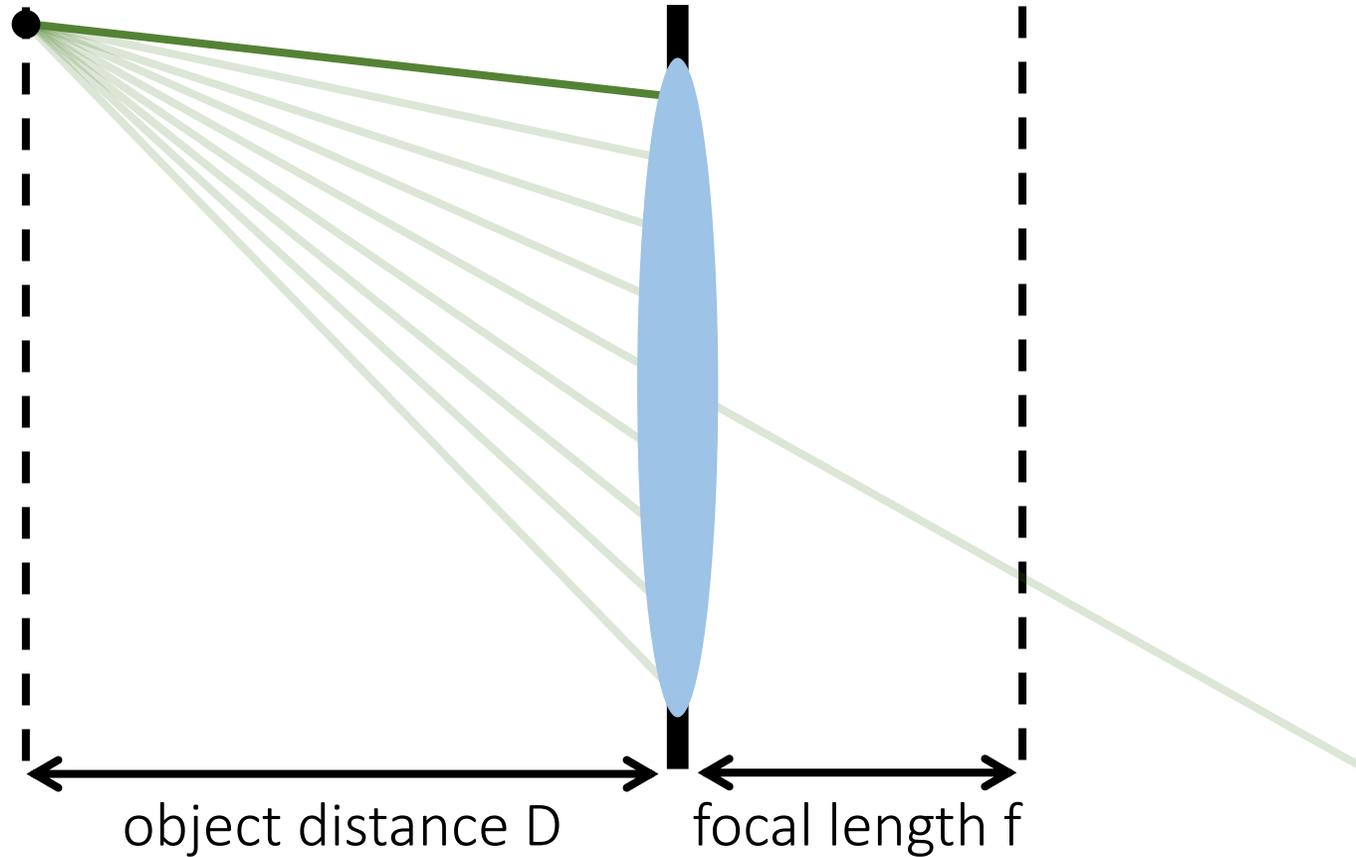
1. Trace rays through lens center.



# Tracing rays through a thin lens

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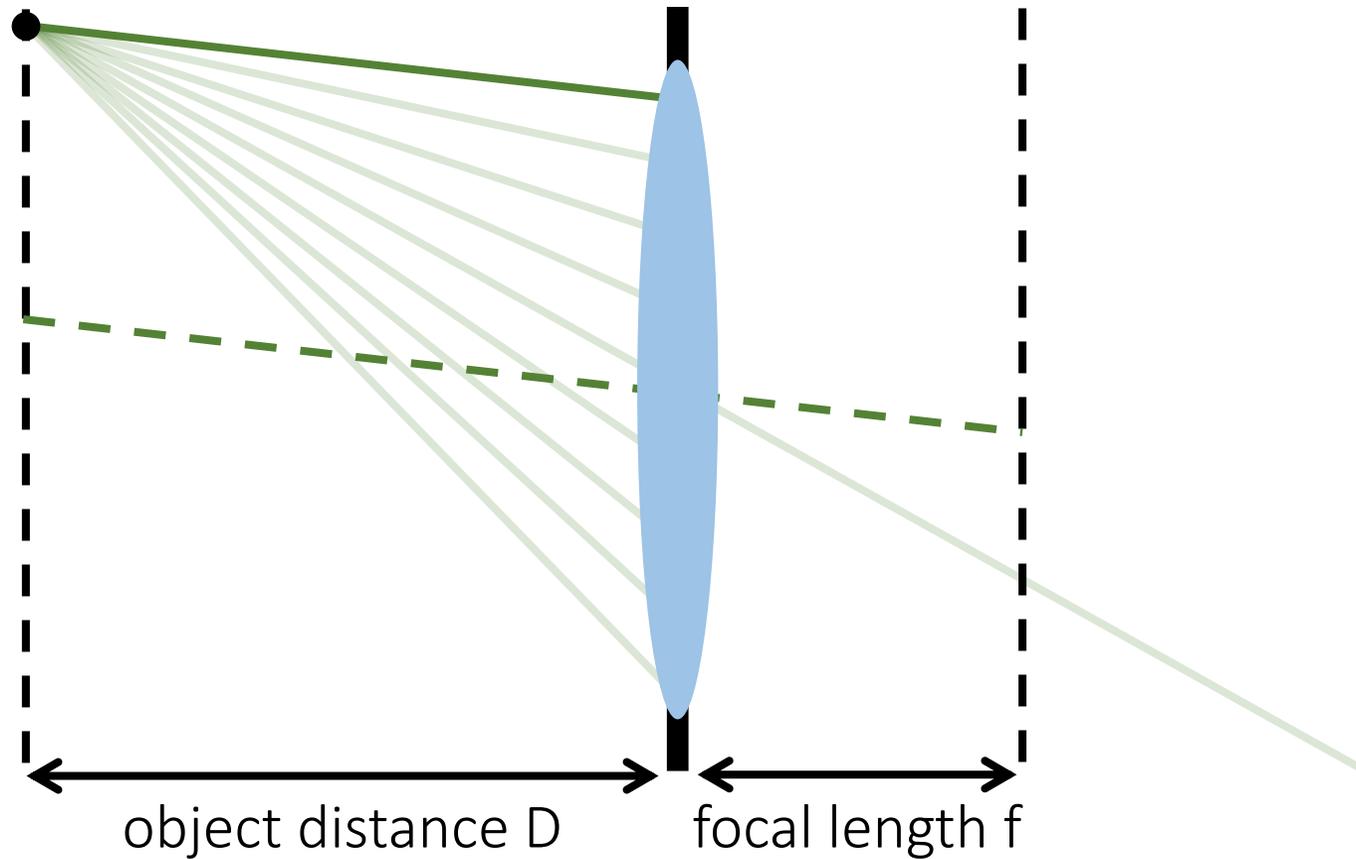
1. Trace rays through lens center.
2. For all other rays:



# Tracing rays through a thin lens

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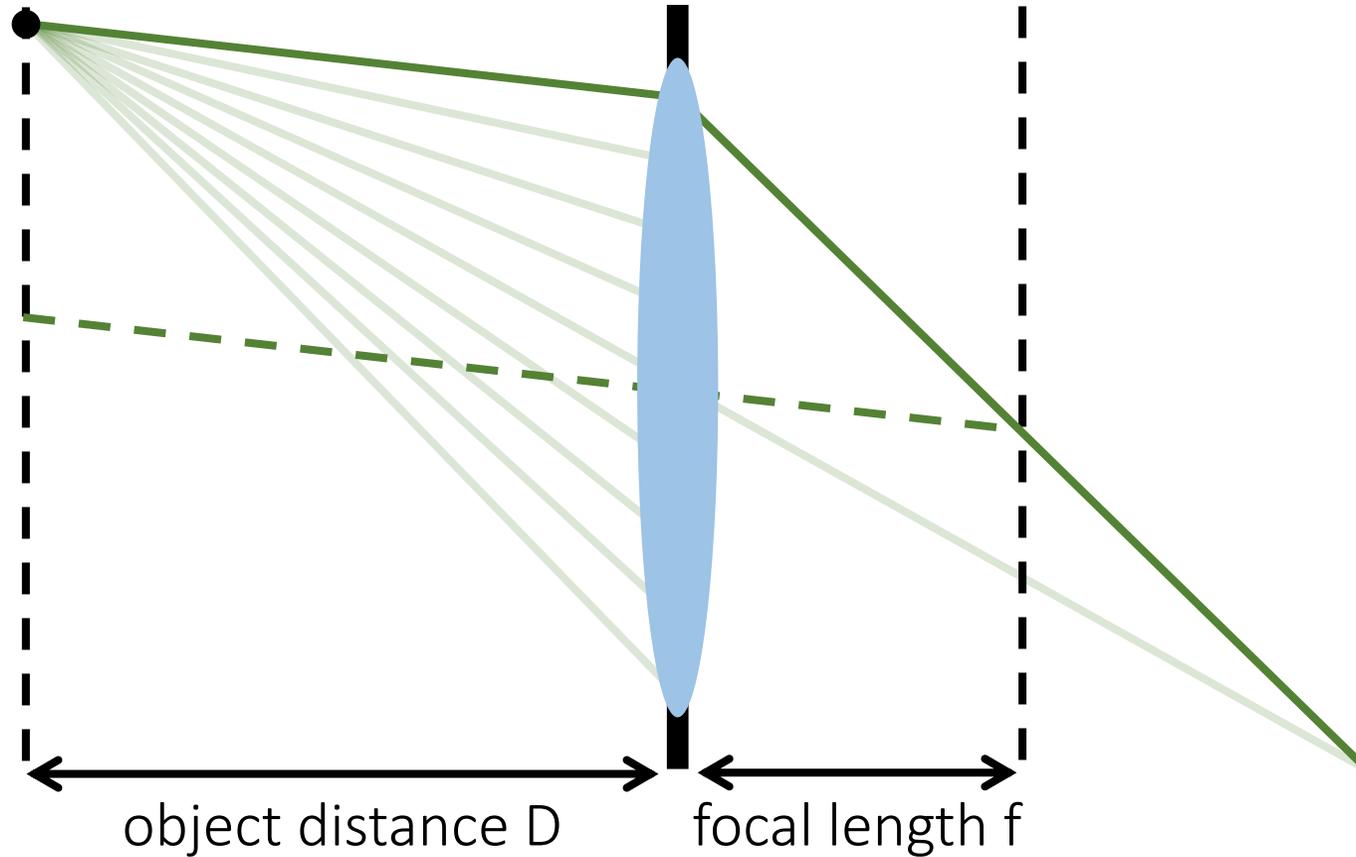
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2. For all other rays:
  - a. Trace their parallel through lens center.



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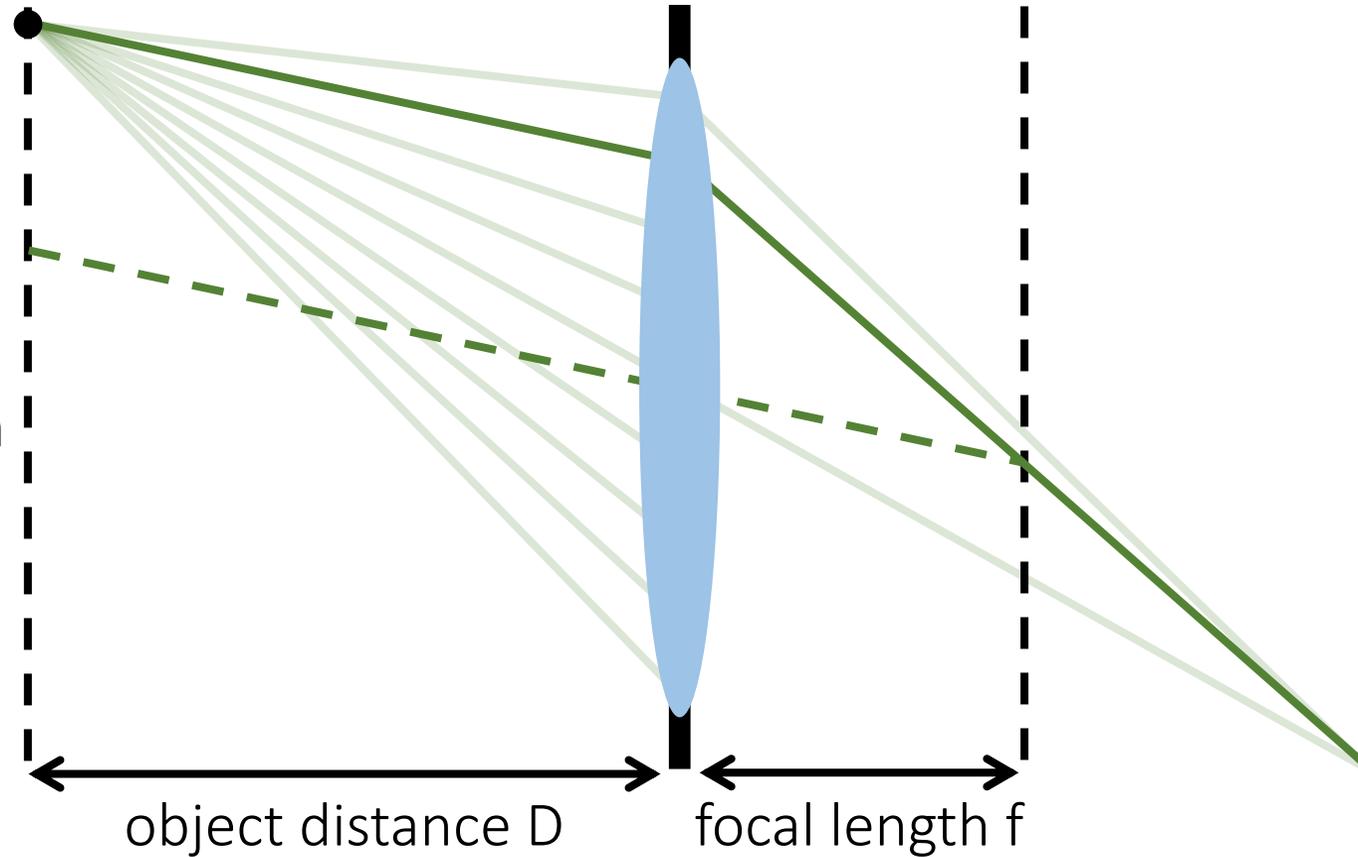
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2. For all other rays:
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  - b. Connect on focal plane.



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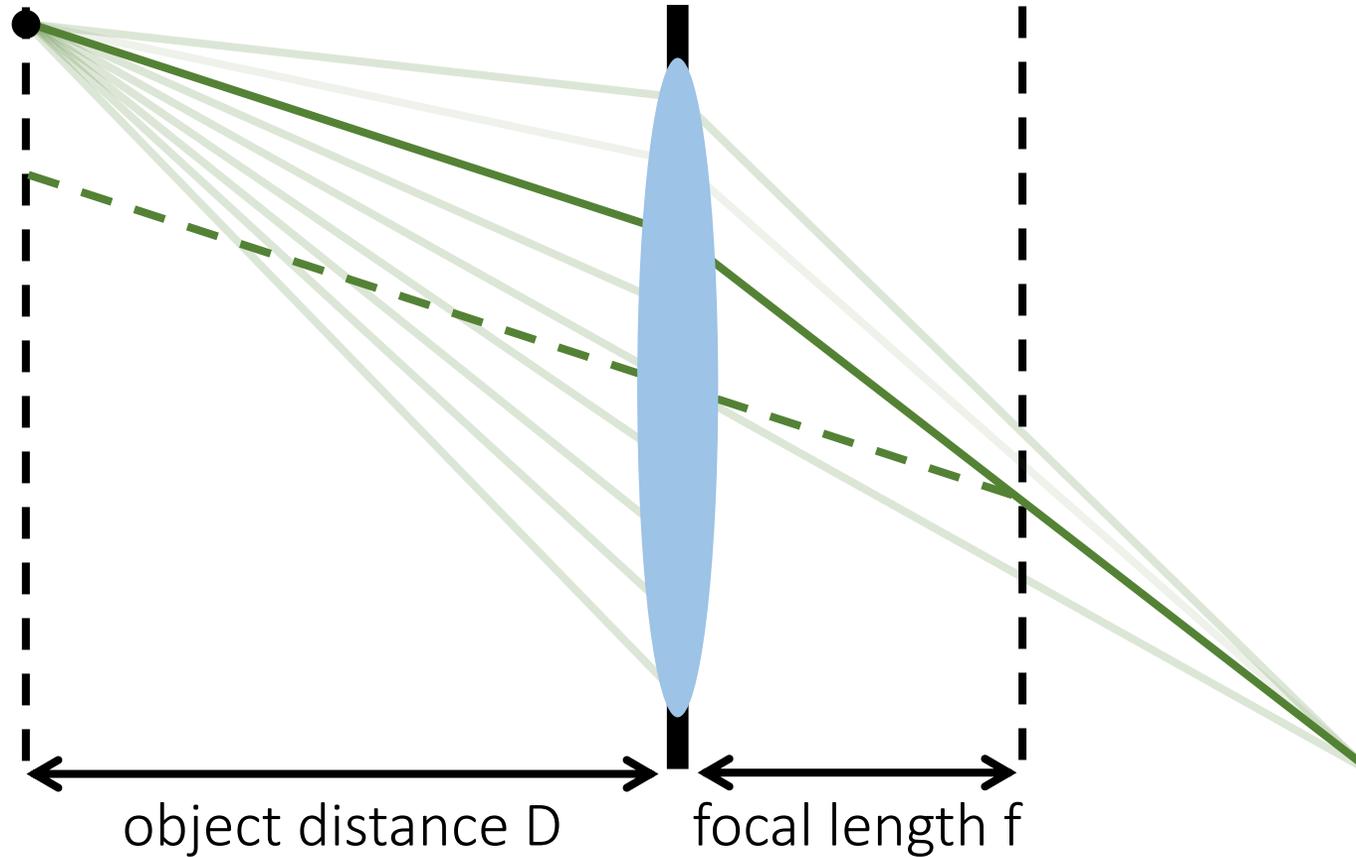
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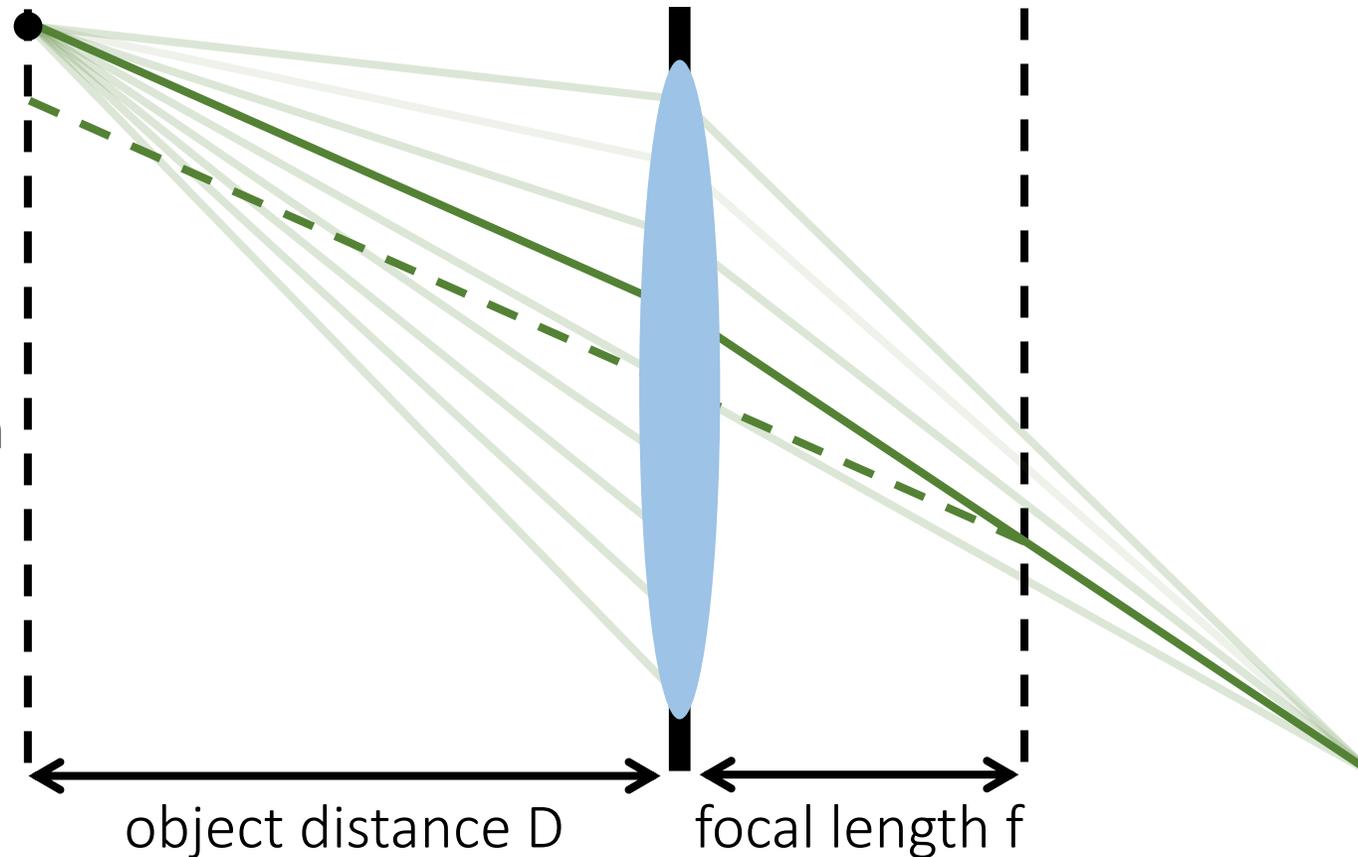
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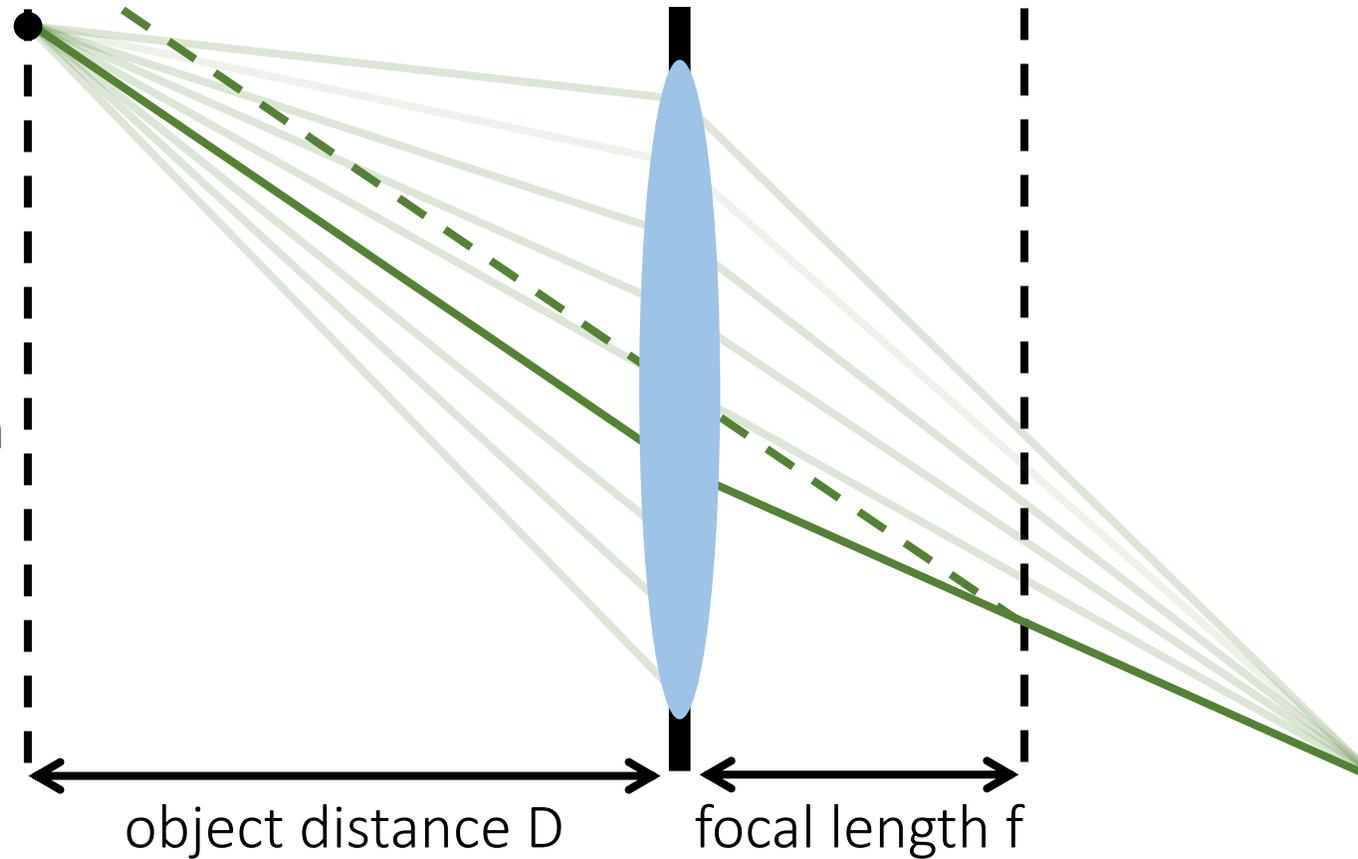
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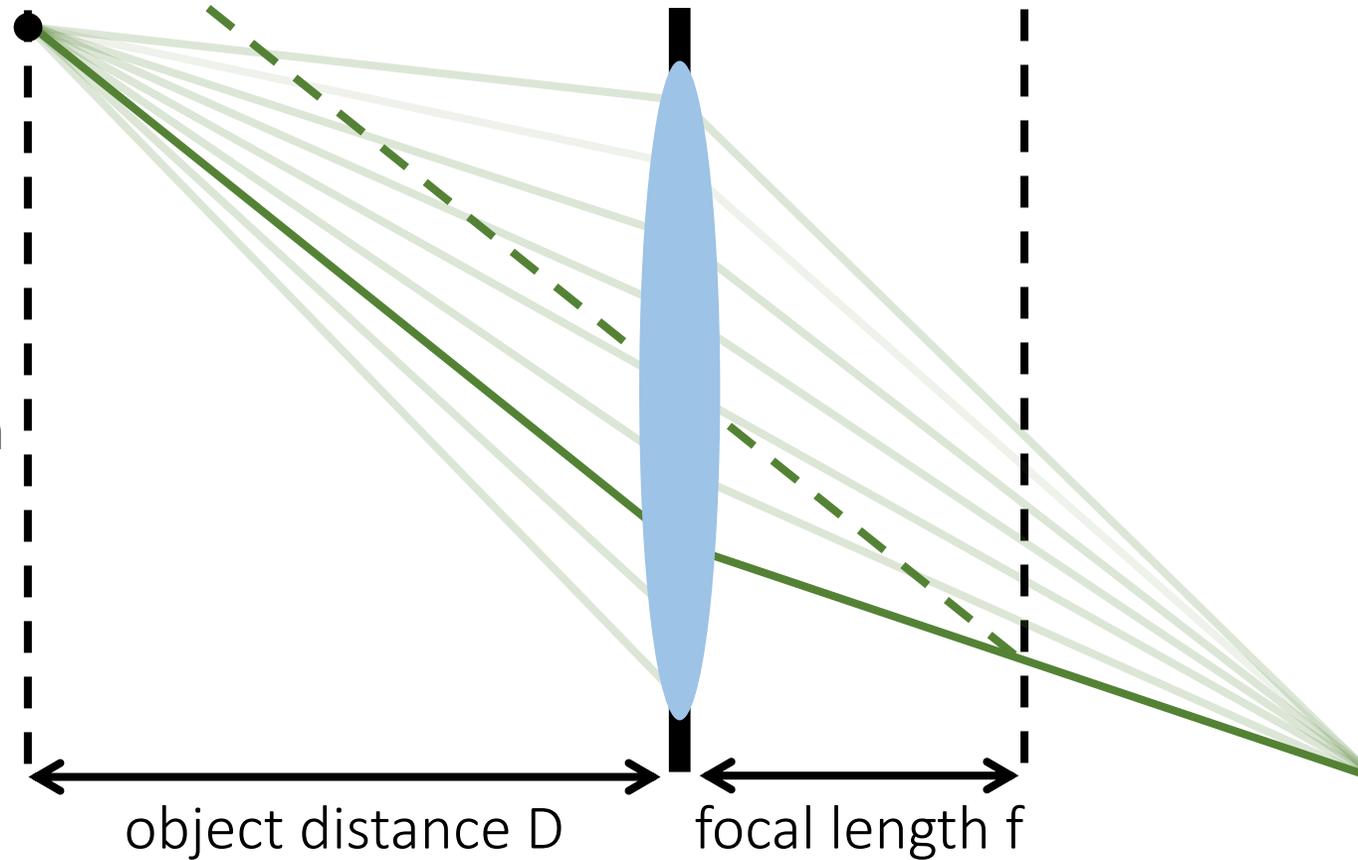
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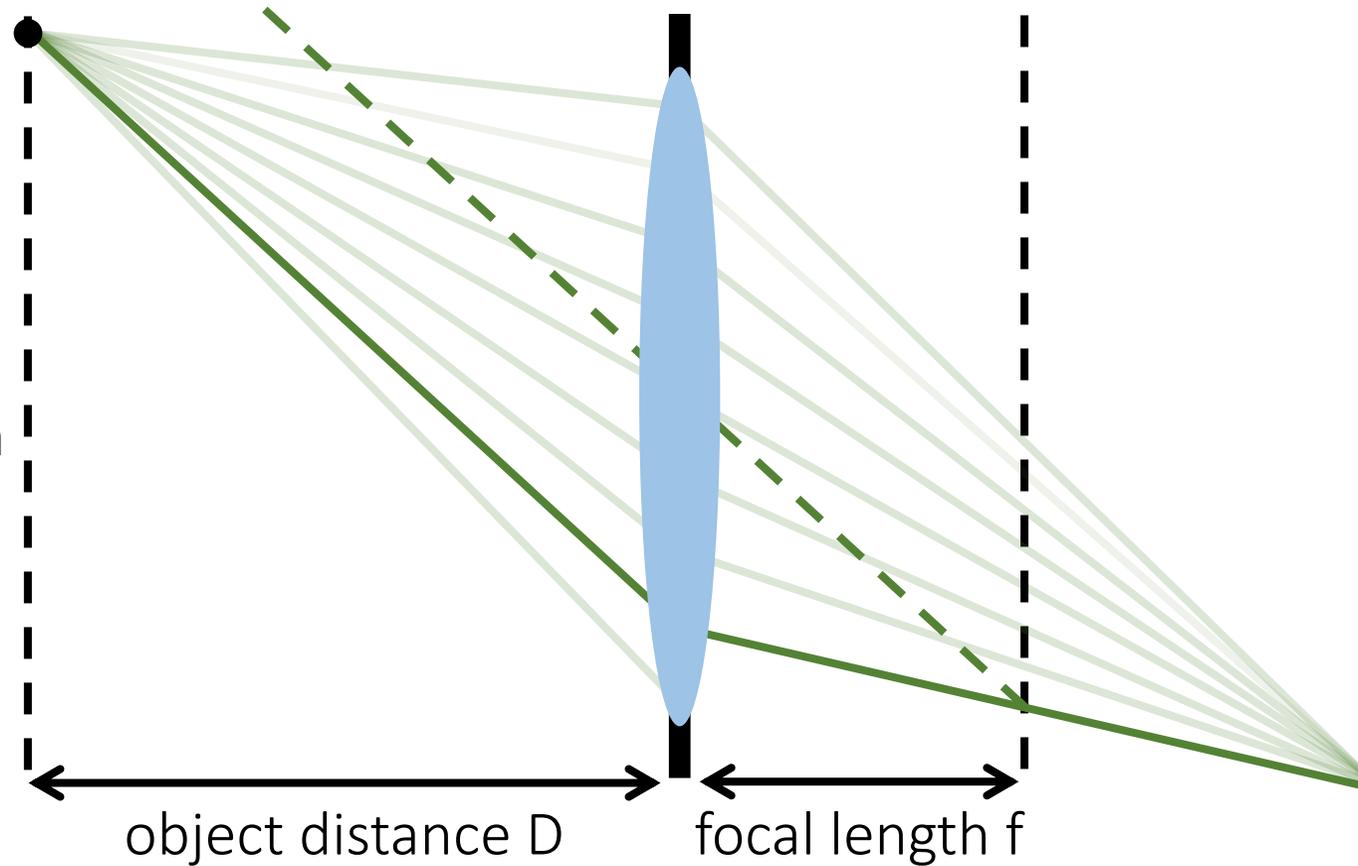
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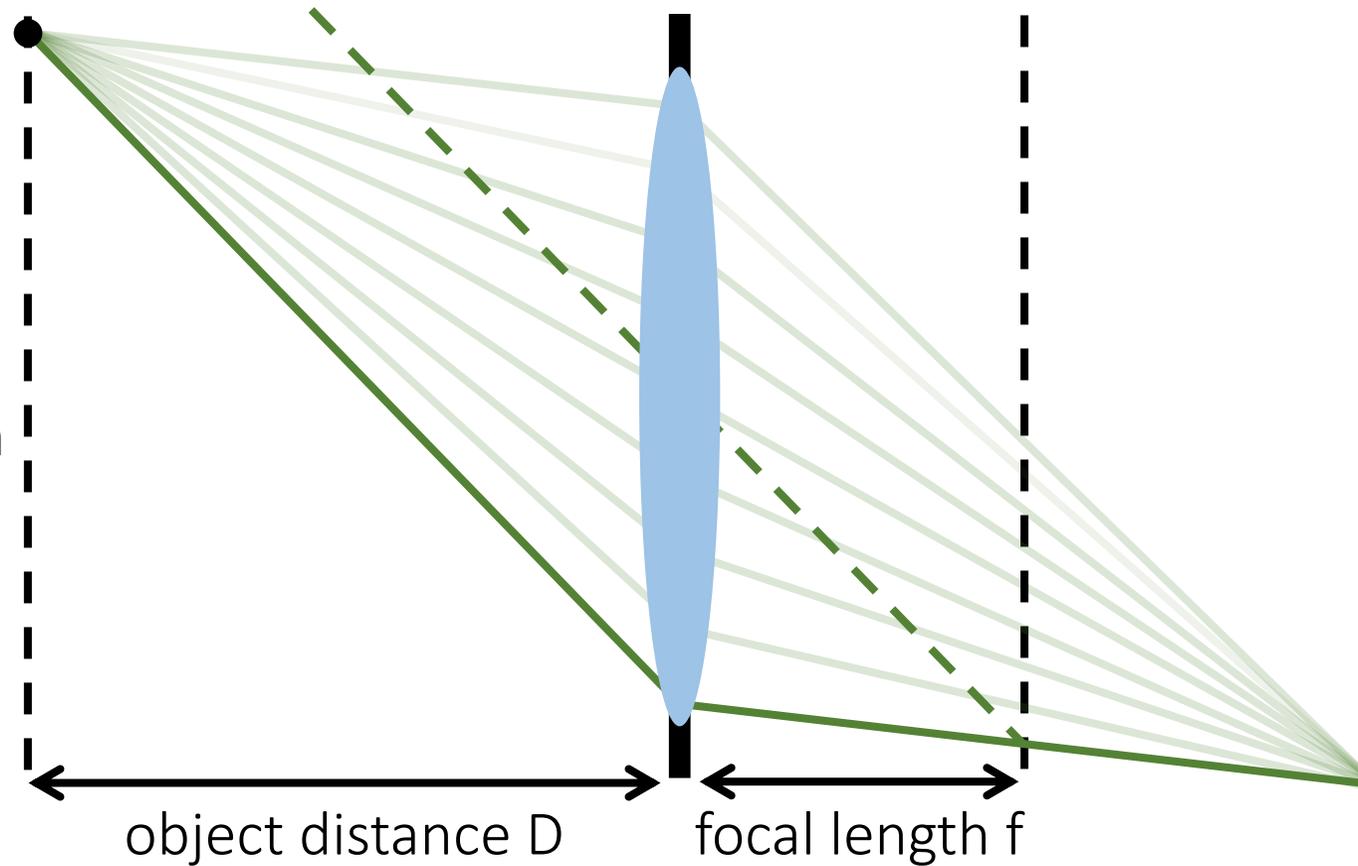
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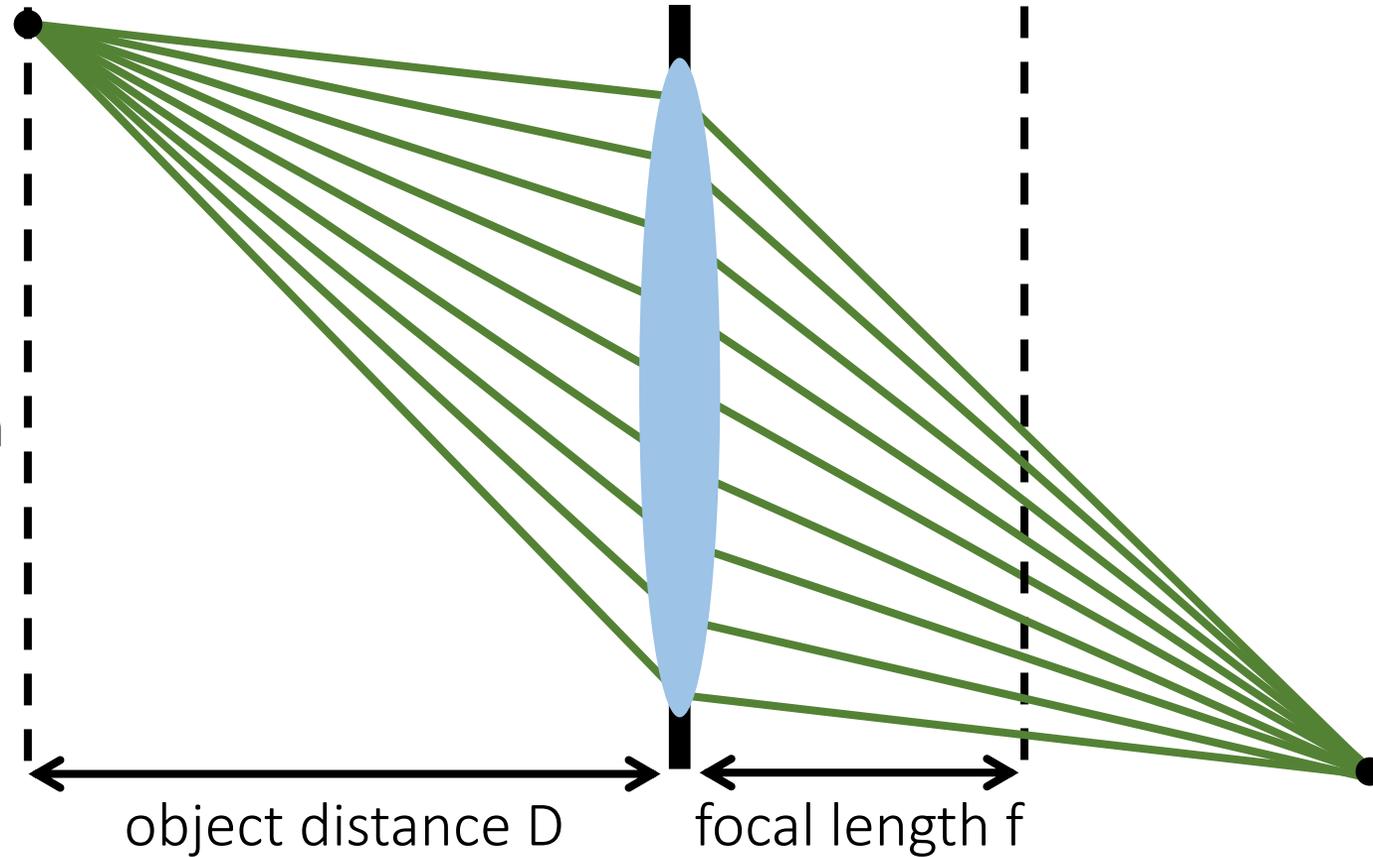
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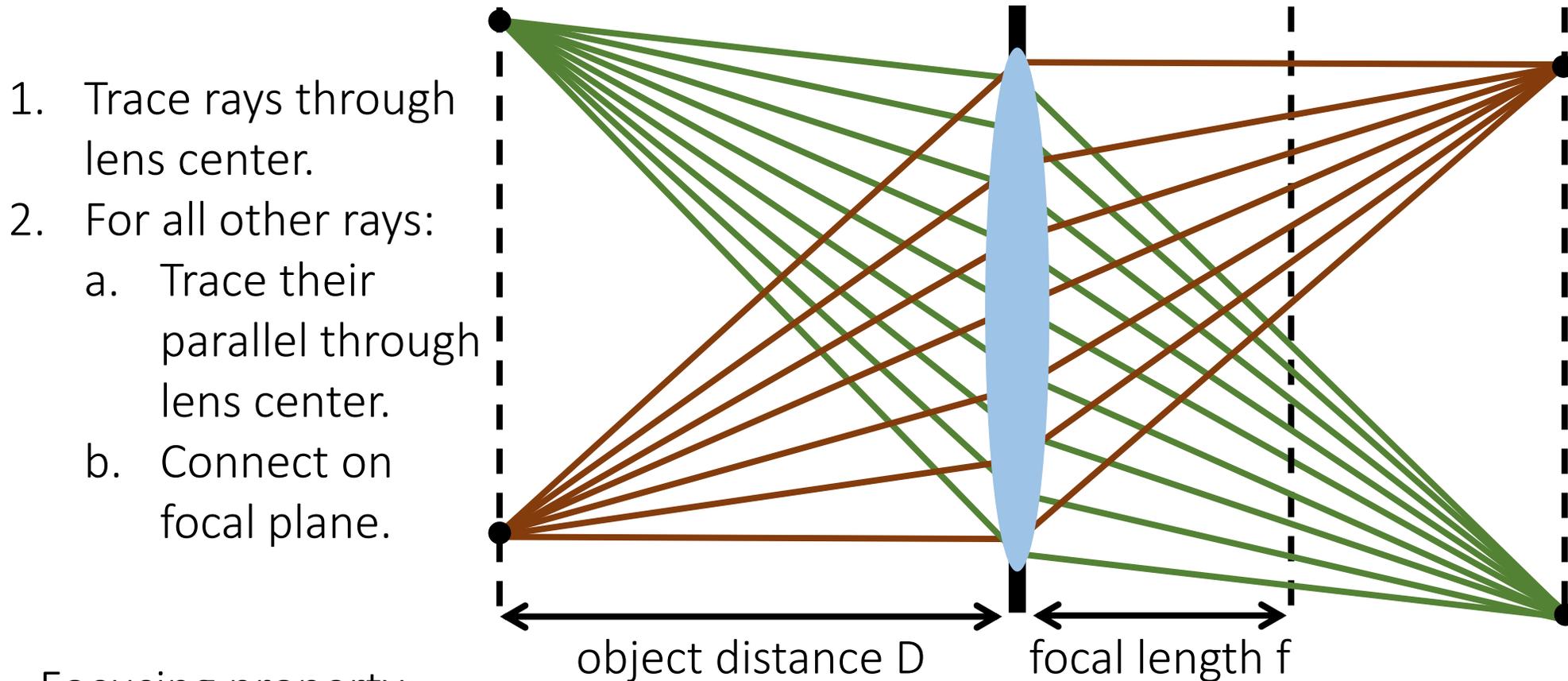


Focusing property:

1. Rays emitted from a point on one side converge to a point on the other side.

# Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?



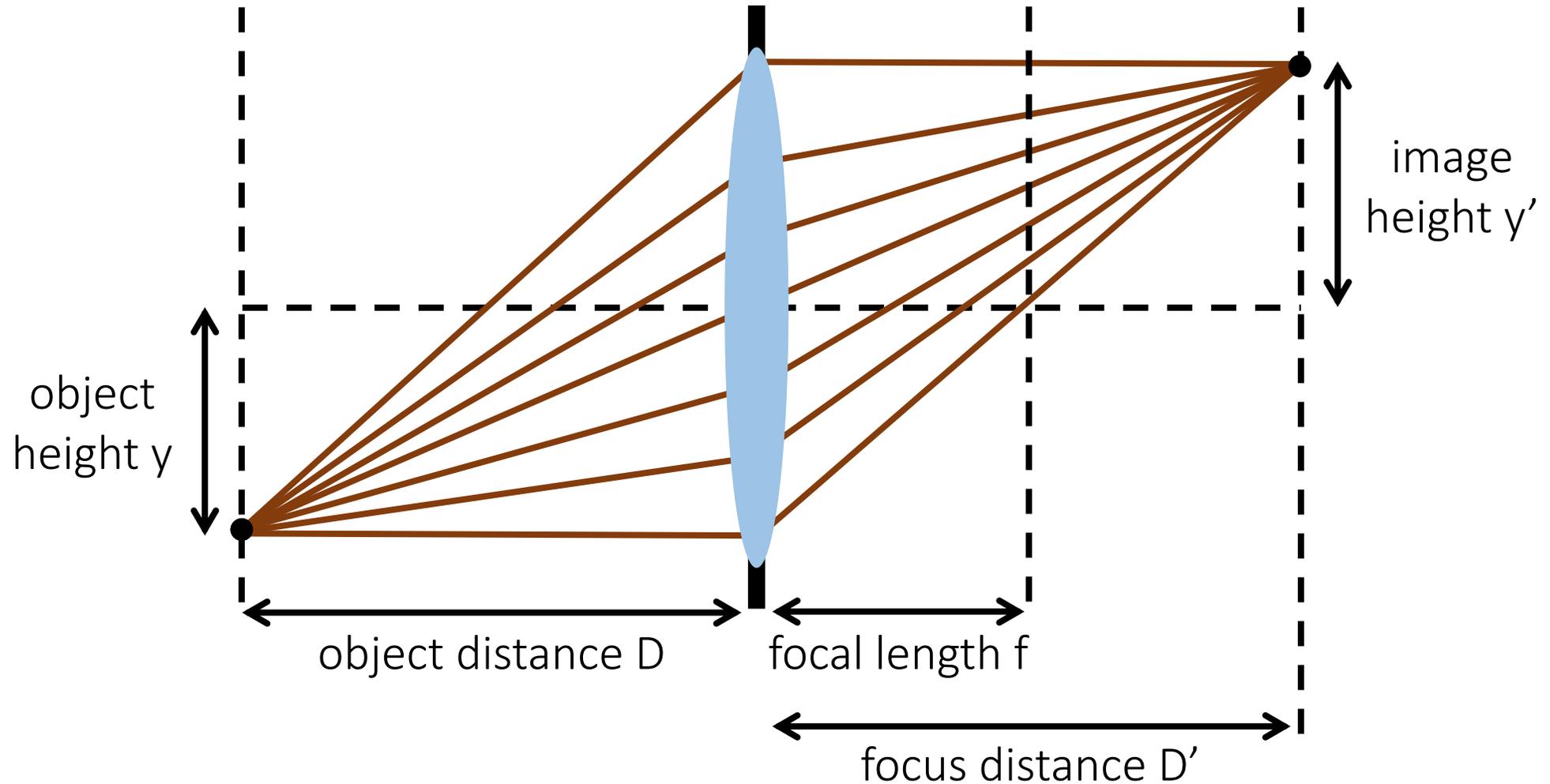
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Focusing property:

1. Rays emitted from a point on one side converge to a point on the other side.
2. Bundles emitted from a plane parallel to the lens converge on a common plane.

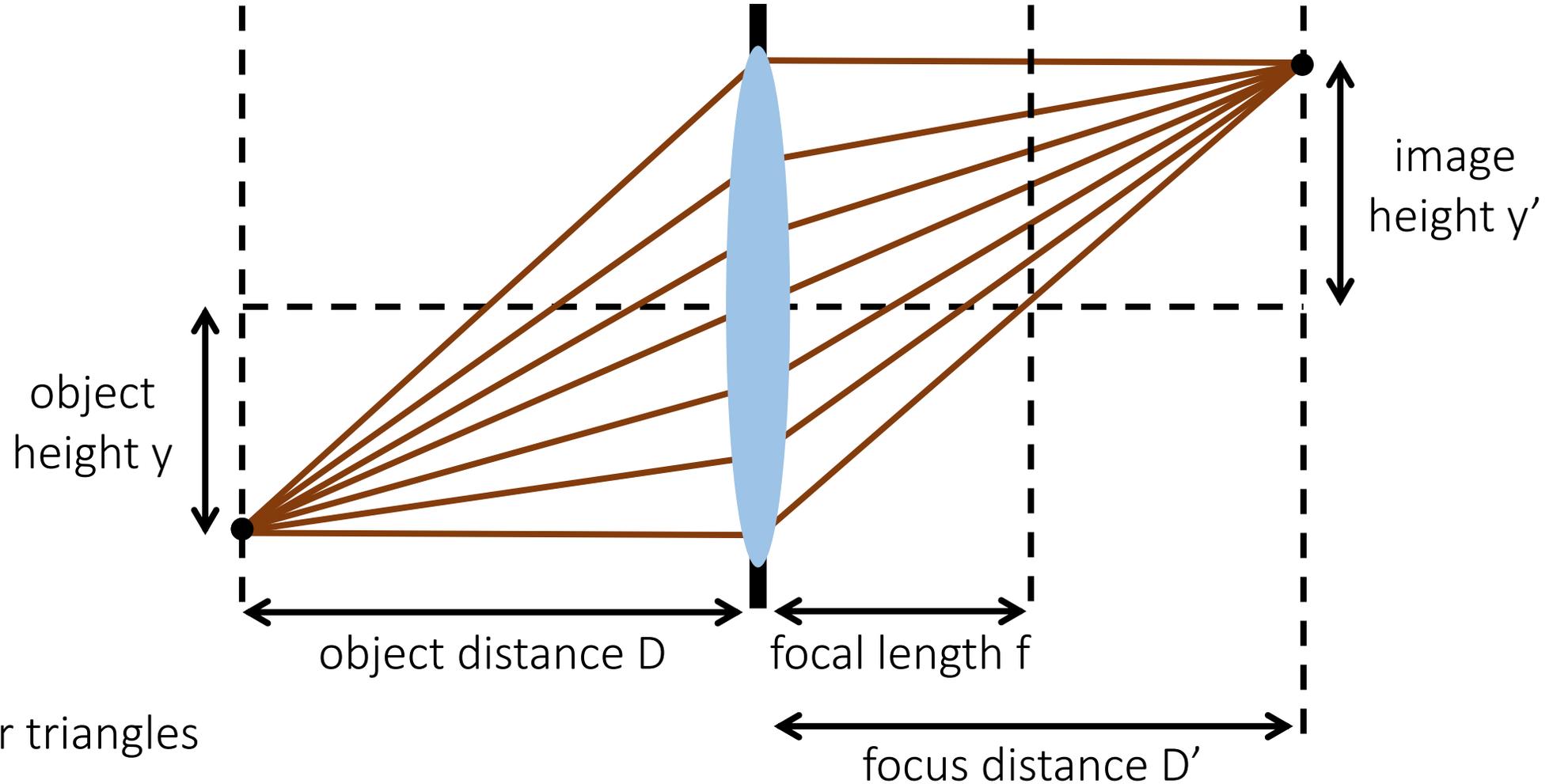
# Thin lens formula

How can we relate scene-space ( $D, y$ ) and image space ( $D', y'$ ) quantities?



# Thin lens formula

How can we relate scene-space ( $D, y$ ) and image space ( $D', y'$ ) quantities?

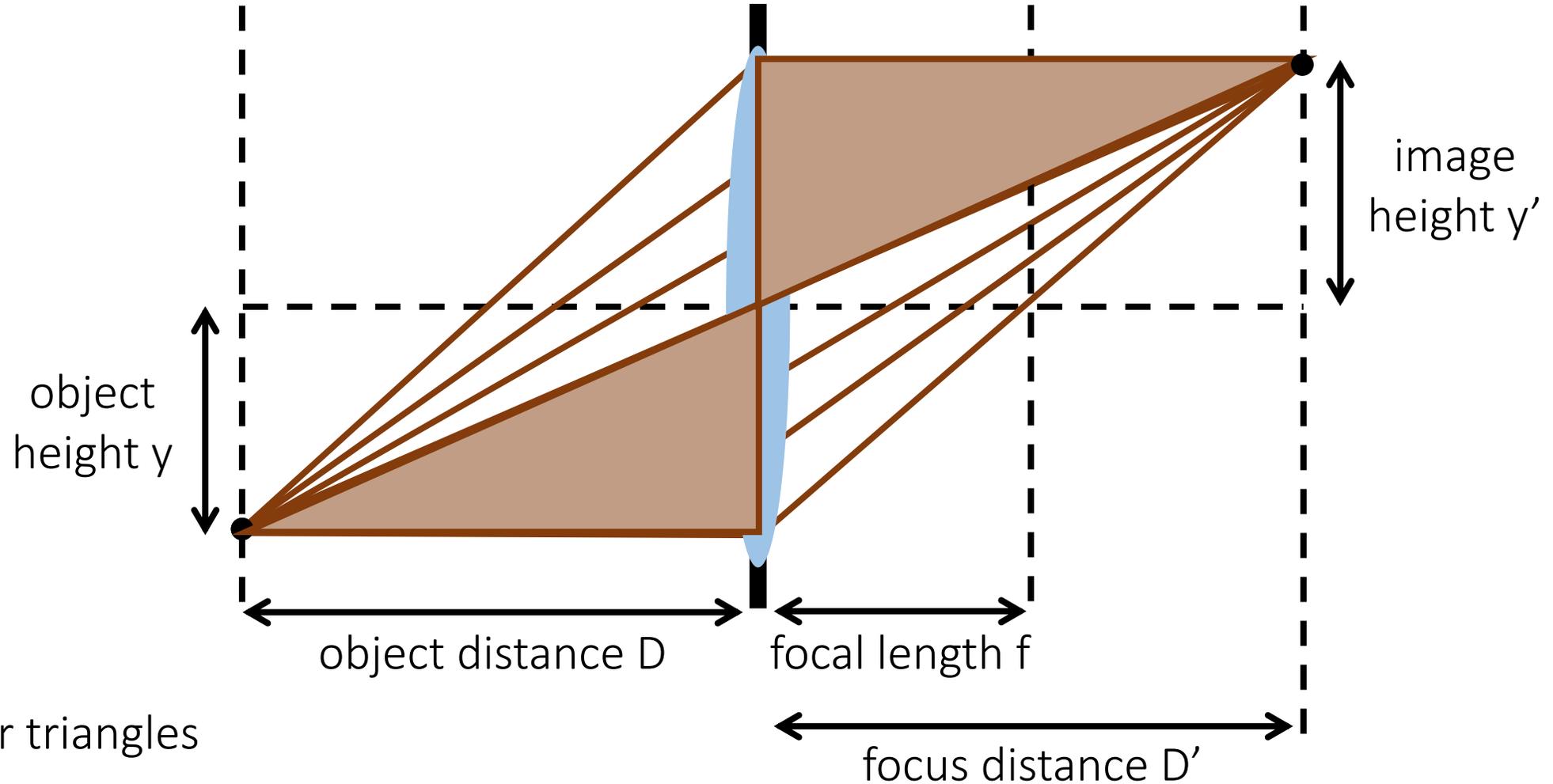


Use similar triangles

# Thin lens formula

How can we relate scene-space ( $D, y$ ) and image space ( $D', y'$ ) quantities?

$$\frac{y}{y'} = ?$$

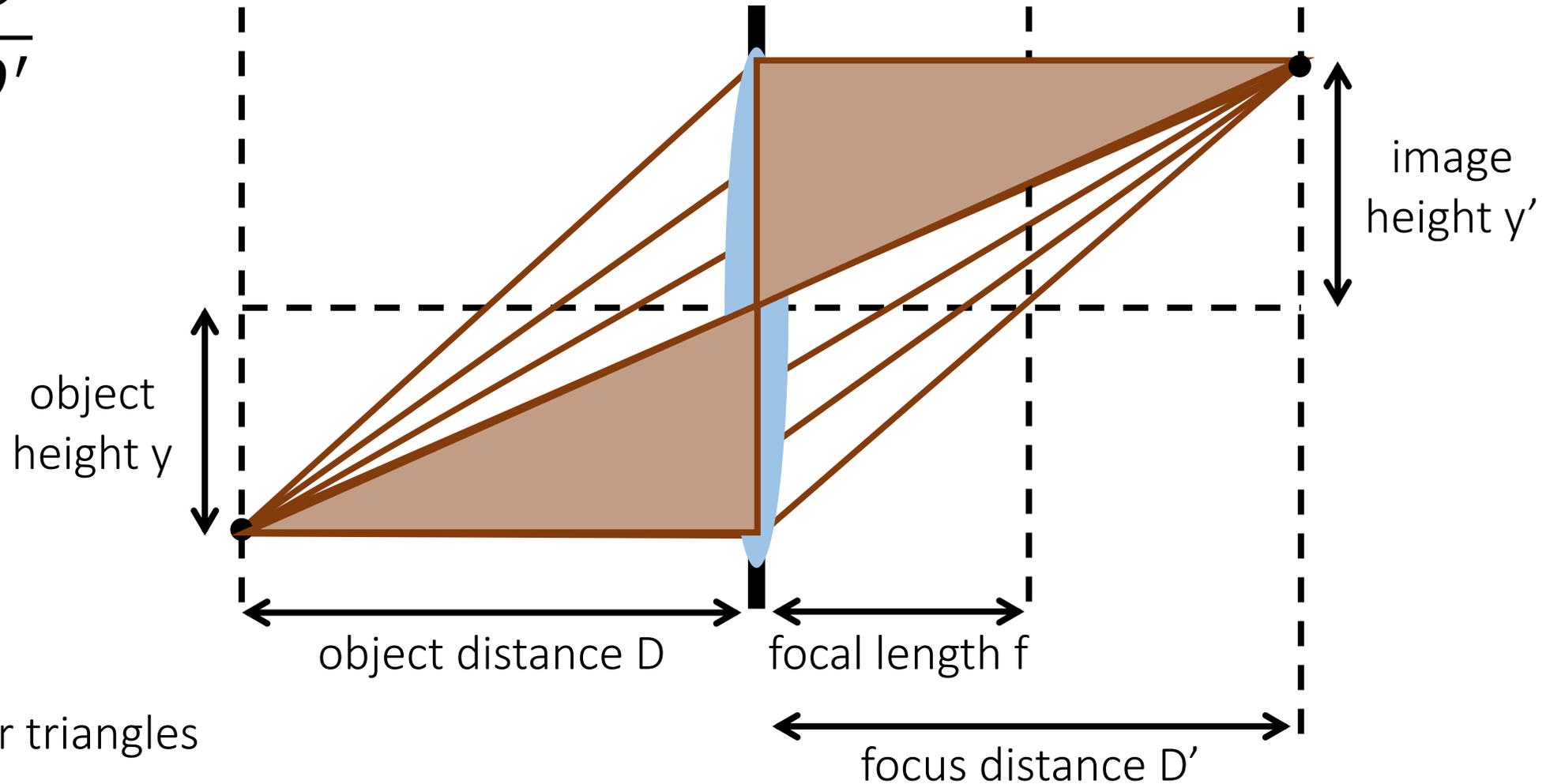


Use similar triangles

# Thin lens formula

How can we relate scene-space ( $D, y$ ) and image space ( $D', y'$ ) quantities?

$$\frac{y}{y'} = \frac{D}{D'}$$



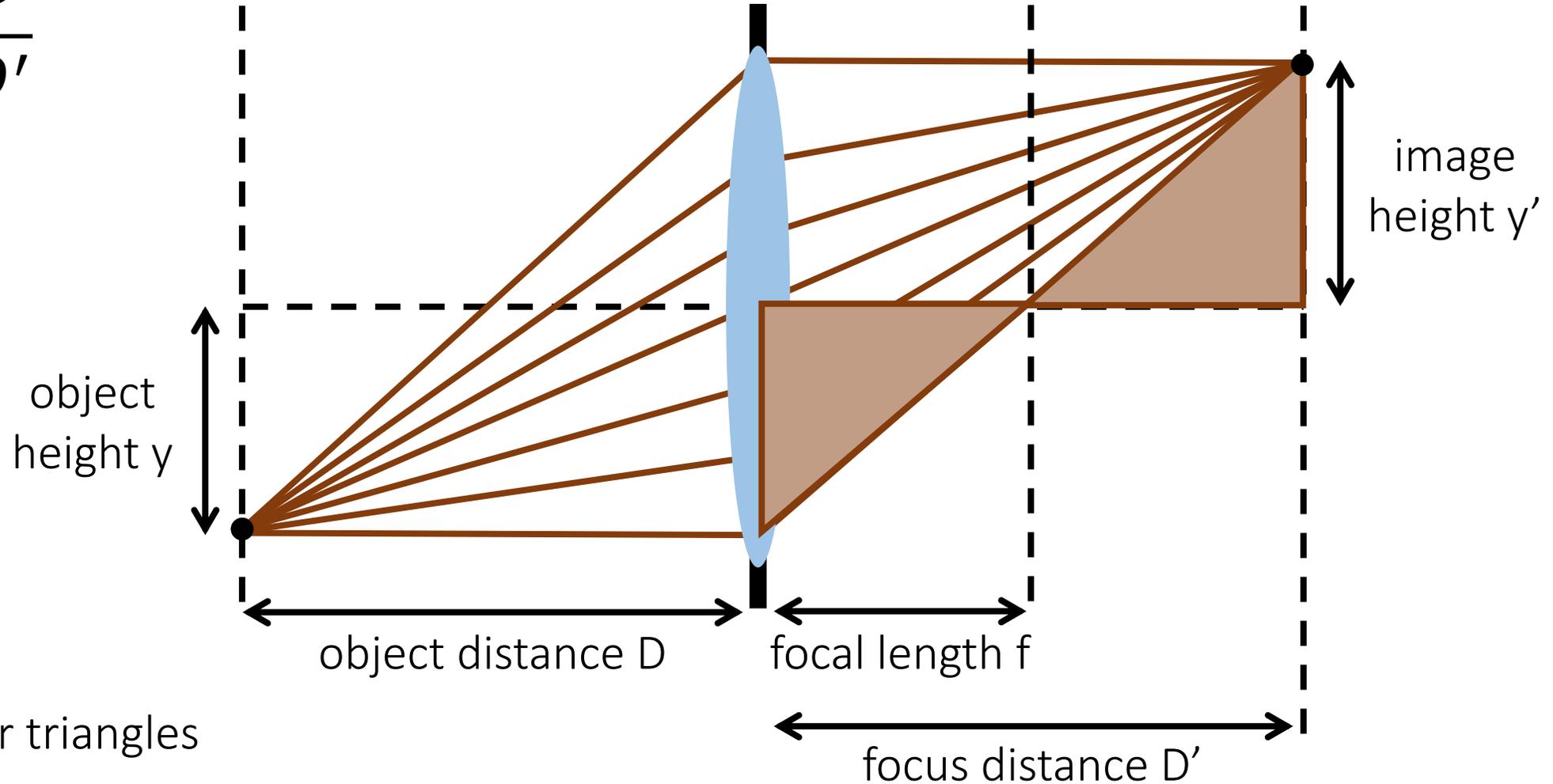
Use similar triangles

# Thin lens formula

How can we relate scene-space ( $D, y$ ) and image space ( $D', y'$ ) quantities?

$$\frac{y}{y'} = \frac{D}{D'}$$

$$\frac{y}{y'} = ?$$



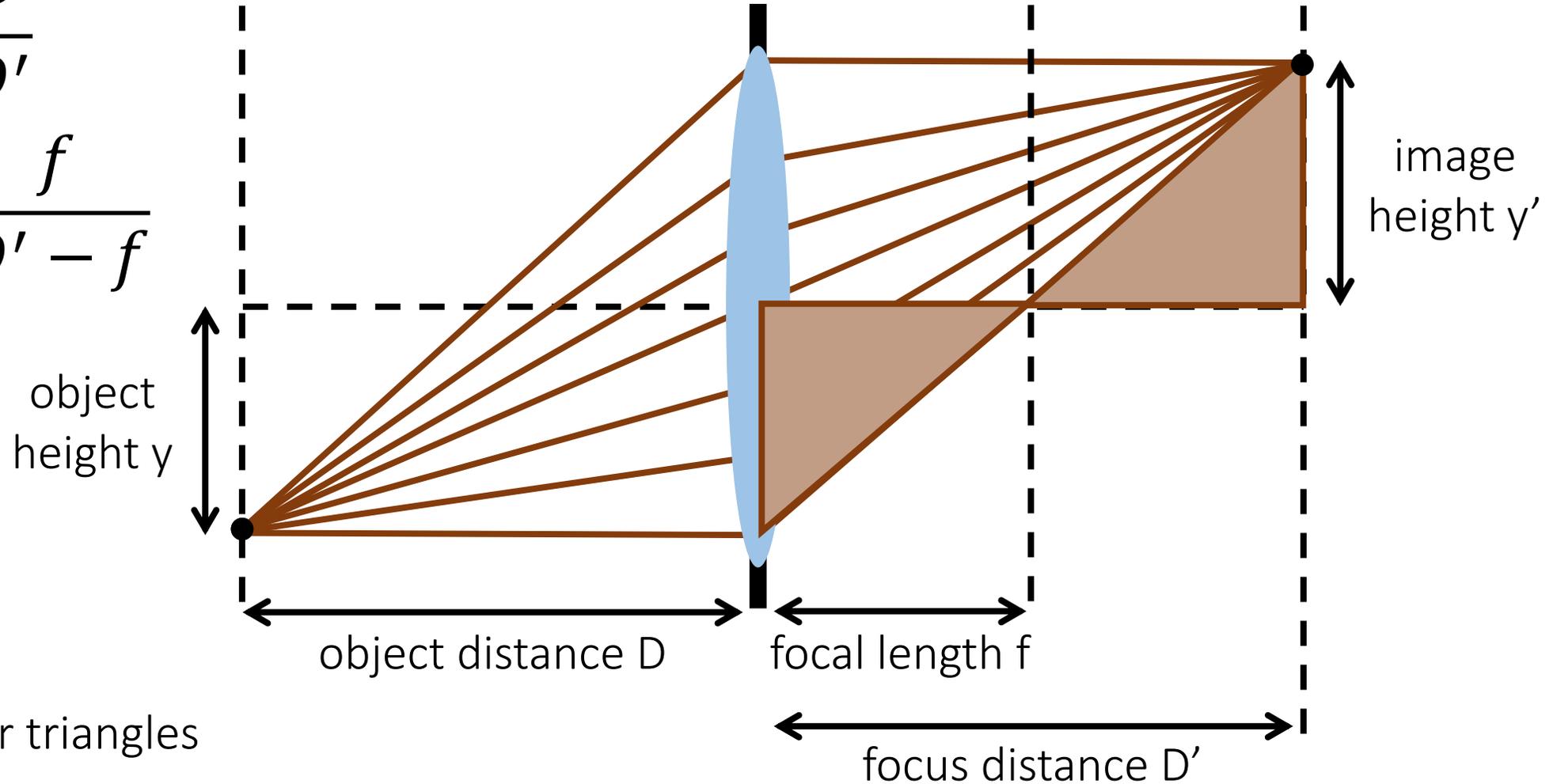
Use similar triangles

# Thin lens formula

How can we relate scene-space ( $D, y$ ) and image space ( $D', y'$ ) quantities?

$$\frac{y}{y'} = \frac{D}{D'}$$

$$\frac{y}{y'} = \frac{f}{D' - f}$$

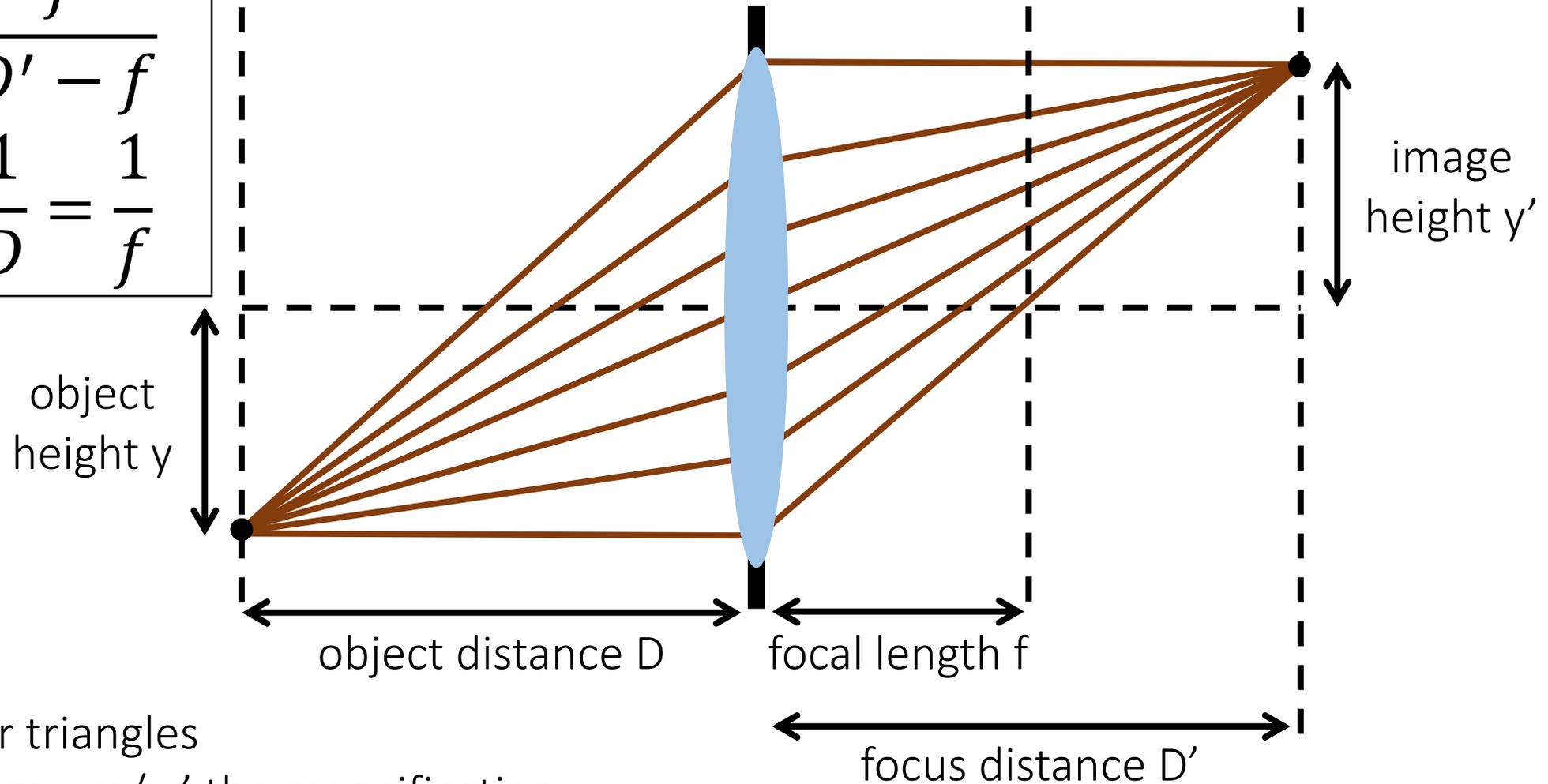


Use similar triangles

# Thin lens formula

How can we relate scene-space ( $D, y$ ) and image space ( $D', y'$ ) quantities?

$$m = \frac{f}{D' - f}$$
$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$



Use similar triangles

- We call  $m = y / y'$  the magnification

# Special focus distances

$$D' = f, D = ?, m = ?$$

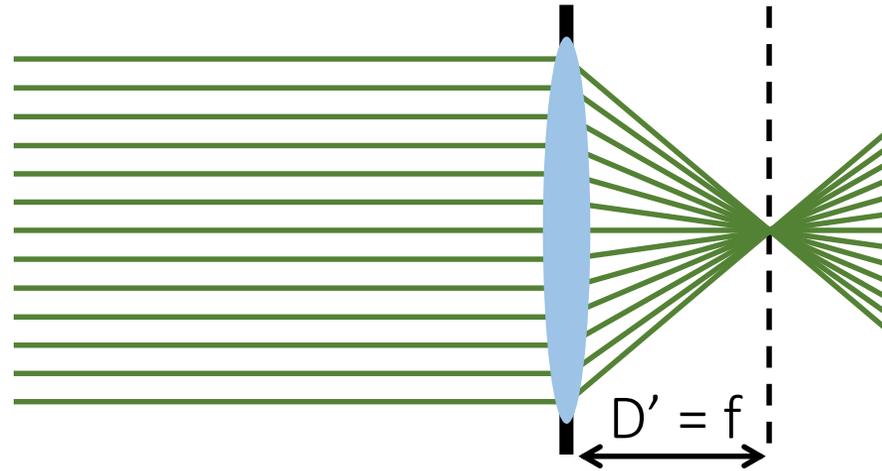
$$m = \frac{f}{D' - f}$$

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

# Special focus distances

$D' = f, D = \infty, m = \infty \rightarrow$  infinity focus (parallel rays)

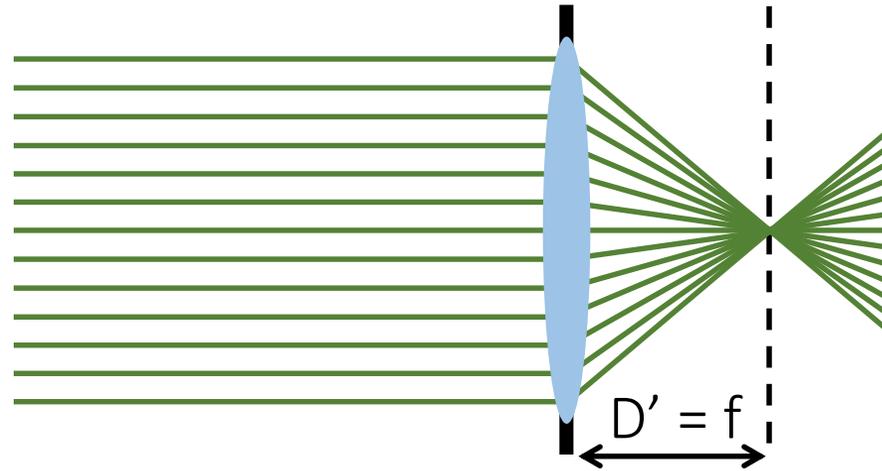
$$m = \frac{f}{D' - f}$$
$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$



$D' = D = ?, m = ?$

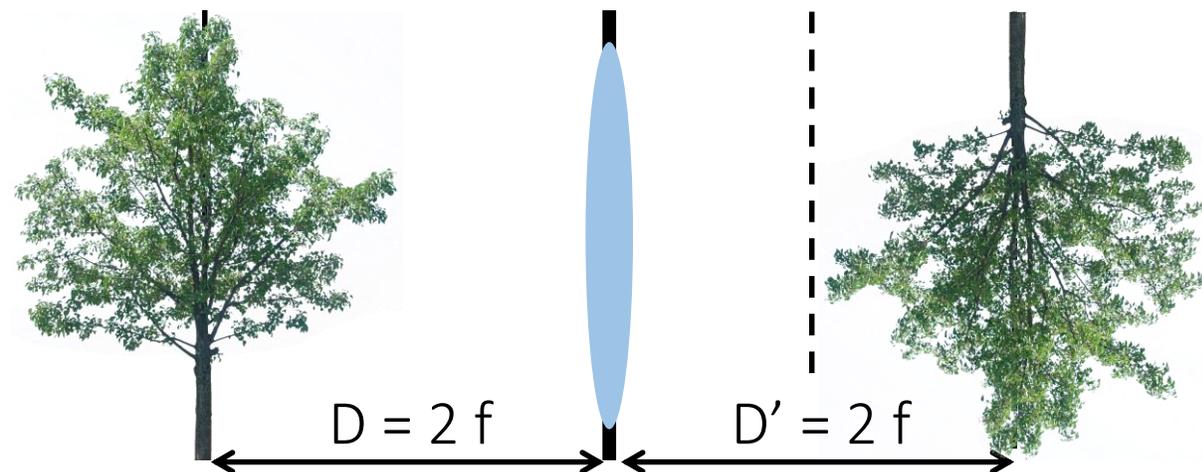
# Special focus distances

$D' = f, D = \infty, m = \infty \rightarrow$  infinity focus (parallel rays)

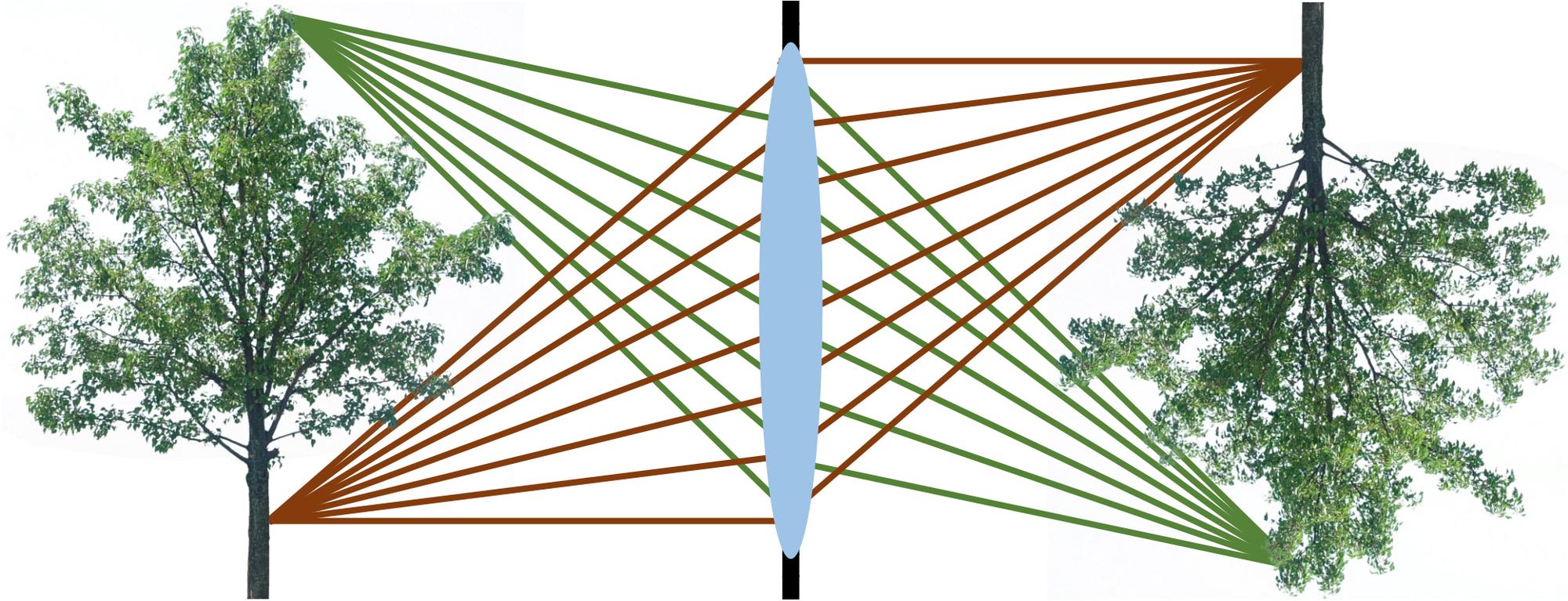


$$m = \frac{f}{D' - f}$$
$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

$D' = D = 2f, m = 1 \rightarrow$  object is reproduced in real-life size



# Free lunch?

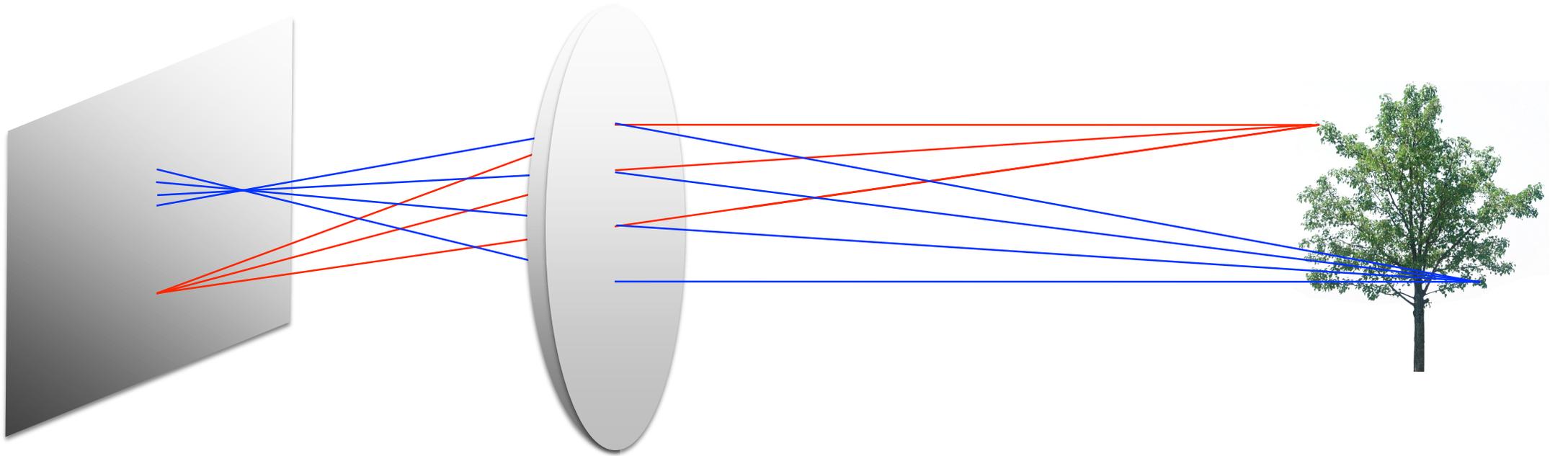


By using a lens we simultaneously achieve:

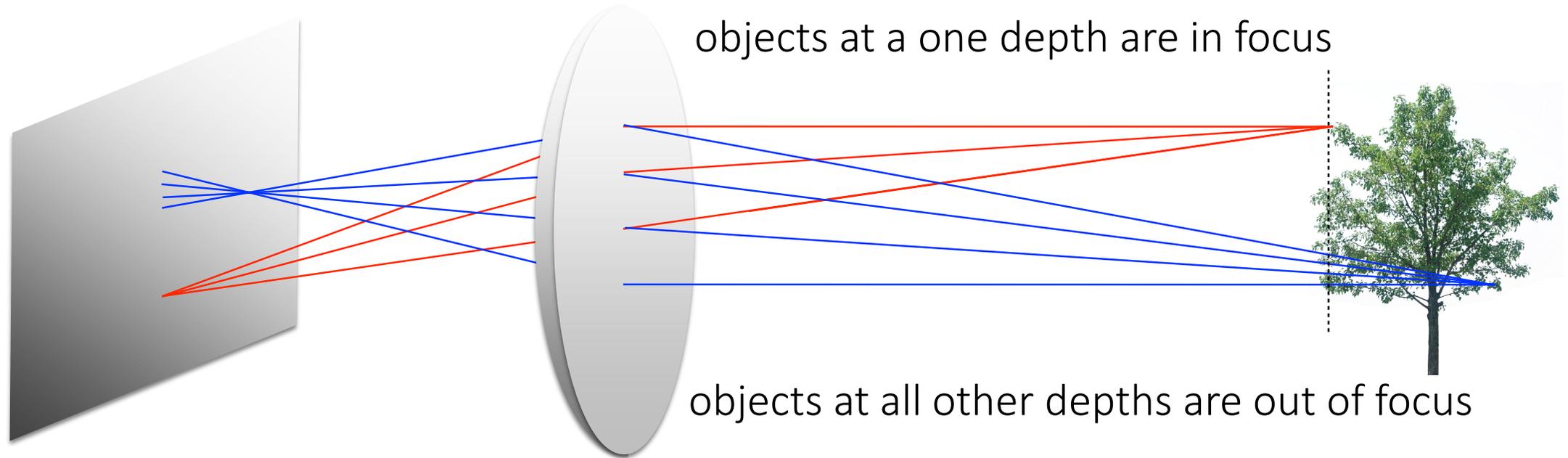
1. Image is sharp.
2. Signal-to-noise ratio is high.

Do we lose anything by using a lens?

# Defocus

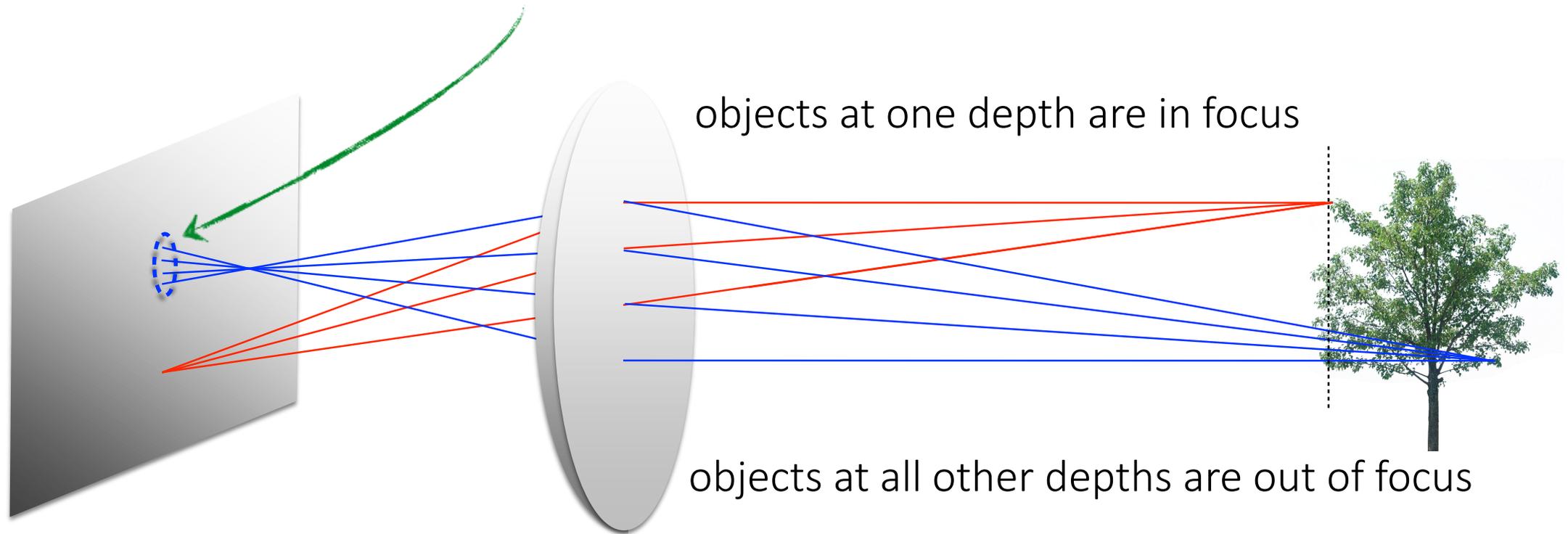


# Defocus



# Defocus

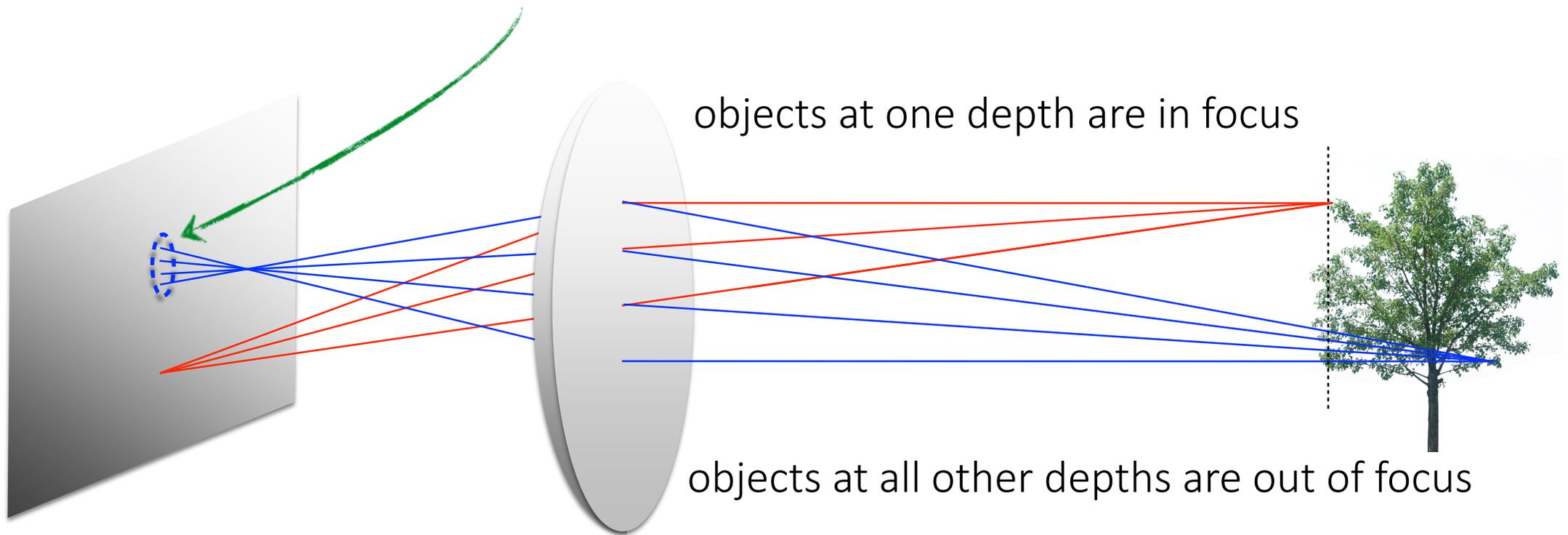
circle of confusion (i.e., blur kernel)



Is the circle of confusion constant?

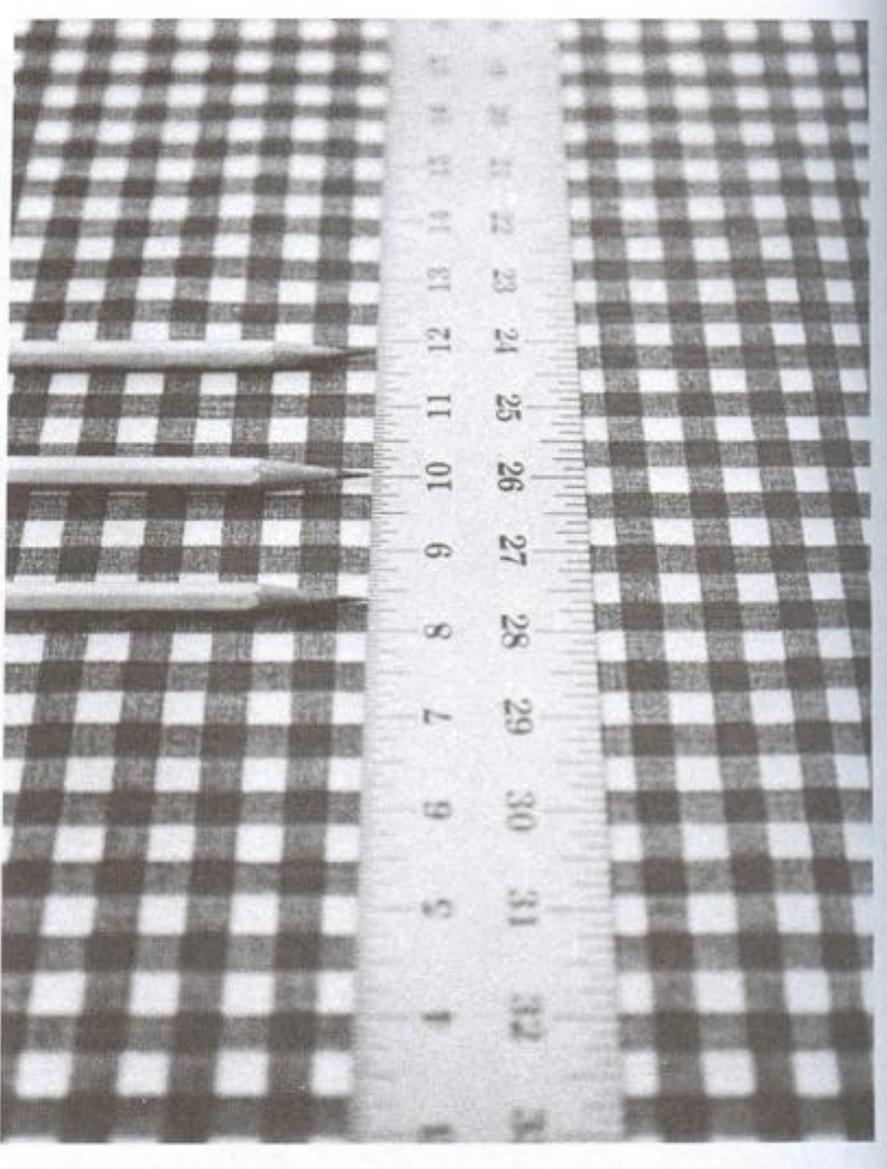
# Defocus

circle of confusion (i.e., blur kernel)



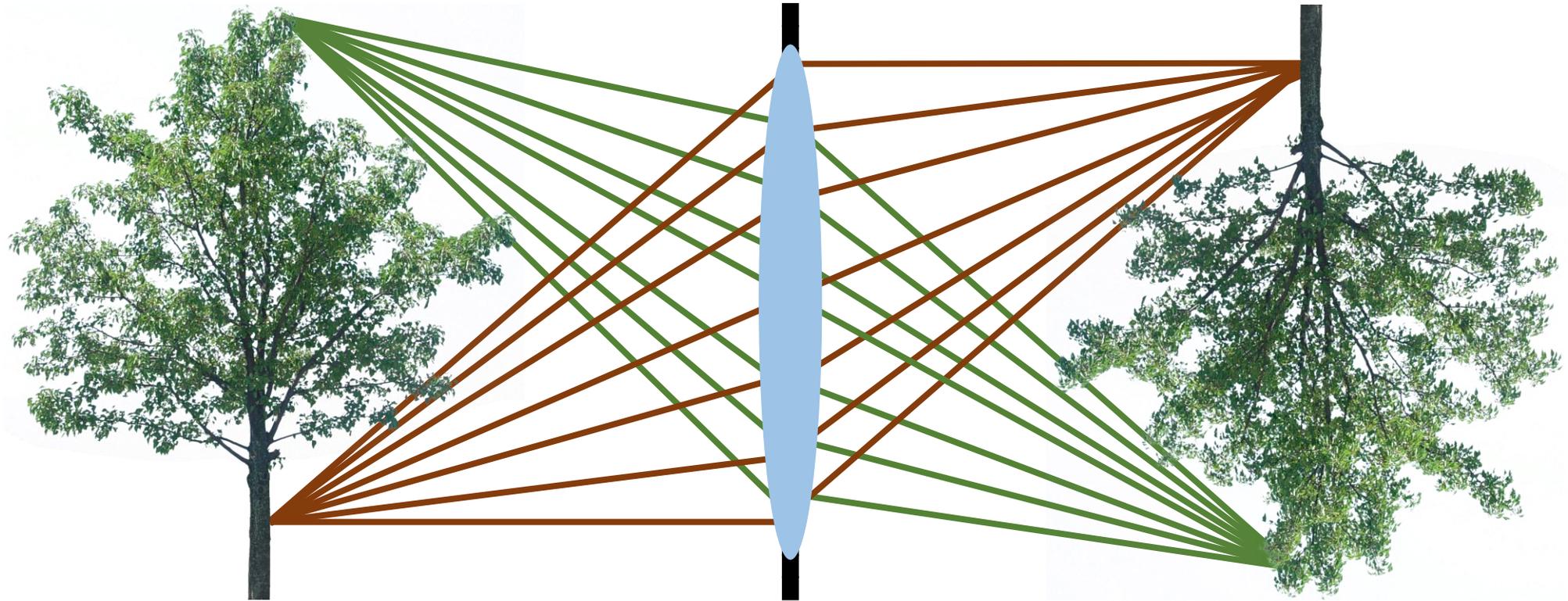
How do we change the depth where objects are in focus?

# Change of focus for different depths

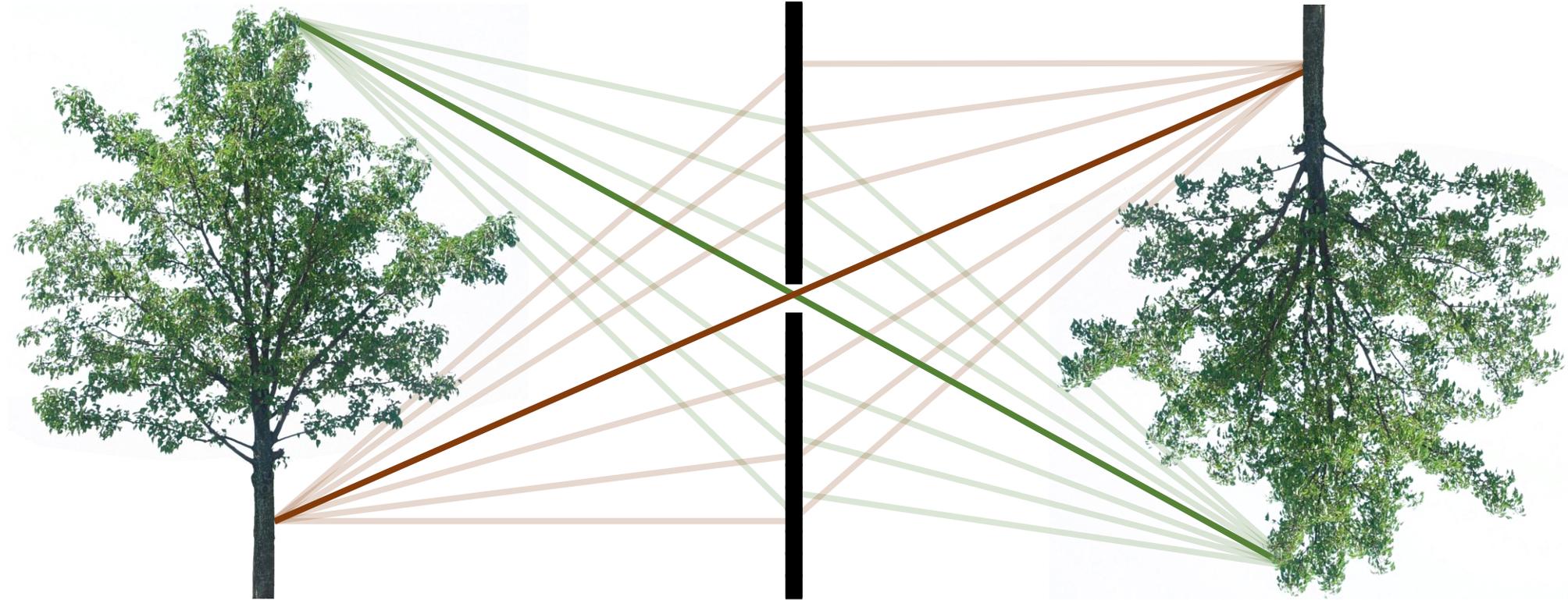


Lens camera and pinhole camera

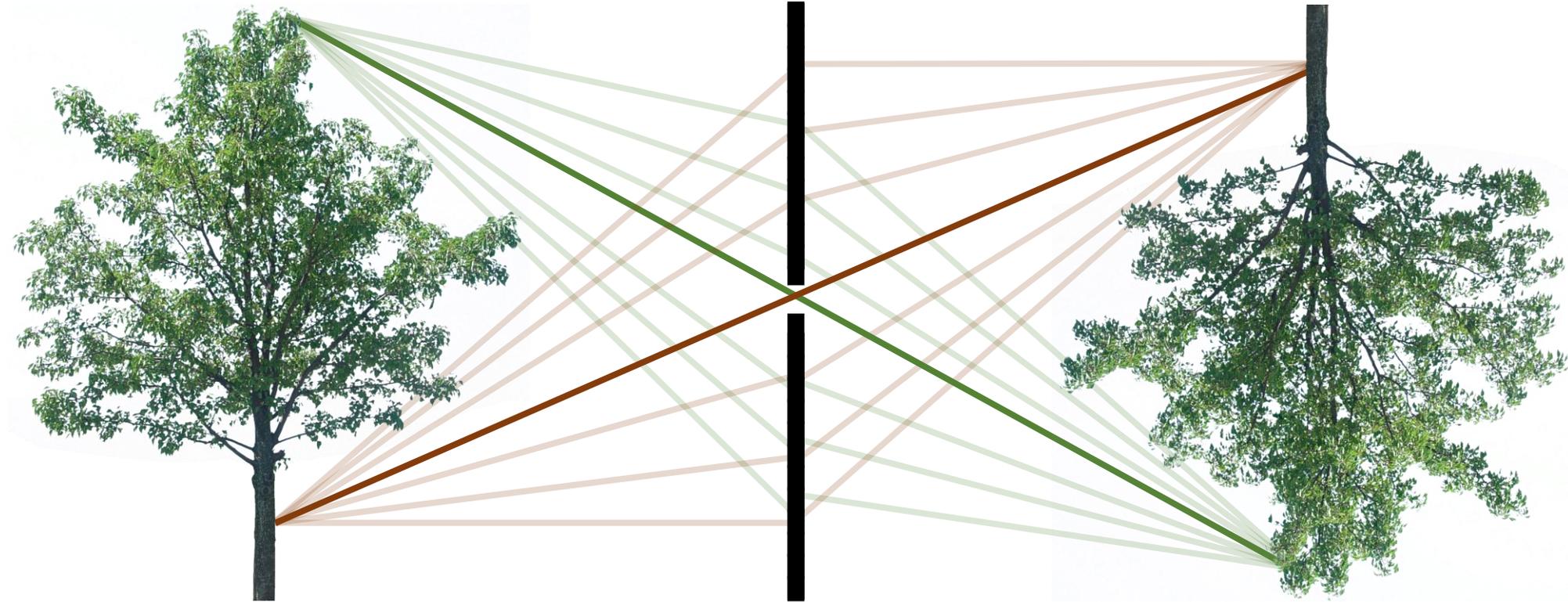
# The lens camera



# The pinhole camera

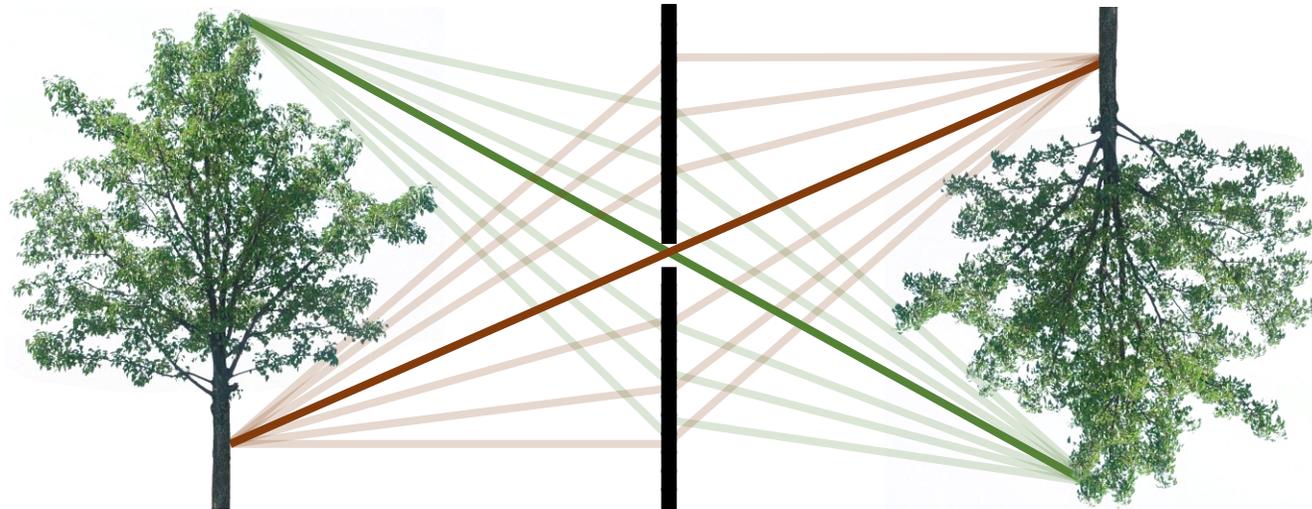
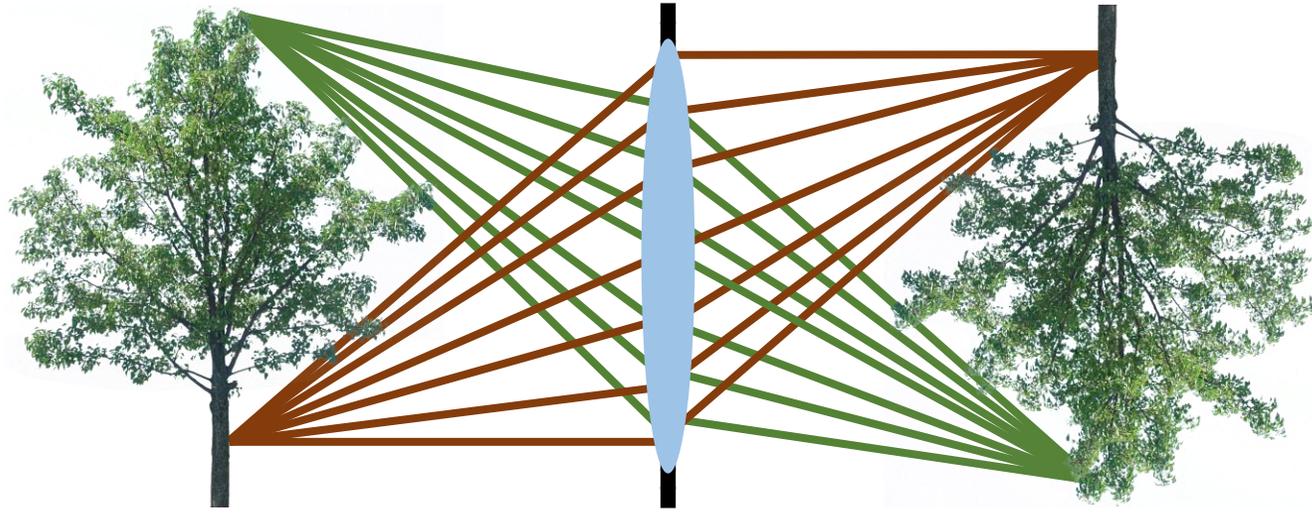


# The pinhole camera



Central rays propagate in the same way for both models!

# Describing both lens and pinhole cameras

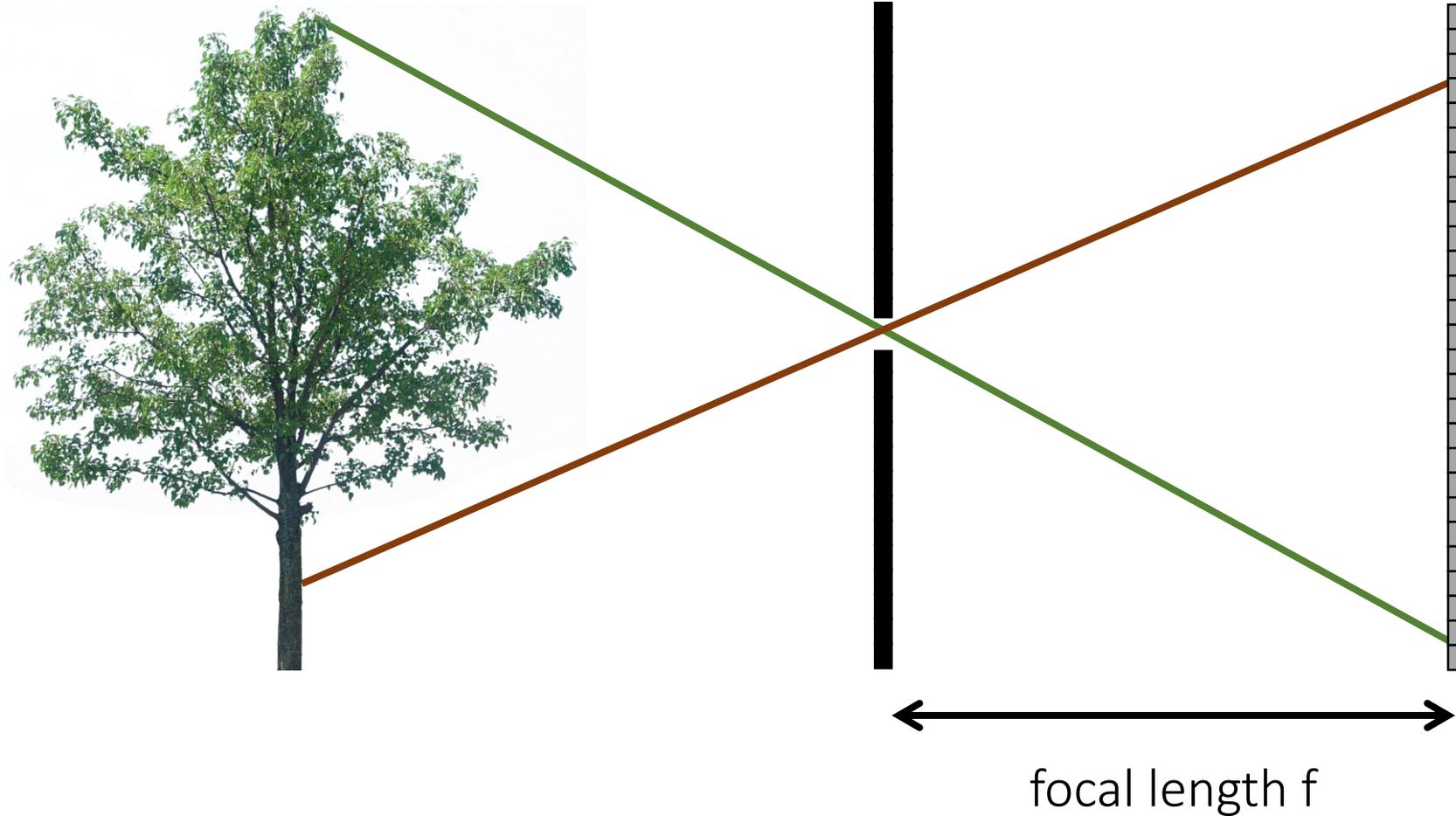


We can derive properties and descriptions that hold for both camera models if:

- We use only central rays.
- We assume the lens camera is in focus.

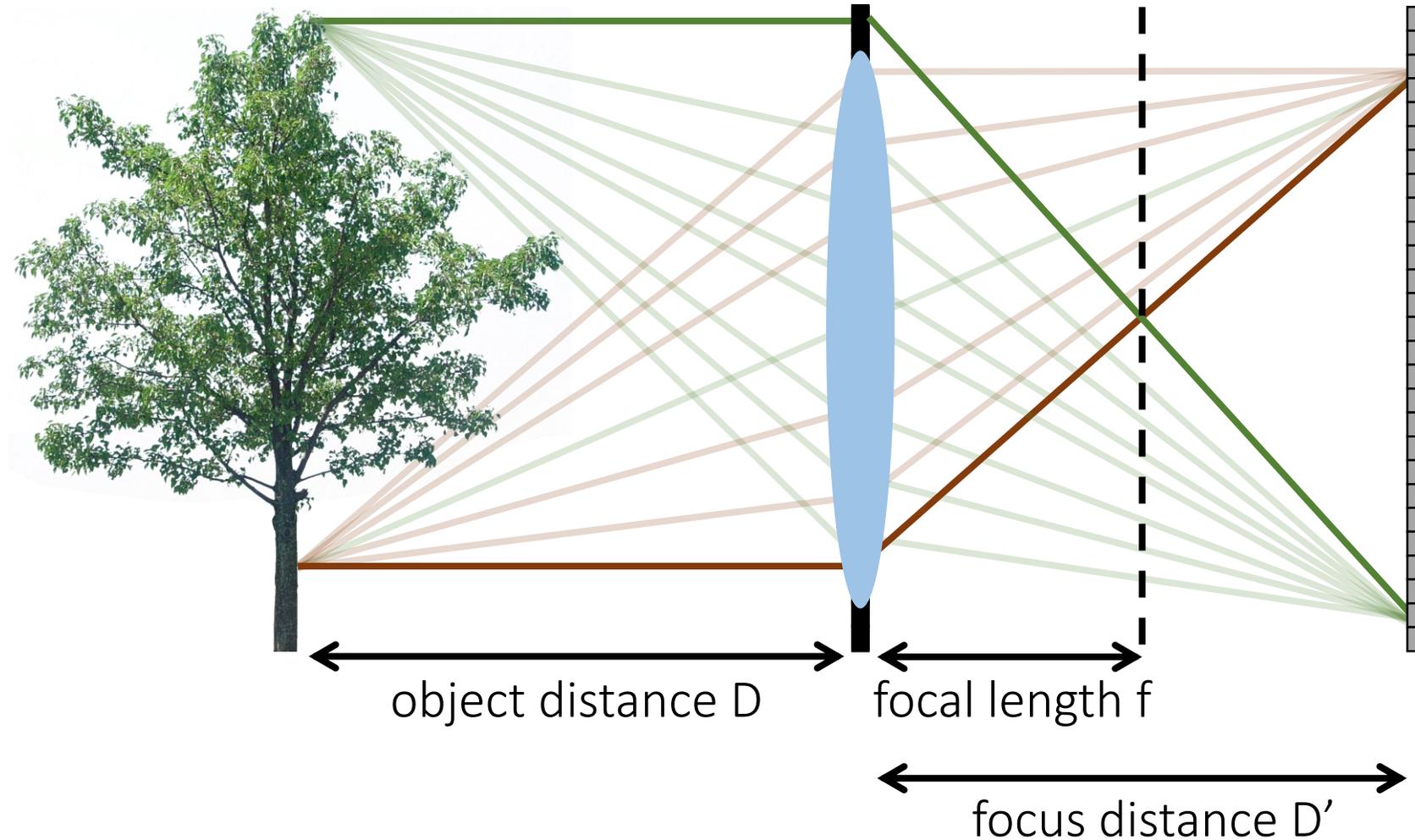
# Important difference: focal length

In a pinhole camera, focal length is distance between aperture and sensor

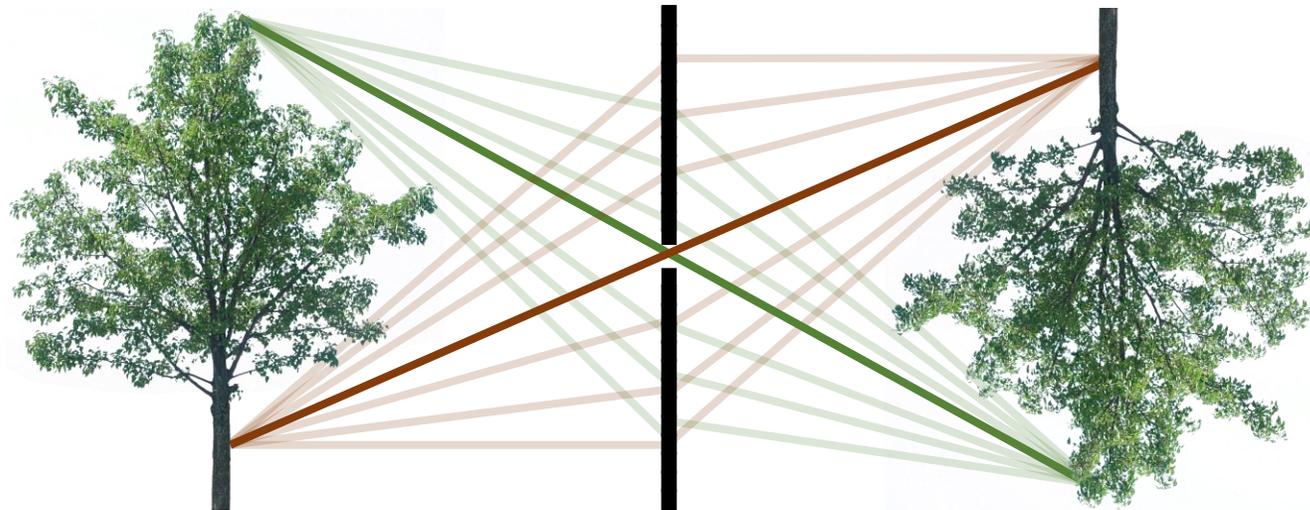
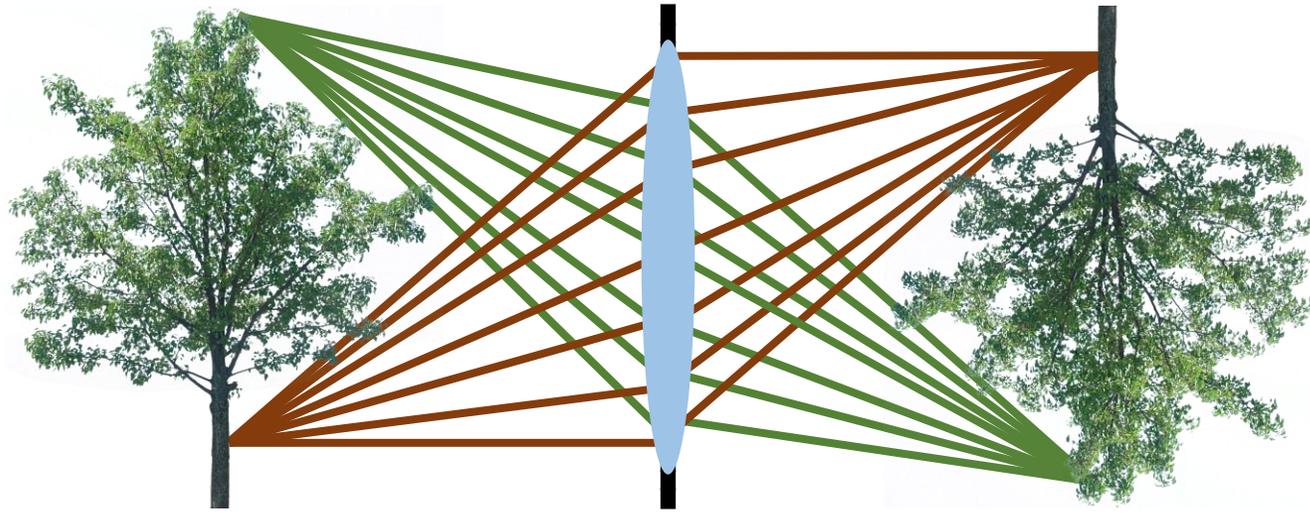


# Important difference: focal length

In a lens camera, focal length is distance where parallel rays intersect



# Describing both lens and pinhole cameras

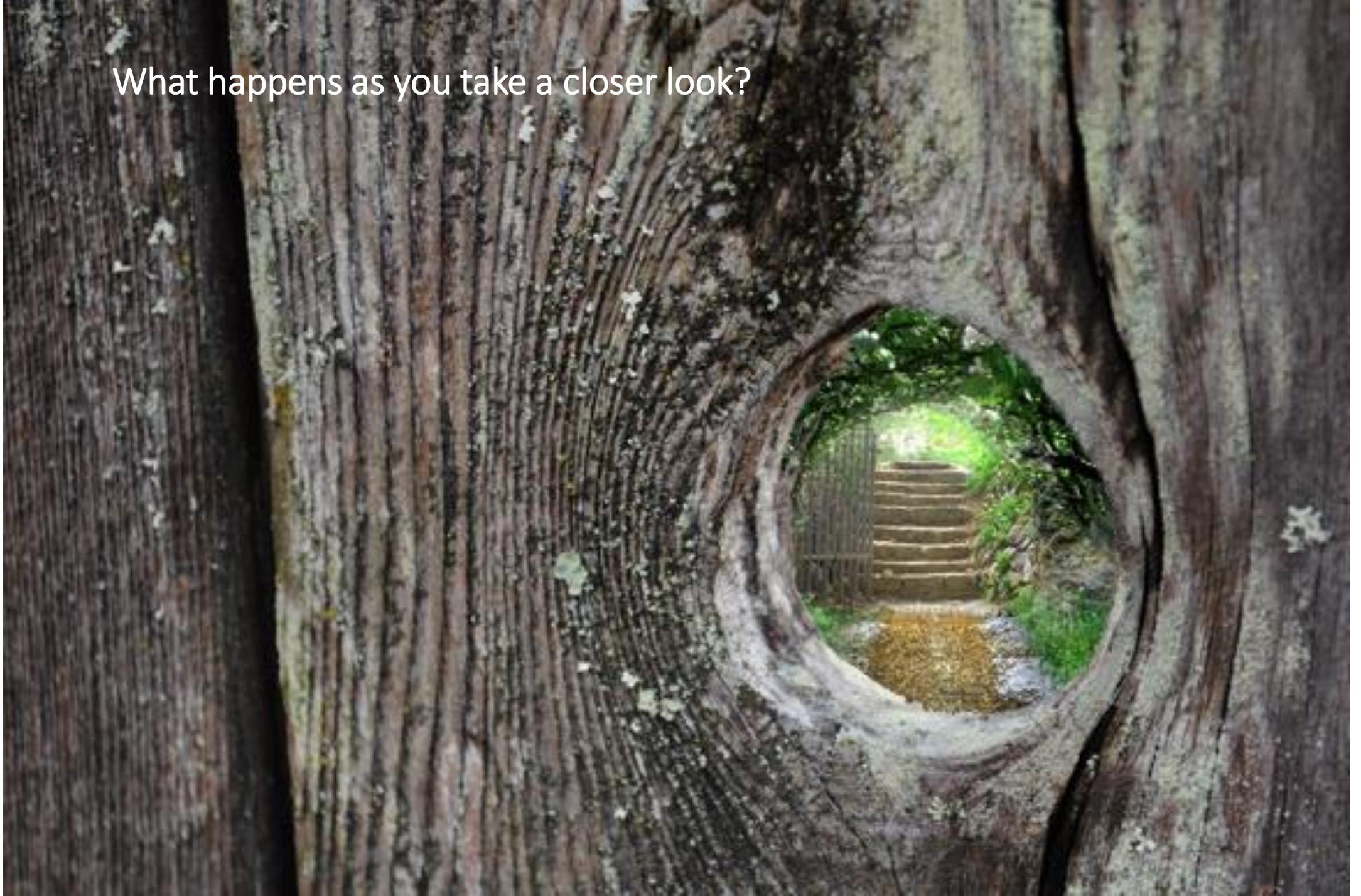


We can derive properties and descriptions that hold for both camera models if:

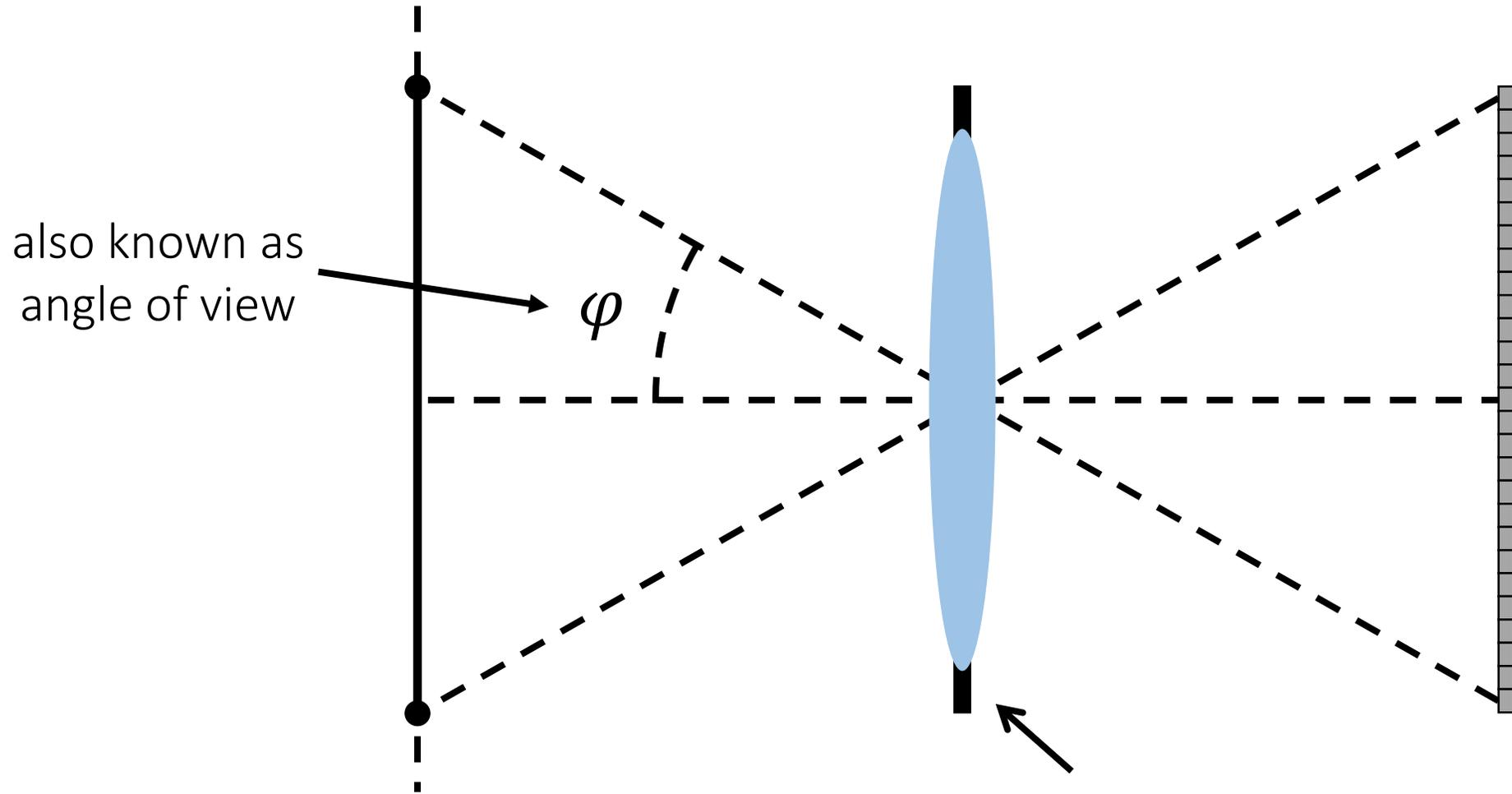
- We use only central rays.
- We assume the lens camera is in focus.
- We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.

Field of view

What happens as you take a closer look?



# Field of view

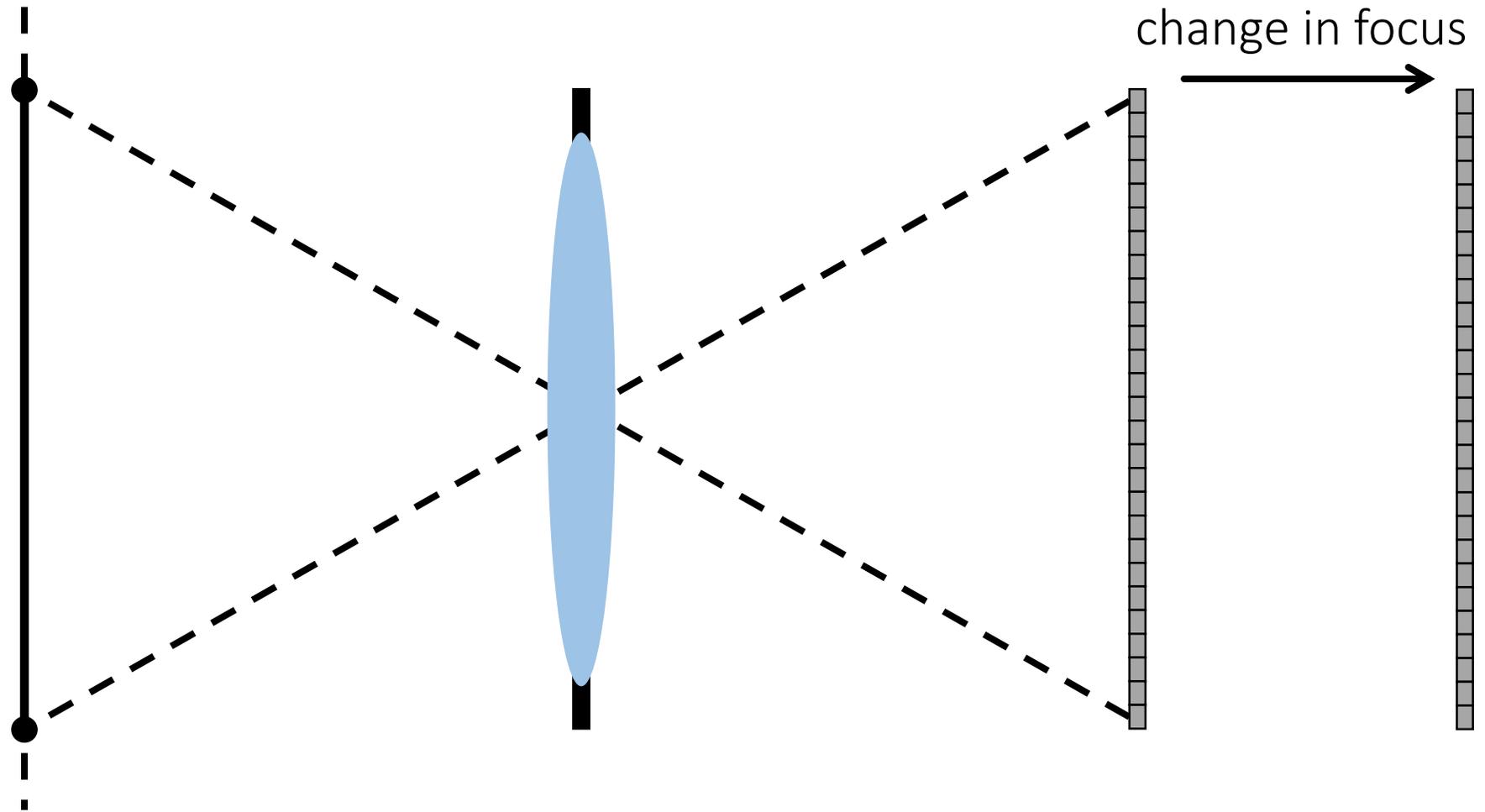


also known as  
angle of view

$\varphi$

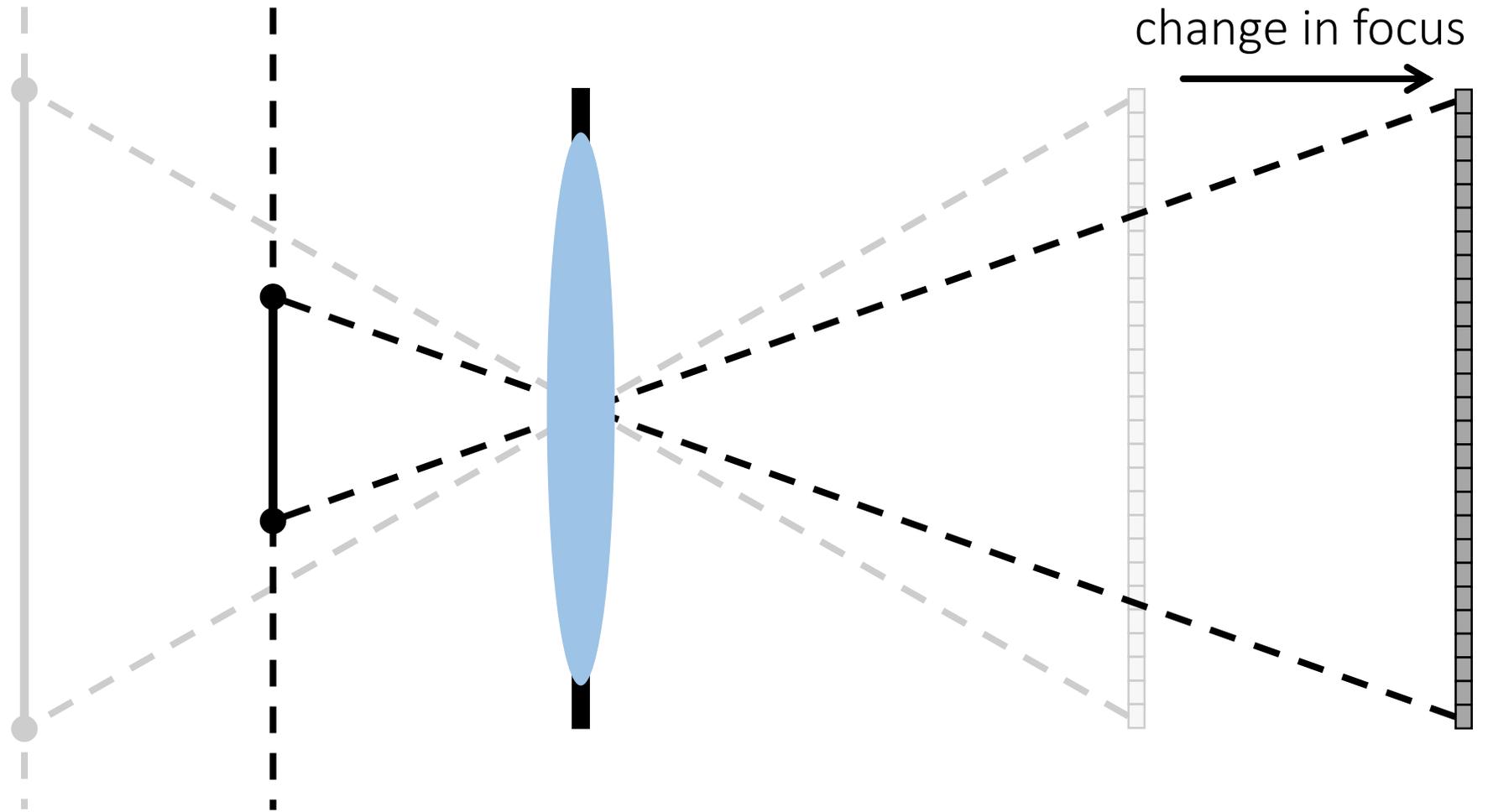
Note: here I drew a lens, but I could  
have just as well drawn a pinhole

# Field of view



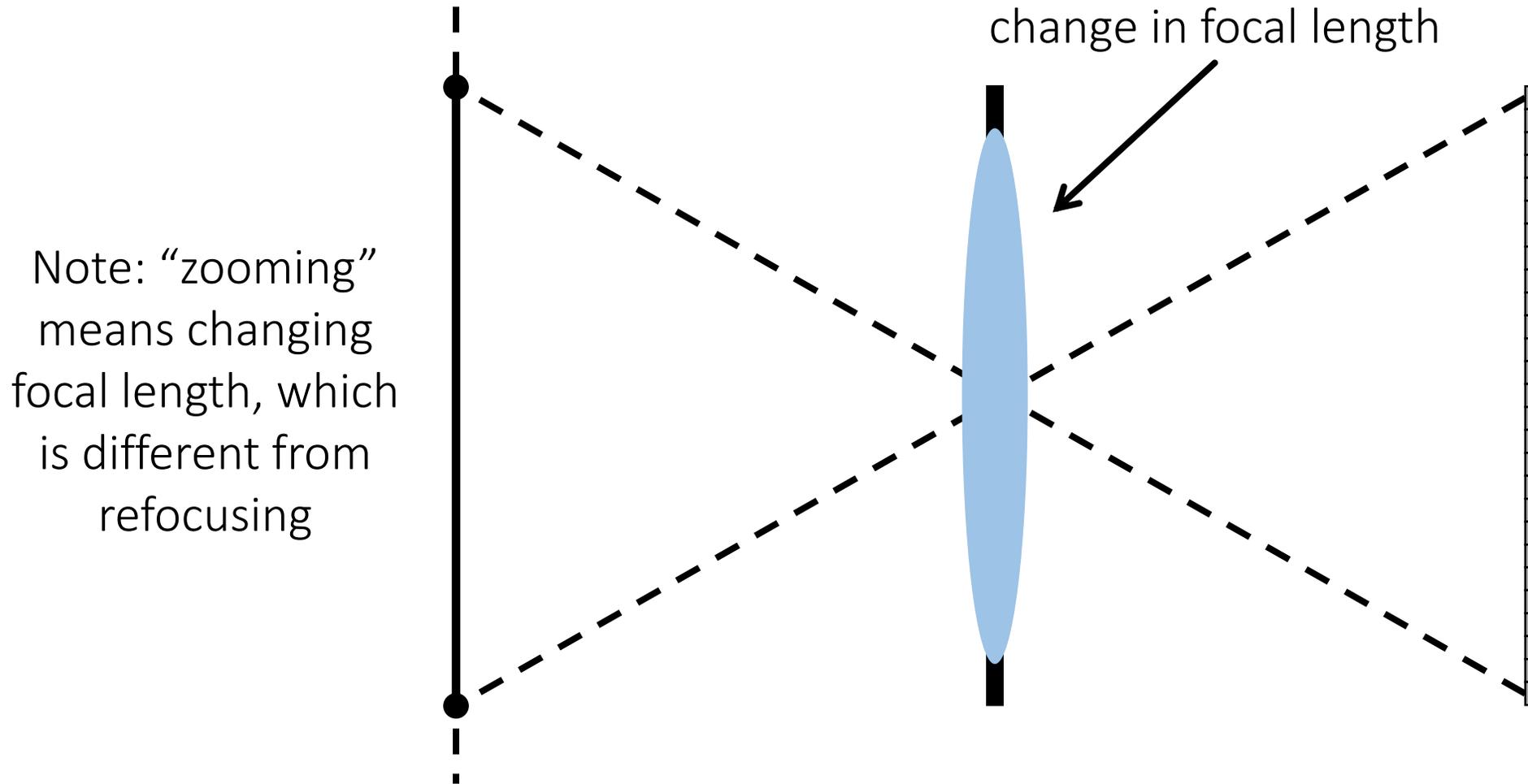
- What happens to field of view when we focus closer?

# Field of view



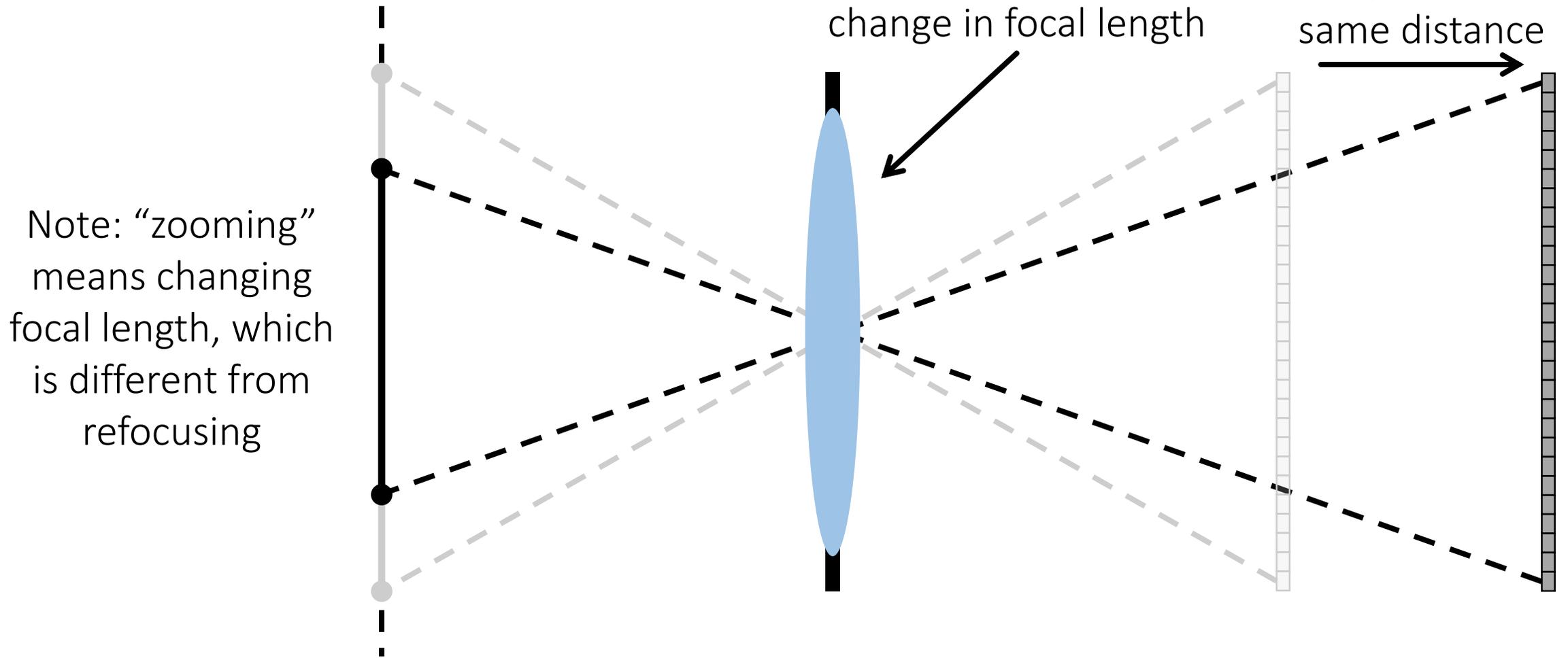
- What happens to field of view when we focus closer? → It decreases.

# Field of view



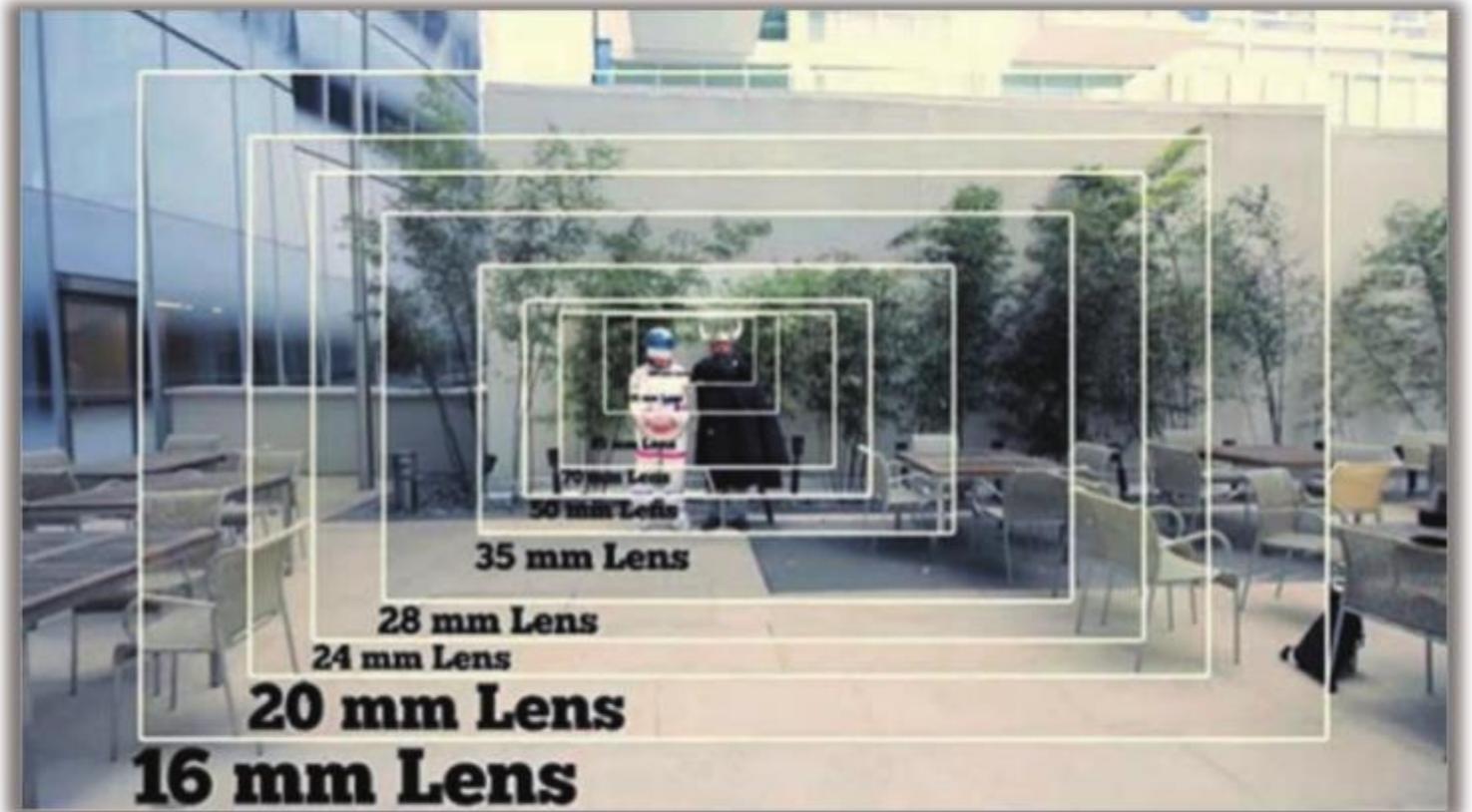
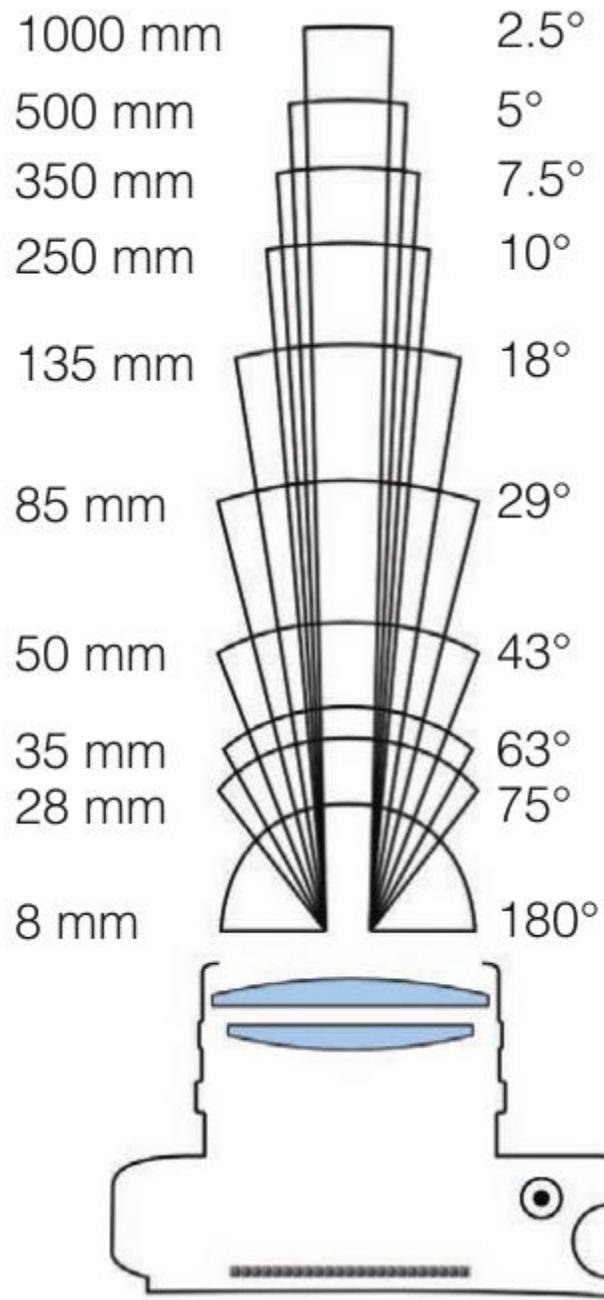
- What happens to field of view when we focus closer? → It decreases.
- What happens to field of view when we increase lens focal length?

# Field of view



- What happens to field of view when we focus closer? → It decreases.
- What happens to field of view when we increase lens focal length? → It decreases.

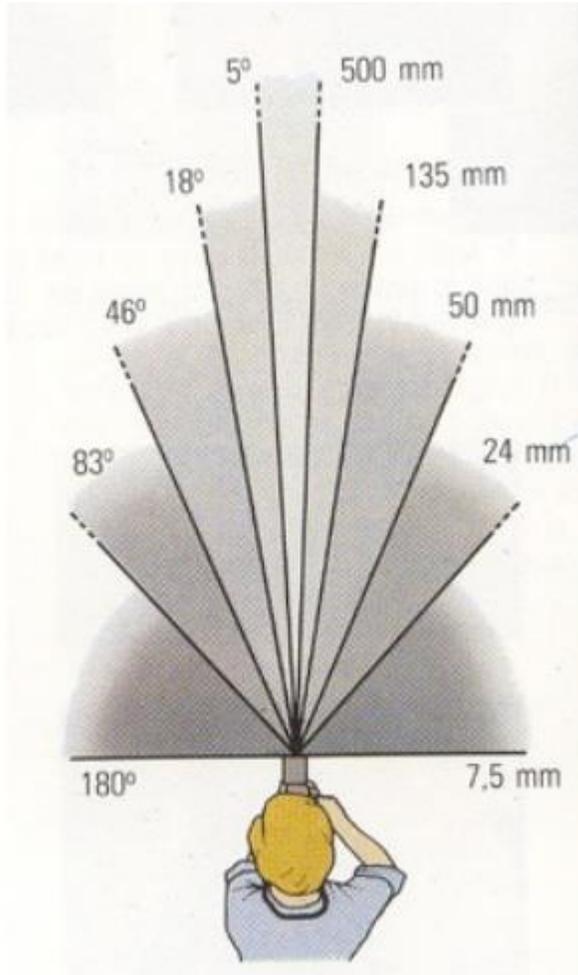
# Field of view



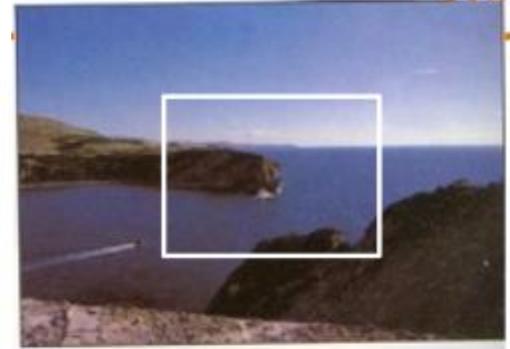
Andrew McWilliams

# Field of view

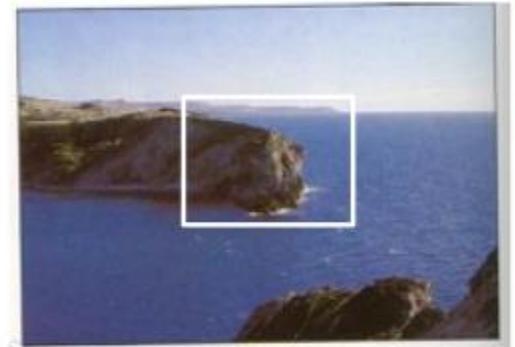
Increasing the lens focal length is similar to cropping



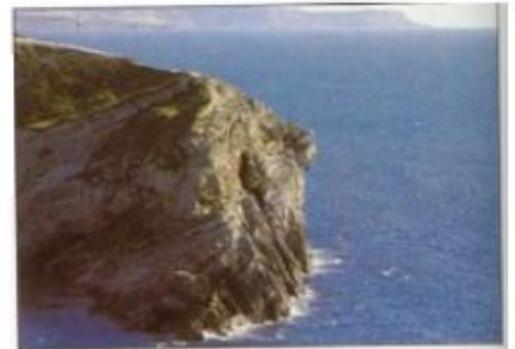
$f = 25 \text{ mm}$



$f = 50 \text{ mm}$

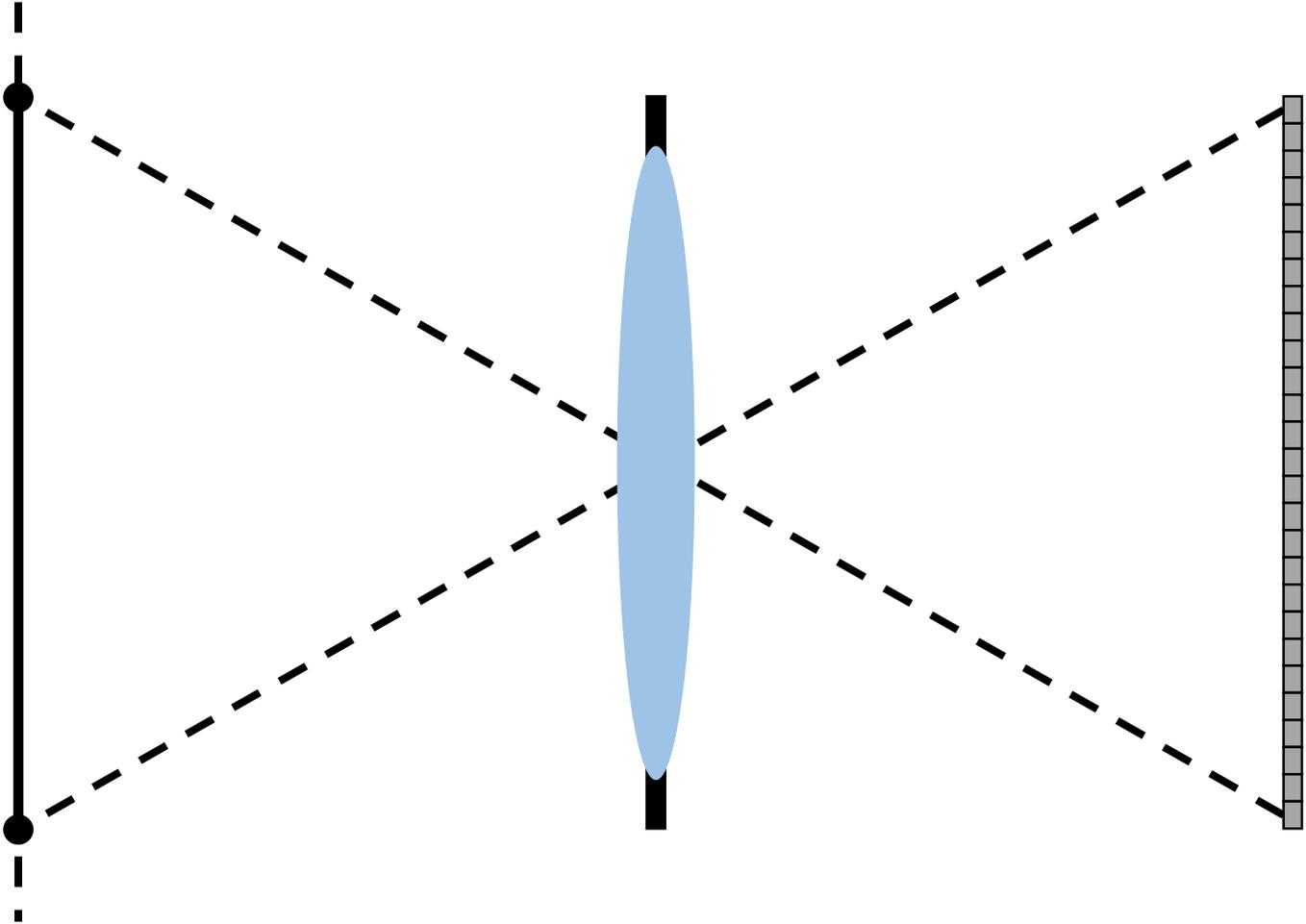


$f = 135 \text{ mm}$



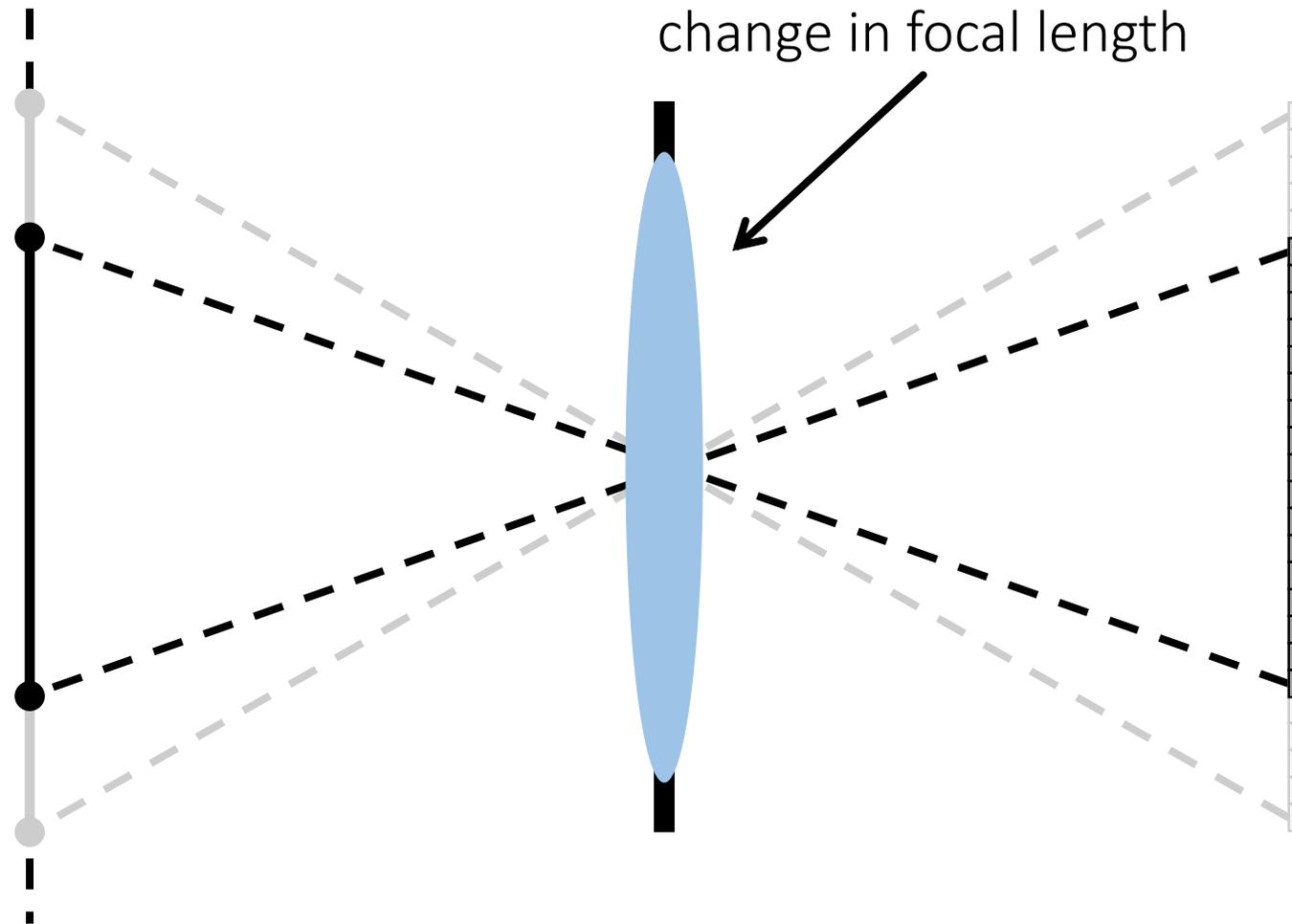
Is this effect identical to cropping?

# Field of view also depends on sensor size



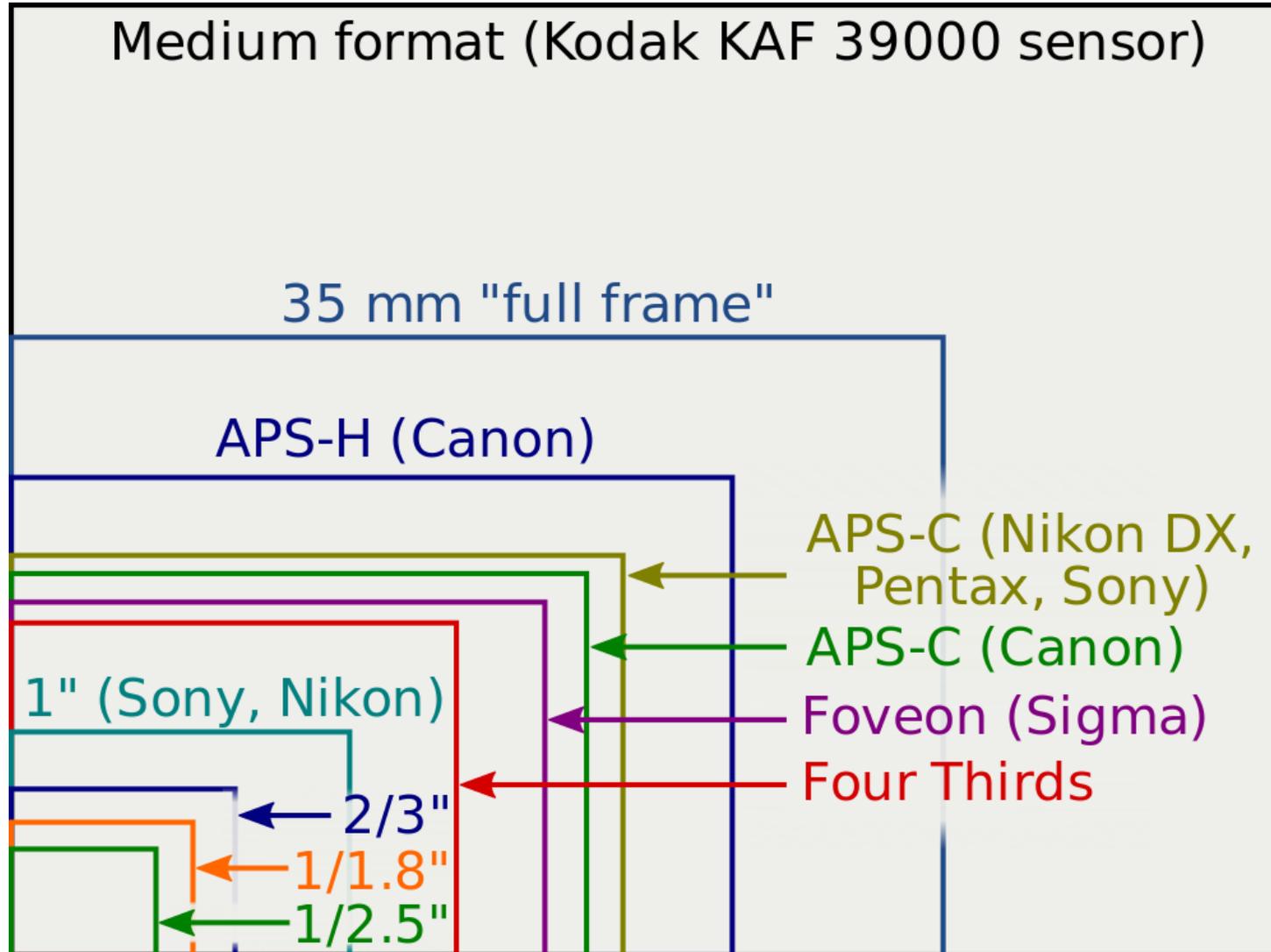
- What happens to field of view when we reduce sensor size?

# Field of view also depends on sensor size



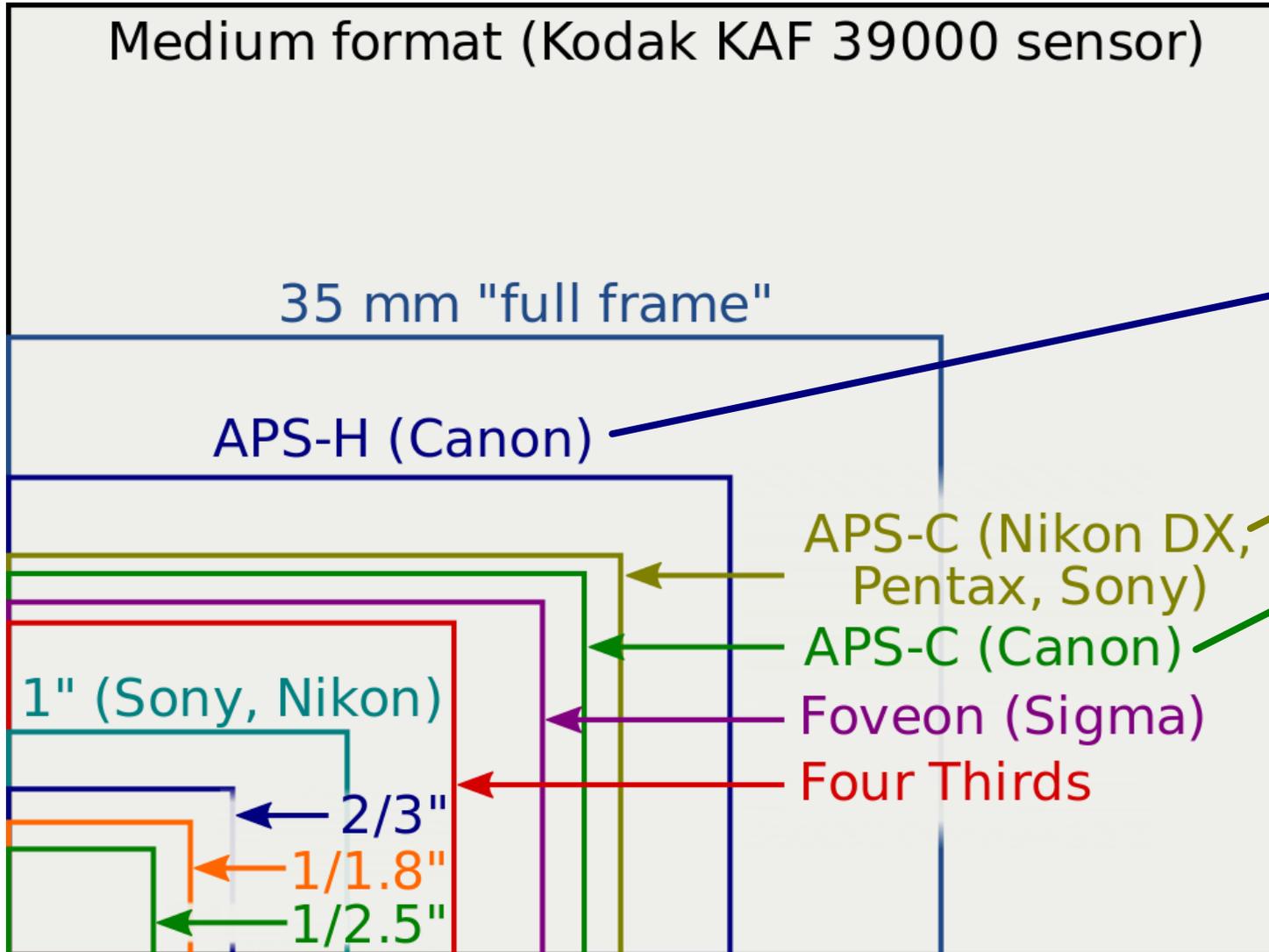
- What happens to field of view when we reduce sensor size? → It decreases.

# Field of view also depends on sensor size



- “Full frame” corresponds to standard film size.
- Digital sensors come in smaller formats due to manufacturing limitations (now mostly overcome).
- Lenses are often described in terms of field of view on film instead of focal length.
- These descriptions are invalid when not using full-frame sensor.

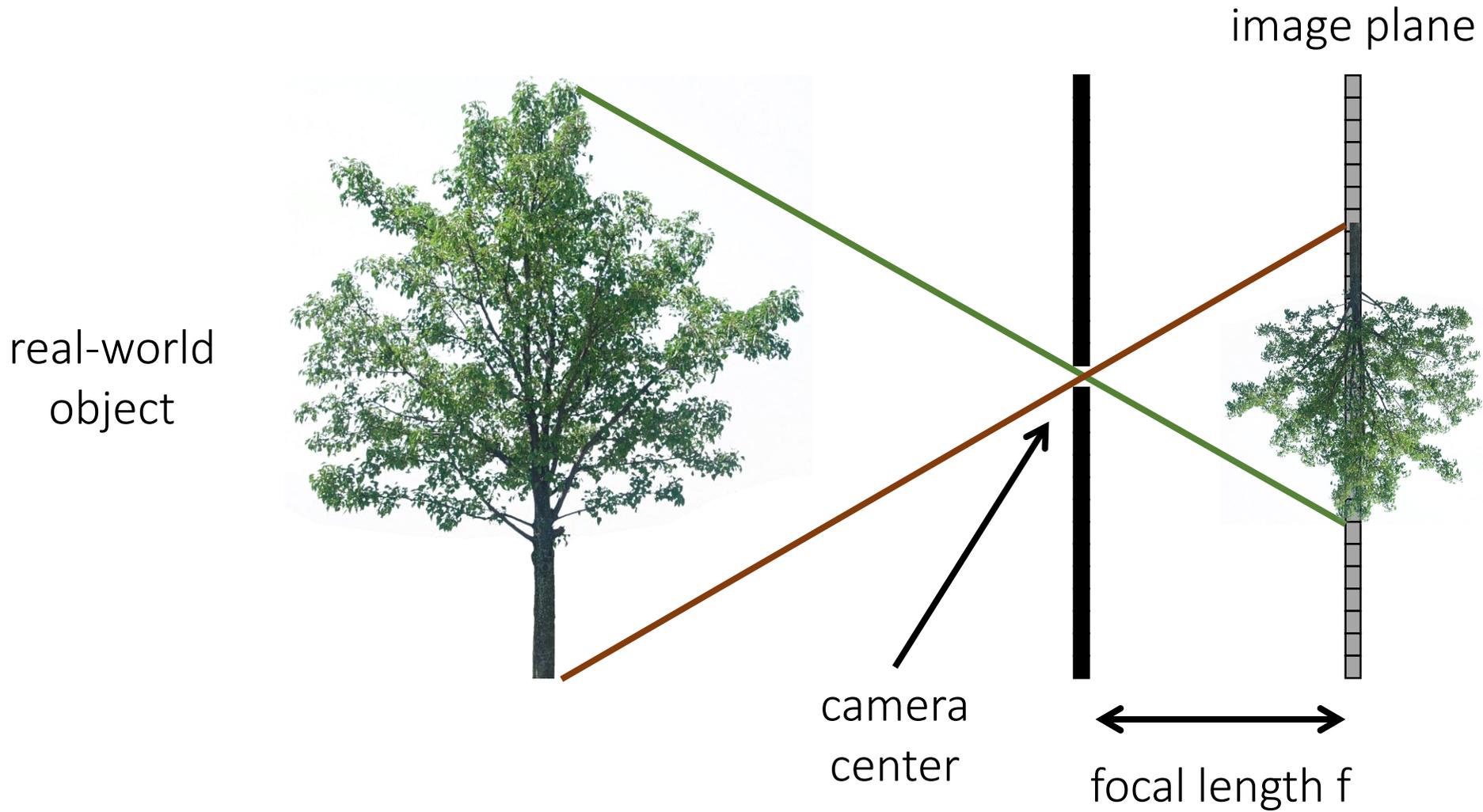
# Crop factor



How much field of view is cropped when using a sensor smaller than full frame.

Perspective

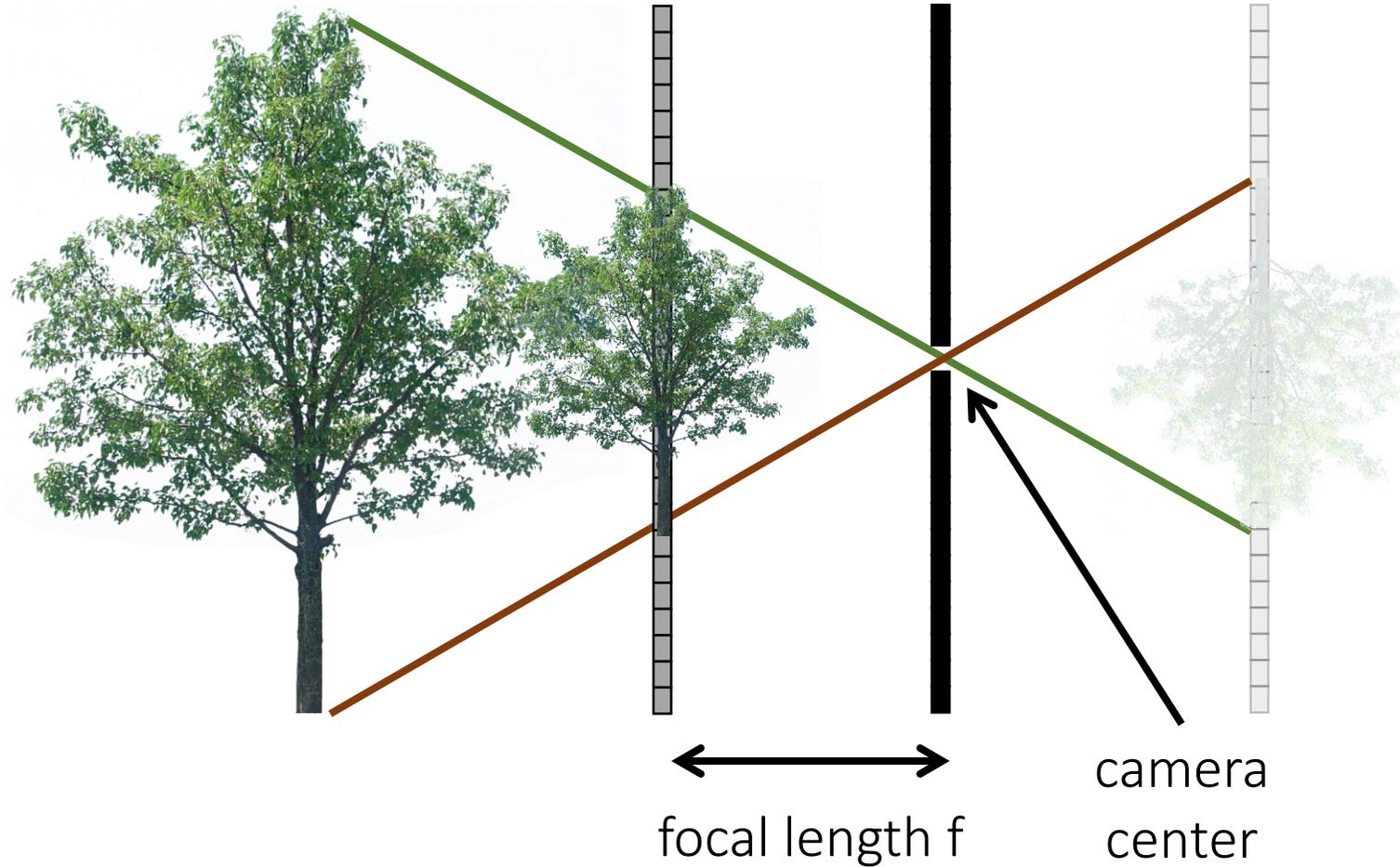
# The pinhole camera



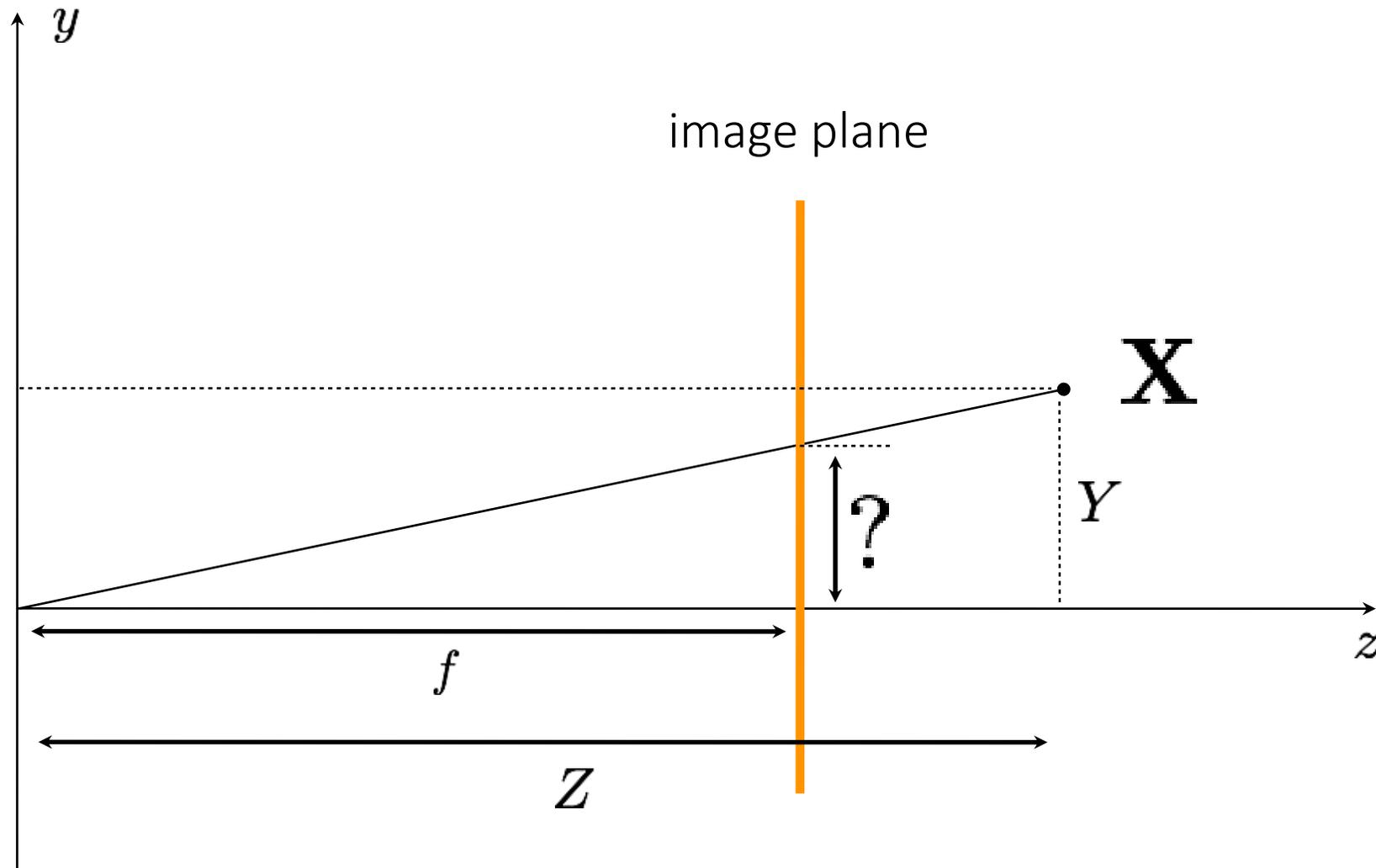
# The (rearranged) pinhole camera

image plane

real-world  
object

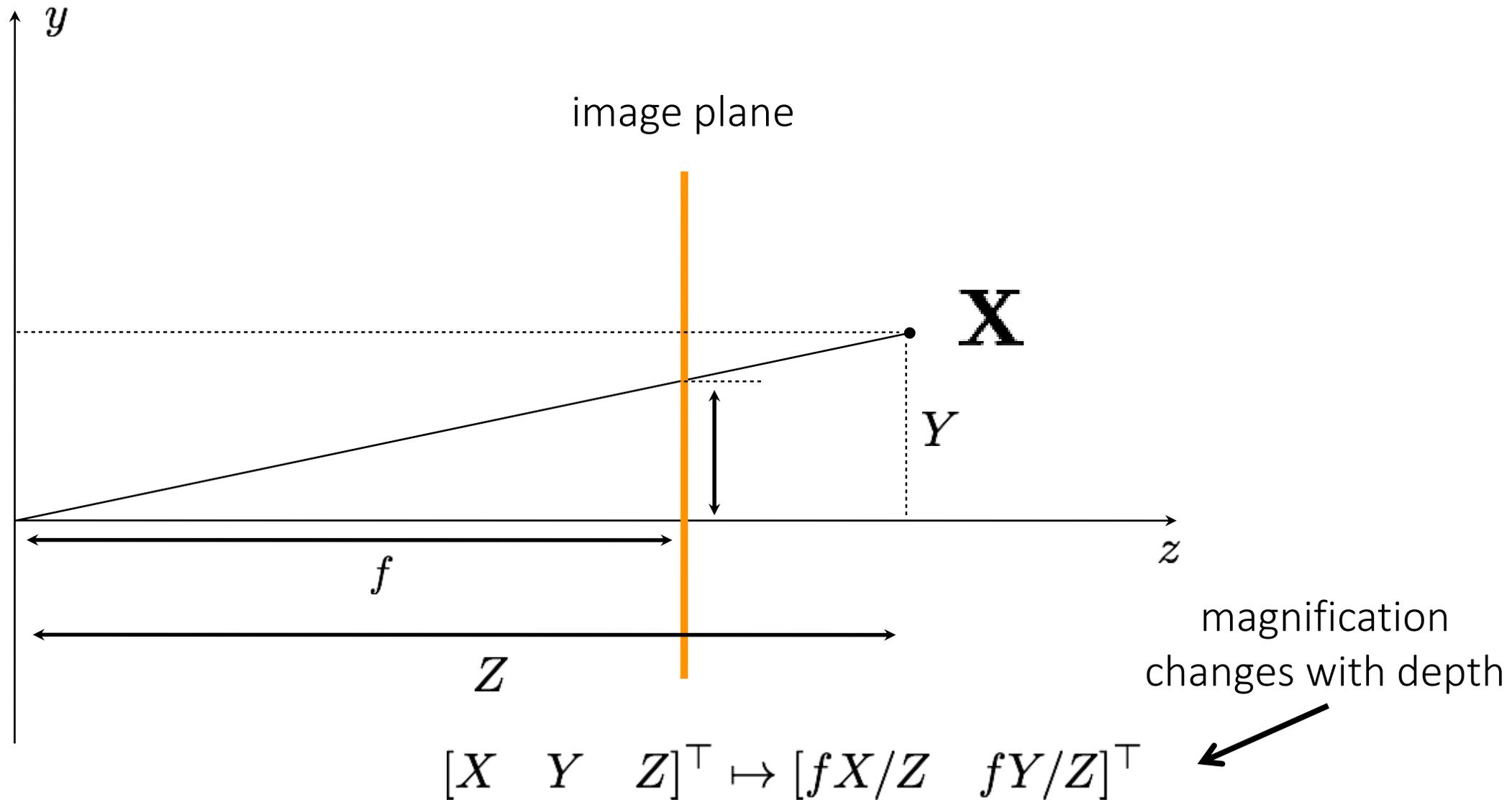


# The 2D view of the (rearranged) pinhole camera



What is the equation for image coordinate  $\mathbf{x}$  in terms of  $\mathbf{X}$ ?

# The 2D view of the (rearranged) pinhole camera



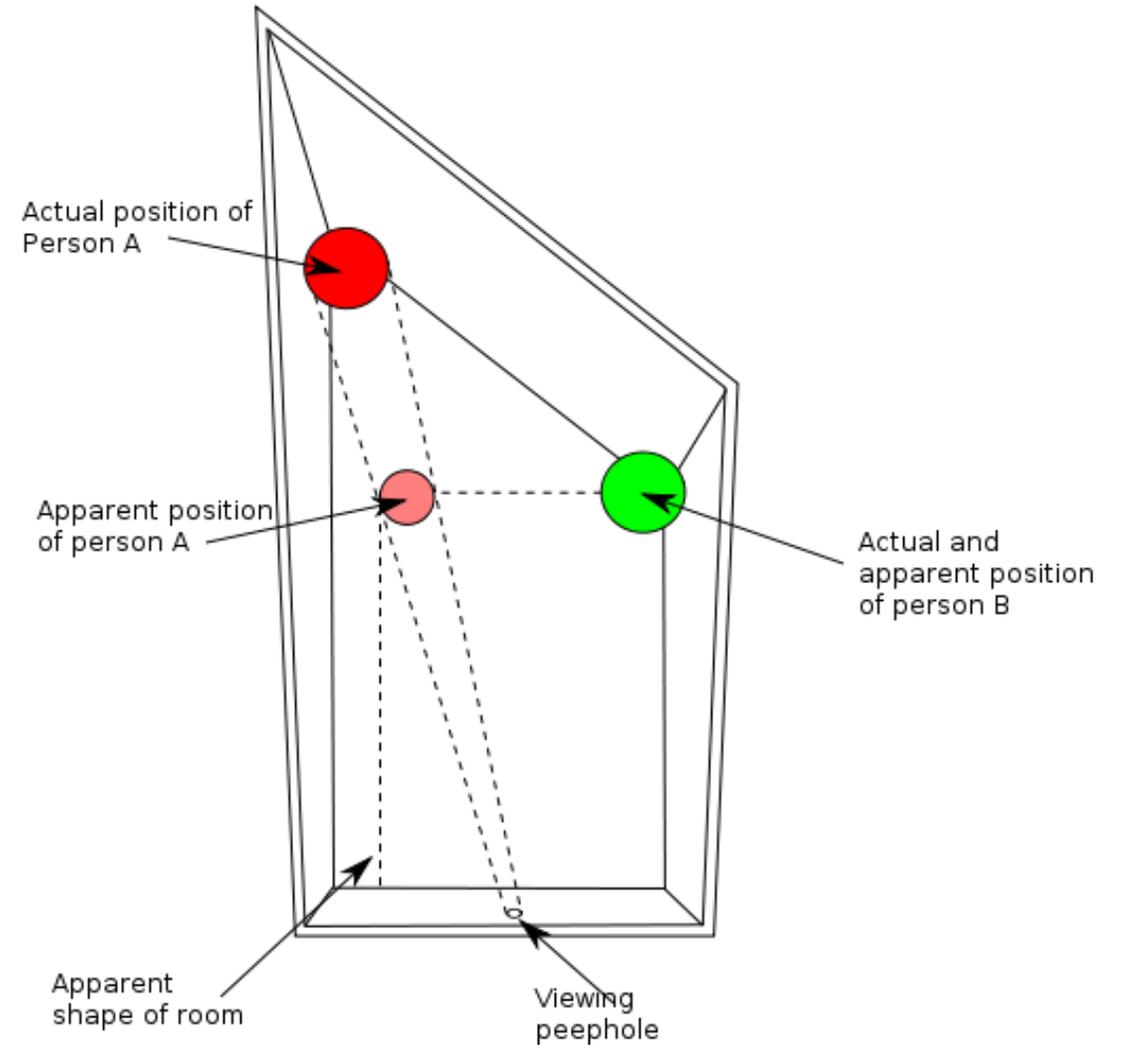
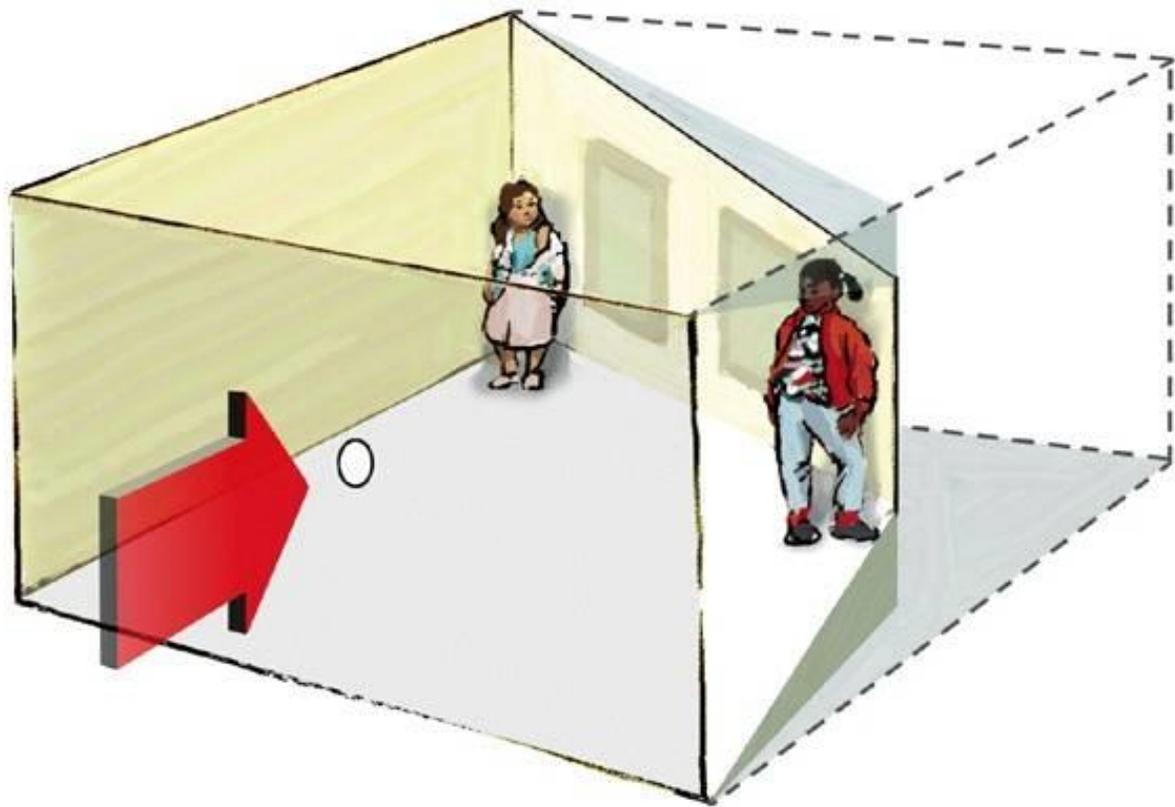
# Forced perspective



# The Ames room illusion

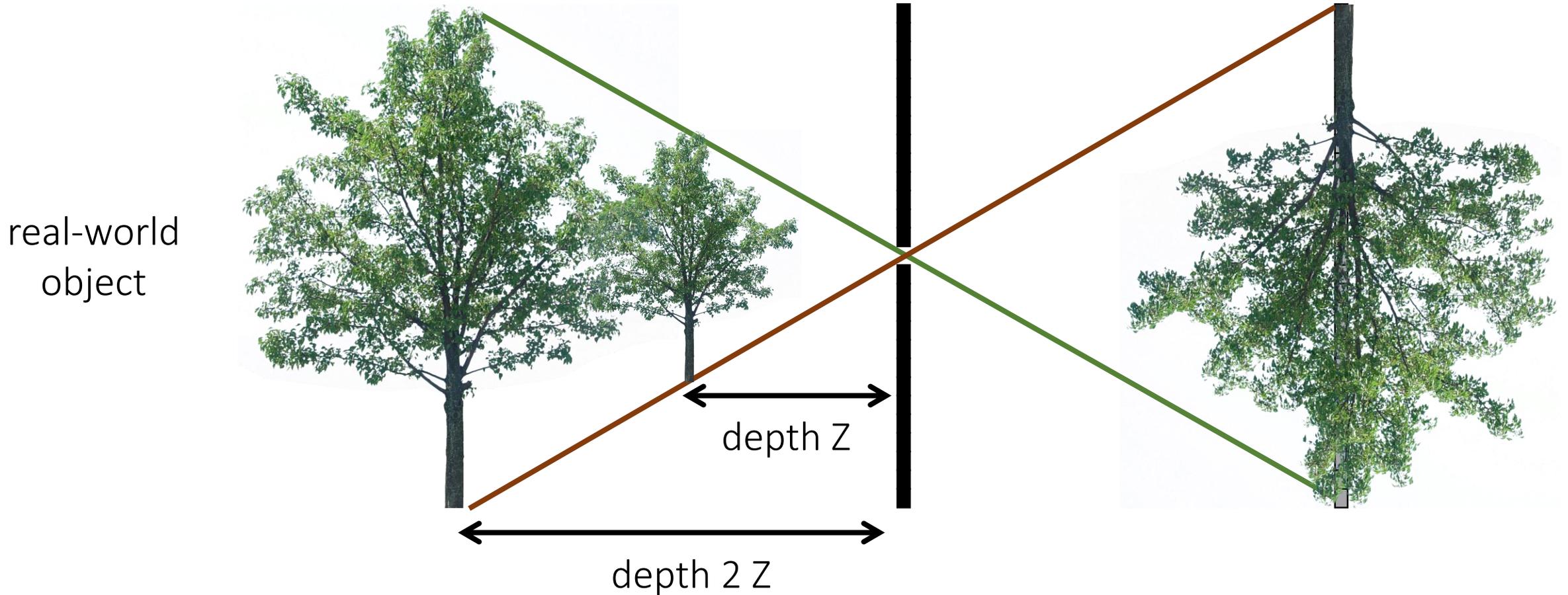


# The Ames room illusion



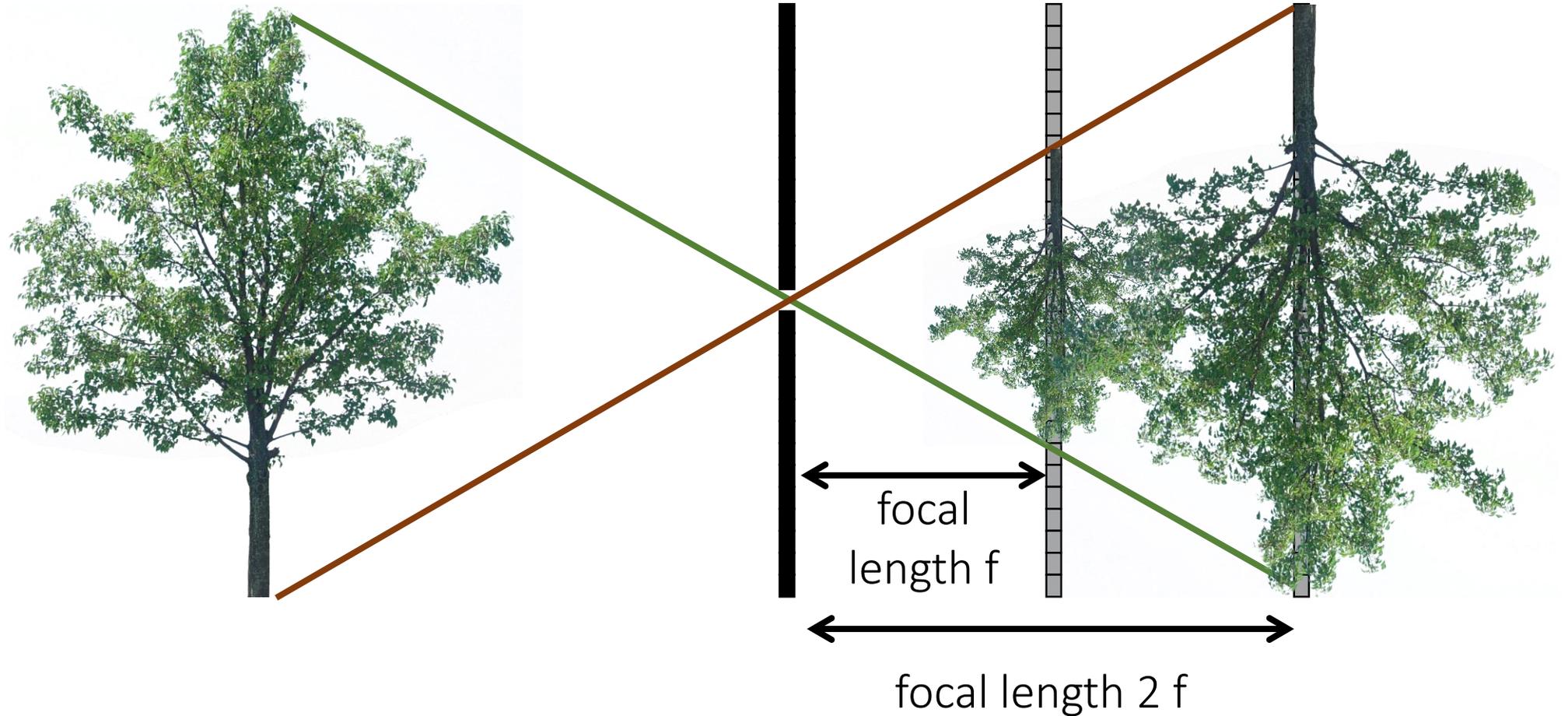
# Magnification depends on depth

What happens as we change the focal length?



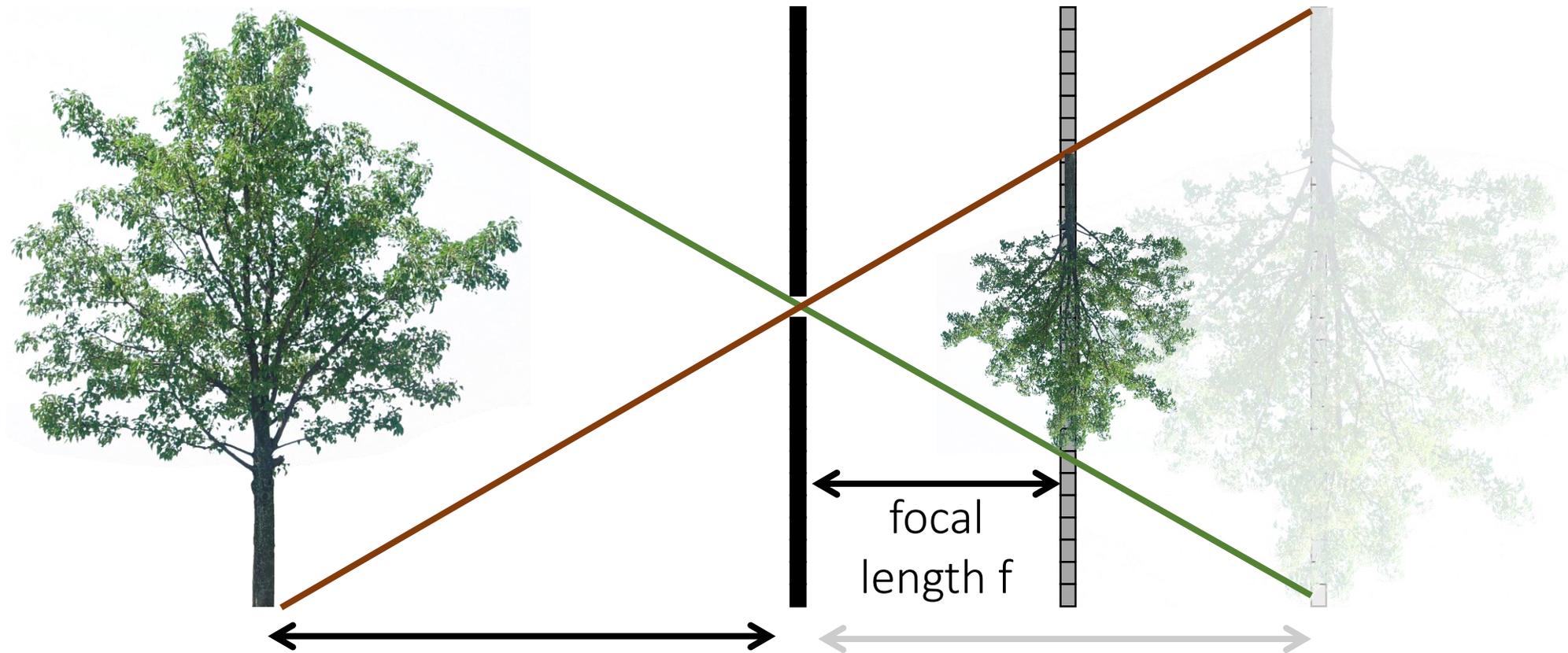
# Magnification depends on focal length

real-world  
object



# What if...

real-world  
object



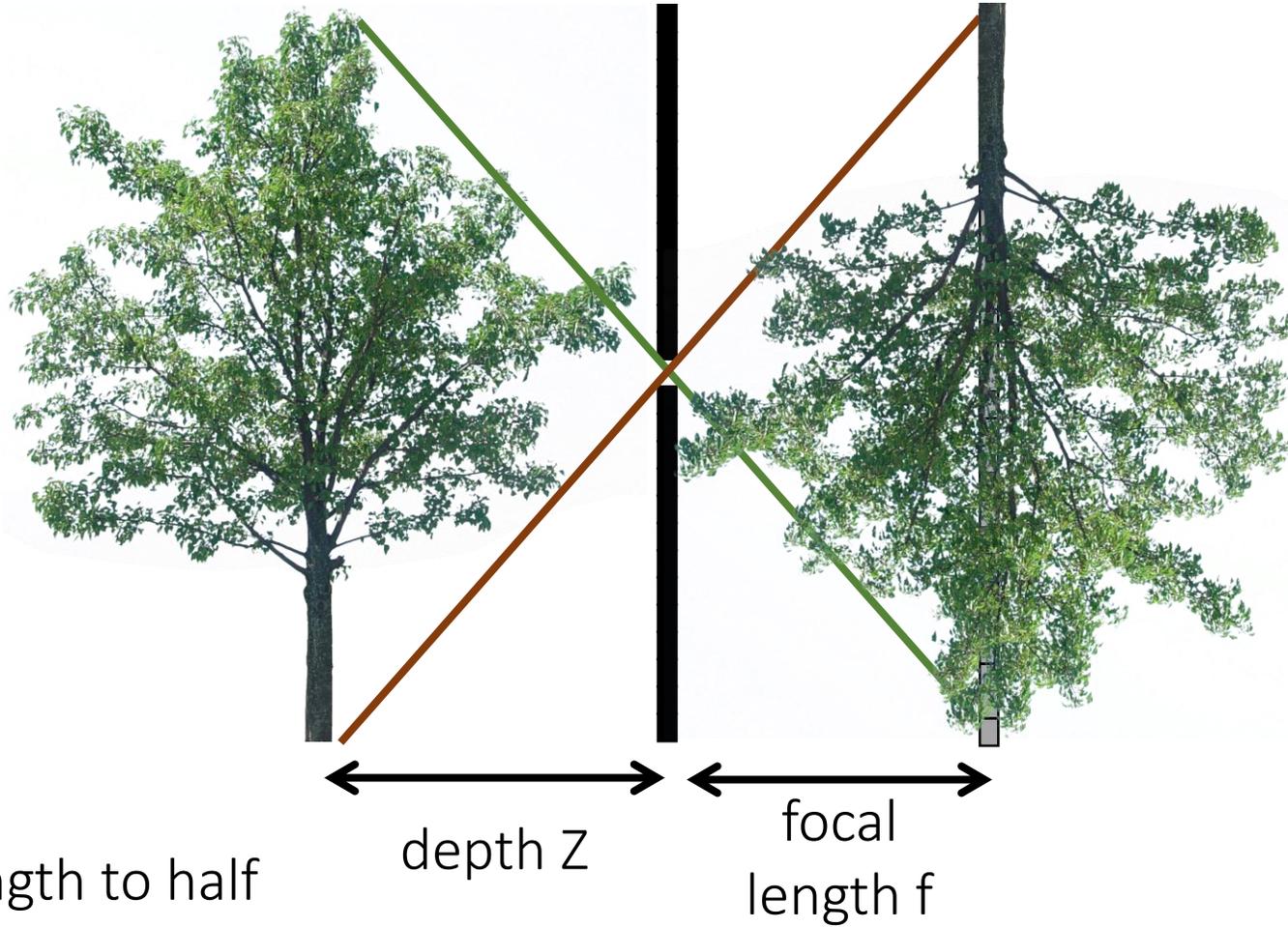
1. Set focal length to half

depth 2 Z

focal length 2 f

# What if...

real-world  
object



Is this the same image as  
the one I had at focal  
length  $2f$  and distance  $2Z$ ?

1. Set focal length to half
2. Set depth to half

# Perspective distortion



long focal length

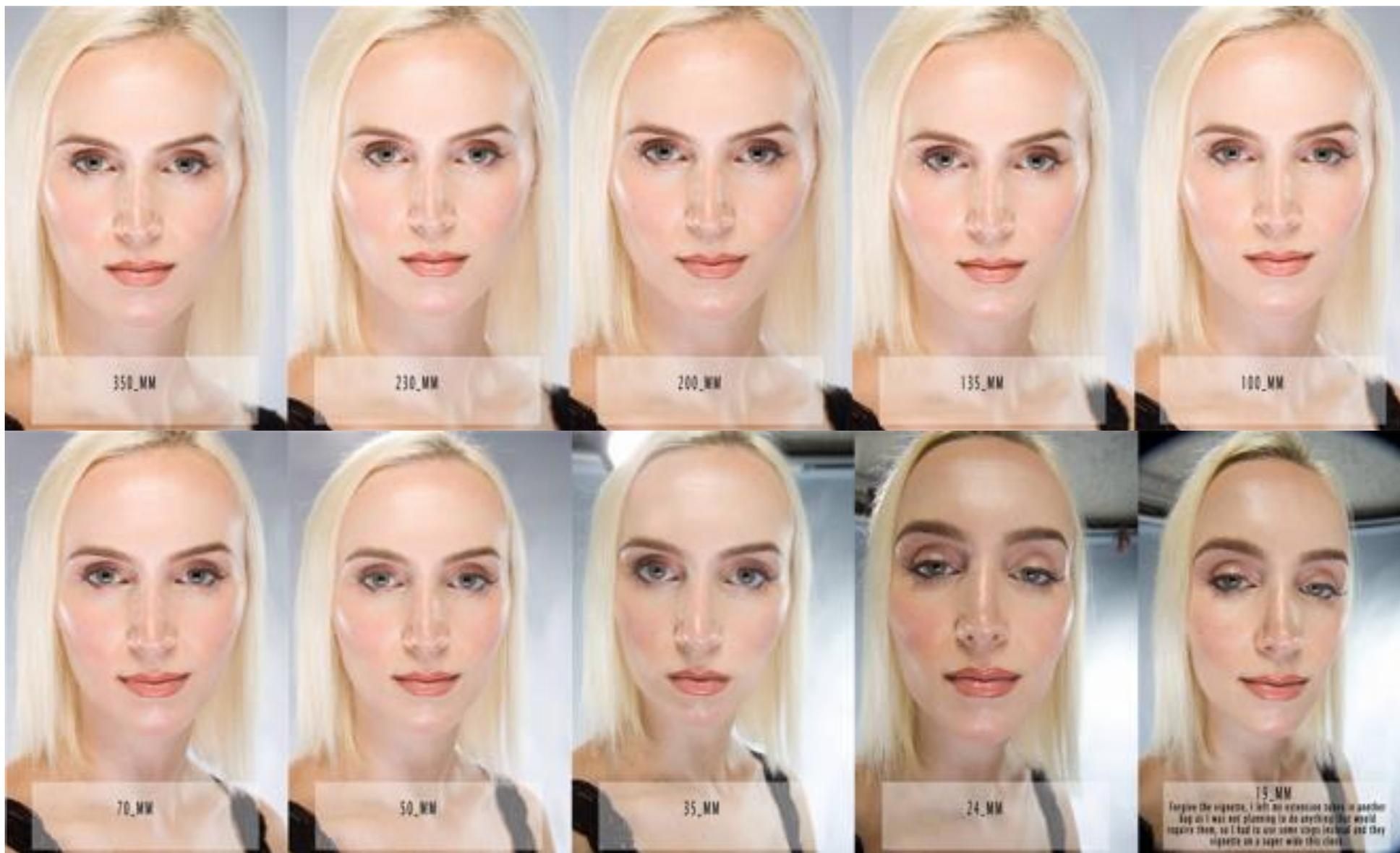


mid focal length



short focal length

# Perspective distortion



# What is the best focal length for portraits?

That's like asking which is better, vi or emacs...



long focal length



mid focal length



short focal length

# Vertigo effect

Named after Alfred Hitchcock's movie

- also known as “dolly zoom”



# Vertigo effect

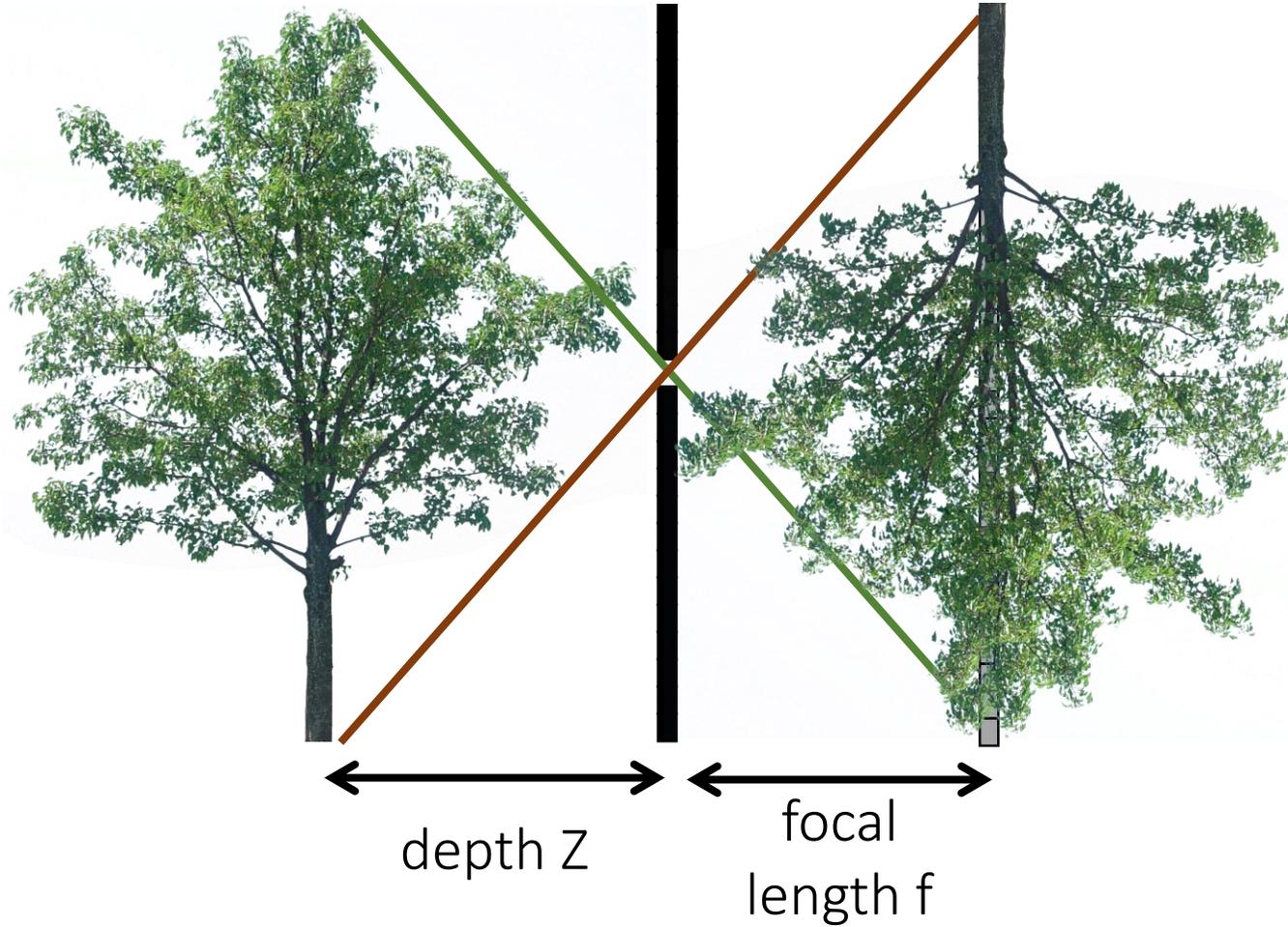


How would you  
create this effect?

Orthographic camera and telecentric lenses

# What if...

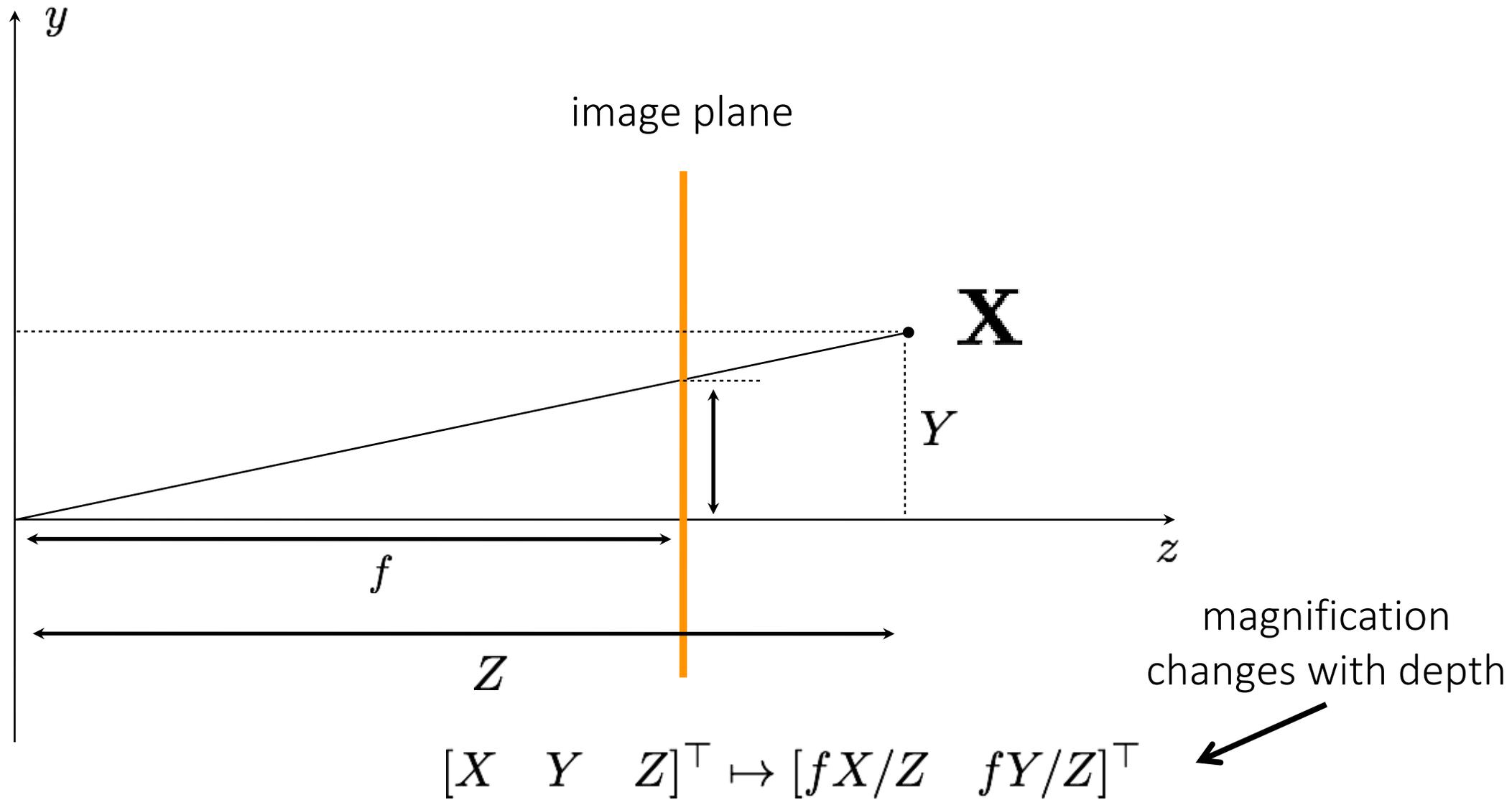
real-world  
object



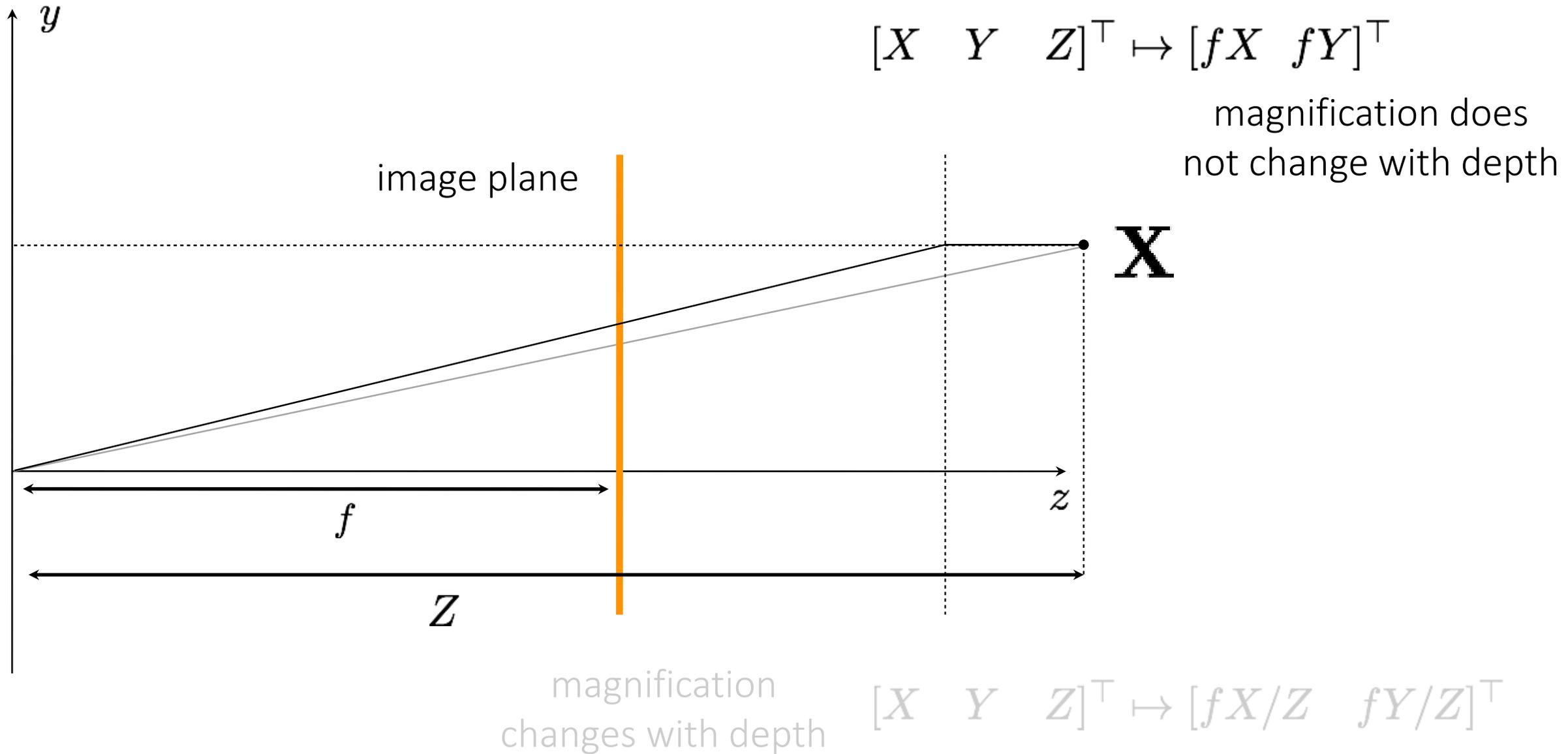
Continue increasing  $Z$  and  $f$  while maintaining same magnification?

$$f \rightarrow \infty \text{ and } \frac{f}{Z} = \text{constant}$$

# The 2D view of the (rearranged) pinhole camera



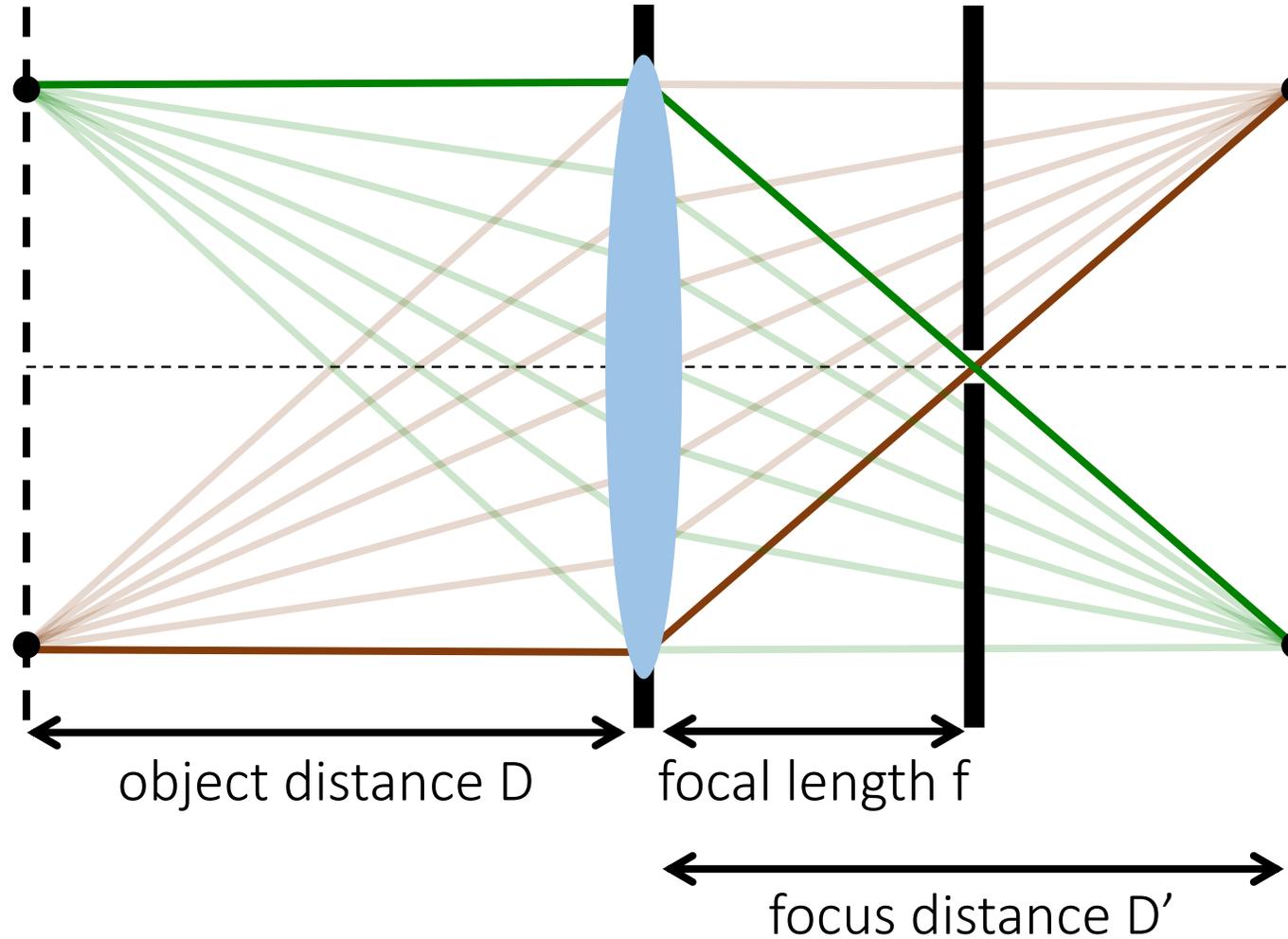
# Orthographic vs pinhole camera



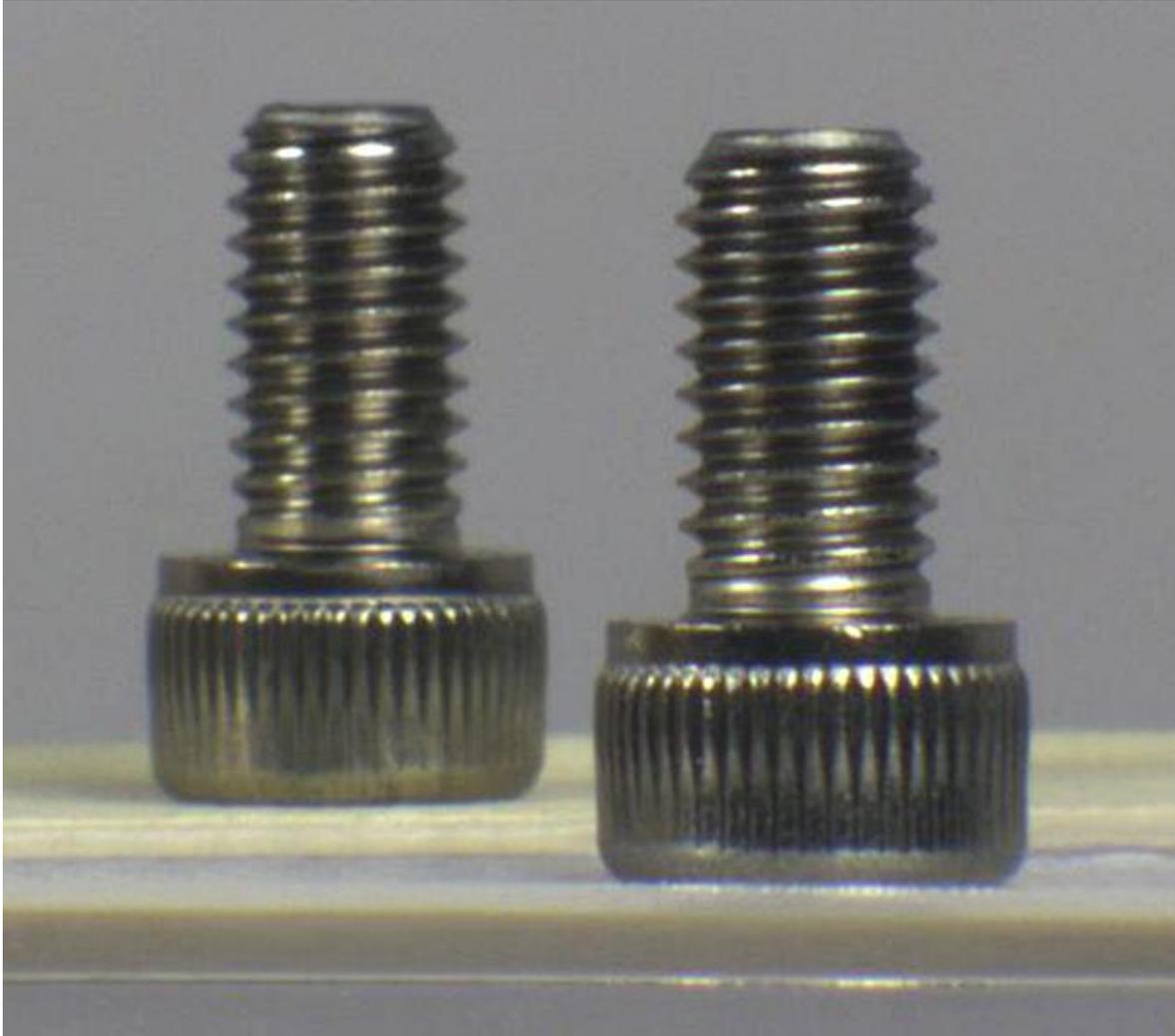
How can we implement such a camera with lenses?

# Telecentric lens

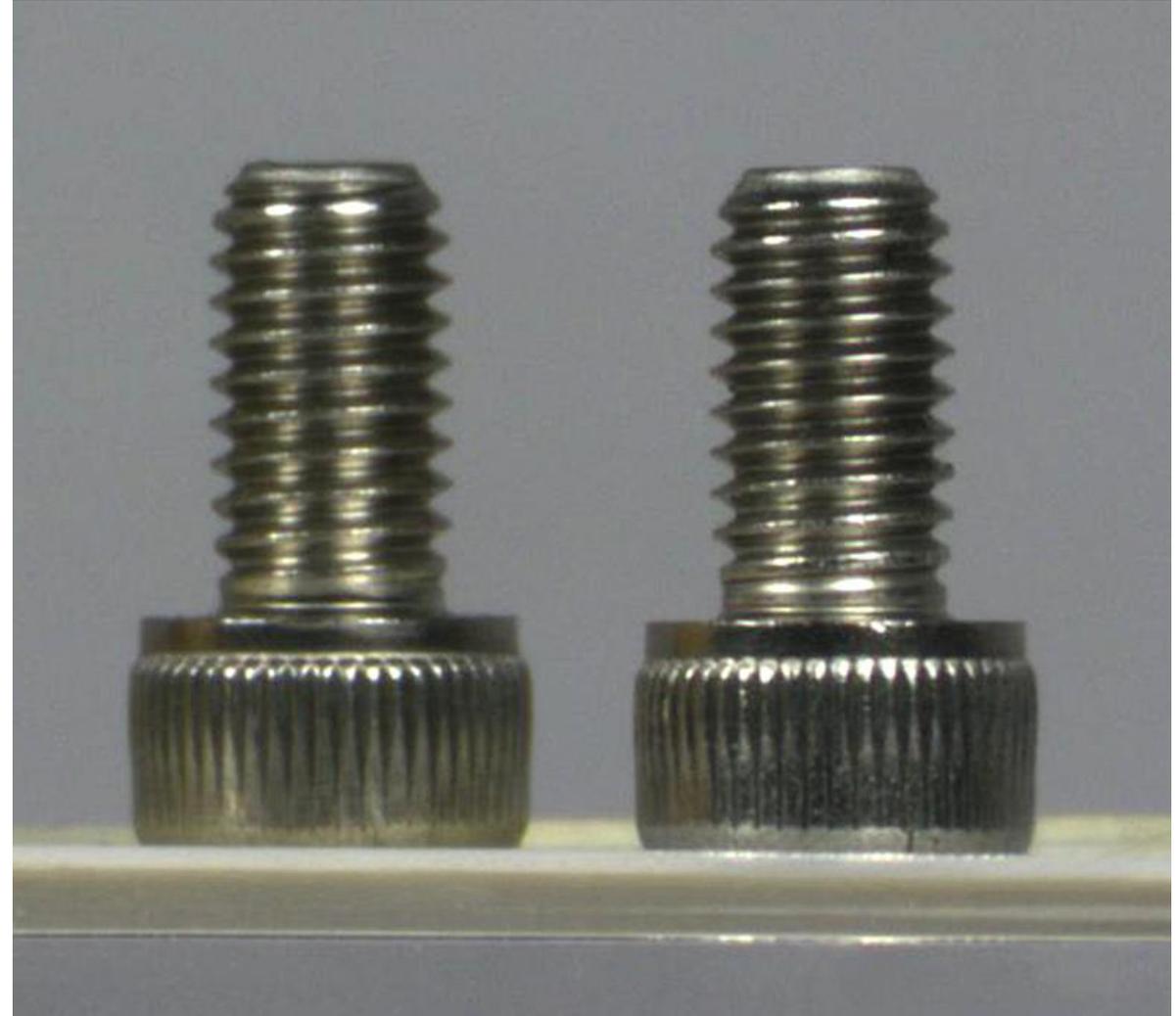
Place a pinhole at focal length, so that only rays parallel to primary ray pass through.



# Regular vs telecentric lens



regular lens



telecentric lens

# References

Basic reading:

- Szeliski textbook, Section 2.1.5, 2.2.3.

Additional reading:

- Hartley and Zisserman, “Multiple View Geometry in Computer Vision,” Cambridge University Press 2004.  
Chapter 6 of this book is a very thorough treatment of camera models.
- Goodman, “Introduction to Fourier Optics,” W.H. Freeman 2004.  
The standard reference on Fourier optics, chapter 4 covers aperture diffraction.
- Ray, “Applied Photographic Optics,” Focal Press 2002.  
A great book covering everything about photographic optics.
- Torralba and Freeman, “Accidental Pinhole and Pinspeck Cameras,” CVPR 2012.  
The eponymous paper discussed in the slides.