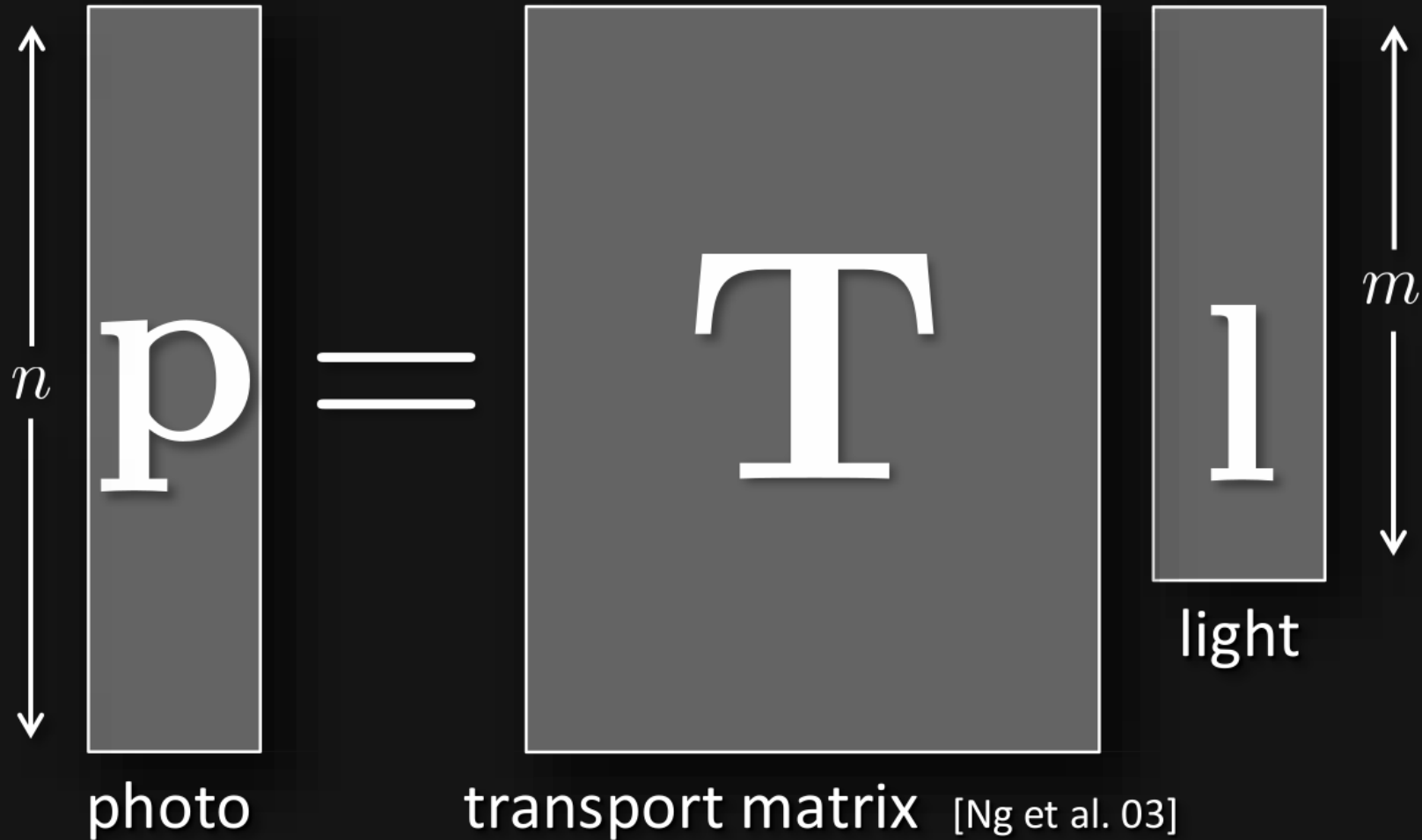


Light transport probing

conventional photography



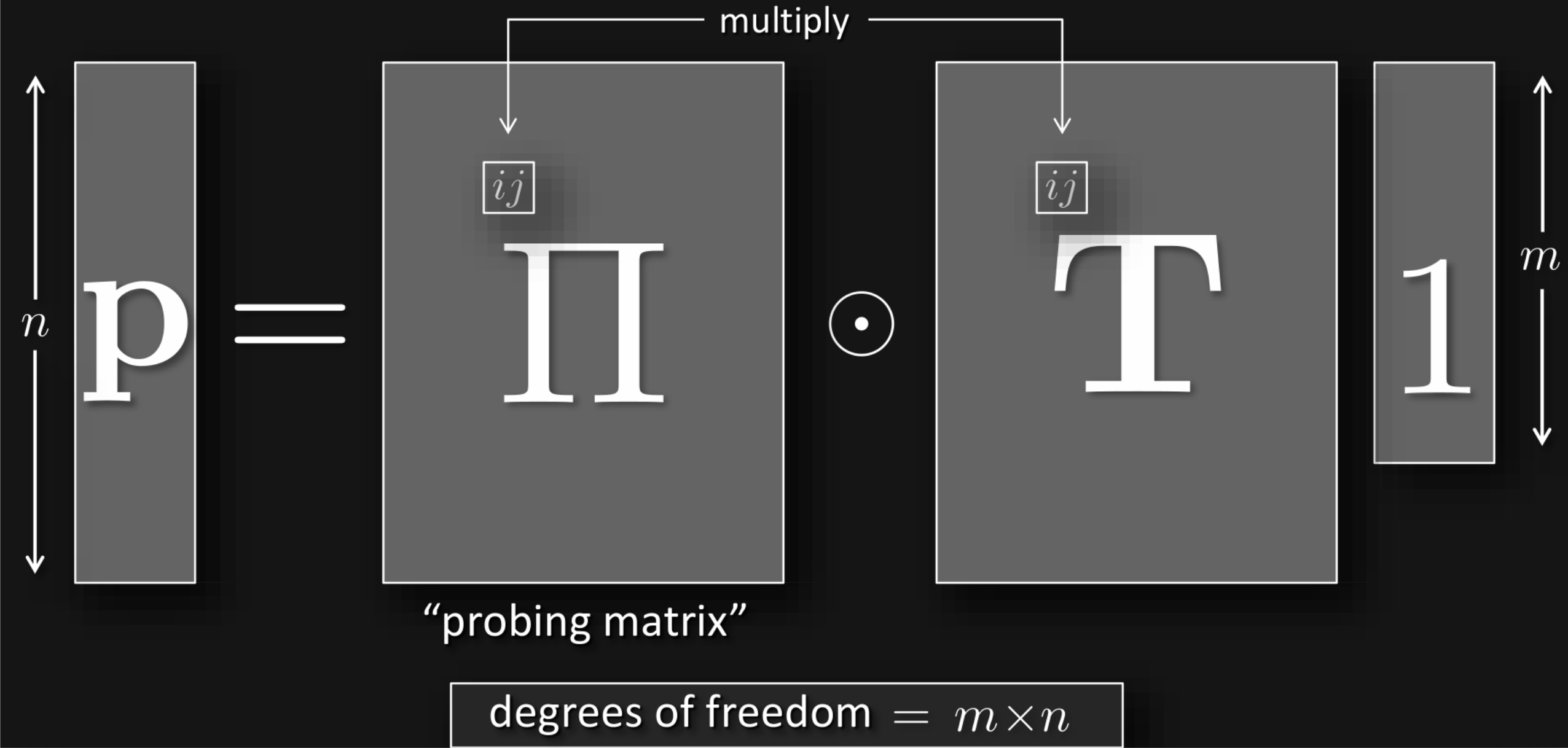
degrees of freedom = m

primal-dual coding photography

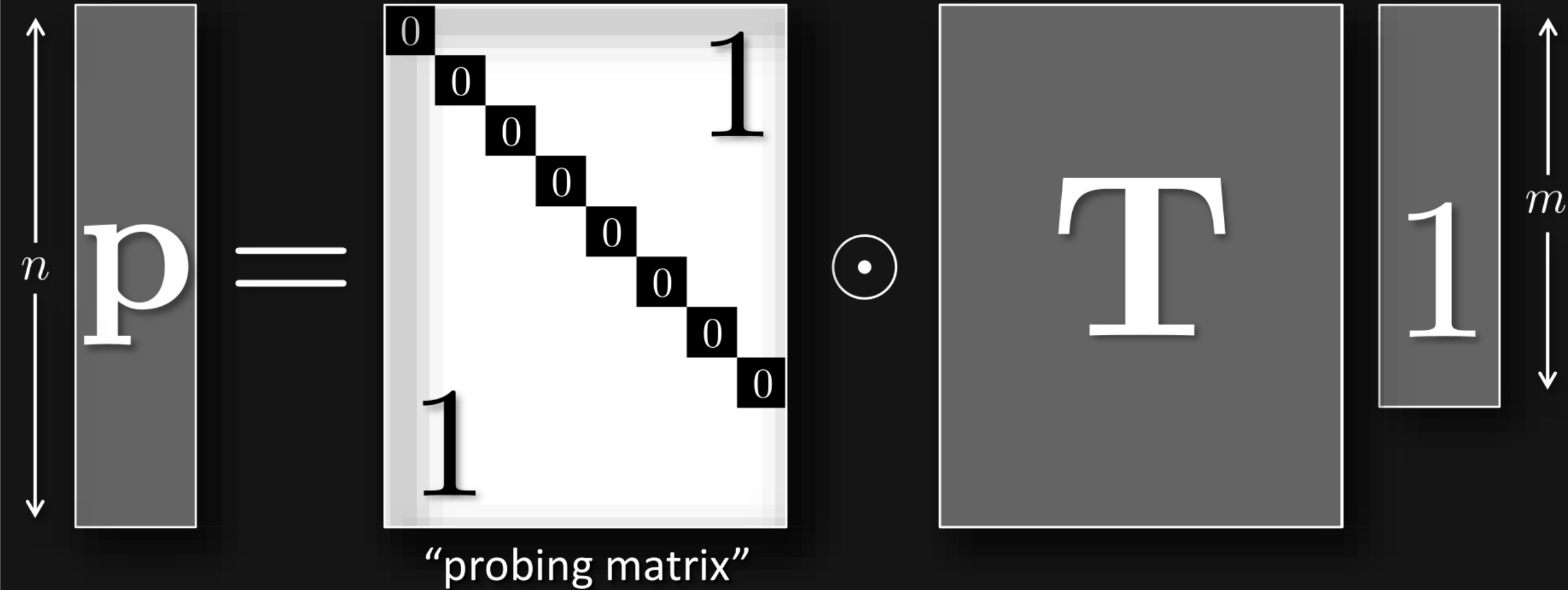
$$\begin{array}{c} \updownarrow n \\ \mathbf{p} \end{array} = \begin{array}{c} \Pi \\ \text{"probing matrix"} \end{array} \odot \begin{array}{c} \mathbf{T} \end{array} \begin{array}{c} \mathbf{1} \\ \updownarrow m \end{array}$$

degrees of freedom = $m \times n$

primal-dual coding photography

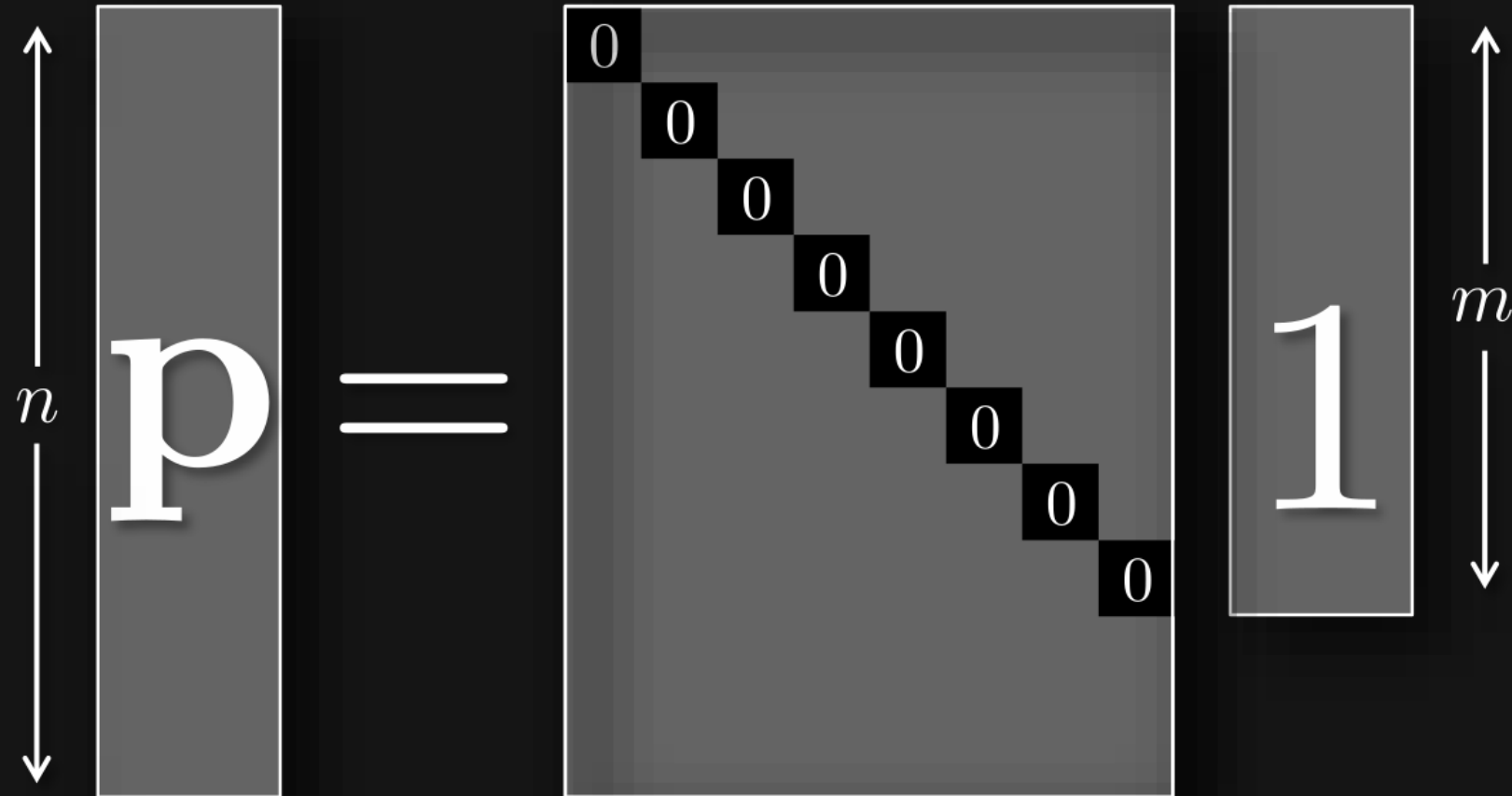


primal-dual coding photography


$$\begin{matrix} \updownarrow n \\ \mathbf{p} \end{matrix} = \begin{matrix} \begin{matrix} 0 & & & & & & 1 \\ & 0 & & & & & \\ & & 0 & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ & & & & & & 0 \end{matrix} \\ \text{"probing matrix"} \end{matrix} \odot \begin{matrix} \mathbf{T} \end{matrix} \begin{matrix} \mathbf{1} \\ \updownarrow m \end{matrix}$$

degrees of freedom = $m \times n$

primal-dual coding photography



The diagram illustrates the primal-dual coding photography equation. On the left, a vertical gray rectangle labeled **p** has a double-headed arrow to its left labeled n . In the center is an equals sign. To the right of the equals sign is a large gray square containing a diagonal sequence of seven black squares, each labeled with the number 0. To the right of this square is another vertical gray rectangle labeled **1**, with a double-headed arrow to its right labeled m .

degrees of freedom = $m \times n$

A diagram illustrating a matrix multiplication operation. It features four gray rectangular blocks with white outlines, arranged horizontally. The first block is a tall vertical rectangle containing the letter 'p'. To its left is a vertical double-headed arrow with the label n next to it, indicating the height of the 'p' block. The second block is a square containing the Greek letter Π . The third block is a square containing the letter 'T'. The fourth block is a tall vertical rectangle containing the number '1'. To its right is a vertical double-headed arrow with the label m next to it, indicating the height of the '1' block. Between the second and third blocks is a small circle with a dot inside, representing the element-wise multiplication operation \odot . An equals sign '=' is placed between the first and second blocks, and another equals sign '=' is placed between the third and fourth blocks, indicating the sequence of operations: $p = \Pi \odot T \cdot 1$.

$$\begin{matrix} \updownarrow n \\ \mathbf{p} \end{matrix} = \Pi \odot \begin{matrix} \mathbf{T} \\ \updownarrow m \\ \mathbf{1} \end{matrix}$$

Diagram illustrating the element-wise multiplication of a vector \mathbf{p} and a matrix \mathbf{T} to produce a matrix $\mathbf{\Pi}$.

- The vector \mathbf{p} has dimension n .
- The matrix \mathbf{T} has dimension m .
- The resulting matrix $\mathbf{\Pi}$ is the element-wise product of \mathbf{p} and \mathbf{T} , denoted by \odot .

$$\mathbf{p} = \mathbf{\Pi} \odot \mathbf{T} \mathbf{1}$$

Diagram illustrating the summation of matrices $\mathbf{\Pi}$ over K elements, weighted by \mathbf{m}_i , to produce a matrix \mathbf{l}_i .

$$\mathbf{\Pi} = \sum_{i=1}^K \mathbf{m}_i \mathbf{l}_i$$

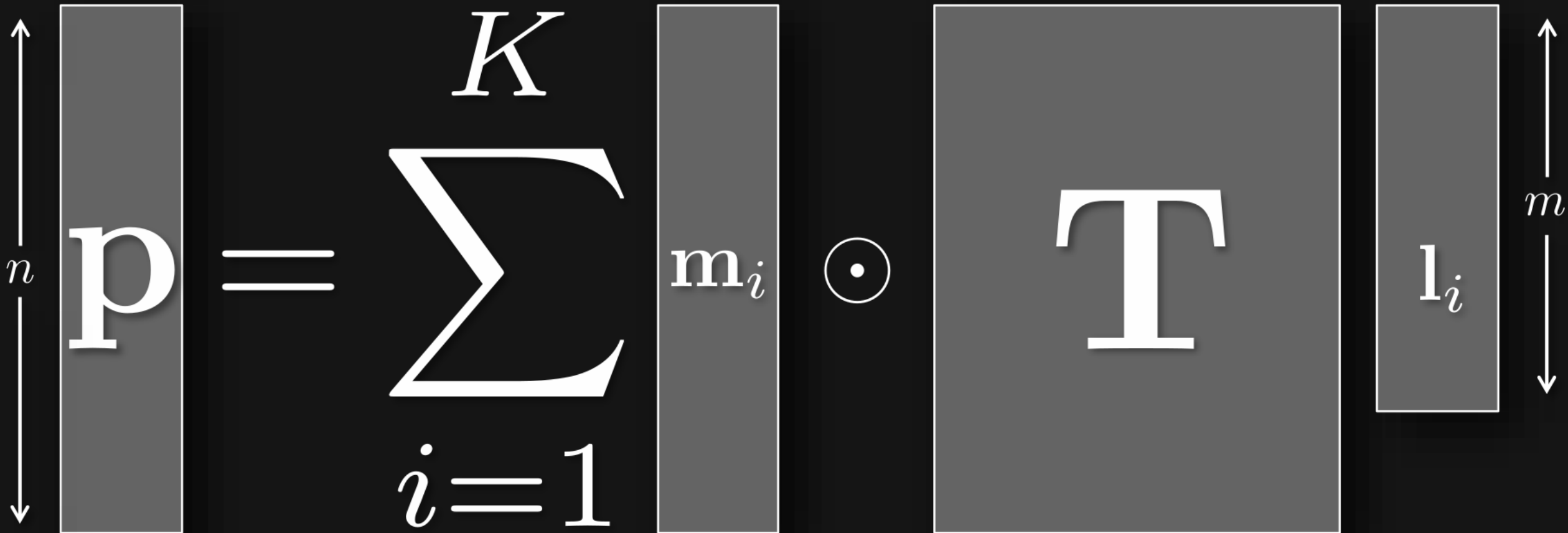


Diagram illustrating a vector equation:

$$\mathbf{p} = \sum_{i=1}^K \mathbf{m}_i \odot \mathbf{T} \mathbf{l}_i$$

The diagram shows the following components:

- \mathbf{p} : A vertical vector of size n .
- $\sum_{i=1}^K$: A summation over K terms.
- \mathbf{m}_i : A vertical vector.
- \odot : An element-wise multiplication operation (Hadamard product).
- \mathbf{T} : A matrix.
- \mathbf{l}_i : A vertical vector of size m .

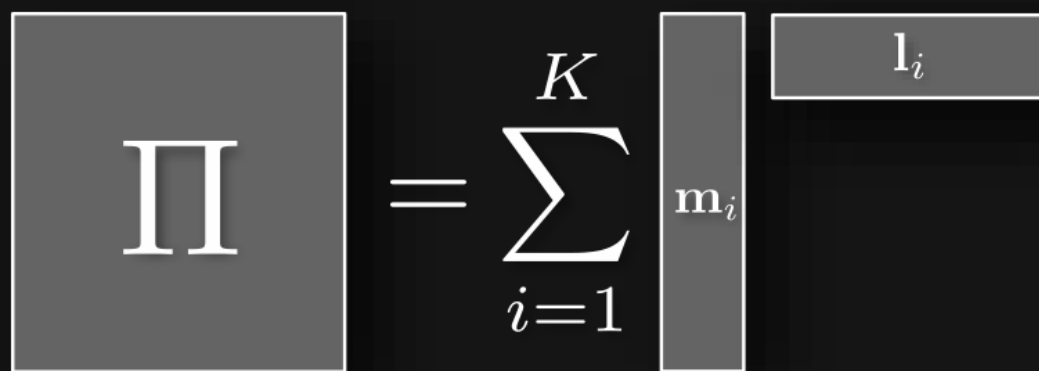


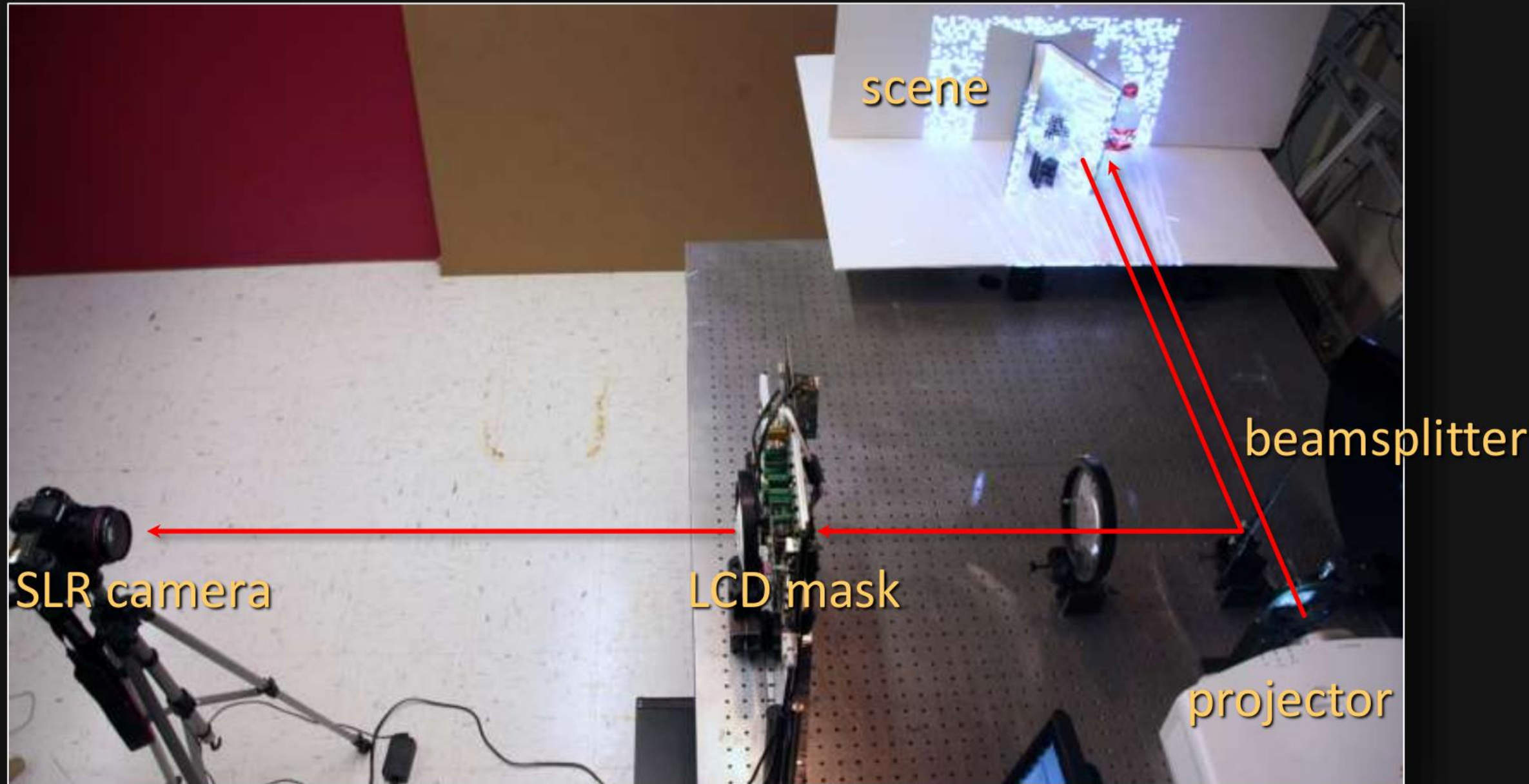
Diagram illustrating a matrix equation:

$$\mathbf{\Pi} = \sum_{i=1}^K \mathbf{m}_i \mathbf{l}_i^T$$

The diagram shows the following components:

- $\mathbf{\Pi}$: A square matrix.
- $\sum_{i=1}^K$: A summation over K terms.
- \mathbf{m}_i : A vertical vector.
- \mathbf{l}_i^T : A horizontal vector (transpose of \mathbf{l}_i).

experimental setup



$$\begin{array}{c} \updownarrow n \\ \mathbf{p} \end{array} = \sum_{i=1}^K \begin{array}{c} \mathbf{m}_i \end{array} \odot \begin{array}{c} \mathbf{T} \end{array} \begin{array}{c} \updownarrow m \\ \mathbf{l}_i \end{array}$$

step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter

$$\begin{array}{c} \updownarrow n \\ \mathbf{p} \end{array} = \sum_{i=1}^K \begin{array}{c} \mathbf{m}_i \end{array} \odot \begin{array}{c} \mathbf{T} \end{array} \begin{array}{c} \mathbf{l}_i \end{array} \begin{array}{c} \updownarrow m \end{array}$$

step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter

$$\begin{array}{c} \text{height } n \\ \downarrow \\ \mathbf{p} \\ \uparrow \\ \text{height } n \end{array} = \sum_{i=1}^K \begin{array}{c} \mathbf{m}_i \end{array} \odot \begin{array}{c} \mathbf{T} \\ \text{width } m \end{array} \begin{array}{c} \mathbf{l}_i \\ \text{height } m \end{array}$$

step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter

$$\begin{array}{c} \text{size } n \end{array} \mathbf{p} = \sum_{i=1}^K \begin{array}{c} \text{size } n \\ \mathbf{m}_i \end{array} \odot \mathbf{T} \quad \begin{array}{c} \text{size } m \\ \mathbf{l}_i \end{array}$$

step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter

$$\begin{array}{c} \text{height } n \\ \text{p} \end{array} = \sum_{i=1}^K \begin{array}{c} \text{m}_i \end{array} \odot \begin{array}{c} \text{T} \end{array} \begin{array}{c} \text{l}_i \\ \text{height } m \end{array}$$

step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter

$$\begin{array}{c} \text{height } n \\ \text{vector } \mathbf{p} \end{array} = \sum_{i=1}^K \begin{array}{c} \text{vector } \mathbf{m}_i \end{array} \odot \begin{array}{c} \text{matrix } \mathbf{T} \end{array} \begin{array}{c} \text{vector } \mathbf{l}_i \\ \text{height } m \end{array}$$

step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter



step 5

close shutter

step 4

repeat K times

step 3

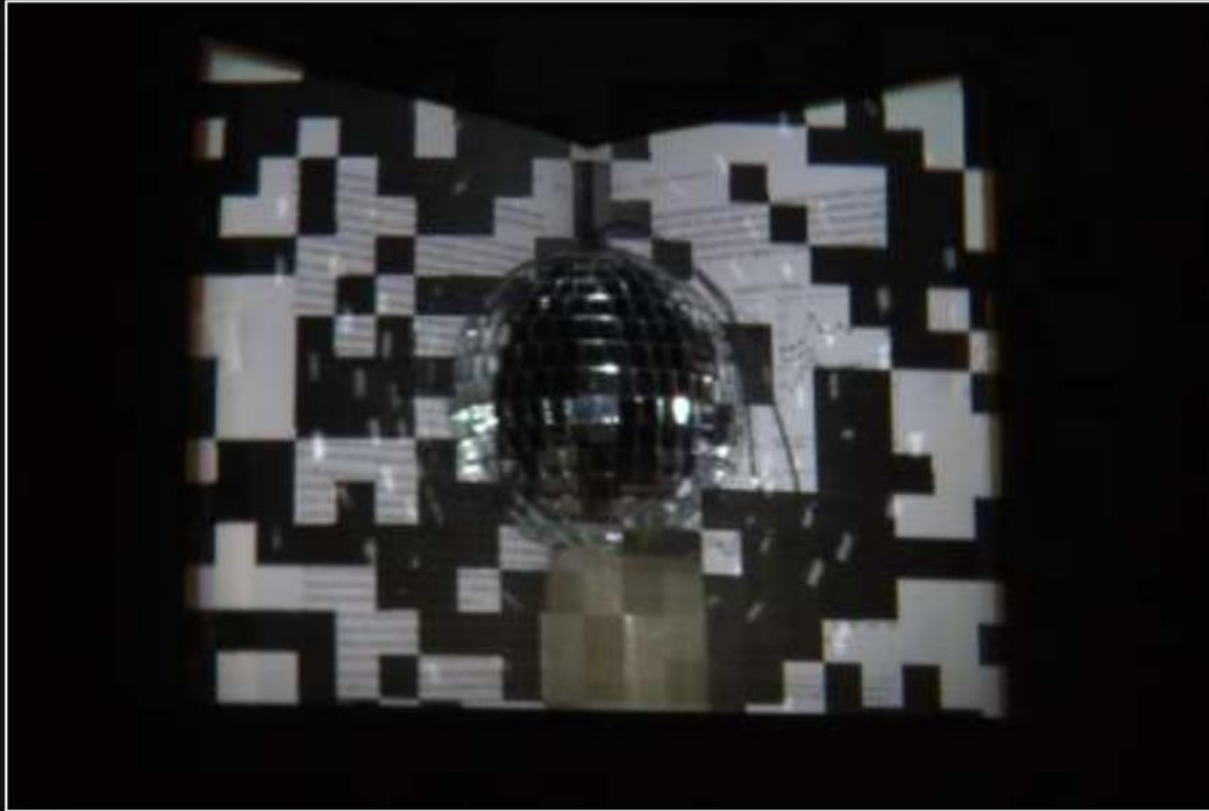
attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter



step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter



step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

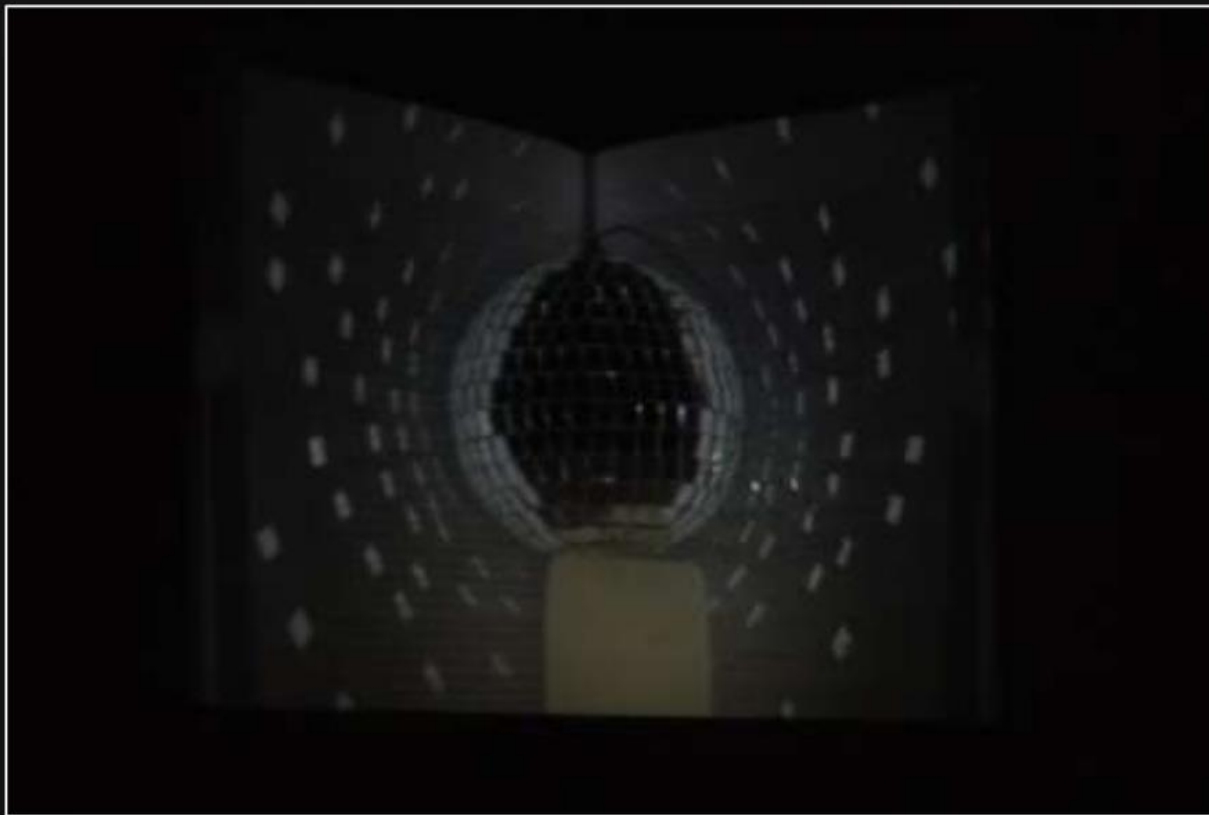
step 2

illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter

$$\sum_{i=1}^K$$



step 5

close shutter

step 4

repeat K times

step 3

attenuate image
with vector \mathbf{m}_i
(dual code)

step 2

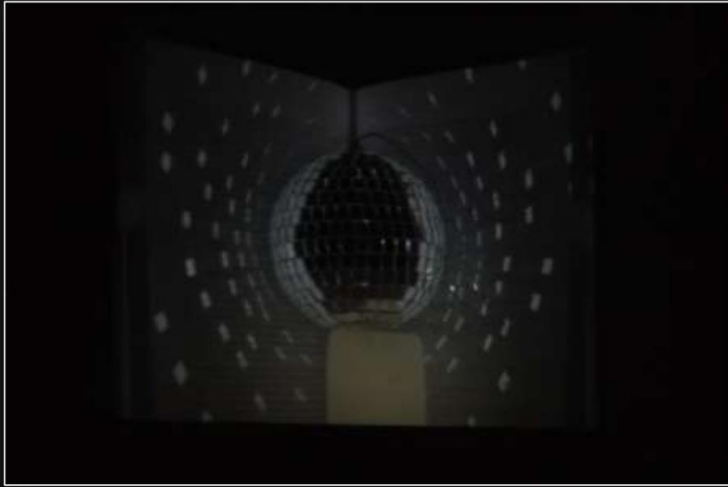
illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1

open shutter

28 of 70

192.79%



step 5
close shutter

step 4
repeat K times

step 3
attenuate image
with vector \mathbf{m}_i
(dual code)

step 2
illuminate scene
with vector \mathbf{l}_i
(primal code)

step 1
open shutter

Rademacher primal-dual codes

stochastic diagonal estimator [Bekas et al. 07]

primal codes are Rademacher random vectors: $\mathbf{l}_i =$ random vector in $\{-1, +1\}^m$

dual codes derive from primal code: $\mathbf{m}_i = \mathbf{l}_i$

codes converge to identity probing matrix: $(\mathbf{I} \odot \mathbf{T})\mathbf{1} \approx \frac{1}{K} \sum_{i=1}^K \mathbf{m}_i \odot \mathbf{T} \mathbf{l}_i$

variance of pixel n for K primal-dual codes $= \frac{1}{K} \sum_{m=1, n \neq m}^M \mathbf{T}_{nm}^2$

aperture correlation (microscopy) is a diagonal estimator [Wilson et al. 96, Levoy et al. 04]

stochastic estimator for general probing

dual codes for general probing matrix $\mathbf{\Pi}$: $\mathbf{m}_i = \mathbf{\Pi} \mathbf{l}_i$

Direct-global separation using diagonal
probing (co-axial case)

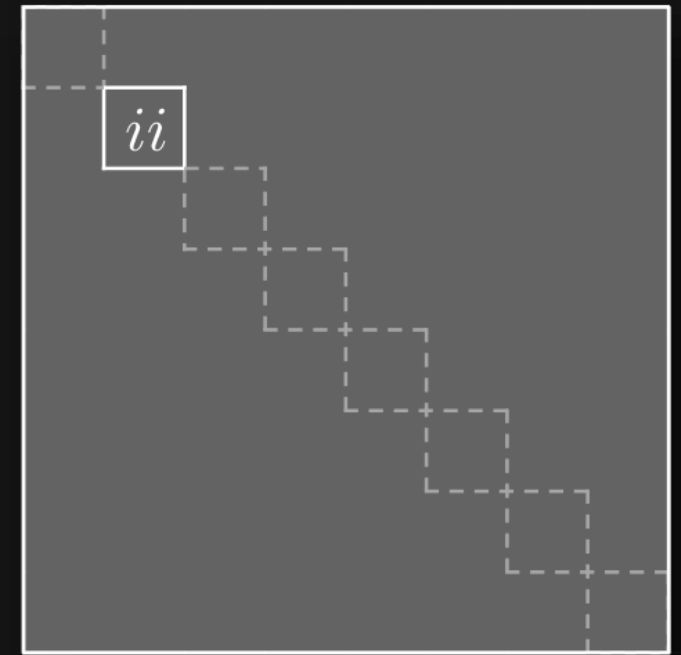
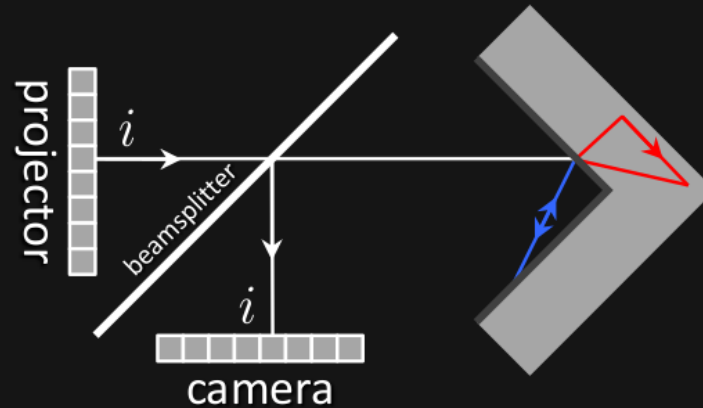
designing probing matrices



transport matrix

designing probing matrices

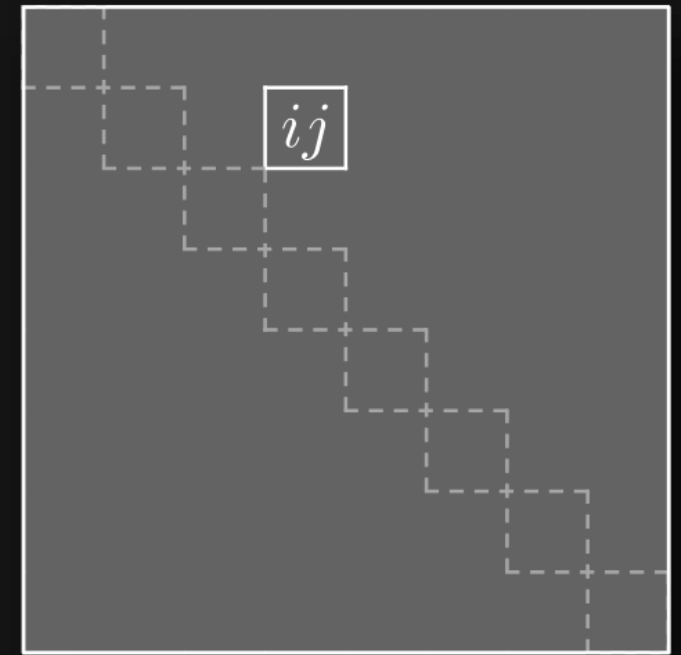
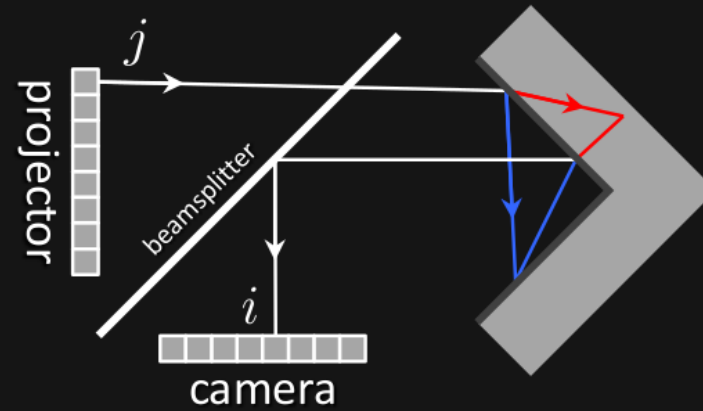
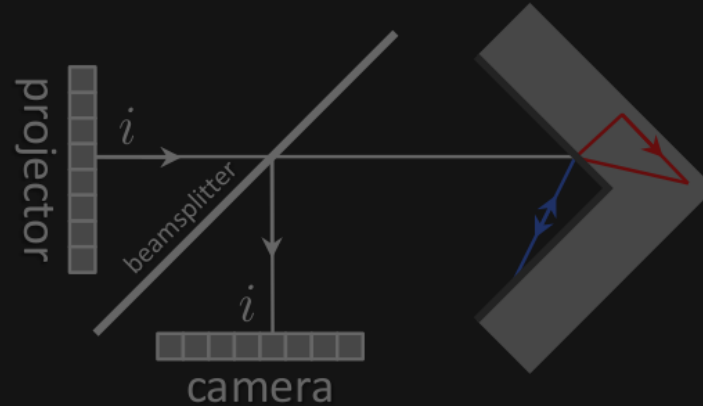
Coaxial configuration:
use a beamsplitter to
make projector and
camera effectively
collocated



transport matrix

designing probing matrices

Coaxial configuration:
use a beamsplitter to
make projector and
camera effectively
collocated



transport matrix

coaxial example: contrast-enhancing direct light



conventional
photo



all light paths

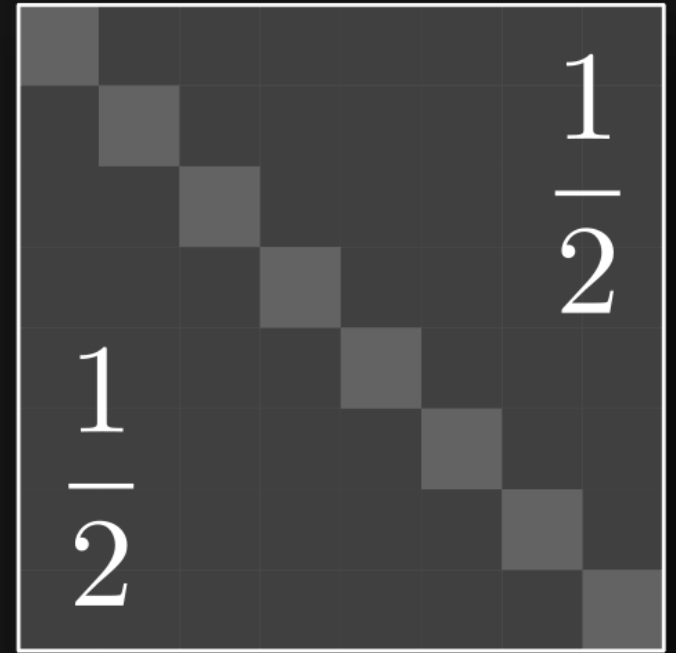
coaxial example: contrast-enhancing direct light



conventional
photo



direct +
 $\frac{1}{2}$ indirect



direct + $\frac{1}{2}$ indirect
light paths

coaxial example: contrast-enhancing direct light

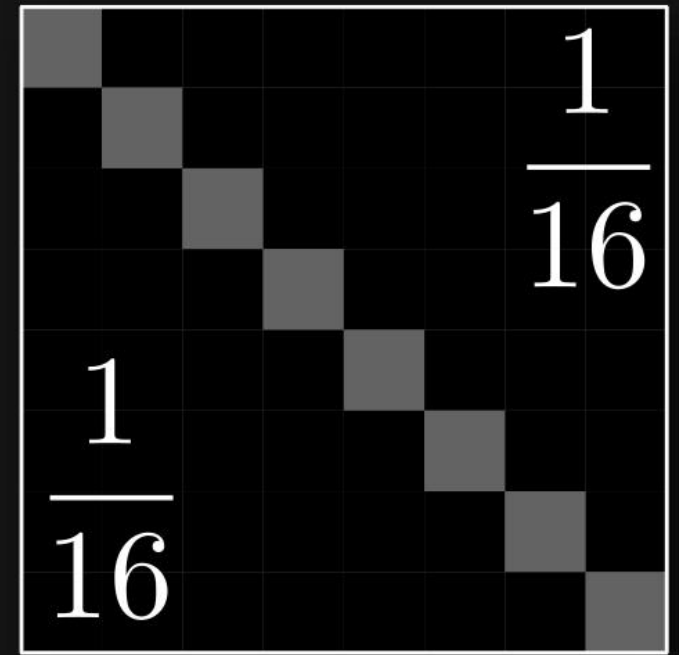


conventional
photo

direct +
 $\frac{1}{2}$ indirect

direct +
 $\frac{1}{4}$ indirect

direct +
 $\frac{1}{16}$ indirect



direct + $\frac{1}{16}$ indirect
light paths

coaxial example: capturing short to long range paths

conventional



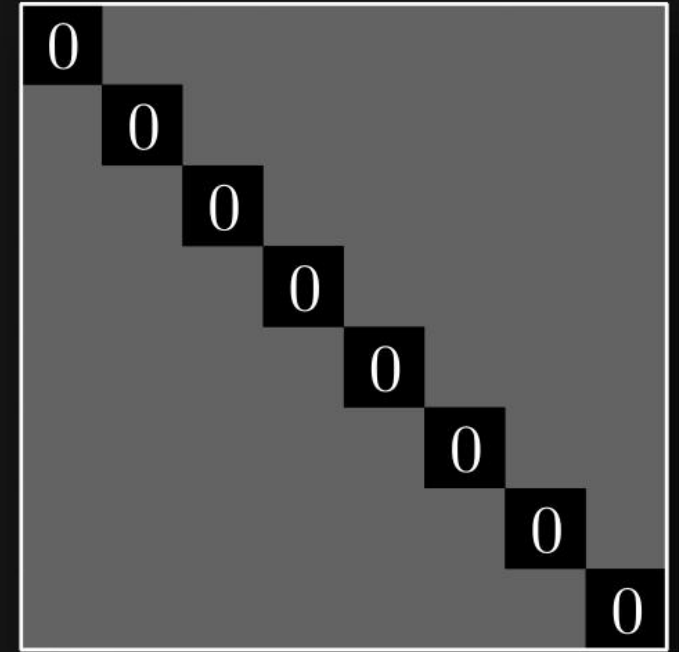
all light paths

coaxial example: capturing short to long range paths

conventional



indirect



indirect light paths

coaxial example: capturing short to long range paths

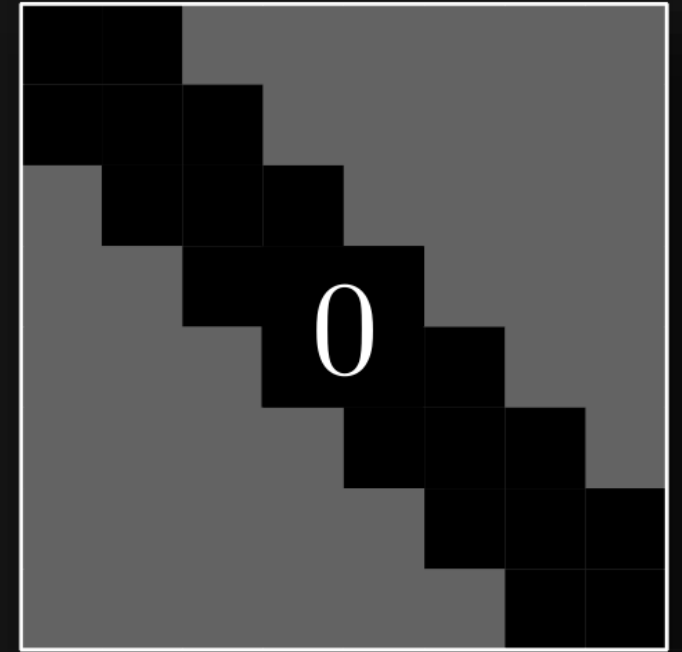
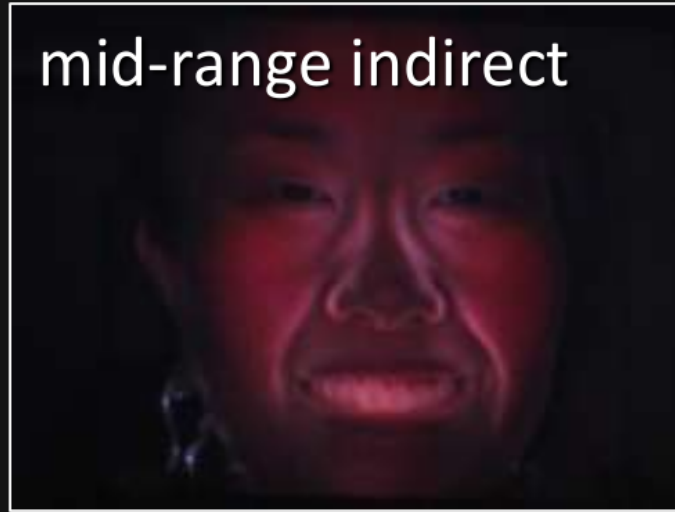
conventional



indirect



mid-range indirect



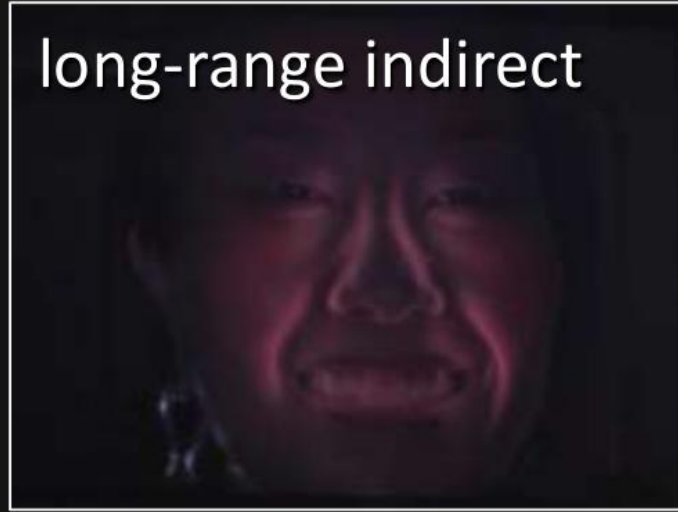
medium to long range
indirect light paths

coaxial example: capturing short to long range paths

conventional



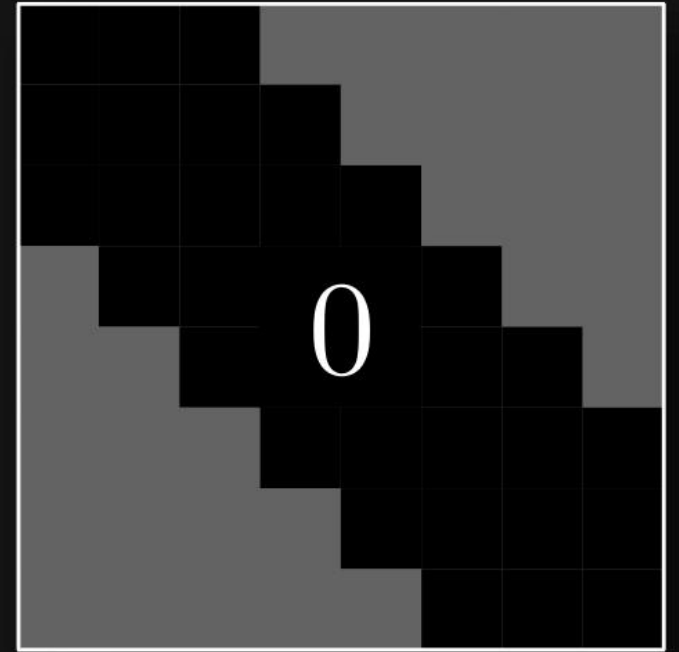
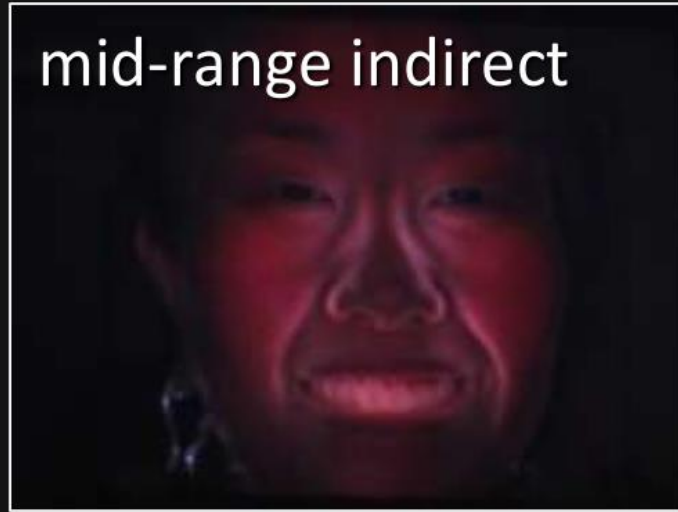
long-range indirect



indirect



mid-range indirect



long range indirect
light paths

coaxial example: separating light transport effects

conventional



T

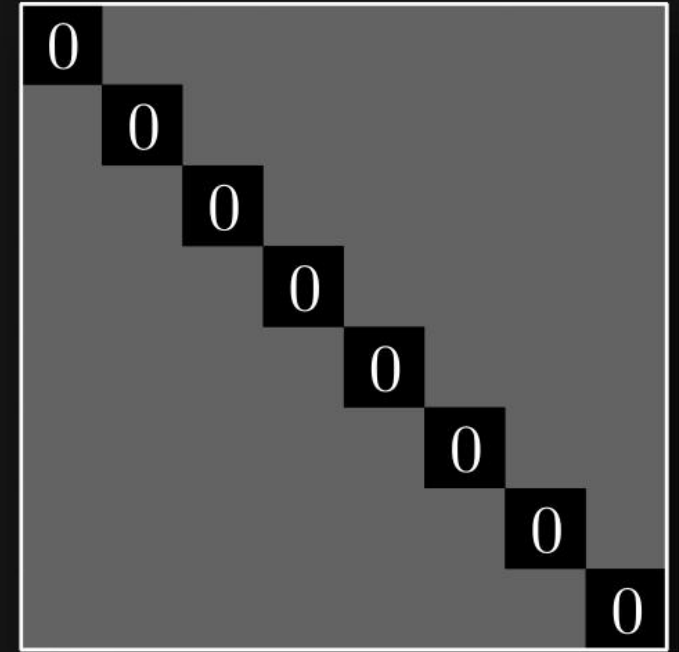
all light paths

coaxial example: separating light transport effects

conventional



indirect



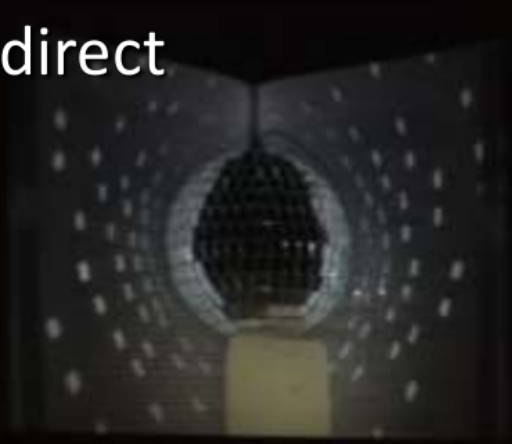
indirect light paths

coaxial example: separating light transport effects

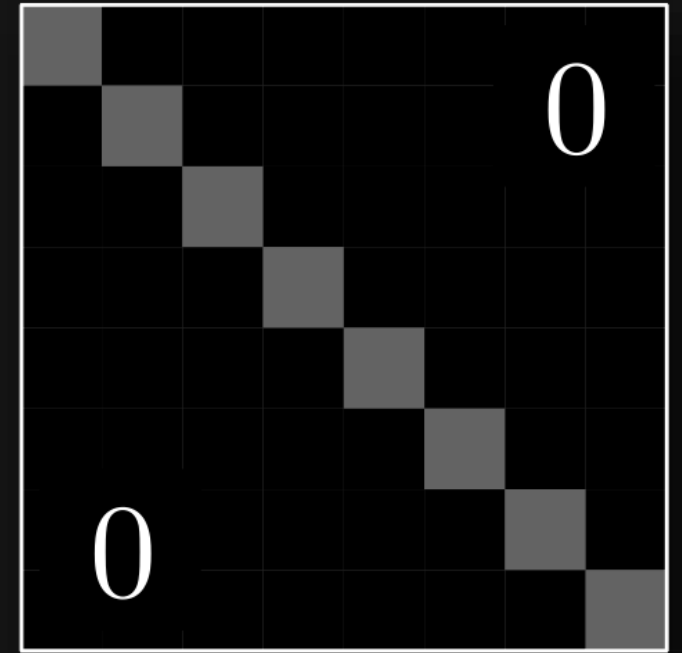
conventional



indirect



direct + backscatter



direct + back-scatter
light paths

coaxial example: separating light transport effects

indirect [Nayar et al. 06]



direct [Nayar et al. 06]



indirect



direct + backscatter



coaxial example: separating light transport effects

low-freq. indirect



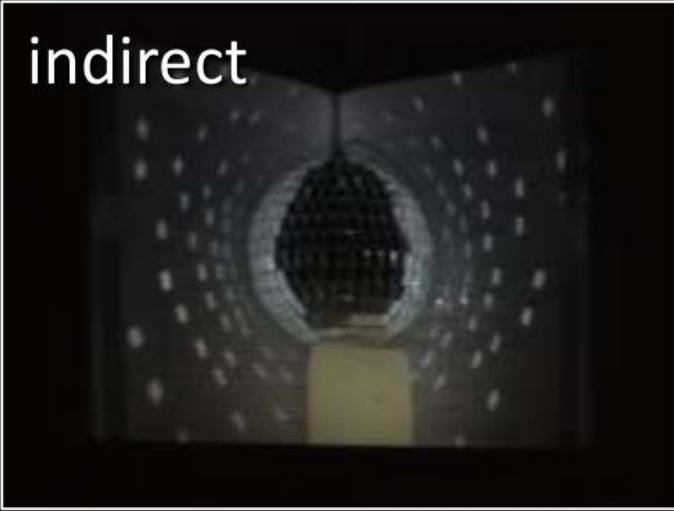
high-freq. indirect



+

||

indirect



direct + backscatter



What if my camera and projector are not co-axial?