Fourier optics

15-463, 15-663, 15-862
Computational Photography
Fall 2017, Lecture 28

http://graphics.cs.cmu.edu/courses/15-463

Course announcements

- Any questions about homework 6?
- Extra office hours today, 3-5pm.
- Make sure to take the three surveys:
  1) faculty course evaluation
  2) TA evaluation survey
  3) end-of-semester class survey
- Monday are project presentations
  - Do you prefer 3 minutes or 6 minutes per person?
  - Will post more details on Piazza.
  - Also please return cameras on Monday!

Overview of today’s lecture

- The scalar wave equation.
- Basic waves and coherence.
- The plane wave spectrum.
- Fraunhofer diffraction and transmission.
- Fresnel lenses.
- Fraunhofer diffraction and reflection.

Slide credits

Some of these slides were directly adapted from:

- Anat Levin (Technion).

Scalar wave equation

Simplifying the EM equations

Scalar wave equation:

- Homogeneous and source-free medium
- No polarization

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)u(r,t) = 0$$

speed of light in medium

Simplifying the EM equations

Helmholtz equation:

- Either assume perfectly monochromatic light at wavelength λ
- Or assume different wavelengths independent of each other

$$(\nabla^2 + k^2)\psi(r) = 0$$

$$u(r,t) = Re\left\{\psi(r)e^{-j\frac{2\pi c}{\lambda}t}\right\}$$

what is this?

$$\psi(r) = A(r)e^{j\varphi(r)}$$

Simplifying the EM equations

Helmholtz equation:

- Either assume perfectly monochromatic light at wavelength λ
- Or assume different wavelengths independent of each other

$$(\nabla^2 + k^2)\psi(r) = 0$$

Wave is a sinusoid at frequency $2\pi/\lambda$:

$$u(r,t) = Re\left\{\psi(r)e^{-j\frac{2\pi c}{\lambda}t}\right\}$$

$$\psi(r) = A(r)e^{j\varphi(r)}$$

what is this?

Simplifying the EM equations

Helmholtz equation:

- Either assume perfectly monochromatic light at wavelength λ
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Wave is a sinusoid at frequency $2\pi/\lambda$:

$$u(r,t) = Re\left\{\psi(r)e^{-j\frac{2\pi c}{\lambda}t}\right\}$$

At every point, wave has amplitude A(r) and phase $\varphi(\mathrm{r})$:

$$\psi(r) = A(r)e^{j\varphi(r)}$$

Simplifying the EM equations

Helmholtz equation:

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At every point, wave has amplitude A(r) and phase $\varphi(\mathrm{r})$:

$$\psi(r) = A(r)e^{j\varphi(r)}$$

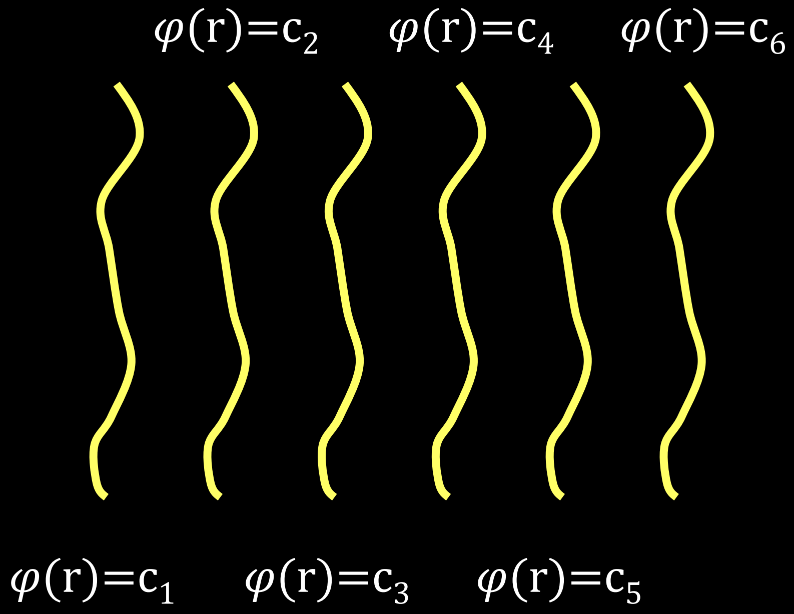
This is how we will describe waves for the rest of lecture

Basic waves and coherence

Visualizing a wave

Wavefront: A set of points that have the same phase

- Points on the wavefront have "travelled" the same distance from wave source
- Gives us "shape" of the wave



At every point, wave has amplitude A(r) and phase $\varphi(\mathrm{r})$:

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Visualizing a wave

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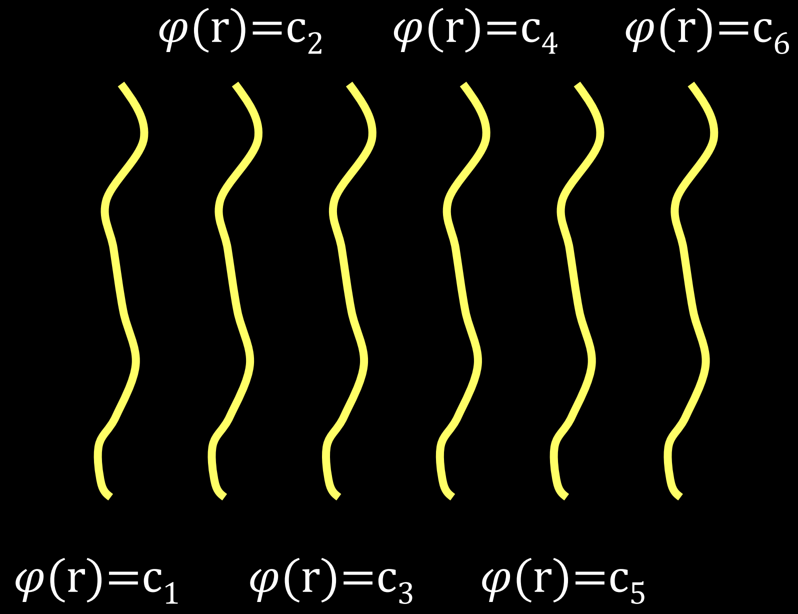


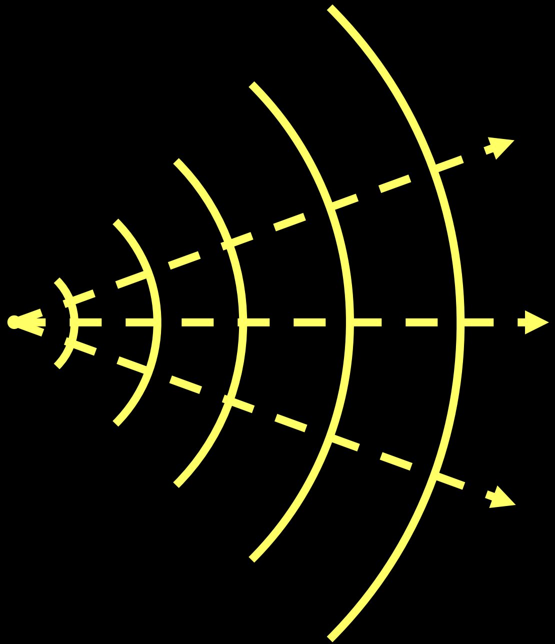
Roughly speaking, in ray optics we replace waves with “rays” that are always normal to wavefront

At every point, wave has amplitude A(r) and phase $\varphi(\mathrm{r})$:

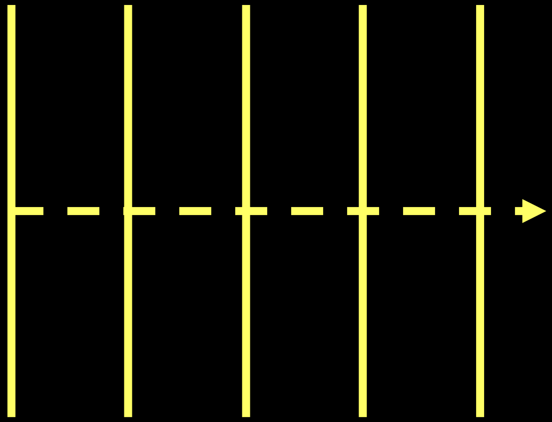
$$\psi(r) = A(r)e^{j\varphi(r)}$$

Two important waves

Spherical wave



Plane wave

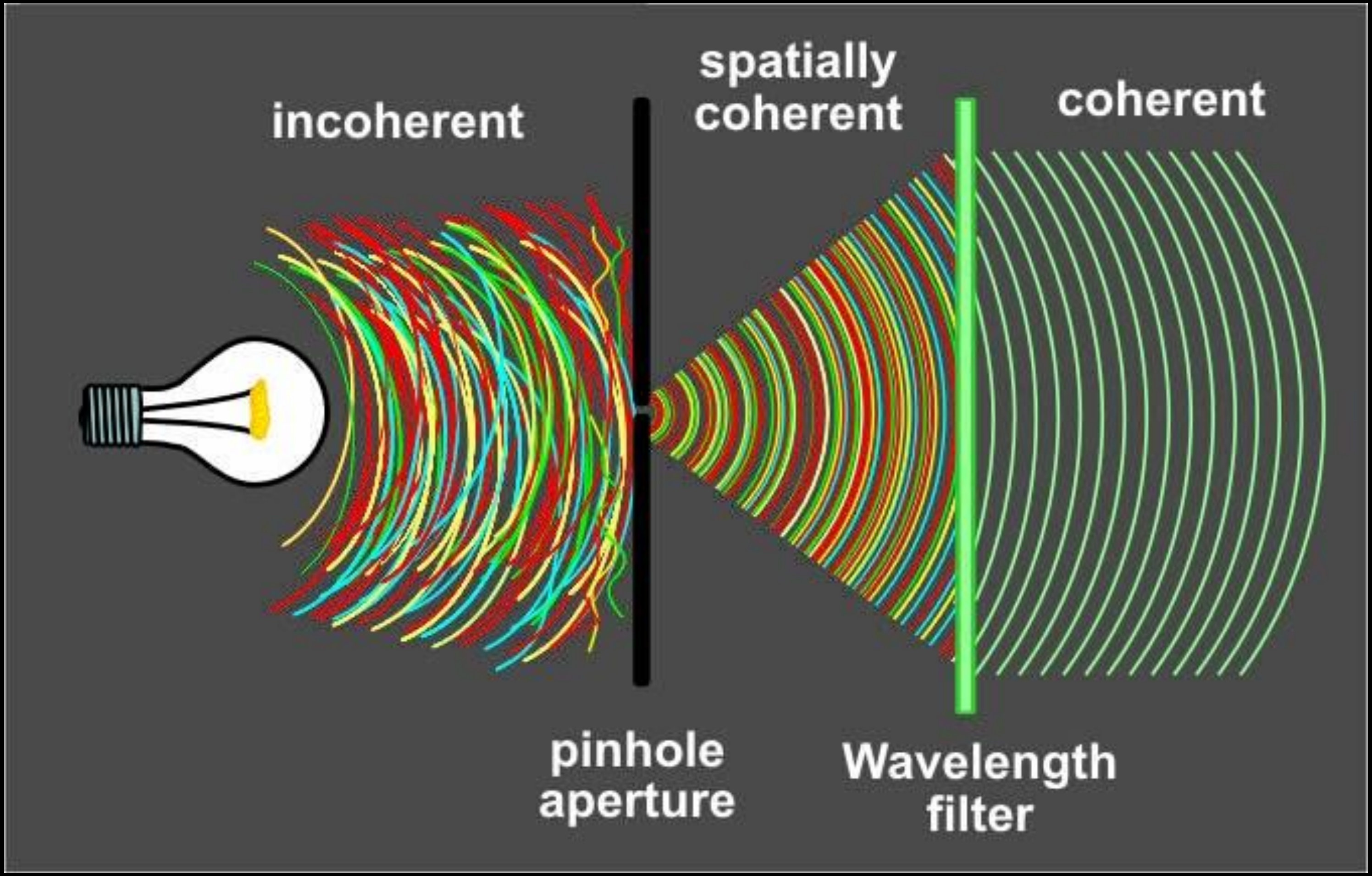


How can you create
a spherical wave?

At every point, wave has amplitude A(r) and phase $\varphi(\mathrm{r})$:

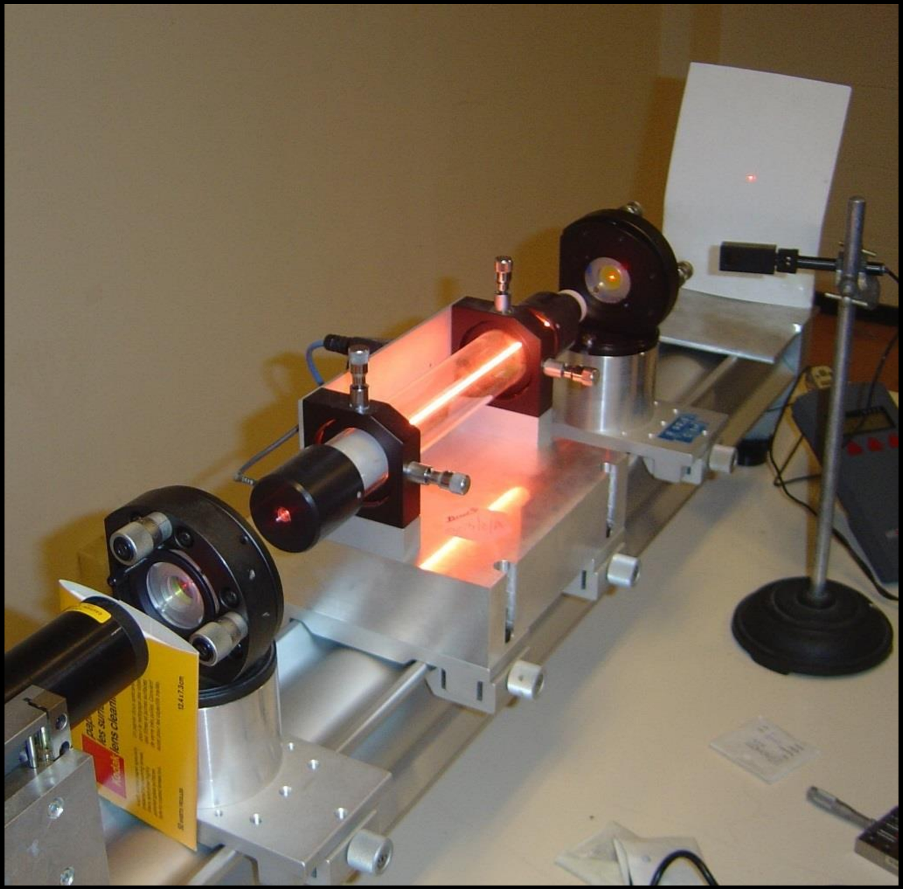
$$\psi(r) = A(r)e^{j\varphi(r)}$$

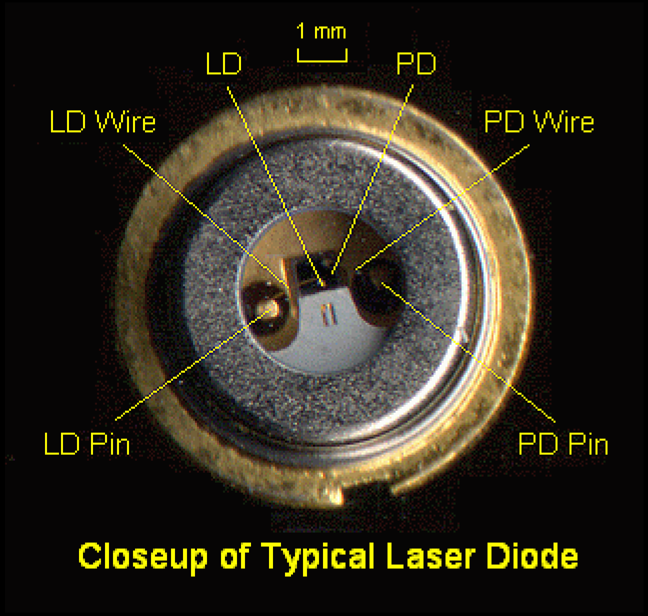
Creating a spherical wave using pinholes



- Any problems with this procedure?
- Do you know of any alternatives?

Creating a spherical wave using lasers



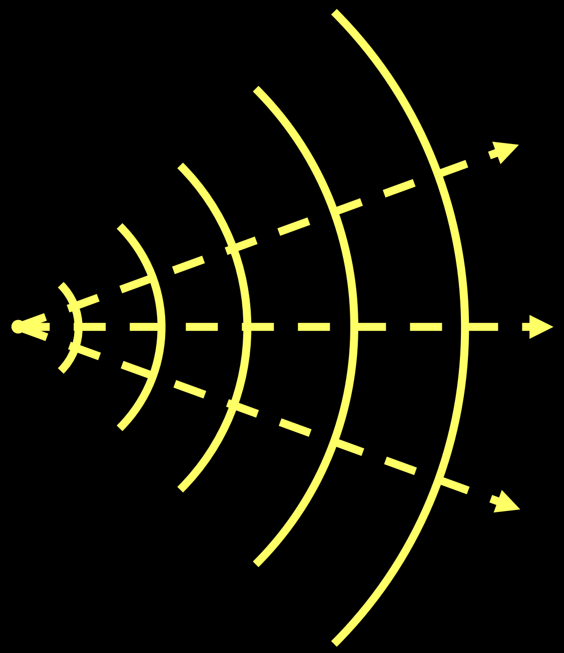


Closeup of Typical Laser Diode

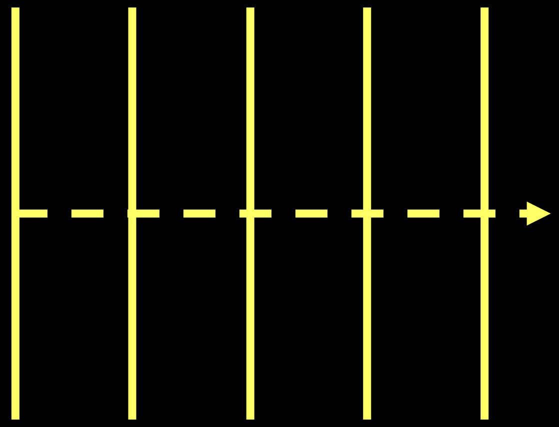
- Lasers are high-power “point” sources
- Standard lasers are also monochromatic (temporally coherent)

Two important waves

Spherical wave



Plane wave



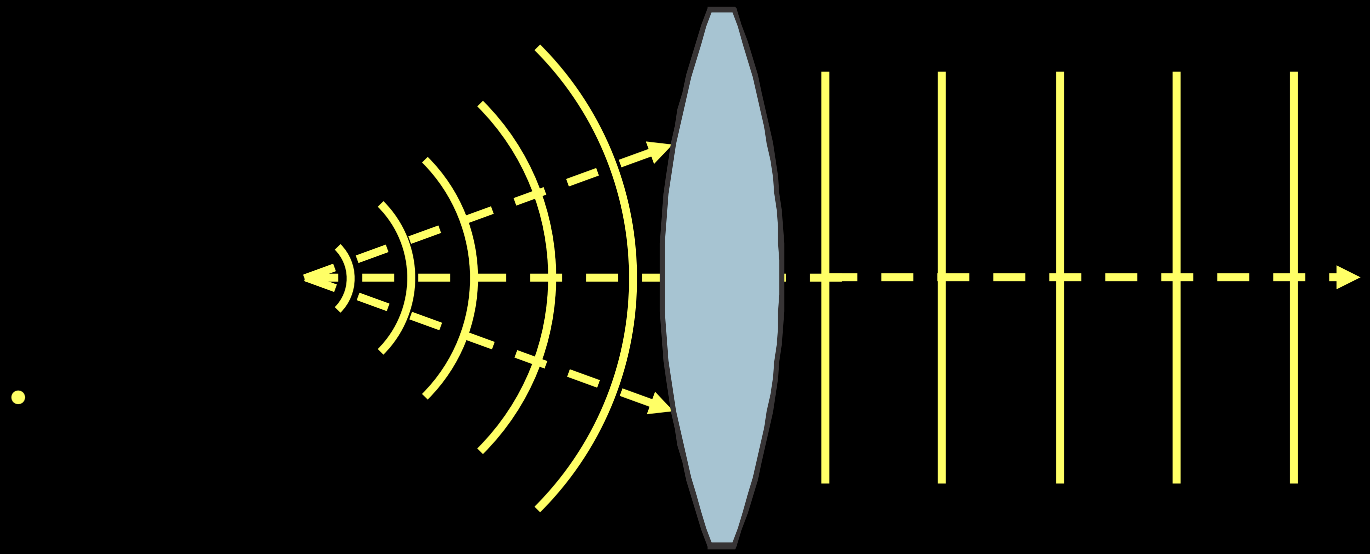
How can you create
a plane wave?

At every point, wave has amplitude A(r) and phase $\varphi(r)$:

$$\psi(r) = A(r)e^{j\varphi(r)}$$

Creating plane waves

1. Use a thin lens:



Creating plane waves

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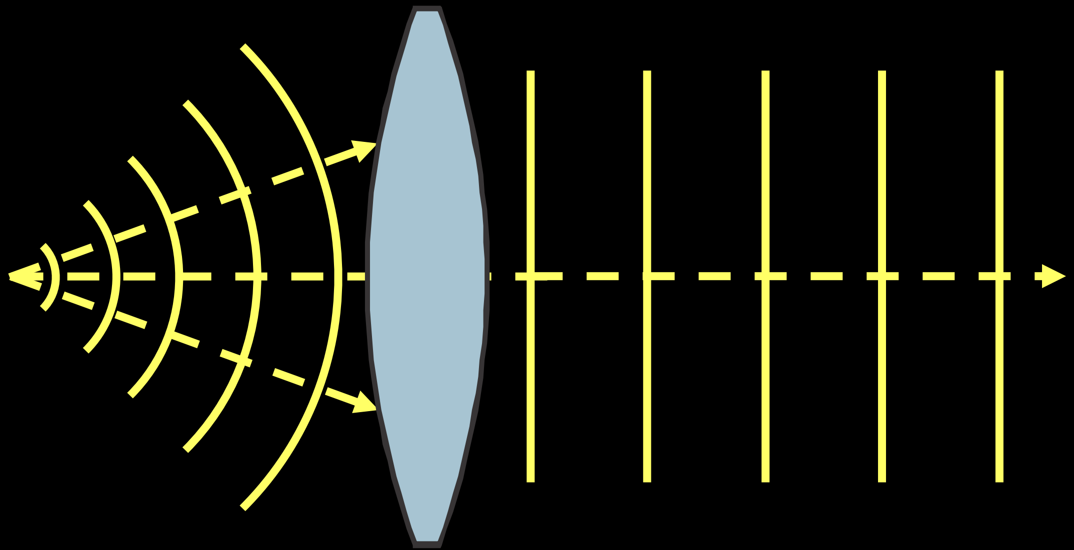
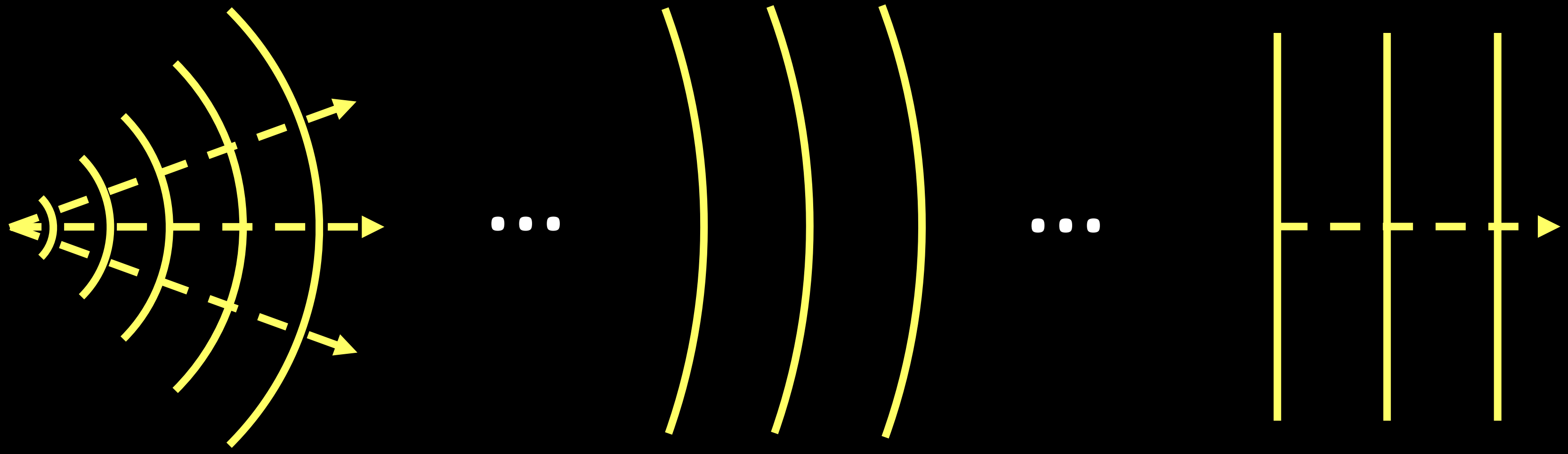
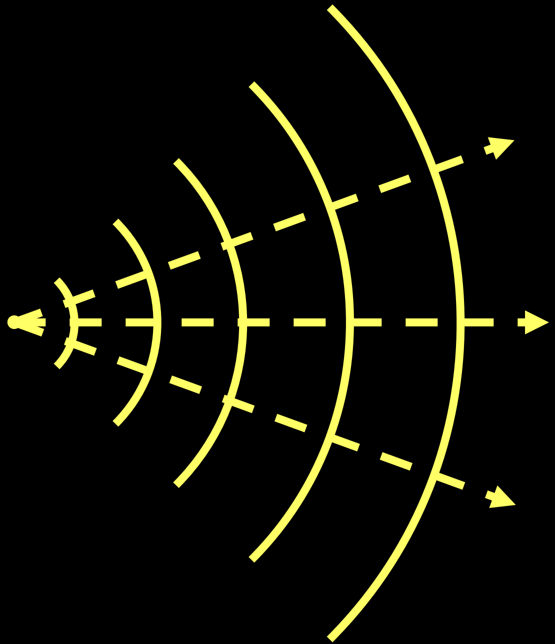


2. Let a spherical wave propagate a very long distance:
   - This is often called the “far-field” assumption.

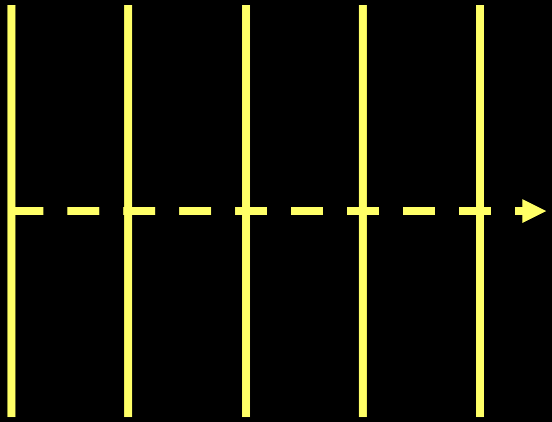


Two important waves

Spherical wave



Plane wave



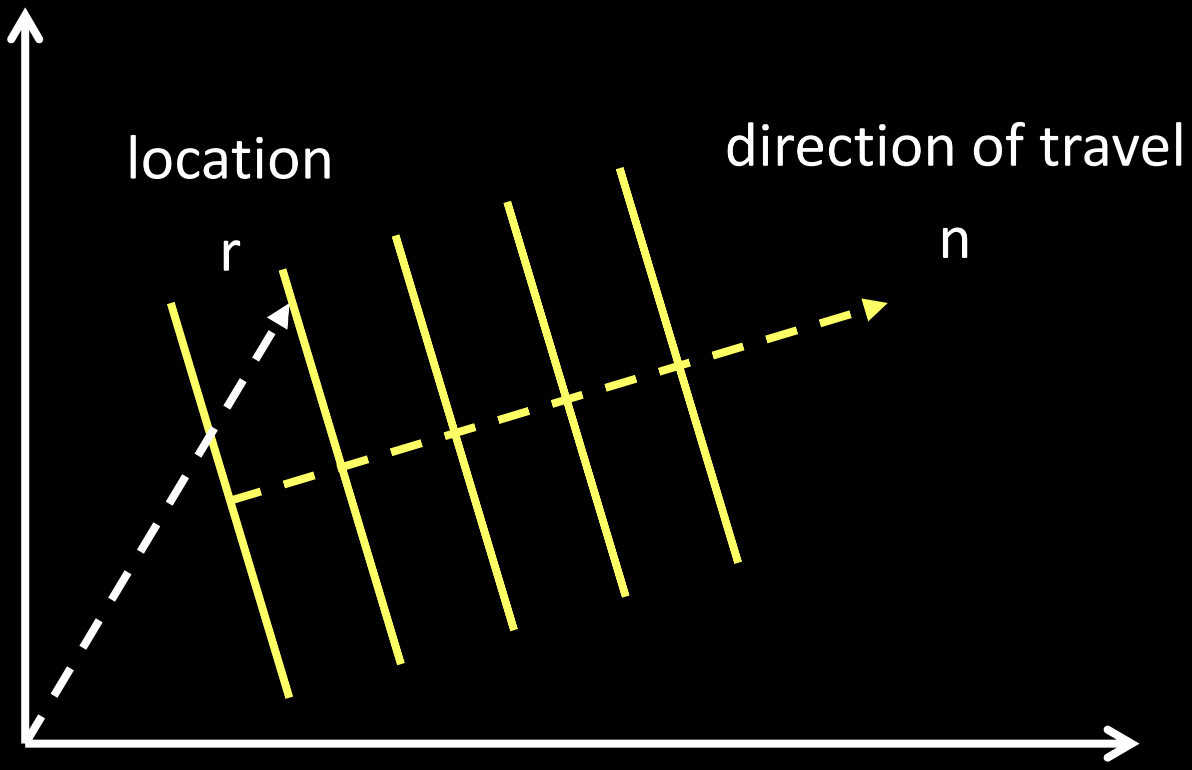
What is the equation of a plane wave?

At every point, wave has amplitude A(r) and phase $\varphi(r)$:

$$\psi(r) = A(r)e^{j\varphi(r)}$$

The plane wave spectrum

Plane wave equation



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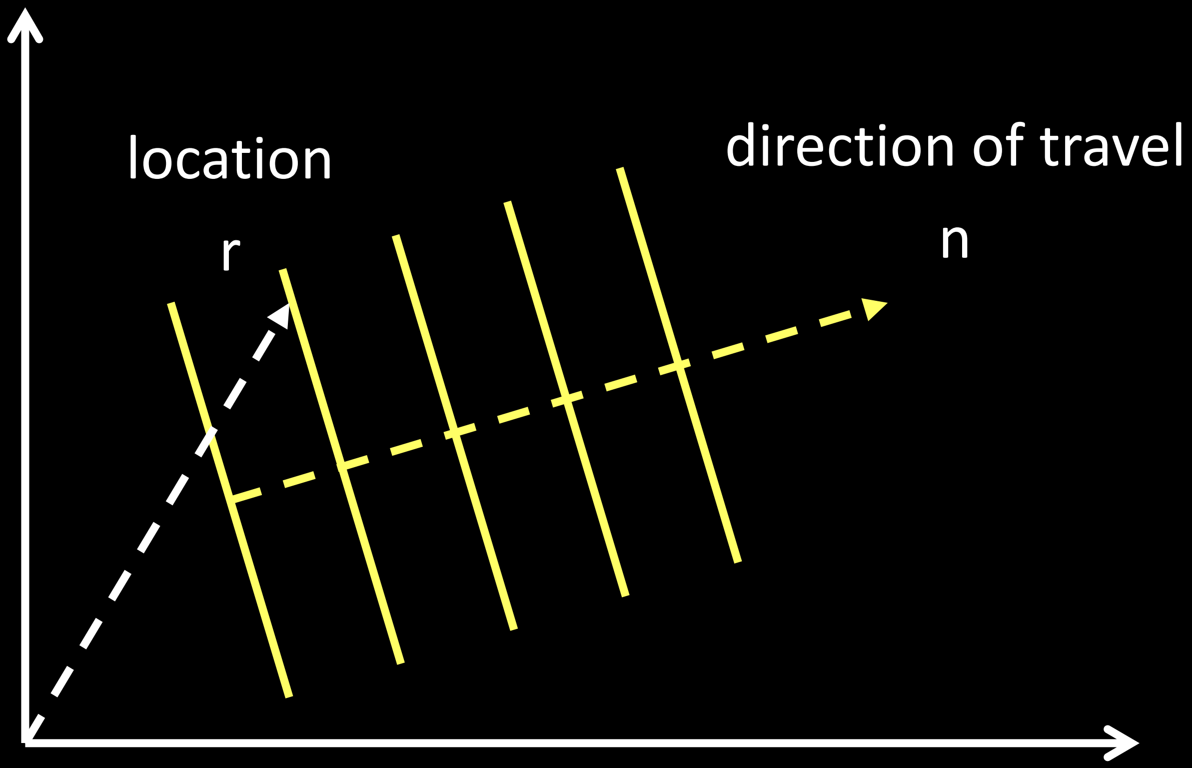
Plane wave equation



Plane wave equation:

$$\psi_{p,k}(r) = e^{jk\cdot r}$$

does this remind you of something?

Wave vector:

$$k = \frac{2\pi c}{\lambda} n$$

Plane wave spectrum

Every wave can be written as the weighted superposition of planar waves at different directions

$$\psi(r) = A(r)e^{j\varphi(r)}$$

$$\psi(r) = \int_k \Psi(k)\psi_{p,k}(r)dk$$

$$\psi(r) = \int_k \underset{\nwarrow}{\Psi(k)}e^{jk\cdot r}dk$$

How are these weights determined?

Plane wave spectrum

Every wave can be written as the weighted superposition of planar waves at different directions

$$\psi(r) = A(r)e^{j\varphi(r)}$$

$$\psi(r) = \int_k \Psi(k)\psi_{p,k}(r)dk$$

$$\psi(r) = \int_k \Psi(k)e^{jk\cdot r}dk$$

$$\Psi\ (k) = \mathrm{Fourier}\{\psi(r)\}$$

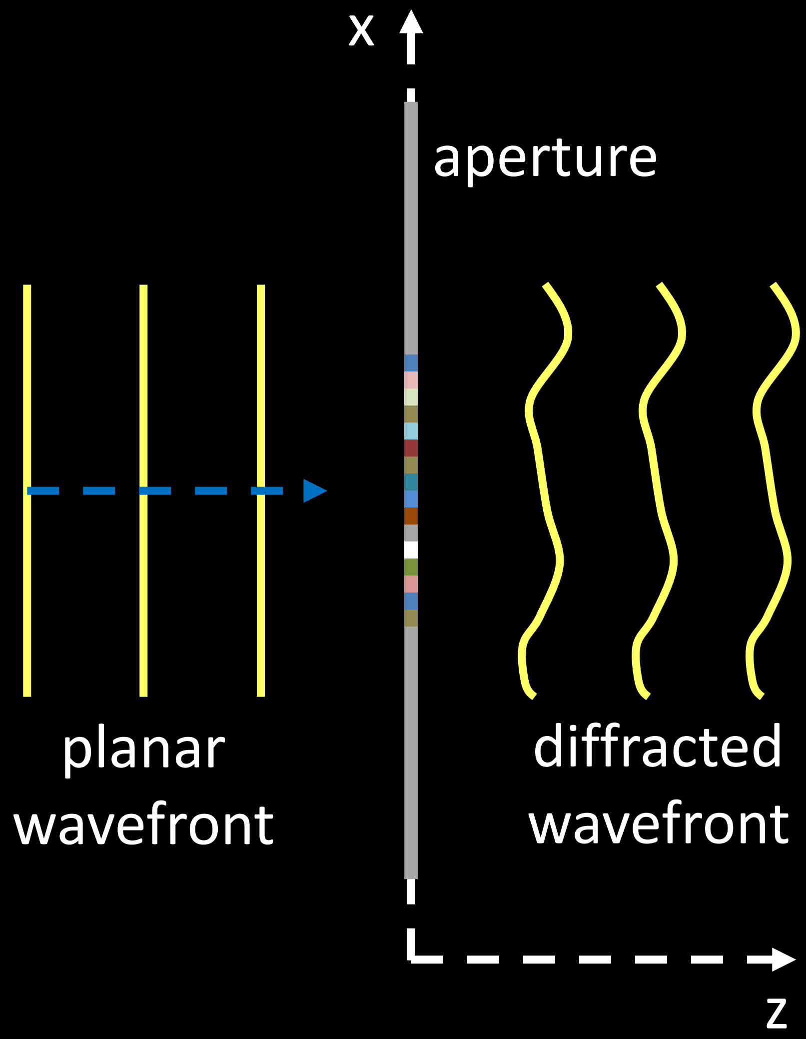
This is the wave's plane wave spectrum

Fraunhofer diffraction and transmission

Fraunhofer diffraction

Wave-optics model for transmission through apertures

- Far-field assumption: Light is coming from and measured



Fraunhofer diffraction

Wave-optics model for transmission through apertures

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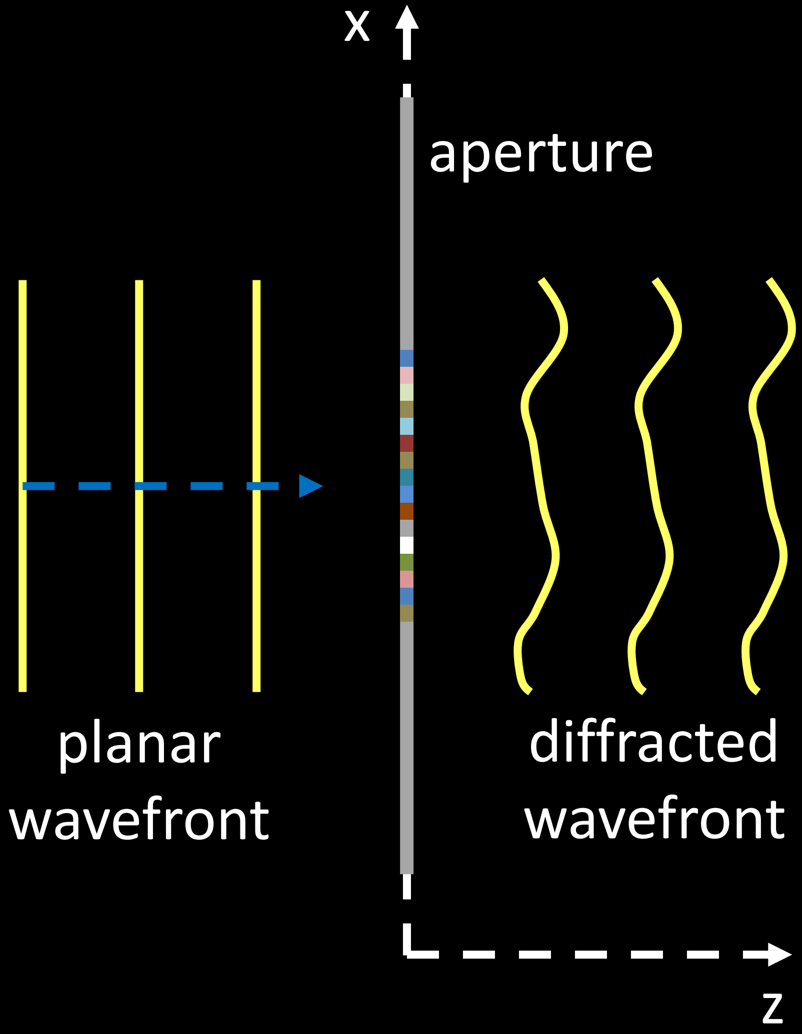


- transmission function:

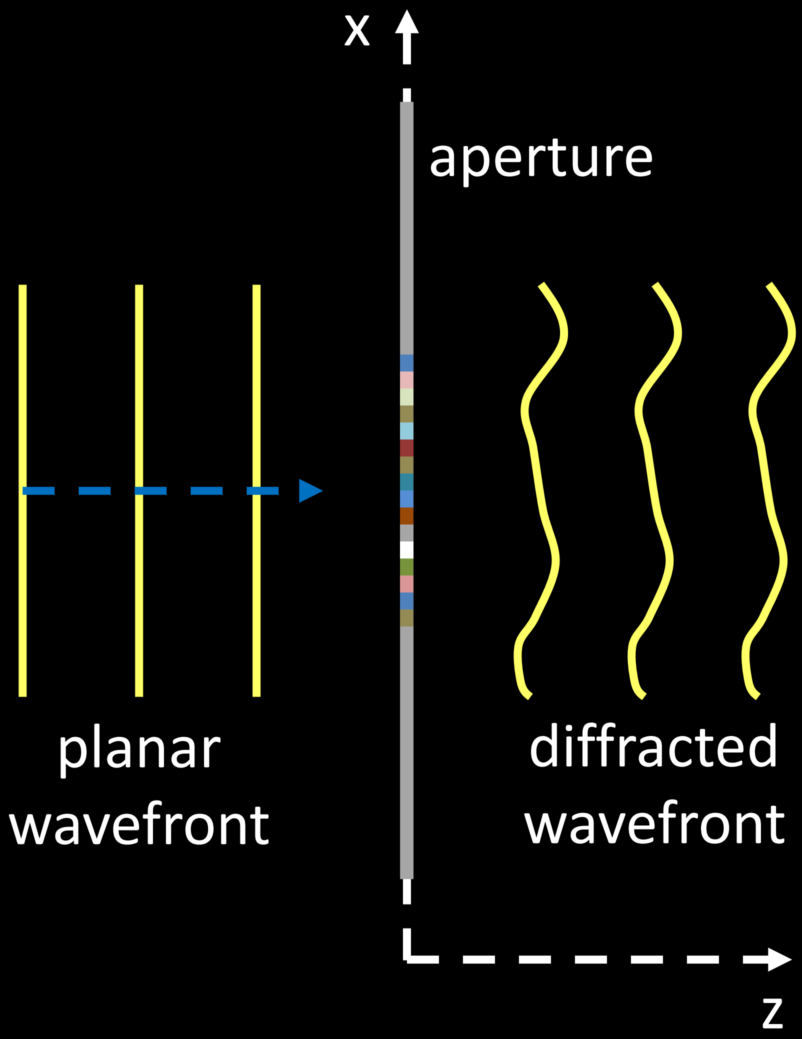
$$p(r) = A(r) \cdot \exp(\, j \cdot \Phi(r)\,)$$

amplitude modulation ↗ phase modulation ↗

Fraunhofer diffraction

Wave-optics model for transmission through apertures

- Far-field assumption: Light is coming from and measured



- transmission function:

$$p(r) = \underbrace{A(r)}_{\text{amplitude modulation}} \cdot \exp(\, j \cdot \underbrace{\Phi(r)}_{\text{phase modulation}}\,)$$

- transfer function:

$$P(k) = \text{Fourier}\{p(r)\}$$

Fraunhofer diffraction

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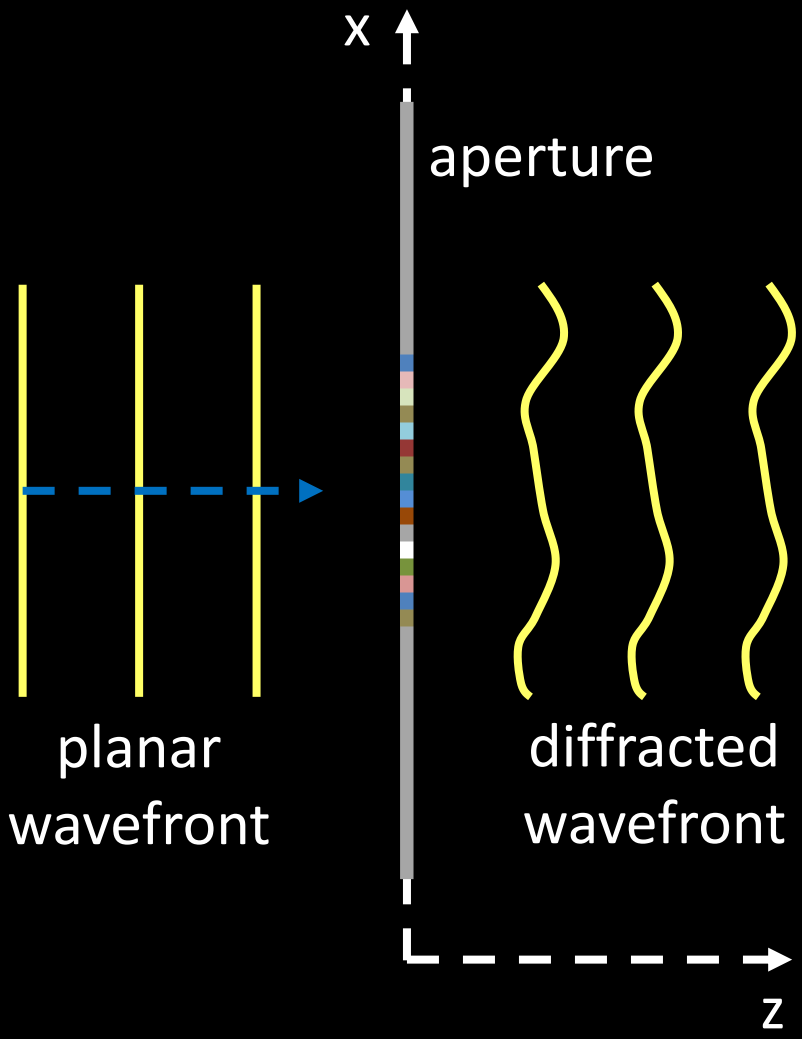

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- transfer function:

$$P(k) = \text{Fourier}\{p(r)\}$$

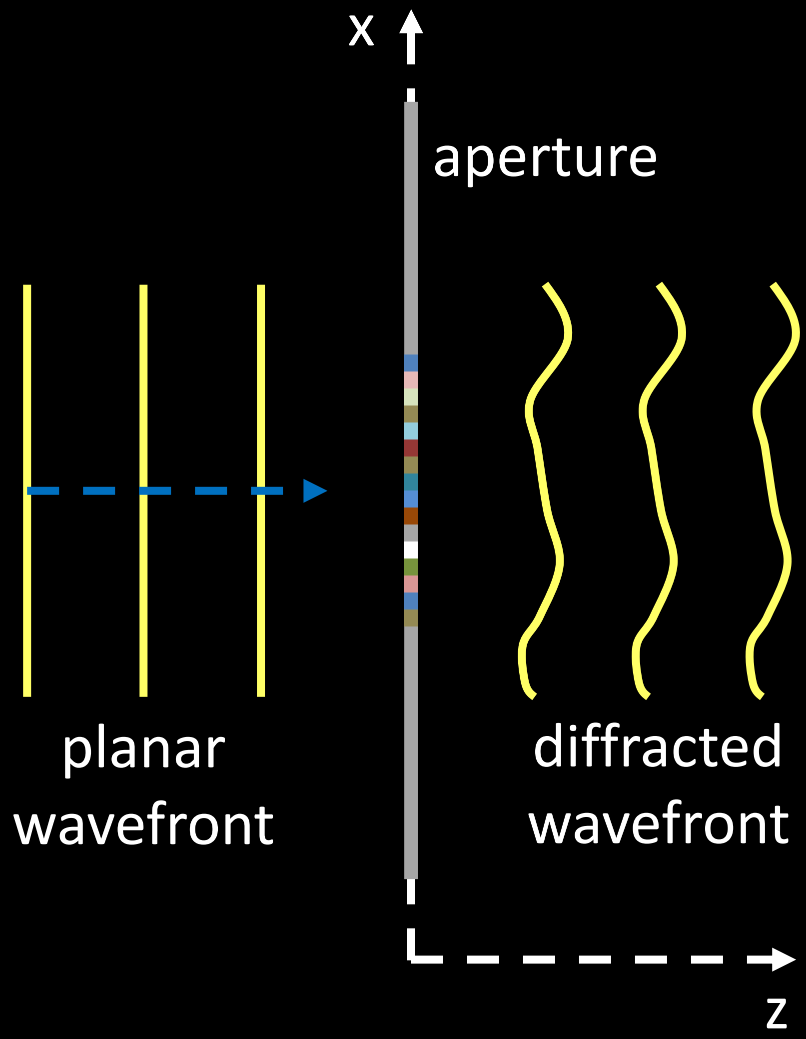
- plane spectrum of outgoing wave:

$$\Psi_{out}(k) = P(k) \cdot \Psi_{in}(k)$$

Fraunhofer diffraction

Wave-optics model for transmission through apertures

- Far-field assumption: Light is coming from and measured



- transmission function:

$$p(r) = A(r) \cdot \exp(\, j \cdot \Phi(r)\,)$$

amplitude modulation ↗ phase modulation ↗

- transfer function:

$$P(k) = \text{Fourier}\{p(r)\}$$

- plane spectrum of outgoing wave:

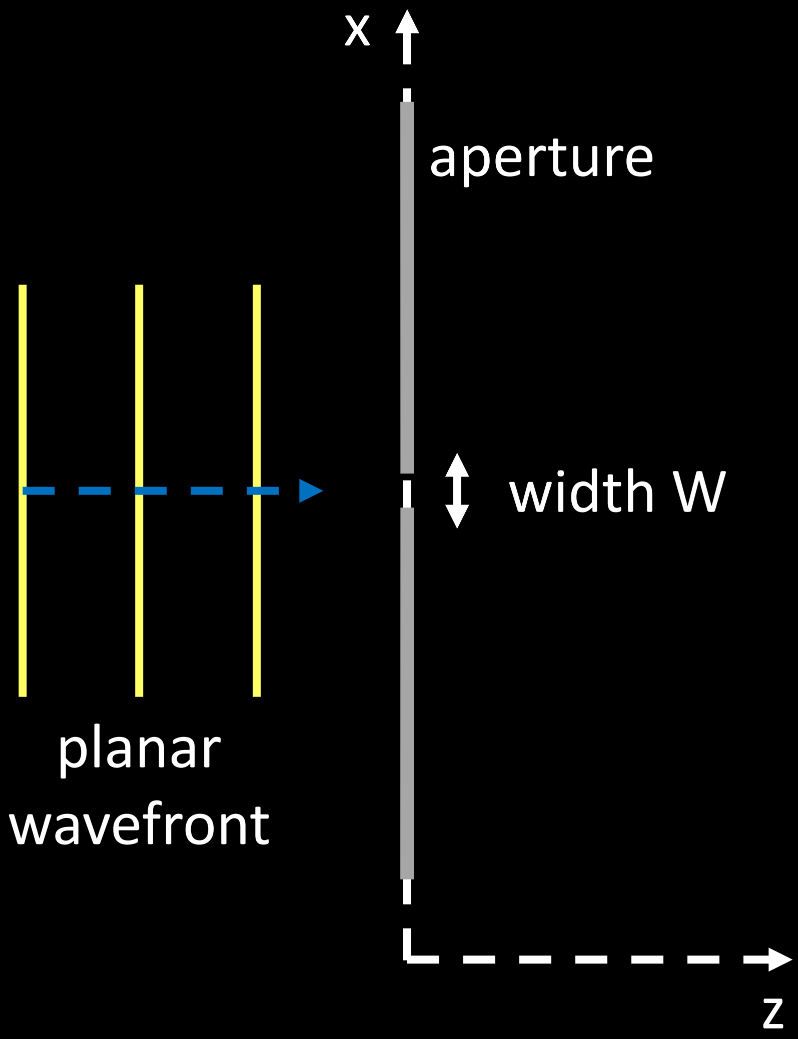
$$\Psi_{out}(k) = P(k) \cdot \Psi_{in}(k)$$

- outgoing wave:

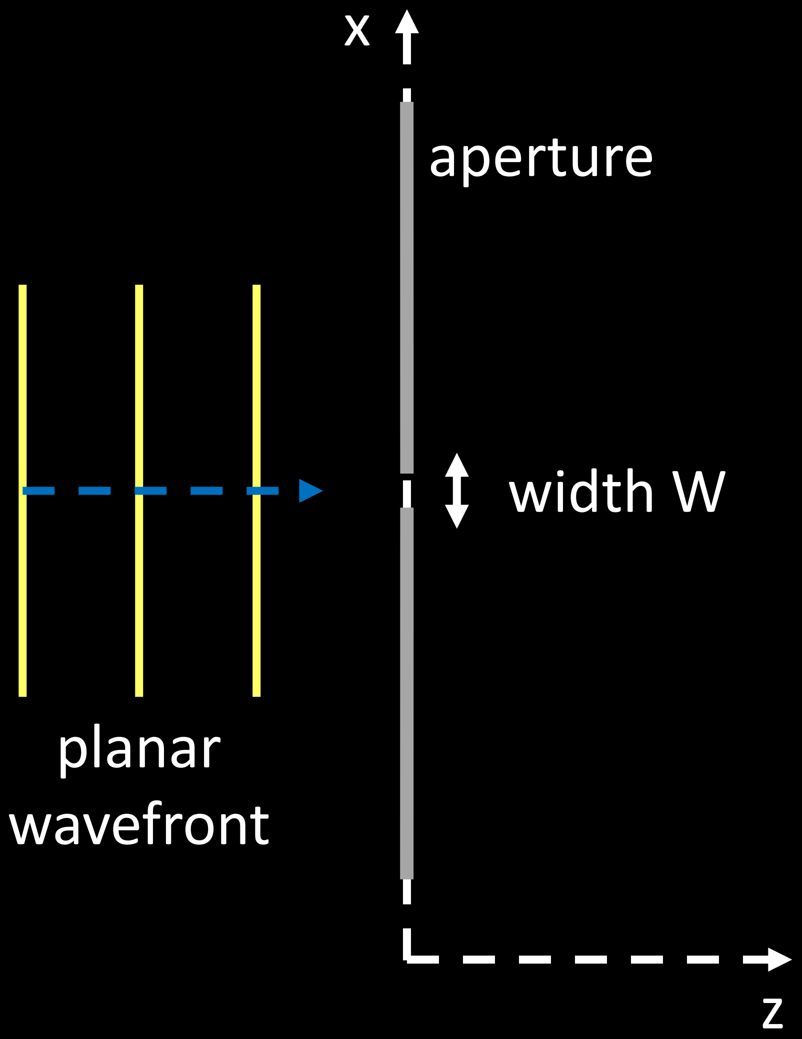
$$\psi_{out}(r) = \text{Fourier}^{-1}\{\, \Psi_{out}(k)\, \}$$

Example: pinhole

What is the transmission function?



Example: pinhole

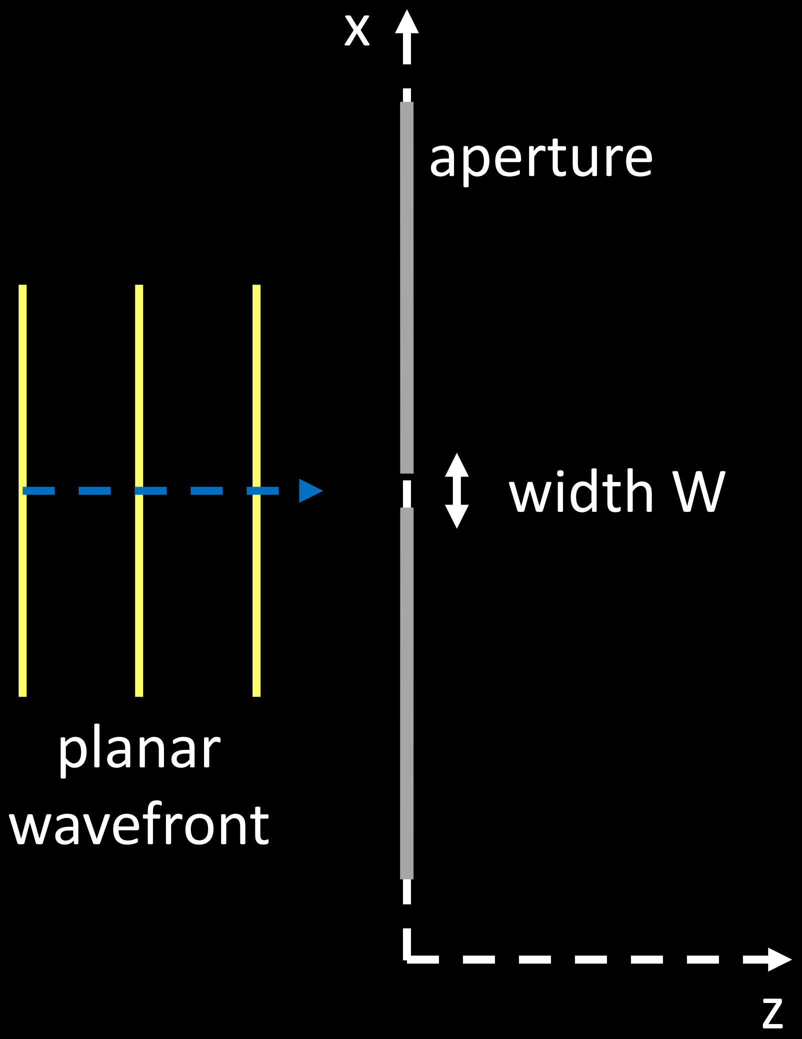


What is the transmission function?

p(r) = rect(W · r)

What is the transfer function?

Example: pinhole



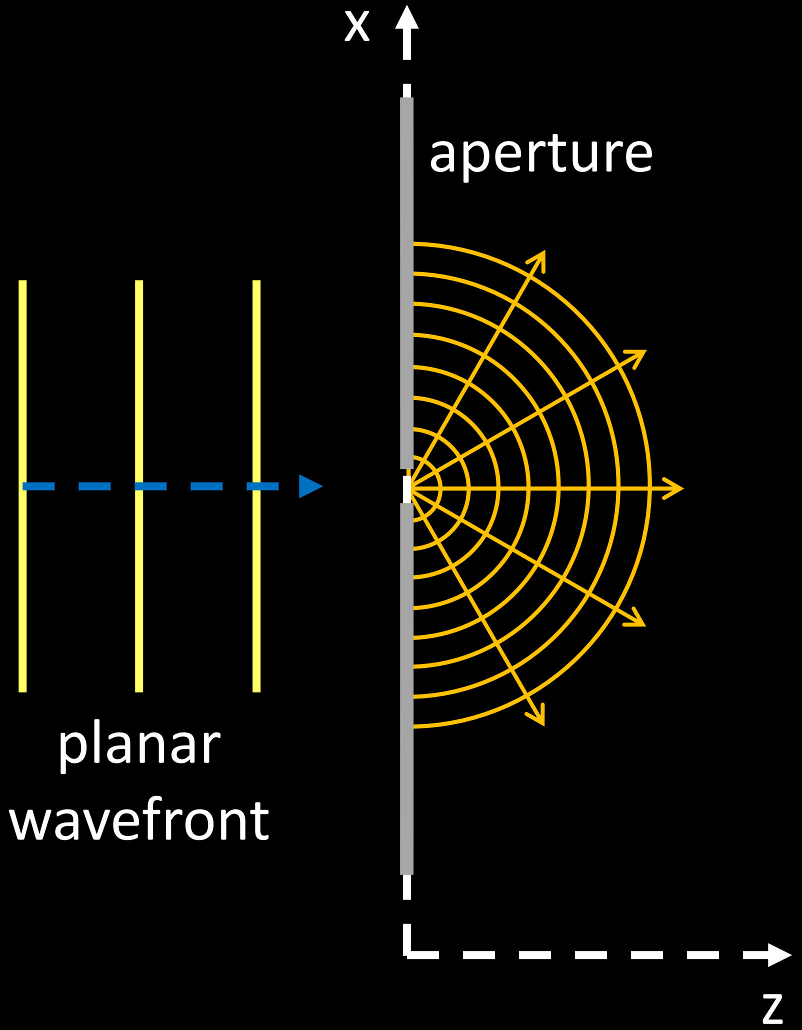
What is the transmission function?

$$p(r) = \text{rect}(W \cdot r)$$

What is the transfer function?

$$P(k) = \text{sinc}(k / W)$$

Example: pinhole



What is the transmission function?

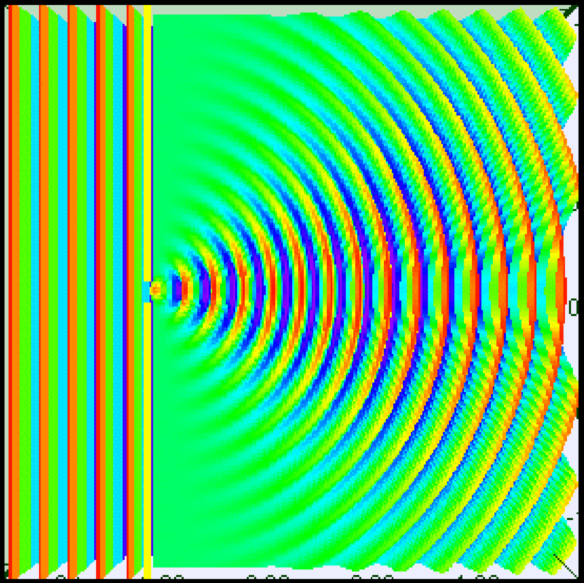
$$p(r) = \text{rect}(W \cdot r)$$

What is the transfer function?

$$P(k) = \text{sinc}(k / W)$$

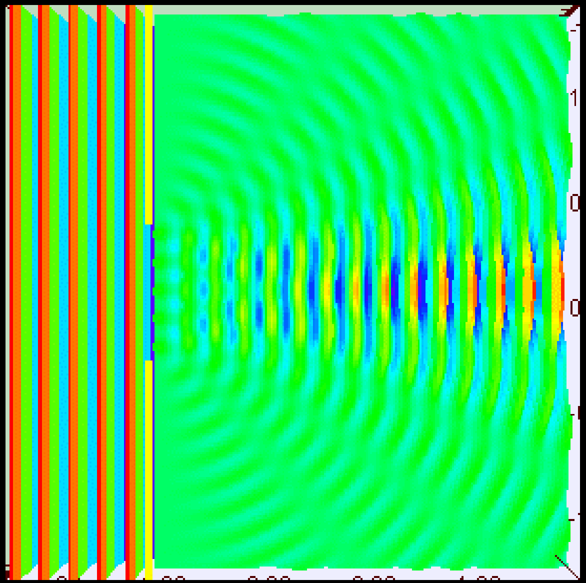
Example: pinhole

Why does the diffraction pattern become wider as we increase width?



wide diffraction pattern

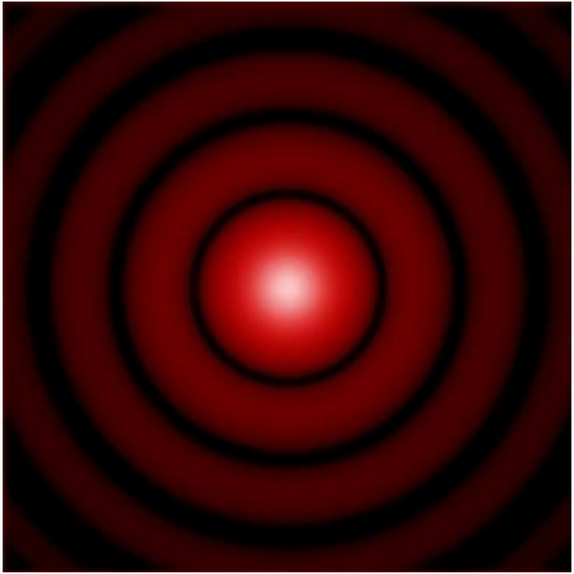
small pinhole



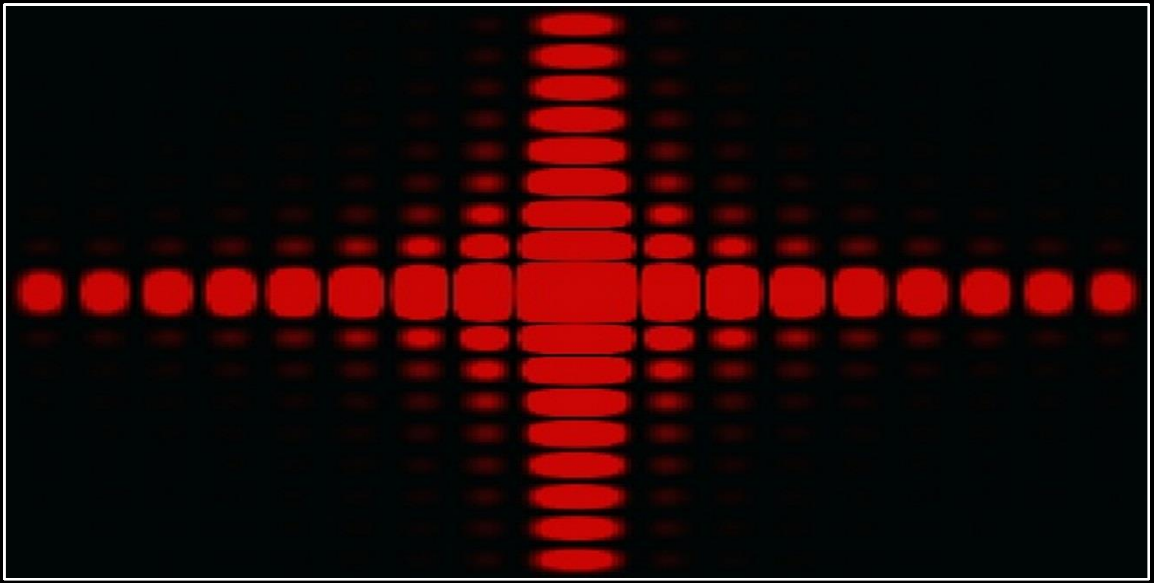
narrow diffraction pattern

large pinhole

Remember: 2D Fourier transform



circular aperture
(Airy disk)

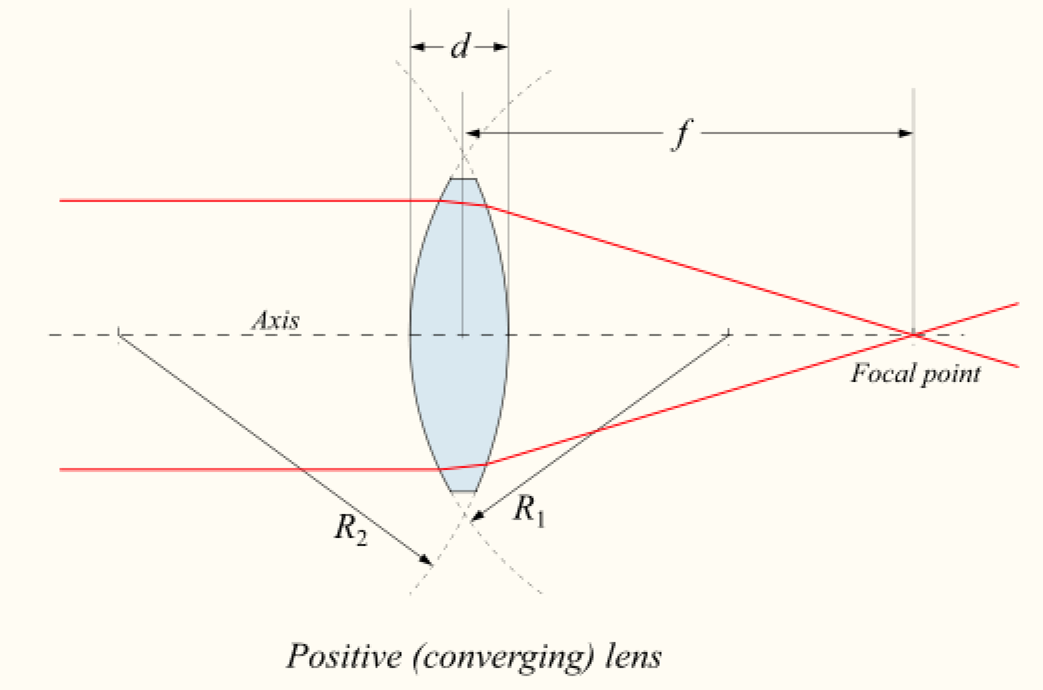


rectangular aperture

Fresnel lenses

Thin lenses

What is the transmission function of a thin lens?



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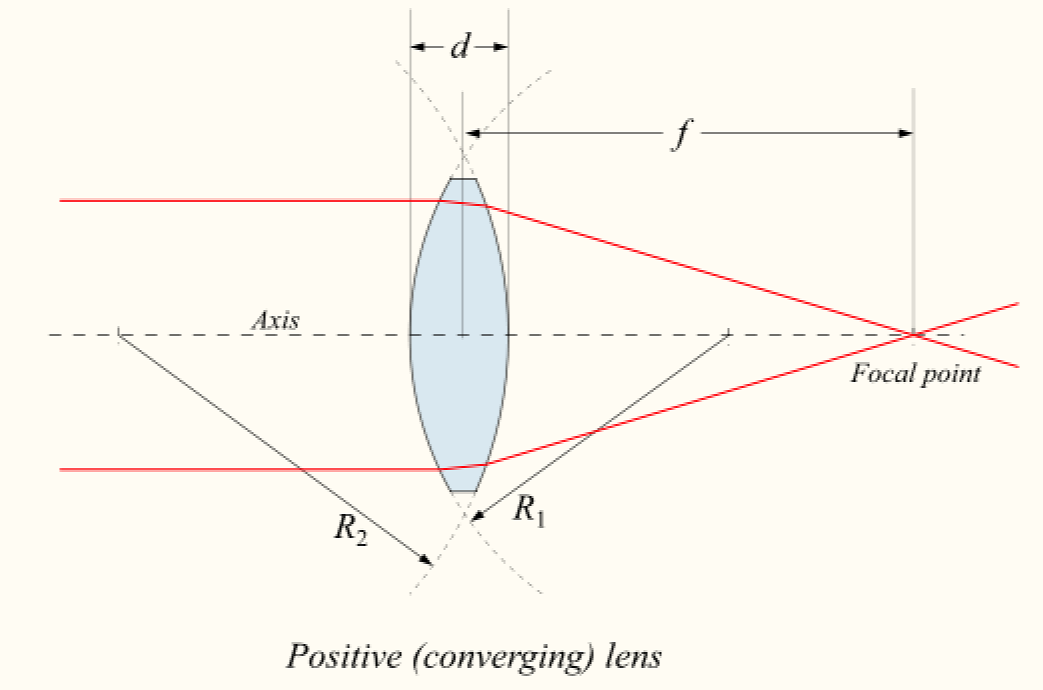


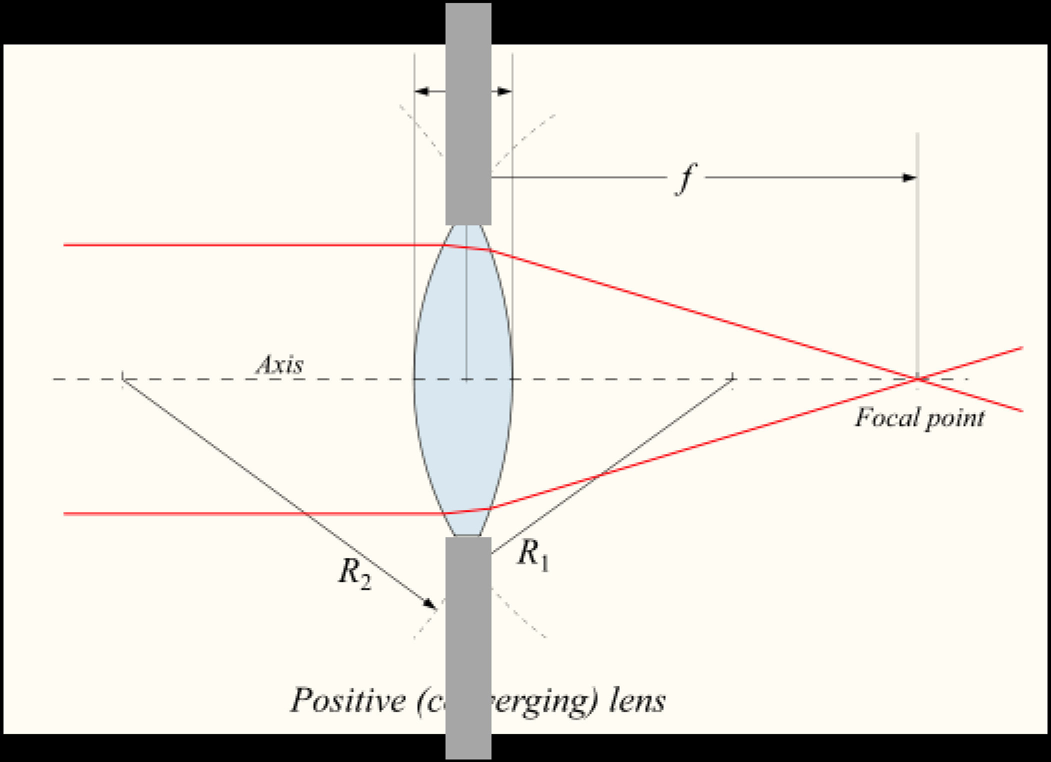
Positive (converging) lens

Complicated expression, but phase-only: $p(r) = \exp(j \cdot \Phi(r))$

- Delay all plane waves so that they have the <u>same phase</u> at focal point

Thin lenses

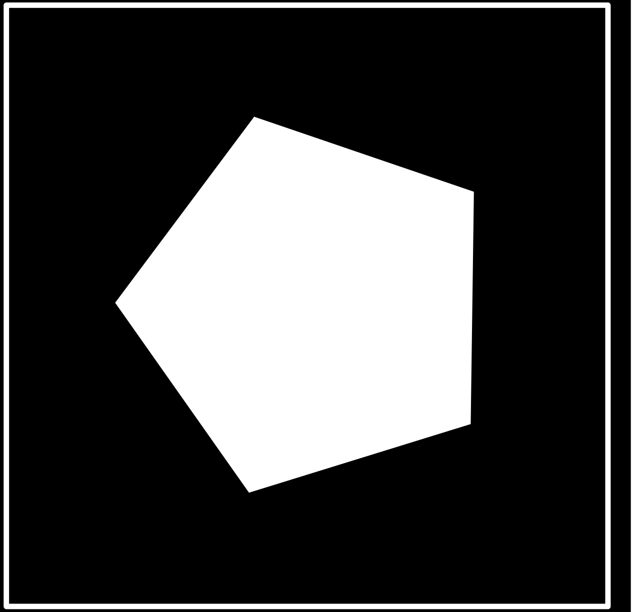
What is the transmission function of a thin lens?

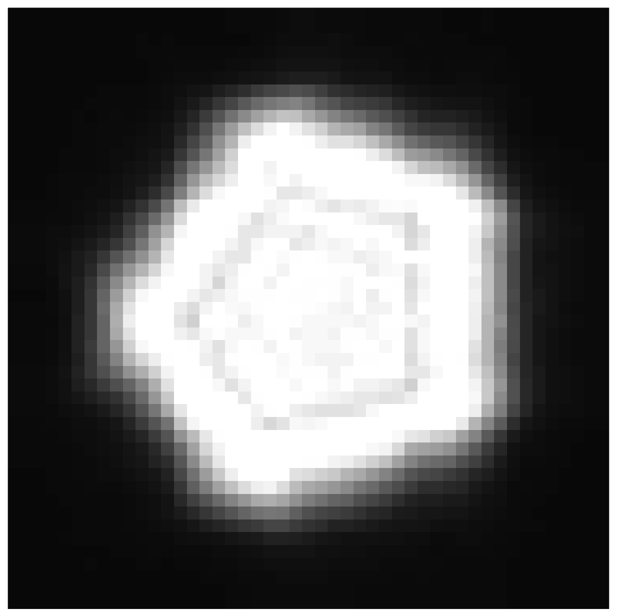


Complicated expression, but phase-only: $p(r) = \exp(\, j \cdot \Phi(r)\,)$

- Delay all plane waves so that they have the same phase at focal point
- The aperture of a real lens creates additional diffraction

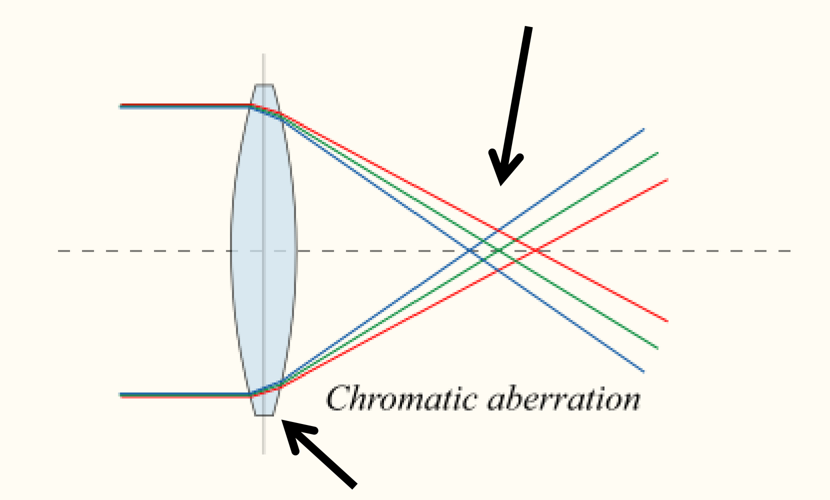
Diffraction in lenses





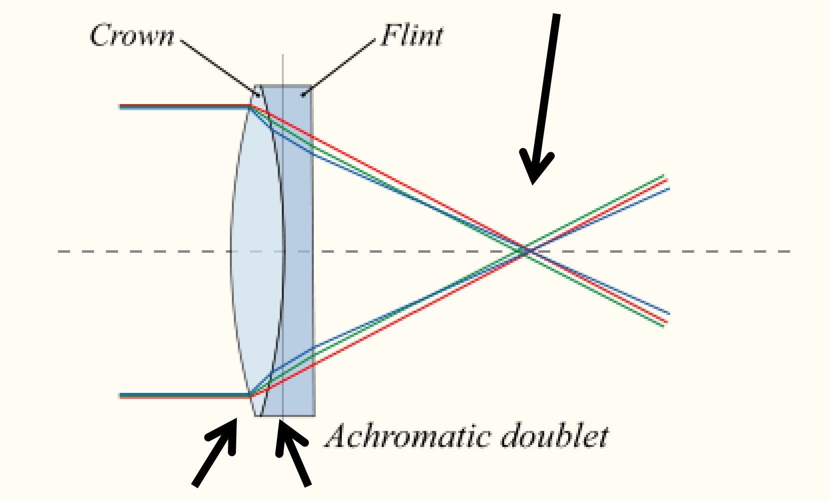
Chromatic aberration

focal length shifts
with wavelength



glass has dispersion
(refractive index changes
with wavelength)

one lens cancels out
dispersion of other

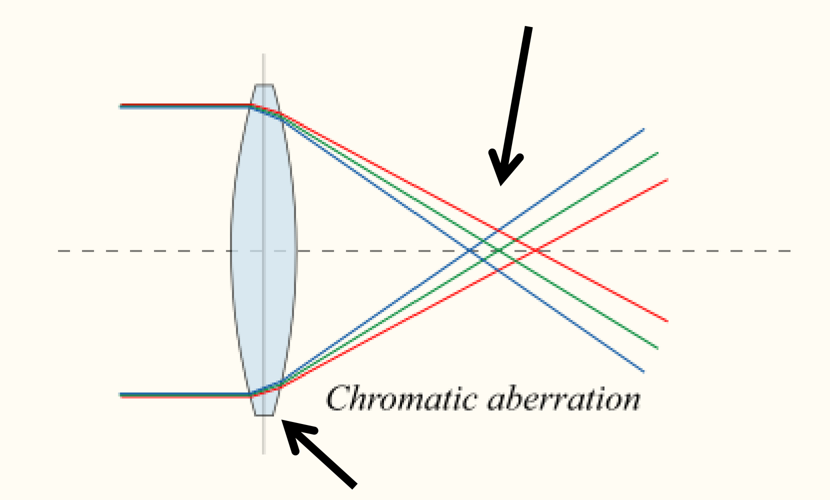


glasses of different
refractive index

How does Fraunhofer diffraction explain chromatic aberration?

Chromatic aberration

focal length shifts
with wavelength



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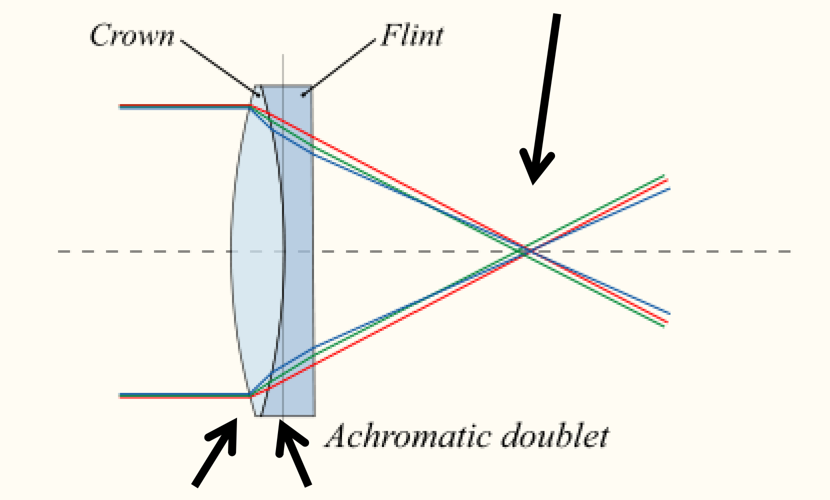


glasses of different
refractive index

How does Fraunhofer diffraction explain chromatic aberration?

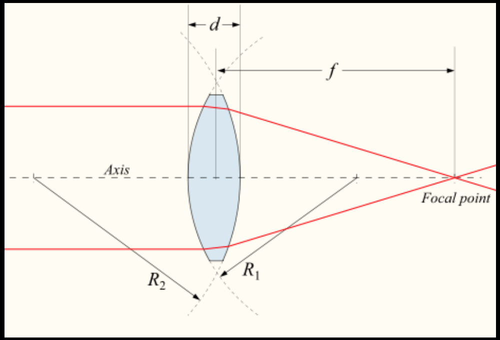
- All our Fourier transforms are wavelength-dependent

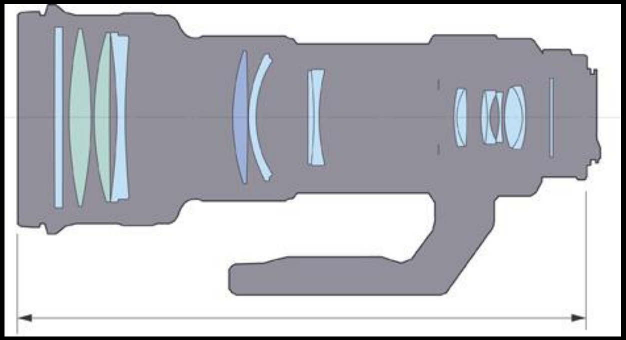
$$P(k) = \mathrm{Fourier}\{p(r)\}$$

$$\Psi\,(k) = \mathrm{Fourier}\{\psi(r)\}$$

$$k = \frac{2\pi c}{\lambda} n$$

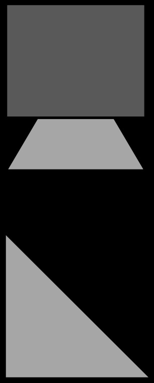
Good “thin” lenses are compound lenses





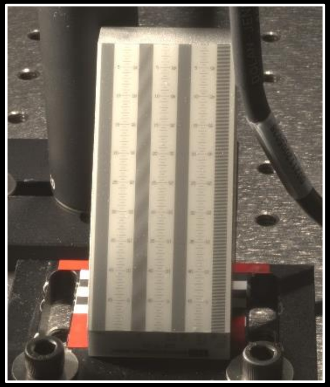
dreaded camera bulge

A small demonstration

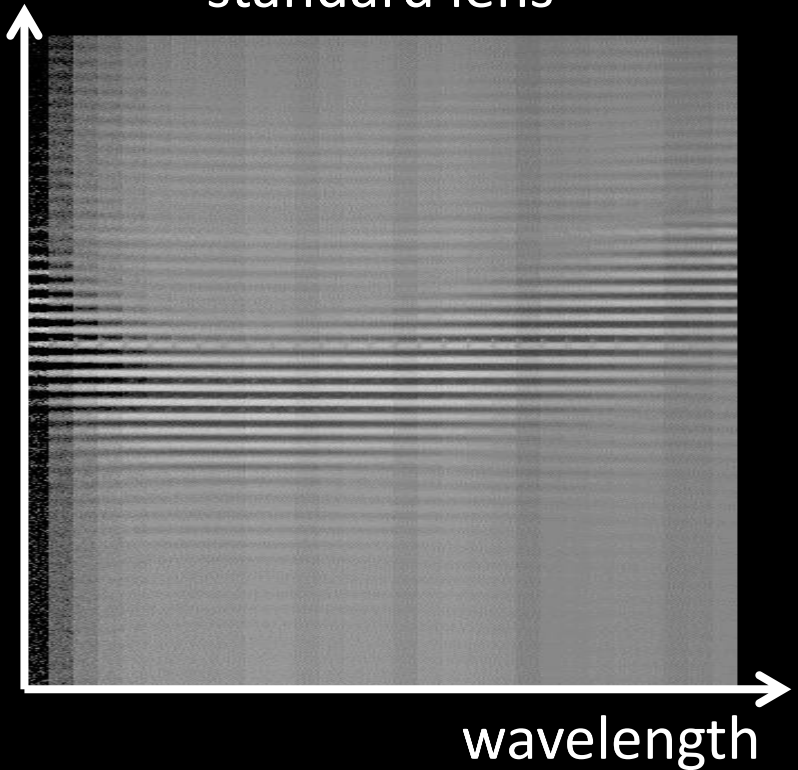


hyperspectral camera

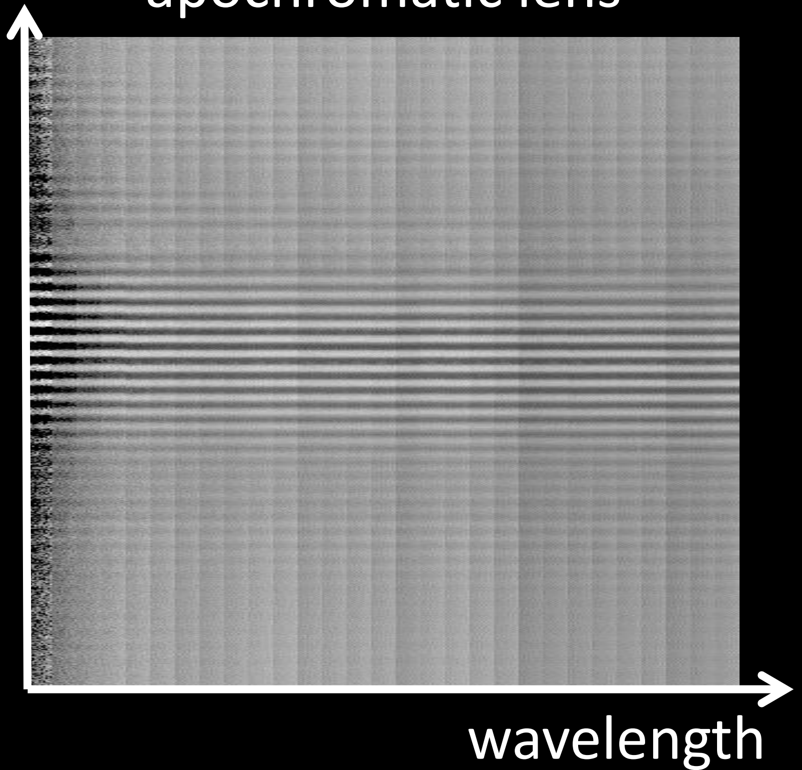
depth-of-field target



standard lens

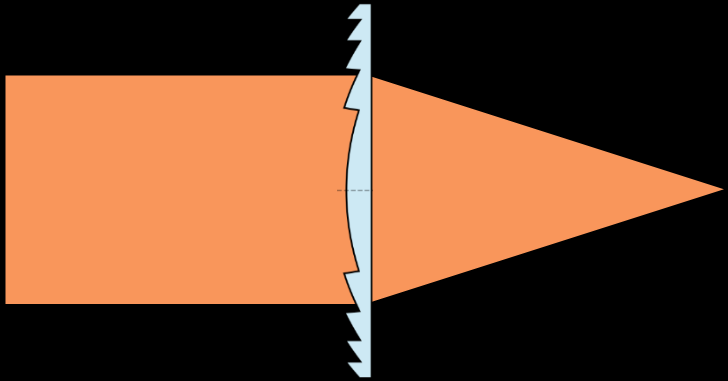


apochromatic lens

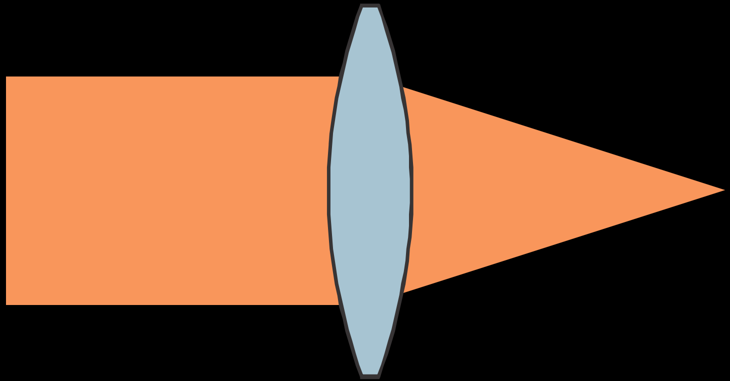


Fresnel lenses

also called diffractive lenses



- operation based on diffraction
- width stays roughly constant with aperture size

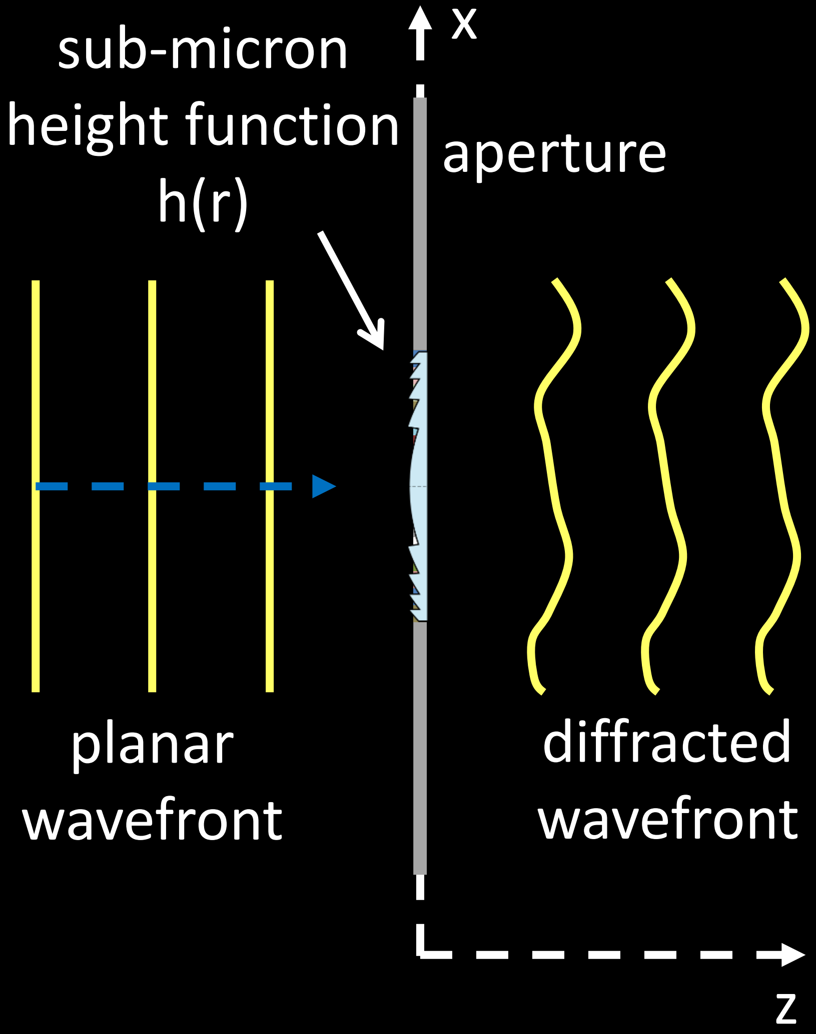


- width scales roughly linearly with aperture size

Fresnel lenses

solar grill

Fresnel lens



- transmission function:

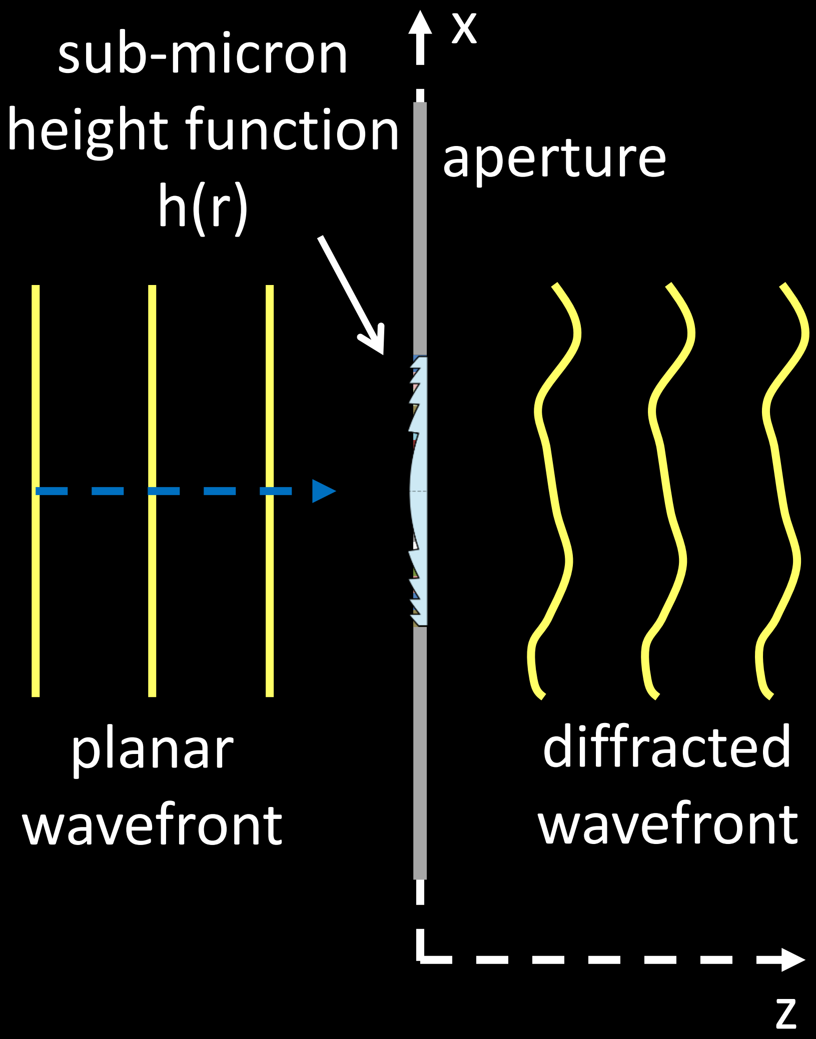
$$p(r) = A(r) \cdot \exp(\, j \cdot \Phi(r)\,)$$

$$A(r) = \text{const}, \; \Phi(r) = c(\lambda) \cdot h(r)$$

Like a standard lens:

- Phase-only modulation.
- Delay all plane waves so that they have the same phase at focal point.

Fresnel lens



- transmission function:

$$p(r) = A(r) \cdot \exp(\, j \cdot \Phi(r)\,)$$

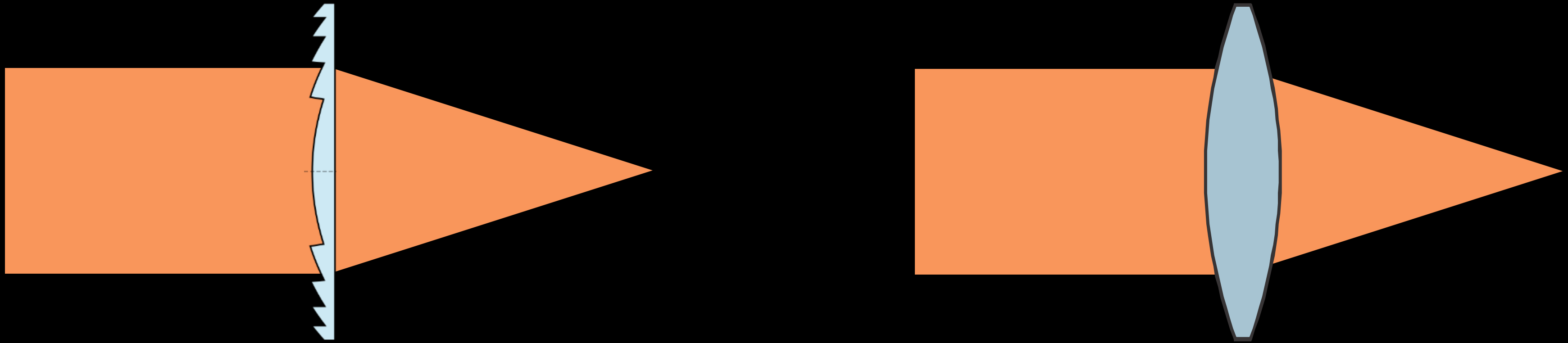
$$A(r) = \text{const},\ \Phi(r) = c\boxed{(\lambda)} \cdot h(r)$$

Like a standard lens:

- Phase-only modulation.
- Delay all plane waves so that they have the <u>same phase</u> at focal point.

Fresnel lenses

also called diffractive lenses



- width stays roughly constant with aperture size
- width scales roughly linearly with aperture size

✓ very thin

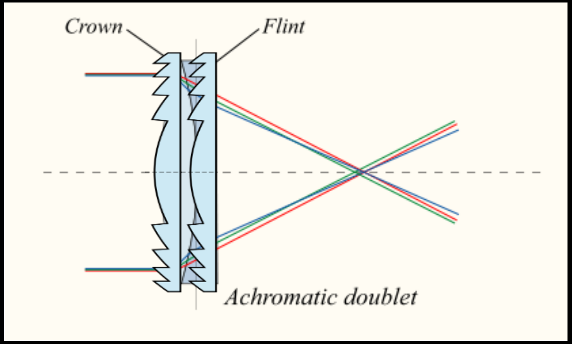
✗ very dispersing

Diffractive achromat

conventional approach:

- multiple layers canceling out each other’s aberration
- same principle as achromatic compound lens

✗ bulky design
(thick and heavy)



Diffractive achromat

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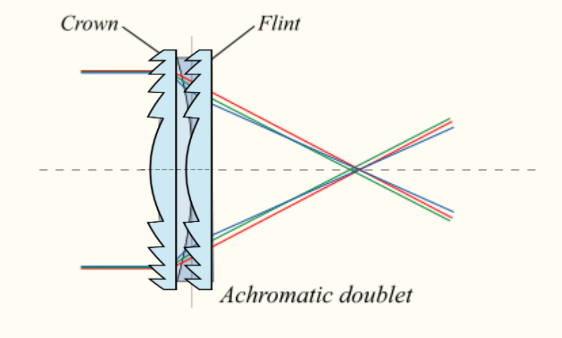
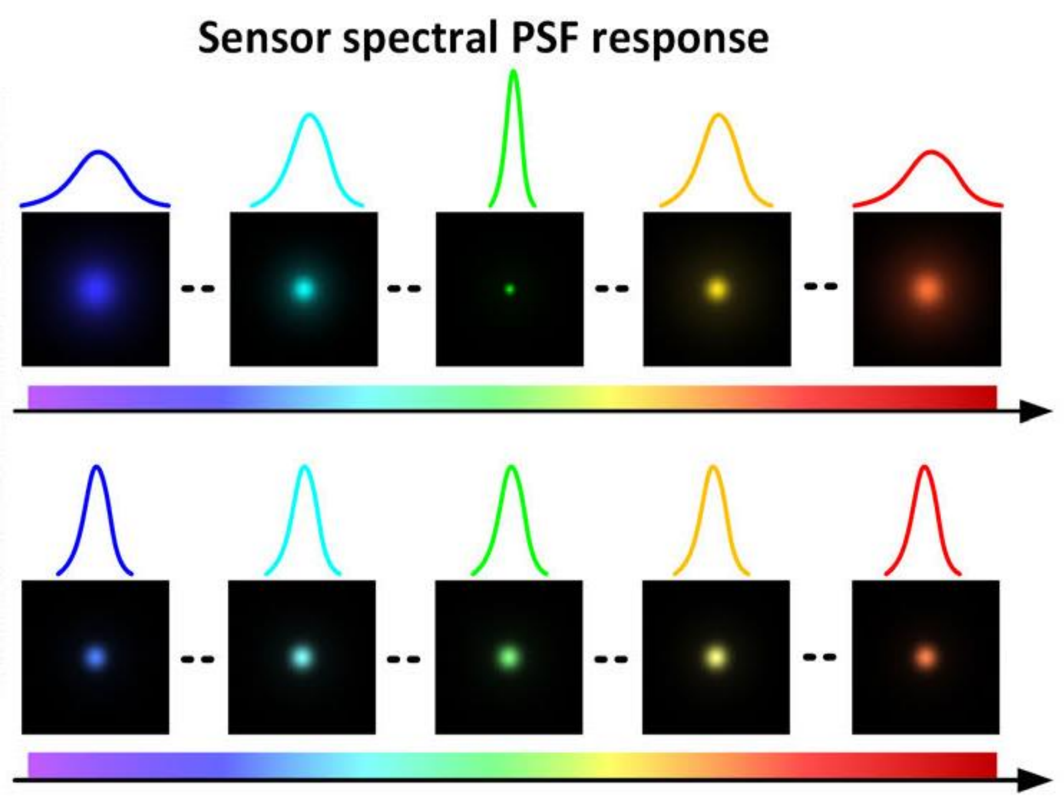


computational imaging approach:

- design single layer that has aberration that can be easily undone computationally
- possible because Fresnel lenses offer a lot more design flexibility (arbitrary height function)

Diffractive achromat



regular Fresnel lens

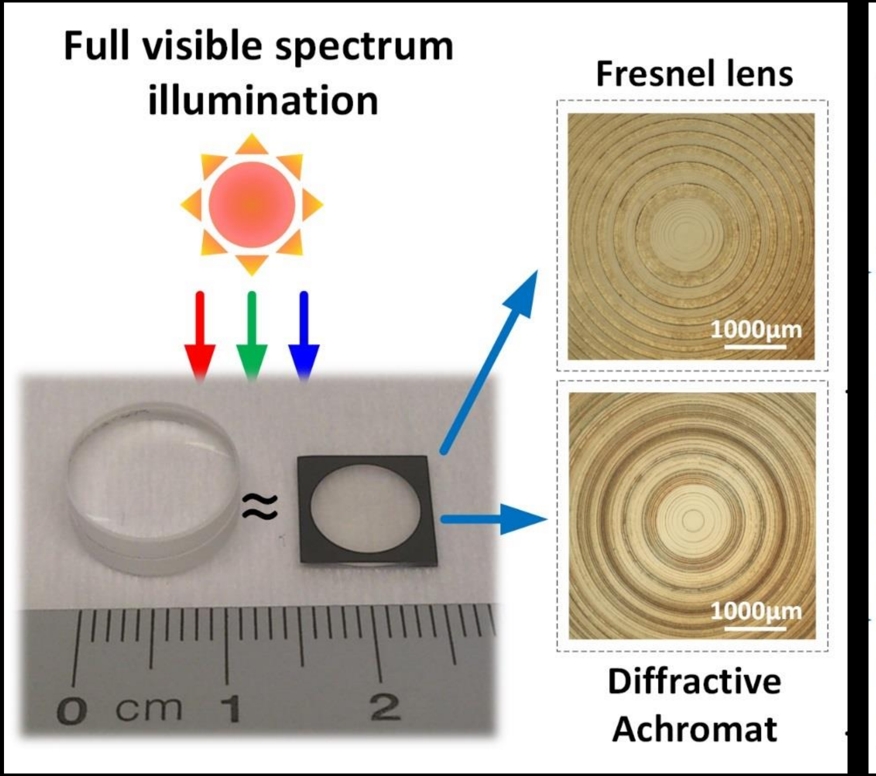
sharp PSF at center wavelength

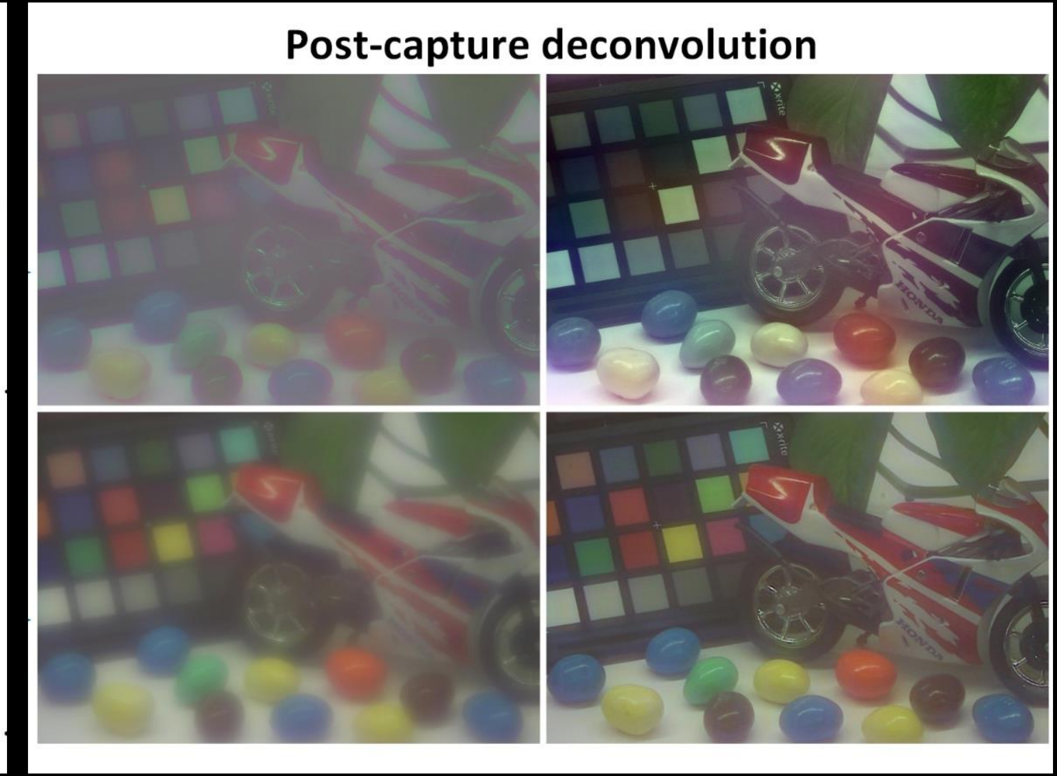
achromatic Fresnel lens

blurry but same PSF at all wavelengths

- Instead of making one wavelength sharp, make all of them equally blurred
- Fix aberration using non-blind deconvolution with same kernel for all wavelengths

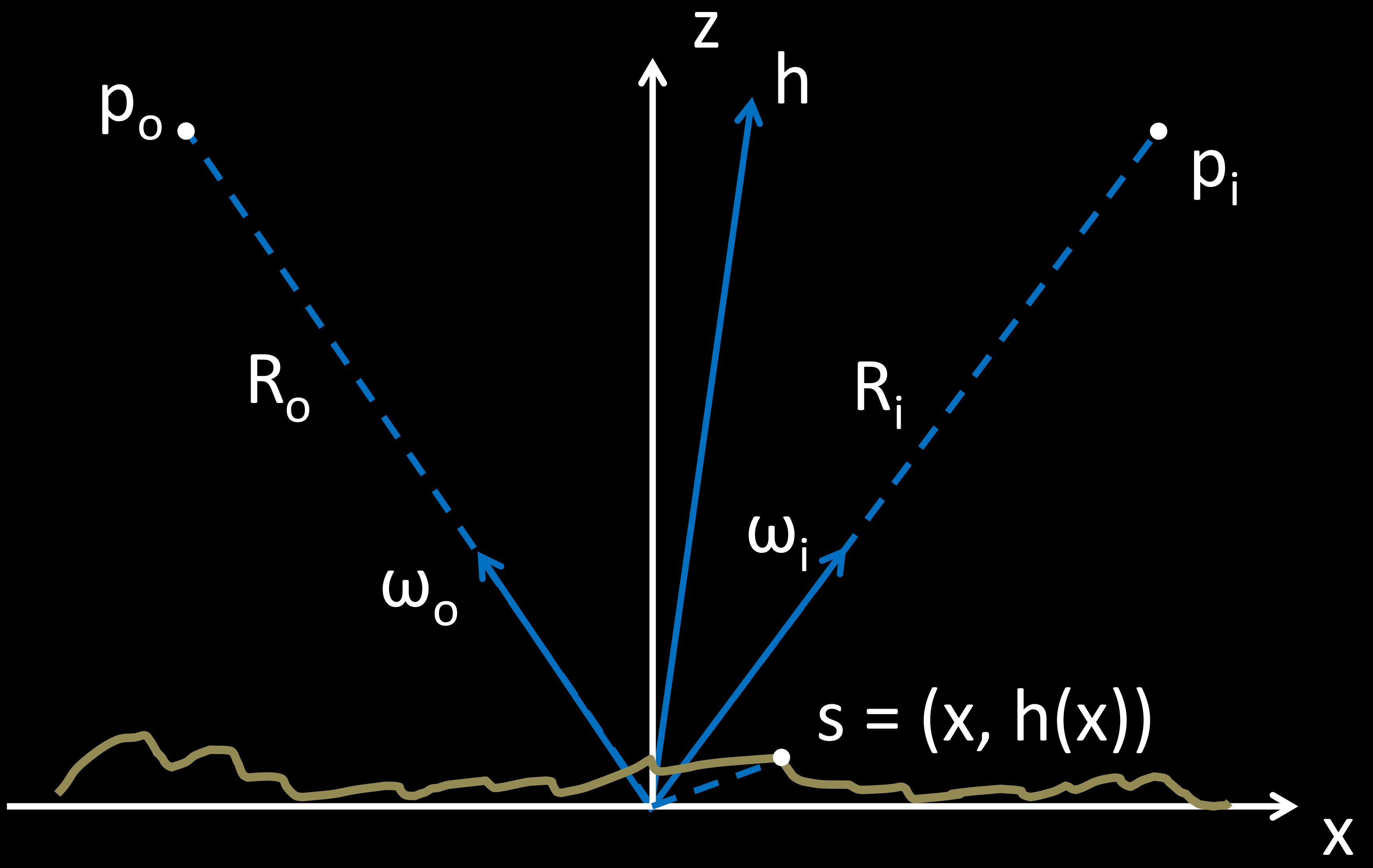
Diffractive achromat





Fraunhofer diffraction and reflection

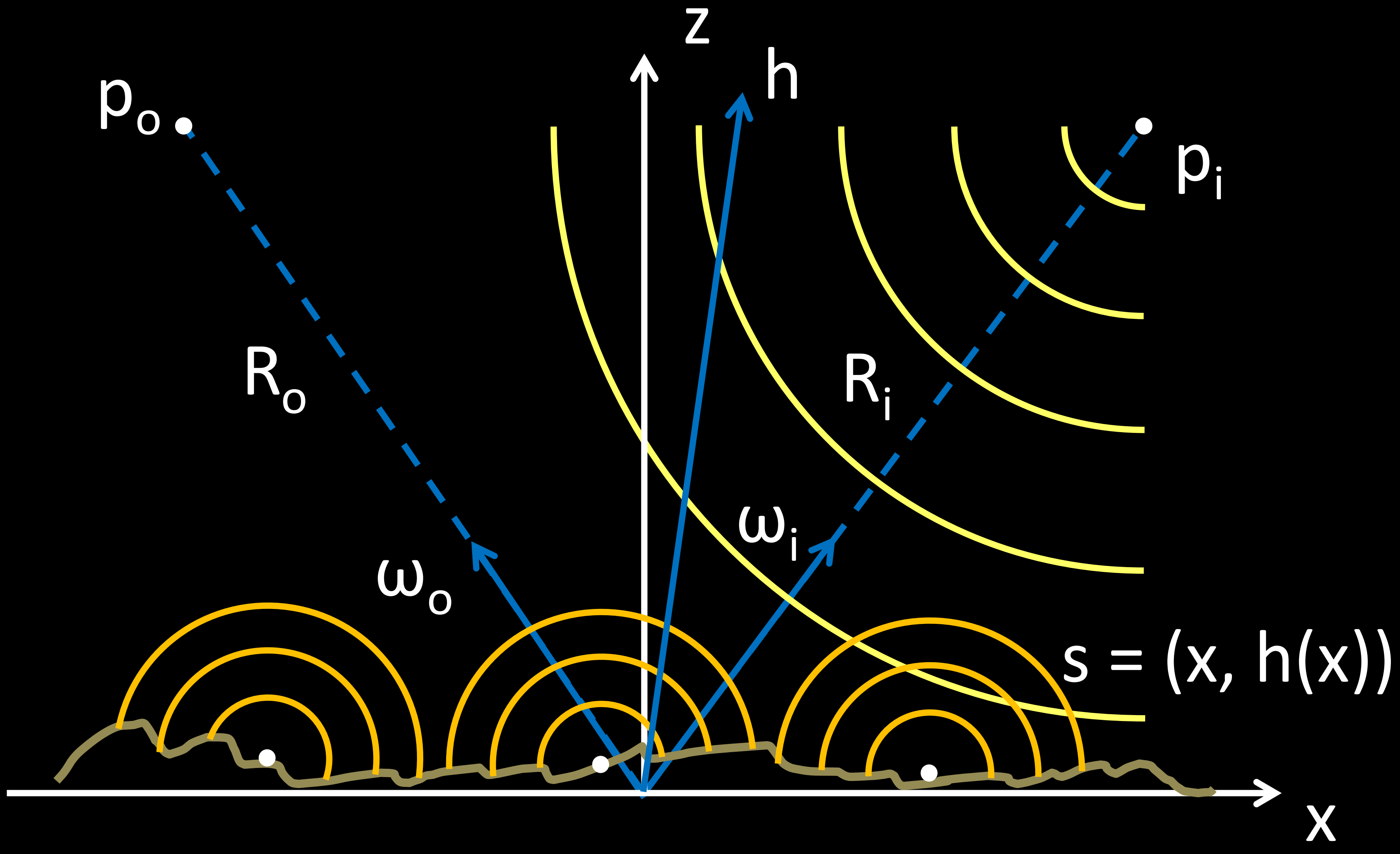
Bidirectional Reflectance Distribution Function (BRDF)



Setting

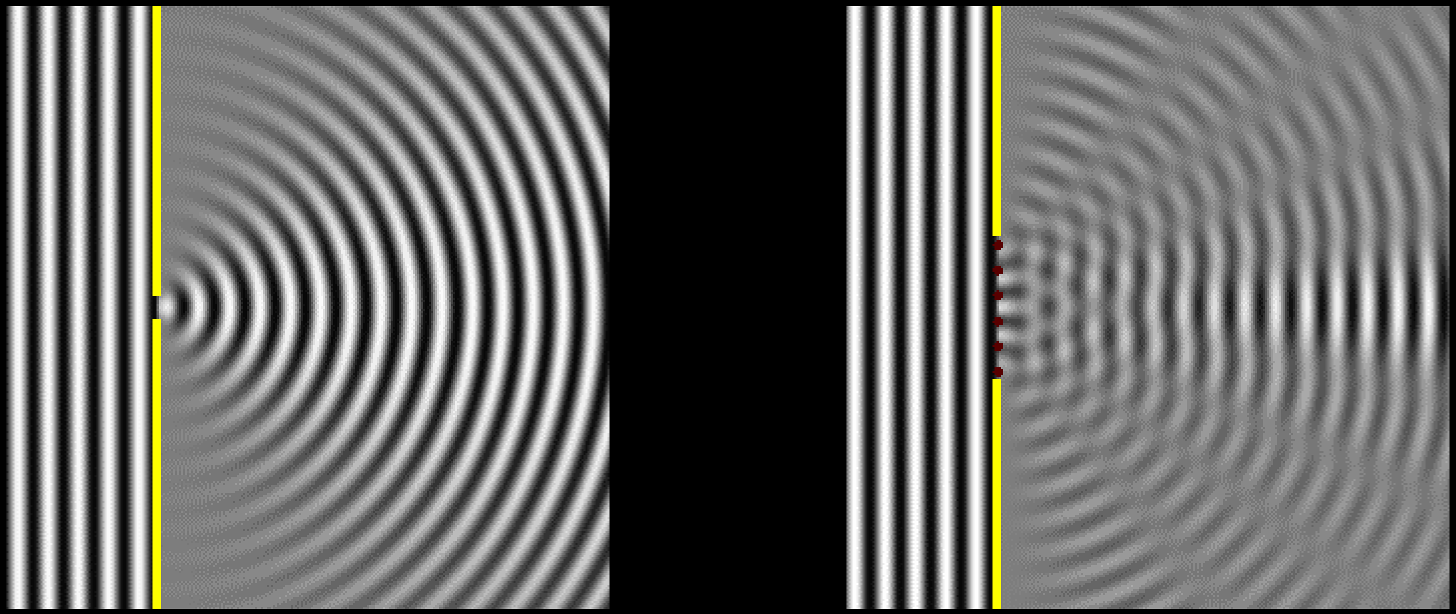
Huygen’s principle

coherent illumination

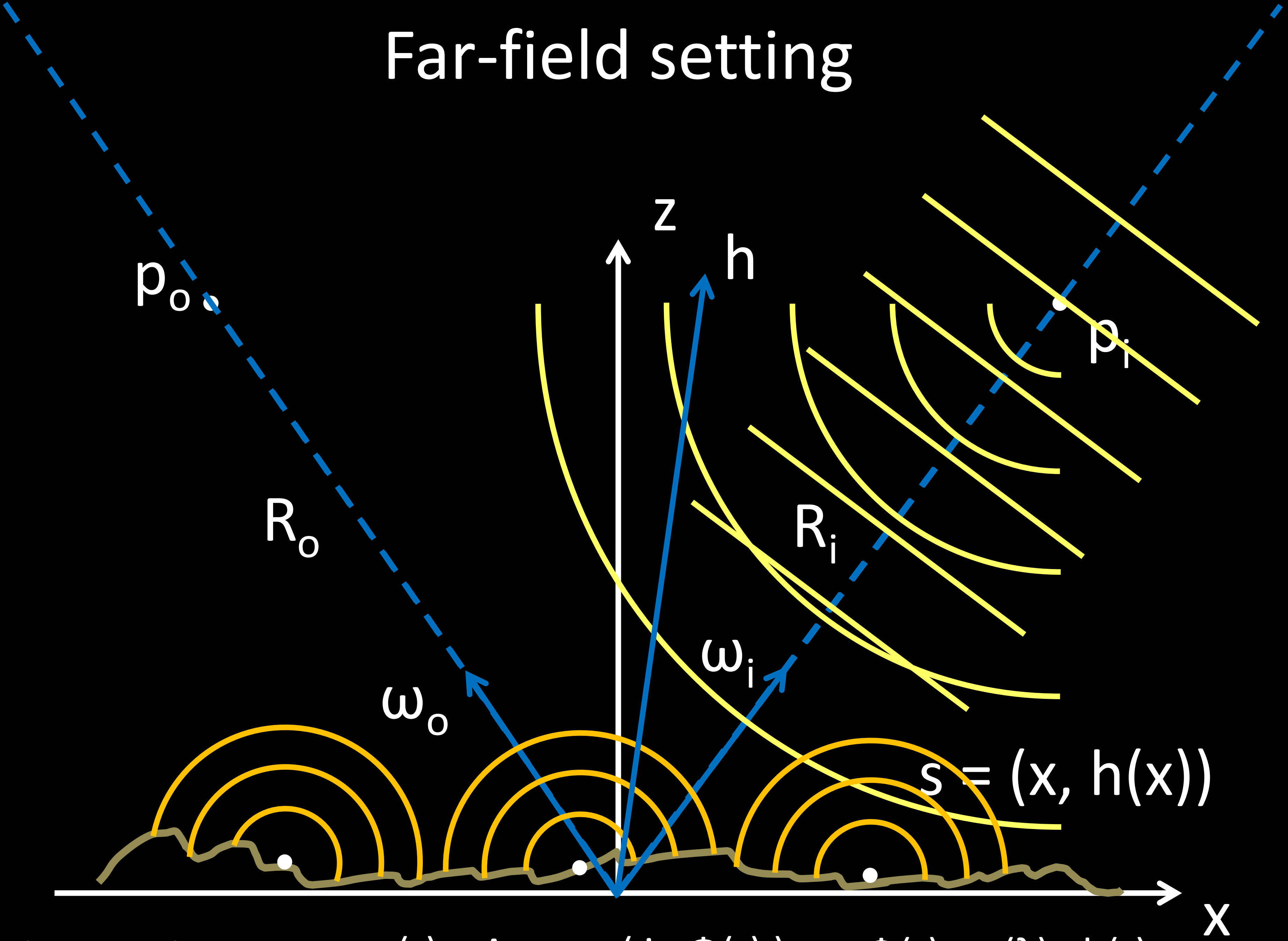


Huygen’s principle

Under far-field approximation, it’s equivalent to Fraunhofer diffraction

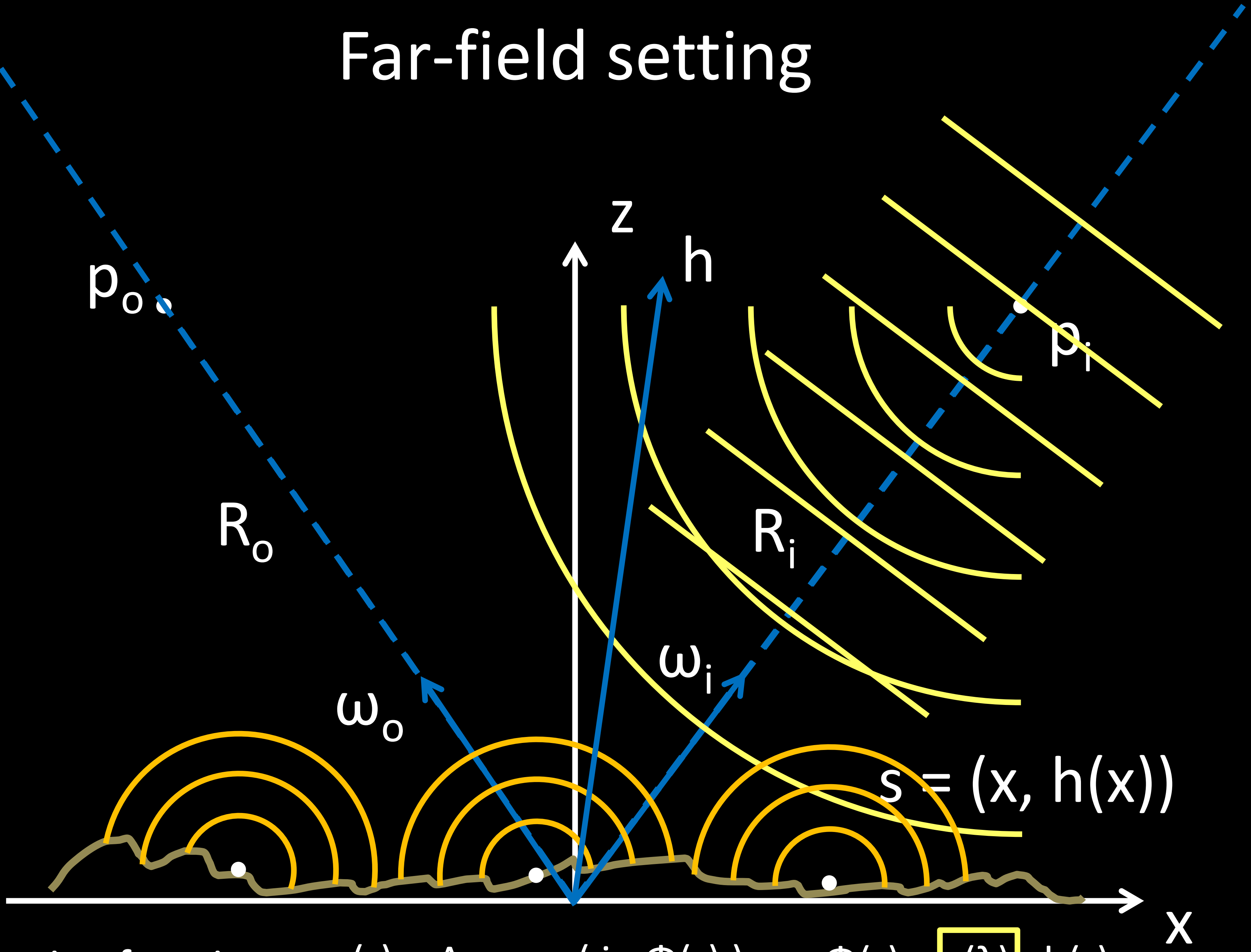


Far-field setting



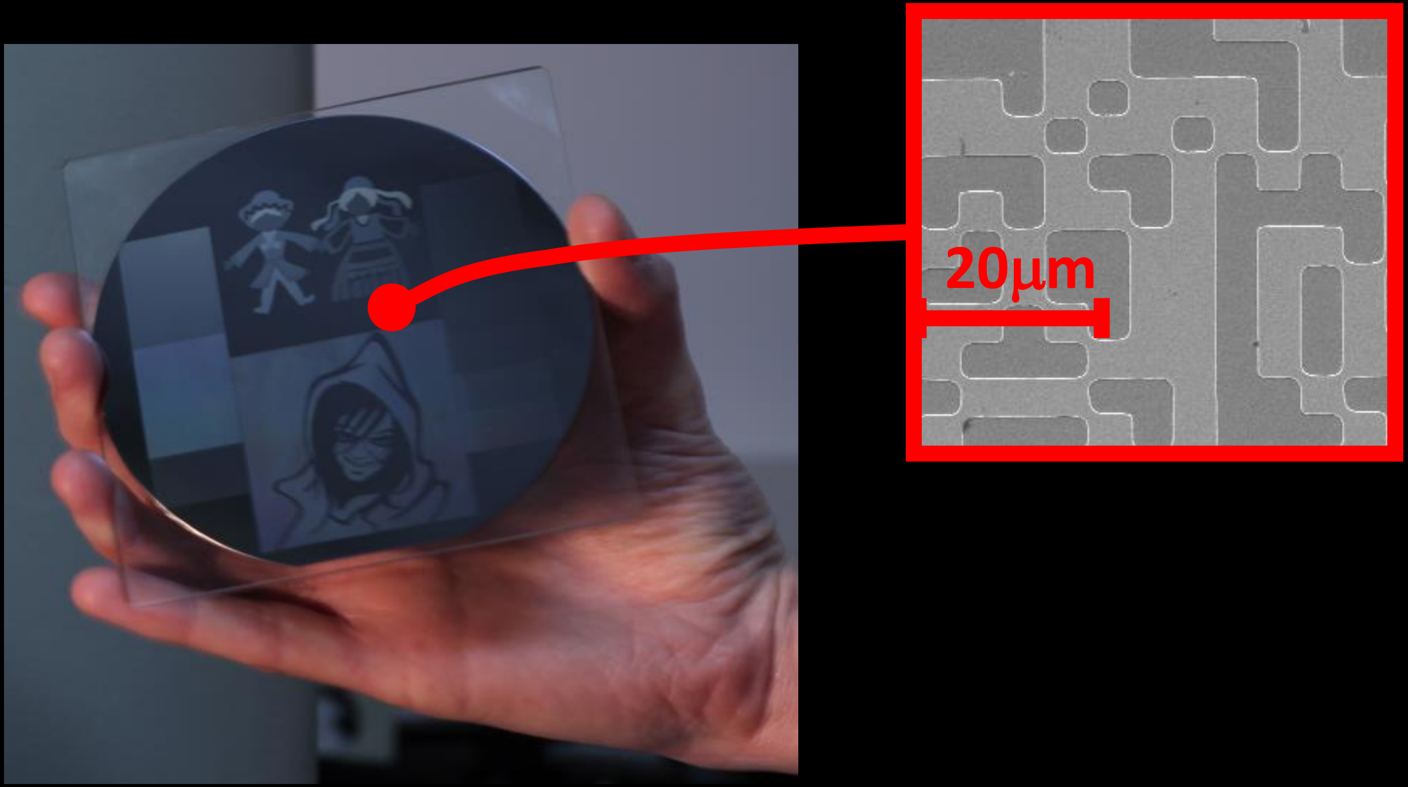
reflection function: $p(r) = A_o \cdot \exp(\, j \cdot \Phi(r) \,)$, $\Phi(r) = c(\lambda) \cdot h(r)$

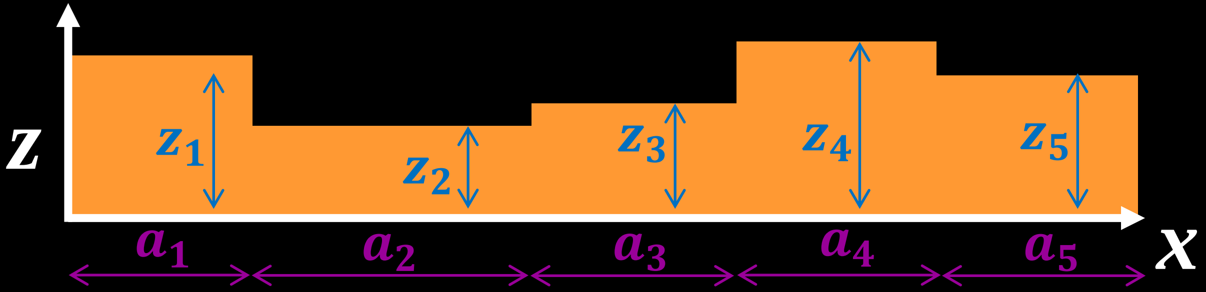
Far-field setting



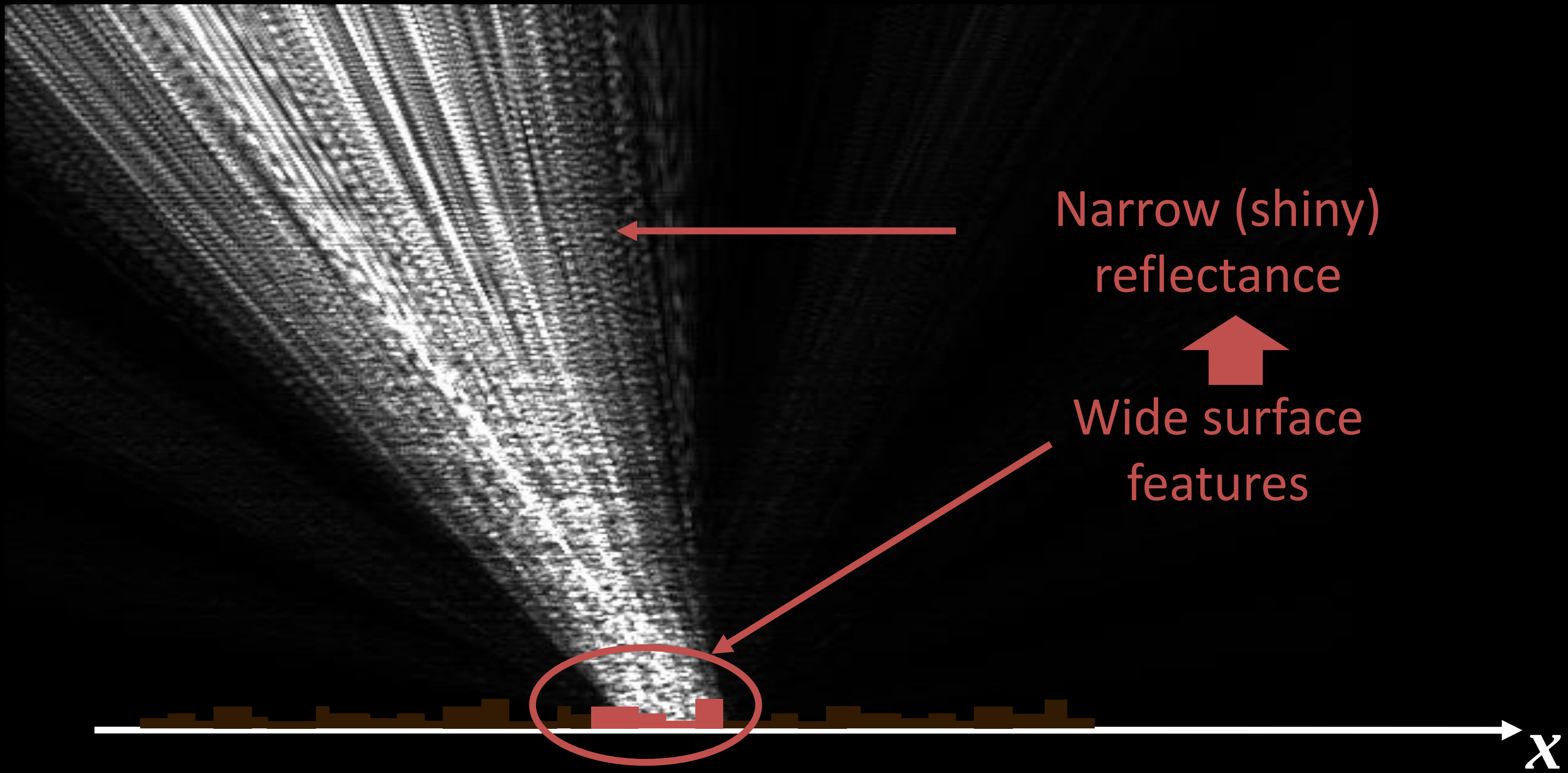
reflection function: $p(r) = A_o \cdot \exp(\, j \cdot \Phi(r)\,)$, $\Phi(r) = \boxed{c(\lambda)} \cdot h(r)$

Photolithography





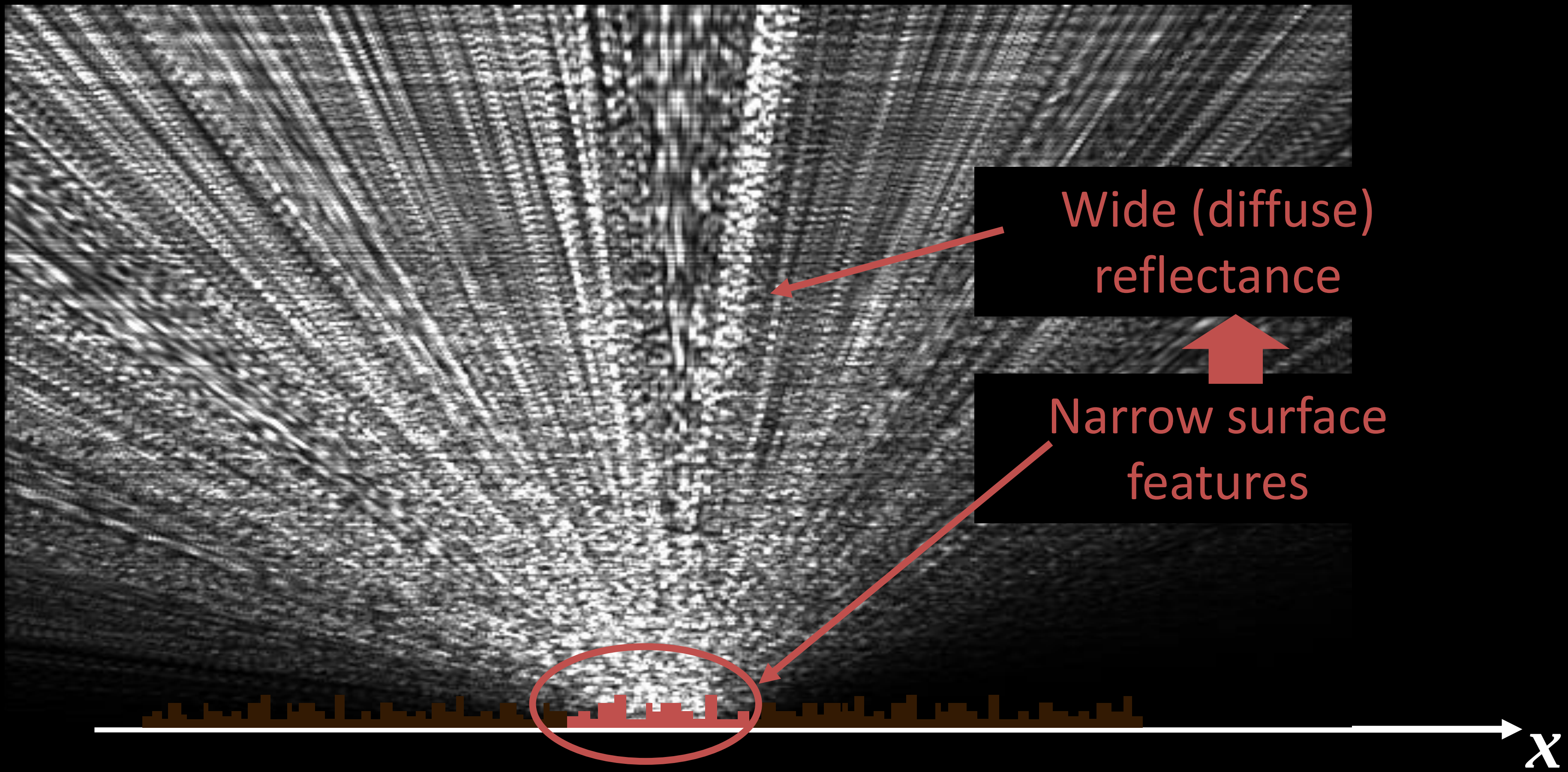
Inverse width relationship



Narrow (shiny) reflectance

Wide surface features

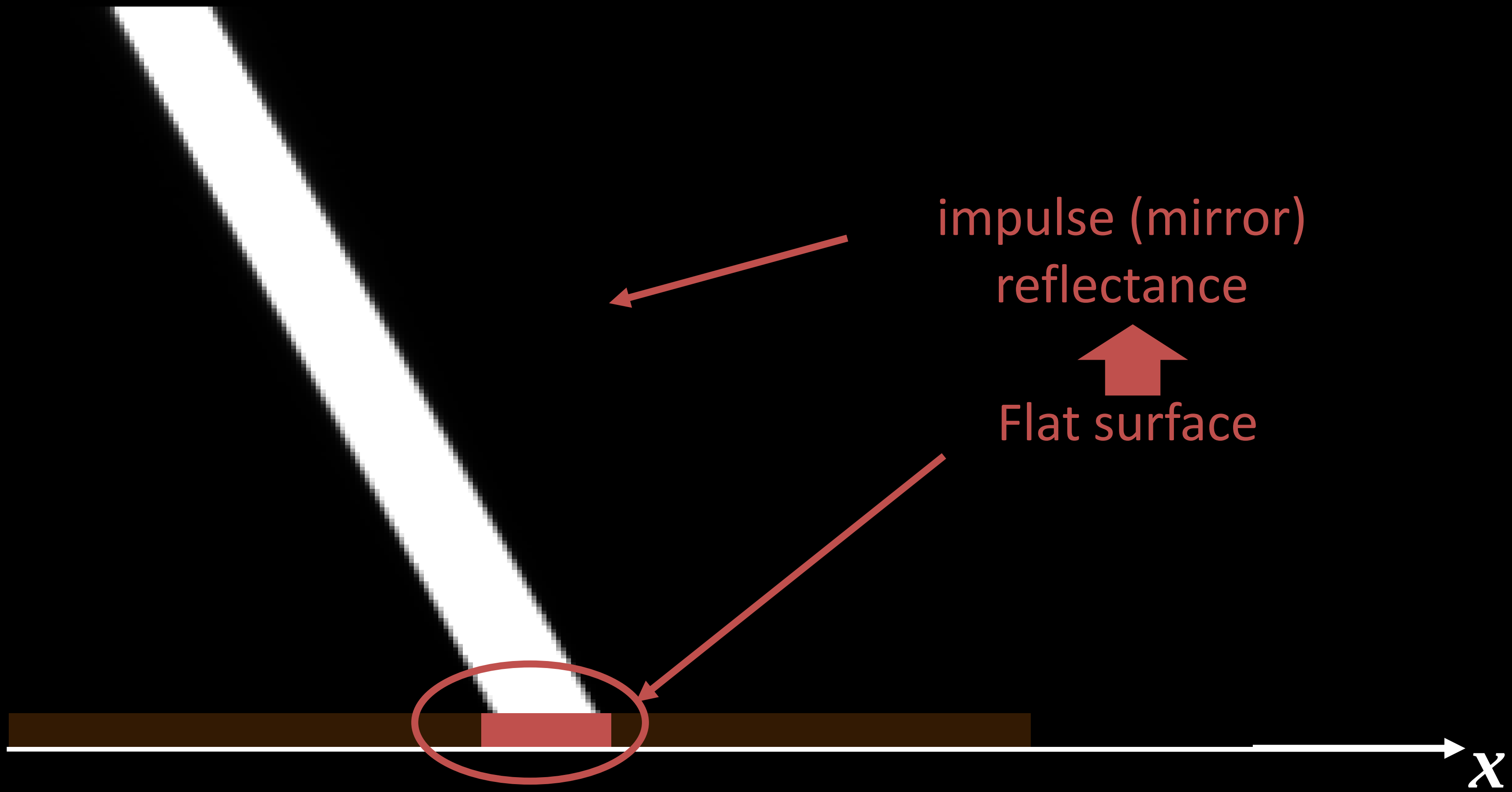
Inverse width relationship



Wide (diffuse) reflectance

Narrow surface features

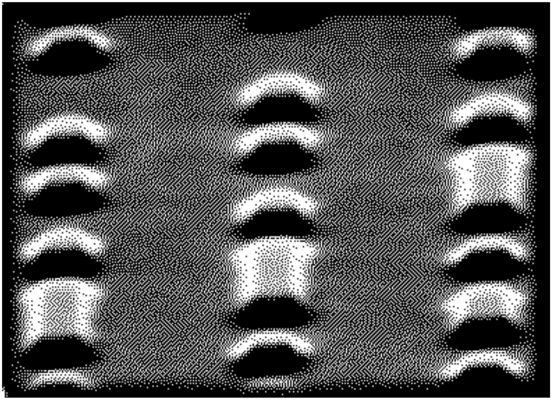
Inverse width relationship



impulse (mirror) reflectance

Flat surface

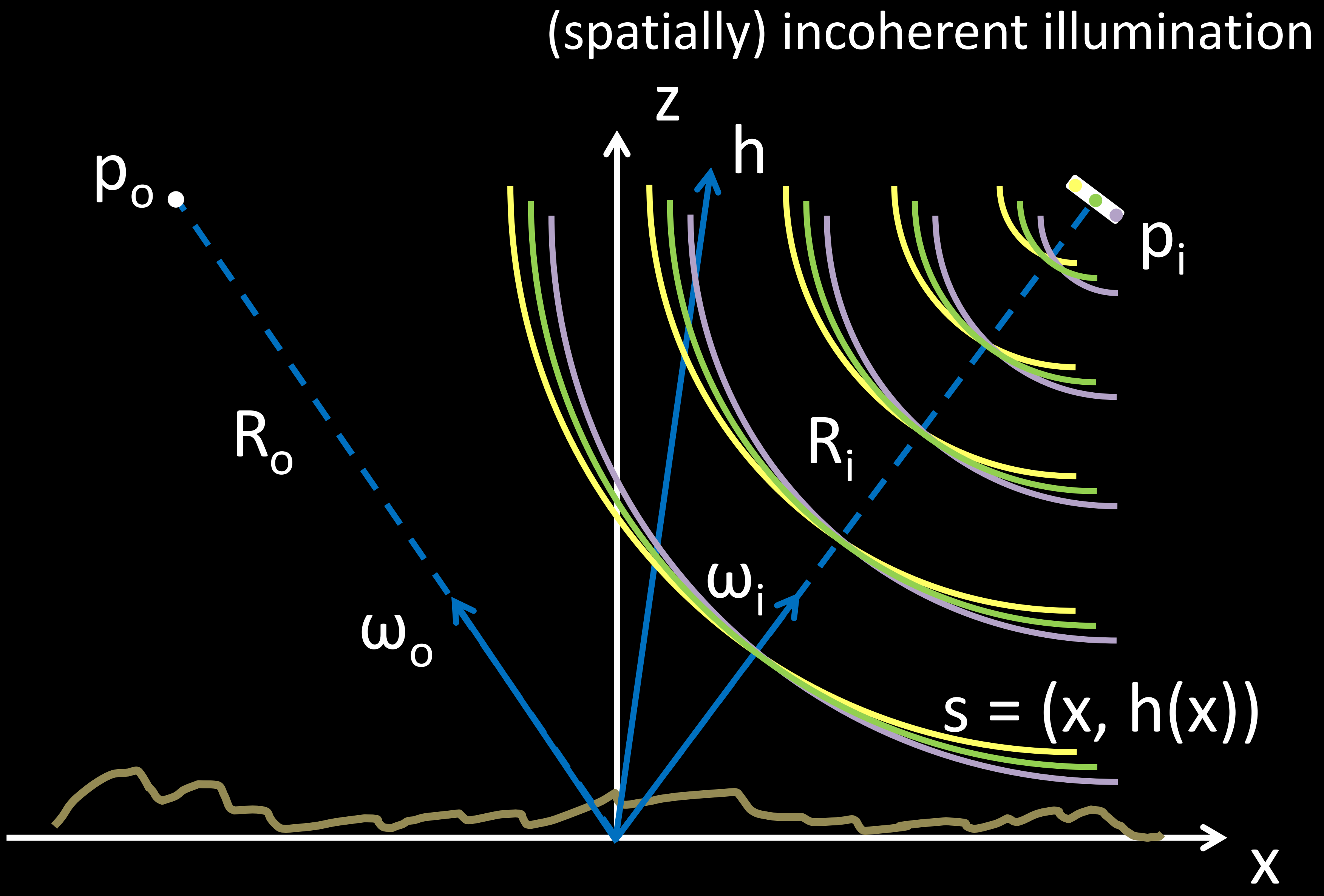
Diffractive BRDF renderings



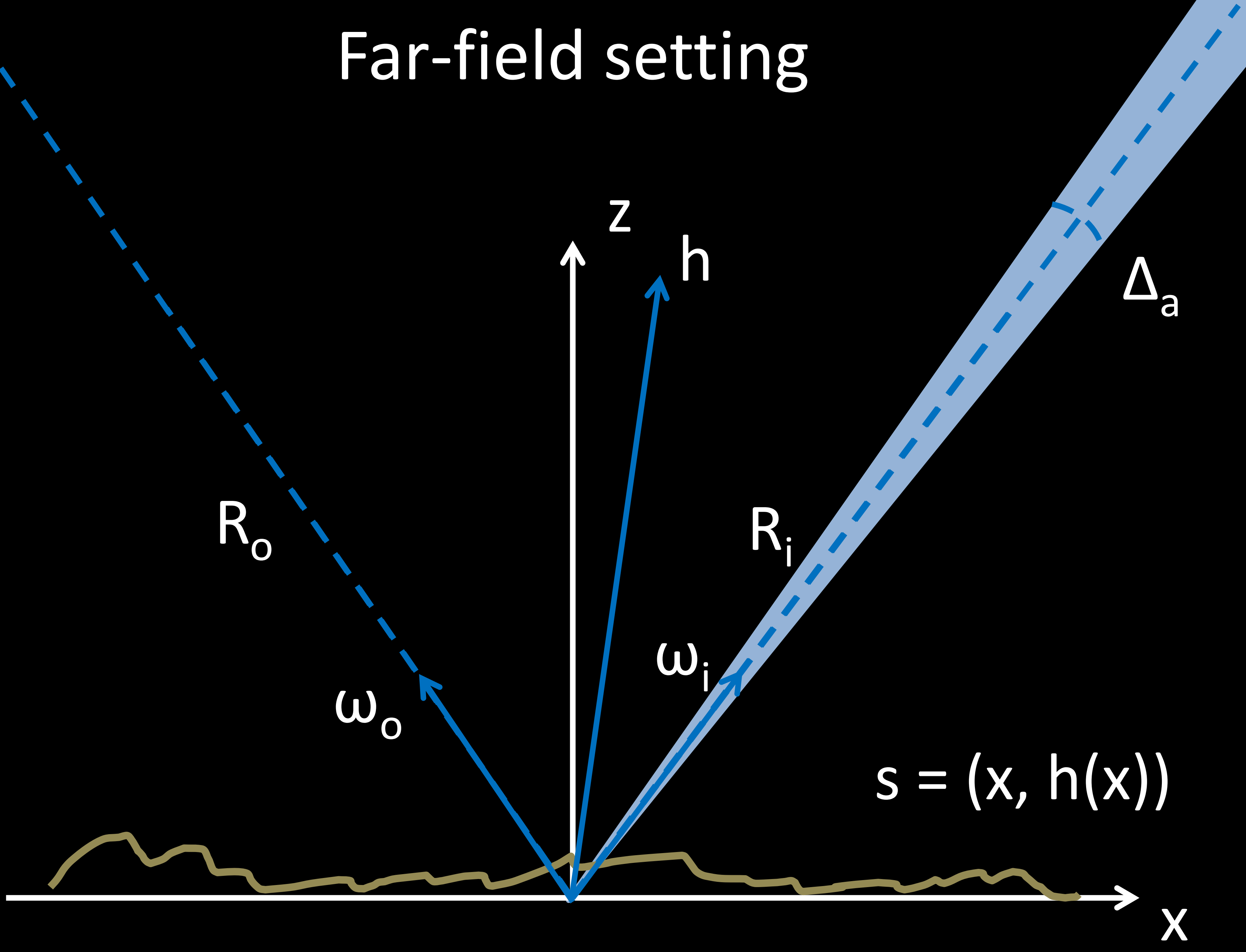
close-up of CD surface

rendering

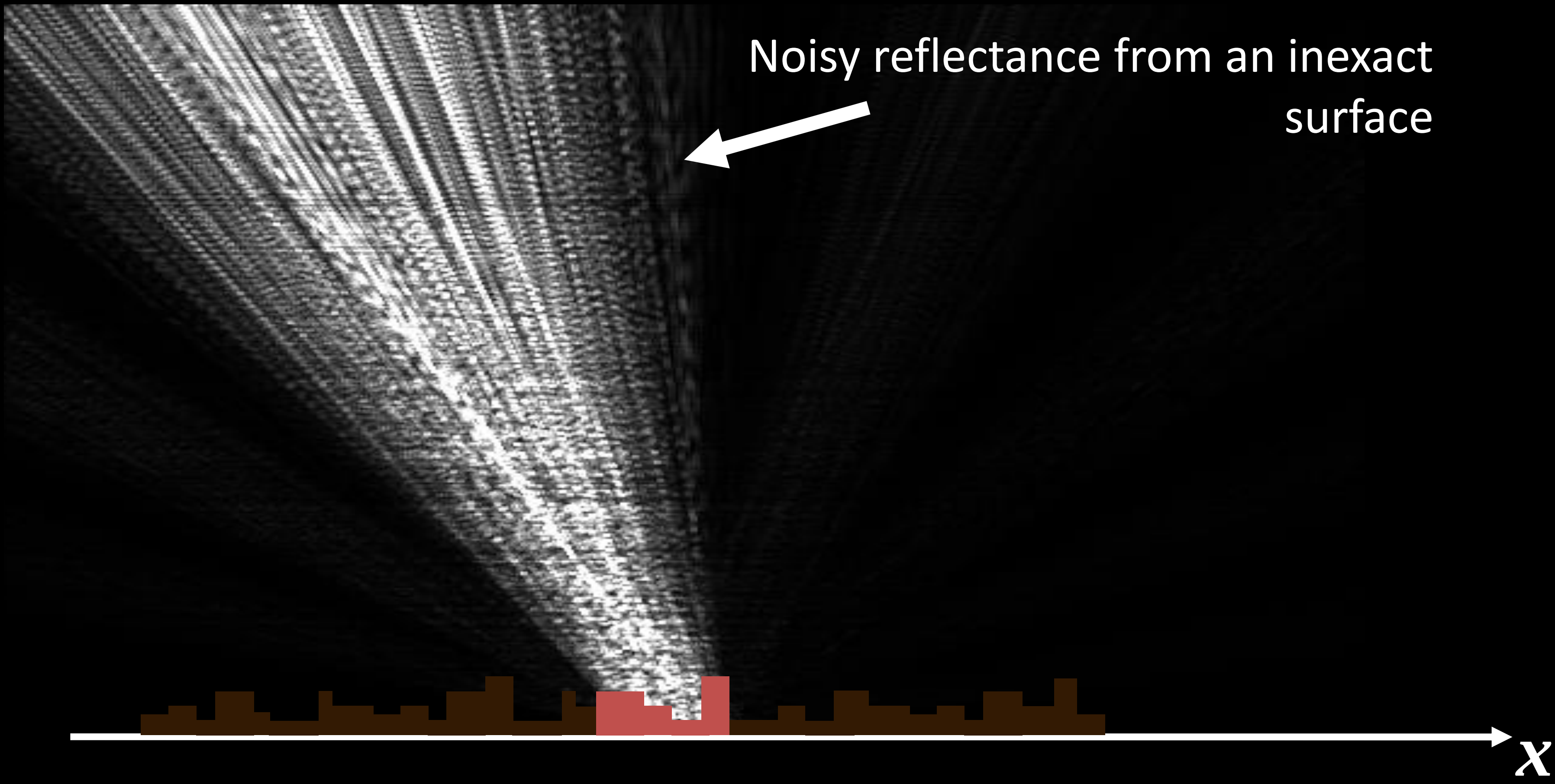
Setting


(spatially) incoherent illumination
z
h
p_o
p_i
R_o
R_i
ω_o
ω_i
$s = (x, h(x))$
x

Far-field setting

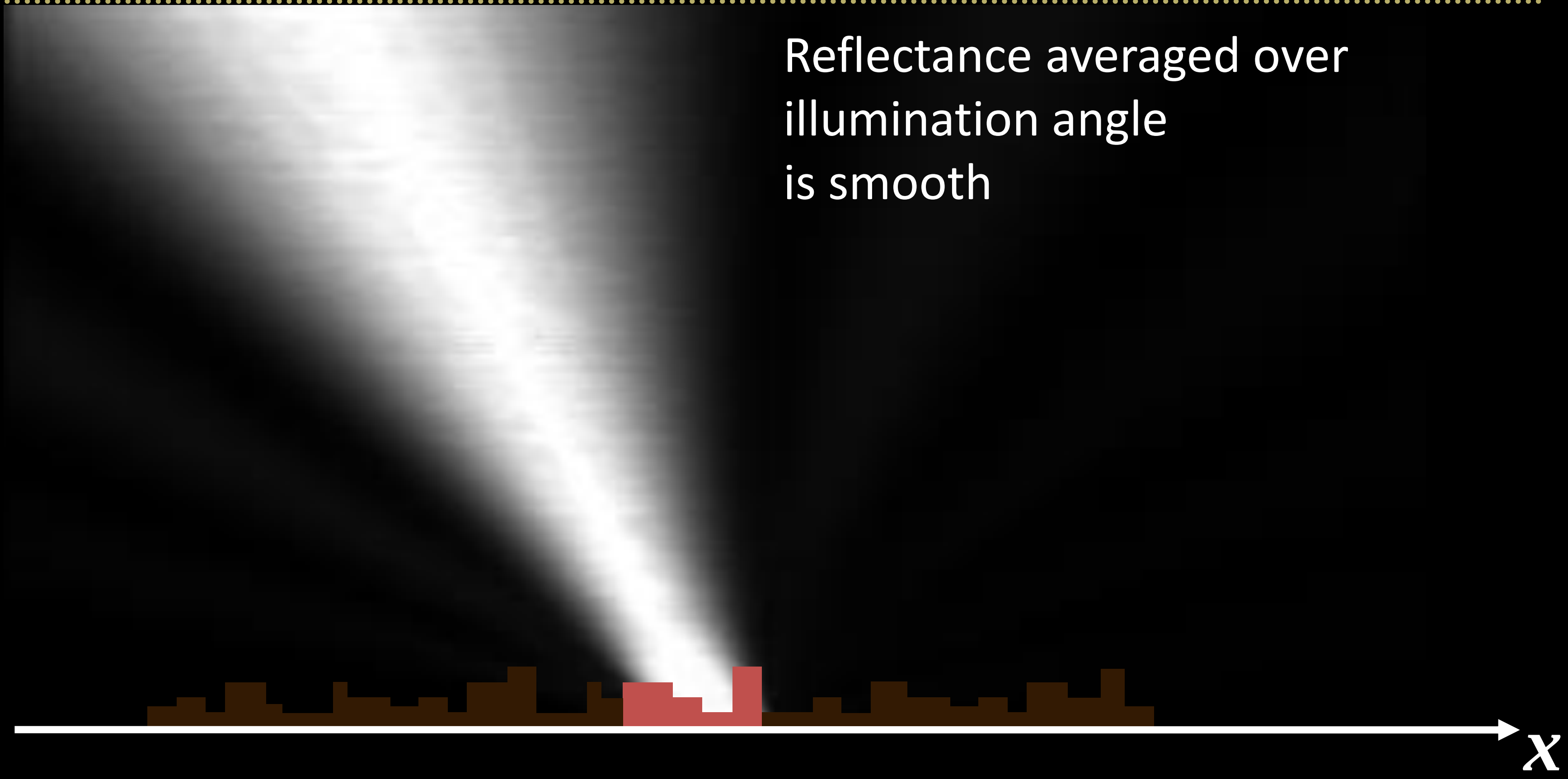


Speckles



Noisy reflectance from an inexact surface

Incoherent reflectance: blurring coherent reflectance by source angle



Reflectance averaged over
illumination angle
is smooth

Fabricating BRDFs at High Spatial Resolution using Wave Optics

Anat Levin, Daniel Glasner, Ying Xiong,
Fredo Durand, William Freeman,
Wojciech Matusik and Todd Zickler.

References

Basic reading:

- Goodman, “Introduction to Fourier Optics,” W. H. Freeman 2004.
  this comprehensive textbook is the standard reference when it comes to Fourier optics.
- Peng et al., “The Diffractive Achromat: Full Spectrum Computational Imaging with Diffractive Optics,” SIGGRAPH 2016.
  this paper discusses Fresnel lenses and how to use computational imaging to deal with chromatic aberration.
- Stam, “Diffractive shaders,” SIGGRAPH 1999.
- Levin et al., “Fabricating BRDFs at high spatial resolution using wave optics,” SIGGRAPH 2013
  these two papers discuss Fraunhofer diffraction for the reflective case.

Additional reading:

- Glasner et al., “A Reflectance Display,” SIGGRAPH 2014.
- Ye et al., “Toward BxDF Display using Multilayer Diffraction,” SIGGRAPH Asia 2014.
- Levin et al., “Passive light and viewpoint sensitive display of 3D content,” ICCP 2016.
  these three papers discuss how to use diffraction to build passive reactive displays.
- Damberg et al., “High Brightness HDR Projection Using Dynamic Freeform Lensing,” TOG 2016
  this paper discusses how to use diffraction to create lenses of arbitrary focusing patterns.
- Matsuda et al., “Focal surface displays,” SIGGRAPH 2017.
  more diffraction-based displays, used for VR headsets.
- Zhang and Levoy, “Wigner Distributions and How They Relate to the Light Field”, ICCP 2009.
- Cuypers et al., “Reflectance Model for Diffraction”, TOG 2012.
  these two papers discuss the relationship between Fourier optics, ray optics and lightfields, and the Wigner transformation.