Deconvolution

15-463, 15-663, 15-862
Computational Photography
Fall 2018, Lecture 12

http://graphics.cs.cmu.edu/courses/15-463

Course announcements

- Homework 3 is out.
    - Due October 12th.
    - Any questions?

- Project logistics on the course website.
    - Next week I'll schedule extra office hours in case you want to discuss project ideas.

- Make-up lecture details: Friday October 12th, 1:30-3:00 pm, GHC 4102 (this room).
    - Next week I'll schedule extra office hours for those of you who cannot make it to the make-up lecture.

- Additional guest lecture next Monday: Anat Levin, "Coded photography."

Overview of today's lecture

- Leftover from lightfield lecture.
- Sources of blur.
- Deconvolution.
- Blind deconvolution.

Slide credits

Most of these slides were adapted from:

- Fredo Durand (MIT).
- Gordon Wetzstein (Stanford).

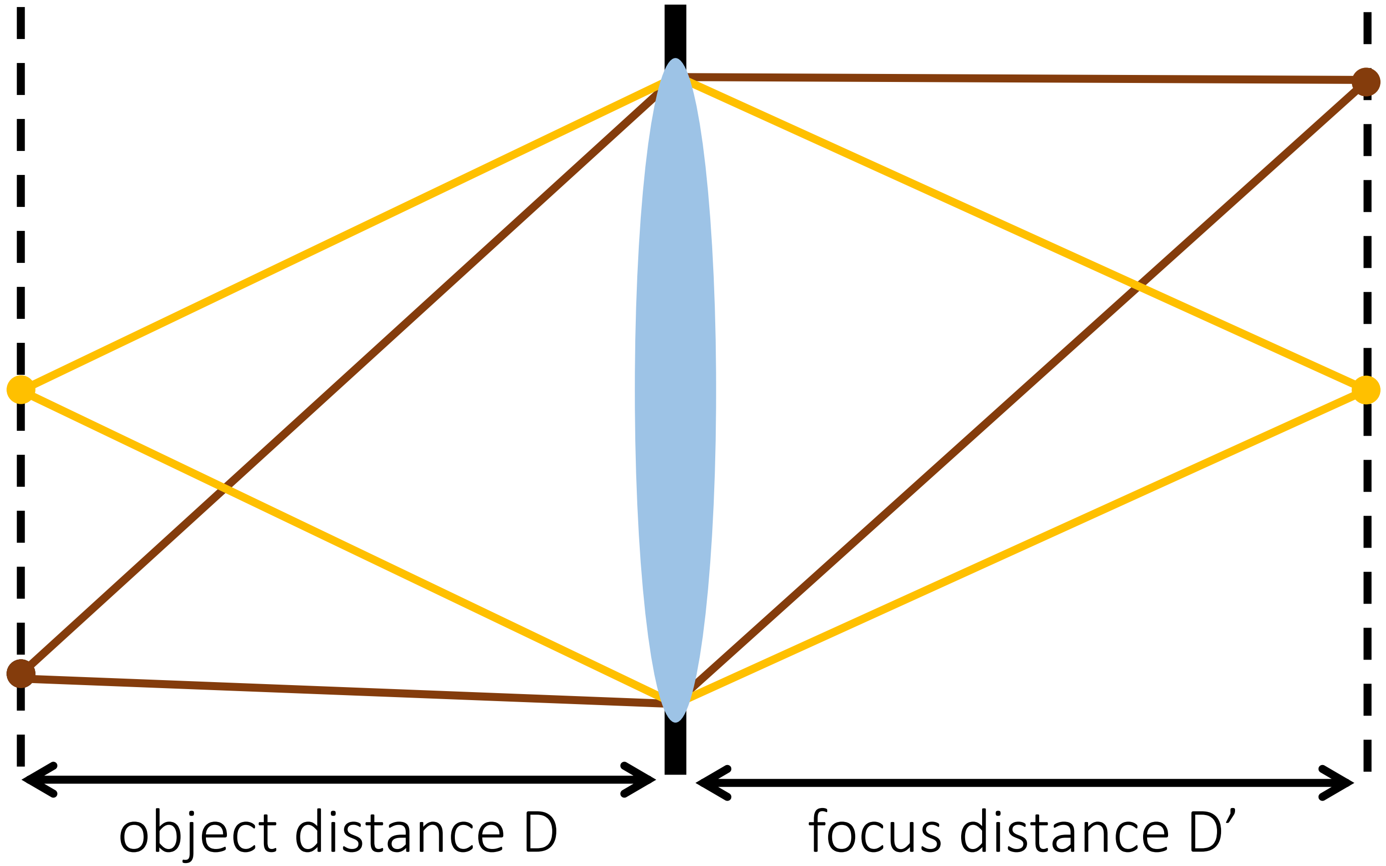
Why are our images blurry?

Why are our images blurry?

- Lens imperfections.
- Camera shake.
- Scene motion.
- Depth defocus.

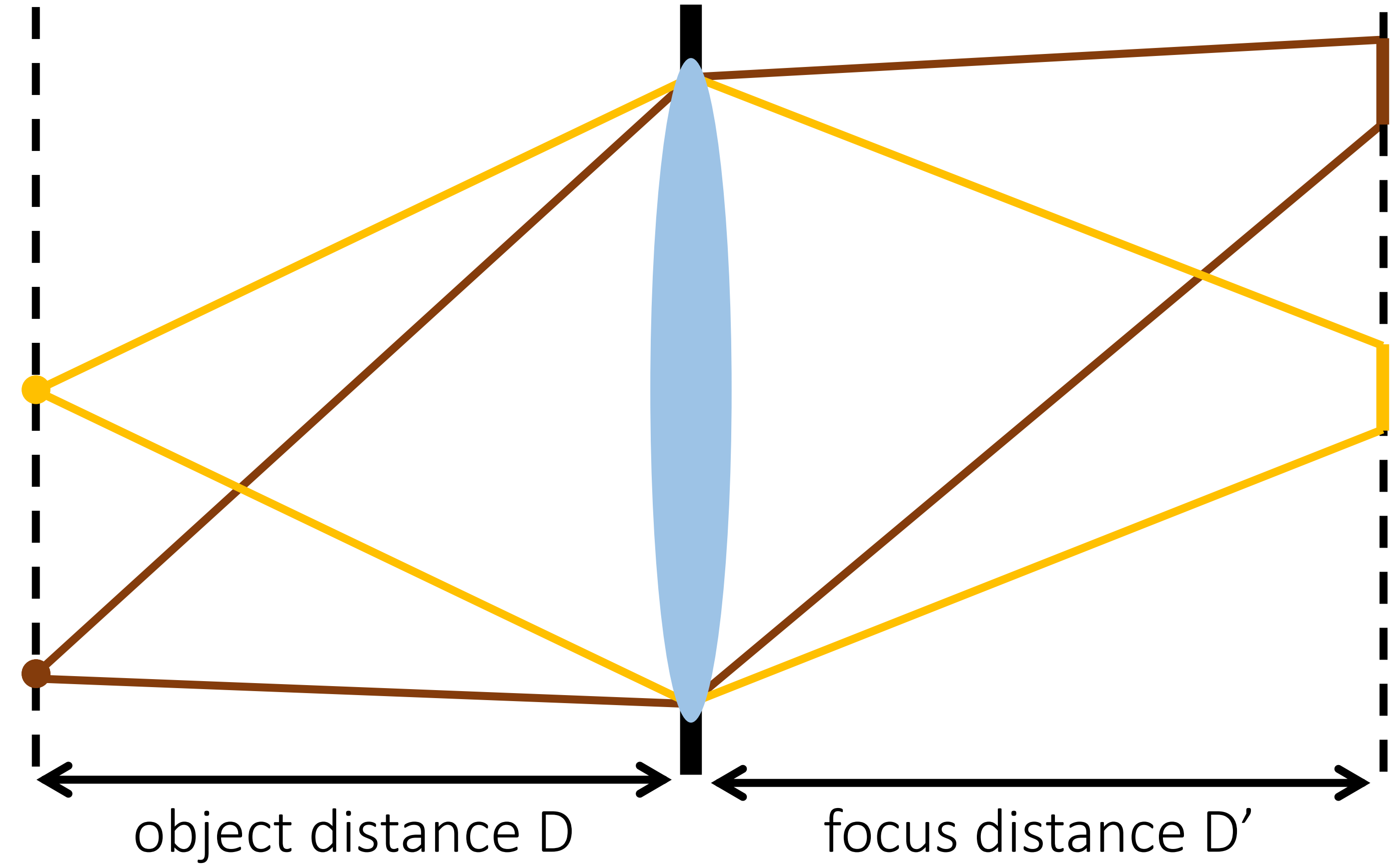
Lens imperfections

- Ideal lens: An point maps to a point at a certain plane.



Lens imperfections

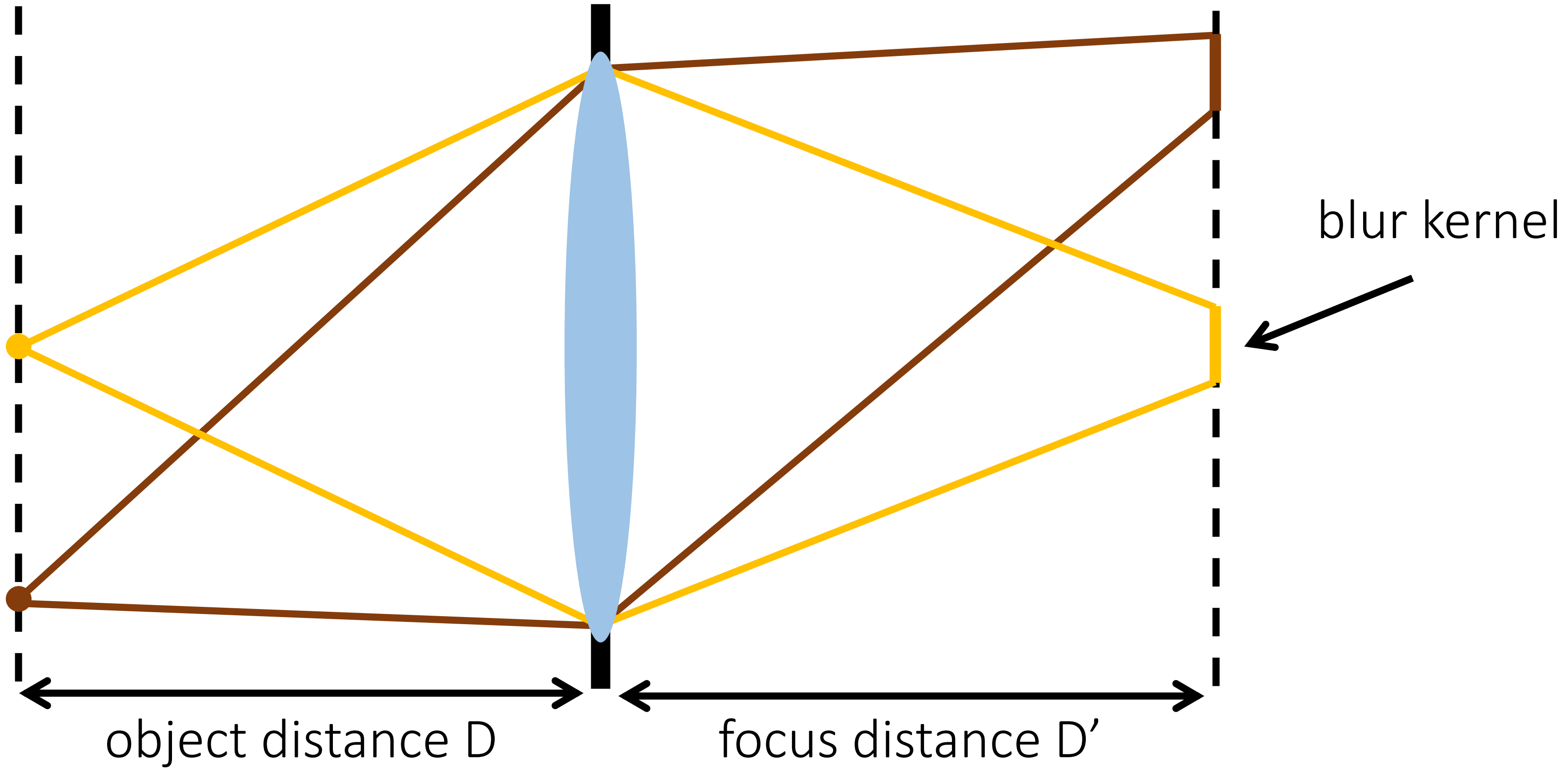
- Ideal lens: An point maps to a point at a certain plane.
- Real lens: A point maps to a circle that has non-zero minimum radius among all planes.



What is the effect of this on the images we capture?

Lens imperfections

- Ideal lens: An point maps to a point at a certain plane.
- Real lens: A point maps to a circle that has non-zero minimum radius among all planes.



Shift-invariant blur.

Lens imperfections

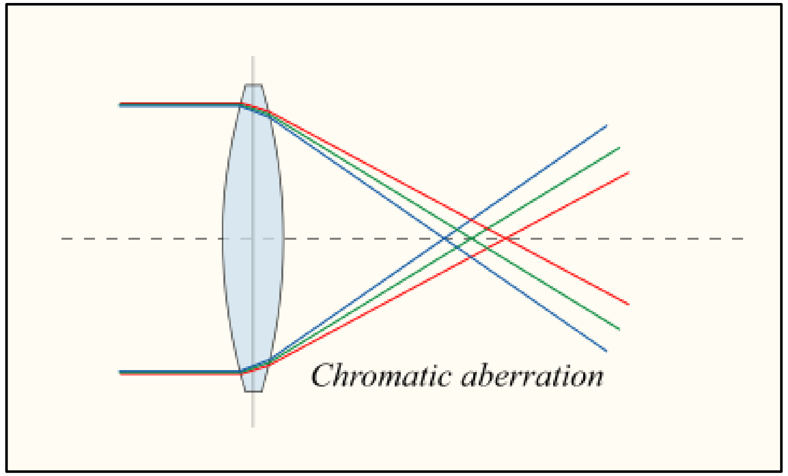
What causes lens imperfections?

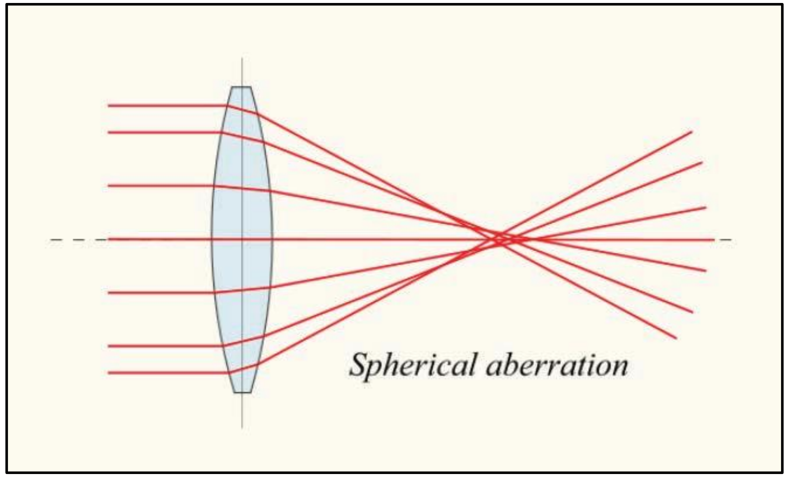
Lens imperfections

What causes lens imperfections?

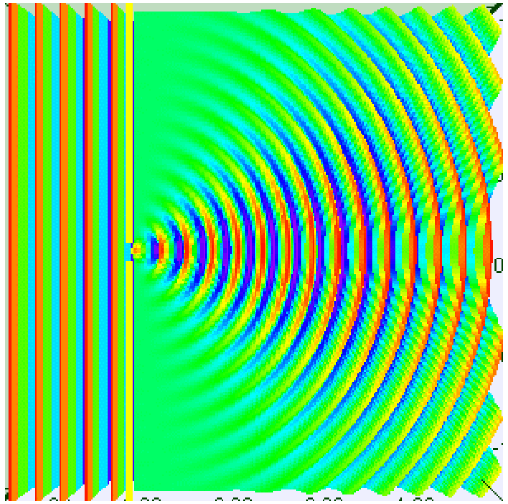
- Aberrations.

(Important note: Oblique aberrations like coma and distortion <u>are not shift-invariant</u> blur and we do not consider them here!)

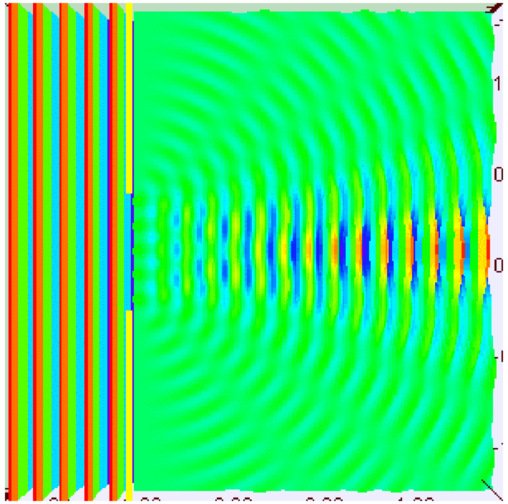




- Diffraction.



small aperture

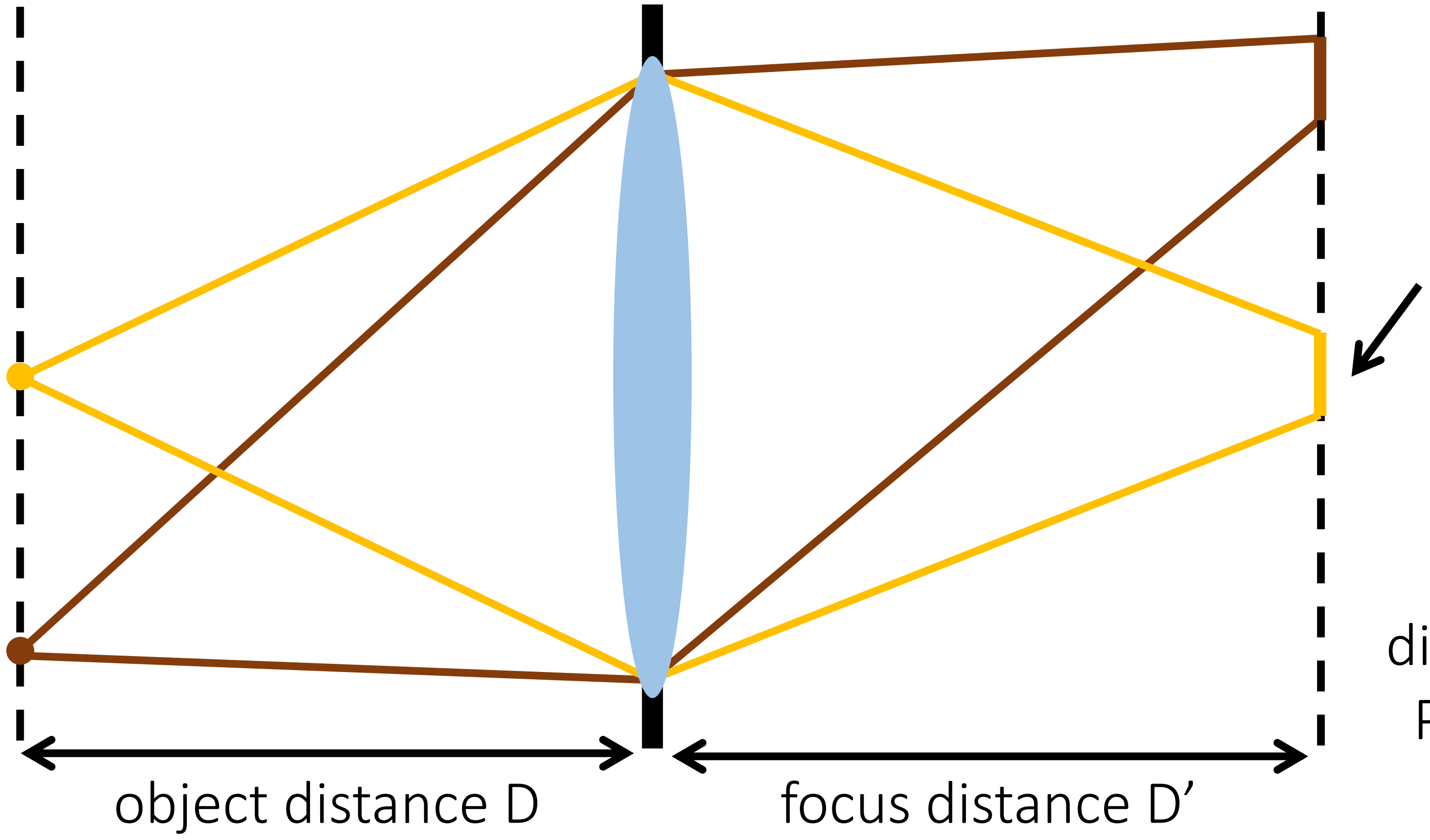


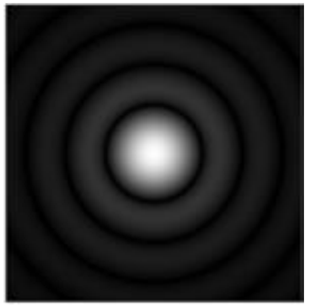
large aperture

Lens as an optical low-pass filter

Point spread function (PSF): The blur kernel of a lens.

- "Diffraction-limited" PSF: No aberrations, only diffraction. Determined by aperture shape.





diffraction-limited PSF of a circular aperture

Lens as an optical low-pass filter

Point spread function (PSF): The blur kernel of a lens.

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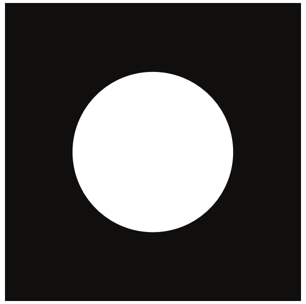
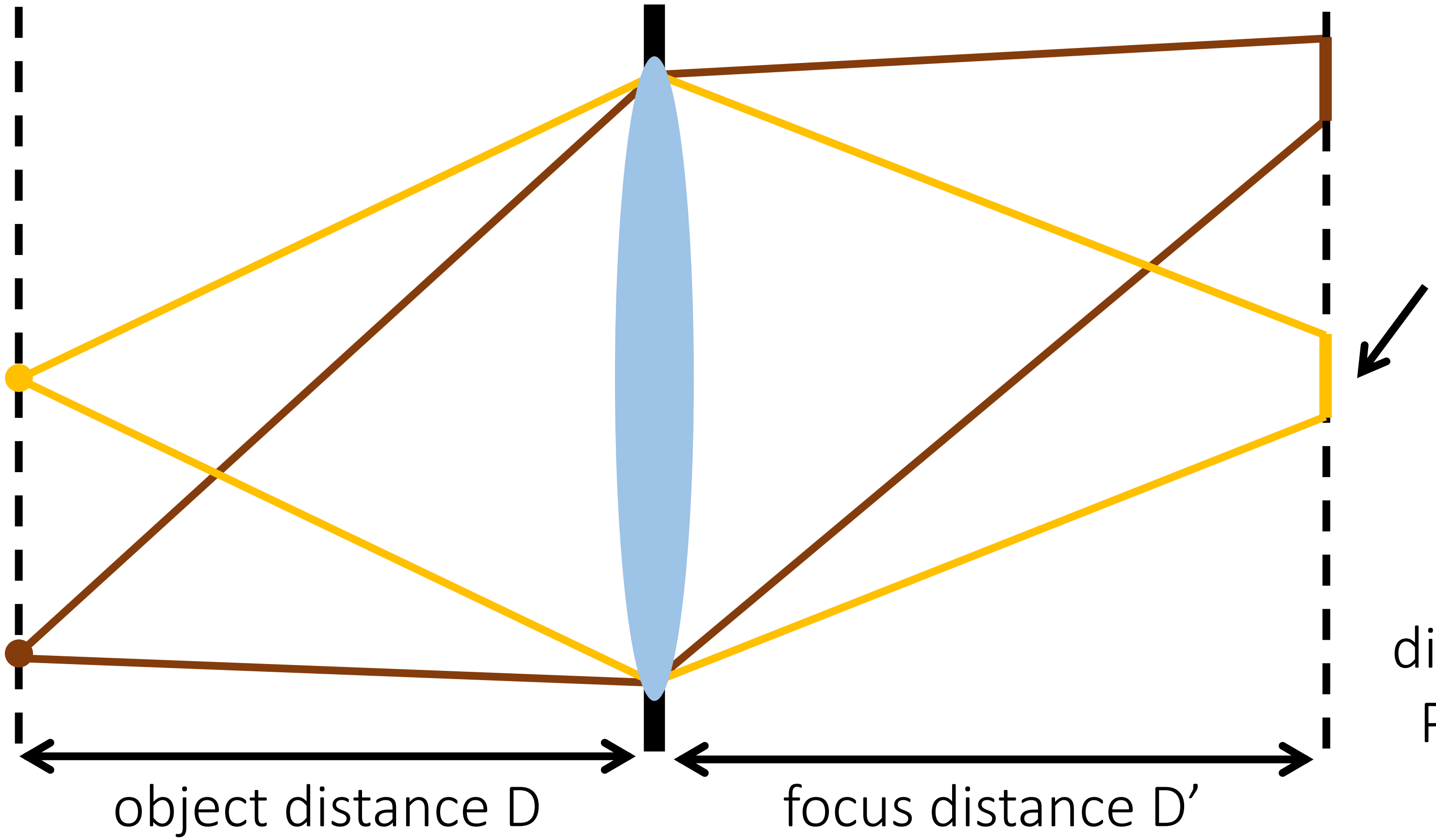
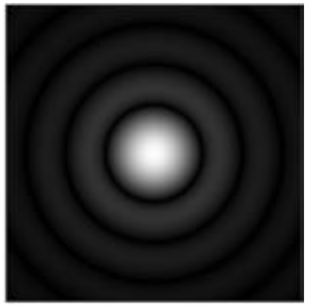


diffraction-limited OTF of a circular aperture





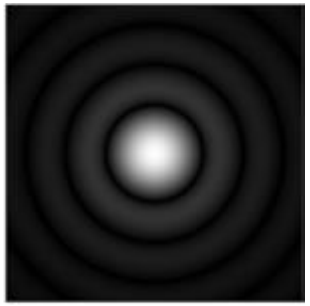
diffraction-limited PSF of a circular aperture

Optical transfer function (OTF): The Fourier transform of the PSF. Equal to aperture shape.

Lens as an optical low-pass filter

image from a perfect lens

$*$



imperfect lens PSF

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image from imperfect lens

$$x \quad * \quad c \quad = \quad b$$

Lens as an optical low-pass filter

If we know c and b, can we recover x?

*

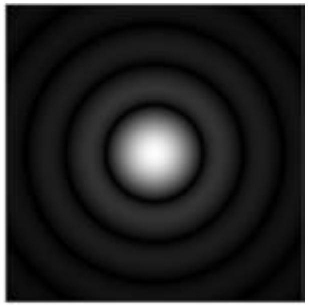

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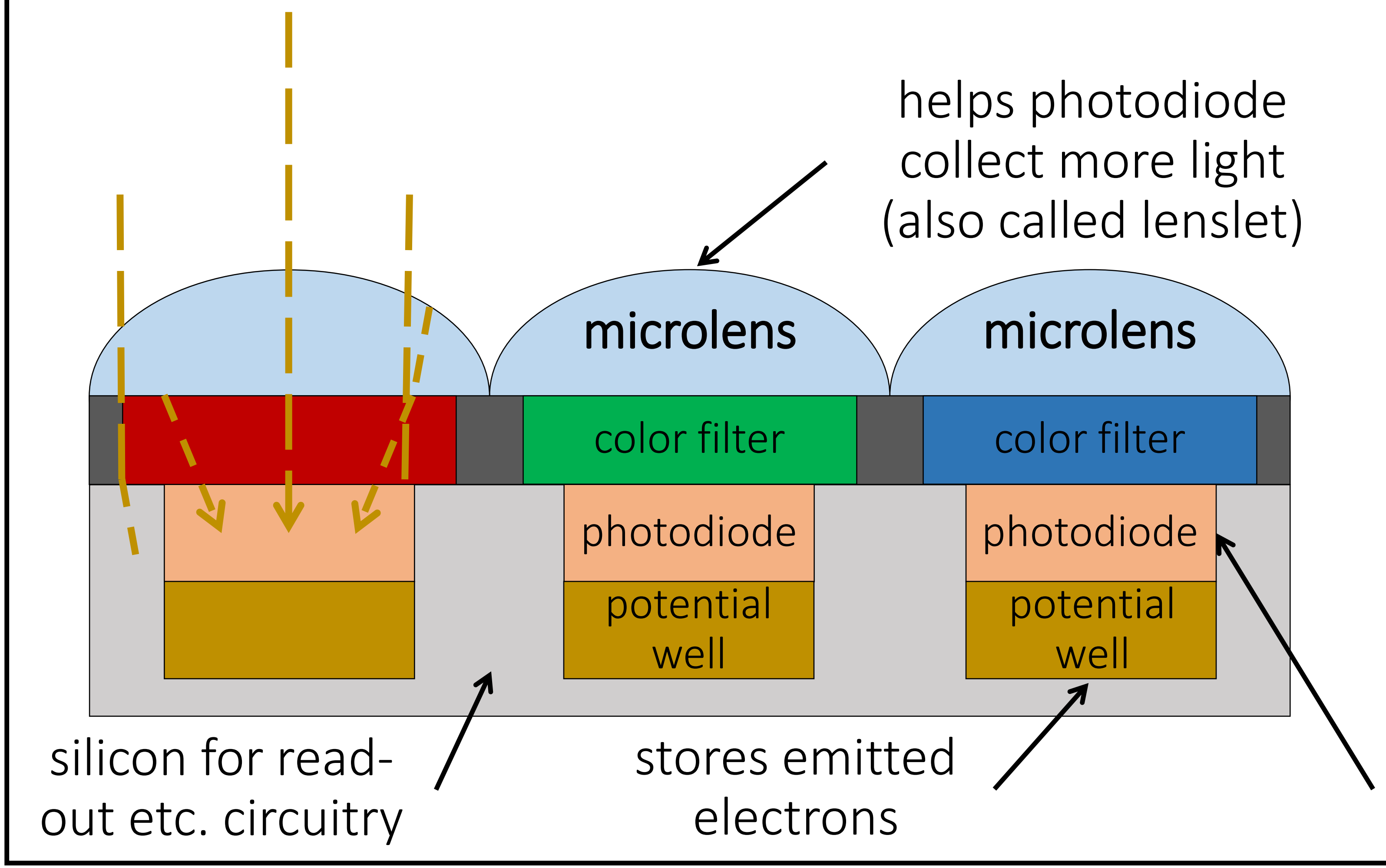
image from a perfect lens | imperfect lens PSF | image from imperfect lens

$$x \quad * \quad c \quad = \quad b$$

Quick aside: optical anti-aliasing

Lenses act as (optical) low-pass filters.

Slide from lecture 2: Basic imaging sensor design



- **Lenslets also filter the image to avoid resolution artifacts.**
- Lenslets are problematic when working with coherent light.
- Many modern cameras do not have lenslet arrays.

We will discuss these issues in more detail at a later lecture.

Quick aside: optical anti-aliasing

Lenses act as (optical) smoothing filters.

- Sensors often have a lenslet array in front of them as an anti-aliasing (AA) filter.
- However, the AA filter means you also lose resolution.
- Nowadays, due the large number of sensor pixels, AA filters are becoming unnecessary.

Photographers often hack their cameras to remove the AA filter, in order to avoid the loss of resolution.

a.k.a. “hot rodding”

Quick aside: optical anti-aliasing

Example where AA filter is needed

without AA filter

with AA filter

Quick aside: optical anti-aliasing

Example where AA filter is unnecessary

without AA filter

with AA filter

Lens as an optical low-pass filter

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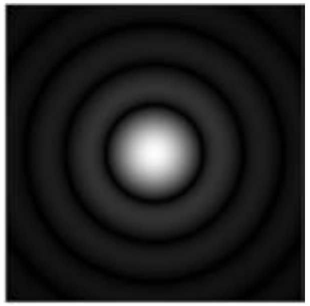

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image from a perfect lens

imperfect lens PSF

image from imperfect lens

$$x \quad * \quad c \quad = \quad b$$

Deconvolution

$$x \quad * \quad c \quad = \quad b$$

If we know c and b, can we recover x?

Deconvolution

$$x \quad * \quad c \quad = \quad b$$

Reminder: convolution is multiplication in Fourier domain:

$$F(x) \quad \cdot \quad F(c) \quad = \quad F(b)$$

If we know c and b, can we recover x?

Deconvolution

$$x \quad * \quad c \quad = \quad b$$

Reminder: convolution is multiplication in Fourier domain:

$$F(x) \cdot F(c) = F(b)$$

Deconvolution is division in Fourier domain:

$$F(x_{est}) = F(b) \setminus F(c)$$

After division, just do inverse Fourier transform:

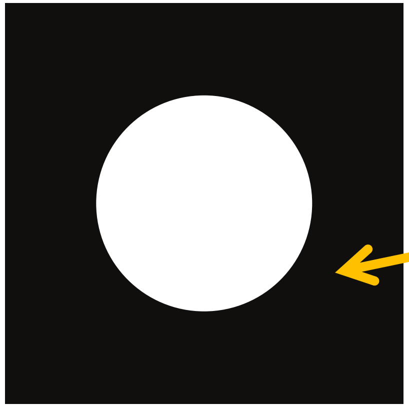
$$x_{est} = F^{-1}\left(F(b) \setminus F(c) \right)$$

Deconvolution

Any problems with this approach?

Deconvolution

- The OTF (Fourier of PSF) is a low-pass filter



zeros at high frequencies

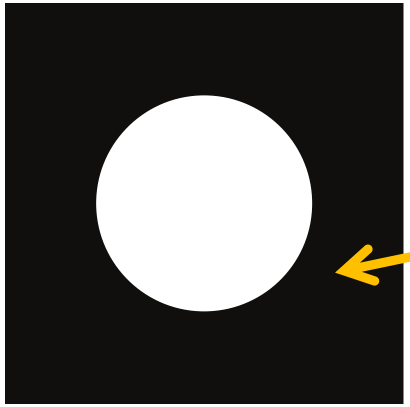
- The measured signal includes noise

$$b = c * x + n$$

noise term

Deconvolution

- The OTF (Fourier of PSF) is a low-pass filter



zeros at high frequencies

- The measured signal includes noise

$$b = c * x + n$$

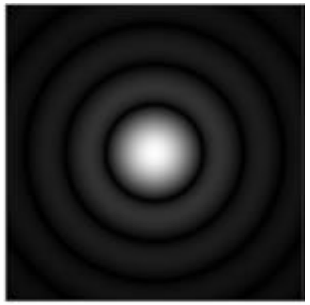
noise term

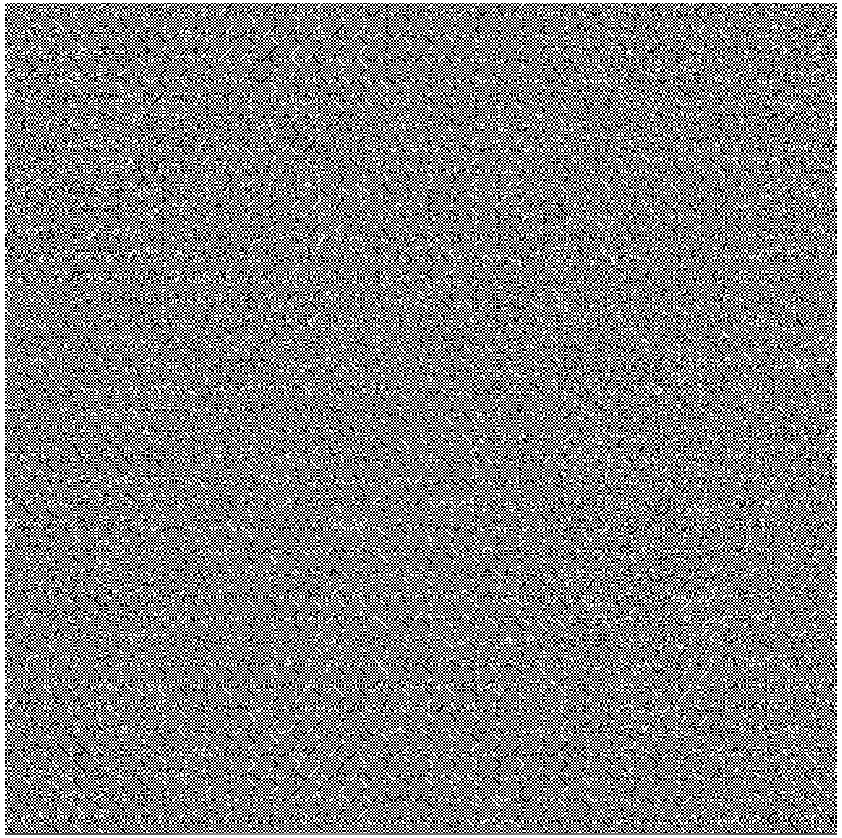
- When we divide by zero, we amplify the high frequency noise

Naïve deconvolution

Even tiny noise can make the results awful.

- Example for Gaussian of σ = 0.05

$*\left(\right.$  $\left.\right)^{-1} =$



$$b \quad * \quad c^{-1} \quad = \quad x_{est}$$

Wiener Deconvolution

Apply inverse kernel and do not divide by zero:

$$x_{est} = F^{-1}\left(\boxed{\frac{|F(c)|^2}{|F(c)|^2 + 1/SNR(\omega)}} \cdot \frac{F(b)}{F(c)} \right)$$

noise-dependent damping factor

- Derived as solution to maximum-likelihood problem under Gaussian noise assumption
- Requires noise of signal-to-noise ratio at each frequency

$$SNR(\omega) = \frac{\text{signal variance at } \omega}{\text{noise variance at } \omega}$$

Wiener Deconvolution

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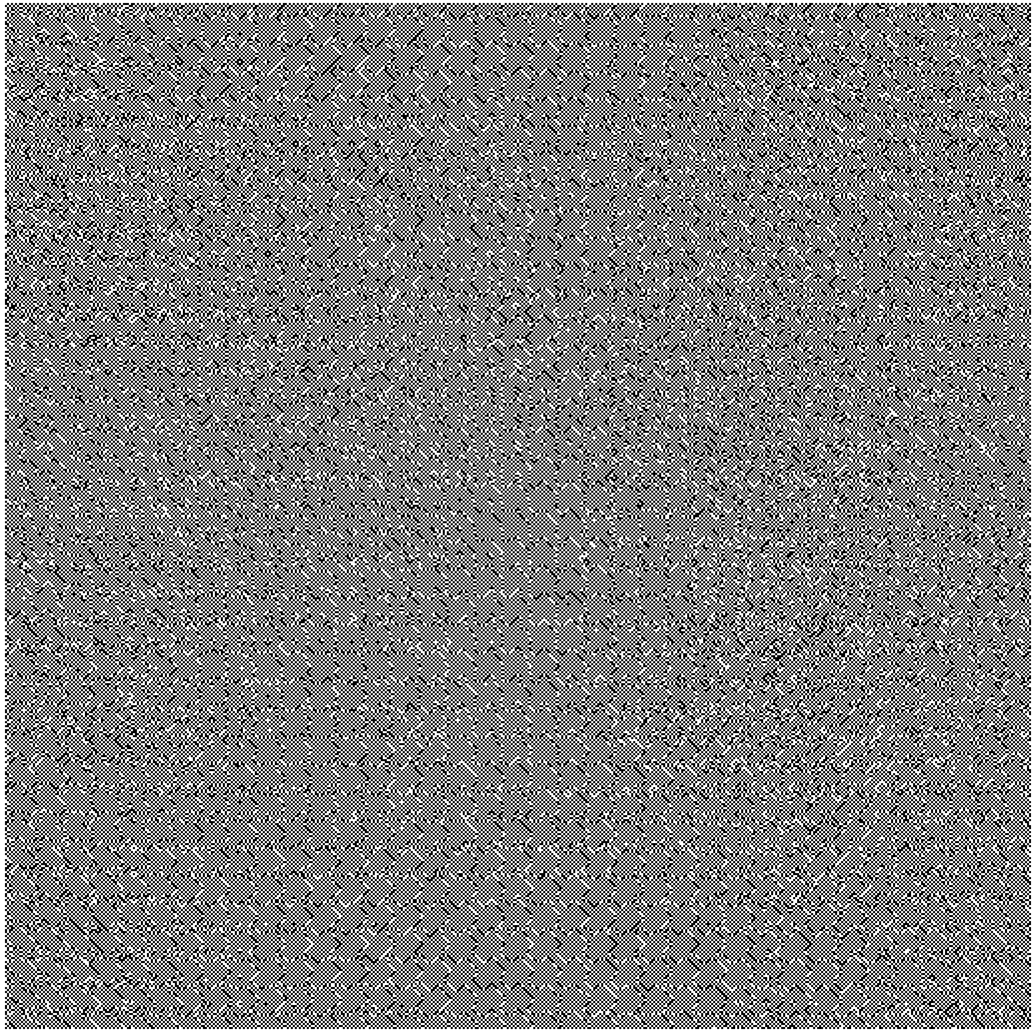
noise-dependent damping factor

Intuitively:

- When SNR is high (low or no noise), just divide by kernel.
- When SNR is low (high noise), just set to zero.

Deconvolution comparisons



naïve deconvolution

Wiener deconvolution

Deconvolution comparisons

σ = 0.01

σ = 0.05

σ = 0.01

Derivation

Sensing model:

$$\tilde{x} = c * x + n$$

Noise $\mathbf{n}$ is assumed to be zero-mean and independent of signal $\mathbf{x}$.

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$$b = c * x + n$$

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Fourier transform:

$$B = C \cdot X + N$$

Why multiplication?

Derivation

Sensing model:

$$b = c * x + n$$

Noise $\mathbf{n}$ is assumed to be zero-mean and independent of signal $\mathbf{x}$.

Fourier transform:

$$B = C \cdot X + N$$

Convolution becomes multiplication.

Problem statement: Find function $\mathrm{H}(\omega)$ that minimizes *expected* error *in Fourier domain*.

$$\min_{H} E\left[\left\|X - H\tilde{B}\right\|^2\right]$$

Derivation

Replace B and re-arrange loss:

$$\min_H E[\|(1 + HC)X - HN\|^2]$$

Expand the squares:

$$\min_H \|1 - HC\|^2 E[\|X\|^2] - 2(1 - HC)E[XN] + \|H\|^2 E[\|N\|^2]$$

Derivation

When handling the cross terms:

- Can I write the following?

$$E[XN] = E[X]E[N]$$

Derivation

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Yes, because X and N are assumed independent.

- What is this expectation product equal to?

Derivation

When handling the cross terms:

- Can I write the following?

$$E[XN] = E[X]E[N]$$

Yes, because X and N are assumed independent.

- What is this expectation product equal to?

Zero, because N has zero mean.

Derivation

Replace B and re-arrange loss:

$$\min_H E[\|(1 + HC)X - HN\|^2]$$

Expand the squares:

$$\min_H \|1 - HC\|^2 E[\|X\|^2] - 2(1 - HC)E[XN] + \|H\|^2 E[\|N\|^2]$$

cross-term is zero

Simplify:

$$\min_H \|1 - HC\|^2 E[\|X\|^2] + \|H\|^2 E[\|N\|^2]$$

How do we solve this optimization problem?

Derivation

Differentiate loss with respect to H, set to zero, and solve for H:

$$\frac{\partial \text{loss}}{\partial H} = 0$$

$$\Rightarrow -2(1 - HC)E[\|X\|^2] + 2HE[\|N\|^2] = 0$$

$$\Rightarrow H = \frac{CE[\|X\|^2]}{C^2E[\|X\|^2] + E[\|N\|^2]}$$

Divide both numerator and denominator with $E[\|X\|^2]$, extract factor $1/\mathrm{C}$, and done!

Wiener Deconvolution

Apply inverse kernel and do not divide by zero:

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noise-dependent damping factor

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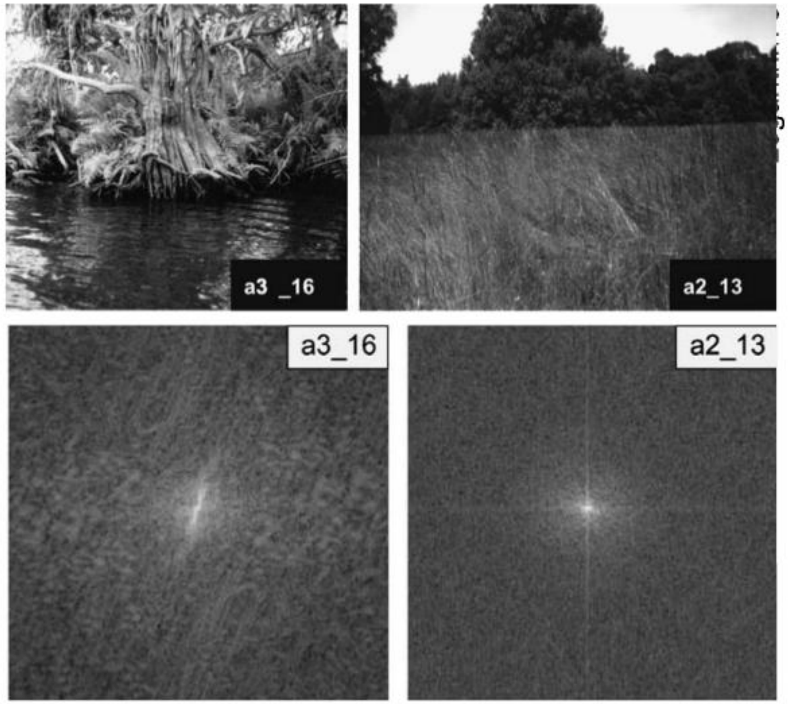
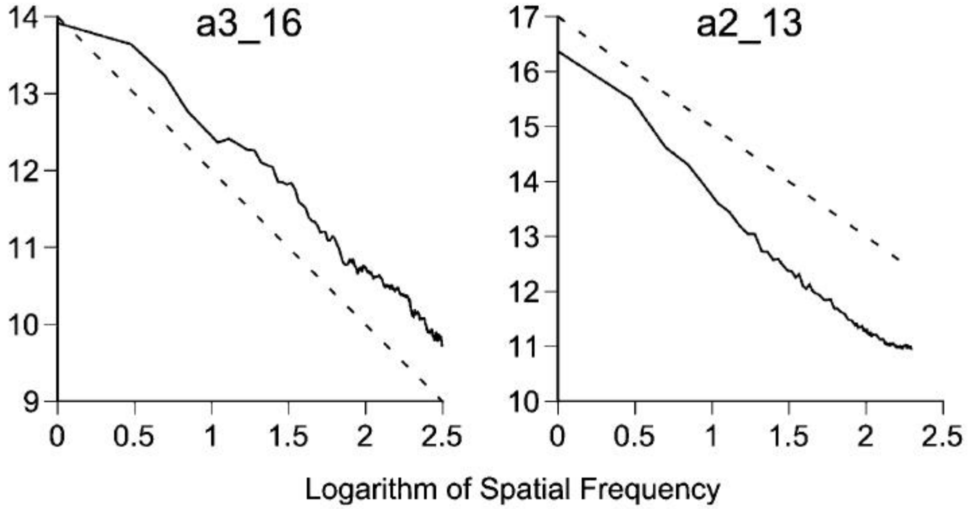
$$SNR(\omega) = \frac{\text{signal variance at } \omega}{\text{noise variance at } \omega}$$

Natural image and noise spectra

Natural images *tend* to have spectrum that scales as $1 / \omega^2$

- This is a *natural image statistic*





http://www.cnbc.cmu.edu/cns/papers/Balboa_PowerSpectra2003.pdf

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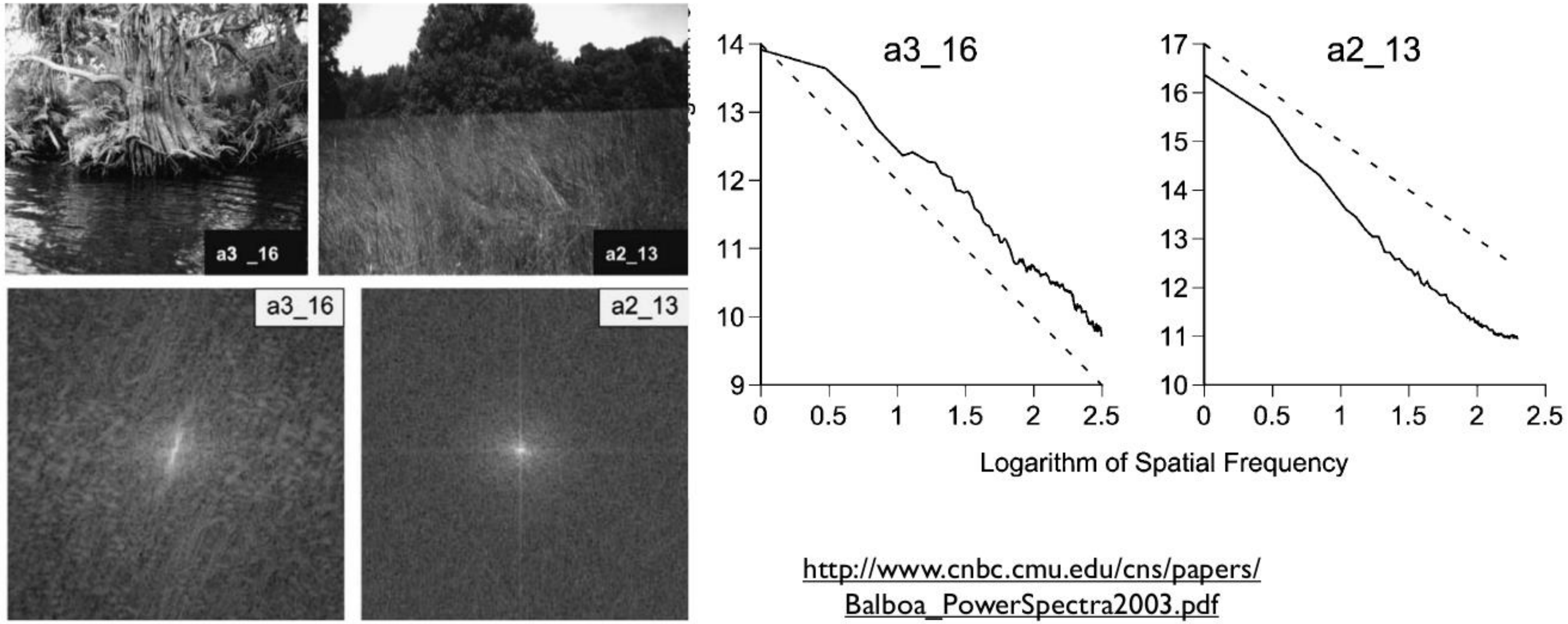

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Noise tends to have flat spectrum, $\sigma(\omega)$ = constant

- We call this white noise

What is the SNR?

Natural image and noise spectra

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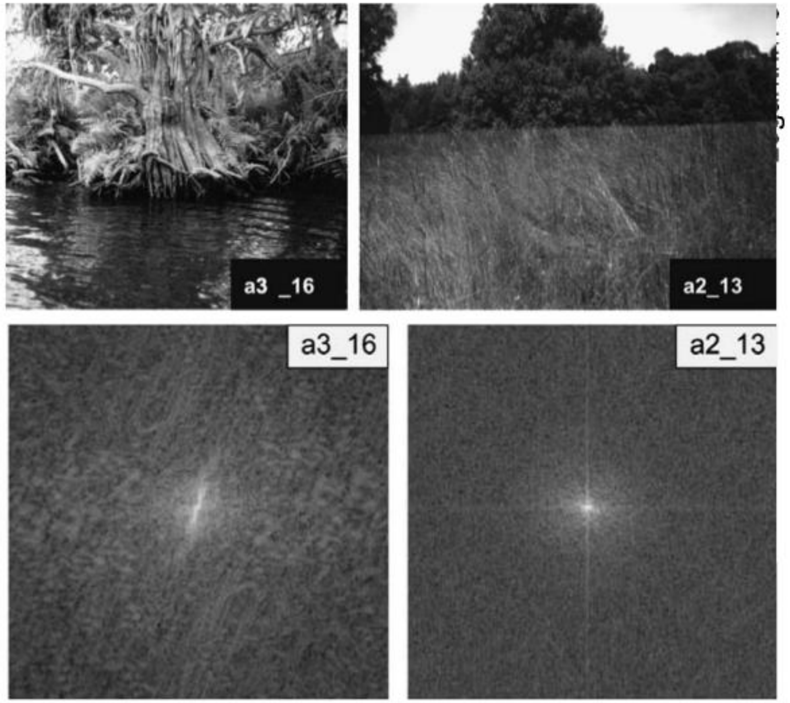

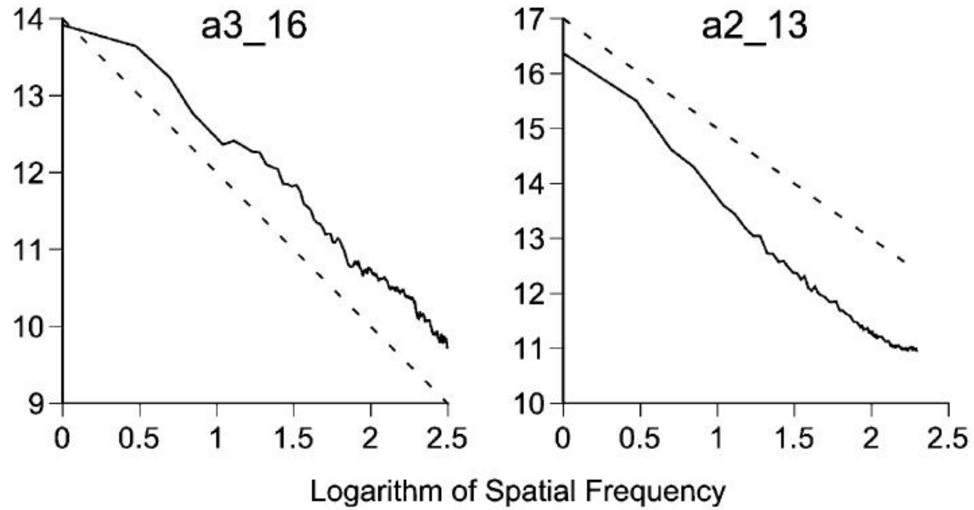

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Noise tends to have flat spectrum, $\sigma(\omega)$ = constant

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Therefore, we have that: $\text{SNR}(\omega) = 1 / \omega^2$

Wiener Deconvolution

Apply inverse kernel and do not divide by zero:

$$x_{est} = F^{-1}\left(\frac{|F(c)|^2}{|F(c)|^2 + \omega^2} \cdot \frac{F(b)}{F(c)} \right)$$

amplitude-dependent damping factor

- Derived as solution to maximum-likelihood problem under Gaussian noise assumption
- Requires noise of signal-to-noise ratio at each frequency

$$\mathrm{SNR}(\omega) = \frac{1}{\omega^2}$$

Wiener Deconvolution

For natural images and white noise, equivalent to the minimization problem:

$$\min_x ||b - c * x||^2 + ||\nabla x||^2$$

gradient *regularization*

How can you prove this equivalence?

Wiener Deconvolution

For natural images and white noise, it can be re-written as the minimization problem

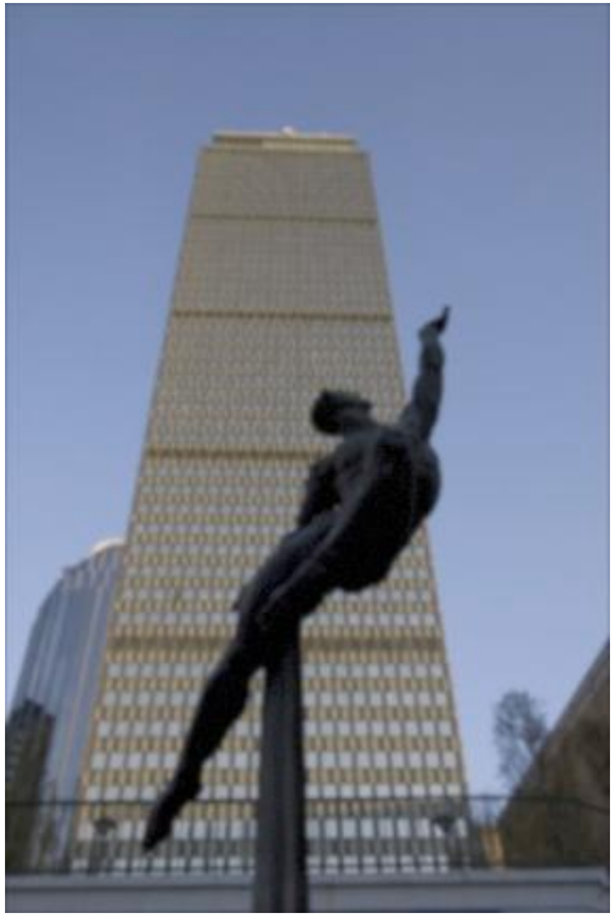
$$\min_x ||b - c * x||^2 + ||\nabla x||^2$$

gradient *regularization*

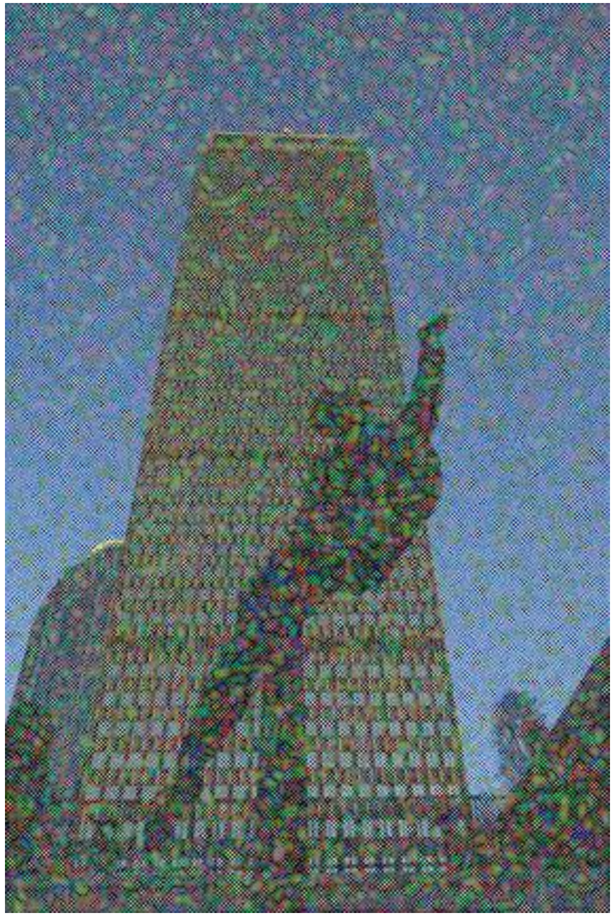
How can you prove this equivalence?

- Convert to Fourier domain and repeat the proof for Wiener deconvolution.
- Intuitively: The ω^2 term in the denominator of the special Wiener filter is the square of the Fourier transform of $\nabla \mathbf{x}$, which is $\mathbf{i} \cdot \omega$.

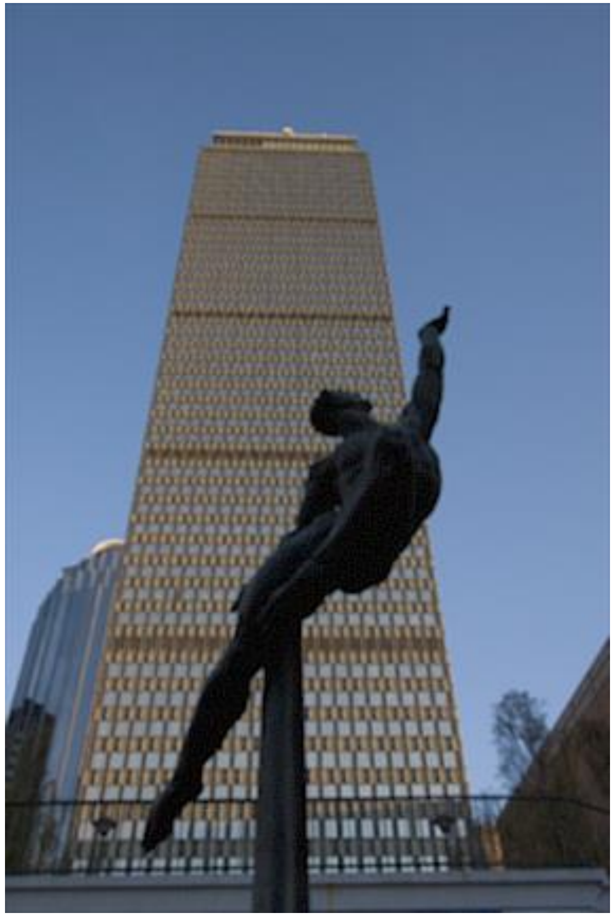
Deconvolution comparisons



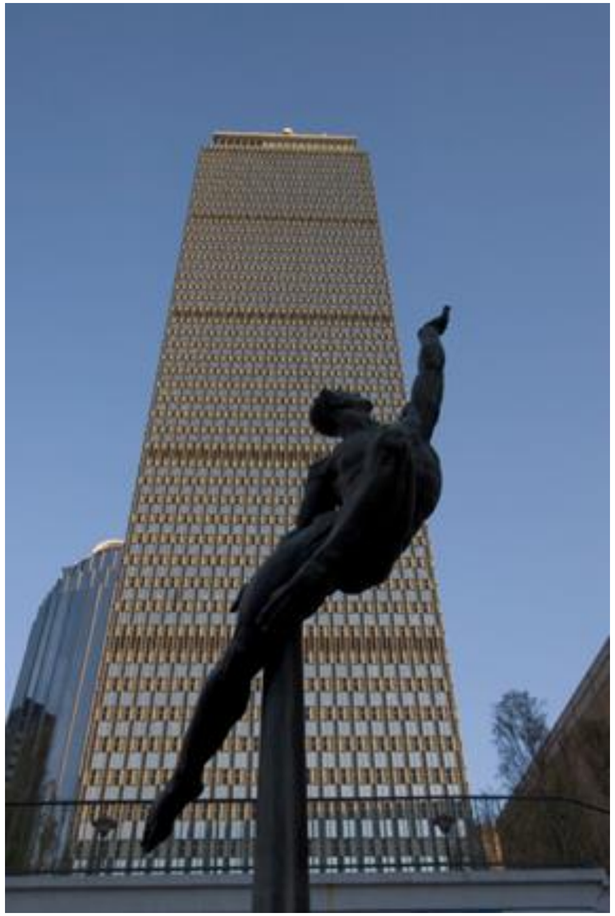
blurry input



naive deconvolution

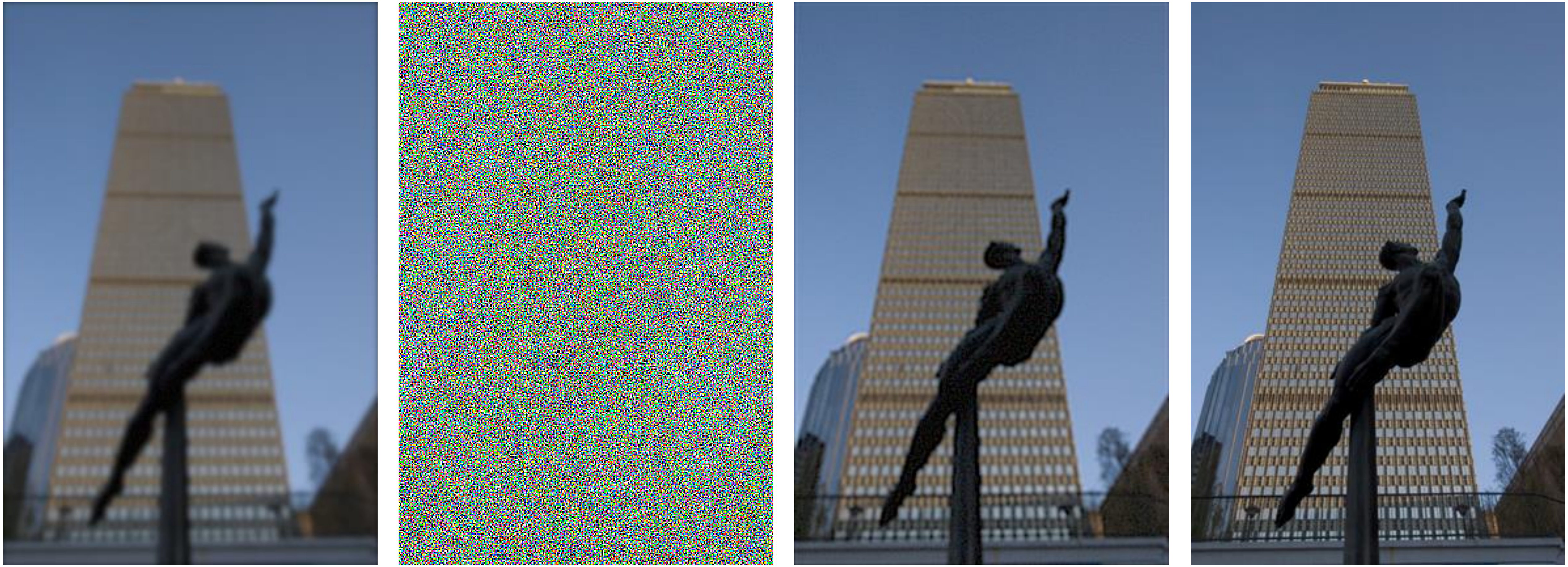


gradient regularization



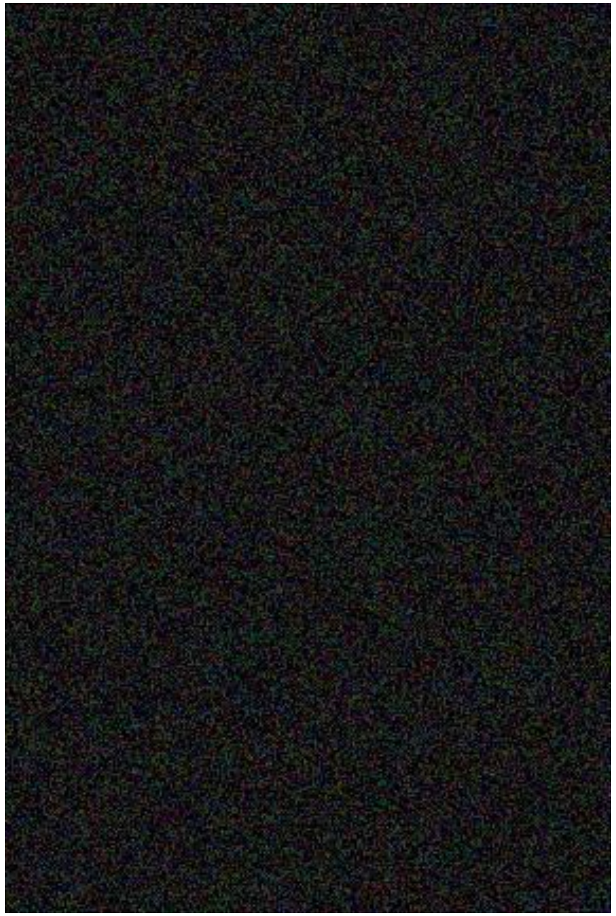
original

Deconvolution comparisons

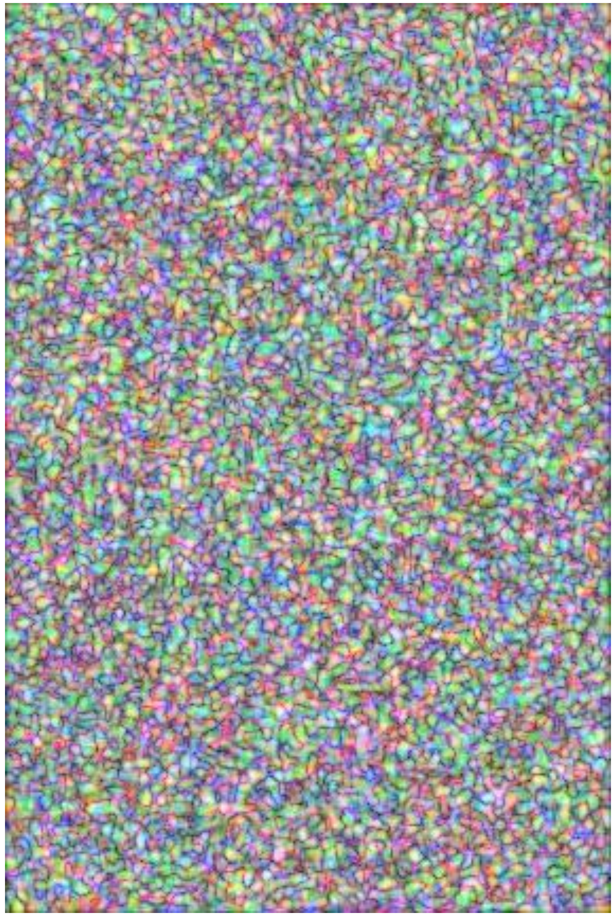


blurry input naive deconvolution gradient regularization original

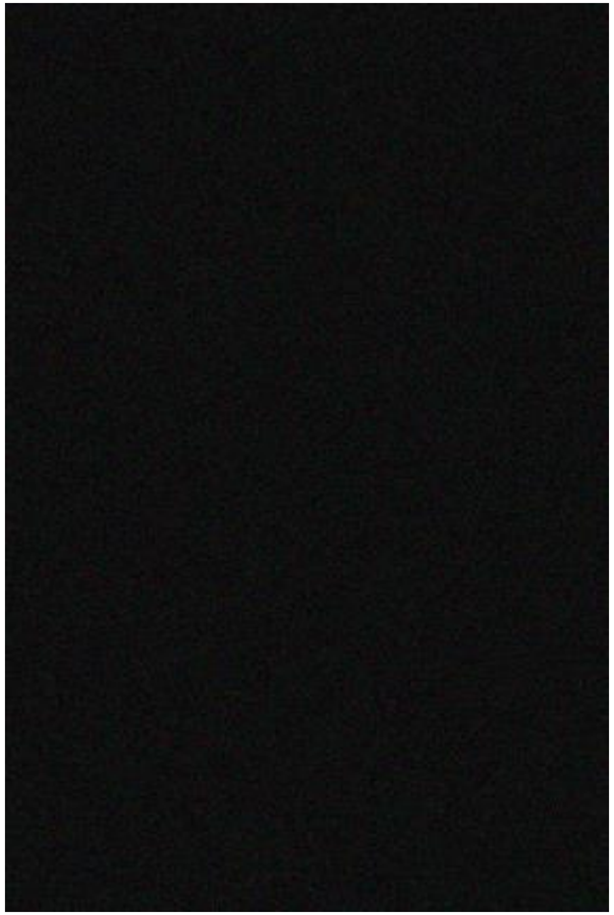
… and a proof-of-concept demonstration



noisy input



naive deconvolution



gradient regularization

Question

Can we undo lens blur by deconvolving a PNG or JPEG image without any preprocessing?

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Can we undo lens blur by deconvolving a PNG or JPEG image without any preprocessing?

- All the blur processes we discuss today happen *optically* (before capture by the sensor).
- Blur model is accurate only if our images are *linear.*

Are PNG or JPEG images linear?

Question

Can we undo lens blur by deconvolving a PNG or JPEG image without any preprocessing?

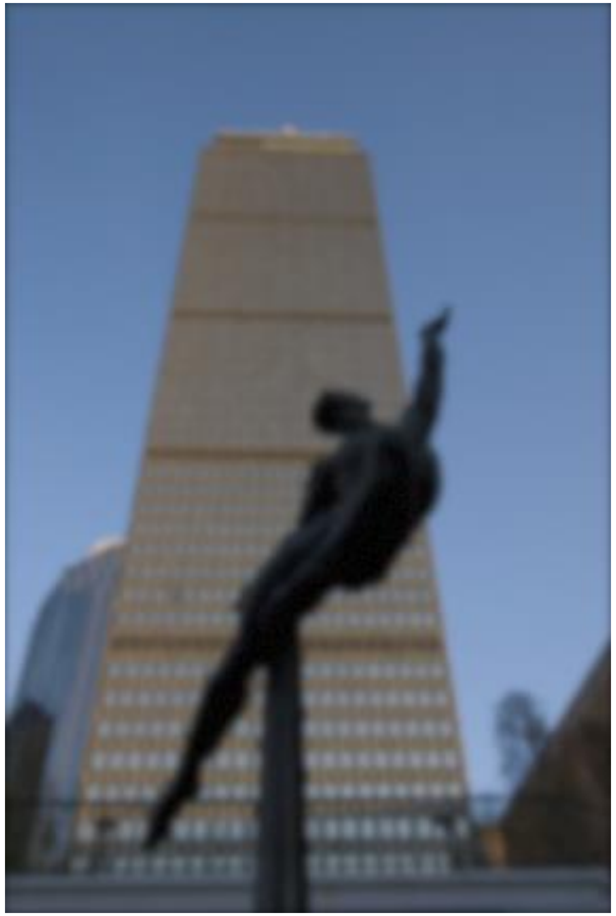
- All the blur processes we discuss today happen *optically* (before capture by the sensor).
- Blur model is accurate only if our images are *linear.*

Are PNG or JPEG images linear?

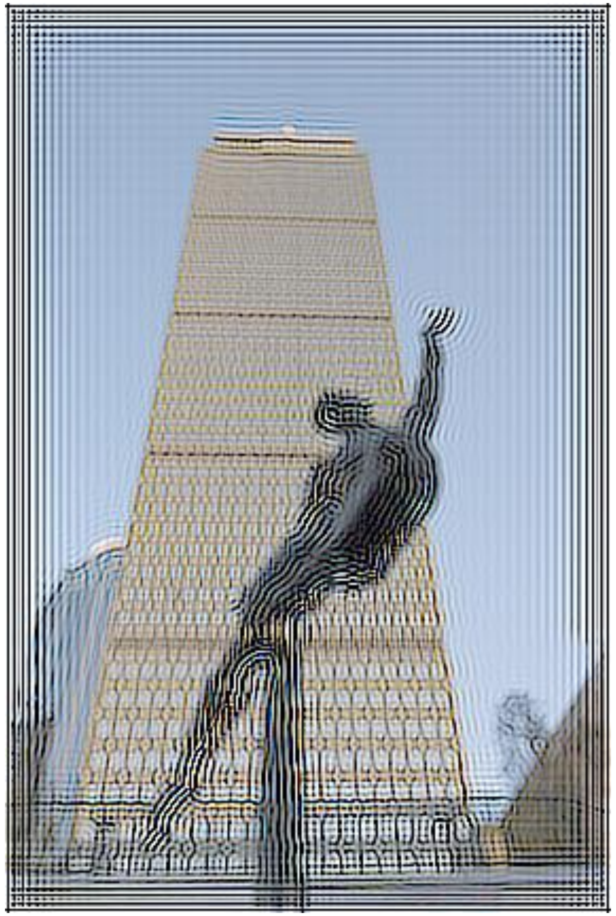
- No, because of gamma encoding.
- Before deblurring, you must linearize your images.

How do we linearize PNG or JPEG images?

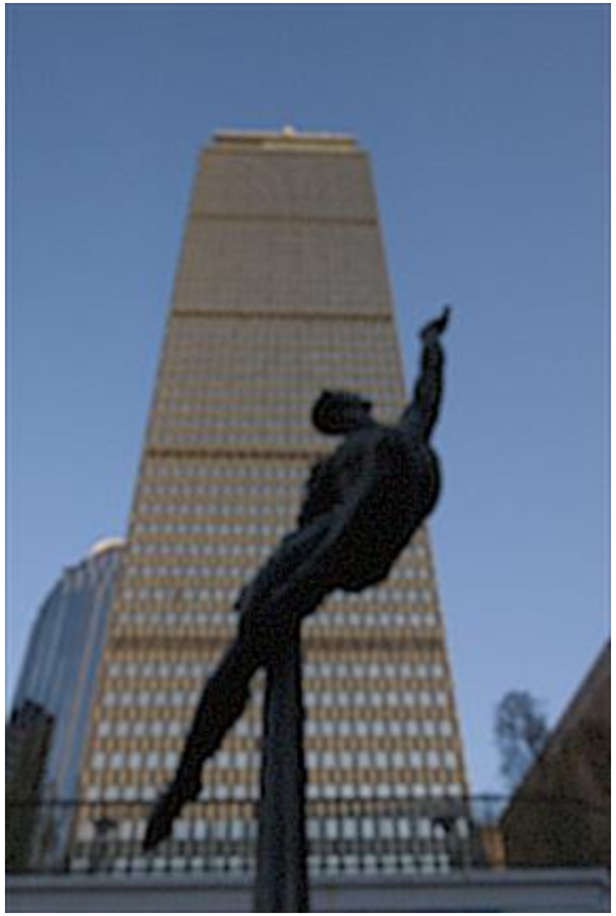
The importance of linearity



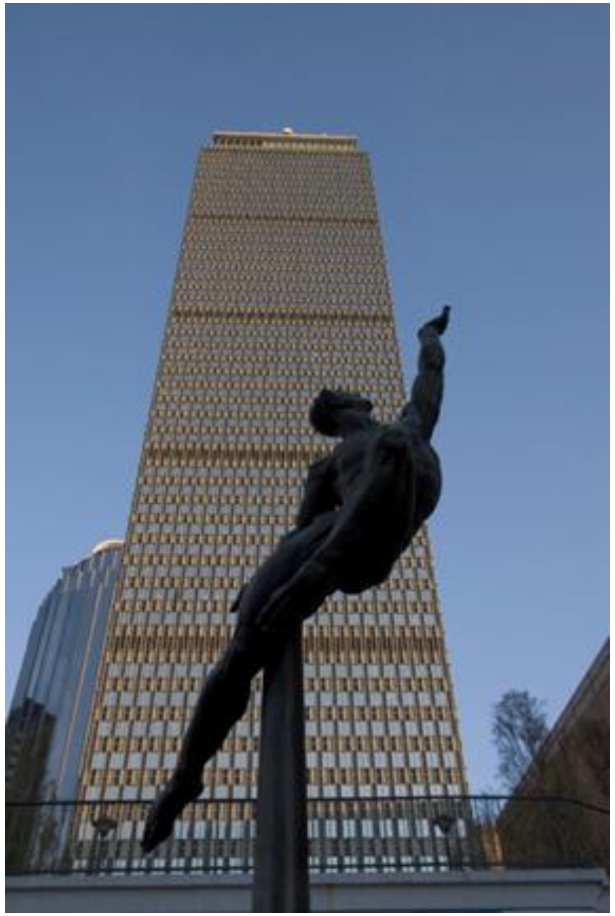
blurry input



deconvolution without linearization



deconvolution after linearization



original

Can we do better than that?

Can we do better than that?

Use different gradient regularizations:

- L_2 gradient regularization (Tikhonov regularization, same as Wiener deconvolution)

$$\min_x \|b - c * x\|^2 + \|\nabla x\|^2$$

- L_1 gradient regularization (sparsity regularization, same as total variation)

$$\min_x \|b - c * x\|^2 + \|\nabla x\|^1$$

- $L_{n<1}$ gradient regularization (fractional regularization)

$$\min_x \|b - c * x\|^2 + \|\nabla x\|^{0.8}$$

How do we solve for these?

All of these are motivated by natural image statistics. Active research area.

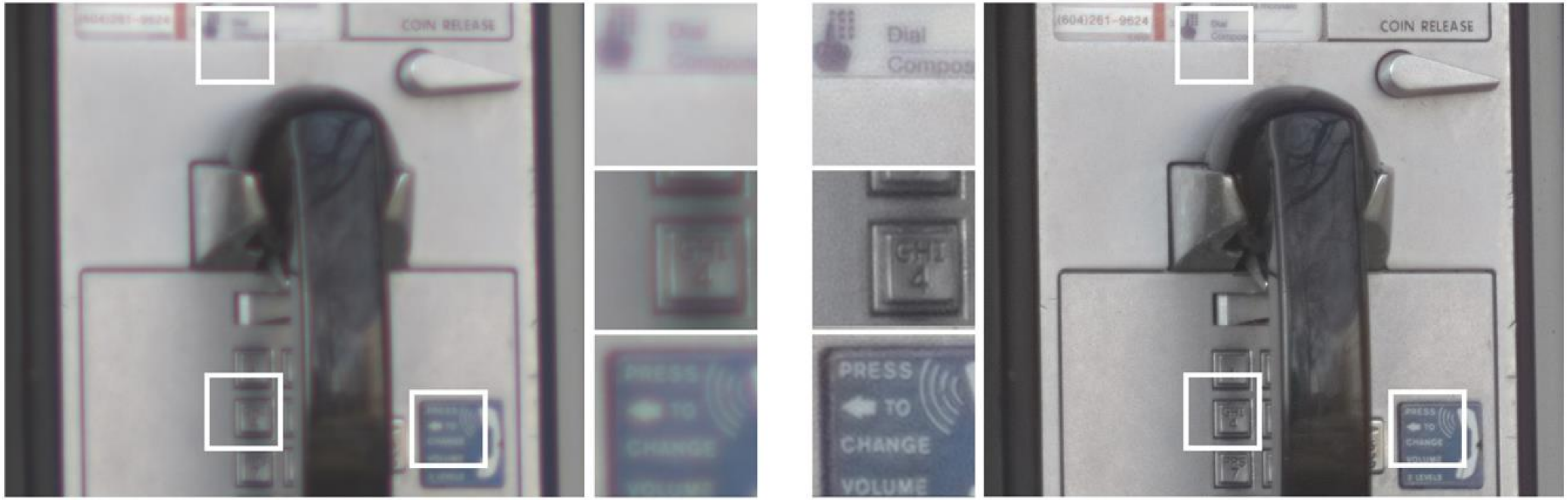
Comparison of gradient regularizations

input

squared gradient regularization

fractional gradient regularization

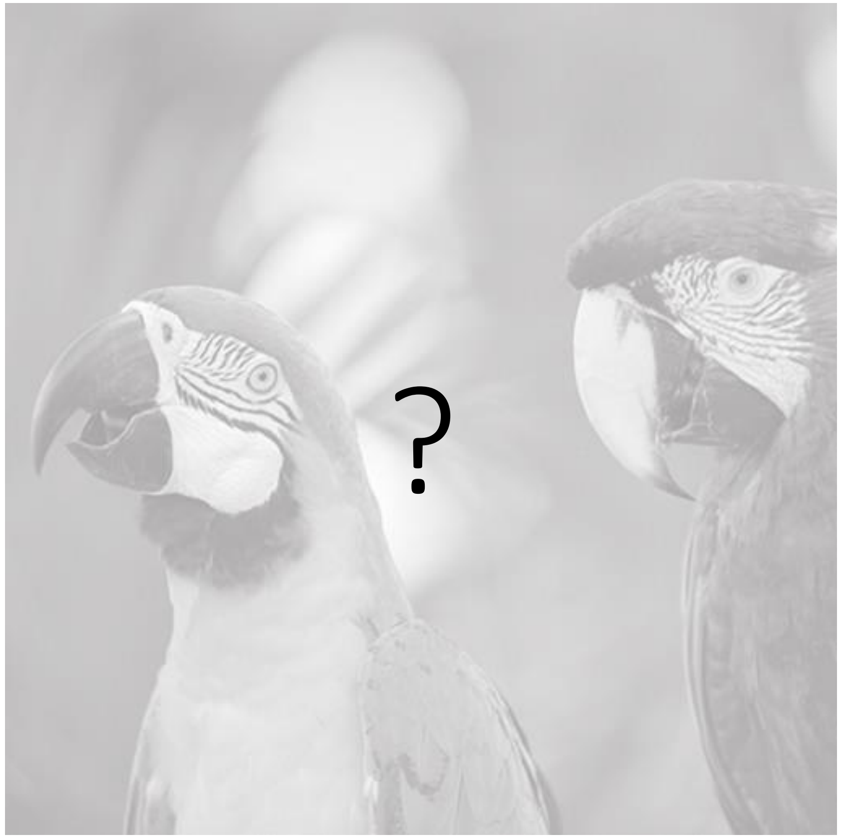
High quality images using cheap lenses



[Heide et al., "High-Quality Computational Imaging Through Simple Lenses," TOG 2013]

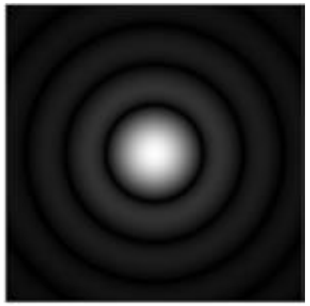
Deconvolution

If we know b and c, can we recover x?



*

How do we measure this?



=

$$x * c = b$$

PSF calibration

Take a photo of a point source

Image of PSF

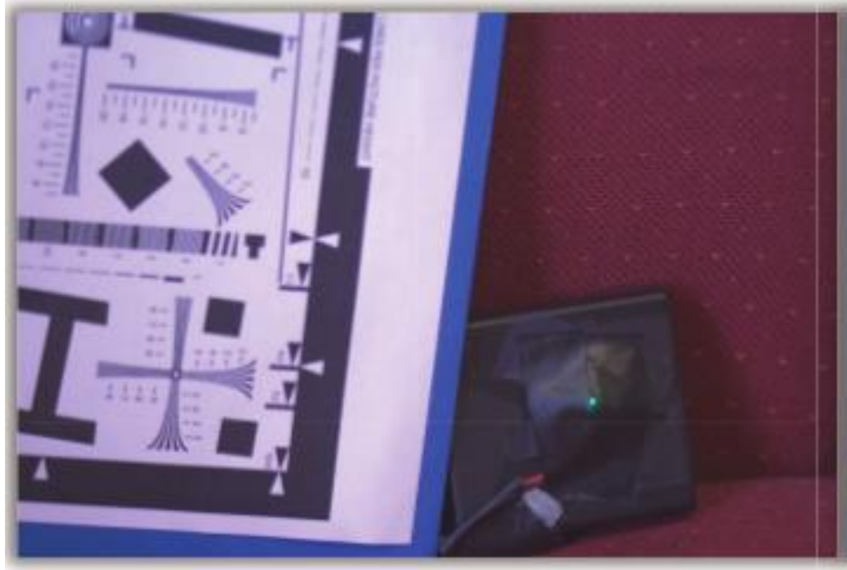
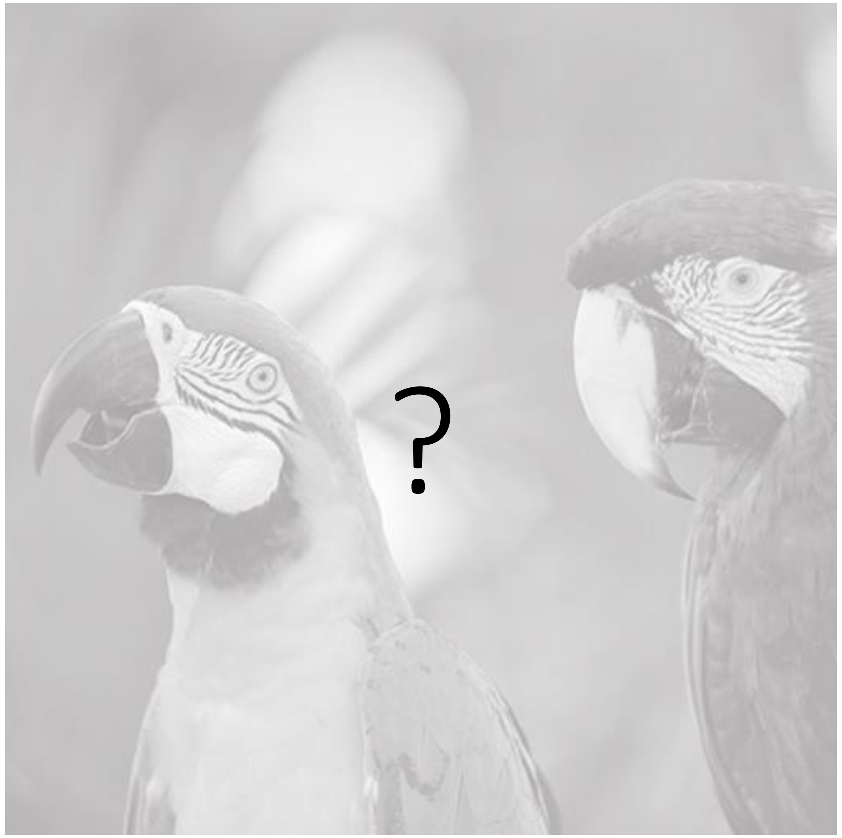


Image with sharp lens

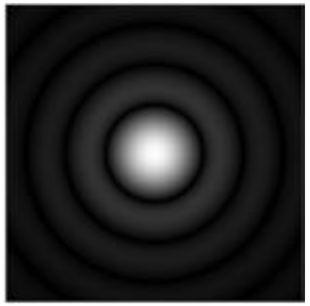
Image with cheap lens

Deconvolution

If we know b and c, can we recover x?



$*$

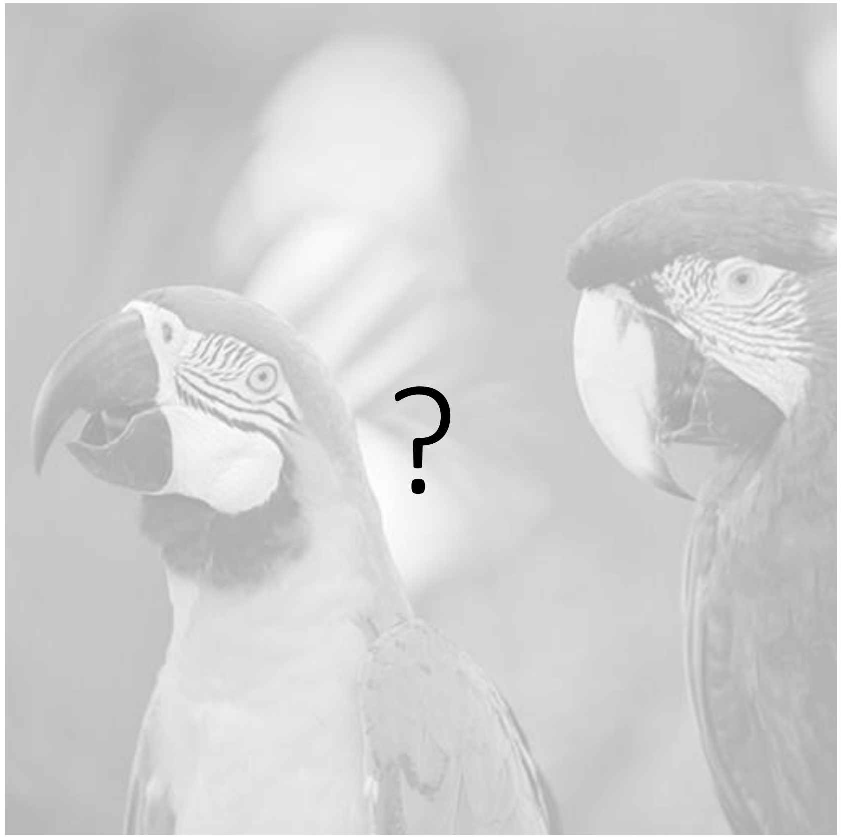


$=$

$$x \quad * \quad c \quad = \quad b$$

Blind deconvolution

If we know b, can we recover x and c?



*

=

x * c = b

Camera shake

Removing Camera Shake from a Single Photograph

Rob Fergus[1] Barun Singh[1] Aaron Hertzmann[2] Sam T. Roweis[2] William T. Freeman[1]

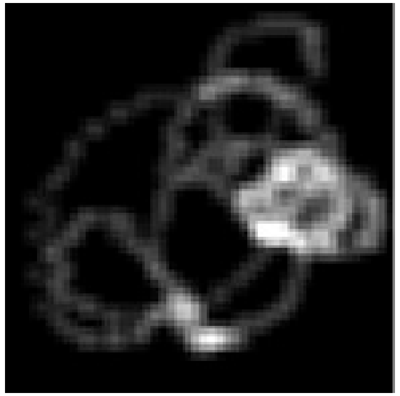
[1]MIT CSAIL [2]University of Toronto

Figure 1: *Left*: An image spoiled by camera shake. *Middle*: result from Photoshop "unsharp mask". *Right*: result from our algorithm.

Camera shake as a filter

If we know b, can we recover x and c?

$*$

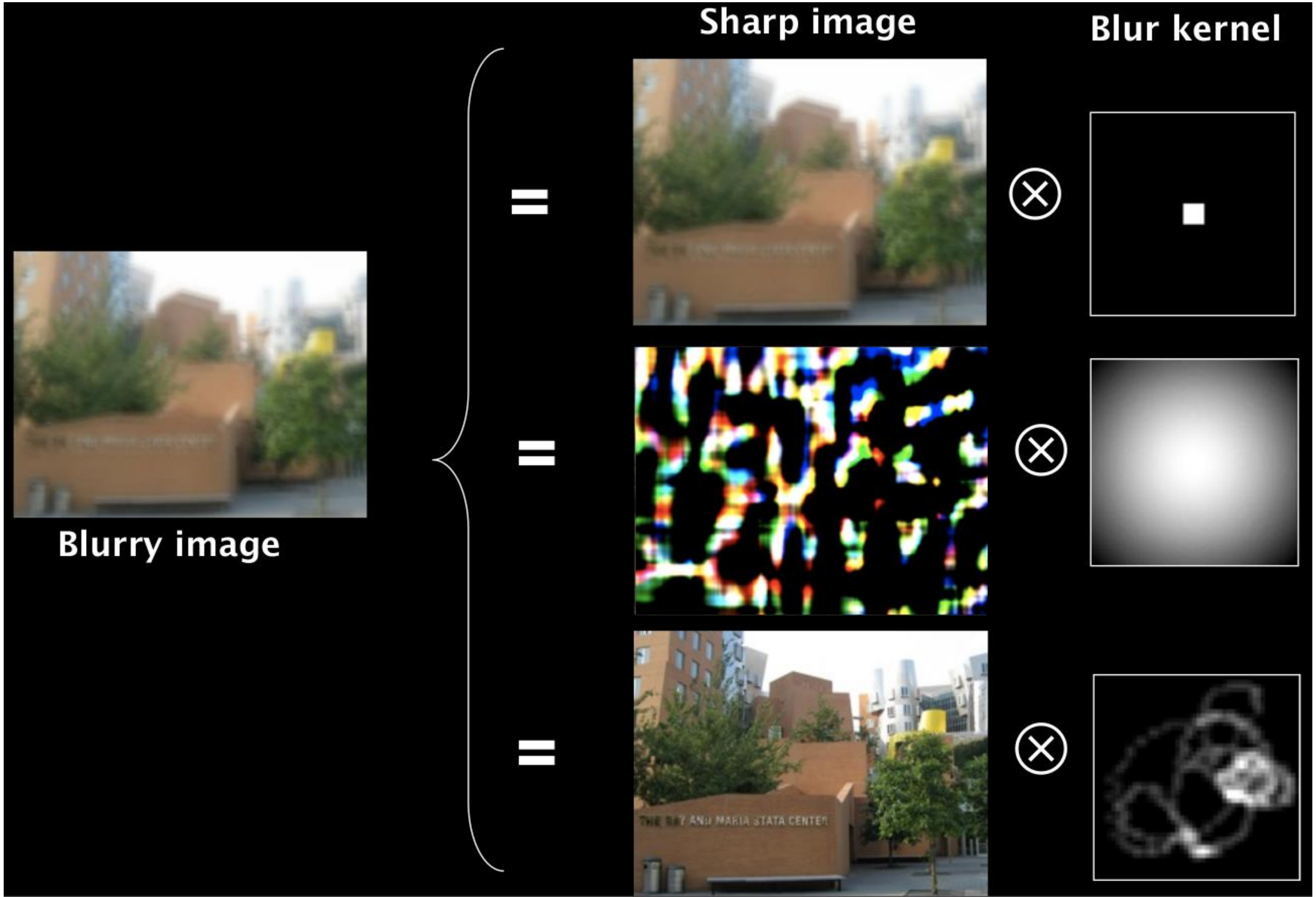


$=$

image from static camera | PSF from camera motion | image from shaky camera

$$x \quad * \quad c \quad = \quad b$$

Multiple possible solutions



How do we detect this one?

Use prior information

Among all the possible pairs of images and blur kernels, select the ones where:

- The image “looks like” a natural image.

- The kernel “looks like” a motion PSF.

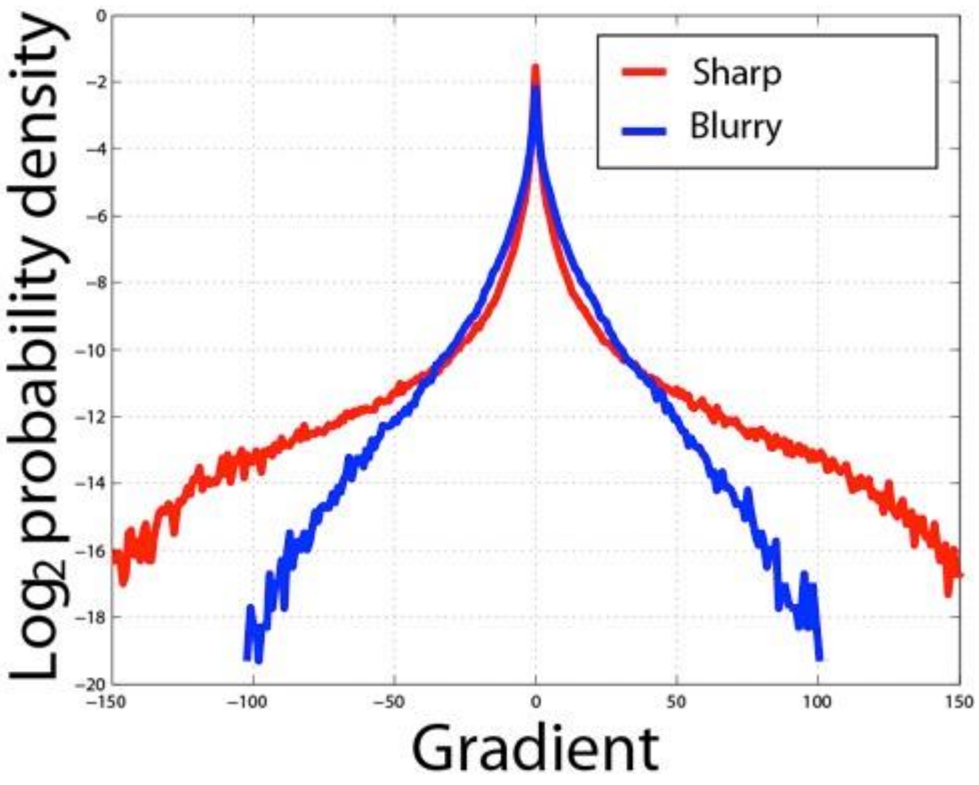
Use prior information

Among all the possible pairs of images and blur kernels, select the ones where:

- The image “looks like” a natural image.
- The kernel “looks like” a motion PSF.

Shake kernel statistics

Gradients in natural images follow a characteristic "heavy-tail" distribution.

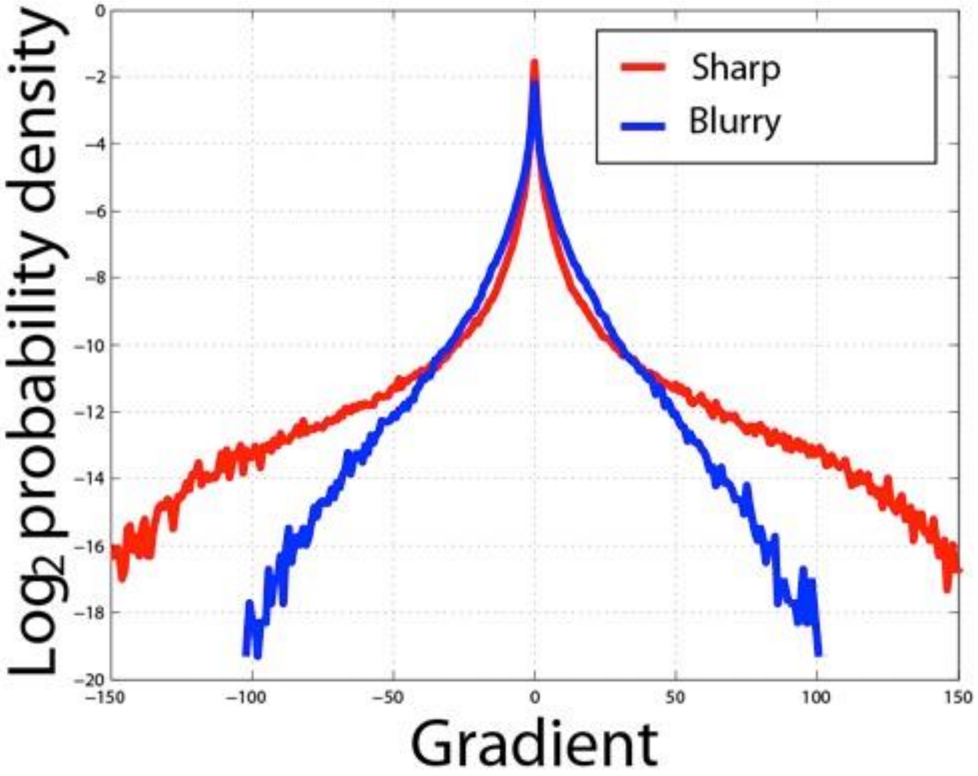


sharp natural image

blurry natural image

Shake kernel statistics

Gradients in natural images follow a characteristic "heavy-tail" distribution.



Can be approximated by $\| \nabla x \|^{0.8}$

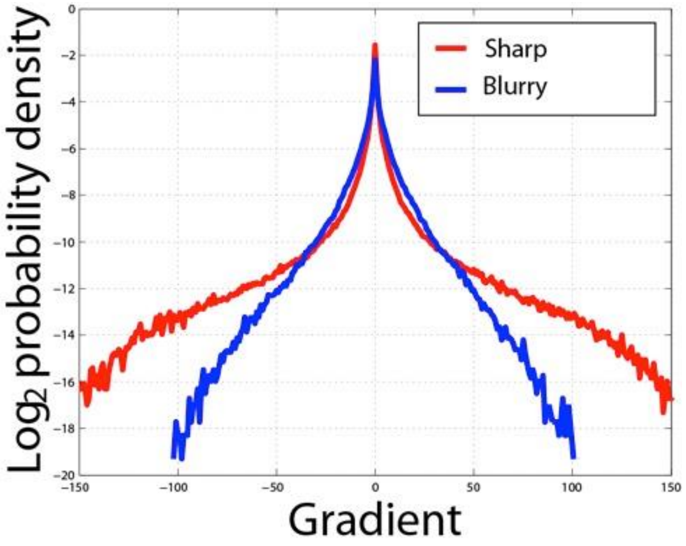
sharp natural image

blurry natural image

Use prior information

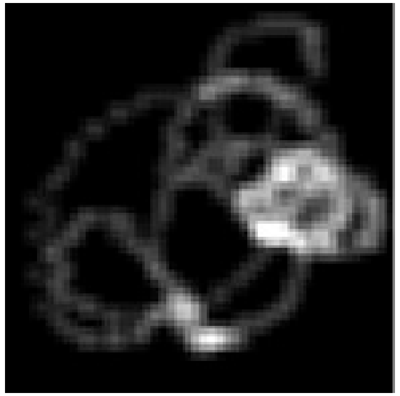
Among all the possible pairs of images and blur kernels, select the ones where:

- The image "looks like" a natural image.



Gradients in natural images follow a characteristic "heavy-tail" distribution.

- The kernel "looks like" a motion PSF.



Shake kernels are very sparse, have continuous contours, and are always positive

How do we use this information for blind deconvolution?

Regularized blind deconvolution

Solve regularized least-squares optimization

$$\min_{x,b} ||b - c * x||^2 + ||\nabla x||^{0.8} + ||c||_1$$

What does each term in this summation correspond to?

Regularized blind deconvolution

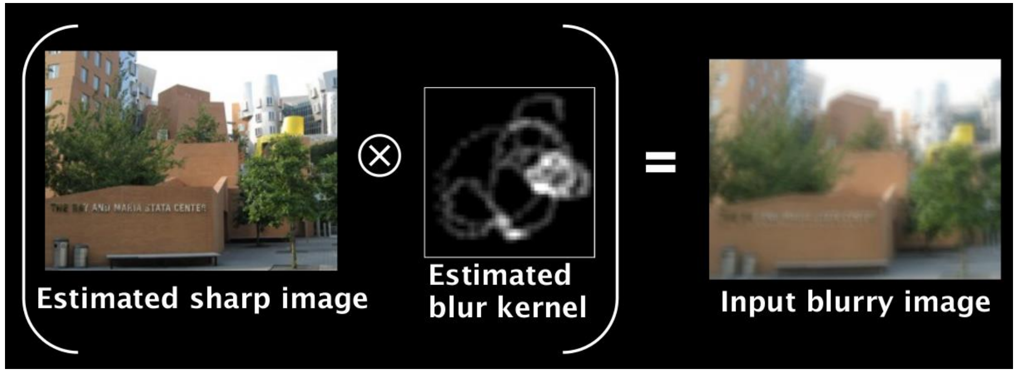
Solve regularized least-squares optimization

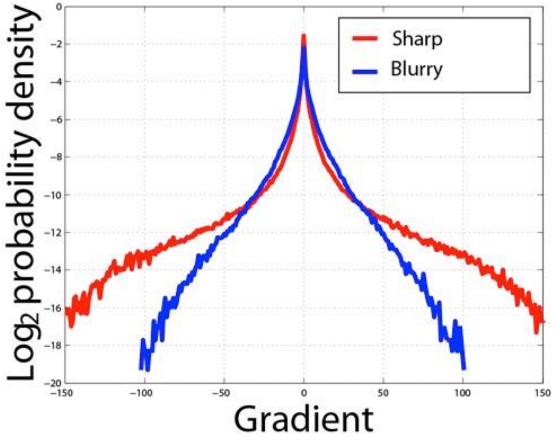
$$\min_{x,b} ||b - c * x||^2 + ||\nabla x||^{0.8} + ||c||_1$$

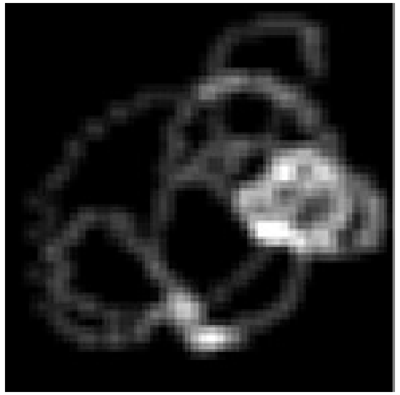
data term

natural image prior

shake kernel prior





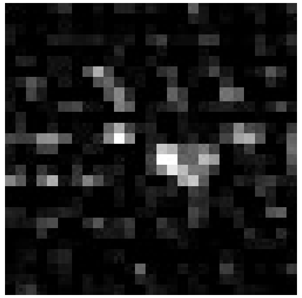


Note: Solving such optimization problems is complicated (no longer *linear* least squares).

A demonstration

input

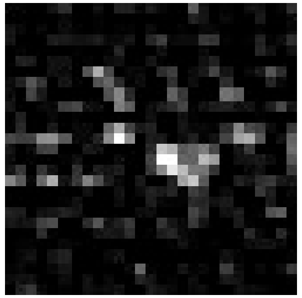
deconvolved image and kernel



A demonstration

input

deconvolved image and kernel



This image looks worse than the original…

This doesn’t look like a plausible shake kernel…

Regularized blind deconvolution

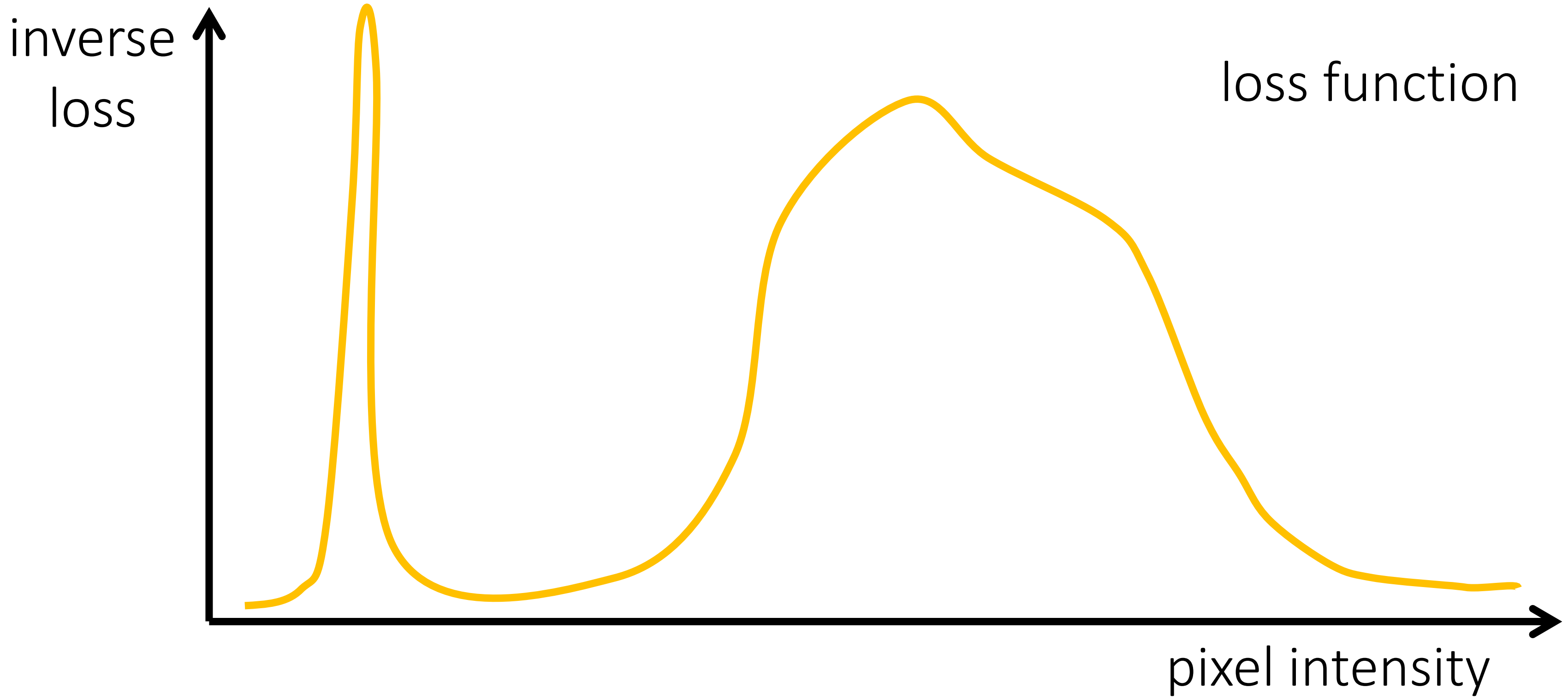
Solve regularized least-squares optimization

$$\min_{x,b} \underbrace{||b - c * x||^2 + ||\nabla x||^{0.8} + ||c||_1}_{\text{loss function}}$$

Regularized blind deconvolution

Solve regularized least-squares optimization

$$\min_{x,b} \underbrace{\|b - c * x\|^2 + \|\nabla x\|^{0.8} + \|c\|_1}_{\text{loss function}}$$

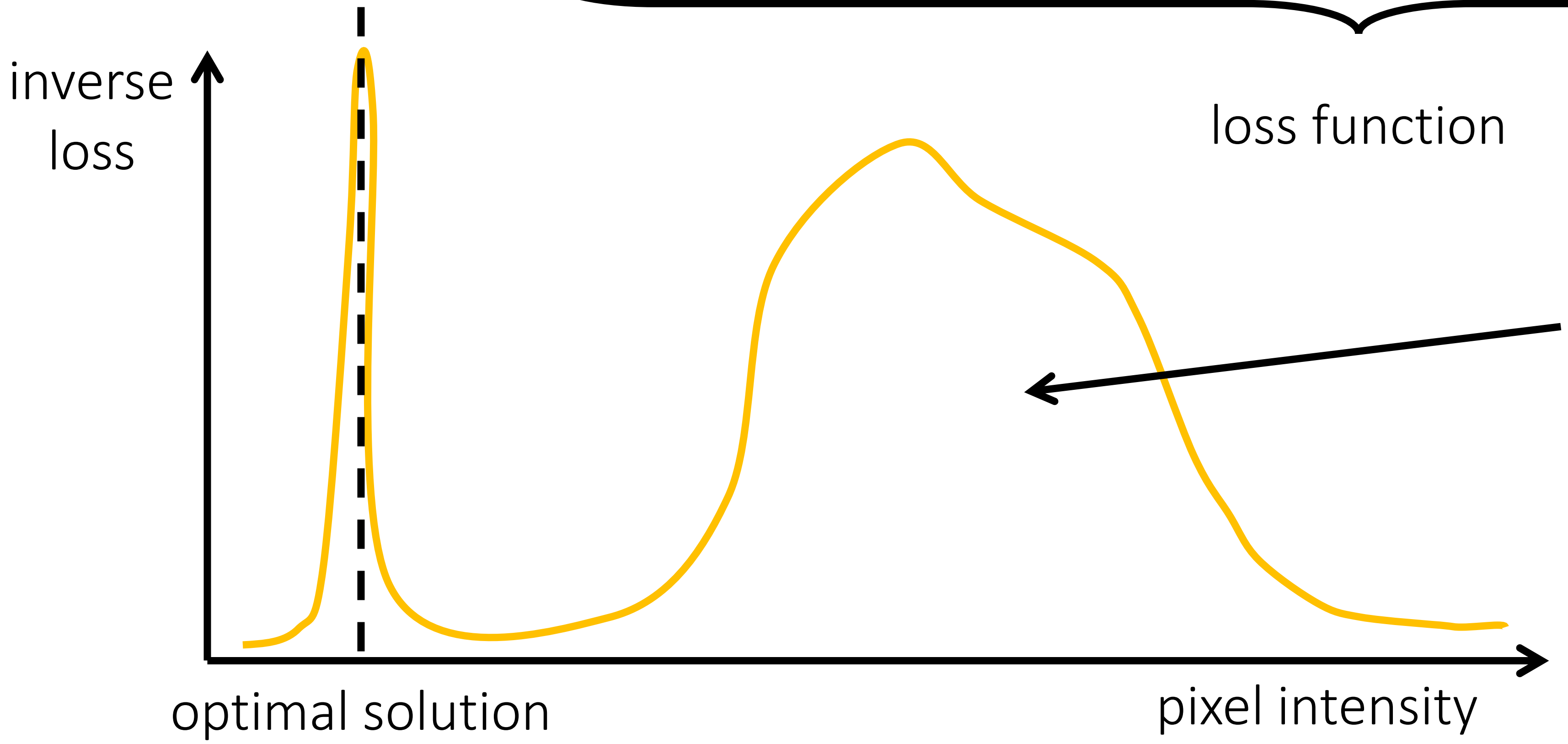


Where in this graph is the solution we find?

Regularized blind deconvolution

Solve regularized least-squares optimization

$$\min_{x,b} \underbrace{||b - c * x||^2 + ||\nabla x||^{0.8} + ||c||_1}_{\text{loss function}}$$

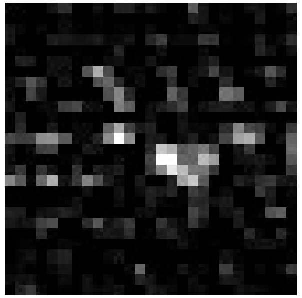


Rather than keep just maximum, do a weighted average of all solutions

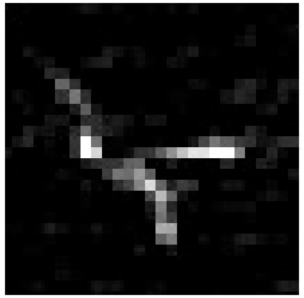
A demonstration

input

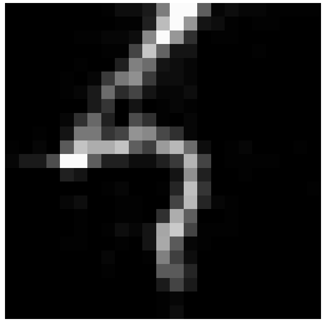
maximum-only

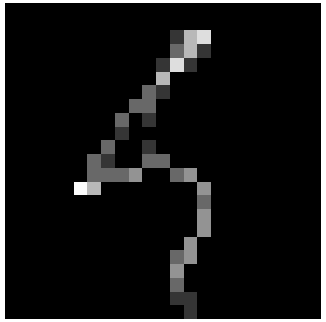


average

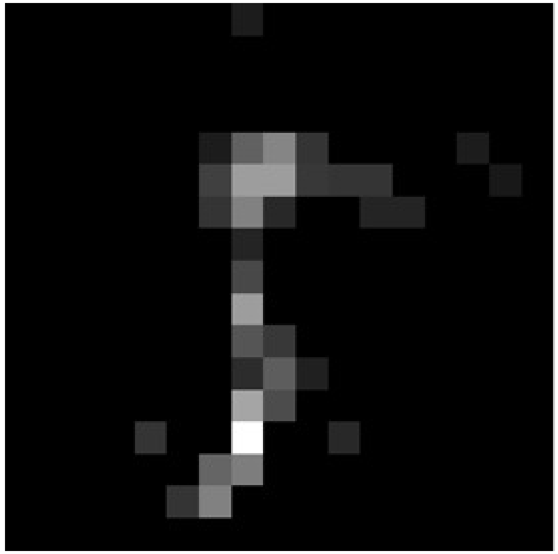


More examples



Results on real shaky images



Results on real shaky images

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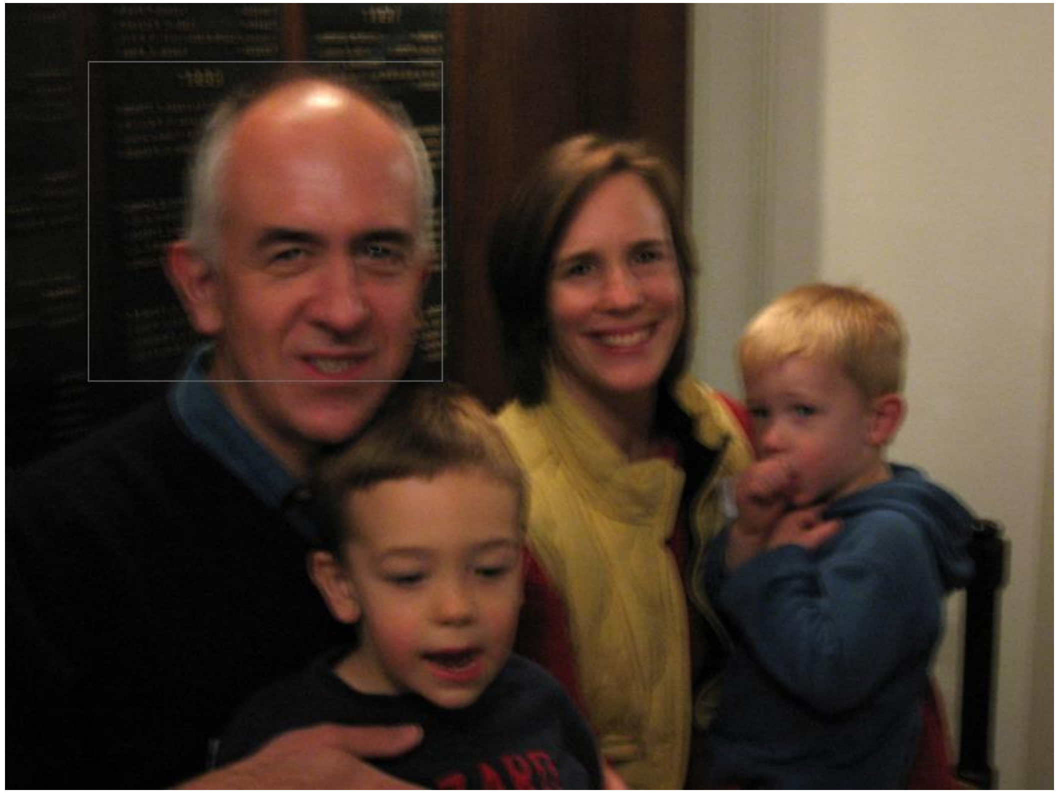
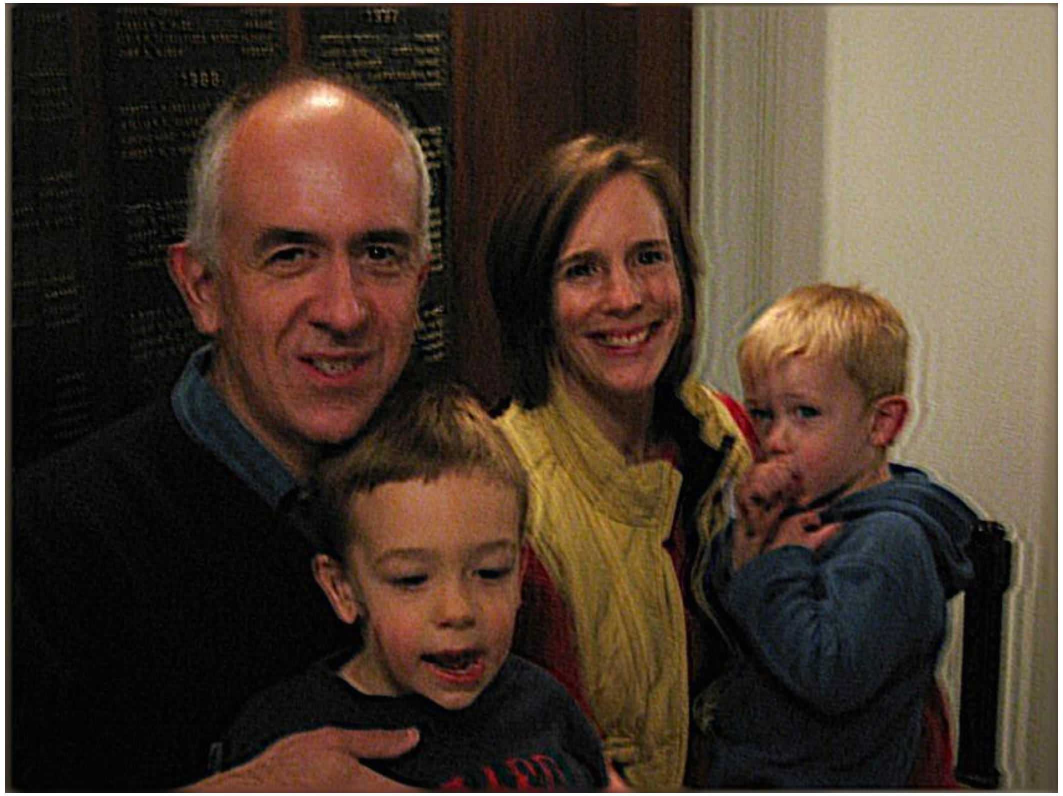
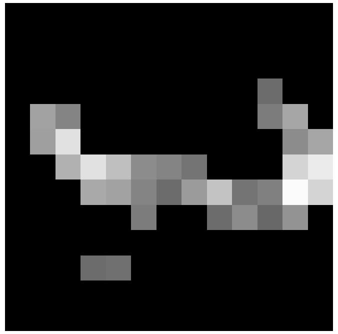
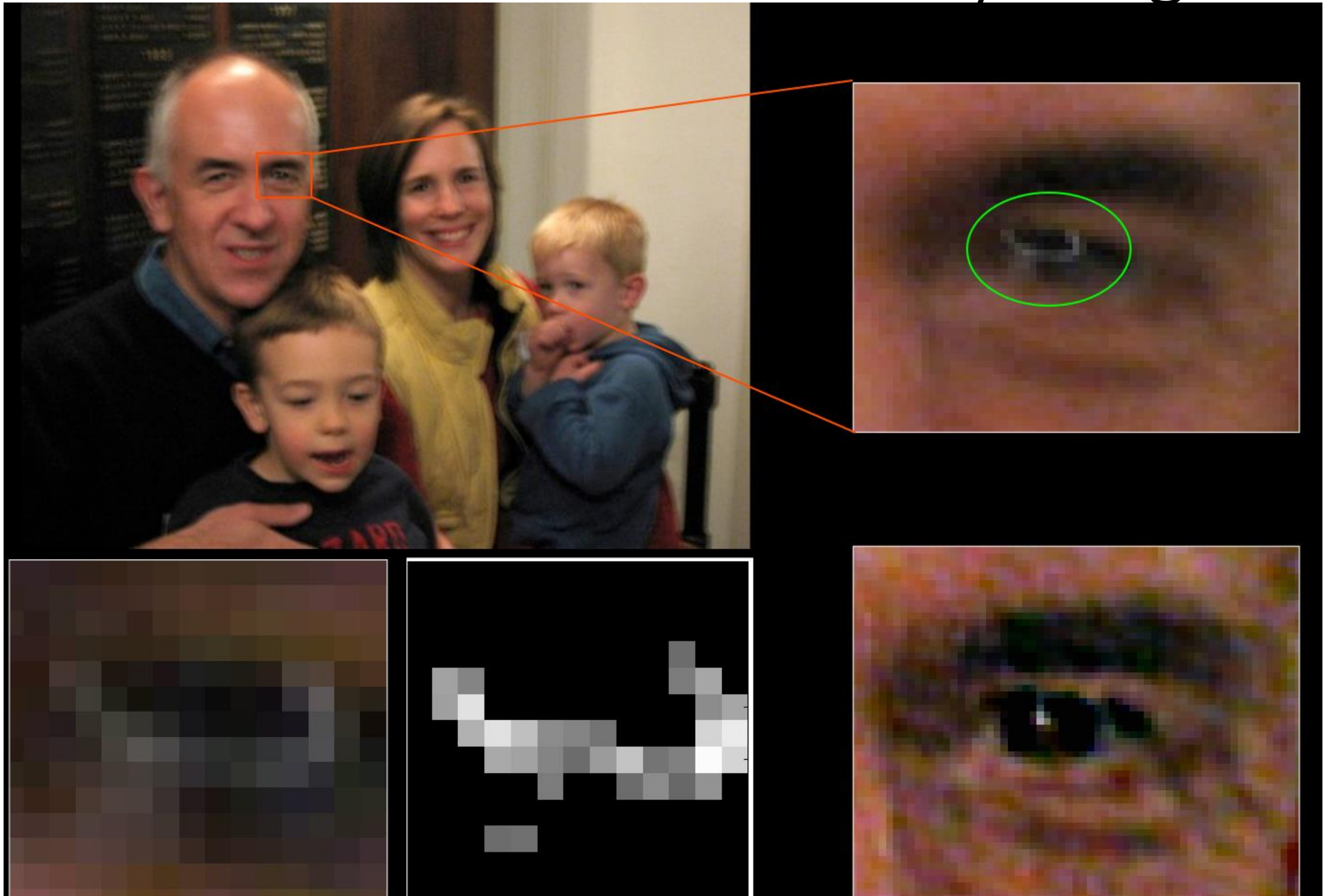
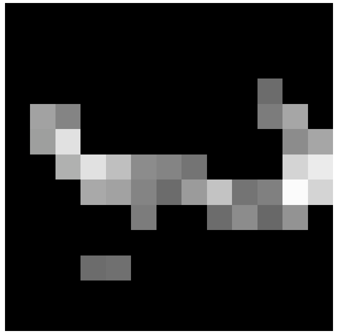






Results on real shaky images





More advanced motion deblurring

[Shah et al., High-quality Motion Deblurring from a Single Image, SIGGRAPH 2008]

Why are our images blurry?

- Lens imperfections.
- Camera shake.
- Scene motion.
- Depth defocus.

Can we solve all of these problems using (blind) deconvolution?

Why are our images blurry?

- Lens imperfections.
- Camera shake.
- Scene motion.
- Depth defocus.

Can we solve all of these problems using (blind) deconvolution?

- We can deal with (some) lens imperfections and camera shake, because their blur is shift invariant.
- We cannot deal with scene motion and depth defocus, because their blur is not shift invariant.
- See coded photography lecture.

References

Basic reading:

- Szeliski textbook, Sections 3.4.3, 3.4.4, 10.1.4, 10.3.
- Fergus et al., "Removing camera shake from a single image," SIGGRAPH 2006.
  the main motion deblurring and blind deconvolution paper we covered in this lecture.

Additional reading:

- Heide et al., "High-Quality Computational Imaging Through Simple Lenses," TOG 2013.
  the paper on high-quality imaging using cheap lenses, which also has a great discussion of all matters relating to blurring from lens aberrations and modern deconvolution algorithms.
- Levin, "Blind Motion Deblurring Using Image Statistics," NIPS 2006.
- Levin et al., "Image and depth from a conventional camera with a coded aperture," SIGGRAPH 2007.
- Levin et al., "Understanding and evaluating blind deconvolution algorithms," CVPR 2009 and PAMI 2011.
- Krishnan and Fergus, "Fast Image Deconvolution using Hyper-Laplacian Priors," NIPS 2009.
- Levin et al., "Efficient Marginal Likelihood Optimization in Blind Deconvolution," CVPR 2011.
  a sequence of papers developing the state of the art in blind deconvolution of natural images, including the use Laplacian (sparsity) and hyper-Laplacian priors on gradients, analysis of different loss functions and maximum a-posteriori versus Bayesian estimates, the use of variational inference, and efficient optimization algorithms.
- Minskin and MacKay, "Ensemble Learning for Blind Image Separation and Deconvolution," AICA 2000.
  the paper explaining the mathematics of how to compute Bayesian estimators using variational inference.
- Shah et al., "High-quality Motion Deblurring from a Single Image," SIGGRAPH 2008.
  a more recent paper on motion deblurring.